Control

Friday, March 8, 2024 7:21 PM

Automation → total system. (Also not using humans)

Control → every single thing in the system.

For example: opening affects → flow
we calculate that in control

and this kind of calculation is affected with time using differential equation

inverter: control electricity in system.

$\boxed{\text{control or regulate}}$ amount of flow into the system

$$\underbrace{1}_{\text{mass}}\ddot{x} + \underbrace{3}_{\text{damper}}\dot{x} + \underbrace{2}_{\text{spring}}x = \underbrace{5}_{\text{most important}}\underbrace{\sin t}_{\text{Force}}$$

We should control in design stage first or changing the component first, if that can't be done, we try this approach.

In Laplace: $t \to s$

$\delta \to$ impulse / impact

$u \to$ on, getting to a specific value.

natural

$e^{-at} \to$ going away with time

$t \to$ at every time period happens something.

$t^n \to$ like (t) but with acceleration/deceleration

$\left.\begin{array}{l} \sin(wt) \\ \cos(wt) \end{array}\right\}$ manmade

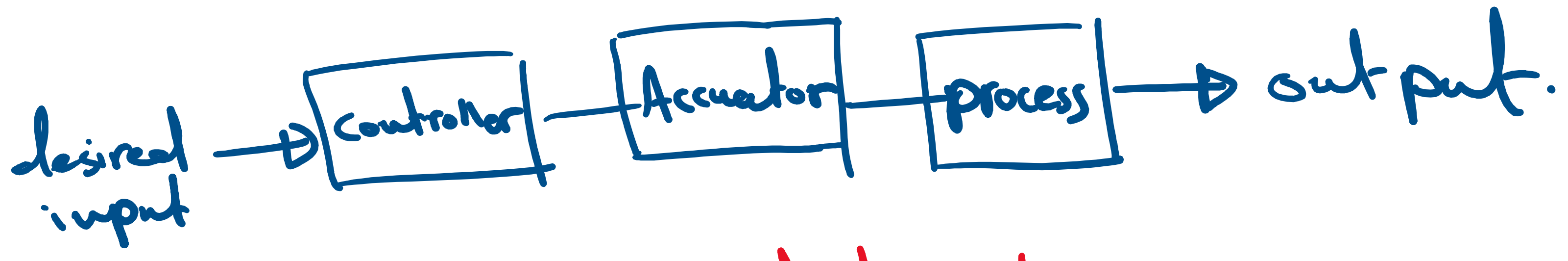
$t \to f(t)$ time domain

$s \to F(t)$ s domain

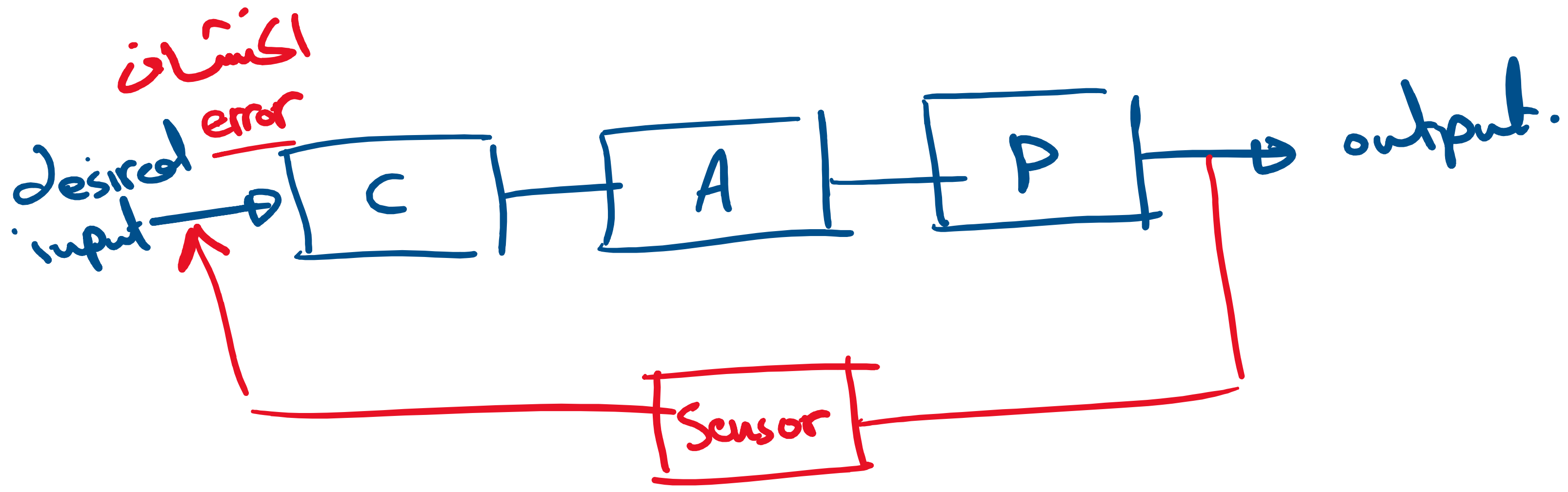
Control systems

Friday, March 15, 2024 4:35 PM

Control Systems :



open loop control system.



closed loop control systems.

Partial fractions

Friday, March 15, 2024 4:50 PM

In slides you should be able to learn how to use:

Laplace, Laplace inverse, Matlab code, Final value theorem, Initial value theorem

$\lim_{s\to 0} sF(s) = f(\infty)$ $\lim_{s\to\infty} sF(s) = f(0)$

* About Partial Fractions:

$$X(s) = \frac{8(s+3)(s+2)}{s(s+2)(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+4}$$

unrepeated real roots

$$K_1 = \left.\frac{8(s+3)(s+2)}{(s+2)(s+4)}\right|_{s=0} \qquad \boxed{K_1 = 24}$$

$$K_2 = \left.\frac{8(s+3)(s+2)}{s(s+4)}\right|_{s=-2} \qquad \boxed{K_2 = -12}$$

$$K_3 = \left.\frac{8(s+3)(s+2)}{s(s+2)}\right|_{s=-4} \qquad \boxed{K_3 = -4}$$

$$X(s) = \frac{24}{s} + \frac{12}{s+2} + \frac{-4}{s+4}$$

$f(t)$	$F(s)$
e^{-at}	$\frac{1}{s+a}$

Laplace inverse:

$$x(t) = 24\,u(t) + 12 \times e^{-2t} - 4 \times e^{-4t}$$

we don't put this

$$X(s) = \frac{8(s+1)}{(s+2)^2} = \frac{K_1}{(s+2)} + \frac{K_2}{(s+2)^2}$$

repeated real roots

$K_1(s+2) + K_2 = 8(s+1)$

$s = -2 \to \quad K_2 = 8 \times -1 = -8$

$s = 0 \to \quad 2K_1 - 8 = 8$

$2K_1 = 16 \qquad \underline{K_1 = 8}$

$$X(s) = \frac{8}{s+2} + \frac{-8}{(s+2)^2}$$

$$x(t) = 8e^{-2t} - 8te^{-2t}$$

$$X(s) = \frac{5.2}{s^2+2s+5} = \frac{5.2}{s^2+2s+1+4-1} = \frac{5.2}{(s+1)^2+2^2} = \frac{5.2}{2} \times \frac{2}{(s+1)^2+2^2} = \frac{5.2}{2} \times e^{-t}\sin(2t) = x(t)$$

حالة البسط رقم

$\left(\frac{2}{2}\right)^2$

$f(t)$	$F(s)$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

unrepeated complex roots

$$X(s) = \frac{K_1 s + K_2}{s^2+2s+5} = \frac{K_1 s + K_2 + K_1 - K_1}{(s+1)^2 + 2^2}$$

$aK_1 - aK_1$

a

$$\frac{K_1 s + K_1}{(s+1)^2+2^2} = \frac{K_1(s+1)}{(s+1)^2+2^2} = K_1 \times \frac{(s+1)}{(s+1)^2+2^2} = K_1 e^{-t}\cos 2t.$$

$+$

$$\frac{K_2 - K_1}{(s+1)^2+2^2} = \frac{K_2-K_1}{2} \times \frac{2}{(s+1)^2+2^2} = \frac{K_2-K_1}{2} \times e^{-t} \times \sin 2t$$

(just like the الي before)

so $K_1 e^{-t}\cos 2t + \frac{K_2-K_1}{2} \times e^{-t}\sin 2t$.

حالة البسط linear.

$$X(s) = \frac{s+2}{(s+1)(s^2+2s+5)(s+3)^2}$$

does not ~~.~~

$$= \frac{K_1}{s+1} + \frac{K_2 s + K_3}{s^2+2s+5} + \frac{K_4}{s+3} + \frac{K_5}{(s+3)^2}$$

بعدها لازم نطلع K_1, K_2, K_3, K_4, K_5

Laplace inverse: $K_1 e^{-t} + \frac{K_2 s + K_3 + K_2 - K_2}{(s+1)^2+2^2} + K_4 e^{-3t} + K_5 te^{-3t}$

مش شرط هاد.

مسألة [illegible] .

Laplace inverse: $k_1 e^{-t} + \dfrac{k_2 s + k_3 + k_2 - k_2}{(s+1)^2 + 2^2} + k_4 e^{-\cdot} + k_5 t e^{-\cdot}$

$$\frac{k_2 s + k_2}{(s+1)^2 + 2^2} + \frac{k_3 - k_2}{(s+1)^2 + 2^2}$$

$$k_2 \frac{(s+1)}{(s+1)^2 + 2^2} \qquad \frac{k_3 - k_2}{2} \times \frac{2}{(s+1)^2 + 2^2}$$

$$k_2 e^{-t} \cos(2t) + \frac{k_3 - k_2}{2} \sin(2t)$$

$$\ddot{x} + 3\dot{x} + 2x = 5\sin t.$$

$$\mathcal{L}(x(t)) = X_{(s)}$$

$$\mathcal{L}(\dot{x}) = s\,X_{(s)} - x(0)$$

$$\mathcal{L}(\ddot{x}) = s^2 X_{(s)} - s\,x(0) - \dot{x}(0)$$

as $x(0) = 1$ $\dot{x}(0) = 0.$

We begin by finding Laplace for $\ddot{x}, \dot{x}, x$ as stated above in order to find $X_{(s)}$:

$$\mathcal{L}\ddot{x} + 3\mathcal{L}\dot{x} + 2\mathcal{L}x = 5\mathcal{L}\sin t.$$

$$\left[s^2 X_{(s)} - s\,X(0) - \cancel{\dot{x}(0)}\right] + 3\left[s X_{(s)} - X(0)\right] + 2X_{(s)} = \frac{5}{s^2+1}$$

$$s^2 X_{(s)} - s + 3sX_{(s)} - 3 + 2X_{(s)} = \frac{5}{s^2+1}$$

$$X_{(s)}\left[s^2 + 3s + 2\right] - s - 3 = \frac{5}{s^2+1}$$

$$X_{(s)}\left[s^2 + 3s + 2\right] = \frac{5}{s^2+1} + s + 3$$

$$X_{(s)} = \underbrace{\frac{5}{(s^2+1)(s^2+3s+2)}}_{X_1(s)} \quad \underbrace{\frac{+s+3}{s^2+3s+2}}_{X_2(s)}$$

$$X_1(s) = \frac{5}{(s^2+1)(s^2+3s+2)} = \frac{K_1s+K_2}{s^2+1} + \frac{K_3}{s+2} + \frac{K_4}{s+1}$$

$$(s^2+3s+2) \to (s+2)(s+1)$$

$$(K_1s+K_2)(s+2)(s+1) + K_3(s^2+1)(s+1) + K_4(s+2)(s^2+1) = 5$$

$s = -1$ $K_4 = \frac{5}{2}$

$s = -2$ $K_3 = -1$

$s = 0$ $2K_1 s + 2K_2 + -1 \times 1 + \frac{5}{2} \times 2 = 5$

$$\boxed{K_2 = \frac{1}{2}}$$

$s=1$
or anything $\rightarrow K_1 = \frac{-3}{2}$

$$\boxed{X(s) = \frac{\frac{-3}{2}s + \frac{1}{2}}{s^2+1} + \frac{-1}{s+2} + \frac{\frac{5}{2}}{s+1}}$$

$$X_2(s) = \frac{s+3}{s^2+3s+2} = \frac{s+3}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$A(s+1) + B(s+2) = s+3$$

$s=-1 \rightarrow B = 2$

$s=-2 \rightarrow A = -1$

$$\boxed{X_2(s) = \frac{-1}{s+2} + \frac{2}{s+1}}$$

$$X(s) = \frac{\frac{-3}{2}s + \frac{1}{2}}{s^2+1} + \frac{-1}{s+2} + \frac{\frac{5}{2}}{s+1} + \frac{-1}{s+2} + \frac{2}{s+1}$$

Finally!

$a=0$
$\omega=1$

$$\frac{\frac{-3}{2}s}{s^2+1} + \frac{\frac{1}{2}}{s^2+1} + \frac{-1}{s+2} + \frac{\frac{5}{2}}{s+1} + \frac{-1}{s+2} + \frac{2}{s+1}$$

$$X(t) = -\frac{3}{2}\cos t + \frac{1}{2}\sin t - e^{-2t} + \frac{5}{2}e^{-t} + -e^{-2t} + 2e^{-t}$$

phew!

Steps:

1) find $\mathcal{L}$ for everything in the equation of $X(t)$

2) You should have $X(s)$ in the left side now, [illegible]

3) divide the left side on the right, you should have $X(s) =$

4) find $K_{1,2,3}$ from partial equations by unrepeated real roots, repeated real roots, complex roots with number or equation كلو.

5) find $X(t)$ applying laplace inverse

Electrical systems

Monday, March 18, 2024 11:58 AM

[1] We have 3 elements in electrical systems:

- resistor —ww— $z_R = R$
- capacitor —||— $z_c = \frac{1}{cs}$
- inductor —ooo— $z_L = Ls$

[2] These elements can be in series or parallel, however we should change them to $z(s)_{equ}$ in order to find what we want

series: $z_{equ} = z_1 + z_2 + \dots$

parallel: $\frac{1}{z_{equ}} = \frac{1}{z_1} + \frac{1}{z_2} + \dots$

[3] we are going to have to find TF (transfer function):

$$TF = \frac{V_o(s)}{V_i(s)} \quad \frac{(\text{output})}{(\text{input})}$$

$V_o(s) = I(s) \times z$ anything at the V_o and its same nodes

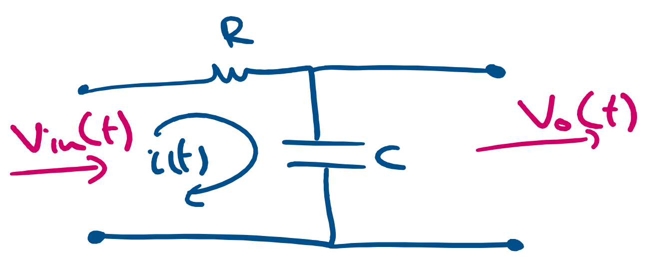
$V_{in}(s) = I(s) \times z_{equ}$.

$V_R = I \times R \longrightarrow V_R(s) = I(s) \times R$.

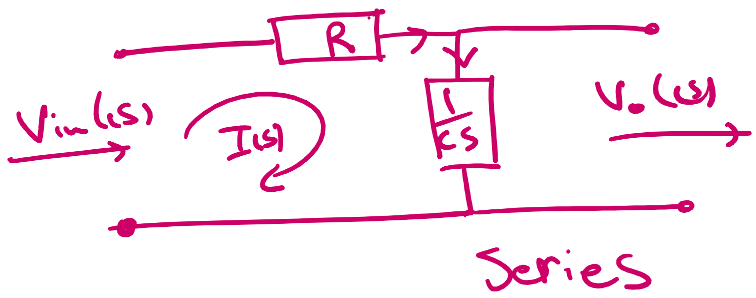
$V_L = L \times \frac{di(t)}{dt} \longrightarrow V_L(s) = I(s) \times s \times L$

$V_c = \frac{1}{C} \times \int i(t)\,dt \longrightarrow V_c(s) = I(s) \times \frac{1}{cs}$

How do we solve stuff?



1. we begin by changing from $t \to s$:



2. we find $V_o(s)$, $V_{in}(s)$

$V_o(s) = I(s) \times z_c = I(s) \times \frac{1}{cs}$

$V_{in}(s) = I(s) \times z_{equ} = I(s) \times \quad = I(s) \times \left(\frac{1}{cs} + R\right)$

$V_{in}(s) = I(s) \times Z_{equ} = I(s) \times$ $= I(s) \times \left(\frac{1}{Cs} + R\right)$

$Z_{equ} = \frac{1}{Cs} + R$

So : $TF = \frac{V_o(s)}{V_{in}(s)} = \frac{\cancel{I(s)} \times \frac{1}{Cs}}{\cancel{I(s)} \times \left(\frac{1}{Cs} + R\right)} = \frac{\frac{1}{\cancel{Cs}}}{\frac{1+RCs}{\cancel{Cs}}} = \frac{1}{1+RCs}$

One more thing : pôle

$1 + RCS = 0$

$RCS = -1 \rightarrow \boxed{S = \frac{-1}{RC}} \rightarrow$ this is called the pole.

So we might get asked : design an electrical system that can place the pole at -3 if you have $R = 1M\Omega$

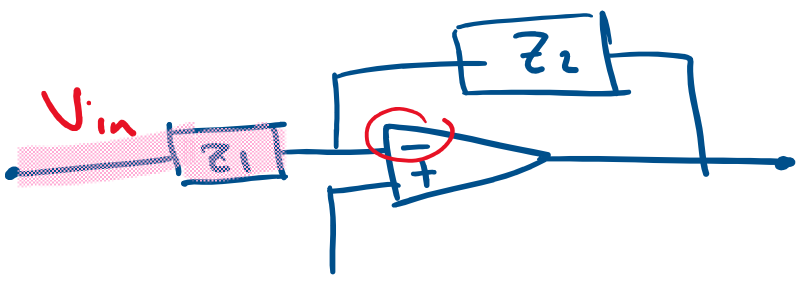
after finding all we did, we :

$\frac{\cancel{-}1}{RC} = \cancel{-}3$ $\quad RC = \frac{1}{3}$

$1 \times 10^6 \, C = \frac{1}{3}$ $\quad \boxed{C = 333 \text{ PF}}$

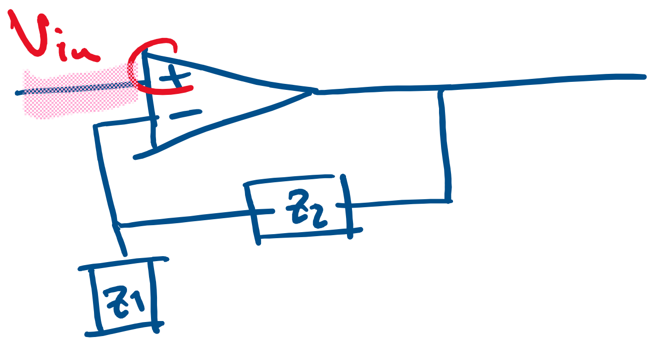
In electronics systems, we have "amplifiers" (مضخم, مصحح),
we have 2 types of amplifiers each has Tf :

① Inverting :



$$Tf = \frac{V_{out}}{V_{in}} = \frac{-Z_2}{Z_1}$$

② Non-inverting :

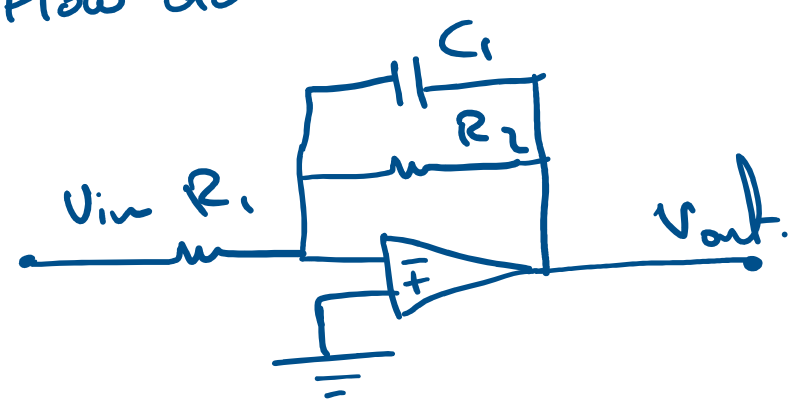


$$Tf = \frac{V_{out}}{V_{in}} = 1 + \frac{Z_2}{Z_1}$$

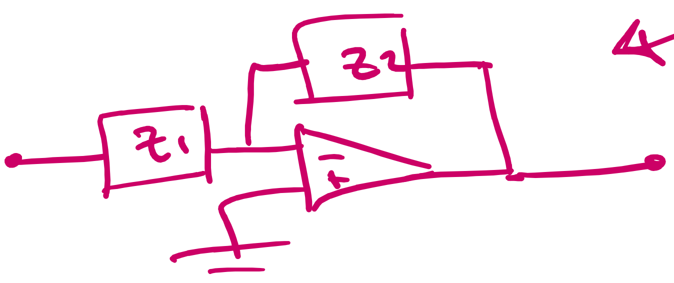
What if we had more than 1 amplifier?

$$Tf = Tf_1 \times Tf_2 \times \dots$$

How do we solve stuff?



1. we determine the type of amplifier
– in this case, inverting.



2. Find Z_1, Z_2 elements.

3. $Tf = \frac{-Z_2}{Z_1}$

$z_1 = R_1$

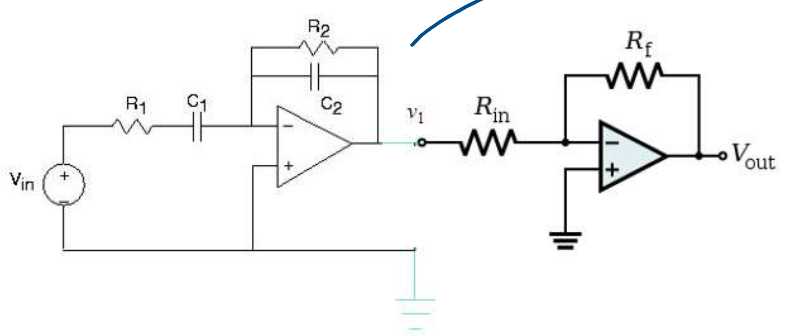
$$\frac{1}{z_2} = \frac{1}{CS} + \frac{1}{R_2} = \frac{R_2 + CS}{CSR_2}$$

$$Tf = \frac{\frac{-R_2 + CS}{CSR_2 + 1}}{R_1}$$

$$z_2 = \frac{CSR_2}{R_2 + CS}$$

Example#8

- Find out the transfer function of the following circuit.



inverting

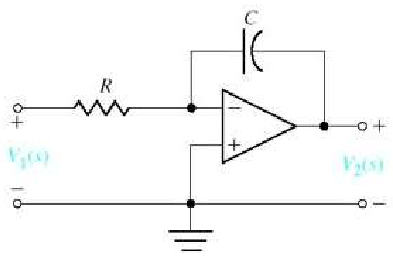
inverting

$$Tf_1 \times Tf_2 = -\frac{\frac{1}{R_2} + \frac{1}{\frac{1}{C_1 S}}}{R_1 + \frac{1}{C_2 S}} \times -\frac{R_f}{R_{in}}$$

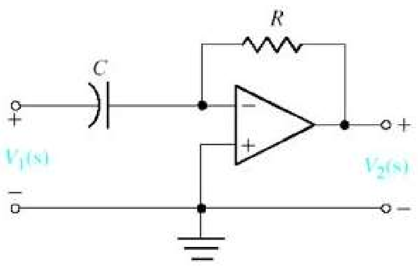
$$-\frac{\frac{1}{R_2} + C_1 S}{R_1 + \frac{1}{C_2 S}} \times -\frac{R_f}{R_{in}}$$

$$\frac{(1 + R_2 C_1 S) C_2 S}{R_1 C_2 S + 1} \times \frac{R_f}{R_{in}}$$

Examples write the transfer function for the following systems



$$\frac{V_2(s)}{V_1(s)} = \frac{-1}{RCs}$$ → integration circuit



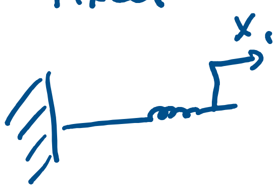
$$\frac{V_2(s)}{V_1(s)} = -RCs$$ → diffrentiation circuit

Mechanical systems

Monday, March 18, 2024 1:37 PM

① We have 3 elements which are:

- **spring**
  - fixed



$F_s = Kx$

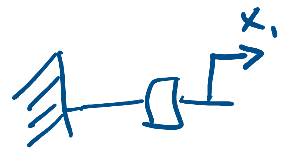
  - free

$F_s = K(x_1 - x_2)$

- **Mass**

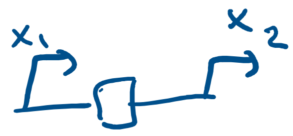
$F_M = M\ddot{x}$

- **Damper.**
  - fixed



$F_B = B\dot{x}$

  - free.



$F_B = B(\dot{x}_1 - \dot{x}_2)$

② we might get asked to find differential equation:

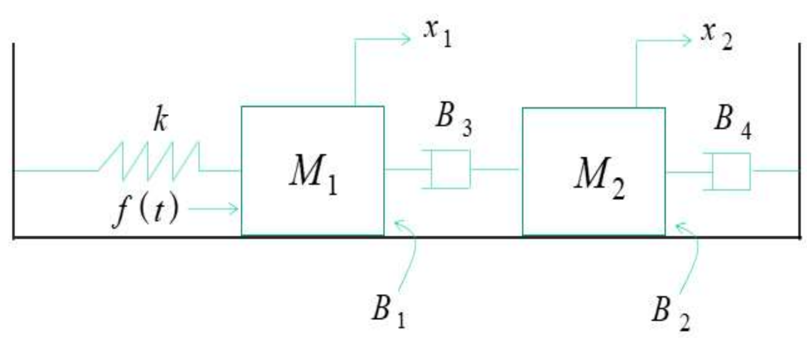
$$\bigcirc = \underbrace{\bigcirc + \bigcirc + \bigcirc}$$

number of differential equations depends on number of $\underline{\underline{x}}$'s from springs and dampers.

- $\bigcirc$ → outside Force.
- $\bigcirc + \bigcirc + \bigcirc$ → Forces above. (F_s, F_B, F_M)

Solving:

Example-6



$$① \quad f(t) = Kx_1 + M_1\ddot{x}_1 + B_1\dot{x}_1 + B_3(\dot{x}_1 - \dot{x}_2)$$

$$② \quad 0 = B_3(\dot{x}_2 - \dot{x}_1) + M_2\ddot{x}_2 + (B_2 + B_4)\dot{x}_2$$

Block Diagrams, Signal Flow Diagrams, and Masons Rule

Monday, March 18, 2024 10:09 PM

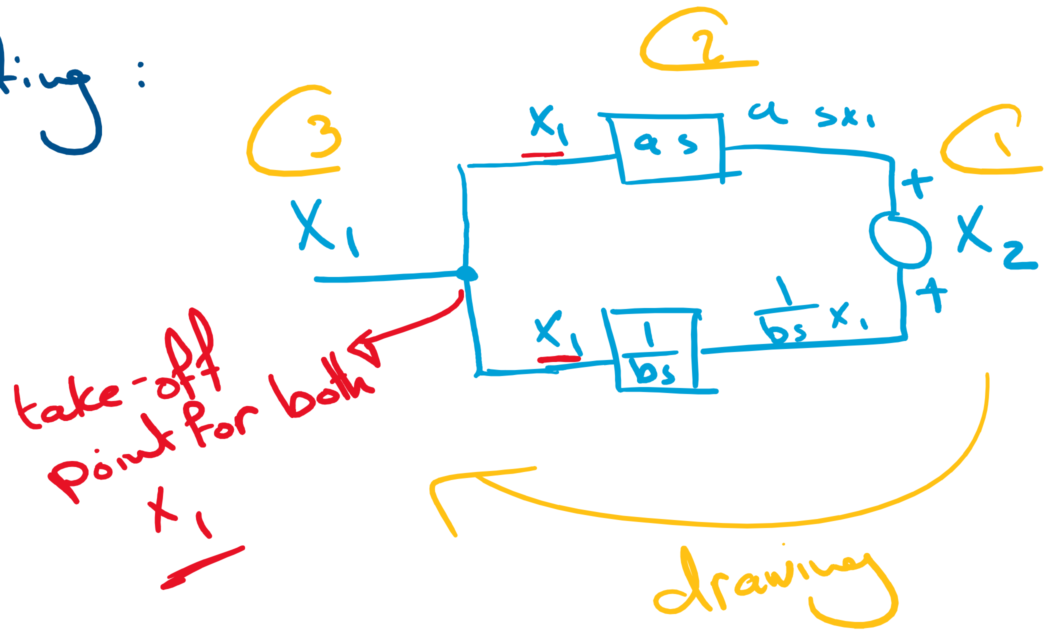
In block diagrams, we consider $\square$ as multiplication, and $\bigcirc$ as $\mp$, so we can use that to represent a system. (also, "division" is considered "multiplication".

Example:

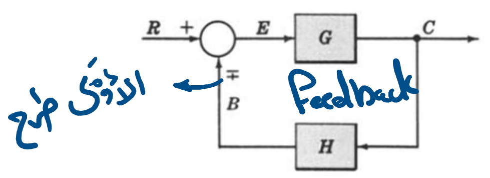
$$x_2 = a_1 \frac{dx_1}{dt} + b \int x_1 \, dt.$$

$\frac{dx_1}{dt} : s x_1$ $\quad\quad$ $\int x_1 \, dt : \frac{x_1}{s}$

representing:



Canonical Form of A Feedback Control System



$G \equiv$ direct transfer function $\equiv$ forward transfer function

$H \equiv$ feedback transfer function

$GH \equiv$ loop transfer function $\equiv$ open-loop transfer function

$C/R \equiv$ closed-loop transfer function $\equiv$ control ratio $\quad \frac{C}{R} = \frac{G}{1 \pm GH}$

$E/R \equiv$ actuating signal ratio $\equiv$ error ratio $\quad \frac{E}{R} = \frac{1}{1 \pm GH}$

$B/R \equiv$ primary feedback ratio $\quad \frac{B}{R} = \frac{GH}{1 \pm GH}$

Characteristic Equation

- The control ratio is the closed loop transfer function of the system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

- The denominator of closed loop transfer function determines the characteristic equation of the system.

- Which is usually determined as:

$$1 \pm G(s)H(s) = 0$$

c

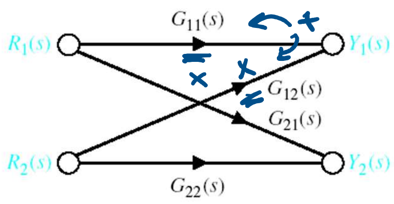
In signal flow diagrams, $(\longrightarrow)$ is considered multiplication, and $\bigcirc$ is still $\pm$.

* Any arrow going INTO a variable is its [illegible], not going out

Signal-Flow Graph Models

$$Y_1(s) = G_{11}(s)\cdot R_1(s) + G_{12}(s)\cdot R_2(s)$$

$$Y_2(s) = G_{21}(s)\cdot R_1(s) + G_{22}(s)\cdot R_2(s)$$



Lastly, when we have many loops and want to find the transfer function (TF), we want to use Mason's Rule.

Mason's Rule:

- The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

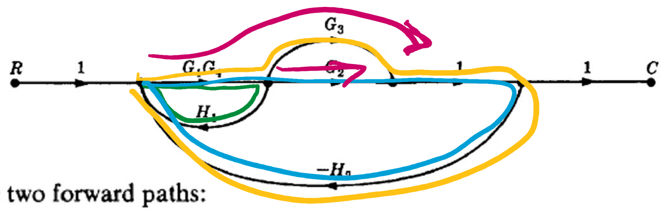
$\Delta = 1 - L_1 + \Sigma L_1 L_2 - \Sigma L_{1,2,3} \ldots$

Where

n = number of forward paths.
P_i = the i^{th} forward-path gain.
Δ = Determinant of the system
Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since Δ=0 is the system characteristic equation.

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$P_1 = G_1G_2G_4$ $\quad$ $P_2 = G_1G_3G_4$

Therefore,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

Example#1: Continue

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

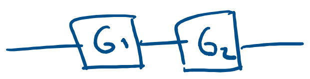
$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

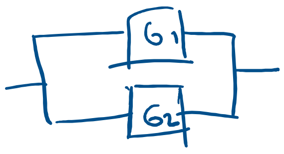
Reducing Block diagrams +TF

Friday, March 22, 2024 5:53 PM

I'm going to summarize 2 ways to reduce block diagrams and finding the transfer function:

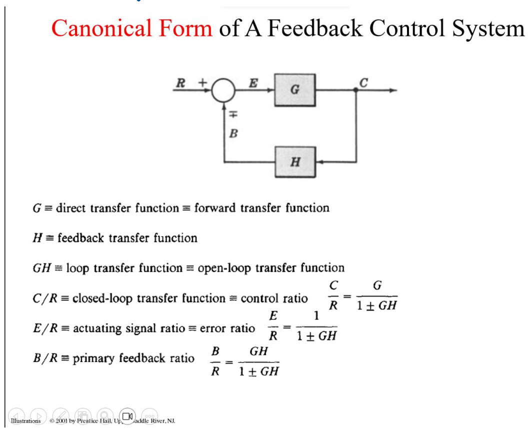
[1] Superposition.

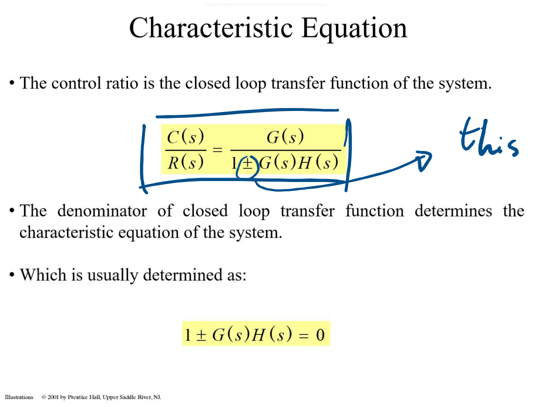
①  $= G_1 G_2$

 $= G_1 + G_2$

② Summing junctions ⊗ can be تبدل مكانها between the same take off points without going out of them, in order to have a better understanding of everything.

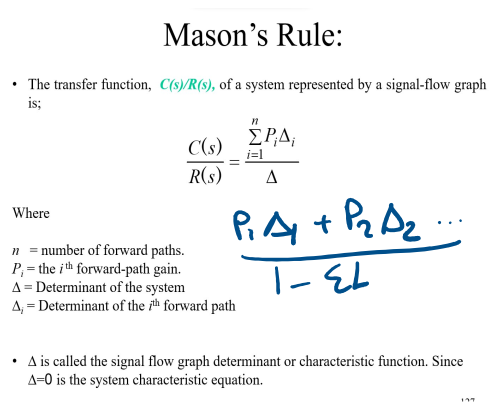
③ TF can then be found by finding the feedback loops little by little just like slides 94-101

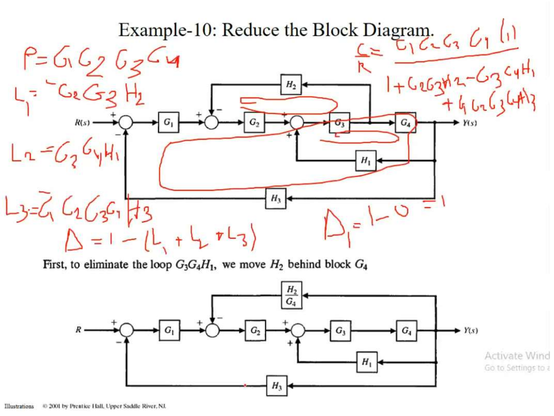


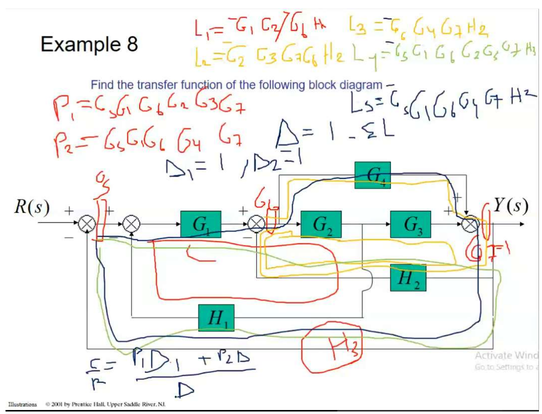


this [illegible] is opposite the summing junction [illegible]

[2] Mason's Rule.







① First, any 2 summing junctions directly after each other, or a summing junction and a take off point, we have to put $\boxed{G_n}$ between them.

② We find ΣG or ΣP going through them as many times to get them all

ΣP_1

ΣP_2

⋮

③ We find all feedback loops without تغيير anything like we did in superposition

④ We find TF using Mason's Rule, Δ is $(1 - \Sigma L_{1,2,3\ldots})$ if the three loops have a G_n in common and Δ_i is $\Delta = 1 - L_1 + \Sigma L_1 L_2 -$, but is usually 1 when L_i has what is in common between it and other loops.

and Δ_i is $\Delta = 1 - L_1 + \Sigma L_1, L_2 - \Sigma L_{1,2,3} \ldots$, but is usually 1 when L_i has what is in common between it and other loops.

First order systems

Monday, April 1, 2024 12:40 PM

Introduction

The time response of a control system consists of two parts:

1. Transient response
   - from initial state to the final state – purpose of control systems is to provide a desired response.

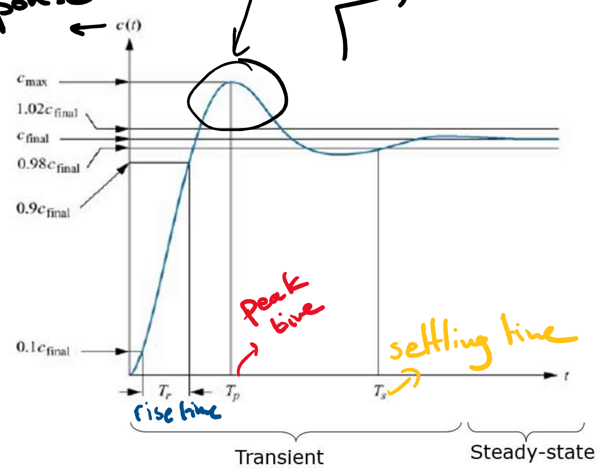
   وقت الوصول الى النتيجة

2. Steady-state response
   - the manner in which the system output behaves as t approaches infinity – the error after the transient response has decayed, leaving only the continuous response.

   كيف يعمل ال system بعد ما يوصل النتيجة

overshoot → second order system.

step response.



rise time: needed time to get from 10%-90% of c_{final} (final value)

Peak time: needed time to get to maximum (c_{max})

settling time: needed time to get to (settle to) c_{final}.

First – order system

A first-order system without zeros can be represented by the following transfer function

$$TF = \frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$$

$$\frac{C}{R} = \frac{K\ (c_{final})}{\tau s + 1}$$

First – order system

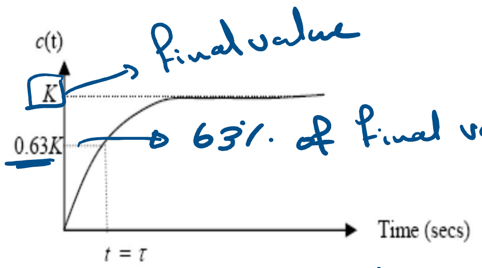
Taking inverse Laplace transform, we have the step response

$$\boxed{c(t) = 1 - e^{-\frac{t}{\tau}}}$$

Time Constant: If $t = \tau$, So the step response is $C(\tau) = (1 - 0.37) = 0.63$

First – order system

Plot $c(t)$ versus time:

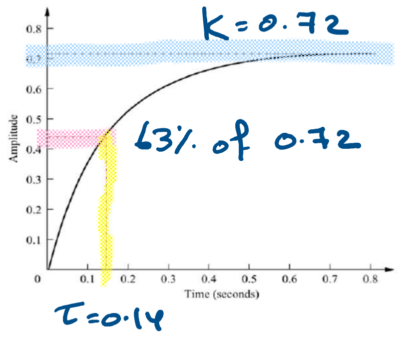


→ τ found at 63% of final value.

First – order system

Example 1

The following figure gives the measurements of the step response of a first-order system, find the transfer function of the system.



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$$TF = \frac{C}{R} = \frac{K}{\tau s + 1}$$

$$\frac{0.72}{0.14s + 1}$$

First – order system
Transient Response Analysis

Rise Time T_r:

The rise-time (symbol T_r units s) is defined as the time taken for the step response to go from **10% to 90%** of the final value.

$$\boxed{T_r = 2.31\tau - 0.11\tau = 2.2\tau}$$

Settling Time T_s:

Defined the settling-time (symbol T_s units s) to be the time taken for the step response to come to within **2% of the final value** of the step response.

$$\boxed{T_s = 4\tau}$$

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$T_p = 0$

Second order systems

Monday, April 1, 2024 1:08 PM

Second – Order System

- A general second-order system is characterized by the following transfer function:

$$G(s)=\frac{b}{s^2+as+b}$$

- We can re-write the above transfer function in the following form (closed loop transfer function):

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

ζ is damping ratio

$\omega_n=\sqrt{b}$

↳ natural frequency.

Second – Order System

ω_n ($\omega_n=\sqrt{b}$) - referred to as the *un-damped natural frequency* of the second order system, which is the frequency of oscillation of the system without damping.

ζ ($\zeta=\frac{a}{2\sqrt{b}}$) - referred to as the *damping ratio* of the second order system, which is a measure of the degree of resistance to change in the system output.

Poles; $-\omega_n\zeta+\omega_n\sqrt{\zeta^2-1}$, $-\omega_n\zeta-\omega_n\sqrt{\zeta^2-1}$ Poles are complex if $\zeta<1!$

Second – Order System

- According the value of ζ, a second-order system can be set into one of the four categories:
1. *Overdamped* - when the system has two real distinct poles ($\zeta>1$).
2. *Underdamped* - when the system has two complex conjugate poles ($0<\zeta<1$)
3. *Undamped* - when the system has two imaginary poles ($\zeta=0$).
4. *Critically damped* - when the system has two real but equal poles ($\zeta=1$).

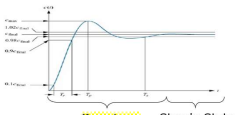
Time-Domain Specification

Given that the closed loop TF

$$T(s)=\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

The system (2nd order system) is parameterized by ζ and ω_n

For $0<\zeta<1$ and $\omega_n>0$, we like to investigate its response due to a unit step input



Transient Steady State

Two types of responses that are of interest:
(A) Transient response
(B) Steady state response

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(A) For transient response, we have 4 specifications:

(a) T_r – rise time $=\frac{\pi-\theta}{\omega_n\sqrt{1-\zeta^2}}$

(b) T_p – peak time $=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\qquad \theta=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

(c) %MP – percentage maximum overshoot $=e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}x100\%$

(d) T_s – settling time (2% error) $=\frac{4}{\zeta\omega_n}$

(B) Steady State Response

(a) Steady State error

Therefore, $$\%MP=e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}x100\%$$

%OS

- For given %*OS*, the damping ratio can be solved from the above equation;

$$\zeta=\frac{-\ln(\%MP/100)}{\sqrt{\pi^2+\ln^2(\%MP/100)}}$$

UNDERDAMPED

Example 2: *Find the natural frequency and damping ratio for the system with transfer function*

Solution: $$G(s)=\frac{36}{s^2+4.2s+36}$$

Compare with general TF

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\boxed{\omega_n=6}$

$\boxed{\zeta=0.35}\rightarrow 2\zeta\omega_n=4.2$

UNDERDAMPED

Example 3: Given the transfer function

$$G(s)=\frac{100}{s^2+15s+100}$$

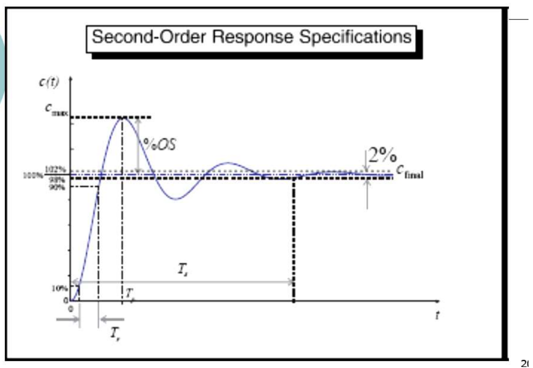
find T_s, $\%OS$, T_p

Solution:

$$\omega_n=10 \qquad \xi=0.75$$

$$T_s=0.533s,\ \%OS=2.838\%,\ T_p=0.475s$$

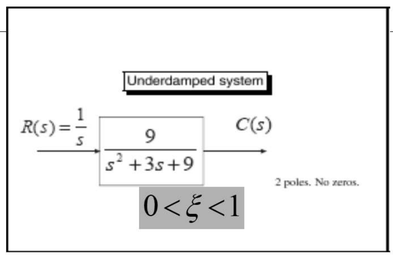
UNDERDAMPED



Underdamped Response

$$G(s)=\frac{b}{s^2+as+b}$$

a = 3



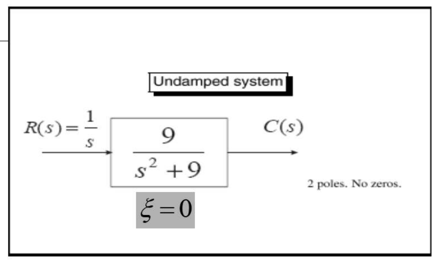
$$c(t)=K_1+e^{-1.5t}(K_2\cos 2.598t+K_3\sin 2.598t)$$

s = 0; s = -1.5 ± j2.598 (*two complex poles*)

Undamped Response

$$G(s)=\frac{b}{s^2+as+b}$$

a = 0



$$c(t)=K_1+K_2\cos 3t$$

s = 0; s = ± j3 (*two imaginary poles*)

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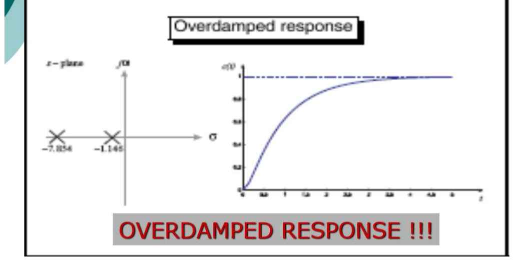
$$c(t)=K_1+K_2e^{-7.854t}+K_3e^{-1.146t}$$

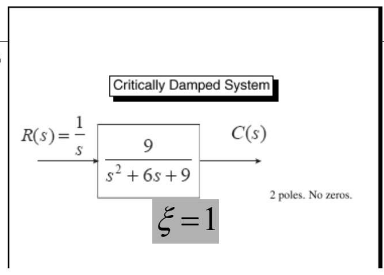
s= 0; s = -7.854; s = -1.146 (*two real poles*)



Critically Damped System

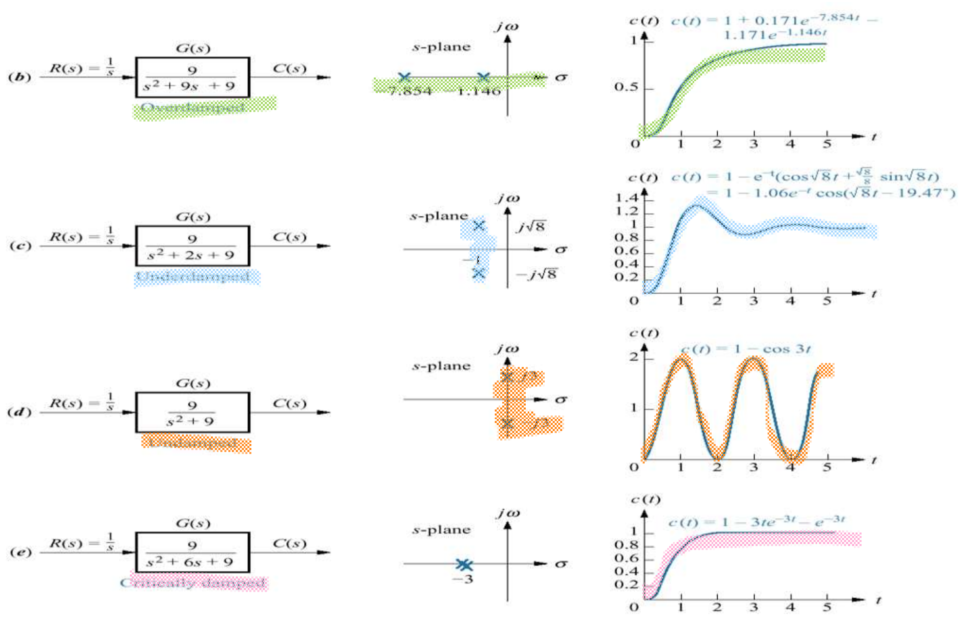
$$G(s)=\frac{b}{s^2+as+b}$$

a = 6

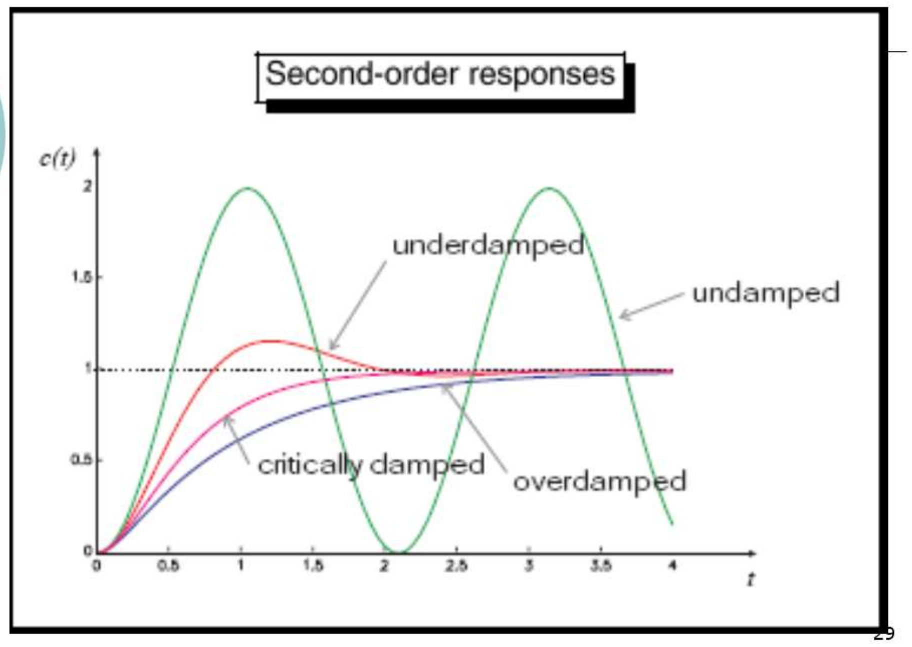


$$c(t)=K_1+K_2e^{-3t}+K_3te^{-3t}$$

S = 0; s = -3,-3 (*two real and equal poles*)



Second – Order System



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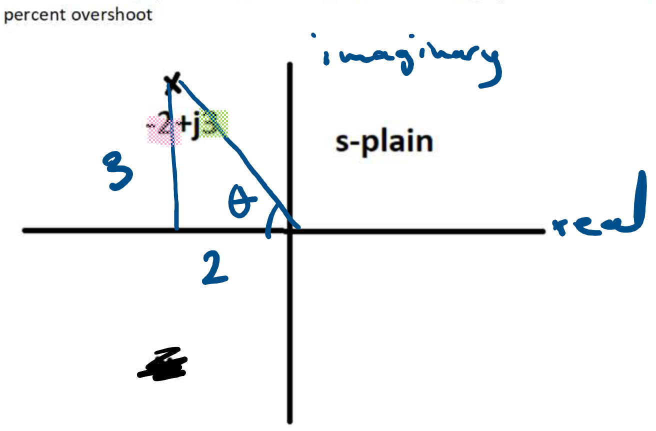
EXAMPLE

Tuesday, April 2, 2024 9:59 PM

For the following system with roots shown in the following s-plane find the rise time, settling time and percent overshoot



$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$\frac{\sqrt{1-\xi^2}}{\xi} = \frac{3}{2}$$

$$\left(2\sqrt{1-\xi^2}\right)^2 = (3\xi)^2$$

$$4(1-\xi^2) = 9\xi^2$$

$$4 - 4\xi^2 = 9\xi^2$$

$$4 = 13\xi^2$$

$$\boxed{\xi = 0.55}$$

So:

$$T_s = \frac{4}{\underbrace{\xi \omega_n}_{\text{real part}}} = \frac{4}{2} = 2\,s$$

$$56.3 \times \frac{\pi}{180} \rightarrow 0.98$$

$$T_r = \frac{\pi - \theta}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi - 0.98}{3} = 0.72\,s$$

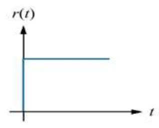
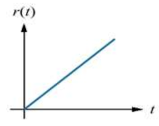
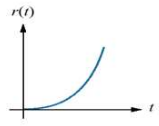
θ in rad

$$OS\% = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\% = e^{\frac{-\pi \times 0.55}{\sqrt{1-0.55^2}}} \times 100\% = 12.6\%$$

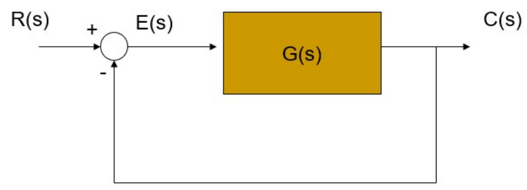
Steady State Error (ess)

Friday, April 5, 2024 10:24 PM

Test Waveform for evaluating steady-state error

Waveform	response Name	Physical interpretation	Time function	Laplace transform R(s)
	Step	Constant position	1 Σ or any num	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

Steady-state error analysis



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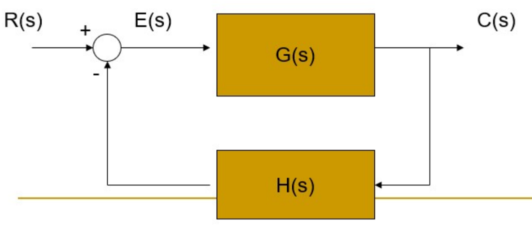
Unity feedback
H(s)=1



Non-unity feedback
H(s)≠1

Substitute (2) into (1)

$$\therefore E(s) = R(s) - \frac{G(s)}{1+G(s)}R(s) = \frac{1}{1+G(s)}R(s) \longrightarrow (3)$$

error

Steady-state error analysis

Based on equation (3), it can be seen that E(s) depends on:

(a) Input signal, R(s)

(b) G(s), open loop transfer function

Now, assume:

$$G(s) = \frac{K \underset{i=1}{\overset{M}{\pi}}(s+z_i)}{S^N \underset{j=1}{\overset{\theta}{\pi}}(s+p_j)}$$

type N

Cases to be considered:

$$(A)\ R(s) = \frac{1}{s}$$ step

$$(B)\ R(s) = \frac{1}{s^2}$$ ramp

$$(C)\ R(s) = \frac{1}{s^3}$$ parabola

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Case (A): Input is a unit step R(s)=1/s

$$E(s) = \frac{1}{1+G(s)}R(s) = \frac{1/s}{1+G(s)}$$

$$e_{ss} = Steady-State\_Error = \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} s\left[\frac{1/s}{1+G(s)}\right] = \lim_{s\to 0}\left[\frac{1}{1+G(s)}\right]$$

$$= \left[\frac{1}{1+\lim_{s\to 0} G(s)}\right] = \left[\frac{1}{1+K_p}\right]$$

where $\boxed{K_p = \lim_{s\to 0} G(s)}$ → "Static Position Error Constant"

If N = 0, K_p = constant $\boxed{e_{ss} = \frac{1}{1+K_p}} = finite$

If N ≥ 1, K_p = infinite $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$

For unit step response, as the type of system increases (N ≥ 1), the steady state error goes to zero

$$E(s)=\frac{1}{1+G(s)}R(s)=\frac{1/s}{1+G(s)}$$

$$e_{ss}=Steady-State\_Error \cong \lim_{s\to 0} sE(s)$$

$$e_{ss}=\lim_{s\to 0} s\left[\frac{1/s}{1+G(s)}\right]=\lim_{s\to 0}\left[\frac{1}{1+G(s)}\right]$$

$$=\left[\frac{1}{1+\lim_{s\to 0} G(s)}\right]=\left[\frac{1}{1+K_p}\right]$$

where $K_p=\lim_{s\to 0} G(s)$ → "Static Position Error Constant"

If N = 0, K_p = constant $\boxed{e_{ss}=\frac{1}{1+K_p}}$ = *finite*

If N ≥ 1, K_p = infinite $e_{ss}=\frac{1}{1+K_p}=\frac{1}{1+\infty}=0$

For unit step response, as the type of system increases (N ≥ 1), the steady state error goes to zero

Case (B): Input is a unit ramp $R(s)=1/s^2$

$$E(s)=\frac{1}{1+G(s)}R(s)=\frac{1/s^2}{1+G(s)}$$

$$e_{ss}=Steady-State\_Error \cong \lim_{s\to 0} sE(s)$$

$$e_{ss}=\lim_{s\to 0} s\left[\frac{1/s^2}{1+G(s)}\right]=\lim_{s\to 0}\left[\frac{1}{s+sG(s)}\right]$$

$$=\left[\frac{1}{0+\lim_{s\to 0} sG(s)}\right]=\left[\frac{1}{\lim_{s\to 0} sG(s)}\right]\cong\frac{1}{K_v}$$

where $K_v=\lim_{s\to 0} sG(s)$ → "Static Velocity Error Constant"

If N = 0, $K_v = s\frac{\pi(s+z_i)}{\pi(s+p_j)}=0$, $\boxed{e_{ss}=\frac{1}{K_v}}=\infty$

If N = 1, K_v = finite $e_{ss}=\frac{1}{K_v}$ = *finite*

If N ≥ 2, K_v = infinite $e_{ss}=\frac{1}{K_v}=\frac{1}{\infty}=0$

For unit ramp response, the steady state error in infinite for system of type zero, finite steady state error for system of type 1, and zero steady state error for systems with type greater or equal to 2.

Case (C): Input is a parabolic, $R(s)=1/s^3$

$$E(s)=\frac{1}{1+G(s)}R(s)=\frac{1/s^3}{1+G(s)}$$

$$e_{ss}=Steady-State\_Error \cong \lim_{s\to 0} sE(s)$$

$$=\lim_{s\to 0} s\left[\frac{1/s^3}{1+G(s)}\right]=\lim_{s\to 0}\left[\frac{1}{s^2+s^2G(s)}\right]$$

$$=\left[\frac{1}{0+\lim_{s\to 0} s^2G(s)}\right]=\left[\frac{1}{\lim_{s\to 0} s^2G(s)}\right]\cong\frac{1}{K_a}$$

where $K_a=\lim_{s\to 0} s^2G(s)$ → "Static Acceler Error Consta

If N = 0, $K_a = s^2\frac{\pi(s+z_i)}{\pi(s+p_j)}=0$, $\boxed{e_{ss}=\frac{1}{K_a}}=\infty$

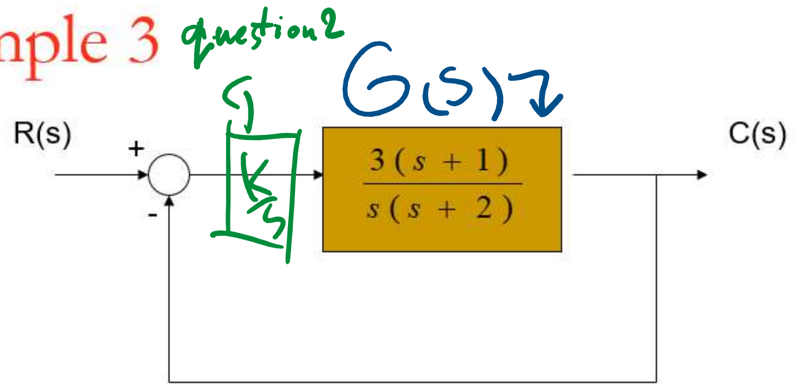
If N = 1, K_a = 0 $e_{ss}=\frac{1}{K_a}=\infty$

If N = 2, K_a = constant $e_{ss}=\frac{1}{K_a}$ = *finite*

If N ≥ 3, K_a = infinite $e_{ss}=\frac{1}{K_a}=\frac{1}{\infty}=0$

→ Increasing system type (N) will accommodate more different inputs.

Example 3 question 2



If r(t) = (2+3t)u(t), find the steady state error (e_{ss}) for the given system.

Solution:

$K_p=\lim_{s\to 0} G(s)=\infty$

$K_v=\lim_{s\to 0} sG(s)=3/2$

$$e_{ss}=\frac{2}{1+K_p}+\frac{3}{K_v}=\frac{2}{1+\infty}+\frac{3}{3/2}=2$$

① what if $G(s)$ was $= \frac{30(s+1)}{s(s+2)}$? $\frac{e_{ss}}{10} = 0.2$

② Add a controller that would make $e_{ss}=0$? #

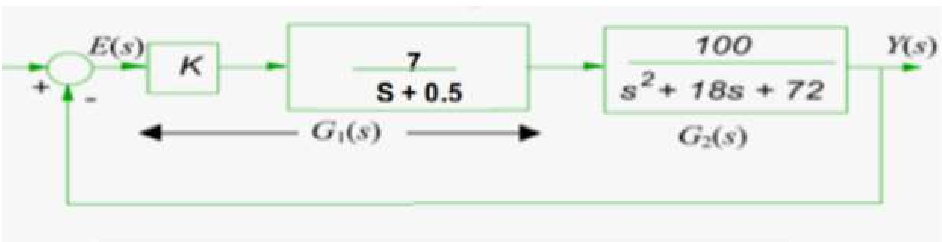
added, so that in K_v $\frac{K}{S}$ would cancel $\lim S\,G(s)$ and ∞ would be the answer so that e_{ss} would become 0

steps:

① look at $r(t)$, see step, ramp, parabola

② Find K_p, K_v, K_a.

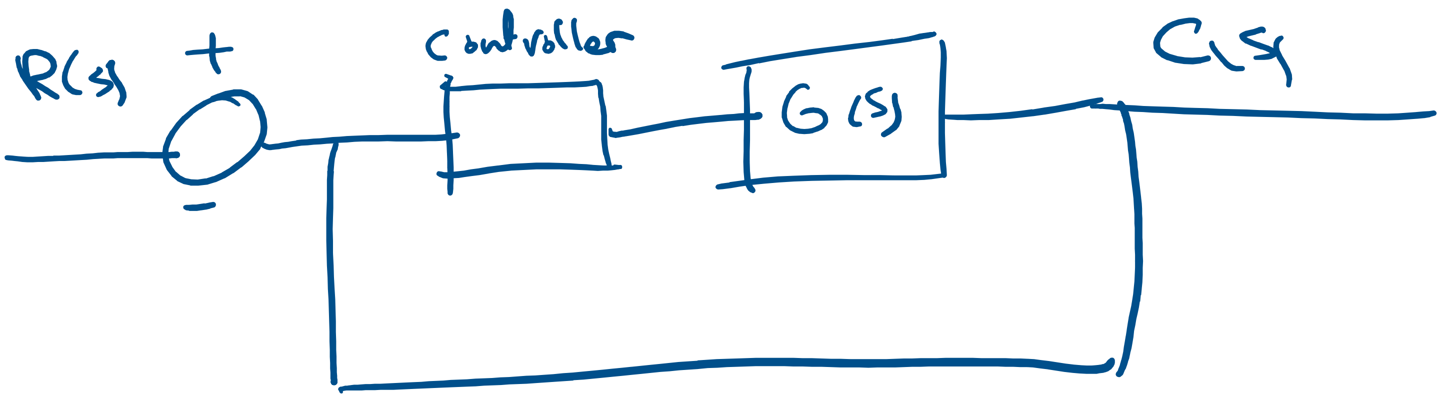
③ Find e_{ss} for each one and add them



$$G(s) = K\,G_1(s)\,G_2(s)$$

EXAMPLE

Monday, April 8, 2024 11:04 PM



given that conroller is K and G(s) = " ? "
find step steady state error e_{ss} that makes it less than 0.5 (no need to calculate K)

$$K_p = \lim_{s \to 0} K \times G(s)$$

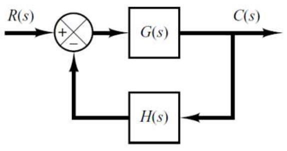
$$e_{ss} = \frac{1}{1 + K_p} < 0.5$$

$$(1 + K_p) \times 0.5 < 1$$

Root Locus

Thursday, April 11, 2024 11:14 PM

- Consider the system shown in following figure.



- The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

- The characteristic equation is obtained by setting the denominator polynomial equal to zero.

- Or
$$1+G(s)H(s)=0$$
$$G(s)H(s)=-1$$

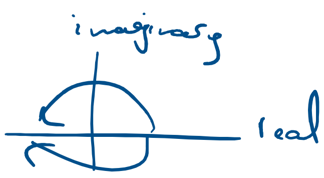
- Where *G(s)H(s)* is a ratio of polynomial in s. $\sigma + j\omega$

- Since *G(s)H(s)* is a complex quantity it can be split into angle and magnitude part.

- Angle Condition

$$\angle G(s)H(s) = \pm 180^{\circ}(2k+1) \qquad (k=1,2,3\ldots)$$

حسب عدد اللفات



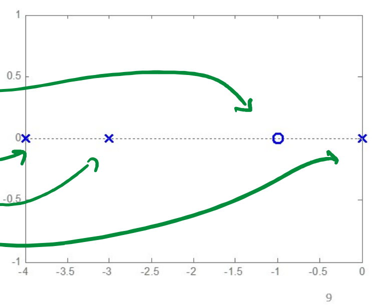
- Magnitude Condition

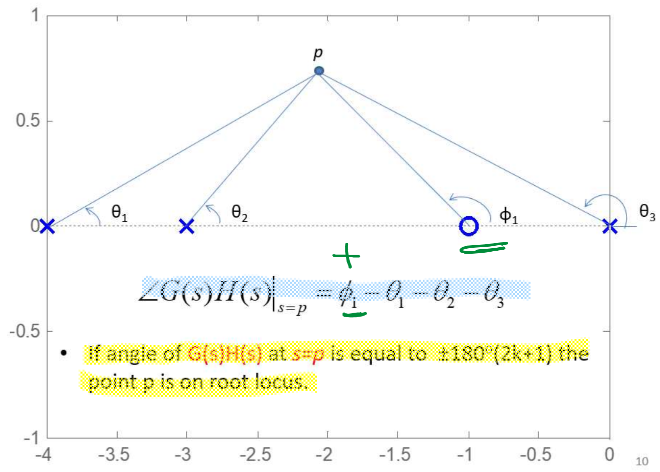
$$|G(s)H(s)| = 1$$

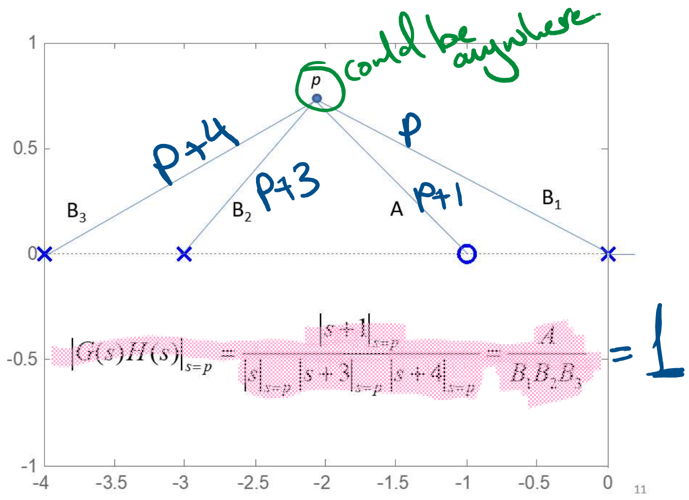
- To apply Angle and magnitude conditions graphically we must first draw the poles and zeros of G(s)H(s) in s-plane.

- For example if G(s)H(s) is given by

$$G(s)H(s) = \frac{s+1}{s(s+3)(s+4)}$$



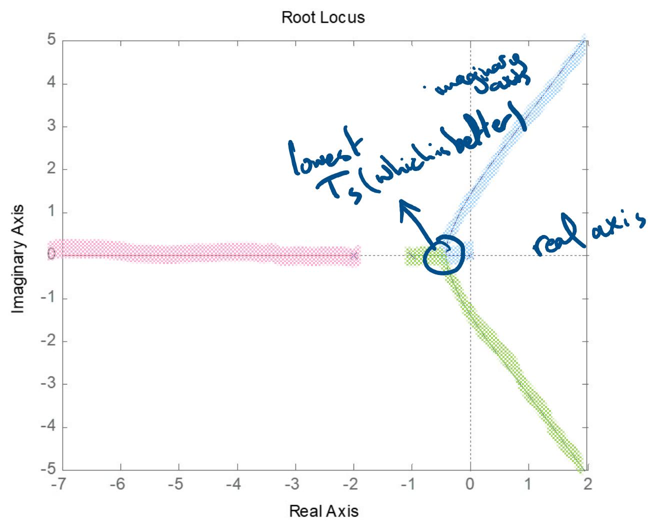




There is illustrative example #1, see it.

Root Locus

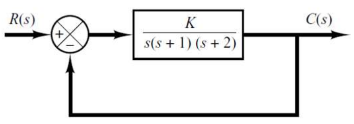
Friday, April 12, 2024 10:37 PM



$$T_s = \frac{4}{\text{real part}}$$

Example#1

- Consider following unity feedback system.



- Determine the value of K such that the damping ratio of a pair of dominant complex-conjugate closed-loop poles is 0.5.

$\xi = 0.5$

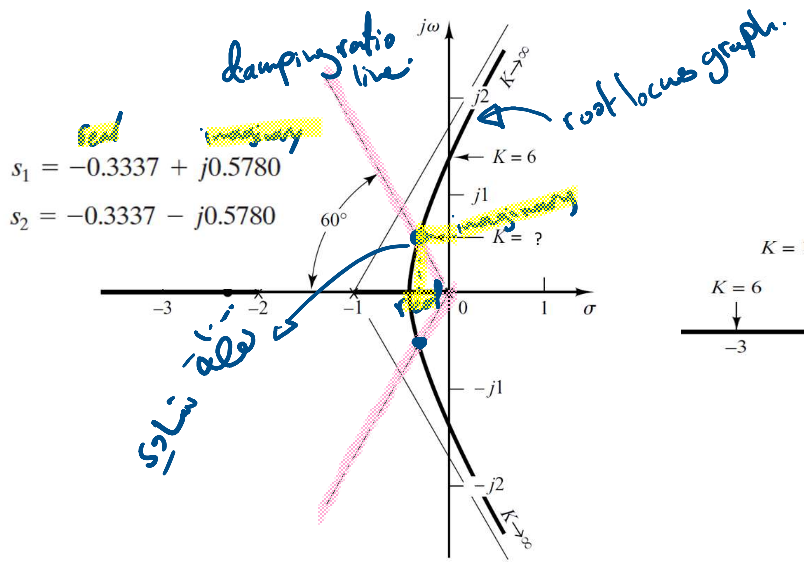
$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

- The damping ratio of 0.5 corresponds to

$$\zeta = \cos\theta$$

$$\theta = \cos^{-1}\zeta$$

$$\theta = \cos^{-1}(0.5) = 60°$$



- The value of K that yields such poles is found from the magnitude condition

$$\left|\frac{K}{s(s+1)(s+2)}\right|_{s=-0.3337+j0.5780} = 1$$

where

نحسبها بال calculator

$$K = |s(s+1)(s+2)|_{s=-0.3337+j0.5780}$$

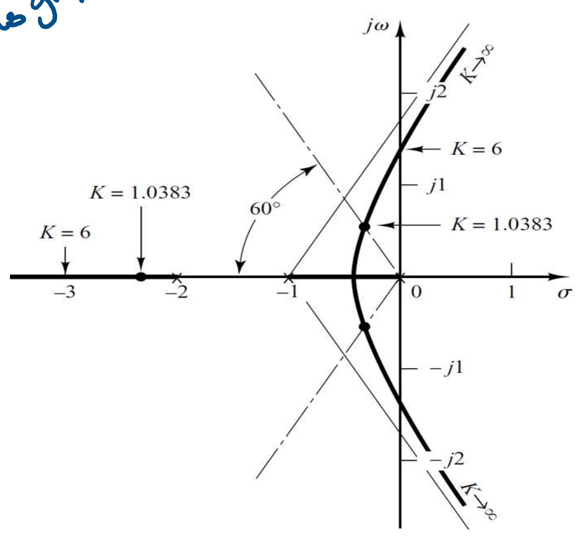
$$= 1.0383$$

- The third closed loop pole at K=1.0383 can be obtained as

$$1 + G(s)H(s) = 1 + \frac{K}{s(s+1)(s+2)} = 0$$

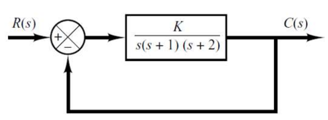
$$1 + \frac{1.0383}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2) + 1.0383 = 0$$



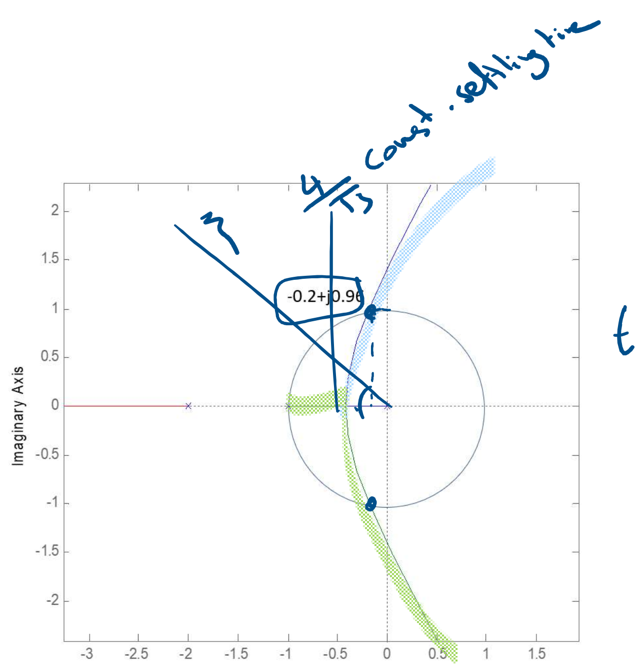
Home Work

- Consider following unity feedback system.



- Determine the value of K such that the natural undamped frequency of dominant complex-conjugate closed-loop poles is 1 rad/sec. $w_n = 1$

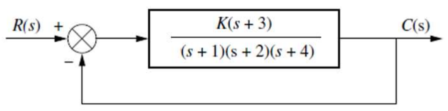
$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

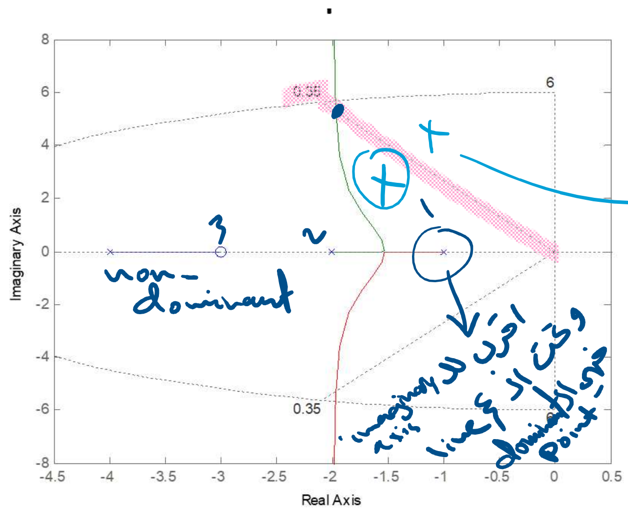


then find K from G(s)

Example#2

- Sketch the root locus of following system and determine the location of dominant closed loop poles to yield maximum overshoot in the step response less than 30%.





anything below ζ line is actually better in damping ratio.

Example#2

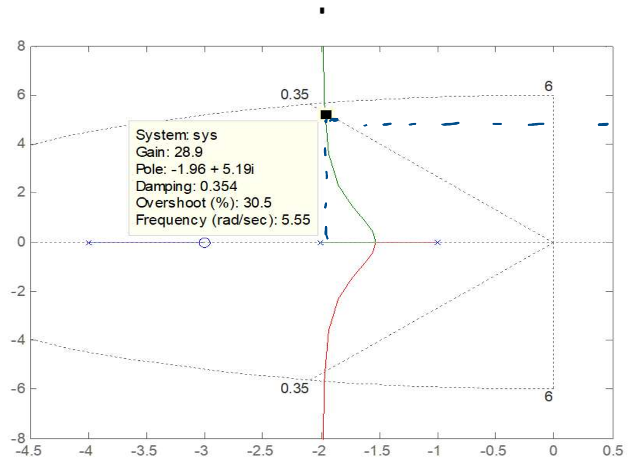
- Mp<30% corresponds to

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

$$30\% = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

$$\zeta > 0.35$$

from ζ of PO



* There is 1st, 2nd order, root locus, see them.

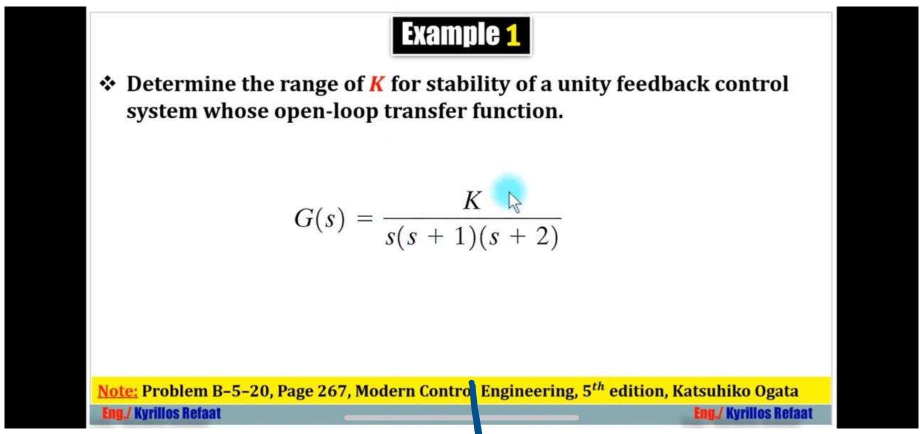
video quiz

EXAMPLE-quiz

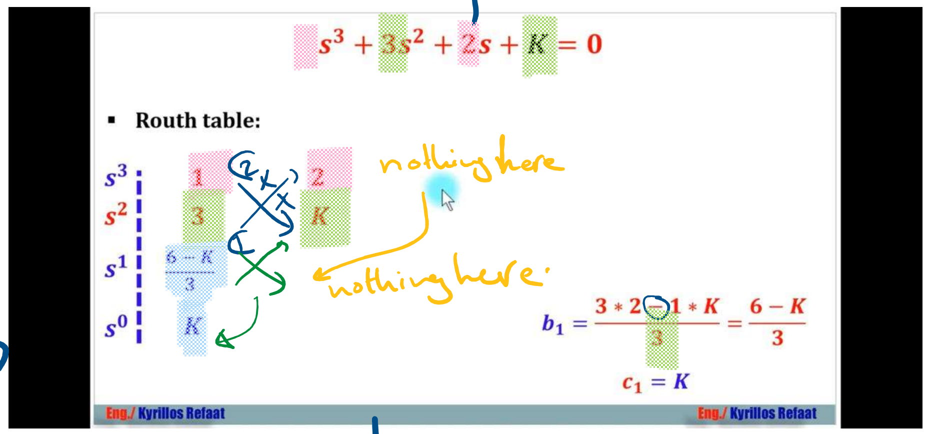
Monday, April 22, 2024 9:28 PM

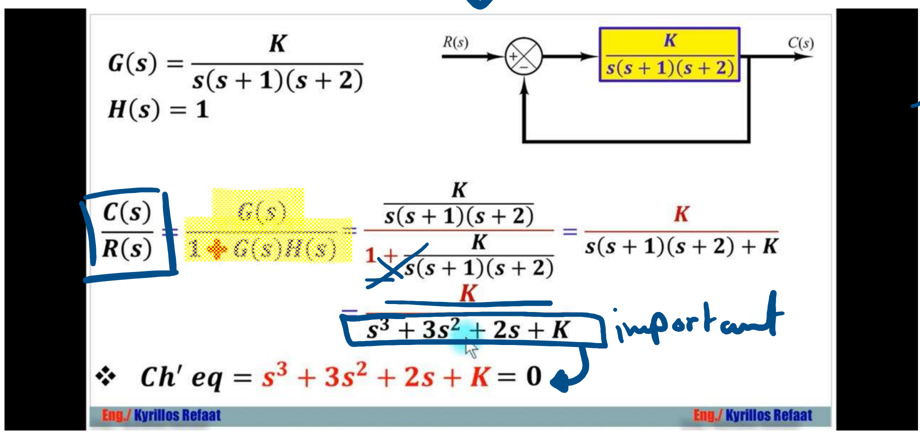
Routh Hurwitz

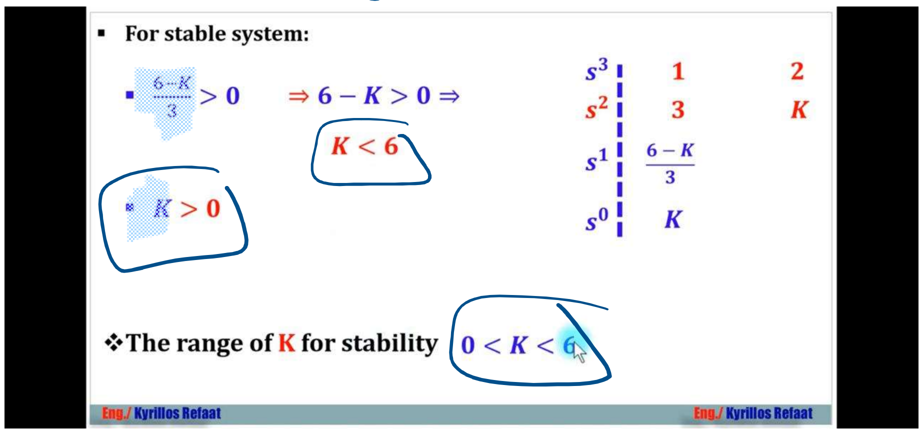
Tuesday, May 14, 2024 9:29 PM



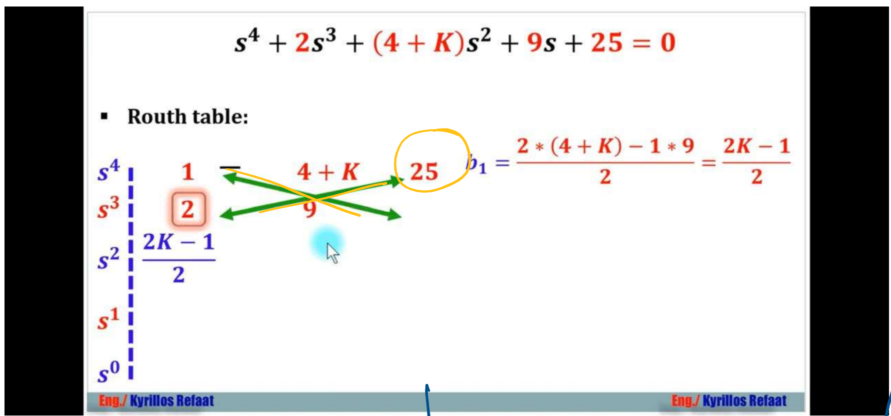
→ characteristic equation.

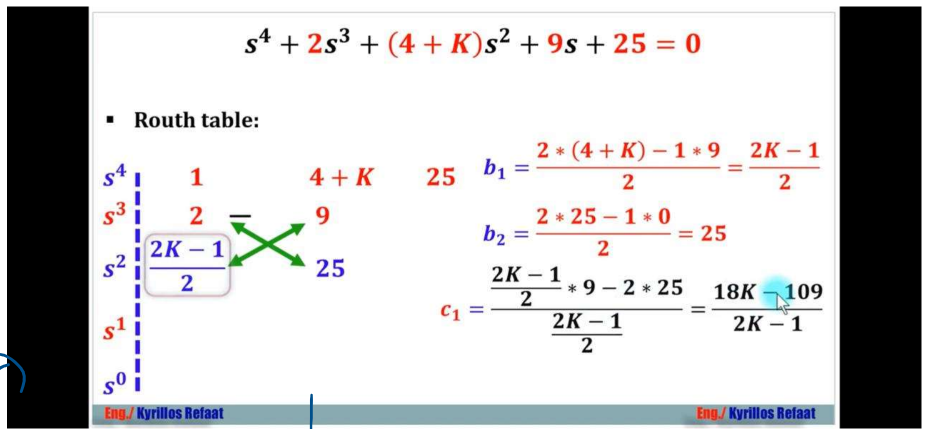


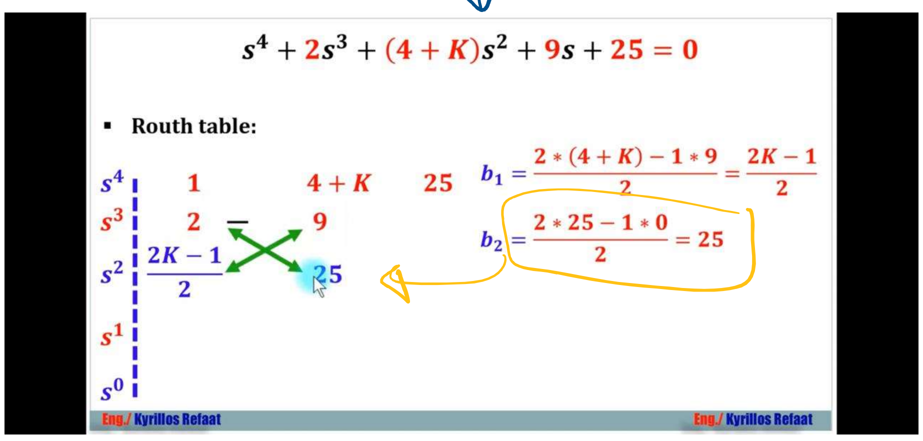


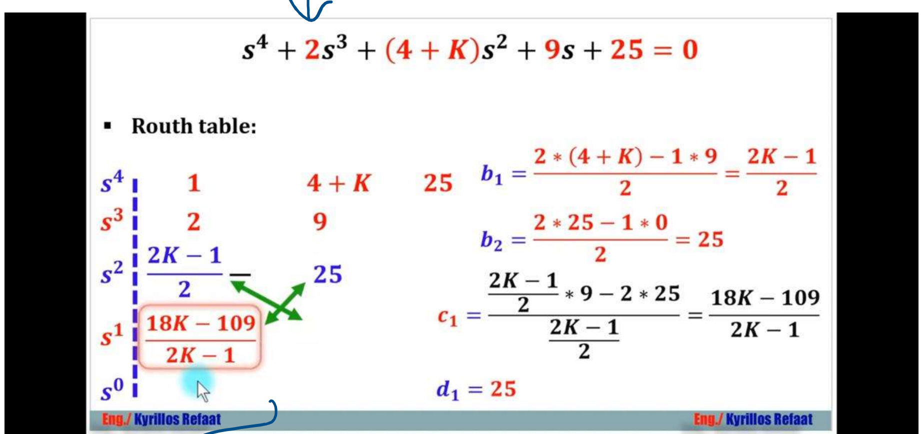


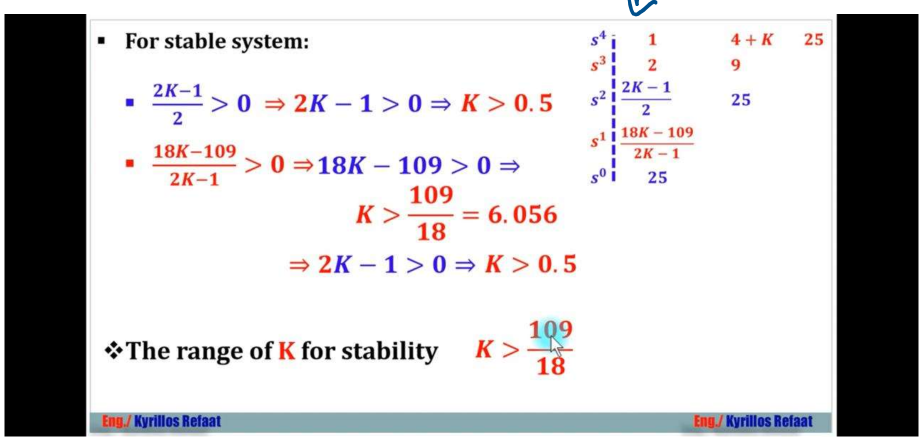
What about more s^n?



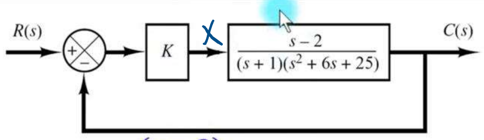








What about more G(s) components?



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{K\dfrac{(s-2)}{(s+1)(s^2+6s+25)}}{1 + K\dfrac{(s-2)}{(s+1)(s^2+6s+25)}}$$

$$= \frac{K(s-2)}{(s+1)(s^2+6s+25) + K(s-2)}$$

$$= \frac{K(s-2)}{s^3 + 7s^2 + (31+K)s + 25 - 2K}$$

Eng./ Kyrillos Refaat

Eng./ Kyrillos Refaat

PID controllers

Tuesday, May 14, 2024 10:50 PM

P → Proportional.

I → Integral

D → Derivative.

The Characteristics of P, I, and D controllers

	T_R	OS%	T_s	e_{ss}
CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp ↑	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

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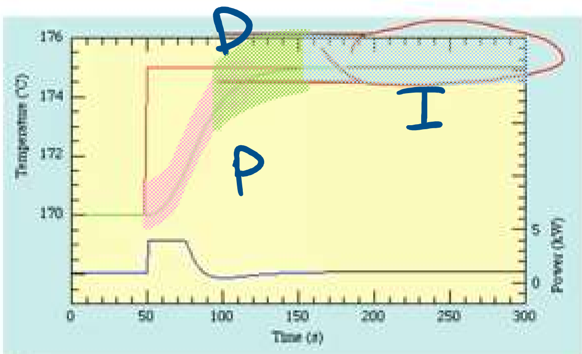
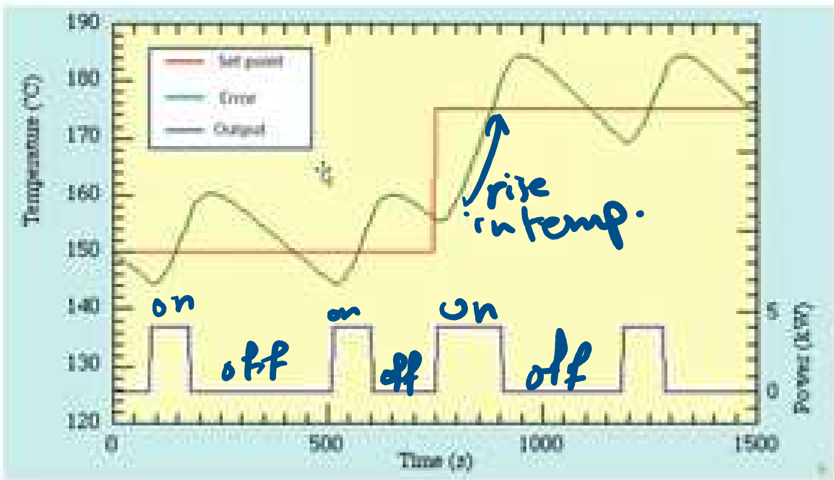
Proportional Plus Integral Plus Derivative Control (PID)

- Although PD control deals neatly with the overshoot and ringing problems associated with proportional control it does not cure the problem with the steady-state error. Fortunately it is possible to eliminate this while using relatively low gain by adding an integral term to the control function which becomes



On-Off Control

- This is the simplest form of control.



*

PID don't work like this, مكواية and on-off things like temperature work like this.

Zeigler Nichols' first method for tuning:

Figure 6.6: *Plant step response*

The suggested parameters are shown in Table 6.2.

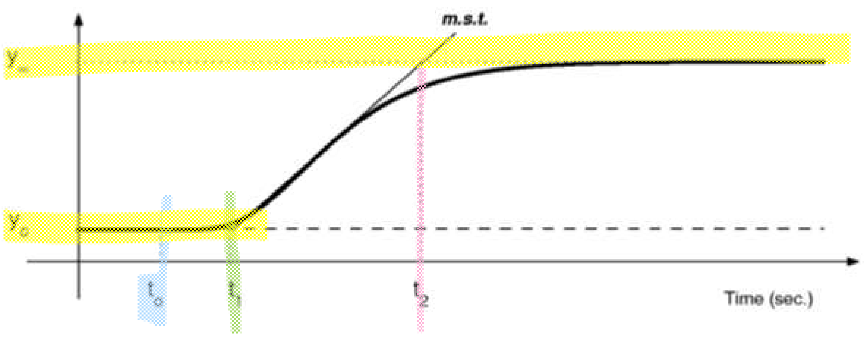


Table 6.2: *Ziegler-Nichols tuning using the reaction curve*

	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

جدول given by doc.

Click to add title

3. Record the plant output until it settles to the new operating point. Assume you obtain the curve shown on the next slide. This curve is known as the *process reaction curve*.

   In Figure 6.6, m.s.t. stands for *maximum slope tangent*.

4. Compute the parameter model as follows

$$K_o = \frac{y_\infty - y_o}{u_\infty - u_o}; \qquad \tau_o = t_1 - t_o; \qquad \nu_o = t_2 - t_1$$

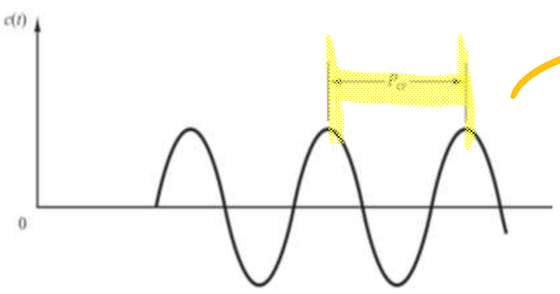
given

فقط

Ziegler Nicholes' second method for tuning:

Zeigler-Nichol's Second Method

- Thus, the critical gain K_{cr} and the corresponding period P_{cr} are determined.



حسب ما فهمت اذا كان معطيني اكتر من وحدة بتختار الأكبر.

Table-2

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

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جدول given by doc.

PID Tuning

- The transfer function of PID controller is given as

$$\frac{C_{pid}(s)}{E(s)} = K_p + K_i \frac{1}{s} + K_d\ s$$

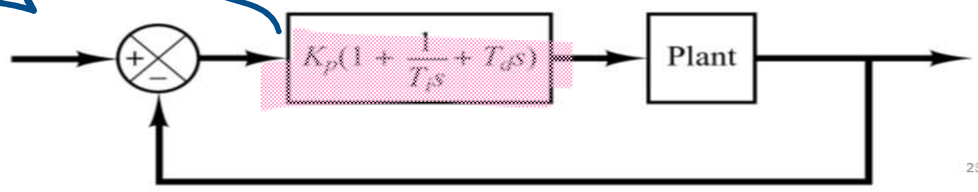
- It can be simplified as

$$\frac{C_{pid}(s)}{E(s)} = K_p(1 + \frac{1}{T_i s} + T_d\ s)$$

- Where

$$T_i = \frac{K_p}{K_i} \qquad T_d = \frac{K_d}{K_p}$$

transfer function



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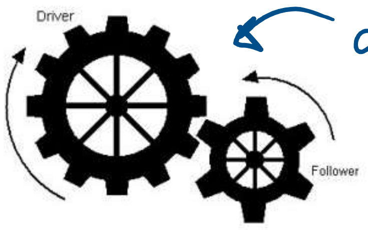
Gears

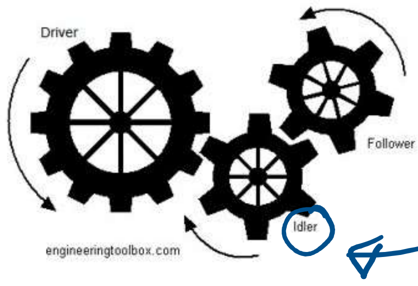
Saturday, May 18, 2024 9:26 PM

Why Gearing is necessary?

- A typical DC motor operates at speeds that are far too high to be useful, and at torques that are far too low.
- *Gear reduction* is the standard method by which a motor is made useful.



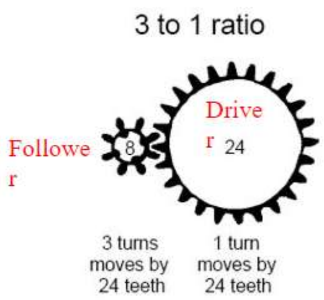
always opposite directions



to keep directions the same

Gear Ratio

- You can calculate the **gear ratio** by using the number of teeth of the *driver* divided by the number of teeth of the *follower*.
- We *gear up* when we increase velocity and decrease torque.
  Ratio: 3:1
- We *gear down* when we increase torque and reduce velocity.
  Ratio: 1:3



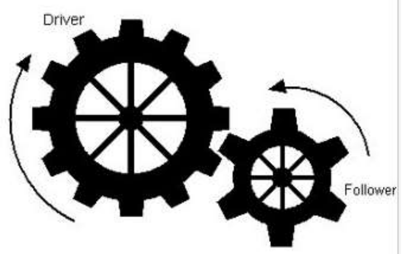
$$Gear\ ratio = \frac{number\ of\ teeth\ of\ input\ gear}{number\ of\ teeth\ of\ ouput\ gear} = \frac{Input\ Torque}{Ouput\ Torque} = \frac{Output\ Speed}{Input\ Speed}$$

Mathematical Modeling of Gear Trains

- Gears increase or descrease angular velocity (while simultaneously decreasing or increasing torque, such that energy is conserved).

Energy of Driving Gear = Energy of Following Gear

$$N_1\theta_1 = N_2\theta_2$$



N_1 → Number of Teeth of Driving Gear

θ_1 → Angular Movement of Driving Gear

N_2 → Number of Teeth of Following Gear

θ_2 → Angular Movement of Following Gear

gear ratio.

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$$J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2$$

$$B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$$

equivalant inertia rotor or motor (resistance)

equivalent damping (resistance)

Mathematical Modelling of Gear Trains

- For three gears connected together

$$J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{N_3}{N_4}\right)^2 J_3$$

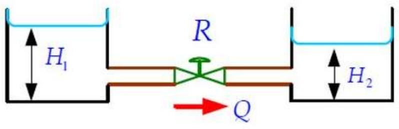
$$B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{N_3}{N_4}\right)^2 B_3$$

Tanks

Friday, May 24, 2024 2:54 PM

Resistance of Liquid-Level Systems

- The resistance for liquid flow in such a pipe is defined as the change in the level difference necessary to cause a unit change inflow rate.



$$Resistance = \frac{change\ in\ level\ difference}{change\ in\ flow\ rate} = \frac{m}{m^3/s}$$

$$R = \frac{\Delta(H_1 - H_2)}{\Delta Q} = \frac{m}{m^3/s}$$

Resistance in Laminar Flow

- For laminar flow, the relationship between the steady-state flow rate and steady state height at the restriction is given by:

$$Q = k_l H$$

- Where Q = steady-state liquid flow rate in m/s^3
- K_l = constant in m/s^2
- and H = steady-state height in m.

- The resistance R_ℓ is

$$R_l = \frac{dH}{dQ}$$

Modelling Example#1

- The rate of change in liquid stored in the tank is equal to the flow in minus flow out.

$$C\frac{dh}{dt} = q_i - q_o \longrightarrow (1)$$

- The resistance R may be written as

$$R = \frac{dH}{dQ} = \frac{h}{q_o} \longrightarrow (2)$$

- Rearranging equation (2)

$$q_o = \frac{h}{R} \longrightarrow (3)$$

Modelling Example#1

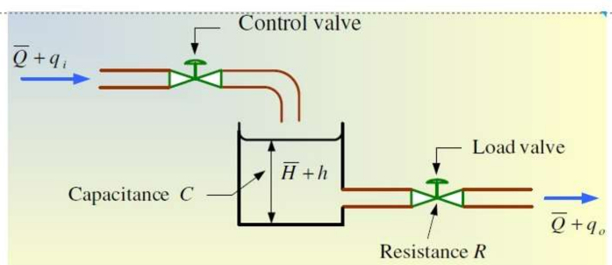
$$RCsH(s) + H(s) = RQ_i(s)$$

- The transfer function can be obtained as

$$\frac{H(s)}{Q_i(s)} = \frac{R}{(RCs+1)}$$

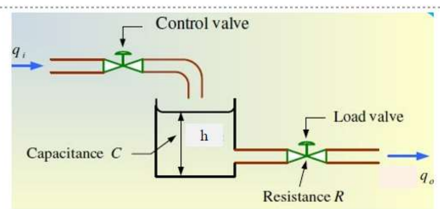
← how is this used ?

Modelling Example#1



$\bar{H}$ = steady-state head (before any change has occurred), m.
h = small deviation of head from its steady-state value, m.
$\bar{Q}$ = steady-state flow rate (before any change has occurred), m^3/s.
q_i = small deviation of inflow rate from its steady-state value, m^3/s.
q_o = small deviation of outflow rate from its steady-state value, m^3/s.

Capacitance of Liquid-Level Systems



$$A\frac{dh}{dt} = q_i - q_o$$

$$C\frac{dh}{dt} = q_i - q_o$$

*Example on Tanks

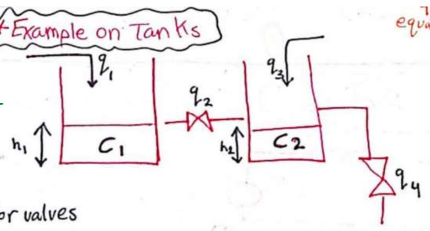
*عدد ال valves = عدد ال equations

for tank ① $q_{in} - q_{out}$

$$\frac{C_1 dh}{dt} = q_1 - q_2$$

for tank ② $q_{in} - q_{out}$

$$\frac{C_2 dh}{dt} = (q_2 + q_3) - q_4$$



*and now the equations for valves

عدد ال q = ← $R_1 = \frac{0 - h_1}{q_1}$ $\quad R_3 = \frac{0 - h_2}{q_3}$

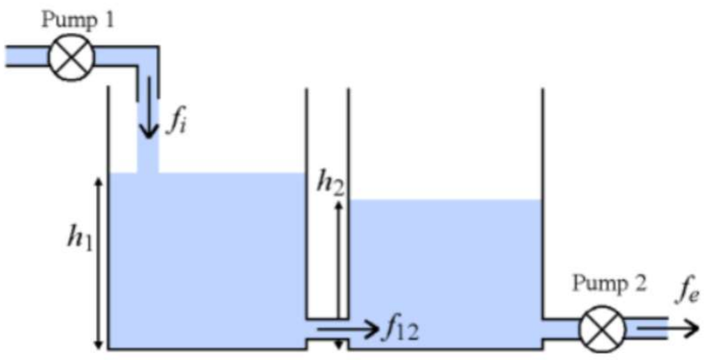
عدد ال R

$R_2 = \frac{h_1 - h_2}{q_2}$ $\quad R_4 = \frac{h_2 - 0}{q_4}$

$$R = \frac{h_{\text{من وين عم تيجي}} - h_{\text{وين عم تروح}}}{q}$$

Name ID

Q1) write the differential equations to describe



① $C_1 \frac{dh}{dt} = f_i - f_{12}$

$C_2 \frac{dh}{dt} = f_{12} - f_e$

② $R_1 = \frac{0 - h_1}{f_i}$

$R_2 = \frac{h_1 - h_2}{f_{12}}$

$R_3 = \frac{h_2 - 0}{f_e}$

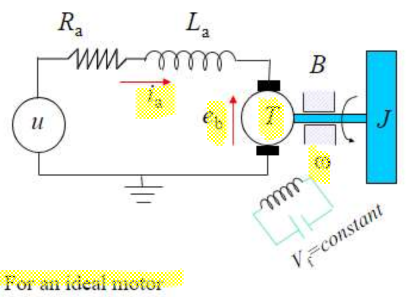
Electromechanical systems

Tuesday, May 28, 2024 9:59 PM

??.

Example-2: Armature Controlled D.C Motor

Power Transformation:



Torque-Current: $T_{motor} = K_t i_a$

Voltage-Speed: $e_b = K_b \omega$

قوة الدفع العكسية

where K_t: torque constant, K_b: velocity constant For an ideal motor

$$K_t = K_b$$

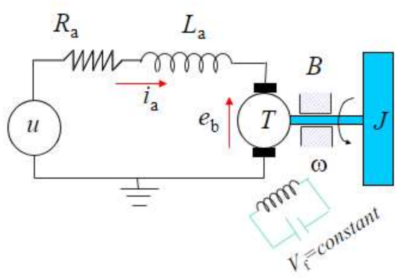
Combing previous equations results in the following mathematical model:

$$\begin{cases} L_a \dfrac{di_a}{dt} + R_a i_a + \overbrace{K_b \omega}^{e_b} = u & \leftarrow \text{electrical subsystem} \\ J\dot{\omega} + B\omega - \underbrace{K_t i_a}_{T_{motor}} = 0 & \leftarrow \text{mechanical subsystem.} \end{cases}$$

Example-2: Armature Controlled D.C Motor

Input: voltage u

Output: Angular velocity ω



Electrical Subsystem (loop method):

$$u = R_a i_a + L_a \frac{di_a}{dt} + e_b, \qquad where \quad e_b = back\text{-}emf\ voltage$$

Mechanical Subsystem

$$T_{motor} = J\underset{\ddot{\theta}}{\dot{\omega}} + B\underset{\dot{\theta}}{\omega}$$

Example-2: Armature Controlled D.C Motor

Taking Laplace transform of the system's differential equations with zero initial conditions gives:

$$\begin{cases} (L_a s + R_a) I_a(s) + K_b \Omega(s) = U(s) \\ (Js + B)\Omega(s) - K_t I_a(s) = 0 \end{cases}$$

Eliminating I_a yields the input-output transfer function

$$\frac{\Omega(s)}{U(s)} = \frac{K_t}{L_a J s^2 + (JR_a + BL_a)s + BR_a + K_t K_b}$$

I have no idea what this is

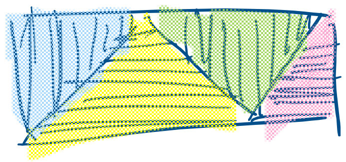
Example-2: Armature Controlled D.C Motor Reduced Order Model

Assuming small inductance, $L_a \approx 0$

speed

Voltage

$$\frac{\Omega(s)}{U(s)} = \frac{\left(K_t / R_a\right)}{Js + \left(B + K_t K_b / R_a\right)}$$



Matlab

Wednesday, May 29, 2024 9:03 PM

*Using Matlab to find:

Control System Toolbox

Transfer Function

- Consider a linear time invariant (LTI) single-input/single-output system

$$y''+6y'+5y=4u'+3u$$

- Applying Laplace Transform to both sides with zero initial conditions → $x(0)=0,\ x'(0)=0$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{4s+3}{s^2+6s+5}$$

Control System Toolbox

Transfer Function → if it was $4s^2+3$ it would be [4 0 3], and if it was 4s only it would be [4 0]

```
>> num = [4 3];
>> den = [1 6 5];
>> sys = tf(num,den)
   Transfer function:
     4 s + 3
   -----------------
   s^2 + 6 s + 5
```

⟺

```
>> [num,den] =
      tfdata(sys,'v')
   num =
      0   4   3
   den =
      1   6   5
```

Control System Toolbox

Zero-pole-gain model (ZPK)

- Consider a Linear time invariant (LTI) single-input/single-output system

$$y''+6y'+5y=4u'+3u$$

- Applying Laplace Transform to both sides with zero initial conditions

$$G(s)=\frac{Y(s)}{U(s)}=\frac{4s+3}{s^2+6s+5}=\frac{4(s+0.75)}{(s+1)(s+5)}$$

Control System Toolbox

Zero-pole-gain model (ZPK)

```
>> sys1 =
 zpk(-0.75,[-1 -5],4)

Zero/pole/gain:
4 (s+0.75)
-----------
(s+1) (s+5)
```

⟺

```
>> [ze,po,k] = zpkdata(sys1,'v')
ze =
   -0.7500
po =
    -1
    -5
k =
     4
```

```
>> s=tf('s') ←
>>G=(4*s+3)/(s^2+6*s+5) ←
```

```
>> s=tf('s')
>>G=(4*s+3)/(s^2+6*s+5)
```

This is the way suggested for the new versions in matlab

MATLAB Control Tool

Control System Toolbox

State-Space Model (SS)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where

x	state vector
u and y	input and output vectors
A, B, C and D	state-space matrices

MATLAB Control Tool

Control System Toolbox

State-Space Models

- Consider a Linear time invariant (LTI) single-input/single-output system

$$y'' + 6y' + 5y = 4u' + 3u$$

- State-space model for this system is

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

How?

laplace $\frac{Y}{R} = \frac{4s+3}{s^2+6s+5} = \frac{1}{s^2+6s+5} * (4s+3) = \frac{X}{R} * \frac{Y}{X}$

$$\frac{1}{s^2+6s+5} = \frac{X}{R}$$

$$(4s+3) = \frac{Y}{X}$$

$R = x_2' + 6x_2 + 5x_1 \quad , \quad Y = 4x_2 + 3x_1$

To differential eq.

$R = X'' + 6X' + 5X;$ $\quad Y = 4X' + 3X$

we don't want this

X1,X2; X1'=X2; X2'=(-5X1-6X2) +R

In Matrix form

$$\begin{bmatrix} X1' \\ X2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} * \begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 3 & 4 \end{bmatrix} * \begin{bmatrix} X1 \\ X2 \end{bmatrix} + [0]R$$

MATLAB Control Tool

Control System Toolbox

State-Space Models

```
>> sys = ss([0 1; -5 -6],[0;1],[3,4],0)
a =
      x1  x2
  x1   0   1
  x2  -5  -6

b =
      u1
  x1   0
  x2   1

c =
      x1  x2
  y1   3   4

d =
      u1
  y1   0
```

MAT

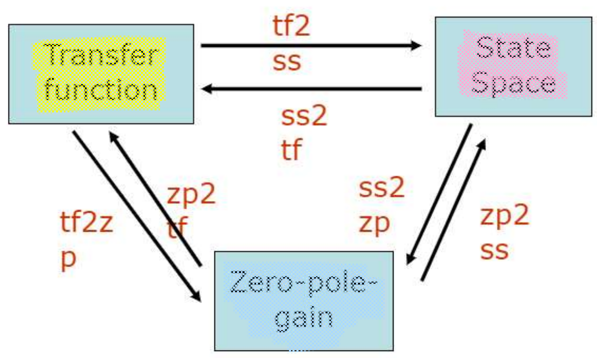
Control System Toolbox

Control System Toolbox

State Space Models

- **rss, drss** - **Random stable state-space models.**
- **ss2ss** - **State coordinate transformation.**
- **canon** - **State-space canonical forms.**
- **ctrb** - **Controllability matrix.**
- **obsv** - **Observability matrix.**
- **gram** - **Controllability and observability gramians.**
- **ssbal** - **Diagonal balancing of state-space realizations.**
- **balreal** - **Gramian-based input/output balancing.**
- **modred** - **Model state reduction.**
- **minreal** - **Minimal realization and pole/zero cancellation.**
- **sminreal** - **Structurally minimal realization.**

Conversion between different models



More Matlab

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Control System Toolbox

Time Response of Systems

- Impulse Response *(impulse)*
- Step Response *(step)*
- General Time Response *(lsim)*
- Polynomial multiplication (*conv)*
- Polynomial division *(deconv)*
- Partial Fraction Expansion *(residue)*
- gensig - Generate input signal for lsim.

Control System Toolbox

Time Response of Systems

Problem Given the LTI system

$$G(s) = \frac{3s+2}{2s^3+4s^2+5s+1}$$

Plot the following responses for:

- The impulse response using the `impulse` command.
- The step response using the `step` command.
- The response to the input $u(t) = \sin(0.5t)$ calculated using both the `lsim` commands

Frequency Domain Analysis and Design

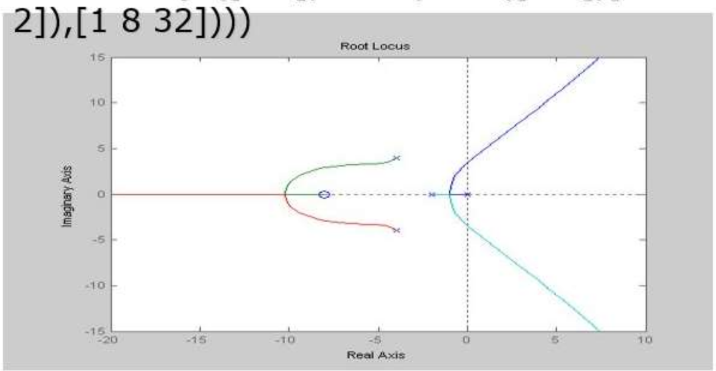
Root Locus

- Plot the root locus of the following system

$$G(s) = \frac{K(s+8)}{s(s+2)(s^2+8s+32)}$$

Root Locus

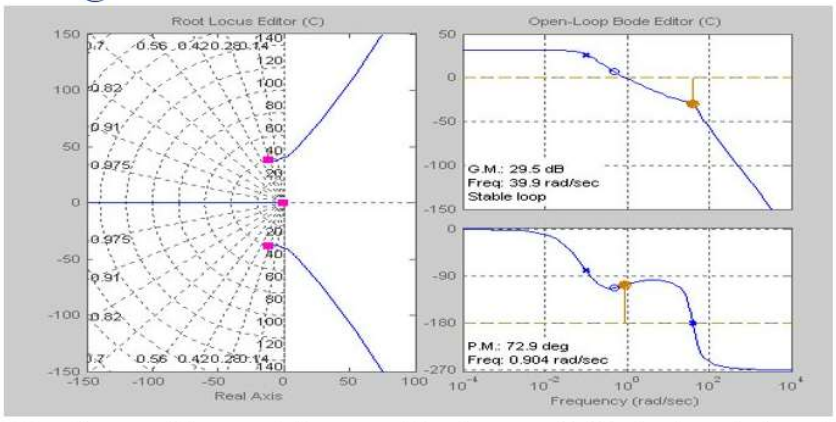
```
>> rlocus(tf([1 8], conv(conv([1 0],[1 2]),[1 8 32])))
```



MATLAB Control Tool

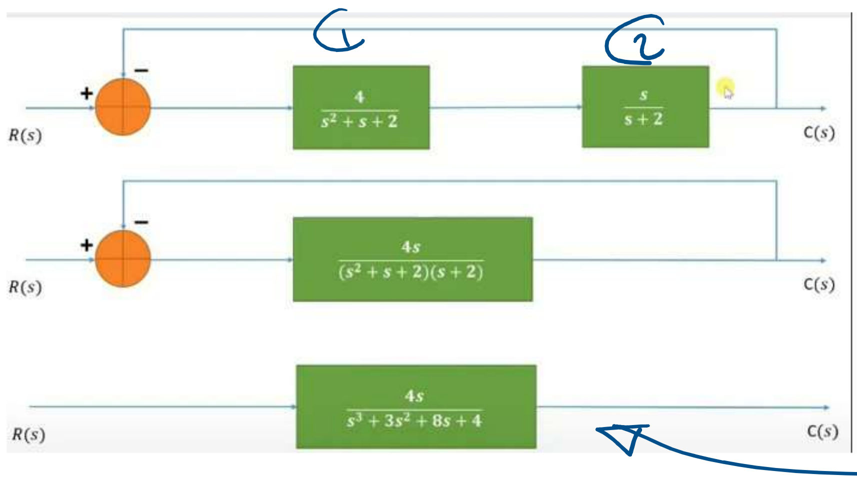
Control System Toolbox

Design Tool: sisotool



Design with root locus, Bode, and Nichols plots of the open-loop system.
Cannot handle continuous models with time delay.

Example:




```
close all;
clear
p = 4
q = [1 1 2]
a = tf(p,q)
r = [1 0]
s = [1 2]
b = tf(r,s)
c = series(a,b)
Ans = feedback(c,1,-1)
```

Matlab- diffrential equations

Wednesday, May 29, 2024 10:05 PM

See them from slide 25-34 numeric way to solve.

47 - 52 symbolic way to solve.

You can see summary from 67-73

Arduino

Wednesday, May 29, 2024 11:45 PM

Reference > Language > Functions > Digital io > Digitalread

digitalRead()

[Digital I/O]

Description

Reads the value from a specified digital pin, either `HIGH` or `LOW`.

Reference > Language > Functions > Analog io > Analogread

analogRead()

[Analog I/O]

Description

Reads the value from the specified analog pin. Arduino boards contain a multichannel, 10-bit analog to digital converter. This means that it will map input voltages between 0 and the operating voltage(5V or 3.3V) into integer values between 0 and 1023. On an Arduino UNO, for example, this yields a resolution between readings of: 5 volts / 1024 units or, 0.0049 volts (4.9 mV) per unit. See the table below for the usable pins, operating voltage and maximum resolution for some Arduino boards.

Reference > Language > Functions > Digital io > Digitalwrite

digitalWrite()

[Digital I/O]

Description

Write a `HIGH` or a `LOW` value to a digital pin.

If the pin has been configured as an `OUTPUT` with `pinMode()`, its voltage will be set to the corresponding value: 5V (or 3.3V on 3.3V boards) for `HIGH`, 0V (ground) for `LOW`.

Reference > Language > Functions > Analog io > Analogwrite

analogWrite()

[Analog I/O]

Description

Writes an analog value (PWM wave) to a pin. Can be used to light a LED at varying brightnesses or drive a motor at various speeds. After a call to `analogWrite()`, the pin will generate a steady rectangular wave of the specified duty cycle until the next call to `analogWrite()` (or a call to `digitalRead()` or `digitalWrite()`) on the same pin.

Reference > Language > Functions > Digital io > Pinmode

pinMode()

[Digital I/O]

Description

Configures the specified pin to behave either as an input or an output. See the Digital Pins page for details on the functionality of the pins.

It is possible to enable the internal pullup resistors with the mode `INPUT_PULLUP`. Additionally, the `INPUT` mode explic disables the internal pullups.

Reference > Language > Functions > Time > Delay

delay()

[Time]

Description

Pauses the program for the amount of time (in milliseconds) specified as parameter. (There are 1000 milliseconds in a second.)

Reference > Language > Functions > Time > Millis

millis()

[Time]

Description

Returns the number of milliseconds passed since the Arduino board began running the current program. This number will overflow (go back to zero), after approximately 50 days.

Reference > Language > Functions > Communication > Serial

Serial

[Communication]

Description

Used for communication between the Arduino board and a computer or other devices. All Arduino boards have at least one serial port (also known as a UART or USART), and some have several.

*These are just for understanding the code.
Everything is in Arduino slides