Center Of Pressure

D

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Team "D"

- Alaa Saqr 0201373
- Omar Sowan 0204768
- Feras Takruri 0201300
- Mohammed AlSaaideh 0202057

Objective:

1. To determine the position of the center of pressure on the rectangular face of the toroid.
2. To compare the measured value with the that predicted from the theoretical analysis.

Apparatus:

A toroid is mounted on a balance and pivoted about the center of curvature of the toroid. Thus, only the vertical face along GC of the toroid has any moment about the balancing point. A rider weight balances the weight of the toroid in the dry, so that the moment of the hydrostatic face on C is measured by the weight at the pan. The toroid is immersed in a tank containing water and depth of immersion is measured by a hook gauge.

Introduction:

Figure 1 shows a plane surface inclined at an angle a to the free surface of the liquid 00. Since there can be no shear stress in a static fluid medium, the force on the plane is due to pressure only and must act normal to the surface. This pressure force is found to be $F = pgA\, y_c\, sin\alpha$ where

A is the area of the surface (m^2)

p is the density of the liquid

(kg/m^3) g is the gravitational

acceleration (m/s^2)

Y_c is the coordinate of the centroid (m)

The force, F, may be taken as acting at the center of pressure CP. Now to determine the position of the center of pressure we take moments about O. After some arrangements and eliminations, we can prove that the position of the center of pressure is given by

$$Y_{cp} = Y_c + \frac{Ixx,c}{\text{Yc A}} \,.$$

Where $I_{xx,c}$ is the second moment of area (also called the moment of inertia) about the axis parallel to the x-axis and passing through the centroid C.

Data Collected:

Partial Immersion

h (m)	M (Kg)	Immersed Area (m^2)	Theoretical Y_{cp} (m)	Experimental Y_{cp} (m)	M/h^2 (Kg/m2)
0.015	0.004	0.00113	0.01	-0.0428	17.78
0.030	0.017	0.00225	0.02	-0.0189	18.89
0.045	0.030	0.00338	0.03	-0.0365	14.81
0.060	0.065	0.00450	0.04	0.0044	18.06
0.075	0.097	0.00563	0.05	0.0130	17.24
0.090	0.132	0.00675	0.06	0.0204	16.30

Total Immersion

h (m)	M (Kg)	Theoretical Y_{cp} (m)	Experimental Y_{cp} (m)	h-d/2 (m)
0.11	0.185	0.0739	0.0333	0.06
0.12	0.210	0.0819	0.0400	0.07
0.13	0.232	0.0904	0.0460	0.08
0.14	0.252	0.0993	0.0520	0.09

Sample Of Calculation:

→ Sample of calculations (partially submerged) :

[for $h = 0.015\,m$ & $M = 0.004\ kg$]

1.) immersed area $= (b)(h) = (0.075\,m)(0.015\,m) = \boxed{1.125 \times 10^{-3}\ m^2}$

2.) theoretical $y_{cp} = y_c + \dfrac{I_{xx,c}}{(y_c)(A)}$

$$\text{"} = \frac{h}{2} + \frac{\frac{1}{12}(b)(h)^3}{\left(\frac{h}{2}\right)(b*h)}$$

$$\text{"} = \frac{0.015}{2} + \frac{\frac{1}{12}(0.075)(0.015)^3}{\left(\frac{0.015}{2}\right)(0.075*0.015)}$$

$$\text{"} = \boxed{0.01\ m}$$

3.) exp. $y_{cp} = \dfrac{(M)(c)}{(\rho)(y_c)(A)} - (a+d-h)$

$$y_{cp} = \frac{(0.004)(0.3)}{(1000)\left(\frac{0.015}{2}\right)(0.075*0.015)} - (0.1 + 0.1 - 0.015)$$

$$y_{cp} = \boxed{-0.043\ m}$$

$\Sigma M_o = 0$

$(F)(a+d-h+y_{cp}) = (Mg)(c)$

$(\rho g y_c A)(a+d-h+y_{cp}) = (Mg)(c)$

$$y_{cp} = \frac{M\cancel{g}c}{\rho \cancel{g} y_c A} - (a+d-h)$$

$$y_{cp} = \frac{Mc}{\rho y_c A} - (a+d-h)$$

4.) $\dfrac{M}{h^2} = \dfrac{0.004}{(0.015)^2} = \boxed{17.78\ kg/m^2}$

→ sample of calculations (totally immerged):

[for $h = 0.11\,m$ & $M = 0.185\,Kg$]

1.) $$y_{cp}\text{ theoretical} = y_c + \frac{I_{xx,c}}{y_c A}$$

$$'' = \left(h-\frac{d}{2}\right) + \frac{\frac{1}{12}(b)(d)^3}{\left(h-\frac{d}{2}\right)(b*d)}$$

$$'' = \left(0.11 - \frac{0.1}{2}\right) + \frac{\frac{1}{12}(0.075)(0.1)^3}{\left(0.11-\frac{0.1}{2}\right)(0.075*0.1)}$$

$$'' = \boxed{0.074\ m}$$

2.) $$y_{cp}\text{ experimental} = \frac{(M)(c)}{\rho\, y_c A} - \left(a + \frac{d}{2} - y_c\right)$$

$$'' = \frac{(M)(c)}{\rho\left(h-\frac{d}{2}\right)(b*d)} - \left(a + \frac{d}{2} - h + \frac{d}{2}\right)$$

$$= \frac{(M)(c)}{\rho\left(h-\frac{d}{2}\right)(b*d)} - (a + d - h)$$

$$'' = \frac{(0.185)(0.3)}{(1000)\left(0.11 - \frac{0.1}{2}\right)(0.075*0.1)} - (0.1 + 0.1 - 0.11)$$

$$'' = \boxed{0.033\ m}$$

$$\sum M_o = 0$$

$$(Mg)(c) = (F)\left(a + \frac{d}{2} - y_c + y_{cp}\right)$$

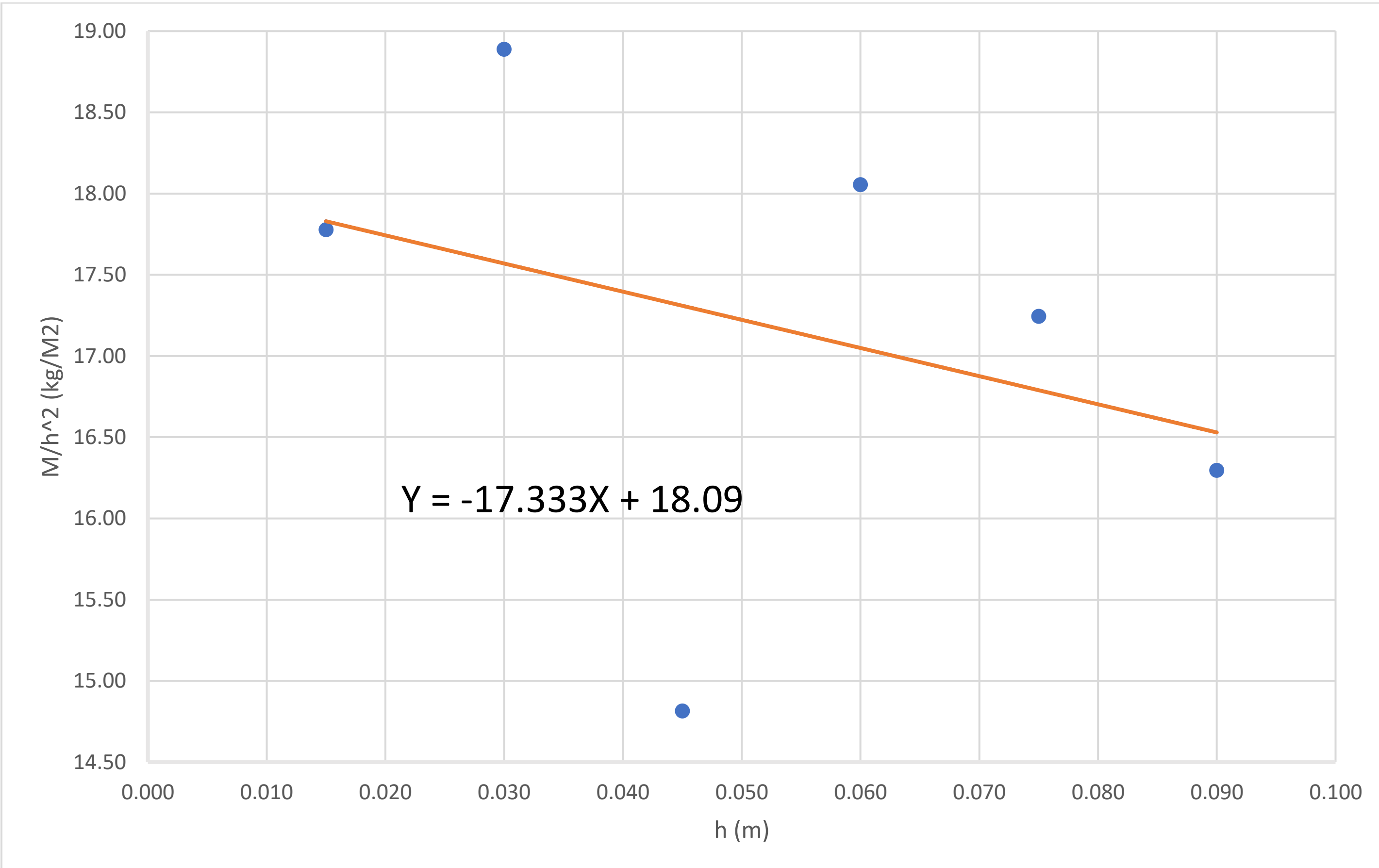
$$(Mg)(c) = (\rho g\, y_c A)\left(a + \frac{d}{2} - y_c + y_{cp}\right)$$

$$y_{cp} = \frac{M\cancel{g}c}{\rho \cancel{g} y_c A} - \left(a + \frac{d}{2} - y_c\right)$$

$$y_{cp} = \frac{Mc}{\rho y_c A} - \left(a + \frac{d}{2} - y_c\right)$$

3.) $$h - \frac{d}{2} = (0.11m) - \left(\frac{0.1m}{2}\right) = \boxed{0.06m}$$

Results:



A.) partial immersion

1. $y_{cp} = y_c + \frac{I_{xx,c}}{y_c \cdot A} \rightarrow y_{cp} = \frac{h}{2} + \frac{\frac{1}{12}(b)(h)^3}{(\frac{h}{2})(b*h)} \rightarrow y_{cp} = \frac{h}{2} + \frac{h}{6} = \frac{2h}{3}$

2. $\Sigma M_o = 0 \rightarrow (Mg)(c) = (F)(a+d-h+y_{cp})$

$$(Mg)(c) = F(a+d-(2y_c)+y_{cp})$$

$$(Mg)(c) = (\rho g y_c A)(a+d-2y_c+y_{cp})$$

$$(\cancel{M}g)(c) = (\rho \cancel{g} [\tfrac{h}{2}]\cdot[b*h])(a+d-2y_c+y_{cp})$$

$$Mc = \frac{1}{2}\rho b h^2 \left(a+d-2\left[\frac{h}{2}\right]+\left[\frac{2h}{3}\right]\right)$$

$$Mc = \frac{1}{2}\rho b h^2 \left(a+d-\frac{1}{3}h\right)$$

$$\frac{M}{h^2} = \frac{Pb}{2c}\left((a+d) - \frac{1}{3}h\right)$$

$$\frac{M}{h^2} = \frac{Pb(a+d)}{2c} - \frac{Pb}{6c}h$$

$$\boxed{\frac{M}{h^2} = \underbrace{-\frac{Pb}{6c}}_{\text{slope}} h + \underbrace{\frac{Pb(a+d)}{2c}}_{\text{y-intercept}}}$$

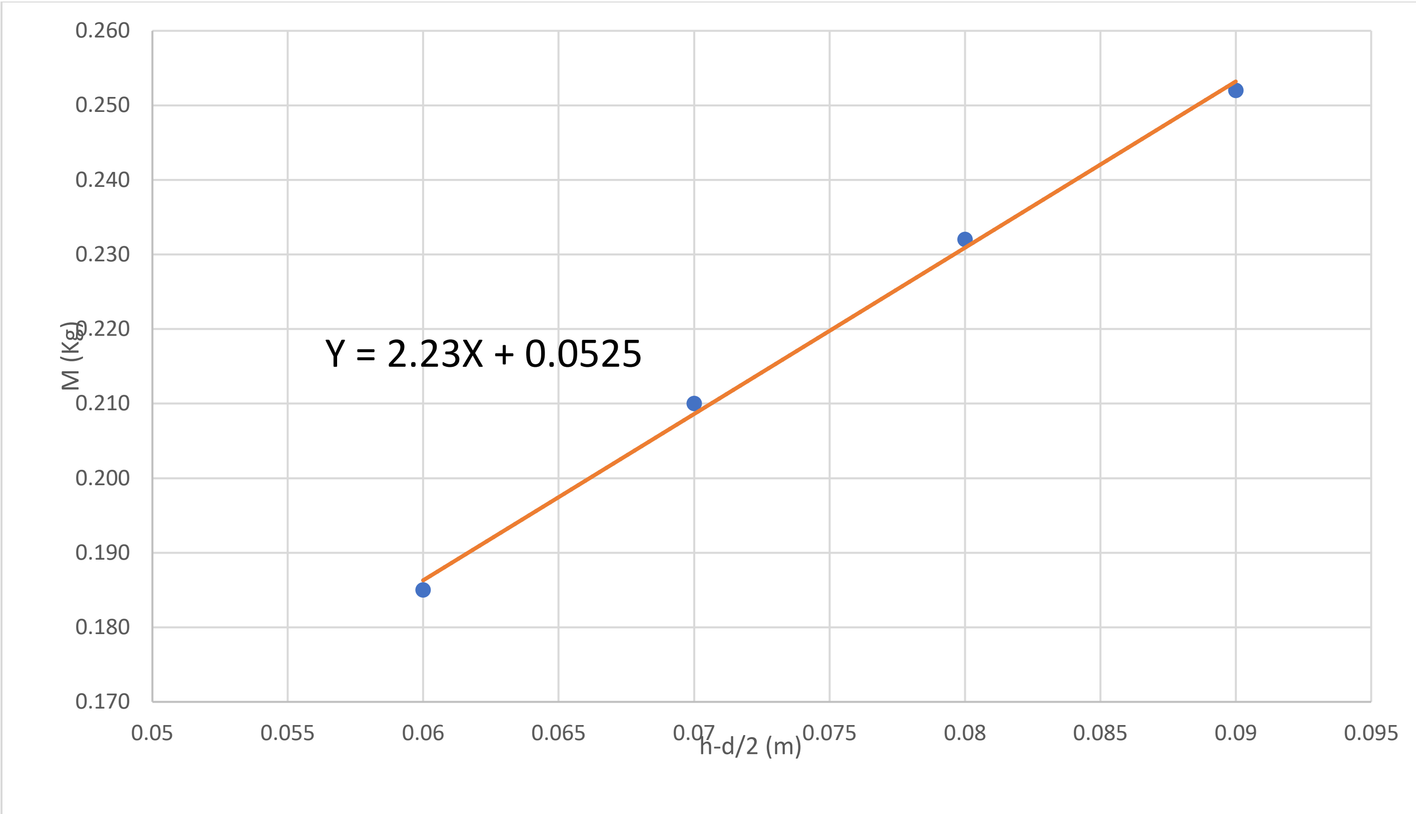
3\. a) slop theoretically $= \frac{Pb}{6c} = \frac{(1000\,\frac{Kg}{m^3})(0.075m)}{6(0.3m)} = \boxed{41.667\ \frac{Kg}{m^3}}$

slope exp. $=$ $\boxed{-17.33 \text{ Kg/m\^{}3}}$

b.) y-intercept theoretically $= \frac{-Pb(a+d)}{2c} = \frac{(-1000\,\frac{Kg}{m^3})(0.075m)(0.1m + 0.1m)}{2(0.3m)}$

$= \boxed{-25\ Kg/m^2}$

y-intercept exp. $=$ $\boxed{18.09 \text{ Kg/m\^{}2}}$



B.) total immersion

1. $$Y_{cp} = Y_c + \frac{I_{xx,c}}{Y_c A} \rightarrow Y_{cp} = \left(h-\frac{d}{2}\right) + \frac{\frac{1}{12}(b)(d)^3}{\left(h-\frac{d}{2}\right)(b*d)}$$

$$Y_{cp} = \left(h-\frac{d}{2}\right) + \frac{(d)^2}{12\left(h-\frac{d}{2}\right)}$$

2. $$\Sigma M_o = 0 \rightarrow (Mg)(c) = (F)\left(a+\frac{d}{2} - Y_c + Y_{cp}\right)$$

$$M\cancel{g}c = (\rho \cancel{g}\, Y_c A)\left(a+\frac{d}{2} - Y_c + Y_{cp}\right)$$

$$Mc = \rho\left(h-\frac{d}{2}\right)(b*d)\left(a+\frac{d}{2} - \left[h-\frac{d}{2}\right] + \left[h-\frac{d}{2} + \frac{d^2}{12\left(h-\frac{d}{2}\right)}\right]\right)$$

$$MC = \rho\left(h-\frac{d}{2}\right)(b*d)\left(a+\frac{d}{2}\cancel{-h}\cancel{+\frac{d}{2}}\cancel{+h}\cancel{-\frac{d}{2}}+\frac{d^2}{12\left(h-\frac{d}{2}\right)}\right)$$

$$MC = \rho bd\left(h-\frac{d}{2}\right)\left(\left[a+\frac{d}{2}\right]+\left[\frac{d^2}{12\left(h-\frac{d}{2}\right)}\right]\right)$$

$$MC = \rho bd\left(a+\frac{d}{2}\right)\left[h-\frac{d}{2}\right]+\frac{\rho bd^3}{12}\cdot\frac{\cancel{\left(h-\frac{d}{2}\right)}}{\cancel{\left(h-\frac{d}{2}\right)}}$$

$$\boxed{M = \left[\underbrace{\frac{\rho bd}{c}\left(a+\frac{d}{2}\right)}_{\text{slope}}\right]\left(h-\frac{d}{2}\right)+\underbrace{\frac{\rho bd^3}{12c}}_{\text{y-intercept}}}$$

3. a) slope theoretically $= \frac{\rho bd}{c}\left(a+\frac{d}{2}\right)$

" $= \frac{(1000)(0.075)(0.1)}{(0.3)}\left[0.1+\frac{0.1}{2}\right]$

" $= \boxed{3.75\ \frac{Kg}{m}}$

slope experimentally = 2.23 Kg/m

b.) y-intercept theoretically $= \frac{\rho bd^3}{12c}$

" $= \frac{(1000)(0.075)(0.1)^3}{12(0.3)}$

" $= \boxed{0.2083\ Kg}$

y-intercept exp. = 0.0525 Kg

Discussion & Conclusions

1- The hydrostatic force exerted on the beam is greater when it is submerged and less when it is partially merged. Therefore, the force is directly proportional to the depth.

2- There is a difference between theoretical and experimental results due to the following:
   - Disregarding the weights of the balance or the pan
   - Errors in reading the depth from the Vernier or in determining the point between the water surface and the pin of the measuring device due to parallax error.

3- The pressure force acts on the four surfaces of the rectangular toroid, but they cancel each other, that is why we did not consider them in the calculations.

4- The location of the center of pressure will change when changing the fluid used (p (rou) depends on the fluid used)