

Q (1: 15 pts) Please state whether each statement is True/False. Please underline the false part then correct it.

1. Quality improvement is the reduction of capability in processes and products. (F) variability
2. Reliability is a critical-to-quality characteristic. (F) Dimension of Quality
3. The upper specification limit is the largest value for the quality characteristic of the product. (T)
4. Acceptance sampling is the least efficient statistical technique for variability reduction. (F) most
5. In Motorola's three-sigma level results in 3.4 parts are defective. (F) 66810? -6
6. Appraisal costs are those costs associated with measuring, evaluating, or auditing products, components, and purchased material. (T)
7. Warranty costs are an example of external production costs. F → (Failure)
8. Training is an example of prevention costs. (T)
9. Box plot is a more compact summary of data than numerical measures. T
10. Histograms are best suited for data sets. (F) large data sets
11. A process that is operating with only assignable causes of variation present is said to be in control. (F) out of control
12. The control charts consists of three parameters; upper control limit, lower control limit, and target. (F) centerline
13. Pareto chart is a simple frequency distribution of numerical data arranged by category. (F) attribute/symbol
14. In phase I, trial control charts are established to monitor future production. T
15. Check sheet is a useful plot for identifying a potential relationship between two variables. (F) Scatter diagram

Q (2:10 pts) Please determine the proper probability distribution for the following cases:

- = A quality control engineer collected 50 samples from a lot of 1000 which contains 50 defectives. The probability of finding 4 defective products in the sample is calculated using Hypergeometric distribution.
- = A quality control engineer monitors the number of surface defects appear on a white board. The probability of product acceptance can be calculated using Poisson distribution.
- = The reliability of electronic component with a constant time to failure is of main interest. The probability of component survival a specific number of hours will be estimated using exponential distribution.
- = An inspector of welded joints decides to stop inspection when he/she finds the first defective weld. The probability of detecting the first defective weld by the fifth sample will be calculated using Geometric distribution.
- = The probability of finding a percentage of broken tubes in a definite sample size is calculated using binomial distribution.



Q (2:20 pts) A product is composed of four identical and independent components. Each component is designed with four critical and independent quality characteristics as follows:

product → system

- Weight distributed as **Normally** distributed with mean 30 gm and variance of 9
- Failure density distributed **Exponentially** with mean failure rate of 200 hr.
- Tensile strength distributed as **Weibull** distribution of shape and scale parameters of 0.5 and 200 N, respectively.
- Hardness distributed as **Gamma** distribution of shape and scale parameters of 0.5 and 150 N, respectively.

$\beta = 0.5$        $\theta = 200$

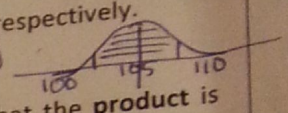
(a) For component's tensile strength, the specification limits are  $105 \pm 5$ , what is the probability that the product is rejected?

Shape parameters = 2

$$1 - P(110 > x > 100) = P(x \leq 100) + P(x \geq 110)$$

$$\left( 1 - e^{-\left(\frac{100}{200}\right)^{0.5}} \right) + e^{-\left(\frac{110}{200}\right)^{0.5}}$$

$$1 - e^{-\left(\frac{1}{4}\right)} + e^{-\frac{11}{40}} = 0.98077$$



(b) If the product requires three out of four components operating, what is the probability that the product will survive before 205 hr?

Prob that ~~the~~ ~~product~~ will survive after 205 hr

$$P(x > 205) = e^{-\lambda t} = e^{-0.005 \times 205} = 0.3587$$

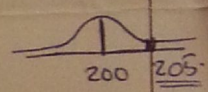
Prob that the product will survive

$$\binom{4}{3} (0.3587)^3 (1 - 0.3587) + \binom{4}{4} (0.3587)^4 (1 - 0.3587)^0 = 0.1183 + 0.0165 = 0.13485$$

$\mu = 200$   
 $\lambda = \frac{1}{\mu} = \frac{1}{200} = 0.005$

(c) What is the probability that the component's hardness exceeds 200 N?

$$P(x > 200) = \sum_{k=0}^{\infty} \frac{e^{-\lambda x} (\lambda x)^k}{k!} = \frac{(150 \times 200)^0}{0!} + \frac{(150 \times 200)^1}{1!} + \dots = \text{Zero}$$



Reliability  
 $(\lambda = 150)$

(d) If the product components are arranged in a standby redundant configuration, what is the probability that the product will survive 150 hrs?

$$P(x > 150) = \sum_{k=0}^3 e^{-\lambda t} \frac{(\lambda t)^k}{k!} = \sum_{k=0}^3 e^{-0.005 \times 150} \frac{(0.005 \times 150)^k}{k!}$$

$$= \frac{(0.005 \times 150)^2}{2} + \frac{(0.005 \times 150)^3}{6} = 0.9927$$

$\mu = 200$   
 $\lambda = 0.005$

Gamma

(f) What is the probability that the component's tensile strength exceeds 250 N?

$$P(x > 250) = e^{-\left(\frac{250}{200}\right)^{0.5}} = e^{-\frac{5}{8}} = 0.5352$$

$\beta = 0.5$        $\theta = 200$

(g) If the product operates if all components operate, what is the probability of product failure?

prob that one component survive after the mean

$$P(x > 200) = e^{-\lambda t} = e^{-0.005 \times 200} = e^{-1} = 0.3678$$

prob that the product operates  $(0.3678)^4 = 0.0183$

14



Q (3:20 pts) A purchaser adopts acceptance sampling procedure to prevent receiving defective items. The purchaser accepts the lot if at most one defective unit is found.  $P(X \leq 1)$

(a) A lot is composed of 50 units in which 4 defective units are found. A sample of five units is selected. Calculate the probability of lot acceptance.  $N=50$   $D=4$   $n=5$   $50 \rightarrow 4 \text{ defective, } 46 \text{ non defective}$

*hyper geometric*

$$P(X \leq 1) = P(X=0) + P(X=1)$$

$$= \frac{\binom{4}{0} \binom{46}{5}}{\binom{50}{5}} + \frac{\binom{4}{1} \binom{46}{4}}{\binom{50}{5}} = \frac{1 \times 15850}{15850} + \frac{4 \times 1585}{15850}$$

$$= 0.9549 = 0.6469 + 0.3080$$

*Binomial*

(b) Previous reports reveal that the probability of nonconforming is 0.01, what is the probability that a sample of 5 units contains less than two nonconforming units (infinite population)?  $(P=0.01)$   $n=5$

$$P(X \leq 2) = P(X=0) + P(X=1)$$

$$\binom{5}{0} (0.01)^0 (1-0.01)^5 + \binom{5}{1} (0.01)^1 (1-0.01)^4$$

$$0.9509 + 5 \times (0.01) (0.99)^4 = 0.9509 + 0.0480 = 0.9989$$

(c) Using proper approximation in part a, calculate the probability of lot acceptance. Is the approximation satisfactory? Why? Why not?

$$\frac{n}{N} < 0.1$$

$$\frac{5}{50} < 0.1$$

$$0.1 < 0.1$$

No, it's not satisfactory

(d) Suppose the variance of surface defects on each unit equals 0.01, calculate the probability that a unit contains at most two surface defects?  $\sigma = 0.01$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-0.1} (0.1)^0}{0!} + \frac{e^{-0.1} \times (0.1)^1}{1} + \frac{e^{-0.1} \times (0.1)^2}{2} = 0.9048 + 0.0904 + 0.0045 = 0.9997$$

$$\lambda = \sqrt{\sigma}$$

$$\lambda = \sqrt{0.01}$$

$$\lambda = 0.1$$

If the inspector decides to continue sampling till the lot is rejected. What is the probability that he will reject the lot the third samples?  $P(X > 1) \rightarrow 1 - (P(X \leq 1))$   $P \geq 0.01$  *negative*

$$1 - \left[ \binom{2}{1} (0.01)^1 (1-0.01)^1 \right]$$

$$1 - \left[ \binom{2}{1} (0.01)^1 (0.99)^1 \right]$$

$$1 - [0.0198] = 0.9802$$

*Binomial*