Fundamentals of Thermal-Fluid Sciences 4th Edition in SI Units Yunus A. Çengel, John M. Cimbala, Robert H. Turner McGraw-Hill, 2012

## Chapter 1 INTRODUCTION AND OVERVIEW

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# **Objectives**

- Be acquainted with the engineering sciences thermodynamics, heat transfer, and fluid mechanics, and understand the basic concepts of thermal-fluid sciences.
- Be comfortable with the metric SI and English units.
- Develop an intuitive systematic problem-solving technique.
- Learn the proper use of software packages in engineering.
- Develop an understanding of accuracy and significant digits in calculations.

## 1-1 INTRODUCTION TO THERMAL-FLUID SCIENCES

### Thermal-fluid sciences:

The physical sciences that deal with energy and the transfer, transport, and conversion of energy.

- Thermal-fluid sciences are studied under the subcategories of
  - ✓ thermodynamics
  - ✓ heat transfer
  - ✓ fluid mechanics



### FIGURE 1-1

The design of many engineering systems, such as this solar hot water system, involves thermal-fluid sciences.

### **Application Areas of Thermal-Fluid Sciences**



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## **1-2 THERMODYNAMICS**

- Thermodynamics: The science of energy.
- Energy: The ability to cause changes.
- The name *thermodynamics* stems from the Greek words *therme* (heat) and *dynamis* (power).
- Conservation of energy principle: During an interaction, energy can change from one form to another but the total amount of energy remains constant.
- Energy cannot be created or destroyed.
- The first law of thermodynamics: An expression of the conservation of energy principle.
- The first law asserts that *energy* is a thermodynamic property.



### FIGURE 1–3

Energy cannot be created or destroyed; it can only change forms (the first law).

- The second law of thermodynamics: It asserts that energy has *quality* as well as *quantity*, and actual processes occur in the direction of decreasing quality of energy.
- Classical thermodynamics: A macroscopic approach to the study of thermodynamics that does not require a knowledge of the behavior of individual particles.
- It provides a direct and easy way to the solution of engineering problems and it is used in this text.
- Statistical thermodynamics: A microscopic approach, based on the average behavior of large groups of individual particles.
- It is used in this text only in the supporting role.



Conservation of energy principle for the human body.



Heat flows in the direction of decreasing temperature.

# **1-3 HEAT TRANSFER**

- Heat: The form of energy that can be transferred from one system to another as a result of temperature difference.
- Heat Transfer: The science that deals with the determination of the rates of such energy transfers and variation of temperature.
- Thermodynamics is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and it gives no indication about how long the process will take. But in engineering, we are often interested in the rate of heat transfer, which is the topic of the science of heat transfer.



### FIGURE 1–6

We are normally interested in how long it takes for the hot coffee in a thermos bottle to cool to a certain temperature, which cannot be determined from a thermodynamic analysis alone.

# **1-4 FLUID MECHANICS**

- Fluid mechanics: The science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.
- Fluid: A substance in the liquid or gas phase.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.



Fluid mechanics deals with liquids and gases in motion or at rest.



Deformation of a rubber block placed between two parallel plates under the influence of a shear force. The shear stress shown is that on the rubber—an equal but opposite shear stress acts on the upper plate.



The normal stress and shear stress at the surface of a fluid element. For fluids at rest, the shear stress is zero and pressure is the only normal stress.



Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space

### **1-5 IMPORTANCE OF DIMENSIONS AND UNITS**

- Any physical quantity can be characterized by dimensions.
- The magnitudes assigned to the dimensions are called units.
- Some basic dimensions such as mass *m*, length *L*, time *t*, and temperature *T* are selected as primary or fundamental dimensions, while others such as velocity *V*, energy *E*, and volume *V* are expressed in terms of the primary dimensions and are called secondary dimensions, or derived dimensions.
- Metric SI system: A simple and logical system based on a decimal relationship between the various units.
- English system: It has no apparent systematic numerical base, and various units in this system are related to each other rather arbitrarily.

#### TABLE 1-1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length Mass	meter (m) kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

#### TABLE 1–2

Standard prefixes in SI units	
Multiple	Prefix
$ \frac{10^{12}}{10^{9}} \\ \frac{10^{6}}{10^{3}} \\ \frac{10^{2}}{10^{1}} \\ \frac{10^{-1}}{10^{-2}} \\ \frac{10^{-3}}{10^{-6}} $	tera, T giga, G mega, M kilo, k hecto, h deka, da deci, d centi, c milli, m micro, μ
10 <sup>-9</sup> 10 <sup>-12</sup>	nano, n pico, p
	. ,

## Some SI and English Units

1 lbm = 0.45359 kg

1 ft = 0.3048 m

Force = (Mass)(Acceleration)

F = ma

 $1 N = 1 \text{ kg} \cdot \text{m/s}^2$  $1 \text{ lbf} = 32.174 \text{ lbm} \cdot \text{ft/s}^2$ 

Work = Force  $\times$  Distance 1 J = 1 N·m 1 cal = 4.1868 J 1 Btu = 1.0551 kJ

$$\begin{array}{c} 1 \text{ M}\Omega \\ 1 \text{ kg} \\ (10^3 \text{ g}) \end{array} \qquad \begin{array}{c} 1 \text{ M}\Omega \\ - \sqrt{10^6 \Omega} \end{array}$$

### FIGURE 1–12

The SI unit prefixes are used in all branches of engineering.

$$m = 1 \text{ kg}$$

$$a = 1 \text{ m/s}^{2}$$

$$F = 1 \text{ N}$$

$$m = 32.174 \text{ lbm}$$

$$a = 1 \text{ ft/s}^{2}$$

$$F = 1 \text{ lbf}$$
Figure 1–13
The definition of the force units.



#### FIGURE 1–14

The relative magnitudes of the force units newton (N), kilogram-force (kgf), and pound-force (lbf).



A body weighing 60 kgf (600 N) on earth will weigh only slightly over 10 kgf (100 N) on the moon.





FIGURE 1–16

The weight of a unit mass at sea level.

A typical match yields about one kJ of energy if completely burned.

**Specific weight**  $\gamma$ : The weight of a unit volume of a substance.

$$\gamma = \rho g$$

## **Dimensional homogeneity**

All equations must be dimensionally **homogeneous**.

## **Unity Conversion Ratios**

All nonprimary units (secondary units) can be formed by combinations of primary units. Force units, for example, can be expressed as

N = kg 
$$\frac{m}{s^2}$$
 and lbf = 32.174 lbm  $\frac{ft}{s^2}$ 

They can also be expressed more conveniently as **unity conversion ratios** as

$$\frac{N}{\text{kg} \cdot \text{m/s}^2} = 1 \quad \text{and} \quad \frac{\text{lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2} = 1$$



### FIGURE 1–18

To be dimensionally homogeneous, all the terms in an equation must have the same dimensions.

Unity conversion ratios are identically equal to 1 and are unitless, and thus such ratios (or their inverses) can be inserted conveniently into any calculation to properly convert units.



### FIGURE 1–21

Always check the units in your calculations.

 $\left(\frac{32.174 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}}\right) \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right)$  $\left(\frac{1 \text{ W}}{1 \text{ J/s}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ N} \cdot \text{m}}\right) \left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right)$  $\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \left(\frac{1 \text{ lbm}}{0.45359 \text{ kg}}\right)$ 

### FIGURE 1–22

Every unity conversion ratio (as well as its inverse) is exactly equal to one. Shown here are a few commonly used unity conversion ratios.



## **1-6 PROBLEM-SOLVING TECHNIQUE**

- Step 1: Problem Statement
- Step 2: Schematic
- Step 3: Assumptions and Approximations
- Step 4: Physical Laws
- Step 5: Properties
- Step 6: Calculations
- Step 7: Reasoning, Verification, and Discussion



### FIGURE 1-25

A step-by-step approach can greatly simplify problem solving.



### FIGURE 1–26

The assumptions made while solving an engineering problem must be reasonable and justifiable.



### FIGURE 1–28

Neatness and organization are highly valued by employers.



### FIGURE 1–27

The results obtained from an engineering analysis must be checked for reasonableness.

## Engineering Software Packages

### FIGURE 1-29

An excellent word-processing program does not make a person a good writer; it simply makes a good writer a more efficient writer.

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# **EES (Engineering Equation Solver)**

(Pronounced as ease)

EES is a program that solves systems of linear or nonlinear algebraic or differential equations numerically.

It has a large library of built-in thermodynamic property functions as well as mathematical functions.

Unlike some software packages, EES does not solve engineering problems; it only solves the equations supplied by the user.

## **A Remark on Significant Digits**

In engineering calculations, the information given is not known to more than a certain number of significant digits, usually three digits.

Consequently, the results obtained cannot possibly be accurate to more significant digits.

Reporting results in more significant digits implies greater accuracy than exists, and it should be avoided.



### FIGURE 1–30

A result with more significant digits than that of given data falsely implies more accuracy.

## **Summary**

- Introduction to Thermal-Fluid Sciences
   ✓ Application areas of thermal-fluid sciences
- Thermodynamics
- Heat Transfer
- Fluid Mechanics
- Importance of Dimensions and Units
  - Some SI and English units
  - ✓ Dimensional homogeneity
  - ✓ Unity conversion ratios
- Problem-Solving Technique
  - Engineering Software Packages
  - Engineering Equation Solver (EES)
  - ✓ A Remark on Significant Digits

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## Chapter 2 BASIC CONCEPTS OF THERMODYNAMICS

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# **Objectives**

- Identify the unique vocabulary associated with thermodynamics through the precise definition of basic concepts to form a sound foundation for the development of the principles of thermodynamics.
- Explain the basic concepts of thermodynamics such as system, state, state postulate, equilibrium, process, and cycle.
- Review concepts of temperature, temperature scales, pressure, and absolute and gage pressure.

## **2-1 SYSTEMS AND CONTROL VOLUMES**

- **System**: A quantity of matter or a region in space chosen for study.
- Surroundings: The mass or region outside the system
- **Boundary**: The real or imaginary surface that separates the system from its surroundings.
- The boundary of a system can be *fixed* or *movable*.
- Systems may be considered to be closed or open.
- Closed system (Control mass): A fixed amount of mass, and no mass can cross its boundary





An open system (a control volume)

with one inlet and one exit.

- Open system (control volume): A properly selected region in space.
- It usually encloses a device that involves mass flow such as a compressor, turbine, or nozzle.
- Both mass and energy can cross the boundary of a control volume.
- **Control surface**: The boundaries of a control volume. It can be real or imaginary.



- (*a*) A control volume with real and imaginary boundaries
- (b) A control volume with fixed and moving boundaries

#### FIGURE 2-4

A control volume can involve fixed, moving, real, and imaginary boundaries.

## **2-2 PROPERTIES OF A SYSTEM**

- **Property:** Any characteristic of a system.
- Some familiar properties are pressure *P*, temperature *T*, volume *V*, and mass *m*.
- Properties are considered to be either *intensive* or *extensive*.
- Intensive properties: Those that are independent of the mass of a system, such as temperature, pressure, and density.
- Extensive properties: Those whose values depend on the size or extent—of the system.
- **Specific properties:** Extensive properties per unit mass.

(v = V/m) (e = E/m)



#### FIGURE 2–6

Criterion to differentiate intensive and extensive properties.

## Continuum

- Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a continuum.
- The continuum idealization allows us to treat properties as point functions and to assume the properties vary continually in space with no jump discontinuities.
- This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.
- This is the case in practically all problems.
- In this text we will limit our consideration to substances that can be modeled as a continuum.



### FIGURE 2–7

Despite the large gaps between molecules, a substance can be treated as a continuum because of the very large number of molecules even in an extremely small volume.

## **2-3 DENSITY AND SPECIFIC GRAVITY**

### Density

$$\rho = \frac{m}{V}$$
 (kg/m<sup>3</sup>)

### **Specific volume**

Specific gravity: The ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C).

$$SG = \frac{\rho}{\rho_{H_2O}}$$

 $v = \frac{V}{m} = \frac{1}{\rho}$ 

 $V = 12 \text{ m}^{3}$ m = 3 kg $\downarrow$  $\rho = 0.25 \text{ kg/m}^{3}$  $v = \frac{1}{\rho} = 4 \text{ m}^{3}/\text{kg}$ 

Specific weight: The

weight of a unit volume of a substance.

 $\gamma_s = \rho g$  (N/m<sup>3</sup>)

Density is mass per unit volume; specific volume is volume per unit mass.

### TABLE 2-1

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7-2.0
Ice	0.92
Air (at 1 atm)	0.0013

### **2-4 STATE AND EQUILIBRIUM**

- Thermodynamics deals with *equilibrium* states.
- Equilibrium: A state of balance.
- In an equilibrium state there are no unbalanced potentials (or driving forces) within the system.
- Thermal equilibrium: If the temperature is the same throughout the entire system.
- Mechanical equilibrium: If there is no change in pressure at any point of the system with time.
- Phase equilibrium: If a system involves two phases and when the mass of each phase reaches an equilibrium level and stays there.
- Chemical equilibrium: If the chemical composition of a system does not change with time, that is, no chemical reactions occur.



## **The State Postulate**

- The number of properties required to fix the state of a system is given by the state postulate:
  - The state of a simple compressible system is completely specified by two independent, intensive properties.
- Simple compressible system: If a system involves no electrical, magnetic, gravitational, motion, and surface tension effects.



The state of nitrogen is fixed by two independent, intensive properties.

## **2-5 PROCESSES AND CYCLES**

- Process: Any change that a system undergoes from one equilibrium state to another.
- Path: The series of states through which a system passes during a process.
- To describe a process completely, one should specify the initial and final states, as well as the path it follows, and the interactions with the surroundings.
- Quasistatic or quasi-equilibrium process: When a process proceeds in such a manner that the system remains infinitesimally close to an equilibrium state at all times.



### FIGURE 2–12

A process between states 1 and 2 and the process path.



- Process diagrams plotted by employing thermodynamic properties as coordinates are very useful in visualizing the processes.
- Some common properties that are used as coordinates are temperature *T*, pressure *P*, and volume *V* (or specific volume *v*).
- The prefix *iso* is often used to designate a process for which a particular property remains constant.
- Isothermal process: A process during which the temperature *T* remains constant.
- Isobaric process: A process during which the pressure *P* remains constant.
- Isochoric (or isometric) process: A process during which the specific volume v remains constant.
- Cycle: A process during which the initial and final states are identical.



## **The Steady-Flow Process**

- The term steady implies no change with time. The opposite of steady is unsteady, or transient.
- A large number of engineering devices operate for long periods of time under the same conditions, and they are classified as steady-flow devices.
- Steady-flow process: A process during which a fluid flows through a control volume steadily.
- Steady-flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers, and heat exchangers or power plants or refrigeration systems.



#### FIGURE 2–16

Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

## 2-6 TEMPERATURE AND THE ZEROTH LAW OF THERMODYNAMICS

- The zeroth law of thermodynamics: If two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other.
- By replacing the third body with a thermometer, the zeroth law can be restated as *two bodies are in thermal equilibrium if both have the same temperature reading even if they are not in contact.*



### FIGURE 2–17

Two bodies reaching thermal equilibrium after being brought into contact in an isolated enclosure.
## **Temperature Scales**

- All temperature scales are based on some easily reproducible states such as the freezing and boiling points of water: the ice point and the steam point.
- Ice point: A mixture of ice and water that is in equilibrium with air saturated with vapor at 1 atm pressure (0°C or 32°F).
- Steam point: A mixture of liquid water and water vapor (with no air) in equilibrium at 1 atm pressure (100°C or 212°F).
- Celsius scale: in SI unit system
- Fahrenheit scale: in English unit system
- Thermodynamic temperature scale: A temperature scale that is independent of the properties of any substance.
- Kelvin scale (SI) Rankine scale (E)
- A temperature scale nearly identical to the Kelvin scale is the ideal-gas temperature scale. The temperatures on this scale are measured using a constant-volume gas thermometer.





A constant-volume gas thermometer would read -273.15°C at absolute zero pressure. <sup>14</sup>



- The reference temperature in the original Kelvin scale was the *ice point*, 273.15 K, which is the temperature at which water freezes (or ice melts).
- The reference point was changed to a much more precisely reproducible point, the *triple point* of water (the state at which all three phases of water coexist in equilibrium), which is assigned the value 273.16 K.

### The International Temperature Scale of 1990 (ITS-90)

The *International Temperature Scale of 1990* supersedes the International Practical Temperature Scale of 1968 (IPTS-68), 1948 (ITPS-48), and 1927 (ITS-27).

The ITS-90 is similar to its predecessors except that it is more refined with updated values of fixed temperatures, has an extended range, and conforms more closely to the thermodynamic temperature scale.

On this scale, the unit of thermodynamic temperature T is again the kelvin (K), defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water, which is sole defining fixed point of both the ITS-90 and the Kelvin scale and is the most important thermometric fixed point used in the calibration of thermometers to ITS-90. The unit of Celsius temperature is the degree Celsius (°C).

The ice point remains the same at 0°C (273.15 K) in both ITS-90 and ITPS-68, but the steam point is 99.975°C in ITS-90 whereas it was 100.000°C in IPTS-68.

The change is due to precise measurements made by gas thermometry by paying particular attention to the effect of sorption (the impurities in a gas absorbed by the walls of the bulb at the reference temperature being desorbed at higher temperatures, causing the measured gas pressure to increase).

### **2-7 PRESSURE**

**Pressure**: A normal force exerted by a fluid per unit area

 $1 \text{ Pa} = 1 \text{ N/m}^2$ 

 $1 \text{ bar} = 10^{5} \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$  1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bars  $1 \text{ kgf/cm}^{2} = 9.807 \text{ N/cm}^{2} = 9.807 \times 10^{4} \text{ N/m}^{2} = 9.807 \times 10^{4} \text{ Pa}$ = 0.9807 bar





= 0.9679 atm

The normal stress (or "pressure") on the feet of a chubby person is much greater than on the feet of a slim person.

- Absolute pressure: The actual pressure at a given position. It is measured relative to absolute vacuum (i.e., absolute zero pressure).
- Gage pressure: The difference between the absolute pressure and the local atmospheric pressure. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and so they indicate gage pressure.
- Vacuum pressures: Pressures below atmospheric pressure.



### **Variation of Pressure with Depth**

$$\Delta P = P_2 - P_1 = \rho g \ \Delta z = \gamma_s \Delta z$$
$$P = P_{\text{atm}} + \rho g h \quad \text{or} \quad P_{\text{gage}} = \rho g h$$

When the variation of density with elevation is known

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g \, dz$$



#### FIGURE 2–25

The pressure of a fluid at rest increases with depth (as a result of added weight).



#### FIGURE 2–26

Free-body diagram of a rectangular fluid element in equilibrium.



In a room filled with a gas, the variation of pressure with height is negligible.

Pressure in a liquid at rest increases linearly with distance from the free surface.





#### FIGURE 2–29

The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

**Pascal's law:** The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \longrightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \longrightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

The area ratio  $A_2/A_1$  is called the *ideal mechanical advantage* of the hydraulic lift.

> Lifting of a large weight by a small force by the application of Pascal's law.



### **2-8 THE MANOMETER**

It is commonly used to measure small and moderate pressure differences. A manometer contains one or more fluids such as mercury, water, alcohol, or oil. Measuring the

pressure drop across

a flow section or a flow

device by a differential

manometer.



#### FIGURE 2–33

In stacked-up fluid layers, the pressure change across a fluid layer of density  $\rho$  and height *h* is  $\rho gh$ .

$$P_{\rm atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

A flow section or flow device Fluid 1  $\rho_1$  $\rho_2$  $P_2$  $P_$ 

$$P_1 + \rho_1 g(a + h) - \rho_2 gh - \rho_1 ga = P_2$$
  
 $P_1 - P_2 = (\rho_2 - \rho_1) gh$ 



### Other Pressure Measurement Devices

- **Bourdon tube**: Consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle.
- Pressure transducers: Use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance.
- Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts.
- Strain-gage pressure transducers: Work by having a diaphragm deflect between two chambers open to the pressure inputs.
- **Piezoelectric transducers**: Also called solidstate pressure transducers, work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure.



### **2-9 THE BAROMETER AND ATMOSPHERIC PRESSURE**

- Atmospheric pressure is measured by a device called a barometer; thus, the atmospheric pressure is often referred to as the barometric pressure.
- A frequently used pressure unit is the standard atmosphere, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ<sub>Hg</sub> = 13,595 kg/m<sup>3</sup>) under standard gravitational acceleration (g = 9.807 m/s<sup>2</sup>).

 $P_{\rm atm} = \rho g h$ 



FIGURE 2–37 The basic barometer. The length or the cross-sectional area of the tube has no effect on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effects.





### FIGURE 2–39

At high altitudes, a car engine generates less power and a person gets less oxygen because of the lower density of air.

## Summary

- Systems and control volumes
- Properties of a system
  Continuum
- Density and specific gravity
- State and equilibrium

✓ The state postulate

Processes and cycles

✓ The steady-flow process

- Temperature and the zeroth law of thermodynamics
  - ✓ Temperature scales
  - ✓ ITS-90
- Pressure

✓ Variation of pressure with depth

The manometer

✓ Other pressure measurement devices

• The barometer and atmospheric pressure

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## Chapter 3 ENERGY, ENERGY TRANSFER, AND GENERAL ENERGY ANALYSIS

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## **Objectives**

- Introduce the concept of energy and define its various forms.
- Discuss the nature of internal energy.
- Define the concept of heat and the terminology associated with energy transfer by heat.
- Discuss the three mechanisms of heat transfer: conduction, convection, and radiation.
- Define the concept of work, including electrical work and several forms of mechanical work.
- Introduce the first law of thermodynamics, energy balances, and mechanisms of energy transfer to or from a system.
- Determine that a fluid flowing across a control surface of a control volume carries energy across the control surface in addition to any energy transfer across the control surface that may be in the form of heat and/or work.
- Define energy conversion efficiencies.

## **3-1 INTRODUCTION**

- If we take the entire room—including the air and the refrigerator (or fan)—as the system, which is an adiabatic closed system since the room is well-sealed and well-insulated, the only energy interaction involved is the electrical energy crossing the system boundary and entering the room.
- As a result of the conversion of electric energy consumed by the device to heat, the room temperature will rise.





## **3-2 FORMS OF ENERGY**

- Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the total energy, *E* of a system.
- Thermodynamics deals only with the *change* of the total energy.
- Macroscopic forms of energy: Those a system possesses as a whole with respect to some outside reference frame, such as kinetic and potential energies.
- Microscopic forms of energy: Those related to the molecular structure of a system and the degree of the molecular activity.
- Internal energy, U: The sum of all the microscopic forms of energy.
- Kinetic energy, KE: The energy that a system possesses as a result of its motion relative to some reference frame.
- Potential energy, PE: The energy that a system possesses as a result of its elevation in a gravitational field.



The macroscopic energy of an object changes with velocity and elevation.

KE = 
$$m \frac{V^2}{2}$$
 (kJ)Kinetic energy  
per unit masske =  $\frac{V^2}{2}$  (kJ/kg)Kinetic energy  
per unit massPE =  $mgz$  (kJ)Potential energy  
per unit massPE =  $mgz$  (kJ/kg)Potential energy  
per unit massPE =  $mgz$  (kJ/kg)Potential energy  
per unit massMass flow rate  
 $\dot{m} = \rho \dot{V} = \rho A_c V_{avg}$  (kg/s  
Energy flow rate  
 $\dot{E} = \dot{m}e$  (kJ/s or kW)E =  $U + KE + PE = U + m \frac{V^2}{2} + mgz$  (kJ)Total energy  
of a systeme =  $u + ke + pe = u + \frac{V^2}{2} + gz$  (kJ/kg)Energy of a system  
per unit masse =  $\frac{E}{m}$  (kJ/kg)Total energy  
per unit mass

## **Some Physical Insight to Internal Energy**



The various forms of microscopic energies that make up *sensible* energy.

Sensible energy: The portion of the internal energy of a system associated with the kinetic energies of the molecules.

Latent energy: The internal energy associated with the phase of a system.

**Chemical energy:** The internal energy associated with the atomic bonds in a molecule.

Nuclear energy: The tremendous amount of energy associated with the strong bonds within the nucleus of the atom itself.

Thermal = Sensible + Latent Internal = Sensible + Latent + Chemical + Nuclear

- The total energy of a system, can be contained or stored in a system, and thus can be viewed as the static forms of energy.
- The forms of energy not stored in a system can be viewed as the dynamic forms of energy or as energy interactions.
- The dynamic forms of energy are recognized at the system boundary as they cross it, and they represent the energy gained or lost by a system during a process.
- The only two forms of energy interactions associated with a closed system are heat transfer and work.



### FIGURE 3–7

The *macroscopic* kinetic energy is an organized form of energy and is much more useful than the disorganized *microscopic* kinetic energies of the molecules.

 The difference between heat transfer and work: An energy interaction is heat transfer if its driving force is a temperature difference. Otherwise it is work.

## More on Nuclear Energy

- The best known fission reaction involves the split of the uranium atom (the U-235 isotope) into other elements and is commonly used to generate electricity in nuclear power plants (440 of them in 2004, generating 363,000 MW worldwide), to power nuclear submarines and aircraft carriers, and even to power spacecraft as well as building nuclear bombs.
- Nuclear energy by fusion is released when two small nuclei combine into a larger one.
- The uncontrolled fusion reaction was achieved in the early 1950s, but all the efforts since then to achieve controlled fusion by massive lasers, powerful magnetic fields, and electric currents to generate power have failed.



The fission of uranium and the fusion of hydrogen during nuclear reactions, and the release of nuclear energy.

### **Mechanical Energy**

Mechanical energy: The form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.

Kinetic and potential energies: The familiar forms of mechanical energy.

$$e_{\rm mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$
 Mechanical energy of a flowing fluid per unit mass

$$\dot{E}_{\rm mech} = \dot{m}e_{\rm mech} = \dot{m}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)$$
 Rate of mechanical energy of a flowing fluid

Mechanical energy change of a fluid during incompressible flow per unit mass

$$\Delta e_{\rm mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad (\rm kJ/kg)$$

Rate of mechanical energy change of a fluid during incompressible flow

$$\Delta \dot{E}_{\rm mech} = \dot{m} \Delta e_{\rm mech} = \dot{m} \left( \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right)$$
(kW)



#### FIGURE 3–10

Mechanical energy is a useful concept for flows that do not involve significant heat transfer or energy conversion, such as the flow of gasoline from an underground tank into a car.

#### FIGURE 3–11

Mechanical energy is illustrated by an ideal hydraulic turbine coupled with an ideal generator. In the absence of irreversible losses, the maximum produced power is proportional to (*a*) the change in water surface elevation from the upstream to the downstream reservoir or (*b*) (close-up view) the drop in water pressure from just upstream to just downstream of the turbine.







## **3-3 ENERGY TRANSFER BY HEAT**

Heat: The form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference.



### FIGURE 3–13

Energy can cross the boundaries of a closed system in the form of heat and work.



### FIGURE 3–14

Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.



exchanges no heat with its surroundings.

### **Historical Background on Heat**

- Kinetic theory: Treats molecules as tiny balls that are in motion and thus possess kinetic energy.
- Heat: The energy associated with the random motion of atoms and molecules.

#### Heat transfer mechanisms:

- Conduction: The transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interaction between particles.
- Convection: The transfer of energy between a solid surface and the adjacent fluid that is in motion, and it involves the combined effects of conduction and fluid motion.
- Radiation: The transfer of energy due to the emission of electromagnetic waves (or photons).



### FIGURE 3–18

In the early nineteenth century, heat was thought to be an invisible fluid called the *caloric* that flowed from warmer bodies to the cooler ones.

## **3-4 ENERGY TRANSFER BY WORK**

- Work: The energy transfer associated with a force acting through a distance.
  - A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions
- Formal sign convention: Heat transfer to a system and work done by a system are positive; heat transfer from a system and work done on a system are negative.
- Alternative to sign convention is to use the subscripts *in* and *out* to indicate direction. This is the primary approach in this text.



## Heat vs. Work

- Both are recognized at the boundaries of a system as they cross the boundaries. That is, both heat and work are *boundary* phenomena.
- Systems possess energy, but not heat or work.
- Both are associated with a *process*, not a state.
- Unlike properties, heat or work has no meaning at a state.
- Both are *path functions* (i.e., their magnitudes depend on the path followed during a process as well as the end states).

Properties are point functions have exact differentials (*d*).

$$\int_{1}^{2} dV = V_2 - V_1 = \Delta V$$

Path functions have inexact differentials  $(\delta)$ 



### FIGURE 3-21

Properties are point functions; but heat and work are path functions (their magnitudes depend on the path followed).

$$\int_{1}^{2} \delta W = W_{12} \qquad (not \ \Delta W)$$

## **Electrical Work**

**Electrical work** 

 $W_e = \mathbf{V}N$ 

Electrical power  $\dot{W}_e = \mathbf{V}I$  (W)

When potential difference and current change with time

$$W_e = \int_1^2 \mathbf{V} I \, dt \qquad (\mathbf{kJ})$$

When potential difference and current remain constant

 $W_e = \mathbf{V}I \ \Delta t \qquad (\mathrm{kJ})$ 



### FIGURE 3–26

Electrical power in terms of resistance R, current I, and potential difference V.

## **3-5 MECHANICAL FORMS OF WORK**

- There are two requirements for a work interaction between a system and its surroundings to exist:
  - ✓ there must be a force acting on the boundary.
  - the boundary must move.



The work done is proportional to the force applied (F) and the distance traveled (s).

#### FIGURE 3–28

If there is no movement, no work is done.

## Shaft Work

A force *F* acting through a moment arm *r* generates a torque T

$$T = Fr \rightarrow F = \frac{T}{r}$$

This force acts through a distance  $s s = (2\pi r)n$ 

Shaft  
work 
$$W_{\rm sh} = Fs = \left(\frac{\mathrm{T}}{r}\right)(2\pi rn) = 2\pi n\mathrm{T}$$
 (kJ)

The power transmitted through the shaft is the shaft work done per unit time  $\dot{W}_{\rm sh} = 2\pi \dot{n} T$  (kW)



### FIGURE 3–29

Energy transmission through rotating shafts is commonly encountered in practice.



#### FIGURE 3–30

Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

When the length of the spring changes by a differential amount dx under the influence of a force F, the work done is

$$\delta W_{\rm spring} = F \, dx$$

F = kx

For linear elastic springs, the displacement *x* is proportional to the force applied

(kN) k: spring constant (kN/m)

# Spring Work

### Substituting and integrating yield $W_{\text{spring}} = \frac{1}{2}k(x_2^2 - x_1^2)$ (kJ)

 $x_1$  and  $x_2$ : the initial and the final displacements



### **Work Done on Elastic Solid Bars**

$$W_{\text{elastic}} = \int_{1}^{2} F \, dx = \int_{1}^{2} \sigma_n A \, dx \qquad (\text{kJ})$$

### Work Associated with the Stretching of a Liquid Film



#### FIGURE 3-34

Solid bars behave as springs under the influence of a force.



### FIGURE 3–35

Stretching a liquid film with a movable wire.

### Work Done to Raise or to Accelerate a Body

- 1. The work transfer needed to raise a body is equal to the change in the potential energy of the body.
- 2. The work transfer needed to accelerate a body is equal to the change in the kinetic energy of the body.

### **Nonmechanical Forms of Work**

**Electrical work**: The generalized force is the *voltage* (the electrical potential) and the generalized displacement is the *electrical charge*.

Magnetic work: The generalized force is the *magnetic field strength* and the generalized displacement is the total *magnetic dipole moment*.

**Electrical polarization work**: The generalized force is the *electric field strength* and the generalized displacement is the *polarization of the medium*.



#### FIGURE 3–36

The energy transferred to a body while being raised is equal to the change in its potential energy.

## **3-6 THE FIRST LAW OF THERMODYNAMICS**

- The first law of thermodynamics (the conservation of energy principle) provides a sound basis for studying the relationships among the various forms of energy and energy interactions.
- The first law states that energy can be neither created nor destroyed during a process; it can only change forms.
- The First Law: For all adiabatic processes between two specified states of a closed system, the net work done is the same regardless of the nature of the closed system and the details of the process.





 $Q_{\rm in} = 15 \,\rm kJ$ 

#### FIGURE 3-41

In the absence of any work interactions, the energy change of a system is equal to the net heat transfer.



#### FIGURE 3-42

The work (electrical) done on an adiabatic system is equal to the increase in the energy of the system.



#### FIGURE 3-43

The work (shaft) done on an adiabatic system is equal to the increase in the energy of the system.
## Energy Balance

The net change (increase or decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process.



### FIGURE 3-44

The work (boundary) done on an adiabatic system is equal to the increase in the energy of the system.

The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

## Energy Change of a System, $\Delta E_{system}$

Energy change = Energy at final state - Energy at initial state

 $\Delta E_{\text{system}} = E_{\text{final}} - E_{\text{initial}} = E_2 - E_1$  $\Delta E = \Delta U + \Delta \text{KE} + \Delta \text{PE}$ 

Internal, kinetic, and potential energy changes

$$\Delta U = m(u_2 - u_1)$$
$$\Delta KE = \frac{1}{2}m(V_2^2 - V_1^2)$$
$$\Delta PE = mg(z_2 - z_1)$$

Stationary Systems  $z_1 = z_2 \rightarrow \Delta PE = 0$   $V_1 = V_2 \rightarrow \Delta KE = 0$  $\Delta E = \Delta U$ 

### FIGURE 3-46

For stationary systems,  $\Delta KE = \Delta PE = 0$ ; thus  $\Delta E = \Delta U$ .

## Mechanisms of Energy Transfer, E<sub>in</sub> and E<sub>out</sub>

- Heat transfer
- Work transfer
- Mass flow

A closed mass involves only *heat transfer* and *work*.

Mass in



## **3-7 ENERGY CONVERSION EFFICIENCIES**

*Efficiency* is one of the most frequently used terms in thermodynamics, and it indicates how well an energy conversion or transfer process is accomplished.



### FIGURE 3–54

The definition of performance is not limited to thermodynamics only.

# Performance = $\frac{\text{Desired output}}{\text{Required input}}$

### Efficiency of a water

heater: The ratio of the energy delivered to the house by hot water to the energy supplied to the water heater.

Туре	Efficiency
Gas, conventional	55%
Gas, high-efficiency	62%
Electric, conventional	90%
Electric, high-efficiency	94%



Water heater

 $\eta_{ ext{combustion}}$ 

ΗV

Amount of heat released during combustion

### Heating value of the fuel burned

Heating value of the fuel: The amount of heat released when a unit amount of fuel at room temperature is completely burned and the combustion products are cooled to the room temperature.

Lower heating value (LHV): When the water leaves as a vapor.

**Higher heating value (HHV)**: When the water in the combustion gases is completely condensed and thus the heat of vaporization is also recovered.



The efficiency of space heating systems of residential and commercial buildings is usually expressed in terms of the annual fuel utilization efficiency (AFUE), which accounts for the combustion efficiency as well as other losses such as heat losses to unheated areas and start-up and cool-down losses.

- Generator: A device that converts mechanical energy to electrical energy.
- Generator efficiency: The ratio of the electrical power output to the mechanical power input.
- Thermal efficiency of a power plant: The ratio of the net electrical power output to the rate of fuel energy input.

#### TABLE 3-1

Theoretical limit

	Efficacy
Type of lighting	lumens/W
Combustion	
Candle	0.3
Kerosene lamp	1–2
Incandescent	
Ordinary	6–20
Halogen	15–35
Fluorescent	
Compact	40-87
Tube	60–120
High-intensity discharge	
Mercury vapor	40–60
Metal halide	65–118
High-pressure sodium	85–140
Low-pressure sodium	70–200
Solid-State	
LED	20–160
OLED	15–60

300\*

officious of different lighting

### Overall efficiency of a power plant



### Lighting efficacy: The amount of light

output in lumens per W of electricity consumed.



### FIGURE 3–57

A 15-W compact fluorescent lamp provides as much light as a 60-W incandescent lamp.

#### TABLE 3-2

Energy costs of cooking a casserole with different appliances\*

[From J. T. Amann, A. Wilson, and K. Ackerly, *Consumer Guide to Home Energy Savings*, 9<sup>th</sup> ed., American Council for an Energy-Efficient Economy, Washington, D.C., 2007, p. 163.]

Cooking appliance	Cooking temperature	Cooking time	Energy used	Cost of energy
Electric oven Convection oven (elect.) Gas oven Frying pan Toaster oven Crockpot	350°F (177°C) 325°F (163°C) 350°F (177°C) 420°F (216°C) 425°F (218°C) 200°F (93°C)	1 h 45 min 1 h 1 h 50 min 7 h	2.0 kWh 1.39 kWh 0.112 therm 0.9 kWh 0.95 kWh 0.7 kWh	\$0.19 \$0.13 \$0.13 \$0.09 \$0.09 \$0.09 \$0.07
Microwave oven	"High"	15 min	0.36 kWh	\$0.03

\*Assumes a unit cost of \$0.095/kWh for electricity and \$1.20/therm for gas.

- Using energy-efficient appliances conserve energy.
- It helps the **environment** by reducing the amount of pollutants emitted to the atmosphere during the combustion of fuel.
- The combustion of fuel produces
  - carbon dioxide, causes global warming
  - nitrogen oxides and hydrocarbons, cause smog
  - carbon monoxide, toxic
  - sulfur dioxide, causes acid rain.





### FIGURE 3–58

The efficiency of a cooking appliance represents the fraction of the energy supplied to the appliance that is transferred to the food.

### **Efficiencies of Mechanical and Electrical Devices**

### Mechanical efficiency

 $\eta_{\text{mech}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}} = \frac{E_{\text{mech,out}}}{E_{\text{mech,in}}} = 1 - \frac{E_{\text{mech,loss}}}{E_{\text{mech,in}}}$ 

The effectiveness of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by the **pump efficiency** and **turbine efficiency**,

$$\eta_{\text{pump}} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{shaft,in}}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{pump}}}$$

$$\Delta \dot{E}_{\text{mech,fluid}} = \dot{E}_{\text{mech,out}} - \dot{E}_{\text{mech,in}}$$

$$\eta_{\text{turbine}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy decrease of the fluid}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta \dot{E}_{\text{mech,fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine},e}}$$

$$|\Delta \dot{E}_{\text{mech,fluid}}| = \dot{E}_{\text{mech,in}} - \dot{E}_{\text{mech,out}}$$



#### FIGURE 3–60

The mechanical efficiency of a fan is the ratio of the rate of increase of the mechanical energy of air to the mechanical power input.



## **Summary**

- Forms of energy
  - Macroscopic = kinetic + potential
  - Microscopic = Internal energy (sensible + latent + chemical + nuclear)
- Energy transfer by heat
- Energy transfer by work
- Mechanical forms of work
- The first law of thermodynamics
  - Energy balance
  - Energy change of a system
  - Mechanisms of energy transfer (heat, work, mass flow)
- Energy conversion efficiencies
  - Efficiencies of mechanical and electrical devices (turbines, pumps)

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## Chapter 4 PROPERTIES OF PURE SUBSTANCES

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Introduce the concept of a pure substance.
- Discuss the physics of phase-change processes.
- Illustrate the P-v, T-v, and P-T property diagrams and P-v-T surfaces of pure substances.
- Demonstrate the procedures for determining thermodynamic properties of pure substances from tables of property data.
- Describe the hypothetical substance "ideal gas" and the ideal-gas equation of state.
- Apply the ideal-gas equation of state in the solution of typical problems.
- Introduce the compressibility factor, which accounts for the deviation of real gases from ideal-gas behavior.

## **4-1 PURE SUBSTANCE**

- **Pure substance**: A substance that has a fixed chemical composition throughout.
- Air is a mixture of several gases, but it is considered to be a pure substance.



## **4-2 PHASES OF A PURE SUBSTANCE**



The molecules in a solid are kept at their positions by the large springlike inter-molecular forces.





#### FIGURE 4–5

The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) molecules move about at random in the gas phase. In a solid, the attractive and repulsive forces between the molecules tend to maintain them at relatively constant distances from each other.

### 4-3 PHASE-CHANGE PROCESSES OF PURE SUBSTANCES

- **Compressed liquid (subcooled liquid)**: A substance that it is *not* about to vaporize.
- Saturated liquid: A liquid that is about to vaporize.



- **Saturated vapor**: A vapor that is *about to condense*. •
- **Saturated liquid–vapor mixture**: The state at which the *liquid and* vapor phases coexist in equilibrium.
- **Superheated vapor:** A vapor that is *not about to condense* (i.e., not a • saturated vapor).



vaporizes (saturated liquidvapor mixture).

constant at 100°C until the last drop of liquid is vaporized (saturated vapor).

(superheated vapor). 6

vapor starts to rise

If the entire process between state 1 and 5 described in the figure is reversed by cooling the water while maintaining the pressure at the same value, the water will go back to state 1, retracing the same path, and in so doing, the amount of heat released will exactly match the amount of heat added during the heating process.



### **Saturation Temperature and Saturation Pressure**

- The temperature at which water starts boiling depends on the pressure; therefore, if the pressure is fixed, so is the boiling temperature.
- Water boils at 100°C at 1 atm pressure.
- Saturation temperature T<sub>sat</sub>: The temperature at which a pure substance changes phase at a given pressure.
- Saturation pressure P<sub>sat</sub>: The pressure at which a pure substance changes phase at a given temperature.



- Latent heat: The amount of energy absorbed or released during a phasechange process.
- Latent heat of fusion: The amount of energy absorbed during melting. It is equivalent to the amount of energy released during freezing.
- Latent heat of vaporization: The amount of energy absorbed during vaporization and it is equivalent to the energy released during condensation.
- The magnitudes of the latent heats depend on the temperature or pressure at which the phase change occurs.
- At 1 atm pressure, the latent heat of fusion of water is 333.7 kJ/kg and the latent heat of vaporization is 2256.5 kJ/kg.
- The atmospheric pressure, and thus the boiling temperature of water, decreases with elevation.

### TABLE 4-2

Variation of the standard atmospheric pressure and the boiling (saturation) temperature of water with altitude

Elevation m	Atmospheric pressure, kPa	Boiling tempera- ture, °C
0	101.33	100.0
1,000	89.55	96.5
2,000	79.50	93.3
5,000	54.05	83.3
10,000	26.50	66.3
20,000	5.53	34.7



### 4-4 PROPERTY DIAGRAMS FOR PHASE-CHANGE PROCESSES

The variations of properties during phase-change processes are best studied and understood with the help of property diagrams such as the T-v, P-v, and P-T diagrams for pure substances.



*T-v* diagram of constant-pressure phase-change processes of a pure substance at various pressures (numerical values are for water).

- saturated liquid line
- saturated vapor line
- compressed liquid region
- superheated vapor region
- saturated liquid–vapor mixture region (wet region)





Property diagrams of a pure substance.

At supercritical pressures ( $P > P_{cr}$ ), there is no distinct phase-change (boiling) process.

**Critical point**: The point at which the saturated liquid and saturated vapor states are identical.



### Extending the Diagrams to Include the Solid Phase



(a) P-v diagram of a substance that contracts on freezing

For water,  $T_{tp} = 0.01^{\circ}C$  $P_{tp} = 0.6117$  kPa

At triple-point pressure and temperature, a substance exists in three phases in equilibrium.





### Sublimation:

Passing from the solid phase directly into the vapor phase.



At low pressures (below the triple-point value), solids evaporate without melting first (*sublimation*).

### **Phase Diagram**



P-T diagram of pure substances.

The *P*-*v*-*T* surfaces present a great deal of information at once, but in a thermodynamic analysis it is more convenient to work with two-dimensional diagrams, such as the *P*-*v* and *T*-*v* diagrams.



## **4-5 PROPERTY TABLES**

- For most substances, the relationships among thermodynamic properties are too complex to be expressed by simple equations.
- Therefore, properties are frequently presented in the form of tables.
- Some thermodynamic properties can be measured easily, but others cannot and are calculated by using the relations between them and measurable properties.
- The results of these measurements and calculations are presented in tables in a convenient format.





### **Saturated Liquid and Saturated Vapor States**

- Table A-4: Saturation properties of water under temperature.
- Table A–5: Saturation properties of water under pressure.

A partial list of Table A-4.

		Sat.	Sat. Specific volume				
	Temp.	press.	Sat.	Sat.			
	°C	kPa	liquid	vapor			
	Т	P <sub>sat</sub>	$V_f$	$V_g$			
	85	57.868	0.001032	2.8261			
	90	70.183	0.001036	2.3593			
	95	84.609	0.001040	1.9808			
		<b>↑</b>	1	1			
Sp	ecific mperature	•	Specific volume of saturated liquid				
Correspondi saturation pressure			ling	Specific volume of saturated vapor			

$v_f$ = specific volume of saturated liquid
$v_g$ = specific volume of saturated vapor
$v_{fg}$ = difference between $v_g$ and $v_f$ (that is, $v_{fg} = v_g - v_f$ )

Enthalpy of vaporization,  $h_{fg}$  (Latent heat of vaporization): The amount of energy needed to vaporize a unit mass of saturated liquid at a given temperature or pressure.





**Examples**: Saturated liquid and saturated vapor states of water on *T-v* and *P-v* diagrams.



#### 19

## **Saturated Liquid–Vapor Mixture**

Quality, *x* : The ratio of the mass of vapor to the total mass of the mixture. Quality is between 0 and 1 → 0: sat. liquid, 1: sat. vapor.

The properties of the saturated liquid are the same whether it exists alone or in a mixture with saturated vapor.





#### FIGURE 4–34

Quality is related to the horizontal distances on P-v and T-v diagrams.



#### FIGURE 4–35

The v value of a saturated liquid–vapor mixture lies between the  $v_f$  and  $v_g$  values at the specified *T* or *P*.

# **Examples**: Saturated liquid-vapor mixture states on *T-v* and *P-v* diagrams.



In the region to the right of the saturated vapor line and at temperatures above the critical point temperature, a substance exists as superheated vapor. In this region, temperature and pressure are independent properties.

	V	U	h	Higher en
T,°C	m³/kg	kJ/kg	kJ/kg	
	P = 0.1	MPa (99	.61°C)	At a sp
Sat.	1.6941	2505.6	2675.0	P, superi
100	1.6959	2506.2	2675.8	vapor ex
150	1.9367	2582.9	2776.6	a higher
	•		:	the sat
1300	7.2605	4687.2	5413.3	
	P = 0.5 ]	MPa (151	.83°C)	
Sat.	0.37483	2560.7	2748.1	A partial
200	0.42503	2643.3	2855.8	listing of
250	0.47443	2723.8	2961.0	Table A-6.

## Superheated Vapor

### Compared to saturated vapor, superheated vapor is characterized by

Lower pressures ( $P < P_{sat}$  at a given T) Higher tempreatures  $(T > T_{sat} \text{ at a given } P)$ Higher specific volumes ( $v > v_g$  at a given P or T) Higher internal energies  $(u > u_g \text{ at a given } P \text{ or } T)$ Higher enthalpies  $(h > h_g \text{ at a given } P \text{ or } T)$ 

At a specified P, superheated vapor exists at a higher h than the saturated vapor.



The compressed liquid properties depend on temperature much more strongly than they do on pressure.

 $y \cong y_{f @ T} \quad \mathbf{y} \to \mathbf{v}, u, \text{ or } h$ 

A more accurate relation for h $h \cong h_{f @ T} + V_{f @ T} (P - P_{sat @ T})$ 

Given: P and T
$u \cong u_{f @ T}$
$h\cong h_{f@T}$

A compressed liquid may be approximated as a saturated liquid at the given temperature.

## **Compressed Liquid**

Compressed liquid is characterized by

Higher pressures  $(P > P_{sat} \text{ at a given } T)$ Lower tempreatures  $(T < T_{sat} \text{ at a given } P)$ Lower specific volumes  $(v < v_f \text{ at a given } P \text{ or } T)$ Lower internal energies  $(u < u_f \text{ at a given } P \text{ or } T)$ Lower enthalpies  $(h < h_f \text{ at a given } P \text{ or } T)$ 



### **Reference State and Reference Values**

- The values of *u*, *h*, and *s* cannot be measured directly, and they are calculated from measurable properties using the relations between properties.
- However, those relations give the *changes* in properties, not the values of properties at specified states.
- Therefore, we need to choose a convenient *reference state* and assign a value of *zero* for a convenient property or properties at that state.
- The referance state for water is 0.01°C and for R-134a is -40°C in tables.
- Some properties may have negative values as a result of the reference state chosen.
- Sometimes different tables list different values for some properties at the same state as a result of using a different reference state.
- However, In thermodynamics we are concerned with the *changes* in properties, and the reference state chosen is of no consequence in calculations.

Saturat	ed water-	—Temperatu	re table										
		<i>Specific volume,</i> m <sup>3</sup> /kg		<i>Internal energy,</i> kJ/kg				<i>Enthalpy,</i> kJ/kg			<i>Entropy,</i> kJ/kg · К		
Temp., <i>T</i> °C	Sat. press., <i>P<sub>sat</sub> kPa</i>	Sat. Iiquid, <i>v<sub>f</sub></i>	Sat. vapor, <i>v<sub>g</sub></i>	Sat. liquio <i>u<sub>f</sub></i>	d, Evap <i>u<sub>fg</sub></i>	Sat. o., vapor <i>u<sub>g</sub></i>	Sat r, liqu <i>h<sub>f</sub></i>	uid,	Evap., <i>h<sub>fg</sub></i>	Sat. vapor, <i>h<sub>g</sub></i>	Sat. liqui <i>s</i> f	d, Evap., <i>s<sub>fg</sub></i>	Sat. vapor, <i>s<sub>g</sub></i>
0.01 5	0.6117 0.8725 ted refrige	0.001000 0.001000	206.00 147.03 -Temperat	0.00 21.03 ure table	00 2374 19 2360	.9 2374 .8 2381	.9 0 .8 21	.001 2 .020 2	2500.9 2489.1	2500.9 2510.1	0.000 0.076	00 9.1556 53 8.9487	9.1556 9.0249
outuru		Specific m <sup>3</sup> /l	volume, <g< td=""><td>Inte</td><td>e<i>rnal ene</i> kJ/kg</td><td>rgy,</td><td></td><td><i>Entha</i> kJ/k</td><td>l<i>py,</i> g</td><td></td><td></td><td><i>Entropy,</i> kJ/kg · K</td><td></td></g<>	Inte	e <i>rnal ene</i> kJ/kg	rgy,		<i>Entha</i> kJ/k	l <i>py,</i> g			<i>Entropy,</i> kJ/kg · K	
Temp., <i>T</i> °C	Sat. press., <i>P<sub>sat</sub></i> kPa	Sat. liquid, v <sub>f</sub>	Sat. vapor, v <sub>g</sub>	Sat. liquid, <i>u<sub>f</sub></i>	Evap., u <sub>fg</sub>	Sat. vapor, <i>u<sub>g</sub></i>	Sat. liquid, <i>h</i> f	Evap h <sub>fg</sub>	Sat. ., vapo <i>h<sub>g</sub></i>	or, lic s <sub>f</sub>	at. quid,	Evap., s <sub>fg</sub>	Sat. vapor, <i>s<sub>g</sub></i>
-40	51.25	0.0007054	0.36081	-0.036	207.40	207.37	0.000	225.8	86 225	.86 0.0	00000	0.96866	0.96866

## **4-6 THE IDEAL-GAS EQUATION OF STATE**

- Equation of state: Any equation that relates the pressure, temperature, and specific volume of a substance.
- The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state. This equation predicts the *P*-*v*-*T* behavior of a gas quite accurately within some properly selected region.

$$P = R\left(\frac{T}{v}\right) \quad Pv = RT \quad \begin{array}{c} \text{Ideal gas equation} \\ \text{of state} \end{array}$$
$$R = \frac{R_u}{M} \quad (kJ/kg \cdot K \text{ or } kPa \cdot m^3/kg \cdot K) \quad \hline \end{array}$$

*R*: gas constant *M*: molar mass (kg/kmol) *R<sub>u</sub>*: universal gas constant

 $R_{u} = \begin{cases} 8.31447 \text{ kJ/kmol} \cdot \text{K} \\ 8.31447 \text{ kPa} \cdot \text{m}^{3}/\text{kmol} \cdot \text{K} \\ 0.0831447 \text{ bar} \cdot \text{m}^{3}/\text{kmol} \cdot \text{K} \\ 1.98588 \text{ Btu/lbmol} \cdot \text{R} \\ 10.7316 \text{ psia} \cdot \text{ft}^{3}/\text{lbmol} \cdot \text{R} \\ 1545.37 \text{ ft} \cdot \text{lbf/lbmol} \cdot \text{R} \end{cases}$ 

Substance	R, kJ/kg∙K	
Air Helium Argon Nitrogen	0.2870 2.0769 0.2081 0.2968	
		• • (J)

Different substances have different gas constants.
Mass = Molar mass × Mole number m = MN (kg)  $V = mV \longrightarrow PV = mRT$   $mR = (MN)R = NR_u \longrightarrow PV = NR_uT$   $V = N\overline{V} \longrightarrow P\overline{V} = R_uT$   $V = N\overline{V} \longrightarrow P\overline{V} = R_uT$  $V = N\overline{V} \longrightarrow V = NR_uT$ 

# Real gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).

<b>}</b>	
Per unit mass	Per unit mole
v, m <sup>3</sup> /kg	⊽, m <sup>3</sup> /kmol
u, kJ/kg	$\overline{u}$ , kJ/kmol
<i>h</i> , kJ/kg	$\overline{h}$ , kJ/kmol
	4

Properties per unit mole are denoted with a bar on the top.

A MALER IN THE REPORT OF A 199 MALE AND A 19



## FIGURE 4-45

The ideal-gas relation often is not applicable to real gases; thus, care should be exercised when using it.

# Is Water Vapor an Ideal Gas?



- At pressures below 10 kPa, water vapor can be treated as an ideal gas, regardless of its temperature, with negligible error (less than 0.1 percent).
- At higher pressures, however, the ideal gas assumption yields unacceptable errors, particularly in the vicinity of the critical point and the saturated vapor line.
- In air-conditioning applications, the water vapor in the air can be treated as an ideal gas. Why?
- In steam power plant applications, however, the pressures involved are usually very high; therefore, ideal-gas relations should not be used.

Percentage of error ([ $|v_{table} - v_{ideal}|/v_{table}$ ] ×100) involved in assuming steam to be an ideal gas, and the region where steam can be treated as an ideal gas with less than 1 percent error.

# **4-7 COMPRESSIBILITY FACTOR—A MEASURE OF DEVIATION FROM IDEAL-GAS BEHAVIOR**

## **Compressibility factor Z**

A factor that accounts for the deviation of real gases from ideal-gas behavior at a given temperature and



## FIGURE 4-48

The compressibility factor is unity for ideal gases.

The farther away Z is from unity, the more the gas deviates from ideal-gas behavior.

Gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).

**Question**: What is the criteria for low pressure and high temperature?

Answer: The pressure or temperature of a gas is high or low relative to its critical temperature or pressure.







Z can also be determined from a knowledge of  $P_R$  and  $v_R$ .



Gases deviate from the ideal-gas behavior the most in the neighborhood of the critical point.

Comparison of Z factors for various gases.

2.5

3.5

Reduced pressure Pp

3.0

4.0

4.5

 $T_R = 1.00$ 

1.5

2.0

0.3

0.2

0.1

0

0.5

1.0

× Methane

O Ethylene

▲ Ethane

o Propane

n-Butane

hydrocarbons

5.0

Iso-pentane

n-Heptane

Carbon dioxide

6.5

7.0

▲ Nitrogen

Water

6.0

Average curve based on data on

5.5

# **Summary**

- Pure substance
- Phases of a pure substance
- Phase-change processes of pure substances
  - ✓ Compressed liquid, Saturated liquid, Saturated vapor, Superheated vapor
  - Saturation temperature and Saturation pressure
- Property diagrams for phase change processes
  - ✓ The *T*-*v* diagram, The *P*-*v* diagram, The *P*-*T* diagram, *The P*-*v*-*T* surface
- Property tables
  - ✓ Enthalpy
  - Saturated liquid, saturated vapor, Saturated liquid vapor mixture, Superheated vapor, compressed liquid
  - ✓ Reference state and reference values
- The ideal gas equation of state
  - ✓ Is water vapor an ideal gas?
- Compressibility factor

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# Chapter 5 ENERGY ANALYSIS OF CLOSED SYSTEMS

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Examine the moving boundary work or P dV work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.

# **5-1 MOVING BOUNDARY WORK**

## Moving boundary work (*P dV* work):

The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F \, ds = PA \, ds = P \, dV$$

 $W_b = \int_{1}^{2} P dV$ (kJ)

## Quasi-equilibrium process:

A process during which the system remains nearly in equilibrium at all times.

## $W_b$ is positive $\rightarrow$ for expansion $W_{h}$ is negative $\rightarrow$ for compression

The work associated with a moving boundary is called boundary work.

A gas does a differential amount of work  $\delta W_{b}$  as it forces the piston to move by a differential amount ds.









#### FIGURE 4–3

The area under the process curve on a P-V diagram represents the boundary work.

Area = 
$$A = \int_{1}^{2} dA = \int_{1}^{2} P dV$$

The area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.  $W_A = 10 \text{ kJ}$   $W_B = 8 \text{ kJ}$   $W_C = 5 \text{ kJ}$  A B C  $V_1$   $V_2$  V

The boundary

during a process

depends on the

path followed as

well as the end

work done

states.



### FIGURE 5–5

The net work done during a cycle is the difference between the work done by the system and the work done on the system.

## Polytropic, Isothermal, and Isobaric processes

 $P = CV^{-n} \text{ Polytropic process: } C, n \text{ (polytropic exponent) constants}$   $W_b = \int_1^2 P \, dV = \int_1^2 CV^{-n} \, dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n} \text{ Polytropic process}$   $W_b = \frac{mR(T_2 - T_1)}{1-n} \text{ Polytropic and for ideal gas}$   $W_b = \int_1^2 P \, dV = \int_1^2 CV^{-1} \, dV = PV \ln\left(\frac{V_2}{V_1}\right) \text{ When } n = 1 \text{ (isothermal process)}$   $W_b = \int_1^2 P \, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \text{ Constant pressure process}$ 

What is the boundary work for a constantvolume process?

> Schematic and P-V diagram for a polytropic process.



## **5-2 ENERGY BALANCE FOR CLOSED SYSTEMS**



The total quantities are related to the quantities per unit time is  $Q = \dot{Q} \Delta t$ ,  $W = \dot{W} \Delta t$ , and  $\Delta E = (dE/dt)\Delta t$  (kJ)  $e_{in} - e_{out} = \Delta e_{system}$  (kJ/kg) Energy balance per unit mass basis  $\delta E_{in} - \delta E_{out} = dE_{system}$  or  $\delta e_{in} - \delta e_{out} = de_{system}$  Energy balance in differential form  $W_{net,out} = Q_{net,in}$  or  $\dot{W}_{net,out} = \dot{Q}_{net,in}$  Energy balance for a cycle

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}}$$
 or  $Q - W = \Delta E$   
 $W = W_{\text{net,out}} = Q_{\text{in}} - Q_{\text{out}}$   
 $W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$ 

Energy balance when sign convention is used: (i.e., heat input and work output are positive; heat output and work input are negative).





Various forms of the first-law relation for closed systems when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

# Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. *Q* is *to* the system and *W* is *from* the system.

 $\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}^{Change in internal, kinetic, potential, etc., energies}$  $Q - W = \Delta U + \Delta K E^{\dagger} + \Delta P E^{\dagger}^{0}$  $Q - W_{\text{other}} - W_{b} = U_{2} - U_{1}$  $Q - W_{\text{other}} - P_{0}(V_{2} - V_{1}) = U_{2} - U_{1}$  $Q - W_{\text{other}} = (U_{2} + P_{2}V_{2}) - (U_{1} + P_{1}V_{1})$ H = U + PV $Q - W_{\text{other}} = H_{2} - H_{1}$ 

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$





## FIGURE 5–14

For a closed system undergoing a quasi-equilibrium, P = constant process,  $\Delta U + W_b = \Delta H$ . Note that this relation is NOT valid for closed systems processes during which pressure DOES NOT remain constant.



## FIGURE 5–16

Expansion against a vacuum involves no work and thus no energy transfer.

# **5-3 SPECIFIC HEATS**

**Specific heat at constant volume,** *c<sub>v</sub>*: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

# **Specific heat at constant pressure,** $c_p$ : The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.



## FIGURE 5–18

Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.



## FIGURE 5–17

It takes different amounts of energy to raise the temperature of different substances by the same amount.





- The equations in the figure are valid for any substance undergoing any process.
- $c_v$  and  $c_p$  are properties.
- $c_v$  is related to the changes in *internal* energy and  $c_p$  to the changes in enthalpy.
- A common unit for specific heats is kJ/kg·°C or kJ/kg·K. Are these units identical?





# **5-4 INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES**

For ideal gases,

temperature only.

 $u, h, c_v, and c_p$ 

vary with



### FIGURE 5-22

Schematic of the experimental apparatus used by Joule.

Joule showed using this experimental apparatus that u=u(T)

$$u = u(T)$$
  

$$h = h(T)$$
  

$$c_v = c_v(T)$$
  

$$c_p = c_p(T)$$

 $\begin{array}{l} h = u + Pv \\ Pv = RT \end{array} \right\} \quad h = u + RT$ 

 $u = u(T) \quad h = h(T)$  $du = c_v(T) \ dT \quad dh = c_p(T) \ dT$ 

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) \, dT$$

$$\Delta h = h_2 - h_1 = \int_{1}^{2} c_p(T) \, dT$$

Internal energy and enthalpy change of an ideal gas

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted  $c_{p0}$  and  $c_{v0}$ .



Ideal-gas constantpressure specific heats for some gases (see Table A– 2c for  $c_p$ equations).

- *u* and *h* data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.



In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature. Internal energy and enthalpy change when specific heat is taken constant at an average value

$$u_{2} - u_{1} = c_{\nu,\text{avg}}(T_{2} - T_{1})$$
  

$$h_{2} - h_{1} = c_{p,\text{avg}}(T_{2} - T_{1})$$
 (kJ/kg)



## FIGURE 5–27

The relation  $\Delta u = c_v \Delta T$  is valid for *any* kind of process, constant-volume or not.



## FIGURE 5–26

For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

## Three ways of calculating $\Delta u$ and $\Delta h$

- By using the tabulated u and h data. This is the easiest and most accurate way when tables are readily available.
- 2. By using the  $c_v$  or  $c_p$  relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are very accurate.
- 3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.



Three ways of calculating  $\Delta u$ .

# **Specific Heat Relations of Ideal Gases**

The relationship between  $c_{p}$ ,  $c_{v}$  and R h = u + RT.  $c_p = c_v + R$  $(kJ/kg \cdot K)$ dh = du + R dT $dh = c_p dT$  and  $du = c_v dT$ On a molar basis  $\overline{c}_p = \overline{c}_v + R_u \qquad (kJ/kmol \cdot K)$  $k = \frac{c_p}{c_p}$  Specific heat ratio Air at 300 K The specific ratio varies with temperature, but this variation is very mild. For monatomic gases (helium, or argon, etc.), its value is essentially  $\overline{c}_{v} = 20.80 \text{ kJ/kmol} \cdot \text{K}$   $R_{u} = 8.314 \text{ kJ/kmol} \cdot \text{K}$   $\overline{c}_{p} = 29.114 \text{ kJ/kmol} \cdot \text{K}$ constant at 1.667. Many diatomic gases, including air, • The  $c_p$  of an ideal gas can be determined have a specific heat ratio of about from a knowledge of  $c_v$  and R. 1.4 at room temperature.

# **5-5 INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS**

Incompressible substance: A substance whose specific volume (or density) is constant. Solids and liquids are incompressible substances.



denoted by c.

incompressible substances remain constant during a process.

## **Internal Energy Changes**

$$du = c_{v} dT = c(T) dT$$
$$\Delta u = u_{2} - u_{1} = \int_{1}^{2} c(T) dT \qquad (kJ/kg)$$

 $\Delta u \cong c_{\mathrm{avg}}(T_2 - T_1)$  (kJ/kg)

# Enthalpy Changes $h = u + P \lor \int_{a}^{0} dh = du + \lor dP + P d\lor = du + \lor dP$ $\Delta h = \Delta u + \lor \Delta P \cong c_{avg} \Delta T + \lor \Delta P \qquad (kJ/kg)$

For *solids*, the term  $\lor \Delta P$  is insignificant and thus  $\Delta h = \Delta u \cong c_{avg} \Delta T$ . For *liquids*, two special cases are commonly encountered:

**1.** Constant-pressure processes, as in heaters  $(\Delta P = 0)$ :  $\Delta h = \Delta u \cong c_{avg} \Delta T$ **2.** Constant-temperature processes, as in pumps  $(\Delta T = 0)$ :  $\Delta h = \lor \Delta P$ 

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{sat @T})$$
 The enthalpy of a compressed liquid

Usually a more accurate relation than  $h_{@ PT} \cong h_{f @ T}$ 

# Summary

- Moving boundary work
  - $\checkmark$  *W*<sub>b</sub> for an isothermal process
  - $\checkmark$  *W*<sup>*b*</sup> for a constant-pressure process
  - $\checkmark$  *W*<sub>b</sub> for a polytropic process
- Energy balance for closed systems
  - Energy balance for a constant-pressure expansion or compression process
- Specific heats
  - ✓ Constant-pressure specific heat,  $c_p$
  - ✓ Constant-volume specific heat,  $c_v$
- Internal energy, enthalpy, and specific heats of ideal gases
  - ✓ Specific heat relations of ideal gases
- Internal energy, enthalpy, and specific heats of incompressible substances (solids and liquids)

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# Chapter 6 MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.

# **6-1 CONSERVATION OF MASS**

**Conservation of mass**: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

*Closed systems*: The mass of the system remain constant during a process.

**Control volumes:** Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

Mass *m* and energy *E* can be converted to each other according to  $E = mc^2$ 

where *c* is the speed of light in a vacuum, which is  $c = 2.9979 \times 10^8$  m/s.

The mass change due to energy change is negligible.



## FIGURE 6–1

Mass is conserved even during chemical reactions.

# **Mass and Volume Flow Rates**





## FIGURE 6–3

The average velocity  $V_{avg}$  is defined as the average speed through a cross section.

$$V_{\text{avg}} = \frac{1}{A_c} \int_{A_c} V_n \, dA_c$$
 Defined aver

Definition of average velocity

Volume flow rate

$$\dot{\mathcal{V}} = \int_{A_c} V_n \, dA_c = V_{\text{avg}} A_c = V A_c \qquad (\text{m}^3/\text{s})$$



## FIGURE 6-4

The volume flow rate is the volume of fluid flowing through a cross section per unit time.

# **Conservation of Mass Principle**

$\langle \text{Total mass entering} \rangle$	_	(Total mass leaving)	_	( Net change of mass )
$\left( \text{ the CV during } \Delta t \right)$	_	the CV during $\Delta t$ /	_	within the CV during $\Delta t$



## FIGURE 6-5

Conservation of mass principle for an ordinary bathtub.

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval  $\Delta t$  is equal to the net change (increase or decrease) in the total mass within the control volume during  $\Delta t$ .

$$m_{\rm in} - m_{\rm out} = \Delta m_{\rm CV}$$
 (kg)

$$\Delta m_{\rm CV} = m_{\rm final} - m_{\rm initial}$$

 $\dot{m}_{\rm in} - \dot{m}_{\rm out} = dm_{\rm CV}/dt$  (kg/s)

These equations are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.



#### FIGURE 6-6

The differential control volume dV and the differential control surface dA used in the derivation of the conservation of mass relation.

General conservation of mass:

Normal component of velocity: 
$$V_n = V \cos \theta = \vec{V} \cdot \vec{n}$$
  
Differential mass flow rate:  $\delta \dot{m} = \rho V_n dA = \rho (V \cos \theta) dA = \rho (\vec{V} \cdot \vec{n}) dA$   
Net mass flow rate:  $\dot{m}_{net} = \int_{CS} \delta \dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA$ 

 $\rho dV$ 

 $\rho \ dV$ 

6

 $m_{\rm CV} =$ 

$$\frac{d}{dt} \int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V} \cdot \vec{n}) \, dA = 0$$

Rate of change of mass within the CV:

Total mass within the CV:

the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt} \int_{CV} \rho \, dV + \sum_{\text{out}} \rho \left| V_n \right| dA - + \sum_{\text{in}} \rho \left| V_n \right| dA = 0$$

$$\frac{d}{dt} \int_{CV} \rho \, dV = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m} \quad \text{or} \quad \frac{dm_{CV}}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m} \quad \text{General conservation of mass in rate form}$$

# **Mass Balance for Steady-Flow Processes**

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ( $m_{CV}$  = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



## FIGURE 6–7

Conservation of mass principle for a two-inlet–one-outlet steady-flow system. For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.* 

$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad (kg/s) \qquad \text{Multiple inlets} \\ and exits \\ \dot{m}_1 = \dot{m}_2 \qquad \rightarrow \qquad \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad \text{Single} \\ \text{stream} \end{cases}$$

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

# **Special Case: Incompressible Flow**

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



## FIGURE 6–8

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

$$\sum_{n} \dot{V} = \sum_{out} \dot{V}$$

(m<sup>3</sup>/s) Steady, incompressible

 $\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$ 

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle.

For steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

# 6-2 FLOW WORK AND THE ENERGY OF A FLOWING FLUID

Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

(kJ)

F = PA  $W_{\text{flow}} = FL = PAL = PV$   $W_{\text{flow}} = PV \quad (kJ/kg)$ 



Schematic for flow work.



#### FIGURE 6–12

In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.



#### FIGURE 6–13

Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to Pv.

# **Total Energy of a Flowing Fluid**

$$e = u + ke + pe = u + \frac{V^2}{2} + gz \qquad (kJ/kg)$$
  

$$\theta = Pv + e = Pv + (u + ke + pe) \qquad h = u + Pv$$
  

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz \qquad (kJ/kg)$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.



The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

# **Energy Transport by Mass**

Amount of energy transport: 
$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right)$$
 (kJ)  
Rate of energy transport:  $\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right)$  (kW)



## FIGURE 6–15

The product  $\dot{m}_i \theta_i$  is the energy transported into control volume by mass per unit time.

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\text{mass}} = mh$$
  $\dot{E}_{\text{mass}} = \dot{m}h$ 

When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\rm in,mass} = \int_{m_i} \theta_i \,\delta m_i = \int_{m_i} \left( h_i + \frac{V_i^2}{2} + g z_i \right) \delta m_i$$

## 6-3 ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS



## FIGURE 6–17

Many engineering systems such as power plants operate under steady conditions.

# **Steady-flow process:** A process during which a fluid flows through a control volume steadily.



## FIGURE 6–18

Under steady-flow conditions, the mass and energy contents of a control volume remain constant.



## FIGURE 6–19

Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).


#### Energy balance relations with sign conventions (i.e., heat input and work output are positive)

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \dot{m} \left( h + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left( h + \frac{V^2}{2} + gz \right)_{\text{for each exit}} - \sum_{\text{for each inlet}} \dot{m} \left( h + \frac{V^2}{2} + gz \right)_{\text{for each inlet}} - \dot{W} = \dot{m} \left[ h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$q - w = h_2 - h_1 + \frac{Q^2 - V_1^2}{2} + g(z_2 - z_1)$$
when kinetic and potential energy

when kinetic and potential energy changes are negligible



Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

$$\frac{J}{kg} \equiv \frac{N \cdot m}{kg} \equiv \left(kg\frac{m}{s^2}\right)\frac{m}{kg} \equiv \frac{m^2}{s^2}$$
$$\left(Also, \frac{Btu}{lbm} \equiv 25,037 \cdot \frac{ft^2}{s^2}\right)$$

The units m<sup>2</sup>/s<sup>2</sup> and J/kg are equivalent.

V <sub>1</sub> m/s	V <sub>2</sub> m/s	∆ke kJ/kg
0	45	1
50	67	1
100	110	1
200	205	1
500	502	1

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.

## **6-4 SOME STEADY-FLOW ENGINEERING DEVICES**

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance. Therefore, these devices can be conveniently analyzed as steady-flow devices.



A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

### **Nozzles and Diffusers**



#### FIGURE 6–25

Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies. Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

Energy balance for a nozzle or diffuser:  $\dot{m}\left(h_{1}+\frac{V_{1}^{2}}{2}\right) = \dot{m}\left(h_{2}+\frac{V_{2}^{2}}{2}\right)$ (since  $\dot{Q} \approx 0, \dot{W} = 0$ , and  $\Delta pe \approx 0$ )

## Turbines and Compressors



Energy balance for the compressor in this figure:

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$
$$(since \Delta ke = \Delta pe \approx 0)$$

**Turbine** drives the electric generator In steam, gas, or hydroelectric power plants.

As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

**Compressors**, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

A *fan* increases the pressure of a gas slightly and is mainly used to mobilize a gas.

A *compressor* is capable of compressing the gas to very high pressures.

*Pumps* work very much like compressors except that they handle liquids instead of gases.

## **Throttling valves**



#### FIGURE 6-31

The temperature of an ideal gas does not change during a throttling (h = constant) process since h = h(T).

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

# What is the difference between a turbine and a throttling valve?

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

Energy  $h_2 \cong h_1$ balance  $u_1 + P_1 v_1 = u_2 + P_2 v_2$ 

Internal energy + Flow energy = Constant



#### FIGURE 6-32

During a throttling process, the enthalpy (flow energy + internal energy) of a fluid remains constant. But internal and flow energies may be converted to each other.

## **Mixing chambers**

In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber**.



#### FIGURE 6–33

The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



Energy balance for the adiabatic mixing chamber in the figure is:

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{n}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

(since  $\dot{Q} \cong 0$ ,  $\dot{W} = 0$ , ke  $\cong$  pe  $\cong 0$ )

### **Heat exchangers**

Heat exchangers are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.



A heat exchanger can be as simple as two concentric pipes.

Fluid B

70°C

50°℃

Heat

Heat

35°C

## Pipe and duct flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.



Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.





Pipe or duct flow may involve more than one form of work at the same time.

## 6-5 ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

Many processes of interest, involve *changes* within the control volume with time. Such processes are called *unsteady-flow*, or *transient-flow*, processes.

Most unsteady-flow processes can be represented reasonably well by the *uniform-flow process*.

Uniform-flow process: The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process. Charging of a rigid tank from a supply line is an unsteadyflow process since it involves changes within the control volume.



Control

volume

The shape and size of a control volume may change during an unsteady-flow process.

#### Mass balance

 $m_{\rm in} - m_{\rm out} = \Delta m_{\rm system} \quad \Delta m_{\rm system} = m_{\rm final} - m_{\rm initial}$  $m_i - m_e = (m_2 - m_1)_{\rm CV}$  i = inlet, e = exit, 1 = initial state, and 2 = final state $\underbrace{E_{\rm in} - E_{\rm out}}_{\rm Net \, energy \, transfer}$  $\Delta E_{\rm system}$ Energy Change in internal, kinetic, balance by heat, work, and mass potential, etc., energies  $\left(Q_{\rm in} + W_{\rm in} + \sum_{\rm in} m\theta\right) - \left(Q_{\rm out} + W_{\rm out} + \sum_{\rm out} m\theta\right) = (m_2 e_2 - m_1 e_1)_{\rm system} \frac{\theta = h + ke + pe}{e = u + ke + pe}$  $Q - W = \sum_{in} mh - \sum_{in} mh + (m_2 u_2 - m_1 u_1)_{system}$   $Q = Q_{net,in} = Q_{in} - Q_{out}$   $W = W_{net,out} = W_{out} - W_{in}$ A uniform-flow system may  $W_b$ involve electrical, shaft, Moving Closed Closed boundary and boundary system W, work all at  $Q - W = \Delta U$ once. Closed The energy equation of a uniform-flow

system reduces to that of a closed system when all the inlets and exits are closed.

# Summary

- Conservation of mass
  - Mass and volume flow rates
  - ✓ Mass balance for a steady-flow process
  - ✓ Mass balance for incompressible flow
- Flow work and the energy of a flowing fluid

✓ Energy transport by mass

- Energy analysis of steady-flow systems
- Some steady-flow engineering devices
  - ✓ Nozzles and Diffusers
  - Turbines and Compressors
  - ✓ Throttling valves
  - Mixing chambers and Heat exchangers
  - ✓ Pipe and Duct flow
- Energy analysis of unsteady-flow processes

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# Chapter 7 THE SECOND LAW OF THERMODYNAMICS

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Introduce the second law of thermodynamics.
- Identify valid processes as those that satisfy both the first and second laws of thermodynamics.
- Discuss thermal energy reservoirs, reversible and irreversible processes, heat engines, refrigerators, and heat pumps.
- Describe the Kelvin–Planck and Clausius statements of the second law of thermodynamics.
- Apply the second law of thermodynamics to cycles and cyclic devices.
- Apply the second law to develop the absolute thermodynamic temperature scale.
- Describe the Carnot cycle.
- Examine the Carnot principles, idealized Carnot heat engines, refrigerators, and heat pumps.
- Determine the expressions for the thermal efficiencies and coefficients of performance for reversible heat engines, heat pumps, and refrigerators.

# **7-1 INTRODUCTION TO THE SECOND LAW**





Transferring heat to a paddle wheel will not cause it to rotate.

A cup of hot coffee does not get hotter in a cooler room.



Transferring heat to a wire will not generate electricity. These processes cannot occur even though they are not in violation of the first law.



Processes occur in a certain direction, and not in the reverse direction.



A process must satisfy both the first and second laws of thermodynamics to proceed.

#### **MAJOR USES OF THE SECOND LAW**

- 1. The second law may be used to identify the direction of processes.
- 2. The second law also asserts that energy has *quality* as well as quantity. The first law is concerned with the quantity of energy and the transformations of energy from one form to another with no regard to its quality. The second law provides the necessary means to determine the quality as well as the degree of degradation of energy during a process.
- 3. The second law of thermodynamics is also used in determining the *theoretical limits* for the performance of commonly used engineering systems, such as heat engines and refrigerators, as well as predicting the *degree of completion* of chemical reactions.

## **7-2 THERMAL ENERGY RESERVOIRS**



- A hypothetical body with a relatively large *thermal energy capacity* (mass x specific heat) that can supply or absorb finite amounts of heat without undergoing any change in temperature is called a thermal energy reservoir, or just a reservoir.
- In practice, large bodies of water such as oceans, lakes, and rivers as well as the atmospheric air can be modeled accurately as thermal energy reservoirs because of their large thermal energy storage capabilities or thermal masses.



## **7-3 HEAT ENGINES**

The devices that convert heat to work.

- 1. They receive heat from a hightemperature source (solar energy, oil furnace, nuclear reactor, etc.).
- 2. They convert part of this heat to work (usually in the form of a rotating shaft.)
- 3. They reject the remaining waste heat to a low-temperature sink (the atmosphere, rivers, etc.).
- 4. They operate on a cycle.

Heat engines and other cyclic devices usually involve a fluid to and from which heat is transferred while undergoing a cycle. This fluid is called the working fluid.



- $Q_{\rm in}$  = amount of heat supplied to steam in boiler from a high-temperature source (furnace)
- $Q_{\text{out}}$  = amount of heat rejected from steam in condenser to a low-temperature sink (the atmosphere, a river, etc.)
- $W_{\rm out}$  = amount of work delivered by steam as it expands in turbine
- $W_{\rm in}$  = amount of work required to compress water to boiler pressure



## **Can we save** $Q_{out}$ **?**



A heat-engine cycle cannot be completed without rejecting some heat to a low-temperature sink.

Every heat engine must *waste* some energy by transferring it to a low-temperature reservoir in order to complete the cycle, even under idealized conditions. In a steam power plant, the condenser is the device where large quantities of waste heat is rejected to rivers, lakes, or the atmosphere.

Can we not just take the condenser out of the plant and save all that waste energy?

The answer is, unfortunately, a firm *no* for the simple reason that without a heat rejection process in a condenser, the cycle cannot be completed.

#### The Second Law of Thermodynamics: Kelvin–Planck Statement

It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.

No heat engine can have a thermal efficiency of 100 percent, or as for a power plant to operate, the working fluid must exchange heat with the environment as well as the furnace.

The impossibility of having a 100% efficient heat engine is not due to friction or other dissipative effects. It is a limitation that applies to both the idealized and the actual heat engines.



A heat engine that violates the Kelvin–Planck statement of the second law.

## **7-4 REFRIGERATORS AND HEAT PUMPS**



Basic components of a refrigeration system and typical operating conditions.

The transfer of heat from a lowtemperature medium to a hightemperature one requires special devices called **refrigerators**.

Refrigerators, like heat engines, are cyclic devices.

The working fluid used in the refrigeration cycle is called a **refrigerant**.

The most frequently used refrigeration cycle is the *vapor-compression refrigeration cycle*.

In a household refrigerator, the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator, and the coils usually behind the refrigerator where heat is dissipated to the kitchen air serve as the condenser.



The objective of a refrigerator is to remove  $Q_L$  from the cooled space.

## **Coefficient of Performance**

The *efficiency* of a refrigerator is expressed in terms of the **coefficient of performance** (COP).

The objective of a refrigerator is to remove heat  $(Q_L)$  from the refrigerated space.

$$COP_{R} = \frac{Desired output}{Required input} = \frac{Q_{L}}{W_{net,in}}$$
$$W_{net,in} = Q_{H} - Q_{L} \quad (kJ)$$
$$COP_{R} = \frac{Q_{L}}{Q_{H} - Q_{L}} = \frac{1}{Q_{H}/Q_{L} - 1}$$

Can the value of COP<sub>R</sub> be greater than unity?



$$COP_{HP} = \frac{Desired output}{Required input} = \frac{Q_H}{W_{net,in}}$$
$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H}$$

 $\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$  for fixed values of  $Q_L$  and  $Q_H$ 

### **Heat Pumps**



Can the value of  $COP_{HP}$ be lower than unity? What does  $COP_{HP}=1$ represent?



When installed backward, an air conditioner functions as a heat pump.

- Most heat pumps in operation today have a seasonally averaged COP of 2 to 3.
- Most existing heat pumps use the cold outside air as the heat source in winter (*air-source* HP).
- In cold climates their efficiency drops considerably when temperatures are below the freezing point.
- In such cases, geothermal (ground-source) HP that use the ground as the heat source can be used.
- Such heat pumps are more expensive to install, but they are also more efficient.
- Air conditioners are basically refrigerators whose refrigerated space is a room or a building instead of the food compartment.
- The COP of a refrigerator decreases with decreasing refrigeration temperature.
- Therefore, it is not economical to refrigerate to a lower temperature than needed.

**Energy efficiency rating (EER):** The amount of heat removed from the cooled space in Btu's for 1 Wh (watthour) of electricity consumed.

 $EER \equiv 3.412 \text{ COP}_R$ 

#### The Second Law of Thermodynamics: Clasius Statement

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lowertemperature body to a higher-temperature body.

It states that a refrigerator cannot operate unless its compressor is driven by an external power source, such as an electric motor.

This way, the net effect on the surroundings involves the consumption of some energy in the form of work, in addition to the transfer of heat from a colder body to a warmer one.

To date, no experiment has been conducted that contradicts the second law, and this should be taken as sufficient proof of its validity.



A refrigerator that violates the Clausius statement of the second law.

### **Equivalence of the Two Statements**



Proof that the violation of the Kelvin–Planck statement leads to the violation of the Clausius statement.

The Kelvin–Planck and the Clausius statements are equivalent in their consequences, and either statement can be used as the expression of the second law of thermodynamics.

Any device that violates the Kelvin–Planck statement also violates the Clausius statement, and vice versa.

### **7-5 REVERSIBLE AND IRREVERSIBLE PROCESSES**

**Reversible process:** A process that can be reversed without leaving any trace on the surroundings.

Irreversible process: A process that is not reversible.



(a) Frictionless pendulum



(b) Quasi-equilibrium expansion and compression of a gas

# Two familiar reversible processes.

- All the processes occurring in nature are irreversible.
- Why are we interested in reversible processes?
- (1) they are easy to analyze and (2) they serve as idealized models (theoretical limits) to which actual processes can be compared.
- Some processes are more irreversible than others.
- We try to approximate reversible processes. Why?



Reversible processes deliver the most and consume the least work.





process is impossible. (b) An impossible heat transfer process

- The factors that cause a process to be • irreversible are called irreversibilities.
- They include friction, unrestrained expansion, • mixing of two fluids, heat transfer across a finite temperature difference, electric resistance, inelastic deformation of solids, and chemical reactions.
- The presence of any of these effects renders a • process irreversible.







#### Internally and Externally Reversible Processes

- Internally reversible process: If no irreversibilities occur within the boundaries of the system during the process.
- Externally reversible: If no irreversibilities occur outside the system boundaries.
- **Totally reversible process:** It involves no irreversibilities within the system or its surroundings.
- A totally reversible process involves no heat transfer through a finite temperature difference, no nonquasi-equilibrium changes, and no friction or other dissipative effects.



external irreversibilities.

Totally and internally reversible heat transfer processes.

# **7-6 THE CARNOT CYCLE**



#### Execution of the Carnot cycle in a closed system.

Reversible Isothermal Expansion (process 1-2,  $T_H$  = constant) Reversible Adiabatic Expansion (process 2-3, temperature drops from  $T_H$  to  $T_L$ ) Reversible Isothermal Compression (process 3-4,  $T_L$  = constant) Reversible Adiabatic Compression (process 4-1, temperature rises from  $T_L$  to  $T_H$ )



P-V diagram of the Carnot cycle.

*P-V* diagram of the reversed Carnot cycle.

#### **The Reversed Carnot Cycle**

The Carnot heat-engine cycle is a totally reversible cycle.

Therefore, all the processes that comprise it can be *reversed*, in which case it becomes the **Carnot refrigeration cycle**.

# **7-7 THE CARNOT PRINCIPLES**



The Carnot principles.

Proof of the first Carnot principle.

reservoirs (the reversible heat engine is then reversed to run as a refrigerator)

- 1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.
- 2. The efficiencies of all reversible heat engines operating between the same two reservoirs are the same.





FIGURE 7–40

All reversible heat engines operating between the same two reservoirs have the same efficiency (the second Carnot principle).

## 7-8 THE THERMODYNAMIC TEMPERATURE SCALE

A temperature scale that is independent of the properties of the substances that are used to measure temperature is called a **thermodynamic temperature scale**.

Such a temperature scale offers great conveniences in thermodynamic calculations.

The arrangement of heat engines used to develop the thermodynamic temperature scale.








A conceptual experimental setup to determine thermodynamic temperatures on the Kelvin scale by measuring heat transfers  $Q_H$  and  $Q_L$ .

$$\left(\frac{Q_H}{Q_L}\right)_{\rm rev} = \frac{T_H}{T_L}$$

This temperature scale is called the Kelvin scale, and the temperatures on this scale are called absolute temperatures.

 $T(^{\circ}C) = T(K) - 273.15$ 

### **7-9 THE CARNOT HEAT ENGINE**



Wnet,out
The Carnot
heat engine
is the most
efficient of
all heat
engines
operating
between the
same highand lowtemperature
reservoirs.

Any heat engine

$$\eta_{\rm th} = 1 - \frac{Q_L}{Q_H}$$

$$\eta_{\rm th,rev} = 1 - \frac{I_L}{T_H}$$

 $\eta_{
m th}$ 



No heat engine can have a higher efficiency than a reversible heat engine operating between the same high- and low-temperature reservoirs.

$$< \eta_{\text{th,rev}} \quad \text{irreversible heat engine} \\ = \eta_{\text{th,rev}} \quad \text{reversible heat engine} \\ > \eta_{\text{th,rev}} \quad \text{impossible heat engine} \\ \end{cases}$$

26





 $\eta_{\rm th,rev} = 1 - \frac{T_L}{T_H}$ 

Can we use °C unit for temperature here?



The higher the temperature of the thermal energy, the higher its quality.

The fraction of heat that can be converted to work as a function of source temperature. How do you increase the thermal efficiency of a Carnot heat engine?

How about for actual heat engines?

### 7-10 THE CARNOT REFRIGERATOR AND HEAT PUMP



No refrigerator can have a higher COP than a reversible refrigerator operating between the same temperature limits. Any refrigerator or heat pump

$$COP_{R} = \frac{1}{Q_{H}/Q_{L} - 1}$$
$$COP_{HP} = \frac{1}{1 - Q_{L}/Q_{H}}$$

Carnot refrigerator or heat pump

$$COP_{HP,rev} = \frac{1}{1 - T_L/T_H}$$
$$COP_{R,rev} = \frac{1}{T_H/T_L - 1}$$

How do you increase the COP of a Carnot refrigerator or heat pump? How about for actual ones?

$$COP_{R} \begin{cases} < COP_{R,rev} & \text{irreversible refrigerator} \\ = \_COP_{R,rev} & \text{reversible refrigerator} \\ > COP_{R,rev} & \text{impossible refrigerator} \end{cases}$$

The COP of a reversible refrigerator or heat pump is the maximum theoretical value for the specified temperature limits.

Actual refrigerators or heat pumps may approach these values as their designs are improved, but they can never reach them.

The COPs of both the refrigerators and the heat pumps decrease as  $T_L$  decreases.

That is, it requires more work to absorb heat from lowertemperature media.

## Summary

- Introduction to the second law
- Thermal energy reservoirs
- Heat engines
  - ✓ Thermal efficiency
  - ✓ The 2<sup>nd</sup> law: Kelvin-Planck statement
- Refrigerators and heat pumps
  - ✓ Coefficient of performance (COP)
  - ✓ The 2<sup>nd</sup> law: Clasius statement
- Reversible and irreversible processes
  - ✓ Irreversibilities, Internally and externally reversible processes
- The Carnot cycle
  - ✓ The reversed Carnot cycle
- The Carnot principles
- The thermodynamic temperature scale
- The Carnot heat engine

✓ The quality of energy

The Carnot refrigerator and heat pump

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## Chapter 10 INTRODUCTION AND PROPERTIES OF FLUIDS

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Understand the basic concepts of Fluid Mechanics.
- Recognize the various types of fluid flow problems encountered in practice.
- Understand the vapor pressure and its role in the occurrence of cavitation.
- Have a working knowledge of the basic properties of fluids and understand the continuum approximation.
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow.
- Calculate the capillary rise (or drop) in tubes due to the surface tension effect.

### **10–1** ■ **THE NO-SLIP CONDITION**



The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.



A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

> **Boundary layer:** The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant.



Flow separation during flow over a curved surface.

## **10–2 CLASSIFICATION OF FLUID FLOWS**

#### **Viscous versus Inviscid Regions of Flow**

Viscous flows: Flows in which the frictional effects are significant.

**Inviscid flow regions:** In many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces.



The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).

#### **Internal versus External Flow**

**External flow:** The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe.

•

•

**Internal flow:** The flow in a pipe or duct if the fluid is completely bounded by solid surfaces.



External flow over a tennis ball, and the turbulent wake region behind.

- Water flow in a pipe is internal flow, and airflow over a ball is external flow .
- The flow of liquids in a duct is called *openchannel flow* if the duct is only partially filled with the liquid and there is a free surface.

#### **Compressible versus Incompressible Flow**

Incompressible flow: If the density of flowing fluid remains nearly constant throughout (e.g., liquid flow).

**Compressible flow:** If the density of fluid changes during flow (e.g., high-speed gas flow)

When analyzing rockets, spacecraft, and other systems that involve highspeed gas flows, the flow speed is often expressed by Mach number

 $Ma = \frac{V}{c} = \frac{Speed of flow}{Speed of sound}$ 

- Ma = 1 Sonic flow
- Ma < 1 Subsonic flow
- Ma > 1 Supersonic flow
- Ma >> 1 Hypersonic flow



Schlieren image of a small model of the space shuttle orbiter being tested at Mach 3 in the supersonic wind tunnel of the Penn State Gas Dynamics Lab. Several *oblique shocks* are seen in the air surrounding the spacecraft.

#### Laminar versus Turbulent Flow

Laminar flow: The highly ordered fluid motion characterized by smooth layers of fluid. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.

Turbulent flow: The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations. The flow of lowviscosity fluids such as air at high velocities is typically turbulent.

**Transitional flow:** A flow that alternates between being laminar and turbulent.



Laminar, transitional, and turbulent flows. 7

#### Natural (or Unforced) versus Forced Flow

Forced flow: A fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan.

Natural flow: Fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid.



In this schlieren image of a girl in a swimming suit, the rise of lighter, warmer air adjacent to her body indicates that humans and warm-blooded animals are surrounded by thermal plumes of rising warm air.

#### **Steady versus Unsteady Flow**

- The term **steady** implies *no change at a point with time*.
- The opposite of steady is **unsteady**.
- The term **uniform** implies *no change with location* over a specified region.
- The term periodic refers to the kind of unsteady flow in which the flow oscillates about a steady mean.
- Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as steady-flow devices.

Oscillating wake of a blunt-based airfoil at Mach number 0.6. Photo (*a*) is an instantaneous image, while photo (*b*) is a long-exposure (time-averaged) image.



(*a*)



#### **One-, Two-, and Three-Dimensional Flows**

- A flow field is best characterized by its velocity distribution.
- A flow is said to be one-, two-, or threedimensional if the flow velocity varies in one, two, or three dimensions, respectively.
- However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored.



Flow over a car antenna is approximately two-dimensional except near the top and bottom of the antenna.



The development of the velocity profile in a circular pipe. V = V(r, z) and thus the flow is two-dimensional in the entrance region, and becomes one-dimensional downstream when the velocity profile fully develops and remains unchanged in <sub>10</sub> the flow direction, V = V(r).

### **10–3** ■ VAPOR PRESSURE AND CAVITATION

- Saturation temperature  $T_{sat}$ : The temperature at which a pure substance changes phase at a given pressure.
- Saturation pressure P<sub>sat</sub>: The pressure at which a pure substance changes phase at a given temperature.
- Vapor pressure ( $P_v$ ): The pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. It is identical to the saturation pressure  $P_{sat}$  of the liquid ( $P_v = P_{sat}$ ).
- Partial pressure: The pressure of a gas or vapor in a mixture with other gases. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor.



The vapor pressure (saturation pressure) of a pure substance (e.g., water) is the pressure exerted by its vapor molecules when the system is in phase equilibrium with its liquid molecules at a given temperature.

Saturation (or vapor) pressure of water at various temperatures		
Temperature <i>T</i> , °C	Saturation Pressure <i>P</i> <sub>sat</sub> , kPa	
-10 -5 0 5 10 15 20 25 30 40 50 100 150 200 250 250	0.26 0.40 0.61 0.87 1.23 1.71 2.34 3.17 4.25 7.39 12.35 101.4 476.2 1555 3976	
150 200 250 300	476.2 1555 3976 8588	

- There is a possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization.
- The vapor bubbles (called cavitation bubbles since they form "cavities" in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.
- This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called cavitation, and it is an important consideration in the design of hydraulic turbines and pumps.



Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 h. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.

### **10–4** ■ VISCOSITY

**Viscosity:** A property that represents the internal resistance of a fluid to motion or the "fluidity".

**Drag force:** The force a flowing fluid exerts on a body in the flow direction. The magnitude of this force depends, in part, on viscosity



The viscosity of a fluid is a measure of its "*resistance to deformation*."

Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.



Newtonian fluids: Fluids for which the rate of deformation is proportional to the shear stress.

$$\tau \propto \frac{d(d\beta)}{dt}$$
 or  $\tau \propto \frac{du}{dy}$ 

$$\tau = \mu \frac{du}{dy}$$
 (N/m<sup>2</sup>) Shear stress

The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

$$\tau = \frac{F}{A} \quad u(y) = \frac{y}{\ell} V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$
$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V \, dt}{\ell} = \frac{du}{dy} \, dt \quad \frac{d\beta}{dt} = \frac{du}{dy}$$

Shear force

$$F = \tau A = \mu A \frac{du}{dy} \qquad (N)$$

 $\mu$  coefficient of viscosity Dynamic (absolute) viscosity kg/m  $\cdot$  s or N  $\cdot$  s/m<sup>2</sup> or Pa  $\cdot$  s 1 poise = 0.1 Pa  $\cdot$  s



Rate of deformation, *du/dy* 

The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.



Rate of deformation, *du/dy* 

Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

#### **Kinematic viscosity**

 $\nu = \mu/\rho$  m<sup>2</sup>/s or stoke 1 stoke = 1 cm<sup>2</sup>/s

For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures.

For gases, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure.

 $\mu = \frac{aT^{1/2}}{1 + b/T} \quad Fo$ 

•  
Air at 20°C and 1 atm:  
$$\mu = 1.83 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$
  
 $\nu = 1.52 \times 10^{-5} \text{ m}^2/\text{s}$   
Air at 20°C and 4 atm:  
 $\mu = 1.83 \times 10^{-5} \text{ kg/m} \cdot \text{s}$   
 $\nu = 0.380 \times 10^{-5} \text{ m}^2/\text{s}$ 

Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.

 $\mu = a 10^{b/(T-c)}$  For liquids



The viscosity of liquids decreases and the viscosity of gases increases with temperature. The viscosity of a fluid is directly related to the pumping power needed to transport a fluid in a pipe or to move a body through a fluid.

Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature.

In a liquid, the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow.



	Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)	
	Fluid	Dynamic Viscosity $\mu$ , kg/m $\cdot$ s
	Glycerin: -20°C 0°C 20°C 40°C	134.0 10.5 1.52 0.31
e variation of namic osolute) cosity of mmon fluids h temperature	Engine oil: SAE 10W SAE 10W30 SAE 30 SAE 50 Mercury Ethyl alcohol Water:	0.10 0.17 0.29 0.86 0.0015 0.0012
1 atm N⋅s/m² = 1kg/m⋅s)	0°C 20°C 100°C (liquid) 100°C (vapor) Blood, 37°C Gasoline Ammonia Air Hydrogen, 0°C	0.0018 0.0010 0.00028 0.000012 0.00040 0.00029 0.00015 0.000018 0.0000088



This equation can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity.

Therefore, two concentric cylinders can be used as a *viscometer*, a device that measures viscosity.

### 10–5 SURFACE TENSION AND CAPILLARY EFFECT

- Liquid droplets behave like small balloons filled with the liquid on a solid surface, and the surface of the liquid acts like a stretched elastic membrane under tension.
- The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.
- The magnitude of this force per unit length is called surface tension (or coefficient of surface tension) and is usually expressed in the unit N/m.
- This effect is also called surface energy [per unit area] and is expressed in the equivalent unit of N · m/m<sup>2</sup>.





Some consequences of surface tension.



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.

$$\sigma_s = \frac{F}{2b}$$

Stretching a liquid film with a Ushaped wire, and the forces acting on the movable wire of length *b*.

Rigid wire frame

Surface of film

$$W =$$
 Force  $\times$  Distance  $= F \Delta x = 2b\sigma_s \Delta x = \sigma_s \Delta A$ 

Liquid film

**Surface tension:** The work done per unit increase in the surface area of the liquid.

Movable

 $\Delta x$ 

Wire

wire

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tensior $\sigma_{\rm s}$ , N/m*	1
<sup>†</sup> Water:		
0°C	0.076	
20°C	0.073	
100°C	0.059	
300°C	0.014	
Glycerin	0.063	
SAE 30 oil	0.035	
Mercury	0.440	
Ethyl alcohol	0.023	
Blood, 37°C	0.058	The free-
Gasoline	0.022	diagram of h
Ammonia	0.021	droplet
Soap solution	0.025	bubble and
Kerosene	0.028	a soap bu
	Droplet or	
	air bubble:	$(2\pi R)\sigma_s = (\pi R^2)$
	Soap	

bubble:



**Capillary effect:** The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

Capillaries: Such narrow tubes or confined flow channels.

The capillary effect is partially responsible for the rise of water to the top of tall trees. **Meniscus:** The curved free surface of a liquid in a capillary tube.

The strength of the capillary effect is quantified by the **contact** (or *wetting*) **angle**, defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.* 



### **Capillary Effect**



The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle. <sup>24</sup>



The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.



The forces acting on a liquid column that has risen in a tube due to the capillary effect.

Capillary rise: 
$$h = \frac{2\sigma_s}{\rho g R} \cos \phi$$
 (R = constant)

Capillary rise is inversely proportional to the radius of the tube and density of the liquid.

## Summary

- The No-Slip Condition
- Classification of Fluid Flows
  - ✓ Viscous versus Inviscid Regions of Flow
  - Internal versus External Flow
  - ✓ Compressible versus Incompressible Flow
  - Laminar versus Turbulent Flow
  - ✓ Natural (or Unforced) versus Forced Flow
  - Steady versus Unsteady Flow
  - ✓ One-, Two-, and Three-Dimensional Flows
- Vapor Pressure and Cavitation
- Viscosity
- Surface Tension and Capillary Effect

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### Chapter 11 FLUID STATICS

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Calculate the forces exerted by a fluid at rest on plane or curved submerged surfaces.
- Analyze the stability of floating and submerged bodies.

## **11–1** INTRODUCTION TO FLUID STATICS

Fluid statics: Deals with problems associated with fluids at rest.

The fluid can be either gaseous or liquid.

Hydrostatics: When the fluid is a liquid.

Aerostatics: When the fluid is a gas.

- In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it.
- The only stress we deal with in fluid statics is the *normal stress*, which is the pressure, and the variation of pressure is due only to the weight of the fluid.

The topic of fluid statics has significance only in gravity fields.

The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics.

### 11–2 HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

A plate, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface when exposed to a liquid.

On a *plane* surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the *magnitude* of the force and its *point of application*, which is called the **center of pressure**.

When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.






Hydrostatic force on an inclined plane surface completely submerged in a liquid.

 $F_R = (P_0 + \rho g y_C \sin \theta) A = (P_0 + \rho g h_C) A = P_C A = P_{avg} A$ 



The pressure at the centroid of a surface is equivalent to the *average* pressure on the surface.



$$y_P = y_C + \frac{I_{xx, C}}{[y_C + P_0/(\rho g \sin \theta)]A}$$

$$y_P = y_C + \frac{I_{xx, C}}{y_C A}$$

$$I_{xx, O} = \int_{A} y^2 dx$$

r

second moment of area (area moment of inertia) about the *x*-axis.

$$I_{xx, O} = I_{xx, C} + y_C^2 A$$



The centroid and the centroidal moments of inertia for some common geometries.

Pressure acts normal to the surface, and the hydrostatic forces acting on a flat plate of any shape form a volume whose base is the plate area and whose length is the linearly varying pressure.

This virtual **pressure prism** has an interesting physical interpretation: its *volume* is equal to the *magnitude* of the resultant hydrostatic force acting on the plate since  $F_R = \int P dA$ , and the line of action of this force passes through the *centroid* of this homogeneous prism.

The projection of the centroid on the plate is the *pressure center*.

Therefore, with the concept of pressure prism, the problem of describing the resultant hydrostatic force on a plane surface reduces to finding the volume and the two coordinates of the centroid of this pressure prism.



The hydrostatic forces acting on a plane surface form a pressure prism whose base (left face) is the surface and whose length is the pressure.

#### **Special Case: Submerged Rectangular Plate**

Hydrostatic force acting on the top surface of a submerged tilted rectangular plate.

$$y_P = s + \frac{b}{2} + \frac{ab^3/12}{[s + b/2 + P_0/(\rho g \sin \theta)]ab}$$
$$= s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin \theta)]}$$

Tilted rectangular plate:  $F_R = P_C A = [P_0 + \rho g(s + b/2) \sin \theta]ab$ 

Tilted rectangular plate (s = 0):  $F_R = [P_0 + \rho g(b \sin \theta)/2]ab$ 



(a) Tilted plate



Hydrostatic force acting on the top surface of a submerged vertical rectangular plate.

(b) Vertical plate

Vertical rectangular plate: $F_R = [P_0 + \rho g(s + b/2)]ab$ Vertical rectangular plate (s = 0): $F_R = (P_0 + \rho gb/2)ab$ 



*Horizontal rectangular plate:*  $F_R = (P_0 + \rho gh)ab$ 

## 11–3 HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES



## $F_R = \sqrt{F_H^2 + F_V^2} \quad \alpha = F_V / F_H$

Determination of the hydrostatic force acting on a submerged curved surface.

*Horizontal force component on curved surface: Vertical force component on curved surface:*   $F_H = F_x$ 

 $F_V = F_y + W$  12



When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.



The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass 13

in a **multilayered fluid** of different densities can be determined by considering different parts of surfaces in different fluids as different surfaces, finding the force on each part, and then adding them using vector addition. For a plane surface, it can be expressed as

*Plane surface in a multilayered fluid:* 

$$F_R = \sum F_{R,i} = \sum P_{C,i} A_i$$

 $P_{C,i} = P_0 + \rho_i g h_{C,i}$ 





# **11–4 BUOYANCY AND STABILITY**

**Buoyant force:** The upward force a fluid exerts on a body immersed in it. The buoyant force is caused by the increase of pressure with depth in a fluid.



The buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.

For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface.

It is also independent of the density of the solid body.

A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface.

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f gsA = \rho_f ghA = \rho_f gV$$



The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The buoyant force  $F_B$  acts upward through the centroid *C* of the displaced volume and is equal in magnitude to the weight W of the displaced fluid, but is opposite in direction. For a solid of uniform density, its weight  $W_s$  also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces. (Here  $W_s > W$  and thus  $W_s > F_B$ ; this solid body would sink.)

Archimedes' principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume. For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body:

$$F_B = W \rightarrow \rho_f g V_{\text{sub}} = \rho_{\text{avg, body}} g V_{\text{total}} \rightarrow \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{avg, body}}}{\rho_f}$$



A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.



The altitude of a hot air balloon is controlled by the temperature difference between the air inside and outside the balloon, since warm air is less dense than cold air. When the balloon is neither rising nor falling, the upward buoyant force exactly balances the downward weight.

#### **Stability of Immersed and Floating Bodies**



For floating bodies such as ships, stability is an important consideration for safety.



A floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.



An immersed neutrally buoyant body is (*a*) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body, (*b*) neutrally stable if *G* and *B* are coincident, and (*c*) unstable if *G* is directly above *B*.



When the center of gravity *G* of an immersed neutrally buoyant body is not vertically aligned with the center of buoyancy *B* of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance.



A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.



A floating body is *stable* if the body is bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or if the metacenter *M* is above point *G*. However, the body is *unstable* if point *M* is below point *G*.

**Metacentric height** *GM:* The distance between the center of gravity *G* and the metacenter *M*—the intersection point of the lines of action of the buoyant force through the body before and after rotation.

The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

# Summary

- Introduction to Fluid Statics
- Hydrostatic Forces on Submerged Plane Surfaces
- Hydrostatic Forces on Submerged Curved Surfaces
- Buoyancy and Stability

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# Chapter 12 BERNOULLI AND ENERGY EQUATIONS

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
- Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

# 12–1 THE BERNOULLI EQUATION

**Bernoulli equation:** An approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

## **Acceleration of a Fluid Particle**

In two-dimensional flow, the acceleration can be decomposed into two components:

streamwise acceleration a<sub>s</sub> along the streamline and

**normal acceleration**  $a_n$  in the direction normal to the streamline, which is given as  $a_n = V^2/R$ .

Streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction.

For particles that move along a *straight path*,  $a_n = 0$  since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \qquad \frac{dV}{dt} = \frac{\partial V}{\partial s} \frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$\partial V/\partial t = 0$$
  $V = V(s)$ 

$$a_{s} = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s}V = V\frac{dV}{ds}$$

V = ds/dt Acceleration in steady flow is due to the change of velocity with position.



During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

## **Derivation of the Bernoulli Equation**



The forces acting on a fluid particle along a streamline.

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible. Steady, incompressible flow:  $\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$ 

The Bernoulli equation between any two points on the same streamline:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

(Steady flow along a streamline) General:

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Incompressible flow ( $\rho = \text{constant}$ ):

T

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.



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The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

- The Bernoulli equation can be viewed as the "conservation of mechanical energy principle."
- This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately.
  - The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.
- There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.
- The Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it.

#### **Force Balance across Streamlines**

Force balance in the direction *n* normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

 $\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \qquad (\text{across streamlines})$ 

For flow along a straight line,  $R \rightarrow \infty$  and this equation reduces to  $P/\rho + gz = \text{constant}$ or  $P = -\rho gz + \text{constant}$ , which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body.

> Pressure decreases towards the center of curvature when streamlines are curved (*a*), but the variation of pressure with elevation in steady, incompressible flow along a straight line (*b*) is the same as that in stationary fluid.



## **Unsteady, Compressible Flow**

The Bernoulli equation for *unsteady, compressible flow:* 

Unsteady, compressible flow:

$$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V_1^2}{2} + gz = \text{constant}$$

## Static, Dynamic, and Stagnation Pressures

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density gives

 $P + \rho \frac{V^2}{2} + \rho gz = \text{constant (along a streamline)}$ 

*P* is the static pressure: It does not incorporate any dynamic effects; it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

 $\rho V^2/2$  is the dynamic pressure: It represents the pressure rise when the fluid in motion is brought to a stop isentropically.

 $\rho gz$  is the hydrostatic pressure: It is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., fluid weight on pressure. (Be careful of the sign—unlike hydrostatic pressure  $\rho gh$  which *increases* with fluid depth *h*, the hydrostatic pressure term  $\rho gz$  decreases with fluid depth.)

**Total pressure:** The sum of the static, dynamic, and hydrostatic pressures. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*.

**Stagnation pressure:** The sum of the static and dynamic pressures. It represents the pressure at a point where the fluid is brought to a complete stop isentropically.



Close-up of a **Pitot-static probe**, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

The static, dynamic, and stagnation pressures measured using **piezometer tubes**.



Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.



Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The **stagnation streamline** is marked.

#### Limitations on the Use of the Bernoulli Equation

- 1. Steady flow The Bernoulli equation is applicable to steady flow.
- 2. Frictionless flow Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible.
- 3. No shaft work The Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles. When these devices exist, the energy equation should be used instead.
- 4. Incompressible flow Density is taken constant in the derivation of the Bernoulli equation. The flow is incompressible for liquids and also by gases at Mach numbers less than about 0.3.
- 5. No heat transfer The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
- 6. Flow along a streamline Strictly speaking, the Bernoulli equation is applicable along a streamline. However, when a region of the flow is *irrotational* and there is negligibly small *vorticity* in the flow field, the Bernoulli equation becomes applicable *across* streamlines as well.







Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

#### Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. Dividing each term of the Bernoulli equation by g gives

 $\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$  (along a streamline)

 $Pl\rho g$  is the pressure head; it represents the height of a fluid column that produces the static pressure P.

 $V^2/2g$  is the velocity head; it represents the elevation needed for a fluid to reach the velocity V during frictionless free fall.

z is the elevation head; it represents the potential energy of the fluid.



An alternative form of the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads is constant along a streamline.*  Hydraulic grade line (HGL),  $Pl\rho g + z$  The line that represents the sum of the static pressure and the elevation heads.

**Energy grade line (EGL)**,  $P/\rho g + V^2/2g + z$  The line that represents the total head of the fluid.

**Dynamic head**, *V*<sup>2</sup>/2*g* The difference between the heights of EGL and HGL.



The hydraulic grade line (HGL) and the energy grade line (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

#### Notes on HGL and EGL

- For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid.
- The EGL is always a distance  $V^2/2g$  above the HGL. These two curves approach each other as the velocity decreases, and they diverge as the velocity increases.
- In an *idealized Bernoulli-type flow*, EGL is horizontal and its height remains constant.
- For open-channel flow, the HGL coincides with the free surface of the liquid, and the EGL is a distance  $V^2/2g$  above the free surface.
- At a *pipe exit*, the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe outlet.
- The mechanical energy loss due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the head loss in the pipe. A component, such as a valve, that generates significant frictional effects causes a sudden drop in both EGL and HGL at that location.
- A steep jump/drop occurs in EGL and HGL whenever mechanical energy is added or removed to or from the fluid (pump, turbine).
- The (gage) pressure of a fluid is zero at locations where the HGL intersects the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive.


In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.





A steep jump occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a steep drop occurs whenever mechanical energy is removed from the fluid by a turbine.

The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid, and the pressure is negative (vacuum) in a flow section that lies above the HGL.





# Example: Velocity Measurement by a Pitot Tube



# Example: The Rise of the Ocean Due to a Hurricane



The eye of hurricane Linda (1997 in the Pacific Ocean near Baja California) is clearly visible in this satellite photo.



$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + \not z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g}^0 + \not z_B \longrightarrow \frac{P_B - P_A}{\rho g} = \frac{V_A^2}{2g}$$

## **12–2 GENERAL ENERGY EQUATION**



$$\begin{split} E_{\rm in} - E_{\rm out} &= \Delta E \\ \dot{Q}_{\rm net\,in} + \dot{W}_{\rm net\,in} = \frac{dE_{\rm sys}}{dt} \\ \dot{Q}_{\rm net\,in} + \dot{W}_{\rm net\,in} = \frac{d}{dt} \int_{\rm sys} \rho e \, dV \\ \dot{Q}_{\rm net\,in} &= \dot{Q}_{\rm in} - \dot{Q}_{\rm out} \\ \dot{W}_{\rm net\,in} = \dot{W}_{\rm in} - \dot{W}_{\rm out} \\ e &= u + ke + pe = u + \frac{V^2}{2} + gz \end{split}$$

The first law of thermodynamics (the conservation of energy principle): Energy cannot be created or destroyed during a process; it can only change forms.



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.<sup>23</sup>

## 12–3 ENERGY ANALYSIS OF STEADY FLOWS

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m} \left( h + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left( h + \frac{V^2}{2} + gz \right)$$

The net rate of energy transfer to a control volume by heat transfer and work during steady flow is equal to the difference between the rates of outgoing and incoming energy flows by mass flow.

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m} \left( h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right)$$

single-stream devices

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h = u + Pv = u + P/\rho.$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$



A control volume with only one inlet and one outlet and energy interactions. Ideal flow (no mechanical energy loss):

 $q_{\text{net in}} = u_2 - u_1$ 

Real flow (with mechanical energy loss):

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech}}$$

$$w_{\text{shaft, net in}} = w_{\text{pump}} - w_{\text{turbine}}$$

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$



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A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.

### Energy equation in terms of *heads*

or turbine.

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump, u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine, e} + h_L$$
where
$$h_{pump, u} = \frac{w_{pump, u}}{g} = \frac{\dot{W}_{pump, u}}{\dot{m}g} = \frac{\eta_{pump} \dot{W}_{pump}}{\dot{m}g}$$
is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, h\_{pump, u} is *less* than  $\dot{W}_{pump}/\dot{m}g$  by the factor  $\eta_{pump}$ .
$$h_{turbine, e} = \frac{w_{turbine, e}}{g} = \frac{\dot{W}_{turbine, e}}{\dot{m}g} = \frac{\dot{W}_{turbine}}{\eta_{turbine}}$$
is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, h\_{turbine, e} = \frac{\dot{W}\_{turbine, e}}{\dot{m}g} = \frac{\dot{W}\_{turbine}}{\eta\_{turbine}}
is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, h\_{turbine, e} is greater than  $\dot{W}_{turbine}/\dot{m}g$  by the factor  $\eta_{turbine}$ .
$$h_L = \frac{e_{mech loss, piping}}{g} = \frac{\dot{E}_{mech loss, piping}}{\dot{m}g}$$
 is the irreversible *head loss* between 1 and 2 due to all components of the piping system other than the pump



Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, i.e., *head*.

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L \quad (5-74)$$

## Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus  $h_L = e_{\text{mech loss, piping}} / g \approx 0$ . Also,  $h_{\text{pump, }u} = h_{\text{turbine, }e} = 0$  when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 5–74 reduces to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \text{or} \quad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

This is the **Bernoulli equation** derived earlier using Newton's second law of motion.

Thus, the Bernoulli equation can be thought of as a degenerate form of the energy equation.

## Kinetic Energy Correction Factor, $\alpha$

The kinetic energy of a fluid stream obtained from  $V^2/2$  is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components.

This error can be corrected by replacing the kinetic energy terms  $V^2/2$  in the energy equation by  $\alpha V_{avg}^2/2$ , where  $\alpha$  is the kinetic energy correction factor.

The correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.



The determination of the *kinetic energy correction* factor using the actual velocity distribution V(r) and the average velocity  $V_{avg}$  at a cross section.

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{pump} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{turbine} + \dot{E}_{mech, loss}$$
$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine, e} + h_L$$
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### **Example: Hydroelectric Power Generation from a Dam**



### **Example: Fan Selection for Air Cooling of a Computer**



## **Summary**

- The Bernoulli Equation
  - ✓ Acceleration of a Fluid Particle
  - ✓ Derivation of the Bernoulli Equation
  - ✓ Force Balance across Streamlines
  - ✓ Unsteady, compressible flow
  - ✓ Static, Dynamic, and Stagnation Pressures
  - ✓ Limitations on the Use of the Bernoulli Equation
  - ✓ Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)
  - ✓ Applications of the Bernouli Equation
- General Energy Equation
- Energy Analysis of Steady Flows
  - Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction
  - ✓ Kinetic Energy Correction Factor,  $\alpha$

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## Chapter 14 INTERNAL FLOW

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements

## 14–1 INTRODUCTION

- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to *friction*, which is directly related to the *pressure* drop and head loss during flow through pipes and ducts.
- The pressure drop is then used to determine the *pumping power requirement*.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot. Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.

Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.

$$\dot{m} = \rho V_{\text{avg}} A_c = \int_{A_c} \rho u(r) \, dA_c$$

The value of the average velocity  $V_{avg}$  at some streamwise cross-section is determined from the requirement that the *conservation of mass* principle be satisfied

$$V_{\text{avg}} = \frac{\int_{A_c} \rho u(r) \, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r \, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r \, dr$$

The average velocity for incompressible flow in a circular pipe of radius *R* 



Average velocity  $V_{avg}$  is defined as the average speed through a cross section. For fully developed laminar pipe flow,  $V_{avg}$  is half of the maximum velocity.

# 14–2 LAMINAR AND TURBULENT FLOWS

Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



Laminar: Smooth streamlines and highly ordered motion. Turbulent: Velocity fluctuations and highly disordered motion. Transition: The flow fluctuates between laminar and turbulent flows. Most flows encountered in practice are turbulent.

Laminar and turbulent flow regimes of candle smoke. The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.



### **Reynolds Number**

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (**Reynolds number**).

$$Re = \frac{Inertial \text{ forces}}{Viscous \text{ forces}} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg}D}{\mu}$$



At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (**turbulent**). At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (**laminar**).

### **Critical Reynolds number, Re**<sub>cr</sub>:

The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element. For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** 

For flow in a circular pipe:

$\text{Re} \lesssim 2300$	laminar flow	
$2300 \lesssim \text{Re} \lesssim 10,000$	transitional flow	
$\text{Re} \gtrsim 10,000$	turbulent flow	

 $D_h =$ 



In the transitional flow region of  $2300 \le \text{Re} \le 10,000$ , the flow switches between laminar and turbulent seemingly randomly.



The hydraulic diameter  $D_h = 4A_c/p$  is defined such that it reduces to ordinary diameter for circular tubes.

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## 14–3 THE ENTRANCE REGION

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

**Boundary layer region:** The viscous effects and the velocity changes are significant.

**Irrotational (core) flow region:** The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

Hydrodynamic entry length  $L_h$ : The length of this region.

Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

**Fully developed:** When both the velocity profile the normalized temperature profile remain unchanged.



### Hydrodynamically fully developed

$$\frac{\partial u(r, x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.



The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

## **Entry Lengths**

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

 $\frac{L_{h,\,\text{laminar}}}{D} \cong 0.05 \text{Re}$ 

$$\frac{L_{h, \text{ turbulent}}}{D} = 1.359 \text{Re}^{1/4}$$

hydrodynamic entry length for turbulent flow

 $\frac{L_{h, \text{ turbulent}}}{D} \approx 10$ 

hydrodynamic entry length for turbulent flow, an approximation The pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe.

This simplistic approach gives *reasonable* results for long pipes but sometimes poor results for short ones since it underpredicts the wall shear stress and thus the friction factor.

## 14–4 LAMINAR FLOW IN PIPES

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile u(r) remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.





Free-body diagram of a fluid disk element of radius *R* and length *dx* in fully developed laminar flow in a horizontal pipe.

### **Pressure Drop and Head Loss**

A quantity of interest in the analysis of pipe flow is the *pressure drop*  $\Delta P$  since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrating from  $x = x_1$  where the pressure is  $P_1$  to  $x = x_1 + L$  where the pressure is  $P_2$  gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \qquad Laminar flow: \qquad \Delta P = P_1 - P_2 = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2}$$

A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure** loss  $\Delta P_L$ .

$$\Delta P_{L} = f \frac{L}{D} \frac{\rho V_{avg}^{2}}{2} \quad \text{pressure loss for all types of fully developed internal flows} \quad f = \frac{64\mu}{\rho V_{avg}} = \frac{64}{Re} \quad \text{Circular pipe, laminar}$$

$$\rho V_{avg}^{2}/2 \quad \text{dynamic pressure} \quad f = \frac{8\tau_{w}}{\rho V_{avg}^{2}} \quad \text{Darcy friction factor} \quad h_{L} = \frac{\Delta P_{L}}{\rho g} = f \frac{L}{D} \frac{V_{avg}^{2}}{2g} \quad \text{Head loss}$$

In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.

$$\dot{W}_{\text{pump},L} = \dot{V} \Delta P_L = \dot{V} \rho g h_L = \dot{m} g h_L$$

$$V_{\text{avg}} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L}$$
Horizontal pipe
$$\dot{V} = V_{\text{avg}} A_c = \frac{(P_1 - P_2)R^2}{8\mu L} \pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P \pi D^4}{128\mu L}$$
Poiseuille's law
For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the fourth power of the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.
$$\dot{W}_{\text{pump}} = 16 \text{ hp}$$

Pressure loss: 
$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$$
  
Head loss:  $h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$   
relation for pressure loss (and

The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and pipes with smooth or rough surfaces.  $\dot{U} \rightarrow V_{avg}$  $\dot{W}_{pump} = 1 \text{ hp}$  $2D \rightarrow V_{avg}/4$ 

The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter. The pressure drop  $\Delta P$  equals the pressure loss  $\Delta P_L$  in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine}, e} - h_{\text{pump}, u} + h_L]$$

Therefore, the pressure drop  $\Delta P = P_1 - P_2$  and pressure loss  $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects  $(z_1 = z_2)$ , (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure  $(h_{pump, u} = h_{turbine, e} = 0)$ , (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant  $(V_1 = V_2)$ , and (4) the velocity profiles at sections 1 and 2 are the same shape  $(\alpha_1 = \alpha_2)$ .

### Effect of Gravity on Velocity and Flow Rate in Laminar Flow



$$W_x = W \sin \theta = \rho g V_{\text{element}} \sin \theta = \rho g (2\pi r \, dr \, dx) \sin \theta$$
$$(2\pi r \, dr \, P)_x - (2\pi r \, dr \, P)_{x+dx} + (2\pi r \, dx \, \tau)_r$$
$$- (2\pi r \, dx \, \tau)_{r+dr} - \rho g (2\pi r \, dr \, dx) \sin \theta = 0$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$
$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{\rm avg} = \frac{(\Delta P - \rho gL \sin \theta)D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho gL \sin \theta)\pi D^4}{128\mu L}$$

Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

$$\begin{array}{c|c} & \underline{\text{Laminar Flow in Circular Pipes}} \\ \hline \\ & (Fully developed flow with no pump or turbine in the flow section, and \\ & \Delta P = P_1 - P_2) \\ \hline \\ & Horizontal \ pipe: \ \dot{V} = \frac{\Delta P \ \pi D^4}{128 \mu L} \\ \hline \\ & \Box \\ & Inclined \ pipe: \ \dot{V} = \frac{(\Delta P - \rho gL \sin \theta) \pi D^4}{128 \mu L} \\ \hline \\ & \Box \\$$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing  $\Delta P$  with  $\Delta P - \rho gL \sin \theta$ .

# Laminar Flow in Noncircular Pipes

The friction factor *f* relations are given in Table 8–1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter  $D_h = 4A_c/p$ , where  $A_c$ is the cross-sectional area of the pipe and *p* is its wetted perimeter

#### TABLE 14-1

Friction factor for fully developed *laminar flow* in pipes of various cross sections  $(D_h = 4A_c/p \text{ and } \text{Re} = V_{avg} D_h/\nu)$ 

Tube Coometry	a/b	Friction Factor
Tube Geometry	OF $\theta^-$	1
Circle	_	64.00/Re
Rectangle	a/b	
	1	56.92/Re
	2	62.20/Re
b	3	68.36/Re
	4	72.92/Re
<i>←a</i> ►	6	78.80/Re
	8	82.32/Re
	00	96.00/Re
Ellipse	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
*a*	16	78.16/Re
Isosceles triangle	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
"A	90°	52.60/Re
	120°	50.96/Re

## 14–5 TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress.

Turbulent flow is a complex mechanism dominated by fluctuations, and it is still not fully understood.

We must rely on experiments and the empirical or semi-empirical correlations developed for various situations.



(a) Before turbulence



(b) After turbulence

The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer. Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients
### **Turbulent Velocity Profile**



The very thin layer next to the wall where viscous effects are dominant is the viscous (or laminar or linear or wall) sublayer.

The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

Next to the viscous sublayer is the **buffer layer**, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

Above the buffer layer is the **overlap** (or **transition**) **layer**, also called the **inertial sublayer**, in which the turbulent effects are much more significant, but still not dominant.

Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow. Note that u(r) in the turbulent case is the *time-averaged* velocity component in the axial direction (the overbar on u has been dropped for simplicity).

### The Moody Chart and the Colebrook Equation

$$\frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right) \qquad \text{(turbulent flow)}$$

**Colebrook equation** (for smooth and rough pipes)

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness**  $\varepsilon$  /D.

Equivalent roughness values for new

1 6.9	$(\varepsilon/D)^{1.11}$ Exp	licit Haaland	commercial pip	pes*	
$\overline{\sqrt{f}} \cong -1.8 \log \left[ \frac{1}{\text{Re}} \right]$	$\left(\frac{1}{3.7}\right) \int equal$	ation		Roughne	ess, ε
			Material	ft	mm
Relative Roughness, <i>ɛ/D</i>	Friction Factor, <i>f</i>		Glass, plastic Concrete Wood stave	0 (smooth) 0.003–0.03	0.9–9
0.0* 0.00001 0.0001	0.0119 0.0119 0.0134		Rubber, smoothed Copper or	0.000033	0.01
0.0005 0.001 0.005 0.01	0.0172 0.0199 0.0305	The friction factor is	brass tubing Cast iron Galvanized	0.000005 0.00085	0.0015 0.26
0.01	0.0380	minimum for a smooth pipe	Wrought iron Stainless steel	0.00015 0.000007	0.046 0.002
and are calculated from th	tues are for $Re = 10^{\circ}$ e Colebrook equation.	with roughness	steel	0.00015	0.045



Reynolds number, Re

### **The Moody Chart**

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### **Observations from the Moody chart**

- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- The friction factor is a minimum for a smooth pipe and increases with roughness. The Colebrook equation in this case ( $\varepsilon = 0$ ) reduces to the **Prandtl equation.**

 $1/\sqrt{f} = 2.0 \log(\text{Re}\sqrt{f}) - 0.8$ 

- The transition region from the laminar to turbulent regime is indicated by the shaded area in the Moody chart. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.
- At very large Reynolds numbers (to the right of the dashed line on the Moody chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. The flow in that region is called *fully rough turbulent flow* or just *fully rough flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The Colebrook equation in the *fully rough* zone reduces to the von Kármán equation.

 $1/\sqrt{f} = -2.0 \log[(\epsilon/D)/3.7]$ 



At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number. See Fig. A–12 for a full-page moody chart. In calculations, we should make sure that we use the actual internal diameter of the pipe, which may be different than the nominal diameter.

Standard sizes for Schedule 40 steel pipes

Nominal Size, in	Actual Inside Diameter, in
18	0.269
$\frac{1}{4}$	0.364
3	0.493
$\frac{1}{2}$	0.622
<u>3</u> 4	0.824
1	1.049
$1\frac{1}{2}$	1.610
2	2.067
$2\frac{1}{2}$	2.469
3	3.068
5	5.047
10	10.02

### **Types of Fluid Flow Problems**

- Determining the pressure drop (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
- 2. Determining the flow rate when the pipe length and diameter are given for a specified pressure drop (or head loss)
- 3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

Problem  
typeGivenFind1
$$L, D, \dot{V}$$
 $\Delta P$  (or  $h_L$ )2 $L, D, \Delta P$  $\dot{V}$ 3 $L, \Delta P, \dot{V}$  $D$ 

The three types of problems encountered in pipe flow.

$$h_{L} = 1.07 \frac{\dot{\nabla}^{2} L}{g D^{5}} \left\{ \ln \left[ \frac{\varepsilon}{3.7D} + 4.62 \left( \frac{\nu D}{\dot{\nabla}} \right)^{0.9} \right] \right\}^{-2} \frac{10^{-6} < \varepsilon/D < 10^{-2}}{3000 < \text{Re} < 3 \times 10^{8}}$$
$$\dot{\nabla} = -0.965 \left( \frac{g D^{5} h_{L}}{L} \right)^{0.5} \ln \left[ \frac{\varepsilon}{3.7D} + \left( \frac{3.17 \nu^{2} L}{g D^{3} h_{L}} \right)^{0.5} \right] \qquad \text{Re} > 2000$$
$$D = 0.66 \left[ \varepsilon^{1.25} \left( \frac{L \dot{\nabla}^{2}}{g h_{L}} \right)^{4.75} + \nu \dot{\nabla}^{9.4} \left( \frac{L}{g h_{L}} \right)^{5.2} \right]^{0.04} \qquad 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3 \times 10^{8} \end{cases}$$

To avoid tedious iterations in head loss, flow rate, and diameter calculations, these explicit relations that are accurate to within 2 percent of the Moody chart may be used.

## 14–6 ■ MINOR LOSSES

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes.

These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.

In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the major losses) and are called minor losses.

Minor losses are usually expressed in terms of the loss coefficient  $K_L$ .

$$K_L = \frac{h_L}{V^2/(2g)}$$

 $h_L = \Delta P_L / \rho g$ Head loss due to component Pipe section with valve:



$$\Delta P_L = (P_1 - P_2)_{\text{valve}} - (P_1 - P_2)_{\text{pipe}}$$

For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe. 27 When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure:

### $K_L = \Delta P_L / (\rho V^2 / 2).$

When the loss coefficient for a component is available, the head loss for that component is

$$h_L = K_L \frac{V^2}{2g} \begin{array}{l} Minor\\ loss \end{array}$$

Minor losses are also expressed in terms of the equivalent length  $L_{equiv}$ .

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{\text{equiv}}}{D} \frac{V^2}{2g} \rightarrow L_{\text{equiv}} = \frac{D}{f} K_L$$



The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

#### Total head loss (general)

$$h_{L, \text{ total}} = h_{L, \text{ major}} + h_{L, \text{ minor}}$$
$$= \sum_{i} f_{i} \frac{L_{i}}{D_{i}} \frac{V_{i}^{2}}{2g} + \sum_{j} K_{L, j} \frac{V_{j}^{2}}{2g}$$

#### Total head loss (D = constant)

$$h_{L, \text{ total}} = \left( f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

 $K_L = 0.50$ Vena contracta Recirculating flow Well-rounded inlet  $K_L = 0.03$ 

Sharp-edged inlet

The head loss at the inlet of a pipe is almost negligible for wellrounded inlets ( $K_L = 0.03$  for r/D >0.2) but increases to about 0.50 for sharp-edged inlets.

#### TABLE 14-4

Loss coefficients  $K_L$  of various pipe components for turbulent flow (for use in the relation  $h_L = K_L V^2 / (2g)$ , where V is the average velocity in the pipe that contains the component)<sup>\*</sup>



Note: The kinetic energy correction factor is  $\alpha = 2$  for fully developed laminar flow, and  $\alpha \approx 1.05$  for fully developed turbulent flow.

Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)





\* These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.

 $A_{\text{small}}$  $K_L = \alpha ($ 

### (sudden expansion)



Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.



The effect of rounding of a pipe inlet on the loss coefficient.



All the kinetic energy of the flow is "lost" (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.

> The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs instead of sharp turns.





*(b)* 



(*a*) The large head loss in a partially closed valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage.

(*b*) The head loss through a fully-open ball valve, on the other hand, is quite small.

## **14–7 ■ PIPING NETWORKS AND PUMP SELECTION**





For pipes *in series*, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.

industrial facility.

For pipes *in parallel*, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.



The relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same.

$$h_{L,1} = h_{L,2} \rightarrow f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2} \left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2}\right)^{1/2}$$

The flow rate in one of the parallel branches is proportional to its diameter to the power 5/2 and is inversely proportional to the square root of its length and friction factor.

#### The analysis of piping networks is based on two simple principles:

- Conservation of mass throughout the system must be satisfied. This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system.
- 2. Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions. This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero.

### **Piping Systems with Pumps and Turbines**

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

u

= (7 - 7) + h

h

Liquid out Pump Liquid in  $\eta_{\text{pump}} = 0.70$  $\eta_{\text{pump-motor}} = \eta_{\text{pump}} \eta_{\text{motor}}$  $= 0.70 \times 0.90 = 0.63$ 

Motor

 $\eta_{\rm motor} = 0.90$ 

When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.

The efficiency of the pump–motor combination is the product of the pump and the motor efficiencies.



Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

# Summary

- Introduction
- Laminar and Turbulent Flows
  - ✓ Reynolds Number
- The Entrance Region
  - ✓ Entry Lengths
- Laminar Flow in Pipes
  - Pressure Drop and Head Loss
  - ✓ Effect of Gravity on Velocity and Flow Rate in Laminar Flow
  - ✓ Laminar Flow in Noncircular Pipes
- Turbulent Flow in Pipes
  - ✓ Turbulent Shear Stress
  - ✓ Turbulent Velocity Profile
  - The Moody Chart and the Colebrook Equation
  - Types of Fluid Flow Problems
- Minor Losses
- Piping Networks and Pump Selection
  - ✓ Serial and Parallel Pipes
  - Piping Systems with Pumps and Turbines

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# Chapter 16 MECHANISMS OF HEAT TRANSFER

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Understand the basic mechanisms of heat transfer, which are conduction, convection, and radiation, and Fourier's law of heat conduction, Newton's law of cooling, and the Stefan–Boltzmann law of radiation
- Identify the mechanisms of heat transfer that occur simultaneously in practice
- Develop an awareness of the cost associated with heat losses
- Solve various heat transfer problems encountered in practice

# **16-1 INTRODUCTION**

- Heat: The form of energy that can be transferred from one system to another as a result of temperature difference.
- Thermodynamics is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- Heat Transfer deals with the determination of the *rates* of such energy transfers as well as variation of temperature.
- The transfer of energy as heat is always from the highertemperature medium to the lower-temperature one.
- Heat transfer stops when the two mediums reach the same temperature.
- Heat can be transferred in three different modes:

#### conduction, convection, radiation

All modes of heat transfer require the existence of a temperature difference.

# **16-2 CONDUCTION**

**Conduction:** The transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion.

In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by free electrons.

The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.



Heat conduction through a large plane wall of thickness  $\Delta x$ and area A.

Rate of heat conduction 
$$\propto \frac{(\text{Area})(\text{Temperature difference})}{\text{Thickness}}$$
  
 $\dot{Q}_{\text{cond}} = kA \frac{T_1 - T_2}{\Lambda r_1} = -kA \frac{\Delta T}{\Lambda r_2}$  (W)

 $\Delta x$ 

 $\Delta x$ 

( \*\* )

When 
$$x \to 0$$
  $\dot{Q}_{cond} = -kA \frac{dT}{dx}$  Fourier's law of heat conduction

Thermal conductivity, k: A measure of the ability of a material to conduct heat.

**Temperature gradient dT/dx**: The slope of the temperature curve on a *T-x* diagram.

Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing x. The *negative sign* in the equation ensures that heat transfer in the positive x direction is a positive quantity.

In heat conduction analysis, A represents the area *normal* to the direction of heat transfer.





(a) Copper ( $k = 401 \text{ W/m} \cdot ^{\circ}\text{C}$ )

of



(b) Silicon ( $k = 148 \text{ W/m} \cdot ^{\circ}\text{C}$ )

The rate of heat conduction through a solid is directly proportional to its thermal 5 conductivity.

### Thermal Conductivity

Thermal conductivity:

The rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator*.



A simple experimental setup to determine the thermal conductivity of a material.

The thermal conductivities of some materials at room temperature			
Material	<i>k</i> , W/m ⋅ °C*		
Diamond	2300		
Silver	429		
Copper	401		
Gold	317		
Aluminum	237		
Iron	80.2		
Mercury (I)	8.54		

Alumnum	237
Iron	80.2
Mercury (I)	8.54
Glass	0.78
Brick	0.72
Water (I)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026



The range of thermal conductivity of various materials at room temperature.



The mechanisms of heat conduction in different phases of a substance.

The thermal conductivities of gases such as air vary by a factor of 10<sup>4</sup> from those of pure metals such as copper.

Pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest.

### **TABLE** 16-2

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	<i>k</i> , W/m⋅K, at 300 K
Copper	401
Nickel	91
Constantan	
(55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze	
(90% Cu, 10% AI)	52



#### TABLE 16-3

Thermal conductivities of materials vary with temperature

	<i>k</i> , W/m⋅K		
<i>T</i> , K	Copper	Aluminum	
100	482	302	
200	413	237	
300	401	237	
400	393	240	
600	379	231	
800	366	218	

The variation of the thermal conductivity of various solids, liquids, and gases with temperature.

## **Thermal Diffusivity**

- c<sub>p</sub> Specific heat, J/kg · °C: Heat capacity per unit mass
- pcp Heat capacity, J/m<sup>3</sup>·°C: Heat capacity per unit volume
- α Thermal diffusivity, m<sup>2</sup>/s: Represents
   how fast heat diffuses through a material

 $\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \qquad (\text{m}^2/\text{s})$ 

A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity.

The larger the thermal diffusivity, the faster the propagation of heat into the medium.

A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

The thermal diffusivities of some			
	temperature		
Material	lpha, m²/s*		
Silver	$149  imes 10^{-6}$		
Gold	$127 imes10^{-6}$		
Copper	$113 imes10^{-6}$		
Aluminum	$97.5 imes10^{-6}$		
Iron	$22.8 imes10^{-6}$		
Mercury (I)	$4.7  imes 10^{-6}$		
Marble	$1.2  imes 10^{-6}$		
Ice	$1.2  imes 10^{-6}$		
Concrete	$0.75  imes 10^{-6}$		
Brick	$0.52  imes 10^{-6}$		
Heavy soil (dry)	$0.52  imes 10^{-6}$		
Glass	$0.34 imes10^{-6}$		
Glass wool	$0.23 imes10^{-6}$		
Water (I)	$0.14 imes10^{-6}$		
Beef	$0.14 imes10^{-6}$		
Wood (oak)	$0.13 imes10^{-6}$		

## **16-3 CONVECTION**

**Convection:** The mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion*.

The faster the fluid motion, the greater the convection heat transfer.

In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.



Heat transfer from a hot surface to air by convection.

### Forced convection: If

the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.

### Natural (or free)

convection: If the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.



The cooling of a boiled egg by forced and natural convection.

Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

## $\dot{Q}_{conv} = hA_s (T_s - T_{\infty})$ (W) Newton's law of cooling

- h A<sub>s</sub> T<sub>s</sub> T<sub>∞</sub>
- convection heat transfer coefficient, W/m<sup>2</sup> · °C
- the surface area through which convection heat transfer takes place
  - the surface temperature
- the temperature of the fluid sufficiently far from the surface.

The convection heat transfer coefficient *h* is not a property of the fluid.

It is an experimentally determined parameter whose value depends on all the variables influencing convection such as

- the surface geometry
- the nature of fluid motion
- the properties of the fluid
- the bulk fluid velocity

Typical values of contransfer coefficient	nvection heat
Type of	
convection	h, W/m² · °C*
Free convection of	
gases	2–25
Free convection of	
liquids	10-1000
Forced convection	
of gases	25–250
Forced convection	
of liquids	50–20,000
Boiling and	
condensation	2500-100,000

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## **16-4 RADIATION**

- Radiation: The energy emitted by matter in the form of *electromagnetic* waves (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.
- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium*.
- In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.
- In heat transfer studies we are interested in *thermal radiation,* which is the form of radiation emitted by bodies because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.
- However, radiation is usually considered to be a surface phenomenon for solids.

## $\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$ (W) Stefan–Boltzmann law

#### $\sigma$ = 5.670 × 10<sup>-8</sup> W/m<sup>2</sup> · K<sup>4</sup> Stefan–Boltzmann constant

Blackbody: The idealized surface that emits radiation at the maximum rate.

$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4$ (W)	Radiation emitted		Emissivities of some mat at 300 K	terials
	by roar of		Material	Emissivity
<b>Emissivity</b> $\varepsilon$ : A measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$ of the surface. $0 \le \varepsilon \le 1$ .		Aluminum foil	0.07	
			Anodized aluminum	0.82
		Polished copper	0.03	
$\dot{Q}_{\text{emit, max}} = \dot{Q}_{\text{emit, max}}$		$\sigma T_s^4$	Polished gold	0.03
			Polished silver	0.02
$T_{\rm s} = 400 \; {\rm K}$	= 1452	$W/m^2$	Polished stainless steel	0.17
* \ 1	1102	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	Black paint	0.98
			White paint	0.90
			White paper	0.92-0.97
Blackbody $(\varepsilon = 1)$			Asphalt pavement	0.85–0.93
Blackbody $(e = 1)$			Red brick	0.93–0.96
			Human skin	0.95
Blackbody radiation represen	ts the ma	ximum	Wood	0.82-0.92
amount of radiation that can I	he emitte	d from	Soil	0.93-0.96

Water

Vegetation

0.96

0.92 - 0.96

a surface at a specified temperature.

Absorptivity  $\alpha$ : The fraction of the radiation energy incident on a surface that is absorbed by the surface.  $0 \le \alpha \le 1$ 

A blackbody absorbs the entire radiation incident on it ( $\alpha = 1$ ).

**Kirchhoff's law:** The emissivity and the absorptivity of a surface at a given temperature and wavelength are equal.



The absorption of radiation incident on an opaque surface of absorptivity .
#### Net radiation heat transfer:

The difference between the rates of radiation emitted by the surface and the radiation absorbed.

The determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on

- the properties of the surfaces
- their orientation relative to each other
- the interaction of the medium between the surfaces with radiation

Radiation is usually significant relative to conduction or natural convection, but negligible relative to forced convection. When a surface is *completely enclosed* by a much larger (or black) surface at temperature  $T_{surr}$  separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

 $\dot{Q}_{\rm rad} = \varepsilon \sigma A_s \left( T_s^4 - T_{\rm surr}^4 \right)$  (W)



$$\dot{Q}_{\rm rad} = \varepsilon \sigma A_s (T_s^4 - T_{\rm surr}^4)$$

Radiation heat transfer between a surface and the surfaces surrounding it.

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When radiation and convection occur simultaneously between a surface and a gas:

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s \left( T_s - T_{\infty} \right)$$
 (W)

### Combined heat transfer coefficient $h_{combined}$ Includes the effects of both convection and radiation

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = h_{\text{conv}} A_s (T_s - T_{\text{surr}}) + \varepsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4)$$
$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_{\infty}) \qquad (W)$$
$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = h_{\text{conv}} + \varepsilon \sigma (T_s + T_{\text{surr}}) (T_s^2 + T_{\text{surr}}^2)$$

### 16-5 SIMULTANEOUS HEAT TRANSFER MECHANISMS

Heat transfer is only by conduction in *opaque solids*, but by conduction and radiation in *semitransparent solids*.

A solid may involve conduction and radiation but not convection. A solid may involve convection and/or radiation on its surfaces exposed to a fluid or other surfaces.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid*.

In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion.

Convection = Conduction + Fluid motion

Heat transfer through a vacuum is by radiation.

Most gases between two solid surfaces do not interfere with radiation.

Liquids are usually strong absorbers of radiation.



Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

## Summary

- Conduction
  - Fourier's law of heat conduction
  - Thermal Conductivity
  - ✓ Thermal Diffusivity
- Convection
  - Newton's law of cooling
- Radiation
  - ✓ Stefan–Boltzmann law
- Simultaneous Heat Transfer Mechanisms

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## Chapter 17 STEADY HEAT CONDUCTION

Lecture slides by Mehmet Kanoğlu

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## **Objectives**

- Understand the concept of thermal resistance and its limitations, and develop thermal resistance networks for practical heat conduction problems
- Solve steady conduction problems that involve multilayer rectangular, cylindrical, or spherical geometries
- Develop an intuitive understanding of thermal contact resistance, and circumstances under which it may be significant
- Identify applications in which insulation may actually increase heat transfer
- Analyze finned surfaces, and assess how efficiently and effectively fins enhance heat transfer

### **17-1 STEADY HEAT CONDUCTION IN PLANE WALLS**

Q



#### FIGURE 17-1

Heat transfer through a wall is onedimensional when the temperature of the wall varies in one direction only. Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*.

The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as T(x).

$$\begin{pmatrix} \text{Rate of} \\ \text{heat transfer} \\ \text{into the wall} \end{pmatrix} - \begin{pmatrix} \text{Rate of} \\ \text{heat transfer} \\ \text{out of the wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of change} \\ \text{of the energy} \\ \text{of the wall} \end{pmatrix}$$

$$dE_{\text{wall}} = \frac{dE_{\text{wall}}}{dt}$$
  $dE_{\text{wall}}/dt = 0$   
for steady operation

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx}$$
 (W) Fourier's law of heat conduction



Under steady conditions, the temperature distribution in a plane wall is a straight line: dT/dx = const.

$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx}$$
$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}} dx = -\int_{T=T_1}^{T_2} kA dT$$
$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L} \qquad (W)$$

The rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

Once the rate of heat conduction is available, the temperature T(x) at any location x can be determined by replacing  $T_2$  by T, and L by x.

### **Thermal Resistance Concept**

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L}$$

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad (W)$$

$$R_{\text{wall}} = \frac{L}{kA} \qquad (^{\circ}\text{C/W})$$

Conduction resistance of the wall: Thermal resistance of the wall against heat conduction.

Thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

 $R_e = L/\sigma_e A$ 

Electrical resistance

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$

Analogy between thermal and electrical resistance concepts.

rate of heat transfer  $\rightarrow$  electric current thermal resistance  $\rightarrow$  electrical resistance temperature difference  $\rightarrow$  voltage difference



**Convection resistance of the surface:** *Thermal resistance* of the surface against heat convection.



Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large  $(h \rightarrow \infty)$ , the convection resistance becomes *zero* and  $T_s \approx T$ .

That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.

This situation is approached in practice at surfaces where boiling and condensation occur.

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4) = h_{rad} A_s (T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}}$$

$$R_{rad} = \frac{1}{h_{rad} A_s} \qquad (K/W)$$
Radiation resistance of the surface: Thermal resistance of the surface against radiation.

$$h_{\rm rad} = \frac{\dot{Q}_{\rm rad}}{A_s(T_s - T_{\rm surr})} = \varepsilon \sigma (T_s^2 + T_{\rm surr}^2)(T_s + T_{\rm surr}) \qquad (W/m^2 \cdot K)$$

Radiation heat transfer coefficient

When 
$$T_{surr} \approx T_{\infty}$$
  
 $h_{combined} = h_{conv} + h_{rad}$   
Combined heat transfer  
coefficient

Schematic for convection and radiation resistances at a surface.





The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{wall}} + R_{\text{conv}, 2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$
 (°C/W)

#### Temperature drop

$$\Delta T = \dot{Q}R \qquad (^{\circ}C)$$
  
$$\dot{Q} = UA \ \Delta T \qquad (W)$$
  
$$UA = \frac{1}{R_{\text{total}}} \qquad (^{\circ}C/K)$$
  
$$U \text{ overall heat transfer coefficient}$$

Once Q is evaluated, the surface temperature  $T_1$  can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$



The temperature drop across a layer is proportional to its thermal resistance.



$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total, }i-j}}$$
$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, }1} + R_{\text{wall, }1}} = \frac{T_{\infty 1} - T_2}{\frac{1}{h_1 A} + \frac{L_1}{k_1 A}}$$

The evaluation of the surface and interface temperatures when  $T_{\infty 1}$  and  $T_{\infty 2}$  are given and  $\dot{Q}$  is calculated.



### **17-2 THERMAL CONTACT RESISTANCE**



Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

- When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air.
- These numerous air gaps of varying sizes act as insulation because of the low thermal conductivity of air.
- Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the thermal contact resistance, R<sub>c</sub>.

A typical experimental setup for the determination of thermal contact resistance



$$\dot{Q} = \dot{Q}_{contact} + \dot{Q}_{gap}$$

$$\dot{Q} = h_c A \Delta T_{interface} \qquad \begin{array}{l} h_c \text{ thermal contact} \\ conductance \end{array}$$

$$h_c = \frac{\dot{Q}/A}{\Delta T_{interface}} \qquad (W/m^2 \cdot {}^{\circ}C)$$

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{interface}}{\dot{Q}/A} \qquad (m^2 \cdot {}^{\circ}C/W)$$

The value of thermal contact resistance depends on:

- surface roughness,
- material properties,
- *temperature* and *pressure* at the interface
- *type of fluid* trapped at the interface.

$$R_{c,\text{ insulation}} = \frac{L}{k} = \frac{0.01 \text{ m}}{0.04 \text{ W/m} \cdot ^{\circ}\text{C}} = 0.25 \text{ m}^2 \cdot ^{\circ}\text{C/W}$$

$$R_{c, \text{ copper}} = \frac{L}{k} = \frac{0.01 \text{ m}}{386 \text{ W/m} \cdot ^{\circ}\text{C}} = 0.000026 \text{ m}^2 \cdot ^{\circ}\text{C/W}$$

Thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be disregarded for poor heat conductors such as insulations.

#### **TABLE 17-1**

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10  $\mu$ m and interface pressure of 1 atm (from Fried, 1969).

Fluid at the interface	Contact conductance, <i>h</i> <sub>c</sub> , W/m²⋅K
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000

#### The thermal contact resistance can be minimized by applying

- a *thermal grease* such as silicon oil
- a better conducting gas such as helium or hydrogen
- a soft metallic foil such as tin, silver, copper, nickel, or aluminum



## Effect of metallic coatings on thermal contact conductance

#### **TABLE 17-2**

#### Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness um	Temperature, °C	Pressure, MPa	<i>h<sub>c</sub>,</i> * ₩/m²⋅K
	condition	neuginieco, µin	Temperature, e	ini u	
Identical Metal Pairs	- ·				
416 Stainless steel	Ground	2.54	90–200	0.17-2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2-2.5	11,400
Copper	Ground	1.27	20	1.2-20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17-7	11,400
Dissimilar Metal Pairs					
Stainless steel-				10	2900
Aluminum		20–30	20	20	3600
Stainless steel-				10	16,400
Aluminum		1.0-2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4-2.0	20	15–35	59,000
Steel Ct-30–				10	4800
Aluminum	Milled	4.5-7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.17-1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4-4.5	20	20–35	22,000

\*Divide the given values by 5.678 to convert to Btu/h-ft<sup>2</sup>.°F.

## The *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure*. 16

### 17-3 GENERALIZED THERMAL RESISTANCE NETWORKS

$$\dot{Q} = \dot{Q}_{1} + \dot{Q}_{2} = \frac{T_{1} - T_{2}}{R_{1}} + \frac{T_{1} - T_{2}}{R_{2}} = (T_{1} - T_{2})\left(\frac{1}{R_{1}} + \frac{1}{R_{2}}\right)$$
Insulation
$$\dot{Q} = \frac{T_{1} - T_{2}}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_{1}} + \frac{1}{R_{2}} \longrightarrow R_{\text{total}} = \frac{R_{1}R_{2}}{R_{1} + R_{2}}$$
Thermal
resistance
network for two
parallel layers.
$$\dot{Q} = \dot{Q}_{1} + \dot{Q}_{2}$$

$$\dot{Q} = \frac{T_1 - T_{\infty}}{R_{\text{total}}}$$
  $R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$ 

$$R_1 = \frac{L_1}{k_1 A_1}$$
  $R_2 = \frac{L_2}{k_2 A_2}$ 

$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{\rm conv} = \frac{1}{h A_3}$$

Two assumptions in solving complex multidimensional heat transfer problems by treating them as onedimensional using the thermal resistance network are

- (1) any plane wall normal to the x-axis is *isothermal* (i.e., to assume the temperature to vary in the x-direction only)
- (2) any plane parallel to the x-axis is adiabatic (i.e., to assume heat transfer to occur in the x-direction only)

Do they give the same result?



Thermal resistance network for combined series-parallel arrangement.

# **17-4 HEAT CONDUCTION IN CYLINDERS AND SPHERES**



Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional. Heat transfer through the pipe can be modeled as *steady* and *one-dimensional*.

The temperature of the pipe depends on one direction only (the radial *r*-direction) and can be expressed as T = T(r).

The temperature is independent of the azimuthal angle or the axial distance.

This situation is approximated in practice in long cylindrical pipes and spherical containers.



$$\dot{Q}_{\text{cond, cyl}} = -kA \frac{dT}{dr} \qquad (W)$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A} dr = -\int_{T=T_1}^{T_2} k dT$$

$$A = 2\pi rL$$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk \frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad (W)$$

A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures  $T_1$  and  $T_2$ .

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}}$$
 (W)

 $R_{\rm cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$ 

Conduction resistance of the cylinder layer



 $R_{\rm sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi (\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$ 

Conduction resistance of the spherical layer



$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{cyl}} + R_{\text{conv}, 2}$$

The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{sph}} + R_{\text{conv},2}$$
$$= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

for a spherical layer

### **Multilayered Cylinders and Spheres**



$$\dot{Q} = \frac{T_{2} - T_{3}}{R_{2}} \qquad \text{The ratio } \Delta T/R \text{ across a equal to } \dot{Q}, \text{ which remains one-dimensional steady}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{1}}{R_{\text{conv},1}}$$

$$= \frac{T_{\alpha 1} - T_{2}}{R_{1} + R_{2}} \qquad \text{Once heat transfer rate calculated, the interface of the interface o$$

 $T_{\infty 1}$ 

cross any layer is h remains constant in steady conduction.

sfer rate Q has been terface temperature ined from any of the lowing two relations:

$$\frac{1}{r_{1} + R_{\text{cyl},1}} = \frac{T_{\infty 1} - T_{2}}{\frac{1}{h_{1}(2\pi r_{1}L)} + \frac{\ln(r_{2}/r_{1})}{2\pi Lk_{1}}}$$

 $-T_{\infty 2}$ 

 $\overline{h_o(2\pi r_4 L)}$ 

## **17-5 CRITICAL RADIUS OF INSULATION**

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

In a a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

The heat transfer from the pipe may increase or decrease, depending on which effect dominates.



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_{\infty}}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_{\infty}}{\frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$

The critical radius of insulation for a cylindrical body:

 $r_{\rm cr, \ cylinder} = \frac{k}{h}$  (m)

The critical radius of insulation for a spherical shell:

 $r_{\rm cr, \ sphere} = \frac{2k}{h}$ 

The largest value of the critical radius we are likely to encounter is

 $r_{\rm cr,\ max} = \frac{\kappa_{\rm max}}{2}$ 

 $\frac{k_{\text{max, insulation}}}{h_{\text{min}}} \approx \frac{0.05 \text{ W/m} \cdot ^{\circ}\text{C}}{5 \text{ W/m}^2 \cdot ^{\circ}\text{C}}$ = 0.01 m = 1 cm

We can insulate hot-water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.



The variation of heat transfer rate with the outer radius of the insulation  $r_2$  when  $r_1 < r_{cr}$ .

### **17-6 HEAT TRANSFER FROM FINNED SURFACES**

 $\dot{Q}_{\rm conv} = hA_s(T_s - T_{\infty})$ 

Newton's law of cooling: The rate of heat transfer from a surface to the surrounding medium

## When $T_s$ and $T_{\infty}$ are fixed, *two ways* to increase the rate of heat transfer are

- To increase the convection heat transfer coefficient h. This may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate.
- To increase the surface area A<sub>s</sub> by attaching to the surface extended surfaces called *fins* made of highly conductive materials such as aluminum.



FIGURE 17–35 Some innovative fin designs.



#### **FIGURE 17–33**

Presumed cooling fins on dinosaur stegosaurus.









having a length of  $\Delta x$ , cross-sectional area of  $A_c$ , and perimeter of p.

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 $\theta = T - T_{\infty}$  Temperature

excess

## The general solution of the differential equation

 $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$ 

Boundary condition at fin base  $\theta(0) = \theta_b = T_b - T_{\infty}$ 

### 1 Infinitely Long Fin ( $T_{\text{fin tip}} = T_{\infty}$ )

Boundary condition at fin tip  $\theta(L) = T(L) - T_{\infty} = 0$   $L \rightarrow \infty$ 



base and the fin tip.

The variation of temperature along the fin

$$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = e^{-mx} = e^{-x\sqrt{hp/kA_c}} \frac{\theta = T - T_{\infty}}{m = \sqrt{hp/kA_c}}$$

The steady rate of heat transfer from the entire fin

$$\dot{Q}_{\text{long fin}} = -kA_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hpkA_c} (T_b - T_\infty)$$





Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

The rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin:

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_{\infty}] \, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x) \, dA_{\text{fin}}$$

A long circular fin of uniform cross section and the variation of temperature along it.

# 2 Negligible Heat Loss from the Fin Tip (Adiabatic fin tip, $Q_{\text{fin tip}} = 0$ )

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic assumption is for heat transfer from the fin tip to be negligible since the surface area of the fin tip is usually a negligible fraction of the total fin area.


## **3 Specified Temperature (** $T_{\text{fin,tip}} = T_L$ **)**

In this case the temperature at the end of the fin (the fin tip) is fixed at a specified temperature  $T_L$ .

This case could be considered as a generalization of the case of *Infinitely Long Fin* where the fin tip temperature was fixed at  $T_{\infty}$ .

Boundary condition at fin tip: 
$$\theta(L) = \theta_L = T_L - T_{\infty}$$

Specified fin tip temperature:

$$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = \frac{\left[(T_L - T_{\infty})/(T_b - T_{\infty})\right]\sinh mx + \sinh m(L - x)}{\sinh mL}$$

Specified fin tip temperature:

$$\dot{Q}_{\text{specified temp.}} = -kA_c \frac{dT}{dx}\Big|_{x=0}$$
$$= \sqrt{hpkA_c}(T_b - T_{\infty}) \frac{\cosh mL - [(T_L - T_{\infty})/(T_b - T_{\infty})]}{\sinh mL}$$

## **4 Convection from Fin Tip**

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that may also include the effects of radiation. Consider the case of convection only at the tip. The condition at the fin tip can be obtained from an energy balance at the fin tip.

 $(\dot{Q}_{\rm cond}=\dot{Q}_{\rm conv})$ 

Boundary condition at fin tip: 
$$-kA_c \frac{dT}{dx}\Big|_{x=L} = hA_c[T(L) - T_{\infty}]$$

 $Convection from fin tip: \frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh m(L - x) + (h/mk) \sinh m(L - x)}{\cosh mL + (h/mk) \sinh mL}$ 

Convection from fin tip:

$$\dot{Q}_{\text{convection}} = -kA_c \frac{dT}{dx}\Big|_{x=0}$$
$$= \sqrt{hpkA_c} (T_b - T_\infty) \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$$

A practical way of accounting for the heat loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case by a **corrected length** defined as

$$L_c = L + \frac{A_c}{p}$$

$$L_{c, \text{ rectangular fin}} = L + \frac{t}{2}$$

$$L_{c, \text{ cylindrical fin}} = L + \frac{D}{4}$$

*t* the thickness of the rectangular fins *D* the diameter of the cylindrical fins



 $\left( b\right)$  Equivalent fin with insulated tip

Corrected fin length  $L_c$  is defined such that heat transfer from a fin of length  $L_c$ with insulated tip is equal to heat transfer from the actual fin of length *L* with convection at the fin tip. 35



## **Fin Efficiency**



Ideal and actual temperature distribution along a fin.

### FIGURE 3-41

Fins enhance heat transfer from a surface by enhancing surface area.

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}} \left(T_b - T_{\infty}\right)$$

Zero thermal resistance or infinite thermal conductivity ( $T_{fin} = T_b$ )

$$\eta_{\rm fin} = \frac{Q_{\rm fin}}{Q_{\rm fin,\,max}} =$$

Actual heat transfer rate from the fin Ideal heat transfer rate from the fin if the entire fin were at base temperature

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_{\infty})$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$$

$$\eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c} (T_b - T_\infty) \tanh aL}{hA_{\text{fin}} (T_b - T_\infty)} = \frac{\tanh mL}{mL}$$



Efficiency of straight fins of rectangular, triangular, and parabolic profiles.



Efficiency of annular fins of constant thickness t.

### Efficiency and surface areas of common fin configurations

### Straight rectangular fins

$$m = \sqrt{2h/kt}$$
$$L_c = L + t/2$$
$$A_{fin} = 2wL_c$$

### Straight triangular fins

$$m = \sqrt{2h/kt}$$
$$A_{\rm fin} = 2w\sqrt{L^2 + (t/2)^2}$$

### Straight parabolic fins

$$m = \sqrt{2h/kt} A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)] C_1 = \sqrt{1 + (t/L)^2}$$

 $\begin{array}{l} m = \sqrt{2h/kt} \\ r_{2c} = r_2 + t/2 \\ A_{fin} = 2\pi (r_{2c}^2 - r_1^2) \end{array}$ 

#### Pin fins of rectangular profile

$$m = \sqrt{4h/kD}$$
$$L_c = L + D/4$$
$$A_{fin} = \pi DL_c$$

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c}$$

$$\eta_{\rm fin} = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

$$\eta_{\mathsf{fin}} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$

$$\eta_{\text{fin}} = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$
$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$

 $\eta_{\rm fin} = \frac{\tanh mL_c}{mL_c}$ 

$$y = (t/2) (1 - x/L)$$



- Fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles.
- The fin efficiency decreases with increasing fin length. Why?
- How to choose fin length? Increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically.
- The efficiency of most fins used in practice is above 90 percent.

 $T_b$ 

 $A_{k}$ 

 $\dot{Q}_{\rm fin}$ 

aluminum, copper, iron.

- The ratio of the *perimeter* to the *cross-sectional area* of the fin *p/A<sub>c</sub>* should be as high as possible. Use slender pin fins.
- Low convection heat transfer coefficient
   h. Place fins on gas (air) side.

# The total rate of heat transfer from a finned surface

$$\begin{split} \dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\ &= h A_{\text{unfin}} \left( T_b - T_{\infty} \right) + \eta_{\text{fin}} h A_{\text{fin}} \left( T_b - T_{\infty} \right) \\ &= h (A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}}) (T_b - T_{\infty}) \end{split}$$

### **Overall effectiveness** for a finned surface

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_{\infty})}{hA_{\text{no fin}}(T_b - T_{\infty})}$$

The overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins.

The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.



Various surface areas associated with a rectangular surface with  $_{43}^{43}$  three fins.



Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.  $mL = 5 \rightarrow$  an infinitely long fin mL = 1 offer a good compromise between heat transfer performance and the fin size. A common approximation used in the analysis of fins is to assume the fin temperature to vary in one direction only (along the fin length) and the temperature variation along other directions is negligible.

Perhaps you are wondering if this one-dimensional approximation is a reasonable one.

This is certainly the case for fins made of thin metal sheets such as the fins on a car radiator, but we wouldn't be so sure for fins made of thick materials.

Studies have shown that the error involved in one-dimensional fin analysis is negligible (less than about 1 percent) when

$$\frac{h\delta}{k} < 0.2$$

where  $\delta$  is the characteristic thickness of the fin, which is taken to be the plate thickness *t* for rectangular fins and the diameter *D* for cylindrical ones.

- Heat sinks: Specially designed finned surfaces which are commonly used in the cooling of electronic equipment, and involve oneof-a-kind complex geometries.
- The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances R.*
- A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

$$\dot{Q}_{\rm fin} = \frac{T_b - T_\infty}{R} = h A_{\rm fin} \, \eta_{\rm fin} \left( T_b - T_\infty \right)$$

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long.

HS 5030	R = 0.9°C/W (vertical) R = 1.2°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm <sup>2</sup>				
HS 6065	$R = 5^{\circ}\text{C/W}$				
	Dimensions: 76 mm $\times$ 38 mm $\times$ 24 mm Surface area: 387 cm <sup>2</sup>				
HS 6071	R = 1.4°C/W (vertical) R = 1.8°C/W (horizontal)				
	Dimensions: 76 mm $\times$ 92 mm $\times$ 26 mm Surface area: 968 cm <sup>2</sup>				
HS 6105	$R = 1.8^{\circ}$ C/W (vertical) $R = 2.1^{\circ}$ C/W (horizontal)				
	Dimensions: 76 mm $\times$ 127 mm $\times$ 91 mm Surface area: 677 cm <sup>2</sup>				
HS 6115	R = 1.1°C/W (vertical) R = 1.3°C/W (horizontal)				
	Dimensions: 76 mm $\times$ 102 mm $\times$ 25 mm Surface area: 929 cm <sup>2</sup>				

# Summary

- Steady Heat Conduction in Plane Walls
  - ✓ Thermal Resistance Concept
  - ✓ Thermal Resistance Network
  - ✓ Multilayer Plane Walls
- Thermal Contact Resistance
- Generalized Thermal Resistance Networks
- Heat Conduction in Cylinders and Spheres
  - ✓ Multilayered Cylinders and Spheres
- Critical Radius of Insulation
- Heat Transfer from Finned Surfaces
  - ✓ Fin Equation
  - ✓ Fin Efficiency
  - ✓ Fin Effectiveness
  - ✓ Proper Length of a Fin

Fundamentals of Thermal-Fluid Sciences 4th Edition in SI Units Yunus A. Çengel, John M. Cimbala, Robert H. Turner McGraw-Hill, 2012

## Chapter 18 TRANSIENT HEAT CONDUCTION

Lecture slides by Mehmet Kanoğlu

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# **Objectives**

- Assess when the spatial variation of temperature is negligible, and temperature varies nearly uniformly with time, making the simplified lumped system analysis applicable
- Obtain analytical solutions for transient one-dimensional conduction problems in rectangular, cylindrical, and spherical geometries using the method of separation of variables, and understand why a one-term solution is usually a reasonable approximation
- Solve the transient conduction problem in large mediums using the similarity variable, and predict the variation of temperature with time and distance from the exposed surface
- Construct solutions for multi-dimensional transient conduction problems using the product solution approach

## **18-1 LUMPED SYSTEM ANALYSIS**

Interior temperature of some bodies remains essentially uniform at all times during a heat transfer process.

The temperature of such bodies can be taken to be a function of time only, T(t).

Heat transfer analysis that utilizes this idealization is known as **lumped system** analysis.

> A small copper ball can be modeled as a lumped system, but a roast beef cannot.



$$\begin{pmatrix} \text{Heat transfer into the body} \\ \text{during } dt \end{pmatrix} = \begin{pmatrix} \text{The increase in the} \\ \text{energy of the body} \\ \text{during } dt \end{pmatrix}$$

$$hA_s(T_{\infty} - T) dt = mc_p dT$$
$$m = \rho \lor \quad dT = d(T - T_{\infty})$$

$$\frac{d(T-T_{\infty})}{T-T_{\infty}} = -\frac{hA_s}{\rho V c_p} dt$$

Integrating with  $T = T_i$  at t = 0T = T(t) at t = t

$$\ln \frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = -\frac{hA_s}{\rho V c_p} t$$

A<sub>s</sub>  
SOLID BODY  

$$m = mass$$
  
 $V = volume$   
 $\rho = density$   
 $T_i = initial temperature$   
 $T = T(t)$   
 $\dot{Q} = hA_s[T_{\infty} - T(t)]$ 

The geometry and parameters involved in the lumped system analysis.

 $\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \quad b = \frac{hA_s}{\rho Vc_p}$ 

(1/s) time constant



The temperature of a lumped system approaches the environment temperature as time gets larger.

- This equation enables us to determine the temperature *T(t)* of a body at time *t*, or alternatively, the time *t* required for the temperature to reach a specified value *T(t)*.
- The temperature of a body approaches the ambient temperature  $T_{\infty}$  exponentially.
- The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of *b* indicates that the body approaches the environment temperature in a short time

$$\dot{Q}(t) = hA_s[T(t) - T_{\infty}] \tag{W}$$

The *rate* of convection heat transfer between the body and its environment at time *t* 

$$Q = mc_p[T(t) - T_i]$$
 (kJ)

The *total amount* of heat transfer between the body and the surrounding medium over the time interval t = 0 to t

$$Q_{\max} = mc_p(T_{\infty} - T_i)$$
 (kJ)

The *maximum* heat transfer between the body and its surroundings



Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

## **Criteria for Lumped System Analysis**



 $L_c = \frac{V}{A_c}$  Characteristic length  $Bi = \frac{hL_c}{\nu}$  Biot number Lumped system analysis is applicable if  $Bi \leq 0.1$ When  $Bi \leq 0.1$ , the temperatures within the body relative to the

surroundings (i.e.,  $T - T_{\infty}$ ) remain within 5 percent of each other.

Bi =	h	$\Delta T$ _	Convection at the surface of the body				
	$k/L_c$	$\Delta T$	Conduction within the body				

Conduction resistance within the body  $Bi = \frac{L_c/k}{m} = 1$ Convection resistance at the surface of the body



Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

$$\operatorname{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$





When the convection coefficient h is high and k is low, large temperature differences occur between the inner and outer regions of a large solid.

Analogy between heat transfer to a solid and passenger traffic to an island.

## 18-2 TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

We will consider the variation of temperature with *time* and *position* in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.





Transient temperature profiles in a plane wall exposed to convection from its surfaces for  $T_i > T_{\infty}$ .

 $T_{\infty}$ 

h

 $r_o$ 

Initially

 $T = T_i$ 

Schematic of the simple geometries in which heat transfer is one-dimensional. 9

## Nondimensionalized One-Dimensional Transient Conduction Problem



 $\theta(X, \tau) = \frac{T(x, t) - T_i}{T_{\infty} - T_i}$  $X = \frac{x}{L}$ Bi =  $\frac{hL}{k}$  $\tau = \frac{\alpha t}{L^2}$ Fo

Dimensionless temperature

Dimensionless distance from the center

Dimensionless heat transfer coefficient (Biot number)

Dimensionless time (Fourier number)

- (a) Original heat conduction problem:
- $$\begin{split} \frac{\partial^2 T}{\partial x^2} &= \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad T(x, 0) = T_i \\ \frac{\partial T(0, t)}{\partial x} &= 0, \quad -k \frac{\partial T(L, t)}{\partial x} = h[T(L, t) T_\infty] \\ T &= F(x, L, t, k, \alpha, h, T_i) \end{split}$$
- (b) Nondimensionalized problem:

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}; \ \theta(X, 0) = 1$$
$$\frac{\partial \theta(0, \tau)}{\partial X} = 0, \quad \frac{\partial \theta(1, \tau)}{\partial X} = -\operatorname{Bi}\theta(1, \tau)$$
$$\theta = f(X, \operatorname{Bi}, \tau)$$

Nondimensionalization reduces the number of independent variables in onedimensional transient conduction problems from 8 to 3, offering great convenience in the presentation of results.

### **TABLE 18-1**

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness 2*L*, a cylinder of radius  $r_o$  and a sphere of radius  $r_o$  subjected to convention from all surfaces.\*

Geometry	Solution	$\lambda_n$ 's are the roots of		
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$	$l_n \tan l_n = Bi$		
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_o)$	$\lambda_n \ \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \mathrm{Bi}$		
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x/L)}{\lambda_n x/L}$	$1 - \lambda_n \cot \lambda_n = Bi$		

\*Here  $\theta = (T - T_{\infty})/(T_i - T_{\infty})$  is the dimensionless temperature, Bi = hL/k or  $hr_o/k$  is the Biot number, Fo =  $\tau = \alpha t/L^2$  or  $\alpha t/r_0^2$  is the Fourier number, and  $J_0$  and  $J_1$  are the Bessel functions of the first kind whose values are given in Table 4–3.

$$\theta_n = A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$$
$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$
$$\lambda_n \tan \lambda_n = \text{Bi}$$

For Bi = 5, 
$$X = 1$$
, and  $t = 0.2$ :

п	$\lambda_n$	$A_n$	$\theta_n$
1	1.3138	1.2402	0.22321
2	4.0336	-0.3442	0.00835
3	6.9096	0.1588	0.00001
4	9.8928	-0.876	0.00000

The analytical solutions of transient conduction problems typically involve infinite series, and thus the evaluation of an infinite number of terms to determine the temperature at a specified location and time.

The term in the series solution of transient conduction problems decline rapidly as *n* and thus  $\lambda_n$  increases because of the exponential decay function with the exponent  $-\lambda_n \tau$ .

## **Approximate Analytical and Graphical Solutions**

The terms in the series solutions converge rapidly with increasing time, and for  $\tau > 0.2$ , keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent.

Solution with one-term approximation

$$\begin{array}{ll} Plane \ wall: & \theta_{\text{wall}} = \frac{T(x, t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\lambda_1 x/L\right), \quad \tau > 0.2 \\ Cylinder: & \theta_{\text{cyl}} = \frac{T(r, t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0.2 \\ Sphere: & \theta_{\text{sph}} = \frac{T(r, t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0.2 \\ \hline Center \ of \ plane \ wall \ (x = 0): & \theta_{0, \text{ wall}} = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \\ \hline Center \ of \ cylinder \ (r = 0): & \theta_{0, \text{cyl}} = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \\ \hline Center \ of \ sphere \ (r = 0): & \theta_{0, \text{sph}} = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \\ \hline \end{array}$$

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#### **TABLE 18-2**

Coefficients used in the one-term approximate solution of transient onedimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/kfor a plane wall of thickness 2L, and  $Bi = hr_o/k$  for a cylinder or sphere of radius r<sub>o</sub>)

radius r <sub>0</sub> /							0.0	1.0000	0.0000
	Plane Wall Cylinder Sphere			0.0	1.0000	0.0000			
Di		Λ		Δ	<u> </u>	10/0	0.1	0.9975	0.0499
DI	Λ1	A1	Λ1	A1	<i>A</i> 1	A1	. 0.2	0.9900	0.0995
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030	0.3	0.9776	0.1483
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060	0.4	0.9604	0.1960
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120	0.5	0.0005	
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179	0.5	0.9385	0.2423
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239	0.6	0.9120	0.2867
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298	0.7	0.8812	0.3290
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592	0.8	0.8463	0.3688
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880	0.9	0.8075	0.4059
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164	1.0	0.7652	0.4400
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441	1.0	0.7652	0.4400
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713	1.1	0.7196	0.4709
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978	1.2	0.6711	0.4983
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236	1.3	0.6201	0.5220
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488	1.4	0.5669	0.5419
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732	1.5	0.5118	0 5579
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793	1.5	0.4554	0.5699
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227	1.0	0.3980	0.5055
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202	1.8	0.3400	0.5815
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870	1.0	0.2818	0.5812
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338	1.5	0.2010	0.0012
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673	2.0	0.2239	0.5767
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920	2.1	0.1666	0.5683
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106	2.2	0.1104	0.5560
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249	2.3	0.0555	0.5399
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781	2.4	0.0025	0.5202
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898			
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942	2.6	-0.0968	-0.4708
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962	2.8	-0.1850	-0.4097
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990	3.0	-0.2601	-0.3391
00	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000	3.2	-0.3202	-0.2613

TABLE 18-3

η

The zeroth- and first-order Bessel

 $J_0(\eta)$ 

 $J_1(\eta)$ 

functions of the first kind

### (a) Midplane temperature



**Transient temperature and heat transfer charts** (Heisler and Grober charts) for a plane wall of thickness 2*L* initially at a uniform temperature  $T_i$  subjected to convection from both sides to an environment at temperature  $T_{\infty}$  with a convection coefficient of *h*.



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$$T_{\infty}$$
Initially
$$T_{\infty}$$

$$T_{-} T_{i}$$

$$D_{-}$$

$$L$$

$$x$$

### (b) Temperature distribution



(c) Heat transfer

$$\begin{array}{c}
T_{\infty} \\
h \\
T = T_{i} \\
0 \\
L \\
x \\
-2L \\
-2L$$

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The dimensionless temperatures anywhere in a plane wall, cylinder, and sphere are related to the center temperature by

$$\frac{\theta_{\text{wall}}}{\theta_{0,\text{ wall}}} = \cos\left(\frac{\lambda_1 x}{L}\right), \quad \frac{\theta_{\text{cyl}}}{\theta_{0,\text{ cyl}}} = J_0\left(\frac{\lambda_1 r}{r_o}\right), \quad \text{and} \quad \frac{\theta_{\text{sph}}}{\theta_{0,\text{ sph}}} = \frac{\sin\left(\lambda_1 r/r_o\right)}{\lambda_1 r/r_o}$$



(a) Finite convection coefficient

(b) Infinite convection coefficient

The specified surface temperature corresponds to the case of convection to an environment at  $T_{\infty}$  with with a convection coefficient *h* that is *infinite*.



(b) Actual heat transfer for time t

### The physical significance of the Fourier number

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2 (1/L)}{\rho c_p L^3 / t} \frac{\Delta T}{\Delta T} =$$

 $= \frac{\text{The rate at which heat is conducted}}{\text{across } L \text{ of a body of volume } L^3}$ The rate at which heat is stored in a body of volume  $L^3$ 

- The Fourier number is a measure of *heat conducted* through a body relative to *heat stored*.
- A large value of the Fourier number indicates faster propagation of heat through a body.

Fourier number at time *t* can be viewed as the ratio of the rate of heat conducted to the rate of heat stored at that time.


## **18-3 TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS**



Schematic of a semi-infinite body.

For short periods of time, most bodies can be modeled as semi-infinite solids since heat does not have sufficient time to penetrate deep into the body. Semi-infinite solid: An idealized body that has a *single plane surface* and extends to infinity in all directions.

The earth can be considered to be a semi-infinite medium in determining the variation of temperature near its surface.

A thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation.

#### Analytical solution for the case of constant temperature $T_s$ on the surface

Differential equation:	$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$	
Boundary conditions:	$T(0, t) = T_s$ and $T(x \to \infty, t)$	$= T_i$
Initial condition:	$T(x,0) = T_i$	
Similarity variable:	$\eta = \frac{x}{\sqrt{4\alpha t}}$	$\frac{\partial^2 T}{\partial t}$
$\frac{d^2T}{d\eta^2} = -2\eta \frac{dT}{d\eta}$		$\frac{\partial x^2}{\partial T}$ $\frac{\partial T}{\partial t}$
$T(0) = T_s$ and $T(\eta \to \infty)$ $\frac{T - T_s}{T_s} = \frac{2}{2} \int_{-u^2}^{u} du = \frac{1}{2} \int_{-u^2}^{u} d$	$T_i = T_i$ = $\operatorname{erf}(n) = 1 - \operatorname{erfc}(n)$	$\frac{\partial T}{\partial x}$
$T_i - T_s  \sqrt{\pi} \int_0^{\pi} e^{-\alpha t} dt$		$\frac{\partial^2 T}{\partial x^2}$
$\operatorname{erf}(\eta) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du$	unction	Tran in th
$\operatorname{erfc}(\eta) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} \varepsilon^{-u^2} dv$	<i>du</i> complementary error function	heat by th

$\frac{\partial^2 T}{\partial x^2} =$	$\frac{1}{\alpha} \frac{\partial T}{\partial t}$ and $\eta = \frac{x}{\sqrt{4\alpha t}}$
$\frac{\partial T}{\partial t} =$	$\frac{dT}{d\eta}\frac{\partial\eta}{\partial t} = \frac{x}{2t\sqrt{4\alpha t}}\frac{dT}{d\eta}$
$\frac{\partial T}{\partial x} =$	$\frac{dT}{d\eta}\frac{\partial\eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}}\frac{dT}{d\eta}$
$\frac{\partial^2 T}{\partial x^2} =$	$\frac{d}{d\eta} \left( \frac{\partial T}{\partial x} \right) \frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t} \frac{d^2 T}{d\eta^2}$

Transformation of variables in the derivatives of the heat conduction equation by the use of chain rule. <sup>23</sup>



Error function is a standard mathematical function, just like the sine and cosine functions, whose value varies between 0 and 1. TABLE 18-4

#### The complementary error function

η	erfc (η)	η	erfc (η)	η	erfc (η)
0.00	1.00000	0.38	0.5910	0.76	0.2825
0.02	0.9774	0.40	0.5716	0.78	0.2700
0.04	0.9549	0.42	0.5525	0.80	0.2579
0.06	0.9324	0.44	0.5338	0.82	0.2462
0.08	0.9099	0.46	0.5153	0.84	0.2349
0.10	0.8875	0.48	0.4973	0.86	0.2239
0.12	0.8652	0.50	0.4795	0.88	0.2133
0.14	0.8431	0.52	0.4621	0.90	0.2031
0.16	0.8210	0.54	0.4451	0.92	0.1932
0.18	0.7991	0.56	0.4284	0.94	0.1837
0.20	0.7773	0.58	0.4121	0.96	0.1746
0.22	0.7557	0.60	0.3961	0.98	0.1658
0.24	0.7343	0.62	0.3806	1.00	0.1573
0.26	0.7131	0.64	0.3654	1.02	0.1492
0.28	0.6921	0.66	0.3506	1.04	0.1413
0.30	0.6714	0.68	0.3362	1.06	0.1339
0.32	0.6509	0.70	0.3222	1.08	0.1267
0.34	0.6306	0.72	0.3086	1.10	0.1198
0.36	0.6107	0.74	0.2953	1.12	0.1132

$$\dot{q}_s = -k \frac{\partial T}{\partial x} \Big|_{x=0} = -k \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial x} \Big|_{\eta=0} = -k C_1 e^{-\eta^2} \frac{1}{\sqrt{4\alpha t}} \Big|_{\eta=0} = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}}$$

Case 1: Specified Surface Temperature,  $T_s = \text{constant}$  $\frac{T(x, t) - T_i}{T_s - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$ 

Analytical solutions for different boundary conditions on the surface

Case 2: Specified Surface Heat Flux,  $\dot{q}_s = \text{constant}$ .

$$T(x, t) - T_i = \frac{\dot{q}_s}{k} \left[ \sqrt{\frac{4\alpha t}{\pi}} \exp\left(-\frac{x^2}{4\alpha t}\right) - x \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \right]$$

Case 3: Convection on the Surface,  $\dot{q}_s(t) = h[T_{\infty} - T(0, t)]$ .

$$\frac{T(x,t) - T_i}{T_{\infty} - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

Case 4: Energy Pulse at Surface,  $e_s = \text{constant}$ .

$$T(x, t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(-\frac{x^2}{4\alpha t}\right)$$

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Variations of temperature with position and time in a large cast iron block ( $\alpha = 2.31 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $k = 80.2 \text{ W/m} \cdot \text{°C}$ ) initially at 0 °C under different thermal conditions on the surface.



Variations of temperature with position and time in a large cast iron block ( $\alpha = 2.31 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $k = 80.2 \text{ W/m} \cdot \text{°C}$ ) initially at 0 °C under different thermal conditions on the surface.



Variation of temperature with position and time in a semi-infinite solid initially at temperature  $T_i$  subjected to convection to an environment at  $T_{\infty}$  with a convection heat transfer coefficient of *h*.

### **Contact of Two Semi-Infinite Solids**

When two large bodies *A* and *B*, initially at uniform temperatures  $T_{A,i}$  and  $T_{B,i}$  are brought into contact, they instantly achieve temperature equality at the contact surface.

If the two bodies are of the same material, the contact surface temperature is the arithmetic average,  $T_s = (T_{A,i} + T_{B,i})/2$ .

If the bodies are of different materials, the surface temperature  $T_s$  will be different than the arithmetic average.



Contact of two semi-infinite solids of different initial temperatures.

$$\dot{q}_{s,A} = \dot{q}_{s,B} \rightarrow -\frac{k_A(T_s - T_{A,i})}{\sqrt{\pi\alpha_A t}} = \frac{k_B(T_s - T_{B,i})}{\sqrt{\pi\alpha_B t}} \rightarrow \frac{T_{A,i} - T_s}{T_s - T_{B,i}} = \sqrt{\frac{(k\rho c_p)_B}{(k\rho c_p)_A}}$$

$$T_s = \frac{\sqrt{(k\rho c_p)_A}T_{A,i} + \sqrt{(k\rho c_p)_B}T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}}$$

The interface temperature of two bodies brought into contact is dominated by the body with the larger  $k\rho c_p$ .

**EXAMPLE:** When a person with a skin temperature of 35°C touches an aluminum block and then a wood block both at 15°C, the contact surface temperature will be 15.9°C in the case of aluminum and 30°C in the case of wood.

### **18-4 TRANSIENT HEAT CONDUCTION IN** MULTIDIMENSIONAL SYSTEMS

- Using a superposition approach called the **product solution**, the transient temperature charts and solutions can be used to construct solutions for the twodimensional and three-dimensional transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, a rectangular prism or a semi-infinite rectangular bar, provided that all surfaces of the solid are subjected to convection to the same fluid at temperature  $T_{\infty}$ , with the same heat transfer coefficient h, and the body involves no heat generation.
- The solution in such multidimensional geometries can be expressed as the • *product* of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.



The temperature in a short cylinder exposed to convection from all surfaces varies in both the radial and axial directions, and thus heat is transferred in both directions. 31

The solution for a multidimensional geometry is the product of the solutions of the one-dimensional geometries whose intersection is the multidimensional body.

The solution for the two-dimensional short cylinder of height *a* and radius  $r_o$  is equal to the *product* of the nondimensionalized solutions for the one-dimensional plane wall of thickness *a* and the long cylinder of radius  $r_o$ .



$$\left(\frac{T(x, y, t) - T_{\infty}}{T_i - T_{\infty}}\right)_{\text{tectangular}} = \theta_{\text{wall}}(x, t)\theta_{\text{wall}}(y, t)$$



$$\begin{split} \theta_{\text{wall}}(x, t) &= \left(\frac{T(x, t) - T_{\infty}}{T_i - T_{\infty}}\right)_{\text{plane}} \\ \theta_{\text{cyl}}(r, t) &= \left(\frac{T(r, t) - T_{\infty}}{T_i - T_{\infty}}\right)_{\text{infinite}} \\ \theta_{\text{semi-inf}}(x, t) &= \left(\frac{T(x, t) - T_{\infty}}{T_i - T_{\infty}}\right)_{\substack{\text{semi-infinite}}} \end{split}$$

A long solid bar of rectangular profile  $a \times b$  is the *intersection* of two plane walls of thicknesses *a* and *b*.

The transient heat transfer for a two-dimensional geometry formed by the intersection of two onedimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\text{max}}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\text{max}}}\right)_1 + \left(\frac{Q}{Q_{\text{max}}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\text{max}}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is

$$\begin{pmatrix} Q \\ Q_{\text{max}} \end{pmatrix}_{\text{total, 3D}} = \begin{pmatrix} Q \\ Q_{\text{max}} \end{pmatrix}_1 + \begin{pmatrix} Q \\ Q_{\text{max}} \end{pmatrix}_2 \left[ 1 - \begin{pmatrix} Q \\ Q_{\text{max}} \end{pmatrix}_1 \right]$$
$$+ \left( \frac{Q}{Q_{\text{max}}} \right)_3 \left[ 1 - \left( \frac{Q}{Q_{\text{max}}} \right)_1 \right] \left[ 1 - \left( \frac{Q}{Q_{\text{max}}} \right)_2 \right]$$

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature  $T_i$  and exposed to convection from all surfaces to a medium at  $T_{\infty}$ 



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Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature  $T_i$  and exposed to convection from all surfaces to a medium at  $T_{\infty}$ 



# **Summary**

- Lumped System Analysis
  - ✓ Criteria for Lumped System Analysis
  - ✓ Some Remarks on Heat Transfer in Lumped Systems
- Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects
  - Nondimensionalized One-Dimensional Transient Conduction Problem
  - ✓ Approximate Analytical and Graphical Solutions
- Transient Heat Conduction in Semi-Infinite Solids

✓ Contact of Two Semi-Infinite Solids

Transient Heat Conduction in Multidimensional Systems