Question 2: Use Cramer's rule to solve for $\cos\gamma$ and $\cos\beta$ in terms of a, b and c.

$$b\times\cos\gamma + c\times\cos\alpha = a$$
$$c\times\cos\beta + a\times\cos\gamma = b$$
$$a\times\cos\alpha + b\times\cos\beta = c$$

$$\begin{array}{cccc} & \cos\gamma & \cos\alpha & \cos\beta \\ A = & \left[\begin{matrix} b \\ a \\ 0 \end{matrix}\right. & \begin{matrix} c \\ 0 \\ a \end{matrix} & \left.\begin{matrix} 0 \\ c \\ b \end{matrix}\right] \end{array}$$

$$RHS = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\det(A) = -b(ac) - c(ab)$$
$$= -2abc$$

$$A_1 = \begin{bmatrix} a & c & 0 \\ b & 0 & c \\ c & a & b \end{bmatrix}$$

$$\det(A_1) = -a(ac) - c(b^2 - c^2)$$

$$\cos\gamma = \frac{+a^2\cancel{c} + \cancel{c}(b^2 - c^2)}{+2ab\cancel{c}} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$A_3 = \begin{bmatrix} b & c & a \\ a & 0 & b \\ 0 & a & c \end{bmatrix}$$

$$\det(A_3) = -b\,ab - c\,ac + a^3$$

$$\cos\beta = \frac{+b^2\cancel{a} + c^2\cancel{a} \mp a^3}{+2\cancel{a}bc}$$

Question 3: Given that A, B and C are 2x2 matrices that are shown below:

$$A=\begin{bmatrix} 2 & x \\ x & 5 \end{bmatrix}, \quad B=\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \quad and \quad C=\begin{bmatrix} 2+x & 2 \\ 4 & x-4 \end{bmatrix}$$

a. Find an expression for AB in terms of x.

$$\begin{bmatrix} 8+3x & 4+x \\ 4x+15 & 2x+5 \end{bmatrix}$$

b. Determine the value(s) of x, given that the determinant of A and C are equal.

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a

$$\begin{bmatrix}8+3x & 4+x\\ 4x+15 & 2x+5\end{bmatrix}$$

b. Determine the value(s) of x, given that the determinant of A and C are equal.

b

$\det(A) = 10 - x^2$

$\det(C) = (2+x)(x-4) - 8$

$= 2x - 8 + x^2 - 4x - 8$

$= x^2 - 2x - 16 = 10 - x^2$

$2x^2 - 2x - 26$

Question 4: Given that A is a 3x3 upper triangular matrix, show that the corresponding cofactor matrix is a lower one.

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{bmatrix}$$

$$C_{11} = (-1)^2 * \begin{vmatrix} a_4 & a_5 \\ 0 & a_6 \end{vmatrix}$$

$$C = \begin{bmatrix} a_4 a_6 & -0 & 0 \\ -a_2 a_6 & a_1 a_6 & 0 \\ a_2 a_5 - a_4 a_3 & -a_1 a_5 & a_1 a_4 \end{bmatrix}$$

Question 6: Show if the following statement is always correct.

"If **u** is orthogonal to (**w**+**v**), then **u** is orthogonal to **v** and **w**"

$$u\cdot(w+v)=0$$

$$(u\cdot w)+(u\cdot V)=0$$

scaler   Scaler

$$w=(2,-2,4)$$

$$V=(-2,2,-4)$$

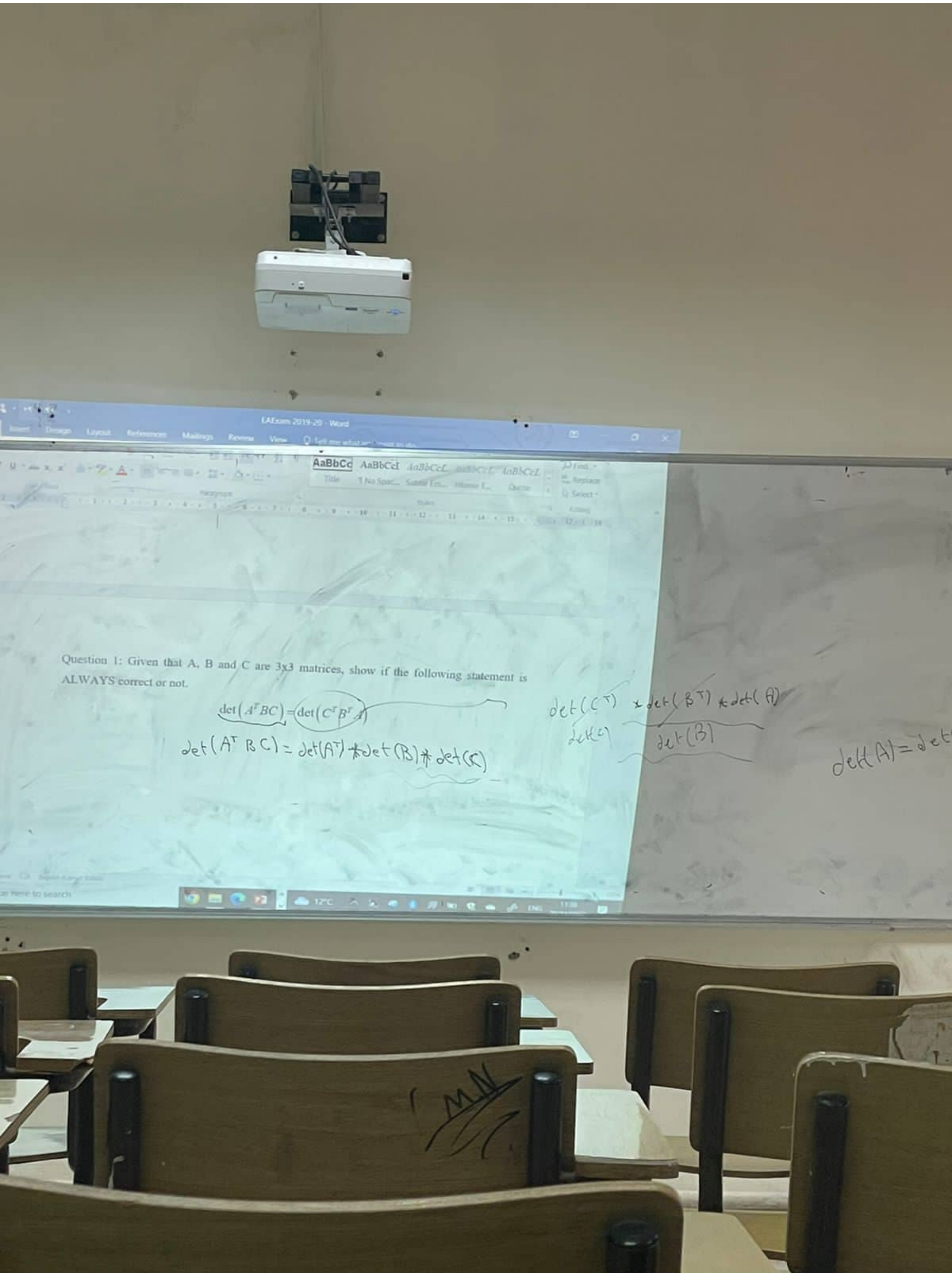
$$u=(4,5,7)$$

Question 5: Show that a matrix A is nonsingular if and only if A^T is nonsingular.

inv $\left[\begin{array}{l} \text{sq} \\ \det \neq 0 \end{array}\right.$ if A^T $\left[\begin{array}{l} \text{sq} \\ \det \neq 0 \end{array}\right.$

if $(A^T$ is sq then A is sq

if $\det(A^T) \neq 0$ then $\det(A) = \det(A^T) \neq$ zero


LAExam 2019-20 - Word
Question 1: Given that A, B and C are 3x3 matrices, show if the following statement is ALWAYS correct or not.
$\det(A^T BC) = \det(C^T B^T A)$
$\det(A^T BC) = \det(A^T) * \det(B) * \det(C)$
$\det(C^T) * \det(B^T) * \det(A)$
$\det(C)$
$\det(B)$
$\det(A) = \det$