

Notebook

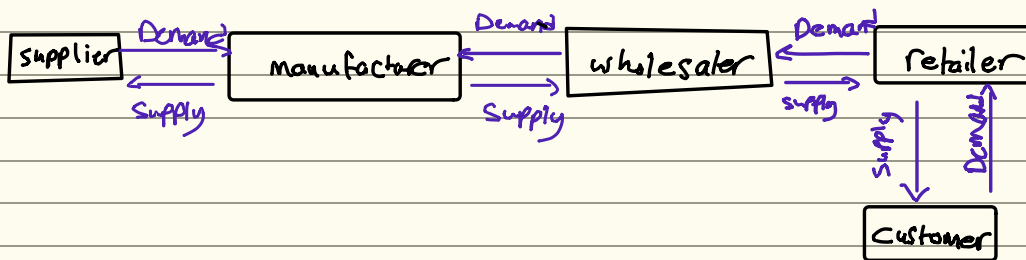
PPC

Sec
Semester
2024

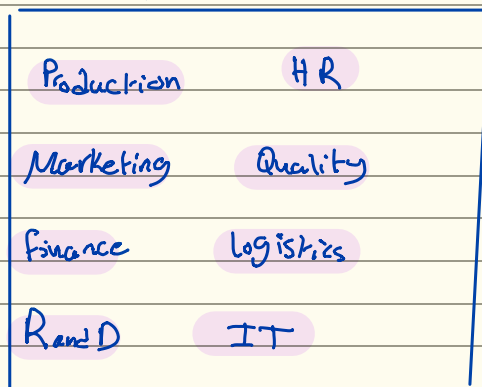


PPC

General Supply chain :



Manufacturer



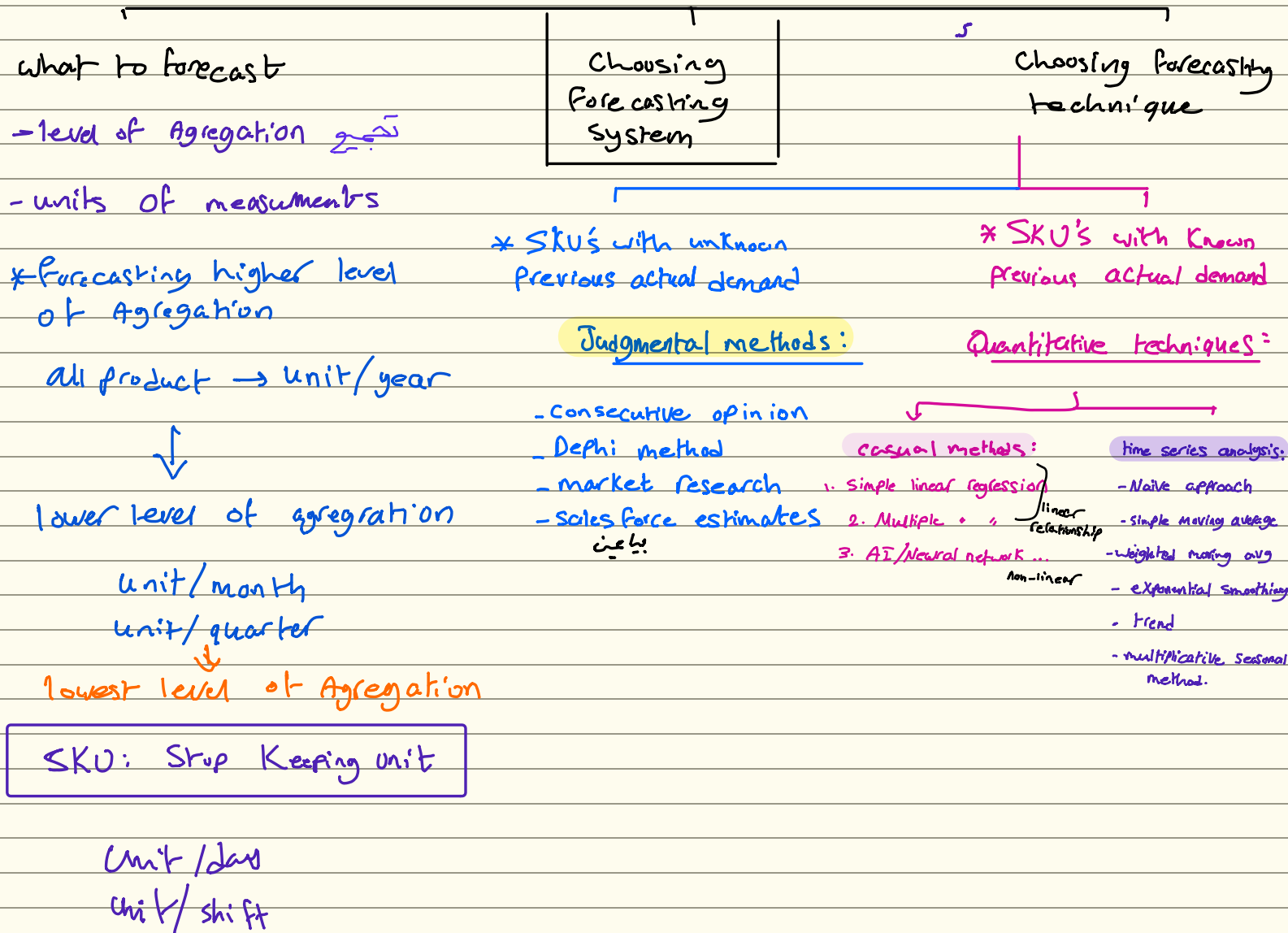
[Involved to produce product]

losing profit is a cost

- * Supply chain management: Matching supply with demand.
- * Logistics management

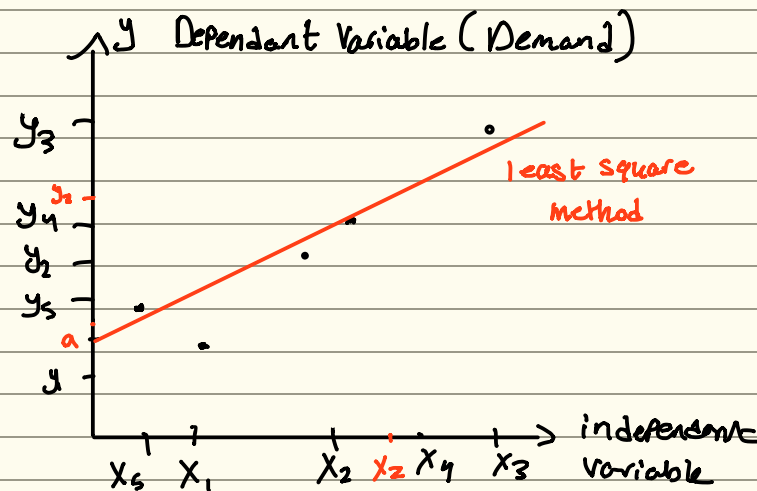
-
- forecasting
 - inventory management
 - Aggregate planning
 - Materials requirements planning
 - scheduling

1. Forecasting: (توقع) (actual activity)



Simple linear regression:

Period	X	y	xy	x^2	y^2
1	x_1	y_1	$x_1 y_1$	$\cdot$	$\cdot$
2	x_2	y_2	$x_2 y_2$	$\cdot$	$\cdot$
3	x_3	y_3	$x_3 y_3$	$\cdot$	$\cdot$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
	ΣX	Σy	Σxy	Σx^2	Σy^2



Linear model:

$$Y = a + bx$$

equations:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{y} = \frac{\sum y}{n}$$

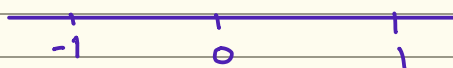
$$a = \bar{y} - b\bar{x}$$

1 Correlation Coefficient (r)

$$-1 \leq r \leq 1$$

direction
strength

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{(n \sum x^2 - (\sum x)^2)(n \sum y^2 - (\sum y)^2)}}$$



كل ما قربنا لـ (1, -1)

more strength (ie)

M_1

M_2

$$r = -0.9$$

$$r = -0.6$$

Stronger

2 Coefficient of determination (r^2): amount of variation

$$0 \leq r^2 \leq 1$$

$$r^2 = 0.9$$

90%

$$\text{if } r = 0.6$$

$$r^2 = 0.36$$

36%

not strong

$$\text{if } r = 0.9$$

$$r^2 = 0.81$$

81%

strong

if $r = 1$ perfect relationship

if $r = 0$ either there is no relationship
or there is relation but not linear

3 Standard error of estimates (S)

$$S = \sqrt{\frac{\sum y^2 - a \sum y - b \sum xy}{n-2}}$$

Ex:

Month	Sales ^y thousand	Advertise ^x	x^2	y^2	xy
1	264	2.5			
2	116	1.3			
3	165	1.4			
4	101	1.0			
5	209	2.0			
	$\sum y$	$\sum x$	$\sum x^2$	$\sum y^2$	$\sum xy$

$\bar{x}, \bar{y}$

$$y = a + bx$$

من القانون $b = 109.229$

$$a = -8.135$$

① $y = -8.135 + 109.229x$

② $r = 0.98 \rightarrow$ Positive, strong

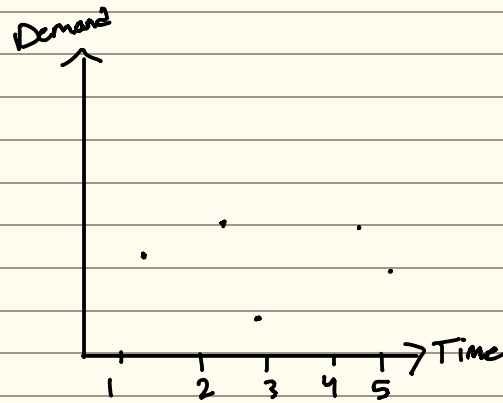
③ $r^2 = 0.96$

④ $S = 15.6$

Forecast demand

ADV expand

Time Series :

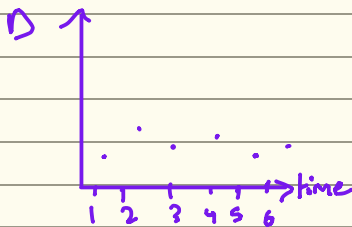


time series Patterns:

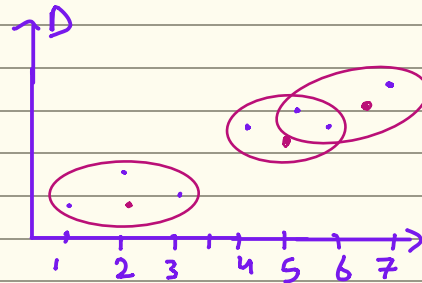
- 1) horizontal
- 2) trend
- 3) Seasonal
- 4) cyclical
- 5) random

What type of time series?

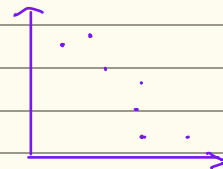
1) horizontal



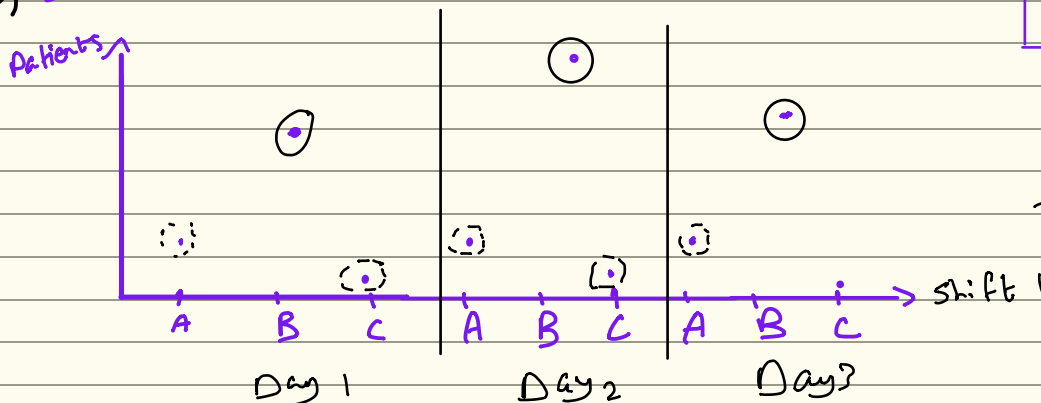
2) Positive trend avg increasing



negative (decreasing)



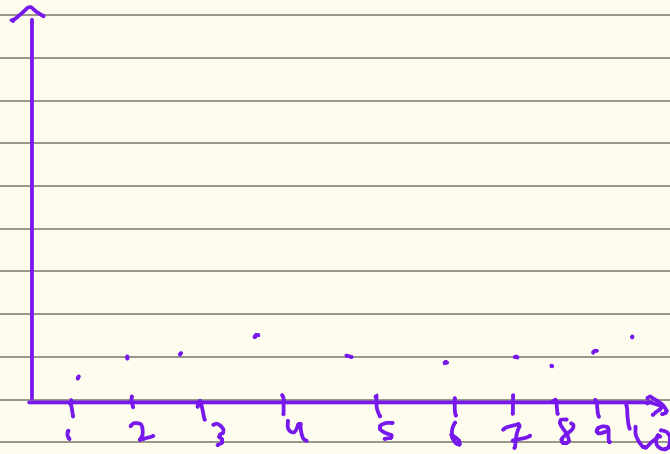
3) Seasonal



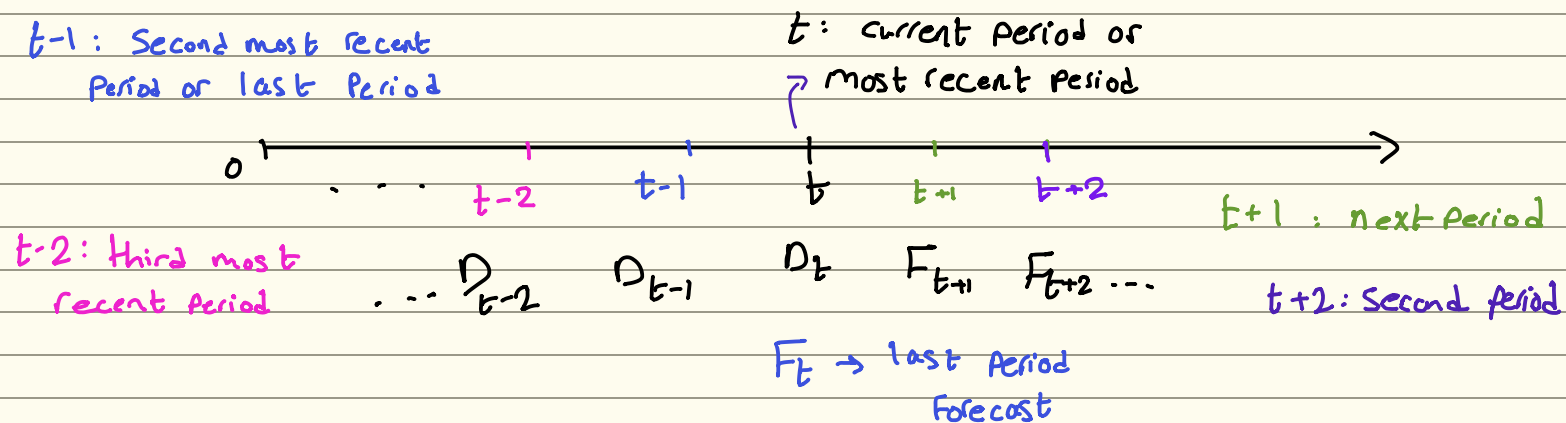
Period → days

Season → shifts

4. Cyclical



* Time Series analysis techniques:



$$E_t = D_t - F_t$$

error

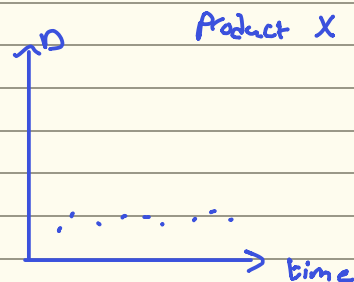
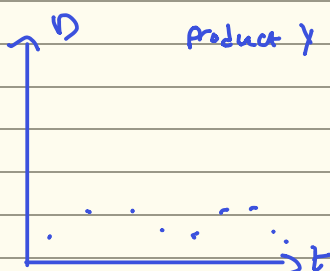
$= +ve \rightarrow F_t < D_t$ underestimated D
 $= -ve \rightarrow F_t > D_t$ overestimated D

Techniques:

القوانين
في التنبؤ

[1] Naive approach (horizontal)

$$F_{t+1} = D_t$$



[2] Simple moving average : (horizontal)

$$F_{t+1} = \frac{D_t + \dots + D_{t-n+1}}{n}$$

3-periods simple moving avg : (t, t-1, t-2) most recent

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2}}{3}$$

	Week	Patient arrived
t-2 ←	1	400
t-1 ←	2	380
t ←	3	411
	4	??

$$F_{w4} = \frac{411 + 380 + 400}{3} = 397 \text{ patient}$$

	Week	Patient arrived
t-2 ←	1	400
t-1 ←	2	380
t ←	3	411
	4	415

↓
if it was 415

error:

$$E_{w4} = 415 - 397 = 18 \text{ patient}$$

We underestimated demand

period	Week	Patient arrived
t-3 ←	1	400
t-2 ←	2	380
t-1 ←	3	411
t ←	4	415
	5	?

only last 3 periods

$$F_{w5} = \frac{415 + 411 + 380}{3} = 402 \text{ patient}$$

[3] Weighted moving average (horizontal)

$$F_{t+1} = w_1 D_t + \dots + w_n D_{t-n+1}$$

$$w_1 + \dots + w_n = 1$$

$$w_1 > \dots > w_n$$

2-Period weighted moving avg

$$F_{t+1} = w_1 D_t + w_2 D_{t-1}$$

$$w_1 + w_2 = 1$$

$$w_1 > w_2$$

3-Period " " "

$$F_{t+1} = w_1 D_t + w_2 D_{t-1} + w_3 D_{t-2}$$

$$w_1 + w_2 + w_3 = 1$$

$$w_1 > w_2 > w_3$$

Ex:

Month	Customer arrival	
1	800	3-Periods weighted moving avg / $w_1 = 0.5$ $w_2 = 0.3$ / $w_3 = 0.2$
$t-2 \leftarrow$ 2	740	
$t-1 \leftarrow$ 3	810	$F_{ms} = (0.5)(790) + (0.3)(810) + (0.2)(790) =$ 786 Customer
$t \leftarrow$ 4	790	
5	805	

if m_s is 805 from question

$$F_{ms} = 805 - 786 = 19 \text{ customer}$$

3 periods only

we underestimated
the demand

$$F_{m6} = (0.5)(805) + (0.3)(790) + (0.2)(810) = 801.5 \approx 802$$

Notes:

উল্লিখিত

Forecast month 4 through 7 $\rightarrow$ উল্লিখিত দেব

if $n=1 \rightarrow$ Naive approach

if n is big $\rightarrow$ simple moving avg

4 Exponential Smoothing (α)

$$F_{t+1} = \alpha D_t + (1-\alpha) F_t$$

D_t : this period demand

F_t : last period Forecast

α : exponential smoothing parameter

$$0 \leq \alpha \leq 1$$

$$(0.1, 0.3, 0.5, 0.7, 0.9)$$

if $\alpha = 1$ Naive approach

$$F_{t2} = (0.1)(400) + (0.9)(400)$$

$$F_{t3} = (0.1)(380) + (0.9)(F_{t2})$$

Ex:

Week	Patients
1	400 $F_{w1} = 400$
2	380 $F_{w2} = ?$
3	411 $F_{w3} = ?$
4	415

$X_t \leftarrow 3$

$t \leftarrow 4$

why like this?

find F_{w4} using exponential technique

if $\alpha = 0.1$

to find $F_3 \rightarrow$

$$F_{w4} = (0.1)(411) + (0.9)(F_{w3})$$

$$F_{w5} = (0.1)(415) + (0.9)(F_{w4}) =$$

Simple moving average technique

Simple moving avg $n=2$

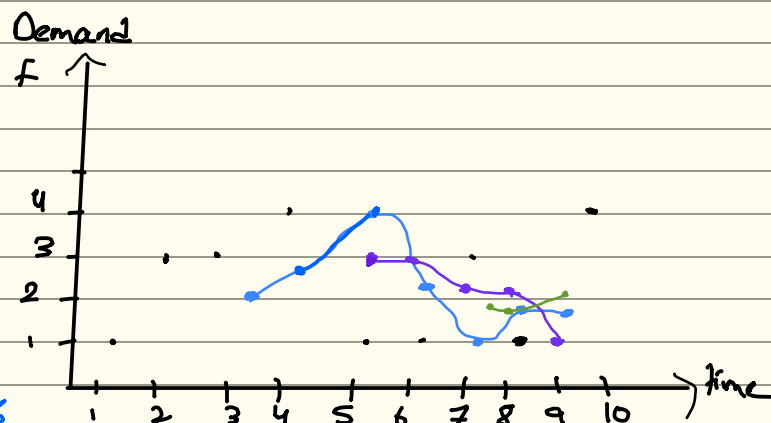
Simple moving average $n=4$

Simple moving avg $n=6$

$$F_{t+1} = \frac{D_t + \dots + D_{t-n+1}}{n}$$

$$F_{t+1} = w_1 D_t + \dots + w_n D_{t-n+1}$$

$$F_{t+1} = \alpha D_t + (1-\alpha) F_t$$



كيف حاسبة

$$\textcircled{1} \frac{3+1}{2} = 2 \rightarrow \text{for period 3}$$

$$\frac{3+2}{2} = 2.5 \rightarrow \text{for period 4}$$

⋮

② $\frac{4+3+3+1}{4} = 2.75 \rightarrow$ from period 5

$\frac{1+4+5+3}{4} = 2.75 \rightarrow$ from period 6

⋮

Rule: α

responsive forecast $\begin{cases} \rightarrow \text{smaller } n \\ \rightarrow \text{larger } \alpha \end{cases}$

smoothed forecast $\begin{cases} \rightarrow \text{larger } n \\ \rightarrow \text{smaller } \alpha \end{cases}$

n smaller, α larger $\rightarrow$ more responsive forecast

n larger, α smaller $\rightarrow$ more smoothed forecast

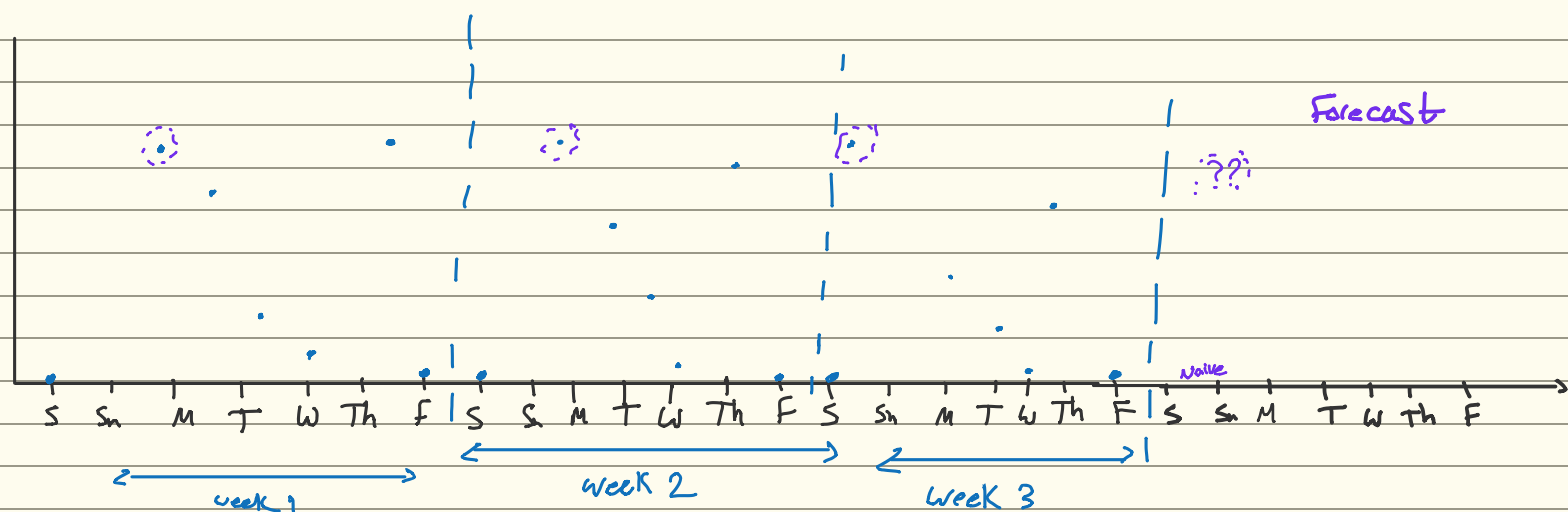
Positive trend

$\downarrow$
linear regression technique

3) Seasonal time Series

Period $\rightarrow$ weeks

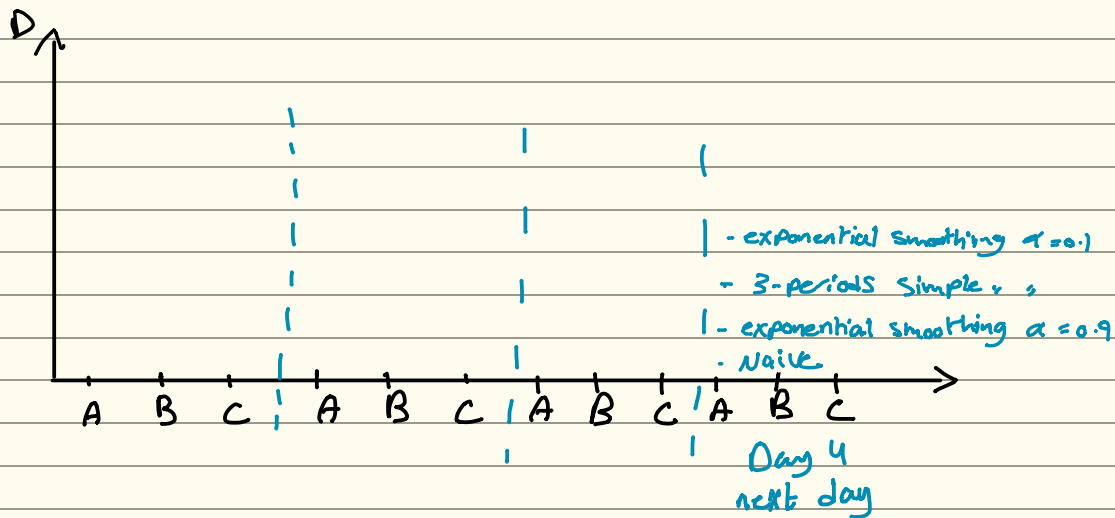
season $\rightarrow$ days



- * horizontal time series :
- 1 - Naive approach
 - 2 - Simple moving average
 - 3 - Weighted moving average
 - 3 - exponential smoothing

* trend → Simple linear regression

Multiplicative Seasonal Method



Season	D	Seasonal Index (I) Period ①	D	Period ②	Avg Seasonal Index	Period 3 (F) next period
S ₁	D _{P1S1}	$SI_{AS1} = \frac{D_{AS1}}{ADSP_1}$	D _{P2S1}	$SI = \frac{D_{P2S1}}{ADSP_2}$	$ASI_{S1} = \frac{SI_{AS1} + SI_{AS2}}{\# \text{ of periods}}$	$F_{P3S1} = ASI_{S1} \times EADSP_3$
S ₂	D _{P1S2}	$SI_{AS2} = \frac{D_{AS2}}{ADSP_1}$	D _{P2S2}	$SI = \frac{D_{P2S2}}{ADSP_2}$	$ASI_{S2} = \frac{SI_{AS2} + SI_{AS3}}{\# \text{ of periods}}$	$F_{P3S2} = ASI_{S2} \times EADSP_3$
S ₃	D _{P1S3}	$SI_{AS3} = \frac{D_{AS3}}{ADSP_1}$	D _{P2S3}	$SI = \frac{D_{P2S3}}{ADSP_3}$	$ASI_{S3} = SI_{P1S3} + SI_{P2S3}$	$F_{P3S3} = ASI_{S3} \times EADSP_3$

Total Demand = $D_{P1S1} + D_{P1S2} + D_{P1S3}$
(TD_{P1})

Avg Demand (P₁) = $\frac{TD_{P1}}{\# \text{ of seasons}}$

Seasonal Index = $\frac{D}{ADP}$ ≥ 1 peak
(SI) ≤ valley

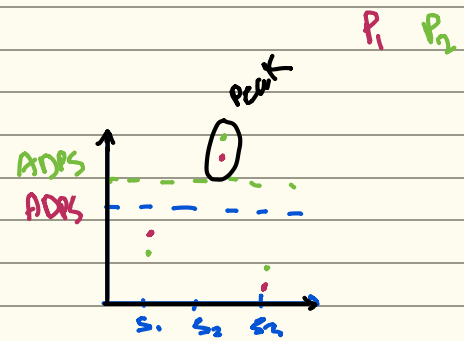
$$TD_{P2} = D_{P2S1} + D_{P2S2} + D_{P2S3}$$

$$ADP_{P2} = \frac{TD_{P2}}{\# \text{ of seasons}}$$

$$SI = \frac{D}{ADP}$$

Estimated total D (ETD_{P3}) = using an appropriate technique to forecast demand

Estimated avg demand per Season = $\frac{ETD_{P3}}{\# \text{ of seasons}}$
(EADP₃)



Example: (Multiplication seasonal method) time series technique

The manager of the Stanley Steemer carpet cleaning company needs a quarterly forecast of the number of customers expected next year. The carpet cleaning business is seasonal, with a peak in the third quarter and a trough in the first quarter. Following are the quarterly demand data from the past 4 years:

Quarter	Year 1	Year 2	Year 3	Year 4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215
Total	1000	1200	1800	2200

Q	D	SI	D	SI	D	SI	D	SI	ASI	Forecast
1	45	$\frac{45}{250} = 0.18$	70	$\frac{70}{300} = 0.23$	100	0.22	100	0.18	$(0.18 + 0.23 + 0.22 + 0.18) / 4 = 0.2043$	$FQ_1 = (0.204)(650) = 132.79$ carpet
2	335	$\frac{335}{250} = 1.34$	370	$\frac{370}{300} = 1.23$	585	1.3	725	1.32	$(1.34 + 1.23 + 1.3 + 1.32) / 4 = 1.2979$	$FQ_2 = (1.2979)(650) = 843.6$
3	520	$\frac{520}{250} = 2.08$	590	$\frac{590}{300} = 1.96$	830	1.84	1160	2.1	$= 2.0001$	$FQ_3 = 1300.1$
4	100	$\frac{100}{250} = 0.4$	170	$\frac{170}{300} = 0.56$	285	0.63	215	0.39	$= 0.4977$	$FQ_4 = 323.5$

① $TD_{Y_1} = 1000$ $TD_{Y_2} = 1200$ $TD_{Y_3} = 1800$ $TD_{Y_4} = 2200$

② $ADPS_{Y_1} = \frac{1000}{4} = 250$ $ADPS_{Y_2} = \frac{1200}{4} = 300$ $ADPS_{Y_3} = \frac{1800}{4} = 450$ $ADPS_{Y_4} = \frac{2200}{4} = 550$

③ $EADPS_{Y_5} = \frac{2600}{4} = 650$

④ $ETR_{Y_5} = 2200 + 400 = 2600$

⑤ $Aug = 400$

Quarters are 1, 2, 3, 4

is 500, 600, 700, 800

$Q_1 \rightarrow$ lowest (Peak) < 1

$Q_2 \rightarrow$ higher (valley) > 1

* Can use linear regression technique

Forecasting technique	Period	Product X	Product Y
	3	$E = 1$	$E = 5$
	4	$E = 30$	$E = 2$
	5	$E = 15$	$E = 4$
	6	$E = 7$	$E = 3$

Periods	Demand	Forecast	$E = D - F$
1	D_1	F_1	E_1
2	D_2	F_2	E_2
3	D_3	F_3	E_3
⋮	⋮	⋮	⋮
n	D_n	F_n	E_n

Measures of central tendency

1 Cumulative sum of Forecast error $CFE = \sum_{t=1}^{t=n} E_t$

Period	Method 1 E	Method 2 E	Method 3
1	-2	+2	+2
2	+4	-4	-4
3	+3	-3	-7
4	+5	-5	-10
CFE	10	-10	-19

$$E = D - F$$

$E(+ve)$ = underestimation of demand

$E(-ve)$ = overestimation of demand

2 Mean bias

$$\bar{E} = \frac{CFE}{n}$$

3 Mean absolute deviation (MAD)

$$MAD = \frac{\sum_{t=1}^{t=n} |E_t|}{n}$$

4 Mean Squared Error (MSE)

P	M ₁ F	M ₂ E
	E^2	E^2
1	1 1	0 0
2	1 1	0 0
3	1 1	0 0
4	1 1	4 16
	$\frac{4}{4} = 1$	$\frac{4}{4} = 1$
	$\frac{4}{4} = 1$	$\frac{16}{4} = 4$

$$MSE = \frac{\sum E_t^2}{n}$$

5 Mean absolute percent error (MAPE)

Method ①

Product	E	D
X	1	1000
Y	1	10

$$MAPE = \sum_{t=1}^{t=n} \frac{(E_t/D_t) \times 100}{n}$$

Error as percent of actual Demand

$$\sigma_E = \sqrt{\frac{\sum (E_t - \bar{E})^2}{n-1}} \quad (\text{standard deviation of error})$$

Ex:

Month	D	F	E _t	E	E ²	$\frac{ E_t }{D} \times 100$	$(E_t - \bar{E})^2$
1	200	225	-25	25	625	$\frac{25}{200} \times 100 = 12.5$	$(-25 - (-1.87))^2$
2	240	220	20	20	400	$20/240 \times 100 = 8.3$	$(20 - (-1.87))^2$
3	300	285	15	15	225	,	,
4	370	290	-20	20	,	,	,
5	230	250	-20	20	,	,	,
6	260	240	20	20	,	,	,
7	210	250	-40	40	,	,	,
8	275	240	35	35	,	,	,
$\sum E = -15$				$\sum = 195$	$= 5275$	$= 81.3\%$	

Sol:

$$CFE = -15$$

$$\bar{E} = \frac{-15}{8} = -1.87$$

$$MAD = \frac{\sum |E|}{n} = \frac{195}{8}$$

$$MSE = \frac{5275}{8} = 659.3$$

$$MAPE = \frac{81.3\%}{8} = 10.2\%$$

$$\sigma_E = 27.4$$

* Combination Forecasting

P	M ₁	M ₂	Forecast decision
10	F _{M₁P₁₀}	F _{M₂P₁₀}	$F_{P_{10}} = \frac{F_{M_1 P_{10}} + F_{M_2 P_{10}}}{2}$

* Focus Forecasting

P	M ₁ D F E	M ₂ D F E	Forecast decision
10	D ₁₀ F _{M₁P₁₀} E _{M₁P₁₀}	D ₁₀ F _{M₂P₁₀} E _{M₂P₁₀}	
11	D ₁₁ F _{M₁P₁₁} E _{M₁P₁₁}	D ₁₁ F _{M₂P₁₁} E _{M₂P₁₁}	F _{P₁₁} = F _{M₂P₁₁}
12	F _{M₁P₁₂}	F _{M₂P₁₂}	F _{P₁₂} = F _{M₁P₁₂}

1 If the SKU has previous data about actual demand

- No → Judgmental methods
- yes → quantitative methods

2 Judgmental methods (executive opinion, Delphi method, market research sales force estimates)

3 Quantitative methods

- Causal method: if we think a variable affects demand.
- time series analysis: if we think that the behaviour of the demand continue the same in future.

4 Causal methods

- linear relation:
 - one variable affects demand → simple linear regression.
 - two or more variables → multiple linear regression
- non-linear relation: AI/

5 Time series analysis: demand pattern → horizontal (naive, simple moving avg, weighted, exponential)

trend (simple linear regression, ARIMA, trend, adjusted exponential smoothing)

Seasonal: (Multiplicative seasonal method, additive seasonal method, SARIMA)

6 To choose between chosen techniques → measures of error

interested that the techniques not to have bias → CFE, $\bar{E}$

" " " " to produce large error → MSE

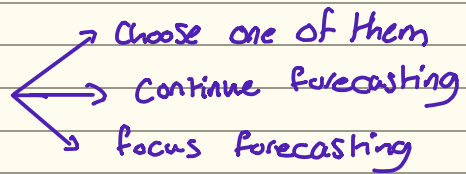
" " " " to produce forecast away from the actual demand → MAD

" " " " as a percent of actual D → MAPE

7

Based on measures of error

there are more than one good technique:



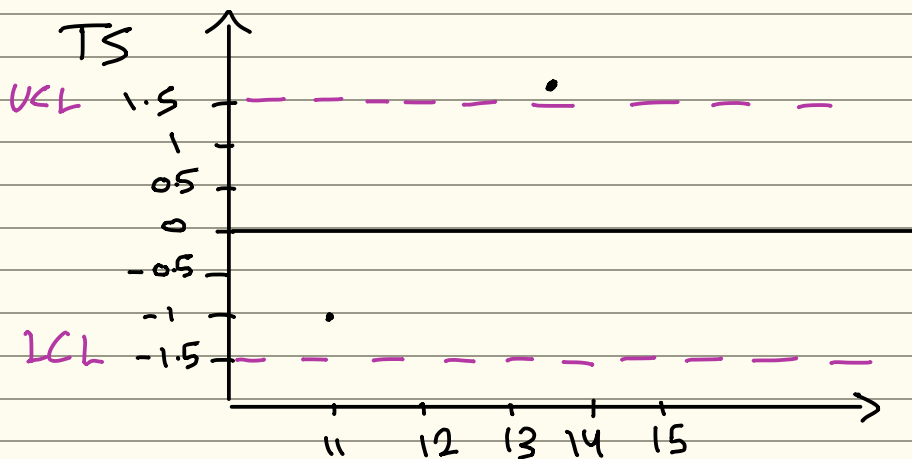
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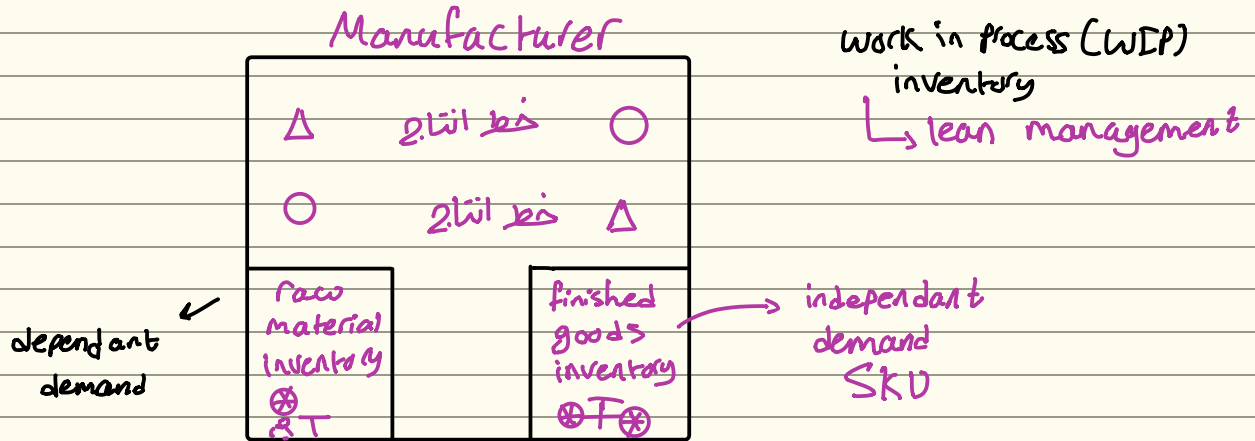
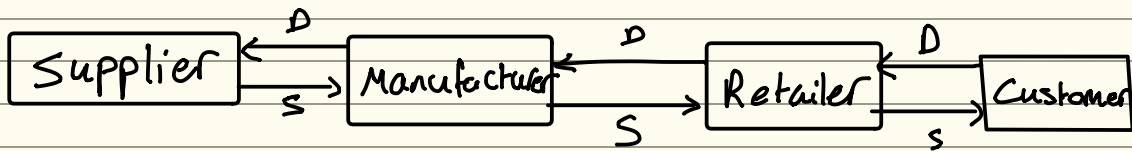
Start using the technique and keep monitoring that the process of using this technique is within control: *process in control*

Tracking Signal

$$TS = \frac{CFE}{MAD}$$

Period	D	F	E	CFE	MAD	TS
1	D_{11}	F_{11}	E_{11}	CFE_{11}	MAD_{11}	TS_{11}
2	D_{12}	F_{12}	E_{12}	CFE_{12}	MAD_{12}	TS_{12}





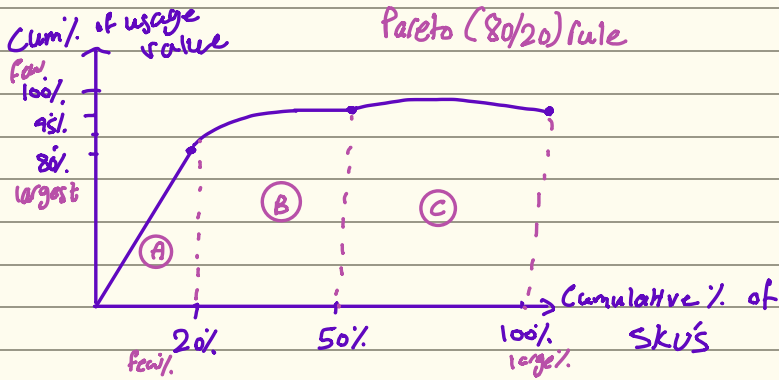
Inventory Management

classification of SKU's

Choosing inventory control system

- ABC classification (usage value)

- when to order?
- how much to order?
- how often to review IP?



class A

few % of SKU's (largest % usage value) → critical SKUs

class C

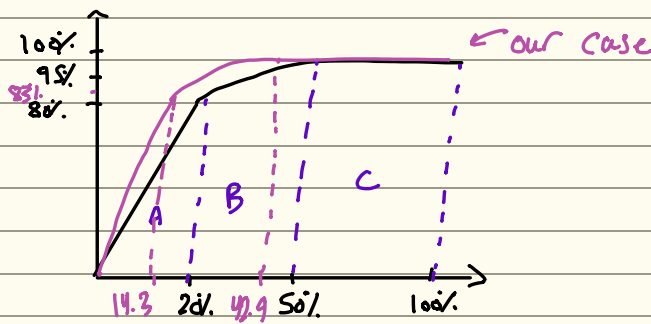
largest % of SKU's (smallest % usage value) → useful many

Ex:

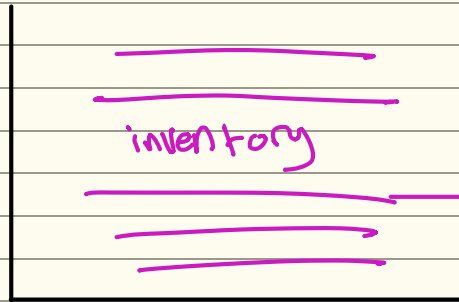
SKU #	Description	Quantity used per year	Unit value \$	Annual usage value \$/year (unit value * quantity per year)
1	Boxes	500	3	1500
2	Cardbord	1800	0.02	360
3	Cover	1000	0.75	7500
4	Glue	75	40	3000
5	Inside cover	20 000	0.05	1000
6	tape	3000	0.15	450
7	Signature	150 000	0.45	6750

ترتيب

SKU	Description	Usage value	% of sku	% of usage value	Cumulative % of SKU	Cumulative % usage value	Class
7	Signature	67500	14.3%	$\frac{67500}{81310} \times 100\% = 83\%$	14.3	83%	A (80%) (less than B)
3	cover	7500	14.3%	$\frac{7500}{81310} \times 100\% = 9.2\%$	28.6%	92.2%	
4	Glue	3000	14.3%	$= 3.7\%$	42.9%	95.9%	
1	Boxes	1500	14.3%	1.8%	57.1%	97.8%	B (num less than C)
5	inside cover	1000	14.3%	1.2%	71.5%	99%	
6	tape	450	14.3%	0.6%	85.7%	99.6%	
2	cardboard	360	14.3%	0.4%	100%	100%	C (more num of sku)
		Σ 81310					few → largest

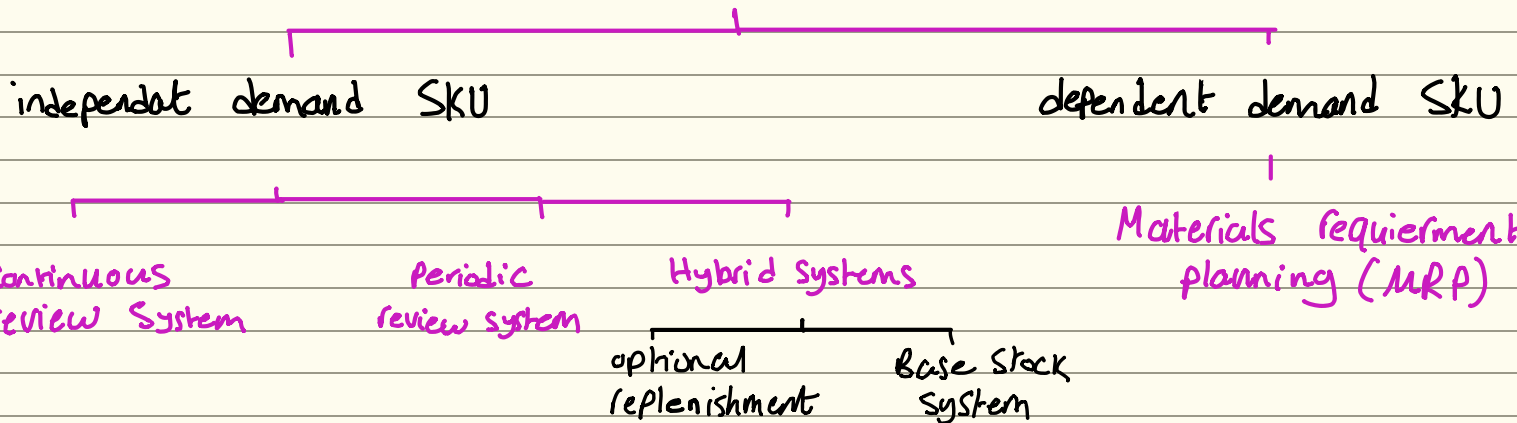


inflow (replenishment) ↓



Decisions related to managing inventory

- 1- when to order?
- 2- how much to order?
- 3- how often to review inventory position?



Inventory Cost

Holding Cost

cost of Capital

Storage and handling cost

Taxes insurance and shrinkage

Pilferage (theft) obsolescence (outdated) deterioration (damaged)

↓
ع
unit cost

H: Unit holding cost \$/unit/year

ordering/setup cost

S: order/setup cost

\$/order

\$/setup

Ordering/setup cost $\times$ Purchasing cost
الفرق

Unit cost $\times Q$ = Purchasing cost

* Inventory position = on-hand + Scheduled Receipts - Back order inventory

$$IP = OH + SR - BO$$

كمية متوفرة

Cycle inventory

product X: $D = 100$ unit/year

Case 1: $Q = 100$

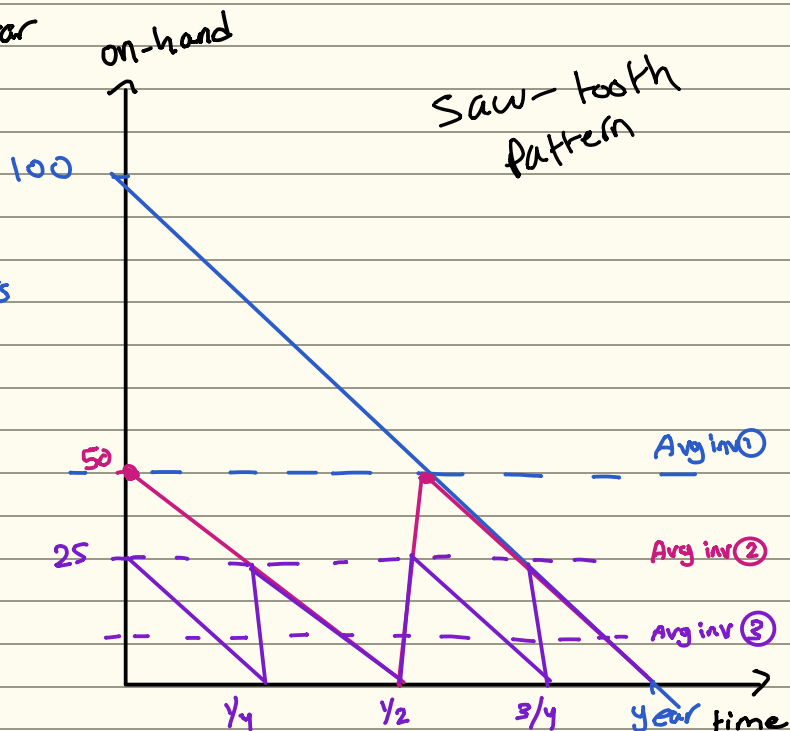
$$\text{Avg inventory} = \frac{100}{2} = 50 \text{ units}$$

$$\text{num of orders} = \frac{D}{Q} = \frac{100}{100} = 1 \text{ orders}$$

Case 2: $Q = 50$

$$\text{Avg inventory} = \frac{50}{2} = 25 \text{ unit}$$

$$\text{num of orders} = \frac{100}{50} = 2 \text{ orders}$$



Case 3: $Q = 25$

$$\text{Avg inventory} = \frac{25}{2} = 12.5$$

$$\text{num of orders} = \frac{100}{25} = 4$$

هو ايجاز
الاجاز

$$\begin{aligned} \text{Total annual Inventory Cost} &= \text{Annual holding cost} + \text{Annual ordering/setup cost} \\ &= \frac{Q}{2} H + \frac{D}{Q} S \end{aligned}$$

$$\text{Annual holding Cost} = \text{Average cycle Inventory } \left(\frac{Q}{2} H\right) \times \text{unit holding cost (H)}$$

$$\text{Annual Ordering Setup cost} = \text{num of orders/year} \times \text{order/setup cost} \left(\frac{D}{Q}\right) (S)$$

Economic order Quantity

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Time between orders

$$TBO = \frac{Q}{D} \text{ --- year}$$

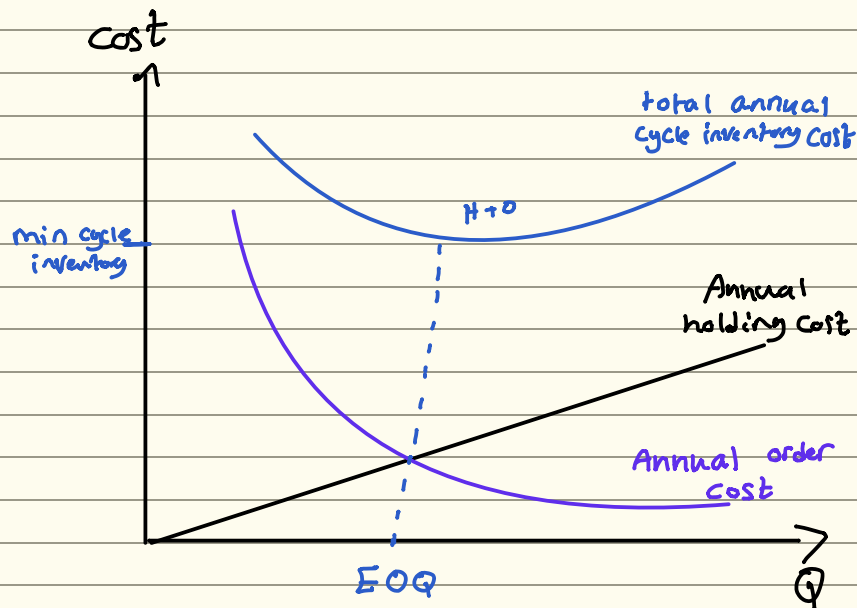
to months:

$$\frac{Q}{D} \times 12 \text{ --- months}$$

to weeks:

$$\frac{Q}{D} \times 52 \text{ --- week}$$

$$\text{to days} = \frac{Q}{D} \times 365 \text{ --- days}$$



Example (bird feeder) : operates 52 weeks

$$d = 18 \text{ units/week}$$

$$\text{unit cost} = 16 \$/\text{unit}$$

$$\text{Ordering Cost (S)} = 45 \$/\text{order}$$

$$\begin{aligned} \text{Annual holding Cost (H)} &= \left(25\% \frac{\text{unit cost}}{\text{cost}} \right) \\ &= 0.25 \times 60 = 15 \$/\text{unit/year} \end{aligned}$$

$$\text{Lot size} \rightarrow Q = 390 \text{ unit}$$

What is the annual cycle inventory cost?

$$C = \frac{Q}{2} H + \frac{D}{Q} S$$

$$= \frac{390}{2} (15) + \frac{18(52)}{390} 45$$

$$\begin{aligned} \text{EOQ} &= \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(18)(52)(45)}{15}} = 74.94 \\ &= 75 \text{ unit} \end{aligned}$$

$$Q = 75$$

annual cycle
inventory cost:

$$C_{\text{EOQ}} = \frac{75}{2} (15) + \frac{18(52)(45)}{75} = 1124 \$$$

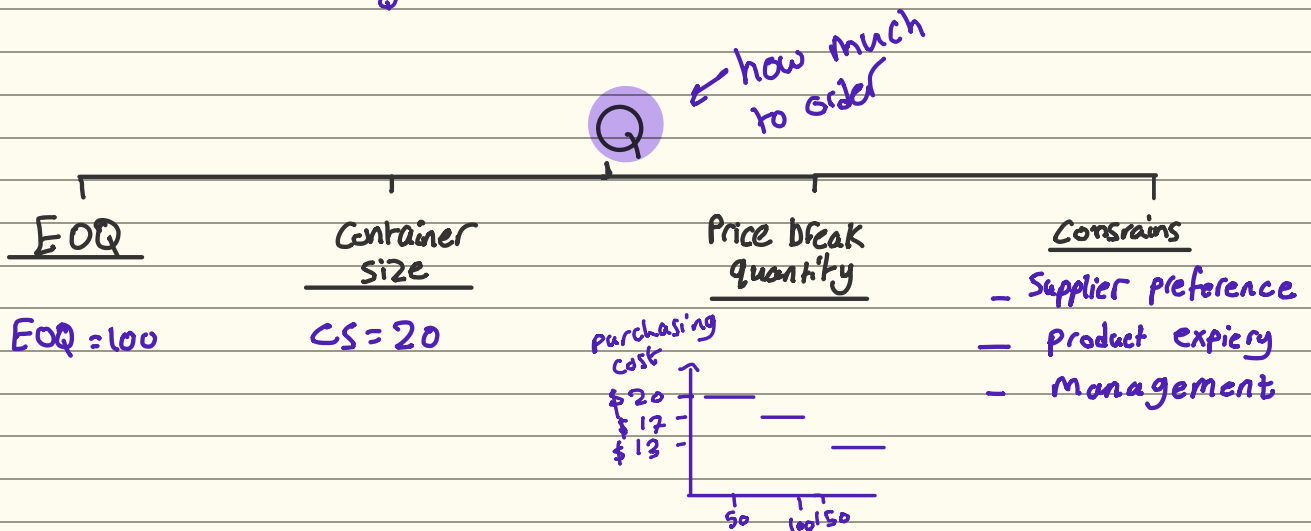
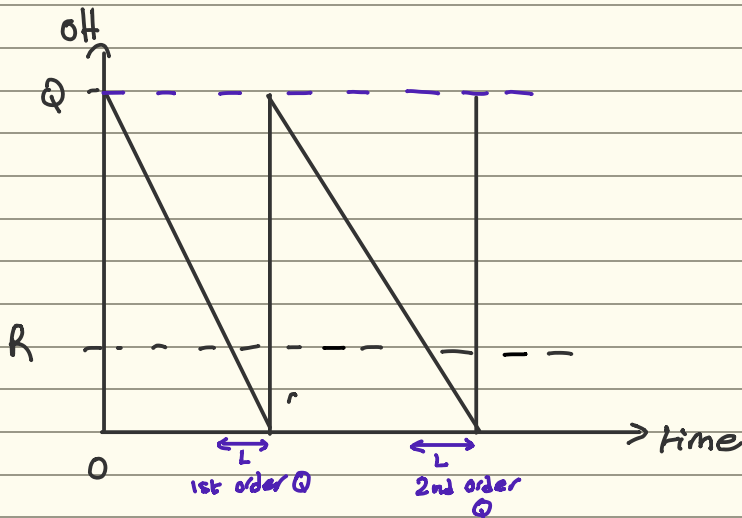
$$\text{TBO} = \frac{Q}{D} = \frac{75}{18(52)} = 0.08 \text{ year}$$

$$(\times 12) = 0.96 \text{ months}$$

$$(\times 52) = 4.17 \text{ week}$$

Continuous Review System / ROP system / fixed order quantity system / Q system

How often to review IP?	When to order?	how much to order?
continuously at the end of each withdrawal	When the IP reaches R	fixed quantity Q

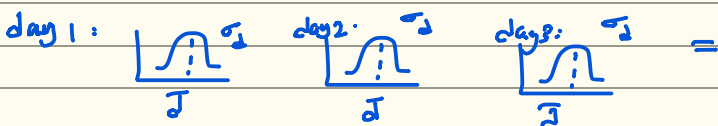


R ← when to order

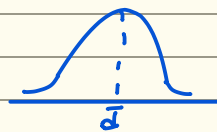
Constant demand (d)
Constant lead time (L)

$$R = (d)(L)$$

if $L = 3$ days



Variable demand ($\bar{d}$)
Constant lead time (L)

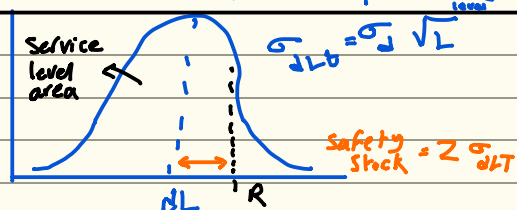


$$R = \bar{d}L + S.S$$

$$S.S = Z \sigma_{\bar{d}L}$$

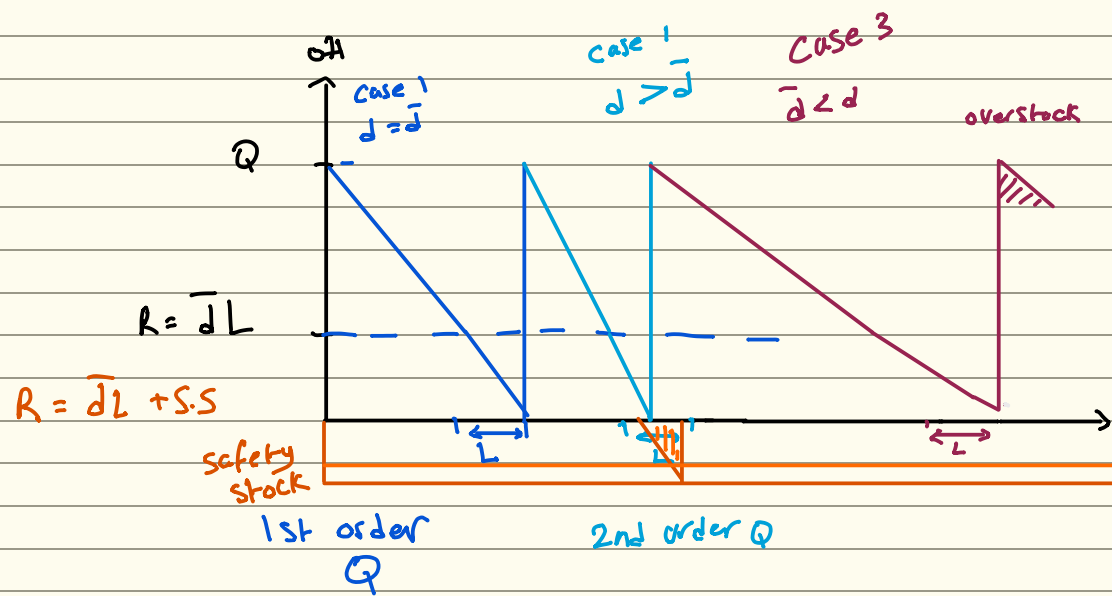
Z = num of std needed to achieve required service level

$$\sigma_{\bar{d}L} = \sigma_d \sqrt{L}$$



$$\text{Safety Stock} = Z \sigma_{\bar{d}L}$$

Variable demand
variable lead time



Stock decision: Service level

- 100%
- 95%
- 80%
- 75%

Bird Feeder Example 12.12 :

- operates 52 weeks
- avg demand $\bar{d} = 18$ unit/week
- $\sigma_d = 5$ units
- lead time (L) = 2 weeks
- $H = 15$ \$/unit/year
- $S = 45$ \$/order
- Q: - EOQ = 75 unit
- 90% Service level
- Q_{system} , $R = ?$, $C = ?$

Sol: $R = \bar{d}L + S.S$

$$S.S = Z \sigma_{dL} = Z \sigma_d \sqrt{L}$$

$$= 1.28 (5) \sqrt{2} = 9.05 \approx 9 \text{ units}$$

Service level 90%
 $\hookrightarrow$ area = 0.9
 $\hookrightarrow$ Z table Appendix
 0.8997
 So $Z = 1.28$

1) $R = 18(2) + 9 = 45$ units

$Q = 75$

2) $C = \frac{Q}{2} H + \frac{\overset{\text{annual}}{D}}{Q} S + S.S(H)$

$$= \frac{75}{2} 15 + \frac{18(52)}{75} (45) + 9(15) = 1259 \$$$

Due to variability we added safety stock

b) The last check of inventory position was 65 units.

A withdrawal of 27 unit was done (No scheduled receipts, no backorders)
 Is it time to order? And if yes how much to order?

$$IP_{current} = OH + SR - BO = \underline{(65 - 27)} + \underline{0 - 0} = 38$$

if current $< R$

$$38 < 45$$

$\Rightarrow$ yes we order

$Q = 75$

if = 45 we order / if > 45 do not order

Continue chart

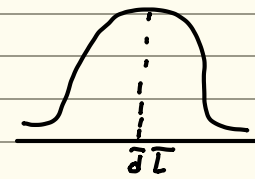
R

variable Demand (σ_d)
variable lead time ($\bar{L}_{LT}$)

$$R = \bar{d}\bar{L} + S.S$$

$$\text{Safety Stock} = Z \sigma_{dLT}$$

Z: number of standard deviations
needed to achieve required service
level.



distribution of demand
during protection interval
(lead time)

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

Ex: Avg demand for pen = 12,000 pen/week

Standard deviation $\sigma_d = 3000$ pens

Current calls for replenishment order of (Q) = 156,000 pens

$\bar{L} = 5$ weeks

$\sigma_{LT} = 2$ weeks

95% service level, Q system

Reorder point (R) = ??

$$R = \bar{d}\bar{L} + S.S$$

$$Z(0.95) = 1.65$$

$$S.S = Z \sigma_{dLT} = Z \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

$$= 1.65 \sqrt{5(3000)^2 + 12000^2(2)^2}$$

$$= 4117.32 \approx 4118 \text{ pens}$$

$$R = (12000)(5) + 4118 = 10118 \text{ pens}$$

IP

Manually

OH $\rightarrow$ Shelf Counting

SR $\rightarrow$ purchasing (order?)

BO $\rightarrow$ marketing (customer order?)

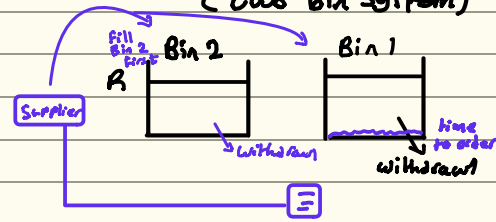
Technology

barcode scanner

OH

SR

Visual Q system (two bin system)



* Periodic Review System / Fixed interval re-order system / Periodic re-order system / P-System

how often to review IP?

Periodically at the end of each review period (P)

when to order?

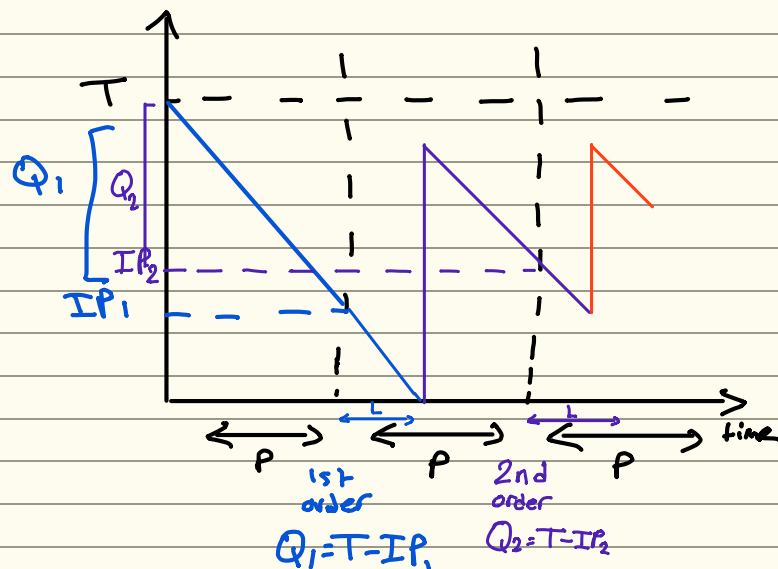
At the end of each review period (P)

how much to order?

Order Quantity = Target inventory (T) - inventory position

$$Q = T - IP$$

$$IP = OH + SR - BO$$



$$P = TBO = \frac{Q}{D}$$

$$TBO_{EOQ} = \frac{EOQ}{D}$$

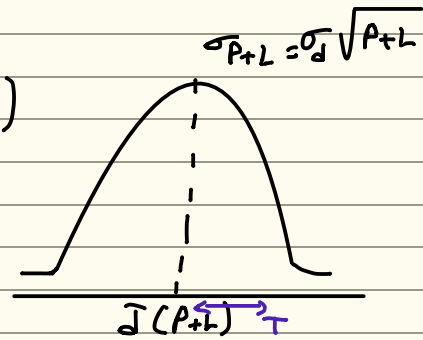
$$TBO_{\text{Container size}} = \frac{Q_{\text{Container}}}{D}$$

$$TBO_{\text{Price break quantity}} = \frac{Q_{Pbq}}{D}$$

$$TBO_{\text{Selected Q}} = \frac{Q_{\text{Selected}}}{D}$$

T

case 1	case 2
Demand Constant (d)	Demand Variable ($\bar{d}, \sigma_d$)
lead time constant (L)	lead time constant (L)
$T = d(P+L)$	$T = \bar{d}(P+L) + S.S$
	$S.S = Z \sigma_{P+L}$
	Z : num of standard deviation needed to achieve required service level.
	$\sigma_{P+L} = \sigma_d \sqrt{P+L}$



Bird Feeder example:

52 weeks
 $\bar{d} = 18$ units/week
 $\sigma_d = 5$ units

$H = 15$ \$/unit/year
 $S = 45$ \$/order

EOQ = 75 units

$L = 2$ weeks

90% service level ($Z = 1.28$)

P-System

a) S.S = ? , T = ? , C = ?

Sol:

$$P = \frac{TBO_{EOQ}}{\bar{d}} = \frac{EOQ \times 52 \text{ weeks}}{\bar{d}} = \frac{75}{18(52)} \times 52 = 4.2 \approx 4 \text{ weeks}$$

$$S.S = Z \sigma_{P+L} = Z \sigma_d \sqrt{P+L} = (1.28)(5) \sqrt{4+2} = 15.68 \approx 16 \text{ units}$$

$$T = \bar{d}(P+L) + S.S = 18(4+2) + 16 = 124 \text{ units}$$

$$C = \frac{\bar{d} P}{2H} + \frac{D}{\bar{d} P} S + S.S \cdot H$$

$$C = \frac{(18)(4)}{2} \cdot 15 + \frac{18(52)(45)}{18(4)} + (16)(15) = 1365 \$$$

b) Recently the $ott = 60$ units (no scheduled receipts/no backorders). Now it's the end of review period and the onhand inventory = 45 units. Is it time to order? And how much?

Yes, we order

We don't order when

$$T = IP$$

$$IP = Ot + SR - BO = 45 + 0 - 0 = 45$$

how much? $\rightarrow Q = T - IP = 124 - 45 = 79$ units

Case 3

Demand variable

lead time variable

Protection interval ($P+L$)

Use simulation to estimate the demand during protection interval ($P+L$) so as to use it to select value of T (under desired service level)

Differences:

continuous

Q-System

- less cost

- fixed Q

periodic

P-System

advantage:

- order $\rightarrow$ fixed interval $\rightarrow$ TBO

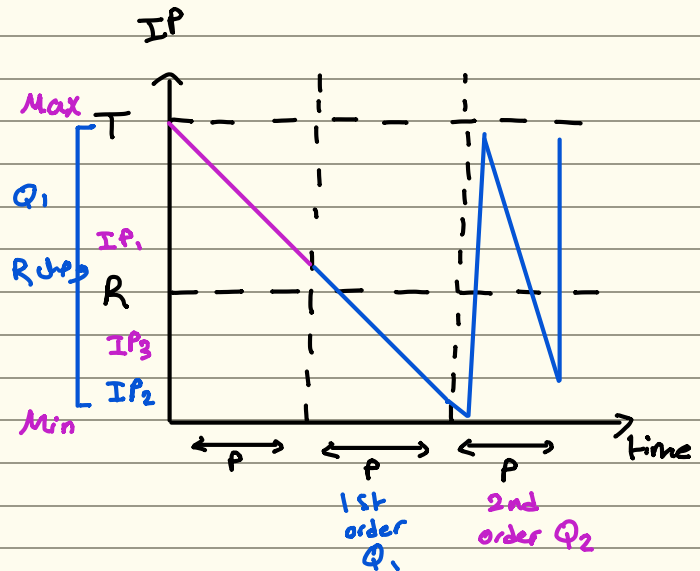
- IP $\rightarrow$ periodic

hybrid system $\Rightarrow$ Optional replenishment

(Max - Min) system
 T^* R^*

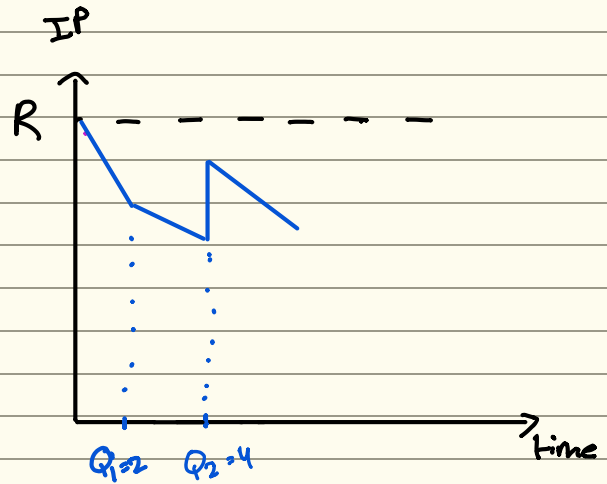
(S-s) system

We use this system for cheap items



Base - Stock system

We use it when we have very expensive items



Continuous - Periodic

supermarket

x_1	x_5	x_9
x_2	x_6	x_{10}
x_3	x_7	
x_4	x_8	

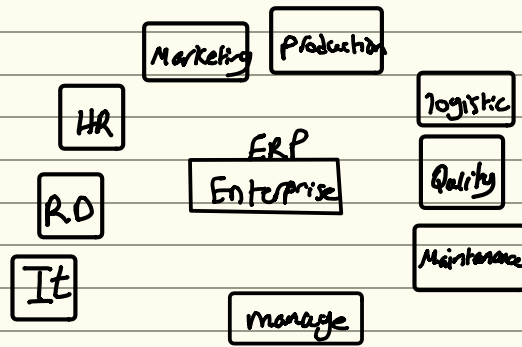
مستودع

x -ray	توليد
x_1	x_{10}
x_2	x_{11}
x_3	x_{12}
OR	
x_4 x_6	
x_5 x_7	
main warehouse	
x_1 x_2 x_3 x_4 x_5	
x_6	

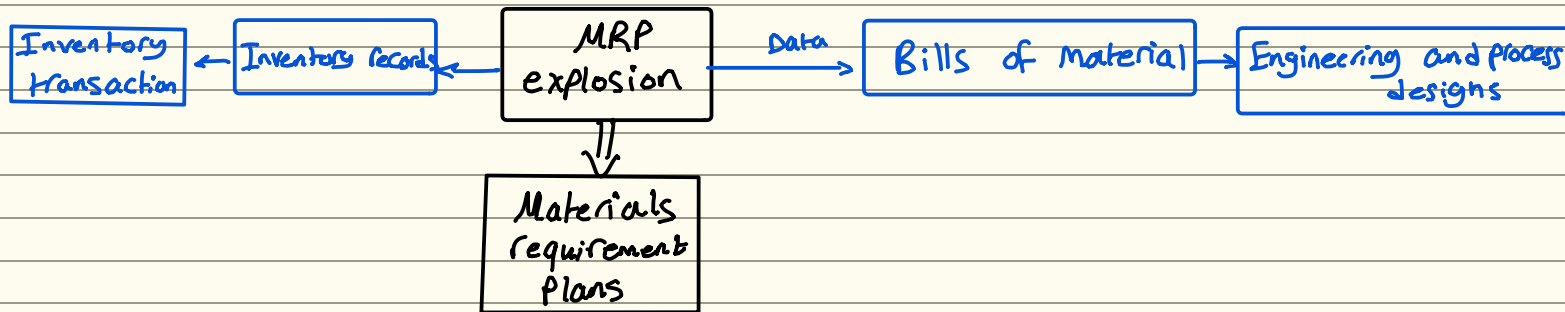
$\rightarrow$ Periodic system

Expensive items $\rightarrow$ Continuous

ERP: enterprise resource planning



MRP : Materials requirements plans



- Dependant demand SKU's
- how much to order? When to order?

if lead time 2 weeks

	1	2	3	4	5
orders					20
MPS start			20		

↑
we start in week 3

⊗ MRP

OH: 15
L: 1 week

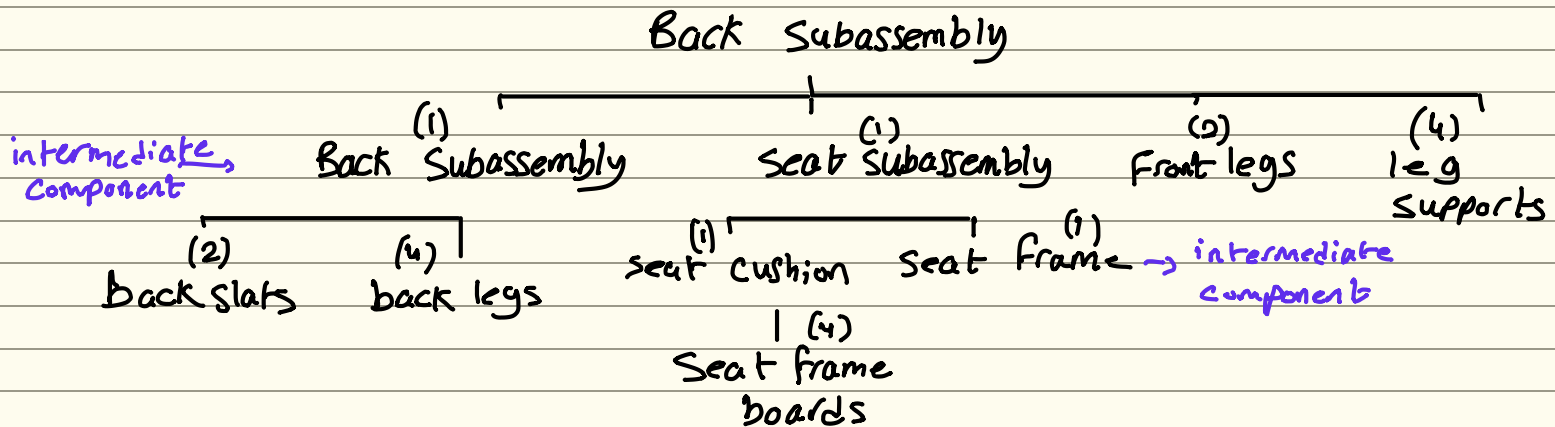
$$40 - 15 = 25$$

	1	2	3	4	5
gross requirement			40		
planned order release		25			

order 25 at week 2

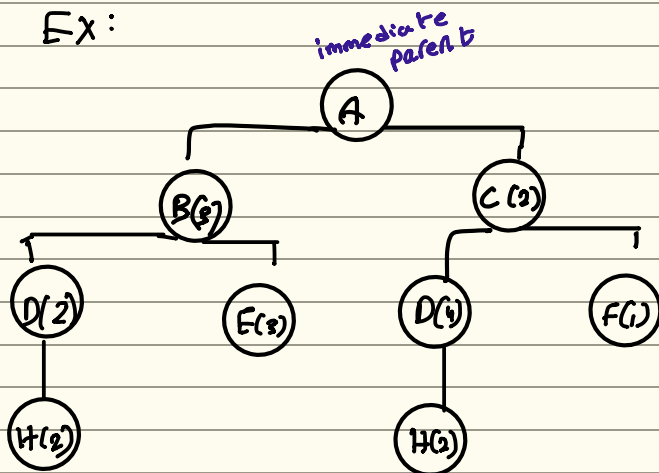
Bill of Material

BOM for a ladder-Back chair



Numbers are the Quantity we need.

Ex:



Given $\leftarrow A = 5 \text{ units}$

$$B = 3 \times 5 = 15$$

$$C = 2 \times 5 = 10$$

$$D = (15 \times 2) + (10 \times 4) = 70$$

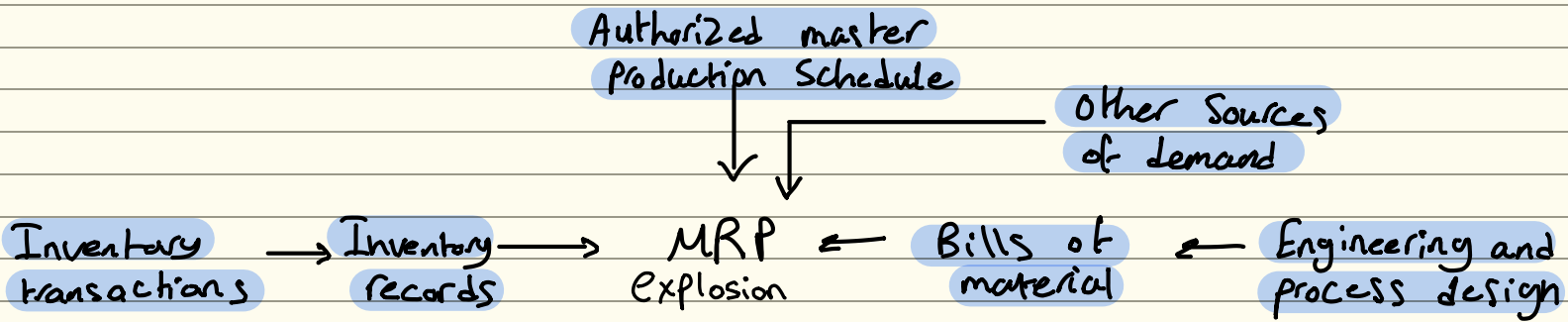
$$E = 15 \times 3 = 45$$

$$F = 10 \times 1 = 10$$

$$H = 70 \times 2 = 140$$

Dot

MRP inputs



Master production schedule

Master production schedule (MPS)

Item: 1adder back chair		Order Policy: 150 units							
		April				May			
Quantity on Hand: 55		1	2	3	4	5	6	7	8
Forecast		30	30	30	30	35	35	35	35
Customer orders booked		38	27	24	8	0	0	0	0
Projected on-hand inventory		55-38=17	17-30=-13	150+17-30=137	107-30=77	77-35=42	42-35=7	7-35=-28	122-35=87
MPS quantity	50 →	0	150	0	0	0	0	150	0
MPS start		150					150		
Available-to-promise (ATP) inventory		17	91	20			150		

Handwritten notes and calculations:

- Forecast**: 150+7-35=122
- Shortage** → if negative only MPS=150
- no shortage** → MPS quantity=0
- when? how much?** → find product MRP explosion
- 17-91=8**
- 150-27-24-8-0-0**
- 71+17=88** can't accept till week 6

if

customer #	Q	Week	Decision
1	20	3	✓ (we have 17+91)
2	100	5	✗ (we have 17+71)
3	80	4	✓ available 71+17 update 71/0 17/8

look at ATP row

EX2:

Item: Product A							Order Policy: 50 units				
							Lead Time: 1 week				
Quantity on Hand	5	1	2	3	4	5	6	7	8	9	10
Forecast		20	10	40	10	0	0	30	20	40	20
Customer orders (booked)		30	20	5	8	0	2	0	0	0	0
Projected on-hand inventory		-25	5	-35	5	5	3	-27	3	-37	-7
MPS quantity		50	0	50	0	0	0	50	0	50	50
MPS start	50		50				50		50	50	
Available-to-promise (ATP) inventory		5		35				50		50	50

$50 + 5 - 30 - 20 = 5$
oh

$50 - 5 - 8 - 0 - 2 = 35$

Customer

Q

w

From ATP

1

50

5

X

(have 35+5)

2

17

3

✓

(35+5)

MRP

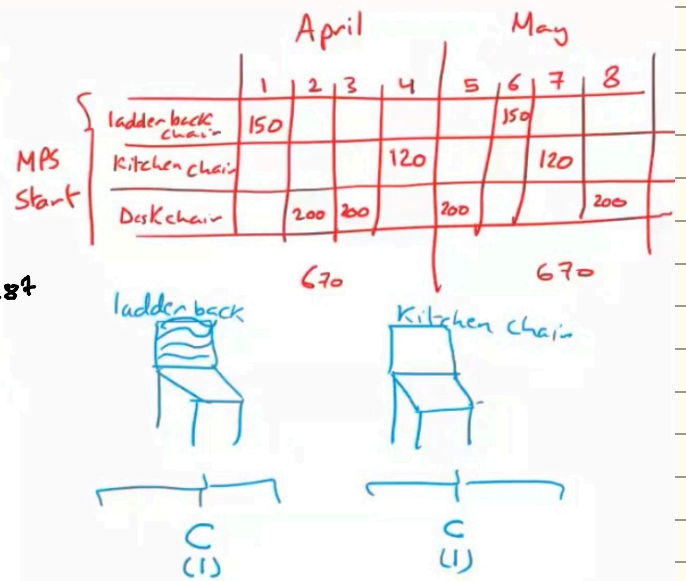
FOQ

Shortage if off less than safety stock

MRP → Dependent demand SKUs

Item: C	Lot Size: FOQ = 230							
Description: Seat subassembly	Lead Time: 2 weeks							
	Safety stock: ZERO							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory	37	117	117	117	227	227	77	187
Planned receipts	0	0	0	230	0	0	230	0
Planned order releases		230			230			

two weeks before



POQ

Item: C	Lot Size: POQ (P = 3)							
Description: Seat subassembly	Lead Time: 2 weeks							
	S.S. = ZERO							
	Week							
	1	2	3	4 ✓	5 ✓	6 ✓	7 ✗	8 ✗
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory	37	117	117	117	150	150	0	0
Planned receipts	0	0	0	153	0	0	120	0
Planned order releases		153			120			

$$120 + 0 + 150 + 0 - 117 = 153$$

S.S. on-hand in the last period

$$120 + 0 - 0 - 0 = 120$$

$$120 + 0 + 0 - 0 = 120$$

3 periods

L4L (Lot for Lot)

Item: C	Lot Size: L4L							
Description: Seat subassembly	Lead Time: 2 weeks							
	Week S.S = zero							
	1	2	3	4 ✓	5	6 ✱	7 ●	8
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory 37	117	117	117	0	0	0	0	0
Planned receipts	0	0	0	3	0	150	120	0
Planned order releases		3		150	120			

Lot for lot

$$120 + 0 - 117 = 3$$

$$150 + 0 - 0 = 150$$

$$120 + 0 - 0 = 120$$

SKUs (cheap) low unit cost

Solve by FOQ

Item: A	Lot Size: FOQ = 80 units									
Description:	Lead Time: 1 week									
	Week S.S = 0									
	31	32	33	34	35	36	37	38	39	40
Gross requirements		60		25		35		45		60
Scheduled receipts										
Projected on-hand inventory 0	0	20	20	75	75	40	40	75	75	15
Planned receipts	0	80	0	80	0	0	0	80	0	0
Planned order releases	80		80				80			

reminant

reminant → disadvantage (holding cost)
→ advantage (work as safety stock)

Q↑ → TBO↓ → # of order ↓

→ ordering cost ↓

$$\text{holding cost} = (0 + 20 + 20 + 75 + 75 + 40 + 75 + 75 + 15) / 10 \times H = 43.5H$$

$$\text{Ordering cost} = \text{num of orders} \times S$$

$$= 3 \times S$$

Solve by POQ

how to choose P : $P = TBO - \frac{Q}{D} = \frac{80}{(22.5)(52)} \times 52$

$= 3.55 \approx 4 \text{ weeks}$

Avg demand = $\frac{60 + 25 + 35 + 45 + 60}{10} = 22.5$

SKU = expensive unit cost

Item: A Description:	Lot Size: POQ(P=4) Lead Time: 1 week SS = Zero									
	Week									
	31	32	33	34	35	36	37	38	39	40
Gross requirements		60		25		35		45		60
Scheduled receipts										
Projected on-hand inventory 0	0	25	25	0	0	45	45	0	0	0
Planned receipts	0	85	0	0	0	80	0	0	0	60
Planned order releases	85				80				60	

$60 + 0 + 25 + 0 = 85$

$35 + 0 + 45 + 0 = 80$

holding cost = $(0 + 25 + 25 + 0 + 0 + 45 + 0 + 0 + 0) / 10 H = 14 H$

Ordering cost = 35

L4L

Item: A Description:	Lot Size: L4L Lead Time: 1 week SS = Zero									
	Week									
	31	32	33	34	35	36	37	38	39	40
Gross requirements		60		25		35		45		60
Scheduled receipts										
Projected on-hand inventory 0	0	0	0	0	0	0	0	0	0	0
Planned receipts	0	60	0	25	0	35	0	45	0	60
Planned order releases	60		25		35		45		60	

→ Special case POQ(P=1)

Shortage ≤ 80

Item: Description:		Lot Size: $P=4$ Lead Time: 1 week									
		Week $SS = 80$									
		31	32	33	34	35	36	37	38	39	40
Gross requirements			60		25		35		45		60
Scheduled receipts											
Projected on-hand inventory		80	80	105	105	80	80	125	125	80	80
Planned receipts		0	85	0	0	0	80	0	0	0	
Planned order releases		85				80					

FOQ

Item: C		Lot Size: FOQ = 230							
Description: Seat sub		Lead Time: 2 weeks							
		Week $SS = \text{zero}$							
		1	2	3	4	5	6	7	8
Gross requirements		150			120		150	120	
Scheduled receipts		230							
Projected on-hand inventory		37	117	117	117	227	227	77	187
Planned receipts		0	0	0	230	0	0	230	0
Planned order releases			230			230			

POQ

Item: C	Lot Size: POQ (P=3)							
Description: Seat sub	Lead Time: 2 weeks							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory 37	117	117	117	150	150	0	0	0
Planned receipts	0	0	0	153	0	0	120	0
Planned order releases		153						

S.S = Zero

لقد عينا كمية
لبننا ودي س.س
نظر 2 OH

$$120 + 150 - 117 = 153$$

we have 117 already

L4L

Item: C	Lot Size: L4L							
Description: Seat sub	Lead Time: 2 weeks							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory 37	117	117	117	0	0	0	0	0
Planned receipts	0	0	0	3	0	150	120	0
Planned order releases		3		150	120			

S.S = Zero

$$120 - 117$$

because oh $\neq$ S.S

POQ with S.S

Item: C	Lot Size: POQ (P=8)							
Description: Seat Sub	Lead Time: 2 weeks							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory	37	117	117	117	170	170	20	20
Planned receipts	0	0	0	173	0	0	120	0
Planned order releases		173			120			

$$120 + 150 - 117 + 20 = 173$$

↙

S.S

$$120 - 20 + 20 = 120$$

↙

S.S

MRP

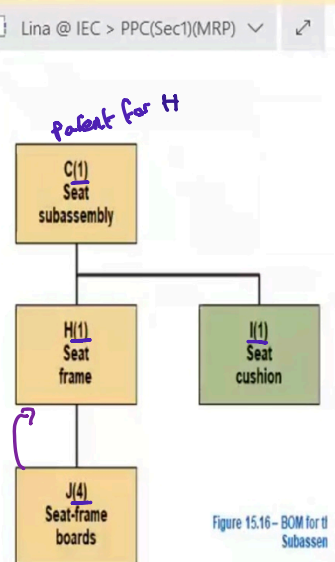
To sync this notebook, sign in to OneNote.

Item: C	Lot Size: FOQ=230							
Description: Seat sub	Lead Time: 2 weeks							
	S-S = Zero							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements	150			120		150	120	
Scheduled receipts	230							
Projected on-hand inventory	37	117	117	117	227	227	77	187
Planned receipts	0	0	0	230	0	0	230	0
Planned order releases		230			230			

Item: I	Lot Size: L4L							
Description: Seat cushion	Lead Time: 2 week							
	SS = Zero							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements		230			230			
Scheduled receipts								
Projected on-hand inventory 	0	0	0	0	0	0	0	0
Planned receipts	0	230	0	0	230	0	0	0
Planned order releases	230			230				

Item: H					Lot Size: FOQ=300			
Description: Seat frame					Lead Time: 1 week			
	Week S-S = Zero							
	1	2	3	4	5	6	7	8
Gross requirements		230			230			
Scheduled receipts		300						
Projected on-hand inventory 40	40	40	110	110	180	180	180	180
Planned receipts	0	0	0	0	300	0	0	0
Planned order releases				300				

Item: J	Lot Size: FOQ=1500							
Description: Seat frame boards	Lead Time: 1 week							
	SS = Zero							
	Week							
	1	2	3	4	5	6	7	8
Gross requirements				1200				
Scheduled receipts								
Projected on-hand inventory	200	200	200	200	500	500	500	500
Planned receipts	0	0	0	1500	0	0	0	0
Planned order releases			1500					



2C → A

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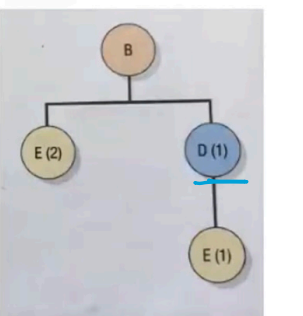
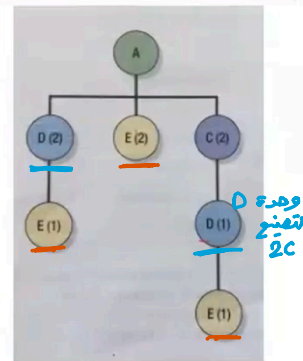
Item: C	MRP								Lot Size: L4L
Description:									Lead Time: 2 weeks
	Week								SS = zero
	1	2	3	4	5	6	7	8	
Gross requirements		250		190		300		260	
Scheduled receipts		200							
Projected on-hand inventory	85	85	35	35	0	0	0	0	
Planned receipts	0	0	0	155	0	300	0	260	
Planned order releases	155			300	260				

Item: D	↑ 140-35								Lot Size: POQ P=3
Description: Parent A, B, C									Lead Time: 2 weeks
									S.S = 260
	Week								
	1	2	3	4	5	6	7	8	
Gross requirements	155	250	80 300	190	260	300 70		260	
Scheduled receipts									
Projected on-hand inventory	625	470	220	450	260	0	260	260	
Planned receipts	0	0	610	0	0	630	0	0	
Planned order releases	610			630					

Product	1	2	3	4	5	6	7	8
A		125		95		150		130
B			80			70		

Data Category	C	D	E
Lot-sizing rule	L4L	POQ (P = 3)	FOQ = 800
Lead time	3 weeks	2 weeks	1 week
Scheduled receipts	200 (week 2)	None	800 (week 1)
Beginning inventory	85	625	350

Item: E									Lot Size: FOQ = 800
Description: Parent A, D, B									Lead Time: 1 week
									S-S = Zero
	Week								
	1	2	3	4	5	6	7	8	
Gross requirements	610	250	160	190 630		300 140		260	
Scheduled receipts	800								
Projected on-hand inventory 350	540	290	130	110	110	470	470	210	
Planned receipts	0	0	0	800	0	800	0	0	
Planned order releases			800		800				



$$80 + 300 + 190 + 260 - 220 = 610$$

