Risk Management

Summer Semester 2023/2024

Lecturer: Dr Mohannad Jreissat

Department of Industrial Engineering
School of Engineering

CHAPTER EIGHT: Decision Theory and the Normal Distribution

Textbooks:

- *Introduction to Risk Management and Insurance, by M. Dorfman and D. Cather, 10th edition, Prentice Hall.*
- *Quantitative Analysis for Management, by B. Render, 14th edition.*
- *Lecturer Handouts, Book Chapters*

Email: *m.jreissat@ju.edu.jo*
Office: *IE Department, 2nd floor*

Introduction

- Decision theory can be extended to handle very large problems with hundreds or thousands of variables
- It is virtually impossible to solve using techniques like decision trees or decision tables

Break-Even Analysis and the Normal Distribution

- **Break-even analysis**, or **cost-volume analysis**, can be used to analyze the effect of a decision on overall revenues or costs
- The normal probability distribution can be used in the decision-making process

Barclay Brothers Company’s New Product Decision (1 of 3)

- Large manufacturer of adult parlor games
- Deciding whether to introduce a new game called Strategy
- Company is concerned with costs, potential demand, and profit

Relevant costs

$\text{Fixed cost } (f) = \$36{,}000$ **(costs that do not vary with volume produced, such as new equipment, insurance, rent, and so on)**

$\text{Variable cost } (v) \text{ per Game produced} = \$4$ **(costs that are proportional to the number of games produced, such as materials and labor)**

Selling price per unit is $10

Barclay Brothers Company's New Product Decision (2 of 3)

- **Break-even point** is the number of games at which total revenues are equal to total costs

$$\text{Break-even point (units)} = \frac{\text{Fixed cost}}{\text{Price / unit} - \text{Variable cost / unit}}$$

$$= \frac{f}{s - v}$$

$$\text{Break-even point (units)} = \frac{\$36{,}000}{\$10 - \$4} = \frac{\$36{,}000}{\$6}$$

$$= 6{,}000 \text{ games of } \textbf{Strategy}$$

Barclay Brothers Company's New Product Decision (3 of 3)

- If demand is 11,000 games

Revenue	(11,000 games $\times$ \$10/game)	\$110,000
Less expenses		
Fixed cost		\$36,000
Variable cost (11,000 games $\times$ \$4/game)		\$44,000
Total expense		\$80,000
Profit		\$30,000

Probability Distribution of Demand (1 of 6)

- Actual demand can range from 0 to many thousands of units
- Need to establish the probability of various levels of demand
- Normal probability distribution is used to estimate the demand

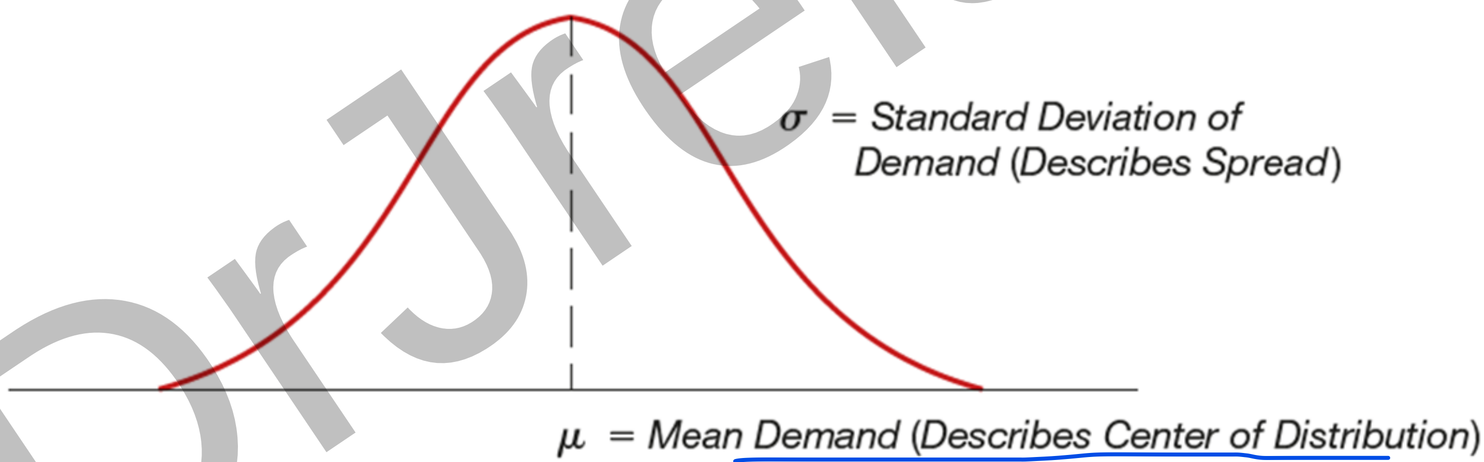


Figure M3.1 Shape of a Typical Normal Distribution

Probability Distribution of Demand (2 of 6)

- Because demand is symmetric a normal curve is appropriate

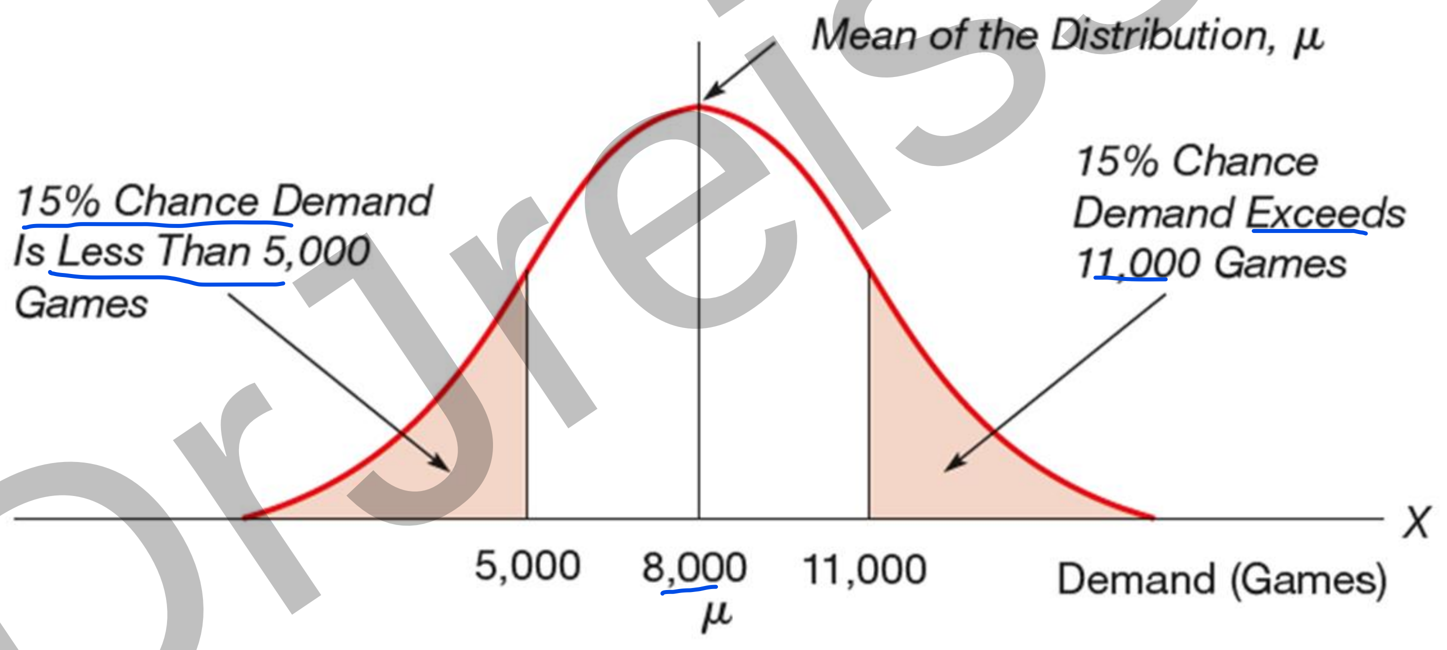


Figure M3.2 Normal Distribution for Barclay's Demand

Probability Distribution of Demand (3 of 6)

- To calculate the number of standard deviations any value of demand is away from the mean

$Z = \frac{d - \mu}{\sigma}$

$$Z = \frac{\text{demand} - \mu}{\sigma}$$

- The area under the curve to the left of 11,000 units demanded is 85% of the total area so $Z = 1.04$

$$1.04 = \frac{11{,}000 - 8{,}000}{\sigma}$$

$$1.04\sigma = 3{,}000$$

$$\sigma = \frac{3{,}000}{1.04} = 2{,}885 \text{ units}$$

Probability Distribution of Demand (4 of 6)

- Probability of breaking even $Z = \dfrac{\text{break-even point} - \mu}{\sigma}$
  - Break-even point = 6,000 $= \dfrac{6{,}000 - 8{,}000}{2{,}885} = \dfrac{-2{,}000}{2{,}885} = -0.69$

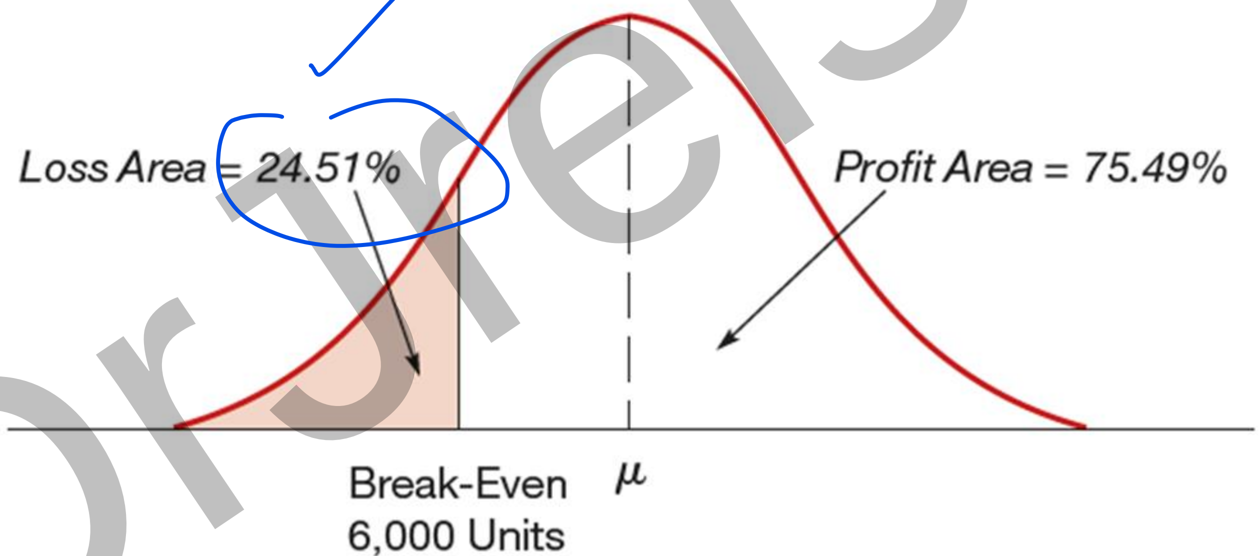


Figure M3.3 Probability of Breaking Even for Barclay's New Game

Probability Distribution of Demand (5 of 6)

- To calculate the probability of making a profit

$$P(\text{loss}) = P(\text{demand} < \text{break-even}) = 0.2451$$
$$= 24.51\%$$

$$P(\text{profit}) = P(\text{demand} > \text{break-even}) = 0.7549$$
$$= 75.49\%$$

Probability Distribution of Demand (6 of 6)

Two caveats

1. **Normally distributed demand**. If we find that this is not reasonable, other distributions may be applied
2. **The only random variable is demand**. If one of the other variables (price, variable cost, or fixed costs) were a random variable, a similar procedure could be followed. If two or more variables are random, the mathematics become very complex

Using Expected Monetary Value to Make a Decision

EMV

- The EMV of not developing Strategy = $0
- Calculate the EMV of producing the game

$$\text{EMV} = \left(\frac{\text{Price}}{\text{unit}} - \frac{\text{Variable cost}}{\text{unit}} \right) \times (\text{Mean demand}) - \text{Fixed costs}$$

$$= (\$10 - \$4)(8{,}000 \text{ units}) - \$36{,}000$$

$$= \$48{,}000 - \$36{,}000 = \$12{,}000$$

R(S - 1) -

Expected Value of Perfect Information and the Normal Distribution

- Compute the expected value of perfect information (EVPI) and expected opportunity loss (EOL)
- Two steps

1. Determine the opportunity loss function
2. Use the opportunity loss function and the unit normal loss integral (given in Appendix M3.2 at the end of this module) to find EOL, which is the same as EVPI

Opportunity Loss Function

- The **opportunity loss function** describes the loss that would be suffered by making the wrong decision
  - For any level of sales, X, Barclay's opportunity loss function can be expressed as

$$\text{Opportunity loss} = \begin{cases} \$6(6{,}000 - X) & \text{for } X \le 6{,}000 \text{ games} \\ \$0 & \text{for } X > 6{,}000 \text{ games} \end{cases}$$

- In **general**

$$\text{Opportunity loss} = \begin{cases} K(\text{break-even point} - X) & \text{for } X \le \text{BEP} \\ \$0 & \text{for } X > \text{BEP} \end{cases}$$

where

K = loss per unit when sales are below the break-even point

X = sales in units

Expected Opportunity Loss (1 of 3)

- Calculating these for many possible values can be a very lengthy and tedious task
  - Calculations are much easier assuming a very large number of normally distributed possible sales values
- Using the unit **normal loss integral**, EOL can be computed using

$$\text{EOL} = K\sigma N(D)$$

where

EOL = expected opportunity loss

K = loss per unit when sales < break-even point   S-V

σ = standard deviation of the distribution

N(D) = value for the unit normal loss integral (Appendix M3.2)

$$D = \left| \frac{\mu - \text{break-even point}}{\sigma} \right|$$

Expected Opportunity Loss (2 of 3)

- EOL for this situation

$K = S - V$

$$K = \$6$$
$$\sigma = 2{,}885$$
$$D = \left|\frac{8{,}000 - 6{,}000}{2{,}885}\right| = 0.69 = 0.60 + 0.09$$

- From Appendix M3.2,

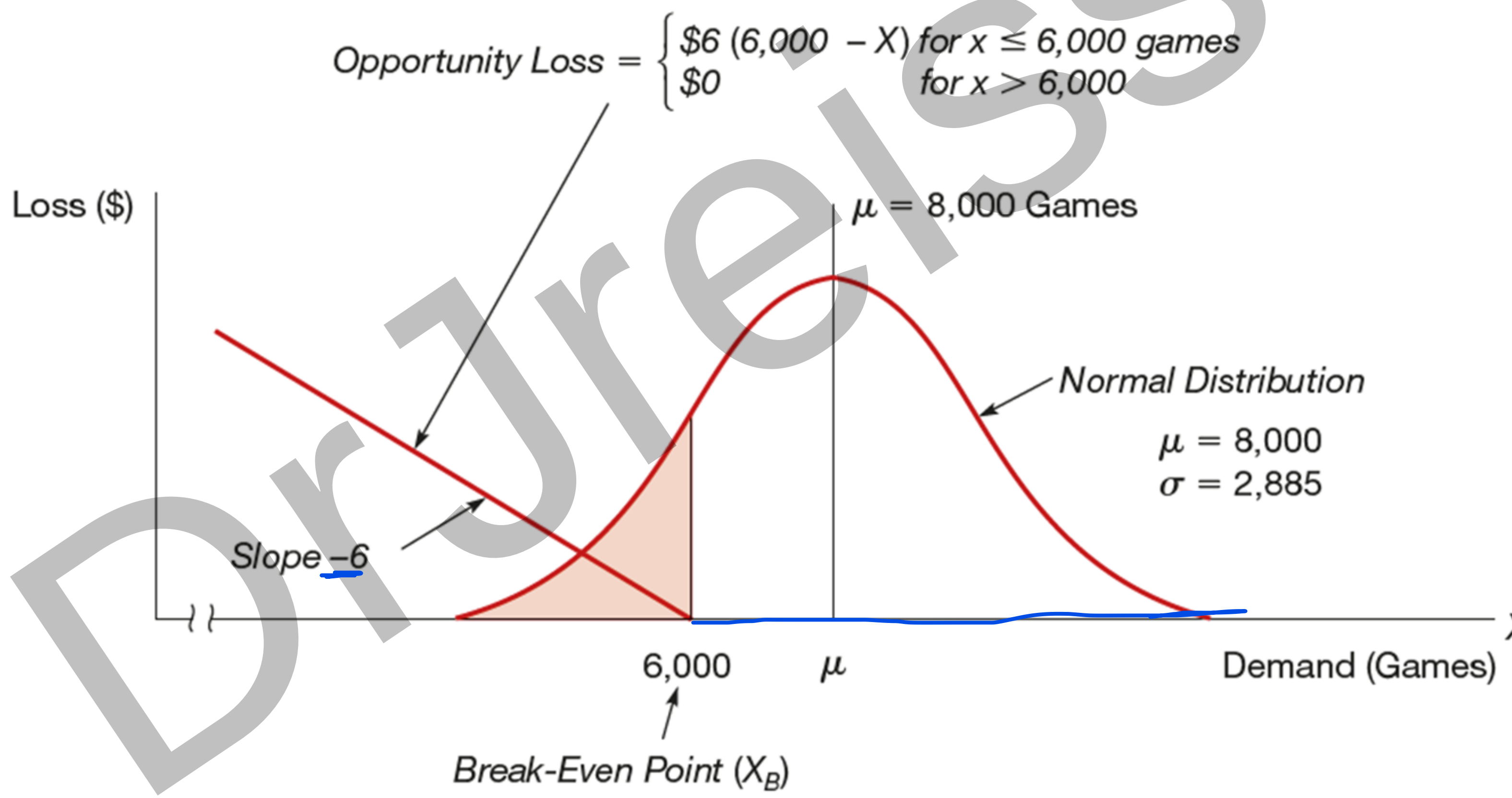
$$N(0.69) = 0.1453$$

- Therefore,

$$EOL = K\sigma N(0.69)$$
$$= (\$6)(2{,}885)(0.1453) = \$2{,}515.14$$

Expected Opportunity Loss (3 of 3)

Figure M3.4 Barclay's Opportunity Loss Function



Example1:

Terry Wagner is considering self-publishing a book on yoga. She has been teaching yoga for more than 20 years. She believes that the fixed costs of publishing the book will be about $10,000. The variable costs are $5.50, and the price of the yoga book to bookstores is expected to be $12.50. What is the break-even point for Terry?

$$= \frac{10000}{7} =$$

Solution:

$$\text{Break-even point in units} = \frac{\$10,000}{\$12.50 - \$5.50}$$

$$= \frac{\$10,000}{\$7}$$

EDL = 5 × N(0) × k = 2000 × 24 × N(7)

0.8833

Example2:

The annual demand for a new electric product is expected to be normally distributed with a mean of 16,000 and a standard deviation of 2,000. The break-even point is 14,000 units. For each unit less than 14,000, the company will lose $24. Find the expected opportunity loss.

Solution:

The expected opportunity loss (EOL) is

$$\text{EOL} = K\sigma N(D)$$

We are given the following:

$$K = \text{loss per unit} = \$24$$
$$\mu = 16{,}000$$
$$\sigma = 2{,}000$$

Using Equation M3-6, we find

$$D = \left|\frac{\mu - \text{break-even point}}{\sigma}\right| = \left|\frac{16{,}000 - 14{,}000}{2{,}000}\right| = 1$$

$$N(D) = N(1) = 0.08332 \text{ from Appendix M3.2}$$

$$\text{EOL} = K\sigma N(1) = 24(2{,}000)(0.08332) = \$3{,}999.36$$