

11/6/2019

NO. ORI

Operation Research: Allocation of Resources to activities in the best way such organization achieve most effective / efficient results
↳ max/min of sth. of profit-cost.

Resources: Raw material, equ., labor, time (scarce)

Activities: many different things

Principle phase of OR Implementation

- ① Problem definition
- ② Model Formulation
- ③ Solu. ~~validation~~ model
- ④ solu. validation
- ⑤ implementation

Translation of verbal statement to mathematical statement (Model)

Model: Representation of problem or situation

Mathematical: Relationship between (variable) & (parameters)

variables: (symbols): $x_1, x_2, \dots, x_n$

$y_1, y_2, \dots, y_m$

Parameters: $\times$ numbers

$3x_1 + 5x_2 \rightarrow$ mathematical statement.

Deterministic Vs Stochastic
 all inf. about ~~are~~ probabilistic
 model are know
 with containing.

OR Techniques.

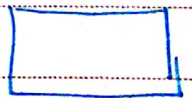
- ① Linear Programming
- ② non linear Programming
- ③ integer Programming
- ④ network programming

*
 Programming =
 planning
 (Allocation)

Q ⇒ Suppose we wish to enclose a rectangle yard
 by atmost (80 meters) of fencing
 formulate the math model find the design of
 max area

Solu:

[atmost / equal / at least ⇒ $\leq$ / $=$ / $\geq$]



① D.v l : length in m
 w : width in m

② $\text{Max } Z = l + w$

③ a. $w, l \geq 0$ (non negative
 const.)

③ b. $2l + 2w \leq 80$

Statement $\Rightarrow$ Mathematical : كيف أتبع في الحل :

Principle components In OR Models

① Define Decision (D.v) variable : unknown Quantities.

$\hookrightarrow$ # of tables // # of labors / length / width

② Define objective of function " criterion "

$\hookrightarrow$ Max Z // Min Z

③ constraints " Restrictions "

$\hookrightarrow$ (a) variable type restriction (non-negative constraint)

(b) Geneml. $\rightarrow$ Resources - Guid - Respe.

④ $\rightarrow$ $\overline{a} \overline{b} \overline{c}$

$$\max Z = l + w$$

(non linear) (amprovi)

Subject to

$$2l + 2w \leq 80$$

$$l, w \geq 0$$

l/w

$\hookrightarrow$ integers

متى سبب



negative sign $\rightarrow$ متى موضح ال
عند التبسيط والسر

LP Properties

لديهم يكونوا موضوعين للأشياء

① Proportionality

(1) وما يكونوا متوازنين ببعضهم

② Additivity

③ Divisibility $\rightarrow$ continuous.

④ Deterministic

mixing-blending problem.

LP Formulation

Example 1: Maximization Problem

Bubba Corporation produces two types of paint (interior and exterior) from three raw materials M1, M2, M3. The following table provides the basic data of the problem:

Raw Material	Tons of raw material per ton of		Maximum daily availability (tons)
	Exterior paint (1)	Interior paint (2)	
M1	2	1	10
M2	3	3	20
M3	2	4	20
Profit per ton (\$1000)	\$20	\$30	

Bubba would like to maximize his corporation's profit from selling these two paint types. He has asked class IE 0906353 to help him determine the number of tons of each paint type he should produce to achieve his goal.

Maximize the profit from selling these two paint types.

Determine the number of tons of each paint type that should be produced to achieve his goal.

Bubba wants to max 2 paint types

- ① D.v $\Rightarrow$ Exterior paint x_1 (num of tons)
Interior paint x_2 (* of tons)

② $\text{Max } Z = 20x_1 + 30x_2$

s.t

③ availability

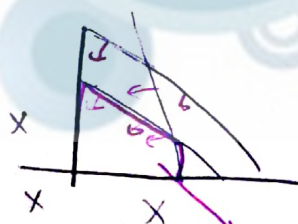
$$2x_1 + x_2 \leq 10$$

$$3x_1 + 3x_2 \leq 20$$

$$2x_1 + 4x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

$\hookrightarrow$ non negativity constraint.



Feasible region
Infeasible

x_1	in	inputs
M_1	in	2
M_2	3	
M_3	2	

food is min and cost

{ corn is Dr is }
soybeanResource
allocation**Example 2: Minimization Problem**

Bubba has a farm. His farm uses at least 800 lb of a special feed daily. The special feed is a mixture of corn and soybean meal with the following compositions:

Feedstuff	Lb per lb of feedstuff		Cost (\$/lb)
	Protein	Fiber	
Corn	0.09	0.02	0.30
Soybean meal	0.6	0.06	0.90

- The dietary requirements of the special feed stipulate at least 30% protein and at most 5% fiber.
- Bubba wishes to determine the daily minimum-cost feed mix.

Dr $x_1 \rightarrow \text{corn}$ $x_2 \rightarrow \text{soybean}$

$$\min z = .3x_1 + .9x_2$$

$$x_1 + x_2 \geq 800$$

S.T

$$.09x_1 + .6x_2 \geq .3(x_1 + x_2)$$

~~also~~

$$.02x_1 + .06x_2 \leq .05(x_1 + x_2)$$

$$x_1, x_2 \geq 0$$

↳ non negative
constraint.

↳ $z(800)$
 $x_1 + x_2 = 800$

↳ $z(800)$
800

LP formulation: A Staff Scheduling Problem

Suppose you run the popular JU fast food Place that is open seven days a week. You hire employees to work a five-day workweek with two consecutive days off. Each employee receives the same weekly salary. Some days of the week are busier than others and, based on past experience, you know how many workers are required on a given day of the week. In particular, your forecast calls for these staffing requirements:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Staff Req'd	20	16	13	16	19	14	12

available $\rightarrow x_1 \geq 20$
 $x_2 \geq 16$

You need to determine how many employees to start each day of the week in order to minimize the total number of required employees, while still meeting or exceeding staffing requirement each day of the week.

* Personal / staff scheduling

Note: All the assumptions of Linear Programming hold.

Formulate a complete linear programming model for this problem.

$x_1 =$ # of starting employees in Monday

$x_7 =$ # of starting employees in Sunday

min (total) cost

$$\min Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

s.t

$$x_1 + x_4 + x_5 + x_6 + x_7 \geq 20$$

$$x_2 + x_3 + x_6 + x_7 + x_1 \geq 16$$

$$x_3 + x_4 + x_7 + x_1 + x_2 \geq 13$$

$$x_4 + x_7 + x_1 + x_2 + x_3 \geq 16$$

1 of 1

So on

$$x_1, x_2, x_3, x_4, x_5, x_6 - x_7 \geq 0$$

$\geq$
 exceeding

NO. Formulating

in General.

$$\max / \min Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$$

s.t

→ Right hand side.

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \dots + a_{1n}X_n \leq b_1$$

$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \dots + a_{2n}X_n \leq b_2$$

 $\leq b_m$ for $i = 1, \dots, m$ $j = 1, \dots, n$ → for all i

(*matlab) compact form:

$$Z = \sum C_j X_j$$

$$\sum a_{ij} X_j \leq b_i \quad \forall i$$

(ex1)

$$\max Z = 20X_1 + 30X_2$$

$$2X_1 + X_2 \leq 10 \rightarrow \textcircled{1}$$

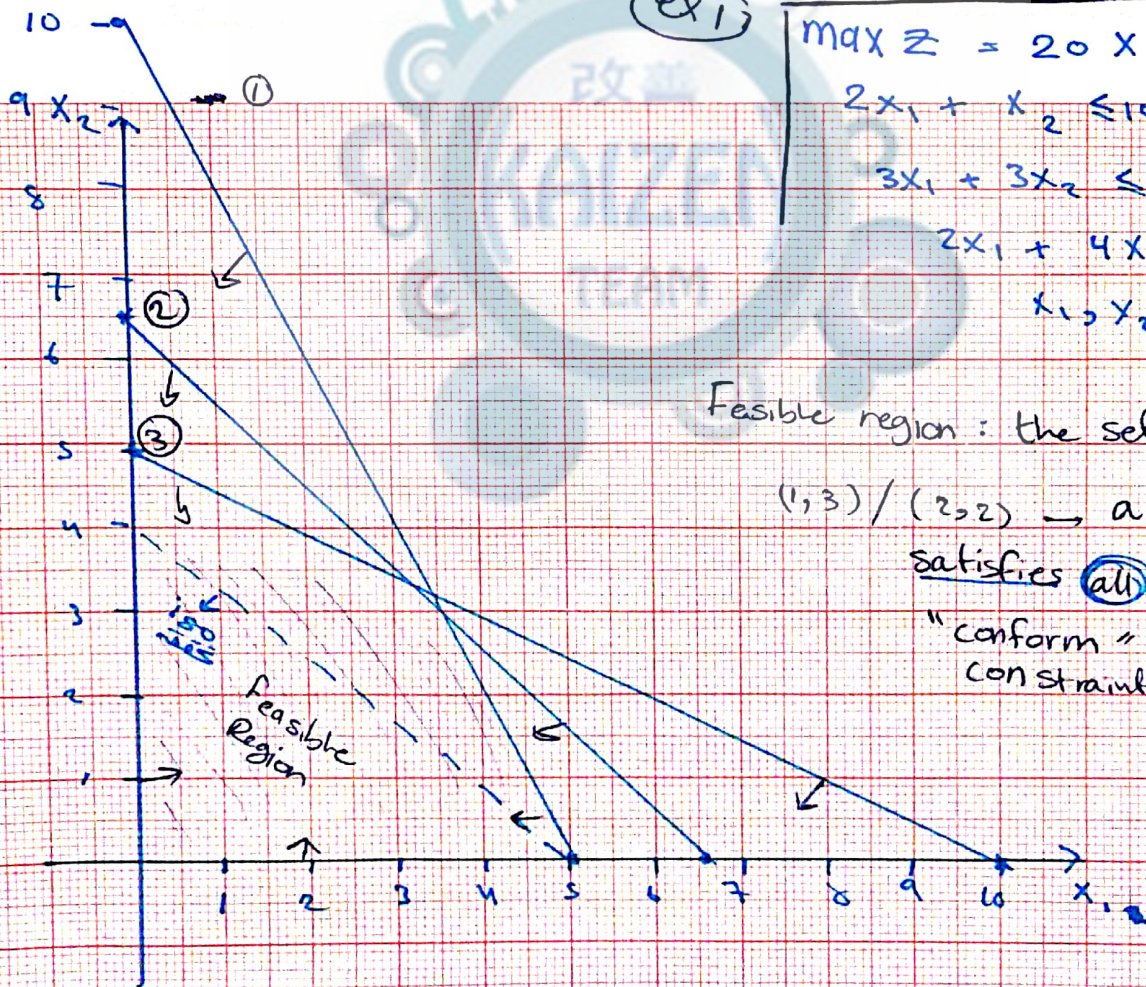
$$3X_1 + 3X_2 \leq 20 \rightarrow \textcircled{2}$$

$$2X_1 + 4X_2 \leq 20 \rightarrow \textcircled{3}$$

$$X_1, X_2 \geq 0$$

Feasible region: the set of all F.S

$(1,3) / (2,2) \rightarrow$ a point that satisfies all constraints
"conform" to all constraints



Find solution $\rightarrow$ Bdo (x_1, x_2)

$$\text{Max } Z = 20x_1 + 30x_2$$

FR : set of all F.s

F.s : per missible value of (x_1, x_2)
: conform to all constraints

ويفر OS $100 = 20x_1 + 30x_2$:

ISO profit line $\rightarrow$ max

ISO cost line $\rightarrow$ min



Binding $\rightarrow$ $2x_1 + x_2 = 10$ $\rightarrow$ ①

$$2x_1 + x_2 = 10 \rightarrow ①$$

$$2x_1 + 4x_2 = 20 \rightarrow ③$$

$$③ - ① \Rightarrow x_2 = 3.33$$

$$x_1 = 3.33$$

optimal solution $\rightarrow (3.33, 3.33)$

$$\text{max } Z = (20x_1 + 30x_2)$$

$$\text{max } Z = 166.5$$

(Profit)

Butba Almas

$$\min Z = .3x_1 + .9x_2$$

$$x_1 + x_2 \geq 800$$

$$.09x_1 + .6x_2 \geq .3(x_1 + x_2)$$

$$.02x_1 + .06x_2 \leq .05(x_1 + x_2)$$

$$x_1, x_2 \geq 0$$

$$-.21x_1 + .3x_2 \geq 0$$

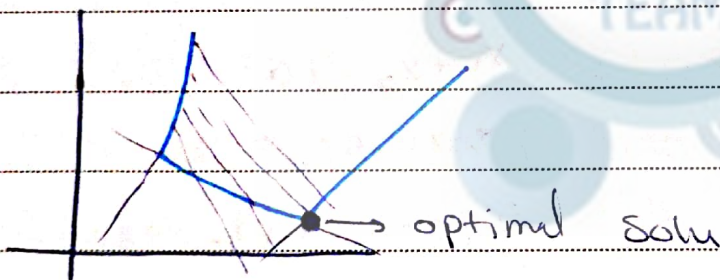
$$-.48x_1 + .01x_2 \leq 0$$

$$x_1, x_2 \geq 0$$

max → كبر المسطرة فوجد

min → كبر المسطرة حتى

لازم المسطرة تقاطع
in كل المساحة



constraint solutions objective to
multiple optimal solu.

can make → تفوق

NO.

Q 3.1.7

Resource allocation.

D.V

x_1 = # of aluminum window

x_2 = # of wood

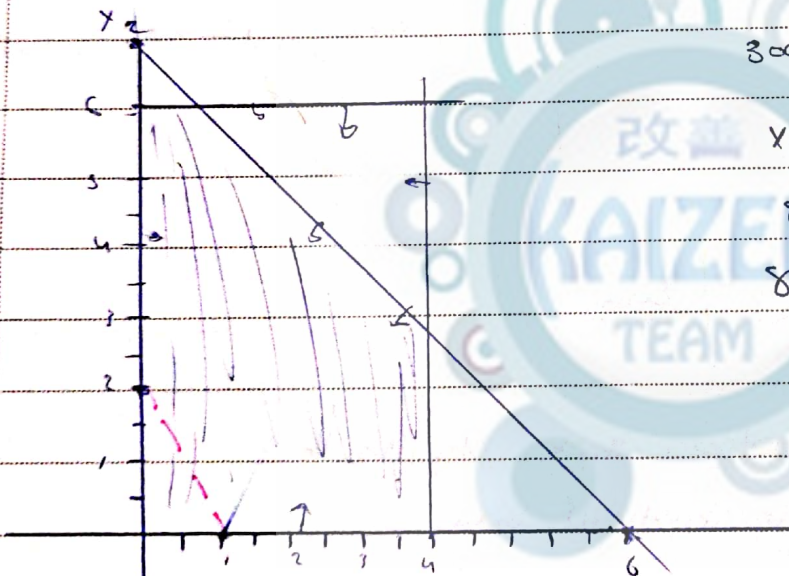
$$\max z = 300x_1 + 150x_2$$

$$x_1 \leq 4 \rightarrow \textcircled{1}$$

$$x_2 \leq 6 \rightarrow \textcircled{2}$$

$$8x_1 + 6x_2 \leq 48$$

$$x_1, x_2 \geq 0$$



$$300 = 300x_1 + 0$$

$$x_1 + x_2 = 10$$

$$8x_1 + 6x_2 = 48$$

$$8(10 - x_2) + 6x_2 = 48$$

$$80 - 2x_2 = 48$$

$$30 = 2x_2$$

$$x_2 = 15$$

$$x_2 - x_1 = 2$$

$$x_2 = 2 + x_1$$

$$8x_1 + 12 + 6x_1 \leq 48$$

$$14x_1 \leq 36$$

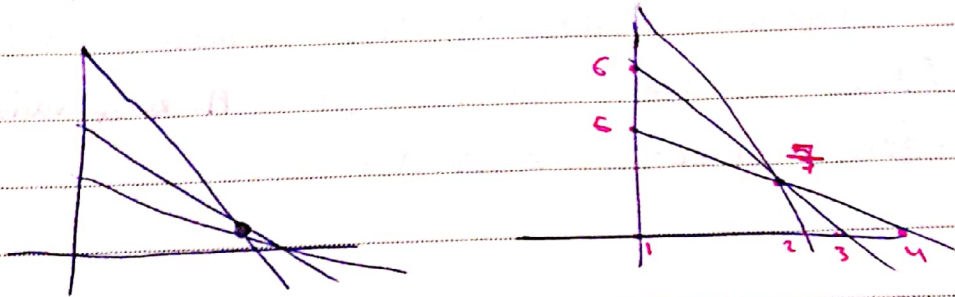
$$x_1 \leq 2.57 \quad x_2 = 4.57$$

$$(2.57, 4.57) \Rightarrow \max z = 1456.7$$

$x_2 = 2 + x_1$
 $8x_1 + 12 + 6x_1 \leq 48$

optimal solution $\Rightarrow x_1/x_2/x_n$

optimal objective $\Rightarrow \min/\max Z$



Corner point: intersection of $n=2$ constraint boundaries.

- * Points 1, 5, 2, 7 corner points feasible soln. (C P F)
- * Points 2, 3, 4, 6 corner point in feasible soln (C P i F)
- * Relationship btwn optimal soln & (C P F) soln.
- Consider any linear program with feasible solution & a bounded feasible region. - The problem must possess (C P F) soln & at least one optimal soln.
- The best (C P F) soln must be optimal soln. $\rightarrow$ if the problem has one optimal soln, it must be a C P F
- $\rightarrow$ if the problem has multiple optimal soln at least 2 must be (C P F) soln.

points $\left\{ \begin{array}{l} (1) \rightarrow (0,0) \\ (5) \rightarrow (0,5) \\ (2) \rightarrow (5,0) \end{array} \right.$

* Point 1 & 5 are adjacent bec they share the $x_1=0$ constraint boundary

" 1 & 2 are - - -

" - - they share $x_2=0$ - - -

- In general two CPFs are adjacent to each other if they share $n-1$ constraint boundary.
- The line segment connecting two adjacent CPF Solu in Edge of the Feasible region. $n = \#$ of variables
- if an edge connects 2 (CPF)

→

Consider any LP that possesses at least one opt solu if a CPF has no adjacent CPFs that are better than it must be an opt solu (simplex method)

(best) solu ← Simplex algorithm

(edge) $CP \leq 1 \equiv$

① → ② & ⑤

⑤ → ① & ⑦

⑦ → ① & ②

② → ① & ⑦

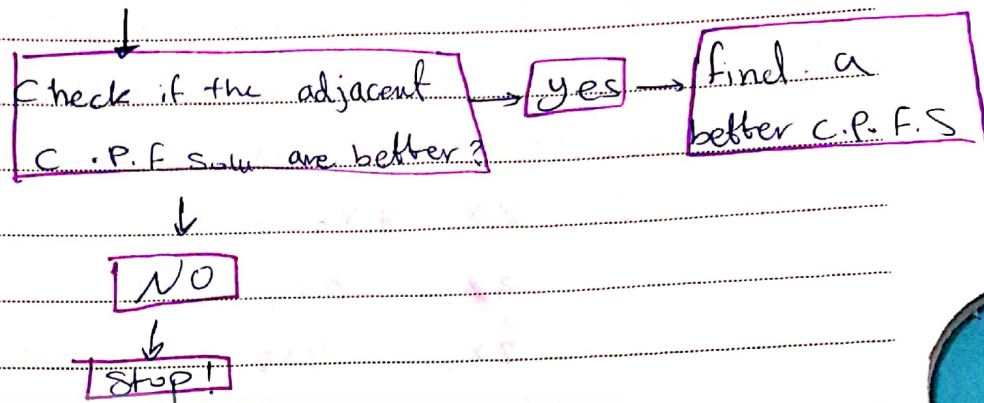
→ F. Region

* Simplex method: to locate the optimum by concentrating on a selected $\#$ of CPFs

* Each iteration of the simplex method moves from the current CPFs to an adjacent one by moving along one of the n edges.

* is an algebraic procedure based on solving systems of equations

* Start of a C.P.F. soln *



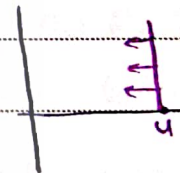
* To use the simplex method, we first need to put the eqns in a special form.

* Augmented form:

convert func "in equality" to equality "to equivalent equality constraints by introducing slack variable

$$x_1 \leq 4 \quad x_1 + x_3 = 4$$

$$x_3, x_1 \geq 0$$



$$\text{if } x_1 = 3$$

$$x_3 = 1 \Rightarrow \text{slack variable}$$

$$\text{if } x_1 = 0$$

$$x_3 = 4 \Rightarrow \text{slack v.}$$

$$\text{if } x_1 = -1 \rightarrow (\text{I.F.R.}) \text{ is } 0$$

Slack variable x_3 is added to the constraint

One slack variable x_3 is added to the constraint

Bubbas ex $\rightarrow 2x_1 + x_2 \leq 10 \rightarrow (1)$

$$3x_1 + 3x_2 \leq 20 \rightarrow (2)$$

$$2x_1 + 4x_2 \leq 20 \rightarrow (3)$$

$$2x_1 + x_2 + s_1 = 10 \rightarrow (1')$$

$$3x_1 + 3x_2 + s_2 = 20 \rightarrow (2')$$

$$2x_1 + 4x_2 + s_3 = 20 \rightarrow (3')$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

$$x_1 = 3.33 = x_2$$

$$6.66 + 13.33 + s_3 = 20$$

$$s_3 = 0$$

(5, 0)

am Ende

$\rightarrow$ Ende

$$s_1 = 0 \rightarrow \text{aus der Basis}$$

bis 1.25

- A basic solu is an augmented C P S
- A basic feasible solu is an augmented C P F S
- The simplex method uses Zeros for the arbitrary values.

Two variables that are set together are called non basic variables N.B.V. & other 3 variables are called Basic variables (B.V)

Defining Equ $\leftarrow$ $\bar{U} \bar{J} \bar{a} \bar{u} \bar{S} \bar{u} \bar{C} \bar{S} \bar{a} \bar{u} \bar{Y}$ is

- The B.V are used in the ~~concurrent~~ simultaneous Solu of the 3 equ (Defining equ) the Solu is called a basic solu.

• To summarize: a basic solu has the following properties

(1) Variable are designated on either basic or non basic

B.V & N.B.V

The set of basic variables is referred to as the basis

(2) * of basic variables

* of functional constraints (m) (no Redundancy)

(3) * of N.B.V = total * of variable - total * of variables - * of functional constraints

(4) non basic variables are set equal to 0

(5) Values of B.V are obtained as the simultaneous solu at the system of equ.

(6) if non negativity constraints are satisfied the basic solu is basic & feasible solu B.F.S

C.P.F.S

Augmented form BFS

$(x_1, x_2, s_1, s_2, s_3)$

نقطه
مقادیر

$(5, 0)$ →
 $(0, 10)$ →
 $(0, 5)$ →
 $(3.33, 3.33)$ →

$(5, 0, 10, 20, 0)$
 $(0, 10, 0, 20, 20)$
 ~~$(0, 5, 5, 5, 10)$~~
 $(3.33, 3.33, 0, 0, 0)$

$\max Z = 20x_1 + 30x_2$

s.t $2x_1 + x_2 + s_1 = 10$

$3x_1 + 3x_2 + s_2 = 20$

$2x_1 + 4x_2 + s_3 = 20$

x_1, x_2

x_1, x_2, s_1, s_2, s_3

U.B.V

Basis (B.V)

(F.P.) $(5, 0, 10, 20, 0)$

$(0, 0)$

$(0, 10, 10, 20, 20)$

~~x_2, s_1~~ ~~x_1, s_2, s_3~~

$(0, 5)$

$(0, 5, 5, 5, 10)$

x_1, x_2 s_1, s_2, s_3

$(3.33, 3.33)$

$(3.33, 3.33, 0, 0, 0)$

x_1, s_3 x_2, s_2, s_1

$(5, 0)$

$(5, 0, 10, 5, 10)$

s_1, s_3 s_2, x_1, x_2

$(0, 10)$

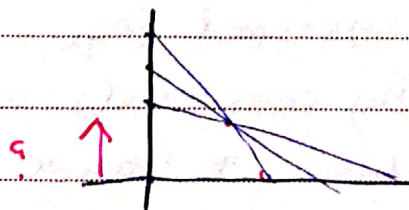
$(0, 10, 10, 20, 20)$

x_2, s_1 x_1, s_2, s_3

x_1, x_2 s_1, s_2, s_3

adjacent

نقطه
مقادیر



Direction of movement

$(0, 5)$?

$(5, 0)$?

$20 + 30x_2$
 $20 + 20x_1$

[23/6/2019]

NO. OR I

① Determine Direction

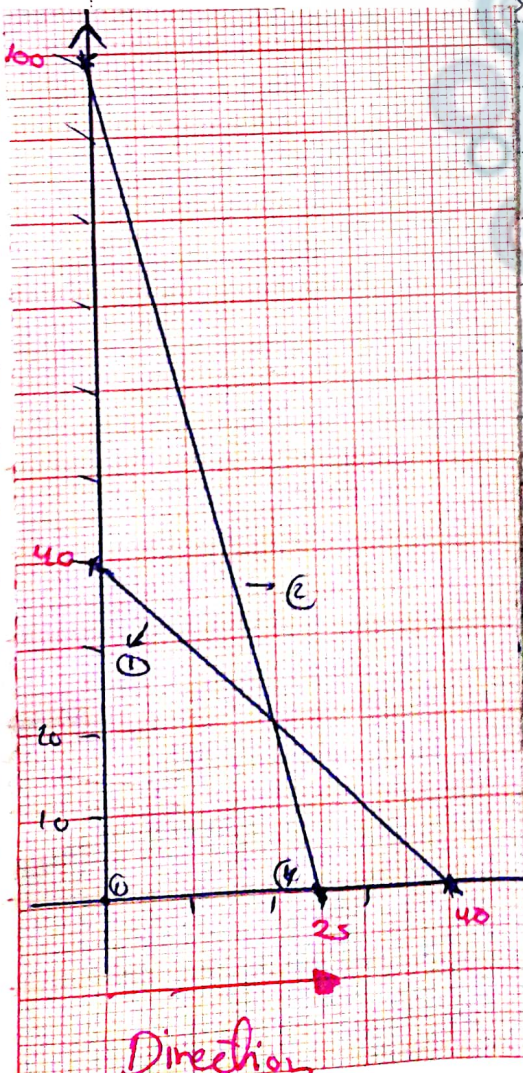
no. of B.S. $\leq$ no. of constraints $\rightarrow$ infeasible or unbounded Region

② How much magnitude

note: - Two basic Feasible solu. are adjacent if all but one of their N.B.V are the same (not their values)

- Two b.f.s are adjacent if all but one of their B.S are the same (not their values)

Ex: $\max Z = 2x_1 + x_2 \Rightarrow * [Z - 2x_1 - x_2 = 0$
 S.T $x_1 + x_2 \leq 40 \Rightarrow x_1 + x_2 + S_1 = 40$
 $4x_1 + x_2 \leq 100 \Rightarrow 4x_1 + x_2 + S_2 = 100$
 $x_1, x_2 \geq 0$ $x_1, x_2, S_1, S_2 \geq 0$



BV	Z	x_1	x_2	S_1	S_2	RHS
Z	0	1	-2	0	0	0
S_1	1	0	1	1	0	40
S_2	2	0	4	1	1	100

Pivot Row

Pivot column

Right hand side

$\Rightarrow$ For (0,0)

Ratio
 $\frac{0}{-2}$
 $\frac{40}{1}$
 $\frac{100}{4}$

most negative $\rightarrow$ direction
 (X)

x_1 : Entering B.V (most negative)

S_2 : leaving B.V (min ratio)

$\uparrow$ (iteration # 1)

FIVE APPLE

$x_1, x_2 \rightarrow$ non Basic = 0

iteration ②

since the coeff of x_2 is $\ominus \frac{1}{2}$
its not optimal solu.

		Z	x_1	x_2	s_1	s_2	RHS	ratio
Z	0	1	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	50	100 100
s_1	1	0	0	$\frac{3}{4}$	1	$-\frac{1}{4}$	15	20
x_1	2	0	1	$\frac{1}{4}$	0	$\frac{1}{4}$	25	100

Leaving s_1
entering x_2
Basic x_1
Pivot column

iteration ③

		Z	x_1	x_2	s_1	s_2	RHS	ratio
Z	0	1	0	0	$\frac{2}{3}$	$\frac{1}{3}$	60	
x_2	1	0	0	1	$\frac{4}{3}$	$-\frac{1}{3}$	20	
x_1	2	0	$-\frac{1}{4}$	0	$\frac{-4}{4+3}$	$\frac{-4}{3+4}$	20	

Basic values

opt objective solu ~~2000~~ $x_1, x_2 = 20$

opt objective $Z = 60$

$$\max Z = 20x_1 + 30x_2$$

$$2x_1 + x_2 + S_1 = 10$$

$$3x_1 + 3x_2 + S_2 = 20$$

$$2x_1 + 4x_2 + S_3 = 20$$

Iteration # 1

	Z	x_1	x_2	S_1	S_2	S_3	RHS	Ratio
Z	0	1	-20	-30	0	0	0	
S_1	1	0	2	1	0	0	10	10
S_2	2	0	3	0	1	0	20	6.66
S_3	3	0	2	0	0	1	20	5

Entering column: x_2

Leaving: S_3

Iteration # 2

	Z	x_1	x_2	S_1	S_2	S_3	RHS	Ratio
Z	0	1	-5	0	0	7.5	150	
S_1	1	0	1.5	0	1	-2.5	5	20/3
S_2	2	0	1.5	0	1	-7.5	5	10/3
x_2	3	0	.5	0	0	.25	5	10

Iteration # 3

	Z	x_1	x_2	S_1	S_2	S_3	RHS	Ratio
Z	1	0	5	0	10/3	5	166.6	
S_1	0	0	0	1	-1	-1/4	0	
x_1	0	1	0	0	2/3	-1/2	10/3	
x_2	0	0	1	0	-1/3	1/2	10/3	

Optimal soln ~~max~~ $x_1, x_2 = 10/3$

optimal objective $\max Z = 166.6$

* Degeneracy

$$\max Z = 5x_1 + 3x_2$$

$$x_1 - x_2 \leq 2$$

$$2x_1 + x_2 \leq 4$$

$$-3x_1 + 2x_2 \leq 6$$

$$Z - 5x_1 + 3x_2 = 0$$

$$x_1 - x_2 + s_1 = 2$$

$$2x_1 + x_2 + s_2 = 4$$

$$s_3 - 3x_1 + 2x_2 = 6$$

1

either s_1 or s_2

	Z	x_1	x_2	s_1	s_2	s_3	RHS	Ratio
Z	1	-5	-3	0	0	0	0	
s_1	0	1	-1	1	0	0	2	2
s_2	0	2	1	0	1	0	4	2
s_3	0	-3	2	0	0	1	6	-2

(integers) $\rightarrow$ s_1 or s_2 (Ratio) $\rightarrow 1$ or 2

* Tie: Break arbitrarily

2

	Z	x_1	x_2	s_1	s_2	s_3	RHS	Ratio
Z	1	0	-1/2	0	5/2	0	10	
s_1	0	0	-3/2	1	-1/2	0	0	0
x_1	0	1	1/2	0	1/2	0	2	4
s_3	0	0	7/2	0	3/2	1	12	3.42

3

	Z	x_1	x_2	s_1	s_2	s_3	RHS	Ratio
Z	1	0	0	0	19/7	2/7	82/7	
s_1	0	0	0	1	11/7	3/7	36/7	
x_1	0	1	0	0	2/7	-1/7	2/7	
x_2	0	0	1	0	3/7	2/7	24/7	

Optimal soln $x_1 = 2/7$ $x_2 = 24/7$

optimal objective max $Z = 82/7$

selection is made in this situation reveals that model has at

least one redundant constraint.

(boundary)

FIVE APPLE

Multiple optimal soln

$$Z = 2x_1 + 4x_2 \rightarrow Z - 2x_1 - 4x_2 = 0$$

$$x_1 + 2x_2 + s_1 = 5$$

$$x_1 + x_2 + s_2 = 4$$

special case

objective func slope =
constraint 1 slope

	Z	x_1	x_2	s_1	s_2	RHS	Ratio
Z	0	-2	-4	0	0	0	
s_1	0	1	2	1	0	5	5/2
s_2	0	1	1	0	1	4	4

XX1

	Z	x_1	x_2	s_1	s_2	RHS	Ratio
Z	1	0	0	2	0	10	
x_2	0	1/2	1	1/2	0	5/2	(*)
s_2	0	1/2	0	-1/2	1	1.5	

$$10 = Z + 0x_1 + 0x_2 + 2s_1 + 0s_2$$

$$\max Z = 10$$

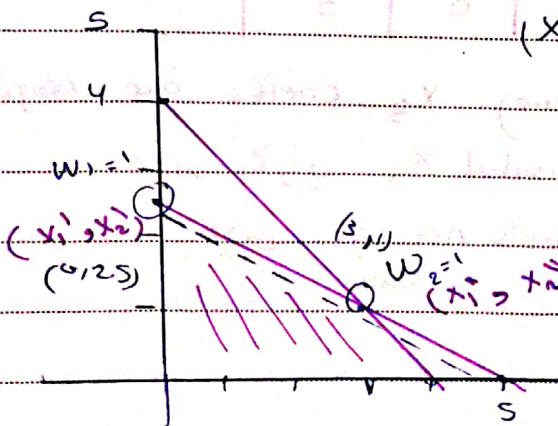
optimal objective

$$x_1 + 2x_2 = 5$$

$$x_1 + x_2 = 4$$

$$x_1 \geq 0$$

optimal soln $\Rightarrow$



$$(x_1, x_2) = w_1 (x_1', x_2') + w_2 (x_1'', x_2'')$$

$$w_1 + w_2 = 1$$

$$* \text{find } w_1 = \frac{1}{2}, w_2 = \frac{1}{2}$$

$$\frac{1}{2} (0, 2.5) + \frac{1}{2} (3, 1)$$

$$(0, \frac{5}{4}) + (\frac{3}{2}, \frac{1}{2}) = (\frac{3}{2}, \frac{7}{4})$$

indication
Multiple soln

$$\Leftrightarrow 0 = \text{N.B.V. \& coeff 16}$$

* * *

$$\max Z = 36x_1 + 30x_2 - 3x_3 - 4x_4$$

$$x_1 + x_2 - x_3 + s_1 = 5$$

$$6x_1 + 5x_2 - x_4 + s_2 = 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	Ratio
Z	1	-36	-30	3	4	0	0	0	
s_1	0	1	1	-1	0	1	0	5	5
s_2	0	6	5	0	-1	0	1	10	5/3

	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	Ratio
Z	1	0	0	3	-2	0	6	60	
s_1	0	0	1/6	-1	1/6	1	-1/6	10/3	20
x_1	0	1	5/6	0	-1/6	0	1/6	10/6	-10

	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	Ratio
Z	1	0	2	-9	0	12	4	100	
x_4	0	0	1	-0	1	6	-1	20	
x_1	0	1	1	-1	0	1	0	5	

سوال
جواب

entering x_3 coeffs are negative
unlimited x_3 $\rightarrow$ ∞

There's no opt soln.

* unbounded Z
no opt soln.

max $\rightarrow$ most negative
min $\rightarrow$ most positive

26/4/2019

NO.

$$1/6 X_2 - X_3 + \frac{1}{6} X_4 + S_1 - \frac{1}{6} S_2 = 10/3$$

$$S_1 = \frac{10}{3} - \frac{1}{6} X_4 \quad \text{bounded.}$$

$$0X_1 + 1X_2 + -6X_3 + 1X_4 + 6S_1 + -1S_2 = 20$$

$$X_4 = 20 + 6X_3 \quad \text{unbounded}$$

* Adapting to other Model forms *

$$\min Z = 4X_1 + X_2 \rightarrow \max(-Z) = -4X_1 - X_2 \quad \min \quad 1)$$

$$\text{s.t.} \quad 3X_1 + X_2 \leq 3 \quad \geq \leq \quad 2)$$

$$-X_1 - 2X_2 \geq -4 \rightarrow X_1 + 2X_2 \leq 4 \quad -1 \quad 3)$$

$$4X_1 + 3X_2 \geq 6$$

$$3X_1 + X_2 + S_1 = 3$$

$$S_1, S_2, S_3, X_1, X_2, \bar{X}_3 \geq 0$$

[Slack] $+X_1 + 2X_2 + S_2 = 4$

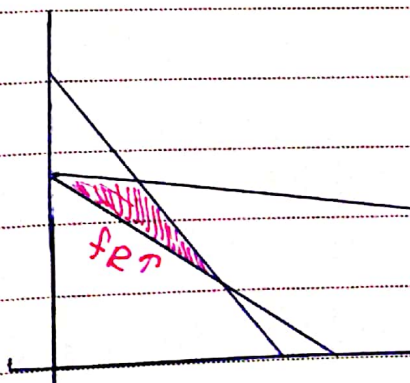
[surplus] $4X_1 + 3X_2 - S_3 = 6 \rightarrow 4X_1 + 3X_2 - S_3 + \bar{X}_3 = 6$

$\rightarrow$ Artificial variable
w.l.o. $\leftarrow S_3$ i.e.

$$X_1, X_2, S_1, S_2, S_3, \bar{X}_3$$

$$(0, 0, 3, 4, 0, 6)$$

$$(NB, NB, B, B, NB, B)$$



Now, to obtain an optimal soln, we must drive out artificial variable. $\bar{x}_3 = 0$

We have two methods to accomplish this:

- ① Big M $\downarrow$ huge num ② 2 phase

Ex: Sol Min $Z = 4x_1 + x_2 + M\bar{x}_3 + M\bar{x}_4$

$$3x_1 + x_2 = 3 \rightarrow \bar{x}_3 \text{ bi p.c.}$$

$$-x_1 - 2x_2 \geq -4$$

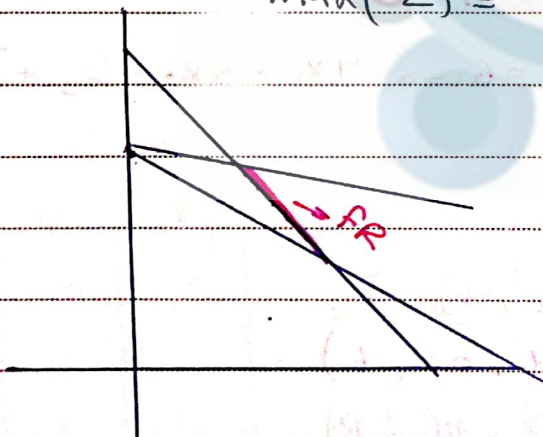
$$4x_1 + 3x_2 \geq 6$$

$$3x_1 + x_1 + \bar{x}_3 = 3$$

$$+x_1 + 2x_2 + S_1 = 4$$

$$4x_1 + 3x_2 - S_2 + \bar{x}_4 = 6$$

$$\max(-Z) = -4x_1 - x_2 - M\bar{x}_3 - M\bar{x}_4$$



Suggested

4.1-6

$$\max Z = 2x_1 + 3x_2$$

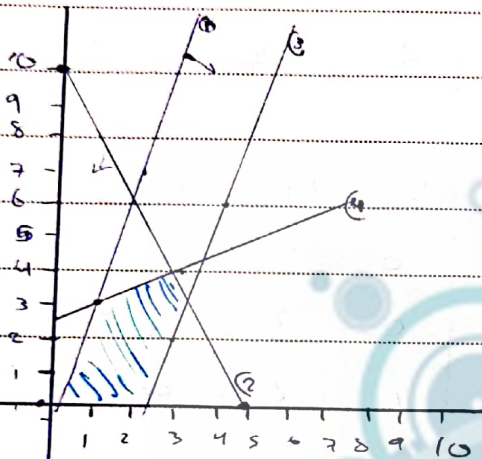
$$-3x_1 + x_2 \leq 1$$

$$4x_1 + 2x_2 \leq 20$$

$$4x_1 - x_2 \leq 10$$

$$-x_1 + 2x_2 \leq 5$$

Graphical



Simplex

#1	Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS	Ratio
Z	1	-2	-3	0	0	0	0	0	
s_1	0	-3	1	1	0	0	0	1	$+1/3 = 1$
s_2	0	4	2	0	1	0	0	20	10
s_3	0	4	-1	0	0	1	0	10	-10
s_4	0	-1	2	0	0	0	1	5	2.5

#2

Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS	Ratio
Z	1	-4	0	3	0	0	3	
s_1	0	-3	1	1	0	0	1	$-1/3 = 1$
s_2	0	10	0	-2	1	0	18	1.8
s_3	0	1	0	1	0	1	11	11
s_4	0	6	0	-2	0	1	3	$1/2$

8
-2
-2

$$\max = 17/2$$

$$x_1 = \frac{1}{2}$$

$$x_2 = 2.5$$

Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS	Ratio
Z	1	0	0	$-\frac{2}{3}$	0	$\frac{11}{6}$	$\frac{17}{2}$	
x_2	0	0	0	0	0	$\frac{1}{2}$	2.5	
s_2	0	0	0	$\frac{4}{3}$	1	$-\frac{10}{6}$	13	
s_3	0	0	0	$\frac{4}{3}$	0	$-\frac{1}{6}$	10.5	
x_1	0	0	1	0	0	$\frac{1}{6}$	$\frac{1}{6}$	

(11)

(3)

-10

FIVE APPLE

4.5-8

$$\max Z = x_1 + x_2 + x_3 + x_4$$

$$x_1 + x_2 \leq 3$$

$$x_3 + x_4 \leq 2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	Ratio
Z	1	-1	-1	-1	-1	0	0	0	
s_1	0	1	1	0	0	1	0	3	
s_2	0	0	0	1	1	0	1	2	2

	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	Ratio
Z	1	-1	-1	0	0	0	0	2	
s_1	0	1	1	0	0	1	0	3	3
x_4	0	0	0	1	1	0	1	2	∞

	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	Ratio
Z	1	0	0	0	0	1	0	5	
x_2	0	1	1	0	0	1	0	3	
x_4	0	0	0	1	1	0	1	2	

$$\max Z = 5$$

$$\text{opt soln} = W_1(x_1, x_2, x_3, x_4) + W_2(x_1, x_2, x_3, x_4)$$

$$W_1 = 1$$

$$(x_1, x_2, x_3, x_4) = (0, 3, 0, 2)$$

$$\max Z = 20x_1 + 30x_2$$

$$st \quad 2x_1 + x_2 \leq 10$$

$$3x_1 + 3x_2 \leq 20$$

$$2x_1 + 4x_2 \leq 20$$

II I
x₁ x₂

cell reference

20 30
50 10

8 DV

constraint R# S

obj func 20 30 50

RHS

add

utilis.
simplex

const	1	2	1	0	10
	2	3	3	0	20
	3	2	4	0	20

* excel

$$2(1) + (1 \times 1) = 3 \text{ units}$$

\$ \rightarrow\$ Excel

Sum [H11 * H8 + L11
D11 * D8

= sum product (\$H\$8 : \$I\$8 , H11 : I11)

↓
copy & paste
into formula bar

) وین ای بس دی جواب *

Box 01

[add

[change just 1 constraint

cell target → Z

cell changing → x₁/x₂/x₃

Suggested

4.1-6

$$\max Z = 2x_1 + 3x_2$$

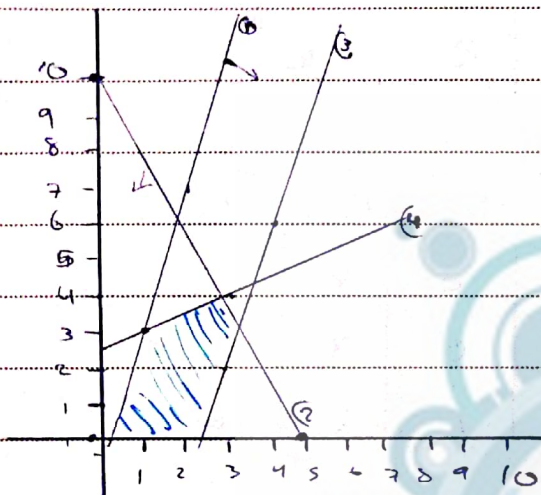
$$-3x_1 + x_2 \leq 1$$

$$4x_1 + 2x_2 \leq 20$$

$$4x_1 - x_2 \leq 10$$

$$-x_1 + 2x_2 \leq 5$$

Graphical



Simplex

#1

	Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS	Ratio
Z	1	-2	-3	0	0	0	0	0	
s_1	0	-3	1	1	0	0	0	1	$+1/3 = 1$
s_2	0	4	2	0	1	0	0	20	10
s_3	0	4	-1	0	0	1	0	10	-10
s_4	0	-1	2	0	0	0	1	5	2.5

#2

#2

	Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS	Ratio
Z	1	-4	0	3	0	0	0	3	
x_2	0	-3	1	1	0	0	0	1	$-1/3 \times$
s_2	0	10	0	-2	1	0	0	18	1.8
s_3	0	1	0	1	0	1	0	11	11
s_4	0	5	0	-2	0	0	1	3	$3/5$

$\frac{18}{1.8}$
 $\frac{3}{3/5}$

$$\max Z = 18$$

$$x_1 = 3$$

$$x_2 = 4$$



#4

	Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
Z	1	0	0	0	$\frac{7}{10}$	0	$\frac{4}{5}$	18
x_2	0	0	1	0	$\frac{1}{10}$	0	$\frac{2}{5}$	4
s_1	0	0	0	1	$\frac{1}{2}$	0	1	6
s_3	0	0	0	0	$\frac{1}{10}$	1	$\frac{1}{5}$	2
x_1	0	1	0	0	$\frac{1}{20}$	0	$\frac{1}{5}$	3

$$\min Z = 4x_1 + x_2 \rightarrow \max -Z + 4x_1 + x_2 + M\bar{x}_3 = 0$$

st.

$$3x_1 + x_2 + \bar{x}_3 = 3$$

$$x_1 + 2x_2 + s_1 = 4$$

$$4x_1 + 3x_2 - s_2 + \bar{x}_4 = 6$$

$$\text{obj. function} \left\{ \begin{array}{l} -M * (3x_1 + x_2 + \bar{x}_3 = 3) \\ -M * (4x_1 + 3x_2 - s_2 + \bar{x}_4 = 6) \end{array} \right.$$

$$-Z + (4 - 7M)x_1 + (1 - 4M)x_2 + Ms_2 = -9M$$

	Z	x_1	x_2	s_1	s_2	$\bar{x}_3$	$\bar{x}_4$	RHS	ratio
Z	-1	$4-7M$	$(1-4M)$	0	M	0	0	$-9M$	1
$\bar{x}_3$	0	3	1	0	0	0	1	3	1
s_1	0	1	2	1	0	0	0	4	4
$\bar{x}_4$	0	4	3	0	-1	1	0	6	1.5

	Z	x_1	x_2	s_1	s_2	$\bar{x}_3$	$\bar{x}_4$	RHS	ratio
$7M-4$	Z	-1	0	$-\frac{5}{3}M-\frac{1}{3}$	0	M	0	$-\frac{7}{3}M-\frac{4}{3}$	$-\frac{2M-4}{3}$
	x_1	0	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	1	3
-4	s_1	0	0	$\frac{2}{3}$	1	0	0	1	$\frac{3}{2}$
	$\bar{x}_4$	0	0	$\frac{2}{3}$	0	-1	$-\frac{4}{3}$	$\frac{14}{3}$	$\frac{14}{3}$

Suggested 4.6-5

$$\max Z - 90x_1 - 70x_2 + Mx_3 = 0$$

$$\max Z = 90x_1 + 70x_2 \rightarrow 90x_1 + 70x_2 - Mx_3 = \max Z$$

$$2x_1 + x_2 \leq 2 \rightarrow 2x_1 + x_2 + s_1 = 2$$

$$x_1 - x_2 \geq 2 \rightarrow -M(x_1 - x_2 - s_2 + \bar{x}_3 = 2)$$

$$-Mx_1 + Mx_2 + Ms_2 - M\bar{x}_3 = -2M$$

$$\max Z + (-90-M)x_1 + (-70+M)x_2 + Ms_2 - M\bar{x}_3 = -2M$$

	Z	x_1	x_2	s_1	s_2	$\bar{x}_3$	RHS	Ratio
Z	1	$-90-M$	$-70+M$	0	M	0	$-2M$	
s_1	0	2	1	1	0	0	2	1
$\bar{x}_3$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0	-1	1	2	2

	Z	x_1	x_2	s_1	s_2	$\bar{x}_3$	RHS	Ratio
Z	1	0	$+95+\frac{3}{2}M$	$45+\frac{1}{2}M$	-M	0	$-90+M$	
x_1	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	2
$\bar{x}_3$	0	0	$-\frac{3}{2}$	0	-1	1	1	$-\frac{2}{3}$

	Z	x_1	x_2	s_1	s_2	$\bar{x}_3$	RHS	Ratio
Z	1		0	$M+20$	-M	$\frac{3}{2}$	$4-25$	$\frac{(6M-4025)}{+90+M}$
x_2	0	2	1	1	0	0	2	∞
$\bar{x}_3$	0	3	0	$\frac{3}{2}$	-1	1	4	-4

$$\frac{3}{2}M - 25$$

non feasible

NO. 2 phases

Obj $\rightarrow$ 6
basic soln
no artificial
artificial

Phase 1 : The purpose of phase 1 is to get us to an external point of the F.R

Phase 2: takes us from this Point in the F.R to the optimal solu.

$$\min Z = 4x_1 + x_2$$

$$\max -Z = -4x_1 - x_2 - \bar{x}_3 - \bar{x}_6$$

$$\max -Z = -\bar{x}_3 - \bar{x}_6$$

$$\rightarrow \max -Z + \bar{x}_3 + \bar{x}_6 = 0$$

$$-1(3x_1 + x_2 + \bar{x}_3 = 3)$$

$$-1(4x_1 + 3x_2 - x_5 + \bar{x}_6 = 6)$$

$$-Z - 7x_1 - 4x_2 + x_5 = -9$$

(X1)

	Z	x_1	x_2	$\bar{x}_3$	x_4	x_5	$\bar{x}_6$	RHS	Ratio
Z	-1	-7	-4	0	0	1	0	-9	
$\bar{x}_3$	0	3	1	1	0	0	0	3	1
x_4	0	1	2	0	1	0	0	4	4
$\bar{x}_6$	0	4	3	0	0	-1	1	6	1.5

	Z	x_1	x_2	$\bar{x}_3$	x_4	x_5	$\bar{x}_6$	RHS	Ratio
Z	-1	0	-5/3	7/3	0	1	0	-2	
x_1	0	1	1/3	1/3	0	0	0	1	
x_4	0	0	5/3	-1/3	1	0	0	3	
$\bar{x}_6$	0	0	5/3	-4/3	0	-1	1	2	

	Z	x_1	x_2	$\bar{x}_3$	x_4	x_5	$\bar{x}_6$	RHS	Ratio
Z	-1	0	0	1	0	0	1	2	
x_1	0	1	0	3/5	0	1/5	1/5	3/5	
x_4	0	0	0	1	1	1	-1	1	
x_2	0	0	1	-4/5	0	-3/5	3/5	6/5	

End of phase 1

we are on
artificial

FIVE APPLE

Phase 2

introduce back original obj fn $\max -Z + 4x_1 + x_2 = 0$

	Z	x_1	x_2	$\bar{x}_3$	x_4	(x_5)	$\bar{x}_6$	RHS	ratio
Z	1	0	0		0	-1/5		-18/5	
x_1	0	1	0		0	1/5		3/5	3
(x_4)	0	0	0		1	1		1	1
x_2	0	0	1		0	-3/5		6/5	

	Z	x_1	x_2	$\bar{x}_3$	x_4	x_5	$\bar{x}_6$	RHS	ratio
Z	1	0	0		1/5	0		-17/5	
x_1	0	1	0		-1/5	0		2/5	
x_3	0	0	0		1	1		1	
x_2	0	0	1		3/5	0		9/5	

$$(2/5, 9/5, 0, 1) = x_1, 2x_2, (x_4, x_5)$$

Post optimality Analysis:

Analysis performed after opt soln is obtained

obj func coeff

constraints

RHS

} how to affect Z

• intering column (-) as $\rightarrow$ unbounded

W/L $\rightarrow$ zero

Ratio $\infty \rightarrow$ infeasible

$\bar{x}$ in $\bar{x}$ list $\rightarrow$ infeasible

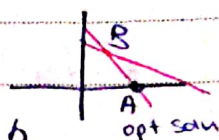
NO post optimal analysis

$$\max Z = 3x_1 + 1x_2$$

$\downarrow$ $\downarrow$
 C_1 C_2

$$\text{st. } 2x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 9$$



Allowable range to stay optimal:
The range of values for this coeff. over which the current opt solu. (Basis) remains opt. assuming no change in other coeff.

Range $\Rightarrow$ max/min

نطاق تغییرات ضرایب
opt solu در A

for $C_1 = 3$ $\rightarrow$ Allowable increase
 $\rightarrow$ Allowable decrease

From equ 0 $x_2 = Z - C_1 x_1$

From equ 1 $x_2 = 6 - 2x_1$

if $C_1 < 2$ A will no longer remain opt solu

equ 1 می شود A در این محدوده می ماند

if $C_1 \geq 2$ A will remain opt solu

$C_1 \rightarrow$ می تواند از 3 تا ∞

if $C_1 = 2$ A, B, multi opt solu

Allowable decrease: current value - lower bound

$$3 - 2 = 1$$

opt solu $\leftarrow$ A می باشد 2 می شود 3 می شود

Ad $\leftarrow$ از این مقدار کمتر می شود

Allowable Increase: upper bound - current value

opt solu A $\leftarrow$ مقدار از این مقدار بیشتر می شود $\leftarrow \infty - 3 = \infty$
(C_1)

**** lower, upper bound mn

$$C_1 \rightarrow (2 - \infty)$$

* for $C_2 = 1$

From eqn 0 $C_2 x_2 = Z - 3x_1 \Rightarrow x_2 = \frac{Z}{C_2} - \frac{3}{C_2} x_1$

from eqn 1 $x_2 = 6 - 2x_1 \Rightarrow x_2 = 6 - 2x_1$

Slope of this line

if Slope < 2 A is no longer opt

$$\frac{3}{C_2} < 2 \Rightarrow \frac{3}{2} < C_2$$

Allowable increase is:

$$UB - CV$$

$$\frac{3}{2} - 1 = .5$$

All owable decrease:

$$CV - LB$$

$$1 - -\infty = \infty$$

* $C_2 = (.5 - \infty)$

Allowable Range to Stay Feasible.

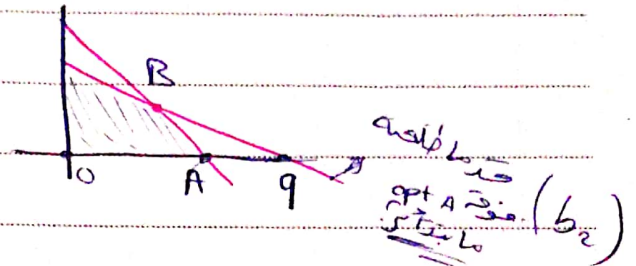
Range of values of the R.H.S over which the current opt soln with adjusted values of the basic variables remains feasible

NO.

$$\text{Max } Z = 3x_1 + x_2$$

$$\text{s.t. } 2x_1 + x_2 \leq b_1 \rightarrow b_1$$

$$x_1 + 3x_2 \leq 9 \rightarrow b_2$$



Allowable increase for b_1

$$b_1 = 6$$

$$2x_1 + x_2 = b_1$$

max value of $x_1 = 9$ *any line*

$$Z(9) + 1(0) = b_1 = 18$$

$$\text{Allowable increase} = \text{UB} - \text{C.V.}$$

$$= 18 - 6 = 12$$

$$\text{min value of } x_1 = 0 \quad b_1 = 0$$

$$\text{C.V.} - \text{LB} \Rightarrow \text{Allowable decrease} = 6 - 0 = 6$$

$$b_2 = 9$$

for b_2 : Allowable increase:

We can Increase b_2 as much as

we want & it won't affect point A

obj func coeff. $\left\{ \begin{array}{l} \text{optimality Range} \\ \text{Feasibility Range} \end{array} \right.$

- Graphically / Algebraically

z x_1 x_2 x_3 x_4 RHS

①	z	1	0	.5	1.5	0	9
②	x_1	0	1	.5	.5	0	3
③	x_4	0	0	2.5	-.5	1	1

Final Table

$$\max = \underbrace{(3+d_1)}_{C_1} x_1 + \underbrace{(1+d_2)}_{C_2} x_2$$

for N.B.V.S

$$\textcircled{1} x_2: (.5)(1) + (.5)(d_1) + (2.5)(0) - d_2 \geq 0$$

$$.5 + .5d_1 - d_2 \geq 0$$

$$\textcircled{2} x_3: (1.5)(1) + (.5)(d_1) + (-.5)(0) - 0 \geq 0$$

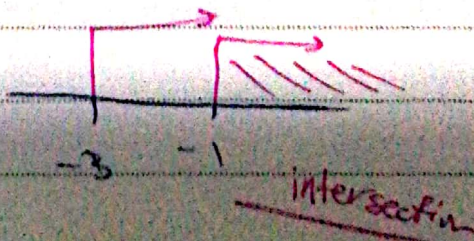
$$1.5d_1 \geq -.5 \Rightarrow d_1 \geq -3$$

if we change one at a time

for $x_1 \Rightarrow d_1$ from 1 $.5 + .5d_1 \geq 0$ $d_2 = 0$

From 2

$d_1 \geq -1$
 $d_1 \geq -3$
 $\Rightarrow$ simply
 allowable decrease for C_1
 to $-\infty$



for $x_2 \Rightarrow d_2$ from 1 $d_2 \leq .5$: allowable Increase

all allowable Increase & all allowable decrease are greater than 0 $\Rightarrow$ the optimal soln is unique
if = 0 $\Rightarrow$ multiple optimal solution

from eqn ① $Z = 9 - 0x_1 - .5x_2 - 1.5x_3 + 0x_4$

$\underbrace{-.5x_2 - 1.5x_3}_{NBV}$

Reduced cost = Cost of consumed - Revenue per unit
Resource per unit

Reduced cost of variable: is the amount by which the profit contribution of the variable must be improved before the variable in question would have take positive value in the opt. solution

max $Z = 3x_1 + x_2$

s.t $2x_1 + x_2 \leq 6 + D_1$

$x_1 + 3x_2 \leq 9 + D_2$ $x_1, x_2 \geq 0$

$\boxed{D_1}$ $\boxed{D_2} \Rightarrow$ Slack var

Z x_1 x_2 x_3 x_4 RHS

$\begin{bmatrix} Z \\ 0 & x_1 \\ D_2 & x_2 \end{bmatrix}$

$$Z + 0 \underset{BV}{X_1} + .5 \underset{NBV}{X_2} + 1.5 \underset{NBV}{X_3} + 0 \underset{BV}{X_4} = 9 + 1.5 D_1 + 0 D_2$$

~~30 + 0 D_1 + 0 D_2 + 0.5 D_1 + 0 D_2 = 30~~

$$X_1 + .5 X_2 + .5 X_3 + 0 X_4 = 3 + .5 D_1 + 0 D_2$$

~~6 + 0 D_1 + 0 D_2 + 0.5 D_1 + 0 D_2 = 6~~

$$0 X_1 + 2.5 X_2 - .5 X_3 + 1 X_4 = 6 - .5 D_1 + 1 D_2$$

current solu remains feasible if all B.V remain non negative

$$3 + .5 D_1 \geq 0 \Rightarrow D_1 \geq -6 \quad \text{allowable decrease}$$

$$6 - .5 D_1 \geq 0 \Rightarrow D_1 \leq 12 \quad \text{allowable increase}$$

$$6 + D_2 \geq 0 \Rightarrow D_2 \geq -6 \quad \text{allowable decrease}$$

X_3	X_4	MUS
1.5	0	9 → 10.5
.5	0	3
-.5	1	6

لوتخیز
const 1
7
منه 6 ال 7
بتزیط ال
9 بقبار
رابط

shadow price: the rate at which Z could be increased by slightly increasing the amount of the resource.

لوتخیز
const 2
10
9
منه 8 ال 10
بتزیط ال
6 ال 9
رابط

* The current solu remains feasible if all B.V remain non negative

We have company that manufactures tables & chairs
 - There are 3 board ft available & 3 hr of finishing
 hr. a Table sells for \$4 & a chair for \$5
 we want to maximize the total revenues

Primal	Resource	table	chair	
Prob 1	lumber	2	3	*suppose we want to know
finish hour	finish hour	4	2	how much Price we'll pay
Tables & chairs				

$$\max Z = 4x_1 + 5x_2$$

$$\text{st. } 2x_1 + 3x_2 \leq 3$$

$$4x_1 + 2x_2 \leq 3$$

$$x_1 = \# \text{ of tables} \quad x_1, x_2 \geq 0$$

$$x_2 = \# \text{ of chairs}$$

y_1 : Price per 1 board ft
of lumber

y_2 : Price per 1 hr of
Finishing

→ making a new problem

$$\min W = 3y_1 + 3y_2$$

st.

$$2y_1 + 4y_2 \geq 4 \quad \text{Tables}$$

$$3y_1 + 2y_2 \geq 5 \quad \text{Chairs}$$

$$y_1, y_2 \geq 0$$

minimize ← أقل تكلفة

لنظم أدفع أقل Raw material

وأيضاً سعر كل

chairs/tables

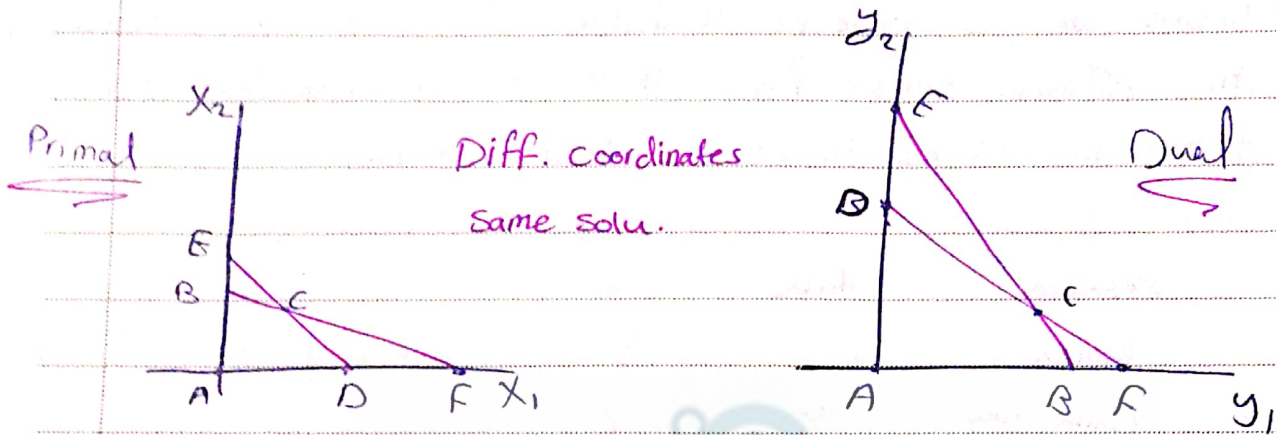
الوحدات
بعض

Dual
Prob 2
Cost
Raw material
Selling price

Sensitivity analysis.

NO. Prime & Dual

اذا احلنا اول Prime ← في احلنا Dual



Primal	Dual
Slack variable	D.V
D.V	Surplus Variable
BV	NBV
NBV	BV
opt basic solu	complementing opt basic solu

Primal

Point A (0, 0, 3, 3)

B.V x_3, x_4 Feasible

NBV x_1, x_2

B (0, 1, 0, 1)

B.V : x_2, y_4 (F)

N.B.V : x_1, x_3

Dual

Point A (0, 0, -4, -5)

الجابج

B.V : y_3, y_4

NBV : y_1, y_2

infeasible
(رقين سالب)

B ($\frac{5}{3}, 0, \frac{2}{3}, 0$)

NBV : y_2, y_4 (F)

B.V : y_1, y_3

Im not solving → artificial var

Primal

$$C \left(\frac{3}{8}, \frac{5}{4}, 0, 0 \right)$$

(F)

$$BV: x_1, x_2$$

$$NBV: x_3, x_4$$

Dual

$$C \left(\frac{6}{9}, \frac{1}{4}, 0, 0 \right)$$

(F)

$$NBV: y_3, y_4$$

$$BV: y_1, y_2$$

opt
soln.

opt soln. = $\frac{1}{4}$ (F) لا يكون النوعين

Variation of the Basic model.

$$\max z = cX$$

$$\text{s.t. } Ax \leq b.$$

$$x \geq 0$$

after changes are made.

$$b_i \rightarrow \bar{b}_i$$

$$b \rightarrow \bar{b}$$

$$c_j \rightarrow \bar{c}_j$$

$$c \rightarrow \bar{c}$$

$$\bar{a}_{ij} \rightarrow \bar{a}_{ij}$$

$$A \rightarrow \bar{A}$$

$$A = A_1 \dots A_j$$

$$\text{ex} \rightarrow \max z = 4x_1 + 5x_2$$

$$\text{s.t. } 2x_1 + 3x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

$$z = \begin{bmatrix} 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\underbrace{\begin{matrix} \downarrow & \downarrow \\ A_1 & A_2 \end{matrix}}_A$$

A

④ $\rightarrow$ opt. is always is

Z	x_1	x_2	x_3	x_4	RHS
Z	1	0	0	1.5	2.5
x_2	0	0	1	.25	.75
x_1	0	1	0	.375	.375

Z	x_1	x_2	x_3	x_4	RHS	
Z	1	0	0	y_1^*	y_2^*	Z^*
x_2	0	0	1			b_1^*
x_1	0	1	0			b_2^*

$[S^*]$

slack coeff.

two cases $\begin{cases} \rightarrow \text{in feasible (obj)} \\ \rightarrow \text{non optimal (constraints)} \end{cases}$

10 additivity

7.2.3 page 270

$$\text{maximize } Z = -5x_1 + 5x_2 + 13x_3$$

$$\text{s.t. } -x_1 + x_2 + 3x_3 \leq 20 \quad +D_1$$

$$12x_1 + 4x_2 + 10x_3 \leq 90 \quad +D_2$$

$$x_1, x_2, x_3 \geq 0 \quad \text{make changes}$$

Final iter

Z	x_1	x_2	x_3	s_1	s_2	RHS
Z	1	0	0	2	0	100
	0	-1	1	3	1	20
	0	16	2	-2	-4	10

* Allowable increase $b_1 = 2.5$

7.2.2changes in right hand side RHS b_i

$$z^*_{\text{new}} = y^* \bar{b}$$

$$b^*_{\text{new}} = s^* \bar{b}$$

$$b_1 \rightarrow \bar{b}_1$$

Q₁ →

$$20 \rightarrow 30$$

one of them is given as a new value

$$\begin{aligned} a_1 x_1 + b_1 x_2 &= c \\ a_2 x_1 + b_2 x_2 &= d \end{aligned} \rightarrow \begin{bmatrix} c \\ d \end{bmatrix} \rightarrow \bar{b}$$

$$z^*_{\text{new}} = [5 \ 0] \begin{bmatrix} 30 \\ 90 \end{bmatrix} = 5 \times 30 + 0 \times 90 = 150$$

$$b^*_{\text{new}} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 90 \end{bmatrix} = \begin{bmatrix} 30 \\ -30 \end{bmatrix}$$

D₁, D₂ (increase 10) 20 → 30 is ok → infeasibleQ₂ →

$$b_2 \rightarrow \bar{b}_2$$

$$90 \rightarrow 70$$

$$z^*_{\text{new}} = [5 \ 0] \begin{bmatrix} 20 \\ 70 \end{bmatrix} = 100$$

$$b^*_{\text{new}} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 70 \end{bmatrix} = \begin{bmatrix} 20 \\ -10 \end{bmatrix} \rightarrow \text{if}$$

20 allowable decrease 20

* Changes in the coeff of a N.B.V x_j . The x_j is the only colm. that has changed / if the current soln is no longer opt, the new value of " $z_j^* - \bar{c}_j$ " will be the negative value in Row 50. Restart the simplex ~~method~~ method with x_1 on the entering B.V

$$z_j^* - \bar{c}_j = y^* \cdot \bar{A}_j - \bar{c}_j$$

$$\bar{A}_j^* = S^* \cdot \bar{A}_j \quad \text{from original Table}$$

* change coeff of x_3 in obj fn

ج $\bar{c}_3 \leftarrow$ $C_3 \rightarrow \bar{C}_3$
 $13 \rightarrow 8$

From constraint

$$z_j^* - \bar{c}_j = [5 \ 0] \begin{bmatrix} 3 \\ 10 \end{bmatrix} - 8 \Rightarrow 15 - 8 = 7$$

$$\bar{A}_j^* = S^* \cdot \bar{A}_j = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$\rightarrow z_j^* - \bar{c}_j = 7$ when
 Table is LE

Change of x_1

$$\begin{bmatrix} C_1 \\ a_{11} \\ a_{21} \end{bmatrix} \rightarrow \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

* Change in $b_i \rightarrow$ Feasibility

$$\bar{b} \Rightarrow \begin{aligned} Z^*_{\text{new}} &= y^* \bar{b} \\ b^*_{\text{new}} &= s^* \bar{b} \end{aligned}$$

* changes in of N.B.V

$$Z_j - \bar{C}_j = y^* \cdot \bar{A}_{ij} - \bar{C}_j$$

$$A_{ij}^* = s^* \cdot \bar{A}_{ij}$$

اذا قلت قيمة $Z_j - \bar{C}_j$ قلت $Z_j - \bar{C}_j$ (iteration) اكبر

$$\begin{aligned} & \rightarrow x_1 \\ & \rightarrow \begin{bmatrix} Z_j - \bar{C}_j \\ A_{ij} \end{bmatrix} \end{aligned}$$

* Changes in coeff of B.V

$$x_2 = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$$

$$Z_0 - C_j = y^* \bar{A}_{ij} - \bar{C}_j$$

$$\begin{bmatrix} 5 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 6$$

$$\begin{bmatrix} 5 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 6$$

$$\begin{aligned} & x_2 \\ & \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix} \rightarrow \bar{C}_j \end{aligned}$$

مساوي جدول

	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	4	4	5	0	100
x_2	0	-1	2	2	1	0	20
x_5	0	16	-3	-3	-4	-1	10

regular source

منه

14/7/2019

NO.

	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	2	0	-4	3	0	60
x_2	0	-1/2	1	3/2	1/2	0	10
x_5	0	14.5	0	2.5	-2.5	1	40

one more table $\rightarrow$ bec $\uparrow$ is not optimal soln.

* Adding new variable

کامیابی کا نتیجہ بڑھانے کے لئے نئے متغیر کا اضافہ کرنا

- 2 ways of solving $\rightarrow$ Pretend that the new variable was in the model but with zero coeff $\Rightarrow$ * 1
 $\rightarrow$ complementing & duality. $\Rightarrow$ * 2

ex on way * 1 $x_6 = \begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$

$$Z_6^* - \bar{C}_6 = y^* \cdot \bar{A}_6 - \bar{C}_6 = \overset{\text{obj. in}}{[5 \ 0]} \begin{bmatrix} 3 \\ 5 \end{bmatrix} - 10 = 5$$

$$A_6^* = S^* \cdot \bar{A}_6$$

$$A_6^* = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$$

Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
						$\begin{bmatrix} 5 \\ 3 \\ -7 \end{bmatrix}$	

chp
journal

Adding New constraint

Check ~~whether~~ whether the optimal soln satisfies the new constraint if yes then its still optimal if not add this constraint into the final tableau as if this was the initial tableau

Add $\Rightarrow 2x_1 + 3x_2 + 5x_3 \leq 50$

From initial tableau $x_1 = x_3 = 0$ $x_2 = 20$

$$2(0) + 3(20) + 5(0) \leq 50$$

$$60 \leq 50 \quad \times \text{ infeasible}$$

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	1	0	2	5	0	100
x_2	0	-1	1	3	1	0	20
x_5	0	16	0	-2	-4	1	10
x_6	0	2	3	5	0	0	50

① $\rightarrow$ x_2 is $\leftarrow$

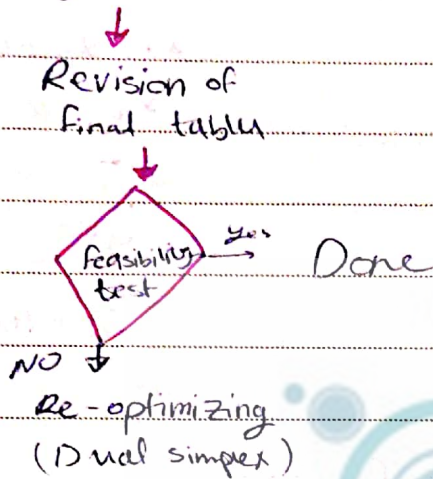
	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_6	0	5	0	-4	-3	0	-10

iteration 2

18/7/2019

NO. Chp 7

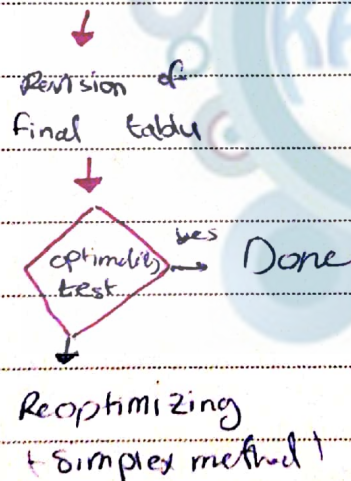
changes in b_i



	Primal	dual
super optimal	inf	f
optimal	f	f
sub optimal	f	inf

x_i change constraint in the dual & check if opt. soln is still feasible.

Change in coeff of N.B.V



$$\begin{aligned} \max Z &= -5x_1 + 5x_2 + 13x_3 \\ -x_1 + x_2 + 3x_3 &\leq 20 \\ 12x_1 + 4x_2 + 10x_3 &\leq 90 \end{aligned}$$

Simplex
primal

Dual Simplex

$$\begin{aligned} \min W &= 20y_1 + 90y_2 \\ \text{s.t. } -y_1 + 12y_2 &\geq -5 \\ y_1 + 4y_2 &\geq 5 \\ 3y_1 + 10y_2 &\geq 13 \end{aligned}$$

Primal into

$$x_1 = x_2 = 0$$

$$x_2 = 20$$

$$y_1^+ = 5$$

$$y_2^+ = 0$$

$$Z = 100$$

$$3(5) + 10(0) \geq 8 \quad C_3 \rightarrow \bar{C}_3$$

its

$$15 \geq 8$$

$$13 \rightarrow 8$$

feasible

change in coeff
of B.V

Revision of final
tableau

Proper
form

no
Correct to
Proper form

yes

feasible
test

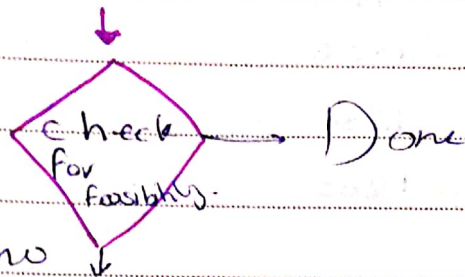
no
Reoptimizing Dual simplex

optimality
test

no
Reoptimizing

yes
done

New constraint



Revision of
Final table

Proper form

reoptimization

*

Find opt solution without
Resolving dual method

[Chp 8

7-2.2

Dual simplex method

	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	0	0	2	5	0	150
x_2	0	-1	1	3	1	0	30
x_3	0	16	0	-2	-4	1	-30

Reduced cost

$$\frac{5}{-4} = \frac{2}{-2} = -1$$

is minimum
absolute
value

x_3 0 -8 0 1 2 -5 15

infeasible dual & primal

* optimal solution $\rightarrow$ 117

	Denver	miami	supply
LA	1000	2600	1000
Detroit	1250	1350	1500
new	1275	850	1200
original demand	2300	1400	

	Denver	miami
LA	80 \$	215 \$
Detroit	100 \$	108 \$
new	102 \$	68 \$

$$\min Z = 80 x_{11} + 215 x_{12} + 100 x_{21} + 108 x_{22} + 102 x_{31} + 68 x_{32}$$

$$x_{11} + x_{12} = 1000$$

$$x_{21} + x_{22} = 1500$$

$$x_{31} + x_{32} = 1200$$

$$x_{11} + x_{22} + x_{31} = 2300$$

$$x_{12} + x_{22} + x_{32} = 1400$$

$$\min \sum \sum C_{ij} x_{ij}$$

$$\left[\begin{array}{l} \sum_{j=1}^n x_{ij} = s_i \\ \sum_{i=1}^m x_{ij} = d_j \end{array} \right]$$

Tran → objective func is to minimize cost

BIN → Binary (decision)

INT = integer

NO. chp 9

Solver →

* minimize cell num cell label

changing cells

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{bmatrix}$$

[constraints

$$\begin{bmatrix} \sum \text{sum} = \text{supply} \\ \sum \text{sum} = \text{Demand} \end{bmatrix}$$

If Demand > supply. Add dummy supply (as it is trick)

Dummy row (0 0 0 0) (supply is not given) & demand

supply is 3

dummy cost 0 Miami 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

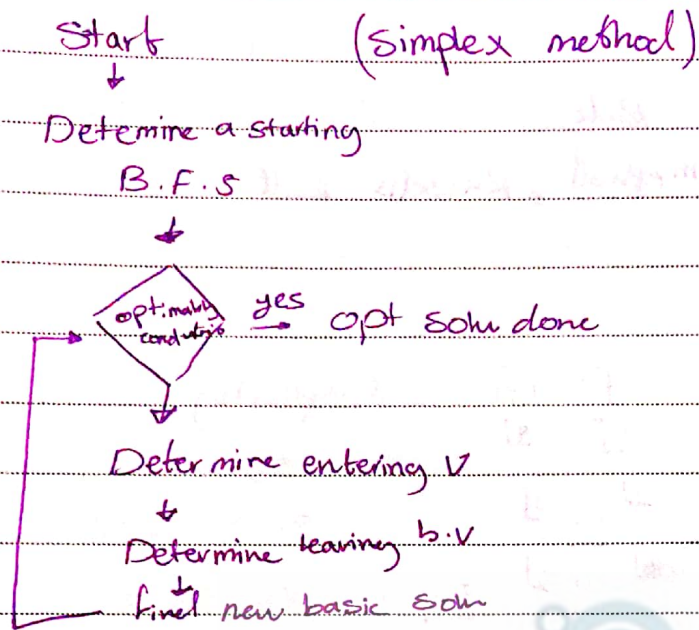
if

$$3300 = D < S = 3700$$

Demand row (Dummy)

0
0
0

Demand 400



Primal Prob
or Dual Prob

max Z (or w)

Constraint :

$\leq$

$=$

$\geq$

variable

$x \geq 0$

unconstrained

$x \leq 0$

Dual prob
or primal prob

min w (or Z)

variable

$y \geq 0$

unconstrained

$x \leq 0$

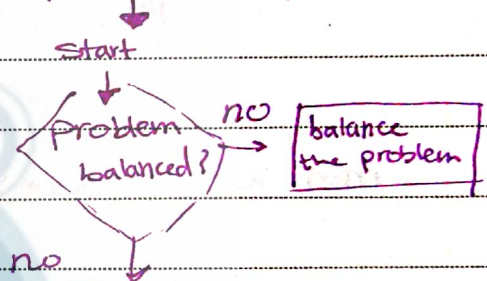
constraint

$\geq$

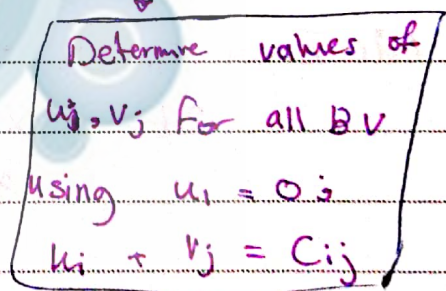
$=$

$\leq$

Transportation prob



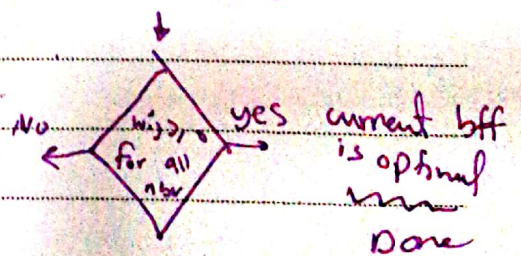
Determine starting BFs



for each NBV compute $w_{ij} = C_{ij} - (u_i + v_j)$

Find leading BV ratio

Find entering BV cell with largest -ve value



FIVE APPLE

Transportation min / =

like small
bait
23/7/2019

NO. chp 9 transportation

methods

- ① North west corner Rule
- ② minimum cost method, Russells rule
- ③ vogels method

* 1

	D	E	F	G	Capacity
A	8 35	6	10	4	35 0
B	9 10	12 20	13 20	7	50 40 20 0
C	14	9	16 10	5 30	40 30 0
Demand	45 10 0	20 0	30 10 0	30 0	125

$$\min Z = 35 \times 8 + 10 \times 9 + 20 \times 12 + 20 \times 13 + 10 \times 16 + 5 \times 30$$

$$\min Z = 1080$$

$$\min Z = C_{11} \times X_{11} + C_{12} \times X_{12} + C_{21} \times X_{21} + C_{22} \times X_{22}$$

$$st \quad X_{11} + X_{12} = S_1 \quad * \quad U_1$$

$$X_{22} + X_{21} = S_2 \quad * \quad U_2$$

$$X_{11} + X_{21} = d_1 \quad * \quad V_1$$

$$X_{12} + X_{22} = d_2 \quad * \quad V_2$$

$$X_{ij} \geq 0$$

dual
form

قویلیا

unconstraint = +ve



NO.

موضوع

Dual form

$$\max W = S_1 u_1 + S_2 u_2 + d_1 v_1 + d_2 v_2$$

$$st \quad u_1 + v_1 \leq c_{11}$$

$$u_1 + v_2 \leq c_{12}$$

$$u_2 + v_1 \leq c_{21}$$

$$u_2 + v_2 \leq c_{22}$$

u_i, v_j ~~unconstrained~~

~~unconstrained~~

unconstrained

$$u_1 + v_1 + w_{11} = c_{11}$$

$$u_1 + v_2 + w_{12} = c_{12}$$

$$u_2 + v_1 + w_{21} = c_{21}$$

$$u_2 + v_2 + w_{22} = c_{22}$$

	D	E	F	G	capacity	u_i
A						0 $\rightarrow \bar{a}_{11}$
B						1
C						4

v_j 8 11 12 1

$$u_1 + v_1 = c_{11}$$

$$0 + v_1 = 8$$

$$u_2 + v_1 = c_{21}$$

$$1 + 8 = 9$$

$$u_2 + v_2 = c_{22}$$

$$1 + 11 = 12$$

$$u_2 + v_3 = c_{23}$$

$$1 + 12 = 13$$

$$v_3 + u_3 = c_{33}$$

$$4 + 12 = 16$$

for B₁

only ←
form B.V

$$W_{ij} = C_{ij} - (u_i + v_j)$$

$$W_{12} = C_{12} - (u_1 + v_2)$$

$$= 6 - (0 + 11) = -5$$

ⓔ -5 (opportunity cost)

حقیقہ W

35	-5	-2	8
10	20	20	5
2	-6	10	30

most negative impl*

loops لازم آئیں ہر اوپا کا ہے



→ Basic variables



→ not loop

35

10

20

20

-6

10

30

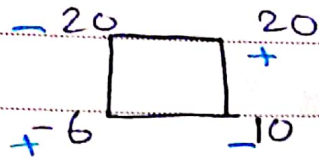
my loop



ما فیہ الدج من 10 ← 30 لائن شرجہ

B.V میرا بال B.V

most negative



-10 / -20 → ~~closed~~
Duration

absolute negative

آقل واحد بیش منه

+10	30
+10	0

نشین بعد
iteration

لے مہ نکالیں ال سوالات و یووقت اکل

$$u_1 + v_1 \leq c_{11}$$

$$u_2 + v_1 \leq c_{21}$$

$$u_2 + v_2 \leq c_{22}$$

$$u_2 + v_3 \leq c_{23}$$

$$u_3 + v_2 \leq c_{32}$$

$$u_3 + v_3 \leq c_{33}$$

$$c_{31} + v_4 \leq c_{34}$$

	D	E	F	G
A	30	12	13	4
B	10	10	30	2
C	14	10	16	30
V	8	11	12	7

u

0

1

-2

unconstrained

only for BL

$$W_{ij} = c_{ij} - (u_i + v_j)$$

$$W_{12} = 6 - (11) = -5$$

$$W_{13} = 10 - (12) = -2$$

$$W_{14} = 9 - (7) = 2$$

$$W_{24} = 7 - (6) = 1$$

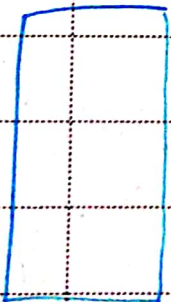
$$W_{31} = 8$$

$$W_{33} = 6$$

$$W_{24} = -1$$

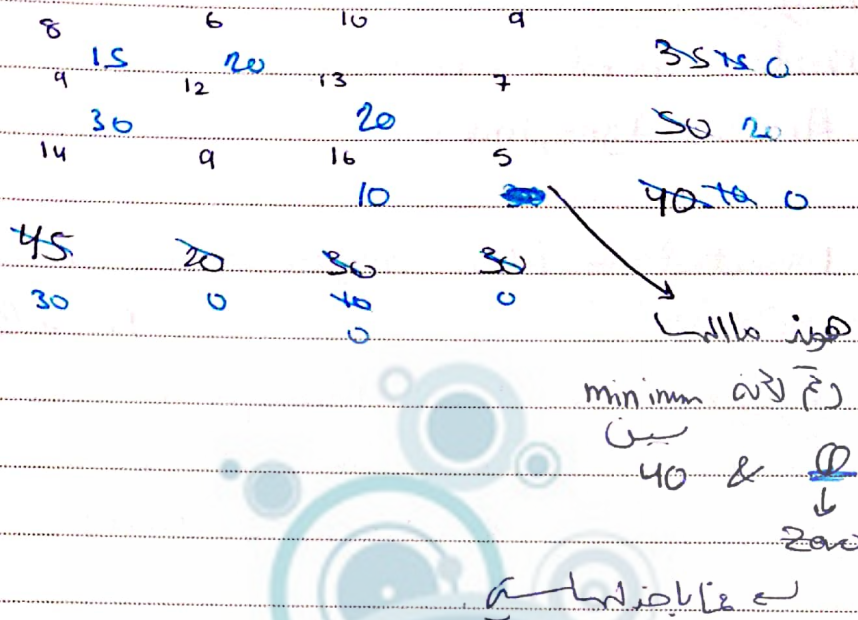
		+		
25	-5	-2	2	
20	10	30	-1	
80	10	6	30	

ماتریس
هزینه



→ هیک
عکس

NO. chp 9 OR



Assignable → formulating problem → minimize cost always
 Binary "Lalla"
 keyword always

[chp 9 network optimization method

Nodes / Arcs

- * Shortest path problem (الهدف بين نقطتين معينة)
- * Minimum spanning tree (بمها توصيل لكل nodes)
 الهدف توصيل لكل nodes
- * maximum flow

Terminology

Nodes: points, vertices

Arcs: edges, links

* Directed $\rightarrow$ flow only one direct (one way street) ex.
(oriental) $A \rightarrow B$

* Undirect $\rightarrow$ flow through arc is allowed in either directions
(link)
ex (two ways street) $A \rightleftarrows B$

* mixed: network has both types

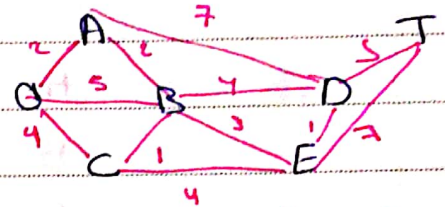


$O \rightarrow A \rightarrow B$ $\sim$ feasible
 $B \rightarrow C \rightarrow B$ α infeasible

28/7/2019

NO. shortest path method

define source & destination



n (iteration)	Solneel nodes directly connected to unsolved nodes	Candidate for the n th nearest node	total distance from <u>origin</u>
1	O	A	2
2	O A	C B	4 4
3	O A C	B B B	5 4 5
4	A B C	D E E	9 7 8

n	n th nearest node to the origin	minimum distance	last connection
1	A 1 st nearest to origin	2	OA
2	C 2 nd nearest to origin	4	OC
3	B 3 rd	4	AB
4	E 4 th	7	BE

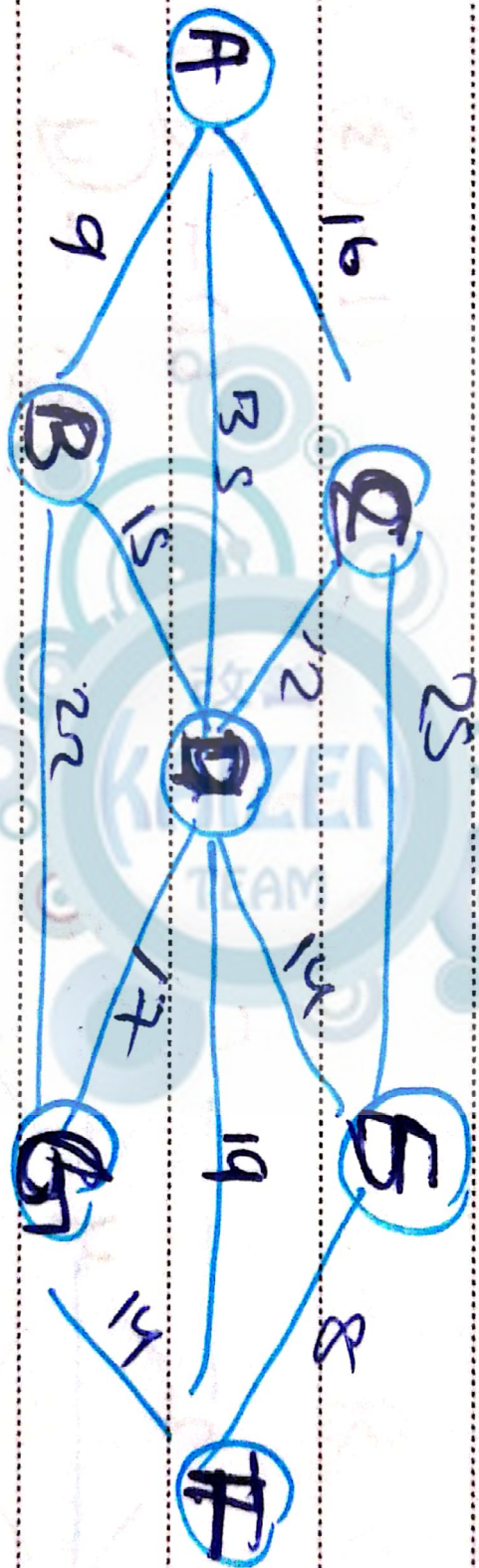
28/7/2019

NO. _____

5	A	D	9			
	B	D	8	13 th	8	BD
	E	D	8		8	ED

6	D	T	13			
	E	T	14	T	13	DT





1

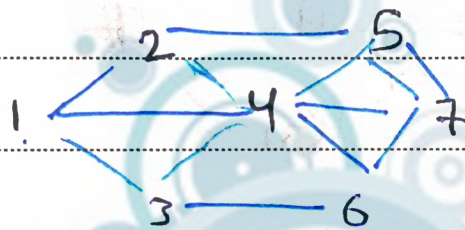
2

3

4

5

6



①-③

①-②

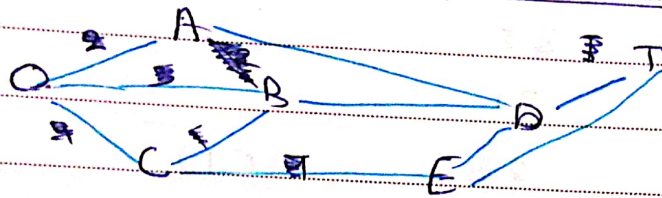
③-④

③-⑥

④-⑤

④-⑦

1 → 3 → 4 → 7



minimum spanning tree

iteration

$$C_k \quad k=0 \quad C_0 = \Phi \quad \bar{C}_0 = \{O, A, B, C, D, E, T\}$$

$\bar{C}_k \quad k=1$ start with any node i in C_0 , choose O

$$C_1 = \{O\} \quad \bar{C}_1 = \bar{C}_{k-1} - \{i\} = \{A, B, C, D, E, T\}$$

$k=2$: select node j^* in $\bar{C}_{k-1}$ that yields shortest arc in C_{k-1}

$$J^* = A \quad C_2 = \{O, A\} \quad \bar{C}_2 = \{B, C, D, E, T\}$$

if $\bar{C}_k = \Phi$ then stop otherwise $k = k+1$

$$k=3 \quad J^* = B \quad C_3 = \{O, A, B\} \quad \bar{C}_3 = \{C, D, E, T\}$$

29/7/2019

NO.

$J^+ = C$

$k=4 \quad C_4 = \{0 \ A \ B \ C\} \quad \bar{C}_4 = \{D \ E \ T\}$

$k=5 \quad J^+ = E \quad C_5 = \{0 \ A \ B \ C \ E\}$
 $\bar{C}_5 = \{D \ T\}$

$k=6 \quad J^+ = D \quad C_6 = \{0 \ A \ B \ C \ E \ D\}$
 $\bar{C}_6 = \{T\}$

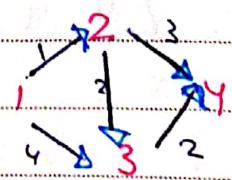
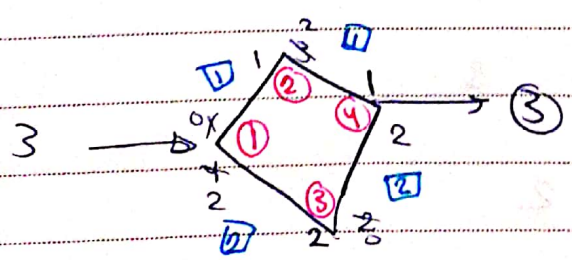
$k=7 \quad J^+ = T \quad C_7 = \{0 \ A \ B \ C \ E \ D \ T\} \quad \bar{C}_7 = \emptyset$

nodes لا تسبلي
 Greedy Algorithms

Simplex method $\rightarrow$ not greedy.

Travelling salesman Problem

max flow prob



- (a) Identify an augmenting path
 (b) Identify residual capacity c^*
 c^* = minimum residual capacities of
 Arcs on this path

(c) Increase the flow in the path by c^*

(d) Decrease residual capacities by c^* of each
 arc

(e) Increase residual cap in opposite direction



① O - B - D - T
 $c^* = \min \{ 7, 4, 9 \} = 4$

② O - C - E - T
 $c^* = \{ 4, 4, 6 \} = 4$

③ O - A - D - T
 $c^* = \{ 3, 5, 5 \} = 3$

④ O - B - E - T
 $c^* = \{ 3, 5, 2 \} = 2$

⑤ O - B - E - D - T
 $\{ 1, 3, 1, 2 \} = 1$