Statistics 1

Notebook

للدكتور : محمد شبول

two directions of reasoning :-

physical laws :

product designs

بالحالة في قوانين فنعمل Design لل product

population :

المجتمع اللي بدنا ندرسو

statistical inference.

بحالة ما عنا قوانين بناخذ sample

بنحلل العينة و بندرسها

لنستنتج عن هذا المجتمع

Basic methods of collecting data :

(1) retrospective study : historical data collected in the past for other purposes.

(2) observational : collected by a (passive) observer

ما بيتدخل بل system بس بيراقب.

(3) designed expriment .

hypothesis tests: -اختبار الفرضية

ناخذ sample ونختبرها لنتأكد من صحة الفرضية

هون احنا نفترض ونختبر عشان اتأكد

mechanistic modal:

→ example : ohm's law

current = voltage / resistanee

تيار جهد مقاومة

تعتمد على نوعية المادة يلي المصنوع منها السلك

$I = E / R$

$I = E / R + \boxed{\epsilon}$ عشان الدقة

السلك مش آمن $\boxed{100\%}$ فيسبب اختلاف بالتيار

في variability

مهندس بدو يصمم (نايلون) :

thickness : $\frac{3}{32}$ inch

رح نسأل إذا القوة كافية ؟

بناخد قطعة للنايلون ونقيس (pull off force) تاعتها حتى نشوف اذا بتتحمل العمل أو لا :

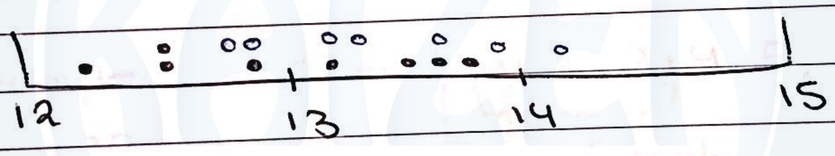
قياسات	12.6	13.1	بنقيس القوة عن طريق connector
pull off	12.6	13.4	
force	12.3	13.5	
للي طلعت :	12.9	13.6	

في اختلاف بالقياسات : لازم نسأل إذا حقيقي أو لا الاختلاف

(data diagram) للقيم الكبيرة

• = $\frac{3}{32}$ inch

○ = $\frac{1}{8}$ inch



نرسم على خط الأعداد point

لو جبنا connecter تاني thickness $\frac{1}{8}$ inch وطلعت معنا القياسات مختلفة

connecter الجديد : أعلى $\boxed{\text{pull off force}}$ يعني صار $\boxed{\text{shift points}}$

$\boxed{\text{variability}}$ في pull off force لكل connecter مختلفة مع انه في نفس design

لما أخذ في variability إذا pull off force نفسها :

random variable متغيرات عشوائية

لذا حسبنا المتوسط المتاح (solid) وطلع معنا 13

كل Data point بعيدة عن 13 بمقدار $\boxed{\text{ع}}$

Deming: a process should not be adjusted in response to random variation.

مثلاً بالتجربة السابقة pull off force الناتج

الي طلع هو 12 وهو بدو 13 فنزود الـ connecter

عشان يوصل لـ 13 . هيك غلط حسب Deming

sample space: set of all possible outcome of experiment $\underline{\underline{S}}$

Sample space
- Discrete
  - finite
  - countable infinte
- continuous interval
  - finite
  - infinte

event: subset of the sample space $\underline{\underline{E}}$

$\underline{\underline{E}} < \underline{\underline{S}}$

elementary event : points of sample space
<u>one</u> outcome

example: حجر نرد

$$S = \{\underbrace{1,2,3}_{\text{Event}},4,5,6\}$$

- elementary event

flibing coin : $S = \{H, T\}$

Discrete (finite)

3 machined part, each is classified of either above or below specifications
a , b
above below

S = { ~~a,a,a~~ aaa , aab , aba , baa , bba ,
abb , bab , bbb }

فكرة التجربة نكمل اختبار ال 3parts مع بعض

outcome = $\boxed{3}$ results

مثلاً aaa : 1 out come

عدد الكلي : 8

Discrete (finite)

نقدر نعدهم و نحصرهم

ex: the number of views recorded at high volume website in a day:

التجربة هي بسجل عدد الي زاروا الموقع خلال يوم

S = { 1, 2, 3, ---, n }
very large

Discrete (countable infinte)

كبير بس بنقدر نحصرهم / نعدهم

يعني وحدة بس (selecting a ball) from urn which has 3 balls green, blue, red

$S = \{G, b, r\}$

rain precipitation = $S = R^+$ real numbers

أو $S = \{x \mid x > 0\}$

continuous infinte

لأن مافي limit

لو عملنا نسبة تقرن 50" : $S = \{x \mid 0 < x < 50''\}$

continuous finite لأنها محدودة (50)

إذا ما أمطرت أو في امكانية ماتمطر ندخل $\boxed{\text{مساواة}}$

two connectors wire : $S = R^+ \otimes R^+$

لأن فيه connector لأن طول الواير حـ - الأول ممكن ياخذ كل الأطوال المحتملة لل connector للثاني

if we are interested in showing if the connector is confirming or not : yes or no $S = \{yy, yn, ny, nn\}$

Discrete finite 1 1

interested of number of confirming:

$S = \{0, 1, 2\}$

measuring thickness until first fail

مثلاً نشتغل على خط انتاج ونفحص parts ونفحص طولهم ونشوف إذا confirm أو لا

$S = \{n, yn, yyn, yyyn, \dots\}$

Discrete countable infinte

مابنعرف متى حيوقف

Graphical representation of sample using tree diagram

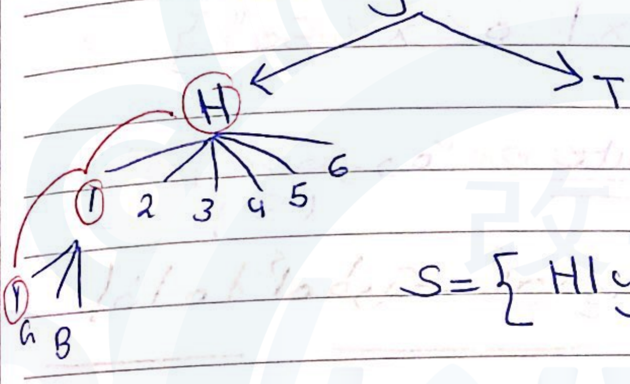
→ when the experiment is done in several steps

n_1 ways of first step → n_1 branches of the tree

n_2 ways of second step → n_2 branches of the tree

Tossing a coin, rolling a dice and choosing a ball from urn (Y G B)

Tossing a coin



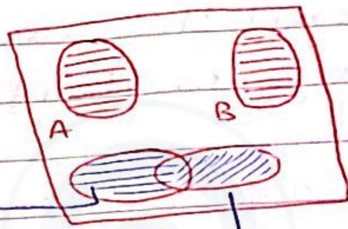
$$2 \times 6 \times 3 = \boxed{36}$$

$S = \{ H1y, H1G, H1B, H2y, H2G, H2B, \ldots$

Basic of set theory :

(1) two events $\underline{A}$ and $\underline{B}$
(venn diagram)

(2) $A \cup B \Rightarrow$ or
<u>union</u>



$A + B - A \cap B$

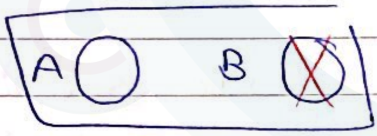
مساحة مشتركة تحسب مرة واحدة
لما ماتحسبت المساحة المتبقية من B

(3) $A \cap B \Rightarrow$ and
intersection

(4) $\phi \Rightarrow$ empty set { }

(5) $A^c \Rightarrow A' \Rightarrow$ complement "not"
any outcome not in $\underline{A}$

if $A \cap B = \phi = \{ \} \Rightarrow$ A ~~anb~~ and B mutually exclusive or (disjoint) مافي بينهم شي مشترك



: share outcome(s) لو حكينا مافي أي
وطلع في $\underline{A}$ معناه حتماً $\underline{B}$
طلعت من الحسبة

Digital scale provides weight to the (nearest gram)

non negative integrams from zero

أحداث events

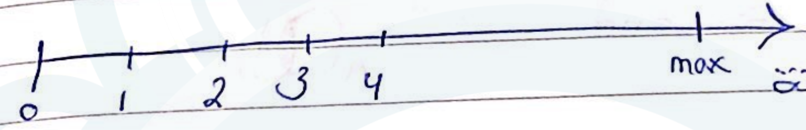
A : event that a weight exceeds 11 gram

B : event that a weight is less than or equal to 15 gram

C : event that a weight is greater than or equal to 8 gram and less than 12 gram

S = non negative integars from 0 to the largest integar that can be displayed by the scale

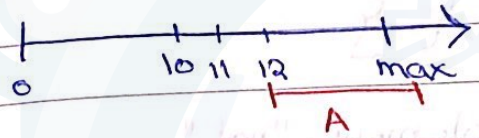
S



discrete (finite)

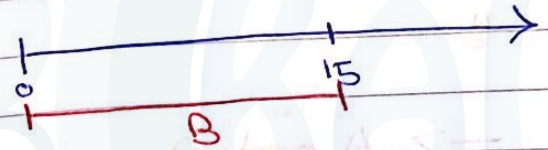
لأنو فيه max معينة

A =



$x > 11$ exceeds means 11 not included

B =



$x \le 15$ included

C =

$8 \le x < 12$ أو $8 \le x \le 11$

(1) $A \cup B$: S

A تبدأ من 12 لل max

B تغطي المنطقة اللي مغطيها A

(2) $A \cap B$: $\{x \mid 12 \le x \le 15\}$ أو $\{12, 13, 14, 15\}$

(3) A' : not included in A

$\{x \mid x \le 11\}$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(4) $A \cup B \cup C$: $A \cup B = S$ $B \cup C =$ ما اتضاف شي جديد
$\underline{S}$

(5) $(A \cup C)' = S - (A \cup C)$
$A \cup C$ في not included كل اللي
$\underline{A}$ غطت من $\underline{12}$ لل max , $\underline{C}$ غطت من $\underline{8}$ لل $\underline{11}$
بتضل من صفر لل $\underline{8}$

$\{S - x \mid x \geq 8\}$
أو $\{S - x \mid x < 8\}$
أو $\{S - x \mid x \leq 7\}$
أو $\{0, 1, 2, 3, 4, 5, 6, 7\}$

(6) $A \cap B \cap C = \phi$ ما في بينهم شي مشترك

(7) $B' \cap C = \phi$
not included in B الأرقام الأكبر من $\underline{15}$
ما في شي مشترك من بعد $\underline{15}$ مع $\underline{C}$

(8) $A \cup B \cap C = \{x \mid x \geq 8\}$
$B \cap C = \{8, 9, 10, 11\}$

counting techniques : طرق العد
number of outcome

(1) multiplication rule
→ operation with $\underline{k}$ steps
→ each step is done in a number of ways
$step_1$ with n_1 $step_2$ with n_2

Total number always $= n_1 \times n_2 \times \text{---} \; n_k$

draw a marble twice from a urn with replacement

| G |
| B |
| R |

first draw $\Rightarrow$ 3 choices
second draw $\Rightarrow$ 3 choices
number of choices $= 3 \times 3 = \boxed{9}$

the same example without replacement

$3 \times 2 = \boxed{6}$

permutations : order of items arangment is important

تباديل

Green red $\neq$ red Green

first second third

$n \times (n-1) \times (n-2) \times \ldots \times (n-k+1)$

$P^n_k = \frac{n!}{(n-k)!}$

$0! = 1$
$1! = 1$

5 student to select (president) (vice president) (secretary) ABCDE

دي احتار 3 طلاب من أول 5 فكل ما احتار طالب نقل العدد

اضربنا أول طالب

$\underline{5}\ \underline{4}\ \underline{3} = 5 \times 4 \times 3 = \boxed{60}$

order important so it's permutation

$P^5_3 = \frac{5!}{(5-3)!} = \frac{120}{2} = \boxed{60}$

iBC

(3) combination : the number of ways of selecting $\underline{\underline{r}}$ object from $\underline{\underline{n}}$ without regard to order

– order is not important

$$C^n_r = \frac{n!}{r!(n-r)!}$$

$\underline{\underline{4}}$ cars (hon) (toyata) (ford) (Bmw) number of way selecting $\underline{\underline{2}}$ of them :

HT , HF , HB , TF , TB , FB , - - -

$$C^4_2 = \frac{4!}{2!\,(4-2)!} = \frac{4\times3\times\cancel{2\times1}}{2\times1\,(\cancel{2\times1})!} = \frac{12}{2} = \boxed{6}$$

group of $\underline{\underline{7}}$ engineers , $\underline{\underline{5}}$ statician we need committee of $\underline{\underline{5}}$

how many ways to select committee

بشرط $\underline{\underline{3}}$ eng $\underline{\underline{2}}$ s :

$$\underset{7}{\underline{E}}\ \underset{6}{\underline{E}}\ \underset{5}{\underline{E}}\ \underset{5}{\underline{S}}\ \underset{4}{\underline{S}}$$: order is not important

number of committee $= \frac{7\times6\times5}{3\times2\times1} \times \frac{5\times4}{2\times1}$

$$C^7_3 \times C^5_2$$

probability: discrete sample space

continuous بنتعامل على -

likelihood or chance of outcome that will occur.

فرصة حدوث outcome معينة

P(A) is an event

P(A) = 0.6 ① student will achieve B in the class

② likelihood that any randomely selected student will achieve B is 60%

③ 60% of student will achieve B

experiment → outcome → events

probabilty: function assign to each event $A \subset S$

(means A جزء من sample space)

$P(A) \in [0,1]$ a real number

$P(A) \geq 0$

$P(S) = 1$ مجموع (probabilty) لكل outcome

if $A_1, A_2, \ldots$ are disjoint

$$P(\cup_n A_n) = \sum_n P(A_n) = P(A_1) + P(A_2)$$

لكل event

(1) Degree of belief
الاعتماد على ارقام معينة

(2) Frequency

↳ how many times the event occur in a very large number of $\underline{S}$

(3) equally likely : $\underline{\underline{N}}$ outcome

P (each outcome) $= \frac{1}{N}$

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urn contains 30 red marble, 20 green marble, 50 blue marble

what is the P (of choosing every marble)

$P(\text{every marble}) = \frac{1}{100} = 0{,}01$

$P(\text{choosing red marble}) = \frac{30}{100}$

$P(\text{red marble } 1) + P(\text{red marble } 2) + P(\text{red marble } 3)$

$\frac{1}{100} + \frac{1}{100} + \text{-----} = \frac{30}{100}$

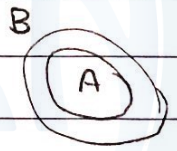
for a discrete sample space

$\underset{\text{event}}{P(E)} = \sum_{1}^{n} P(\text{all outcome in event})$

properties :

$P(\phi) = 0$

if $A \subset B = P(A) \leq P(B)$



A جزء من B

$0 \leq P(A) \leq 1$

$P(A^c) = 1 - P(A)$

if $A \cap B = \phi \implies P(A \cup B) = P(A) + P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Batch contains 6 part {a, b, c, D, E, F}
2 parts are selected randomly with replacement

what is the probability that F is in the sample ?

event يذكرون : E = F is in the sample

$P(E) = \frac{\text{number of sample contain F}}{\text{total number of possible sample}}$

$S = \{ab, ac, ad, aE, aF, bc, \ldots, bF\}$

outcome

total number of possible sample choosing 2 from 6

$C^6_2 = \frac{6 \times 5}{2} = 15$ order not important

$E = \frac{F}{6} \quad \frac{5}{\quad}$ متاحين 5 / خيارات من أصل 6 أو $1 \times C^5_1 = 5$

$P(E) = \frac{5}{15} = \frac{1}{3}$

ex 65: Nichel battery, charge state and probability as the following:

charge	prob
0	0.17
+2	0.35
+3	0.33
+4	0.15

what is the probability (P) at least one of the positive charges:

$E = \{+2, +3, +4\}$

$P(E) = P(+2) + P(+3) + P(+4)$
$0.35 + 0.33 + 0.15 = 0.83$

what is the probability that a cell has not a nichel charge greater than +3

$E = \{\text{charge is not greater than } +3\}$

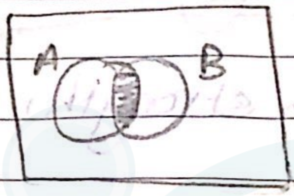
$E = \{\text{charge is less than or equal } (\leq) +3\}$
$\{0, +2, +3\}$

$P(E) = 0.85$ $\quad 0.17 + 0.35 + 0.33 = 0.85$

Addition rule : A and B

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

مرة اجمع A   مرة نجمع B   نطرح المنطقة المشتركة



نعتبرها مثل $\rightarrow A \cup (B \cap A^c)$

- probability of raining today : 20%
- " " " tomorrow : 30%
- " " " today + tomorrow : 6%

probability of raining today or tomorrow ?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

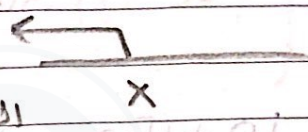
A → today, B → tomorrow

$$= 0.2 + 0.3 - 0.06 = \boxed{0.44}$$

$\{\text{at least } x\} = x \text{ or more}$

$\{X \geq x\}^c$

متممة



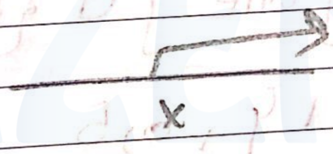
الأقل من $\leq$

$\{\text{at least } x\}^c = \{\text{less than } x\}$

$\{\text{at most } x\} = \{x \text{ or less}\}$

وما قبلها

$\{X \leq x\}^c$



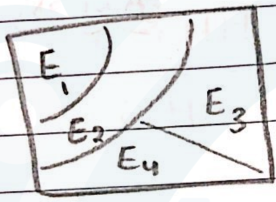
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$E_1, E_2, ----, E_k$

↳ collection of mutually exclusive event

$$P(E_1 \cup E_2 \cup E_3 \cup --- \cup E_k) = P(E_1) + P(E_2) + P(E_k)$$

$$= \sum_{i=1}^{k} P(E)$$



E_1, E_2, E_3, E_4

↳ mutually exculsive

$E_i \cap E_j = \phi$ for all i and j

$E_1 \cup E_2 \cup E_3 \cup E_4 = S$

conditional probability:

when the probability change as additional information becomes available = conditional probability

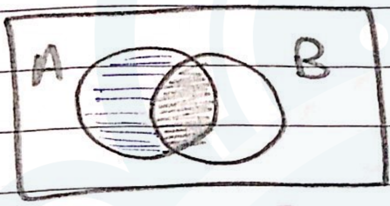
knowledge of the occurrence of an event A will change the probability of other event B

$P(B | A)$

given

narrowing a sample space upon happening of an event.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$



$A = \{1, 3, 5\}$

$B = \{3, \cancel{4}, \cancel{6}\}$

اذا صارت A
تلاقي B
الجزء الي بين
المشترك مع A

$P(A) = \frac{3}{6} = 0{,}5$

$P(B) = 0{,}5$

$P(A \cap B) = \frac{1}{6}$ مع رقم واحد بس 3

$$P(B|A) = \frac{\frac{1}{\cancel{6}}}{\frac{3}{\cancel{6}}} = \frac{1}{3}$$

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Fliping 2 coins $\Longrightarrow$ {HH, HT, TH, TT}

E_1 : at least one head = {HT, TH, HH}

E_2 : 2 heads = {HH}

what is the P that 2 heads are tossed giving that at least one head is tossed :

$$P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)}$$

$$\frac{P\{HH\}}{P\{HH, HT, TH\}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

100 disk are analysed for shock and scratch resistance

		shock resistance H	shock resistance L
scratch resistance	H	70	9
	L	16	5

A : event disk has high shock resistance

B : event disk has high scratch resistance

$$P(A) = \frac{70+16}{100} \qquad P(B) = \frac{70+9}{100}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{70/100}{\frac{70+9}{100}} = \frac{70}{79}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{70}{86}$$

- multiplication and total prob rule :-

$$P(A\cap B) = P(A|B) \times P(B) = P(B|A) \times P(A)$$

ex : 3 airlines

$A_1 \rightarrow$ airline 1

$A_2 \rightarrow$ airline 2

$A_3 \rightarrow$ airline 3

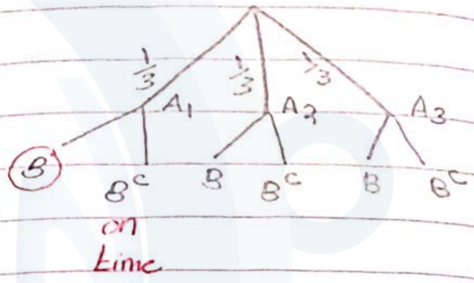
$P(A_1) = \frac{1}{3} = P(A_2) = P(A_3)$

B = late take off

$P(B|A_1) = 40\%$ $\quad P(B|A_2) = 50\%$ $\quad P(B|A_3) = 70\%$

احتمالية تأخر الطيارة بالعادة

(1) what is the prob that i selected airline 1 and I'm late taking off ?



$P(B|A_1) = 40\%$

$$P(A_1 \cap B) = P(B|A_1) \cdot P(A_1)$$

طيران

تأخير

$$= 40\% \times \frac{1}{3}$$

$$= \frac{2}{15}$$

(2) what is the prob that I'm late taking off ?

التأخير بغض النظر عن الطيارة

$P(B) = P(\text{late and airline } 1) + P(\text{late and } A_2) + P(\text{late and } A_3)$

$$= P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$$

$$\frac{2}{15} + \left(50\% \times \frac{1}{3}\right) + \left(70\% \times \frac{1}{3}\right) = \frac{8}{15}$$

(3) Given that i was late, what is the prob that i selected airline 1 ?

السؤال : في تأخير ايش احتمالية تأخرت عن طريق airline 1 ؟

$$P(A_1 | B_1) = \frac{P(A_1 \cap B)}{P(B)} = \frac{\frac{4}{30}}{\frac{8}{15}} = \frac{1}{4}$$

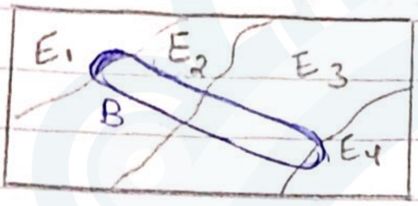
بنعرف انو في تأخير

law of total probability:

let $E_1, E_2, \dots E_4$ be a collection of mutually exclusive and collectively exhaustive events:

exclusive : disjoint : مافي شي مشترك بينهم -

exhaustive : بدخلوا كل ال samplespace اذا جمعناهم -



$E_1, E_2 \rightarrow$ mutually exclusive

$E_1, E_2, E_3 \rightarrow$ mutually exclusive

$E_1, E_2, E_3, E_4 \rightarrow$ mutually

exclusive exhaustive

$E_1 + E_2 + E_3 + E_4 = (\underline{\underline{S}})$

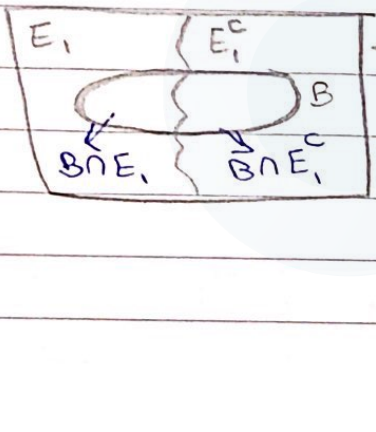
استهلكوا كل ال (sample space)

$P(B|E_1) = \dfrac{P(B \cap E_1)}{P(E_1)}$ outcome طلع بس في $\underline{\underline{E_1}}$

$P(B) = \sum_{i=1}^{n} P(B|E_i) \cdot P(E_i)$

$= P(E_1 \cap B) + P(E_2 \cap B) + P(E_3 \cap B) + P(E_4 \cap B)$

$= P(B|E_1) P(E_1) + \dots + P(B|E_4) \cdot P(E_4)$

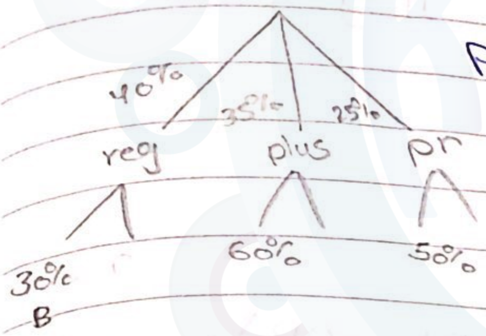


او E_2 : $P(B) = P(B|E_1) P(E_1) + P(B|E_1^c) P(E_1^c)$

at gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2), 25% use a premium gas (A_3):

(1) regular gas : 30% fill their tank B
(2) plus gas : 60%
(3) premium gas : 50%

what is the prob that the next customers will fill the tank?



$P(B) =$
$P(B|A_1)\,P(A_1) + P(B|A_2)\,P(A_2) + P(B|A_3)\,P(A_3)$

$= (0.3 \times 0.4) + (0.6 \times 0.35) + (0.5 \times 0.25)$

$= 0.455$

Independence :-

two events are said to be independent if the occurance of (first, second) event does not affect prob of another event.

$P(B|A) = P(B)$

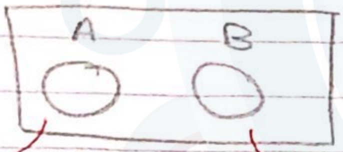
A does not affect the prob of B it's still $P(B)$

$P(A \cap B) = P(B|A)\,P(A) = P(B)\,P(A)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,\cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

for $E_1, E_2, \dots, E_n$ independent event

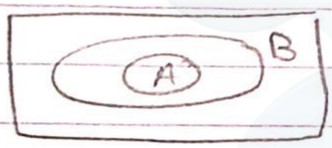
$$P\left(\bigcap_{i=1}^{n} E_i\right) = \prod_{i=1}^{n} P(E_i) = P(E_1)\,P(E_2)$$ المضاريب



: $P(A \cap B) = P(\emptyset) = 0$ they are not independent

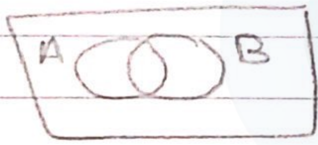
$P(A)\,.\,P(B) > 0$

dependent أو not indepedent



$A \subset B \rightarrow P(A \cap B) = P(A) \neq P(A)\,.\,P(B)$

they are dependent



they may or may not

we have to check

Independence can be :

as a assumption

ex: coin

↳ $P(H) = P(T) = \frac{1}{2}$

$P(HHHT \dots H) = \left(\frac{1}{2}\right)^k$

as a property

ex: toss a fair dice

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$

$B = \{1, 2, 3, 4\}$

dependent : B

↳ we have to check

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{2}{3}$

$P(A \cap B) = 2$ out comes 2, 4

$= \frac{2}{6} = \frac{1}{3}$

$P(A).P(B) = \frac{1}{3} = (A \cap B)$

they are (independent)

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2-126

Shock, scratch resistance example:

		shock resistance: H	L
scratch resistance:	H	70	9
	L	16	5

A: event that disk has high shock resistance

B: event that disk has high scratch resistance

are A and B independent or not?

$$P(A \cap B) = \frac{70}{100}$$

$$P(A) = \frac{86}{100} \qquad P(B) = \frac{79}{100}$$

$$P(A \cap B) \overset{?}{=} P(A) . P(B)$$

لسا ما بنعرف اذا بيتساوا أو لا

$$P(A \cap B) \neq P(A) . P(B)$$

they are not independent

$$\frac{70}{100} \neq \frac{86}{100} \times \frac{79}{100}$$

application of prob concepts:
reliability → الموثوقية

↳ prob that a system or product will perform it's intended function properly

system → series
system → parallel

(1) series system a → [1] — [2] — [3] — [4] — b
(connection) لازم يكون شغال عشان يعطينا ؟
أي واحد بصير عطله Failure بينقطع الـ connection

- each component works / fails independently than others
- system will work if there is a path

A_i : event that a component i works
P_i : prob that a component i works

$$P(\text{system to work}) = P(A_1 \text{ and } A_2 \text{ and } A_3 \text{ and } A_n)$$
$$= P(A_1 \cap A_2 \cap A_3 \dots \cap A_n)$$
$$= P(A_1)\, P(A_2)\, P(A_3) \dots P(A_n)$$
$$= P_1 \times P_2 \dots P_n$$

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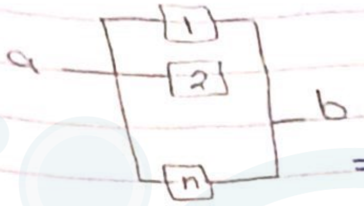
(2) parallel system



P(system works)

$= P(A_1 \text{ or } A_2 \text{ or } \cdots \text{ or } A_n)$

$= P(A_1 \cup A_2 \cup A_3 \cdots \cup A_n)$

$= 1 - P(\text{system does not work})$

$= P(A_1^c \cap A_2^c \cap \cdots \cap A_n^c)$

$P(A_1) = P_1 \quad , \quad P(A_1^c) = P_1^c$

$= (1 - P_1)(1 - P_2)\cdots(1 - P_n)$

$P(\text{system work}) = 1 - [(1-P_1)(1-P_2)\cdots(1-P_n)]$

series: $P(\text{system work}) = \prod_{i=1}^{n} P_i$

parallel: $P(\text{system work}) = 1 - \prod_{i=1}^{n} (1 - P_i)$

ex: circuit operate if there is a function path from left to the right :

as 1 component

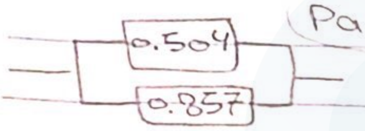
A : denotes the upper devices

B : denotes the lower devices

$P(A) = 0.9 \times 0.8 \times 0.7 = 0.504$

$P(B) = 0.95 \times 0.95 \times 0.95 = 0.8574$

Parallel



$1 - [(1 - 0.504) \times (1 - 0.8574)] = 0.9293$

$P(A \cap B) = 0.504 \times 0.8574 = 0.4321$

$P(\text{circuit operates}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.9293$

Bayes theorem:
events interms of conditional prob

- if condition was present when event occurrence:

$$P(A \cap B) = P(A|B) \cdot P(B)$$
$$= P(B|A) \cdot P(A)$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(E_j|B) = \frac{P(B|E_j)\,P(E_j)}{\sum_{i=1}^{n} P(B|E_i)\,P(E_i)}$$

ex: 2_142

$P(A|B) = 0.7$ $P(A) = 0.5$ $P(B) = 0.2$

$P(B|A) = ?$

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)} = 0.28$$

"لو فيها" $P(A) = P(A|B)\,P(B) + P(A|B^c)\,P(B^c)$

لو

	A	A^c	
B	$P(A \cap B)$	$P(A^c \cap B)$	$P(B)$
B^c	$P(A \cap B^c)$	$P(A^c \cap B^c)$	$P(B^c)$
	$P(A)$	$P(A^c)$	

chapter 3

Discrete random variable (R.V.) and prob distribution :-

R.V : function defined on the sample space
function that assigns a real number to each outcome

$X \Rightarrow x$, $Y \Rightarrow y$ قيمة معينة لل X

ex: throw 2 four-face dice:
x : sum of values of the 2 dice
$S = \{(1,1), (1,2), \dots, (4,4)\}$

1st dice \ second dice	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

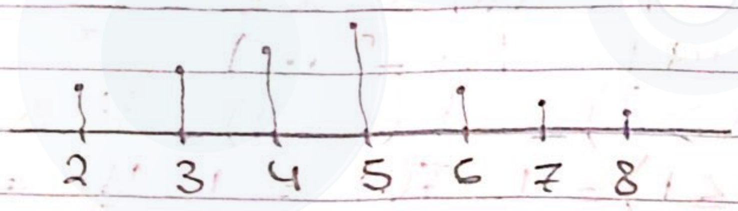
$x = \{2, 3, 4, 5, 6, 7, 8\}$

$P(x=2) = \frac{1}{16}$ تكرار / عدد كلي $P(x=3) = \frac{2}{16}$

$P(x=4) = \frac{3}{16}$ $P(x=5) = \frac{4}{16}$

$P(x=6) = \frac{3}{16}$ $P(x=7) = \frac{2}{16}$

$P(x=8) = \frac{1}{16}$



500 machine parts contains 10 (non-conforming)

R.V → number of parts in a sample of 5 parts that do not conform to customer requirement

$X = \{0, 1, 2, 3, 4, 5\}$

Discrete R.V x with possible value $x_1, x_2, x_3 \ldots x_n$

prob mass function PMF

$P(X=2) = F(2)$ لرمزلها

$F(x_i) = P(X = x_i)$

الشروط:

(1) $F(x_i) \geq 0$

(2) $\sum_{i=1}^{n} F(x_i) = 1$ مجموع Prob للقيم = ١

(3) $F(x_i) = P(X = x_i)$

3_16 : $F(x) = \frac{8}{7} \times \left(\frac{1}{2}\right)^x \quad x = 1, 2, 3$

$F(1) = P(x=1) = \frac{8}{7} \times \left(\frac{1}{2}\right)^1 = 8/14$ 8/14

$F(2) = P(x=2) = \frac{8}{7} \times \left(\frac{1}{2}\right)^2 = 4/14$ 4/14

$F(3) = P(x=3) = \frac{8}{7} \times \left(\frac{1}{2}\right)^3 = 2/14$ 2/14

$\sum_{i=1}^{3} F(x_i) = \frac{8}{14} + \frac{4}{14} + \frac{2}{14} = \frac{14}{14} = 1$ PMF

(1) $P(X \leq 1) = P(x=1)$: $F(1)\ \frac{8}{7} \times \frac{1}{2} = \frac{4}{7}$

(2) $P(X > 1) = P(x \geq 2) = P(x=2) + P(X=3)$

or : $1 - P(x \leq 1) = 1 - F(1) = 1 - \frac{4}{7} = \frac{3}{7}$

تحقق الشرط

(3) $P(2 < x < 6) = P(3 \le x \le 5)$

$F(3) = 0.1428$

(4) $P(x \le 1 \text{ or } x > 1) = 1$

$= P(x = 1) + F(2) + F(3)$

Cumulative distribution function CDF

Find prob of having a value of the random V

less or equal ($\le$) to specific value

$P(X \le x) = F(x) = \sum_{x_i \le x} f(x)$

x_1, x_2, x_i : mutually exclusive

$\rightarrow P(X = x_i) = P(X \le x_i) - P(x \le x_{i-1})$

2 3 4

3-35 :

x	-2	-1	0	1	2
f(x)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{2}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

(1) is this a PMF ?

نجمعهم كلهم : $\frac{8}{8} = 1$

CDF ?

$F(-2) = P(X \le -2) = P(x = -2) = f(-2) = \frac{1}{8}$

$F(-1) = P(X \le -1) = P(= -2) + P(x = -1)$

$P((x = -2) \cup (x = -1))$ $f(-2) + f(-1) = \frac{3}{8}$

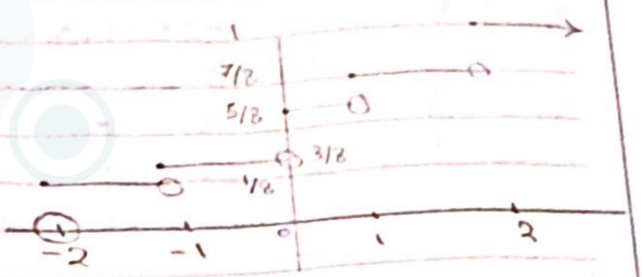
$F(0) = P(X \le 0) = f(-2) + f(-1) + f(0) = \frac{5}{8}$

$F(1) = P(X \le 1) = f(-2) + f(-1) + f(0) + f(1) = \frac{7}{8}$

$F(2) = P(X \le 2) = f(-2) + f(-1) + f(0) + f(1) + f(2) = \frac{8}{8}$

3-35 :

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{8} & -2 \le x < -1 \\ \frac{3}{8} & -1 \le x < 0 \\ \frac{5}{8} & 0 \le x < 1 \\ \frac{7}{8} & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



(1) $P(X \le 1.25) = \frac{7}{8}$

(2) $P(X \le 2.2) = 1$

(3) $P(-1.1 < X \le 1) = \frac{7}{8} - \frac{1}{8} = \frac{6}{8}$

(4) $P(X > 0) = 1 - P(X \le 0) = 1 - \frac{5}{8} = \frac{3}{8}$

3_42 :

$$F(x) = \begin{cases} 0 & x < \frac{1}{8} \\ 0.2 & \frac{1}{8} \le x < \frac{1}{4} \\ 0.9 & \frac{1}{4} \le x < \frac{3}{8} \\ 1 & \frac{3}{8} \le x \end{cases}$$

(1) $P(X \le \frac{1}{18})$: PMF : $F(\frac{1}{8}) = 0.2$

$F(\frac{1}{4}) = 0.9 - 0.2 = 0.7$

$F(\frac{3}{8}) = 1 - 0.9 = 0.1$

$\frac{1}{18} \leftarrow 0$ أقل من الصفر

(2) $P(X \le \frac{1}{4}) = 0.9$

(3) $P(X \le \frac{5}{16}) = 0.9$ $\quad > 0.3125$

(4) $P(X > \frac{1}{4}) = 1 - P(X \le \frac{1}{4}) = 0.1$

(5) $P(X \le \frac{1}{2}) = 1$

mean and variance and discrete variable 2

measur used to summarize of data Prob distribution of R.V

the mean and expected value of adiscrete r.v x denoted as μ or expected value (X)

$\rightarrow \mu \; E(x) = \sum_{x} x F(x)$

Central tendency of distribution :-

variance of x, denoted as σ^2 or $V(x)$

$\sigma^2 = V(x) = E\left[(x-\mu)^2\right] = \sum (x-\mu)^2 F(x)$

$= \sum_{x} x^2 F(x) - \mu^2$

$V[x] = E[x^2] - (E[x])^2$

$(x-\mu)^2 = x^2 - 2\mu x + \mu^2$

$\sum (x-\mu)^2 F(x) = \sum x^2 F(x) - 2\mu \sum x F(x) + \mu^2 \sum F(x)$

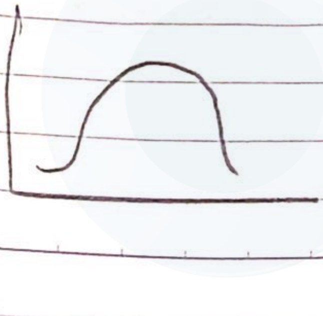
$= \sum x^2 F(x) - 2\mu^2 + \mu^2$

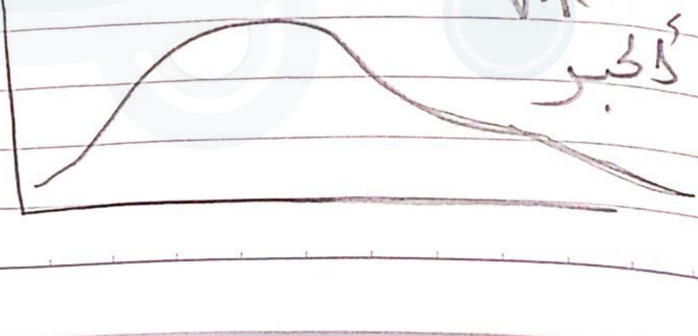
$\sum x^2 F(x) - \mu^2$

variance is ameasure of dispersion or scatter in the value of x

↑ V أكبر

مبعثر أكثر





V↑

أكبر

3-48

outcome	a	b	c	d	e	F
x	0	0	1.5	1.5	2	3
F(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$F(0) = \frac{2}{6}$ $\quad F(1.5) = \frac{2}{6}$ $\quad F(2) = \frac{1}{6}$ $\quad F(3) = \frac{1}{6}$

$\mu = E(x) = 0 \times \frac{2}{6} + 1.5 \times \frac{2}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6}$
$= 1.333$

$V(x) = \sum x^2 F(x) - \mu^2$
$(0)^2 \times \frac{2}{6} + (1.5)^2 \times \frac{2}{6} + (2)^2 \times \frac{1}{6} + (3)^2 \times \frac{1}{6} - (1.333)^2$
$= 1.139$ $\underline{\underline{\text{unit}^2}}$ حسب x

unit of measure of variance is the $\underline{\underline{\text{R.V}}}$ unit square
standard deviation of a R.V
$\sigma = \sqrt{\sigma^2} = \sqrt{V(x)} = \sqrt{1.139} \longrightarrow 1.0067$

expected value of a function R.V x with $\underline{\underline{\text{PMF}}}$ F(x)
$h(x) \Longrightarrow$ Function
$E[h(x)] = \sum h(x) F(x)$
x: time in hours to produce part
$h(x) = 0.5x + 25$ = cost of production
variance $= [E(x-\mu)^2]$

2 product designs to be evaluated based on profit

Design A revenue 3 million

Design B revenue 7 million with prob = 0.3

2 million with prob = 0.7

prob of (x = 3million) = 1

Design B

x	7	2
$F(x)$	0.3	0.7

profit is 0.4 of revenue $h(x) = 0.4x$

Design A : $E(h(x)) = (0.4 \times 3) \times 1 = 1.2$ million

Design B : $E(h(x)) = (0.4 \times 7) \times 0.3 + (0.4 \times 2) \times 0.7$

$= 1.4$ million

$\sigma_B^2 = (0.4 \times 7 - 1.4)^2 \times 0.3 + (0.4 \times 2 - 1.4)^2 \times 0.7$

$= 0.84 \rightarrow \sqrt{0.84} = 0.9165$

$\sigma_A = 0 \rightarrow (0.4 \times 3 - 1.2)^2 \times 1 = 0$

rules for $E(x)$ and $V(x)$

$Y = ax + b$ a, b const

$\mu_y = E[Y] = E[ax+b] = \sum (ax+b) F x$

$\sum_x ax\, F(x) + \sum_x b\, F(x) \rightarrow a \underbrace{\sum x F(x)}_{\mu_x = E[x]} + b \underbrace{\sum F(x)}_{1}$

$= a\, E[x] + b$, $\mu_y = a\mu_x + b$

$V[y] = E[(y - \mu_y)^2] = E[((ax+b) - (a\mu_x + b))^2]$ $(ax + \cancel{b} - a\mu_x - \cancel{b})$

$= E[(a(x - \mu_x))^2] = E[a^2 (x - \mu_x)^2] = a^2 E[(x - \mu_x)]$

$= a^2 V[x]$

$Y = \sum_{i=1}^{n} a_i x_i$

$E[Y] = \sum a_i E[x_i]$

$V[Y] = \sum a_i^2 V[x_i]$

IBC

Common discrete distribution :

→ Discrete uniform dist

equal prob for each of the finite $\underline{n}$ x- values

مجرد نرد

$$F(x_i) = \frac{1}{n}$$

$$\underset{a}{1} \quad 2 \quad 3 \quad 4 \quad 5 \quad \underset{b}{6}$$

For a range of a, $a+1$, ---- b

أول قيمة لل x

$$\frac{1}{6-1+1} = \frac{1}{6}$$

range $b - a + 1$

$$\text{prob} = \frac{1}{b-a+1}, \quad \mu = E(x) = \frac{b+a}{2}$$

$$\sigma^2 = \frac{(b-a+1)^2 - 1}{12}$$