Risk Management

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CHAPTER THREE: Risk Assessment and Pooling

Textbooks:

- ***Introduction to Risk Management and Insurance, by M. Dorfman and D. Cather, 10th edition, Prentice Hall.***
- ***Lecturer Handouts, Book Chapters***

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Insurable Loss Exposures

- Estimation of financial impact of each risk identified previously.
- Two key statistical measures:
  - Frequency with which losses occur.
  - Their severity.

Basic Statistical Concepts - 1

- **Random Variable:** Future value is not known with certainty.
- **Probability Distribution**: Shows all possible outcomes for a Random Variable.

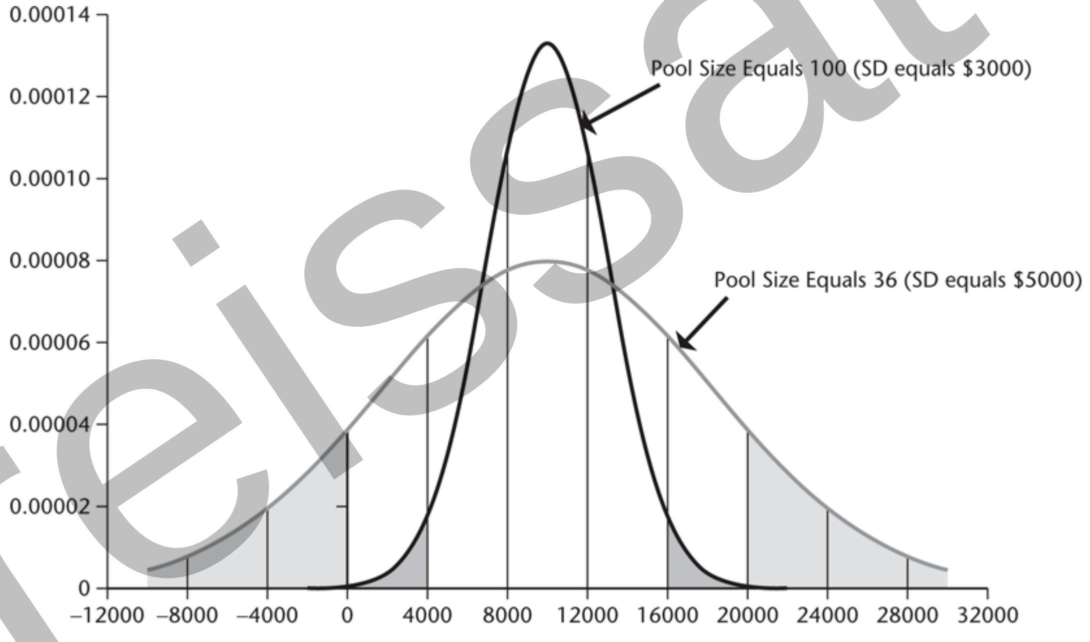

FIGURE 3-1 Normal Curves of the Mean Loss Distribution Based on Pool Sizes of 36 and 100 Exposure Units (SD denotes standard deviation)

Basic Statistical Concepts - 2

- **Expected Value:**
  Sum of the multiplication of each possible outcome of the variable with its probability.

$$E[R] = \Sigma R_i * P_i$$

- **Variance and Standard Deviation:**

$$\sigma = \sum_{i=1}^{N} \sqrt{\left(R_i - E[R]\right)^2 * P_i}$$

The Expected Value

- Can be calculated by **multiplying** the expected losses with their probability and **calculating** the sum of all outcomes

- Is a starting point for calculating an insurance premium or how much a firm should set aside each year to cover the losses

Average Loss

- Estimating:
  Loss Frequency (= Total Amount of Losses **divided by** Total Number of Accidents)
  Loss Severity (= Total Number of Accidents **divided by** Total Units Analyzed).

- Average Loss = Average Loss Frequency **multiplied** with Average Loss Severity.

Convolution

- **A process** of **charting** all possible combinations of frequency and severity to establish the **probable maximum loss**
- Calculates ***all possible combinations of losses*** indicated by the frequency and severity loss distributions, as well as their corresponding probabilities of occurring.
- Often done by computer simulation due to complexity of calculations.

Risk Pooling

- Risk can be reduced through diversification.
- The creation of a pool of many (exposure) units helps the insurer to better predict any individual unit's risk of loss.
- The Probability Distribution matters!

Normal Probability Distribution

- "Bell Curve":

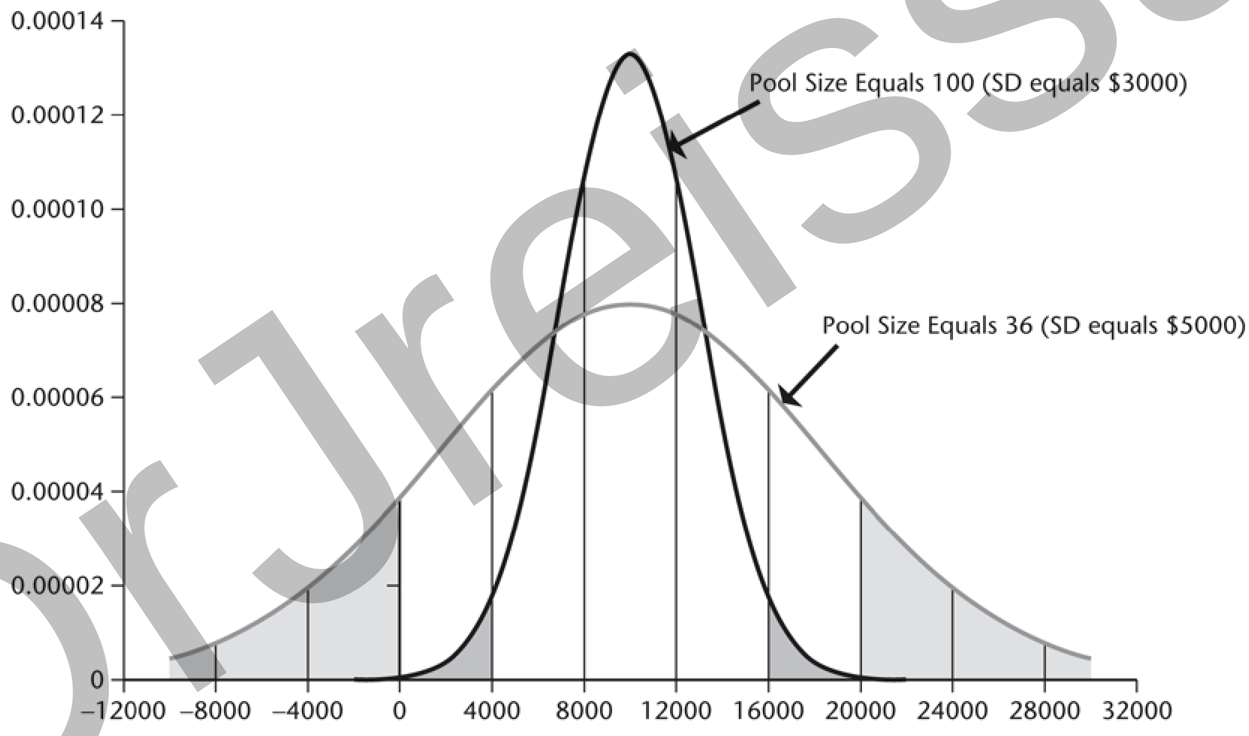

FIGURE 3-1 **Normal Curves of the Mean Loss Distribution Based on Pool Sizes of 36 and 100 Exposure Units (SD denotes standard deviation)**

Confidence Interval

- Assuming Normal Distribution:

Estimated Mean Loss ± (k) * Estimated σ

- Where:
  - **(k)** = Specified number of standard deviations which reflect the uncertainty.
  - **σ** = Standard Deviation calculated using loss data from past.
- This is **the Confidence Interval**.

- **[(k) * Estimated σ]** is also called the **Risk Charge.**
- It represents the **margin of error**.

Practical Considerations

- Insurers sort consumers into homogeneous categories:
  - Age.
  - Gender.
  - Etc.
- Yet still independent of each other.
- Insurers will not insure when these assumptions are violated.

TABLE 3-1 Calculating the Variance Using Data from a Probability Distribution

Column 1	Column 2	Column 3	Column 4	Column 5
Loss Outcomes	*Probabilities*	*Loss Outcome − Expected Loss (Col. 1 − $10000)*	*Squared Differences (Col. 3 × Col. 3)*	*Squared Difference × Prob. (Col 2 × Col 4)*
$0	0.90	−$10,000	100,000,000	90,000,000
$100,000	0.10	$90,000	8,100,000,000	810,000,000
	1.00			900,000,000

E Loss
= 0 × 0.9 + 100k × 0.1 = 10K

TABLE 3-2 Loss Frequency Data and Estimated Probability Distribution

Column 1	*Column 2*	*Column 3*	*Column 4*
No. of Losses per Car	***Number of Cars***	***Total Number of Losses***	***Estimated Probability***
0	910	0	0.91
1	80	80	0.08
2	10	20	0.01
	1000	100	1.00

TABLE 3-3 Loss Severity Data and Estimated Probability Distribution

Column 1	Column 2	Column 3	Column 4	Column 5
Range of Loss Amount	*Midpoint Dollar Amount of Loss*	*Number of Losses*	*Total $ Amt. of Losses*	*Estimated Probability*
$1–4,000	$2,000	75	$150,000	0.75
$4,001–8,000	$6,000	20	$120,000	0.20
$8,001–12,000	$10,000	5	$50,000	0.05
TOTAL		100	$320,000	1.00

TABLE 3-4 All Possible Loss Combinations Calculated Using Convolution

Row	*Loss 1*	*Loss 2*	*Total Loss*	*Probability*	*Total Loss × Probability*	*Joint Probabilities*
A	–	–	0	0.910000	0.0	
B	2,000	–	2,000	0.060000	120.0	.08 × .75
C	6,000	–	6,000	0.016000	96.0	.08 × .20
D	10,000	–	10,000	0.004000	40.0	.08 × .05
E	2,000	2,000	4,000	0.005625	22.5	.01 × .75 × .75
F	2,000	6,000	8,000	0.001500	12.0	.01 × .75 × .20
G	2,000	10,000	12,000	0.000375	4.5	.01 × .75 × .05
H	6,000	2,000	8,000	0.001500	12.0	.01 × .20 × .75
I	6,000	6,000	12,000	0.000400	4.8	.01 × .20 × .20
J	6,000	10,000	16,000	0.000100	1.6	.01 × .20 × .05
K	10,000	2,000	12,000	0.000375	4.5	.01 × .05 × .75
L	10,000	6,000	16,000	0.000100	1.6	.01 × .05 × .20
M	10,000	10,000	20,000	0.000025	0.5	.01 × .05 × .05
TOTAL				1.000000	320.0	

TABLE 3-5 Calculation of All Possible Loss Combinations (Pool Size of 2)

Rick's Loss	*Vic's Loss*	*Total Loss*	*Mean Loss*	*Probability*
0	0	0	0	$.9 \times .9 = .81$
100,000	0	100,000	50,000	$.1 \times .9 = .09$
0	100,000	100,000	50,000	$.9 \times .1 = .09$
100,000	100,000	200,000	100,000	$.1 \times .1 = .01$

TABLE 3-6 Calculation of Variance of Mean Loss Distribution (Pool Size of Two)

Column 1	*Column 2*	*Column 3*	*Column 4*	*Column 5*
Mean Loss Outcomes	***Probabilities***	***Loss Outcome − Expected Loss (Col. 1 − $10,000)***	***Squared Differences (Col. 3 × Col. 3)***	***Squared Differences × Prob. (Col 2 × Col 4)***
$0	0.81	−$10,000	100,000,000	81,000,000
$50,000	0.18	$40,000	1,600,000,000	288,000,000
$100,000	0.01	$90,000	8,100,000,000	81,000,000
	1.00			450,000,000

TABLE 3-7 Risk Reduction Through Pooling as the Size of the Pool Increases

Pool Size	*Mean*	*Standard Deviation*	*Normal Distribution*
1	\$10,000	\$30,000	No
2	\$10,000	$\$30{,}000/2^{0.5} = \$21{,}213$	No
4	\$10,000	$\$30{,}000/4^{0.5} = \$15{,}000$	No
36	\$10,000	$\$30{,}000/36^{0.5} = \$5{,}000$	Yes
100	\$10,000	$\$30{,}000/100^{0.5} = \$3{,}000$	Yes
900	\$10,000	$\$30{,}000/900^{0.5} = \$1{,}000$	Yes