12+9 = 21

The University of Jordan

Quality Control (~~First~~ Exam 25 %) **Instructor: Dr. Abbas Al-Refaie**

Name: [redacted] **ID:** [redacted] **Absence:** لايوجد **Extra:** ------

Q1 (5 pts) Please ~~select~~ state the right answer: 3.5

Sensitizing Rules من
Western Rules

Quality is inversely proportional to waste. (~~X~~ variability)

Sensitizing rules are used to decide whether or not one side of the control chart is in control. (X Out of control diagram)

Pareto chart is used to identify potential causes for a specific effect. (X Cause and Effect diagram)

Histograms are used to describe variations of a quality response with large data. (~~X frequencies~~ data Quality characteristic)

Failure analysis is an example of prevention costs. (Internal Failure costs)

"Indirect costs" is an example of appraisal costs. (External Failure costs) (T)

Phase I of the control chart is used to establish trial control chart. (✓)

Scatter diagram is used to examine a relationship between two variables. (✓)

Control charts are effective in defect detection. (X Assignable causes) → prevention

Assignable variability is caused by chance causes. (X Chance variability)

Q2 (5 pts) Please state the proper distribution in each of the following cases: 5

Surface defect produced on an electronic appliance. (Poisson distribution (discrete))

The life time for a system of independent and exponentially distributed components connected in a standby configuration. (Gamma distribution)

The life time of a system of n components which requires at least some components operating. (Binomial distribution)

The weight of a system of independent components when each is normally distributed with different means and standard deviations. (Normal distribution)

The fraction of nonconforming components in a sample of components. (Binomial distribution) $\hat{P} = \frac{X}{n}$

Q3 (20 pts) A product is composed of five identical and independent components. The time to failure is the main quality response. Consider the following cases:

5-identical and indep.

(a:2pts) If the time to failure is normally distributed with mean 500 hours and standard deviation of 49 hour. Calculate the probability that the component survive 500 hours.

$P(x=500) = P(z \leq 500) = \cancel{P\left(z = \frac{\mu - 500}{}\right)} = \boxed{P\left(z = \frac{X-\mu}{s}\right)}$

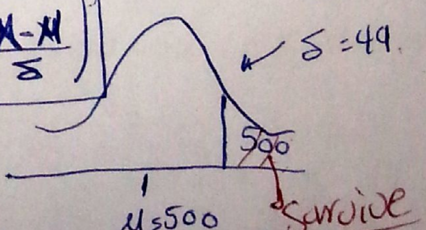
$= P\left(z = \frac{500-500}{s}\right)$

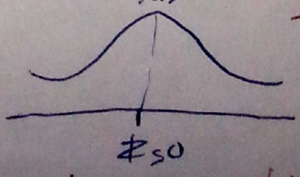
↓ By Normal distribution

$= P\left(z \leq \frac{500-500}{49}\right)$

$= P(z \leq 0) = \boxed{0.5}$ 1/2

$P(z=0) = 0.5$





$P(z \leq)$ failure $P(z >) = 1 - P(z <)$

$p(x>150) = 1 - p(x<150) = 1 - \left[1 - e^{-\lambda a}\right]$

$= e^{-(10^{-2})(150)} = 0.2231 \rightarrow \boxed{5}$ وباقيهم حوّل

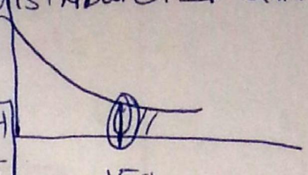
بيشتغلوا 5 او 4 يكونوا كلهم شرط مش يشتغل مرة = p5

(b:3pts) If the time to failure of each component is exponentially distributed with mean equals to 10^2 hour. Calculate the probability that the **product** survive $\boxed{\text{150 hours}}$. Assume components are connected such that if any component fails, the whole product fails.

$\mu = \cancel{\lambda}$

$\mu_{exponential} = \frac{1}{\lambda} = \frac{1}{10^2} = 10^{-2}$/hours.

→ product survive 150 hours → system (standby Redundant) of exponentially distributed → Gamma



$p(x=150) = \cancel{\exp} \sum_{k=0}^{\cancel{n}-1} \frac{e^{-\lambda a}(\lambda a)^k}{\cancel{k}!\ k!} = \sum_{k=0}^{4} e^{-\lambda a}\frac{(\lambda a)^k}{k!}$

$= e^{-10^{-2} \times 150}\left[1 + (10^{-2} \times 150) + \frac{(10^{-2} \times 150)^2}{2! = 2} + \frac{(10^{-2} \times 150)^3}{3! = 6} + \frac{(10^{-2} \times 150)^4}{4! = 24}\right]$

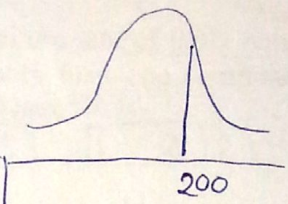
$= e^{-10^{-2} \times 150}\left[1 + 1.5 + 1.125 + 0.5625 + 0.2109375\right]$ $\quad 3.2734$

$= 0.2231\left[\ 3.2734\ \right] = \cancel{0.98129}\ \boxed{0.73039}$

(c:2pts) If the component's time to failure is distributed as Weibull with scale parameter of 400 (θ) and shape parameter of 0.5 (β). Calculate the probability that a **component** will survive 200 hour.

$\cancel{\lambda = 400,\ \gamma = 0.5}$ $\quad \theta = 400,\ \beta = 0.5$

$\underline{p(x = 200)}$ survive $= \exp\left[-\left(\frac{a}{\theta}\right)^{\beta}\right]$



$p(x = 200) = \exp\left[-\left(\frac{200}{400}\right)^{0.5}\right] = \boxed{0.4930686691}$

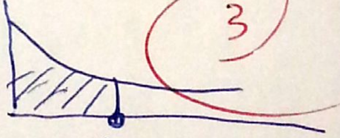
2

(d:3pts) If all the five components are $\boxed{\text{distributed as exponential}}$, each with failure rate of 0.001/hour. Suppose that the components are connected in a standby configuration. Calculate the probability of **product** failure before 100 hour.

$[\lambda = 0.001$ /hour$]$

product failure before 100 hours → Gamma distribution (System of Exponential combonents)

$\cancel{n} \rightarrow$ 5 combonents $(r-1)$



3

$F(100) = 1 - \sum_{k=0} e^{-\lambda a}\frac{(\lambda a)^k}{k!}$

$F(100) = 1 - \sum_{k=0} e^{-\lambda a} \cdot \frac{(\lambda a)^k}{k!}$

$= 1 - \left[e^{-\lambda a} + e^{-\lambda a} \cdot (\lambda a) + e^{-\lambda a} \cdot \frac{(\lambda a)^2}{2!} + e^{-\lambda a} \cdot \frac{(\lambda a)^3}{3!} + e^{-\lambda a} \cdot \frac{(\lambda a)^4}{4!}\right]$

(e:3pts) If the weight of each component is normally distributed with mean 100 gm and variance of 9. Calculate the probability that the **product** weight will exceed 310 gm. $\boxed{y = x_1 + x_2 + x_3 + x_4 + x_5}$

$\mu_y = \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 = 100 + 100 + 100 + 100 + 100 = \boxed{500}$, $s_y^2 = s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 = 9 + 9 + 9 + 9 + 9 = 45$

$\downarrow s_y = \sqrt{45} = \boxed{6.71}$

$p(\cancel{y} > 310) = p\left(z > \frac{310 - \mu_y}{s_y}\right) = 1 - p\left(z < \frac{310 - \mu_y}{s_y}\right)$

$= 1 - p\left(z < \frac{310 - 500}{6.71}\right)$

$= 1 - p(z < -28.3) \simeq \boxed{1}$

3

2

$1 - \left[e^{-\lambda a}\left(1 + \lambda a + \frac{(\lambda a)^2}{2!} + \frac{(\lambda a)^3}{3!} + \frac{(\lambda a)^4}{4!}\right)\right] = 1 - \left[e^{-0.001 \times 100}\left(1 + (0.001 \times 100) + \frac{(0.001 \times 100)^2}{2} + \frac{(0.001 \times 100)^3}{6} + \frac{(0.001 \times 100)^4}{24}\right)\right]$

$= 1 - \left[0.9048\left(1 + 0.1 + 0.005 + 0.000167 + 0.000004167\right)\right]$

$= \boxed{0.0000411284}$

8

from Gamma — $P = 0.73039$.

$\boxed{P = \frac{X}{n}}$ $0.001424 + 0.0193 + 0.1045$

$P(X\le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \binom{5}{0}\cdot(0.7304)^0\cdot(0.2696)^5 + \binom{5}{1}\cdot(0.7304)^1\cdot(0.2696)^4 + \binom{5}{2}\cdot(0.7304)^2\cdot(0.2696)^3$

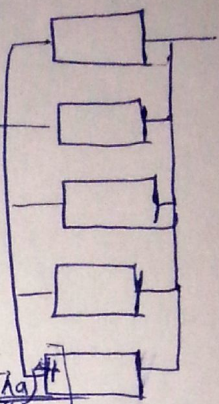
$= 0.001424 + 5*(0.7304)(0.2696)^4 + 10*(0.7304)^2(0.2696)^3$

$= \boxed{0.22976}$

(h:3pts) If all five components are connected as a parallel redundant configuration. Each component is exponentially distributed with mean failure time of 100 hour. Assume that the product survive when at most two components fail. Calculate the probability that the product will survive 150 hour.

$\boxed{\mu = 100 = \frac{1}{\lambda} \quad \lambda = \frac{1}{100} = 10^{-2}}$ exponential

product survive when at most two components fail



$P(X=150)$: * Firstly Gamma distribution

$\boxed{\text{Binomial}}$ ثم $\boxed{P}$ بنوجد

$\cancel{P(X\le 2)}$ $P(X\le 2) = \binom{n}{x}\cdot p^x\cdot(1-p)^{n-x}$

$\cancel{P(X=150)} =$ $= \cancel{\binom{5}{3}\cdot(0.7}$ * $\boxed{0.22976}$ *

* Gamma distribution

$R(150) = \sum_{k=0}^{4} e^{-\lambda a}\frac{(\lambda a)^k}{k!} = 1 - \left[e^{-\lambda a}\left[1 + \lambda a + \frac{(\lambda a)^2}{2!} + \frac{(\lambda a)^3}{3!} + \frac{(\lambda a)^4}{4!}\right]\right]$ $\boxed{n=5}$

Survive $\boxed{N=20}$ $= \left[e^{-10^{-2}*150}\left[1 + (1.5) + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!}\right]\right] P = ??$

$= \boxed{0.73039}$

$R(x) = 0.223$

$P_{Binomial} = 0.077$

(i:4pts) If a lot of twenty components; each is exponentially distributed with a failure rate of 0.02/ hour, are selected. The process engineer selected a sample of five components. If no more than one component is rejected, the lot will be accepted. If the lot contains two components are defective. Then,

$\boxed{\lambda = 0.02/\text{hour}}$, $\boxed{n=5}$, $\boxed{D=2}$, $\boxed{N=20}$

(a) Calculate the probability of lot acceptance.

hyper geometric   142506

$\boxed{P(X\le 1)} = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}} = P(X=0) + P(X=1)$

Accept lots

$= \frac{\binom{2}{0}\binom{28}{5}}{\binom{30}{5}} + \frac{\binom{2}{1}\binom{28}{4}}{\binom{30}{5}} = \frac{\left(\frac{2!}{0!\,2!}\right)\left(\frac{28!}{5!\,23!}\right)}{\left(\frac{30!}{5!\,25!}\right)} + \frac{\left(\frac{2!}{1!\,1!}\right)\left(\frac{28!}{24!\,4!}\right)}{\left(\frac{30!}{5!\,25!}\right)}$

2

$= 0.6897 + 0.2874 = \boxed{0.977}$

(b) Use appropriate approximation to calculate the probability in part (a). Is this approximation satisfactory? Why? Why not?

→ Binomial distribution → $\boxed{p = \frac{D}{N} = \frac{2}{20} = 0.1}$, $\boxed{n=5}$

$P(X\le 1) = P(X=0) + P(X=1)$

$= \binom{5}{0}\cdot p^x*(1-p)^{n-x} + \binom{n}{x}\cdot p^x + (1-p)^{n-x}$

$= \binom{5}{0}\cdot(0.1)^0*(0.9)^5 + \binom{5}{1}\cdot(0.1)^1\cdot(0.9)^4$

$= \frac{5!}{5!\,0!}*(0.1)^0*(0.9)^5 + \frac{5!}{1!\,4!}(0.1)^1\cdot(0.9)^4$

$= 1*1*0.59049 + 5*0.1*0.6561 = \boxed{0.91854}$

2

* This approximation is non satisfactory because $\left(\frac{n}{N} = 0.25 \ge 0.1\right)$ and should be (0.1) to be satisfactory