Events

* commutative laws

$A \cup B = B \cup A$, $A \cap B = B \cap A$

* De Morgan's laws:

$(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

* distributive law:-

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$, $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

counting Techniques:-

* multiplication Rule:

if the operation can be discribed in steps(K)

$n_1 * n_2 * n_3 * \cdots * n_K$

# of ways ~~of~~ completing step 1 ...

* combinations: (order isn't important)

$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

* permutations (order is important)

perm of subset:

$P_K^n = \frac{n!}{(n-K)!}$

perm of similar object:

$P_K^n = \frac{n!}{n_1!\, n_2! \cdots n_K!}$

Baye's Theorem

* $P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}$

* $P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots$

probability:-

For two events with $E_1 \cap E_2 = \emptyset$ mutually exclusive

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$P(E^c) = 1 - P(E)$

Addition Rules:-

* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

* $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

* if A, B, C mutually exclusive $\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C)$

conditional prob:-

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

multi Rule:

* $P(A \cap B) = P(B|A)\, P(A)$
$= P(A|B)\, P(B)$

total prob: (two events):

* $P(B) = P(B \cap A) + P(B \cap A')$
$= P(B|A)P(A) + P(B|A')P(A')$

* $P(B) = P(B \cap E_1) + P(B \cap E_2) + \cdots$
(multiple events)

independence

2 events are independent if:

* $P(A|B) = P(A)$
* $P(B|A) = P(B)$
* $\boxed{P(A \cap B) = P(A)P(B)}$

multiple events $\Rightarrow P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3)$

parallel and series

* series

P(system to work)
$= \prod_{i=1}^{n} P_i$

* parallel

P(system work) $= 1 - \prod_{i=1}^{n} (1 - P_i)$

$P(C) = P(C|A \cap B)\, P(A \cap B)$
$P(C|A \cap B)\, P(B|A)\, P(A)$

1 prob mass Function (pmF)

* $F(x_i) \geq 0$
* $\sum_{i=1}^{n} F(x_i) = 1$
* $F(x_i) = P(X = x_i)$

2 cumulative dist. Function (CDF)

* $F(x) = P(X \leq x) = \sum_{x_i \leq x} F(x_i)$
* $0 \leq F(x) \leq 1$
* $\boxed{P(X = x_i) = F(x_i) - F(x_{i-1})}$
  $= \boxed{P(X \leq x_i) - P(X \leq x_{i-1})}$
* $P(X > x) = 1 - P(X \leq x)$

Ex:- $P(1 \leq X \leq 2)$
$= P(X \leq 2) - P(X \leq 1)$
$= F(2) - F(1)$

3 mean & variance

* $\mu = E(X) = \sum_x x F(x)$
* $\sigma^2 = V(X) = E(X - \mu)^2 = \sum (x - \mu)^2 F(x)$
* standard deviation $\Rightarrow \sigma = \sqrt{\sigma^2}$
* Expected value of Function $\Rightarrow E(h(X)) = \sum_x h(x) F(x)$
* if $Y = aX + b$
  $\hookrightarrow \mu = E[Y] = aE[X] + b$
  $V[Y] = a^2 V(X)$

4 Discrete uniform dist.

* $F(x_i) = \frac{1}{n}$
* $a \leq x \leq b \Rightarrow \mu = E(X) = \frac{b+a}{2}$
* $\sigma^2 = \frac{(b-a+1)^2 - 1}{12}$

5 Bernoulli trials:

(1) independent
(2) two outcome.
(3) p constant

binomial dist.

* $F(x) = \binom{n}{x} p^x (1-p)^{n-x}$

total # of trials.

# of success

$x = 0, 1, \dots, n$

* $\mu = np$
* $\sigma^2 = np(1-p)$

6 geometric dis → # of trials until the First success

* $F(x) = (1-p)^{x-1} (p)$
  $\boxed{x = 1, 2, \dots}$
* $\mu = E(X) = \frac{1}{p}$
* $\sigma^2 = \frac{(1-p)}{p^2}$

7 Negative Bin. dist.

total # of trials

* $F(x) = \binom{x-1}{r-1} (1-p)^{x-r} \cdot (p)^r$

# of success

$x = r, r+1, r+2, \dots$ / $r = 1, 2, 3, \dots$

* $\mu = \frac{r}{p}$
* $\sigma^2 = \frac{r(1-p)}{p^2}$

8 Hyp. dist.

* $F(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

K

x

$x = \max\{0, n+K-N\}$ to $\min\{K, n\}$

X = # of successes in the sample

* $\mu = np$
* $V(X) = np(1-p)\left(\frac{N-n}{N-1}\right)$
* Finite population Correction Factor $= \frac{N-n}{N-1}$
* $\boxed{P = \frac{K}{N}}$

9 poisson dist.

* $F(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$x = 0, 1, 2, \dots$

* $\mu = E(x) = \boxed{\lambda}$
* $V(X) = \lambda$

$\boxed{\sigma = \lambda}$

* $\lambda = np$

$p = \frac{\lambda}{n}$

$\boxed{\lambda_{new} = \lambda_{old} * t}$

Continuous Random Variables

* is a random variable with an interval (either Finite or inFinite) of real numbers For its range.

prob. Density Function (PDF)

* 1) $F(x) \geq 0$
  2) $\int_{-\infty}^{\infty} F(x)\,dx = 1$
  3) $P(a \leq X \leq b) = \int_a^b F(x)\,dx$
     = area under $F(x)$ From "a" to "b"

* $p(x_1 \leq X \leq x_2) = p(x_1 < X < x_2)$
* $p(X = x) = 0$

Cumulative dist. Function (CDF)

* $F(x) = p(X \leq x) = \int_{-\infty}^{x} F(u)\,du$
* $p(a < X < b) = F(b) - F(a)$

Mean & variance

* $\mu = E(X) = \int_{-\infty}^{\infty} x\,F(x)\,dx$
* $\sigma^2 = V(x) = \int_{-\infty}^{\infty} (x-\mu)^2 F(x)\,dx = \boxed{\int_{-\infty}^{\infty} x^2 F(x)\,dx - \mu^2}$
* standard deviation $= \sigma = \sqrt{\sigma^2}$
* Expected value of Function:-
  $E(h(x)) = \int_{-\infty}^{\infty} h(x) F(x)\,dx$

continuous uniform dist

* $F(x) = \frac{1}{b-a}$, $a \leq x \leq b$
* $\mu = \frac{(a+b)}{2}$ * $\sigma^2 = \frac{(b-a)^2}{12}$
* CDF of a continuous uniform random variable is:-

$$F(x) = \begin{cases} 0 & x < a \\ \frac{(x-a)}{(b-a)} & a \leq x < b \\ 1 & b \leq x \end{cases}$$

* pdF $\Rightarrow$ $F(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$

Normal Distribution:-

$X \sim N(\mu, \sigma^2)$

* $P(\mu - \sigma < X < \mu + \sigma) = 0.6827$
  $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9545$
  $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$
* $P(X > \mu) = P(X < \mu) = 0.5$
* Standard normal R.V:-
  $\mu = 0$, $\sigma^2 = 1$
  $Z = \frac{X - \mu}{\sigma}$
* CDF $\Rightarrow$ $\Phi(z) = P(Z \leq z)$
* standardizing:-
  $P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right)$
  $= P(Z \leq z)$

Exponential distribution

* $F(x) = \lambda e^{-\lambda x}$, $0 \leq x < \infty$
  $\lambda > 0$
* $\mu = \frac{1}{\lambda}$, $\sigma^2 = \frac{1}{\lambda^2}$
* CDF $\Rightarrow$ $F(x) = 1 - e^{-\lambda x}$ $x \geq 0$

$F(x) = P(X \leq x)$

* lack of memory property:
  $p(X < t_1 + t_2 \mid X > t_1) = p(X < t_2)$