

Subject Logistics :: video 1

الموضوع

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➔ Costs in Logistics system:

* Four essential types of costs

1. transportation costs. نقل
2. handling costs تفريغ/تحميل
3. holding costs → Cost of Capital
opportunity cost
4. Facility rent cost

معايير التقييم

* Normalizing the cost ↗

- cost per time period.
- " " item

Comparing the cost of carriers or modes

- **objective** :- minimize total cost when selecting mode of transportation.

➔ often requires trading off inventory and transportation.

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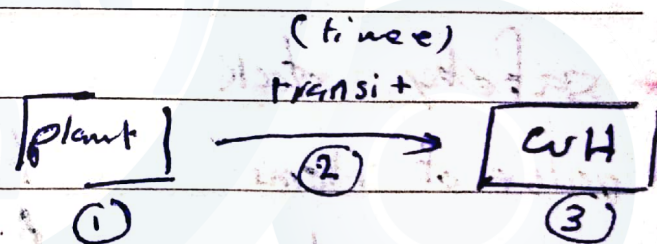
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→ Inventory costs, 2 types:

- 1) holding / cost of capital
 - 2) rent cost for space.
- i.e. f (value held, invested).

→ Inventory held in 3 places:

- 1) at plant
- 2) at warehouse
- 3) in transit



problem:

A company needs to select between 2 transportation mode, (rail / truck), to move goods from plant to WH, with obj of minimizing total transportation ~~cost~~ and Inventory costs on an annual bases:

Data:**plant****WH**

* safety stock

0 demand 100 units

* value of items

\$ 500/unit

Free cost, \$ (512)/unit

* space cost

\$ 20/unit.year

\$ 25/unit.year

* **Rail**:

Car capacity = 400 units

transit time = 20 days

cost / shipment = 1200 \$

* **truck**:

truck capacity = 100 units

transit time = 3 days

cost / shipment = 700 \$

1

1

1

1

التاريخ:

قرآن بتع باليوم

* production rate at plant = 10 units/day

Demand; or $W.H. = 10$ units / day.

inventory holding cost rate: 25% of value/yr. hdd.

Goal:- to determine the optimal shipment size $\rightarrow q$ (# of units); For each mode, and the total annual cost associated with their shipment sizes.

Assumption: the shippers will never ship more than a single car load (Rail) or a single truck load (truck).

steps:-

✓ ~~✓~~ Determine annual inventory space cost at the plant

2// Determine annual inventory holding cost at the plant

3// Determine annual inventory space cost at the WH.

4// Determine annual inventory holding cost at the WH.

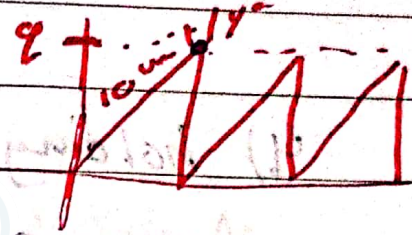
5// Determine intransit inventory cost (Rail / truck)

6// Determine transportation cost (Rail / truck)

7// total annual cost expression. (Rail / truck)

8// Determine optimal q for each mode.

9// choose a mode.

Solution:

1) space cost at plant

$$\text{space cost} = \frac{\$20}{\text{unit} \cdot \text{year}} \times 9 \text{ units} = \$20.9/\text{year}$$

2) holding cost at plant

rate of charge per item of inventory held at plant for a year

$$= \$500 \times 25\% = 125 \$/\text{unit} \cdot \text{year}$$

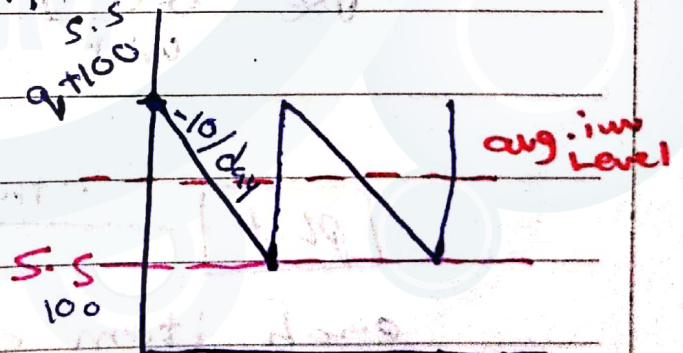
$$\text{Avg inventory level} = \frac{9}{2}$$

$$\text{So, } (125 \$/\text{unit} \cdot \text{year}) \times \frac{9}{2} \text{ units} = 62.59 \$/\text{year}$$

3) space at wh

 $\Rightarrow$ max inv level

$$\text{max inventory level} = 9 + 100 \text{ unit}$$



space cost =

$$(\$25/\text{unit} \cdot \text{year}) (9 + 100) \text{ units}$$

$$= 259 + 2500 \$/\text{year}$$

4) holding cost at wh

Average inventory level

$$\text{Aug. inv level} = \frac{100 + 9}{2} \text{ unit}$$

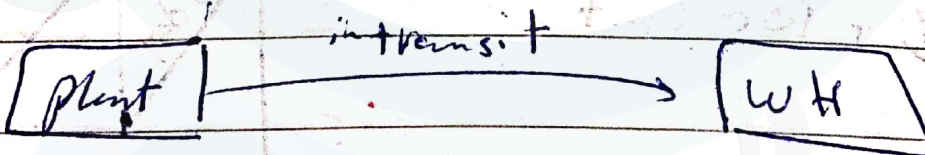
each item held on yr charged at rate of 512 * 25% = 128 \$ / unit / yr

$$\text{holding cost} = \frac{128}{\text{unit. yr}} \times \left(100 + \frac{9}{2}\right) \text{ unit}$$

$$= (12800 + 649) \$ / \text{yr}$$

5) Determin in transit inventory cost :

$$\text{use } \frac{506 \$}{\text{unit}} = \frac{500 + 512}{2}$$



each item charged at $\frac{506 \$}{\text{unit}} \times 25\% / \text{yr}$

$$= \$126.5 / \text{unit. yr}$$

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Rail $\Rightarrow$

each item spends 20 days.

* amount to be shipped in whole
year = production Rate $\times$ period

$$= \frac{10 \text{ unit}}{\text{day}} \times \frac{365 \text{ day}}{\text{yr}} = 3650 \text{ unit/year}$$

Intransit cost = amount shipped yearly $\times$
intransit time $\times$ intransit

holding cost

$$= \frac{3650 \text{ unit}}{\text{yr}} \times 20 \text{ days} \times \frac{1 \text{ yr}}{365 \text{ day}}$$

$$\times \frac{126.5 \$}{\text{unit} \cdot \text{yr}} = 25300 \$/\text{year}.$$

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Truck !:

In transit cost of truck

$$= 3650 \text{ unit} \times \frac{3 \text{ day}}{\text{yr}} \times \frac{1 \text{ yr}}{365 \text{ day}}$$

$$= 126.5 \text{ \$} \times \frac{1 \text{ yr}}{\text{unit}} = 3795 \text{ \$/yr}$$

⑥ Transportation cost:

Rail $\Rightarrow$ every shipment will cost \$1200

$$\# \text{ of shipment} = \frac{3650}{q}$$

$$\text{transportation cost} = \frac{\$1200}{\text{shipment}} \times \frac{3650}{q} \frac{\text{shipment}}{\text{yr}}$$

$$= \frac{4380000}{q} \$/\text{yr}$$

Truck $\Rightarrow$ every shipment will cost \$700

$$\# \text{ of shipment} = \frac{3650}{q}$$

$$\text{transportation cost} = \frac{700}{\text{shipment}} \times \frac{3650}{q} \frac{\text{shipment}}{\text{yr}}$$

$$= \frac{2555000}{q} \$/\text{yr}$$

⑦ Total annual cost

$$\begin{aligned} \text{TCR}(q) = & 20q + 62.5q + 25q + 2500 + \\ & 64q + 12800 + 25300 \\ & + \frac{4380000}{q} = \end{aligned}$$

$$= 171.5q + 15600 + \frac{4380000}{q}$$

inventory cost
transit cost

ABC

Truck $\Rightarrow$

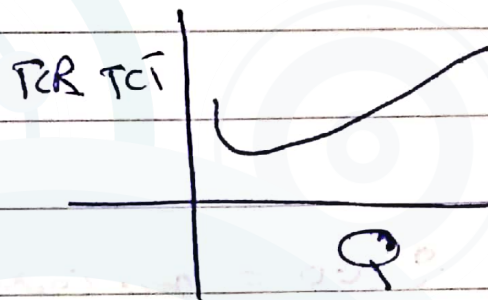
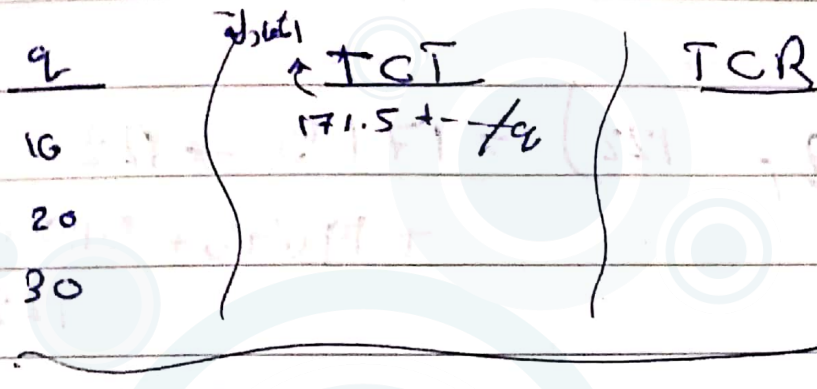
$$TCT_{(q)} = 20q + 62.5q + 25q \\ + 2500 + 64q + 12800$$

$$+ 3795 + \underline{2555000}$$

$$= 171.5q + 19095 + \underline{2555000}$$

setup on excel

وہی چیز جو
ہر چیز کا قیمت



8) to find the optimal q for each mode

المعادلة

$$\textcircled{1} TCR'(q) = 171.5 + 0 - \frac{4380000}{q^2} = 0$$

$$q_R = 160 \text{ units}$$

$$\text{when } q_R = 160 \Rightarrow TCR = \$95415/\text{yr}$$

$$\textcircled{2} TCT'(q) = 171.5 + 0 - \frac{2555000}{q^2}$$

$$q(T) = 122 \text{ units}$$

منه في مشكلة الـ q الـ TCR
يكون بين 160 بالسنة
في 2 فترات
~~الـ q الـ TCT~~

$$\rightarrow \text{shipment cost} = 2 \times \frac{700\$}{\text{ship}} \times \frac{3650}{2}$$

$$= \frac{5110000}{2}$$

ABC

$$TCT(q_T = 122) = 171.5 \times 122 + 19095 + \frac{5110000}{122}$$

$$= 81,903$$

100 = max capacity was selected

* if maximum capacity was selected

$$(TCT_{q_T=100}) = 171.5 \times 100 + 19095 + \frac{2555000}{2}$$

$$TCT(q) = \$61,795$$

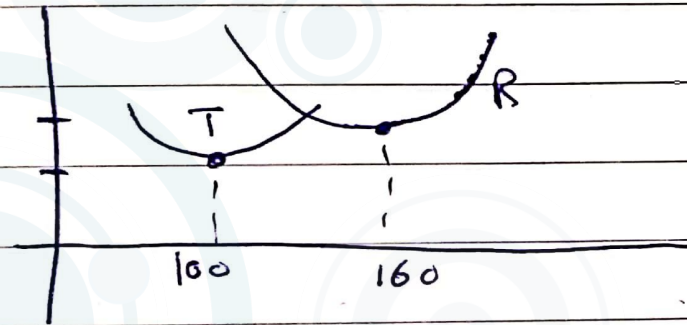
↳ The min possible cost

$$(TCT_{q=99}) = 61,881.584$$

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If you want to draw the two functions



	shipment	inventory cost	Transp cost	Total
<u>Rail</u>	160	- 68040	- 27375	95415
	4100	ج.↑ 109200	ج.↓ 10,950	120150
	140	ج.↓ 64610	ج.↑ 31286	95896

Transportation Rate and cost by mode:-

1) Fixed cost :- (infrastructure :-
roadsapplies Right of way
vehicles

equipment

Terminal

2) Variable cost :- operating cost :- Fuel
maintenance
driver

mode	Fixed	variable	Fixed cost comp	variable Cost comp.
1) water	medium-height	Low	vehicles	- Low speed - high capacity - Fuel efficiency
2) Rail	high	Low	Rail Roads control sys	Fuel efficiency small crew few delays
3) Truck	L	H	Trucks Terminals cross dock	drivers fuel maintenance
4) Air	H	M-H	vehicles terminals facilities	- Fuel inefficient - low capacity

Fixed cost comp

variable
cost
comp.

vehicles

- Low speed
- high capacity
- Fuel efficiency

Rail Roads
control sys

Fuel efficient
small crew
fewer cars

Trucks
Terminals
cross dock

drivers
fuel

maintenance

vehicles
terminals
facilities

- Fuels
- inefficient
- maintenance
- Low capacity

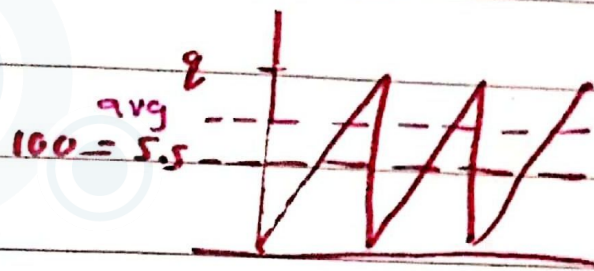
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* space at wh.



$$\text{max} = Q + 100$$

$$\text{min} = 100$$

$$\rightarrow \text{avg} = \frac{Q + 100 + 100}{2} = \frac{Q}{2} + 100$$

* transit + transportation cost

• assumption: إذا ما فرضنا (assumption)

why the value of item become 512 rather than 500 after transit?

- ① (inflation) لأنه الزمن اختلوي من الاعتبار.
- ② تكلفة النقل + handling cost →



* إذا كانت ال wh جوال plant يعني handling cost

(→ forklift driver) ، وبتكفي ال transportation

short-haul-transportation ::

- pickup / delivery of goods in relatively small area ~~is~~ using fleet of vehicles

(international facility) (جوانب دولية)

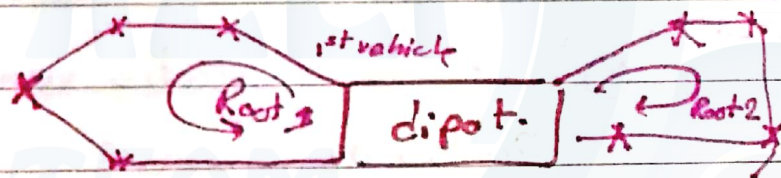
- unless stated otherwise, vehicles are stationed at single depot.

هذا المركبات يمكن أن تكون واحدة.

- Routes includes several pickup and/or delivery points.

- Routes performed in a single work shift.

x :: pickup or delivery points



• depot (مركز التوزيع)

* Decisions: in short haul transportation location problem - strategic :: where to locate the depot/wh/terminal

- Tactical :: fleet sizing → how many trucks / cars to have.

✓ - Operational :: vehicle routing (VRSP) and schedul problem.

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Root → وین اودی دل سارے
وین لکرم بین

we will focus on the operational
problem:

Important:

before hand

الذات الموزونة
مستطاب

what data known in your hands?



we have 2 types of problems

For operational level:

موزونة نوع
الذات

① Static: static vehicles Routing
problem

"ما یقصر الذات به"
"عن planning"

all customers requests (location amount
of goods)

(Food delivery)

wh
Distribution
centers
مواقع اخرى

are known in
advance

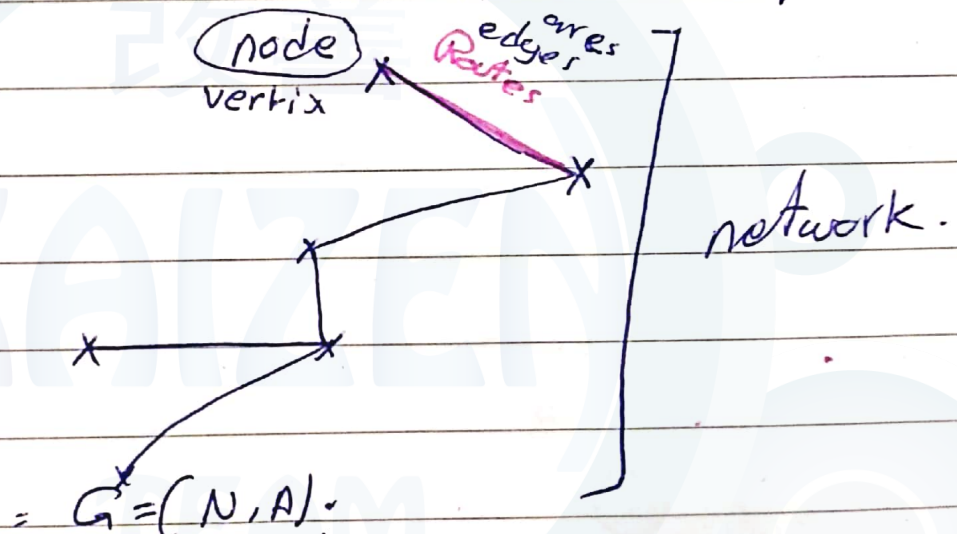
في ذات بتكون
اشارة عملية
Routing

② Dynamic VRP: not all information
is known in advance

ممكن تغير الذات
Flexible

①
 Routes planned for set of request
 ②
 known in advance, dynamically updated
 to incorporate new request.

Transportation problems are defined
 on graphs, which are symbolic representation
 of networks and their connectivity.



$G = (N, A)$
 Graph = $G = (V, E)$ is a set
 of (vertices/nodes) connected by a set
 (V) (n)

of (edges/arcs) •
 (E) (q)

vertex: a terminal point, intersection point of graph

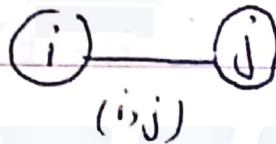


$\Rightarrow$ representation: City, customer location, depot, road intersection.

first, final vertex

Edges: the link between two vertices

Edges / arcs often referred to as (i, j) connected vertex (i) to vertex (j) .



Arrows representation direction

$\rightarrow$ Head node j

$\rightarrow$ Tail node i



directional connected
بالاتجاه، متصلة

• Assumed bi directional if no arrows



لا اتجاه، متصلة

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⇒ Edges / arcs have number of characteristics.

① cost - C_{ij}

(time, distance, risk, Fuel, Denesisation)
associated with (i, j)

② capacity - U_{ij}

maximum flow on edge (i, j)

↳ traffic, amount of goods

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order of a graph :: number of vertices, $|V|$

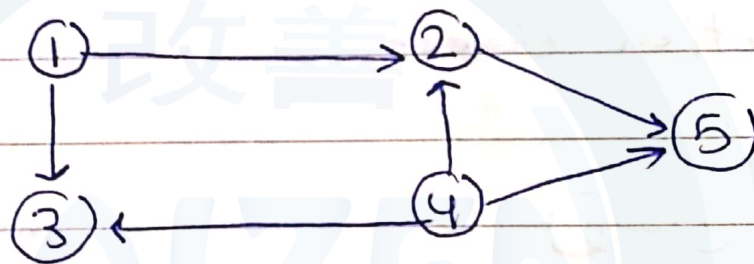
$$|V| = 5$$

$$V = \{1, 2, 3, 4, 5\}$$

$$V = \{\text{Amman, Irbid, madaba, ...}\}$$

size of graph :: number of edges, $|E|$

example ::



$$G = (V, E) ::$$

$$V = \{1, 2, 3, 4, 5\}$$

$$E = \{(1, 2), (1, 3), (2, 5), (4, 3), (4, 2), (4, 5)\}$$

$$|V| = 5, \text{ order of } G$$

$$|E| = 6, \text{ size of } G$$

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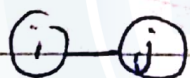
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Structural properties of graphs:

- Undirected vs directed :-

network without
specify direction

a connection between
 i and j implies a
connection between
 j and i



undirected

Also, referred to as a symmetrical
network



in transportation problems, sometimes symmetry used to represent whether cost is same when travelling from i to j as when travelling from j to i .

• A symmetry example:

* traffic may be higher in one direction resulting in higher travelling time.

• إذا كان cost متماثلًا في اتجاهين مختلفين

• $\checkmark$ a symmetric ~~is not~~ a symmetric

→ symmetric → cost is same on two directions

→ a symmetric → " isn't same "

-? Conclusion

• symmetric $\leftarrow$ Bi-directional is also *

symmetric or a symmetric → Bi-directional

Subject

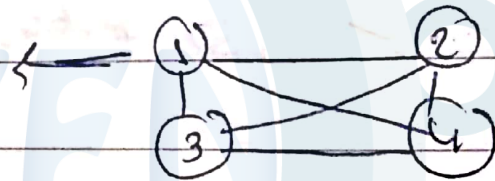
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* Completeness :- a graph is complete if an edge exists between every pair of vertices.
(regardless of direction)



كل زوج من العقد متصل
بالحافة في كلا الاتجاهين
المع كل node متصلة مع كل ال nodes
الباقيين

Complete
undirected



مفاهيم
→ paths, cycles, Trees.
for networks

For the following definitions:
assume there are n vertices in V , and
 m edges in E

① path: a path p in G is a sequence of vertices $\{i_1, i_2, \dots, i_n\}$, such that:

• $i_k \in V \quad \forall \quad k = 1, \dots, p \sim \text{nodes}$

• i_k unique $\forall \quad k = 1, \dots, p$.

• no vertex visited more than once.

• (i_k, i_{k+1}) or $(i_{k+1}, i_k) \in E$

⇒ find path from 1 to 5:

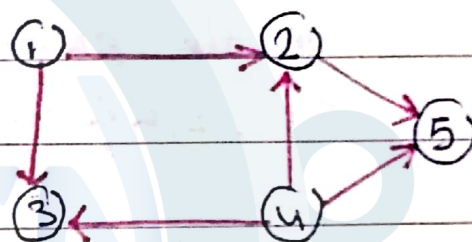
$\{1, 2, 3\}$ feasible

$\{1, 2, 4, 5\}$ → not feasible
but still path.

$\{1, 3, 4, 5\}$

$\{1, 4, 5\}$ → not a path

$\{1, 3, 5\}$



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Directed path:

* additional conditions:

$$(i_k, i_{k+1}) \in E \quad \forall k = 1, \dots, (P-1)$$

is a Feasible sequence of

decisions For users.

Path is Feasible

In the previous example:

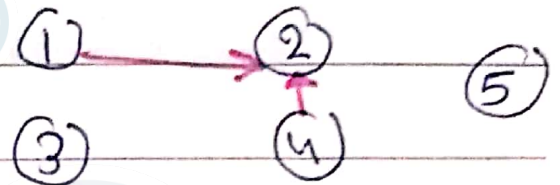
$\{1, 2, 5\}$ is a directed path

- Related concept :-

Connectedness.

* connected $\leftrightarrow$ two vertices i and j are connected in G if there exists path From i to j regardless of direction

* adjacency : connected ~~with~~ by a path
of length ~~1~~
one edge



nodes 1 and 4

are they connected?

yes, they are connected but not adjacent

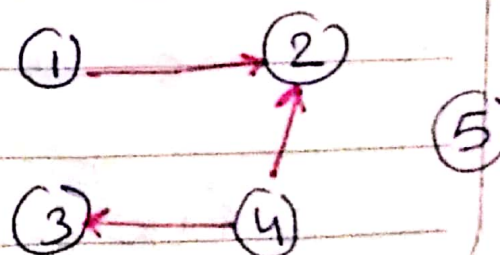
nodes 1 and 2 are they connected

yes, they are connected and they are adjacent

adjacent = connect
but connect $\neq$ adjacent

- A graph is connected if every pair of vertices in the graph are connected

Not connected $\leftarrow$



path minimal size 3

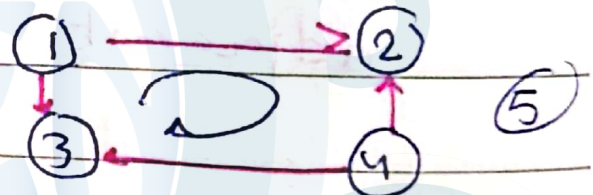
* Cycle:

المسار من نقطة وارجع لنفس النقطة.

set of edges, that form a closed loop in G , you can begin walking at some vertex follow only edges in the cycle. regardless of their direction, then return to original vertex.

* cycle but not feasible

لبنني هاد
ياكل



Cycle is a sequence of vertices

$\{i_1, i_2, \dots, i_p, i_1\}$ such that $\exists$

• $i_k \in V \quad \forall k=1, \dots, p$

• i_k unique $\forall k=1, \dots, p$

• $\{i_k, i_{k+1}\}$ or $\{i_{k+1}, i_k\} \in E \quad \forall k=1 \sim p$

→ • either (ip, i_1) or $(i_1, ip) \in E$

لأنه لازم
من P

• feasible or not *

undirected by default feasible.

Tree:-

is a connection of edges, such that vertices are connected and no cycle exist.

= # of edges.

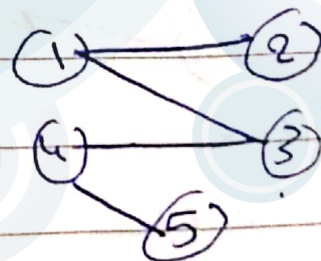
equivalent definition:-

For n vertices in the network, A tree is a set of $(n-1)$ edges, such that all vertices are connected.

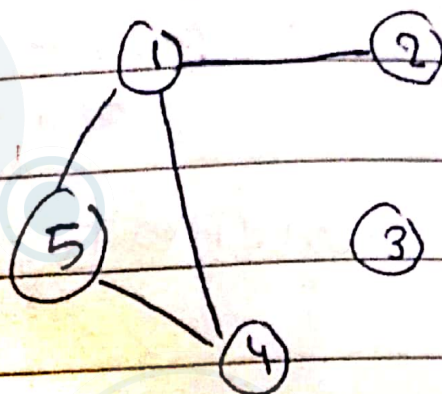
vertices = 5

edges = 4

• connected also

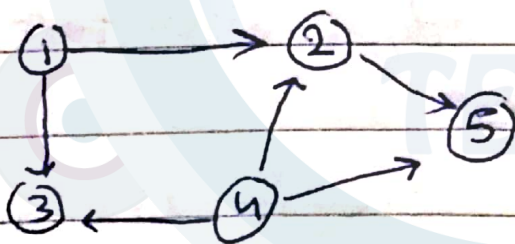


3 isn't connected
 + there is a cycle
 between {1, 4, 5}
 so, it's not a tree.



important property of tree: define
 For each pair of vertices, a unique
 (not a necessary directed) path between
 them.

directed network



⇒ not tree

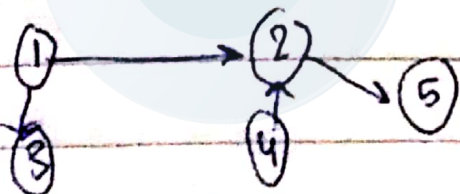
edges = $n+1$

undirected network



⇒ not tree

there is a cycle.



⇒ directed tree



yes, tree

what happen if we remove one edge
From tree?

- ⇒ Some vertices become unconnected
- ⇒ So, the graph is not connected
- ⇒ So, tree - one edge = unconnected graph.

if we add an edge?

creat a cycle ⇒ no longer a tree.

first problem :-minimum cost path models :-

* many transportation decision models require finding the min cost path(s) between pair(s) of vertices.

⇒ cost can be (time, distance, \$, risk, ----.

* min cost المسار ذو التكلفة الدنيا

* Applications :-

- 1 - personal travel: road network ⇒ need to determine the min time path between origine and destination.

import .

- 2 - Global supply chain: Given the network model representing shipper and its full set of transportation services. for a particular commodity.

and need to determine the min cost (usually time) transport option from the origin to the destination.

min cost path problem types:-

(A) Single-pair shortest path problem.

goal $\Rightarrow$ Find the shortest path between

a single source ^{node} ~~node~~ (S), and a destination node (t).

(B) Single-Source shortest path problem.

goal $\Rightarrow$ to find the shortest path between a single Source node (s) and all other vertices.

(C) Single-destination shortest path problem,
 $\Rightarrow$ to find the min path from all the vertices to a given destination (t).
node,

④ All pairs shortest path problem

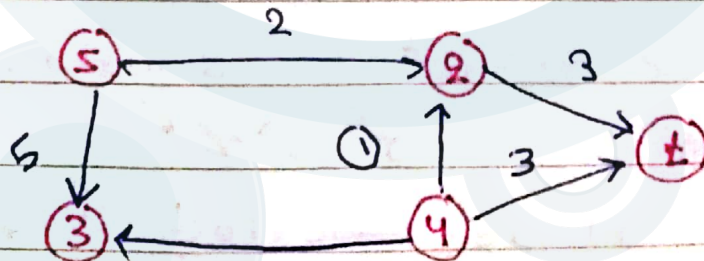
$\Rightarrow$ Find the shortest path between all pairs of vertices.

to the next link source $\rightarrow$ as destination

Formulation definition ...

Given a graph $G=(V,E)$ with costs C_{ij}
 For each $(i,j) \in E$, a source node (S),
 and a destination node (t), Find the
 path(P) From (S) to (t) that minimizes total
 cost for the path ($\sum C_{kl}$)
 $\downarrow$
 in General $(k,l) \in p$

$$\text{obj} \Rightarrow \min \sum C_{kl}$$



path from S to t $\Rightarrow \{S, 2, t\}$ cost = 5

$\{s, 3, 4, t\}$

cost = 9

 $\{s, 2, 4, t\}$

cost = 6

 $\{s, 1\}$

• not feasible / feasible path.
 ممكن / ممكن

Solving minimum cost path problems.

1. Dijkstra's algorithm : Single - pair

Single - Source

Single - destination.

Find the single source shortest path (SP).

- requires $C_{ij} \geq 0$

(+ve)

advantage

* Can't deal with (-ve)

* pros :: * Fast For dense networks.

$\Rightarrow m = n^2 \Leftarrow$ dense network.

* Can be truncated if the path of interest has been found.

Can be truncated if the path of interest has been found.

* Cons :- dis adv

- Alternative methods Faster for ~~source~~ sparse networks.

edges كثرة في الشبكات الكثيفة dense networks

- programming is needed for efficient implementation.

2. Bellman-Ford :-

single-source

allows $C_{ij} \leq 0$

allows negative modification.

* pros :- Fast for sparse network.

- Simple to implement.

* Cons :- cannot be truncated.

لا يمكن التوقف عن العمل إذا كان هناك حل.

- may be slow in dense networks.

كيف لا يكون cost (-ve) ؟
if negative has negative.

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Revenue = - Cost

بما أن Cost هو المطلوب الحد من Cost

$\begin{matrix} \nearrow \text{max} \\ = \\ \searrow \text{min} \end{matrix}$

$$Z = x_1 + 2x_2$$

$$-Z = -x_1 - 2x_2$$

Dijkstra Algorithm :-

it is a Labelling algorithm in which, at each iteration's a different vertex P is given a permanent Label $V(i)$ that indicates the cost of the shortest path from $\underline{S}$ to $\underline{P}$.

Assumption :-

$\Rightarrow C_{ij} > 0$ $\forall (i,j) \in E$
 $\downarrow$ $\downarrow$
 can not be negative belong edges.

Notations :-

• $V(i)$:- Label for vertex i that denotes cost of path from $\underline{S}$ to $\underline{i}$

min cost from each node
 • $Pred(i)$:- predecessor of node i on shortest path from $\underline{S}$ to $\underline{i}$

• $PERM$:- set of permanently labeled nodes

$\Rightarrow$ complement of $PERM$:- not permanently labeled nodes.
 For Perm

Initialization :

min($\bar{z}$)

$$v(j) = \begin{cases} 0^{\text{cost}}, & j = s \\ \infty, & j \neq s \end{cases}$$

بغير صلا
ولوح كل قبة
اقل بطلع هذا

$$pred(j) = \begin{cases} 0 & j = s \\ -1 & j \neq s \end{cases}$$

node
لها صلا

$$PERM = \{ \}, \quad \overline{PERM} = V$$

Iterations:

while $\overline{PERM} \neq \emptyset$
in programming

بلاكاره قدم تقدر نجيب ال node

1. Let $P \in \overline{PERM}$ be node for which $v(P) = \min \{ v(j), j \in \overline{PERM} \}$
نوجد النقطة التي لها اقل كوست

$$2. \overline{PERM} \leftarrow \overline{PERM} + \{P\}$$

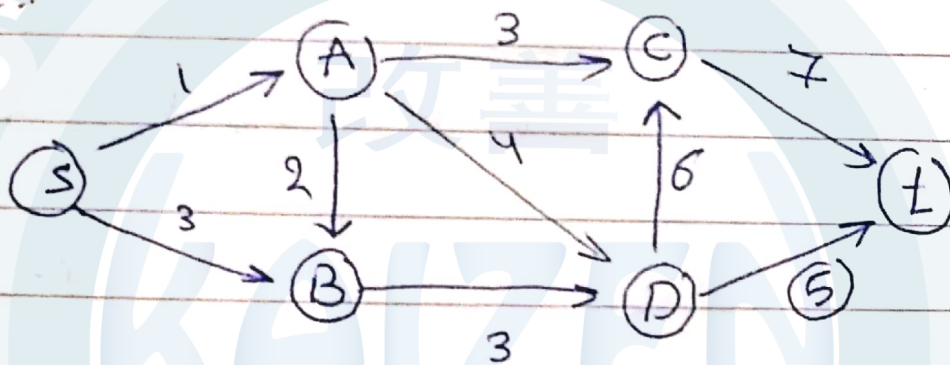
$$3. \overline{PERM} \leftarrow \overline{PERM} \setminus \{P\}$$

باستثناء

4. For $\forall (p, j) \in E$,
 $j \in \text{PERM}$ leading from p
 if $v(p) + C_{pj} < v(j)$ then
 $v(j) = v(p) + C_{pj}$
 $\text{pred}(j) = p$

else, do nothing.

example:



$v(j) \rightarrow$ cost of shortest path from
 S to j

$\text{pred}(j) \rightarrow$ node before j in path.

$v(j) [\text{pred}(j)]$

iteration $\rightarrow$

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iteration

s A B C D t

initialization 0[0] 0[1] 0[2] 0[3] 0[4]

1 0[0] 1[5] 3[5] — ما بين كوسين س الـ

$$C_{SB} + V(s) < U(B) \rightarrow \text{Yes}$$

احصائيه الجايه

2 3[A] 4[A] 5[A]

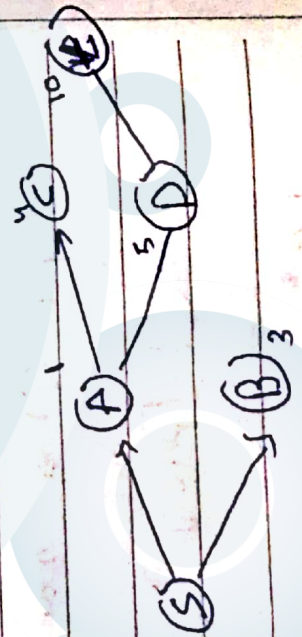
$$V(A) + C_{AC} = 1 + 3 = 4 < U(C)$$

3 ————— يصل فيها شيء بلعبها لأنه في أقل منها —

4. 5[A] 11[C]

10[D]

0[6] 1[6] 3[5] 4[A] 5[A] 10[D]



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بوحدة الحاسب والبرمجة

لغات برمجة

P PERM PERM

- { } S, A, B, C, D, t

S { } A, B, C, D, t

A S, A B, C, D, t

min على الحد الأدنى من الترتيب على PERM واختار 1

B S, A, B C, D, t

C S, A, B, C D t

D S, A, B, C, D t

t S, A, B, C, D, t

$$\Rightarrow S - A - D = 5$$

$$S - A - D - t = 10$$

ABC

Bilman - Ford Algorithm

This is sometimes referred to as a ~~Label~~ algorithm $\Rightarrow$ ~~no~~
label correcting

There is no requirement for the non-negative edge cost ≤ 0

Initialization:

$$u(j) = \begin{cases} 0 & j = s \\ \infty & j \neq s \end{cases}$$

$$\text{pred}(j) = \begin{cases} 0 & j = s \\ -1 & j \neq s \end{cases}$$

$$\text{LIST} = \{s\}$$

Iterations:

• while $\text{LIST} \neq \emptyset$ isn't empty

1. Remove node p from top of LIST

2. For every (p, j) leading from node p

a. if $v(p) + c_{pj} < v(j)$, then

i - $v(j) = v(p) + c_{pj}$

ii - $\text{pred}(j) = p$

iii - if $j \notin \text{LIST}$, $\text{LIST} = \text{LIST} + \{j\}$

الحالة

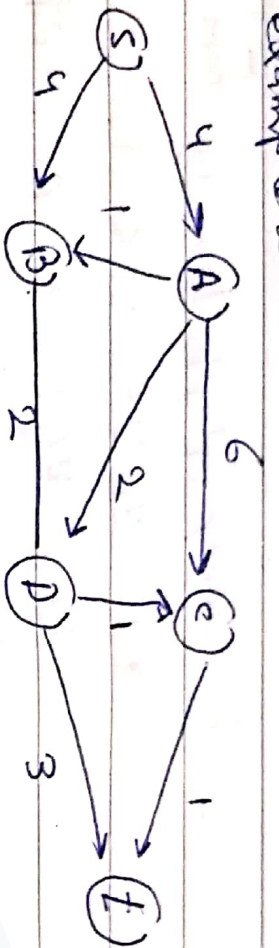
Pape's modification:

if $j \notin \text{LIST}$, then

- if j has never been in LIST, Put j at end

- else, put j at beginning at LIST.

example -



Iteration S A B C D

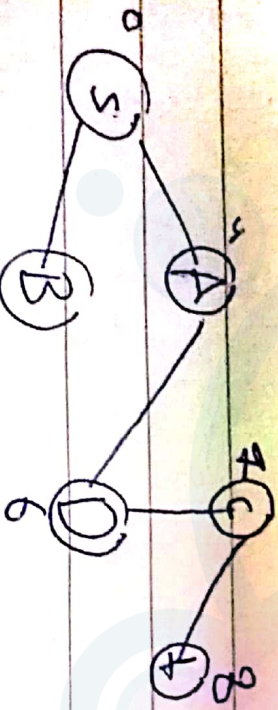
∞ [S] ∞ [A] ∞ [B] ∞ [C] ∞ [D]

1 4[S] 4[S]

2 10[A] 6[A]

5 7[B]

7 6[C] 4[S] 4[S] 7[D] 6[A]



T P List

∞ [S] - {S}

- S A, B

- A B, C, D

- B C, D

11[C] C D, F

9[D] D C, F

8[C] C F

8[C] F {F}

Shortest path from S to

* Multi-label - Algorithm For shortest paths:

- multiple objectives $\therefore$ (distance - time).

- vertices given Labels with two Components (d_k, t_k) .

- d_k - first cost component.

$\Rightarrow$ total distance required to reach this vertex along some path.

From S

- t_k - second cost component,

$\Rightarrow$ total time to reach k

- vertices may have multiple Labels resulting From different paths

min cost $\Rightarrow$ d_k

min dist $\Rightarrow$ t_k

- one path to a vertex may have larger distance but shortest time than other path (is reverse)

- now, instead of a vertex having a predecessor Label, each cost component label (D_k, t_k) has an associated predecessor Label, $\text{pred}(k)$.

* we only maintain the non-dominated Labels at each vertex.

- A Label (d_q, t_q) is dominated ^{by} (d_k, t_k) , if both $d_q > d_k$ and $t_q > t_k$.

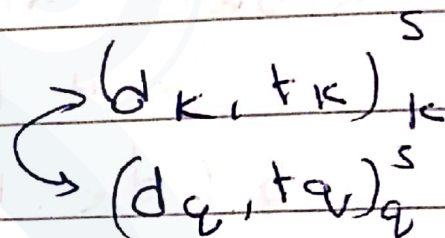
• maintain a LIST of Labels instead of vertices.

Remove Label From LIST and examine edges leaving From the vertex associated with the Label.

Initialization:

1. origin vertex $\underline{=}$ given the Label $(0,0)_1^s$ where subscript indicates Label $L \# (k=1)$ and superscript indicates the vertex where the Label exists (s) .

the predecessor for this Label is 0,
 $\text{pred}(k) = \text{pred}(1) = 0$



2. NO Labels are given to other vertices.

$$3. \text{LIST} = \left\{ (0,0)_1^s \right\}$$

$r = 2$ (keep track of # of next Label generated).

Iterations:

1. while LIST $\neq \{ \}$

1. Remove Label k From the beginning of LIST. suppose label k has components such that it exists vertex p .

2. For every edge (p, j) Leaving From vertex p :

A) Create a new Label r at vertex j where $(d_r, t_r) = (d_k + d_{pj}, t_k + t_{pj})$
Cost Cost

B) Define $\text{pred}(r) = k$

C) Add Label r to end of LIST.

1D) Compare Label r to each existing Label (d_q, t_q) . currently at vertex j

i) if Label r is dominated by Label q , discard the new Label r and remove it from LIST.

if $d_r \geq d_q$ and $t_r \geq t_q$

ii)) else , if Label q is dominated by r discard q , and if necessary remove it from LIST

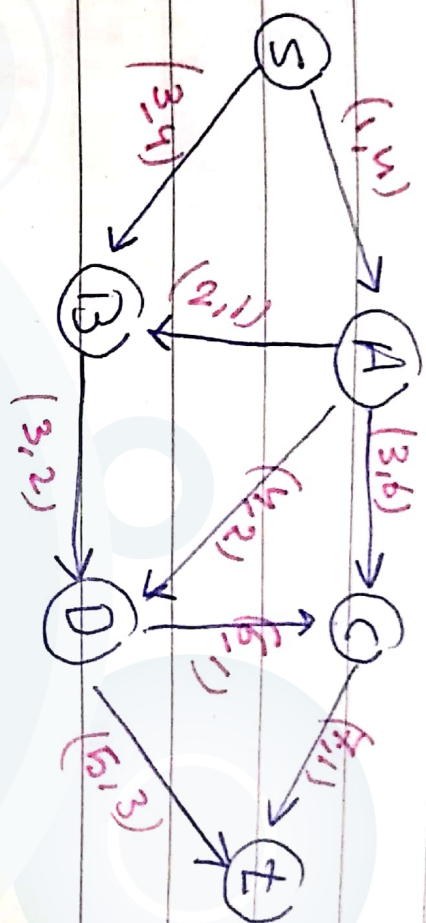
if $d_r \leq d_q$ and $t_r \leq t_q$

iii)) else , retain both Labels .

E) $r = r + 1$

- Stopping criteria : when no Labels remain in the LIST , no remaining Label corrections ~~can~~ can be found , All non dominated paths to all vertices have been identified

ex:



Label
1. cost

S	A	B	C	D	E	F	LIST	r
(0, 0)	(1, 1)	(3, 4)	(4, 10)	(5, 6)	(11, 11)	(6, 0)		2

(3, 6)	(1, 2)	(4, 2)	(5, 1)	(18, 8)	(3, 4)	(4, 10)	(5, 6)	(13, 5)	(6, 0)	(11, 11)	(18, 8)
--------	--------	--------	--------	---------	--------	---------	--------	---------	--------	----------	---------

12

10

8

7

6

5

4

3

2

1

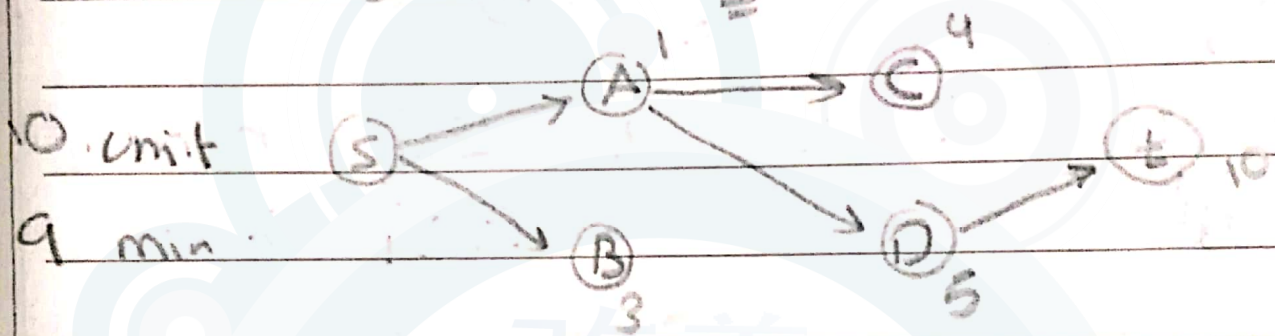
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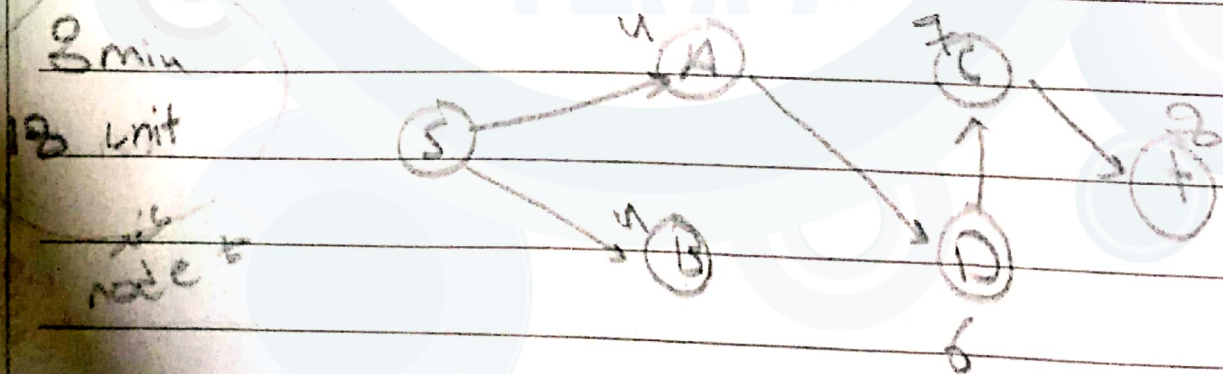
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* First cost component : distance
shortest distance path to all
nodes From S



* Second cost component : time
shortest path to all nodes
From S



minimum spanning tree models v.

tree: a collection of $n-1$ edges such that all vertices in G are connected no cycles exists.

A minimum spanning tree T^* in $G = (V, E)$ is a tree such that total cost

Q: is MST the same as SP tree?
No; bc SP tree minimizes length of shortest paths from some origin node to an other nodes.

while the MST minimizes the total cost of all arcs in the tree.

Kruskal's algorithm:

- Iteration

• $TREE = E$

- Iteration

• sort arcs in order increasing C_{ij}
into LIST, w
while $|TREE| < n-1$

• Remove (i,j) from top of LIST

• Add (i,j) to TREE : if it doesn't make a cycle.

At completion of algo, $T^* = TREE$.

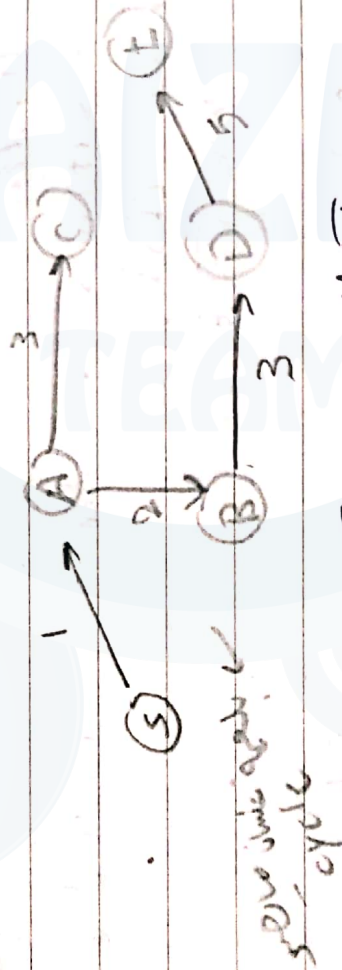
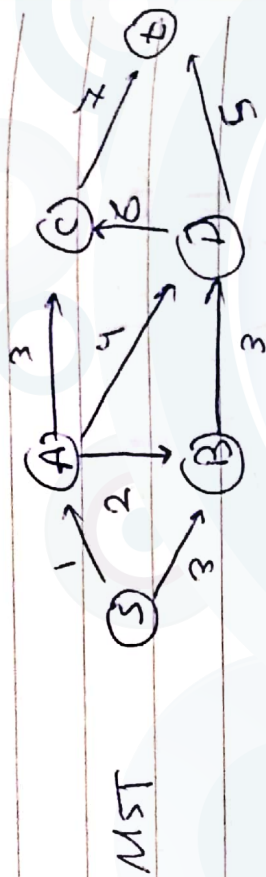
Algorithm: steps or procedure to solve a problem

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Cost for MST = $\sum c_{ij} = 14$

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TREE $\{ \}$

$n-1 \rightarrow$ stopping

$$S-A = C_{SA} = 1$$

$$A-B = C_{AB} = 2$$

$$A-C \rightarrow C_{AC} = 3$$

$$B-D = C_{BD} = 3$$

$$S-B = C_{SB} = 3$$

$$A-D = 4$$

$$D-E = 5$$

$$D-C = 6$$

$$E-C = 7$$

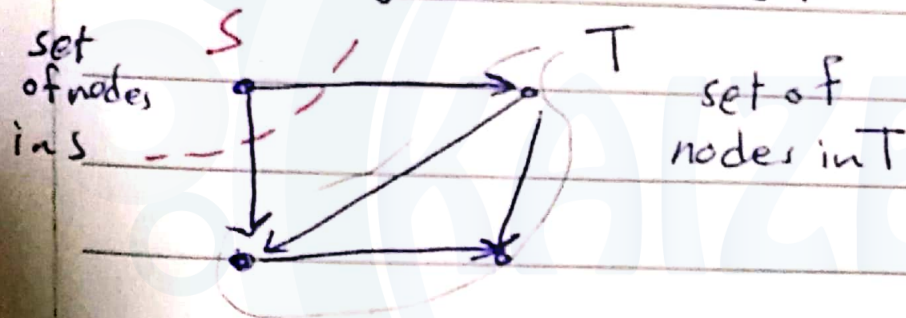
Prims Algorithm:

we must first understand what a cut is in a network.

→ suppose vertices in G partitioned into two independent subsets (S) and (T) .

The cut set (S, T) is given by all arcs (i, j) such that...

- $i \in S$ and $j \in T$ or.
- $j \in S$ and $i \in T$



cut S	cut T
10	A0
20	B0
30	C0
3	—

$$S = \{1, 2, 3\}$$

$$T = \{a, b, c\}$$

$$\{(1, A), (1, C), (2, B), (3, A), (3, B), (3, C)\}$$

* algorithm

Initialization

$$Tree = \{ \}$$

$$S = \{s\}, T = V \setminus S$$

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Iterations:

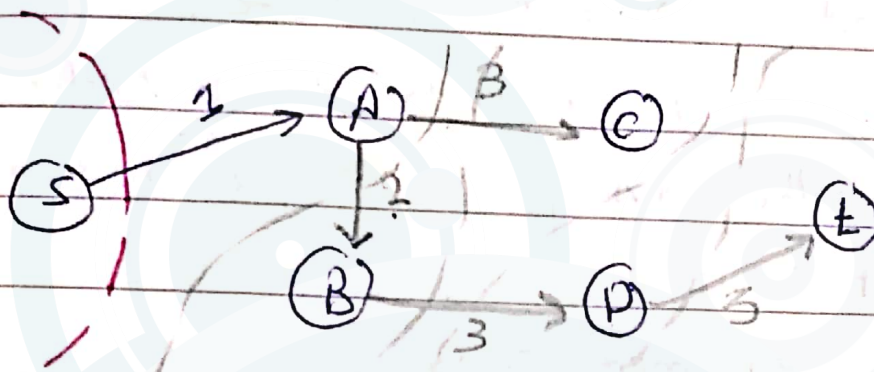
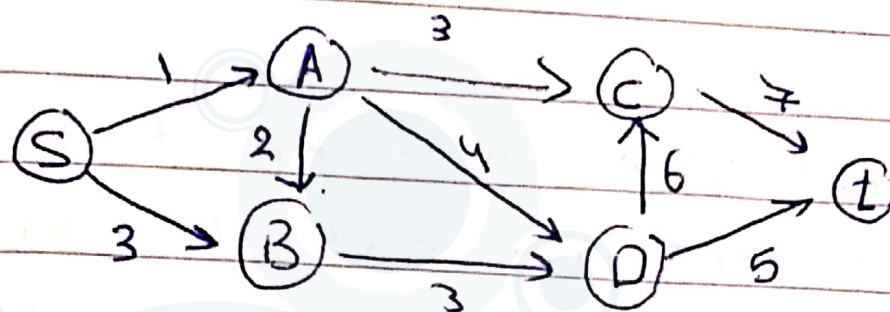
- While $|T_{\text{ree}}| < n-1$

 - ↳ number of edges in tree.

- Find the arc $(i, j) \in (S, T)$ with min cost C_{ij}

 - i - Add (i, j) to tree.

 - i - Remove j from T and add to S



cut 1

cut 2

cut 3

cut 4

cut 5

$$C_{SA} = 1$$

$$C_T = 7$$

$$C_{SB} = 3$$

$$C_D = 6$$

$$C_{BD} = 3$$

$$C_{AC} = 3$$

$$C_{AD} = 4$$

$$C_{AD} = 4$$

$$C_{AB} = 3$$

$$C_{DB} = 5$$

$$C_{SB} = 3$$

$$C_{AC} = 3$$

$$C_{AD} = 4$$

$$C_{BD} = 3$$

* minimum spanning TREE Applications:

standard min cost paths (shortest path)

are called (mini-sum paths) ($\sum c_{ij} = \min$)

because among all paths P connecting $\underline{s}$ to $\underline{t}$ the min path $\neq P_s^*$ has minimum total cost $\therefore$

$$P_s^* = \arg \min_{P \in G} \sum_{(i,j) \in P} c_{ij}$$

* minimax paths:

the minimax path P_m^* encounters the smallest maximum arc cost $\therefore$

$$P_m^* = \arg \min_{P \in G} \max_{(i,j) \in P} c_{ij}$$

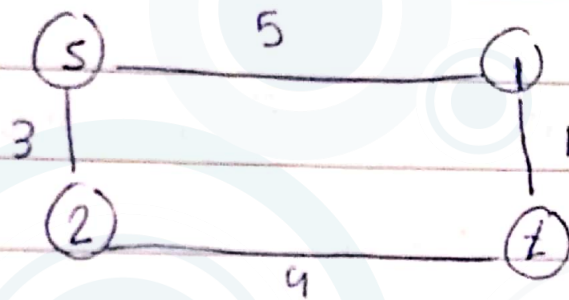
- * finding all P
- * $\max c_{ij}$ for each path
- * min of these

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path from (s-t) $\sum c_{ij}$

$\max c_{ij}$

P_s^*

$\leftarrow s-1-t$

min cost [6]

5

$s-2-t$

7

[4] min max.

P_m^*

min cost $\rightarrow$ أقل التكلفة

min max $\rightarrow$ • Two disjoint arcs

Useful application of min max paths:

• Elevations change:

very steep road segment, $\uparrow$ fuel $\uparrow$ $\downarrow$ $\uparrow$

maintenance cost, so you want to avoid segment with maximum elevation change.

* الارتفاعات تكون بغير حد الارتفاع $\rightarrow$ انما بي اتجنبها.
الحل elevation يكون اقل ما ممكن
 $\rightarrow$ MST.

* Hazardous materials:

if spilled, population harmed, avoid densely populated road ways

$\rightarrow$ min max expected population exposed

يتوجب الامانة الاقل الاحتياط والى ما فيها الناس كثير.