CHAPTER 9

Section 9-1

9-1 a) $H_0 : \mu = 25, H_1 : \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0 : \sigma > 10, H_1 : \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0 : \bar{x} = 50, H_1 : \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0 : p = 0.1, H_1 : p = 0.3$ No, the values in the null and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0 : s = 20, H_1 : s < 20$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-2 The conclusion does not provide strong evidence that the critical dimension mean equals 80nm. There is not sufficient evidence to reject the null hypothesis.

9-3 a) $H_0 : \sigma = 25\text{nm}, \; H_1 : \sigma < 25\text{nm}$

b) This result does not provide strong evidence that the standard deviation has not been reduced. There is insufficient evidence to reject the null hypothesis but this is not strong support for the null hypothesis.

9-4 a) $H_0 : \mu = 30\text{newtons}, H_1 : \mu < 30\text{newtons}$

b) No, this result only implies that we do not have enough evidence to support H_1.

9-5 α = P(reject H_0 when H_0 is true)

= P($\overline{X}$ ≤ 10.5 when μ = 12) = $P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{10.5-12}{1.5/\sqrt{4}}\right)$ = P(Z ≤ –2) = 0.02275.

The probability of rejecting the null hypothesis when it is true is 0.02275.

b) β = P(accept H_0 when μ = 10.25) = $P\left(\overline{X} > 10.5 | \mu = 10.25\right)$

= $P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{10.5-10.25}{1.5/\sqrt{4}}\right)$ = P(Z > 0.33) = 1 – P(Z ≤ 0.33) = 1 – 0.6293 = 0.3707

The probability of failing to reject the null hypothesis when it is false is 0.3707

c) β = P(accept H_0 when μ = 10.5) =

= $P\left(\overline{X} > 10.5 | \mu = 10.5\right)$= $P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{10.5-10.5}{1.5/\sqrt{4}}\right)$

= P(Z > 0) = 1 – P(Z ≤ 0) = 1 – 0.5 = 0.5

The probability of failing to reject the null hypothesis when it is false is 0.5

9-6 a) α = P($\overline{X}$ ≤ 11.5 | μ = 12) = $P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{11.5-12}{0.5/\sqrt{20}}\right)$ = P(Z ≤ –4.47) = 0.

The probability of rejecting the null hypothesis when it is true is approximately 0 with a sample size of 20.

b) β = P($\overline{X}$ > 11.5 | μ = 11.25) = $P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{20}}\right)$

= P(Z > 2.236) = 1 – P(Z ≤ 2.236) = 1– 0.987314 = 0.01269

The probability of accepting the null hypothesis when it is false is 0.01269.

c) $\beta = P(\bar{X} > 11.5 \mid \mu = 11.5) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.5}{0.5/\sqrt{20}}\right)$

$= P(Z > 0) = 1 - P(Z \le 0) = 1 - 0.5 = 0.5$

The probability of accepting the null hypothesis when it is false is 0.5.

9-7 The critical values of $\bar{x}$ are $12 \pm Z_\alpha 0.5/\sqrt{n}$. For the one-sided test with $\bar{X} \le 12$, we obtain

a) $\alpha = 0.01$, n = 5, from Table III $-2.33 = z_\alpha$ and $\bar{X} \le 11.48$

b) $\alpha = 0.05$, n = 5, from Table III $-1.65 = z_\alpha$ and $\bar{X} \le 11.63$

c) $\alpha = 0.01$, n = 20, from Table III $-2.33 = z_\alpha$ and $\bar{X} \le 11.74$

d) $\alpha = 0.05$, n = 20, from Table III $-1.65 = z_\alpha$ and $\bar{X} \le 11.816$

9-8 a) $\beta = P(\bar{X} > 11.59 \mid \mu = 11.5) = P(Z > 0.36) = 1 - 0.6406 = 0.3594$

b) $\beta = P(\bar{X} > 11.79 \mid \mu = 11.5) = P(Z > 2.32) = 1 - 0.9898 = 0.0102$

c) Notice that the value of β decreases as n increases

9-9 a) $\bar{x} = 11.25$, then P-value = $P\left(Z \le \frac{11.25-12}{0.5/\sqrt{4}}\right) = P(Z \le -3) = 0.00135$

b) $\bar{x} = 11.0$, then P-value = $P\left(Z \le \frac{11.0-12}{0.5/\sqrt{4}}\right) = P(Z \le -4) \le 0.000033$

c) $\bar{x} = 11.75$, then P-value = $P\left(Z \le \frac{11.75-12}{0.5/\sqrt{4}}\right) = P(Z \le -1) = 0.158655$

9-10 a) $\alpha = P(\bar{X} \le 98.5) + P(\bar{X} > 101.5)$

$$= P\left(\frac{\bar{X}-100}{3/\sqrt{9}} \le \frac{98.5-100}{3/\sqrt{9}}\right) + P\left(\frac{\bar{X}-100}{3/\sqrt{9}} > \frac{101.5-100}{3/\sqrt{9}}\right)$$

$= P(Z \le -1.5) + P(Z > 1.5) = (P(Z \le -1.5)) + (1 - P(Z \le 1.5))$

$= 0.0668 + 1 - 0.9332 = 0.0668 + 0.0668 = 0.1336$

b) $\beta = P(98.5 \le \bar{X} \le 101.5$ when $\mu = 103)$

$$= P\left(\frac{98.5-103}{3/\sqrt{9}} \le \frac{\bar{X}-103}{3/\sqrt{9}} \le \frac{101.5-103}{3/\sqrt{9}}\right)$$

$= P(-4.5 \le Z \le -1.5) = P(Z \le -1.5) - P(Z \le -4.5) = 0.0668 - 0 = 0.0668$

c) $\beta = P(98.5 \le \bar{X} \le 101.5 \mid \mu = 105)$

$$= P\left(\frac{98.5-105}{3/\sqrt{9}} \le \frac{\bar{X}-105}{3/\sqrt{9}} \le \frac{101.5-105}{3/\sqrt{9}}\right)$$

$= P(-6.5 \le Z \le -3.5) = P(Z \le -3.5) - P(Z \le -6.5) = 0.0002 - 0 = 0.0002$

The probability of failing to reject the null hypothesis when it is actually false is smaller in part (c) because the true mean, μ = 105, is further from the acceptance region. That is, there is a greater difference between the true mean and the hypothesized mean.

9-11 Use *n* = 5, everything else held constant:

a) $P(\bar{X} \le 98.5) + P(\bar{X} > 101.5)$

$$= P\left(\frac{\bar{X}-100}{3/\sqrt{5}} \le \frac{98.5-100}{3/\sqrt{5}}\right) + P\left(\frac{\bar{X}-100}{3/\sqrt{5}} > \frac{101.5-100}{3/\sqrt{5}}\right)$$

$= P(Z \le -1.12) + P(Z > 1.12)$

$= P(Z \le -1.12) + (1 - P(Z \le 1.12)) = 0.1314 + (1 - 0.8686) = 0.2627$

b) β = P(98.5 ≤ $\overline{X}$ ≤ 101.5 when μ = 103)

$$= P\left(\frac{98.5-103}{3/\sqrt{5}} \le \frac{\overline{X}-103}{3/\sqrt{5}} \le \frac{101.5-103}{3/\sqrt{5}}\right)$$

= P(–3.35 ≤ Z ≤ –1.12) = P(Z ≤ –1.12) – P(Z ≤ –3.35) = 0.1314 – 0.0004 = 0.1310

c) β = P(98.5 ≤ $\bar{x}$ ≤ 101.5 when μ = 105)

$$= P\left(\frac{98.5-105}{3/\sqrt{5}} \le \frac{\overline{X}-105}{3/\sqrt{5}} \le \frac{101.5-105}{3/\sqrt{5}}\right)$$

= P(–4.84≤ Z ≤ –2.61)
= P(Z ≤ –2.61) – P(Z ≤ –4.84) = 0.0045 – 0 = 0.0045

It is smaller because it is not likely to accept the product when the true mean is as high as 105.

9-12 $\mu_0 - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \le \overline{X} \le \mu_0 + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$, where σ =3

a) α = 0.01, n = 9, then $z_{\alpha/2}$ =2.58, then 97.42 ,102.58

b) α = 0.05, n = 9, then $z_{\alpha/2}$ =1.96, then 98.04,101.96

c) α = 0.01, n = 5, then $z_{\alpha/2}$ = 2.58, then 96.54,103.46

d) α = 0.05, n = 5, then $z_{\alpha/2}$ =1.96, then 97.37,102.63

9-13 δ = 103 - 100 = 3

δ > 0 then $\beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$, where σ = 3

a) β = P(98.04< $\overline{X}$ <101.96|μ=103) = P(–4.96 ≤ Z ≤ –1.04) = 0.1492 – 0 = 0.1492

b) β = P(97.37< $\overline{X}$ <102.63|μ=103) = P(-5.63<Z<-0.37) = 0.3557

c) As n increases, β decreases

9-14 a) P-value = 2(1 - $\Phi(|Z_0|)$) = 2(1 - $\Phi\left(\left|\frac{98-100}{3/\sqrt{9}}\right|\right)$) = 2(1 - $\Phi(2)$) = 2(1 - 0.9772) = 0.0455

b) P-value = 2(1 - $\Phi(|Z_0|)$) = 2(1 - $\Phi\left(\left|\frac{101-100}{3/\sqrt{9}}\right|\right)$) = 2(1 - $\Phi(1)$) = 2(1 - 0.8413) = 0.3173

c) P-value = 2(1 - $\Phi(|Z_0|)$) = 2(1 - $\Phi\left(\left|\frac{102-100}{3/\sqrt{9}}\right|\right)$) = 2(1 - $\Phi(2)$) = 2(1 - 0.9772) = 0.0455

9-15 a) α = P($\overline{X}$ > 185 when μ = 175)

$$= P\left(\frac{\overline{X}-175}{25/\sqrt{15}} > \frac{185-175}{25/\sqrt{15}}\right)$$

= P(Z > 1.55)
= 1 – P(Z ≤ 1.55)
= 1 – 0.93943 = 0.06057

b) β = P($\bar{X}$ ≤ 185 when μ = 185)

$$= P\left(\frac{\bar{X}-185}{25/\sqrt{15}} \le \frac{185-185}{25/\sqrt{15}}\right)$$

= P(Z ≤ 0) = 0.5

c) β = P($\bar{X}$ ≤ 185 when μ = 195)

$$= P\left(\frac{\bar{X}-195}{25/\sqrt{15}} \le \frac{185-195}{25/\sqrt{15}}\right)$$

= P(Z ≤ –1.55) = 0.06057

9-16 Using n = 16:

a) α = P($\bar{X}$ > 185 when μ = 175)

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{16}} > \frac{185-175}{20/\sqrt{16}}\right)$$

= P(Z > 2)

= 1 – P(Z ≤ 2) = 1 – 0.97725 = 0.02275

b) β = P($\bar{X}$ ≤ 185 when μ = 185)

$$= P\left(\frac{\bar{X}-195}{20/\sqrt{16}} \le \frac{185-185}{20/\sqrt{16}}\right)$$

= P(Z ≤ 0) = 0.5

c) β = P($\bar{X}$ ≤ 185 when μ = 195)

$$= P\left(\frac{\bar{X}-195}{20/\sqrt{16}} \le \frac{185-195}{20/\sqrt{16}}\right)$$

= P(Z ≤ –2) = 0.02275

9-17 $$\bar{X} \ge 175 + Z_\alpha\left(\frac{20}{\sqrt{n}}\right)$$

a) α = 0.01, n = 10, then z_α = 2.32 and critical value is 189.67

b) α = 0.05, n = 10, then z_α = 1.64 and critical value is 185.93

c) α = 0.01, n = 16, then z_α = 2.32 and critical value is 186.6

d) α = 0.05, n = 16, then z_α = 1.64 and critical value is 183.2

9-18 a) α = 0.05, n = 10, then the critical value 185.93 (from 9-17 part (b))

β = P($\bar{X}$ ≤ 185.37 when μ = 185)

$$= P\left(\frac{\bar{X}-185}{20/\sqrt{10}} \le \frac{185.93-185}{20/\sqrt{10}}\right)$$

= P(Z ≤ 0.147) = 0.5584

b) α = 0.05, n = 16, then the critical value 183.2 (from 9-17(d)), then

β = P($\bar{X}$ ≤ 183.2 when μ = 185)

$$= P\left(\frac{\bar{X}-185}{20/\sqrt{16}} \le \frac{183.2-185}{20/\sqrt{16}}\right)$$

= P(Z ≤ –0.36) = 0.3594

c) as n increases, β decreases

9-19 P-value = 1 – $\Phi(Z_0)$) where $Z_0 = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

a) $\overline{X}$ = 180 then $Z_0 = \dfrac{180-175}{20/\sqrt{10}} = 0.79$

P-value = 1 – $\Phi(0.79) = 1 - 0.7852 = 0.2148$

b) $\overline{X}$ = 190 then $Z_0 = \dfrac{190-175}{20/\sqrt{10}} = 2.37$

P-value = 1 – $\Phi(2.37) = 1 - 0.991106 = 0.008894$

c) $\overline{X}$ = 170 then $Z_0 = \dfrac{170-175}{20/\sqrt{10}} = -0.79$

P-value = 1 – $\Phi(-0.79) = 1 - 0.214764 = 0.785236$

9-20 a) α = P($\overline{X} \le 4.85$ when $\mu = 5$) + P($\overline{X} > 5.15$ when $\mu = 5$)

$= P\left(\dfrac{\overline{X}-5}{0.20/\sqrt{8}} \le \dfrac{4.85-5}{0.20/\sqrt{8}}\right) + P\left(\dfrac{\overline{X}-5}{0.20/\sqrt{8}} > \dfrac{5.15-5}{0.20/\sqrt{8}}\right)$

= P(Z ≤ –1.697) + P(Z > 1.697)

= P(Z ≤ –1.697) + (1 – P(Z ≤ 1.697)

= 0.04475 + (1 – 0.95525) = 0.0895

b) Power = 1 – β

β = P(4.85 ≤ $\overline{X}$ ≤ 5.15 when μ = 5.1)

$= P\left(\dfrac{4.85-5.1}{0.20/\sqrt{8}} \le \dfrac{\overline{X}-5.1}{0.20/\sqrt{8}} \le \dfrac{5.15-5.1}{0.20/\sqrt{8}}\right)$

= P(–2.83 ≤ Z ≤ 0.566)

= P(Z ≤ 0.566) – P(Z ≤ –2.83)

= 0.71566 – 0.00233 = 0.71333

1 – β = 0.2867

9-21 Using n = 16:

a) α = P($\overline{X}$ ≤ 4.85 | μ = 5) + P($\overline{X}$ > 5.15 | μ = 5)

$= P\left(\dfrac{\overline{X}-5}{0.25/\sqrt{16}} \le \dfrac{4.85-5}{0.25/\sqrt{16}}\right) + P\left(\dfrac{\overline{X}-5}{0.25/\sqrt{16}} > \dfrac{5.15-5}{0.25/\sqrt{16}}\right)$

= P(Z ≤ –2.4) + P(Z > 2.4)

= P(Z ≤ –2.4) +(1 – P(Z ≤ 2.4))

= 2(1 – P(Z ≤ 2.4))

= 2(1 – 0.99180) = 2(0.0082) = 0.0164

b) β = P(4.85 ≤ $\overline{X}$ ≤ 5.15 | μ = 5.1)

$= P\left(\dfrac{4.85-5.1}{0.25/\sqrt{16}} \le \dfrac{\overline{X}-5.1}{0.25/\sqrt{16}} \le \dfrac{5.15-5.1}{0.25/\sqrt{16}}\right)$

= P(–4 ≤ Z ≤ 0.8) = P(Z ≤ 0.8) – P(Z ≤ –4)

= 0.78814 – 0 = 0.78814

1 – β = 0.21186

c) With larger sample size, the value of α decreased from approximately 0.089 to 0.016. The power declined modestly from 0.287 to 0.211 while the value for α declined substantially. If the test with n = 16 were conducted at the α value of 0.089, then it would have greater power than the test with n = 8.

9-22 $\sigma = 0.25,\ \mu_0 = 5$

a) α = 0.01, n = 8 then

a = $\mu_0 + z_{\alpha/2}\sigma/\sqrt{n}$ = 5 + 2.57*.25/ $\sqrt{8}$ = 5.22 and

b = $\mu_0 - z_{\alpha/2}\sigma/\sqrt{n}$ = 5-2.57*.25/ $\sqrt{8}$ = 4.77

b) α = 0.05, n = 8 then

a = $\mu_0 + Z_{\alpha/2} * \sigma/\sqrt{n}$ = 5 + 1.96*.25/ $\sqrt{8}$ = 5.1732 and

b = $\mu_0 - z_{\alpha/2}\sigma/\sqrt{n}$ = 5-1.96*.25/ $\sqrt{8}$ = 4.8267

c) α = 0.01, n = 16 then

a = $\mu_0 + z_{\alpha/2}\sigma/\sqrt{n}$ = 5 + 2.57*.25/ $\sqrt{16}$ = 5.1606 and

b = $\mu_0 - z_{\alpha/2}\sigma/\sqrt{n}$ = 5-2.57*.25/ $\sqrt{16}$ = 4.8393

d) α = 0.05, n = 16 then

a = $\mu_0 + z_{\alpha/2}\sigma/\sqrt{n}$ = 5 + 1.96*.25/ $\sqrt{16}$ = 5.1225 and

b = $\mu_0 - z_{\alpha/2}\sigma/\sqrt{n}$ = 5-1.96*.25/ $\sqrt{16}$ = 4.8775

9-23 P-value = 2(1 − $\Phi(|Z_0|)$) where $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

a) $\bar{x}$ = 5.2 then $z_0 = \dfrac{5.2-5}{.25/\sqrt{8}} = 2.26$

P-value = 2(1 − $\Phi(2.26)) = 2(1-0.988089) = 0.0238$

b) $\bar{x}$ = 4.7 then $z_0 = \dfrac{4.7-5}{.25/\sqrt{8}} = -3.39$

P-value = 2(1 − $\Phi(3.39)) = 2(1-0.99965) = 0.0007$

c) $\bar{x}$ = 5.1 then $z_0 = \dfrac{5.1-5}{.25/\sqrt{8}} = 1.1313$

P-value = 2(1 − $\Phi(1.1313)) = 2(1-0.870762) = 0.2585$

9-24 a) β = P(4.845 < $\bar{X}$ < 5.155|μ = 5.05) = P(−2.59 < Z < 1.33) = 0.9034

b) β = P(4.8775 < $\bar{X}$ < 5.1225|μ = 5.05) = P(−2.76 < Z < 1.16) = 0.8741

c) As n increases, β decreases

9-25 X ~ bin(20, 0.4) H_0: p = 0.4 and H_1: $p \neq 0.4$

p_1 = 6/20 = 0.3 p_2 = 11/20 = 0.55

Accept Region: $0.3 \le \hat{p} \le 0.55$

Reject Region: $\hat{p} < 0.3$ or $\hat{p} > 0.55$

Use the normal approximation for parts a) and b)

a) When p = 0.4, $\alpha = P(\hat{p} < 0.3) + P(\hat{p} > 0.55)$

$$= P\left(Z < \frac{0.3-0.4}{\sqrt{\frac{0.4(0.6)}{20}}} \right) + P\left(Z > \frac{0.55-0.4}{\sqrt{\frac{0.4(0.6)}{20}}} \right)$$

$$= P(Z < -0.9129) + P(Z > 1.3693)$$

$$= P(Z < -0.9129) + (1 - P(Z < 1.3693))$$

$$= 0.181411 + 1 - 0.914657$$

$$= 0.26675$$

b) When p = 0.2

$$\beta = P(0.3 \le \hat{p} \le 0.55) = P\left(\frac{0.3-0.2}{\sqrt{\frac{0.2(0.8)}{20}}} \le Z \le \frac{0.55-0.2}{\sqrt{\frac{0.2(0.8)}{20}}}\right)$$
$$= P(1.118 \le Z \le 2.4749)$$
$$= P(Z \le 2.4749) - P(Z \le 1.118)$$
$$= 0.99334 - 0.86821$$
$$= 0.12513$$

9-26 $X \sim$ Bin(10, 0.4) Implicitly, H_0: $p = 0.4$ and H_1: $p < 0.4$
$n = 10$
Accept region: $\hat{p} > 0.1$
Reject region: $\hat{p} \le 0.1$
Use the normal approximation for parts a), b) and c):

a) When p = 0.4 α = $P(\hat{p} < 0.1) = P\left(Z \le \frac{0.1-0.4}{\sqrt{\frac{0.4(0.6)}{10}}}\right)$
$$= P(Z \le -1.38)$$
$$= 0.08379$$

b) When p = 0.2 β = $P(\hat{p} > 0.1) = P\left(Z > \frac{0.1-0.2}{\sqrt{\frac{0.2(0.8)}{10}}}\right)$
$$= P(Z > -0.79)$$
$$= 1 - P(Z < -0.79)$$
$$= 0.78524$$

c) Power = 1 – β = 1 – 0.78524 = 0.21476

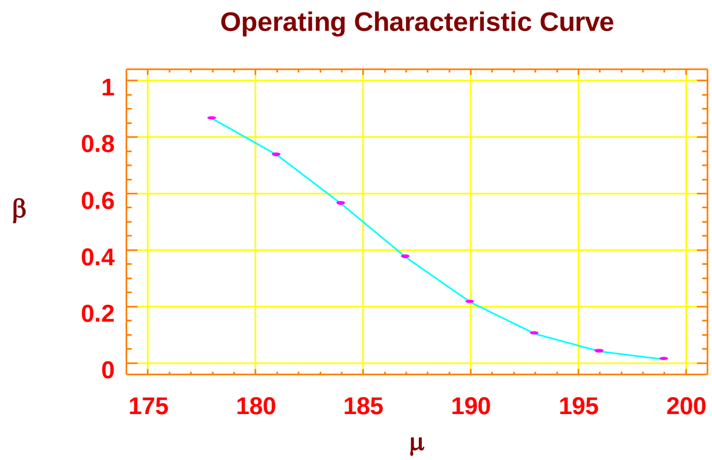
9-27 The problem statement implies H_0: $p = 0.6$, H_1: $p > 0.6$ and defines an acceptance region as $\hat{p} \le \frac{600}{750} = 0.80$ and rejection region as $\hat{p} > 0.80$

a) α=P($\hat{p}$ > 0.80 | p = 0.60) = P$\left(Z > \frac{0.80-0.60}{\sqrt{\frac{0.6(0.4)}{750}}}\right)$ = P(Z > 11.18)=1 - P(Z ≤ 11.18) ≈ 0

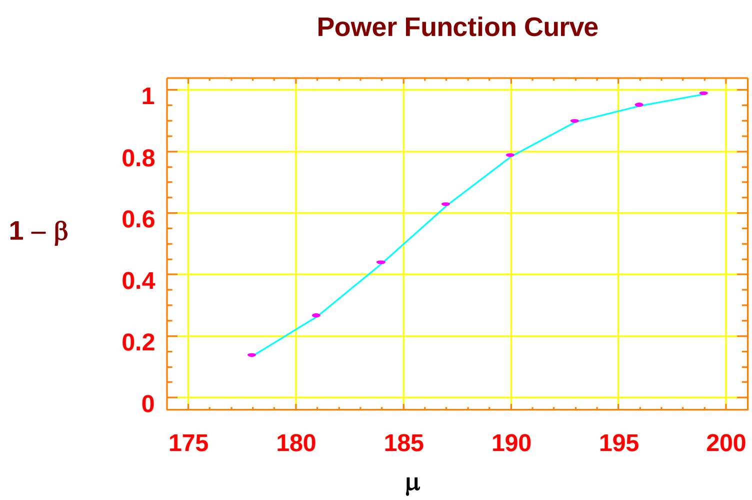
b) β = P($\hat{p}$ ≤ 0.8 when p = 0.75) = P(Z ≤ 2.80) = 0.9974

9-28 a) Operating characteristic curve:
$\bar{x} = 185$
$$\beta = P\left(Z \le \frac{\bar{x}-\mu}{20/\sqrt{10}}\right) = P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right)$$

μ	$P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right) =$	β	$1-\beta$
178	P(Z ≤ 1.11) =	0.8665	0.1335
181	P(Z ≤ 0.63) =	0.7357	0.2643
184	P(Z ≤ 0.16) =	0.5636	0.4364
187	P(Z ≤ –0.32) =	0.3745	0.6255
190	P(Z ≤ –0.79) =	0.2148	0.7852
193	P(Z ≤ –1.26) =	0.1038	0.8962
196	P(Z ≤ –1.74) =	0.0409	0.9591
199	P(Z ≤ –2.21) =	0.0136	0.9864



b)



Section 9-2

9-29 a) $H_0: \mu = 6, H_1: \mu > 6$

b) $H_0: \mu = 8, H_1: \mu \neq 8$

c) $H_0: \mu = 9, H_1: \mu < 9$

9-30 a) $\alpha = 0.01$, then a = $z_{\alpha/2}$ = 2.58 and b = - $z_{\alpha/2}$ = -2.58

b) $\alpha = 0.05$, then a = $z_{\alpha/2}$ = 1.96 and b = - $z_{\alpha/2}$ = -1.96

c) $\alpha = 0.1$, then a = $z_{\alpha/2}$ = 1.65 and b = - $z_{\alpha/2}$ = -1.65

9-31 a) $\alpha = 0.01$, then a = $z_{\alpha} \cong 2.33$

b) $\alpha = 0.05$, then a = $z_{\alpha} \cong 1.65$

c) $\alpha = 0.1$, then a = $z_{\alpha} \cong 1.29$

9-32 a) $\alpha = 0.01$, then a = $z_{1-\alpha} \cong -2.33$

b) $\alpha = 0.05$, then a = $z_{1-\alpha} \cong -1.65$

c) $\alpha = 0.1$, then a = $z_{1-\alpha} \cong -1.29$

9-33 a) P-value = $2(1 - \Phi(|Z_0|)) = 2(1 - \Phi(2.05)) \cong 0.04$

b) P-value = $2(1 - \Phi(|Z_0|)) = 2(1 - \Phi(1.84)) \cong 0.066$

c) P-value = $2(1 - \Phi(|Z_0|)) = 2(1 - \Phi(0.4)) \cong 0.69$

9-34 a) P-value = $1 - \Phi(Z_0) = 1 - \Phi(2.05) \cong 0.02$

b) P-value = $1 - \Phi(Z_0) = 1 - \Phi(-1.84) \cong 0.97$

c) P-value = $1 - \Phi(Z_0) = 1 - \Phi(0.4) \cong 0.34$

9-35 a) P-value = $\Phi(Z_0) = \Phi(2.05) \cong 0.98$

b) P-value = $\Phi(Z_0) = \Phi(-1.84) \cong 0.03$

c) P-value = $\Phi(Z_0) = \Phi(0.4) \cong 0.65$

9-36 a) SE Mean from the sample standard deviation $= \dfrac{\sigma}{\sqrt{N}} = \dfrac{1.475}{\sqrt{25}} = 0.295$

$$z_0 = \frac{30.421 - 30}{1.8/\sqrt{25}} = 1.17$$

P-value = $2[1 - \Phi(Z_0)] = 2[1 - \Phi(1.17)] = 2[1 - 0.8790] = 0.2420$

Because the P-value $> \alpha = 0.05$, fail to reject the null hypothesis that $\mu = 30$ at the 0.05 level of significance.

b) A two-sided test because the alternative hypothesis is mu not = 30.

c) 95% CI of the mean is $\bar{x} - z_{0.025}\dfrac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{0.025}\dfrac{\sigma}{\sqrt{n}}$

$$30.421 - (1.96)\frac{1.8}{\sqrt{25}} < \mu < 30.421 + (1.96)\frac{1.8}{\sqrt{25}}$$

$$29.7154 < \mu < 31.1266$$

d) P-value = $1 - \Phi(Z_0) = 1 - \Phi(1.17) = 1 - 0.8790 = 0.1210$

9-37 a) StDev = $\sqrt{N}$ (SE Mean) = $\sqrt{20} \times 0.237 = 1.0599$

$$z_0 = \frac{19.889 - 20}{0.75/\sqrt{20}} = -0.03$$

P-value = $1 - \Phi(Z_0) = 1 - \Phi(-0.03) = 1 - 0.4880 = 0.5120$

Because the P-value > α = 0.05, we fail to reject the null hypothesis that μ = 20 at the 0.05 level of significance.

b) A one-sided test because the alternative hypothesis is μ > 20

c) 95% CI of the mean is $\bar{x} - z_{0.025}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{0.025}\frac{\sigma}{\sqrt{n}}$

$$19.889 - (1.96)\frac{0.75}{\sqrt{20}} < \mu < 19.889 + (1.96)\frac{0.75}{\sqrt{20}}$$

$$19.5603 < \mu < 20.2177$$

d) P-value = $2[1-\Phi(Z_0)] = 2[1-\Phi(0.03)] = 2[1-0.5120] = 0.9761$

9-38 a) SE Mean from the sample standard deviation $= \frac{s}{\sqrt{N}} = \frac{1.015}{\sqrt{16}} = 0.2538$

$$z_0 = \frac{15.016 - 14.5}{1.1/\sqrt{16}} = 1.8764$$

P-value= $1-\Phi(Z_0) = 1-\Phi(1.8764) = 1-0.9697 = 0.0303$

Because the P-value < α = 0.05, reject the null hypothesis that μ = 14.5 at the 0.05 level of significance.

b) A one-sided test because the alternative hypothesis is μ > 14.5

c) 95% lower CI of the mean is $\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu$

$$15.016 - (1.645)\frac{1.1}{\sqrt{16}} \le \mu$$

$$14.5636 \le \mu$$

d) P-value = $2[1-\Phi(Z_0)] = 2[1-\Phi(1.8764)] = 2[1-0.9697] = 0.0606$

9-39 a) SE Mean from the sample standard deviation $= \frac{s}{\sqrt{N}} = \frac{2.365}{\sqrt{12}} = 0.6827$

b) A one-sided test because the alternative hypothesis is μ > 99.

c) If the null hypothesis is changed to the μ = 98, $z_0 = \frac{100.039 - 98}{2.5/\sqrt{12}} = 2.8253$

Because $\Phi(2.8253)$ is close to 1, the P-value = $1-\Phi(2.8253)$ = 0.002 is very small and close to 0. Thus, the P-value < α = 0.05, and we reject the null hypothesis at the 0.05 level of significance.

d) 95% lower CI of the mean is $\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu$

$$100.039 - (1.645)\frac{2.5}{\sqrt{12}} \le \mu$$

$$98.8518 \le \mu$$

e) If the alternative hypothesis is changed to the mu $\neq$ 99, $z_0 = \frac{100.039 - 99}{2.5/\sqrt{12}} = 1.4397$

P-value = $2[1-\Phi(Z_0)] = 2[1-\Phi(1.4397)] = 2[1-0.9250] = 0.15$

Because the P-value > α = 0.05, we fail to reject the null hypothesis at the 0.05 level of significance.

9-40 a) 1) The parameter of interest is the true mean water temperature, μ.
2) H_0 : μ = 38
3) H_1 : μ > 38

4) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

5) Reject H_0 if $z_0 > z_\alpha$ where $\alpha = 0.05$ and $z_{0.05} = 1.65$

6) $\bar{x} = 37$, $\sigma = 1.1$

$$z_0 = \frac{37 - 38}{1.1/\sqrt{9}} = -2.73$$

7) Because –2.73 < 1.65 fail to reject H_0. The water temperature is not significantly greater than 38 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(-2.73) = 1 - 0.003167 = 0.9968$

c) $\beta = \Phi\left(z_{0.05} + \dfrac{38 - 40}{1.1/\sqrt{9}} \right)$

$= \Phi(1.65 + -5.45)$

$= \Phi(-3.80) \cong 0.000072$

9-41 a) 1) The parameter of interest is the true mean crankshaft wear, μ.

2) $H_0 : \mu = 3$

3) $H_1 : \mu \neq 3$

4) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.05$ and $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.05$ and $z_{0.025} = 1.96$

6) $\bar{x} = 2.78$, $\sigma = 0.9$

$$z_0 = \frac{2.78 - 3}{0.9/\sqrt{15}} = -0.95$$

7) Because –0.95 > –1.96 fail to reject the null hypothesis. There is not sufficient evidence to support the claim the mean crankshaft wear differs from 3 at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.025} + \dfrac{3 - 3.25}{0.9/\sqrt{15}} \right) - \Phi\left(-z_{0.025} + \dfrac{3 - 3.25}{0.9/\sqrt{15}} \right)$

$= \Phi(1.96 + -1.08) - \Phi(-1.96 + -1.08)$

$= \Phi(0.88) - \Phi(-3.04)$

$= 0.81057 - (0.00118)$

$= 0.80939$

c) $n = \dfrac{\left(z_{\alpha/2} + z_\beta\right)^2 \sigma^2}{\delta^2} = \dfrac{\left(z_{0.025} + z_{0.10}\right)^2 \sigma^2}{(3.75 - 3)^2} = \dfrac{(1.96 + 1.29)^2 (0.9)^2}{(0.75)^2} = 15.21,$ $\qquad n \cong 16$

9-42 a) 1) The parameter of interest is the true mean melting point, μ.

2) $H_0 : \mu = 155$

3) $H_1 : \mu \neq 155$

4) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.01$ and $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.01$ and $z_{0.005} = 2.58$

6) $\bar{x} = 68$, $\sigma = 0.83$

$$z_0 = \frac{68 - 68.3}{0.83\sqrt{15}} = -1.4$$

7) Because –1.4 > –2.58 fail to reject the null hypothesis. There is not sufficient evidence to support the claim the mean melting point differs from 68.3°C at $\alpha = 0.01$.

b) P-value = 2*P(Z < –1.4) = 2* 0.080757 = 0.161514

c) $\beta = \Phi\left(z_{0.005} - \dfrac{\delta\sqrt{n}}{\sigma} \right) - \Phi\left(-z_{0.005} - \dfrac{\delta\sqrt{n}}{\sigma} \right)$

$$= \Phi\left(2.58 - \frac{(68.3-65.5)\sqrt{15}}{0.83}\right) - \Phi\left(-2.58 - \frac{(68.3-65.5)\sqrt{15}}{0.83}\right)$$

= Φ(2.58 – 13.065) – Φ (–2.58 – 13.065)
= Φ(10.485) – Φ (15.645) = 0 – 0 = 0

d) $n = \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \sigma^2}{\delta^2} = \frac{\left(z_{0.005} + z_{0.10}\right)^2 \sigma^2}{(150-155)^2} = \frac{(2.58+1.29)^2 (0.83)^2}{(2.8)^2} = 1.316$

$n \cong 2$.

9-43 a) 1) The parameter of interest is the true mean battery life in hours, μ.
2) $H_0 : \mu = 40$
3) $H_1 : \mu > 40$
4) $z_0 = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$
5) Reject H_0 if $z_0 > z_\alpha$ where α = 0.05 and $z_{0.05} = 1.65$
6) $\overline{x} = 40.5$, σ = 1.25

$$z_0 = \frac{40.5-40}{1.25/\sqrt{20}} = 1.79$$

7) Because 1.79 > 1.65 reject H_0 and conclude the battery life is significantly greater than 40 at α = 0.05.

b) P-value = $1 - \Phi(1.79) = 1 - 0.963273 = 0.036727$

c) $\beta = \Phi\left(z_{0.05} + \frac{40-42}{1.25/\sqrt{20}}\right)$
= Φ(1.65 + –7.155)
= Φ(–5.505)
≅ 0

d) $n = \frac{\left(z_\alpha + z_\beta\right)^2 \sigma^2}{\delta^2} = \frac{\left(z_{0.05} + z_{0.10}\right)^2 \sigma^2}{(40-44)^2} = \frac{(1.65+1.29)^2 (1.25)^2}{(4)^2} = 0.844, \quad n \cong 1$

e) 95% Confidence Interval

$$\overline{x} - z_{0.05}\sigma/\sqrt{n} \le \mu$$
$$40.5 - 1.65(1.25)/\sqrt{20} \le \mu$$
$$40.04 \le \mu$$

The lower bound of the 90 % confidence interval must be greater than 40 to verify that the true mean exceeds 40 hours. As the lower bound is greater than 40, the true mean exceeds 40 hours.

9-44 a) 1) The parameter of interest is the true mean tensile strength, μ.
2) $H_0 : \mu = 24{,}150$
3) $H_1 : \mu \neq 24{,}150$
4) $z_0 = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$
5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where α = 0.01 and $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where α = 0.01 and $z_{0.005} = 2.58$
6) $\overline{x} = 23{,}800$, σ = 415

$$z_0 = \frac{23800-24150}{415/\sqrt{12}} = -2.92$$

7) Because –2.92 < –2.58, reject the null hypothesis and conclude the true mean tensile strength is significantly different from 24,150 at α = 0.01.

b) Smallest level of significance =
P-value = 2[1 – Φ (2.92)] = 2[1 – 0.998250] = 0.0035
The smallest level of significance at which we are willing to reject the null hypothesis is 0.0035.

c) δ = 23,925 – 24,150 = –225

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(23,925 - 24,150)\sqrt{12}}{415}\right) - \Phi\left(-2.58 - \frac{(23,925 - 24150)\sqrt{12}}{415}\right)$$

= Φ(4.458) – Φ(–0.702) = 1 – 0.2413 = 0.7586

d) $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$23,800 - 2.58\left(\frac{415}{\sqrt{12}}\right) \le \mu \le 23,800 + 2.58\left(\frac{415}{\sqrt{12}}\right)$$

$23,490.916 \le \mu \le 24,109.08$

With 99% confidence, the true mean tensile strength is between 23,490.916 kN/m^2 and 24,109.08 kN/m^2. We can test the hypotheses that the true mean tensile strength is not equal to 24,150 by noting that the value is not within the confidence interval. Hence we reject the null hypothesis.

9-45 a)

1) The parameter of interest is the true mean speed, μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu < 100$

4) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

5) Reject H_0 if $z_0 < -z_\alpha$ where $\alpha = 0.05$ and $-z_{0.05} = -1.65$

6) $\bar{x} = 102.2$, $\sigma = 4$

$$z_0 = \frac{102.2 - 100}{4/\sqrt{8}} = 1.56$$

7) Because 1.56 > –1.65 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean speed is less than 100 at $\alpha = 0.05$.

b) $z_0 = 1.56$, then P-value= $\Phi(z_0) \cong 0.94$

c) $\beta = 1 - \Phi\left(-z_{0.05} - \dfrac{(95 - 100)\sqrt{8}}{4}\right)$ = 1-Φ(-1.65 - –3.54) = 1-Φ(1.89) = 0.02938

Power = 1-β = 1-0.0294 = 0.9706

d) $n = \dfrac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.15})^2\sigma^2}{(95 - 100)^2} = \dfrac{(1.65 + 1.03)^2(4)^2}{(5)^2} = 4.60,$ $n \cong 5$

e) 95% Confidence Interval

$$\mu \le \bar{x} + z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\mu \le 102.2 + 1.65\left(\frac{4}{\sqrt{8}}\right)$$

$$\mu \le 104.53$$

Because 100 is included in the CI, there is not sufficient evidence to reject the null hypothesis.

9-46 a) 1) The parameter of interest is the true mean hole diameter, μ.

2) $H_0 : \mu = 3.81$

3) $H_1 : \mu \ne 3.81$

4) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.01$ and $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

6) $\bar{x} = 3.8037$, $\sigma = 0.03$

$$z_0 = \frac{3.8037 - 3.81}{0.03 / \sqrt{25}} = -1.05$$

7) Because $-2.58 < -1.25 < 2.58$ fail to reject the null hypothesis. The true mean hole diameter is not significantly different from 3.81 cm at $\alpha = 0.01$.

b) P-value = $2(1 - \Phi(|Z_0|)) = 2(1 - \Phi(1.05)) \cong 0.2937$

c) $\beta = \Phi\left(z_{0.005} - \dfrac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \dfrac{\delta\sqrt{n}}{\sigma}\right)$

$= \Phi\left(2.58 - \dfrac{(3.797 - 3.81)\sqrt{25}}{0.03}\right) - \Phi\left(-2.58 - \dfrac{(3.797 - 3.81)\sqrt{25}}{0.03}\right)$

$= \Phi(4.747) - \Phi(-0.413) = 1 - 0.34348 = 0.65652$

power = $1 - \beta = 0.34348$

d) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2 \sigma^2}{(3.797 - 3.81)^2} \cong \frac{(2.58 + 1.29)^2 (0.03)^2}{(0.013)^2} = 79.759$$

$n \cong 80.$

e) For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3.8037 - 2.58\left(\frac{0.03}{\sqrt{25}}\right) \le \mu \le 3.8037 + 2.58\left(\frac{0.03}{\sqrt{25}}\right)$$

$3.7882 \le \mu \le 3.8192$

The confidence interval constructed contains the value 3.81. Therefore, there is not strong evidence that true mean hole diameter differs from 3.81 cm. using a 99% level of confidence. Because a two-sided 99% confidence interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$, the conclusions necessarily must be consistent.

9-47 a)

1) The parameter of interest is the true average battery life, μ.

2) $H_0 : \mu = 4$

3) $H_1 : \mu > 4$

4) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

5) Reject H_0 if $z_0 > z_\alpha$ where $\alpha = 0.05$ and $z_{0.05} = 1.65$

6) $\bar{x} = 4.05$, $\sigma = 0.2$

$$z_0 = \frac{4.05 - 4}{0.2 / \sqrt{50}} = 1.77$$

7) Because $1.77 > 1.65$, reject the null hypothesis. Conclude that the true average battery life exceeds 4 hours at $\alpha = 0.05$.

b) P-value=$1 - \Phi(Z_0) = 1 - \Phi(1.77) \cong 0.04$

c) $\beta = \Phi\left(z_{0.05} - \frac{(4.5-4)\sqrt{50}}{0.2} \right)$ = Φ(1.65 – 17.68) = Φ(-16.03) = 0

Power = 1-β = 1-0 = 1

d) n = $\frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.1})^2 \sigma^2}{(4.5-4)^2} = \frac{(1.65+1.29)^2 (0.2)^2}{(0.5)^2} = 1.38,$

n ≅ 2

e) 95% Confidence Interval

$$\bar{x} - z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu$$

$$4.05 - 1.65\left(\frac{0.2}{\sqrt{50}}\right) \le \mu$$

$$4.003 \le \mu$$

Because the lower limit of the CI is greater than 4, we conclude that average life is greater than 4 hours at α = 0.05.

Section 9-3

9-48 From Table V of Appendix A, we obtain
a) α = 0.01, n = 18, the critical values are ± 2.898
b) α = 0.05, n = 15, the critical values are ± 2.145
c) α = 0.1, n = 14, the critical values are ± 1.771
Note, ν = n – 1 and the 'α' in Table V is the half of the 'α' value given here.

9-49 a) α = 0.01, n = 18, the critical value = 2.567
b) α = 0.05, n = 15, the critical value = 1.761
c) α = 0.1, n = 14, the critical value = 1.350

9-50 a) α = 0.01, n = 18, the critical value = -2.567
b) α = 0.05, n = 15, the critical value = -1.761
c) α = 0.1, n = 19, the critical value = -1.330

9-51 From Table V of Appendix A,
a) $2*0.025 \le p \le 2*0.05$ then $0.05 \le p \le 0.1$
b) $2*0.025 \le p \le 2*0.05$ then $0.05 \le p \le 0.1$
c) $2*0.25 \le p \le 2*0.4$ then $0.5 \le p \le 0.8$

9-52 a) $0.025 \le p \le 0.05$
b) $1-0.05 \le p \le 1-0.025$ then $0.95 \le p \le 0.975$
c) $0.25 \le p \le 0.4$

9-53 a) $1-0.05 \le p \le 1-0.025$ then $0.95 \le p \le 0.975$
b) $0.025 \le p \le 0.05$
c) $1-0.4 \le p \le 1-0.25$ then $0.6 \le p \le 0.75$

9-54 a) SE Mean $= \frac{S}{\sqrt{N}} = \frac{0.717}{\sqrt{10}} = 0.2267$

$$t_0 = \frac{92.379 - 91}{0.717/\sqrt{10}} = 6.0820$$

$t_0 = 6.820$ with df = 10 – 1 = 9, so the P-value < 0.0005. Because the P-value < α = 0.05 we reject the null hypothesis that μ = 91at the 0.05 level of significance.

95% lower CI of the mean is $\bar{x} - t_{0.05,9}\frac{S}{\sqrt{n}} \le \mu$

$$92.379 - (1.833)\frac{0.717}{\sqrt{10}} \le \mu$$

$$91.9634 \le \mu$$

b) A one-sided test because the alternative hypothesis is μ > 91.

c) If the alternative hypothesis is changed to μ > 90, then $t_0 = \frac{92.379 - 90}{0.717/\sqrt{10}} = 10.4924$

$t_0 = 10.4924$ with df = 10 – 1 = 9, so the P-value < 0.0005. The P-value < α = 0.05 and we reject the null hypothesis at the 0.05 level of significance.

9-55 a) degrees of freedom = n - 1 = 10 - 1 = 9

b) SE Mean $= \frac{s}{\sqrt{N}} = \frac{s}{\sqrt{10}} = 0.375$, then s = 1.1859.

$$t_0 = \frac{12.564 - 12}{0.375} = 1.504$$

$t_0 = 1.504$ with df = 10 – 1 = 9.

The P-value falls between two values: 1.383 (for α = 0.10) and 1.833 (for α = 0.05), so 0.10 = 2(0.05) < P-value < 2(0.10) = 0.2. The P-value > α = 0.05, so we fail to reject the null hypothesis at the 0.05 level of significance.

c) A two-sided test because the alternative hypothesis is $\mu \ne 12$.

d) 95% two-sided CI

$$\bar{x} - t_{0.025,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,9}\left(\frac{s}{\sqrt{n}}\right)$$

$$12.564 - 2.262\left(\frac{1.1859}{\sqrt{10}}\right) \le \mu \le 12.564 + 2.262\left(\frac{1.1859}{\sqrt{10}}\right)$$

$$11.7157 \le \mu \le 13.4123$$

e) Suppose that the alternative hypothesis is changed to μ > 12. Because $t_0 = 1.504 < t_{0.05,9} = 1.833$ we fail to reject the null hypothesis at the 0.05 level of significance.

f) Reject the null hypothesis that μ = 11.5 versus the alternative hypothesis ($\mu \ne 11.5$) at the 0.05 level of significance because the μ = 11.5 is not included in the 95% two-sided CI on the mean.

9-56 a) degrees of freedom = N – 1 = 16 – 1 = 15

b) SE Mean $= \frac{S}{\sqrt{N}} = \frac{1.783}{\sqrt{16}} = 0.4458$

$$t_0 = \frac{35.274 - 34}{1.783/\sqrt{16}} = 2.8581$$

c) P-value = 2P(t > 2.8581) = 0.012. We reject the null hypothesis if the P-value < α. Thus, we can reject the null hypothesis at significance levels greater than 0.012.

d) If the alternative hypothesis is changed to the one-sided alternative μ > 34, the P-value = 0.5(0.12) = 0.006.

e) If the null hypothesis is changed to μ = 35 versus the alternative hypothesis (μ ≠ 35) the t statistic is reduced. In particular, $t_0 = \frac{35.274 - 35}{1.783/\sqrt{16}} = 0.6147$ and $t_{0.025,15} = 2.131$.

Because $t_0 = 0.6147 < t_{0.025,15}$, we fail to reject the null hypothesis at the 0.05 level of significance.

9-57 a)

1) The parameter of interest is the true mean of body weight, μ.
2) H_0: μ = 310
3) H_1: μ ≠ 310
4) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where α = 0.05 and $t_{\alpha/2,n-1} = 2.056$ for n = 27
6) $\bar{x} = 325.496$, s = 198.786, n = 27

$$t_0 = \frac{325.496 - 310}{198.786/\sqrt{27}} = 0.4051$$

7) Because 0.4051 < 2.056 we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true mean body weight differs from 310 at α = 0.05. We have 2(0.25) < P-value < 2(0.4). That is, 0.5 < P-value < 0.8

b) We reject the null hypothesis if P-value < α. The P-value = 2(0.2554) = 0.5108. Therefore, the smallest level of significance at which we can reject the null hypothesis is approximately 0.51.

c) 95% two sided confidence interval

$$\bar{x} - t_{0.025,26}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,26}\left(\frac{s}{\sqrt{n}}\right)$$

$$325.496 - 2.056\left(\frac{198.786}{\sqrt{27}}\right) \le \mu \le 325.496 + 2.056\left(\frac{198.786}{\sqrt{27}}\right)$$

$$246.8409 \le \mu \le 404.1511$$

We fail to reject the null hypothesis because the hypothesized value of 310 is included within the confidence interval.

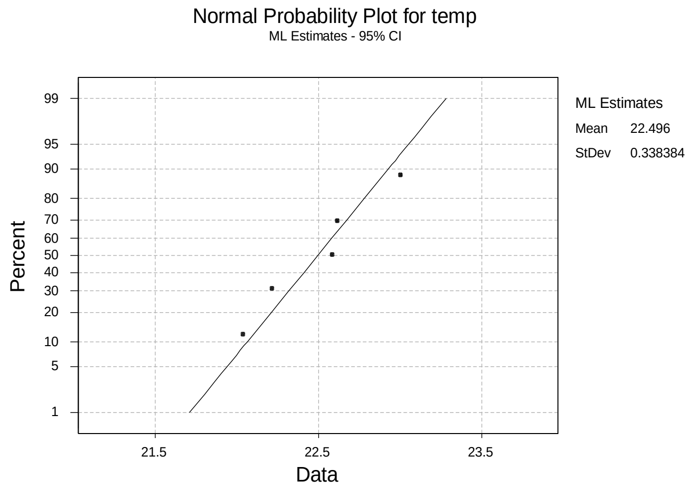
9-58 a)

1) The parameter of interest is the true mean interior temperature life, μ.
2) H_0 : μ = 22.5
3) H_1 : μ ≠ 22.5
4) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where α = 0.05 and $t_{\alpha/2,n-1} = 2.776$ for n = 5
6) $\bar{x} = 22.496$, s = 0.378, n = 5

$$t_0 = \frac{22.496 - 22.5}{0.378/\sqrt{5}} = -0.00237$$

7) Because –0.00237 > –2.776, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true mean interior temperature differs from 22.5 °C at α = 0.05.
Also, 2(0.4) < *P*-value < 2(0.5). That is, 0.8 < *P*-value < 1.0.

b) The points on the normal probability plot fall along a line. Therefore, the normality assumption is reasonable.



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|22.80 - 22.5|}{0.378} = 0.79$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, $d = 0.79$, and n = 5, we obtain $\beta \cong 0.5$ and power of 1–0.5 = 0.5

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|22.80 - 22.5|}{0.378} = 0.79$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, $d = 0.79$, and $\beta \cong 0.1$ (Power=0.9), n = 40

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right)$$

$$22.496 - 2.776\left(\frac{0.378}{\sqrt{5}}\right) \le \mu \le 22.496 + 2.776\left(\frac{0.378}{\sqrt{5}}\right)$$

$$22.027 \le \mu \le 22.965$$

We cannot conclude that the mean interior temperature differs from 22.5 at $\alpha = 0.05$ because the value is included in the confidence interval.

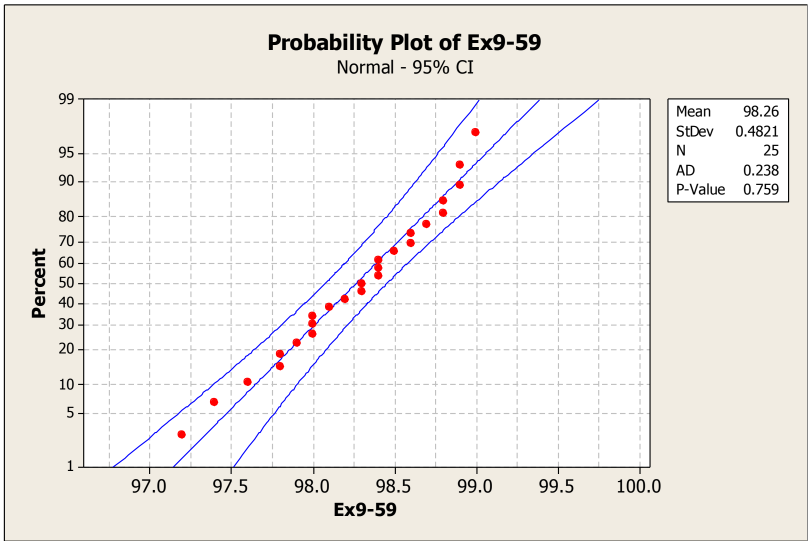
9-59 a)

1) The parameter of interest is the true mean female body temperature, μ.
2) $H_0 : \mu = 98.6$
3) $H_1 : \mu \neq 98.6$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{\alpha/2,n-1} = 2.064$ for n = 25
6) $\bar{x} = 98.264$, s = 0.4821, n = 25

$$t_0 = \frac{98.264 - 98.6}{0.4821/\sqrt{25}} = -3.48$$

7) Because 3.48 > 2.064, reject the null hypothesis. Conclude that the true mean female body temperature differs from 98.6 °F at $\alpha = 0.05$.
P-value = 2(0.001) = 0.002

b) The data on the normal probability plot falls along a line. The normality assumption is reasonable.



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|98 - 98.6|}{0.4821} = 1.24$

Using the OC curve, Chart VII*e* for $\alpha = 0.05$, d = 1.24, and n = 25, obtain $\beta \cong 0$ and power of $1 - 0 \cong 1$

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|98.2 - 98.6|}{0.4821} = 0.83$

Using the OC curve, Chart VII*e* for $\alpha = 0.05$, d = 0.83, and $\beta \cong 0.1$ (Power=0.9), n =20

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$98.264 - 2.064\left(\frac{0.4821}{\sqrt{25}}\right) \le \mu \le 98.264 + 2.064\left(\frac{0.4821}{\sqrt{25}}\right)$$

$$98.065 \le \mu \le 98.463$$

We conclude that the mean female body temperature differs from 98.6 at $\alpha = 0.05$ because the value is not included inside the confidence interval.

9-60 a)

1) The parameter of interest is the true mean rainfall, μ.

2) $H_0 : \mu = 30 \times 10^3$

3) $H_1 : \mu > 30 \times 10^3$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.01$ and $t_{0.01,19} = 2.539$ for n = 20

6) $\bar{x} = 32.096 \times 10^3$ s = 5894.63 n = 20

$$t_0 = \frac{32096 - 30000}{5894.63/\sqrt{20}} = 1.5902$$

7) Because 1.5902 < 2.539 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean rainfall is greater than 30×10^3 m^3 at $\alpha = 0.01$. The 0.05 < P-value < 0.1.

b) The data on the normal probability plot fall along the straight line. Therefore, the normality assumption is reasonable.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|33,000 - 30,000|}{5894.63} = 0.5089$

⊔ 0.51

Using the OC curve, Chart VII h) for α = 0.01, d = 0.51, and n = 20, obtain β ≅ 0.6 and power of 1 − 0.6 = 0.4.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|33,500 - 30,000|}{5894.63} = 0.59$

Using the OC curve, Chart VII h) for α = 0.01, d = 0.59, and β ⊔ 0.1 (Power = 0.9), n = 40.

e) 99% lower confidence bound on the mean diameter

$$\bar{x} - t_{0.01,19}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$32096 - 2.539\left(\frac{5894.63}{\sqrt{20}}\right) \le \mu$$

$$28749.4 \le \mu$$

Because the lower limit of the CI is less than 30,000 there is insufficient evidence to conclude that the true mean rainfall is greater than 30,000 m^3 at α = 0.01.

9-61 a) 1) The parameter of interest is the true mean sodium content, μ.

2) H_0 : μ = 130

3) H_1 : μ ≠ 130

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where α = 0.05 and $t_{\alpha/2,n-1}$ = 2.064 for n = 25

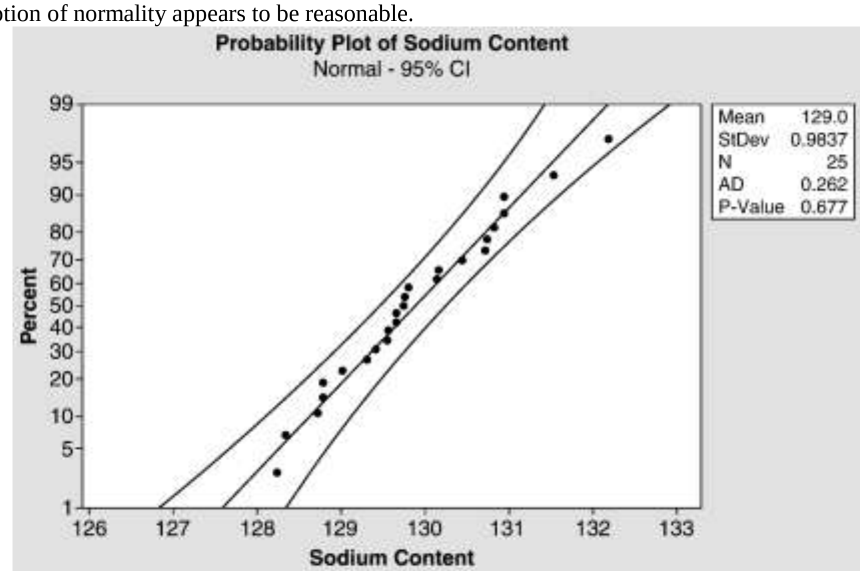
6) $\bar{x} = 129.859$, s = 0.984 n = 25

$$t_0 = \frac{129.859 - 130}{0.984/\sqrt{25}} = -0.716$$

7) Because 0.716 < 2.064 we fail to reject the null hypothesis. There is not sufficient evidence that the true mean sodium content is different from 130mg at α = 0.05.

From the *t* table (Table V) the t_0 value is between the values of 0.1 and 0.25 with 24 degrees of freedom. Therefore, 2(0.1) < P-value < 2(0.25) and 0.2 < P-value < 0.5.

b) The assumption of normality appears to be reasonable.



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|130.5 - 130|}{0.984} = 0.508$

We have to use the OC curve, Chart VII e) for α = 0.05 and d = 0.51. As there is no curve corresponding to n = 25, we assume that the curve for n = 25 will be equidistant from the curves for n = 20 and n = 30 and obtain $\beta \cong 0.3$ and the power of 1 – 0.30 = 0.70

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|130.1 - 130|}{0.984} = 0.1016$

Using the OC curve, Chart VII e) for α = 0.05, d = 0.10, and $\beta \cong 0.25$ (Power = 0.75), the sample sizes do not extend to the point *d* = 0.10 and β = 0.25. We can conclude that n > 100

e) 95% two-sided confidence interval

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$129.859 - 2.064\left(\frac{0.984}{\sqrt{25}}\right) \le \mu \le 129.859 + 2.064\left(\frac{0.984}{\sqrt{25}}\right)$$

$$129.453 \le \mu \le 130.265$$

There is no evidence that the mean differs from 130 because that value is inside the confidence interval. The result is the same as part (a).

9-62 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean coefficient of restitution, μ.

2) $H_0 : \mu = 0.635$

3) $H_1 : \mu > 0.635$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where α = 0.05 and $t_{0.05,39} = 1.685$ for n = 40

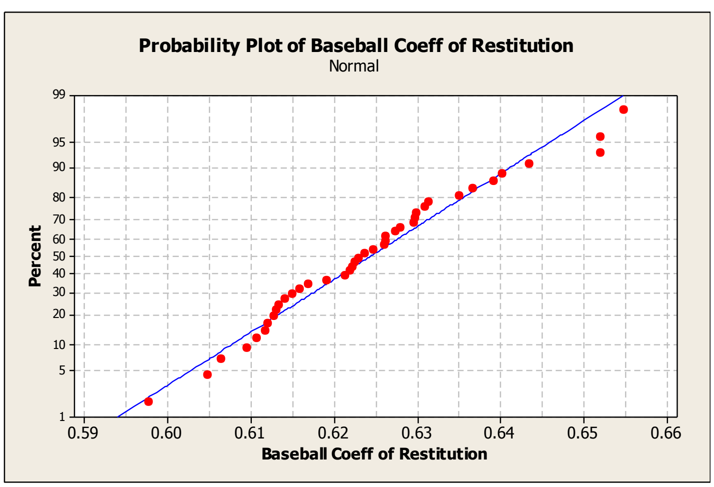
6) $\bar{x} = 0.624$ s = 0.013 n = 40

$$t_0 = \frac{0.624 - 0.635}{0.013/\sqrt{40}} = -5.35$$

7) Because –5.25 < 1.685 fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true mean coefficient of restitution is greater than 0.635 at α = 0.05.

The area to right of -5.35 under the *t* distribution is greater than 0.9995 from Table V.

b) From the normal probability plot, the normality assumption seems reasonable:



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.64 - 0.635|}{0.013} = 0.38$

Using the OC curve, Chart VII g) for α = 0.05, d = 0.38, and n = 40, obtain $\beta \cong 0.25$ and power of 1 – 0.25 = 0.75.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.638 - 0.635|}{0.013} = 0.23$

Using the OC curve, Chart VII g) for $\alpha = 0.05$, $d = 0.23$, and $\beta \cong 0.25$ (Power = 0.75), $n = 40$

e) 95% lower confidence bound is $\bar{x} - t_{\alpha,n-1}\left(\dfrac{s}{\sqrt{n}}\right) = 0.6205$

Because 0.635 > 0.6205, we fail to reject the null hypothesis.

9-63 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean oxygen concentration, μ.

2) $H_0 : \mu = 4$

3) $H_1 : \mu \neq 4$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

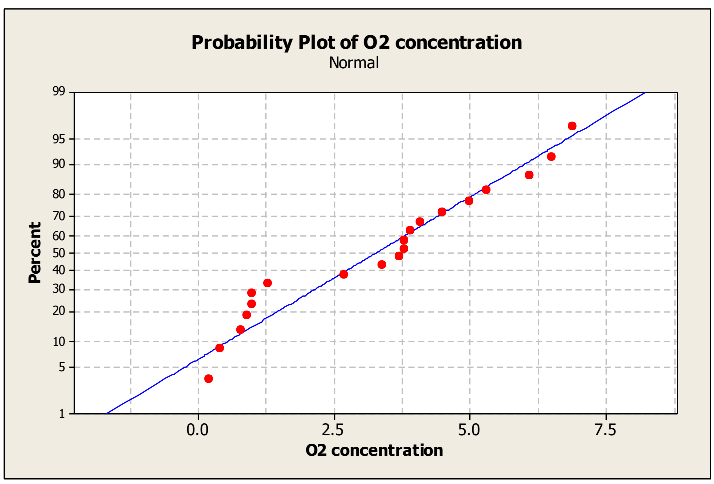
5) Reject H_0 if $|t_0| > t_{\alpha/2,\, n-1}$ where $\alpha = 0.01$ and $t_{0.005,\, 19} = 2.861$ for $n = 20$

6) $\bar{x} = 3.265$, $s = 2.127$, $n = 20$

$$t_0 = \frac{3.265 - 4}{2.127/\sqrt{20}} = -1.55$$

7) Because -2.861< -1.55, fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean oxygen differs from 4 at $\alpha = 0.01$. Also $2(0.05) < P\text{-value} < 2(0.10)$. Therefore $0.10 < P\text{-value} < 0.20$

b) From the normal probability plot, the normality assumption seems reasonable.



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|3 - 4|}{2.127} = 0.47$

Using the OC curve, Chart VII f) for $\alpha = 0.01$, $d = 0.47$, and $n = 20$, we obtain $\beta \cong 0.70$ and power of $1-0.70 = 0.30$.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|2.5 - 4|}{2.127} = 0.71$

Using the OC curve, Chart VII f) for $\alpha = 0.01$, $d = 0.71$, and $\beta \cong 0.10$ (Power=0.90), $n = 40$.

e) The 95% confidence interval is

$$\bar{x} - t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) = 1.9 \le \mu \le 4.62$$

Because 4 is within the confidence interval, we fail to reject the null hypothesis.

9-64 a)

1) The parameter of interest is the true mean sodium content, μ.
2) H_0 : μ = 300
3) H_1 : μ > 300
4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where α = 0.05 and $t_{\alpha,n-1}$ = 1.943 for n = 7
6) $\bar{x} = 315$, s = 16 n=7

$$t_0 = \frac{315 - 300}{16 / \sqrt{7}} = 2.48$$

7) Because 2.48 > 1.943 reject the null hypothesis and conclude that the leg strength exceeds 300 watts at α = 0.05.

The P-value is between 0.01 and 0.025

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|304 - 300|}{16} = 0.25$

Using the OC curve, Chart VII g) for α = 0.05, d = 0.25, and n = 7, β ≅ 0.9 and power = 1–0.9 = 0.1.

c) If 1 - β > 0.9 then β < 0.1 and n is approximately 100

d) Lower confidence bound is $\bar{x} - t_{\alpha,n-1}\left(\dfrac{s}{\sqrt{n}}\right) = 303.2 < \mu$

Because 300 is not include in the interval, reject the null hypothesis

9-65 a)

1) The parameter of interest is the true mean tire life, μ.
2) H_0 : μ = 60000
3) H_1 : μ > 60000
4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where α = 0.05 and $t_{0.05,15} = 1.753$ for n = 16
6) $n = 16$ $\bar{x} = 60{,}139.7$ $s = 3645.94$

$$t_0 = \frac{60139.7 - 60000}{3645.94 / \sqrt{16}} = 0.15$$

7) Because 0.15 < 1.753 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean tire life is greater than 60,000 kilometers at α = 0.05. The P-value > 0.40.

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|61000 - 60000|}{3645.94} = 0.27$

Using the OC curve, Chart VII g) for α = 0.05, d = 0.27, and β ≅ 0.1 (Power = 0.9), n = 4.
Yes, the sample size of 16 was sufficient.

9-66 In order to use *t* statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean impact strength, μ.
2) H_0 : μ = 1.0
3) H_1 : μ > 1.0
4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where α = 0.05 and $t_{0.05,19}$ = 1.729 for n = 20
6) $\bar{x}$ = 1.25 s = 0.25 n = 20

$$t_0 = \frac{1.25 - 1.0}{0.25 / \sqrt{20}} = 4.47$$

7) Because 4.47 > 1.729 reject the null hypothesis. There is sufficient evidence to conclude that the true mean impact strength is greater than 1.0 ft-lb/in at α = 0.05. The P-value < 0.0005

9-67 In order to use a t statistic in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean current, μ.
2) $H_0 : \mu = 300$
3) $H_1 : \mu > 300$
4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,19} = 1.729$ for n = 20
6) $n = 20 \quad \bar{x} = 317.2 \quad s = 15.7$

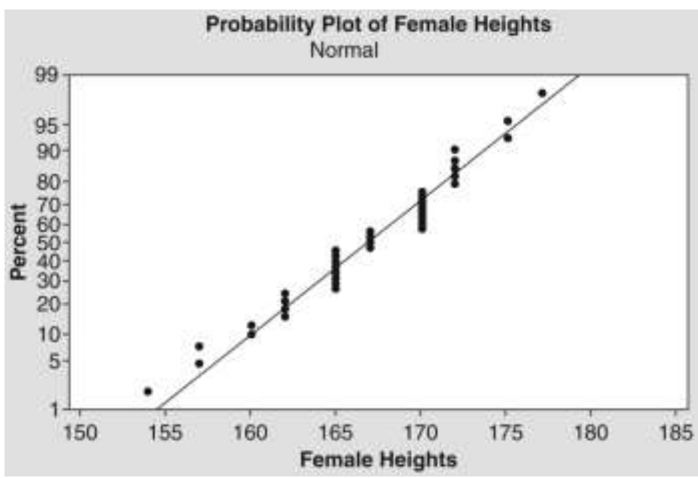
$$t_0 = \frac{317.2 - 300}{15.7/\sqrt{20}} = 4.90$$

7) Because 4.90 > 1.729, reject the null hypothesis. There is sufficient evidence to indicate that the true mean current is greater than 300 microamps at $\alpha = 0.05$. The *P*-value < 0.0005

9-68 a)
1) The parameter of interest is the true mean height of female engineering students, μ.
2) $H_0 : \mu = 165$ cm
3) $H_1 : \mu > 165$
4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,36} = 1.68$ for n = 37
6) $\bar{x} = 166.78$ cm $s = 5.329$ cm inches n = 37

$$t_0 = \frac{166.78 - 165}{5.329/\sqrt{37}} = 2.032$$

7) Because 2.032 > 1.68 reject the null hypothesis. There is sufficient evidence to conclude that the true mean height of female engineering students is greater than 165 at $\alpha = 0.05$.
Since n = 37 is closer to 40, rather than n = 30, we conclude
P-value: 0.01<*P*-value < 0.025.
b) From the normal probability plot, the normality assumption seems reasonable:



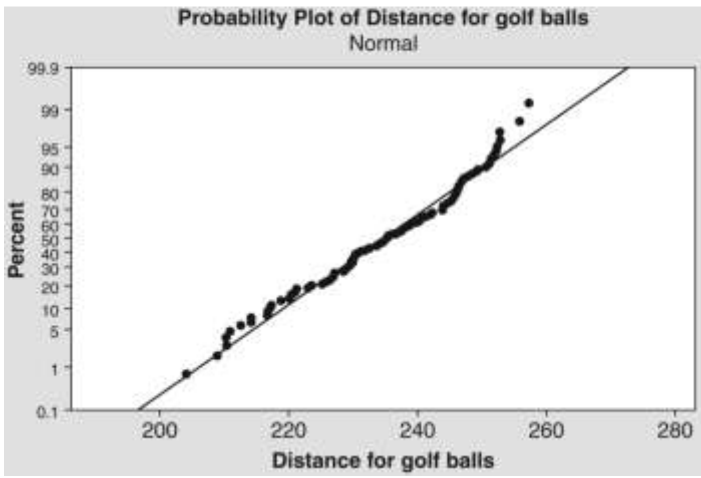
c) $d = \dfrac{|157 - 165|}{5.329} = 1.50$, n = 37 so, from the OC Chart VII g) for $\alpha = 0.05$, we find that $\beta \cong 0$.
Therefore, the power ≅ 1.

d) $d = \dfrac{|162 - 165|}{5.329} = 0.563$ so, from the OC Chart VII g) for $\alpha = 0.05$, and $\beta \cong 0.2$ (Power = 0.8).
$n^* = 20.$

9-69 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean distance, μ.
2) $H_0 : \mu = 270$
3) $H_1 : \mu > 270$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,99} = 1.6604$ for n = 100
6) $\bar{x} = 234.2$ s = 12.07 n = 100

$$t_0 = \frac{234.2 - 270}{12.07/\sqrt{100}} = -29.66$$

7) Because –29.66 < 1.6604 fail to reject the null hypothesis. There is insufficient evidence to indicate that the true mean distance is greater than 270 at α = 0.05.
From Table V the t_0 value in absolute value is greater than the value corresponding to 0.0005.
Therefore, the P-value > 0.9995.
b) From the normal probability plot, the normality assumption seems reasonable:



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|261-270|}{12.07} = 0.75$

Using the OC curve, Chart VII g) for α = 0.05, d = 0.75, and n = 100, obtain β ≅ 0 and power ≈ 1

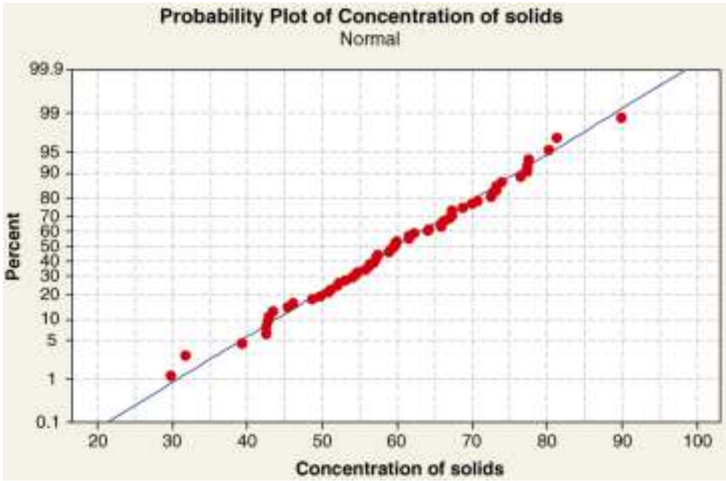
d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|261-270|}{12.07} = 0.75$

Using the OC curve, Chart VII g) for α = 0.05, d = 0.75, and β ≅ 0.20 (Power = 0.80), n = 15

9-70 a) In order to use *t* statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean concentration of suspended solids, μ.
2) $H_0 : \mu = 55$
3) $H_1 : \mu \neq 55$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{0.025,59} = 2.000$ for n = 60
6) $\bar{x} = 59.87$ s = 12.50 n = 60

$$t_0 = \frac{59.87 - 55}{12.50/\sqrt{60}} = 3.018$$

7) Because 3.018 > 2.000, reject the null hypothesis. There is sufficient evidence to conclude that the true mean concentration of suspended solids is not equal to 55 at α = 0.05.
From Table V the t_0 value is between the values of 0.001 and 0.0025 with 59 degrees of freedom. Therefore, 2*0.001 < P-value < 2* 0.0025 and 0.002 < P-value < 0.005. Minitab gives a P-value of 0.0038.
b) From the normal probability plot, the normality assumption seems reasonable:



c) $d = \dfrac{|52-55|}{12.50} = 0.24$, n = 60 so, from the OC Chart VII e) for α = 0.05, d = 0.4 and n = 60 obtain β ≅ 0.2.
Therefore, the power = 1 – 0.2 = 0.8.
d) From the same OC chart, and for the specified power, we would need approximately 75 observations.

$$d = \frac{|50-55|}{12.50} = 0.4$$

Using the OC Chart VII e) for α = 0.05, d = 0.4, and β ≅ 0.10 so that the power = 0.90, n = 75

Section 9-4

9-71 a) α = 0.01, n = 18, from table IV we find the following critical values 5.70 and 35.72
b) α = 0.05, n = 15, from table IV we find the following critical values 5.63 and 26.12
c) α = 0.10, n = 14, from table IV we find the following critical values 5.89 and 22.36

9-72 a) α = 0.01, n = 18, from Table V we find $\chi^2_{\alpha,n-1} = 33.41$

b) α = 0.05, n = 10, from Table V we find $\chi^2_{\alpha,n-1} = 16.92$

c) α = 0.10, n = 14, from Table V we find $\chi^2_{\alpha,n-1} = 19.81$

9-73 a) α = 0.01, n = 20, from Table V we find $\chi^2_{1-\alpha,n-1} = 7.63$

b) α = 0.05, n = 15, from Table V we find $\chi^2_{1-\alpha,n-1} = 6.57$

c) α = 0.10, n = 18, from Table V we find $\chi^2_{1-\alpha,n-1} = 10.09$

9-74 a) 2(0.05) < *P*-value < 2(0.1), then 0.1 < *P*-value < 0.2
b) 2(0.05) < *P*-value < 2(0.1), then 0.1 < *P*-value < 0.2
c) 2(0.025) < *P*-value < 2(0.05), then 0.05 < *P*-value < 0.1

9-75 a) 0.05 < 1-*P* < 0.1 then 0.9 < *P*-value < 0.95
b) 0.05 < 1-*P*< 0.1 then 0.9 < *P*-value < 0.95
c) 0.975 <1-*P* <0.99 then 0.01 < *P*-value < 0.025

9-76 a) 0.05 < *P*-value < 0.1
b) 0.05 < *P*-value < 0.1
c) 0.975 < *P*-value < 0.99

9-77 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of performance time σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) $H_0: \sigma^2 = 0.75^2$

3) $H_1: \sigma^2 > 0.75^2$

4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 > \chi^2_{\alpha,n-1}$ where α = 0.05 and $\chi^2_{0.05,19} = 30.14$

6) n = 20, s = 0.09

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.09)^2}{.75^2} = 0.27$$

7) Because 0.27 < 30.14 fail to reject H_0. There is insufficient evidence to conclude that the true variance of performance time content exceeds 0.75^2 at α = 0.05.

Because $\chi_0^2 = 0.27$ the *P*-value > 0.995

b) The 95% one-sided confidence interval given below includes the value 0.75. Therefore, we are not be able to conclude that the standard deviation is greater than 0.75.

$$\frac{19(.09)^2}{30.14} \le \sigma^2$$
$$0.07 \le \sigma$$

9-78 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true measurement standard deviation σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) $H_0: \sigma^2 = .01^2$

3) $H_1: \sigma^2 \neq .01^2$

4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where α = 0.05 and $\chi^2_{0.975,14} = 5.63$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where α = 0.05 and $\chi^2_{0.025,14} = 26.12$ for n = 15

6) n = 15, s = 0.0083

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(.0083)^2}{.01^2} = 9.6446$$

7) Because 5.63 < 9.64 < 26.12 fail to reject H_0. 0.1 < *P*-value/2 < 0.5. Therefore, 0.2 < *P*-value < 1

b) The 95% confidence interval includes the value 0.01. Therefore, there is not enough evidence to reject the null hypothesis.

$$\frac{14(.0083)^2}{26.12} \le \sigma^2 \le \frac{14(.0083)^2}{5.63}$$
$$0.00607 \le \sigma^2 \le 0.013$$

9-79 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of titanium percentage, σ. However, the solution can be found by performing a hypothesis test on σ^2.

2) $H_0: \sigma^2 = (0.25)^2$

3) $H_1: \sigma^2 \neq (0.25)^2$

4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.995,50} = 32.36$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.005,50} = 71.42$ for n = 51

6) n = 51, s = 0.28

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{50(0.28)^2}{(0.25)^2} = 62.72$$

7) Because 32.36<62.72 <71.42, we fail to reject H_0. There is not enough evidence that the standard deviation of titanium percentage is significantly different from 0.25 at $\alpha = 0.05$.
0.1< P-value/2 < 0.5, then 0.2<P-value < 1.

b) 95% confidence interval for σ:
First find the confidence interval for σ^2 :
For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.28)^2}{71.42} \le \sigma^2 \le \frac{50(0.28)^2}{32.36}$$

$$0.0549 \le \sigma^2 \le 0.1211$$

Taking the square root of the endpoints of this interval we obtain, $0.2343 \le \sigma^2 \le 0.3480$

Because 0.25 falls in the confidence bound we conclude that the population standard deviation is equal to 0.25.

9-80 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true standard deviation of Izod impact strength, σ. However, the solution can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = (0.10)^2$
3) $H_1: \sigma^2 \ne (0.10)^2$
4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.01$ and $\chi^2_{0.995,29} = 13.12$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\alpha = 0.01$ and $\chi^2_{0.005,29} = 52.34$ for n = 30

6) n = 30, s = 0.25

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{29(0.25)^2}{(0.10)^2} = 181.25$$

7) Because 181.25 > 52.34 reject H_0. The true standard deviation of Izod impact strength differs from 0.10 at $\alpha = 0.01$.

b) P-value < 0.005

c) 99% confidence interval for σ. First find the confidence interval for σ^2 :
For $\alpha = 0.01$ and n = 30, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.995,29} = 13.12$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.005,29} = 52.34$

$$\frac{29(0.25)^2}{52.34} \le \sigma^2 \le \frac{29(0.25)^2}{13.12}$$

$$0.0346 \le \sigma^2 \le 0.1381$$

$$0.1861 < \sigma < 0.3717$$

Because 0.10 falls below the lower confidence bound, we conclude that the population standard deviation is not equal to 0.10.

9-81 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the standard deviation of tire life, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = 4000^2$
3) $H_1: \sigma^2 < 4000^2$
4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$
5) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha,n-1}^2$ where α = 0.05 and $\chi_{0.95,19}^2 = 10.12$ for n = 20
6) n = 20, $s^2 = (3645.94)^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(3645.94)^2}{4000^2} = 15.785$$

7) Because 15.785 > 7.26 fail to reject H_0. There is not sufficient evidence to conclude the true standard deviation of tire life is less than 4000 km at α = 0.05.
P-value = P(χ^2 < 15.785) for 19 degrees of freedom. Thus, 0.5 < 1-P-value < 0.9 and 0.1 < P-value < 0.5

b) The 95% one sided confidence interval below includes the value 4000. Therefore, we are not able to conclude that the variance is less than 4000^2.

$$\sigma^2 \le \frac{19(3645.94)^2}{7.26} = 34788525$$

$$\sigma \le 5898$$

9-82 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true standard deviation of the diameter,σ. However, the solution can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = 0.0001$
3) $H_1: \sigma^2 > 0.0001$
4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$
5) Reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$ where α = 0.01 and $\chi_{0.01,14}^2 = 29.14$ for n = 15
6) n = 15, $s^2 = 0.008$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.008)^2}{0.0001} = 8.96$$

7) Because 8.96 < 29.14 fail to reject H_0. There is insufficient evidence to conclude that the true standard deviation of the diameter exceeds 0.0001 at α = 0.01.

P-value = P(χ^2 > 8.96) for 14 degrees of freedom: 0.5 < P-value < 0.9

b) Using the chart in the Appendix, with $\lambda = \frac{0.015}{0.01} = 1.5$ and n = 15 we find β = 0.50.

c) $\lambda = \frac{\sigma}{\sigma_0} = \frac{0.0125}{0.01} = 1.25$, power = 0.8, β = 0.2, using Chart VII k) the required sample size is 50

9-83 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true variance of sugar content, σ^2. The answer can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = 18$
3) $H_1: \sigma^2 \ne 18$
4) $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where α = 0.05 and $\chi^2_{0.975,9} = 2.70$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where α = 0.05 and $\chi^2_{0.025,9} = 19.02$ for n = 10

6) n = 10, s = 4.8

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(4.8)^2}{18} = 11.52$$

7) Because 11.52 < 19.02 fail to reject H_0. There is insufficient evidence to conclude that the true variance of sugar content is significantly different from 18 at α = 0.01.

P-value: The χ_0^2 is between 0.10 and 0.50. Therefore, 0.2 < *P*-value < 1

b) Using the chart in the Appendix, with $\lambda = 2$ and n = 10 we find β = 0.45.

c) Using the chart in the Appendix, with $\lambda = \sqrt{\frac{40}{18}} = 1.49$ and β = 0.10, n = 30.

Section 9-5

9-84 a) A two-sided test because the alternative hypothesis is p not = 0.4

b) sample p = $\frac{X}{N} = \frac{102}{275} = 0.3709$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{102 - 275(0.4)}{\sqrt{275(0.4)(0.6)}} = -0.9847$$

P-value = 2(1 – Φ(0.9847)) = 2(1 – 0.8376) = 0.3248

c) The normal approximation is appropriate because np > 5 and n(1 - p) > 5.

9-85 a) A one-sided test because the alternative hypothesis is p < 0.6

b) The test is based on the normal approximation. It is appropriate because np > 5and n(1 - p) > 5.

c) sample p = $\frac{X}{N} = \frac{287}{450} = 0.638$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{287 - 450(0.6)}{\sqrt{450(0.6)(0.4)}} = 1.6358$$

P-value = Φ(1.6358) = 0.9491

The 95% upper confidence interval is:

$$p \le \hat{p} + z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le 0.638 + 1.65\sqrt{\frac{0.638(0.362)}{450}}$$

$$p \le 0.6754$$

d) P-value = 2[1 - Φ(1.6358)] = 2(1 - 0.9491) = 0.1018

9-86 a)

1) The parameter of interest is the true fraction of satisfied customers.

2) $H_0 : p = 0.9$

3) $H_1 : p \neq 0.9$

4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.05$ and $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.05$ and $z_{\alpha/2} = z_{0.025} = 1.96$

6) $x = 950$ n = 1100 $\hat{p} = \dfrac{950}{1100} = 0.86$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{950 - 1100(0.9)}{\sqrt{1100(0.9)(0.1)}} = -4.02$$

7) Because –4.02 < –1.96 reject the null hypothesis and conclude the true fraction of satisfied customers is significantly different from 0.9 at $\alpha = 0.05$.
The P-value: $2(1 - \Phi(4.02)) \le 2(1 - 1) \approx 0$

b) The 95% confidence interval for the fraction of surveyed customers is:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.86 - 1.96\sqrt{\frac{0.86(0.14)}{1100}} \le p \le 0.86 + 1.96\sqrt{\frac{0.86(0.14)}{1100}}$$

$$0.84 \le p \le 0.88$$

Because 0.9 is not included in the confidence interval, we reject the null hypothesis at $\alpha = 0.05$.

9-87 a)

1) The parameter of interest is the true fraction of rejected parts
2) $H_0 : p = 0.03$
3) $H_1 : p < 0.03$

4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

5) Reject H_0 if $z_0 < -z_\alpha$ where $\alpha = 0.05$ and $-z_\alpha = -z_{0.05} = -1.65$

6) $x = 12$ $n = 600$ $\hat{p} = \dfrac{12}{600} = 0.02$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{12 - 600(0.03)}{\sqrt{600(0.03)(0.97)}} = -1.197$$

7) Because –1.197 > –1.65 fail to reject the null hypothesis. There is not enough evidence to conclude that the true fraction of rejected parts is less than 0.03 at $\alpha = 0.05$.
P-value = $\Phi(-1.197) = 0.115$

b) The upper one-sided 95% confidence interval for the fraction of rejected parts is:

$$p \le \hat{p} - z_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le .02 + 1.65\sqrt{\frac{0.02(0.98)}{600}}$$

$$p \le 0.0294$$

Because 0.03 > 0.0294 we reject the null hypothesis

9-88 a)

1) The parameter of interest is the true fraction defective integrated circuits
2) $H_0 : p = 0.05$
3) $H_1 : p \ne 0.05$

4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.05$ and $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.05$ and $z_{\alpha/2} = z_{0.025} = 1.96$

6) $x = 13$ $n = 200$ $\hat{p} = \dfrac{13}{200} = 0.065$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{13 - 200(0.05)}{\sqrt{200(0.05)(0.95)}} = -0.97$$

7) Because $-0.97 > -1.65$ fail to reject null hypothesis. The true fraction of defective integrated circuits is not significantly different from 0.05, at $\alpha = 0.05$.

P-value = $2(1 - \Phi(0.97)) = 2(1 - 0.833977) = 0.332$

b) The 95% confidence interval is:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.065 - 1.96\sqrt{\frac{0.065(0.935)}{200}} \le p \le 0.065 + 1.96\sqrt{\frac{0.065(0.935)}{200}}$$

$$0.031 \le p \le 0.09917$$

Because the hypothesized value ($p = 0.05$) is contained in the confidence interval we fail to reject the null hypothesis.

9-89 a)

1) The parameter of interest is the true success rate

2) $H_0 : p = 0.78$

3) $H_1 : p > 0.78$

4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

5) Reject H_0 if $z_0 > z_\alpha$. Since the value for α is not given. We assume $\alpha = 0.05$ and $z_\alpha = z_{0.05} = 1.65$

6) $x = 289$ $n = 350$ $\hat{p} = \dfrac{289}{350} \cong 0.83$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{289 - 350(0.78)}{\sqrt{350(0.78)(0.22)}} = 2.06$$

7) Because $2.06 > 1.65$ reject the null hypothesis and conclude the true success rate is greater than 0.78, at $\alpha = 0.05$.

P-value = 1 - 0.9803 = 0.0197

b) The 95% lower confidence interval:

$$\hat{p} - z_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$

$$.83 - 1.65\sqrt{\frac{0.83(0.17)}{350}} \le p$$

$$0.7969 \le p$$

Because the hypothesized value is not in the confidence interval ($0.78 < 0.7969$), reject the null hypothesis.

9-90 a)

1) The parameter of interest is the true percentage of polished lenses that contain surface defects, p.
2) $H_0 : p = 0.02$
3) $H_1 : p < 0.02$
4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
5) Reject H_0 if $z_0 < -z_\alpha$ where $\alpha = 0.05$ and $-z_\alpha = -z_{0.05} = -1.65$
6) x = 7 n = 250 $\hat{p} = \dfrac{7}{250} = 0.028$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.028 - 0.02}{\sqrt{\frac{0.02(1-0.02)}{250}}} = 0.9035$$

7) Because 0.9035 > –1.65 fail to reject the null hypothesis. There is not sufficient evidence to qualify the machine at the 0.05 level of significance. P-value = $\Phi(0.9035) = 0.8169$

b) The upper 95% confidence interval is:

$$p \le \hat{p} + z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le .028 + 1.65\sqrt{\frac{0.028(0.972)}{250}}$$

$$p \le 0.0452$$

Because the confidence interval contains the hypothesized value ($p = 0.02 \le 0.0452$) we fail to reject the null hypothesis.

9-91 a)

1) The parameter of interest is the true percentage of football helmets that contain flaws, p.
2) $H_0 : p = 0.1$
3) $H_1 : p > 0.1$
4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
5) Reject H_0 if $z_0 > z_\alpha$ where $\alpha = 0.01$ and $z_\alpha = z_{0.01} = 2.33$
6) x = 19 n = 200 $\hat{p} = \dfrac{19}{200} = 0.095$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.095 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{200}}} = -0.2357$$

7) Because –0.2357 < 2.33 fail to reject the null hypothesis. There is not enough evidence to conclude that the proportion of football helmets with flaws exceeds 10%.
P-value = 1- $\Phi(-0.2357)$ =0.5932

b) The 99% lower confidence interval

$$\hat{p} - z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$

$$0.095 - 2.33\sqrt{\frac{0.095(0.905)}{200}} \le p$$

$$0.0467 \le p$$

Because the confidence interval contains the hypothesized value ($0.0467 \le p = 0.1$) we fail to reject the null hypothesis.

9-92 a)

1) The parameter of interest is the true proportion of engineering students planning graduate studies
2) $H_0 : p = 0.50$
3) $H_1 : p \neq 0.50$
4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.01$ and $-z_{\alpha/2} = -z_{0.025} = -2.58$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.01$ and $z_{\alpha/2} = z_{0.025} = 2.58$
6) x = 117 n = 484 $\hat{p} = \dfrac{117}{484} = 0.2423$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{117 - 484(0.5)}{\sqrt{484(0.5)(0.5)}} = -11.36$$

7) Because –11.36 > –2.58 reject the null hypothesis and conclude that the true proportion of engineering students planning graduate studies differs from 0.5, at $\alpha = 0.01$.

P-value = $2[1 - \Phi(11.36)] \cong 0$

b) $\hat{p} = \dfrac{117}{484} = 0.242$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.242 - 2.58\sqrt{\frac{0.242(0.758)}{484}} \le p \le 0.242 + 2.58\sqrt{\frac{0.242(0.758)}{484}}$$

$$0.1918 \le p \le 0.2922$$

Because the 99% confidence interval does not contain the value 0.5 we conclude that the true proportion of engineering students planning graduate studies differs from 0.5.

9-93 1) The parameter of interest is the true proportion of batteries that fail before 48 hours, *p*.
2) $H_0 : p = 0.002$
3) $H_1 : p < 0.002$
4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
5) Reject H_0 if $z_0 < -z_{\alpha}$ where $\alpha = 0.01$ and $-z_{\alpha} = -z_{0.01} = -2.33$

6) $x = 15$ $n = 5500$ $\hat{p} = \frac{15}{5500} = 0.0027$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0027 - 0.002}{\sqrt{\frac{0.002(1-0.002)}{5500}}} = 1.162$$

7) Because 1.62 > –2.33 fail to reject the null hypothesis. There is not sufficient evidence to conclude that the proportion of cell phone batteries that fail is less than 0.2% at $\alpha = 0.01$.

9-94 The problem statement implies that H_0: p = 0.6, H_1: p > 0.6 and defines an acceptance region as $\hat{p} \le \frac{320}{500} = 0.64$ and rejection region as $\hat{p} > 0.64$

a) The probability of a type 1 error is

$$\alpha = P(\hat{p} \ge 0.64 \mid p = 0.6) = P\left(Z \ge \frac{0.64 - 0.6}{\sqrt{\frac{0.6(0.4)}{500}}}\right) = P(Z \ge 1.83) = 1 - P(Z < 1.83) = 0.0336.$$

b) $\beta = P(\hat{P} \le 0.64 \mid p = 0.75) = P(Z \le -5.68) = 0$.

9-95 a)

1) The parameter of interest is the true proportion of engine crankshaft bearings exhibiting surface roughness.

2) $H_0 : p = 0.10$

3) $H_1 : p > 0.10$

4) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

5) Reject H_0 if $z_0 > z_\alpha$ where $\alpha = 0.05$ and $z_\alpha = z_{0.05} = 1.65$

6) $x = 10$ $n = 85$ $\hat{p} = \frac{10}{85} = 0.118$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{10 - 85(0.10)}{\sqrt{85(0.10)(0.90)}} = 0.54$$

7) Because 0.54 < 1.65 fail to reject the null hypothesis. There is not enough evidence to conclude that the true proportion of crankshaft bearings exhibiting surface roughness exceeds 0.10, at $\alpha = 0.05$.

P-value = $1 - \Phi(0.54) = 0.295$

b) $p = 0.15$, $p_0 = 0.10$, $n = 85$, and $z_{\alpha/2} = 1.96$

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

$$= \Phi\left(\frac{0.10 - 0.15 + 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right) - \Phi\left(\frac{0.10 - 0.15 - 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right)$$

$$= \Phi(0.36) - \Phi(-2.94) = 0.6406 - 0.0016 = 0.639$$

c) $n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p - p_0}\right)^2$

$$n = \left(\frac{1.96\sqrt{0.10(1-0.10)} - 1.28\sqrt{0.15(1-0.15)}}{0.15-0.10} \right)^2$$

$$= (10.85)^2 = 117.63 \cong 118$$

Section 9-7

9-96 Expected Frequency is found by using the Poisson distribution

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where $\lambda = [0(24)+1(30)+2(31)+3(11)+4(4)]/100 = 1.41$

Value	0	1	2	3	4
Observed Frequency	24	30	31	11	4
Expected Frequency	30.12	36.14	21.69	8.67	2.60

Since value 4 has an expected frequency less than 3, combine this category with the previous category:

Value	0	1	2	3-4
Observed Frequency	24	30	31	15
Expected Frequency	30.12	36.14	21.69	11.67

The degrees of freedom are $k - p - 1 = 4 - 0 - 1 = 3$

a)

1) The variable of interest is the form of the distribution for X.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$ for $\alpha = 0.05$

6) $\chi_0^2 = \frac{(24-30.12)^2}{30.12} + \frac{(30-36.14)^2}{36.14} + \frac{(31-21.69)^2}{21.69} + \frac{(15-11.67)^2}{11.67} = 7.23$

7) Because 7.23 < 7.81 fail to reject H_0. We are unable to reject the null hypothesis that the distribution of X is Poisson.

b) The P-value is between 0.05 and 0.1 using Table IV. From Minitab the P-value = 0.0649.

9-97 The expected frequency is found from the Poisson distribution

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where $\lambda = [1(1) + 2(11) + \ldots + 7(10) + 8(9)]/80 = 4.7375$ is the estimated mean.

Value	1 or less	2	3	4	5	6	7	8 or more
Observed Frequency	3	11	11	13	11	12	10	9
Expected Frequency	3.320	7.865	12.420	14.710	13.937	11.005	7.448	4.411

Value	1 or less	2	3	4	5	6	7	8	9 or more
Observed Frequency	3	11	11	13	11	12	10	9	0

Expected Frequency	4.0211	7.8648	12.4198	14.7098	13.9375	11.0048	7.4479	4.4106	4.1837

Because the expected frequencies of all categories are greater than 3, there is no need to combine categories.

The degrees of freedom are k – p – 1 = 9 – 1 – 1 = 7
a)
1) Interest is on the form of the distribution for the number of flaws.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.01,7}^2 = 18.48$ for $\alpha = 0.01$
6)

$$\chi_0^2 = \frac{(3-4.0211)^2}{4.0211} + \frac{(11-7.8648)^2}{7.8648} + \cdots + \frac{(0-4.1837)^2}{4.1837} = 12.4129$$

7) Because 12.4129 < 18.48 fail to reject H_0.

b) P-value = 0.0534 (from computer software)

9-98 Estimated mean = 10.131

Value	5 or less	6	7	8	9	10	11	12	13	14	15 or more
Rel. Freq	0.067	0.067	0	0.1	0.133	0.2	0.133	0.133	0.067	0.033	0.067
Observed (Days)	2	2	0	3	4	6	4	4	2	1	2
Expected (Days)	1.7752	1.01504	1.72912	2.52476	3.2257	3.66332	3.74428	3.47912	2.96334	2.32987	2.86433

Because there are several cells with expected frequencies less than 3, a revised table follows. We note that there are other reasonable alternatives to combine cells.

Value	8 or less	9	10	11	12	13 or more
Observed (Days)	7	4	6	4	4	5
Expected (Days)	7.0441	3.2257	3.6633	3.7443	3.4791	8.1575

The degrees of freedom are k – p – 1 = 6 – 1 – 1 = 4

a)
1) Interest is on the form of the distribution for the number of calls arriving to a switchboard from noon to 1pm during business days.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,4}^2 = 9.49$ for $\alpha = 0.05$

6)

$$\chi_0^2 = \frac{(7-7.0441)^2}{7.0441} + ... + \frac{(5-8.1575)^2}{8.1575} = 2.9943$$

7) Because 2.9943 < 9.49, fail to reject H_0.

b) The P-value is between 0.5 and 0.9 using Table IV. P-value = 0.5588 (from computer software)

9-99 Under the null hypothesis there are 50 observations from a binomial distribution with n = 6 and p = 0.25. Use the binomial distribution to obtain the expected frequencies from the 50 observations.

Value	0	1	2	3	4 or more
Observed	4	14	10	20	2
Expected	8.8989	17.7979	14.8315	6.5918	1.8799

The expected frequency for cell "4 or more" is less than 3. Combine this cell with its neighboring cell to obtain the following table.

Value	0	1	2	3 or more
Observed	4	14	10	22
Expected	8.8989	17.7979	14.8315	8.4717

The degrees of freedom are k – p – 1 = 4 – 0 – 1 = 3

a)
1) The variable of interest is the form of the distribution for the random variable X.
2) H_0: The form of the distribution is binomial with n = 6 and p = 0.25
3) H_1: The form of the distribution is not binomial with n = 6 and p = 0.25
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$ for $\alpha = 0.05$

6)

$$\chi_0^2 = \frac{(4-8.8989)^2}{8.8989} + \cdots + \frac{(22-8.4717)^2}{8.4717} = 26.6843$$

7) Because 26.6843> 7.81 reject H_0. We conclude that the distribution is not binomial with n = 6 and p = 0.25 at $\alpha = 0.05$.

b) P-value = 6.856304e-06 (from computer software)

9-100 Under the null hypothesis there are 75 observations from a binomial distribution with n = 24. The value of *p* must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$. The sample mean $= \frac{0(43)+1(23)+2(8)+3(1)}{75} = 0.56$ and the mean of a binomial distribution is np. Therefore,
$\hat{p}_{sample} = \frac{sample\ \ mean}{n} = \frac{0.56}{24} = 0.0233$. From the binomial distribution with n = 24 and p = 0.0233 the expected frequencies follow

Value	0	1	2	3 or more
Observed	39	23	12	1
Expected	38.1426	26.1571	8.5952	2.1051

Value	0	1	2	3 or more
Observed	43	23	8	1
Expected	42.5922	24.3857	6.6900	1.3321

Because the cell "3 or more" has an expected frequency less than 3, combine this category with that of the neighboring cell to obtain the following table.

Value	0	1	2 or more
Observed	39	23	13
Expected	38.1426	26.1571	10.7003

Value	0	1	2 or more
Observed	43	23	9
Expected	42.5922	24.3857	8.0221

The degrees of freedom are k – p – 1 = 3 – 1 – 1 = 1

a)
1) Interest is the form of the distribution for the number of underfilled cartons, X.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,1}^2 = 3.84$ for $\alpha = 0.05$

6) $$\chi_0^2 = \frac{(43-42.5922)^2}{42.5922} + \frac{(23-24.3857)^2}{24.3857} + \frac{(9-8.0221)^2}{8.0221} = 0.2019$$

7) Because 0.2019 < 3.84 fail to reject H_0.

b) The P-value is between 0.5 and 0.9 using Table IV. From Minitab the P-value = 0.6532.

9-101 The estimated mean = 49.6741. Based on a Poisson distribution with $\lambda = 49.674$ the expected frequencies are shown in the following table. All expected frequencies are greater than 3.

The degrees of freedom are k – p – 1 = 26 – 1 – 1 = 24

Vechicles per minute	Frequency	Expected Frequency
40 or less	14	277.6847033
41	24	82.66977895
42	57	97.77492539
43	111	112.9507307
44	194	127.5164976
45	256	140.7614945
46	296	152.0043599
47	378	160.6527611
48	250	166.2558608
49	185	168.5430665
50	171	167.4445028
51	150	163.091274
52	110	155.7963895

53	102	146.0197251
54	96	134.3221931
55	90	121.3151646
56	81	107.6111003
57	73	93.78043085
58	64	80.31825
59	61	67.62265733
60	59	55.98491071
61	50	45.5901648
62	42	36.52661944
63	29	28.80042773
64	18	22.35367698
65 or more	15	62.60833394

a)
1) Interest is the form of the distribution for the number of cars passing through the intersection.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,24}^2 = 36.42$ for $\alpha = 0.05$
6) Estimated mean = 49.6741

$$\chi_0^2 = 1012.8044$$

7) Because 1012.804351 >> 36.42, reject H_0. We can conclude that the distribution is not a Poisson distribution at α = 0.05.

b) P-value ≈ 0 (from computer software)

9-102 The expected frequency is determined by using the Poisson distribution

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where $\lambda = [6(1) + 7(1) + \ldots + 39(1) + 41(1)]/110 = 19.2455$ is the estimated mean.

The expected frequencies are shown in the following table.

Number of Earthquakes	Frequency	Expected Frequency
6 or less	1	0.048065
7	1	0.093554
8	4	0.225062
9	0	0.4812708
10	3	0.926225
11	4	1.620512
12	3	2.598957
13	6	3.847547
14	5	5.289128
15	11	6.786111
16	8	8.162612
17	3	9.240775

18	9	9.880162
19	4	10.0078
20	4	9.630233
21	7	8.82563
22	8	7.720602
23	4	6.460283
24	3	5.180462
25	2	3.988013
26	4	2.951967
27	4	2.104146
28	1	1.446259
29	1	0.95979
30	1	0.61572
31	1	0.382252
32	2	0.229894
33	0	0.134073
34	1	0.075891
35	1	0.04173
36	2	0.022309
37	0	0.011604
38	0	0.005877
39	1	0.0029
40	0	.001395
41 or more	1	0.001191

After combining categories with frequencies less than 3, we obtain the following table. We note that there are other reasonable alternatives to combine cells. For example, cell 12 could also be combined with cell 13.

Number of earthquakes	Frequency	Expected Frequency	Chi squared
12 or less	16	5.993646	16.70555
13	6	3.847547	1.204158
14	5	5.289128	0.015805
15	11	6.786111	2.616648
16	8	8.162612	0.003239
17	3	9.240775	4.214719
18	9	9.880162	0.078408
19	4	10.0078	3.606553
20	4	9.630233	3.291667
21	7	8.82563	0.377642
22	8	7.720602	0.010111
23	4	6.460283	0.936955
24	3	5.180462	0.917759
25	2	3.988013	0.991019

26 or more	20	8.986998	13.49574

The degrees of freedom are k – p – 1 = 15 – 1 – 1 = 13

a)
1) Interest is the form of the distribution for the number of earthquakes per year of magnitude 7.0 and greater.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,13}^2 = 22.36$ for α = 0.05
6)

$$\chi_0^2 = \frac{(16-5.9936)^2}{5.9936} + \cdots + \frac{(20-8.9856)^2}{8.9856} = 48.4660$$

7) Because 48.4660 > 22.36 reject H_0. We conclude that the distribution of the number of earthquakes is not a Poisson distribution.

b) P-value ≈ 0 (from computer software)

Section 9-8

9-103 1) Interest is on the distribution of breakdowns among shift.
2) H_0: Breakdowns are independent of shift.
3) H_1: Breakdowns are not independent of shift.
4) The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

5) The critical value is $\chi_{.05,6}^2 = 12.592$ for α = 0.05

6) The calculated test statistic is $\chi_0^2 = 11.65$

7) Because $\chi_0^2 \ngtr \chi_{0.05,6}^2$ fail to reject H_0. The evidence is not sufficient to claim that machine breakdown and shift are dependent at α = 0.05. P-value = $P(\chi_0^2 > 11.65)$ = 0.070 (from computer software)

9-104 1) The variable of interest is calls by surgical-medical patients.
2) H_0: Calls by surgical-medical patients are independent of Medicare status.
3) H_1: Calls by surgical-medical patients are not independent of Medicare status.
4) The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

5) The critical value is $\chi_{.01,1}^2 = 6.637$ for α = 0.01

6) The calculated test statistic is $\chi_0^2 = 0.068$

7) Because $\chi_0^2 \ngtr \chi_{0.01,1}^2$ fail to reject H_0. The evidence is not sufficient to claim that surgical-medical patients and Medicare status are dependent. *P*-value = 0.79 for $\chi_0^2 = 0.068$. The P-value was obtained from a computer software.

9-105 1) Interest is on the distribution of statistics and OR grades.
2) H_0: Statistics grades are independent of OR grades.
3) H_1: Statistics and OR grades are not independent.
4) The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$$

5) The critical value is $\chi^2_{.01,9} = 21.665$ for α = 0.01

6) The calculated test statistic is $\chi^2_0 = 25.55$

7) $\chi_0^2 > \chi^2_{0.01,9}$ Therefore, reject H_0 and conclude that the grades are not independent at α = 0.01.

P-value = $P(\chi_0^2 > 25.55)$ = 0.002 (from computer software)

9-106 1. The variable of interest is characteristic among deflections and ranges.
2. H_0: Deflection and range are independent.
3. H_1: Deflection and range are not independent.
4. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$$

5. The critical value is $\chi^2_{0.05,4} = 9.488$ for α = 0.05

6. The calculated test statistic is $\chi^2_0 = 2.46$

7. Because $\chi_0^2 \not> \chi^2_{0.05,4}$ fail to reject H_0. The evidence is not sufficient to claim that the data are dependent at α = 0.05. The *P*-value = 0.652

9-107 1. The variable of interest is failures of an electronic component.
2. H_0: Type of failure is independent of mounting position.
3. H_1: Type of failure is not independent of mounting position.
4. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$$

5. The critical value is $\chi^2_{0.01,3} = 11.344$ for α = 0.01

6. The calculated test statistic is $\chi^2_0 = 14.69$

7. Because $\chi_0^2 > \chi^2_{0.01,3}$ reject H_0. The evidence is sufficient to claim that the type of failure is not independent of the mounting position at α = 0.01. *P*-value = 0.021.

9-108 1) Interest is on the distribution of opinion on core curriculum change.
2) H_0: Opinion of the change is independent of the class standing.
3) H_1: Opinion of the change is not independent of the class standing.
4) The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$$

5) The critical value is $\chi^2_{0.05,3} = 7.815$ for α = 0.05

6) The calculated test statistic is $\chi^2_{.0} = 26.97$.

7) $\chi_0^2 >>> \chi^2_{0.05,3}$, reject H_0 and conclude that opinion on the change and class standing are not independent. *P*-value = $P(\chi_0^2 > 26.97) \approx 0$

9-109 a)
1) Interest if on the distribution of successes.
2) H_0: successes are independent of size of stone.
3) H_1: successes are not independent of size of stone.
4) The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

5) The critical value is $\chi^2_{.05,1} = 3.84$ for α = 0.05

6) The calculated test statistic χ_0^2 = 13.766 with details below.

7) $\chi_0^2 > \chi^2_{0.05,1}$, reject H_0 and conclude that the number of successes and the stone size are not independent.

```
             1       2      All
1           55      25       80
         66.06   13.94    80.00
2          234      36      270
        222.94   47.06   270.00
All        289      61      350
        289.00   61.00   350.00
Cell Contents:       Count
                     Expected count
Pearson Chi-Square = 13.766, DF = 1, P-Value = 0.000
```

b) *P*-value = $P(\chi_0^2 > 13.766)$ < 0.005

Section 9-9

9-110 a)

1. The parameter of interest is the median of pH.
2. $H_0 : \tilde{\mu} = 7.0$
3. $H_1 : \tilde{\mu} \neq 7.0$
4. The test statistic is the observed number of plus differences or $r^+ = 8$ for α = 0.05.
5. We reject H_0 if the P-value corresponding to $r^+ = 8$ is less than or equal to α = 0.05.
6. Using the binomial distribution with $n = 10$ and $p = 0.5$, the P-value = $2P(R^+ \geq 8 \mid p = 0.5) = 0.1$
7. Conclusion: we fail to reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the pH is 7.0

b)

1. The parameter of interest is median of pH.
2. $H_0 : \tilde{\mu} = 7.0$
3. $H_1 : \tilde{\mu} \neq 7.0$
4. The test statistic is $z_0 = \dfrac{r^* - 0.5n}{0.5\sqrt{n}}$
5. We reject H_0 if $|Z_0| > 1.96$ for α = 0.05.
6. r* = 8 and $z_0 = \dfrac{r^* - 0.5n}{0.5\sqrt{n}} = \dfrac{8 - 0.5(10)}{0.5\sqrt{10}} = 1.90$
7. Conclusion: we fail to reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the pH is 7.0

P-value = $2[1 - P(|Z_0| < 1.90)] = 2(0.0287) = 0.0574$

9-111 a)

1. The parameter of interest is median titanium content.
2. $H_0 : \tilde{\mu} = 8.5$
3. $H_1 : \tilde{\mu} \neq 8.5$
4. The test statistic is the observed number of plus differences or $r^+ = 8$ for α = 0.05.
5. We reject H_0 if the *P*-value corresponding to $r^+ = 8$ is less than or equal to α = 0.05.
6. Using the binomial distribution with $n = 20$ and $p = 0.5$, *P*-value = $2P(R^* \leq 8 \mid p = 0.5) = 0.5$

7. Conclusion: we fail to reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the titanium content is 8.5.

b)
1. Parameter of interest is the median titanium content
2. $H_0 : \tilde{\mu} = 8.5$
3. $H_1 : \tilde{\mu} \neq 8.5$
4. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
5. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$ for $\alpha = 0.05$
6. Computation: $z_0 = \dfrac{8 - 0.5(20)}{0.5\sqrt{20}} = -0.89$
7. Conclusion: we fail to reject H_0. There is not enough evidence to conclude that the median titanium content differs from 8.5. The P-value = 2*P($Z < -0.89$) = 0.3735.

9-112 a)
1. Parameter of interest is the median impurity level.
2. $H_0 : \tilde{\mu} = 2.5$
3. $H_1 : \tilde{\mu} < 2.5$
4. The test statistic is the observed number of plus differences or $r^+ = 3$ for $\alpha = 0.05$.
5. We reject H_0 if the *P-value* corresponding to $r^+ = 3$ is less than or equal to $\alpha = 0.05$.
6. Using the binomial distribution with $n = 22$ and $p = 0.5$, the P-value = $P(R^+ \leq 3 \mid p = 0.5) = 0.00086$
7. Conclusion, reject H_0. The data supports the claim that the median is impurity level is less than 2.5.

b)
1. Parameter of interest is the median impurity level
2. $H_0 : \tilde{\mu} = 2.5$
3. $H_1 : \tilde{\mu} < 2.5$
4. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
5. We reject H_0 if the $Z_0 < Z_{0.05} = -1.65$ for $\alpha = 0.05$
6. Computation: $z_0 = \dfrac{3 - 0.5(22)}{0.5\sqrt{22}} = -3.41$
7. Conclusion: reject H_0 and conclude that the median impurity level is less than 2.5.

The P-value = P($Z < -3.41$) = 0.000325

9-113 a)
1) Parameter of interest is the median margarine fat content
2) $H_0 : \tilde{\mu} = 17.0$
3) $H_1 : \tilde{\mu} \neq 17.0$
4) $\alpha = 0.05$
5) The test statistic is the observed number of plus differences or $r^+ = 3$.
6) Reject H_0 if the *P*-value corresponding to $r^+ = 3$ is less than or equal to α=0.05.
7) Using the binomial distribution with n = 6 and p = 0.5, the P-value = $2 \cdot P(R^+ \geq 3 | p=0.5, n=6) \approx 1$.
8) Fail to reject H_0. There is not enough evidence to conclude that the median fat content differs from 17.0.

b)
1) Parameter of interest is the median margarine fat content
2) $H_0 : \tilde{\mu} = 17.0$
3) $H_1 : \tilde{\mu} \neq 17.0$
4) Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$

5) Reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$ for $\alpha=0.05$

6) Computation: $z_0 = \dfrac{3-0.5(6)}{0.5\sqrt{6}} = 0$

7) Fail to reject H_0. The P-value = $2[1 - \Phi(0)] = 2(1 - 0.5) = 1$. There is not enough evidence to conclude that the median fat content differs from 17.0.

9-114 a)

1. Parameter of interest is the median compressive strength
2. $H_0 : \tilde{\mu} = 2250$
3. $H_1 : \tilde{\mu} > 2250$
4. The test statistic is the observed number of plus differences or $r^+ = 7$ for $\alpha = 0.05$
5. We reject H_0 if the P-value corresponding to $r^+ = 7$ is less than or equal to $\alpha = 0.05$.
6. Using the binomial distribution with $n = 12$ and $p = 0.5$, the P-value = $P(R^+ \geq 7 \mid p = 0.5) = 0.3872$
7. Conclusion: fail to reject H_0. There is not enough evidence to conclude that the median compressive strength is greater than 2250.

b)

1. Parameter of interest is the median compressive strength
2. $H_0 : \tilde{\mu} = 2250$
3. $H_1 : \tilde{\mu} > 2250$
4. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
5. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$ for $\alpha = 0.05$
6. Computation: $z_0 = \dfrac{7-0.5(12)}{0.5\sqrt{12}} = 0.577$
7. Conclusion: fail to reject H_0. The P-value = $1 - \Phi(0.58) = 1 - 0.7190 = 0.281$. There is not enough evidence to conclude that the median compressive strength is greater than 2250.

9-115 a)

1) The parameter of interest is the mean ball diameter

2) H_0: $\mu_0 = 0.265$

3) H_0: $\mu_0 \neq 0.265$

4) w = min(w^+, w^-)

5) Reject H_0 if $w \leq w^*_{0.05,n=9} = 5$ for $\alpha = 0.05$

6) The data are ranked from the smallest to the largest value. Usually zeros are dropped from the ranking and the sample size is reduced. In the case of a tie, the ranks are added and divided by the number of tries. For example, in these data, there are two instances of the difference 0.001. The ranks corresponding to this value are 1 and 2. The sum of the ranks is 3. After dividing by the number of ties, We set a mean of 1.5 and this value is assigned to both. The sum of the positive ranks is $w^+ = (1.5 + 1.5 + 4.5 + 4.5 + 4.5 + 7.5 + 7.5 + 9) = 40.5$. There is only one negative rank $w^- = 4.5$. Therefore, w = min(40.5, 4.5) = 4.5.

Observation	Difference $x_i - 0.265$	Signed Rank
6	0	-
9	0	-
12	0	-
1	0.001	1.5
3	0.001	1.5
2	–0.002	–4.5
4	0.002	4.5
5	0.002	4.5
7	0.002	4.5
10	0.003	7.5

11	0.003	7.5
8	0.004	9

7) Conclusion: because w^{-} = 4.5 is less than the critical value $w^{*}_{0.05,n=9} = 5$, we reject the null hypothesis that the mean ball diameter is 0.265 at the 0.05 level of significance.

b) $Z_0 = \dfrac{W^{+} - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}} = \dfrac{40.5 - 9(10)/4}{\sqrt{9(10)(19)/24}} = 2.132$

and $Z_{0.025}$ = 1.96. Because Z_0 = 2.132 > $Z_{0.025}$ = 1.96 we reject the null hypothesis that the mean ball diameter is 0.265 at the 0.05 level of significance. Also, the P-value = 2[1 – P(Z_0 < 2.132)] = 0.033.

9-116 1) The parameter of interest is hardness reading

2) H_0: $\mu_0 = 60$

3) H_0: $\mu_0 > 60$

4) w^{-}

5) Reject H_0 if $w^{-} \le w^{*}_{0.05,n=7} = 3$ for α = 0.05

6) The sum of the positive rank is w^{+} = (4.5 + 7 + 1) = 12.5. The sum of the negative rank is w^{-} = (+2 + 6 + 4.5 + 8) = 20.5.

Observation	Difference x_i – 60	Sign Rank
4	3	4.5
7	6	7
3	–2	–2
1	0	–
6	–5	–6
5	–3	–4.5
8	–7	–8
2	1	1

7) Conclusion: Because w^{-} = 20.5 is not less than or equal to the critical value $w^{*}_{0.05,n=7} = 3$, we fail to reject the null hypothesis that the mean hardness reading is greater than 60.

9-117 1) The parameter of interest is the mean dying time of the primer

2) H_0: $\mu_0 = 1.5$

3) H_0: $\mu_0 > 1.5$

4) w^{-}

5) Reject H_0 if $w^{-} \le w^{*}_{0.05,n=17} = 41$ for α = 0.05

6) The sum of the positive rank is w^{+} = (4+4+4+4+4+9.5+9.5+13.5+13.5+13.5+13.5+16.5+16.5) = 126. The sum of the negative rank is w^{-} = (4+4+9.5+9.5) = 27.

Observation	Difference xi - 1.5	Sign Rank
1.5	0	-
1.5	0	-
1.5	0	-
1.6	0.1	4
1.6	0.1	4
1.6	0.1	4
1.4	-0.1	-4
1.6	0.1	4
1.4	-0.1	-4

1.6	0.1	4
1.3	-0.2	-9.5
1.7	0.2	9.5
1.7	0.2	9.5
1.3	-0.2	-9.5
1.8	0.3	13.5
1.8	0.3	13.5
1.8	0.3	13.5
1.8	0.3	13.5
1.9	0.4	16.5
1.9	0.4	16.5

7) Conclusion: Because w^- = 27 is less than the critical value $w^*_{0.05,n=17} = 41$, we reject the null hypothesis that the mean dying time of the primer exceeds 1.5.

Section 9-10

9-118 a)

1) The parameter of interest is the mean absorption rate of the new product, μ.

2, 3) The null and alternative hypotheses that must be tested are as follows (δ = 0.50):

$H_0 : \mu = 18.50$ and $H_0 : \mu = 17.50$

$H_1 : \mu \le 18.50$ $H_1 : \mu \ge 17.50$

b)

Test the first hypothesis ($H_0 : \mu = 18.50$ vs. $H_1 : \mu \le 18.50$):

4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

5) Reject H_0 if $t_0 < -t_{\alpha,n-1}$ where $t_{0.05,19} = 1.729$ for n = 20 and $\alpha = 0.05$.

6) $\bar{x} = 18.22$, $s = 0.92$, n = 20

$$t_0 = \frac{18.22 - 18.50}{0.92/\sqrt{20}} = -1.361$$

7) Because -1.361 > -1.729, fail to reject H_0.

Test the second hypothesis ($H_0 : \mu = 17.50$ vs. $H_1 : \mu \ge 17.50$):

4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,19} = 1.729$ for n = 20 and $\alpha = 0.05$.

6) $\bar{x} = 18.22$, $s = 0.92$, n = 20

$$t_0 = \frac{18.22 - 17.50}{0.92/\sqrt{20}} = 3.450$$

7) Because 3.450 > 1.729, reject H_0.

There is enough evidence to conclude that the mean absorption rate is greater than 17.50. However, there is not enough evidence to conclude that it is less than 18.50. As a result, we cannot conclude that the new product has an absorption rate that is equivalent to the absorption rate of the current one at $\alpha = 0.05$.

9-119 a)

1) The parameter of interest is the mean molecular weight of a raw material from a new supplier, μ.

2, 3) The null and alternative hypotheses that must be tested are as follows (δ=50):

$H_0 : \mu = 3550$ and $H_0 : \mu = 3450$

$H_1 : \mu \le 3550$ $H_1 : \mu \ge 3450$

b)

Test the first hypothesis ($H_0 : \mu = 3550$ vs. $H_1 : \mu \le 3550$):

4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

5) Reject H_0 if $t_0 < -t_{\alpha,n-1}$ where $t_{0.05,9} = 1.833$ for n = 10 and $\alpha = 0.05$.

6) $\bar{x} = 3550$, $s = 25$, n = 10

$$t_0 = \frac{3550 - 3550}{25/\sqrt{10}} = 0$$

7) Because 0 > -1.833, fail to reject H_0.

Test the second hypothesis ($H_0 : \mu = 3450$ vs. $H_1 : \mu \ge 3450$):

4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,9} = 1.833$ for n = 10 and $\alpha = 0.05$.

6) $\bar{x} = 3550$, $s = 25$, n = 10

$$t_0 = \frac{3550 - 3450}{25/\sqrt{10}} = 12.65$$

7) Because 12.65 > 1.833, reject H_0.

There is enough evidence to conclude that the mean molecular weight is greater than 3450. However, there is not enough evidence to conclude that it is less than 3550. As a result, we cannot conclude that the new supplier provides a molecular weight that is equivalent to the current one at $\alpha = 0.05$.

9-120 a)

1) The parameter of interest is the mean breaking strength , μ.

2) $H_0 : \mu = 65.5$

3) $H_1 : \mu \ge 65.5$

b)

4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,49} = 1.677$ for n = 50 and $\alpha = 0.05$.

6) $\bar{x} = 64.2$, $s = 1.52$, n = 50

$$t_0 = \frac{64.2 - 65.5}{1.52/\sqrt{50}} = -6.048$$

7) Because -6.048 < 1.677, fail to reject H_0.

There is not enough evidence to conclude that the mean breaking strength of the insulators is at least 65.5 kPa and that the process by which the insulators are manufactured is equivalent to the standard.

9-121 a)

1) The parameter of interest is the mean bond strength , μ.
2) H_0 : μ = 67224
3) H_1 : μ ≥ 67224

b)
4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,5} = 2.015$ for n = 6 and α = 0.05.

6) $\bar{x} = 64535$, $s = 294$, n = 6

$$t_0 = \frac{64535 - 67224}{294/\sqrt{6}} = -22.404$$

7) Because -22.404 < 2.015, fail to reject H_0.

There is not enough evidence to conclude that the mean bond strength of cement product is at least 67224 kPa and that the process by which the cement product is manufactured is equivalent to the standard.

9-122 $\chi_0^2 = -2[\ln(0.12) + \ln(0.08) + ... + \ln(0.06)] = 46.22$ with $2m = 2(10) = 20$ degrees of freedom.

The *P*-value for this statistic is less than 0.01 (≈ 0.0007). In conclusion, we reject the shared null hypothesis.

9-123 $\chi_0^2 = -2[\ln(0.15) + \ln(0.83) + ... + \ln(0.13)] = 37.40$ with $2m = 2(8) = 16$ degrees of freedom.

The *P*-value for this statistic is less than 0.01 (≈ 0.0018). In conclusion, we reject the shared null hypothesis.

<u>Section 9-11</u>

9-124 H_0 : σ = 5.7
H_1 : σ < 5.7

$\chi_0^2 = -2[\ln(0.15) + \ln(0.091) + ... + \ln(0.06)] = 33.66$ with $2m = 2(6) = 12$ degrees of freedom.

The *P*-value for this statistic is less than 0.01 (≈ 0.0007). As a result, we reject the shared null hypothesis. There is sufficient evidence to conclude that the standard deviation of fill volume is less than 5.7g.

9-125 H_0 : μ = 624
H_1 : μ ≠ 624

$\chi_0^2 = -2[\ln(0.065) + \ln(0.0924) + ... + \ln(0.021)] = 30.57$ with $2m = 2(5) = 10$ degrees of freedom.

The *P*-value for this statistic is less than 0.01 (≈ 0.0006). As a result, we reject the shared null hypothesis. There is sufficient evidence to conclude that the mean package weight is not equal to 624 g.

Supplemental Exercises

9-126 a) SE Mean $= \frac{\sigma}{\sqrt{N}} = \frac{1.5}{\sqrt{N}} = 0.401$, so n = 18

$$z_0 = \frac{26.541 - 26}{1.7/\sqrt{18}} = 1.3502$$

P-value = $1 - \Phi(Z_0) = 1 - \Phi(1.3502) = 1 - 0.9115 = 0.0885$

b) A one-sided test because the alternative hypothesis is $\mu > 26$.

c) Because $z_0 < 1.65$ and the P-value = 0.0885 > α = 0.05, we fail to reject the null hypothesis at the 0.05 level of significance.

d) 95% CI of the mean is $\bar{x} - z_{0.025}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{0.025}\frac{\sigma}{\sqrt{n}}$

$$26.541 - (1.96)\frac{1.7}{\sqrt{18}} < \mu < 26.541 + (1.96)\frac{1.7}{\sqrt{18}}$$
$$25.7556 < \mu < 27.3264$$

9-127 a) Degrees of freedom = n – 1 = 16 – 1 = 15.

b) SE Mean $= \frac{S}{\sqrt{N}} = \frac{4.61}{\sqrt{16}} = 1.1525$

$$t_0 = \frac{98.33 - 100}{4.61/\sqrt{16}} = -1.4490$$

$t_0 = -1.4490$ with df = 15, so 2(0.05) < P-value < 2(0.1). That is, 0.1 < P-value < 0.2.

99% CI of the mean is $\bar{x} - t_{0.005,15}\frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{0.005,15}\frac{S}{\sqrt{n}}$

$$98.33 - (2.947)\frac{4.61}{\sqrt{16}} < \mu < 98.33 + (2.947)\frac{4.61}{\sqrt{16}}$$
$$94.9336 < \mu < 101.7264$$

c) Because the P-value > α = 0.01 we fail to reject the null hypothesis at the 0.01 level of significance.

d) $t_{0.01,15} = 2.602$. Because t_0 = -1.4490 < $t_{0.01,15} = 2.602$ we fail to reject the null hypothesis at the 0.01level of significance.

9-128 a) Degree of freedom = n – 1 = 31 – 1 = 30.

b) SE Mean $= \frac{s}{\sqrt{N}} = \frac{s}{\sqrt{31}} = 0.631$, so s = 3.513

$$t_0 = \frac{84.331 - 85}{3.513/\sqrt{31}} = -1.0603$$

$t_0 = -1.06$ with df = 30, so 0.1 < P-value < 0.25

c) Because the P-value > α = 0.05 we fail to reject the null hypothesis at the 0.05 level of significance.

d) 95% upper CI of the mean is $\mu < \bar{x} + t_{0.05,30}\frac{S}{\sqrt{n}}$

$$\mu < 84.331 + (1.697)\frac{3.513}{\sqrt{31}}$$
$$\mu < 85.4017$$

e) If the null hypothesis is changed to mu = 100 versus mu > 100,

$$t_0 = \frac{84.331 - 100}{3.513/\sqrt{31}} = -24.8339$$

$t_0 = -24.8339$ and $t_{0.05,30} = 1.697$ with df = 30.

Because $t_0 << t_{0.05,30}$ we fail to reject the null hypothesis at the 0.05 level of significance.

9-129 a) The null hypothesis is μ = 12 versus μ > 12

$\bar{x} = 12.4737$, S = 3.6266, and N = 19

$t_0 = \frac{12.4737 - 13}{3.6266/\sqrt{19}} = -0.6326$ with df = 19 – 1 = 18.

The P-value falls between two values 0.257 (α = 0.4) and 0.688 (α = 0.25). Thus, 0.25 < P-value < 0.4.
Because the P-value > α = 0.05 we fail to reject the null hypothesis at the 0.05 level of significance.

b) 95% two-sided CI of the mean is $\bar{x} - t_{0.025,18}\frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{0.025,18}\frac{S}{\sqrt{n}}$

$$12.4737 - (2.101)\frac{3.6266}{\sqrt{19}} < \mu < 12.4737 + (2.101)\frac{3.6266}{\sqrt{19}}$$
$$10.7257 < \mu < 14.2217$$

9-130 a) The null hypothesis is μ = 300 versus μ < 300

$\bar{x} = 275.333$, s = 42.665, and n = 6

$$t_0 = \frac{275.333 - 285}{42.665/\sqrt{6}} = -0.5550$$

and $t_{0.05,5} = 2.015$. Because $t_0 > -t_{0.05,5} = -2.015$ we fail to reject the null hypothesis at the 0.05 level of significance.

b) Yes, because the sample size is very small the central limit theorem's conclusion that the distribution of the sample mean is approximately normally distributed is a concern.

c) 95% two-sided CI of the mean is $\bar{x} - t_{0.025,5}\frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{0.025,5}\frac{S}{\sqrt{n}}$

$$275.333 - (2.571)\frac{42.665}{\sqrt{6}} < \mu < 275.333 + (2.571)\frac{42.665}{\sqrt{6}}$$
$$230.5515 < \mu < 320.1145$$

9-131 For $\alpha = 0.01$

a) $n = 30$, $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{30}}\right) = \Phi(2.33-0.31) = \Phi(2.02) = 0.9783$

$n = 100$, $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{100}}\right) = \Phi(2.33-0.63) = \Phi(1.70) = 0.9554$

$n = 400$, $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{400}}\right) = \Phi(2.33-1.25) = \Phi(1.08) = 0.8599$

$n = 2550$, $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{2550}}\right) = \Phi(2.33-3.156) = \Phi(-0.826) = 0.2078$

b) $n = 30$ $z_0 = \frac{86-85}{16/\sqrt{30}} = 0.31$ *P*-value: $1-\Phi(0.31) = 1-0.6217 = 0.3783$

$n = 100$ $z_0 = \frac{86-85}{16/\sqrt{100}} = 0.63$ *P*-value: $1-\Phi(0.63) = 1-0.7357 = 0.2643$

$n = 400$ $z_0 = \frac{86-85}{16/\sqrt{400}} = 1.25$ *P*-value: $1-\Phi(1.25) = 1-0.8944 = 0.1056$

$n = 2550$ $z_0 = \frac{86-85}{16/\sqrt{2550}} = 3.156$ *P*-value: $1-\Phi(3.13) = 1-0.9993 = 0.0007$

The data would be statistically significant when $n = 2550$ at $\alpha = 0.01$

9-132 Sample Mean = $\hat{p}$ Sample Variance = $\frac{\hat{p}(1-\hat{p})}{n}$

	Sample Size, *n*	Sampling Distribution	Sample Mean	Sample Variance
a.	60	Normal	$\hat{p}$	$\frac{\hat{p}(1-\hat{p})}{60}$
b.	90	Normal	$\hat{p}$	$\frac{\hat{p}(1-\hat{p})}{90}$
c.	110	Normal	$\hat{p}$	$\frac{\hat{p}(1-\hat{p})}{110}$

d) As the sample size increases, the variance of the sampling distribution decreases.

9-133

	n	Test statistic	P-value	conclusion
a.	50	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/50}} = -0.12$	0.4522	Fail to reject H_0
b.	100	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/100}} = -0.15$	0.4404	Fail to reject H_0
c.	500	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/500}} = -0.37$	0.3557	Fail to reject H_0
d.	1000	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/1000}} = -0.53$	0.2981	Fail to reject H_0

e) The P-value decreases as the sample size increases.

9-134 $\sigma = 12$, $\delta = 206 - 200 = 6$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$,

a) $n = 20$: $\beta = \Phi\left(1.96 - \frac{6\sqrt{20}}{12}\right) = \Phi(-0.276) = 1 - \Phi(0.276) = 1 - 0.6087 = 0.39$

b) $n = 50$: $\beta = \Phi\left(1.96 - \frac{6\sqrt{50}}{12}\right) = \Phi(-1.576) = 1 - \Phi(1.576) = 1 - 0.943 = 0.057$

c) $n = 100$: $\beta = \Phi\left(1.96 - \frac{6\sqrt{100}}{12}\right) = \Phi(-3.04) = 1 - \Phi(3.04) = 1 - 0.9988 = 0.0012$

d) β (probability of a Type II error) decreases as the sample size increases because the variance of the sample mean decreases. Consequently, the probability of observing a sample mean in the acceptance region centered about the incorrect value of 200 ml/h decreases with larger *n*.

9-135 $\sigma = 14$, $\delta = 205 - 200 = 5$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$,

a) $n = 20$: $\beta = \Phi\left(1.96 - \frac{5\sqrt{20}}{14}\right) = \Phi(0.362) = 0.6406$

b) $n = 50$: $\beta = \Phi\left(1.96 - \frac{5\sqrt{50}}{14}\right) = \Phi(-0.565) = 1 - \Phi(0.565) = 1 - 0.7123 = 0.2877$

c) $n = 100$: $\beta = \Phi\left(1.96 - \frac{5\sqrt{100}}{14}\right) = \Phi(-1.611) = 1 - \Phi(1.611) = 1 - 0.9463 = 0.0537$

d) The probability of a Type II error increases with an increase in the standard deviation.

9-136 $\sigma = 8$, $= 210 - 204 = 6$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$.

a) $n = 20$: $\beta = \Phi\left(1.96 - \frac{6\sqrt{20}}{8}\right) = \Phi(-1.39) = 1 - \Phi(1.39) = 1 - 0.917736 = 0.0823$

Therefore, power = $1 - \beta = 0.9177$

b) $n = 50$: $\beta = \Phi\left(1.96 - \frac{6\sqrt{50}}{8}\right) = \Phi(-3.34) = 1 - \Phi(3.34) = 1 - 0.999581 = 0.00042$

Therefore, power = $1 - \beta = 0.9996$

c) $n = 100$: $\beta = \Phi\left(1.96 - \frac{6\sqrt{100}}{8}\right) = \Phi(-5.54) = 1 - \Phi(5.54) = 1 - 1 = 0$

Therefore, power = $1 - \beta = 1$

d) As sample size increases, and all other values are held constant, the power increases because the variance of the sample mean decreases. Consequently, the probability of a Type II error decreases, which implies the power increases.

9-137 a) $\alpha = 0.05$

$n = 120$ $\beta = \Phi\left(z_{0.05} + \frac{0.5 - 0.6}{\sqrt{0.5(0.5)/120}}\right) = \Phi(1.65 - 2.19) = \Phi(-0.54) = 0.2946$

$\text{Power} = 1 - \beta = 1 - 0.2946 = 0.7054$

$n = 140$ $\beta = \Phi\left(z_{0.05} + \frac{0.5 - 0.6}{\sqrt{0.5(0.5)/140}}\right) = \Phi(1.65 - 2.37) = \Phi(-0.72) = 0.2357$

$\text{Power} = 1 - \beta = 1 - 0.2357 = 0.7643$

$$n = 300 \quad \beta = \Phi\left(z_{0.05} + \frac{0.5 - 0.6}{\sqrt{0.5(0.5)/300}} \right) = \Phi(1.65 - 3.46) = \Phi(-1.81) = 0.03515$$

$$\text{Power} = 1 - \beta = 1 - 0.03515 = 0.96485$$

b) $\alpha = 0.01$

$$n = 100 \quad \beta = \Phi\left(z_{0.01} + \frac{0.5 - 0.6}{\sqrt{0.5(0.5)/100}} \right) = \Phi(2.33 - 2.0) = \Phi(0.33) = 0.6293$$

$$\text{Power} = 1 - \beta = 1 - 0.6293 = 0.3707$$

$$n = 150 \quad \beta = \Phi\left(z_{0.01} + \frac{0.5 - 0.6}{\sqrt{0.5(0.5)/100}} \right) = \Phi(2.33 - 2.45) = \Phi(-0.12) = 0.4522$$

$$\text{Power} = 1 - \beta = 1 - 0.4522 = 0.5478$$

$$n = 300 \quad \beta = \Phi\left(z_{0.01} + \frac{0.5 - 0.6}{\sqrt{0.5(0.5)/300}} \right) = \Phi(2.33 - 3.46) = \Phi(-1.13) = 0.1292$$

$$\text{Power} = 1 - \beta = 1 - 0.1292 = 0.8702$$

Decreasing the value of α decreases the power of the test for the different sample sizes.

c) $\alpha = 0.05$

$$n = 100 \quad \beta = \Phi\left(z_{0.05} + \frac{0.5 - 0.8}{\sqrt{0.5(0.5)/100}} \right) = \Phi(1.65 - 6.0) = \Phi(-4.35) \cong 0.0$$

$$\text{Power} = 1 - \beta = 1 - 0 \cong 1$$

The true value of p has a large effect on the power. The greater is the difference of p from p_0 the larger is the power of the test.

d)

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p - p_0} \right)^2$$

$$= \left(\frac{2.58\sqrt{0.5(1-0.50)} - 1.65\sqrt{0.6(1-0.6)}}{0.6 - 0.5} \right)^2 = (4.82)^2 = 23.2 \cong 24$$

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p - p_0} \right)^2$$

$$= \left(\frac{2.58\sqrt{0.5(1-0.50)} - 1.65\sqrt{0.8(1-0.8)}}{0.8 - 0.5} \right)^2 = (2.1)^2 = 4.41 \cong 5$$

The true value of p has a large effect on the sample size. The greater is the distance of p from p_0 the smaller is the sample size that is required.

9-138 a) Rejecting a null hypothesis provides a *stronger conclusion* than failing to reject a null hypothesis. Therefore, place what we are trying to demonstrate in the alternative hypothesis.

Assume that the data follow a normal distribution.

b)

1) The parameter of interest is the mean weld strength, μ.

2) $H_0 : \mu = 1035$

3) $H_1 : \mu > 1035$

4) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

5) Since no given value of alpha, so no critical value is given. We will calculate the P-value

6) $\bar{x} = 1059.7$, $s = 77.9$, $n = 20$

$$t_0 = \frac{1059.7 - 1035}{77.9/\sqrt{20}} = 1.42$$

P-value = P(t > 1.42) = 0.05 < P-value < 0.10

7) There is some modest evidence to support the claim that the weld strength exceeds 1035 kN/m^2.
If we used α = 0.01 or 0.05, we would fail to reject the null hypothesis, thus the claim would not be supported. If we used α = 0.10, we would reject the null in favor of the alternative and conclude the weld strength exceeds 1035 kN/m^2.

9-139 a) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|325 - 330|}{4.4} = \frac{5}{4.4} = 1.14$

Using the OC curve for α = 0.05, d = 1.14, and n = 10, $\beta \cong 0.0$ and power of $1 - 0.0 \cong 1$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|320 - 330|}{4.4} = 2.27$$

Using the OC curve for α = 0.05, d = 2.27, and n = 10, $\beta \cong 0.0$ and power of $1 - 0.0 \cong 1$.

b) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|325 - 330|}{4.4} = 1.14$

Using the OC curve, Chart VII e) for α = 0.05, d = 1.14, and $\beta \cong 0.1$ (Power = 0.9), $n^* = 10$.

Therefore, $n = \frac{n^* + 1}{2} = \frac{10 + 1}{2} = 5.5$ □ 6

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|320 - 330|}{4.4} = 2.27$$

Using the OC curve, Chart VII e) for α = 0.05, d = 2.27, and $\beta \cong 0.1$ (Power = 0.9), $n^* = 5$.

Therefore, $n = \frac{n^* + 1}{2} = \frac{5 + 1}{2} = 3$

c) $\sigma = 8.8$

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|125 - 130|}{8.8} = 0.57$$

Using the OC curve for α = 0.05, d = 0.57, and n = 10, $\beta \cong 0.52$ and power of $1 - 0.52 \cong 0.48$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|120 - 130|}{8.8} = 1.14$$

Using the OC curve for α = 0.05, d = 1.14, and n = 10, $\beta \cong 0.12$ and power of $1 - 0.12 \cong 0.88$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|125 - 130|}{8.8} = 0.57$$

Using the OC curve, Chart VII e) for α = 0.05, d = 0.57, and $\beta \cong 0.1$ (Power = 0.9), $n^* = 30$.

Therefore, $n = \frac{n^* + 1}{2} = \frac{30 + 1}{2} = 15.5$ □ 16

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|120 - 130|}{8.8} = 1.14$$

Using the OC curve, Chart VII e) for α = 0.05, d = 1.14, and $\beta \cong 0.1$ (Power = 0.9), $n^* = 13$.

Therefore, $n = \frac{n^* + 1}{2} = \frac{13 + 1}{2} = 7$

Increasing the standard deviation decreases the power of the test and increases the sample size required to obtain a certain power.

9-140 Assume the data follow a normal distribution.

a) 1) The parameter of interest is the standard deviation, σ.

2) $H_0 : \sigma^2 = (0.00003)^2$

3) $H_1 : \sigma^2 < (0.00003)^2$

4) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) $\chi^2_{0.99,7} = 1.24$ reject H_0 if $\chi_0^2 < 1.24$ for $\alpha = 0.01$

6) $s = 0.00001$ and $\alpha = 0.01$

$$\chi_0^2 = \frac{7(0.00001)^2}{(0.00003)^2} = 0.78$$

7) Conclusion: Because 0.78 < 1.24 we reject the null hypothesis. That is, there is sufficient evidence to conclude the standard deviation is at most 0.00003 mm.

b) Although the sample standard deviation is less than the hypothesized value of 0.00003, it is *not significantly less* (when $\alpha = 0.01$) than 0.00003. The value of 0.00001 could have occurred as a result of sampling variation.

9-141 Assume the data follow a normal distribution.

1) The parameter of interest is the standard deviation of the concentration, σ.

2) $H_0 : \sigma^2 = 3^2$

3) $H_1 : \sigma^2 < 3^2$

4) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Because no value of alpha is specified we calculate the P-value

6) s = 0.004 and n = 10

$$\chi_0^2 = \frac{9(0.004)^2}{(3)^2} = 0.000016$$

P-value = $P\left(\chi^2 < 0.000016\right)$ $\quad P-value \cong 0$

7) Conclusion: The P-value is approximately 0. Therefore we reject the null hypothesis and conclude that the standard deviation of the concentration is less than 3 grams per liter.

9-142 The null hypothesis is that these are 40 observations from a binomial distribution with n = 50 and p must be estimated. Create a table for the number of nonconforming coil springs (value) and the observed frequency. A possible table follows.

Value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Frequency	0	0	0	1	4	3	4	5	5	2	1	3	3	1	1	0	2	2	1	2

The value of *p* must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$. The

$$\text{sample mean} = \frac{0(0)+1(0)+2(0)+\cdots+19(2)}{40} = 9.475$$ and the mean of a binomial distribution is np.

Therefore,

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{9.475}{50} = 0.1895$$

The expected frequencies are obtained from the binomial distribution with n = 50 and p = 0.1895 for 40 observations.

Value	Observed	Expected
0	0	0.0112
1	0	0.1047
2	0	0.4774
3	1	1.4139
4	4	3.0578
5	3	5.1476
6	4	7.0206

7	5	7.9728
8	5	7.6894
9	2	6.3923
10	1	4.6331
11	3	2.9543
12	3	1.6693
13	1	0.8406
14	1	0.3790
15	0	0.1536
16	2	0.0561
17	2	0.0185
18	1	0.0055
19 or more	2	0.0020

Because several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	4 or less	5	6	7	8	9	10	11 or more
Observed	4	3	4	5	5	2	1	3
Expected	5.0651	5.1476	7.0206	7.9728	7.6894	6.3923	4.6331	6.0791

The degrees of freedom are k – p – 1 = 8 – 1 – 1 = 6

a)
1) Interest is on the form of the distribution for the number of nonconforming coil springs.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_0^2 > \chi_{0.01,6}^2 = 16.81$ for $\alpha = 0.01$
6)

$$\chi_0^2 = \frac{(4 - 5.0651)^2}{5.0651} + \frac{(3 - 5.1476)^2}{5.1476} + \cdots + \frac{(3 - 6.0791)^2}{6.0791} = 11.8952$$

7) Because 11.8952<16.81 fail to reject H_0. We conclude that the distribution of nonconforming springs is binomial at $\alpha = 0.01$.

b) P-value = 0.0643 (from computer software)

9-143 The null hypothesis is that these are 20 observations from a binomial distribution with n = 1000 and p must be estimated. Create a table for the number of errors in a string of 1000 bits (value) and the observed frequency. A possible table follows.

Value	0	1	2	3	4	5
Frequency	3	8	4	2	3	0

The value of *p* must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$. The

sample mean $= \dfrac{0(3) + 1(8) + 2(4) + 3(2) + 4(3) + 5(0)}{20} = 1.7$ and the mean of a binomial distribution is np.

Therefore,

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{1.7}{1000} = 0.0017$$

Value	0	1	2	3	4	5 or more
Observed	3	8	4	2	3	0
Expected	3.64839	6.21282	5.28460	2.99371	1.27067	0.58981

Because several of the expected values are less than 3, some cells are combined resulting in the following table:

Value	0	1	2	3 or more
Observed	3	8	4	5
Expected	3.6484	6.2128	5.2846	4.8542

The degrees of freedom are k – p – 1 = 4 – 1 – 1 = 2

a)
1) Interest is on the form of the distribution for the number of errors in a string of 1000 bits.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_0^2 > \chi_{0.01,2}^2 = 9.21$ for $\alpha = 0.01$
6)

$$\chi_0^2 = \frac{(3-3.6484)^2}{3.6484} + \cdots + \frac{(5-4.8542)^2}{4.8542} = 0.9460$$

7) Because 0.9460 < 9.21 fail to reject H_0. We are unable to reject the null hypothesis that the distribution of the number of errors is binomial at $\alpha = 0.01$.

b) *P*-value = 0.623 (from computer software)

9-144 Divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0, 0.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = 0.125 so the expected cell frequencies are E = *np* = (100) (0.125) = 12.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency	Exp. Frequency
x ≤ 5332.5	1	12.5
5332.5< x ≤ 5357.5	4	12.5
5357.5< x ≤ 5382.5	7	12.5
5382.5< x ≤ 5407.5	24	12.5
5407.5< x ≤ 5432.5	30	12.5
5432.5< x ≤ 5457.5	20	12.5
5457.5< x ≤ 5482.5	15	12.5
x ≥ 5482.5	5	12.5

The test statistic is:

$$\chi_0^2 = \frac{(1-12.5)^2}{12.5} + \frac{(4-12.5)^2}{12.5} + \cdots + \frac{(15-12.5)^2}{12.5} + \frac{(5-12.5)^2}{12.5} = 63.36$$

and we would reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Because $\chi_o^2 > \chi_{0.05,5}^2$, reject the hypothesis that the data are normally distributed

9-145 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean concentration of suspended solids, μ.
2) $H_0 : \mu = 50$
3) $H_1 : \mu < 50$
4) Because n >> 30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

5) Reject H_0 if $z_0 < -1.65$ for $\alpha = 0.05$
6) $\bar{x} = 59.87$ s = 12.50   n = 60

$$z_0 = \frac{59.87 - 50}{12.50 / \sqrt{60}} = 6.12$$

7) Because 6.12 > –1.65 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean concentration of suspended solids is less than 50 ppm at α = 0.05.

b) P-value = $\Phi(6.12) \cong 1$

c) We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0, 0.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = 0.125 so that the expected cell frequencies are E = np = (60) (0.125) = 7.5.

The sample mean and standard deviation are 59.86667 and 12.49778, respectively, and these are used to determine the boundaries of the intervals. That is, the first interval is from negative infinity to 59.86667 + 12.49778(-1.15) = 45.50. The second interval is from this value to 59.86667 + 12.49778(-0.675) = 51.43. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 45.50$	9	7.5
$45.50 < x \le 51.43$	5	7.5
$51.43 < x \le 55.87$	7	7.5
$55.87 < x \le 59.87$	11	7.5
$59.87 < x \le 63.87$	4	7.5
$63.87 < x \le 68.31$	9	7.5
$68.31 < x \le 74.24$	8	7.5
$x \ge 74.24$	6	7.5

The test statistic is:

$$\chi^2_o = \frac{(9-7.5)^2}{7.5} + \frac{(5-7.5)^2}{7.5} + \cdots + \frac{(8-7.5)^2}{7.5} + \frac{(6-7.5)^2}{7.5} = 5.06$$

and we reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Because it does not, we fail to reject the hypothesis that the data are normally distributed.

9-146 a) In order to use t statistic in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean overall distance for this brand of golf ball, μ.
2) $H_0 : \mu = 265$
3) $H_1 : \mu < 265$
4) Since n >> 30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

5) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.05} = 1.65$ for $\alpha = 0.05$
6) $\bar{x} = 1.25$ s = 0.25   n = 100

$$z_0 = \frac{260.30 - 265.0}{13.41 / \sqrt{100}} = -3.50$$

7) Because –3.50 < –1.65 reject the null hypothesis. There is sufficient evidence to indicate that the true mean distance is less than 265 yard at $\alpha = 0.05$.

b) P-value =0.0002

c) We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0,.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = .125 so the expected cell frequencies are E = np = (100) (0.125) = 12.5.

The sample mean and standard deviation are 260.302 and 13.40828, respectively, and these are used to determine the boundaries of the intervals. That is, the first interval is from negative infinity to 260.302 + 13.40828(-1.15) = 244.88. The second interval is from this value to 260.302 + 13.40828(-0.675) = 251.25. The table of ranges and their corresponding frequencies is completed as follows.

The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 244.88$	16	12.5
$244.88 < x \le 251.25$	6	12.5
$251.25 < x \le 256.01$	17	12.5
$256.01 < x \le 260.30$	9	12.5
$260.30 < x \le 264.59$	13	12.5
$264.59 < x \le 269.35$	8	12.5
$269.35 < x \le 275.72$	19	12.5
$x \ge 275.72$	12	12.5

The test statistic is:

$$\chi^2_o = \frac{(16-12.5)^2}{12.5} + \frac{(6-12.5)^2}{12.5} + \cdots + \frac{(19-12.5)^2}{12.5} + \frac{(12-12.5)^2}{12.5} = 12$$

and we reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Because it does, we reject the hypothesis that the data are normally distributed.

9-147 a) Assume the data are normally distributed.
1) The parameter of interest is the true mean coefficient of restitution, μ.
2) H_0: $\mu = 0.635$
3) H_1: $\mu > 0.635$
4) Becasue n > 30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

5) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05}$ =2.33 for $\alpha = 0.01$
6) $\bar{x} = 0.624$ s = 0.0131 n = 40

$$z_0 = \frac{0.624 - 0.635}{0.0131/\sqrt{40}} = -5.31$$

7) Because –5.31< 2.33 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean coefficient of restitution is greater than 0.635 at $\alpha = 0.01$.

c) P-value $\Phi(5.31) \cong 1$

d) If the lower bound of the one-sided CI is greater than the value 0.635 then we can conclude that the mean coefficient of restitution is greater than 0.635. Furthermore, a confidence interval provides a range of values for the true mean coefficient that generates information on how much the true mean coefficient differs from 0.635.

9-148 a)
In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal. Use the t-test to test the hypothesis that the true mean is 2.5 mg/L.

1) State the parameter of interest: The parameter of interest is the true mean dissolved oxygen level, μ.
2) State the null hypothesis H_0: $\mu = 2.5$
3) State the alternative hypothesis H_1: $\mu \neq 2.5$
4) Give the statistic

$$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

5) Reject H_0 if $|t_0| < t_{\alpha/2,n-1}$ for $\alpha = 0.05$
6) Sample statistic $\bar{x} = 3.265$ s =2.127 n = 20

$$\text{t-statistic } t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

7) Draw your conclusion and find the P-value.

b) Assume the data are normally distributed.
1) The parameter of interest is the true mean dissolved oxygen level, μ.
2) H_0: $\mu = 2.5$
3) H_1: $\mu \neq 2.5$
4) Test statistic

$$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = t_{0.025,19b} = 2.093$ for $\alpha = 0.05$
6) $\bar{x} = 3.265$ s =2.127 n = 20

$$t_0 = \frac{3.265 - 2.5}{2.127/\sqrt{20}} = 1.608$$

7) Because 1.608 < 2.093 fail to reject the null hypotheses. The sample mean is not significantly different from 2.5 mg/L.

c) The value of 1.608 is found between the columns of 0.05 and 0.1 of Table V. Therefore, 0.1 < P-value < 0.2. Minitab provides a value of 0.124.

d) The confidence interval found in the previous exercise agrees with the hypothesis test above. The value of 2.5 is within the 95% confidence limits. The confidence interval shows that the interval is quite wide due to the large sample standard deviation.

$$\bar{x} - t_{0.025,19}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{0.025,19}\frac{s}{\sqrt{n}}$$

$$3.265 - 2.093\frac{2.127}{\sqrt{20}} \le \mu \le 3.265 + 2.093\frac{2.127}{\sqrt{20}}$$

$$2.270 \le \mu \le 4.260$$

9-149 a)

1) The parameter of interest is the true mean sugar concentration, μ.
2) $H_0 : \mu = 11.5$
3) $H_1 : \mu \neq 11.5$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.093$ for $\alpha = 0.05$
6) $\bar{x} = 11.47$, s = 0.022 n=20

$$t_0 = \frac{11.47 - 11.5}{0.022/\sqrt{20}} = -6.10$$

7) Because 6.10 > 2.093 reject the null hypothesis. There is sufficient evidence that the true mean sugar concentration is different from 11.5 at $\alpha = 0.05$.

From Table V the t_0 value in absolute value is greater than the value corresponding to 0.0005 with 19 degrees of freedom. Therefore 2(0.0005) = 0.001 > P-value

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|11.3 - 11.5|}{0.022} = 9.09$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 9.09, and n =20 we find $\beta \cong 0$ and Power $\cong 1$.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|11.45 - 11.5|}{0.022} = 2.27$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 2.27, and 1 - β > 0.9 (β < 0.1), we find that n should be at least 5.

d) 95% two sided confidence interval

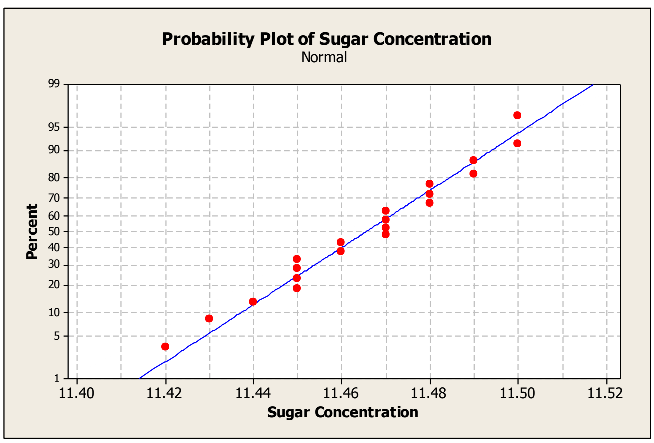
$$\bar{x} - t_{0.025,19}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,19}\left(\frac{s}{\sqrt{n}}\right)$$

$$11.47 - 2.093\left(\frac{0.022}{\sqrt{20}}\right) \le \mu \le 11.47 + 2.093\left(\frac{0.022}{\sqrt{20}}\right)$$

$$11.46 \le \mu \le 11.48$$

We conclude that the mean sugar concentration content is not equal to 11.5 because that value is not inside the confidence interval.

e) The normality plot below indicates that the normality assumption is reasonable.



9-150 a) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1 - p_0)}} = \dfrac{58 - 250(0.25)}{\sqrt{250(0.25)(0.75)}} = -0.6573$

The P-value = $\Phi(-0.6573)$ = 0.2555

b) Because the P-value = 0.2555 > α = 0.05 we fail to reject the null hypothesis at the 0.05 level of significance.

c) The normal approximation is appropriate because np > 5 and n(p-1) > 5.

d) $\hat{p} = \dfrac{58}{250} = 0.232$

The 95% upper confidence interval is:

$$p \le \hat{p} + z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le 0.232 + 1.65\sqrt{\frac{0.232(0.768)}{250}}$$

$$p \le 0.2760$$

e) P-value = 2(1 - Φ(0.6573)) = 2(1 - 0.7445) = 0.511.

9-151 a)

1) The parameter of interest is the true mean percent protein, μ.

2) $H_0 : \mu = 80$

3) $H_1 : \mu > 80$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

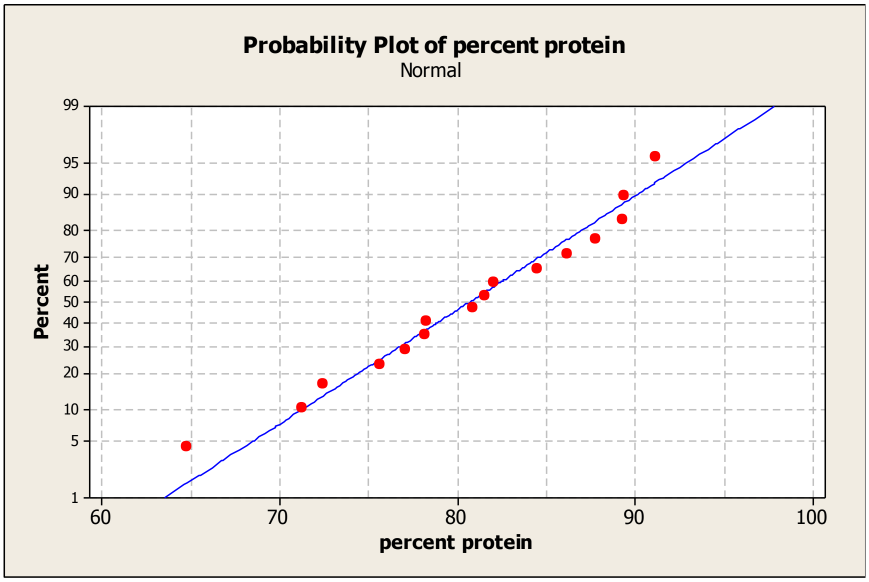
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,15} = 1.753$ for $\alpha = 0.05$

6) $\bar{x} = 80.68$ s = 7.38 n = 16

$$t_0 = \frac{80.68 - 80}{7.38/\sqrt{16}} = 0.37$$

7) Because 0.37 < 1.753 fail to reject the null hypothesis. There is not sufficient evidence to indicate that the true mean percent protein is greater than 80 at α = 0.05.

b) From the normal probability plot, the normality assumption seems reasonable:



c) From Table V, 0.25 < P-value < 0.4

9-152 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true variance of tissue assay, σ^2.

2) $H_0 : \sigma^2 = 0.6$

3) $H_1 : \sigma^2 \ne 0.6$

4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.01$ and $\chi^2_{0.995,11} = 2.60$ or $\chi_0^2 > \chi^2_{\alpha/2,n-1}$ where $\alpha = 0.01$ and $\chi^2_{0.005,11} = 26.76$ for n = 12

6) n = 12, s = 0.758

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{11(0.758)^2}{0.6} = 10.53$$

7) Because 2.6 <10.53 < 26.76 we fail to reject H_0. There is not sufficient evidence to conclude the true variance of tissue assay differs from 0.6 at $\alpha = 0.01$.

b) 0.1 < P-value/2 < 0.5, so that 0.2 < P-value < 1

c) 99% confidence interval for σ, first find the confidence interval for σ^2

For $\alpha = 0.05$ and n = 12, $\chi^2_{0.995,11} = 2.60$ and $\chi^2_{0.005,11} = 26.76$

$$\frac{11(0.758)^2}{26.76} \le \sigma^2 \le \frac{11(0.758)^2}{2.60}$$

$$0.236 \le \sigma^2 \le 2.43$$

$$0.486 \le \sigma \le 1.559$$

Because 0.6 falls within the 99% confidence bound there is not sufficient evidence to conclude that the population variance differs from 0.6

9-153 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true variance of the ratio between the numbers of symmetrical and total synapses, σ^2.

2) $H_0 : \sigma^2 = 0.02$

3) $H_1 : \sigma^2 \ne 0.02$

4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.975,30} = 16.79$ or $\chi_0^2 > \chi^2_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.025,30} = 46.98$ for n = 31

6) n = 31, s = 0.198

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{30(0.198)^2}{0.02} = 58.81$$

7) Because 58.81 > 46.98 reject H_0. The true variance of the ratio between the numbers of symmetrical and total synapses is different from 0.02 at $\alpha = 0.05$.

b) P-value/2 < 0.005 so that P-value < 0.01

9-154 a)

1) The parameter of interest is the true mean of cut-on wave length, μ.

2) $H_0 : \mu = 6.5$

3) $H_1 : \mu \ne 6.5$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$. Since no value of α is given, we will assume that $\alpha = 0.05$. So $t_{\alpha/2,n-1} = 2.228$

6) $\bar{x} = 6.55$, $s = 0.35$ $n = 11$

$$t_0 = \frac{6.55 - 6.5}{0.35/\sqrt{11}} = 0.47$$

7) Because 0.47< 2.228, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true mean of cut-on wave length differs from 6.5 at α = 0.05.

b) From Table V the t_0 value is found between the values of 0.25 and 0.4 with 10 degrees of freedom, so 0.5 < *P*-value < 0.8

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|6.30 - 6.5|}{0.35} = 0.57$

Using the OC curve, Chart VII e) for α = 0.05, d = 0.57, and 1 – β > 0.95 (β < 0.05). We find that n should be at least 50.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|6.80 - 6.5|}{0.35} = 0.86$

Using the OC curve, Chart VII e) for α = 0.05, n = 11, d = 0.86, we find $\beta \approx 0.35$.

9-155 a)

1) the parameter of interest is the variance of fatty acid measurements, σ^2
2) $H_0 : \sigma^2 = 1.0$
3) $H_1 : \sigma^2 \neq 1.0$
4) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
5) Reject H_0 if $\chi_0^2 < \chi^2_{0.995,5} = 0.41$ or reject H_0 if $\chi_0^2 > \chi^2_{0.005,5} = 16.75$ for α=0.01 and n = 6
6) n = 6, s = 0.343

$$\chi_0^2 = \frac{5(0.343)^2}{1^2} = 0.588$$

P-value: 0.01 < P-value/2 < 0.025 so that 0.02 < P-value < 0.05

7) Because the statistic 0.588 > 0.41from the table, fail to reject the null hypothesis at α = 0.01. There is insufficient evidence to conclude that the variance differs from 1.0.

b)

1) the parameter of interest is the variance of fatty acid measurements, σ^2 (now n=51)
2) $H_0 : \sigma^2 = 1.0$
3) $H_1 : \sigma^2 \neq 1.0$
4) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
5) Reject H_0 if $\chi_0^2 < \chi^2_{0.995,50} = 27.99$ or reject H_0 if $\chi_0^2 > \chi^2_{0.005,50} = 79.49$ for α=0.01 and n = 51

6) n = 51, s = 0.343

$$\chi_0^2 = \frac{50(0.343)^2}{1^2} = 5.88$$

P-value/2 < 0.005 so that P-value < 0.01

7) Because 5.88 < 27.99 reject the null hypothesis. There is sufficient evidence to conclude that the variance differs from1.0 at α = 0.01.

c) The sample size changes the conclusion that is drawn. With a small sample size, we fail to reject the null hypothesis. However, a larger sample size allows us to conclude the null hypothesis is false.

9-156 a) 1) the parameter of interest is the standard deviation, σ
2) $H_0 : \sigma^2 = 400$
3) $H_1 : \sigma^2 < 400$
4) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
5) No value of α is given, so that no critical value is given. We will calculate the P-value.
6) n = 14, s = 15.7

$$\chi_0^2 = \frac{15(15.7)^2}{400} = 8.63$$

P-value = $P\left(\chi^2 < 8.63\right)$; $0.1 < P < 0.5$

7) The P-value is greater than a common significance level α (such as 0.05). Therefore, we fail to reject the null hypothesis. There is insufficient evidence to support the claim that the standard deviation is less than 20 microamps.

b) 7) n = 51, s = 20

$$\chi_0^2 = \frac{50(15.7)^2}{400} = 30.81$$

P-value = $P\left(\chi^2 < 30.81\right)$; $0.01 < \text{P-value} < 0.025$

The P-value is less than 0.05. Therefore we reject the null hypothesis and conclude that the standard deviation is significantly less than 20 microamps.

c) Increasing the sample size increases the test statistic χ_0^2 and therefore decreases the P-value, providing more evidence against the null hypothesis.

Mind Expanding Exercises

9-157 a)

H_0: $\mu = \mu_0$

H_1 $\mu \neq \mu_0$

$P(Z > z_\varepsilon) = \varepsilon$ and $P(Z < -z_{\alpha-\varepsilon}) = (\alpha - \varepsilon)$. Therefore $P(Z > z_\varepsilon$ or $Z < -z_{\alpha-\varepsilon}) = (\alpha - \varepsilon) + \varepsilon = \alpha$

b) $\beta = P(-z_{\alpha-\varepsilon} < Z < z_\varepsilon \mid \mu_0 + \delta)$

9-158 a) Reject H_0 if $z_0 < -z_{\alpha-\varepsilon}$ or $z_0 > z_\varepsilon$

$$P\left(\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} < -\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} = \mid \mu = \mu_0\right) + P\left(\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} > \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} \mid \mu = \mu_0\right) \text{P}$$

$$P(z_0 < -z_{\alpha-\varepsilon}) + P(z_0 > z_\varepsilon) = \Phi(-z_{\alpha-\varepsilon}) + 1 - \Phi(z_\varepsilon)$$

$$= ((\alpha - \varepsilon)) + (1 - (1 - \varepsilon)) = \alpha$$

b) $\beta = P(z_\varepsilon \le \overline{X} \le z_\varepsilon$ when $\mu_1 = \mu_0 + d$)

$$\beta = P(-z_{\alpha-\varepsilon} < Z_0 < z_\varepsilon \mid \mu_1 = \mu_0 + \delta)$$

$$\beta = P\left(-z_{\alpha-\varepsilon} < \frac{\bar{x}-\mu_0}{\sqrt{\sigma^2/n}} < z_\varepsilon \mid \mu_1 = \mu_0 + \delta\right)$$

$$= P\left(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}} < Z < z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}}\right)$$

$$= \Phi\left(z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}}\right) - \Phi\left(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}}\right)$$

9-159 1) The parameter of interest is the true mean number of open circuits, λ.

2) H_0 : $\lambda = 2$

3) H_1 : $\lambda > 2$

4) Because n > 30 we can use the normal distribution

$$z_0 = \frac{\overline{X} - \lambda}{\sqrt{\lambda / n}}$$

5) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$ for $\alpha = 0.05$

6) $\bar{x}$ = 1038/500=2.076 n = 500

$$z_0 = \frac{2.076 - 2}{\sqrt{2/500}} = 1.202$$

7) Because 1.202 < 1.65 fail to reject the null hypothesis. There is insufficient evidence to indicate that the true mean number of open circuits is greater than 2 at $\alpha = 0.01$

9-160 a)

1) The parameter of interest is the true standard deviation of the golf ball distance σ.

2) H_0: $\sigma = 10$

3) H_1: $\sigma < 10$

4) Because $n > 30$ we can use the normal distribution

$$z_0 = \frac{S - \sigma_0}{\sqrt{\sigma_0^2/(2n)}}$$

5) Reject H_0 if $z_0 < z_\alpha$ where $z_{0.05} = -1.65$ for $\alpha = 0.05$

6) $s = 12.07$ $n = 100$

$$z_0 = \frac{12.07 - 10}{\sqrt{10^2/(200)}} = 2.93$$

7) Because 2.93 > –1.65 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true standard deviation is less than 10 at $\alpha = 0.05$.

b) 95% percentile: $\theta = \mu + 1.645\sigma$

95% percentile estimator: $\hat{\theta} = \bar{X} + 1.645S$

From the independence

$$SE(\hat{\theta}) \cong \sqrt{\sigma^2/n + 1.645^2\sigma^2/(2n)}$$

The statistic S can be used as an estimator for σ in the standard error formula.

c) 1) The parameter of interest is the true 95th percentile of the golf ball distance θ.

2) H_0: $\theta = 260$

3) H_1: $\theta < 260$

4) Since $n > 30$ we can use the normal distribution

$$z_0 = \frac{\hat{\theta} - \theta_0}{\hat{S}E(\hat{\theta})}$$

5) Reject H_0 if $z_0 < -1.65$ for $\alpha = 0.05$

6) $\hat{\theta} = 254.06$, $s = 12.07$, $n = 100$

$$z_0 = \frac{254.06 - 260}{\sqrt{(12.07)^2/100 + (1.645)^2(12.07)^2/200}} = -3.21$$

7) Because –1.283 < –1.65 reject the null hypothesis. There is sufficient evidence to conclude that the true θ is less than 285 at $\alpha = 0.05$.

9-161 1) The parameter of interest is the parameter of an exponential distribution, λ.

2) $H_0 : \lambda = \lambda_0$

3) $H_1 : \lambda \neq \lambda_0$

4) test statistic

$$\chi_0^2 = \frac{2\lambda\sum_{i=1}^{n} X_i - \lambda_0}{\sqrt{2\lambda\sum_{i=1}^{n} X_i}}$$

5) Reject H_0 if $\chi_0^2 > \chi^2_{a/2,2n}$ or $\chi_0^2 < \chi^2_{1-a/2,2n}$ for $\alpha = 0.05$

6) Compute $2\lambda\sum_{i=1}^{n} X_i$ and plug into

$$\chi_0^2 = \frac{2\lambda\sum_{i=1}^{n} X_i - \lambda_0}{\sqrt{2\lambda\sum_{i=1}^{n} X_i}}$$

7) Draw Conclusions

The one-sided hypotheses below can also be tested with the derived test statistic as follows:

1) $H_0 : \lambda = \lambda_0$ $H_1 : \lambda > \lambda_0$

Reject H_0 if $\chi_0^2 > \chi_{a,2n}^2$

2) $H_0 : \lambda = \lambda_0$ $H_1 : \lambda < \lambda_0$

Reject H_0 if $\chi_0^2 < \chi_{a,2n}^2$