CHAPTER 10

Section 10-2

10-1 a)

1) The parameter of interest is the difference in means $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$ for $\alpha = 0.05$

6) $\bar{x}_1 = 4.7$ $\quad \bar{x}_2 = 7.8$

$\sigma_1 = 10$ $\quad \sigma_2 = 8$

$n_1 = 10$ $\quad n_2 = 15$

$$z_0 = \frac{(4.7 - 7.8)}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}} = -0.82$$

7) Conclusion: Because –1.96 < –0.82 < 1.96, do not reject the null hypothesis. There is not sufficient evidence to conclude that the two means differ at $\alpha = 0.05$.

P-value = $2(1 - \Phi(0.82)) = 2(1 - 0.7939) = 0.4122$

b) $$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(4.7 - 7.8) - 1.96\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}} \le \mu_1 - \mu_2 \le (4.7 - 7.8) + 1.96\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}$$

$$-10.50 \le \mu_1 - \mu_2 \le 4.30$$

With 95% confidence, the true difference in the means is between −10.50 and 4.30. Because zero is contained in this interval, we conclude there is no significant difference between the means. We fail to reject the null hypothesis.

c)

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{3}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}}\right) - \Phi\left(-1.96 - \frac{3}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}}\right) = \Phi(1.17) - \Phi(-2.75) = 0.8790 - 0.0030 = 0.8760$$

Power = 1 – 0.876 = 0.124

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.05$, and $\delta = 3$

$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{\delta^2} = \frac{(1.96 + 1.645)^2\left(10^2 + 8^2\right)}{(3)^2} = 236.8$$

Use $n_1 = n_2 = 237$

10-2 a)

1) The parameter of interest is the difference in means $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_\alpha = -1.645$ for $\alpha = 0.05$

6) $\bar{x}_1 = 14.2$ $\bar{x}_2 = 19.7$

$\sigma_1 = 10$ $\sigma_2 = 8$

$n_1 = 10$ $n_2 = 15$

$$z_0 = \frac{(14.2 - 19.7)}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}} = -1.46$$

7) Conclusion: Because –1.46 > -1.645, do not reject the null hypothesis. There is not sufficient evidence to conclude that the two means differ at $\alpha = 0.05$.

P-value $= \Phi(-1.46) = 0.0721$

b) $$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (14.2 - 19.7) + 1.645\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}$$

$$\mu_1 - \mu_2 \le 0.71$$

With 95% confidence, the true difference in the means is less than 0.71. Because zero is contained in this interval, we fail to reject the null hypothesis.

c)

$$\beta = 1 - \Phi\left(-z_\alpha - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= 1 - \Phi\left(-1.65 - \frac{-4}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}}\right) = 1 - \Phi(-0.586) = 0.721$$

Power = 1 – 0.721 = 0.279

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.05$, and $\delta = \Delta - \Delta_0 = 4$

$$n \cong \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{\delta^2} = \frac{(1.645 + 1.645)^2(10^2 + 5^2)}{(4)^2} = 110.947$$

Use $n_1 = n_2 = 111$

10-3 a)

1) The parameter of interest is the difference in means $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 > z_\alpha = 2.325$ for $\alpha = 0.01$

6) $\bar{x}_1 = 24.5$ $\bar{x}_2 = 21.3$

$\sigma_1 = 10$ $\sigma_2 = 8$

$n_1 = 10$ $n_2 = 15$

$$z_0 = \frac{(24.5 - 21.3)}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}} = 0.85$$

7) Conclusion: Because 0.85 < 2.325, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the two means differ at $\alpha = 0.01$.

P-value = $1 - \Phi(0.85) = 1 - 0.8023 = 0.1977$

b) $\mu_1 - \mu_2 \geq (\bar{x}_1 - \bar{x}_2) - z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$\mu_1 - \mu_2 \geq (24.5 - 21.3) - 2.325\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}$

$\mu_1 - \mu_2 \geq -5.58$

The true difference in the means is greater than -5.58 with 99% confidence. Because zero is contained in this interval, we fail to reject the null hypothesis.

c)

$$\beta = \Phi\left(z_\alpha - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) = \Phi\left(2.325 - \frac{2}{\sqrt{\frac{(10)^2}{10} + \frac{(8)^2}{15}}}\right) = \Phi(1.80) = 0.96$$

Power = 1 – 0.96 = 0.04

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.05$, and $\Delta = 2$

$$n \cong \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{\delta^2} = \frac{(1.645 + 1.645)^2(10^2 + 8^2)}{(2)^2} = 443.8$$

Use $n_1 = n_2 = 444$

10-4 a)

1) The parameter of interest is the difference in fill volume $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$ for $\alpha = 0.05$

6) $\bar{x}_1 = 473.581$ $\bar{x}_2 = 473.324$

$\sigma_1 = 0.6$ $\sigma_2 = 0.75$

$n_1 = 10$ $n_2 = 10$

$$z_0 = \frac{(473.581 - 473.324)}{\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}} = 0.85$$

7) Conclusion: Because –1.96 < 0.85 < 1.96, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the two machine fill volumes differ at $\alpha = 0.05$.
P-value = $2(1 - \Phi(0.85)) = 2(1 - 0.8023) = 0.395$

b) $$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\left(473.581 - 473.324\right) - 1.96\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}} \le \mu_1 - \mu_2 \le \left(473.581 - 473.324\right) + 1.96\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}$$

$$-0.3383 \le \mu_1 - \mu_2 \le 0.8523$$

With 95% confidence, we believe the true difference in the mean fill volumes is between −0.3383 and 0.8523. Because 0 is contained in this interval, we can conclude there is no significant difference between the means.

c) $$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{1.2}{\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}}\right) - \Phi\left(-1.96 - \frac{1.2}{\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}}\right)$$

$$= \Phi\left(1.96 - 3.95\right) - \Phi\left(-1.96 - 3.95\right) = \Phi\left(-1.99\right) - \Phi\left(-5.91\right) = 0.0233 - 0 = 0.0233$$

Power = 1 – 0.9481 = 0.0519

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.05$, and $\Delta = 0.04$

$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{\delta^2} = \frac{(1.96 + 1.645)^2\left(\left(0.60\right)^2 + \left(0.75\right)^2\right)}{(1.2)^2} = 8.326$$

Use $n_1 = n_2 = 9$

10-5 a)

1) The parameter of interest is the difference in breaking strengths $\mu_1 - \mu_2$ and $\Delta_0 = 70$

2) $H_0: \mu_1 - \mu_2 = 10$

3) $H_1: \mu_1 - \mu_2 > 10$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 > z_\alpha = 1.645$ for $\alpha = 0.05$

6) $\bar{x}_1 = 1120$ $\bar{x}_2 = 1070$ $\delta = 70$
$\sigma_1 = 7$ $\sigma_2 = 7$
$n_1 = 10$ $n_2 = 12$

$$z_0 = \frac{(1120 - 1070) - 70}{\sqrt{\frac{(7)^2}{10} + \frac{(7)^2}{12}}} = -6.67$$

7) Conclusion: Because –6.67 < 1.645 fail to reject the null hypothesis. There is insufficient evidence to support the use of plastic 1 at $\alpha = 0.05$.

P-value = $1-\Phi\left(-6.67\right)$ = 1 – 0 = 1

b) $$\mu_1-\mu_2 \ge \left(\bar{x}_1-\bar{x}_2\right)-z_{\alpha}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$\mu_1-\mu_2 \ge \left(1120-1070\right)-1.645\sqrt{\frac{(7)^2}{10}+\frac{(7)^2}{12}}$$

$$\mu_1-\mu_2 \ge 45.07$$

c) $$\beta = \Phi\left(1.645-\frac{(84-70)}{\sqrt{\frac{7}{10}+\frac{7}{12}}}\right)=\Phi\left(-10.715\right)=0$$

Power = 1 – 0 = 1

d) $$n=\frac{(z_{\alpha/2}+z_{\beta})^2(\sigma_1^2+\sigma_2^2)}{(\Delta-\Delta_0)^2}=\frac{(1.645+1.645)^2(1+1)}{(12-10)^2}=5.41\cong 6$$

Yes, the sample size is adequate

10-6 a)

1) The parameter of interest is the difference in mean burning rate, $\mu_1-\mu_2$

2) $H_0: \mu_1-\mu_2=0$ or $\mu_1=\mu_2$

3) $H_1: \mu_1-\mu_2\neq 0$ or $\mu_1\neq\mu_2$

4) The test statistic is

$$z_0=\frac{(\bar{x}_1-\bar{x}_2)-\Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$ for $\alpha = 0.05$

6) $\bar{x}_1 = 18$ $\bar{x}_2 = 24$

$\sigma_1 = 3$ $\sigma_2 = 3$

$n_1 = 10$ $n_2 = 10$

$$z_0=\frac{(18-24)}{\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}}=-4.47$$

7) Conclusion: Because –4.47 < –1.96 reject the null hypothesis and conclude the mean burning rates differ significantly at $\alpha = 0.05$.

P-value $=2(1-\Phi(4.47))=2(1-1)=0$

b) $$\left(\bar{x}_1-\bar{x}_2\right)-z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\le\mu_1-\mu_2\le\left(\bar{x}_1-\bar{x}_2\right)+z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$\left(18-24\right)-1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}\le\mu_1-\mu_2\le\left(18-24\right)+1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}$$

$$-8.63\le\mu_1-\mu_2\le-3.37$$

We are 95% confident that the mean burning rate for solid fuel propellant 2 exceeds that of propellant 1 by between 3.37 and 8.63 cm/s.

c) $\beta = \Phi\left(z_{\alpha/2} - \dfrac{\Delta - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \right) - \Phi\left(-z_{\alpha/2} - \dfrac{\Delta - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \right)$

$$= \Phi\left(1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}}} \right) - \Phi\left(-1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}}} \right)$$

$$= = \Phi(0.47) - \Phi(-3.45) = 0.68 - 0 = 0.68$$

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, β = 1-power=0.1, and $\Delta = 4$

$$n \cong \frac{(z_{\alpha/2} + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{\delta^2} = \frac{(1.96 + 1.28)^2(3^2 + 3^2)}{(4)^2} = 11.8$$

Use $n_1 = n_2 = 12$

10-7 $\bar{x}_1 = 75.6 \quad \bar{x}_2 = 77.9$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

a)

1) The parameter of interest is the difference in mean road octane number $\mu_1 - \mu_2$ and $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_\alpha = -1.645$ for $\alpha = 0.05$

6) $\bar{x}_1 = 75.6 \quad \bar{x}_2 = 77.9$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

$$z_0 = \frac{(75.6 - 77.9)}{\sqrt{\frac{1.5}{15} + \frac{1.2}{20}}} = -5.75$$

7) Conclusion: Because $-5.75 < -1.645$ reject the null hypothesis and conclude the mean road octane number for formulation 2 exceeds that of formulation 1 using $\alpha = 0.05$.

P-value $\cong P(z \le -5.75) = 1 - P(z \le 5.75) = 1 - 1 \cong 0$

b) 95% confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(75.6 - 77.9) - 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}} \le \mu_1 - \mu_2 \le (75.6 - 77.9) + 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}}$$

$$-3.084 \le \mu_1 - \mu_2 \le -1.516$$

With 95% confidence, the mean road octane number for formulation 2 exceeds that of formulation 1 by between 1.516 and 3.084.

c) 95% level of confidence, E = 1, and $z_{0.025}$ =1.96

$$n \cong \left(\frac{z_{0.025}}{E}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right) = \left(\frac{1.96}{1}\right)^2 (1.5 + 1.2) = 10.37,$$

Use $n_1 = n_2 = 11$

10-8 a)

1) The parameter of interest is the difference in mean batch viscosity before and after the process change, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 10$

3) $H_1: \mu_1 - \mu_2 < 10$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.1} = -1.28$ for $\alpha = 0.10$

6) $\bar{x}_1 = 750.2$ $\bar{x}_2 = 756.88$ $\Delta_0 = 10$

$\sigma_1 = 20$ $\sigma_2 = 20$

$n_1 = 15$ $n_2 = 8$

$$z_0 = \frac{(750.2 - 756.88) - 10}{\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}}} = -1.90$$

7) Conclusion: Because $-1.90 < -1.28$ reject the null hypothesis and conclude the process change has increased the mean by less than 10.

P-value = $P(Z \le -1.90) = 1 - P(Z \le 1.90) = 1 - 0.97128 = 0.02872$

b) <u>Case 1: Before Process Change</u>

μ_1 = mean batch viscosity before change

$\bar{x}_1 = 750.2$

$\sigma_1 = 20$

$n_1 = 15$

<u>Case 2: After Process Change</u>

μ_2 = mean batch viscosity after change

$\bar{x}_2 = 756.88$

$\sigma_2 = 20$

$n_2 = 8$

90% confidence on $\mu_1 - \mu_2$, the difference in mean batch viscosity before and after process change:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(750.2 - 756.88) - 1.645\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}} \le \mu_1 - \mu_2 \le \left(750.2 - 756.88\right) + 1.645\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}}$$

$$-21.08 \le \mu_1 - \mu_2 \le 7.72$$

We are 90% confident that the difference in mean batch viscosity before and after the process change lies within −21.08 and 7.72. Because zero is contained in this interval, we fail to detect a difference in the mean batch viscosity from the process change.

c) Parts (a) and (b) conclude that the mean batch viscosity change is less than 10. This conclusion is obtained from the confidence interval because the interval does not contain the value 10. The upper endpoint of the confidence interval is only 7.72.

10-9 Catalyst 1 | Catalyst 2

$\bar{x}_1 = 65.22$ | $\bar{x}_2 = 68.42$

$\sigma_1 = 3$ | $\sigma_2 = 3$

$n_1 = 10$ | $n_2 = 10$

a) 95% confidence interval on $\mu_1 - \mu_2$, the difference in mean active concentration

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(65.22 - 68.42) - 1.96\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}} \le \mu_1 - \mu_2 \le \left(65.22 - 68.42\right) + 1.96\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}}$$

$$-5.83 \le \mu_1 - \mu_2 \le -0.57$$

We are 95% confident that the mean active concentration of catalyst 2 exceeds that of catalyst 1 by between 0.57 and 5.83 g/l.

P-value:

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(65.22 - 68.42)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}} = -2.38$$

Then P-value = 2(0.008656) = 0.0173

b) Yes, because the 95% confidence interval does not contain the value zero. We conclude that the mean active concentration depends on the choice of catalyst.

c)

$$\beta = \Phi\left(1.96 - \frac{(5)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right) - \Phi\left(-1.96 - \frac{(5)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right)$$

$$= \Phi(-1.77) - \Phi(-5.69) = 0.038364 - 0$$

$$= 0.038364$$

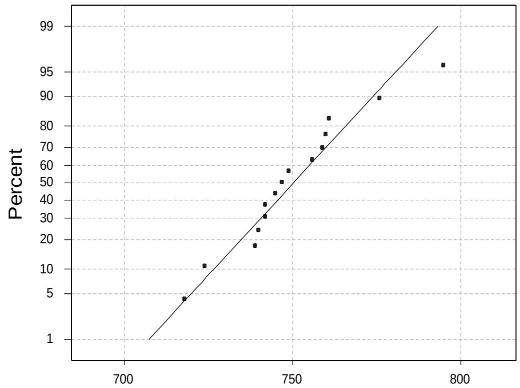
Power = 1 – β = 1– 0.038364 = 0.9616.

d) Calculate the value of n using α and β.

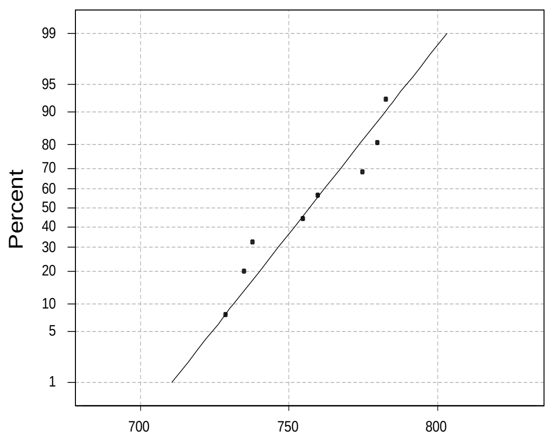
$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(\Delta - \Delta_0)^2} = \frac{(1.96 + 1.77)^2(9 + 9)}{(5)^2} = 10.02,$$

Therefore, 10 is only slightly too few samples. The sample sizes are adequate to detect the difference of 5.

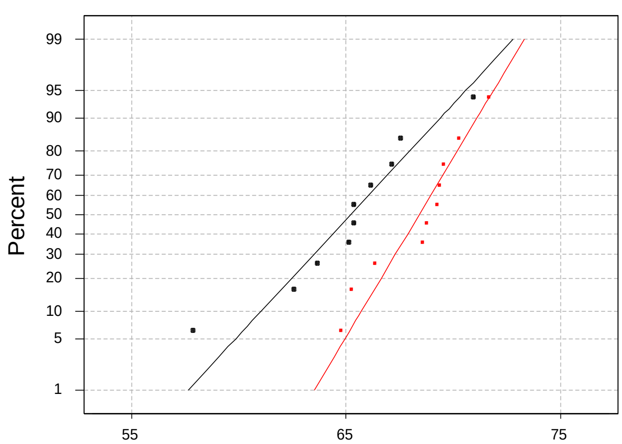
The data from the first sample n = 15 appear to be normally distributed.



The data from the second sample n = 8 appear to be normally distributed



Plots for both samples are shown in the following figure.



Section 10-2

10-10 a) $\bar{x}_1 = 8.74$ $\bar{x}_2 = 9.95$ $s_1^2 = 1.26^2$ $s_2^2 = 1.99^2$ $n_1 = 12$ $n_2 = 16$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(12-1)1.26^2 + (16-1)1.99^2}{12+16-2}} = 1.7194$$

Degree of freedom = $n_1 + n_2 - 2 = 12 + 16 - 2 = 26$.

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(-1.21)}{1.7194\sqrt{\frac{1}{12}+\frac{1}{16}}} = -1.8428$$

P-value = 2[P(t > 1.8428)] and 2(0.025) < P-value < 2(0.05) = 0.05 < P-value < 0.1

This is a two-sided test because the hypotheses are mu1 – mu2 = 0 versus not equal to 0.

b) Because 0.05 < P-value < 0.1 the P-value is greater than $\alpha = 0.05$. Therefore, we fail to reject the null hypothesis of $\mu_1 - \mu_2 = 0$ at the 0.05 and 0.01 levels of significance.

c) Yes, the sample standard deviations are somewhat different, but not excessively different. Consequently, the assumption that the two population variances are equal is reasonable.

d) P-value = P (t < -1.8428) and 0.025 < P-value < 0.05
Because 0.025 < P-value < 0.05, the P-value is less than $\alpha = 0.05$. Therefore, we reject the null hypothesis of $\mu_1 - \mu_2 = 0$ at the 0.05 level of significance.

10-11 a) $\bar{x}_1 = 68.39$ $\bar{x}_2 = 72.30$ $s_1^2 = 2.13^2$ $s_2^2 = 5.28^2$ $n_1 = 15$ $n_2 = 20$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1-1} + \frac{\left(s_2^2/n_2\right)^2}{n_2-1}} = \frac{\left(\frac{2.13^2}{15} + \frac{5.28^2}{20}\right)^2}{\frac{\left(2.13^2/15\right)^2}{15-1} + \frac{\left(5.28^2/20\right)^2}{20-1}} = 26.45 \approx 26 \text{ (truncated)}$$

The 95% upper one-sided confidence interval: $t_{0.05,26} = 1.706$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (68.39 - 72.30) + 1.706\sqrt{\frac{(2.13)^2}{15} + \frac{(5.28)^2}{20}}$$

$$\mu_1 - \mu_2 \le -1.6880$$

P-value = P(t < –3.00): 0.0025 < P-value < 0.005

This is one-sided test because the hypotheses are mu1 – mu2 = 0 versus less than 0.

b) Because 0.0025 < P-value < 0.005 the P-value < $\alpha = 0.05$. Therefore, we reject the null hypothesis of mu1 – mu2 = 0 at the 0.05 or the 0.01 level of significance.

c) Yes, the sample standard deviations are quite different. Consequently, one would not want to assume that the population variances are equal.

d) If the alternative hypothesis were changed to mu1 – mu2 ≠ 0, then the P-value = 2P (t < –3.00) and 0.005 < P-value < 0.01. Because the P-value < $\alpha = 0.05$, we reject the null hypothesis of mu1 – mu2 = 0 at the 0.05 level of significance.

10-12 a)

1) The parameter of interest is the difference in mean, $\mu_1 - \mu_2$

2) $\text{H}_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $\text{H}_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,28} = -2.048$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,28} = 2.048$ for $\alpha = 0.05$

6) $\overline{x}_1 = 5.7 \qquad \overline{x}_2 = 8.3 \qquad s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1^2 = 4 \qquad s_2^2 = 6.25 \qquad = \sqrt{\dfrac{14(4)+14(6.25)}{28}} = 2.26$

$n_1 = 15 \qquad n_2 = 15$

$$t_0 = \frac{(5.7 - 8.3)}{2.26\sqrt{\frac{1}{15} + \frac{1}{15}}} = -3.15$$

7) Conclusion: Because $-3.15 < -2.048$, reject the null hypothesis at $\alpha = 0.05$.

P-value = $P\left(t > 3.15\right) <$ 2(0.0025), P-value < 0.005

b) 95% confidence interval: $t_{0.025,28} = 2.048$

$$\left(\overline{x}_1 - \overline{x}_2\right) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq \left(\overline{x}_1 - \overline{x}_2\right) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(5.7-8.3) - 2.048(2.26)\sqrt{\frac{1}{15} + \frac{1}{15}} \leq \mu_1 - \mu_2 \leq (5.7-8.3) + 2.048(2.26)\sqrt{\frac{1}{15} + \frac{1}{15}}$$

$$-4.29 \leq \mu_1 - \mu_2 \leq -0.91$$

Because zero is not contained in this interval, we are 95% confident that the means are different.

c) $\Delta = 3$ Use s_p as an estimate of σ:

d = $\dfrac{\mu_2 - \mu_1}{2s_p} = \dfrac{3}{2(2.26)} = 0.66$

Using Chart VII (e) with $d = 0.66$ and $n = n_1 = n_2$ we obtain $n^* = 2n - 1 = 29$ and $\alpha = 0.05$. Therefore, $\beta = 0.1$ and the power is $1 - \beta = 0.9$

d) $\beta = 0.05$, $d = \dfrac{2}{2(2.26)} = 0.44$, therefore $n^* \cong 75$ then $n = \dfrac{n*+1}{2} = 38$, then $n = n_1 = n_2 = 38$

10-13 a)

1) The parameter of interest is the difference in means, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $\text{H}_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $\text{H}_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha, n_1+n_2-2}$ where $-t_{0.05,28} = -1.701$ for $\alpha = 0.05$

6) $\bar{x}_1 = 7.2 \quad \bar{x}_2 = 7.9$ $\qquad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1^2 = 4 \quad s_2^2 = 6.25$ $\qquad = \sqrt{\frac{14(4)+14(6.25)}{28}} = 2.26$

$n_1 = 15 \quad n_2 = 15$

$$t_0 = \frac{(7.2-7.9)}{2.26\sqrt{\frac{1}{15}+\frac{1}{15}}} = -0.85$$

7) Conclusion: Because $-0.85 > -1.701$ we fail to reject the null hypothesis at the 0.05 level of significance.
P-value = P $(t > 0.85)$, $0.1 < P\text{-value} < 0.25$

b) 95% confidence interval: $t_{0.05,28} = 1.701$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\mu_1 - \mu_2 \le (7.2-7.9) + 1.701(2.26)\sqrt{\frac{1}{15}+\frac{1}{15}}$$

$$\mu_1 - \mu_2 \le 0.704$$

Because zero is contained in this interval, we are 95% confident that $\mu_1 > \mu_2$

c) $\Delta = 3$ Use s_p as an estimate of σ:

d = $\frac{\mu_2 - \mu_1}{2s_p} = \frac{3}{2(2.26)} = 0.66$

Using Chart VII (g) with d = 0.66 and $n = n_1 = n_2$ we get $n^* = 2n - 1 = 29$ and $\alpha = 0.05$. Therefore, $\beta = 0.05$ and the power is $1 - \beta = 0.95$

d) $\beta = 0.05$, d = $\frac{2.5}{2(2.26)} = 0.55$. Therefore $n^* \cong 40$ and $n = \frac{n*+1}{2} \cong 21$. Thus, $n = n_1 = n_2 = 21$

10-14 a)

1) The parameter of interest is the difference in means, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{\alpha, n_1+n_2-2}$ where $t_{0.05,18} = 1.734$ for $\alpha = 0.05$

6)

$\bar{x}_1 = 7.8 \qquad \bar{x}_2 = 5.6$ $\qquad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1^2 = 4 \qquad s_2^2 = 6.25$ $\qquad = \sqrt{\dfrac{9(4)+9(6.25)}{18}} = 2.26$

$n_1 = 10 \qquad n_2 = 10$

$$t_0 = \frac{(7.8-5.6)}{2.26\sqrt{\frac{1}{10}+\frac{1}{10}}} = 2.17$$

7) Conclusion: Because 2.17 > 1.734 reject the null hypothesis at the 0.05 level of significance.

P-value = $P(t > 2.17)$ and 0.001 < P-value < 0.025

b) 95% confidence interval:

$$\mu_1 - \mu_2 \geq (\bar{x}_1 - \bar{x}_2) - t_{\alpha, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\mu_1 - \mu_2 \geq (7.8-5.6) - 1.734(2.26)\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$\mu_1 - \mu_2 \geq 0.45$$

Because zero is not contained in this interval, we reject the null hypothesis.

c) $\Delta = 3$ Use s_p as an estimate of σ:

$$d = \frac{\mu_2 - \mu_1}{2s_p} = \frac{3}{2(2.26)} = 0.66$$

Using Chart VII (g) with d = 0.66 and $n = n_1 = n_2 = 10$ we obtain $n^* = 2n - 1 = 19$ and α = 0.05. Therefore, β ≈ 0.17 and the power is 1-β =0.83

d) β = 0.05, d = $\dfrac{3}{2(2.26)}$ = 0.66, therefore $n^* \cong 30$. Finally, $n = \dfrac{n^*+1}{2} \cong 16$, and $n = n_1 = n_2 = 16$

10-15 a)

1) The parameter of interest is the difference in mean rod diameter, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,31} = -2.04$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,31} = 2.04$ for α = 0.05

6) $\bar{x}_1 = 8.73 \qquad \bar{x}_2 = 8.68$ $\qquad s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1^2 = 0.35 \qquad s_2^2 = 0.90$ $\qquad = \sqrt{\dfrac{14(0.35)+17(0.90)}{31}} = 0.807$

$n_1 = 15 \qquad n_2 = 18$

$$t_0 = \frac{(8.73-8.68)}{0.807\sqrt{\frac{1}{15}+\frac{1}{18}}} = 0.177$$

7) Conclusion: Because –2.04 < 0.177 < 2.04, we fail to reject the null hypothesis. There is insufficient evidence to conclude that the two machines produce different mean diameters at $\alpha = 0.05$.

P-value = $2P\left(t > 0.177\right) > 2(0.40)$, P-value > 0.80

b) 95% confidence interval: $t_{0.025,31} = 2.04$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(8.73-8.68) - 2.04(0.807)\sqrt{\frac{1}{15}+\frac{1}{18}} \le \mu_1 - \mu_2 \le \left(8.73-8.68\right) + 2.04(0.807)\sqrt{\frac{1}{15}+\frac{1}{18}}$$

$$-0.526 \le \mu_1 - \mu_2 \le 0.626$$

Because zero is contained in this interval, there is insufficient evidence to conclude that the two machines produce rods with different mean diameters.

10-16 a) Assume the populations follow normal distributions and $\sigma_1^2 = \sigma_2^2$. The assumption of equal variances may be relaxed in this case because it is known that the t-test and confidence intervals involving the t-distribution are robust to the assumption of equal variances when sample sizes are equal.

Case 1: AFFF	Case 2: ATC
μ_1 = mean foam expansion for AFFF	μ_2 = mean foam expansion for ATC
$\bar{x}_1 = 5.0$	$\bar{x}_2 = 7.2$
$s_1 = 0.6$	$s_2 = 0.8$
$n_1 = 5$	$n_2 = 5$

95% confidence interval: $t_{0.025,8} = 2.306$ $\qquad s_p = \sqrt{\frac{4(0.60^2)+4(0.80^2)}{8}} = 0.7071$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(5.0-7.2) - 2.306(0.7071)\sqrt{\frac{1}{5}+\frac{1}{5}} \le \mu_1 - \mu_2 \le \left(5.0-7.2\right) + 2.306(0.7071)\sqrt{\frac{1}{5}+\frac{1}{5}}$$

$$-3.23 \le \mu_1 - \mu_2 \le -1.17$$

b) Yes, with 95% confidence, the mean foam expansion for ATC exceeds that of AFFF by between 1.17 and 3.23 units.

10-17 a) 1) The parameter of interest is the difference in mean catalyst yield, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha,n_1+n_2-2}$ where $-t_{0.01,25} = -2.485$ for $\alpha = 0.01$

6) $\bar{x}_1 = 86$ $\quad \bar{x}_2 = 89$

$s_1 = 3$ $\quad s_2 = 2$

$n_1 = 12$ $\quad n_2 = 15$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{11(3)^2 + 14(2)^2}{25}} = 2.4899$$

$$t_0 = \frac{(86-89)}{2.4899\sqrt{\frac{1}{12}+\frac{1}{15}}} = -3.11$$

7) Conclusion: Because $-3.11 < -2.485$, reject the null hypothesis and conclude that the mean yield of catalyst 2 exceeds that of catalyst 1 at $\alpha = 0.01$.

b) 99% upper confidence interval $\mu_1 - \mu_2$: $t_{0.01,25} = 2.485$

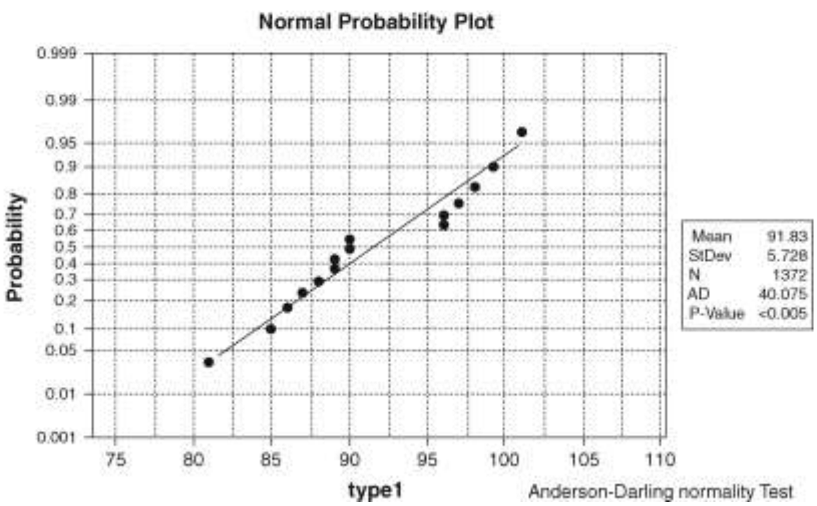
$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

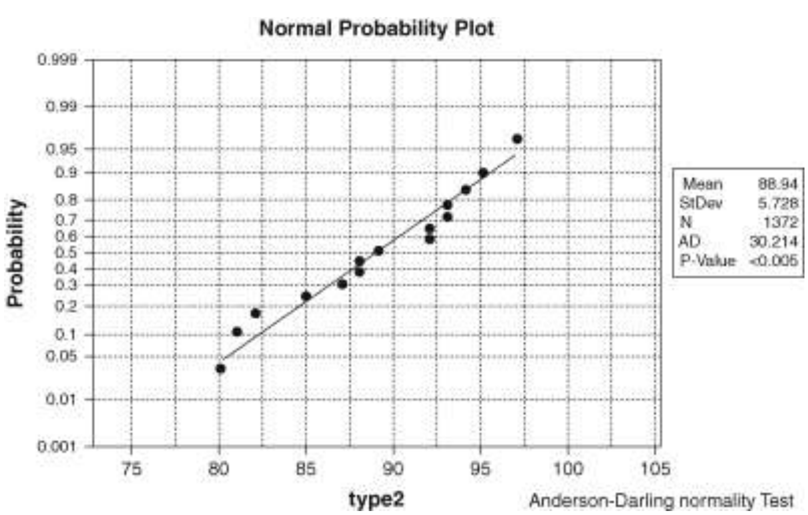
$$\mu_1 - \mu_2 \le (86-89) + 2.485(2.4899)\sqrt{\frac{1}{12}+\frac{1}{15}}$$

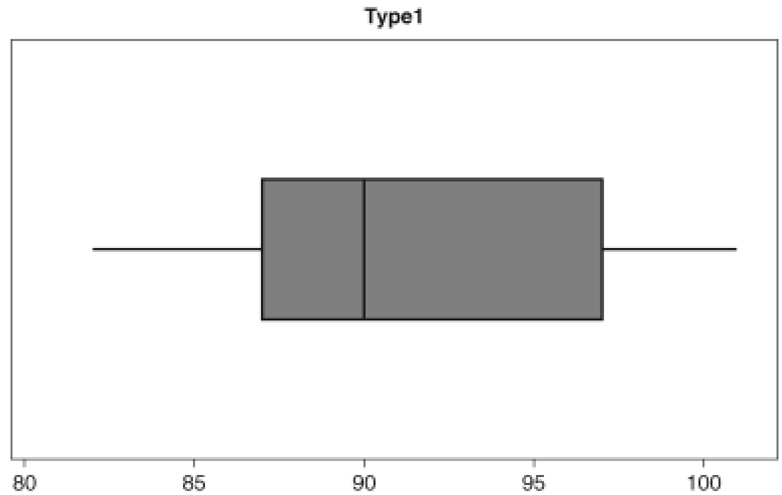
$\mu_1 - \mu_2 \le -0.603$ or equivalently $\mu_1 + 0.603 \le \mu_2$

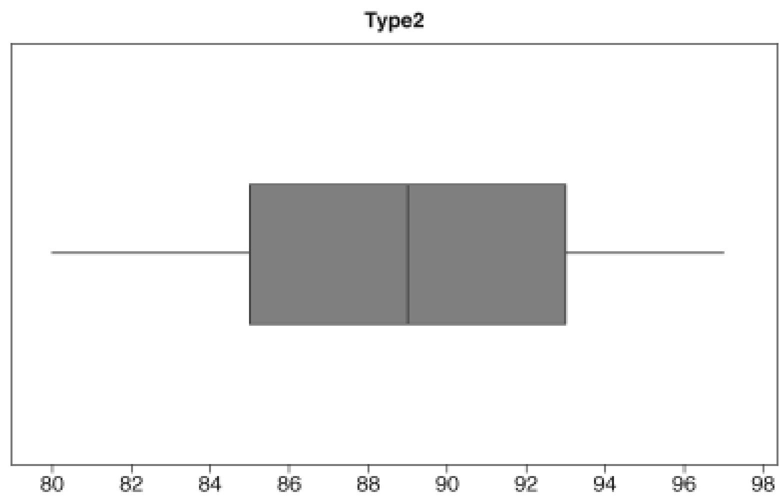
We are 99% confident that the mean yield of catalyst 2 exceeds that of catalyst 1 by at least 0.603 units.

10-18 a) According to the normal probability plots, the assumption of normality is reasonable because the data fall approximately along straight lines. The equality of variances does not appear to be severely violated either because the slopes are approximately the same for both samples.









b) 1) The parameter of interest is the difference in deflection temperature under load, $\mu_1 - \mu_2$, with $\Delta_0 = 0$ 2) H_0 : $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1 : $\mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{\alpha, n_1+n_2-2}$ where $t_{0.05,28} = 1.701$ for $\alpha = 0.05$

6) Type 1 Type 2

$\bar{x}_1 = 91.47$ $\bar{x}_2 = 89.07$ $$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$s_1 = 5.93$ $s_2 = 5.28$ $$s_p = \sqrt{\frac{14(5.93)^2 + 14(5.28)^2}{28}} = 5.61$$

$n_1 = 15$ $n_2 = 15$

$$t_0 = \frac{(91.47 - 89.07)}{5.61\sqrt{\frac{1}{15} + \frac{1}{15}}} = 1.17$$

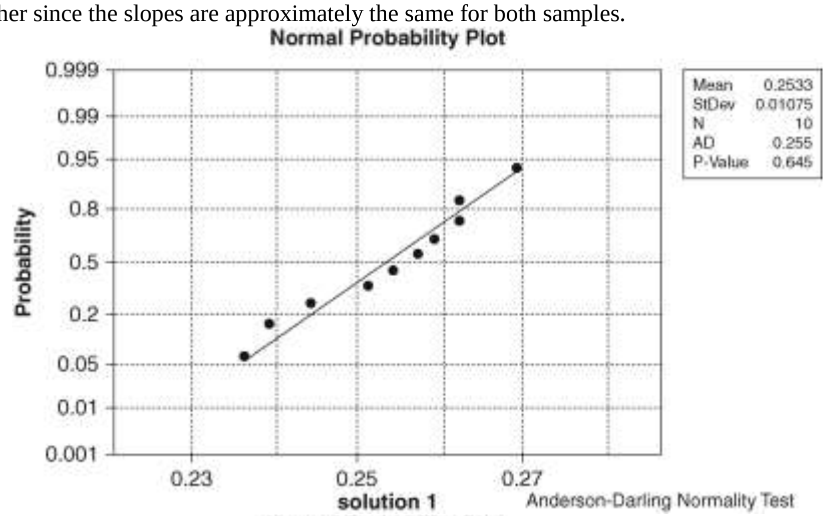
7) Conclusion: Because 1.17 < 1.701 we fail to reject the null hypothesis. There is insufficient evidence to conclude that the mean deflection temperature under load for Type 1 exceeds the mean for Type 2 at the 0.05 level of significance.
P-value = $P\left(t > 1.17\right)$, $0.1 < P\text{-value} < 0.25$

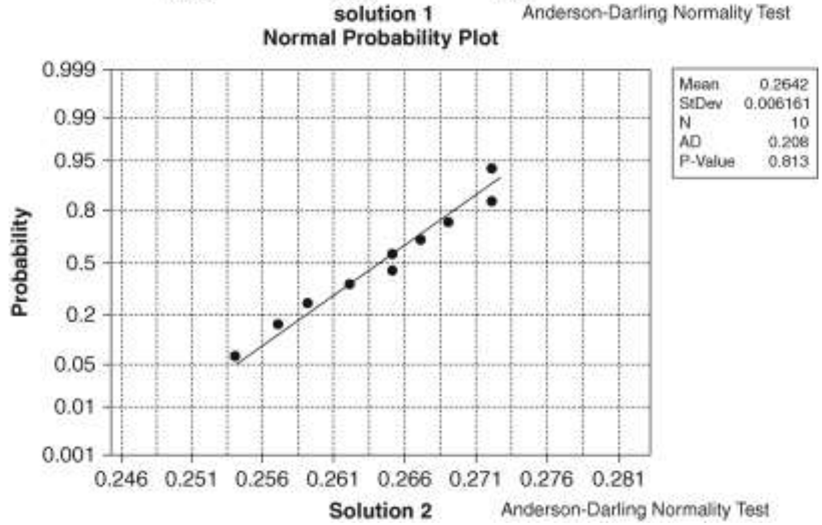
c) $\Delta = 5$ Use s_p as an estimate of σ:

$$d = \frac{\mu_1 - \mu_2}{2s_p} = \frac{5}{2(5.61)} = 0.446$$

Using Chart VII (g) with $\beta = 0.10$, $d = 0.446$ we get $n^* \cong 40$. Because $n^* = 2n - 1$, $n_1 = n_2 = 21$. Therefore, the sample sizes of 15 are not adequate to meet the given probability of detection.

10-19 a) According to the normal probability plots, the assumption of normality appears to be reasonable because the data from both the samples fall approximately along a straight line. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.





b)

1) The parameter of interest is the difference in mean etch rate, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,18} = -2.101$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,18} = 2.101$ for $\alpha = 0.05$

6) $\bar{x}_1 = 0.2533$ $\quad \bar{x}_2 = 0.2642$ $\quad s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 0.011$ $\quad s_2 = 0.006$ $\quad s_p = \sqrt{\dfrac{9(0.011)^2 + 9(0.006)^2}{18}} = 0.0089$

$n_1 = 10$ $n_2 = 10$

$$t_0 = \frac{(0.2533 - 0.2642)}{0.0089\sqrt{\frac{1}{10}+\frac{1}{10}}} = -2.74$$

7) Conclusion: Because $-2.74 < -2.101$ reject the null hypothesis and conclude the two machines mean etch rates differ at $\alpha = 0.05$.

P-value = $2P\left(t < -2.74\right)$ $\quad 2(0.005) < P\text{-value} < 2(0.010) = 0.010 < P\text{-value} < 0.020$

c) 95% confidence interval: $t_{0.025,18} = 2.101$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(0.2533 - 0.2642) - 2.101(0.0089)\sqrt{\frac{1}{10}+\frac{1}{10}} \le \mu_1 - \mu_2 \le (0.2533 - 0.2642) + 2.101(0.0089)\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$-0.01926 \le \mu_1 - \mu_2 \le -0.00254$$

We are 95% confident that the mean etch rate for solution 2 exceeds the mean etch rate for solution 1 by between 0.00254 and 0.01926.

10-20 a)

1) The parameter of interest is the difference in mean impact strength, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,23} = 1.714$ for $\alpha = 0.05$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)}{n_2-1}} = 23.21$$

$\nu \cong 23$

(truncated)

6) $\bar{x}_1 = 395$ $\quad \bar{x}_2 = 435$

$s_1 = 15$ $\quad s_2 = 30$

$n_1 = 10$ $\quad n_2 = 16$

$$t_0 = \frac{(395 - 435)}{\sqrt{\frac{15^2}{10}+\frac{30^2}{16}}} = -4.51$$

7) Conclusion: Because –4.51 < –1.714 reject the null hypothesis and conclude that supplier 2 provides gears with higher mean impact strength at the 0.05 level of significance.
P-value = P(t < –4.51): *P*-value < 0.0005

b)
1) The parameter of interest is the difference in mean impact strength, $\mu_2 - \mu_1$

2) H$_0$: $\mu_2 - \mu_1 = 25$

3) H$_1$: $\mu_2 - \mu_1 > 25$ or $\mu_2 > \mu_1 + 25$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_2 - \bar{x}_1) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{\alpha,\nu} = 1.714$ for $\alpha = 0.05$ where

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 23.21$$

$$\nu \cong 23$$

6) $\bar{x}_1 = 395$ $\bar{x}_2 = 435$ $\Delta_0 = 35$ $s_1 = 15$ $s_2 = 30$ $n_1 = 10$ $n_2 = 16$

$$t_0 = \frac{(435 - 395) - 35}{\sqrt{\frac{15^2}{10} + \frac{30^2}{16}}} = 0.563$$

7) Conclusion: Because 0.563 < 1.714, fail to reject the null hypothesis. There is insufficient evidence to conclude that the mean impact strength from supplier 2 is at least 35 Nm higher than from supplier 1 using $\alpha = 0.05$.

c) Using the information provided in part (a), and $t_{0.025,25} = 2.069$, a 95% confidence interval on the difference $\mu_2 - \mu_1$ is

$$(\bar{x}_2 - \bar{x}_1) - t_{0.025,25}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_2 - \mu_1 \le (\bar{x}_2 - \bar{x}_1) + t_{0.025,25}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$40 - 2.069(8.874) \le \mu_2 - \mu_1 \le 40 + 2.069(8.874)$$

$$21.64 \le \mu_2 - \mu_1 \le 58.36$$

Because zero is not contained in the confidence interval, we conclude that supplier 2 provides gears with a higher mean impact strength than supplier 1 with 95% confidence.

10-21 a)
1) The parameter of interest is the difference in mean melting point, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) H$_0$: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H$_1$: $\mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.0025,40} = -2.021$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,40} =$ 2.021 for $\alpha = 0.05$

6) $\bar{x}_1 = 215$ $\bar{x}_2 = 219$ $s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 2$ $s_2 = 1.7$ $= \sqrt{\dfrac{20(2)^2 + 20(1.7)^2}{40}} = 1.856$

$n_1 = 21$ $n_2 = 21$

$$t_0 = \frac{(215-219)}{1.856\sqrt{\frac{1}{21}+\frac{1}{21}}} = -6.984$$

7) Conclusion: Because –6.984 < –2.021 reject the null hypothesis. The alloys differ significantly in mean melting point at α = 0.05.

P-value = $2P(t < -6.984)$ P-value < 0.0010

b) $d = \dfrac{|\mu_1-\mu_2|}{2\sigma} = \dfrac{1.7}{2(2)} = 0.425$

Using the appropriate chart in the Appendix, with β = 0.10 and α = 0.05 we have $n^* = 75$.

Therefore, $n = \dfrac{n^*+1}{2} = 38$, $n_1 = n_2 = 38$

10-22 a)

1) The parameter of interest is the difference in mean speed, $\mu_1 - \mu_2$, $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{\alpha, n_1+n_2-2}$ where $t_{0.10,14} = 1.345$ for α = 0.10

6) <u>Case 0: 65 mm</u> <u>Case 2: 0.5 mm</u>

$\bar{x}_1 = 0.03$ $\bar{x}_2 = 0.027$ $s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 0.0028$ $s_2 = 0.0023$ $= \sqrt{\dfrac{7(0.0028)^2 + 7(0.0023)^2}{14}} = 2.56\times10^{-3}$

$n_1 = 8$ $n_2 = 8$

$$t_0 = \frac{(0.03-0.027)}{2.56\times10^{-3}\sqrt{\frac{1}{8}+\frac{1}{8}}} = (0.03-0.027)2.34$$

7) Because 2.34 > 1.345 reject the null hypothesis and conclude that reducing the film thickness from 0.65 mm to 0.5 mm significantly increases the mean speed of the film at the 0.10 level of significance (Note: an increase in film speed will result in *lower* values of observations).

P-value = $P(t > 2.34)$ 0.01 < P-value < 0.025

b) 95% confidence interval: $t_{0.025,14} = 2.145$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(0.03-0.027) - 2.145(2.56\times10^{-3})\sqrt{\frac{1}{8}+\frac{1}{8}} \le \mu_1 - \mu_2 \le (0.03-0.027) + 2.145(2.56\times10^{-3})\sqrt{\frac{1}{8}+\frac{1}{8}}$$

$$-0.00025 \le \mu_1 - \mu_2 \le 0.00575$$

We are 95% confident the difference in mean speed of the film is between 0.00025 and 0.00575 μJ/mm².

10-23 a)

1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.025,26}$ or $t_0 > t_{0.025,26}$ where $t_{0.025,26} = 2.056$ for $\alpha = 0.05$ because

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 26.98$$

$$\nu \cong 26$$

6) $\bar{x}_1 = 25$ $\bar{x}_2 = 20$

$s_1 = 2$ $s_2 = 8$

$n_1 = 25$ $n_2 = 25$

$$t_0 = \frac{(25 - 20)}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

7) Conclusion: Because 3.03 > 2.056 reject the null hypothesis. The data support the claim that the two companies produce material with significantly different wear at the 0.05 level of significance.

P-value = 2P(t > 3.03), 2(0.0025) < P-value < 2(0.005), 0.005 < P-value < 0.010

b)

1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{0.05,27}$ where $t_{0.05,26} = 1.706$ for $\alpha = 0.05$ since

6) $\bar{x}_1 = 25$ $\bar{x}_2 = 20$

$s_1 = 2$ $s_2 = 8$

$n_1 = 25$ $n_2 = 25$

$$t_0 = \frac{(25 - 20)}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

7) Conclusion: Because 3.03 > 1.706 reject the null hypothesis. The data support the claim that the material from company 1 has a higher mean wear than the material from company 2 at a 0.05 level of significance.

c) For part (a) use a 95% two-sided confidence interval:
$t_{0.025,26} = 2.056$

$$\left(\overline{x}_1-\overline{x}_2\right)-t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\le\mu_1-\mu_2\le\left(\overline{x}_1-\overline{x}_2\right)+t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$
$$(25-20)-2.056\sqrt{\frac{(2)^2}{25}+\frac{(8)^2}{25}}\le\mu_1-\mu_2\le(25-20)+2.056\sqrt{\frac{(2)^2}{25}+\frac{(8)^2}{25}}$$
$$1.609\le\mu_1-\mu_2\le 8.391$$

For part (b) use a 95% lower one-sided confidence interval:

$$t_{0.05,26}=1.706$$
$$\left(\overline{x}_1-\overline{x}_2\right)-t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\le\mu_1-\mu_2$$
$$(25-20)-1.706\sqrt{\frac{(2)^2}{25}+\frac{(8)^2}{25}}\le\mu_1-\mu_2$$
$$2.186\le\mu_1-\mu_2$$

For part a) we are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by between 1.609 and 8.391 mg/1000.

For part b) we are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by at least 2.186 mg/1000.

10-24 a)

1) The parameter of interest is the difference in mean coating thickness, $\mu_1-\mu_2$, with $\Delta_0 = 0$.

2) $H_0: \mu_1-\mu_2=0$

3) $H_1: \mu_1-\mu_2>0$

4) The test statistic is

$$t_0=\frac{(\overline{x}_1-\overline{x}_2)-\delta}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{0.01,18}$ where $t_{0.01,18}$ = 2.552 for α = 0.01 since

$$\nu=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)}{n_2-1}}=18.23$$

$$\nu\cong 18$$

(truncated)

6) $\overline{x}_1=2.65$ $\quad\overline{x}_2=2.55$

$s_1=0.25$ $\quad s_2=0.5$ 20.1

$n_1 = 11$ $n_2 = 13$

$$t_0=\frac{(2.65-2.55)}{\sqrt{\frac{(0.25)^2}{11}+\frac{(0.5)^2}{13}}}=0.634$$

7) Conclusion: Because 0.634 < 2.552, fail to reject the null hypothesis. There is insufficient evidence to conclude that increasing the temperature reduces the mean coating thickness at α = 0.01.

P-value = P(t > 0.634), 0.25 < *P*-value < 0.40

b) If α = 0.01, construct a 99% two-sided confidence interval on the difference in means. Here,

$t_{0.005,19} = 2.878$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(2.65 - 2.55) - 2.878\sqrt{\frac{(0.25)^2}{11} + \frac{(0.5)^2}{13}} \le \mu_1 - \mu_2 \le (2.65 - 2.55) - 2.878\sqrt{\frac{(0.25)^2}{11} + \frac{(0.5)^2}{13}}$$

$$-0.354 \le \mu_1 - \mu_2 \le 0.554$$

Because the interval contains zero, there is no significant difference in the mean coating thickness between the two temperatures.

10-25 a)

1) The parameter of interest is the difference in mean width of the backside chip-outs for the single spindle saw process versus the dual spindle saw process , $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,28} = -2.048$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$

where $t_{0.025,28} = 2.048$ for $\alpha = 0.05$

6) $\bar{x}_1 = 66.385$ $\bar{x}_2 = 45.278$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$s_1^2 = 7.895^2$ $s_2^2 = 8.612^2$

$$= \sqrt{\frac{14(7.895)^2 + 14(8.612)^2}{28}} = 8.26$$

$n_1 = 15$ $n_2 = 15$

$$t_0 = \frac{(66.385 - 45.278)}{8.26\sqrt{\frac{1}{15} + \frac{1}{15}}} = 7.00$$

7) Conclusion: Because 7.00 > 2.048, we reject the null hypothesis at $\alpha = 0.05$. *P*-value $\cong 0$

b) 95% confidence interval: $t_{0.025,28} = 2.048$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(66.385 - 45.278) - 2.048(8.26)\sqrt{\frac{1}{15} + \frac{1}{15}} \le \mu_1 - \mu_2 \le (66.385 - 45.278) + 2.048(8.26)\sqrt{\frac{1}{15} + \frac{1}{15}}$$

$$14.93 \le \mu_1 - \mu_2 \le 27.28$$

Because zero is not contained in this interval, we reject the null hypothesis.

c) For $\beta < 0.01$ and d = $\frac{15}{2(8.26)} = 0.91$, with $\alpha = 0.05$ then using Chart VII (e) we find $n^* > 15$. Then $n > \frac{15+1}{2} = 8$

10-26 a)

1) The parameter of interest is the difference in mean blood pressure between the test and control groups, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,12} = -1.782$ for $\alpha = 0.05$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 12$$

$$\nu \cong 12$$

6)

$\bar{x}_1 = 90$ $\bar{x}_2 = 115$

$s_1 = 5$ $s_2 = 10$

$n_1 = 8$ $n_2 = 9$

$$t_0 = \frac{(90 - 115)}{\sqrt{\frac{(5)^2}{8} + \frac{(10)^2}{9}}} = -6.63$$

7) Conclusion: Because –6.62 < –1.782 reject the null hypothesis and conclude that the test group has higher mean arterial blood pressure than the control group at the 0.05 level of significance.

P-value = P(t < –6.62): P-value ≅ 0

b) 95% confidence interval: $t_{0.05,12} = 1.782$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (90 - 115) + 1.782\sqrt{\frac{5^2}{8} + \frac{10^2}{9}}$$

$$\mu_1 - \mu_2 \le -18.28$$

Because zero is not contained in this interval, we reject the null hypothesis.

c)

1) The parameter of interest is the difference in mean blood pressure between the test and control groups, $\mu_1 - \mu_2$, with $\Delta_0 = -15$

2) $H_0: \mu_1 - \mu_2 = -15$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < -15$ or $\mu_1 < \mu_2 - 15$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,12} = -1.782$ for $\alpha = 0.05$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 12$$

$$\nu \cong 12$$

6) $\bar{x}_1 = 90 \quad \bar{x}_2 = 115$

$s_1 = 5 \quad s_2 = 10$
$n_1 = 8 \quad n_2 = 9$

$$t_0 = \frac{(90 - 115) + 15}{\sqrt{\frac{(5)^2}{8} + \frac{(10)^2}{9}}} = -2.65$$

7) Conclusion: Because $-2.65 < -1.782$ reject the null hypothesis and conclude that the test group has higher mean arterial blood pressure than the control group at the 0.05 level of significance.

d) 95% confidence interval: $t_{0.05,12} = 1.782$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le -18.28$$

Because -15 is greater than the values in this interval, we are 95% confident that the mean for the test group is at least 15 mmHg higher than the control group.

10-27 a)

1) The parameter of interest is the difference in mean number of periods in a sample of 200 trains for two different levels of noise voltage, 100mv and 150mv

$\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{\alpha, n_1 + n_2 - 2}$ where $t_{0.05,198} = 1.645$ for $\alpha = 0.05$

6)

$\bar{x}_1 = 7.9 \quad \bar{x}_2 = 6.9$

$s_1 = 2.6 \quad s_2 = 2.4$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{99(2.6)^2 + 99(2.4)^2}{198}} = 2.5$$

$n_1 = 100 \qquad n_2 = 100$

$$t_0 = \frac{(7.9 - 6.9)}{2.5\sqrt{\frac{1}{100} + \frac{1}{100}}} = 2.82$$

7) Conclusion: Because 2.82 > 1.645, reject the null hypothesis at the 0.05 level of significance.

P-value = $P(t > 2.82)$ $\quad$ P-value $\cong 0.0025$

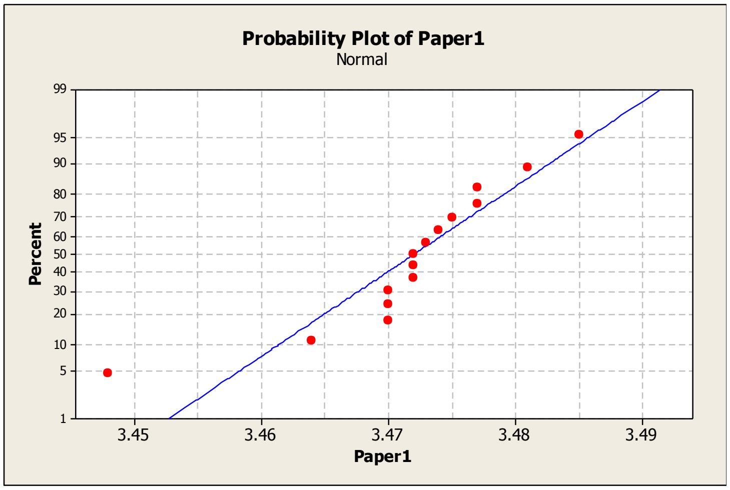
b) 95% confidence interval: $t_{0.05,198} = 1.645$

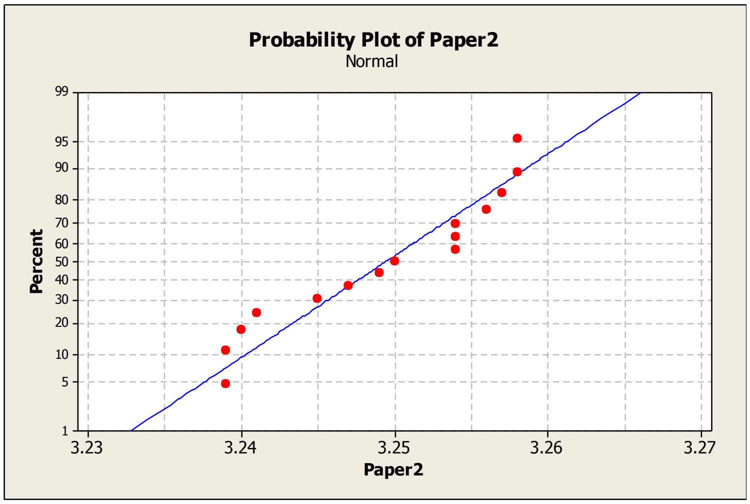
$$\mu_1 - \mu_2 \geq (\bar{x}_1 - \bar{x}_2) - t_{\alpha, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\mu_1 - \mu_2 \geq 0.418$$

Because zero is not contained in this interval, reject the null hypothesis.

10-28 a) The probability plots below show that the normality assumptions are reasonable for both data sets.





b)

1) The parameter of interest is the difference in mean weight of two sheets of paper, $\mu_1 - \mu_2$. Assume equal variances.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,28} = -2.048$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,28} = 2.048$ for $\alpha = 0.05$

6) $\bar{x}_1 = 3.472$ $\quad \bar{x}_2 = 3.2494$

$s_1^2 = 0.00831^2 \quad s_2^2 = .00714^2$

$n_1 = 15 \quad n_2 = 15$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{14(.00831)^2 + 14(0.00714)^2}{28}} = .00775$$

$t_0 = 78.66$

7) Conclusion: Because 78.66 >2.048, reject the null hypothesis at $\alpha = 0.05$.

P-value $\cong 0$

c)

1) The parameter of interest is the difference in mean weight of two sheets of paper, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.005,28} = -2.763$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.005,28} = 2.763$ for $\alpha = 0.01$

6) $\bar{x}_1 = 3.472$ $\quad \bar{x}_2 = 3.2494$

$s_1^2 = 0.00831^2 \quad s_2^2 = .00714^2$

$n_1 = 15 \quad n_2 = 15$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{14(.00831)^2 + 14(0.00714)^2}{28}} = .00775$$

$t_0 = 78.66$

7) Conclusion: Because 78.66 > 2.763, reject the null hypothesis at $\alpha = 0.01$. *P*-value $\cong 0$

d) The answer is the same because the decision to reject the null hypothesis made in part (b) was at a lower level of significance than the test in (c). Therefore, the decision is the same for any value of α larger than that used in part (b).

Alternatively, the P-value from part (b) is essentially 0, meaning that for any level of α greater than or equal to the P-value, the decision is to reject the null hypothesis.

e) 95% confidence interval for part (b): $t_{0.025,28} = 2.048$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$0.216 \le \mu_1 - \mu_2 \le 0.228$$

Because zero is not contained in this interval we reject the null hypothesis.

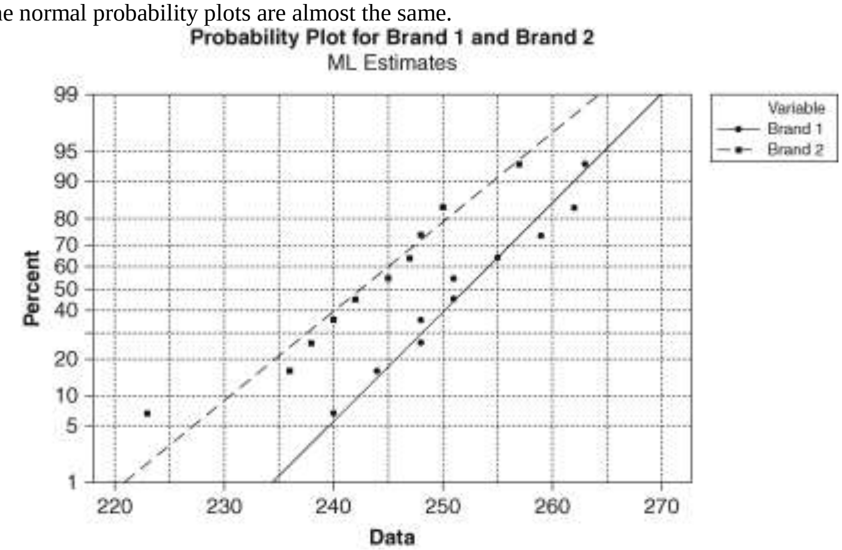
99% confidence interval for part (c): $t_{0.005,28} = 2.763$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$0.215 \le \mu_1 - \mu_2 \le 0.230$$

Because zero is not contained in this interval we reject the null hypothesis.

10-29 a) The data appear to be normally distributed and the variances appear to be approximately equal. The slopes of the lines on the normal probability plots are almost the same.



b)

1) The parameter of interest is the difference in mean overall distance, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,18} = 2.101$ for $\alpha = 0.05$

6) $\bar{x}_1 = 252.1$ $\quad \bar{x}_2 = 242.6$ $\quad s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 7.61$ $\quad s_2 = 9.26$ $\quad = \sqrt{\dfrac{9(7.61)^2 + 9(9.26)^2}{20}} = 8.04$

$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(252.1 - 242.6)}{8.04\sqrt{\frac{1}{10}+\frac{1}{10}}} = 2.642$$

7) Conclusion: Because 2.642 > 2.101 reject the null hypothesis. The data support the claim that the means differ at α = 0.05.

P-value = 2P $(t > 2.642)$ *P*-value ≈ 2(0.01) = 0.02

c) $$(\bar{x}_1 - \bar{x}_2) - t_{\alpha,\nu} s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha,\nu} s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

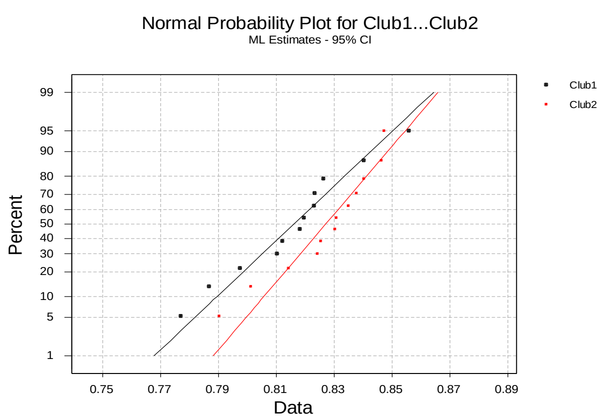
$$(252.1 - 242.6) - 2.101(8.04)\sqrt{\frac{1}{10}+\frac{1}{10}} \le \mu_1 - \mu_2 \le (252.1 - 242.6) + 2.101(8.04)\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$1.94 \le \mu_1 - \mu_2 \le 17.05$$

d) $d = \frac{4.5}{2(8.04)} = 0.28$ $\quad \beta = 0.95$ $\quad$ Power = 1 – 0.95 = 0.05

e) $\beta = 0.25$ $\quad d = \frac{2.75}{2(8.04)} = 0.171$ $\quad$ *n** = 100 Therefore, *n* = 51

10-30 a) The data appear to be normally distributed and the variances appear to be approximately equal. The slopes of the lines on the normal probability plots are almost the same.



b)

1) The parameter of interest is the difference in mean coefficient of restitution, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,22} = 2.074$ for α = 0.05

6) $\bar{x}_1 = 0.8161$ $\quad \bar{x}_2 = 0.8271$ $\quad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 0.0217$ $\quad s_2 = 0.0175$ $\quad = \sqrt{\frac{11(0.0217)^2 + 11(0.0175)^2}{22}} = 0.01971$

n_1 = 12 n_2 = 12

$$t_0 = \frac{(0.8161 - 0.8271)}{0.01971\sqrt{\frac{1}{12} + \frac{1}{12}}} = -1.367$$

7) Conclusion: Because –1.367 > –2.074 fail to reject the null hypothesis. The data do not support the claim that there is a difference in the mean coefficients of restitution for club1 and club2 at α = 0.05

P-value = 2P$\left(t < -1.36\right)$, P-value ≈ 2(0.1) = 0.2

c) 95% confidence interval

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,\nu} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha,\nu} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(0.8161 - 0.8271) - 2.074(0.01971)\sqrt{\frac{1}{12} + \frac{1}{12}} \le \mu_1 - \mu_2 \le (0.8161 - 0.8271) + 2.074(0.01971)\sqrt{\frac{1}{12} + \frac{1}{12}}$$

$$-0.0277 \le \mu_1 - \mu_2 \le 0.0057$$

Because zero is included in the confidence interval there is not a significant difference in the mean coefficients of restitution at α = 0.05.

d) $d = \frac{0.2}{2(0.01971)} = 5.07$ $\beta \cong 0$, Power ≅1

e) 1 - β = 0.8 β = 0.2 $d = \frac{0.1}{2(0.01971)} = 2.53$ n^* = 4 , $n = \frac{n^*+1}{2} = 2.5$ $n \cong 3$

Section 10-3

10-31 a)

1) The parameters of interest are the mean current (note: set circuit 1 equal to sample 2 so that Table X can be used. Therefore, μ_1 = mean of circuit 2 and μ_2 = mean of circuit 1)

2) $H_0 : \mu_1 = \mu_2$

3) $H_1 : \mu_1 > \mu_2$

4) The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

5) Reject H_0 if $w_2 \le w^*_{0.01}$ = 45. Because α = 0.01 and n_1 = 8 and n_2 = 9, Appendix A, Table X gives the critical value.

6) w_1 = 78 and w_2 = 75 and because 75 is less than 45, fail to reject H_0

7) Conclusion, fail to reject H_0. There is not enough evidence to conclude that the mean of circuit 2 exceeds the mean of circuit 1.

b)

1) The parameters of interest are the mean image brightness of the two tubes

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 > \mu_2$

4) The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

5) We reject H_0 if $Z_0 > Z_{0.025}$ = 1.96 for α = 0.025

6) w_1 = 78, μ_{w_1} =72 and $\sigma^2_{w_1}$ =108

$$z_0 = \frac{78 - 72}{10.39} = 0.58$$

Because Z_0 < 1.96, fail to reject H_0

7) Conclusion: fail to reject H_0. There is not a significant difference in the heat gain for the heating units at α = 0.05.
P-value = 2[1 - P(Z < 0.58)] = 0.5619

10-32 1) The parameters of interest are the mean flight delays

2) $H_0 : \mu_1 = \mu_2$

3) $H_1 : \mu_1 \neq \mu_2$

4) The test statistic is $w_2 = \dfrac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

5) Reject H_0 if $w \leq w^*_{0.05} = 26$. Because $\alpha = 0.05$ and $n_1 = 6$ and $n_2 = 6$, Appendix A, Table X gives the critical value.

6) $w_1 = 40$ and $w_2 = 38$ and because 40 and 38 are greater than 26, fail to reject H_0

7) Conclusion: fail to reject H_0. There is no significant difference in the flight delays at $\alpha = 0.05$.

10-33 a)

1) The parameters of interest are the mean heat gains for heating units

2) $H_0 : \mu_1 = \mu_2$

3) $H_1 : \mu_1 \neq \mu_2$

4) The test statistic is $w_2 = \dfrac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

6) We reject H_0 if $w \leq w^*_{0.01} = 78$, because $\alpha = 0.01$ and $n_1 = 10$ and $n_2 = 10$, Appendix A, Table X gives the critical value.

7. $w_1 = 77$ and $w_2 = 133$ and because 77 is less than 78, we can reject H_0

8. Conclusion: reject H_0 and conclude that there is a significant difference in the heating units at $\alpha = 0.05$.

b)

1) The parameters of interest are the mean heat gain for heating units

2) $H_0 : \mu_1 = \mu_2$

3) $H_1 : \mu_1 \neq \mu_2$

4) The test statistic is $z_0 = \dfrac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

5) Reject H_0 if $|Z_0| > Z_{0.025} = 1.96$ for $\alpha = 0.05$

6) $w_1 = 77$, $\mu_{w_1} = 105$ and $\sigma^2_{w_1} = 175$

$$z_0 = \frac{77 - 105}{13.23} = -2.12$$

Because $|Z_0| > 1.96$, reject H_0

7. Conclusion: reject H_0 and conclude that there is a difference in the heat gain for the heating units at $\alpha = 0.05$.

P-value $= 2[1 - P(Z < 2.19)] = 0.034$

10-34 a)

1) The parameters of interest are the mean etch rates

2) $H_0 : \mu_1 = \mu_2$

3) $H_1 : \mu_1 \neq \mu_2$

4) The test statistic is $w_2 = \dfrac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

5) We reject H_0 if $w \leq w^*_{0.05} = 78$, because $\alpha = 0.05$ and $n_1 = 10$ and $n_2 = 10$, Appendix A, Table X gives the critical value.

6. $w_1 = 74$ and $w_2 = 136$ and because 74 is less than 78, we reject H_0

7. Conclusion: reject H_0 and conclude that there is a significant difference in the mean etch rate at $\alpha = 0.05$.

b) 1) The parameters of interest are the mean temperatures

2) $H_0 : \mu_1 = \mu_2$

3) $H_1 : \mu_1 \neq \mu_2$

4) The test statistic is $z_0 = \dfrac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

5) We reject H_0 if $|Z_0| > Z_{0.025} = 1.96$ for $\alpha = 0.05$
6) $w_1 = 74$, $\mu_{w_1} = 105$ and $\sigma^2_{w_1} = 175$

$$z_0 = \frac{74-105}{13.229} = -2.343$$

Because $|Z_0| > 1.96$, it rejects H_0
7) Conclusion: It rejects H_0. There is a difference in the pipe deflection temperatures at $\alpha = 0.05$.
P-value $= 2[P(Z < -2.343)] = 0.019$

10-35 a)
1) The parameters of interest are the mean temperatures
2) $H_0 : \mu_1 = \mu_2$
3) $H_1 : \mu_1 \neq \mu_2$
4) The test statistic is $w_2 = \frac{(n_1+n_2)(n_1+n_2+1)}{2} - w_1$
5) We reject H_0 if $w \leq w^*_{0.05} = 185$, because $\alpha = 0.05$ and $n_1 = 15$ and $n_2 = 15$, Appendix A, Table X gives the critical value.
6) $w_1 = 259$ and $w_2 = 206$ and because both 259 and 206 are greater than 185, we fail to reject H_0
7) Conclusion: fail to reject H_0. There is not a significant difference in the mean pipe deflection temperature at $\alpha = 0.05$.

b)
1) The parameters of interest are the mean etch rates
2) $H_0 : \mu_1 = \mu_2$
3) $H_1 : \mu_1 \neq \mu_2$
4) The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$
5) We reject H_0 if $|Z_0| > Z_{0.025} = 1.96$ for $\alpha = 0.05$
6) $w_1 = 259$, $\mu_{w_1} = 232.5$ and $\sigma^2_{w_1} = 581.25$

$$z_0 = \frac{259-232.5}{24.11} = 1.1$$

Because $|Z_0| < 1.96$, do not reject H_0
7) Conclusion: Fail to reject H_0. There is not a significant difference between the mean etch rates.
P-value = 0.2713

10-36 a) The data are analyzed in ascending order and ranked as follows:

Group	Distance	Rank
2	223	1.0
2	236	2.0
2	238	3.0
1	240	4.5
2	240	4.5
2	242	6.0
1	244	7.0
2	245	8.0
2	247	9.0
1	248	11.0
1	248	11.0
2	248	11.0
2	250	13.0
1	251	14.5

1	251	14.5
1	255	16.0
2	257	17.0
1	259	18.0
1	262	19.0
1	263	20.0

The sum of the ranks for group 1 is $w_1 = 135.5$ and for group 2, $w_2 = 74.5$. Because w_2 is less than $w_{0.05} = 78$, we reject the null hypothesis that both groups have the same mean.

b) When the sample sizes are equal it does not matter which group we select for w_1

$$\mu_{W_1} = \frac{10(10+10+1)}{2} = 105$$

$$\sigma^2_{W_1} = \frac{10*10(10+10+1)}{12} = 175$$

$$Z_0 = \frac{135.5-105}{\sqrt{175}} = 2.31$$

Because $z_0 > z_{0.025} = 1.96$, reject H_0 and conclude that the sample means for the two groups are different.
When $z_0 = 2.31$, P-value = 0.021

Section 10-4

10-37 a) $\bar{d} = 0.2738$ $s_d = 0.1351$, n = 9
95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

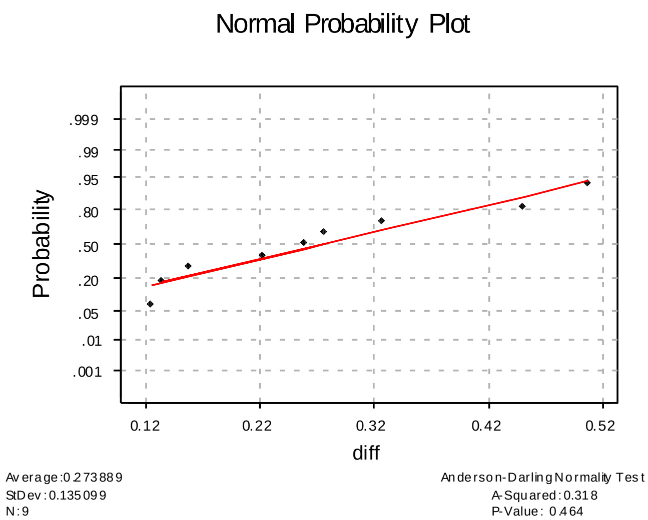
$$0.2738 - 2.306\left(\frac{0.1351}{\sqrt{9}}\right) \le \mu_d \le 0.2738 + 2.306\left(\frac{0.1351}{\sqrt{9}}\right)$$

$0.1699 \le \mu_d \le 0.3776$

With 95% confidence, the mean shear strength of Karlsruhe method exceeds the mean shear strength of the Lehigh method by between 0.1699 and 0.3776. Because zero is not included in this interval, the interval is consistent with rejecting the null hypothesis that the means are equal.

The 95% confidence interval is directly related to a test of hypothesis with 0.05 level of significance and the conclusions reached are identical.

b) It is only necessary for the differences to be normally distributed for the paired *t*-test to be appropriate and reliable. Therefore, the t-test is appropriate.



10-38 a)

1) The parameter of interest is the difference between the mean parking times, μ_d.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.05,13}$ where $-t_{0.05,13} = -1.771$ or $t_0 > t_{0.05,13}$ where $t_{0.05,13} = 1.771$ for $\alpha = 0.10$

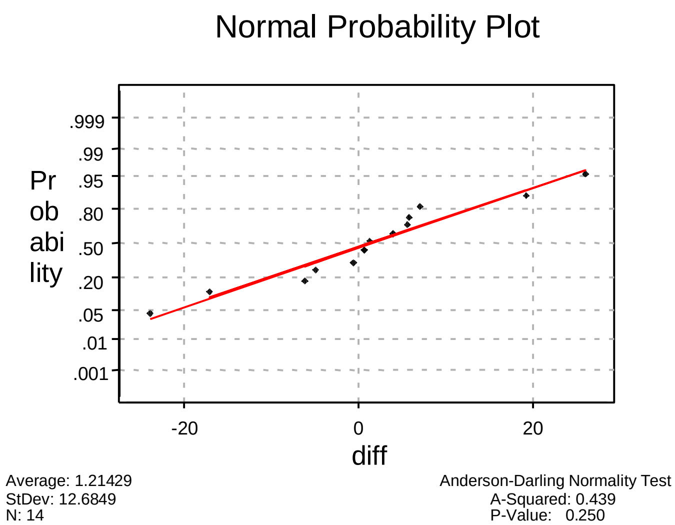
6) $\bar{d} = 1.21$
$s_d = 12.68$
$n = 14$

$$t_0 = \frac{1.21}{12.68/\sqrt{14}} = 0.357$$

7) Conclusion: Because $-1.771 < 0.357 < 1.771$, fail to reject the null. The data fail to support the claim that the two cars have different mean parking times at the 0.10 level of significance.

b) The result is consistent with the confidence interval constructed because zero is included in the 90% confidence interval.

c) The data fall approximately along a line in the normal probability plots. Therefore, the assumption of normality does not appear to be violated.



10-39 $\bar{d}$ = 868.375 s_d = 1290, n = 8 where d_i = brand 1 – brand 2

95% confidence interval:

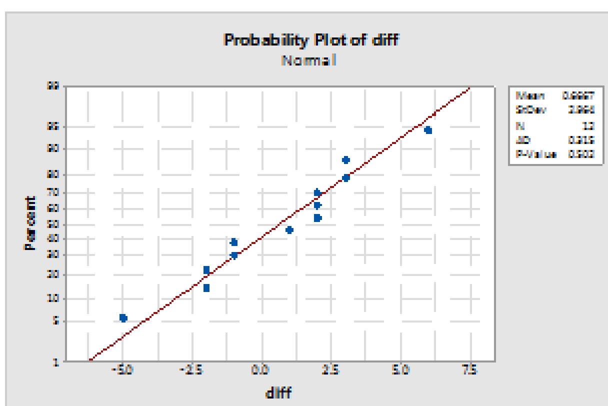
$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$868.375 - 2.365\left(\frac{1290}{\sqrt{8}}\right) \le \mu_d \le 868.375 + 2.365\left(\frac{1290}{\sqrt{8}}\right)$$

$$-210.26 \le \mu_d \le 1947.01$$

Because this confidence interval contains zero, there is no significant difference between the two brands of tire at a 5% significance level.

10-40 a) The data fall approximately along a line in the normal probability plots. Therefore, the assumption of normality does not appear to be violated.



b) $\bar{d}$ = 0.667 s_d = 2.964, n = 12

95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$0.667 - 2.201\left(\frac{2.964}{\sqrt{12}}\right) \le \mu_d \le 0.667 + 2.201\left(\frac{2.964}{\sqrt{12}}\right)$$

$$-1.216 \le \mu_d \le 2.55$$

Because zero is contained within this interval, one cannot conclude that one design language is preferable at a 5% significance level

10-41 a)

1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before − After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 > t_{0.05,14}$ where $t_{0.05,14} = 1.761$ for $\alpha = 0.05$

6) $\bar{d} = 25.53$
$s_d = 18.75$
$n = 15$

$$t_0 = \frac{25.53}{18.75/\sqrt{15}} = 5.273$$

7) Conclusion: Because 5.273 > 1.761, reject the null hypothesis. The data support the claim that the mean difference in cholesterol levels is significantly less after diet and an aerobic exercise program at the 0.05 level of significance.

P-value = $P(t > 5.273) \cong 0$

b) 95% confidence interval:

$$\bar{d} - t_{\alpha,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d$$

$$25.53 - 1.761\left(\frac{18.75}{\sqrt{15}}\right) \le \mu_d$$

$$17.00 \le \mu_d$$

Because the lower bound is positive, the mean difference in blood cholesterol level is significantly less after the diet and aerobic exercise program.

10-42 a)

1) The parameter of interest is the mean difference in natural vibration frequencies, μ_d where d_i = finite element – equivalent plate.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.025,6}$ or $t_0 > t_{0.025,6}$ where $t_{0.005,6} = 2.447$ for $\alpha = 0.05$

6) $\bar{d} = -5.49$
$s_d = 5.924$
$n = 7$

$$t_0 = \frac{-5.49}{5.924/\sqrt{7}} = -2.45$$

7) Conclusion: Because −2.45 < −2.447, reject the null hypothesis. The two methods have different mean values for natural vibration frequency at the 0.05 level of significance.

b) 95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$-5.49-2.447\left(\frac{5.924}{\sqrt{7}}\right)\le \mu_d \le -5.49+2.447\left(\frac{5.924}{\sqrt{7}}\right)$$
$$-10.969 \le \mu_d \le -0.011$$

With 95% confidence, the mean difference between the natural vibration frequency from the equivalent plate method and the finite element method is between −10.969 and −0.011 cycles.

10-43 a)
1) The parameter of interest is the difference in mean weight, μd where di = Weight Before – Weight After.
2) $H_0: \mu_d = 0$
3) $H_1: \mu_d > 0$
4) The test statistic is
$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}}$$
5) Reject the null hypothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$ for $\alpha = 0.05$
6) $\bar{d} = 8$
$s_d = 3.2$
$n = 10$
$$t_0 = \frac{8}{3.2/\sqrt{10}} = 7.906$$
7) Conclusion: Because 7.906 > 1.833 reject the null hypothesis and conclude that the mean weight loss is significantly greater than zero. That is, the data support the claim that this particular diet modification program is effective in reducing weight at the 0.05 level of significance.

b)
1) The parameter of interest is the difference in mean weight loss, μ_d where d_i = Before – After.
2) $H_0: \mu_d = 10$
3) $H_1: \mu_d > 10$
4) The test statistic is
$$t_0 = \frac{\bar{d}-\Delta_0}{s_d/\sqrt{n}}$$
5) Reject the null hypothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$ for $\alpha = 0.05$
6) $\bar{d} = 8$
$s_d = 3.2$
$n = 10$
$$t_0 = \frac{8-4.5}{3.2/\sqrt{10}} = 3.46$$
7) Conclusion: Because 3.46 > 1.833 reject the null hypothesis. There is evidence to support the claim that this particular diet modification program is effective in producing a mean weight loss of at least 4.5 kg at the 0.05 level of significance.

c) Use s_d as an estimate for σ:
$$n = \left(\frac{(z_\alpha + z_\beta)\sigma_d}{10}\right)^2 = \left(\frac{(1.645+1.29)3.2}{10}\right)^2 = 0.88,\ n = 1$$
Yes, the sample size of 10 is adequate for this test.

10-44 a)
1) The parameter of interest is the mean difference in impurity level, μ_d, where d_i = Test 1 – Test 2.
2) $H_0: \mu_d = 0$
3) $H_1: \mu_d \neq 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.005,7}$ or $t_0 > t_{0.005,7}$ where $t_{0.005,7} = 3.499$ for $\alpha = 0.01$

6) $\bar{d} = -0.2125$
$s_d = 0.1727$
$n = 8$

$$t_0 = \frac{-0.2125}{0.1727/\sqrt{8}} = -3.48$$

7) Conclusion: Because –3.499 < –3.48 < 3.499, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the tests generate different mean impurity levels at $\alpha = 0.01$.

b)
1) The parameter of interest is the mean difference in impurity level, μ_d, where d_i = Test 1 – Test 2.
2) $H_0: \mu_d + 0.1 = 0$
3) $H_1: \mu_d + 0.1 < 0$
4) The test statistic is

$$t_0 = \frac{\bar{d} + 0.1}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.05,7}$ where $t_{0.05,7} = 1.895$ for $\alpha = 0.05$

6) $\bar{d} = -0.2125$
$s_d = 0.1727$
$n = 8$

$$t_0 = \frac{-0.2125 + 0.1}{0.1727/\sqrt{8}} = -1.8424$$

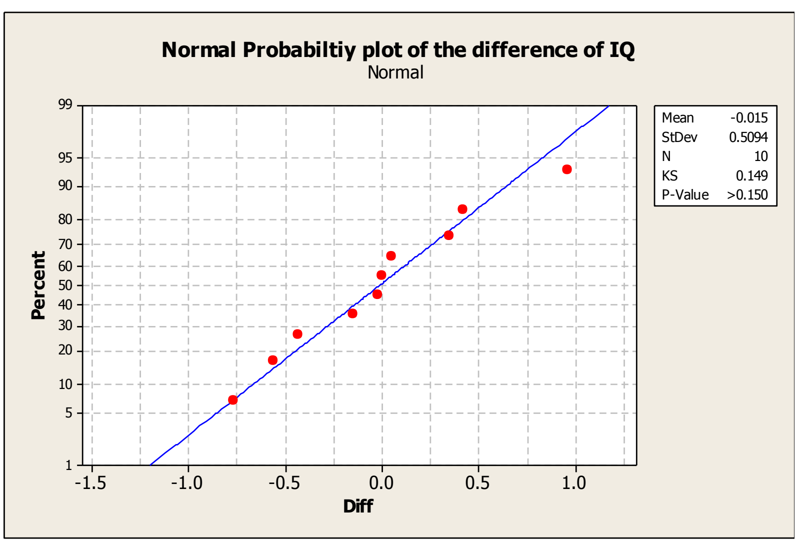
7) Conclusion: Because $-1.895 < -1.8424$, fail to reject the null hypothesis at the 0.05 level of significance.

c) $\beta = 1 - 0.9 = 0.1$

$$d = \frac{|0.1|}{0.1727} = 0.579$$

n = 8 is not an adequate sample size. From the chart VII*g*, $n \approx 30$

10-45 a) The data in the probability plot fall approximately along a line. Therefore, the normality assumption is reasonable.



b)
$\bar{d} = -0.015$
$s_d = 0.5093$
$n = 10$
$t_{0.025,9} = 2.262$

95% confidence interval:
$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$
$$-0.379 \le \mu_d \le 0.3493$$

Because zero is contained in the confidence interval, there is not sufficient evidence that the mean IQ depends on birth order.

c) β = 1- 0.9 = 0.1
d = $d = \frac{|\delta|}{\sigma} = \frac{1}{s_d} = 1.96$
Thus $6 \le n$ would be enough.

10-46 a) Let $x_{12} = x_2 - x_1$ and $x_{23} = x_3 - x_2$ and $x_d = x_{23} - x_{12}$

1) The parameter of interest is the mean difference in circumference μ_d where $x_d = x_{23} - x_{12}$

2) H_0 : $\mu_d = 0$

3) H_1 : $\mu_d \ne 0$

4) The test statistic is
$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.025,4}$ or $t_0 > t_{0.025,4}$ where $t_{0.025,4}$ = 2.776 for α = 0.05

6) $\bar{d} = 8.6$
$s_d = 7.829$
$n = 5$
$$t_0 = \frac{8.6}{7.829/\sqrt{5}} = 2.456$$

7) Conclusion: Because -2.776< 2.456< 2.776, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the means are significantly different at α = 0.05.

b) Let $x_{67} = x_7 - x_6$

Let $x_d = x_{12} - x_{67}$

1) The parameter of interest is the mean difference in circumference μ_d where $x_d = x_{12} - x_{67}$

2) H_0 : $\mu_d = 0$

3) H_1 : $\mu_d > 0$

4) The test statistic is
$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}}$$

5) Reject the null hypothesis if $t_0 > t_{0.05,4}$ where $t_{0.05,4}$ = 2.132 for α = 0.05

6) $\bar{d}$ = -24.4

$s_d = 7.5$

$n = 5$ $\qquad t_0 = \dfrac{-24.4}{7.5/\sqrt{5}} = 7.27$

7) Conclusion: Because 7.27 > 2.132, reject the null hypothesis. The means are significantly different at $\alpha = 0.1$.

P-value = P(t > 7.27) ≈ 0

c) No, the paired t test uses the differences to conduct the inference.

10-47 1) Parameters of interest are the median cholesterol levels for two activities.

2) $H_0 : \tilde{\mu}_D = 0$ or 2) $H_0 : \tilde{\mu}_1 - \tilde{\mu}_2 = 0$

3) $H_1 : \tilde{\mu}_D > 0$ 3) $H_1 : \tilde{\mu}_1 - \tilde{\mu}_2 > 0$

4) r^-

5) Because $\alpha = 0.05$ and $n = 15$, Appendix A, Table VIII gives the critical value of $r^*_{0.05} = 3$. We reject H_0 in favor of H_1 if $r^- \le 3$.

6) The test statistic is $r^- = 2$.

Observation	Before	After	Difference	Sign
1	265	229	36	+
2	240	231	9	+
3	258	227	31	+
4	295	240	55	+
5	251	238	13	+
6	245	241	4	+
7	287	234	53	+
8	314	256	58	+
9	260	247	13	+
10	260	240	20	+
11	283	246	37	+
12	240	218	22	+
13	238	219	19	+
14	225	226	-1	-
15	258	244	14	+

P-value = $P(R^+ \ge r^+ = 14 \mid p = 0.5) = \sum_{r=13}^{15} \binom{15}{r} (0.5)^r (0.5)^{20-r} = 0.00049$

7) Conclusion: Because the P-value = 0.00049 is less than $\alpha = 0.05$, reject the null hypothesis. There is a significant difference in the median cholesterol levels after diet and exercise at $\alpha = 0.05$.

10-48 1) The parameters of interest are the median cholesterol levels for two activities.

2) and 3) $H_0 : \mu_D = 0$ or $H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_D > 0$ $\quad H_1 : \mu_1 - \mu_2 > 0$

4) w^-

5) Reject H_0 if $w^- \le w^*_{0.05, n=15} = 30$ for $\alpha = 0.05$

6) The sum of the negative ranks is $w^- = 1$.

Observation	Before	After	Difference	Signed Rank
14	225	226	-1	-1
6	245	241	4	2
2	240	231	9	3
5	251	238	13	4.5

9	260	247	13	4.5
15	258	244	14	6
13	238	219	19	7
10	260	240	20	8
12	240	218	22	9
3	258	227	31	10
1	265	229	36	11
11	283	246	37	12
7	287	234	53	13
4	295	240	55	14
8	314	256	58	15

7) Conclusion: Because w^- = 1 is less than the critical value $w^*_{0.05,n=15} = 30$, reject the null hypothesis. There is a significant difference in the mean cholesterol levels after diet and exercise at α = 0.05.

The previous exercise tests the difference in the median cholesterol levels after diet and exercise while this exercise tests the difference in the mean cholesterol levels after diet and exercise.

Section 10-5

10-49 a) $f_{0.25,10,5}$ = 1.89

d) $f_{0.75,5,10} = \dfrac{1}{f_{0.25,10,5}} = \dfrac{1}{1.89} = 0.529$

b) $f_{0.10,24,9}$ = 2.28

e) $f_{0.90,6,10} = \dfrac{1}{f_{0.10,10,6}} = \dfrac{1}{2.94} = 0.34$

c) $f_{0.05,8,15}$ = 2.64

f) $f_{0.95,8,15} = \dfrac{1}{f_{0.05,15,8}} = \dfrac{1}{3.22} = 0.311$

10-50 a) $f_{0.25,7,15}$ = 1.47

d) $f_{0.75,15,7} = \dfrac{1}{f_{0.25,7,15}} = \dfrac{1}{1.47} = 0.68$

b) $f_{0.10,10,12}$ = 2.19

e) $f_{0.90,10,12} = \dfrac{1}{f_{0.10,12,10}} = \dfrac{1}{2.28} = 0.438$

c) $f_{0.01,5,15}$ = 2.27

f) $f_{0.99,20,10} = \dfrac{1}{f_{0.01,10,20}} = \dfrac{1}{3.37} = 0.297$

10-51 1) The parameters of interest are the standard deviations σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 < \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{.0.95,4,9} = 1/f_{.0.05,9,4}$ = 1/6 = 0.1666 for α = 0.05

6) $n_1 = 5$ $n_2 = 10$ $s_1^2 = 29.8$ $s_2^2 = 37.5$

$$f_0 = \frac{(29.8)}{(37.5)} = 0.795$$

7) Conclusion: Because 0.1666 < 0.795 do not reject the null hypothesis.

95% confidence interval:

$$\frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha,n_2-1,n_1-1}$$

$\frac{\sigma_1^2}{\sigma_2^2} \le (\frac{29.8}{37.5}) f_{0.05,9,4}$ where $f_{.05,9,4} = 6.00$ $\quad \frac{\sigma_1^2}{\sigma_2^2} \le 4.768$ or $\frac{\sigma_1}{\sigma_2} \le 2.184$

Because the value one is contained within this interval, there is no significant difference in the variances.

10-52 1) The parameters of interest are the standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 > \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 > f_{.0.01,19,7} \cong 6.16$ for $\alpha = 0.01$

6) $n_1 = 20$ $\quad n_2 = 8$ $\quad s^2{}_1 = 12.3$ $\quad s_2{}^2 = 8.5$

$$f_0 = \frac{12.3}{8.5} = 1.447$$

7) Conclusion: Because 6.16 > 1.447, fail to reject the null hypothesis.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{0.99,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$1.447(1/6.16) \le \frac{\sigma_1^2}{\sigma_2^2}$ $\qquad 0.235 \le \frac{\sigma_1^2}{\sigma_2^2}$

Because the value one is contained within this interval, there is no significant difference in the variances.

10-53 a)

1) The parameters of interest are the standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{.0.975,14,14} = 0.33$ or $f_0 > f_{0.025,14,14} = 3$ for $\alpha = 0.05$

6) $n_1 = 15$ $\quad n_2 = 15$ $\quad s_1^2 = 2.5$ $\quad s_2^2 = 2.2$

$$f_0 = \frac{2.5}{2.2} = 1.14$$

7) Conclusion: Because 0.333 < 1.14 < 3, we fail to reject the null hypothesis. There is no sufficient evidence that there is a difference in the standard deviation.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_2-1,n_1-1}$$

$$(1.14)0.333 \le \frac{\sigma_1^2}{\sigma_2^2} \le (1.14)3 \qquad 0.38 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.42$$

Because the value one is contained within this interval, there is no significant difference in the variances.

b) $\lambda = \frac{\sigma_1}{\sigma_2} = 2$

$n_1 = n_2 = 5$

$\alpha = 0.05$

Chart VII (o) we find $\beta = 0.35$ then the power $1 - \beta = 0.65$

c) $\beta = 0.05$ and $\sigma_2 = \sigma_1 / 2$ so that $\frac{\sigma_1}{\sigma_2} = 2$ and $n \approx 31$

10-54 1) The parameters of interest are the variances of concentration, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$ for $\alpha = 0.05$

6) $n_1 = 10$ $\quad n_2 = 16$

$s_1 = 6.4$ $\quad s_2 = 4.8$

$$f_0 = \frac{(6.4)^2}{(4.8)^2} = 1.778$$

7) Conclusion: Because 0.265 < 1.778 < 3.12, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the two population variances differ at the 0.05 level of significance.

10-55 a)

1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2 where Group 1 = men and Group 2 = women

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{1-\alpha/2, n_1-1, n_2-1} = 0.365$ or $f_0 > f_{\alpha/2, n_1-1, n_2-1} = 2.86$ for $\alpha = 0.02$

6) $n_1 = 25$ $n_2 = 21$ $s_1 = 0.98$ $s_2 = 1.02$

$$f_0 = \frac{(0.98)^2}{(1.02)^2} = 0.923$$

7) Conclusion: Because 0.365 < 0.923 < 2.86, fail to reject the null hypothesis. There is not sufficient evidence to support the claim that men and women differ in repeatability for this assembly task at the 0.02 level of significance.

ASSUMPTIONS: Assume random samples from two normal distributions.

b) 98% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_2-1, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_2-1, n_1-1}$$

$$f_{1-\alpha/2,n_2-1,n_1-1} = \frac{1}{f_{\alpha/2,n_1-1,n_2-1}} = \frac{1}{f_{0.01,24,20}} = \frac{1}{2.86} = 0.350$$

$$(0.923)0.350 \le \frac{\sigma_1^2}{\sigma_2^2} \le (0.923)2.73$$

$$0.323 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.527$$

Because the value one is contained within this interval, there is no significant difference between the variance of the repeatability of men and women for the assembly task at a 2% significance level.

10-56 a) 90% confidence interval for the ratio of variances:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_2-1,n_1-1}$$

$$\left(\frac{0.6^2}{0.8^2}\right)0.156 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{0.6^2}{0.8^2}\right)6.39 \qquad 0.08775 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.594$$

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_2-1,n_1-1}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right)0.104 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.6)^2}{(0.8)^2}\right)9.60 \qquad 0.0585 \le \frac{\sigma_1^2}{\sigma_2^2} \le 5.4$$

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right)0.243 \le \frac{\sigma_1^2}{\sigma_2^2} \qquad 0.137 \le \frac{\sigma_1^2}{\sigma_2^2}$$

A 90% lower confidence bound on $\frac{\sigma_1}{\sigma_2}$ is given by $0.370 \le \frac{\sigma_1}{\sigma_2}$

10-57 a) 90% confidence interval for the ratio of variances:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_1-1,n_2-1}$$

$$\left(\frac{(0.35)}{(0.90)}\right)0.412 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.35)}{(0.90)}\right)2.33 \qquad 0.1602 \le \frac{\sigma_1^2}{\sigma_2^2} \le 0.9061 \quad 0.4002 \le \frac{\sigma_1}{\sigma_2} \le 0.9519$$

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_1-1,n_2-1}$$

$$\left(\frac{(0.35)}{(0.90)}\right)0.342 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.35)}{(0.90)}\right)2.82 \qquad 0.133 \le \frac{\sigma_1^2}{\sigma_2^2} \le 1.097 \qquad 0.3647 \le \frac{\sigma_1}{\sigma_2} \le 1.047$$

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.35)}{(0.90)}\right) 0.500 \le \frac{\sigma_1^2}{\sigma_2^2} \qquad 0.194 \le \frac{\sigma_1^2}{\sigma_2^2} \qquad 0.441 \le \frac{\sigma_1}{\sigma_2}$$

10-58 1) The parameters of interest are the strength variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$ for $\alpha = 0.05$

6) $n_1 = 10$ $\quad n_2 = 16$

$s_1 = 15$ $\quad s_2 = 30$

$$f_0 = \frac{(15)^2}{(30)^2} = 0.25$$

7) Conclusion: Because 0.25 < 0.265 reject the null hypothesis. The population variances differ at the 0.05 level of significance for the two suppliers.

10-59 1) The parameters of interest are the melting variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.975,20,20}$ where $f_{0.975,20,20} = 0.4058$ or $f_0 > f_{0.025,20,20}$ where $f_{0.025,20,20} = 2.46$ for $\alpha = 0.05$

6) $n_1 = 21$ $\quad n_2 = 21$

$s_1 = 2$ $\quad s_2 = 1.7$

$$f_0 = \frac{(2)^2}{(1.7)^2} = 1.384$$

7) Conclusion: Because 0.4058 < 1.384 < 2.46 fail to reject the null hypothesis. The population variances do not differ at the 0.05 level of significance.

10-60 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.995,10,12}$ where $f_{0.995,10,12} = 0.1766$ or $f_0 > f_{0.005,10,12}$ where $f_{0.005,10,12} = 5.0855$ for $\alpha = 0.01$

6) $n_1 = 11$ $\quad n_2 = 13$

$s_1 = 0.25$ $\quad s_2 = 0.5$

$$f_0 = \frac{(0.25)^2}{(0.5)^2} = 0.25$$

7) Conclusion: Because 0.1766 < 0.25 < 5.0855 fail to reject the null hypothesis. The thickness variances are not significantly different at the 0.01 level of significance.

10-61 1) The parameters of interest are the overall distance standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{.0.975,9,9} = 0.248$ or $f_0 > f_{0.025,9,9} = 4.03$ for $\alpha = 0.05$

6) $n_1 = 10$ $n_2 = 10$ $s_1 = 7.61$ $s_2 = 9.26$

$$f_0 = \frac{(7.61)^2}{(9.26)^2} = 0.6754$$

7) Conclusion: Because 0.248 < 0.6754 < 4.04 fail to reject the null hypothesis. There is not sufficient evidence that the standard deviations of the overall distances of the two brands differ at the 0.05 level of significance.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$(0.6754)0.248 \le \frac{\sigma_1^2}{\sigma_2^2} \le (0.6754)4.03 \qquad 0.168 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.723$$

A 95% lower confidence bound on the ratio of standard deviations is given by $0.41 \le \frac{\sigma_1}{\sigma_2} \le 1.65$

Because the value one is contained within this interval, there is no significant difference in the variances of the distances at a 5% significance level.

10-62 1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{.0.975,11,11} = 0.288$ or $f_0 > f_{0.025,11,11} = 3.474$ for $\alpha = 0.05$

6) $n_1 = 12$ $n_2 = 12$ $s_1 = 0.0217$ $s_2 = 0.0175$

$$f_0 = \frac{(0.0217)^2}{(0.0175)^2} = 1.538$$

7) Conclusion: Because 0.288 < 1.538 < 3.474, fail to reject the null hypothesis. There is not sufficient evidence that there is a difference in the standard deviations of the coefficients of restitution between the two clubs at the 0.05 level of significance.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_2-1, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_2-1, n_1-1}$$

$$(1.538)0.288 \le \frac{\sigma_1^2}{\sigma_2^2} \le (1.538)3.474 \qquad 0.443 \le \frac{\sigma_1^2}{\sigma_2^2} \le 5.343$$

A 95% lower confidence bound the ratio of standard deviations is given by $0.666 \le \frac{\sigma_1}{\sigma_2} \le 2.311$

Because the value one is contained within this interval, there is no significant difference in the variances of the coefficient of restitution at a 5% significance level.

10-63 1) The parameters of interest are the variances of the weight measurements between the two sheets of paper, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{.0.975,14,14} = 0.33$ or $f_0 > f_{0.025,14,14} = 3$ for $\alpha = 0.05$

6) $n_1 = 15$ $\qquad n_2 = 15$ $\qquad s_1^{\;2} = 0.00831^2$ $\qquad s_2^{\;2} = 0.00714^2$

$$f_0 = 1.35$$

7) Conclusion: Because 0.333 < 1.35 < 3, fail to reject the null hypothesis. There is not sufficient evidence that there is a difference in the variances of the weight measurements between the two sheets of paper at $\alpha = 0.05$.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_2-1, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_2-1, n_1-1}$$

$$(1.35)0.333 \le \frac{\sigma_1^2}{\sigma_2^2} \le (1.35)3 \qquad 0.45 \le \frac{\sigma_1^2}{\sigma_2^2} \le 4.05$$

Because the value one is contained within this interval, there is no significant difference in the variances.

10-64 a)

1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.99,7,7}$ where $f_{0.99,7,7} = 0.143$ or $f_0 > f_{0.01,7,7}$ where $f_{0.01,7,7} = 6.99$ for $\alpha = 0.02$

6) $n_1 = 8$ $\qquad n_2 = 8$

$s_1 = 0.0028$ $\qquad s_2 = 0.0023$

$$f_0 = \frac{(0.0028)^2}{(0.0023)^2} = 1.48$$

7) Conclusion: Because 0.143 < 1.48 < 6.99 fail to reject the null hypothesis. The thickness variances do not significantly differ at the 0.02 level of significance.

b) If one population standard deviation is to be 50% larger than the other, then $\lambda = 2$. Using $n = 8$, $\alpha = 0.01$ and Chart VII (p), we obtain $\beta \cong 0.85$. Therefore, $n = n_1 = n_2 = 8$ is not adequate to detect this difference with high probability.

10-65 a)

1) The parameters of interest are the etch-rate variances, σ_1^2, σ_2^2.

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.975,9,9} = 0.248$ or $f_0 > f_{0.025,9,9} = 4.03$ for $\alpha = 0.05$

6) $n_1 = 10$ $\quad n_2 = 10$

$s_1 = 0.011$ $\quad s_2 = 0.006$

$$f_0 = \frac{(0.011)^2}{(0.006)^2} = 3.361$$

7) Conclusion: Because 0.248 < 3.361 < 4.03 fail to reject the null hypothesis. There is not sufficient evidence that the etch rate variances differ at the 0.05 level of significance.

b) With $\lambda = \sqrt{2} = 1.4$ $\quad \beta = 0.10$ and $\alpha = 0.05$, we find from Chart VII (o) that $n_1^* = n_2^* = 100$. Therefore, the samples of size 10 would not be adequate.

Section 10-6

10-66 a) This is a two-sided test because the hypotheses are $p_1 - p_2 = 0$ versus not equal to 0.

b) $\hat{p}_1 = \frac{54}{250} = 0.216$ $\quad \hat{p}_2 = \frac{60}{290} = 0.207$ $\quad \hat{p} = \frac{54+60}{250+290} = 0.2111$

Test statistic is $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$$z_0 = \frac{0.0091}{\sqrt{(0.2111)(1-0.2111)\left(\frac{1}{250} + \frac{1}{290}\right)}} = 0.2584$$

P-value = 2[1 – P(Z < 0.2584)] = 2[1-0.6020] = 0.796

c) Because the P-value is greater than $\alpha = 0.05$, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the proportions differ at the 0.05 level of significance.

d) 90% two sided confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \leq p_1 - p_2 \leq (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.0091) - 1.65\sqrt{\frac{0.216(1-0.216)}{250} + \frac{0.207(1-0.207)}{290}} \leq p_1 - p_2 \leq (0.0091) + 1.65\sqrt{\frac{0.216(1-0.216)}{250} + \frac{0.207(1-0.207)}{290}}$$

$$-0.0491 \leq p_1 - p_2 \leq 0.0673$$

10-67 a) This is one-sided test because the hypotheses are $p_1 - p_2 = 0$ versus greater than 0.

b) $\hat{p}_1 = \frac{188}{250} = 0.752$ $\quad \hat{p}_2 = \frac{245}{350} = 0.7$ $\quad \hat{p} = \frac{188+245}{250+350} = 0.7217$

Test statistic is $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$$z_0 = \frac{0.052}{\sqrt{(0.7217)(1-0.7217)\left(\frac{1}{250}+\frac{1}{350}\right)}} = 1.4012$$

P-value = [1- P(*Z* < 1.4012)] = 1 - 0.9194 = 0.0806

95% lower confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_\alpha \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2$$

$$(0.052) - 1.65\sqrt{\frac{0.752(1-0.752)}{250} + \frac{0.7(1-0.7)}{350}} \le p_1 - p_2$$

$$-0.0085 \le p_1 - p_2$$

c) The P-value = 0.0806 is less than $\alpha = 0.10$. Therefore, we reject the null hypothesis that $p_1 - p_2 = 0$ at the 0.1 level of significance. If $\alpha = 0.05$, the P-value = 0.0806 is greater than $\alpha = 0.05$ and we fail to reject the null hypothesis.

10-68 a)

1) The parameters of interest are the proportion of successes of surgical repairs for different tears, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 > p_2$

4) Test statistic is $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

5) Reject the null hypothesis if $z_0 > z_{0.1}$ where $z_{0.1} = 1.29$ for $\alpha = 0.1$

6) $n_1 = 18$ $n_2 = 30$

$x_1 = 14$ $x_2 = 22$

$\hat{p}_1 = 0.78$ $\hat{p}_2 = 0.73$ $\hat{p} = \dfrac{14+22}{18+30} = 0.75$

$$z_0 = \frac{0.78 - 0.73}{\sqrt{0.75(1-0.75)\left(\frac{1}{18}+\frac{1}{30}\right)}} = 0.387$$

7) Conclusion: Because 0.387 < 1.29, we fail to reject the null hypothesis at the 0.1 level of significance.

P-value = [1 – P(*Z* < 0.387)] = 1 - 0.6517 ≈ 0.35

b) 90% confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_\alpha \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2$$

$$(0.78 - 0.73) - 1.29\sqrt{\frac{0.78(1-0.78)}{18} + \frac{0.73(1-0.73)}{30}} \le p_1 - p_2$$

$$-0.114 \le p_1 - p_2$$

Because this interval contains the value zero, there is not enough evidence to conclude that the success rate p_1 exceeds p_2.

10-69 a)

1) The parameters of interest are the proportion of voters in favor of Bush vs those in favor of Kerry, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) Test statistic is $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

5) Reject the null hypothesis if $z_0 < -z_{0.005}$ or $z_0 > z_{0.005}$ where $z_{0.005} = 2.58$ for $\alpha = 0.01$

6) $n_1 = 2020$ $n_2 = 2020$

$x_1 = 1071$ $x_2 = 930$

$\hat{p}_1 = 0.53$ $\hat{p}_2 = 0.46$ $\hat{p} = \dfrac{1071+930}{2020+2020} = 0.495$

$$z_0 = \frac{0.53 - 0.46}{\sqrt{0.495(1-0.495)\left(\dfrac{1}{2020}+\dfrac{1}{2020}\right)}} = 4.45$$

7) Conclusion: Because 4.45 > 2.58 reject the null hypothesis and conclude yes there is a significant difference in the proportions at the 0.05 level of significance.
P-value = 2[1 – P(*Z* < 4.45)] ≈ 0

b) 99% confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2 \le (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$0.029 \le p_1 - p_2 \le 0.11$$

Because this interval does not contain the value zero, we are 99% confident there is a difference in the proportions.

10-70 a)

1) The parameters of interest are the proportion of defective parts, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \ne p_2$

4) Test statistic is $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

5) Reject the null hypothesis if $z_0 < -z_{0.025}$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$ for $\alpha = 0.05$

6) $n_1 = 300$ $n_2 = 300$

$x_1 = 20$ $x_2 = 10$

$\hat{p}_1 = 0.067$ $\hat{p}_2 = 0.033$ $\hat{p} = \dfrac{20+10}{300+300} = 0.05$

$$z_0 = \frac{0.067 - 0.033}{\sqrt{0.05(1-0.05)\left(\dfrac{1}{300}+\dfrac{1}{300}\right)}} = 1.91$$

7) Conclusion: Because –1.96 < 1.91 < 1.96, we fail to reject the null hypothesis. There is no significant difference in the fraction of defective parts produced by the two machines at the 0.05 level of significance.
P-value = 2[1 – P(*Z* < 1.91)] = 0.05613

b) 95% confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2 \le (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.067-0.033)-1.96\sqrt{\frac{0.067(1-0.067)}{300}+\frac{0.033(1-0.033)}{300}} \le p_1 - p_2 \le (0.067-0.033)+1.96\sqrt{\frac{0.067(1-0.067)}{300}+\frac{0.033(1-0.033)}{300}}$$

$$-0.00077 \le p_1 - p_2 \le 0.06877$$

Because this interval contains the value zero, there is no significant difference in the fraction of defective parts produced by the two machines. We have 95% confidence that the difference in proportions is between −0.00077 and 0.06877.

c) Power = 1 − β

$$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)-\Phi\left(\frac{-z_{\alpha/2}\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$$\bar{p}=\frac{300(0.05)+300(0.01)}{300+300}=0.03 \qquad \bar{q}=0.97$$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2}=\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.01(1-0.01)}{300}}=0.014$$

$$\beta=\Phi\left(\frac{1.96\sqrt{0.03(0.97)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.01)}{0.014}\right)-\Phi\left(\frac{-1.96\sqrt{0.03(0.97)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.01)}{0.014}\right)$$

$$=\Phi(-0.91)-\Phi(-4.81)=0.18141-0=0.18141$$

Power = 1 − 0.18141 = 0.81859

d) $$n=\frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}}+z_\beta\sqrt{p_1q_1+p_2q_2}\right)^2}{(p_1-p_2)^2}$$

$$=\frac{\left(1.96\sqrt{\frac{(0.05+0.01)(0.95+0.99)}{2}}+1.29\sqrt{0.05(0.95)+0.01(0.99)}\right)^2}{(0.05-0.01)^2}=382.11$$

n = 383

e) $$\beta=\Phi\left(\frac{z_{\alpha/2}\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)-\Phi\left(\frac{-z_{\alpha/2}\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$$\bar{p}=\frac{300(0.05)+300(0.02)}{300+300}=0.035 \quad \bar{q}=0.965$$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2}=\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.02(1-0.02)}{300}}=0.015$$

$$\beta=\Phi\left(\frac{1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.02)}{0.015}\right)-\Phi\left(\frac{-1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.02)}{0.015}\right)$$

$$=\Phi(-0.04)-\Phi(-3.96)=0.48405-0.00004=0.48401$$

Power = 1 − 0.48401 = 0.51599

f) $$n = \frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}} + z_\beta\sqrt{p_1q_1+p_2q_2} \right)^2}{(p_1-p_2)^2}$$

$$= \frac{\left(1.96\sqrt{\frac{(0.05+0.02)(0.95+0.98)}{2}} + 1.29\sqrt{0.05(0.95)+0.02(0.98)} \right)^2}{(0.05-0.02)^2} = 790.67$$

$n = 791$

10-71 a)

1) The parameters of interest are the proportion of satisfactory lenses, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{x_1+x_2}{n_1+n_2}$$

5) Reject the null hypothesis if $z_0 < -z_{0.005}$ or $z_0 > z_{0.005}$ where $z_{0.005} = 2.58$ for $\alpha = 0.01$

6) $n_1 = 400$ $\quad n_2 = 400$

$x_1 = 253$ $\quad x_2 = 196$

$\hat{p}_1 = 0.633$ $\quad \hat{p}_2 = 0.49$ $\quad \hat{p} = \frac{253+196}{400+400} = 0.561$

$$z_0 = \frac{0.633-0.49}{\sqrt{0.561(1-0.561)\left(\frac{1}{400}+\frac{1}{400}\right)}} = 4.075$$

7) Conclusion: Because 4.075 > 2.58, reject the null hypothesis and conclude that there is a difference in the fraction of polishing-induced defects produced by the two polishing solutions at the 0.01 level of significance.

P-value = $2[1 - P(Z < 4.075)] \approx 0$

b) By constructing a 99% confidence interval on the difference in proportions, the same question can be answered by whether or not zero is contained in the interval.

10-72 a)

1) The parameters of interest are the proportion of residents in favor of an increase, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) Test statistic is $$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{x_1+x_2}{n_1+n_2}$$

5) Reject the null hypothesis if $z_0 < -z_{0.025}$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$ for $\alpha = 0.05$

6) $n_1 = 500$ $\quad n_2 = 400$

$x_1 = 385$ $\quad x_2 = 267$

$\hat{p}_1 = 0.77$ $\quad \hat{p}_2 = 0.6675$ $\quad \hat{p} = \frac{385+267}{500+400} = 0.724$

$$z_0 = \frac{0.77 - 0.6675}{\sqrt{0.724(1-0.724)\left(\frac{1}{500} + \frac{1}{400}\right)}} = 3.42$$

7) Conclusion: Because 3.42 > 1.96 reject the null hypothesis and conclude yes there is a significant difference in the proportions of support for increasing the speed limit between residents of the two counties at the 0.05 level of significance.
P-value = 2[1 – P(Z < 3.42)] = 0.00062

b) 95% confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2 \le (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.77 - 0.6675) - 1.96\sqrt{\frac{0.77(1-0.77)}{500} + \frac{0.6675(1-0.6675)}{400}} \le p_1 - p_2 \le (0.77 - 0.6675) + 1.96\sqrt{\frac{0.77(1-0.77)}{500} + \frac{0.6675(1-0.6675)}{400}}$$

$$0.0434 \le p_1 - p_2 \le 0.1616$$

We are 95% confident that the difference in proportions is between 0.0434 and 0.1616. Because the interval does not contain zero there is evidence that the counties differ in support of the change.

Supplemental Exercises

10-73 a) SE Mean$_1$ $= \frac{s_1}{\sqrt{n_1}} = \frac{2.23}{\sqrt{25}} = 0.446$

$\overline{x}_1 = 11.87$ $\overline{x}_2 = 12.73$ $s_1^2 = 2.23^2$ $s_2^2 = 3.19^2$ $n_1 = 25$ $n_2 = 25$

Degrees of freedom = $n_1 + n_2$ - 2 = 25 +25 - 2 = 48.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(25-1)2.23^2 + (25-1)3.19^2}{25+25-2}} = 2.752$$

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(-0.86)}{2.752\sqrt{\frac{1}{25} + \frac{1}{25}}} = -1.105$$

P-value = 2 [P(t < –1.105)] and 2(0.10) <*P*-value < 2(0.25) = 0.20 < *P*-value < 0.5

The 95% two-sided confidence interval: $t_{\alpha/2, n_1+n_2-2} = t_{0.025,48} = 2.01$

$$(\overline{x}_1 - \overline{x}_2) - t_{\alpha/2, n_1+n_2-2}\, s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\overline{x}_1 - \overline{x}_2) + t_{\alpha/2, n_1+n_2-2}\, s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(-0.86) - (2.01)(2.752)\sqrt{\frac{1}{20} + \frac{1}{20}} \le \mu_1 - \mu_2 \le (-0.86) + (2.01)(2.752)\sqrt{\frac{1}{20} + \frac{1}{20}}$$

$$-2.609 \le \mu_1 - \mu_2 \le 0.889$$

b) This is two-sided test because the alternative hypothesis is $\mu_1 - \mu_2$ not = 0.

c) Because the 0.20 < *P*-value < 0.5 and the P-value > α = 0.05, we fail to reject the null hypothesis at the 0.05 level of significance. If α = 0.01, we also fail to reject the null hypothesis.

10-74 a) This is one-sided test because the alternative hypothesis is $\mu_1 - \mu_2 < 0$.

b) SE Mean$_1$ $= \frac{s_1}{\sqrt{n_1}} = \frac{2.98}{\sqrt{16}} = 0.745$

SE Mean$_2$ $= \frac{s_2}{\sqrt{n_2}} = \frac{5.36}{\sqrt{25}} = 1.072$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}} = \frac{\left(\frac{2.98^2}{16} + \frac{5.36^2}{25}\right)^2}{\frac{\left(2.98^2/16\right)^2}{16-1} + \frac{\left(5.36^2/25\right)^2}{25-1}} = 38.44 \approx 38 \text{ (truncated)}$$

Degrees of freedom = 38
P-value = P($t < -1.65$) and $0.05 < P\text{-value} < 0.1$

c) Because $0.05 < P$-value < 0.1 and the P-value $> \alpha = 0.05$, we fail to reject the null hypothesis of $\mu_1 - \mu_2 = 0$ at the 0.05 level of significance. If $\alpha = 0.1$, we reject the null hypothesis because the P-value < 0.1.

d) The 95% upper one-sided confidence interval: $t_{0.05,38} = 1.686$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (-2.16) + 1.686\sqrt{\frac{(2.98)^2}{16} + \frac{(5.36)^2}{25}}$$

$$\mu_1 - \mu_2 \le 0.0410$$

10-75 a) Assumptions that must be met are normality, equality of variance, and independence of the observations. Normality and equality of variances appear to be reasonable from the normal probability plots. The data appear to fall along lines and the slopes appear to be the same. Independence of the observations for each sample is obtained if random samples are selected.

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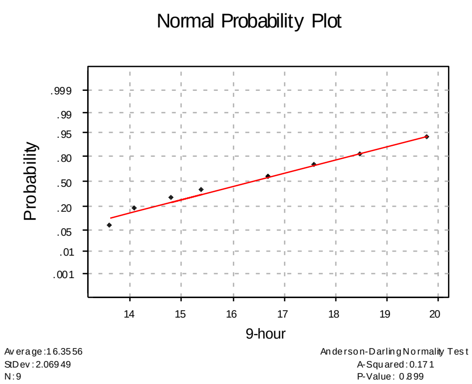
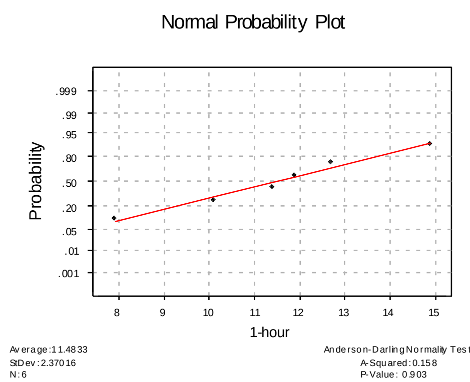




b) $\bar{x}_1 = 16.36$ $\quad \bar{x}_2 = 11.483$
$s_1 = 2.07$ $\quad s_2 = 2.37$
$n_1 = 9$ $\quad n_2 = 6$

95% confidence interval: $t_{\alpha/2, n_1+n_2-2} = t_{0.025,13}$ where $t_{0.025,13} = 2.160$

$$s_p = \sqrt{\frac{8(2.07)^2 + 5(2.37)^2}{13}} = 2.19$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(16.36 - 11.483) - 2.160(2.19)\sqrt{\frac{1}{9} + \frac{1}{6}} \le \mu_1 - \mu_2 \le (16.36 - 11.483) + 2.160(2.19)\sqrt{\frac{1}{9} + \frac{1}{6}}$$

$$2.38 \le \mu_1 - \mu_2 \le 7.37$$

c) Yes, we are 95% confident the results from the first test condition exceed the results of the second test condition because the confidence interval contains only positive values.

d) 95% confidence interval for σ_1^2 / σ_2^2

$$f_{0.975,8,5} = \frac{1}{f_{0.025,5,8}} = \frac{1}{4.82} = 0.2075, \qquad f_{0.025,8,5} = 6.76$$

$$\frac{s_1^2}{s_2^2} f_{0.975,5,8} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{0.025,5,8}$$

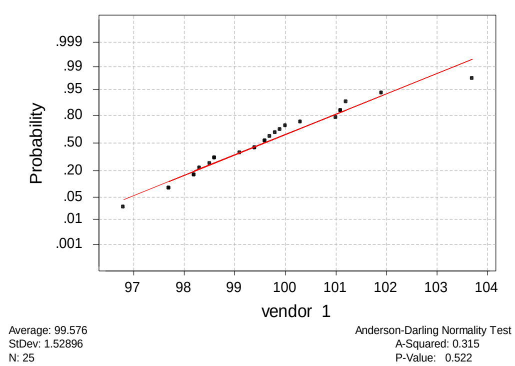
$$\left(\frac{4.283}{5.617}\right)(0.148) \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{4.283}{5.617}\right)(4.817)$$

$$0.113 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.673$$

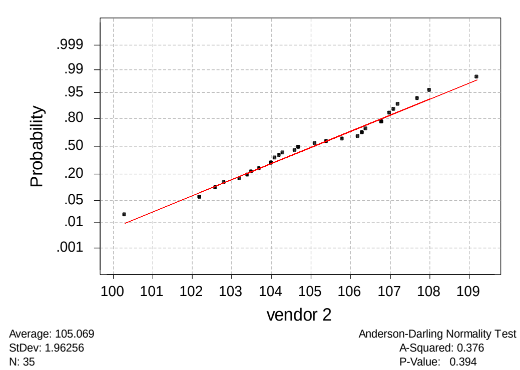
e) Because the value one is contained within this interval, the population variances do not differ at a 5% significance level.

10-76 a) Assumptions that must be met are normality and independence of the observations. Normality appears to be reasonable.

Normal Probability Plot



Normal Probability Plot



The data appear to fall along lines in the normal probability plots. Because the slopes appear to be the same, it appears the population standard deviations are similar. Independence of the observations for each sample is obtained if random samples are selected.

b)

1) The parameters of interest are the variances of resistance of products, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject H_0 if $f_0 < f_{0.975,24,34}$ where $f_{0.975,24,34} = \dfrac{1}{f_{0.025,34,24}} = \dfrac{1}{2.18} = 0.459$ for $\alpha = 0.05$

or $f_0 > f_{0.025,24,34}$ where $f_{0.025,24,34} = 2.07$ for $\alpha = 0.05$

6) $s_1 = 1.53$ $s_2 = 1.96$

$n_1 = 25$ $n_2 = 35$

$$f_0 = \frac{(1.53)^2}{(1.96)^2} = 0.609$$

7) Conclusion: Because 0.459 < 0.609 < 2.07, fail to reject H_0. There is not sufficient evidence to conclude that the variances are different at $\alpha = 0.05$.

10-77 a) 1) The parameter of interest is the mean weight loss, μ_d where d_i = Initial Weight – Final Weight.

2) $H_0: \mu_d = 1.5$

3) $H_1: \mu_d > 1.5$

4) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$ for $\alpha = 0.05$.

6) $\bar{d} = 1.875$

$s_d = 0.641$

$n = 8$

$$t_0 = \frac{1.875 - 1.5}{0.641/\sqrt{8}} = 1.655$$

7) Conclusion: Because 1.655 > 1.895, fail to reject the null hypothesis and conclude the average weight loss is significantly less than 1.5 at $\alpha = 0.05$.

b) 2) $H_0: \mu_d = 1.5$

3) $H_1: \mu_d > 1.5$

4) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,7} = 2.998$ for $\alpha = 0.01$.

6) $\bar{d} = 1.875$

$s_d = 0.641$

$n = 8$

$$t_0 = \frac{1.875 - 1.5}{0.641/\sqrt{8}} = 1.655$$

7) Conclusion: Because 1.655 < 2.998, fail to reject the null hypothesis. The average weight loss is not significantly greater than 1.5 at $\alpha = 0.01$.

c) 2) $H_0: \mu_d = 2.2$

3) $H_1: \mu_d > 2.2$

4) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$ for $\alpha = 0.05$

6) $\bar{d} = 1.875$

$s_d = 0.641$
$n = 8$

$$t_0 = \frac{1.875 - 2.2}{0.641/\sqrt{8}} = -1.434$$

7) Conclusion: Because –1.434 < 1.895, fail to reject the null hypothesis and conclude that the average weight loss is not significantly greater than 2.2 at α = 0.05.

d) 2) $H_0: \mu_d = 2.2$
3) $H_1: \mu_d > 2.2$
4) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,7} = 2.998$ for α = 0.01.
6) $\bar{d} = 1.875$
$s_d = 0.641$
$n = 8$

$$t_0 = \frac{1.875 - 2.2}{0.641/\sqrt{8}} = -1.434$$

7) Conclusion: Because –1.434 < 2.998, fail to reject the null hypothesis and conclude the average weight loss is not significantly greater than 2.2 at α = 0.01.

10-78 $$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

a) 90% confidence interval: $z_{\alpha/2} = 1.65$

$$\left(600 - 625\right) - 1.65\sqrt{\frac{35^2}{20} + \frac{30^2}{20}} \le \mu_1 - \mu_2 \le \left(600 - 625\right) + 1.65\sqrt{\frac{35^2}{20} + \frac{30^2}{20}}$$

$$-42.01 \le \mu_1 - \mu_2 \le -7.99$$

Yes, the data indicate that the mean breaking strength of the yarn of manufacturer 2 exceeds that of manufacturer 1 by between 42.01 and 7.99 with 90% confidence.

b) 98% confidence interval: $z_{\alpha/2} = 2.33$

$$\left(600 - 625\right) - 2.33\sqrt{\frac{35^2}{20} + \frac{30^2}{20}} \le \mu_1 - \mu_2 \le \left(600 - 625\right) + 2.33\sqrt{\frac{35^2}{20} + \frac{30^2}{20}}$$

$$-49.02 \le \mu_1 - \mu_2 \le 0.98$$

Yes, we can again conclude that yarn of manufacturer 2 has greater mean breaking strength than that of manufacturer 1 by between 49.02 and 0.98 with 98% confidence.

c) The results of parts (a) and (b) are same although the confidence level used is different. The appropriate interval depends upon the level of confidence considered acceptable.

10-79 a)
1) The parameters of interest are the proportions of children who contract polio, p_1 , p_2
2) $H_0: p_1 = p_2$
3) $H_1: p_1 \neq p_2$
4) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 1.96$ for α = 0.05

6) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{110}{201299} = 0.00055$ (Placebo) $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.000356$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{33}{200745} = 0.00016$ (Vaccine)

$$z_0 = \frac{0.00055 - 0.00016}{\sqrt{0.000356(1-0.000356)\left(\frac{1}{201299} + \frac{1}{200745}\right)}} = 6.55$$

7) Because 6.55 > 1.96, reject H_0 and conclude the proportions of children who contracted polio differ at $\alpha = 0.05$.

b) $\alpha = 0.01$
Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 2.58$. Here, still $z_0 = 6.55$.
Because 6.55 > 2.58, reject H_0 and conclude the proportions of children who contracted polio differ at $\alpha = 0.01$.

c) The conclusions are the same because z_0 is large enough to exceed $z_{\alpha/2}$ in both cases.

10-80 a) $\alpha = 0.10$ $z_{\alpha/2} = 1.65$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(E)^2} \cong \frac{(1.65)^2\,(1225+900)}{(10)^2} = 57.85,\ n = 58$$

b) $\alpha = 0.02$ $z_{\alpha/2} = 2.33$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(E)^2} \cong \frac{(2.33)^2\,(1225+900)}{(10)^2} = 115.36,\ n = 116$$

c) As the confidence level increases, sample size also increases.

d) $\alpha = 0.10$ $z_{\alpha/2} = 1.65$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(E)^2} \cong \frac{(1.65)^2\,(1225+900)}{(5)^2} = 231.41,\ n = 232$$

$\alpha = 0.02$ $z_{\alpha/2} = 2.33$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(E)^2} \cong \frac{(2.33)^2\,(1225+900)}{(5)^2} = 461.5,\ n = 462$$

e) As the error decreases, the required sample size increases.

10-81 $\hat{p}_1 = \frac{x_1}{n_1} = \frac{516}{2000} = 0.258$ $\hat{p}_2 = \frac{x_2}{n_2} = \frac{310}{1200} = 0.2583$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

a) $z_{\alpha/2} = z_{0.025} = 1.96$

$$(0.258 - 0.2583) \pm 1.96\sqrt{\frac{0.258(0.742)}{1500} + \frac{0.2583(0.7417)}{1200}}$$

$$-0.0335 \le p_1 - p_2 \le 0.0329$$

Because zero is contained in this interval, there is no significant difference between the proportions of unlisted numbers in the two cities at a 5% significance level.

b) $z_{\alpha/2} = z_{0.05} = 1.65$

$$(0.258 - 0.2583) \pm 1.65\sqrt{\frac{0.258(0.742)}{1500} + \frac{0.2583(0.7417)}{1200}}$$

$$-0.0282 \le p_1 - p_2 \le 0.0276$$

The proportions of unlisted numbers in the two cities do not significantly differ at a 5% significance level.

c) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{1032}{4000} = 0.258$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{620}{2400} = 0.2583$

95% confidence interval:

$$(0.258 - 0.2583) \pm 1.96\sqrt{\frac{0.258(0.742)}{3000} + \frac{0.2583(0.7417)}{2400}}$$

$-0.0238 \le p_1 - p_2 \le 0.0232$

90% confidence interval:

$$(0.258 - 0.2583) \pm 1.65\sqrt{\frac{0.258(0.742)}{3000} + \frac{0.2583(0.7417)}{2400}} \qquad -0.0201 \le p_1 - p_2 \le 0.0195$$

Increasing the sample size decreased the width of the confidence interval, but did not change the conclusions drawn. The conclusion remains that there is no significant difference.

10-82 a)

1) The parameters of interest are the proportions of those residents who wear a seat belt regularly, p_1 , p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$ for $\alpha = 0.05$

6) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{205}{250} = 0.82$ $\qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.778$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{192}{260} = 0.738$

$$z_0 = \frac{0.82 - 0.738}{\sqrt{0.778(1-0.778)\left(\frac{1}{250} + \frac{1}{260}\right)}} = 2.228$$

7) Conclusion: Because 2.228>1.96, reject H_0. There is a difference in seat belt usage at $\alpha = 0.05$.

b) $\alpha = 0.10$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.05} = 1.65$ $\qquad z_0 = 2.228$

Because 2.228>1.65, reject H_0. There is a difference in seat belt usage at $\alpha = 0.10$.

c) The conclusions are the same, but with different levels of significance.

d) $n_1 = 500$, $n_2 = 520$

$\alpha = 0.05$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

$$z_0 = \frac{0.82 - 0.738}{\sqrt{0.778(1-0.778)\left(\frac{1}{500} + \frac{1}{520}\right)}} = 3.15$$

Because 3.15> 1.96, reject H_0. There is a difference in seat belt usage at $\alpha = 0.05$.

$\alpha = 0.10$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.05} = 1.65$ $\qquad z_0 = 1.246$

Because 3.15> 1.65, reject H_0. There is a difference in seat belt usage at $\alpha = 0.10$.

As the sample size increased, the test statistic also increased (because the denominator of z_0 decreased). However, the sample size increase was not enough to change our conclusion.

10-83 a) Yes, there could be some bias in the results due to the telephone survey.
b) If it could be shown that these populations are similar to the respondents, the results may be extended.

10-84 The parameter of interest is $\mu_1 - 2\mu_2$

$$H_0: \mu_1 = 2\mu_2 \quad \rightarrow \quad H_0: \mu_1 - 2\mu_2 = 0$$
$$H_1: \mu_1 > 2\mu_2 \qquad\quad H_1: \mu_1 - 2\mu_2 > 0$$

Let n_1 = size of sample 1 $\qquad \overline{X}_1$ estimate for μ_1

Let n_2 = size of sample 2 $\qquad \overline{X}_2$ estimate for μ_2

$\overline{X}_1 - 2\overline{X}_2$ is an estimate for $\mu_1 - 2\mu_2$

The variance is $V(\overline{X}_1 - 2\overline{X}_2) = V(\overline{X}_1) + V(2\overline{X}_2) = \dfrac{\sigma_1^2}{n_1} + \dfrac{4\sigma_2^2}{n_2}$

The test statistic for this hypothesis is:

$$Z_0 = \frac{(\overline{X}_1 - 2\overline{X}_2) - 0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{4\sigma_2^2}{n_2}}}$$

We reject the null hypothesis if $z_0 > z_{\alpha/2}$ for a given level of significance. *P*-value = $P(Z \geq z_0)$.

10-85 $\bar{x}_1 = 910 \quad \bar{x}_2 = 905$

$\sigma_1 = 3 \quad \sigma_2 = 4.5$

$n_1 = 12 \quad n_2 = 10$

a) 90% two-sided confidence interval:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(910 - 905) - 1.645\sqrt{\frac{3^2}{12} + \frac{4.5^2}{10}} \leq \mu_1 - \mu_2 \leq (910 - 905) + 1.645\sqrt{\frac{3^2}{12} + \frac{4.5^2}{10}}$$

$$2.259 \leq \mu_1 - \mu_2 \leq 7.741$$

We are 90% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 2.259 and 7.741 ml.

b) 95% two-sided confidence interval:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(910 - 905) - 1.96\sqrt{\frac{3^2}{12} + \frac{4.5^2}{10}} \leq \mu_1 - \mu_2 \leq (910 - 905) + 1.96\sqrt{\frac{3^2}{12} + \frac{4.5^2}{10}}$$

$$1.735 \leq \mu_1 - \mu_2 \leq 8.265$$

We are 95% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 1.735 and 8.265 ml.

Comparison of parts (a) and (b): As the level of confidence increases, the interval width also increases (with all other values held constant).

c) 95% upper-sided confidence interval:

$$\mu_1 - \mu_2 \leq \left(\bar{x}_1 - \bar{x}_2\right) + z_\alpha\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1-\mu_2 \le (910-905)+1.645\sqrt{\frac{3^2}{12}+\frac{4.5^2}{10}}$$

$$\mu_1-\mu_2 \le 7.741$$

With 95% confidence, the fill volume for machine 1 exceeds the fill volume of machine 2 by no more than 7.741 ml.

d) 1) The parameter of interest is the difference in mean fill volume $\mu_1-\mu_2$

2) H_0 : $\mu_1-\mu_2=0$ or $\mu_1=\mu_2$

3) H_1 : $\mu_1-\mu_2\neq 0$ or $\mu_1\neq\mu_2$

4) The test statistic is

$$z_0=\frac{(\bar{x}_1-\bar{x}_2)-\Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$ for $\alpha = 0.05$

6) $\bar{x}_1 = 910$ $\quad\bar{x}_2 = 905$

$\sigma_1 = 3$ $\quad\sigma_2 = 4.5$

$n_1 = 12$ $\quad n_2 = 10$

$$z_0=\frac{(910-905)}{\sqrt{\frac{3^2}{12}+\frac{4.5^2}{10}}}=3$$

7) Because 3 > 1.96 reject the null hypothesis and conclude the mean fill volumes of machine 1 and machine 2 differ significantly at $\alpha = 0.05$.

P-value = $2[1-\Phi(3)] = 2(1-0.998650) = 0.0027$

e) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.10$, and $\Delta = 5$

$$n \cong \frac{\left(z_{\alpha/2}+z_\beta\right)^2\left(\sigma_1^2+\sigma_2^2\right)}{\left(\Delta-\Delta_0\right)^2}=\frac{(1.96+1.28)^2\left(3^2+4.5^2\right)}{(-5)^2}=12.3,\quad n = 12\text{, use } n_1 = n_2 = 12$$

10-86 $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$n_1 = n_2 = n$

$\beta = 0.10$

$\alpha = 0.05$

Assume normal distribution and $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$\mu_1 = \mu_2 + \sigma$

$$d = \frac{|\mu_1-\mu_2|}{2\sigma}=\frac{\sigma}{2\sigma}=\frac{1}{2}$$

From Chart VII*e*, $n^* = 50$ and $n=\frac{n^*+1}{2}=\frac{50+1}{2}=25.5$ and $n_1 = n_2 = 26$

10-87 a)

1) The parameters of interest are: the proportion of lenses that are unsatisfactory after tumble-polishing, p_1, p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) The test statistic is

$$z_0=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 2.58$ for $\alpha = 0.01$.

6) x_1 = number of defective lenses

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{147}{400} = 0.3675 \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.439$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{204}{400} = 0.51$$

$$z_0 = \frac{0.3675 - 0.51}{\sqrt{0.439(1-0.439)\left(\frac{1}{400} + \frac{1}{400}\right)}} = -4.06$$

7) Conclusion: Because –4.06 < –2.58, reject H_0 and conclude that the proportions from the two polishing fluids are different at $\alpha = 0.01$.

b) The conclusions are the same whether we analyze the data using the proportion unsatisfactory or proportion satisfactory.

c)

$$n = \frac{\left(2.575\sqrt{\frac{(0.9+0.6)(0.1+0.4)}{2}} + 1.28\sqrt{0.9(0.1)+0.6(0.4)}\right)^2}{(0.9-0.6)^2}$$

$$= \frac{5.346}{0.09} = 59.4$$

$$n = 60$$

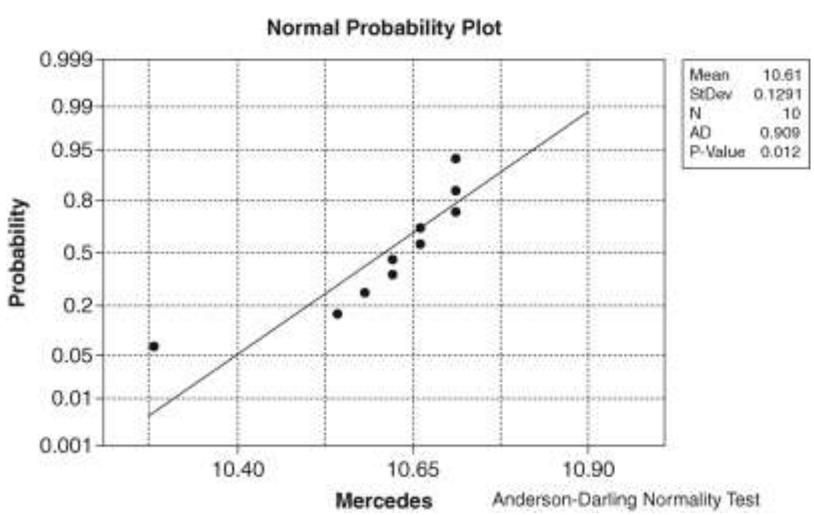
10-88 a) $\alpha = 0.05$, $\beta = 0.05$, $\Delta = 1.5$. Use $s_p = 0.7071$ to approximate σ.

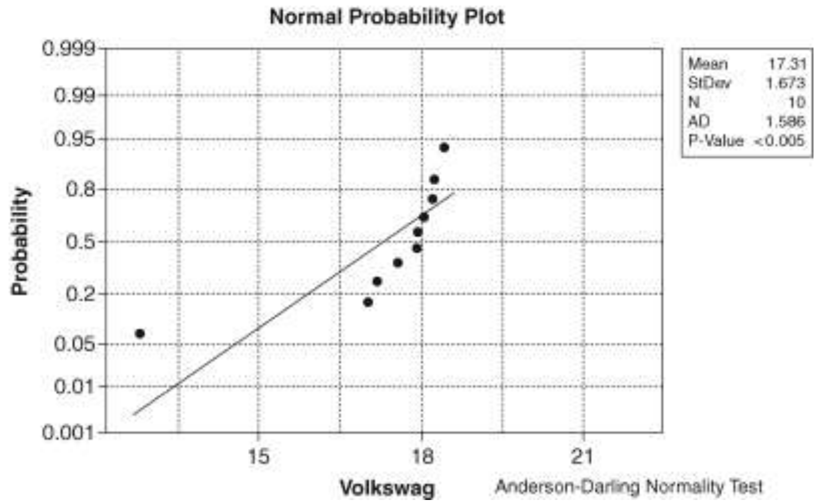
$$d = \frac{\Delta}{2(s_p)} = \frac{1.5}{2(.7071)} = 1.06 \cong 1$$

From Chart VII*e*, $n^* = 20$ $n = \frac{n^* + 1}{2} = \frac{20+1}{2} = 10.5$ n = 11 is needed to detect that the two agents differ by 0.5 with probability of at least 0.95.

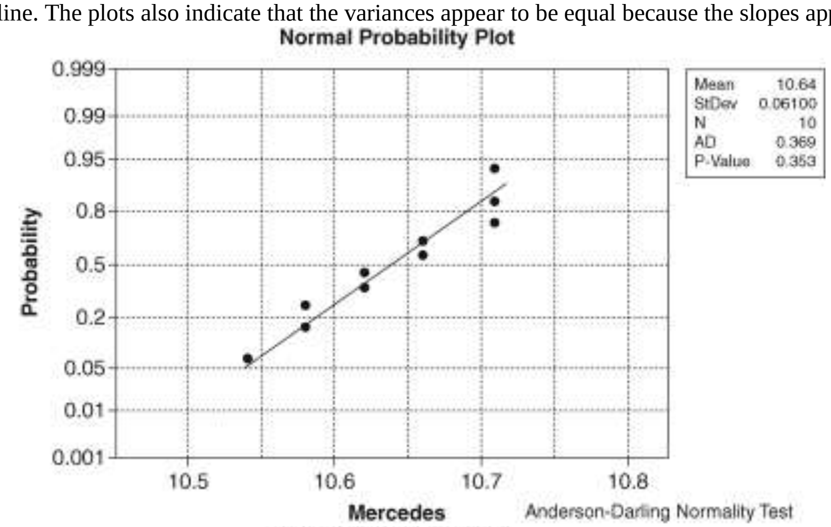
b) The original size of n = 5 was not appropriate to detect the difference because a sample size of 11 is needed to detect that the two agents differ by 1.5 with probability of at least 0.95.

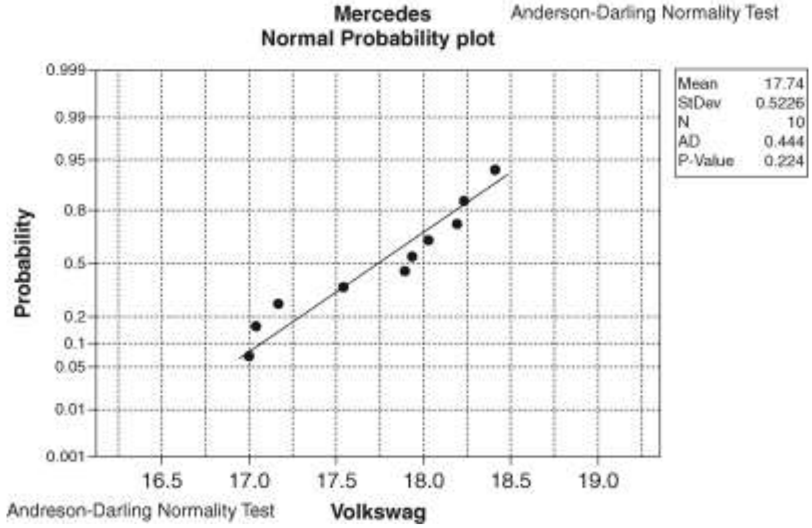
10-89 a) No





b) The normal probability plots indicate that the data follow normal distributions because the data appear to fall along a straight line. The plots also indicate that the variances appear to be equal because the slopes appear to be the same.





c) By correcting the data points, it is more apparent the data follow normal distributions. Note that one unusual observation can cause an analyst to reject the normality assumption.

d) 95% confidence interval on the ratio of the variances, σ_V^2/σ_M^2

$$s_V^2 = 0.27 \qquad f_{9,9,0.025} = 4.03$$

$$s_M^2 = 0.0037 \qquad f_{9,9,0.975} = \frac{1}{f_{9,9,0.025}} = \frac{1}{4.03} = 0.248$$

$$\left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.975} < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.025}$$

$$\left(\frac{0.27}{0.0037}\right) 0.248 < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{0.27}{0.0037}\right) 4.03$$

$$18.097 < \frac{\sigma_V^2}{\sigma_M^2} < 294.08$$

Because the interval does not include the value one, we reject the hypothesis that variability in mileage performance is the same for the two types of vehicles. There is evidence that the variability is greater for a Volkswagen than for a Mercedes.

e)

1) The parameters of interest are the variances in mileage performance, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$ Where Volkswagen is represented by variance 1, Mercedes by variance 2.

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

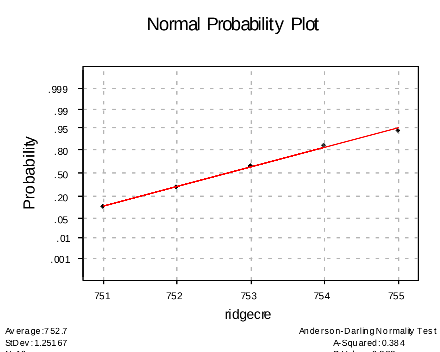
5) Reject H_0 if $f_0 < f_{0.975,9,9}$ where $f_{0.975,9,9} = \frac{1}{f_{0.025,9,9}} = \frac{1}{4.03} = 0.248$ for $\alpha = 0.05$ or $f_0 > f_{0.025,9,9}$ where $f_{0.025,9,9} =$ 4.03 for $\alpha = 0.05$

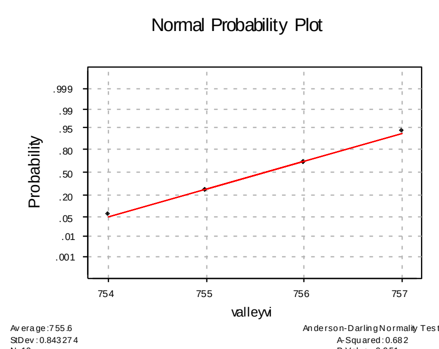
6) $s_1 = 0.5226$ $\qquad$ $s_2 = 0.061$
$n_1 = 10$ $\qquad$ $n_2 = 10$

$$f_0 = \frac{(0.5226)^2}{(0.061)^2} = 73.4$$

7) Conclusion: Because 72.78 > 4.03 reject H_0 and conclude that there is a significant difference between Volkswagen and Mercedes in terms of mileage variability. The same conclusions are reached in part (d).

10-90 a) Underlying distributions appear to be normally distributed because the data fall along a straight line on the normal probability plots. The slopes appear to be similar so it is reasonable to assume that $\sigma_1^2 = \sigma_2^2$.





b) 1) The parameter of interest is the difference in mean volumes, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject H_0 if $t_0 < -t_{\alpha/2,\nu}$ or $z_0 > t_{\alpha/2,\nu}$ where $t_{\alpha/2,\nu} = t_{0.025,18} = 2.101$ for $\alpha = 0.05$

6) $\bar{x}_1 = 752.7$ $\quad \bar{x}_2 = 755.6$ $\quad s_p = \sqrt{\frac{9(1.252)^2 + 9(0.843)^2}{18}} = 1.07$

$s_1 = 1.252$ $\quad s_2 = 0.843$

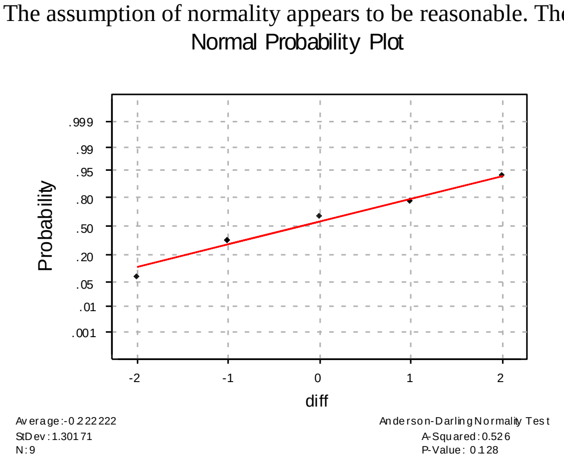
$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(752.7 - 755.6)}{1.07\sqrt{\frac{1}{10} + \frac{1}{10}}} = -6.06$$

7) Conclusion: Because $-6.06 < -2.101$ reject H_0 and conclude there is a significant difference between the two wineries with respect to mean fill volumes at a 5% significance level.

c) From Section 10-3.3, $d = 2/2(1.07) = 0.93$, giving a power of just under 80%. Because the power is relatively low, an increase in the sample size would improve the power of the test.

10-91 a) The assumption of normality appears to be reasonable. The data lie along a line in the normal probability plot.



b)

1) The parameter of interest is the mean difference in tip hardness, μ_d

2) $H_0 : \mu_d = 0$

3) $H_1 : \mu_d \neq 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Since no significance level is given, we calculate the *P*-value. Reject H_0 if the *P*-value is sufficiently small.

6) $\bar{d} = -0.222$

$s_d = 1.30$

$n = 9$

$$t_0 = \frac{-0.222}{1.30/\sqrt{9}} = -0.512$$

P-value = $2P(T < -0.512) = 2P(T > 0.512)$ and $2(0.25) <$ *P*-value $< 2(0.40)$. Thus, $0.50 <$ *P*-value < 0.80

7) Conclusion: Because the P-value is greater than common levels of significance, fail to reject H_0 and conclude there is no difference in mean tip hardness.

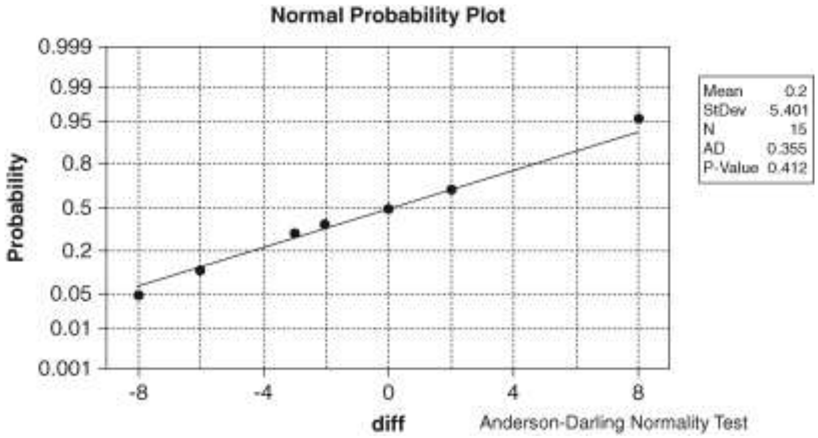
c) $\beta = 0.10$

$\mu_d = 1$

$d = \dfrac{1}{\sigma_d} = \dfrac{1}{1.3} = 0.769$

From Chart VII*f* with $\alpha = 0.01$, $n = 30$

10-92 a) From the normal probability plot the data fall along a line and consequently they appear to follow a normal distribution.



b) 1) The parameter of interest is the mean difference in depth using the two gauges, μ_d

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Since no significance level is given, we will calculate *P*-value. Reject H_0 if the *P*-value is significantly small.

6) $\bar{d} = 0.2$

$s_d = 5.401$

$n = 15$

$$t_0 = \frac{0.2}{5.401/\sqrt{15}} = 0.14$$

P-value = 2P(T > 0.14), 2(0.44) < *P*-value, 0.88 < *P*-value

7) Conclusion: Because the P-value is larger than common levels of significance, fail to reject H_0 and conclude there is no significant difference in mean depth measurements for the two gauges.

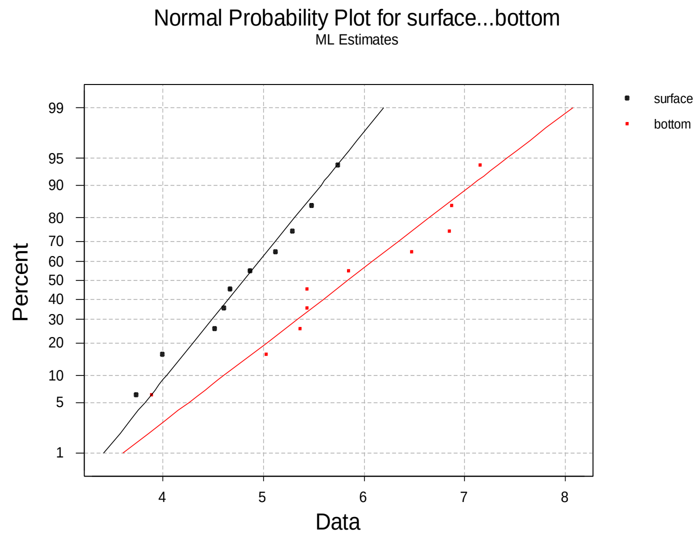
c) Power = 0.8. Because Power = 1 – β , β = 0.20

$\mu_d = 4.2$

$d = \dfrac{4.2}{\sigma_d} = \dfrac{4.2}{(5.401)} = 0.778$

From Chart VII (f) with α = 0.01 and β = 0.20, we find n = 30.

10-93 a) Because the data fall along lines, the data from both depths appear to be normally distributed, but the slopes do not appear to be equal. Therefore, it is not reasonable to assume that $\sigma_1^2 = \sigma_2^2$.



b)
1) The parameter of interest is the difference in mean HCB concentration, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.025,15}$ or $t_0 > t_{0.025,15}$ where $t_{0.025,15} = 2.131$ for $\alpha = 0.05$. Also

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 15.06$$

$$\nu \cong 15$$

6) $\bar{x}_1 = 4.804$ $\quad \bar{x}_2 = 5.839$ $\quad s_1 = 0.631$ $\quad s_2 = 1.014$

$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(4.804 - 5.839)}{\sqrt{\frac{(0.631)^2}{10} + \frac{(1.014)^2}{10}}} = -2.74$$

7) Conclusion: Because –2.74 < –2.131, reject the null hypothesis. Conclude that the mean HCB concentration is different at the two depths at a 0.05 level of significance.

c) Assume the variances are equal. Then $\Delta = 2$, $\alpha = 0.05$, $n = n_1 = n_2 = 10$, $n^* = 2n - 1 = 19$, $s_p = 0.84$

and $d = \dfrac{2}{2(0.84)} = 1.2$.

From Chart VII*e*, we find $\beta \approx 0.05$, and then calculate the power $= 1 - \beta = 0.95$

d) Assume the variances are equal. Then $\Delta = 1$, $\alpha = 0.05$, $n = n_1 = n_2$, $n^* = 2n - 1$, $\beta = 0.1$, $s_p = 0.84 \approx 1$

and $d = \dfrac{1}{2(0.84)} = 0.6$.

From Chart VII*e*, we find *n** = 50 and $n = \dfrac{50+1}{2} = 25.5$, so $n = 26$.

Mind-Expanding Exercises

10-94 The estimate of μ is given by $\hat{\mu} = \frac{1}{2}(\overline{X}_1 + \overline{X}_2) - \overline{X}_3$. From the independence, the variance of $\hat{\mu}$ can be shown to be

$$V(\hat{\mu}) = \frac{1}{4}\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right) + \frac{\sigma_3^2}{n_3}.$$

Use s_1, s_2, and s_3 as estimates for σ_1, σ_2, and σ_3, respectively. One may also used a pooled estimate of variability.

a) An approximate 100(1 – α)% confidence interval on μ is then:

$$\hat{\mu} - Z_{\alpha/2}\sqrt{\frac{1}{4}\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right) + \frac{s_3^2}{n_3}} \le \mu \le \hat{\mu} + Z_{\alpha/2}\sqrt{\frac{1}{4}\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right) + \frac{s_3^2}{n_3}}$$

$$\left(\frac{1}{2}(4.6+5.2)-6.1\right)-1.96\sqrt{\frac{1}{4}\left(\frac{0.7^2}{100}+\frac{0.6^2}{120}\right)+\frac{0.8^2}{130}} \le \mu \le \left(\frac{1}{2}(4.6+5.2)-6.1\right)+1.96\sqrt{\frac{1}{4}\left(\frac{0.7^2}{100}+\frac{0.6^2}{120}\right)+\frac{0.8^2}{130}}$$

$$-1.2-0.163 \le \mu \le -1.2+0.163$$

$$-1.363 \le \mu \le -1.037$$

b) An approximate one-sided 95% confidence interval for $\hat{\mu}$ is

$$\mu \le \left(\frac{1}{2}(4.6+5.2)-6.1\right)+1.64\sqrt{\frac{1}{4}\left(\frac{0.7^2}{100}+\frac{0.6^2}{120}\right)+\frac{0.8^2}{130}}$$

$$\mu \le -1.2+0.136$$

$$\mu \le -1.064$$

Because the interval is negative and does not contain zero, we can conclude that that pesticide three is more effective.

10-95 The $V(\overline{X}_1 - \overline{X}_2) = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$ and suppose this is to equal a constant *k*. Then, we are to minimize $C_1 n_1 + C_2 n_2$ subject to $\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} = k$. Using a Lagrange multiplier, we minimize by setting the partial derivatives of

$f(n_1, n_2, \lambda) = C_1 n_1 + C_2 n_2 + \lambda\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} - k\right)$ with respect to n_1, n_2 and λ equal to zero.

These equations are

$$\frac{\partial}{\partial n_1} f(n_1, n_2, \lambda) = C_1 - \frac{\lambda\sigma_1^2}{n_1^2} = 0 \qquad (1)$$

$$\frac{\partial}{\partial n_2} f(n_1, n_2, \lambda) = C_2 - \frac{\lambda\sigma_2^2}{n_2^2} = 0 \qquad (2)$$

$$\frac{\partial}{\partial \lambda} f(n_1, n_2, \lambda) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = k \qquad (3)$$

Upon adding equations (1) and (2), we obtain $C_1 + C_2 - \lambda\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}\right) = 0$

Substituting from equation (3) enables us to solve for λ to obtain $\dfrac{C_1 + C_2}{k} = \lambda$

Then, equations (1) and (2) are solved for n_1 and n_2 to obtain

$$n_1 = \frac{\sigma_1^2(C_1 + C_2)}{kC_1} \qquad n_2 = \frac{\sigma_2^2(C_1 + C_2)}{kC_2}$$

It can be verified that this is a minimum. With these choices for n_1 and n_2

$$V(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}.$$

10-96 Maximizing the probability of rejecting H_0 is equivalent to minimizing

$$P\left(-z_{\alpha/2} < \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < z_{\alpha/2} \mid \mu_1 - \mu_2 = \delta \right) = P\left(-z_{\alpha/2} - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < Z < z_{\alpha/2} - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right)$$

where z is a standard normal random variable. This probability is minimized by maximizing $\frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$.

Therefore, we are to minimize $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ subject to $n_1 + n_2 = N$.

From the constraint, $n_2 = N - n_1$, we are to minimize $f(n_1) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{N - n_1}$.

Take the derivative of $f(n_1)$ with respect to n_1 and set it equal to zero results in the equation $\frac{-\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{(N - n_1)^2} = 0$.

Solve for n_1 to obtain $n_1 = \frac{\sigma_1 N}{\sigma_1 + \sigma_2}$ and $n_2 = \frac{\sigma_2 N}{\sigma_1 + \sigma_2}$

Also, it can be verified that the solution minimizes $f(n_1)$.

10-97 a) $\alpha = P(Z > z_\varepsilon \ \ or \ \ Z < -z_{\alpha-\varepsilon})$ where Z has a standard normal distribution.

Then, $\alpha = P(Z > z_\varepsilon) + P(Z < -z_{\alpha-\varepsilon}) = \varepsilon + \alpha - \varepsilon = \alpha$

b) $\beta = P(-z_{\alpha-\varepsilon} < Z_0 < z_\varepsilon | \mu_1 = \mu_0 + \delta)$

$$\begin{aligned}
\beta &= P(-z_{\alpha-\varepsilon} < \tfrac{\bar{x}-\mu_0}{\sqrt{\sigma^2/n}} < z_\varepsilon \mid \mu_1 = \mu_0 + \delta) \\
&= P(-z_{\alpha-\varepsilon} - \tfrac{\delta}{\sqrt{\sigma^2/n}} < Z < z_\varepsilon - \tfrac{\delta}{\sqrt{\sigma^2/n}}) \\
&= \Phi(z_\varepsilon - \tfrac{\delta}{\sqrt{\sigma^2/n}}) - \Phi(-z_{\alpha-\varepsilon} - \tfrac{\delta}{\sqrt{\sigma^2/n}})
\end{aligned}$$

10-98 The requested result can be obtained from data in which the pairs are very different. Example:

pair	1	2	3	4	5
sample 1	100	10	50	20	70
sample 2	110	20	59	31	80

$\bar{x}_1 = 50$ $\qquad \bar{x}_2 = 60$

$s_1 = 36.74$ $\qquad s_2 = 36.54$ $\qquad s_{pooled} = 36.64$

Two-sample t-test : $t_0 = -0.43$ $\qquad$ *P*-value = 0.68

$\bar{x}_d = -10$ $\qquad s_d = 0.707$

Paired t-test: $\qquad t_0 = -3162$ $\qquad$ *P*-value ≈ 0

10-99 a) $\theta = \dfrac{p_1}{p_2}$ and $\hat{\theta} = \dfrac{\hat{p}_1}{\hat{p}_2}$ and $\ln(\hat{\theta}) \sim N[\ln(\theta), \sqrt{(n_1 - x_1)/n_1x_1 + (n_2 - x_2)/n_2x_2}\,]$

The (1 – α) confidence interval for ln(θ) can use the relationship $Z = \dfrac{\ln(\hat{\theta}) - \ln(\theta)}{\left(\left(\dfrac{n_1 - x_1}{n_1x_1}\right) + \left(\dfrac{n_2 - x_2}{n_2x_2}\right)\right)^{1/4}}$

$$\ln(\hat{\theta}) - Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1x_1}\right) + \left(\frac{n_2 - x_2}{n_2x_2}\right)\right)^{1/4} \le \ln(\theta) \le \ln(\hat{\theta}) + Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1x_1}\right) + \left(\frac{n_2 - x_2}{n_2x_2}\right)\right)^{1/4}$$

b) The (1 – α) confidence interval for θ can use the CI developed in part (a) where θ = e^(ln(θ))

$$\hat{\theta}e^{-Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1x_1}\right) + \left(\frac{n_2 - x_2}{n_2x_2}\right)\right)^{1/4}} \le \theta \le \hat{\theta}e^{Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1x_1}\right) + \left(\frac{n_2 - x_2}{n_2x_2}\right)\right)^{1/4}}$$

c)

$$\hat{\theta}e^{-Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1x_1}\right) + \left(\frac{n_2 - x_2}{n_2x_2}\right)\right).25} \le \theta \le \hat{\theta}e^{Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1x_1}\right) + \left(\frac{n_2 - x_2}{n_2x_2}\right)\right).25}$$

$$1.42e^{-1.96\left(\left(\frac{100-27}{2700}\right) + \left(\frac{100-19}{1900}\right)\right)^{1/4}} \le \theta \le 1.42e^{1.96\left(\left(\frac{100-27}{2700}\right) + \left(\frac{100-19}{1900}\right)\right)^{1/4}}$$

$$0.519 \le \theta \le 3.887$$

Because the confidence interval contains the value one, we conclude that there is no significant difference in the proportions at the 95% level of significance.

10-100 $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \ne \sigma_2^2$

$$\beta = P\left(f^{\,2}_{1-\alpha/2, n_1-1, n_2-1} < \frac{S_1^2}{S_2^2} < f^{\,2}_{\alpha/2, n_1-1, n_2-1} \mid \frac{\sigma_1^2}{\sigma_2^2} = \delta \ne 1 \right)$$

$$= P\left(\frac{\sigma_2^2}{\sigma_1^2} f_{1-\alpha/2, n_1-1, n_2-1} < \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} < \frac{\sigma_2^2}{\sigma_1^2} f_{\alpha/2, n_1-1, n_2-1} \mid \frac{\sigma_1^2}{\sigma_2^2} = \delta \right)$$

where $\dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$ has an *F* distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.