Risk Management

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CHAPTER SEVEN: Decision Analysis

Textbooks:

- *Introduction to Risk Management and Insurance, by M. Dorfman and D. Cather, 10th edition, Prentice Hall.*
- *Quantitative Analysis for Management, by B. Render, 14th edition.*
- *Lecturer Handouts, Book Chapters*

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Introduction

Decision theory is an analytic and systematic approach to the study of decision making

What is involved in making a good decision?

A good decision is one that is based on logic, considers all available data and possible alternatives, and applies a quantitative approach

Decision Analysis

- **Quantitative methods**
  - a set of tools for ………..
- **Decision analysis**
  - a set of quantitative decision-making techniques for decision situations in which uncertainty exists
  - Example of an uncertain situation
    - demand for a product may vary between 0 and 200 units, depending on the state of market

Decision Making Without Probabilities

- **States of nature**
  - Events that may occur in the future
  - Examples of states of nature:
    - high or low demand for a product
    - good or bad economic conditions
- **Decision making under risk**
- **Decision making under uncertainty**

Payoff Table

- Payoff table
  - method for organizing and illustrating payoffs from different decisions given various states of nature
- Payoff
  - outcome of a decision

	States Of Nature	
Decision	a	b
1	Payoff 1a	Payoff 1b
2	Payoff 2a	Payoff 2b

The Six Steps in Decision Making

1. Clearly define the problem at hand
2. List the possible alternatives
3. Identify the possible outcomes or states of nature
4. List the payoff (typically profit) of each combination of alternatives and outcomes
5. Select one of the mathematical decision theory models
6. Apply the model and make your decision

Thompson Lumber Company (1 of 2)

- **Step 1** – Define the problem
  - Consider manufacturing and marketing new product
- **Step 2** – List alternatives
  - Large plant, small plant, do nothing
- **Step 3** – Identify possible outcomes, states of nature
  - Market is favorable or unfavorable
- **Step 4** – List the payoffs
  - Identify conditional values for each alternative
- **Step 5** – Select the decision model
  - Depends on environment & risk uncertainty
- **Step 6** – Apply the model to the data

Thompson Lumber Company (2 of 2)

Table 1 Decision Table with Conditional Values for Thompson Lumber

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)
Construct a large plant	200,000	–180, 000
Construct a small plant	100,000	–20, 000
Do nothing	0	0

Note: It is important to include all alternatives, including “do nothing.”

Types of Decision-Making Environments

- **Decision making under certainty**
  - The decision maker knows with certainty the consequences of every alternative or decision choice
- **Decision making under uncertainty**
  - The decision maker does not know the probabilities of the various outcomes
  - probabilities can NOT be assigned to the occurrence of states of nature in the future
- **Decision making under risk**
  - The decision maker knows the probabilities of the various outcomes
  - Probabilities can be assigned to the occurrence of states of nature in the future

Decision Making Under Uncertainty

- Criteria for making decisions under uncertainty

1. Maximax (optimistic)
2. Maximin (pessimistic)
3. Criterion of realism (Hurwicz)
4. Equally likely (Laplace)
5. Minimax regret

Max problem

Optimistic = Maximax

- Used to find the alternative that maximizes the maximum payoff—**maximax** criterion
  1. Locate the maximum payoff for each alternative
  2. Select the alternative with the maximum number

Table 2 Thompson's Maximax Decision

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	Maximum in a Row ($)
Construct a large plant	200,000	–180, 000	200,000 Maximax
Construct a small plant	100,000	–20, 000	100,000
Do nothing	0	0	0

Pessimistic = Maximin

- Used to find the alternative that maximizes the minimum payoff—**maximin** criterion
  1. Locate the minimum payoff for each alternative
  2. Select the alternative with the maximum number

Table 3 Thompson’s Maximin Decision

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	Maximum in a Row ($)
Construct a large plant	200,000	–180, 000	–180, 000
Construct a small plant	100,000	–20, 000	–20, 000
Do nothing	0	0	0 Maximin

Criterion of Realism (Hurwicz Criterion) (1 of 2)

- Often called **weighted average**
  - Compromise between optimism and pessimism
  - Select a coefficient of realism α, with $0 \le \alpha \le 1$

    $\alpha = 1$ is perfectly optimistic

    $\alpha = 0$ is perfectly pessimistic
  - Compute the weighted averages for each alternative
  - Select the alternative with the highest value

$$\text{Weighted average} = \alpha(\text{best payoff}) + (1-\alpha)(\text{worst payoff})$$

Criterion of Realism (Hurwicz Criterion) (2 of 2)

For the large plant alternative using $\alpha = 0.8$

$$(0.8)(200{,}000) + (1-0.8)(-180{,}000) = 124{,}000$$

For the small plant alternative using $\alpha = 0.8$

$$(0.8)(100{,}000) + (1-0.8)(-20{,}000) = 76{,}000$$

Table 4 Thompson's Criterion of Realism Decision

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	Criterion of Realism or Weighted Average ($\alpha = 0.8$) ($)
Construct a large plant	200,000 160k	−180, 000 −36 =128K ✓	124,000 Realism
Construct a small plant	100,000 80	−20, 000 76	76,000
Do nothing	0	0 7	0

Equally Likely (Laplace Criterion)

α 0.5

- Considers all the payoffs for each alternative
  - Find the average payoff for each alternative
  - Select the alternative with the highest average

Table 5 Thompson's Equally Likely Decision

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	Row Average ($)
Construct a large plant	200,000 0	–180, 000 180k	180 10,000
Construct a small plant	100,000 100k	–20, 000 20K	100k 40,000 Equally likely
Do nothing	0 200	0 0	200k 0

Minimax Regret (1 of 3)

- Based on **opportunity loss** or **regret**
  - The difference between the optimal profit and actual payoff for a decision

    Step 1—Create an opportunity loss table by determining the opportunity loss from not choosing the best alternative

    Step 2—Calculate opportunity loss by subtracting each payoff in the column from the best payoff in the column

    Step 3—Find the maximum opportunity loss for each alternative, and pick the alternative with the minimum number

Creating an opportunity loss table, also known as a regret table, is a valuable tool in decision-making analysis, particularly in decision theory. It helps to evaluate the potential losses of not choosing the best option for each state of nature. Here's a step-by-step guide to create one:

1. Identify the Decision Alternatives and States of Nature
Decision Alternatives: List all possible actions or decisions you could make.
States of Nature: Identify all possible scenarios or outcomes that could occur as a result of each decision.
2. Create a Payoff Table
Construct a Table: For each decision alternative and state of nature, determine the payoff (or benefit) for that combination. The payoff table will look like this:

Decision Alternatives State of Nature 1 State of Nature 2 ...
Decision 1 Payoff 1 Payoff 2 ...
Decision 2 Payoff 3 Payoff 4 ...
... ...
3. Determine the Best Payoff for Each State of Nature
For each state of nature, identify the maximum payoff. This is the best possible outcome for that state of nature.
4. Calculate Opportunity Losses
Opportunity Loss Calculation: For each cell in the payoff table, subtract the payoff from the maximum payoff for the corresponding state of nature. This gives you the opportunity loss for each combination. The opportunity loss table looks like this:

Decision Alternatives State of Nature 1 State of Nature 2 ...
Decision 1 Loss 1 Loss 2 ...
Decision 2 Loss 3 Loss 4 ...
... ...
5. Analyze the Opportunity Loss Table
Use this table to evaluate which decision minimizes the potential opportunity loss across all states of nature. This can help in making a more informed decision by considering the regret associated with not choosing the optimal option for each state of nature.
Example
Let's say you have two decisions (A and B) and two states of nature (S1 and S2). The payoffs are:

Decision S1 S2
A 30 20
B 40 15
The best payoffs for S1 and S2 are 40 and 20, respectively. So, the opportunity loss table is:

Minimax Regret (2 of 3)

Table 6 Determining Opportunity Losses for Thompson Lumber

State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)
200,000 – 200,000	0 – (– 180,000)
200,000 – 100,000	0 – (– 20,000)
200,000 – 0	0 – 0

Table 7 Opportunity Loss Table for Thompson Lumber

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0

Minimax Regret (3 of 3)

Table 8 Thompson's Minimax Decision Using Opportunity Loss

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	Maximum in a Row ($)
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000 Minimax
Do nothing	200,000	0	200,000

Decision Making Under Risk (1 of 2)

- When there are several possible states of nature and the probabilities associated with each possible state are known
  - Most popular method—choose the alternative with the highest expected monetary value (EMV)

$$\text{EMV(alternative)} = \sum X_i P(X_i)$$

you will get EMV values based on the number of alternatives

where

$X_i =$ payoff for the alternative in state of nature i

$P(X_i) =$ probability of achieving payoff X_i (i.e., probability of state of nature i)

$\sum =$ summation symbol

Decision Making Under Risk (2 of 2)

- Expanding the equation

$$
\begin{aligned}
\text{EMV (alternative } i) = & (\text{payoff of first state of nature}) \\
& \times (\text{probability of first state of nature}) \\
& + (\text{payoff of second state of nature}) \\
& \times (\text{probability of second state of nature}) \\
& + \ldots + (\text{payoff of last state of nature}) \\
& \times (\text{probability of last state of nature})
\end{aligned}
$$

EMV for Thompson Lumber

Each market outcome has a probability of occurrence of 0.50 Which alternative would give the highest EMV?

Table 9 Decision Table with Probabilities and EMVs for Thompson Lumber

Alternative	**State of Nature Favorable Market ($)**	**State of Nature Unfavorable Market ($)**	**EMV ($)**
Construct a large plant	200,000	–180, 000	10,000
Construct a small plant	100,000	–20, 000	40,000 Best EMV
Do nothing	0	0	0
Probabilities	0.50	0.50	

Expected Value of Perfect Information (EVPI) (1 of 6)

- **EVPI** places an upper bound on what you should pay for additional information
- **EVwPI** is the long-run average return if we have perfect information before a decision is made

$$\text{EVwPI} = \sum_{i=1} (\text{best payoff in state of nature } i)$$
$$\times (\text{probability of state of nature } i)$$

Expected Value of Perfect Information (EVPI) (2 of 6)

- Expanded EVwPI becomes

$$\begin{aligned}\text{EVwPI} = &(\text{best payoff for first state of nature}) \\ &\times(\text{probability of first state of nature}) \\ &+(\text{best payoff for second state of nature}) \\ &\times(\text{probability of second state of nature}) \\ &+\ldots+(\text{best payoff for last state of nature}) \\ &\times(\text{probability of last state of nature})\end{aligned}$$

And

$$\text{EVPI} = \text{EVwPI} - \text{Best EMV}$$

Expected Value of Perfect Information (EVPI) (3 of 6)

- Scientific Marketing, Inc. offers analysis that will provide certainty about market conditions (favorable)
- Additional information will cost $65,000
- Should Thompson Lumber purchase the information?

EMV = 40000

EVwPI = 0.5

Expected Value of Perfect Information (EVPI) (4 of 6)

Table 10 Decision Table with Perfect Information

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	EMV ($)
Construct a large plant	200,000	-180, 000	10,000
Construct a small plant	100,000	-20, 000	40,000
Do nothing	0	0	0
With perfect information	200,000	0 $(1-P)x - 188$	100,000 EVwPI
Probabilities	P 0.50	$(1-P)$ 0.50	

$200kP + 180kP - 180k$

$360kP - 180k$

$100kP + 20kP - 20k$

$120kP - 20k$

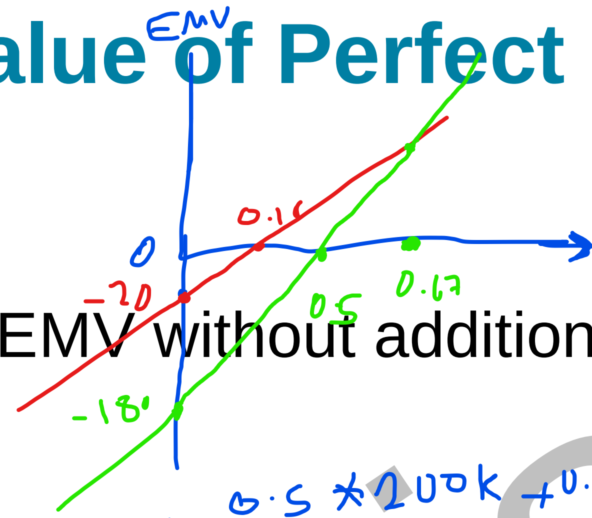
0

$P = 0.5$

$P = 0.167$

$P =$

Expected Value of Perfect Information (EVPI) (5 of 6)



0 – 0.167 (nothing)
0.167 – 0.67 (small)
0 0.67 – 1 (large)

0.5 * 200k + 0.5 * 0

- The maximum EMV without additional information is \$40,000
  - Therefore

$$\begin{aligned} \text{EVPI} &= \text{EVwPI} - \text{Maximum EMV} \\ &= \$100{,}000 - \$40{,}000 \\ &= \$60{,}000 \end{aligned}$$

So, the maximum Thompson should pay for the additional information is \$60,000

Expected Value of Perfect Information (EVPI) (6 of 6)

- The maximum EMV without additional information is $40,000
  - Therefore

Thompson should not pay $65,000 for this information

$$\begin{aligned} \text{EVPI} &= \text{EVwPI} - \text{Maximum EMV} \\ &= \$100{,}000 - \$40{,}000 \\ &= \$60{,}000 \end{aligned}$$

So the maximum Thompson should pay for the additional information is $60,000

Expected Opportunity Loss (1 of 2)

- Expected opportunity loss (EOL) is the cost of not picking the best solution
  - Construct an opportunity loss table
  - For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together
  - Minimum EOL will always result in the same decision as maximum EMV
  - Minimum EOL will always equal EVPI

Expected Opportunity Loss (2 of 2)

$$\text{EOL (large plant)} = (0.50)(\$0) + (0.50)(\$180{,}000) = \$90{,}000$$

$$\text{EOL (small plant)} = (0.50)(\$100{,}000) + (0.50)(\$20{,}000) = \$60{,}000$$

$$\text{EOL (do nothing)} = (0.50)(\$200{,}000) + (0.50)(\$0) = \$100{,}000$$

Table 11 EOL Table for Thompson Lumber

Alternative	State of Nature Favorable Market ($)	State of Nature Unfavorable Market ($)	EOL ($)
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	same as EVwPI 60,000 Best EOL
Do nothing	200,000	0	100,00
Probabilities	0.50	0.50	

Sensitivity Analysis (1 of 3)

Define P = probability of a favorable market

$$\text{EMV}\left(\text{large plant}\right) = \$200{,}000P - \$180{,}000)(1 - P)$$

$$= \$200{,}000P - \$180{,}000 + \$180{,}000P$$

$$= \$380{,}000P - \$180{,}000$$

$$\text{EMV}\left(\text{small plant}\right) = \$100{,}000P - \$20{,}000)(1 - P)$$

$$= \$100{,}000P - \$20{,}000 + \$20{,}000P$$

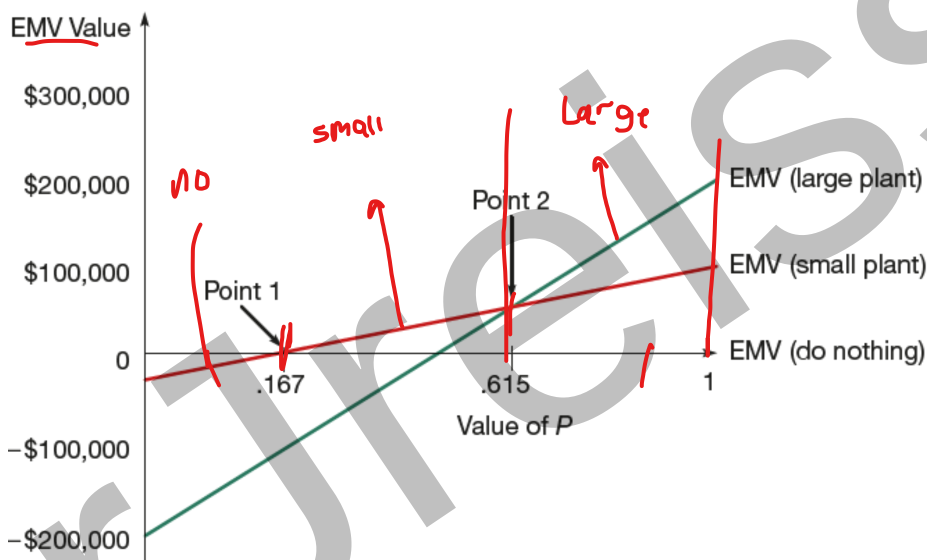
$$= \$120{,}000P - \$20{,}000$$

$$\text{EMV}\left(\text{do nothing}\right) = \$0P + 0(1 - P)$$

$$= \$0$$

Sensitivity Analysis (2 of 3)

Figure 1 Sensitivity Analysis



Best Alternative	**Range of *P* Values**
Do nothing	Less than 0.167
Construct a small plant	0.167–0.615
Construct a large plant	Greater than 0.615

Sensitivity Analysis (3 of 3)

Point 1: $\text{EMV(do nothing)} = \text{EMV(small plant)}$

$$0 = \$120{,}000P - \$20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

Point 2: $\text{EMV(small plant)} = \text{EMV(large plant)}$

$$\$120{,}000P - \$20{,}000 = \$380{,}000P - \$180{,}000$$

$$P = \frac{160{,}000}{260{,}000} = 0.615$$

A Minimization Example (1 of 4)

Three-year lease for a copy machine—which cost is lowest?

Table 12 Payoff Table with Monthly Copy Costs for Business Analytics Department

	10,000 Copies per Month ($)	20,000 Copies per Month ($)	30,000 Copies per Month ($)
Machine A	950	1,050	1,150
Machine B	850	1,100	1,350
Machine C	700	1,000	1,300

Table 13 Best and Worst Payoffs (Costs) for Business Analytics Department

	10,000 Copies per Month ($)	20,000 Copies per Month ($)	30,000 Copies per Month ($)	Best Payoff (Minimum) ($)	Worst Payoff (Maximum) ($)
Machine A	950	1,050	1,150	950	1,150
Machine B	850	1,100	1,350	850	1,350
Machine C	700	1,000	1,300	700	1,300

A Minimization Example (2 of 4)

1. Determine the best alternative using Hurwicz criteria with 70% coefficient. $\alpha = 0.7$
2. Determine the best alternative using the equally likely criteria.
3. Determine the best alternative using the EMV criterion using $P(10{,}000) = 0.4,\ P(20{,}000) = 0.3,\ P(30{,}000) = 0.3$

A Minimization Example (3 of 4)

Table 14 Expected Monetary Values and Expected Values with Perfect Information for Business Analytics Department

	10,000 Copies per Month ($)	20,000 Copies per Month ($)	30,000 Copies per Month ($)	EMV ($)
Machine A	950	1,050	1,150	1,040
Machine B	850	1,100	1,350	1,075
Machine C	700	1,000	1,300	970
With perfect information	700	1,000	1,150	925
Probability	0.4	0.3	0.3	

$$\text{EVwPI} = \$925$$

$$\text{Best EMV without perfect information} = \$970$$

$$\text{EVPI} = 970 - 925 = \$45$$

A Minimization Example (4 of 4)

Table 15 Opportunity Loss Table for Business Analytics Department

	10,000 Copies per Month ($)	20,000 Copies per Month ($)	30,000 Copies per Month ($)	Maximum ($)	EOL ($)
Machine A	250	50	0	250	115
Machine B	150	100	200	200	150
Machine C	0	0	150	150	45
Probability	0.4	0.3	0.3		

Using Excel (1 of 4)

Program 2A Excel QM Results for Thompson Lumber Example

	A	B	C	D	E	F	G	H	I
1	**Thompson Lumber**								
2									
3	**Decision Tables**								
4	Enter the profits in the main body of the data table. Enter probabilities in the first row if you want to compute the expected value.								
5									
6	Data				Results				
7	Profit	Scenario 1	Scenario 2		EMV	Minimum	Maximum		
8	Probability	0.5	0.5						
9	Decision 1	200000	-180000		10000	-180000	200000		
10	Decision 2	100000	-20000		40000	-20000	100000		
11	Decision 3	0	0		0	0	0		
12				Maximum	40000	0	200000		
13									
14	Expected Value of Perfect Information								
15	Column best	200000	0		100000	<-Expected value WITH perfect information			
16					40000	<-Best expected value			
17					60000	<-Expected value OF perfect information			
18									
19	Regret								
20		Scenario 1	Scenario 2		Expected	Maximum			
21	Probability	0.5	0.5						
22	Decision 1	0	180000		90000	180000			
23	Decision 2	100000	20000		60000	100000			
24	Decision 3	200000	0		100000	200000			
25				Minimum	60000	100000			

To see the formulas, hold down the control key (Ctrl) and press the ` (grave accent) key, which is usually found above the Tab key.

Using Excel (2 of 4)

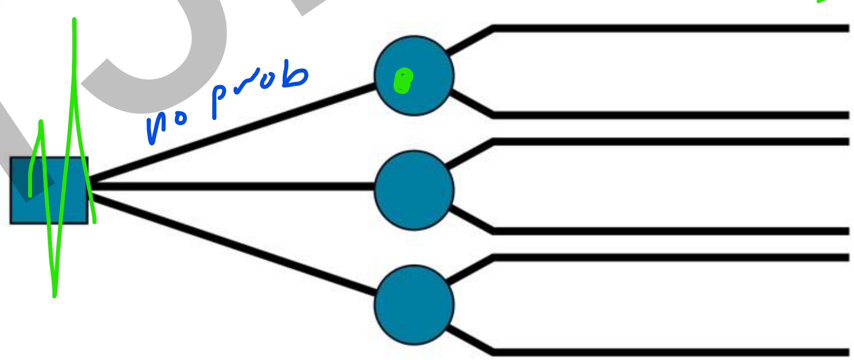
Program 2B Key Formulas in Excel QM for Thompson Lumber Example

9	=SUMPRODUCT($B$8:$C$8,B9:C9)	=MIN(B9:C9)	=MAX(B9:C9)
10	=SUMPRODUCT($B$8:$C$8,B10:C10)	=MIN(B10:C10)	=MAX(B10:C10)
11	=SUMPRODUCT($B$8:$C$8,B11:C11)	=MIN(B11:C11)	=MAX(B11:C11)
12	**=MAX(E9:E11)**	**=MAX(F9:F11)**	**=MAX(G9:G11)**
13			
14			
15	**=SUMPRODUCT($B$8:$C$8,B15:C15)**	**<-Expected value WITH perfect information**	
16	**=E12**	**<-Best expected value**	
17	**=E15-E12**	**<-Expected value OF perfect information**	
18			
19			
20	Expected	Maximum	
21			
22	=SUMPRODUCT($B$8:$C$8,B22:C22)	=MAX(B22:C22)	
23	=SUMPRODUCT($B$8:$C$8,B23:C23)	=MAX(B23:C23)	
24	=SUMPRODUCT($B$8:$C$8,B24:C24)	=MAX(B24:C24)	
25	**=MIN(E22:E24)**	**=MIN(F22:F24)**	

Decision Trees

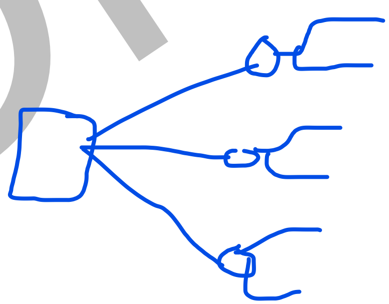
Any problem presented in a decision table can be graphically represented in a **decision tree**

- Most beneficial with a sequence of decisions
- All decision trees contain **decision points/nodes** and **state-of-nature points/nodes**
- At decision nodes (the squares), one of several alternatives may be chosen
- At state-of-nature nodes (the circles), one state of nature will occur



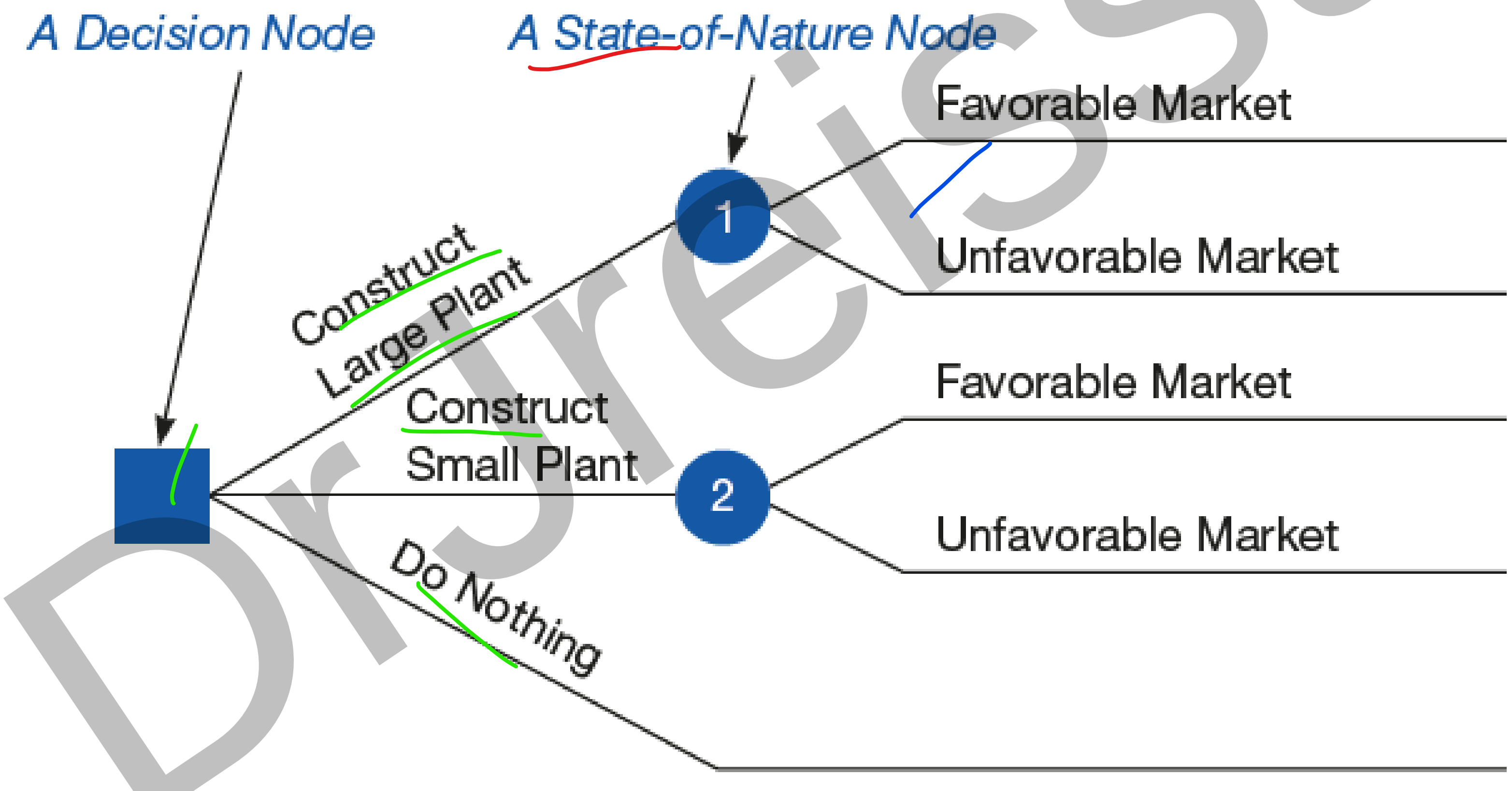
Five Steps of Decision Tree Analysis

1. Define the problem
2. Structure or draw the decision tree
3. Assign probabilities to the states of nature
4. Estimate payoffs for each possible combination of alternatives and states of nature
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node



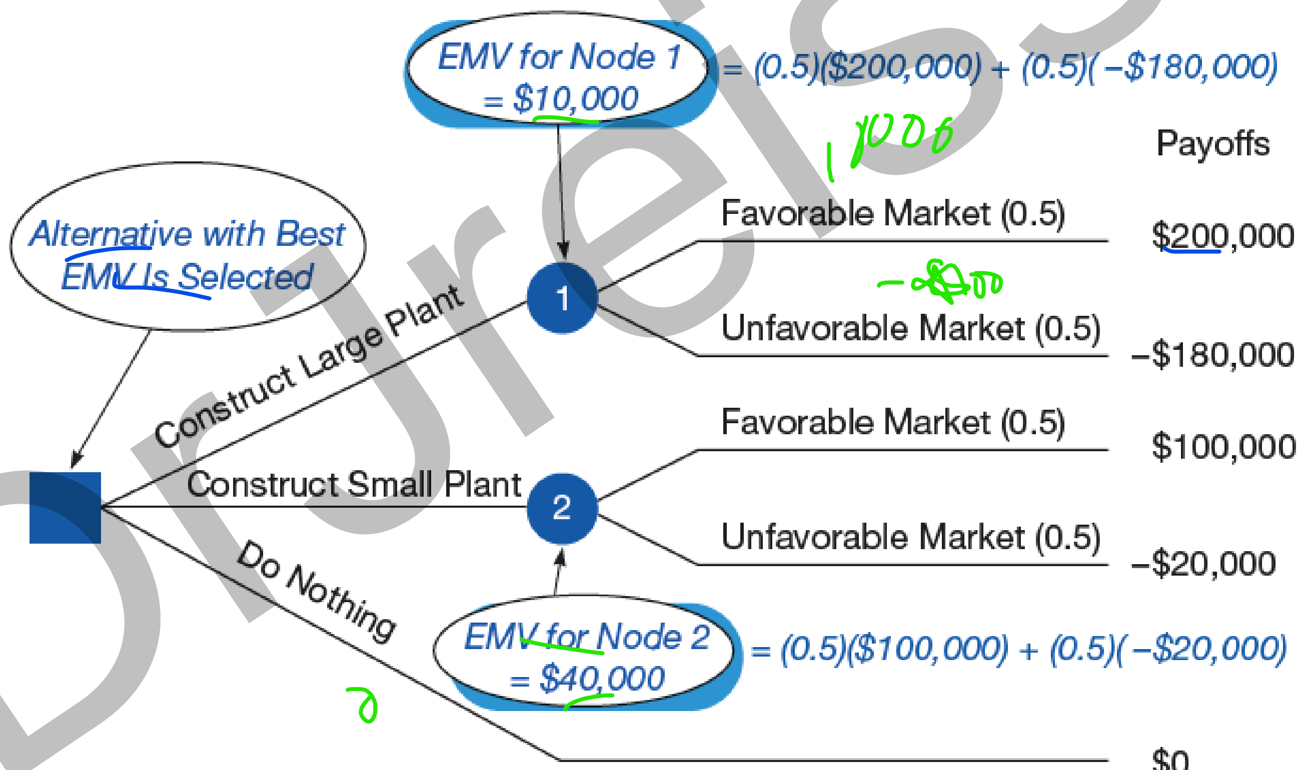
Thompson's Decision Tree (1 of 2)

Figure 2 Thompson Lumber Decision Tree



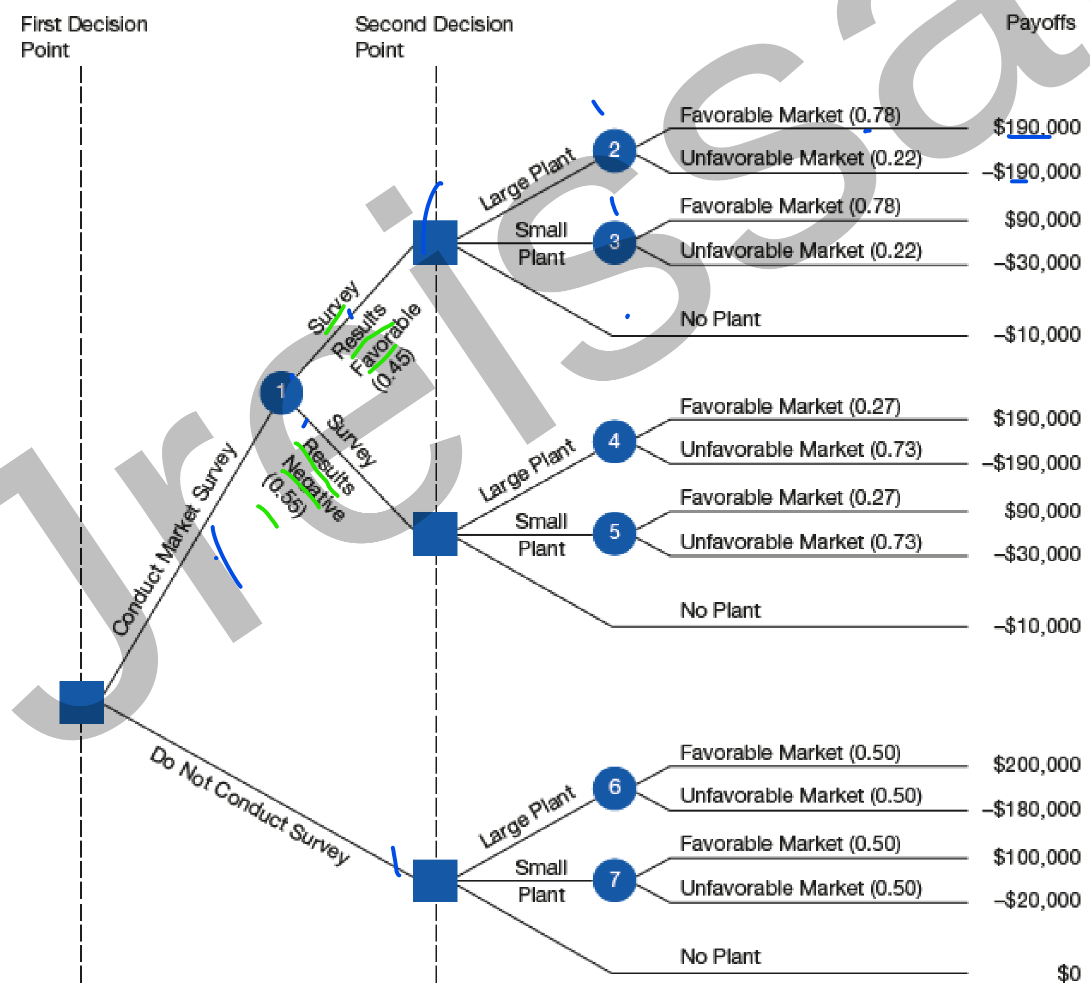
Thompson's Decision Tree (2 of 2)

Figure 3 Completed and Solved Decision Tree for Thompson Lumber



Thompson's Complex Decision Tree (1 of 5)

Figure 4 Larger Decision Tree with Payoffs and Probabilities for Thompson Lumber



Thompson's Complex Decision Tree (2 of 5)

1. Given favorable survey results

$$\begin{aligned}
\text{EMV(node 2)} &= \text{EMV}(\text{large plant} \mid \text{positive survey}) \\
&= (0.78)(\$190{,}000) + (0.22)(-\$190{,}000) \\
&= \$106{,}400 \\
\text{EMV}(\text{node 3}) &= \text{EMV}(\text{small plant} \mid \text{positive survey}) \\
&= (0.78)(\$90{,}000) + (0.22)(-\$30{,}000) \\
&= \$63{,}600 \\
\text{EMV for no plant} &= -\$10{,}000
\end{aligned}$$

Thompson's Complex Decision Tree (3 of 5)

2. Given negative survey results

$$\text{EMV}(\text{node }4) = \text{EMV}(\text{large plant}\,|\,\text{negative survey})$$
$$= (0.27)(\$190{,}000) + (0.73)(-\$190{,}000)$$
$$= -\$87{,}400$$
$$\text{EMV}(\text{node }5) = \text{EMV}(\text{small plant}\,|\,\text{negative survey})$$
$$= (0.27)(\$90{,}000) + (0.73)(-\$30{,}000)$$
$$= \$2{,}400$$
$$\text{EMV for no plant} = -\$10{,}000$$

Thompson's Complex Decision Tree (4 of 5)

The best choice is to seek marketing information

3. Expected value of the market survey

$$\begin{aligned}\text{EMV(node 1)} &= \text{EMV(conduct survey)}\\ &= (0.45)(\$106{,}400) + (0.55)(\$2{,}400)\\ &= \$47{,}880 + \$1{,}320 = \$49{,}200\end{aligned}$$

4. Expected value no market survey

$$\begin{aligned}\text{EMV}(\text{node } 6) &= \text{EMV}(\text{large plant})\\ &= (0.50)(\$200{,}000) + (0.50)(-\$180{,}000)\\ &= \$10{,}000\\ \text{EMV}(\text{node } 7) &= \text{EMV}(\text{small plant})\\ &= (0.50)(\$100{,}000) + (0.50)(-\$20{,}000)\\ &= \$40{,}000\\ \text{EMV for no plant} &= \$0\end{aligned}$$

Thompson's Complex Decision Tree (5 of 5)

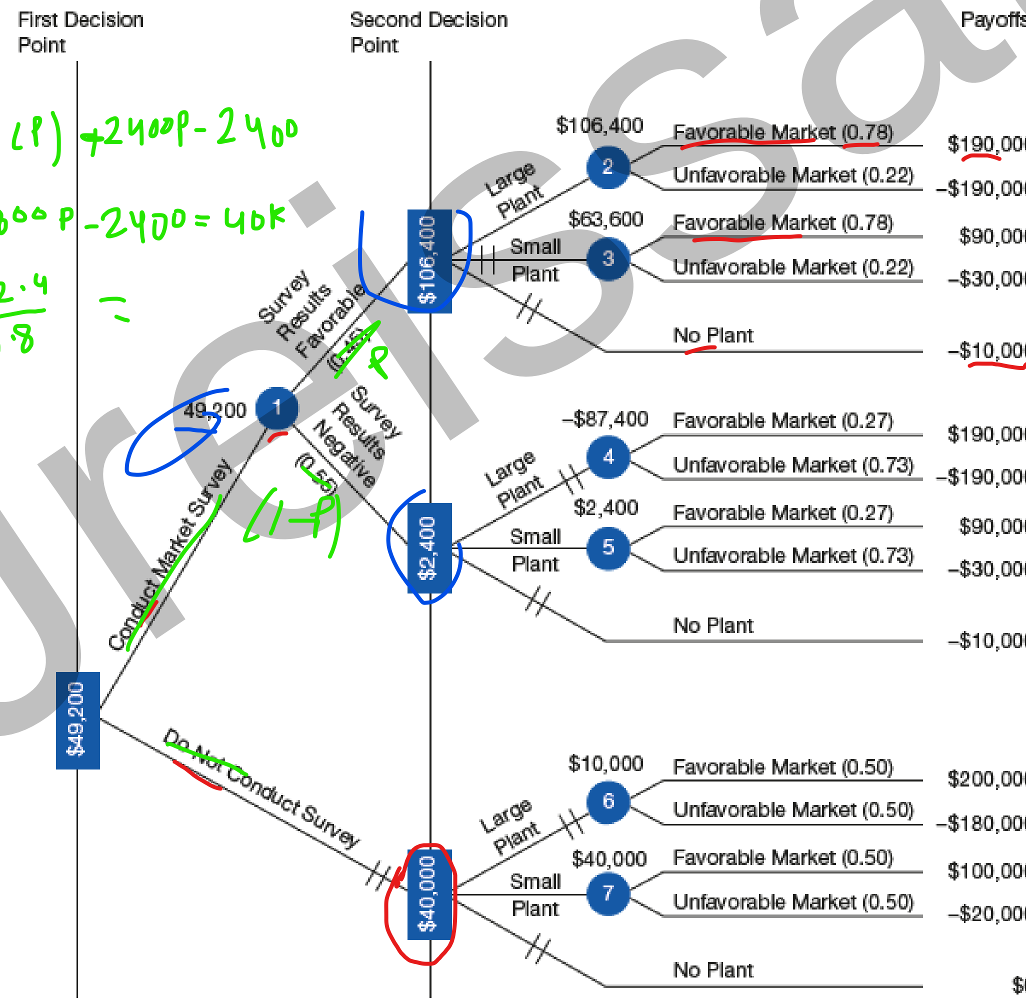
Figure 5 Thompson's Decision Tree with EMVs Shown

What is the value of doing the survey?

$106400(P) + 2400P - 2400$

$108800P - 2400 = 40k$

$P = \frac{40 - 2.4}{108.8} =$



Efficiency of Sample Information

Market survey is only 32% as efficient as perfect information

- Possibly many types of sample information available
- Different sources can be evaluated

$$\text{Efficiency of sample information} = \frac{\text{EVSI}}{\text{EVPI}} 100\%$$

- For Thompson

$$\text{Efficiency of sample information} = \frac{19{,}200}{60{,}000} 100\% = 32\%$$

Sensitivity Analysis (1 of 2)

- How sensitive are the decisions to changes in the probabilities?
- How sensitive is our decision to the probability of a favorable survey result?
- If the probability of a favorable result (p = .45) were to change, would we make the same decision?
- How much could it change before we would make a different decision?

Sample information is data gathered from a subset of a population to inform decision-making, but it is imperfect and subject to errors or biases, leading to some level of uncertainty. Perfect information, on the other hand, refers to having complete, accurate, and reliable knowledge about all relevant factors, eliminating uncertainty and allowing for optimal decision-making.

Sensitivity Analysis (2 of 2)

If $p < 0.36$, do not conduct the survey

If $p > 0.36$, conduct the survey

p = probability of a favorable survey result

$(1-p) =$ probability of a negative survey result

$$\begin{aligned}\text{EMV}(\text{node 1}) &= (\$106{,}400)p + (\$2{,}400)(1-p)\\ &= \$104{,}000p + \$2{,}400\end{aligned}$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey

$$\begin{aligned}\$104{,}000p + \$2{,}400 &= \$40{,}000\\ \$104{,}000p &= \$37{,}600\\ p &= \$37{,}600 \div \$104{,}000 = 0.36\end{aligned}$$

Bayesian Analysis

- Many ways of getting probability data
  - Management's experience and intuition
  - Historical data
  - Computed from other data using Bayes' theorem
- Bayes' theorem incorporates initial estimates and information about the accuracy of the sources
- Allows the revision of initial estimates based on new information

Calculating Revised Probabilities

- Four conditional probabilities for Thompson Lumber

$$P\left(\text{favorable market}\,(\text{FM})\,|\,\text{survey results positive}\right) = 0.78$$

$$P\left(\text{unfavorable market}\,(\text{UM})\,|\,\text{survey results positive}\right) = 0.22$$

$$P\left(\text{favorable market}\,(\text{FM})\,|\,\text{survey results negative}\right) = 0.27$$

$$P\left(\text{unfavorable market}\,(\text{UM})\,|\,\text{survey results negative}\right) = 0.73$$

- Prior probabilities

$$P\left(\text{FM}\right) = 0.50$$

$$P\left(\text{UM}\right) = 0.50$$

Calculating Revised Probabilities (2 of 5)

Table 16 Market Survey Reliability in Predicting States of Nature

Result of Survey	State of Nature Favorable Market (FM)	State of Nature Unfavorable Market (UM)
Positive (predicts favorable market for product)	$P(\text{survey positive} \mid \text{FM}) = 0.70$	$P(\text{survey positive} \mid \text{UM}) = 0.20$
Negative (predicts unfavorable market for product)	$P(\text{survey negative} \mid \text{FM}) = 0.30$	$P(\text{survey negative} \mid \text{UM}) = 0.80$

Calculating Revised Probabilities

- Calculating posterior probabilities

$= \frac{0.78 * 0.5}{0.78 * 0.5}$

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid A') \times P(A')}$$

P(B)

where

A, B = any two events

A' = complement of A

A = favorable market

B = positive survey

Calculating Revised Probabilities (4 of 5)

Table 17 Probability Revisions Given a Positive Survey

State of Nature	Conditional Probability P(Survey Positive | State of Nature)	Prior Probability	Posterior Probability Joint Probability	Posterior Probability P(State of Nature | Survey Positive)
FM	0.70	$\times 0.50$	= 0.35	$0.35/0.45 = 0.78$
UM	0.20	$\times 0.50$	= 0.10	$0.10/0.45 = 0.22$
		P(survey results positive)	= 0.45	1.00

Calculating Revised Probabilities (5 of 5)

Table 18 Probability Revisions Given a Negative Survey

State of Nature	Conditional Probability *P*(Survey Negative | State of Nature)	Prior Probability	Posterior Probability Joint Probability	Posterior Probability *P*(State of Nature | Survey Negative)
~~FM~~	0.30	×0.50	= 0.15	0.15/0.55 = 0.27
UM	0.80	×0.50	= 0.40	0.40/0.55 = 0.73
		P(survey results negative)	= 0.55	1.00

Using Excel (3 of 4)

Program 3A Results of Bayes' Calculations in Excel 2016

	A	B	C	D	E
1	**Bayes Theorem for Thompson Lumber Example**				
2					
3	**Fill in cells B7, B8, and C7**				
4					
5	**Probability Revisions Given a Positive Survey**				
6	State of Nature	P(Sur.Pos.|state of nature)	Prior Prob.	Joint Prob.	Posterior Probability
7	FM	0.7	0.5	0.35	0.78
8	UM	0.2	0.5	0.1	0.22
9			P(Sur.pos.)=	**0.45**	
10					
11	**Probability Revisions Given a Negative Survey**				
12	State of Nature	P(Sur.Pos.|state of nature)	Prior Prob.	Joint Prob.	Posterior Probability
13	FM	0.3	0.5	0.15	0.27
14	UM	0.8	0.5	0.4	0.73
15			P(Sur.neg.)=	**0.55**	

Using Excel (4 of 4)

Program 3 B Formulas Used for Bayes' Calculations

6	P(Sur.Pos.|state of nature)	Prior Prob.	Joint Prob.	Posterior Probability
7	0.7	0.5	=B7*C7	=D7/$D$9
8	0.2	=1-C7	=B8*C8	=D8/$D$9
9		P(Sur.pos.)=	**=SUM(D7:D8)**	
10				
11				
12	P(Sur.Pos.|state of nature)	Prior Prob.	Joint Prob.	Posterior Probability
13	=1-B7	=C7	=B13*C13	=D13/$D$15
14	=1-B8	=C8	=B14*C14	=D14/$D$15
15		P(Sur.neg.)=	**=SUM(D13:D14)**	

Potential Problems Using Survey Results

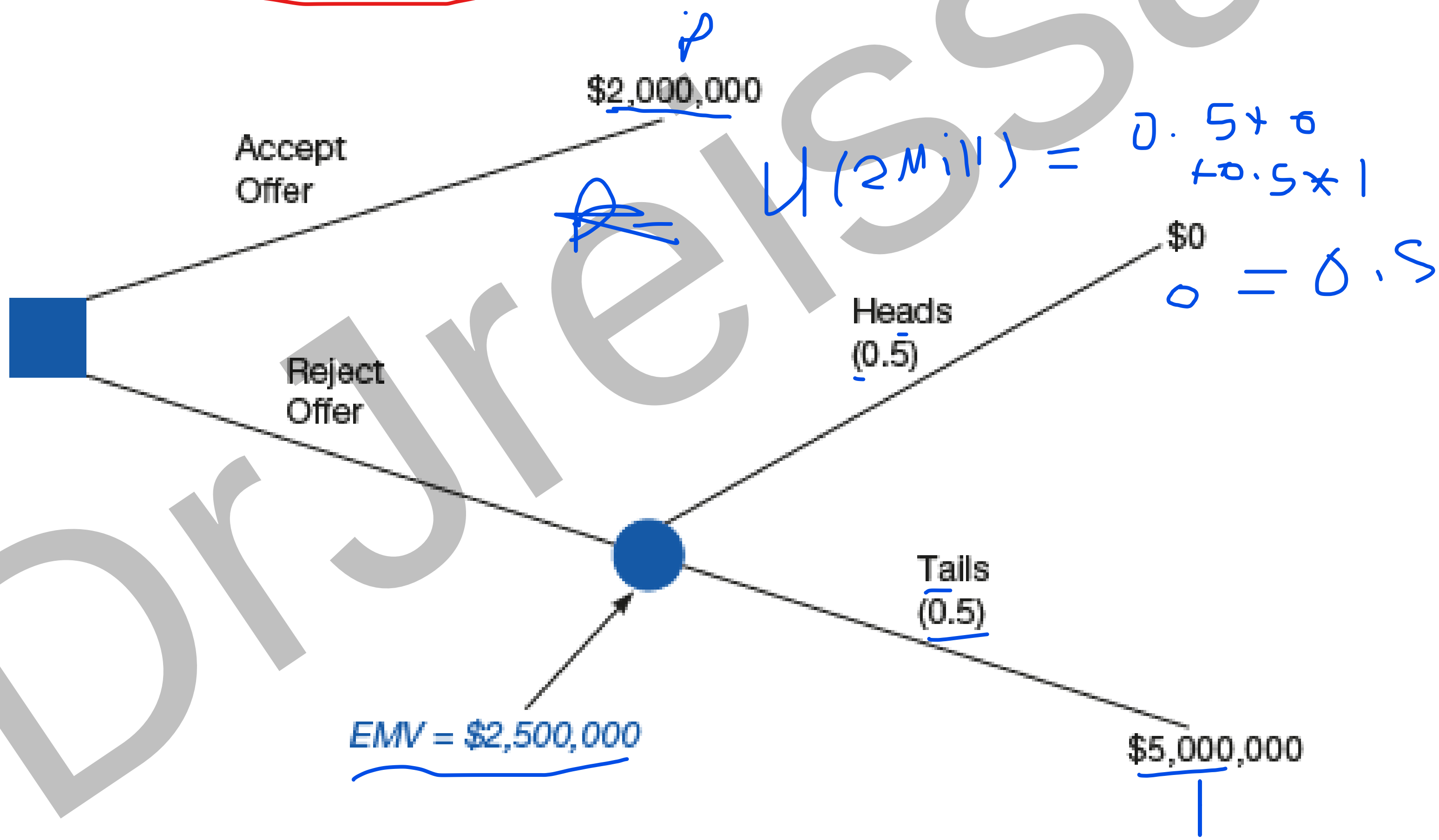
- We can not always get the necessary data for analysis
- Survey results may be based on cases where an action was taken
- Conditional probability information may not be as accurate as we would like

Utility Theory (1 of 3)

- Monetary value is not always a true indicator of the overall value of the result of a decision
- The overall value of a decision is called utility
- Economists assume that rational people make decisions to maximize their utility
- **Utility assessment** assigns the worst outcome a utility of 0 and the best outcome a utility of 1
- A **standard gamble** is used to determine utility values
- When you are indifferent, your utility values are equal

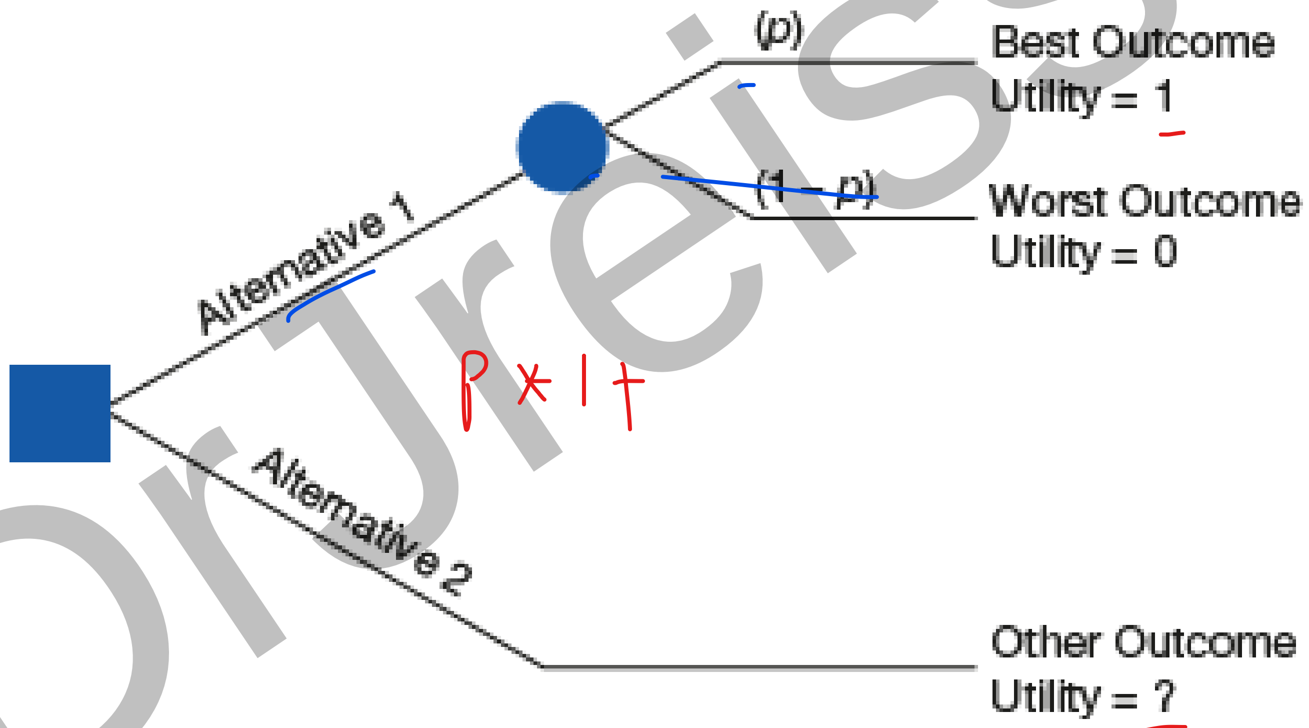
Utility Theory (2 of 3)

Figure 6 Your Decision Tree for the Lottery Ticket



Utility Theory (3 of 3)

Figure 7 Standard Gamble for a Utility Assessment



Investment Example (1 of 2)

- Construct a utility curve revealing preference for money between $0 and $10,000
- A **utility curve** plots the utility value versus the monetary value
  - An investment in a bank will result in $5,000
  - An investment in real estate will result in $0 or $10,000
  - Unless there is an 80% chance of getting $10,000 from the real estate deal, prefer to have her money in the bank
  - If *p* = 0.80, Jane is indifferent between the bank or the real estate investment

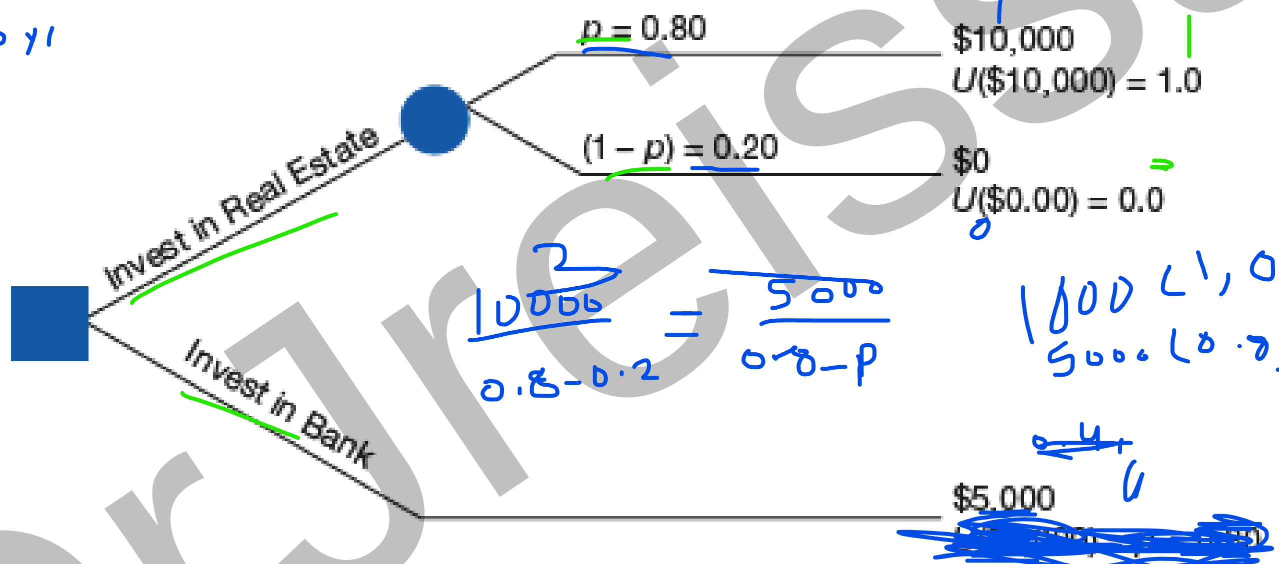
Investment Example (2 of 2)

$0.8 \times 1 = 0.8$

Figure 8 Utility of $5,000

$U = 0.8(1) + 0(1)$

$= 0.8$



$\frac{10000}{0.8-0.2} = \frac{5000}{0.8-p}$

$10000\ (1, 0.8)$

$5000\ (0.8, \frac{1}{2})$

Utility for $5,000 = U($5,000) = pU($10,000) + (1 – p) U($0) = (0.8)(1) + (0.2)(0) = 0.8

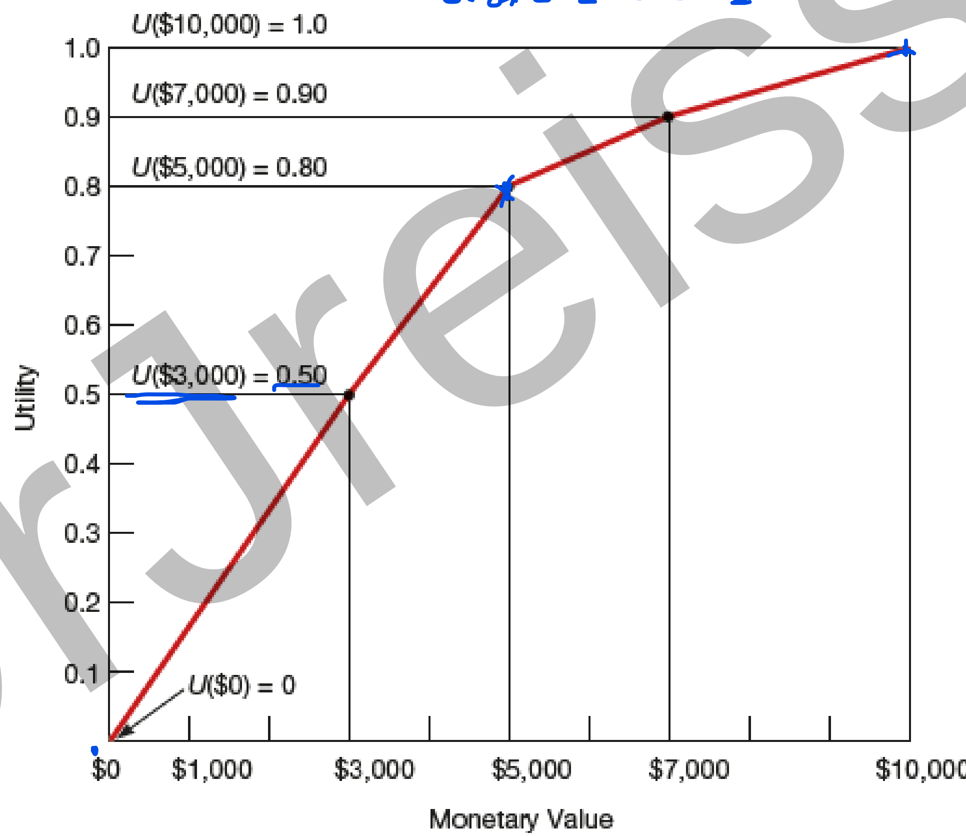
What are the utility values for $3,000 and for $7,000?

$\frac{10-5}{6.2} = \frac{15-7}{1-x} \to x = 0.85$

Utility Curve

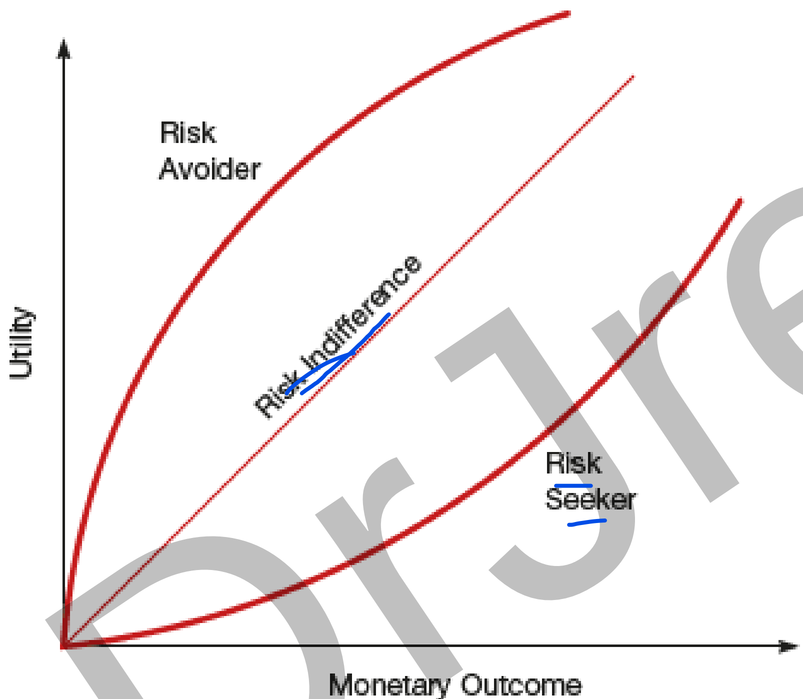
Figure 9 Utility Curve for Jane Dickson

0.8*0.2+0.8*1=



Preferences for Risk

Figure 10 Preferences for Risk



Risk Avoider

- Avoids high losses
- Less utility from greater risk

Risk Seeker

- Utility curve increases faster than payoff
- More utility from greater risk

Risk Indifferent

- Linear utility

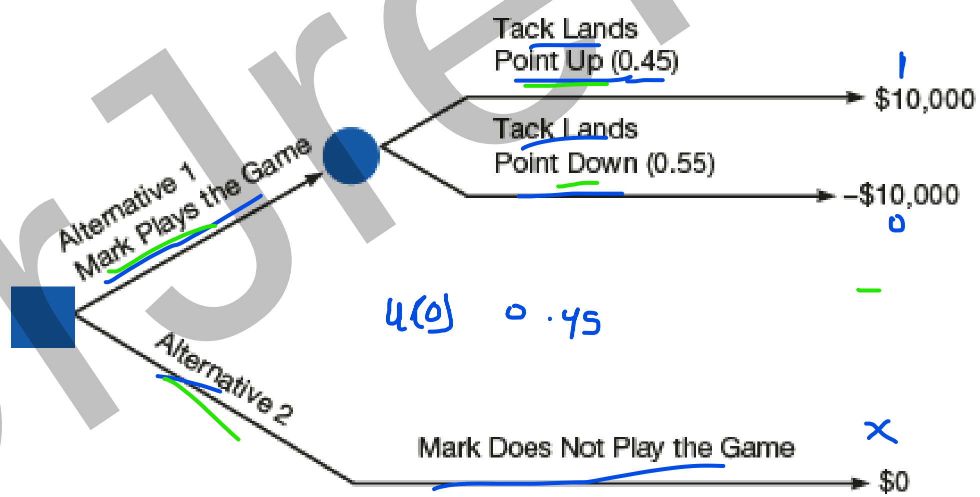
Utility from الى الاعلى

Utility as a Decision-Making Criteria (1 of 3)

Mark Simkin's thumbtack game

Lands Point Up	Lands Point Down
Mark wins $10,000	Mark loses $10,000

Figure 11 Decision Facing Mark Simkin



Should Mark play the game is $P(\text{point up}) = 0.45$?

Utility as a Decision-Making Criteria (2 of 3)

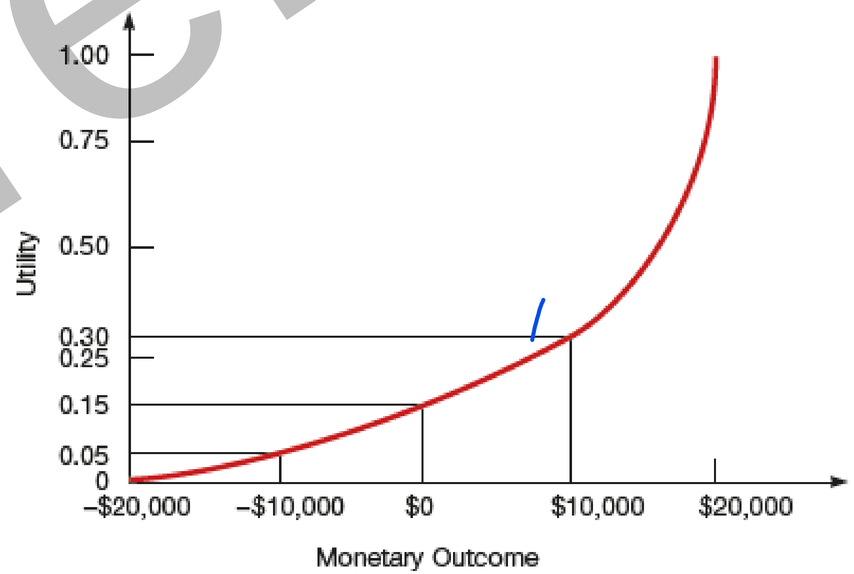
Step 1 – Define Mark's utilities

$$U(-\$10{,}000) = 0.05$$

$$U(\$0) = 0.15$$

$$U(\$10{,}000) = 0.30$$

Figure 12 Utility Curve for Mark Simkin



Utility as a Decision-Making Criteria (3 of 3)

Step 2 – Replace monetary values with utility values

$$E(\text{alternative 1: play the game}) = (0.45)(0.30) + (0.55)(0.05)$$
$$= 0.135 + 0.027 = 0.162$$

$$E(\text{alternative 2: don't play the game}) = 0.15$$

Figure 13 Using Expected Utilities in Decision Making