Queuing Network Example

An assembly line has five stations through which every unit being produced must pass in the same fixed order. The arrival of units follows a Poisson process for all stations. The mean time between arrivals is 8 minutes. The stations operate independently, and queues are allowed to form in front of each station. All stations have same mean processing time of 7 minutes. If processing time at each station has an exponential distribution Analyze this situation

Queuing Network Example

2

i station index $i = 1, 2, 3, 4, 5$

$$\lambda_i = \frac{1}{8} = 0.125 \text{ part /Minute}, \forall\ i = 1, 2, 3, 4, 5$$

$$\mu_i = \frac{1}{7} = 0.143 \text{ part /Minute}, \forall\ i = 1, 2, 3, 4, 5$$

$$Utilization = \rho_i = \frac{\lambda_i}{\mu_i} = \frac{0.125}{0.143} = 0.874, \forall \text{station } i = 1, 2, 3, 4, 5$$

i station index $i = 1, 2, 3, 4, 5$

$$Ls_i = \frac{\rho_i}{1-\rho_i} = \frac{0.874}{1-0.874} = 7 \text{ Part } \forall\ i = 1, 2, 3, 4, 5$$

$$Lq_i = \frac{(\rho_i)^2}{1-\rho_i} = \frac{(0.874)^2}{1-0.874} = 6 \text{ Part} \forall\ i = 1, 2, 3, 4, 5$$

$$Ws_i = \frac{ls_i}{\lambda} = \frac{7}{0.125} = 56 \text{ Min.}$$

$$Wq_i = \frac{lq_i}{\lambda} = \frac{6}{0.125} = 48 \text{ Min.}$$

Queuing Network Example

	Station i					Total Assembly line
	1 M/M/1:GD/∞/∞	2 M/M/1:GD/∞/∞	3 M/M/1:GD/∞/∞	4 M/M/1:GD/∞/∞	5 M/M/1:GD/∞/∞	
λ	0.125	0.125	0.125	0.125	0.125	
μ	0.143	0.143	0.143	0.143	0.143	
c	1	1	1	1	1	
ρ	0.874	0.874	0.874	0.874	0.874	
L_s	7	7	7	7	7	35 Part
L_q	6	6	6	6	6	30 Part
W_s	56	56	56	56	56	280 Min
W_q	48	48	48	48	48	240 Min
$\hat{C}$	0.874	0.874	0.874	0.874	0.874	**0.874**

3

Queuing Network Example

Batch jobs submitted to a computer center

Batch jobs submitted to a computer center pass through three steps: an input processor, the central processor, and an output printer. Because of errors, only 80% of the jobs go through the central processor; the remaining 20% are rejected. Of the jobs that pass through the central processor successfully, 40% are routed to a printer station. Jobs arrive to the computer center with an average rate of 10 per minute. To handle the load of jobs submitted, the computer center may have several of the three types of processors operating in parallel. The times for the three steps have exponential distributions with mean values as follows: input processor 10 seconds, central processor 3 seconds, and printer 70 seconds. When processors are not immediately available for a job, the job must wait in a queue. There is an unlimited queue for each processor. Our task is to find

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1. The minimum number of each type of processor and
2. The average time required for a job to pass through the system.

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Queuing Network Example
Batch jobs submitted to a computer center

λ_1= 10 /Min → **1** Input processor ts =10 second → $\varnothing_{1,2}$=80% → **2** Central proc. ts =3 second → $\varnothing_{2,3}$=40% → **3** Printer ts=70 second

m = 3 , **serial system,**

$\lambda_1 = 10/\text{Min}$ $\quad\quad$ $\lambda_2 = 0.8(10) = 8/\text{Min}$ $\quad\quad$ $\lambda_3 = 0.4(8) = 3.2/\text{Min}$

$\mu_1 = 60/10 = 6/\text{Min}$ $\quad\quad$ $\mu_2 = 60/3 = 20/\text{Min}$ $\quad\quad$ $\mu_3 = 60/70 = 0.857/\text{Min}$

$$c_i = s_i = \text{Smallest integer greater than or equal } \frac{\lambda_i}{\mu_i}$$

$s_1 = \frac{\lambda_1}{\mu_1} = \frac{10}{6} = 2,$ $\quad\quad$ $s_2 = 1,$ $\quad\quad$ $s_3 = 4$

$\rho_1 = \frac{10}{(6)},$ $\quad\quad$ $\rho_2 = \frac{8}{20} = 0.4,$ $\quad\quad$ $\rho_3 = \frac{3.2}{(0.857)},$

M/M/2: GD/∞ /∞ $\quad\quad$ M/M/1: GD/∞ /∞ $\quad\quad$ M/M/4: GD/∞ /∞

Multiple Servers Model M/M/c:GD/∞ /∞

Poisson arrival rate λ Poison Procesing rate μ

multiple servers c $\rho = \dfrac{\lambda}{\mu}$

6

Average number of customer in queue : $L_q = \dfrac{\rho^{c+1}}{(c-1)!(c-\rho)^2} p_0$

Average number of customer in system : $L_s = L_q + \rho$

Average waiting time in system : $W_s = \dfrac{L_s}{\lambda}$

Average waiting time in queue : $W_q = \dfrac{L_q}{\lambda}$

Average number of busy servers : $\bar{c} = L_s - L_q$

$$p_0 = \left\{ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1-\frac{\rho}{c}} \right) \right\}^{-1} , \frac{\rho}{c} < 1$$

Queuing Network Example
Batch jobs submitted to a computer center

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	B	C	D	E	F	G
16	**Queue Station**	Input	Central	Printer	**Queueing Network**	Comp2
17	Independent Arrival Rate	10	0	0		
18	Arrival Rate	10	8	3.2		
19	Service Rate/Channel	6	20	0.857		
20	Number of Servers	2	1	4		
21	Type	M/M/2	M/M/1	M/M/4	Type	General
22	Mean Number at Station	5.454545	0.666667	15.75681	Mean Number in System	21.87802
23	Mean Time at Station	0.545455	0.083333	4.924004	Mean Time in System	5.552791
24	Mean Number in Queue	3.787878	0.266667	12.02286	Mean Number in System Queues	16.0774
25	Mean Time in Queue	0.378788	0.033333	3.757143	Mean Time in System Queues	4.169264
26	Mean Number in Service	1.666667	0.4	3.733956	Mean Number in System Service	5.800622
27	Mean Time in Service	0.166667	0.05	1.166861	Mean Time in System Service	1.383528
28	Efficiency	0.833333	0.4	0.933489		

Queuing Network Example
A Job shop Example

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Measure	A	B	C	D	E	F
	Machine					
Model	M/M/3 ;	M/M/2	M/M/1	M/M/3	M/M/2	M/M/4
γ	60	0	0	0	0	0
μ	25	22	29	11	23	20
s	3	2	1	3	2	4
λ	60	40	20	30	30	60
L_s	4.989	10.476	2.222	11.059	2.270	4.528
W_s	0.083	0.262	0.111	0.369	0.076	0.075
L_q	2.589	8.658	1.533	8.332	0.965	1.528
W_q	0.043	0.216	0.077	0.278	0.032	0.025

WIP=Order rate *Lead time

Product	Order rate (per month)	Route	Lead time (months) $=\sum_{i=1} W_{s_i}$	Queue time (months) $=\sum_{i=1}^{n} W_{q_i}$	WIP (units)
1	30	A B D F	0.789	0.563	23.67
2	10	A B E F	0.496	0.317	4.96
3	20	A C E F	0.345	0.177	6.91

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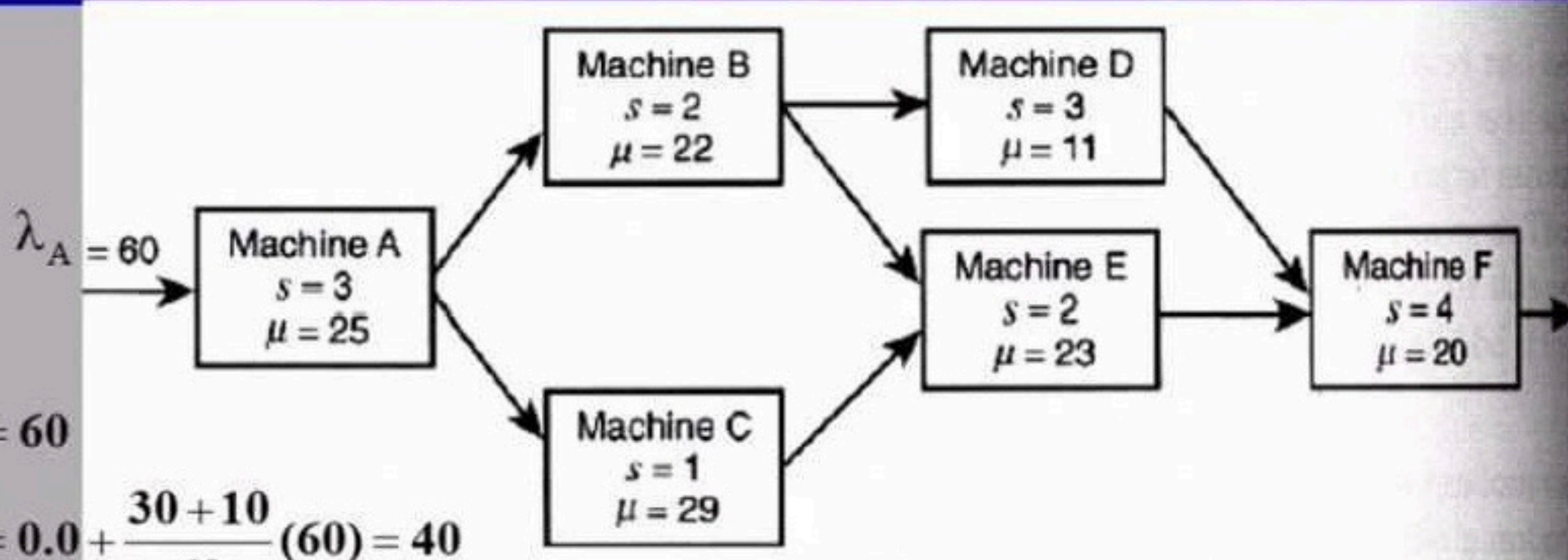
Queuing Network Example
A Job shop Example



$$\lambda_A = 60$$

$$\lambda_B = 0.0 + \frac{30+10}{60}(60) = 40$$

$$\lambda_C = 0.0 + \frac{20}{60}(60) = 20$$

$$\lambda_D = 0.0 + \frac{30}{30+10}(40) = 30$$

$$\lambda_E = 0.0 + \frac{10}{30+10}(40) + 1.0(20) = 30$$

$$\lambda_F = 0.0 + 1.0(30) + 1.0(30) = 60$$

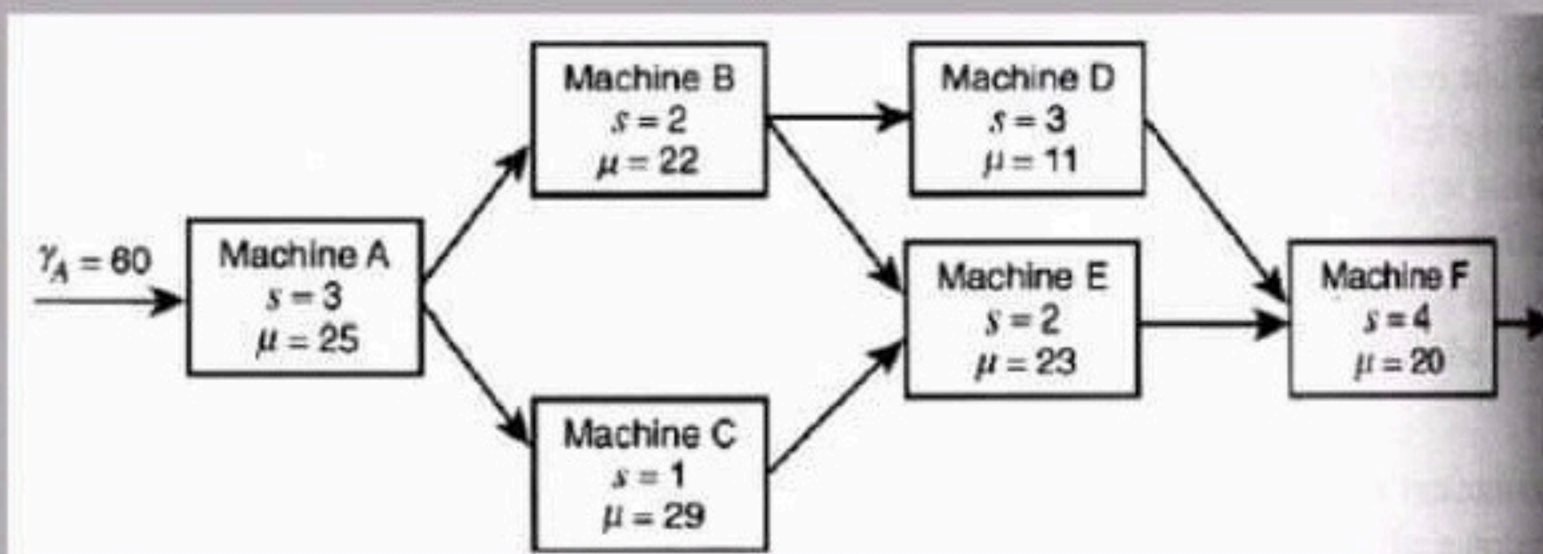
Data for the Job Shop Example

Product class	Order rate	Route
1	30/month	A B D F
2	10/month	A B E F
3	20/month	A C E F

Applying the formulas of M/M/c for each station we get the following

Queuing Network Example
A Job shop Example

An electronics company assembles three classes of products in a job shop environment that is represented by the directed network shown. The rectangles identify workstations that comprise a specific machine type. The other data in the rectangles indicate the number of parallel machines (s) at the station and the average service rate (μ)of each.The order rate for each class of products along with the routing information is given in Table. For example, products in class 1 must visit machines A, B, D, and F in that order. It will be assumed that orders for the products arrive according to Poisson processes characterized by the given rates, and that processing times at the machines are exponentially distributed.



Data for the Job Shop Example

Product class	Order rate	Route
1	30/month	A B D F
2	10/month	A B E F
3	20/month	A C E F

6

Queuing Network Example
A Job shop Example

Measure	Machining Center					
	A	B	C	D	E	F
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
γ External	60	0	0	0	0	0
μ	25	22	29	11	23	20
c (s)	3	2	1	3	2	4
λ	60	40	20	30	30	60
L_s	**4.989**	**10.476**	**2.222**	**11.059**	**2.270**	**4.528**
W_s	0.083	0.262	0.111	0.369	0.076	0.075
L_q	2.589	8.658	1.533	8.332	0.965	1.528
W_q	0.043	0.216	0.077	0.278	0.032	0.025

General total average number of WIP units of all types of products in the job shop:

$= \sum_{i=A,B,C,D,E,F} Ls_i = 4$[illegible]$89 + 10.476 + 2.222 + 11.059 + 2.270 + 4.528 = 35.544$

Queuing Network Example
A Job shop Example

Measure	Machining Center					
	A	**B**	**C**	**D**	**E**	**F**
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
γ **External**	60	0	0	0	0	0
μ	25	22	29	11	23	20
c (s)	3	2	1	3	2	4
λ	60	40	20	30	30	60
L_s	4.989	10.476	2.222	11.059	2.270	4.528
W_s	**0.083**	**0.262**	**0.111**	**0.369**	**0.076**	**0.075**
L_q	2.589	8.658	1.533	8.332	0.965	1.528
W_q	0.043	0.216	0.077	0.278	0.032	0.025

General average time spent in the job shop per each order of all types of products:

$$= \sum_{i=A,B,C,D,E,F} Ws_i = 0.083 + 0.262 + 0.111 + 0.369 + 0.076 + 0.075 = 0.976$$

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Queuing Network Example
A Job shop Example

Measure	Machining Center					
	A	B	C	D	E	F
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
γ External	60	0	0	0	0	0
μ	25	22	29	11	23	20
c (s)	3	2	1	3	2	4
λ	60	40	20	30	30	60
L_s	4.989	10.476	2.222	11.059	2.270	4.528
W_s	0.083	0.262	0.111	0.369	0.076	0.075
L_q	**2.589**	**8.658**	**1.533**	**8.332**	**0.965**	**1.528**
W_q	0.043	0.216	0.077	0.278	0.032	0.025

General average number of WIP units of all types of products waiting in the job shop is: $= \sum_{i=A,B,C,D,E,F} Lq_i = 2.589 + 8.658 + 1.533 + 8.332 + 0.965 + 1.528 = 23.605$

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Queuing Network Example
A Job shop Example

Measure	Machining Center					
	A	**B**	**C**	**D**	**E**	**F**
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
γ **External**	60	0	0	0	0	0
μ	25	22	29	11	23	20
c (s)	3	2	1	3	2	4
λ	60	40	20	30	30	60
L_s	4.989	10.476	2.222	11.059	2.270	4.528
W_s	0.083	0.262	0.111	0.369	0.076	0.075
L_q	2.589	8.658	1.533	8.332	0.965	1.528
W_q	**0.043**	**0.216**	**0.077**	**0.278**	**0.032**	**0.025**

General average waiting time in the shop per each order of all types of products:

$$= \sum_{i=A,B,C,D,E,F} Wq_i = 0.043 + 0.216 + 0.077 + 0.278 + 0.032 + 0.025 = 0.671$$

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Queuing Network Example
A Job shop Example

Measure	Machining Center					
	A	B	C	D	E	F
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
L_s	4.989	10.476	2.222	11.059	2.270	4.528
W_s	**0.083**	**0.262**	**0.111**	**0.369**	**0.076**	**0.075**
L_q	2.589	8.658	1.533	8.332	0.965	1.528
W_q	0.043	0.216	0.077	0.278	0.032	0.025

Average number of WIP and average time of each product class in the whole shop

Product	Order rate per month	Route	W_s Lead Time Months	$Ls = \lambda Ws$
1	30	A B D F	= 0.083 + 0.262 + 0.369 + 0.075 = 0.789	= 30(0.789) = 23.67
2	10	A B E F	= 0.083 + 0.262 + 0.076 + 0.075 = 0.496	= 10(0.496) = 4.96
3	20	A C E F	= 0.083 + 0.111 + 0.076 + 0.075 = 0.345	= 20(0.345) = 6.91

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Queuing Network Example
A Job shop Example

Measure	Machining Center					
	A	B	C	D	E	F
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
L_s	4.989	10.476	2.222	11.059	2.270	4.528
W_s	0.083	0.262	0.111	0.369	0.076	0.075
L_q	2.589	8.658	1.533	8.332	0.965	1.528
W_q	**0.043**	**0.216**	**0.077**	**0.278**	**0.032**	**0.025**

Average number of waiting WIP and average waiting time of each product class

Product	Order rate per month	Route	W_q Waiting Time Months	$Lq = \lambda Wq$
1	30	A B D F	$= 0.043 + 0.216 + 0.278 + 0.025 = 0.563$	$= 30(0.563) = 16.89$
2	10	A B E F	$= 0.043 + 0.216 + 0.032 + 0.025 = 0.317$	$= 10(0.317) = 3.17$
3	20	A C E F	$= 0.043 + 0.077 + 0.032 + 0.025 = 0.177$	$= 20(0.177) = 3.54$

Queuing Network Example
A Job shop Example

Average number of WIP of each product class at each machining center

Measure	Machining Center					
	A	B	C	D	E	F
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
$\lambda_{class\ 1(ABDF)}$	30	30	-	30	-	30
$\lambda_{class\ 2(ABEF)}$	10	10	-	-	10	10
$\lambda_{class\ 3(ACEF)}$	20	-	20	-	20	20
λ	60	40	20	30	30	60
$L_{s\ :\ All\ classes}$	4.989	10.476	2.222	11.059	2.270	4.528
Ls : Class 1	(30/60)(4.989) = 2.4945	(30/40)(10.476) = 7.857	-	(30/30)(11.059) = 11.059	-	(30/60)(4.528) = 2.264
Ls : Class 2	(10/60)(4.989) = 0.8315	(10/40)(10.476) = 2.619	-	-	(10/30)(2.270) = 0.7565	(10/60)(4.528) = 0.7546
: Class 3	(20/60)(4.989) = 1.663	-	(20/20)(2.222) = 2.222	-	(20/30)(2.270) = 1.5135	(20/60)(4.528) = 1.5094

Queuing Network Example
A Job shop Example

Average number of WIP of each product class waiting in front of each machining center

Measure	Machining Center					
	A	B	C	D	E	F
Model	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/1:GD/∞/∞	M/M/3:GD/∞/∞	M/M/2:GD/∞/∞	M/M/4:GD/∞/∞
$\lambda_{class\ 1(ABDF)}$	30	30	-	30	-	30
$\lambda_{class\ 2(ABEF)}$	10	10	-	-	10	10
$\lambda_{class\ 3(ACEF)}$	20	-	20	-	20	20
λ	60	40	20	30	30	60
L_q : All classes	2.589	8.658	1.533	8.332	0.965	1.528
Lq : Class 1	(30/60)(2.589) = 1.2945	(30/40)(8.58) = 6.435	-	(30/30)(8.332) = 8.332	-	(30/60)(1.528) = 0.764
Lq : Class 2	(10/60)(2.589) = 0.4315	(10/40)(8.58) = 2.145	-	-	(10/30)(0.965) = 0.3215	(10/60)(1.528) = 0.255
Lq : Class 3	(20/60)(2.589) = 0.863	-	(20/20)(1.533) = 1.533	-	(20/30)(0.965) = 0.6435	(20/60)(1.528) = 0.509

Queuing Network Example
A Job shop Example

1. General total average number of WIP units of all types of products in the job shop.
2. General average time spent in the job shop per each order of all types of products
3. General average number of waiting WIP units of all types of products in the job shop
4. General average waiting time in the shop per each order of all types of products
5. Average number of WIP of product class1, class2, and class 3 in the whole shop
6. Average time of product class1, class2, class3 in the whole shop
7. Average number of waiting WIP of product class1, class2, and class 3 in the whole shop
8. Average waiting time of product class1, class2, and class 3 in the whole shop
9. Average number of WIP of product class1, class2, and class 3 at machining center A, B, C, D, E, and F
10. Average number of WIP of product class1, class2, and class 3 at machining center A, B, C, D, E, and F