CHAPTER 14

Section 14-3

14-1 a) 1. $H_0 : \tau_1 = \tau_2 = 0$ H_1 : at least one $\tau_i \neq 0$

2. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ H_1 : at least one $\beta_j \neq 0$

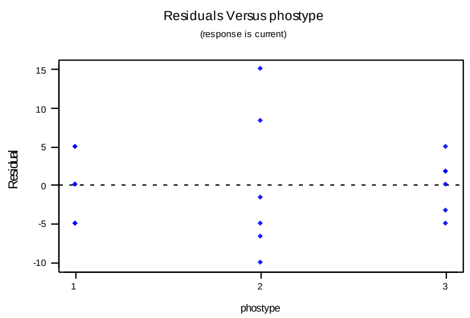
3. $H_0 : \tau\beta_{11} = \tau\beta_{12} = ... = \tau\beta_{23} = 0$ H_1 : at least one $\tau\beta_{ij} \neq 0$

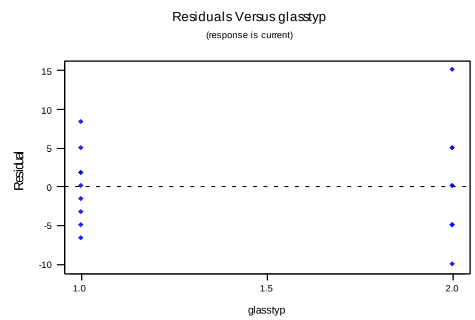
b) Analysis of Variance for current

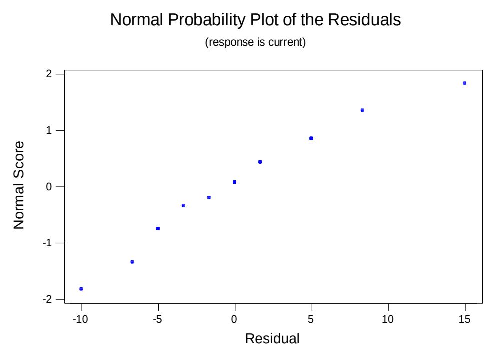
Source	DF	SS	MS	F	P
glasstyp	1	14450.0	14450.0	273.79	0.000
phostype	2	933.3	466.7	8.84	0.004
glasstyp*phostype	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

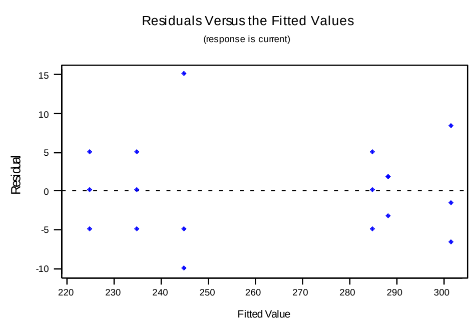
Main factors are significant, but the interaction is not significant. Glass type 1 and phosphor type 2 lead to the highest mean current (brightness).

c) There appears to be more slightly variability at phosphor type 2 and glass type 2. The normal plot of the residuals indicates that the assumption of normality is reasonable.









14-2 a) $H_0 : \tau_1 = \tau_2 = 0$

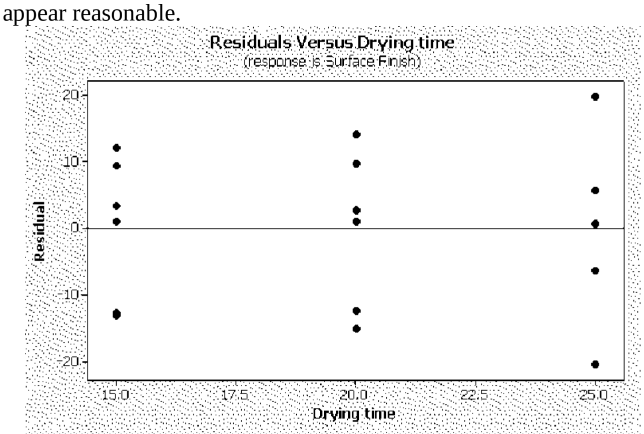
H_1 : at least one $\tau_j \neq 0$

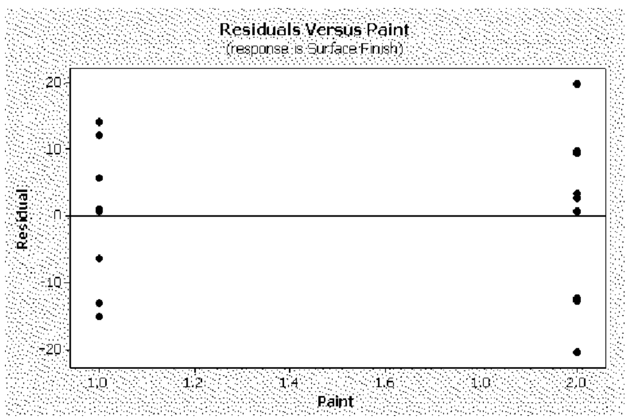
Analysis of variance for SURFACE FINISH

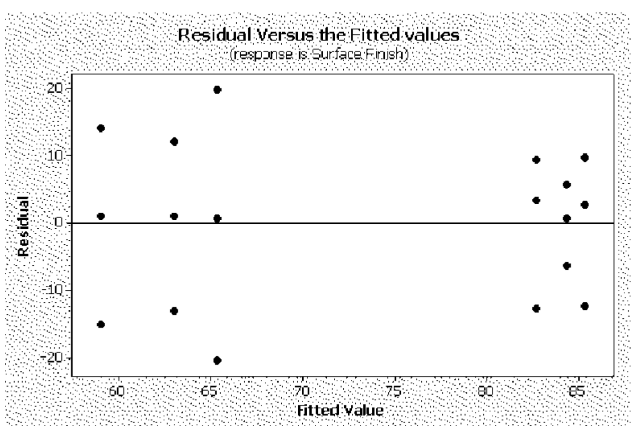
Source	DF	SS	MS	F	P
Drying time	2	23.11	11.556	0.07	0.937
Paint	1	364.50	364.500	2.06	0.177
Paint*drying	2	1797.33	898.667	5.09	0.025
Error	12	2120.67	176.722		
Total	17	4305.61			

Only the interaction between the paint and drying time is significant.

b)The residual plots appear reasonable.







14-3 a) $H_0: \tau_1 = \tau_2 = \tau_3 = 0$ $H_1:$ at least one $\tau_j \neq 0$

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$ $H_1:$ at least one $\beta_j \neq 0$

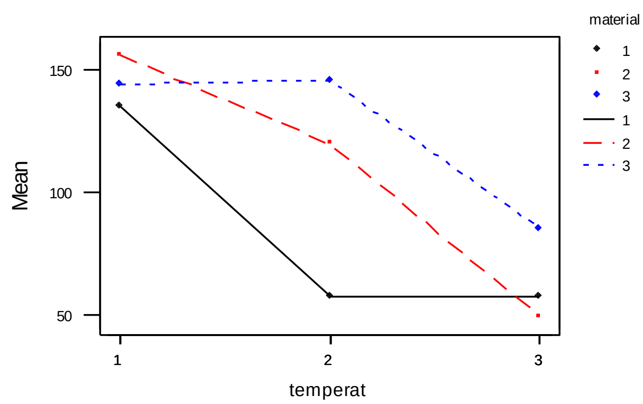
Analysis of Variance for life

Source	DF	SS	MS	F	P
material	2	10683.7	5341.9	7.91	0.002
temperat	2	39118.7	19559.4	28.97	0.000
material*temperat	4	9613.8	2403.4	3.56	0.019
Error	27	18230.7	675.2		
Total	35	77647.0			

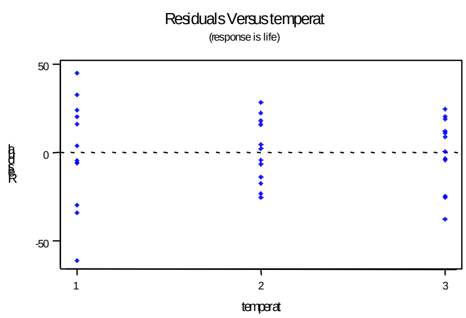
Main factors are significant, but the interaction is not significant.

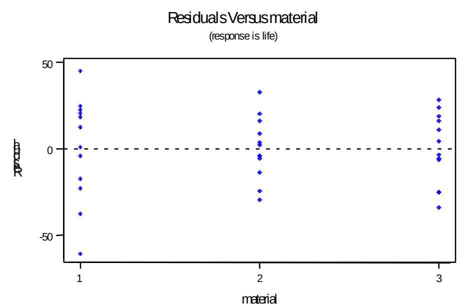
b) The mean life for material 2 is the highest at temperature level 1, in the middle at temperature level 2 and the lowest at temperature level 3. This shows that there is an interaction.

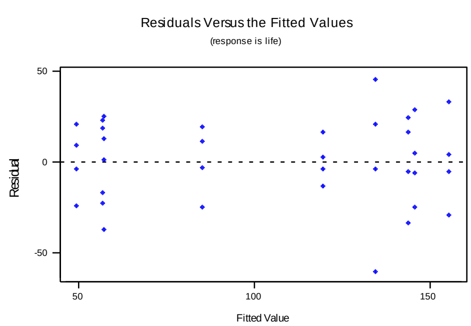
Interaction Plot - Means for life

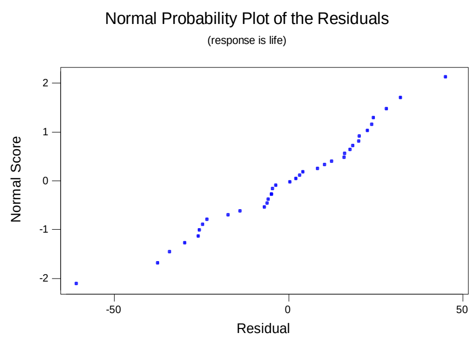


c) There appears to be slightly more variability at temperature 1 and material 1. The normal probability plot shows that the assumption of normality is reasonable.









14-4 a) 1. $H_0: \tau_1 = \tau_2 = 0$

H_1: at least one $\tau_j \neq 0$

2. $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

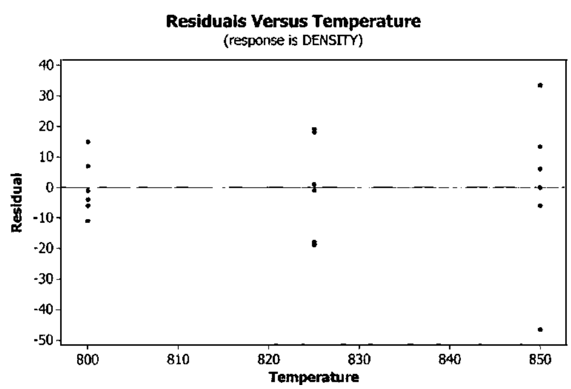
H_1: at least one $\beta_j \neq 0$

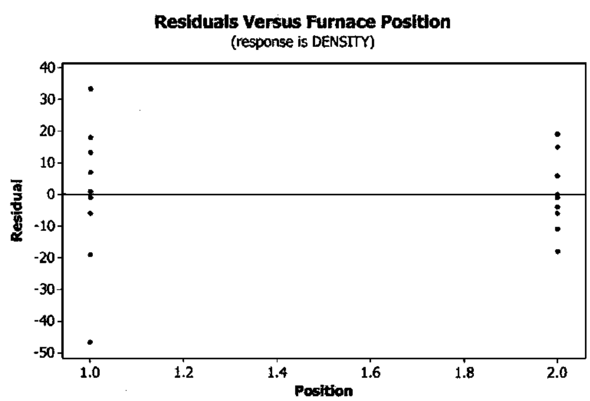
b) Analysis of Variance for DENSITY

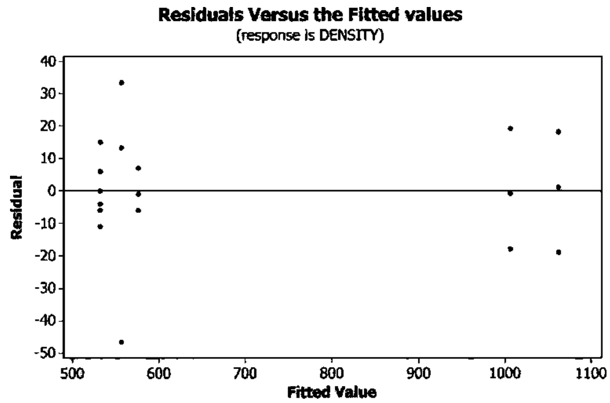
Source	DF	SS	MS	F	P
Temperat	2	940534	470267	1053.10	0.000
furnacep	1	7771	7771	17.40	0.001
furnacep*temp	2	750	375	0.84	0.456
Error	12	5359	447		
Total	17	954413			

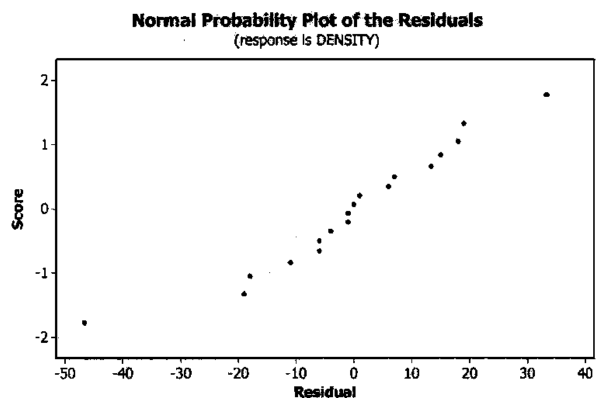
Reject H_0 for both main effects and conclude that both factors are significant.

c) There appears to be more variability at position 1 and temperature and the highest temperature level. There are two unusual points in the data.









d)
```
Fisher's pairwise comparisons
 Family error rate = 0.1187
 Individual error rate = 0.0500
 Critical value = 2.131

 Intervals for (column level mean) - (row level mean)
               800         825
  825      -517.43
           -442.57
  850       -27.77      452.23
             47.10      527.10
```

There are significant differences in the temperature levels 800 and 825, and 825 and 850. Therefore, temperature level 825 is different from the other two levels.

14-5

a) $Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \in_{ijk} \begin{cases} i = 1,2,3 \\ j = 1,2,3,4 \\ k = 1,2,3,4,5,6 \end{cases}$

b) $H_0 : \tau_1 = \tau_2 = \tau_3 = 0$ $\quad H_1$: at least one $\tau_j \neq 0$

$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ $\quad H_1$: at least one $\beta_j \neq 0$

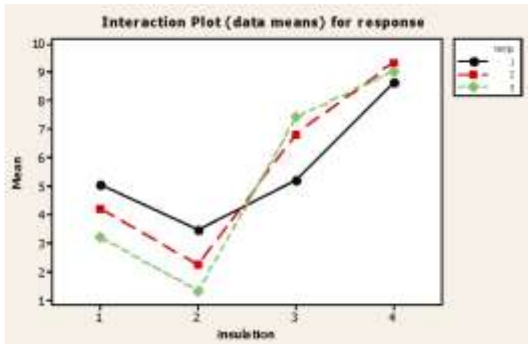
$H_0 : (\tau\beta)_{11} = \ldots = (\tau\beta)_{ab} = 0$ $\quad H_1$: at least one $(\tau\beta)_{ij} \neq 0$

Source	DF	SS	MS	F	P
insulation	3	453.608	151.203	40.07	0.000
temp	2	2.443	1.222	0.32	0.725
insulation*temp	6	38.536	6.423	1.70	0.136
Error	60	226.432	3.774		

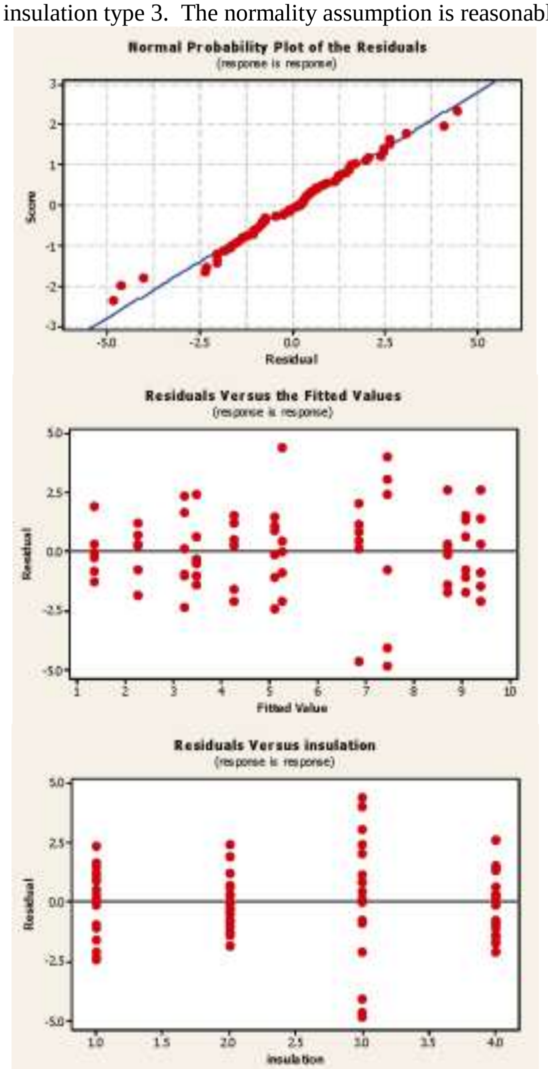
```
Total              71  721.019

S = 1.94264   R-Sq = 68.60%   R-Sq(adj) = 62.84%
```

There is only one significant main effect, insulation. Temperature and interaction effect are not significant.

c) Although there is some crossing of the lines, the interaction effect is minimal and was not found to be statistically significant in part (b).



d) There is more variability for insulation type 3. The normality assumption is reasonable.



e) Here, since only one of the main effects was significant, a model which included only insulation type was fit and LSD comparisons are made from that model:

```
Source      DF      SS      MS      F      P
Insulation   3  453.61  151.20  38.45  0.000
Error       68  267.41    3.93
Total       71  721.02

S = 1.983   R-Sq = 62.91%   R-Sq(adj) = 61.28%

Fisher 99% Individual Confidence Intervals
All Pairwise Comparisons among Levels of Insulation
Simultaneous confidence level = 95.20%

Insulation = 1 subtracted from:

Insulation   Lower  Center   Upper  ------+--------+---------+---------+---
2           -3.585  -1.833  -0.082           (-*--)
3            0.576   2.328   4.080                   (--*-)
4            3.104   4.856   6.607                        (--*-)
                                   ------+--------+---------+---------+---
                                       -5.0      0.0       5.0      10.0

Insulation = 2 subtracted from:

Insulation  Lower  Center  Upper  ------+---------+---------+---------+---
3           2.409   4.161  5.913                        (-*--)
4           4.937   6.689  8.441                             (-*--)
                                  ------+---------+---------+---------+---
                                      -5.0       0.0       5.0      10.0

Insulation = 3 subtracted from:

Insulation  Lower  Center  Upper  ------+---------+---------+---------+---
4           0.776   2.528  4.280                    (--*--)
                                  ------+---------+---------+---------+---
                                      -5.0       0.0       5.0      10.0
```

Because none of the intervals contain 0, all 4 insulation types are significantly different.

14-6 1. $H_0 : \tau_1 = \tau_2 = 0$

$H_1 :$ at least one $\tau_j \neq 0$

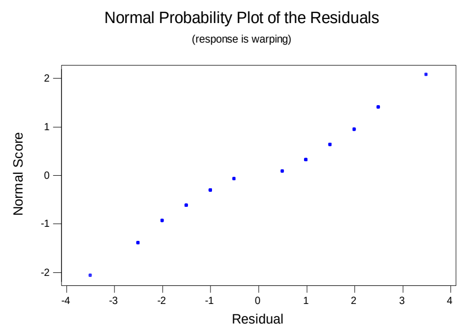
2. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

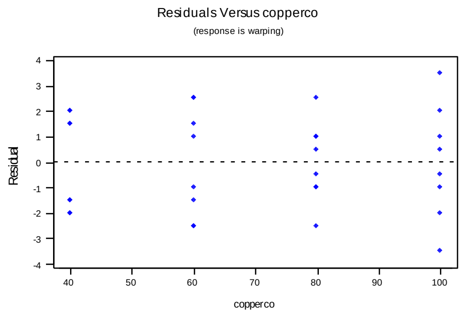
$H_1 :$ at least one $\beta_j \neq 0$

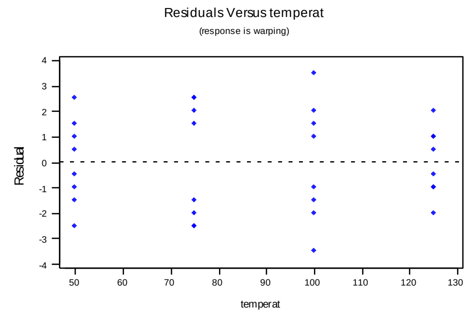
```
Analysis of Variance for warping
Source         DF       SS       MS       F       P
temp            3    156.09    52.03    7.67    0.002
copper          3    698.34   232.78   34.33    0.000
Interaction     9    113.78    12.64    1.86    0.133
Error          16    108.50     6.78
Total          31   1076.72
```

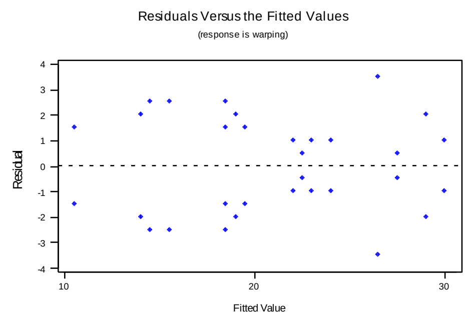
Reject H_0 for both of the main effects and conclude that both temperature and copper content have an effect on the mean warping. The interaction is not significant.

b) The residuals for this experiment appear reasonable.

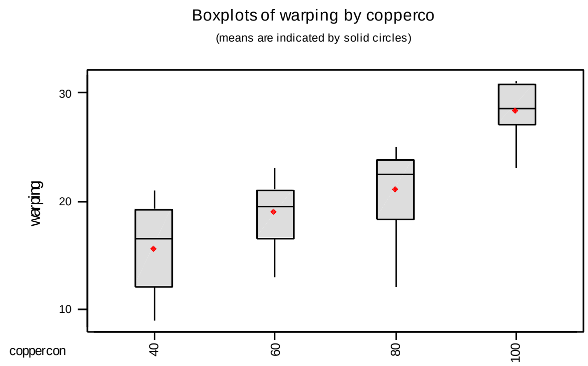








c)




```
Fisher's pairwise comparisons
Family error rate = 0.195
Individual error rate = 0.0500
Critical value = 2.048
Intervals for (column level mean) - (row level mean)
```

	40	60	80
60	-7.139		
	0.389		
80	-9.264	-5.889	
	-1.736	1.639	
100	-16.514	-13.139	-11.014
	-8.986	-5.611	-3.486

There are significant differences in the following temperature levels:
40 and 80, 40 and 100,
60 and 100,
80 and 100
This difference is apparent on the boxplot and using Fisher's LSD method. If low warping is desired, temperature level 40 is most desirable.

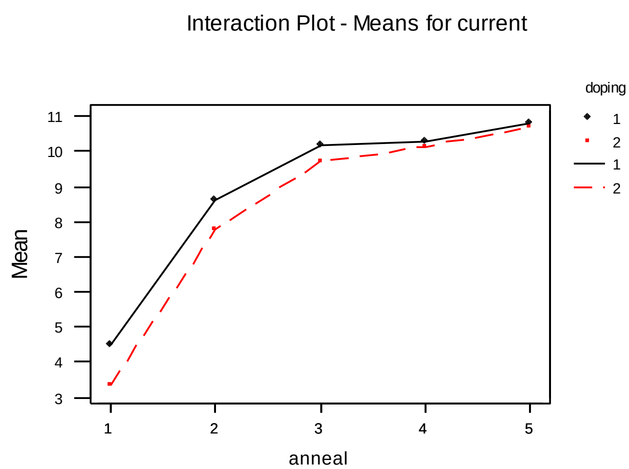
d) No, because the factors do not interact.

14-7 a) Analysis of Variance for current

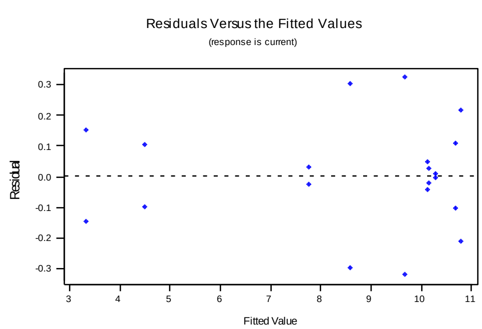
Source	DF	SS	MS	F	P
doping	1	1.442	1.442	25.23	0.000
anneal	4	124.238	31.059	543.52	0.000
doping*anneal	4	0.809	0.202	3.54	0.048
Error	10	0.571	0.057		
Total	19	127.060			

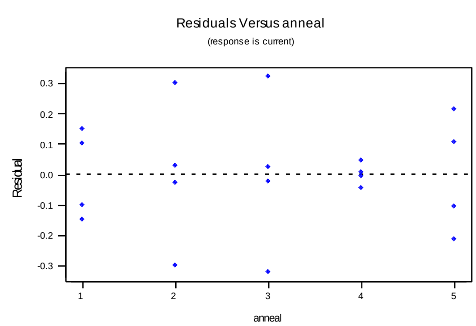
Both main factors are highly significant. The interaction is not significant.

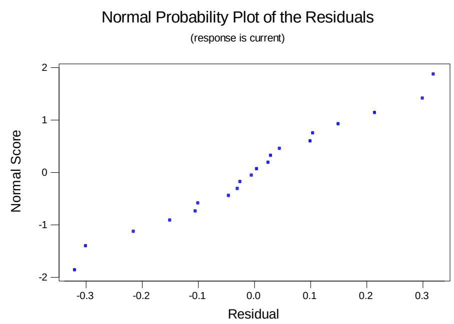
b) The interaction plot shows that there is a slight interaction because the lines are not parallel.

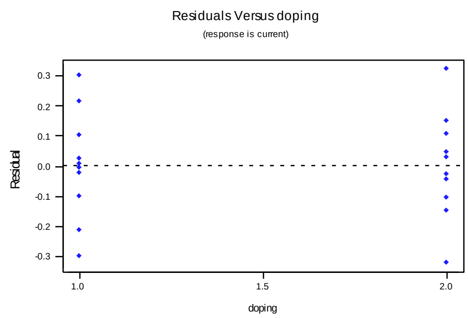


c) Analysis of the residual plots shows that all there is no problem with the model adequacy or the assumptions necessary to build the model.









d)

```
Fisher's pairwise comparisons
Family error rate = 0.258
Individual error rate = 0.0500
Critical value = 2.131
Intervals for (column level mean) - (row level mean)
```

	1	2	3	4
2	-4.9187			
	-3.6113			
3	-6.6662	-2.4012		
	-5.3588	-1.0938		
4	-6.9487	-2.6837	-0.9362	
	-5.6413	-1.3763	0.3712	
5	-7.4787	-3.2137	-1.4662	-1.1837
	-6.1713	-1.9063	-0.1588	0.1237

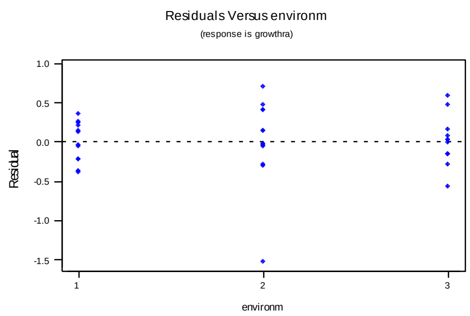
There are significant differences in the annealing levels
1 and 2, 1 and 3, 1 and 4, 1 and 5,
2 and 3, 2 and 4, 2 and 5.
Therefore levels 1 and 2 are different from the other three.

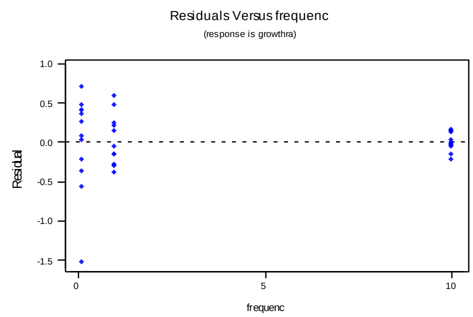
14-8 a) Analysis of Variance for Crack Growth

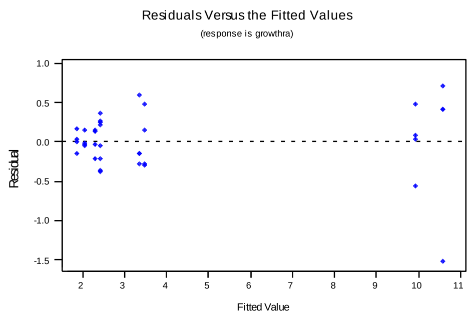
Source	DF	SS	MS	F	P
frequenc	2	209.893	104.946	522.40	0.000
environm	2	64.252	32.126	159.92	0.000
frequenc*environm	4	101.966	25.491	126.89	0.000
Error	27	5.424	0.201		
Total	35	381.535			

Both main factors and the interaction are significant.

b) There appear to be some problems with constant variance in the residual plots.





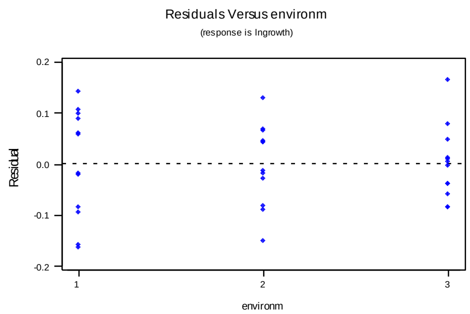


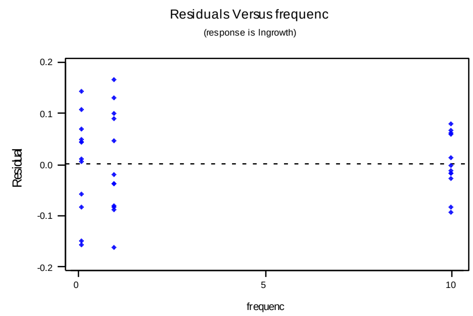
c) Analysis of Variance of Ln(Crack Growth)

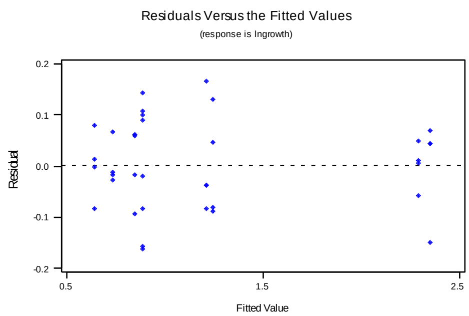
Source	DF	SS	MS	F	P
frequenc	2	7.5702	3.7851	404.09	0.000
environm	2	2.3576	1.1788	125.85	0.000
frequenc*environm	4	3.5284	0.8821	94.17	0.000
Error	27	0.2529	0.0094		
Total	35	13.7092			

The factors frequency, environment, and their interaction are all significant using the log of the data in the ANOVA

Residual plots on the log scale are improved. The variance appears to be more constant.







14-9 The ratio $T = \dfrac{\bar{y}_{\cdot i \cdot} - \bar{y}_{\cdot j \cdot} - (\mu_i - \mu_j)}{\sqrt{2MS_E / n}}$ has a t distribution with *ab*(*n*-1) degrees of freedom

Therefore, the (1-α)% confidence interval on the difference in two treatment means is

$$\bar{y}_{\cdot i \cdot} - \bar{y}_{\cdot j \cdot} - t_{a/2,ab(n-1)}\sqrt{\frac{2MS_E}{n}} \le \mu_i - \mu_j \le \bar{y}_{\cdot i \cdot} - \bar{y}_{\cdot j \cdot} + t_{a/2,ab(n-1)}\sqrt{\frac{2MS_E}{n}}$$

For exercise 14-6, the mean warping at 80% copper concentration is 21.0 and the mean warping at 100% copper concentration is 28.25 $a = 4$, $b = 4$, $n = 2$ and $MS_E = 6.78$. The degrees of freedom are (4)(4)(1) = 16

$$(21.0 - 28.25) - 2.921\sqrt{\frac{2(6.78)}{2}} \le \mu_3 - \mu_2 \le (21.0 - 28.25) + 2.921\sqrt{\frac{2(6.78)}{2}}$$

$$-14.86 \le \mu_3 - \mu_2 \le 0.356$$

Therefore, there is no significant difference between the mean warping values at 80% and 100% copper concentration.

14-10 a) Hypotheses

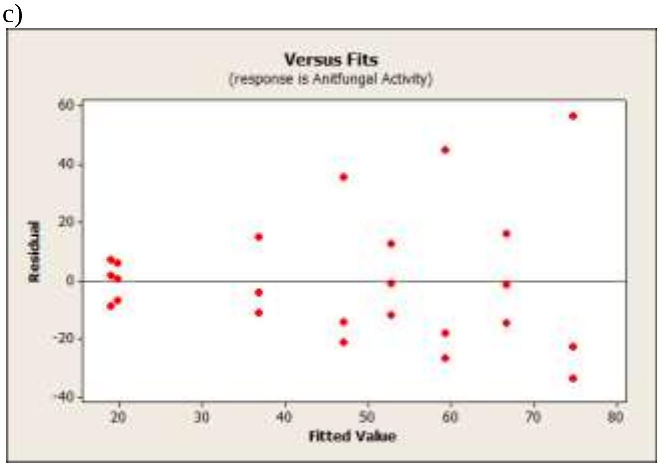
$$H_0: \tau_1 = \tau_2 = \tau_3 = 0,\ \ H_1: at\ least\ one\ \tau_i \ne 0$$
$$H_0: \beta_1 = \beta_2 = \beta_3 = 0,\ \ H_1: at\ least\ one\ \beta_i \ne 0$$

b) ANOVA Table

```
Analysis of Variance for Anitfungal Activity, using Adjusted SS for Tests
Source                DF   Seq SS   Adj SS   Adj MS     F      P
Carbon                 2   1072.8   1072.8    536.4  0.68  0.520
Temperature            2   8146.7   8146.7   4073.4  5.15  0.017
Carbon*Temperature     4    126.4    126.4     31.6  0.04  0.997
Error                 18  14224.3  14224.3    790.2
Total                 26  23570.3

S = 28.1112   R-Sq = 39.65%   R-Sq(adj) = 12.83%
```

The only P-value < 0 .05 is for *Temperature*. Therefore, only the main effect of *Temperature* is significant at α = 0.05.

c)



The variance of the residuals increases as the fitted value increases.

d) Here $t_{0.05/2,\,24} = 2.064$ and the pooled standard deviation is 28.1112. Therefore, the standard error of the difference between two mean is $28.1112(1/9 + 1/9)^{1/2} = 13.252$.

```
Means

Carbon Level  N      Mean Yield
2.0           9          56.933
5.0           9          41.996
7.5           9          46.080
```

The largest difference in means is between Carbon 2.0 and Carbon 5.0 and this difference is 56.933 – 41.996 = 14.937. This is only slightly greater than the standard error of the difference. Therefore, there are no significant differences among the levels of carbon. This result agrees with the conclusions from the ANOVA.

Section 14-4

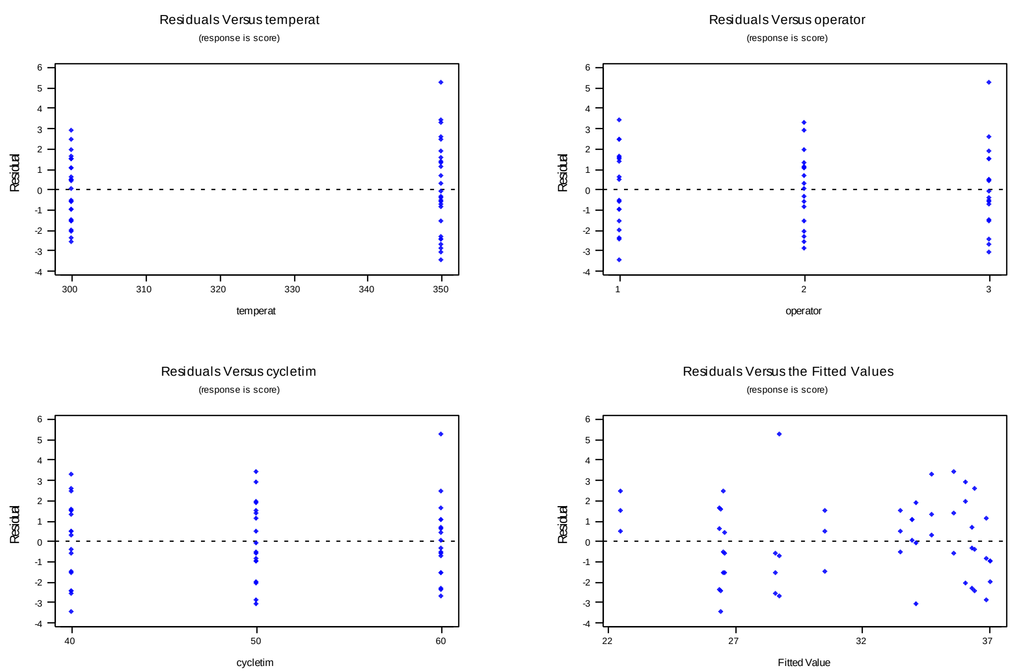
14-11 a) Analysis of Variance for dying score, using Adjusted SS for Tests

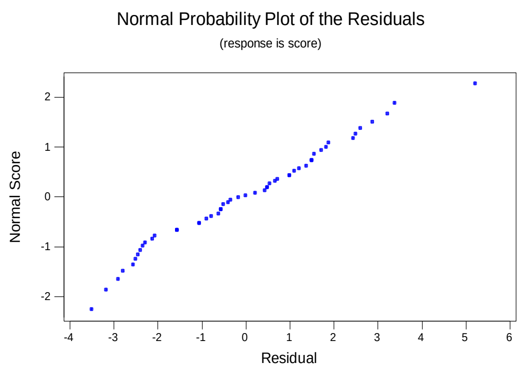
Source	DF	SS	MS	F	P
Time	2	396.778	198.389	39.85	0.000
Temp	1	73.500	73.500	14.77	0.000
Oper	2	256.333	128.167	25.75	0.000
Time*Temp	2	70.778	35.389	7.11	0.002
Time*Oper	4	300.222	75.056	15.08	0.000
Temp*Oper	2	14.111	7.056	1.42	0.254
Error	40	199.111	4.978		
Total	53	1310.833			

S = 2.23109   R-Sq = 84.81%   R-Sq(adj) = 79.87%

Only the operator*temperature interaction is *insignificant*.

b)





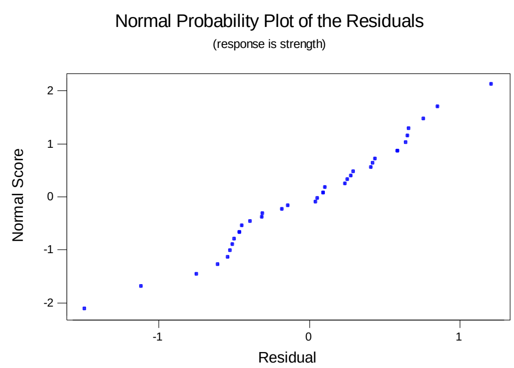
The residuals are acceptable.

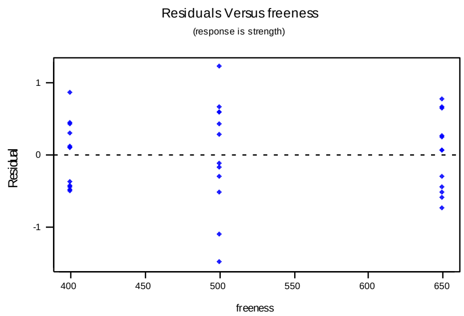
14-12 Parts a) and b)

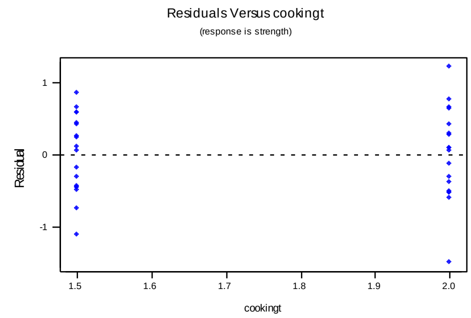
```
Analysis of Variance for strength
  Source                DF        SS         MS        F       P
  hardwood               2     8.3750     4.1875     7.64  0.003
  cookingtime            1    17.3611    17.3611    31.66  0.000
  freeness               2    21.8517    10.9258    19.92  0.000
  hardwood*cookingtime   2     3.2039     1.6019     2.92  0.075
  hardwood*freeness      4     6.5133     1.6283     2.97  0.042
  cookingtime*freeness   2     1.0506     0.5253     0.96  0.399
  Error                 22    12.0644     0.5484
  Total                 35    70.4200
```

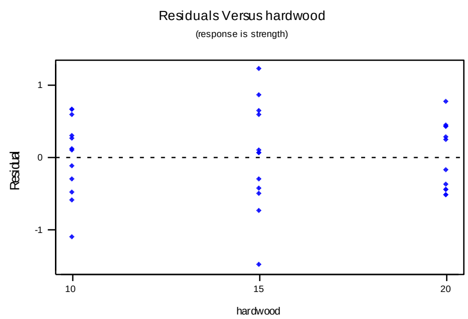
All main factors are significant. The interaction of hardwood*freeness is also significant.

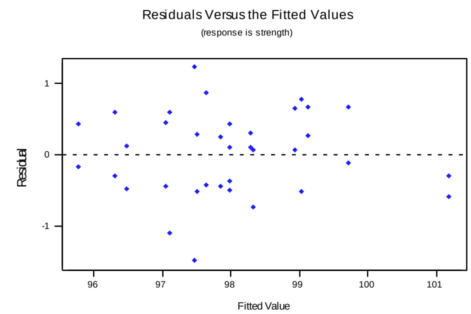
c) The residual plots do not indicate serious problems with normality or equality of variance.











<u>Section 14-5</u>

14-13 a) Analysis of Variance for Life (coded units)

Source	DF	SS	MS	F	P
A	1	1024	1024	0.39	0.547
B	1	28224	28224	10.88	0.011
AB	1	484	484	0.19	0.677
C	1	19600	19600	7.55	0.025
AC	1	55225	55225	21.28	0.002
BC	1	2401	2401	0.93	0.364
ABC	1	4761	4761	1.83	0.213
Error	8	20760	2595		
Total	15	132479			

S = 50.9411   R-Sq = 84.33%   R-Sq(adj) = 70.62%

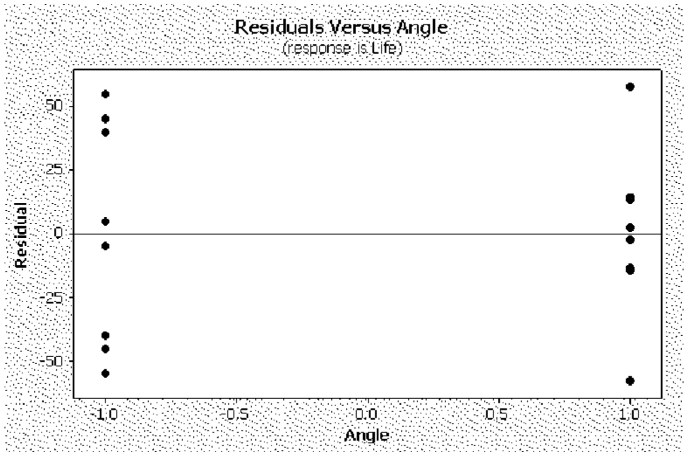
Hardness, angle and the speed-angle interaction are significant.

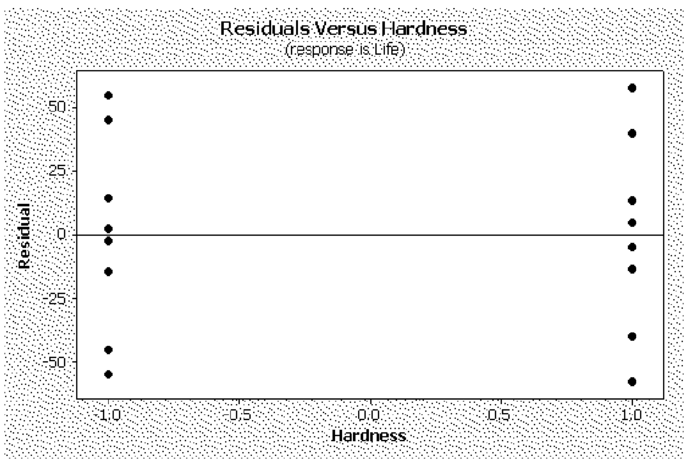
b) Estimated Effects and Coefficients for life (coded units)

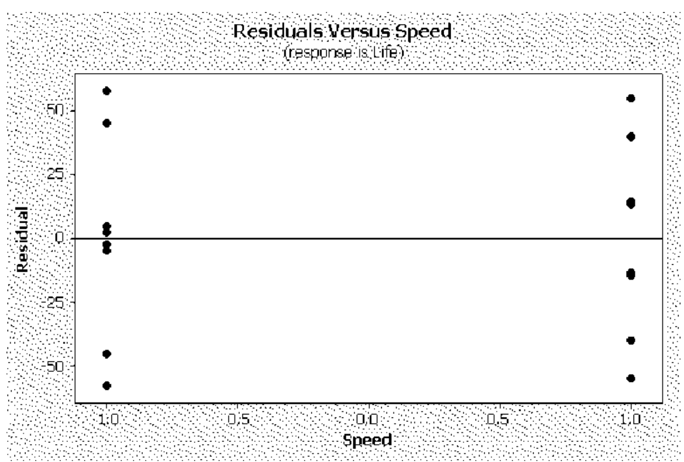
Term	Coef	Effect	SE Coef	T	P
Constant	414.25		12.74	32.53	0.000
Speed	8.00	16.0	12.74	0.63	0.547
Hardness	42.00	84.0	12.74	3.30	0.011
Angle	35.00	70.0	12.74	2.75	0.025
Speed*Hardness	-5.50	-11.0	12.74	-0.43	0.677
Speed*angle	-58.75	-117.5	12.74	-4.61	0.002
Hardness*angle	-12.25	-24.5	12.74	-0.96	0.364
Speed*hardness*angle	-17.25	-34.5	12.74	-1.35	0.213

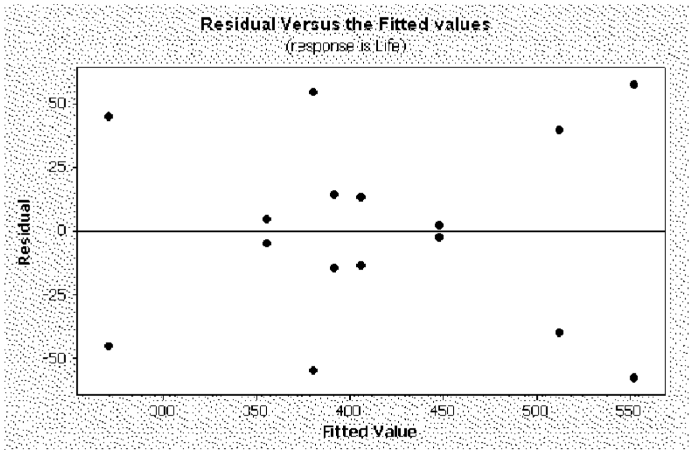
$$\hat{y} = 414.25 + 8x_1 + 42x_2 + 35x_3 - 58.75x$$

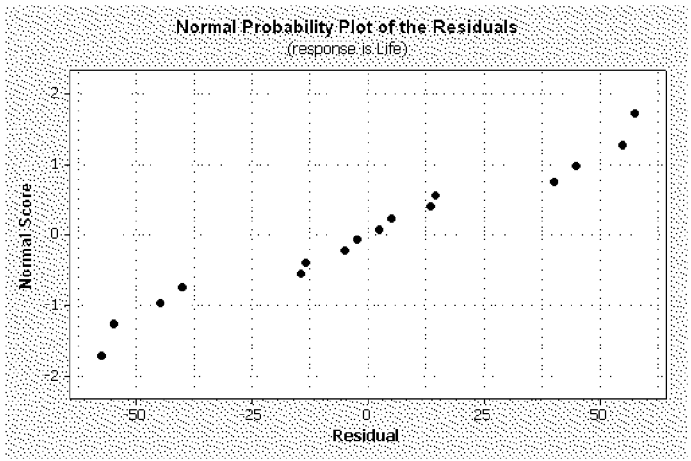
c) Analysis of the residuals shows that all assumptions are reasonable.


Residuals Versus Angle
(response is Life)
Residual
Angle


Residuals Versus Hardness
(response is Life)
Residual
Hardness


Residuals Versus Speed
(response is Life)
Residual
Speed





14-14

Term	Effect	Coef	SE Coef	T	P
Constant		175.250	0.5467	320.59	0.000
A	17.000	8.500	0.5467	15.55	0.000
B	-1.625	-0.812	0.5467	-1.49	0.157
C	10.875	5.438	0.5467	9.95	0.000
D	8.375	4.187	0.5467	7.66	0.000
A*B	-0.125	-0.063	0.5467	-0.11	0.910
A*C	-0.625	-0.313	0.5467	-0.57	0.575
A*D	9.125	4.562	0.5467	8.35	0.000
B*C	-0.250	-0.125	0.5467	-0.23	0.822
B*D	1.250	0.625	0.5467	1.14	0.270
C*D	-1.250	-0.625	0.5467	-1.14	0.270
A*B*C	0.750	0.375	0.5467	0.69	0.503
A*B*D	-0.500	-0.250	0.5467	-0.46	0.654
A*C*D	-0.000	-0.000	0.5467	-0.00	1.000
B*C*D	0.125	0.063	0.5467	0.11	0.910
A*B*C*D	-1.625	-0.812	0.5467	-1.49	0.157

Factors A, C, and D are significant as well as the interaction AD.

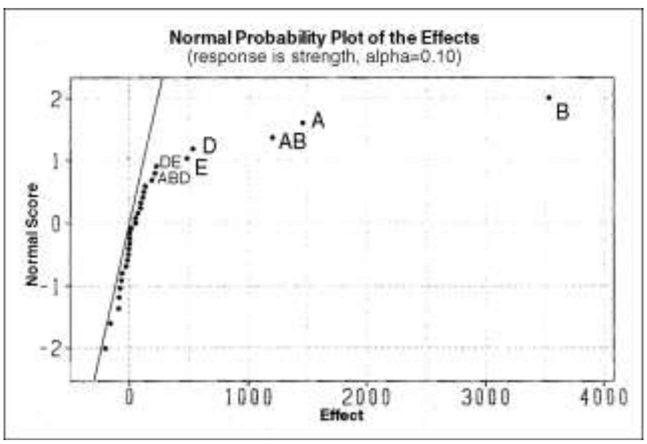
14-15 a)

Estimated Effects and Coefficients for strength

Term	Effect
A	1458.75
B	3535
C	-202.5
D	533.75
E	485

A*B	1202.5
A*C	190
A*D	3.75
A*E	2.5
B*C	-158.75
B*D	52.5
B*E	53.75
C*D	115
C*E	-58.75
D*E	227.5
A*B*C	-3.75
A*B*D	135
A*B*E	-16.25
A*C*D	72.5
A*C*E	96.25
A*D*E	-90
B*C*D	96.25
B*C*E	-77.5
B*D*E	-66.25
C*D*E	-28.75
A*B*C*D	-8.75
A*B*C*E	15
A*B*D*E	3.75
A*C*D*E	216.25
B*C*D*E	-85
A*B*C*D*E	122.5

b)



The effects that appear to be important are A, B, D, E, and the interactions AB, DE, and ABD.

c) To maximize strength, the variables A, B, D, and E should be increased. Variable C is not significant. Thus, any level of C would be acceptable.

The regression equation is

$$\hat{y} = 2889.38 + 729.37x_1 + 1767.5x_2 + 266.87x_4 + 242.5x_5 + 601.25x_1x_2 + 113.75x_4x_5 + 67.5x_1x_2x_4$$

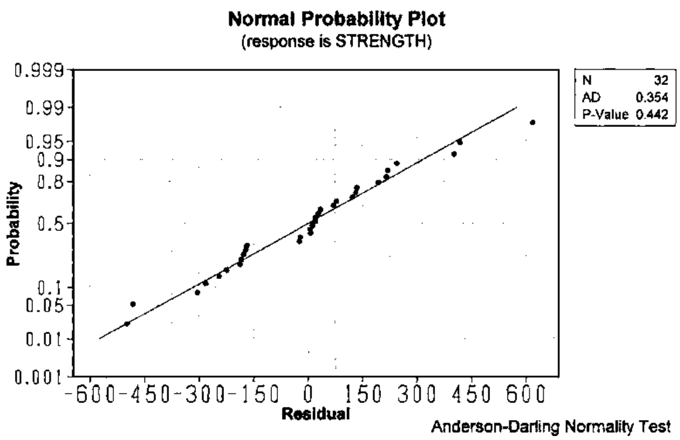
Predictor	Coef	SE Coef	T	P
Constant	2889.38	49.72	58.11	0.000
A	729.37	49.72	14.67	0.000
B	1767.50	49.72	35.55	0.000
D	266.87	49.72	5.37	0.000
E	242.50	49.72	4.88	0.000
A*B	601.25	49.72	12.09	0.000
D*E	113.75	49.72	2.29	0.031
A*B*D	67.50	49.72	1.36	0.187

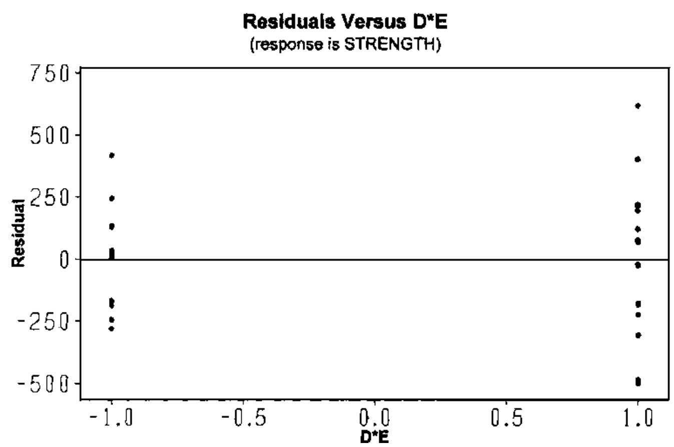
S = 281.274   R-Sq = 98.6%   R-Sq(adj) = 98.2%

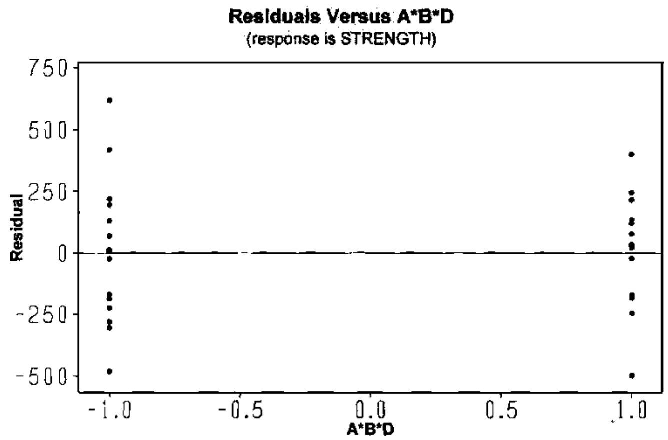
Analysis of Variance

```
Source           DF          SS          MS        F        P
Regression        7   133282225    19040318   240.67    0.000
Residual Error   24     1898763       79115
Total            31   135180987

Source        DF      Seq SS
A              1    17023612
B              1    99969800
D              1     2279112
E              1     1881800
A*B            1    11568050
D*E            1      414050
A*B*D          1      145800
```

d)







The normal probability plot of the residuals indicates the assumption of normality is reasonable. The model appears to be adequate.

14-16 a) Estimated Effects and Coefficients for charge

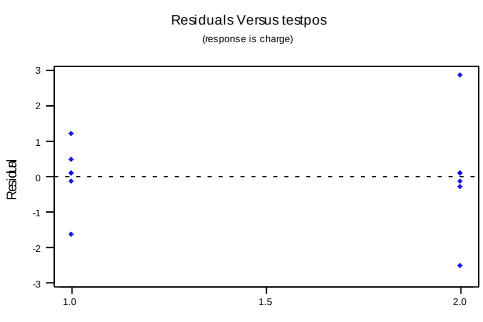
Term	Effect	Coef	StDev Coef	T	P
Constant		-1.000	0.4462	-2.24	0.055
cleanmet	-5.593	-2.797	0.4462	-6.27	0.000
testpos	-1.280	-0.640	0.4462	-1.43	0.189
cleanmet*testpos	-1.220	-0.610	0.4462	-1.37	0.209

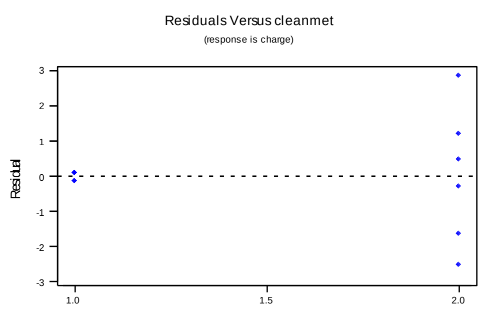
b) Analysis of Variance for charge

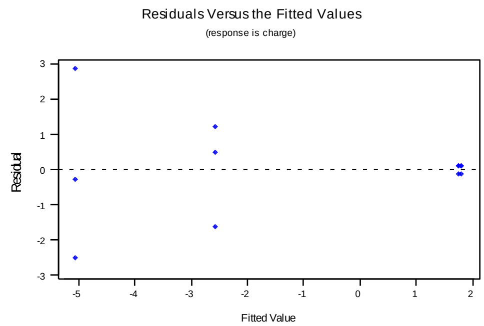
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	98.771	98.7713	49.386	20.67	0.001
2-Way Interactions	1	4.465	4.4652	4.465	1.87	0.209
Residual Error	8	19.110	19.1101	2.389		
Pure Error	8	19.110	19.1101	2.389		
Total	11	122.347				

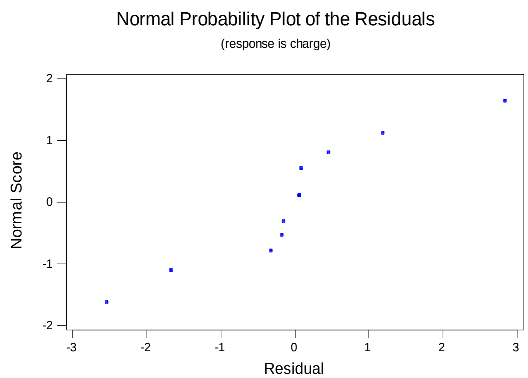
b) Cleaning Method is the only significant factor.

c) Analysis of the residuals shows that there is more variability at test position R and cleaning material SRD. In the case of the cleaning material, the difference in the variances is very large. The variation decreases with increased fitted values. The normal probability plot appears to have some variations from the straight line.









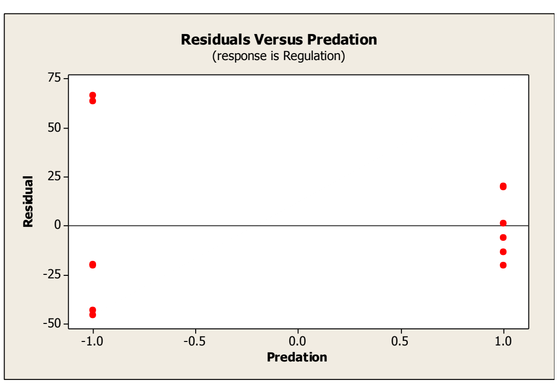
14-17

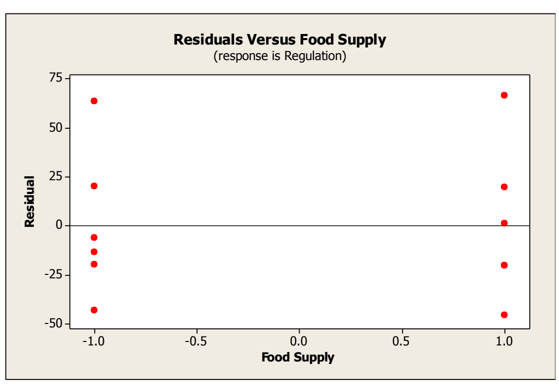
a) $Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \begin{cases} i = 1,2 \\ j = 1,2 \\ k = 1,2,3 \end{cases}$

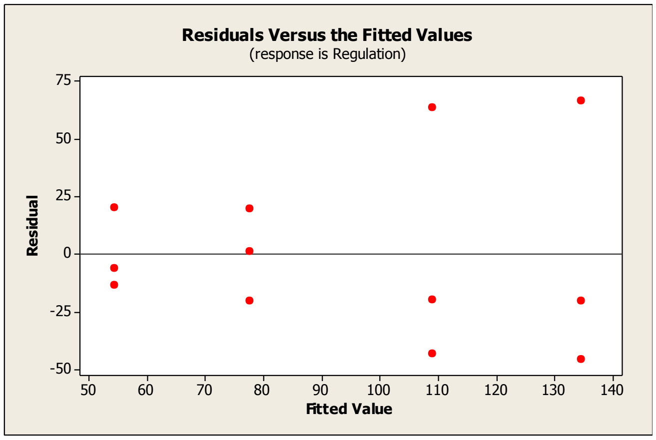
b) There is no significant effect in the model.

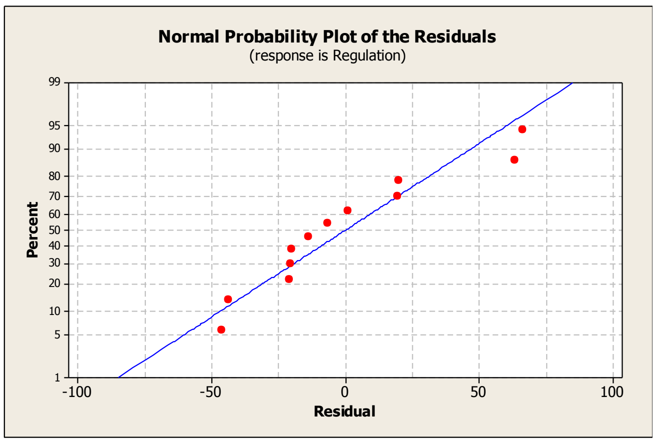
```
Analysis of Variance for Regulation (coded units)
Source            DF      SS       MS     F      P
Food Sup           1   1784.9  1784.86  0.97  0.353
Predatio           1   9324.7  9324.75  5.08  0.054
Interaction        1      4.3     4.28  0.00  0.963
Error              8  14686.2  1835.78
Total             11  25800.1
```

c) There appear to be some problems with constant variance in the residual plots. The normal probability plot shows that the assumption of normality is reasonable.

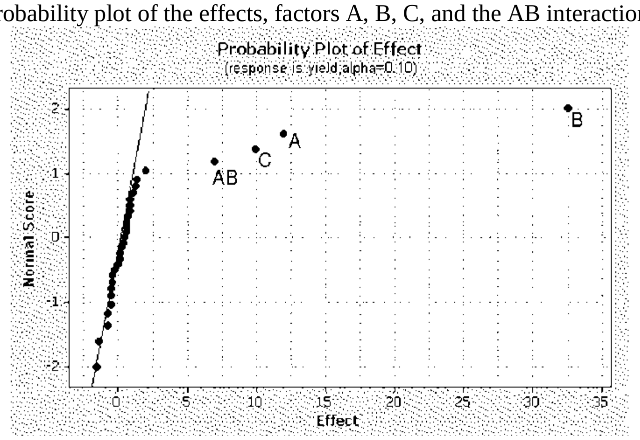








14-18 a) From the normal probability plot of the effects, factors A, B, C, and the AB interaction appear to be significant.



b)

```
Analysis of Variance for yield
Term                  Effects      Coef   SE Coef        T       P
Constant                        30.4063    0.3164    96.11   0.000
factor_A                11.94    5.9687    0.3164    18.87   0.000
factor_B                32.56   16.2813    0.3164    51.46   0.000
factor_C                 9.94    4.9688    0.3164    15.70   0.000
factor_D                -0.56   -0.2813    0.3164    -0.89   0.387
factor_E                 0.69    0.3437    0.3164     1.09   0.293
factor_A*factor_B        6.94    3.4688    0.3164    10.96   0.000
```

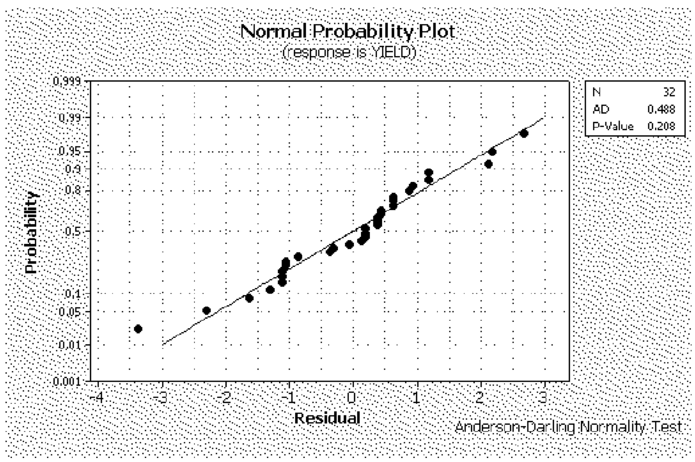
factor_A*factor_C	1.31	0.6562	0.3164	2.07	0.055
factor_A*factor_D	0.81	0.4063	0.3164	1.28	0.217
factor_A*factor_E	0.81	0.4063	0.3164	1.28	0.217
factor_B*factor_C	0.44	0.2187	0.3164	0.69	0.499
factor_B*factor_D	-1.56	-0.7813	0.3164	-2.47	0.025
factor_B*factor_E	1.94	0.9687	0.3164	3.06	0.007
factor_C*factor_D	0.56	0.2813	0.3164	0.89	0.387
factor_C*factor_E	0.06	0.0312	0.3164	0.10	0.923
factor_D*factor_E	-1.44	-0.7187	0.3164	-2.27	0.037

Analysis of Variance for YIELD

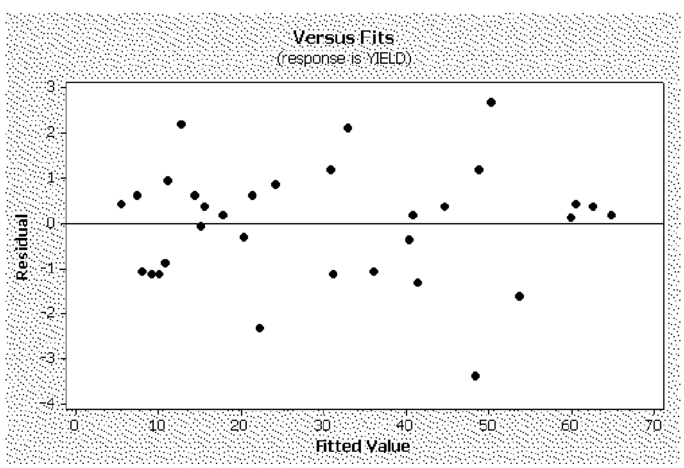
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main effects	5	10418.9	10418.9	2083.78	650.54	0.000
2-Way Interactions	10	479.6	479.6	47.96	14.97	0.000
Residual Error	16	51.3	51.25	3.20		
Total	31	10949.8				

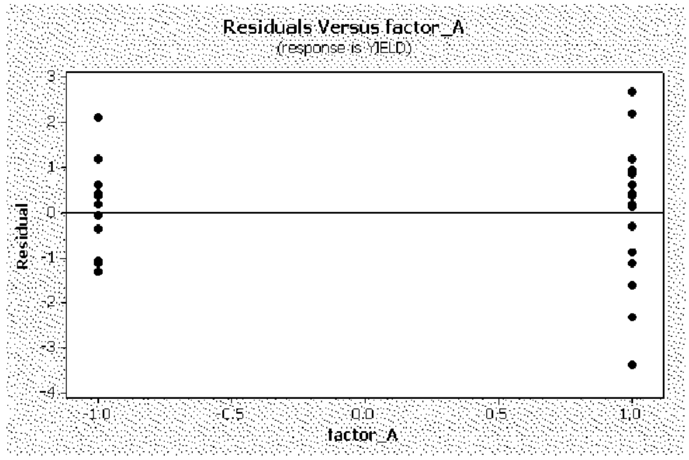
The analysis confirms our findings from part a)

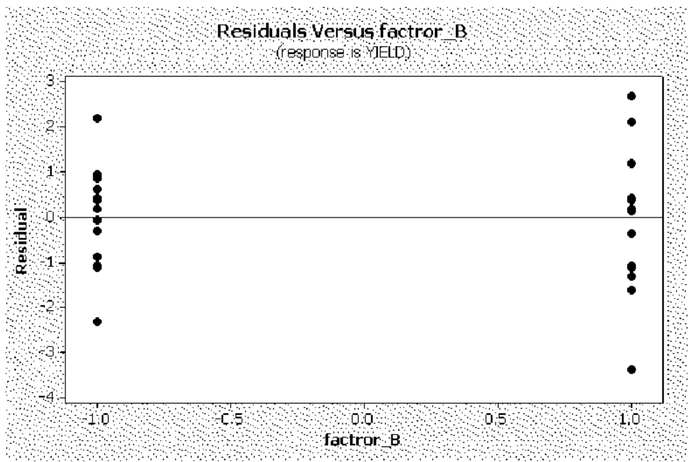
c) The normal probability plot of the residuals is satisfactory. However their variance appears to increase as the fitted value increases.

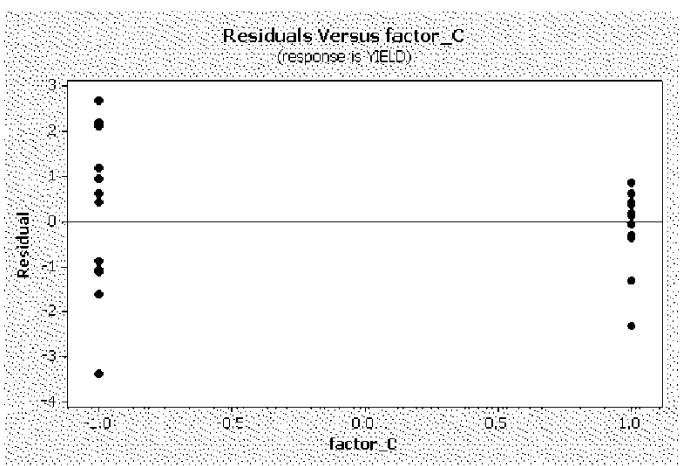


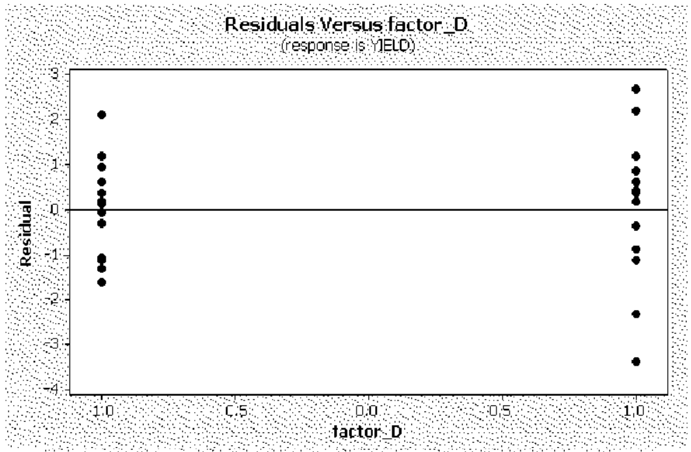
d) All plots support the constant variance assumption, although there is a very slight indication that variability is greater at the high level of factor B.

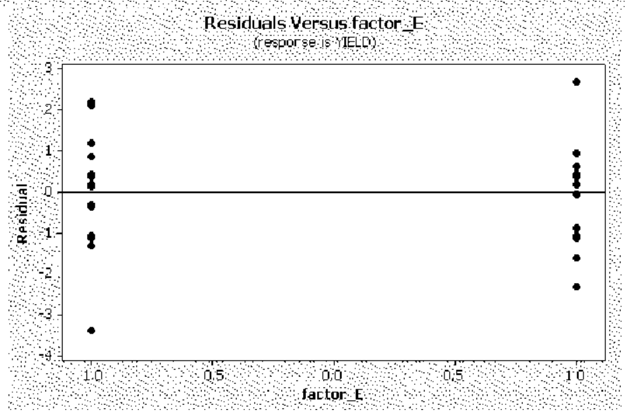



Residuals Versus factor_A
(response is YIELD)
Residual
factor_A
3
2
1
0
-1
-2
-3
-4
-1.0
-0.5
0.0
0.5
1.0

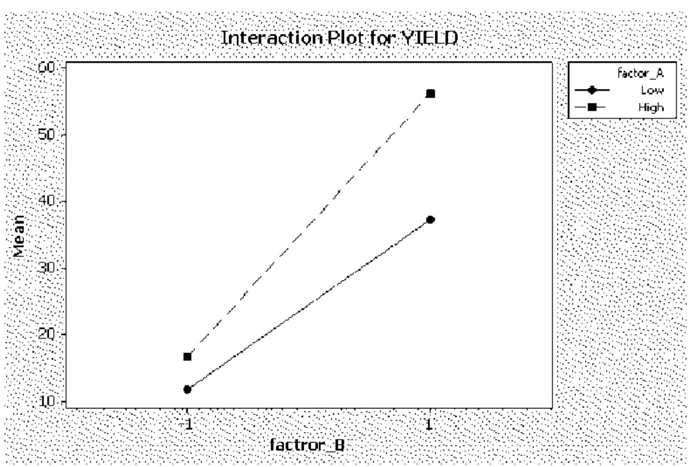

Residuals Versus factror_B
(response is YIELD)
Residual
factror_B
3
2
1
0
-1
-2
-3
-4
-1.0
-0.5
0.0
0.5
1.0


Residuals Versus factor_C
(response is YIELD)
Residual
factor_C
3
2
1
0
-1
-2
-3
-4
-1.0
-0.5
0.0
0.5
1.0



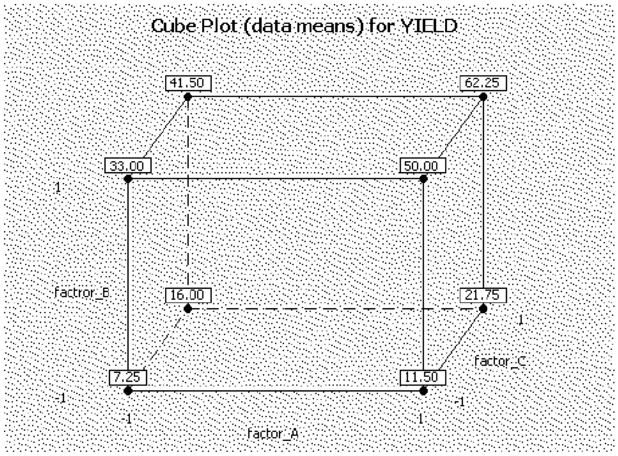


e) The AB interaction appears to be significant. The interaction plot from MINITAB indicates that a high level of A and of B increases the mean yield, while low levels of both factors would lead to a reduction in the mean yield.

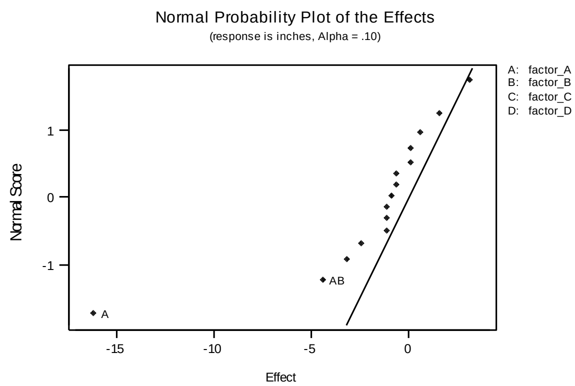


f) To increase yield and optimize the process, we would want to set A, B, and C at their high levels.

g) It is evident from the cube plot that we should run the process with all factors set at their high levels.



14-19 a)



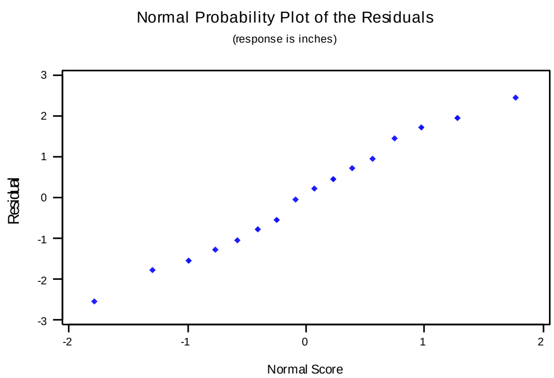

```
Estimated Effects and Coefficients for inches
Term                              Effect      Coef
Constant                                    35.938
factor_A                         -16.125    -8.063
factor_B                           3.125     1.562
factor_C                          -1.125    -0.562
factor_D                          -1.125    -0.562
factor_A*factor_B                 -4.375    -2.187
factor_A*factor_C                 -0.625    -0.312
factor_A*factor_D                 -3.125    -1.562
factor_B*factor_C                  1.625     0.812
factor_B*factor_D                  0.125     0.063
factor_C*factor_D                 -0.625    -0.313
factor_A*factor_B*factor_C         0.625     0.313
factor_A*factor_B*factor_D        -2.375    -1.188
factor_A*factor_C*factor_D        -1.125    -0.563
factor_B*factor_C*factor_D        -0.875    -0.437
factor_A*factor_B*factor_C
*factor_D                          0.125     0.063
```

According to the normal probability plot, factors A, B, and AB appear to be significant.

Parts b) and c)

Remove the three- and four-factor interactions to generate the following analysis:

```
Term                      Effect       Coef  StDev Coef        T       P
Constant                             35.938      0.6355    56.55   0.000
factor_A                 -16.125     -8.063      0.6355   -12.69   0.000
factor_B                   3.125      1.562      0.6355     2.46   0.057
factor_C                  -1.125     -0.562      0.6355    -0.89   0.417
factor_D                  -1.125     -0.562      0.6355    -0.89   0.417
factor_A*factor_B         -4.375     -2.187      0.6355    -3.44   0.018
factor_A*factor_C         -0.625     -0.312      0.6355    -0.49   0.644
factor_A*factor_D         -3.125     -1.562      0.6355    -2.46   0.057
factor_B*factor_C          1.625      0.812      0.6355     1.28   0.257
factor_B*factor_D          0.125      0.063      0.6355     0.10   0.925
factor_C*factor_D         -0.625     -0.313      0.6355    -0.49   0.644

Analysis of Variance for resp, using Adjusted SS for Tests

Source      DF     Seq SS     Adj SS     Adj MS        F       P
A            1    1040.06    1040.06    1040.06   131.03   0.000
B            1      39.06      39.06      39.06     4.92   0.047
A*B          1      76.56      76.56      76.56     9.65   0.009
Error       12      95.25      95.25       7.94
Total       15    1250.94
```



14-20 With only one replicate, the full factorial cannot be analyzed without using the 3-way interaction for error.

Estimated Effects and Coefficients for resp

Parameter	Effect	Estimate	Standard Error	t Value	Pr > |t|
Intercept		414.125	0.625	662.6	0.001
A	23.25	11.625	0.625	18.6	0.0342
B	39.75	19.875	0.625	31.8	0.02
C	42.25	21.125	0.625	33.8	0.0188
AB	-0.25	-0.125	0.625	-0.2	0.8743
AC	-97.75	-48.875	0.625	-78.2	0.0081
BC	3.75	1.875	0.625	3	0.2048

Analysis of Variance for resp

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	26949.75	4491.625	1437.32	0.0202
Error	1	3.125	3.125		
Corrected Total	7	26952.88			

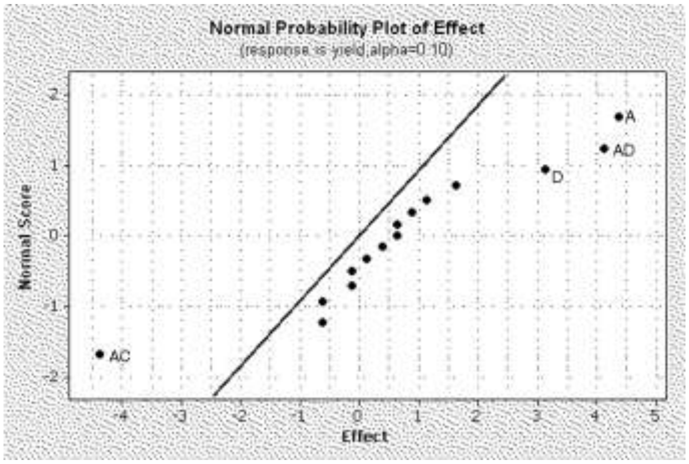
R-Square	Coeff Var	Root MSE	Y Mean
0.999884	0.426868	1.767767	414.125

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	1081.125	1081.125	345.96	0.0342
B	1	3160.125	3160.125	1011.24	0.02
C	1	3570.125	3570.125	1142.44	0.0188
AB	1	0.125	0.125	0.04	0.8743
AC	1	19110.13	19110.13	6115.24	0.0081
BC	1	28.125	28.125	9	0.2048

The results indicate that speed, hardness, angle and speed*angle interaction are significant.

14-21 a)

```
Term          Effect        Coef
Constant                  17.4375
A              4.375      2.18750
B              0.375     0.187500
C              1.625     0.812500
D              3.125      1.56250
A*B           -0.125   -0.0625000
A*C           -4.375     -2.18750
A*D            4.125      2.06250
B*C            0.125    0.0625000
B*D            0.625     0.312500
C*D           -0.125   -0.0625000
A*B*C          1.125     0.562500
A*B*D          0.625     0.312500
A*C*D         -0.625    -0.312500
B*C*D         -0.625    -0.312500
A*B*C*D        0.875     0.437500
```



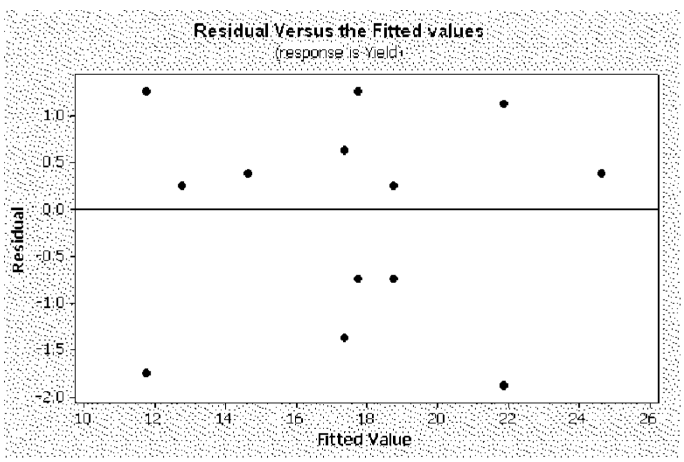
Factors A and D and interactions AC and AD are significant. Factor C should also be included in the model.

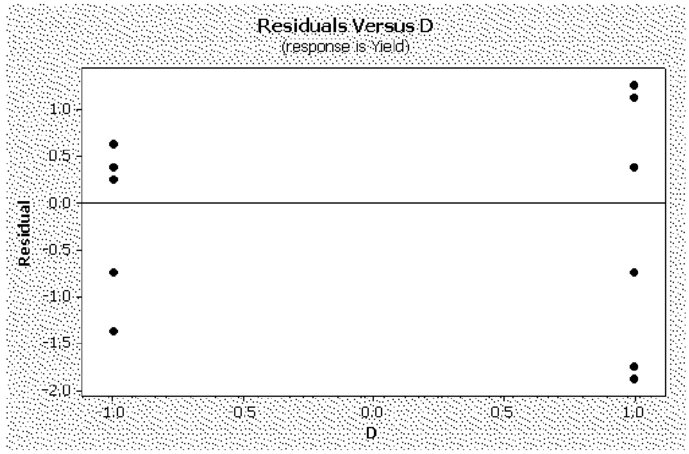
b) Analysis of Variance for Yield, using Adjusted SS for Tests

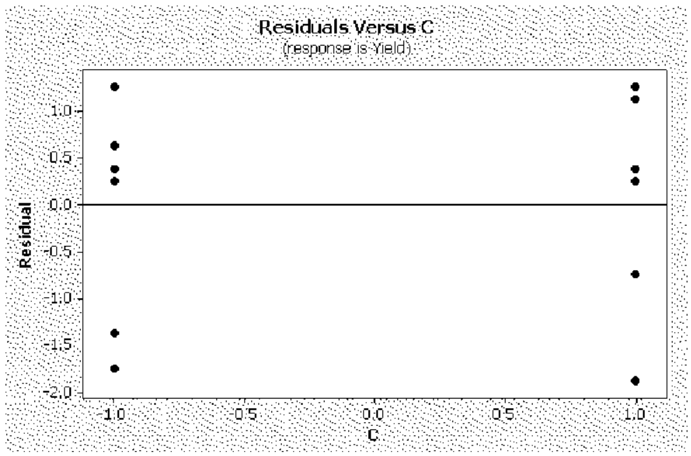
Source	DF	Seq SS	Adj SS	Adj MS	F	P
A	1	76.563	76.563	76.563	50.62	0.000
C	1	10.563	10.563	10.563	6.98	0.025
D	1	39.063	39.063	39.063	25.83	0.000
A*C	1	76.563	76.563	76.563	50.62	0.000
A*D	1	68.063	68.063	68.063	45.00	0.000
Error	10	15.125	15.125	1.513		
Total	15	285.938				

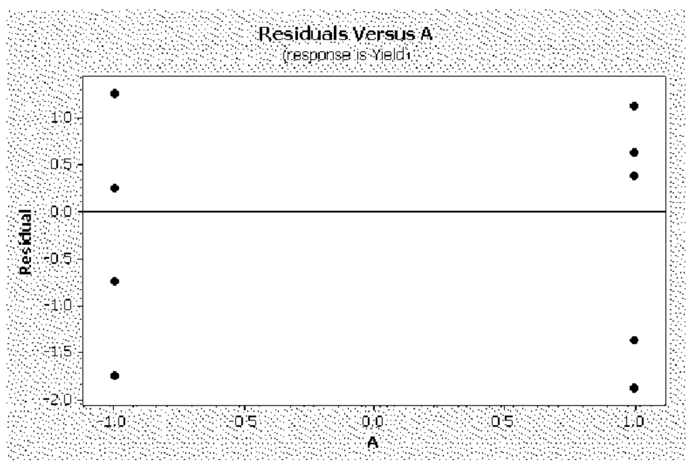
All of the factors and interactions in this table are significant at $\alpha = 0.05$.

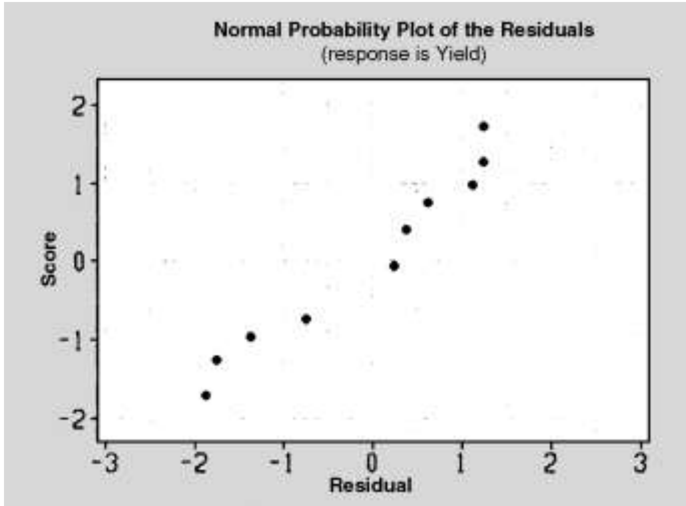
c) The analysis of the residuals shows that the assumptions of normality and constant variance are reasonable.




Residuals Versus D
(response is Yield)
Residual
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-1.0
-0.5
0.0
0.5
1.0
D


Residuals Versus C
(response is Yield)
Residual
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-1.0
-0.5
0.0
0.5
1.0
C

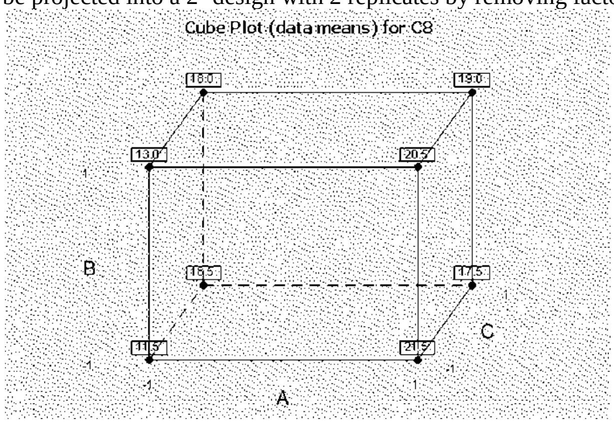

Residuals Versus A
(response is Yield)
Residual
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-1.0
-0.5
0.0
0.5
1.0
A



d) The regression equation is

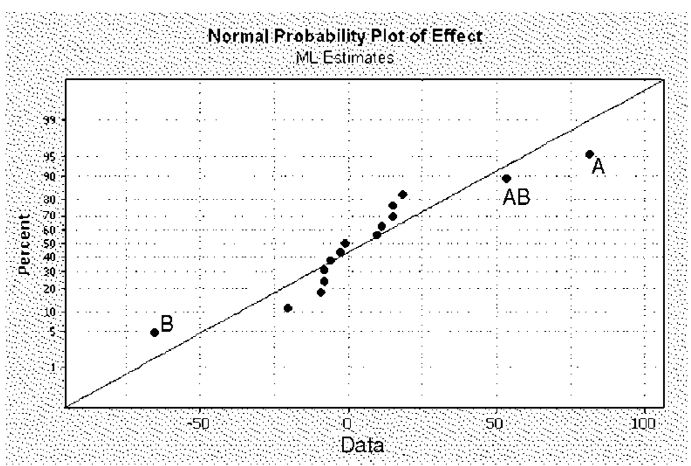
$$\hat{y} = 17.4 + 2.19x_1 + 0.813x_3 + 1.56x_4 - 2.19x_1x_3 + 2.06x_1x_4$$

e) Yes, this design can be projected into a 2^3 design with 2 replicates by removing factor B.



The cube plot shows the means at the high and low of each level. It can also be used to identify the interactions.

14-22 a)

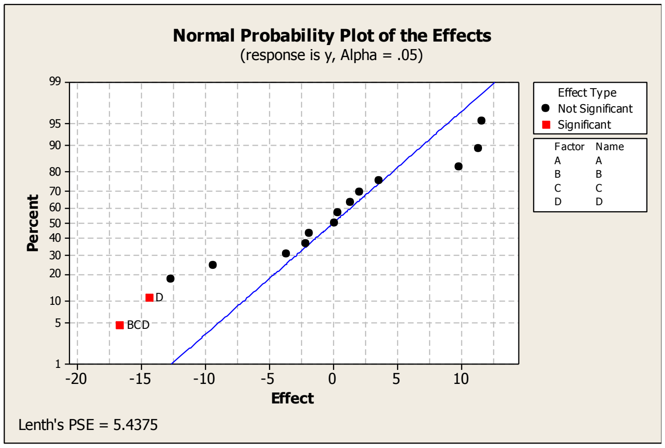


b) Based on the normal probability plot of the effects, factors A, B and AB are significant.

c) The estimated model is: $\hat{y} = 380 + 40.625x_1 - 32.75x_2 + 26.625x_1x_2$

14-23 a) Estimated Effects and Coefficients for y (coded units)

Term	Effect	Coef
Constant		10.063
A	-9.375	-4.688
B	-1.875	-0.937
C	-3.625	-1.813
D	-14.375	-7.188
A*B	0.125	0.063
A*C	-2.125	-1.062
A*D	11.625	5.812
B*C	11.375	5.688
B*D	3.625	1.813
C*D	1.375	0.688
A*B*C	-12.625	-6.313
A*B*D	2.125	1.062
A*C*D	0.375	0.188
B*C*D	-16.625	-8.313
A*B*C*D	9.875	4.938

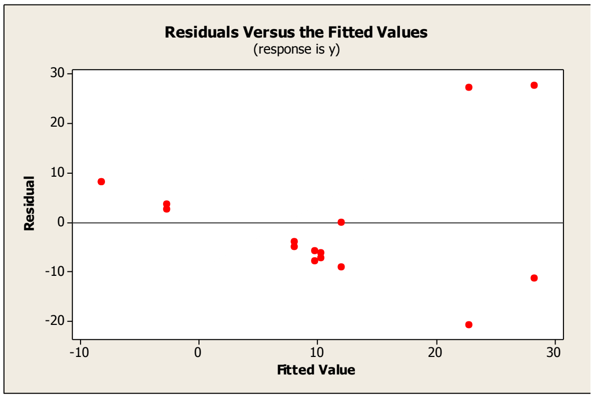


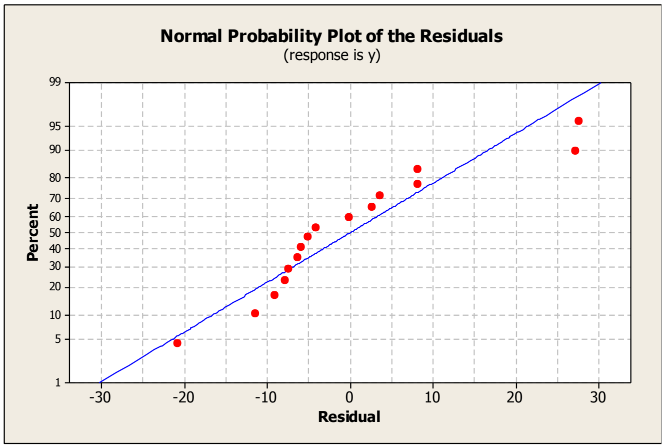
Effects D and BCD are significant effects.

b) The model based on result from (a) is $\hat{y} = 10.063 - 7.188x_4 - 8.313x_2x_3x_4$

The hierarchical model is $\hat{y} = 10.063 - 0.937x_2 - 1.812x_3 - 7.188x_4 - 8.313x_2x_3x_4$

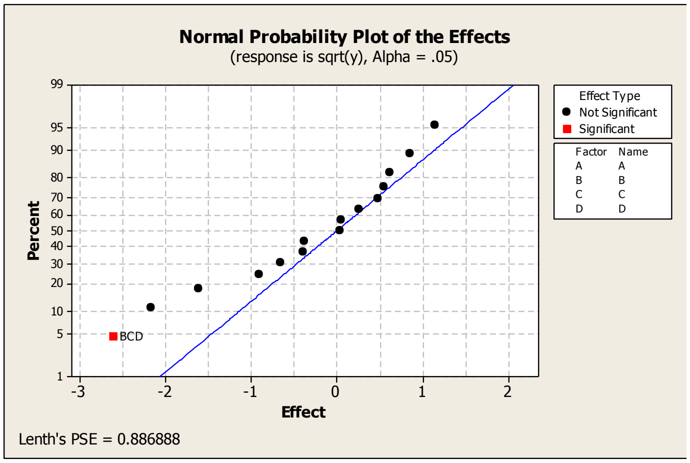
c) The residual versus predicted (fitted) value shows that the model is inadequate. There is also a problem with the normality assumption as shown in normal probability plot.





d)

```
Estimated Effects and Coefficients for sqrt(y)  (coded units)

Term        Effect    Coef
Constant              2.323
A           -0.895   -0.448
B           -0.372   -0.186
C           -0.658   -0.329
D           -2.164   -1.082
A*B          0.061    0.030
A*C         -0.385   -0.192
A*D          1.145    0.573
B*C          0.627    0.314
B*D          0.488    0.244
C*D          0.042    0.021
A*B*C       -1.609   -0.804
A*B*D        0.555    0.278
A*C*D        0.269    0.134
B*C*D       -2.609   -1.305
A*B*C*D      0.859    0.429
```



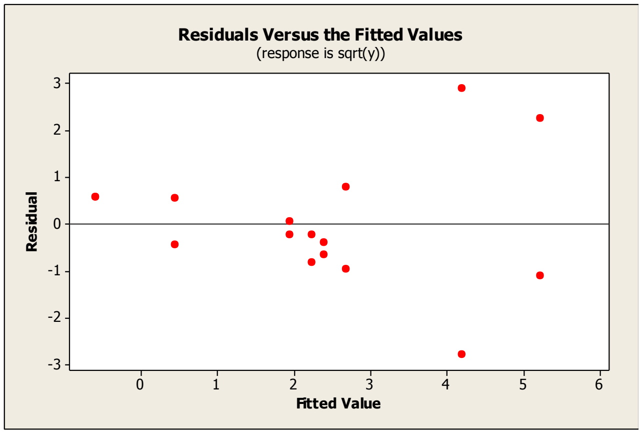
From the normal probability plot, only BCD is a significant effect.

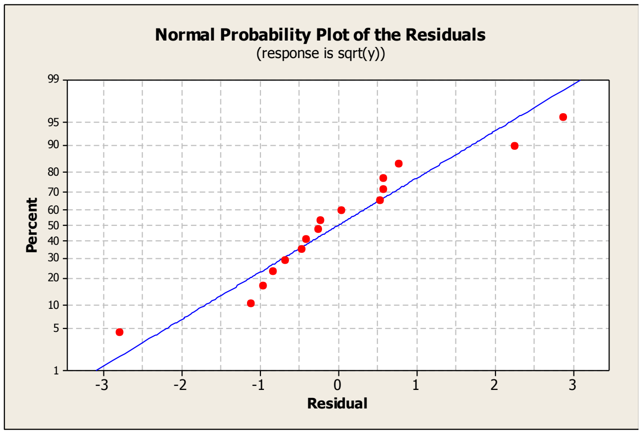
b) The model based on result from (a) is $\hat{y} = 2.323 - 1.305x_2x_3x_4$

A model with main effects, but without the two-factor interactions, is

$$\hat{y} = 2.323 - 0.186x_2 - 0.329x_3 - 1.082x_4 - 1.305x_2x_3x_4$$

c) The plots look very similar to the residual plots from the untransformed data.





14-24 Only main effect are significant. Interaction effects and curvature are not significant at α = 0.05.

Analysis of Variance for Response (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	55201	55201	27600.5	34.85	0.008
2-Way Interactions	1	2209	2209	2209.0	2.79	0.193
Curvature	1	4050	4050	4050.0	5.11	0.109
Residual Error	3	2376	2376	792.0		
Pure Error	3	2376	2376	792.0		
Total	7	63836				

For the curvature, since $F_0 = 5.11 < F_{0.01,1,3} = 34.12$, there is no evidence to conclude that curvature is significant at α = 0.05.

14-25 Analysis of Variance for Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	32	143543530	4485735	3.72	0.1043
Error	4	4828000	1207000		
Corrected Total	36	148371530			

R-Square	Coeff Var	Root MSE	Y Mean
0.96746	35.70131	1098.636	3077.297

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	17023612.5	17023613	14.1	0.0199
B	1	99969800	99969800	82.83	0.0008
C	1	328050	328050	0.27	0.6297
D	1	2279112.5	2279113	1.89	0.2414
E	1	1881800	1881800	1.56	0.2799
AB	1	11568050	11568050	9.58	0.0364
AC	1	288800	288800	0.24	0.6503
AD	1	112.5	112.5	0	0.9928
AE	1	50	50	0	0.9952
BC	1	201612.5	201612.5	0.17	0.7037
BD	1	22050	22050	0.02	0.899
BE	1	23112.5	23112.5	0.02	0.8966
CD	1	105800	105800	0.09	0.7819
CE	1	27612.5	27612.5	0.02	0.8871
DE	1	414050	414050	0.34	0.5895
ABC	1	112.5	112.5	0	0.9928
ABD	1	145800	145800	0.12	0.7457
ABE	1	2112.5	2112.5	0	0.9686
ACD	1	42050	42050	0.03	0.861
ACE	1	74112.5	74112.5	0.06	0.8165
ADE	1	64800	64800	0.05	0.8281
BCD	1	74112.5	74112.5	0.06	0.8165
BCE	1	48050	48050	0.04	0.8516
BDE	1	35112.5	35112.5	0.03	0.8728
CDE	1	6612.5	6612.5	0.01	0.9446
ABCD	1	612.5	612.5	0	0.9831
ABCE	1	1800	1800	0	0.971
ABDE	1	112.5	112.5	0	0.9928
ACDE	1	374112.5	374112.5	0.31	0.6074
BCDE	1	57800	57800	0.05	0.8375
ABCDE	1	120050	120050	0.1	0.7682
Curvature	1	8362542.23	8362542	6.93	0.0581

For the curvature, since $F_0 = 6.93 < F_{0.05,1,4} = 7.71$, there is no evidence that curvature is significant at α = 0.05.

14-26 a) Original: From original analysis terms A, B, C, and AB are significant

Estimated Effects and Coefficients for y

Parameter	Effect	Estimate	Standard Error	t Value	Pr > |t|
Intercept		30.40625	0.419565	72.47	<.0001
A	11.9375	5.96875	0.419565	14.23	<.0001
B	32.5625	16.28125	0.419565	38.81	<.0001
C	9.9375	4.96875	0.419565	11.84	<.0001
AB	6.9375	3.46875	0.419565	8.27	<.0001

Analysis of Variance for y.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	10797.63	2699.406	479.2	<.0001
Error	27	152.0938	5.6331		
Corrected Total	31	10949.72			

R-Square	Coeff Var	Root MSE	Y Mean
0.98611	7.805684	2.373416	30.40625

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	1140.031	1140.031	202.38	<.0001
B	1	8482.531	8482.531	1505.84	<.0001
C	1	790.0313	790.0313	140.25	<.0001
AB	1	385.0313	385.0313	68.35	<.0001

With 5 center points:
The standard deviation of the 5 center points is 2.70 and this is an estimate of experimental error. This is similar to the estimate s =2.37 from the original ANOVA.

If the original ANOVA is combined with these center points, the ANOVA below is generated. The center points contribute an additional 4 degrees of freedom to the residual error.

Estimated Effects and Coefficients for y

Parameter	Effect	Estimate	Standard Error	t Value	Pr > |t|
Intercept		30.40625	0.427499	71.13	<.0001
A	11.9375	5.96875	0.427499	13.96	<.0001
B	32.5625	16.28125	0.427499	38.08	<.0001
C	9.9375	4.96875	0.427499	11.62	<.0001
AB	6.9375	3.46875	0.427499	8.11	<.0001
Ct Pt	25.9875	12.99375	1.162924	11.17	<.0001

Analysis of Variance for y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	11527.73	2305.547	394.23	<.0001
Error	31	181.2938	5.84819		
Corrected Total	36	11709.03			

R-Square	Coeff Var	Root MSE	Y Mean
0.984517	7.519091	2.418302	32.16216

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	1140.031	1140.031	194.94	<.0001
B	1	8482.531	8482.531	1450.46	<.0001
C	1	790.0313	790.0313	135.09	<.0001
AB	1	385.0313	385.0313	65.84	<.0001
Curvature	1	730.1083	730.1083	124.84	<.0001

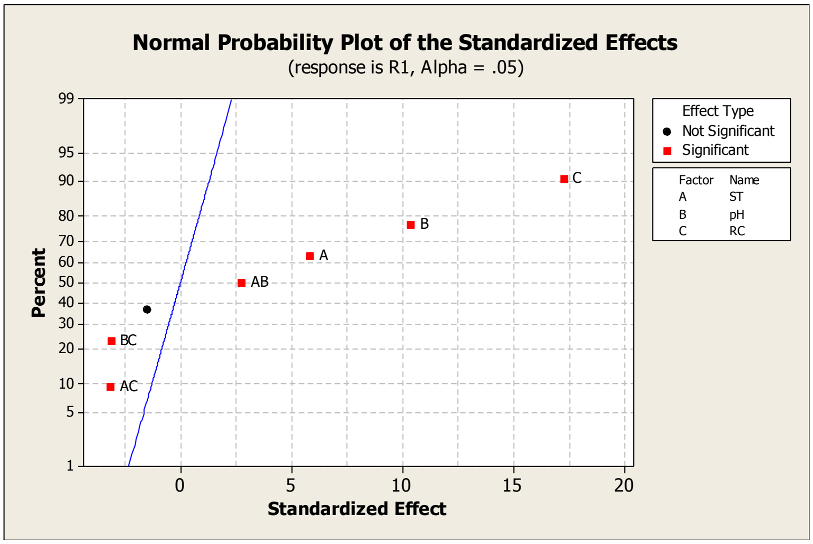
b) From the ANOVA with 5 center points, the *P*-value of curvature is <0.0001. There is strong evidence of curvature from this data.

14-27 a) The plot below shows standardized effects. A standardized effect equals the effect divided by its standard error estimate. Because an effect equals two times its coefficient, the standard error estimate for an effect equals two times the standard error estimate for its coefficient and these are provided in the table. The standardized effect for ST = 10.775/[2(0.9226)] = 17.2.

The effects of all main factors (ST, pH, and RC) and two-factor interaction terms (ST*pH, ST*RC, and pH*RC) are large.

Factorial Fit: R1 versus ST, pH, RC

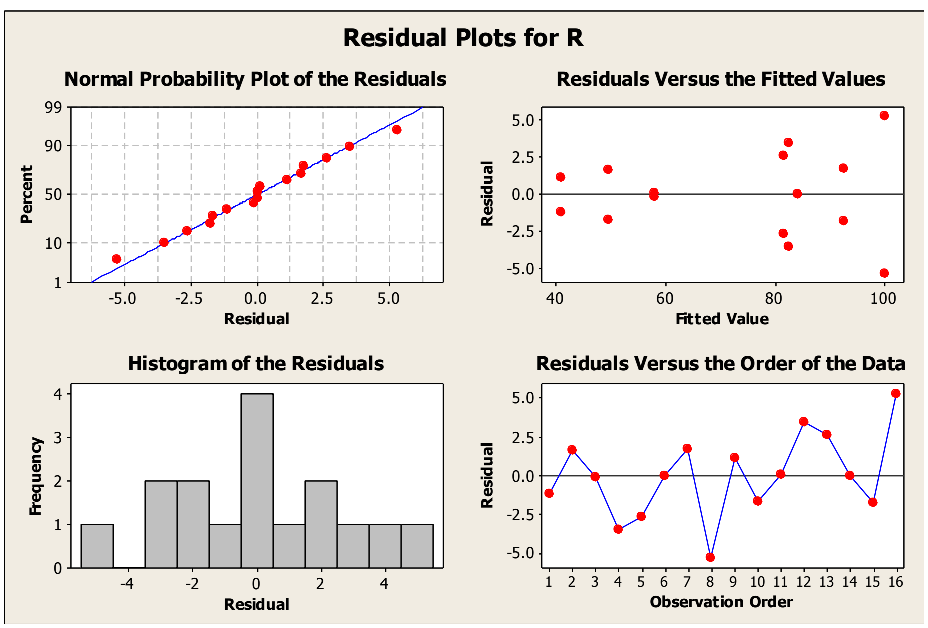
```
Estimated Effects and Coefficients for R1 (coded units)

Term       Effect    Coef  SE Coef      T      P
Constant           73.675   0.9226  79.85  0.000
ST         10.775   5.387   0.9226   5.84  0.000
pH         19.175   9.587   0.9226  10.39  0.000
RC         31.850  15.925   0.9226  17.26  0.000
ST*pH       5.100   2.550   0.9226   2.76  0.025
ST*RC      -5.775  -2.888   0.9226  -3.13  0.014
pH*RC      -5.725  -2.863   0.9226  -3.10  0.015
ST*pH*RC   -2.750  -1.375   0.9226  -1.49  0.174

S = 3.69053   R-Sq = 98.32%   R-Sq(adj) = 96.86%
```



b) Computer output below combines the sum of squares and the degrees of freedom for the main effects, the two-factor effects, and the three factor effects. An F statistic for each individual effect can be obtained from the square of the t statistic in the previous table. That is, the F statistic for ST = $79.85^2 = 6376.02$. However, because the *P*-value for each F test is the same as the P-value for the t test, the test for each effect is already provided with the t statistics.

```
Analysis of Variance for R1 (coded units)

Source                DF   Seq SS   Adj SS   Adj MS       F      P
Main Effects           3  5992.81  5992.81  1997.60  146.67  0.000
2-Way Interactions     3   368.55   368.55   122.85    9.02  0.006
3-Way Interactions     1    30.25    30.25    30.25    2.22  0.174
Residual Error         8   108.96   108.96    13.62
  Pure Error           8   108.96   108.96    13.62
Total                 15  6500.57
```

c) The normality assumption is reasonable. The plot of residuals versus the predicted values indicates some greater variability for larger fitted values so that some departure from assumptions is indicated. The actual time order of the observations was not provided so the plot versus observation order is not relevant.

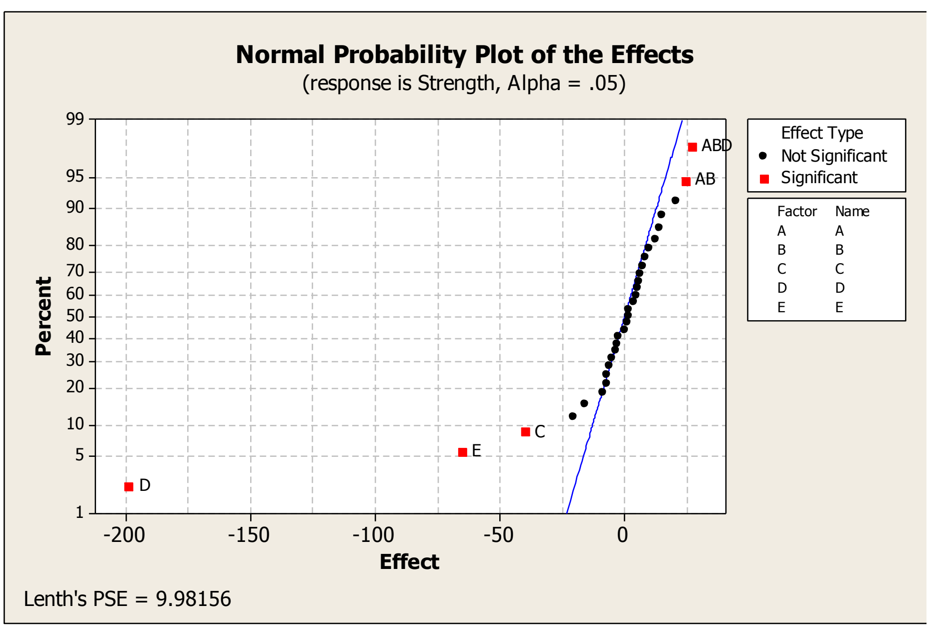


14-28 a)

Factorial Fit: Strength versus A, B, C, D, E

Estimated Effects and Coefficients for Strength (coded units)

Term	Effect	Coef
Constant		546.90
A	10.57	5.29
B	20.91	10.45
C	-39.79	-19.89
D	-198.47	-99.24
E	-64.86	-32.43
A*B	24.68	12.34
A*C	-15.16	-7.58
A*D	14.31	7.15
A*E	7.62	3.81
B*C	6.20	3.10
B*D	15.70	7.85
B*E	4.99	2.49
C*D	-19.87	-9.93
C*E	-1.92	-0.96
D*E	12.89	6.44
A*B*C	6.68	3.34
A*B*D	27.15	13.57
A*C*D	0.51	0.26
A*B*E	4.25	2.13
A*C*E	1.95	0.97
A*D*E	-8.25	-4.13
B*C*D	-2.36	-1.18
B*C*E	1.79	0.89
B*D*E	-4.57	-2.29
C*D*E	2.01	1.01
A*B*C*D	-6.65	-3.33
A*B*C*E	-6.67	-3.34
A*B*D*E	-3.14	-1.57
A*C*D*E	-5.39	-2.70
B*C*D*E	5.80	2.90
A*B*C*D*E	8.75	4.38



The effects for factors C, D, E are large. The AB and ABD effects are next largest in magnitude and might be considered significant.

b) Factorial Fit: Strength versus C, D, E

Estimated Effects and Coefficients for Strength (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		546.90	5.723	95.56	0.000
C	-39.79	-19.89	5.723	-3.48	0.002
D	-198.47	-99.24	5.723	-17.34	0.000
E	-64.86	-32.43	5.723	-5.67	0.000

S = 32.3760   R-Sq = 92.49%   R-Sq(adj) = 91.69%

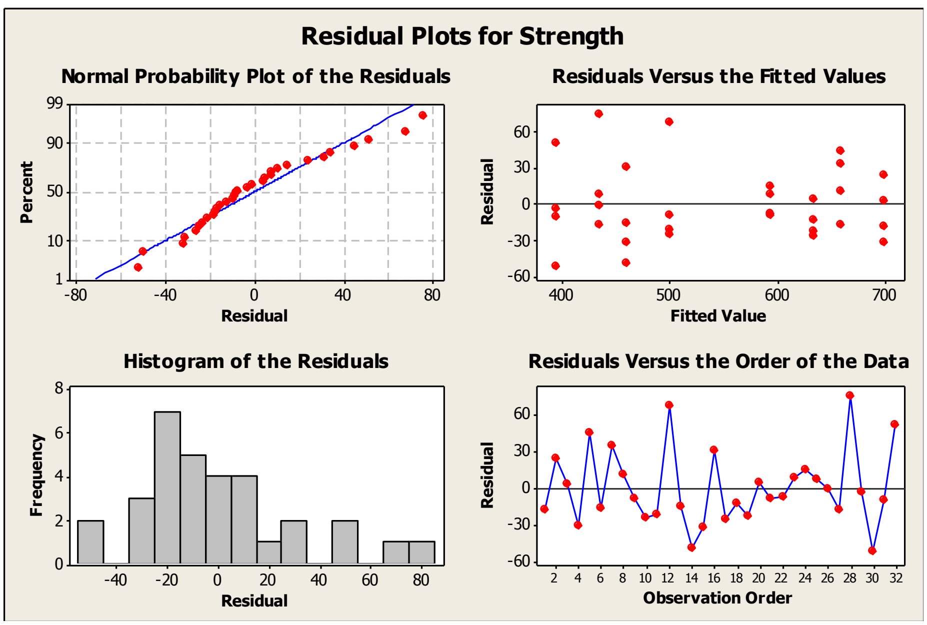
Analysis of Variance for Strength (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	361451	361451	120484	114.94	0.000
Residual Error	28	29350	29350	1048		
Lack of Fit	4	4549	4549	1137	1.10	0.379
Pure Error	24	24801	24801	1033		
Total	31	390800				

The average of the ceramic strength is given by

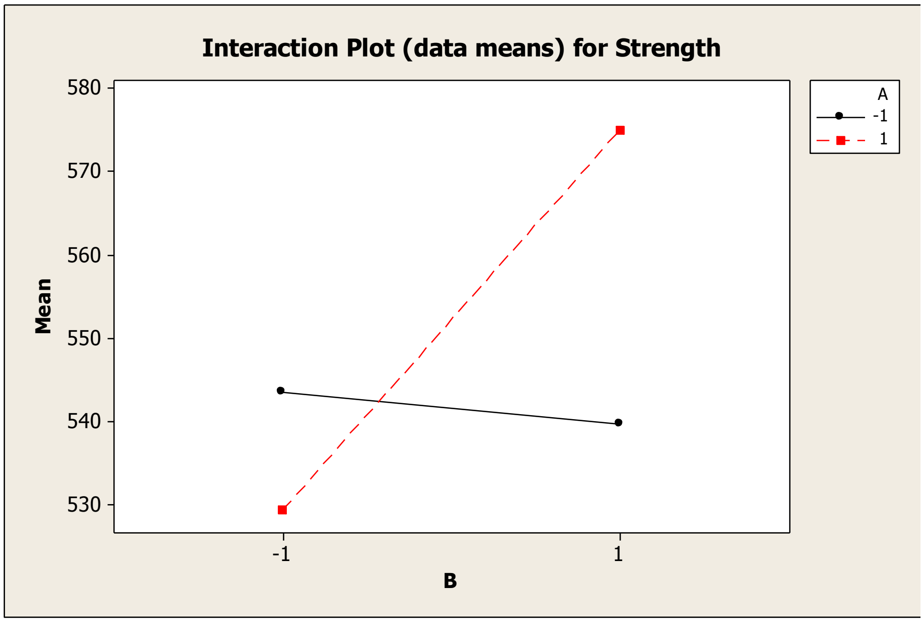
$$\hat{y} = 546.90 - 19.89C - 99.24D - 32.43E$$

c)



The normality assumption is reasonable. The plot of residuals versus the predicted values indicates some greater variability for smaller fitted values, but a strong departure from assumptions is not indicated. The actual time order of the observations was not provided so the plot versus observation order is not relevant.

d) The two interaction terms in this model AB and ABD are large, but not considered significant. We illustrate plots for these effects below. The plot of the AB interaction shows that the effect of changing factor B at low level of A is small, but increasing factor B at high level of A has a greater effect on the average of the ceramic strength (but still not significant).



e) Use the model with only effects C, D, and E and assume that the objective of the process is to maximize the average of the ceramic strength. It can be seen from the equation $\hat{y} = 546.90 - 19.89C - 99.24D - 32.43E$ that the settings for the factors should be C = -1, D = -1, and E = -1. Factors A and B are not important either as main effects or interactions, so these may be set at any convenient levels.

14-29 a) Because the degrees of freedom total = 15, there are 16 trials. There are three factors: A, B, and C with two levels each. Therefore, there are 2 replicates used in this experiment.

b) SE Coef = $\hat{\sigma}\sqrt{\frac{1}{n2^k}} = \sqrt{23664.2}\sqrt{\frac{1}{2(2^3)}} = 38.46$

c) Coef of B = $\frac{Effect}{2} = \frac{15.92}{2} = 7.96$

T statistics for A*B = $\frac{\hat{\beta}}{standard\ error\ \hat{\beta}} = \frac{10.21}{38.46} = 0.27$

SE Coef of A*B*C = $\hat{\sigma}\sqrt{\frac{1}{n2^k}} = \sqrt{23664.2}\sqrt{\frac{1}{2(2^3)}} = 38.46$

F for Main Effects = (Adj MS of Main effects)/ (Adj MS of Error) = 2261.8/23664.2 = 0.09558

Seq SS of 2-way interactions = Adj SS of 2-way interactions = 2918
Seq SS of 3-way interactions = Adj SS of 3-way interactions = 713

14-30 a) Interaction effects with more than two factors are used to estimate error.

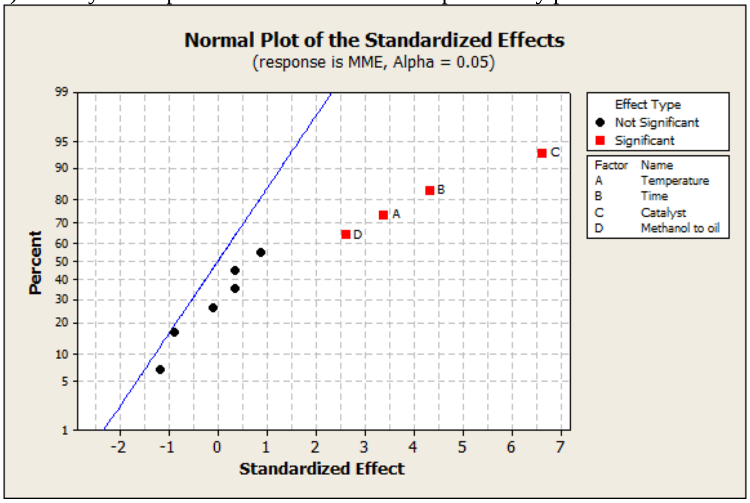
b) Because this is a single replicate of a 2^4 experiment, there are 16 tests total. Therefore, df(Total) = 15. Because there are 10 effects in the model df(Error) = 15 – 10 = 5 and MS(Error) = 64.50/5 = 12.9.
Therefore, the standard error for a coefficient = $0.5[12.9(1/8 + 1/8)]^{1/2} = 0.90$

The *t* test for A is 1.125/0.90 = 1.25. From the *t* distribution with 5 degrees of freedom, this corresponds to a two-sided probability of 0.267.

Because the set of sums of squares needs to add to SS(Total), SS_A = 20.25, with 1 degree of freedom. Therefore, MS_A = 20.25 and F = 20.25/12.9 = 1.57

```
Estimated Effects and Coefficients
Term         Effect     Coef  SE Coef  t       P
Constant              35.250   0.90   39.26  0.000
A             2.250    1.125    0.90   1.25  0.267
B            24.750   12.375   0.90   13.78  0.000
C             1.000    0.500   0.90    0.56  0.602
D            10.750    5.375   0.90    5.99  0.002
A*B         -10.500   -5.250   0.90   -5.85  0.002
A*C           4.250    2.125   0.90    2.37  0.064
A*D          -5.000   -2.500   0.90   -2.78  0.039
B*C           5.250    2.625   0.90    2.92  0.033
B*D           4.000    2.000   0.90    2.23  0.076
C*D          -0.750   -0.375   0.90   -0.42  0.694

S = 3.59166

Analysis of Variance
Source                  DF    SS         MS        F       P
  A                      1     20.25      20.25    1.57  0.266
  B                      1   2450.25    2450.25  189.94  0.000
  C                      1      4.00       4.00    0.31  0.602
  D                      1    462.25     462.25   35.83  0.002
  A*B                    1    441.00     441.00   34.19  0.002
  A*C                    1     72.25      72.25    5.60  0.064
  A*D                    1    100.00     100.00    7.75  0.039
  B*C                    1    110.25     110.25    8.55  0.033
  B*D                    1     64.00      64.00    4.96  0.076
  C*D                    1      2.25       2.25    0.17  0.694
Residual Error           5     64.50      12.9
Total                   15   3791.00
```

14-31 a) Identify the important factor with a normal probability plot based on the corner points.



b) Compare the results in the previous part with results that use an error term based on the center points.

Estimated Effects and Coefficients for MME (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		86.3775	0.1125	767.93	0.000
Temperature	3.7075	1.8537	0.1125	16.48	0.004
Time	4.7175	2.3587	0.1125	20.97	0.002
Catalyst	7.2350	3.6175	0.1125	32.16	0.001
Ratio	2.8550	1.4275	0.1125	12.69	0.006
Temperature*Time	-0.1000	-0.0500	0.1125	-0.44	0.700
Temperature*Catalyst	0.3775	0.1888	0.1125	1.68	0.235
Temperature*Ratio	0.9475	0.4737	0.1125	4.21	0.052
Time*Catalyst	-1.2725	-0.6363	0.1125	-5.66	0.030
Time*Ratio	0.3975	0.1987	0.1125	1.77	0.219
Catalyst*Ratio	-0.9750	-0.4875	0.1125	-4.33	0.049
Temperature*Time*Catalyst	-0.4200	-0.2100	0.1125	-1.87	0.203
Temperature*Time*Ratio	0.1600	0.0800	0.1125	0.71	0.551
Temperature*Catalyst*Ratio	0.7475	0.3738	0.1125	3.32	0.080
Time*Catalyst*Ratio	1.9275	0.9637	0.1125	8.57	0.013
Temperature*Time*Catalyst*Ratio	1.2200	0.6100	0.1125	5.42	0.032
Ct Pt		5.0992	0.2831	18.01	0.003

S = 0.449926   PRESS = *
R-Sq = 99.92%   R-Sq(pred) = *%   R-Sq(adj) = 99.26%

Analysis of Variance for MME (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F
Main Effects	4	385.986	385.986	96.497	476.68
Temperature	1	54.982	54.982	54.982	271.61
Time	1	89.019	89.019	89.019	439.75
Catalyst	1	209.381	209.381	209.381	1034.32
Ratio	1	32.604	32.604	32.604	161.06
2-Way Interactions	6	15.113	15.113	2.519	12.44
Temperature*Time	1	0.040	0.040	0.040	0.20
Temperature*Catalyst	1	0.570	0.570	0.570	2.82
Temperature*Ratio	1	3.591	3.591	3.591	17.74
Time*Catalyst	1	6.477	6.477	6.477	32.00
Time*Ratio	1	0.632	0.632	0.632	3.12
Catalyst*Ratio	1	3.803	3.803	3.803	18.78
3-Way Interactions	4	17.904	17.904	4.476	22.11
Temperature*Time*Catalyst	1	0.706	0.706	0.706	3.49
Temperature*Time*Ratio	1	0.102	0.102	0.102	0.51
Temperature*Catalyst*Ratio	1	2.235	2.235	2.235	11.04
Time*Catalyst*Ratio	1	14.861	14.861	14.861	73.41
4-Way Interactions	1	5.954	5.954	5.954	29.41
Temperature*Time*Catalyst*Ratio	1	5.954	5.954	5.954	29.41
Curvature	1	65.688	65.688	65.688	324.49
Residual Error	2	0.405	0.405	0.202	
Pure Error	2	0.405	0.405	0.202	
Total	18	491.050			

Source	P
Main Effects	0.002
Temperature	0.004
Time	0.002
Catalyst	0.001
Ratio	0.006
2-Way Interactions	0.076
Temperature*Time	0.700
Temperature*Catalyst	0.235
Temperature*Ratio	0.052
Time*Catalyst	0.030
Time*Ratio	0.219

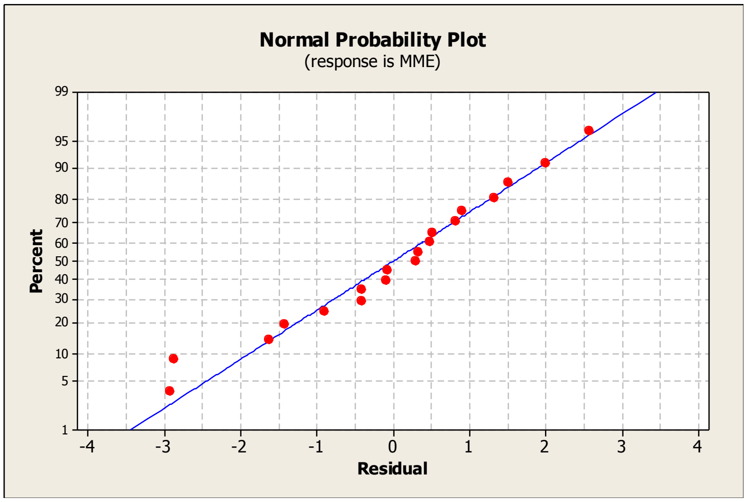
```
  Catalyst*Ratio                       0.049
3-Way Interactions                     0.044
  Temperature*Time*Catalyst            0.203
  Temperature*Time*Ratio               0.551
  Temperature*Catalyst*Ratio           0.080
  Time*Catalyst*Ratio                  0.013
4-Way Interactions                     0.032
  Temperature*Time*Catalyst*Ratio      0.032
  Curvature                            0.003
Residual Error
  Pure Error
Total
```

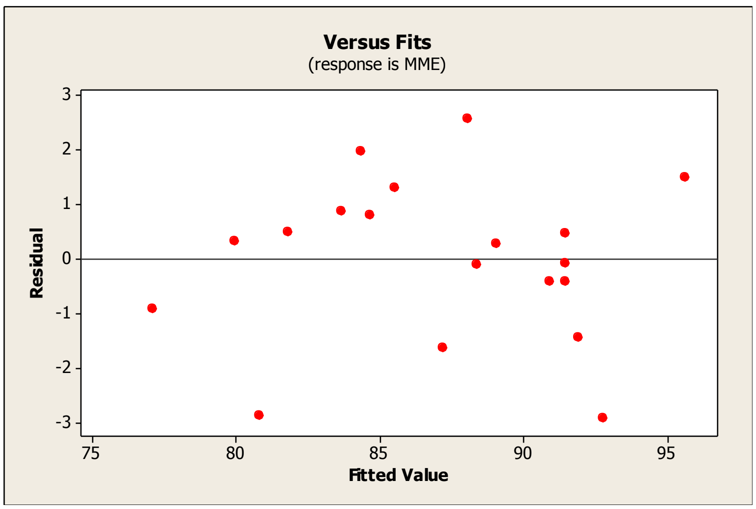
The results based on the error estimate from the center points and the normal probability plot are similar. Some interaction effects have p-values less than 0.05, but the magnitudes of these effects are much smaller than the four main effects (with much lower p values).

c) For the curvature test, the P-value = 0.003. Therefore, significant curvature is present.

d) A model with only the four main effects and the center point term is used to generate residuals.

```
Estimated Effects and Coefficients for MME (coded units)

Term          Effect    Coef  SE Coef       T      P
Constant              86.377   0.4351  198.53  0.000
Temperature    3.707   1.854   0.4351    4.26  0.001
Time           4.717   2.359   0.4351    5.42  0.000
Catalyst       7.235   3.617   0.4351    8.31  0.000
Ratio          2.855   1.428   0.4351    3.28  0.006
Ct Pt                  5.099   1.0950    4.66  0.000

S = 1.74036      PRESS = 83.3604
R-Sq = 91.98%   R-Sq(pred) = 83.02%   R-Sq(adj) = 88.90%

Analysis of Variance for MME (coded units)

Source          DF   Seq SS   Adj SS   Adj MS      F      P
Main Effects     4  385.986  385.986   96.497  31.86  0.000
  Temperature    1   54.982   54.982   54.982  18.15  0.001
  Time           1   89.019   89.019   89.019  29.39  0.000
  Catalyst       1  209.381  209.381  209.381  69.13  0.000
  Ratio          1   32.604   32.604   32.604  10.76  0.006
  Curvature      1   65.688   65.688   65.688  21.69  0.000
Residual Error  13   39.375   39.375    3.029
  Lack of Fit   11   38.970   38.970    3.543  17.50  0.055
  Pure Error     2    0.405    0.405    0.202
Total           18  491.050
```

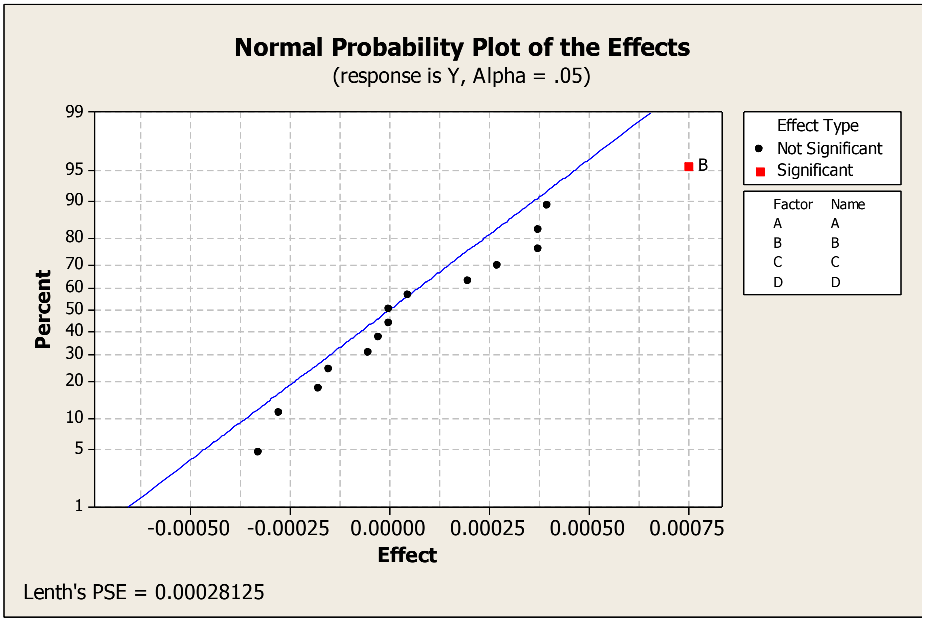




There are no obvious departures from assumptions seen in these plots.

14-32 a)

```
Factorial Fit: Y versus A, B, C, D

Estimated Effects and Coefficients for Y (coded units)
Term          Effect        Coef
Constant                0.119288
A          -0.000275   -0.000138
B           0.000750    0.000375
C           0.000375    0.000188
D           0.000400    0.000200
A*B         0.000000    0.000000
A*C        -0.000325   -0.000162
A*D        -0.000050   -0.000025
B*C         0.000200    0.000100
B*D         0.000375    0.000188
C*D         0.000050    0.000025
A*B*C       0.000000    0.000000
A*B*D      -0.000175   -0.000088
A*C*D      -0.000150   -0.000075
B*C*D       0.000275    0.000138
A*B*C*D    -0.000025   -0.000013
```



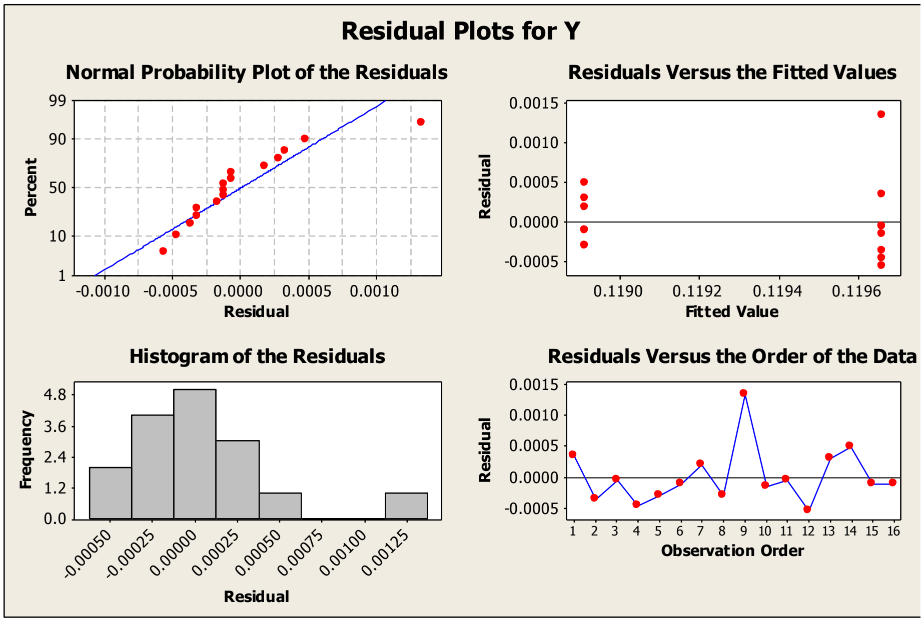
The effect of factor B is large, so this factor is included in the model.

b) Consider the following computer output. Because the *P*-value of factor B is less than α = 0.05, we reject the null hypothesis and conclude that the main factor of factor B is significant.

Factorial Fit: Y versus B

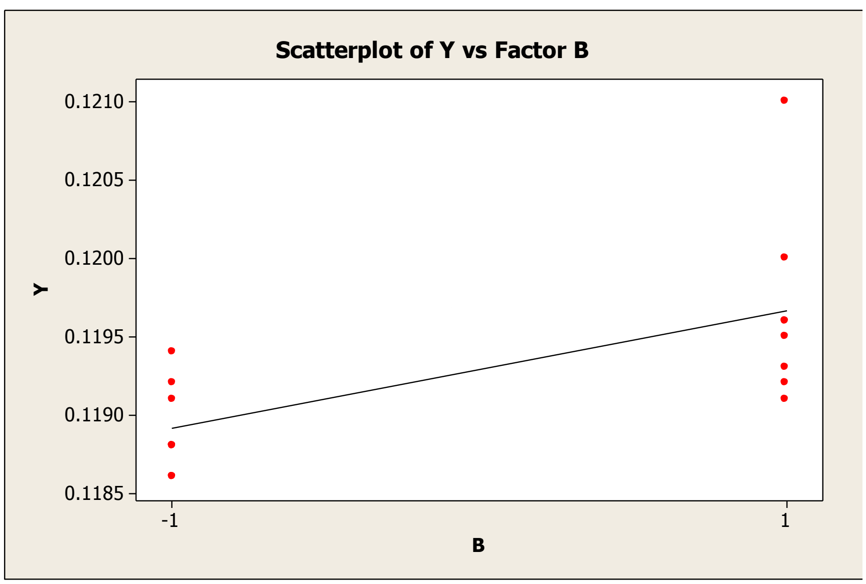
```
Estimated Effects and Coefficients for Y (coded units)
Term          Effect      Coef    SE Coef       T      P
Constant                0.119288  0.000119  999.99  0.000
B           0.000750  0.000375  0.000119    3.14  0.007

S = 0.000477157   R-Sq = 41.38%   R-Sq(adj) = 37.19%

Analysis of Variance for Y (coded units)
Source          DF      Seq SS       Adj SS       Adj MS      F      P
Main Effects     1  0.00000225   0.00000225   0.00000225   9.88  0.007
Residual Error  14  0.00000319   0.00000319   0.00000023
  Pure Error    14  0.00000319   0.00000319   0.00000023
Total           15  0.00000544
```

c)



The normal probability plot does not indicate any serious concerns with assumptions. The plot of residuals versus the predicted values shows a potential problem of non-constant of variance. The actual time order of the observations was not provided, so the plot versus observation order is not relevant.

d) Only one main factor B is significant. The design is reduced to 8 replicates of an experiment with a single factor with two levels. The scatter plot of Y and factor B indicates that an increase to factor B increases the response.



Section 14-6

14-33 a)

BLOCK	A	B	C	y
1	–1	–1	–1	225
1	1	1	–1	552
1	1	–1	1	406
1	–1	1	1	610
2	1	–1	–1	325
2	–1	1	–1	360
2	–1	–1	1	445
2	1	1	1	392

```
Term                   Effect       Coef
Constant                         516.000
Block                            -67.750
factor_A                 8.75      4.375
factor_B               128.25     64.125
factor_C                97.75     48.875
factor_A*factor_B      -21.75    -10.875
factor_A*factor_C     -137.25    -68.625
factor_B*factor_C      -52.75    -26.375

Term                  Effect       Coef     SE Coef       T       P
Constant                         516.00       63.79    8.09   0.015
blocks                           -67.75       40.35   -1.68   0.235
Factor a                8.75       4.37       20.17    0.22   0.848
factor b              128.25      64.13       20.17    3.18   0.086
factor C               97.75      48.88       20.17    2.42   0.136
a*c                  -137.25     -68.63       20.17   -3.40   0.077

Analysis of Variance for life
Source               DF    Seq SS    Adj SS    Adj MS       F       P
blocks                1      9180      9180      9180    2.82   0.235
Main effects          3     52159     52159     17386    5.34   0.357
2-way interaction     1     37675     37675     37675   11.57   0.077
Error                 2      6511      6511      3256
Total                 7    105525
```

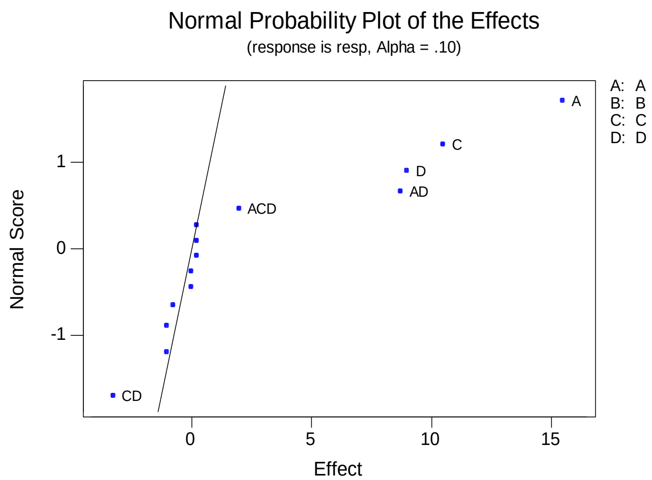
b) In this model with blocking there are no significant factors.

14-34 Design with 2 blocks

Blocks	A	B	C	D	Rep I
1	-1	-1	-1	-1	159
1	1	1	-1	-1	166
1	1	-1	1	-1	179
1	-1	1	1	-1	173
1	1	-1	-1	1	187
1	-1	1	-1	1	163
1	-1	-1	1	1	168
1	1	1	1	1	194
2	1	-1	-1	-1	168
2	-1	1	-1	-1	158
2	-1	-1	1	-1	175
2	1	1	1	-1	179
2	-1	-1	-1	1	164
2	1	1	-1	1	185

2	1	-1	1	1	197
2	-1	1	1	1	170

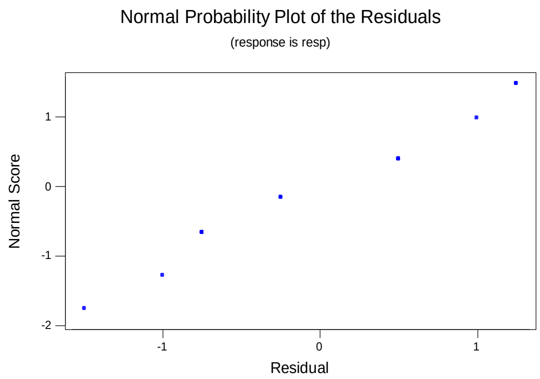
Term	Effect	Coef
Constant		174.063
Block		-0.438
A	15.625	7.813
B	-1.125	-0.563
C	10.625	5.312
D	8.875	4.437
A*B	-0.625	-0.313
A*C	0.125	0.062
A*D	8.875	4.437
B*C	0.375	0.188
B*D	0.125	0.063
C*D	-3.125	-1.562
A*B*C	-0.125	-0.062
A*B*D	-0.875	-0.437
A*C*D	1.875	0.937
B*C*D	0.125	0.063



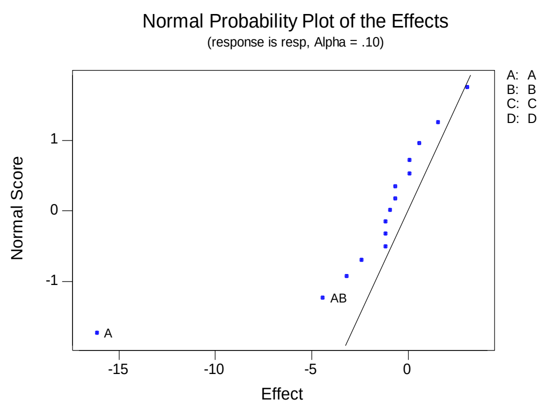
Factors A, C, and D, and interactions AD, CD and ACD appear to be significant.

```
Estimated Effects and Coefficients for resp (coded units)

Term        Effect     Coef  SE Coef       T      P
Constant            174.063   0.2864  607.74  0.000
Block                 0.438   0.2864    1.53  0.165
A           15.625    7.812   0.2864   27.28  0.000
C           10.625    5.313   0.2864   18.55  0.000
D            8.875    4.438   0.2864   15.49  0.000
A*D          8.875    4.437   0.2864   15.49  0.000
C*D         -3.125   -1.563   0.2864   -5.46  0.001
A*C*D        1.875    0.937   0.2864    3.27  0.011

S = 1.14564   R-Sq = 99.51%   R-Sq(adj) = 99.07%
```

The main effects and interactions are all significant in a model that includes the factors listed above. The normal probability plot appears to support the assumption of normality.



14-35



Factor A and interaction AB are significant. Factor B is included in the model to make the model hierarchical.

Term	Effect	Coef
Constant		35.938
BLOCK		-0.063
A	-16.125	-8.062
B	3.125	1.562
C	-1.125	-0.562
D	-1.125	-0.562
A*B	-4.375	-2.188
A*C	-0.625	-0.313
A*D	-3.125	-1.563
B*C	1.625	0.813
B*D	0.125	0.063
C*D	-0.625	-0.312
A*B*C	0.625	0.312
A*B*D	-2.375	-1.187
A*C*D	-1.125	-0.562
B*C*D	-0.875	-0.438

Estimated Effects and Coefficients for resp (coded units)

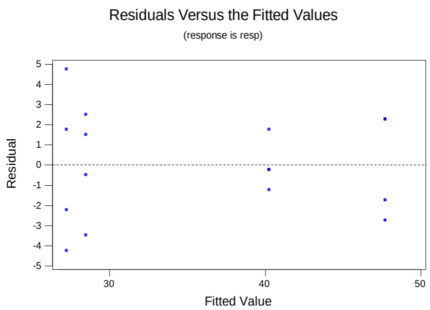
Term	Effect	Coef	SE Coef	T	P
Constant		35.938	0.7043	51.02	0.000
A	-16.125	-8.062	0.7043	-11.45	0.000
B	3.125	1.562	0.7043	2.22	0.047
A*B	-4.375	-2.188	0.7043	-3.11	0.009

Analysis of Variance for resp (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	1079.12	1079.12	539.562	67.98	0.000
2-Way Interactions	1	76.56	76.56	76.563	9.65	0.009
Residual Error	12	95.25	95.25	7.938		
Pure Error	12	95.25	95.25	7.938		
Total	15	1250.94				

Effects A, B, and the AB interaction are significant at $\alpha = 0.05$. The residual analysis shows some slight differences in variability in the data.

14-36 2^5 Design in 2 Blocks with ABCDE confounded with blocks.

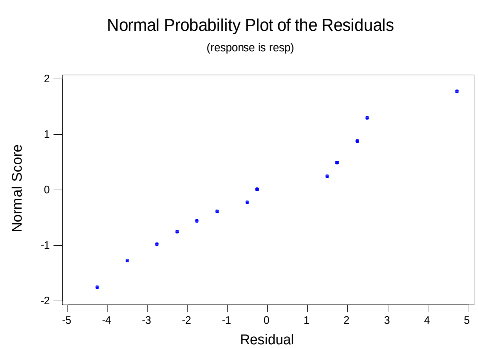


Run	Block	A	B	C	D	E
1	1	−	−	−	−	−
2	1	+	+	−	−	−
3	1	+	−	+	−	−
4	1	−	+	+	−	−
5	1	+	−	−	+	−
6	1	−	+	−	+	−
7	1	−	−	+	+	−
8	1	+	+	+	+	−
9	1	+	−	−	−	+
10	1	−	+	−	−	+
11	1	−	−	+	−	+
12	1	+	+	+	−	+
13	1	−	−	−	+	+
14	1	+	+	−	+	+
15	1	+	−	+	+	+
16	1	−	+	+	+	+
17	2	+	−	−	−	−
18	2	−	+	−	−	−
19	2	−	−	+	−	−
20	2	+	+	+	−	−
21	2	−	−	−	+	−
22	2	+	+	−	+	−
23	2	+	−	+	+	−
24	2	−	+	+	+	−
25	2	−	−	−	−	+
26	2	+	+	−	−	+
27	2	+	−	+	−	+
28	2	−	+	+	−	+
29	2	+	−	−	+	+
30	2	−	+	−	+	+
31	2	−	−	+	+	+
32	2	+	+	+	+	+

14-37 a) The design with four blocks

Blocks	A	B	C	D	Score
1	-1	1	1	1	170
1	1	1	-1	-1	166
1	-1	-1	-1	-1	159
1	1	-1	1	1	197
2	-1	1	1	-1	173
2	-1	-1	-1	1	164
2	1	1	-1	1	185
2	1	-1	1	-1	179
3	1	1	1	-1	179
3	1	-1	-1	1	187
3	-1	1	-1	1	163
3	-1	-1	1	-1	175
4	1	-1	-1	-1	168
4	-1	1	-1	-1	158
4	-1	-1	1	1	168
4	1	1	1	1	194

b)
```
Estimated Effects and Coefficients for Score (coded units)

Term       Effect      Coef   SE Coef       T       P
Constant            174.063    0.5984  290.88   0.000
Block 1              -1.063    1.0364   -1.03   0.381
Block 2               1.188    1.0364    1.15   0.335
Block 3               1.937    1.0364    1.87   0.158
A          15.625     7.812    0.5984   13.06   0.001
B          -1.125    -0.563    0.5984   -0.94   0.417
C          10.625     5.313    0.5984    8.88   0.003
D           8.875     4.438    0.5984    7.42   0.005
A*B        -0.625    -0.313    0.5984   -0.52   0.638
A*C         0.125     0.063    0.5984    0.10   0.923
A*D         8.875     4.438    0.5984    7.42   0.005
B*C         0.375     0.187    0.5984    0.31   0.775
B*D         0.125     0.063    0.5984    0.10   0.923

S = 2.39357   R-Sq = 99.19%   R-Sq(adj) = 95.96%

Analysis of Variance for Score (coded units)

Source              DF   Seq SS   Adj SS   Adj MS       F       P
Blocks               3    42.19    42.19   14.063    2.45   0.240
Main Effects         4  1748.25  1748.25  437.063   76.29   0.002
2-Way Interactions   5   317.31   317.31   63.463   11.08   0.038
Residual Error       3    17.19    17.19    5.729
Total               15  2124.94
```



In this model with blocking there are no significant factors.

14-38 2^4 Design in 4 Blocks.

Run	Blocks	A	B	C	D
1	1	+	+	−	−
2	1	−	−	+	−
3	1	−	+	−	−
4	1	+	−	+	+
5	2	−	−	−	−
6	2	+	+	+	−
7	2	+	−	−	+
8	2	−	+	+	+
9	3	−	+	−	−
10	3	+	−	+	−
11	3	+	+	−	+
12	3	−	−	+	+
13	4	+	−	−	−
14	4	−	+	+	−
15	4	−	−	−	+
16	4	+	+	+	+

14-39 2^5 in 4 blocks.

Run	Blocks	A	B	C	D	E
1	1	−	−	−	−	−
2	1	+	+	−	−	−
3	1	+	−	+	+	−
4	1	−	+	+	+	−
5	1	+	−	+	−	+
6	1	−	+	+	−	+
7	1	−	−	−	+	+
8	1	+	+	−	+	+
9	2	+	−	−	−	−
10	2	−	+	−	−	−
11	2	−	−	+	+	−
12	2	+	+	+	+	−
13	2	−	−	+	−	+
14	2	+	+	+	−	+
15	2	+	−	−	+	+
16	2	−	+	−	+	+
17	3	+	−	+	−	−
18	3	−	+	+	−	−
19	3	−	−	−	+	−
20	3	+	+	−	+	−
21	3	−	−	−	−	+
22	3	+	+	−	−	+
23	3	+	−	+	+	+
24	3	−	+	+	+	+
25	4	−	−	+	−	−
26	4	+	+	+	−	−
27	4	+	−	−	+	−
28	4	−	+	−	+	−
29	4	+	−	−	−	+
30	4	−	+	−	−	+
31	4	−	−	+	+	+
32	4	+	+	+	+	+

14-40 a) Effect ABC is confounded with blocks, where A= juice, B = exercise, C = delay

b) In this model with blocking, there are no significant factors.

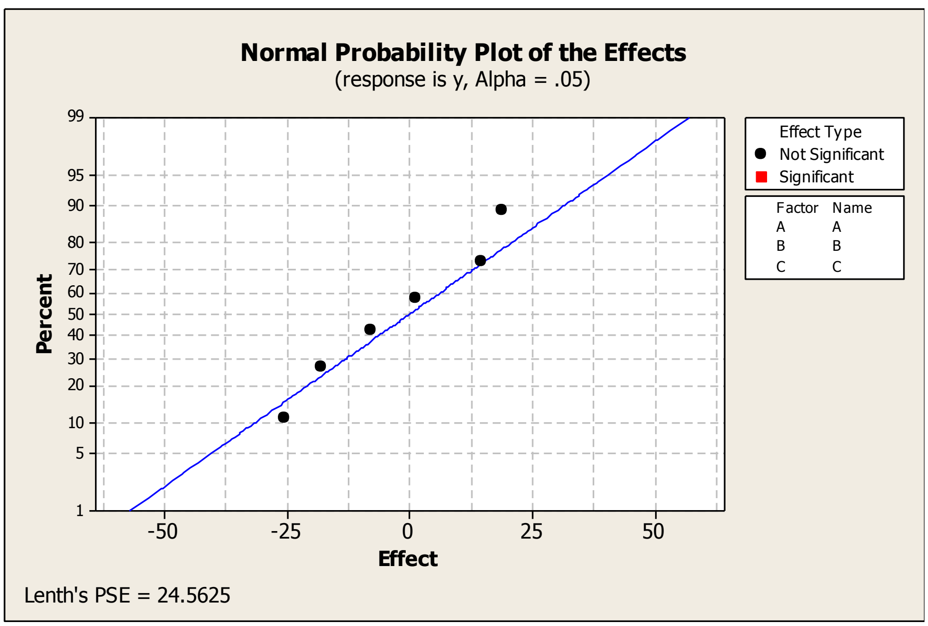
Factorial Fit: y versus Block, A, B, C

Estimated Effects and Coefficients for y (coded units)

Term	Effect	Coef
Constant		103.38
Block		2.13
A	19.00	9.50
B	-8.00	-4.00
C	14.75	7.38
A*B	1.25	0.62
A*C	-18.00	-9.00
B*C	-25.50	-12.75

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Block	36.13	1	36.13		
Model	3233.63	5	646.73	206.95	0.0527
A	722.00	1	722.00	231.04	0.0418
B	128.00	1	128.00	40.96	0.0987
C	435.13	1	435.13	139.24	0.0538
AC	648.00	1	648.00	207.36	0.0441
BC	1300.50	1	1300.50	416.16	0.0312
Residual	3.13	1	3.13		
Cor Total	3272.88	7			

Thus, the effects of *juice* as well as the interactions between *juice* and *delay* and *exercise* and *delay* were marginally significant. Additional degrees of freedom for error are needed and the normal probability plot of the effects does not indicate significant effects.



14-41 a) Estimated Effects and Coefficients for y

Term		Effect	Coef	StDev Coef	T	P
Constant			56.37	2.633	21.41	0.000
Block	1		15.63	4.560	3.43	0.014
	2		-3.38	4.560	-0.74	0.487
	3		-10.88	4.560	-2.38	0.054
A		-45.25	-22.62	2.633	-8.59	0.000
B		-1.50	-0.75	2.633	-0.28	0.785

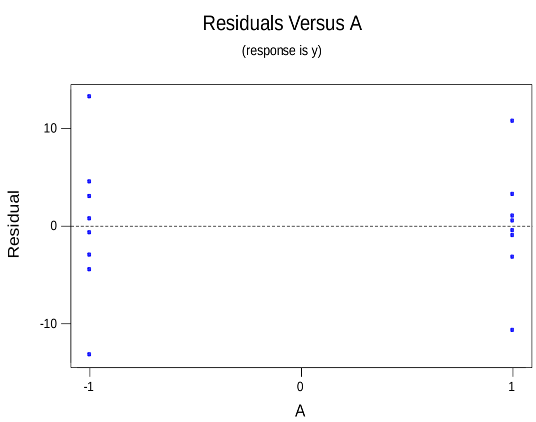
C	14.50	7.25	2.633	2.75	0.033
A*B	19.00	9.50	2.633	3.61	0.011
A*C	-14.50	-7.25	2.633	-2.75	0.033
B*C	-9.25	-4.63	2.633	-1.76	0.130

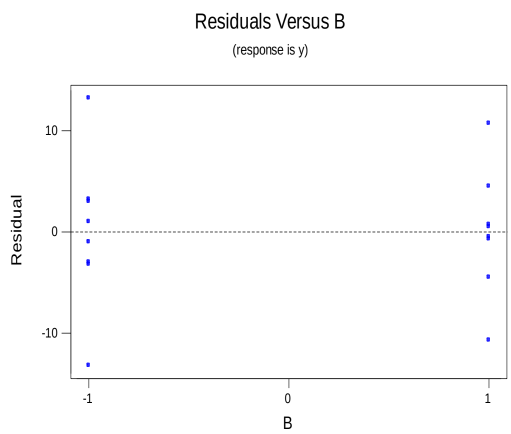
Analysis of Variance for y

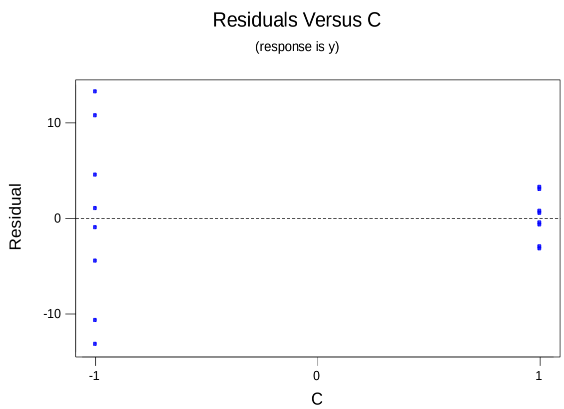
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Blocks	3	1502.8	1502.8	500.9	4.52	0.055
Main Effects	3	9040.2	9040.2	3013.4	27.17	0.001
2-Way Interactions	3	2627.2	2627.2	875.7	7.90	0.017
Residual Error	6	665.5	665.5	110.9		
Total	15	13835.7				

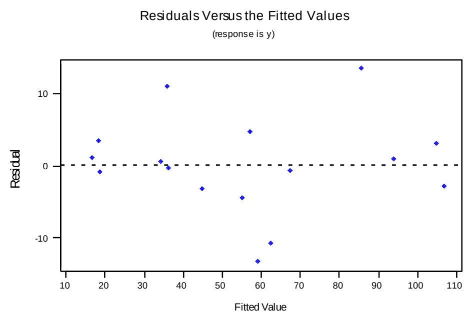
Factors A, C, AB, and AC are significant.

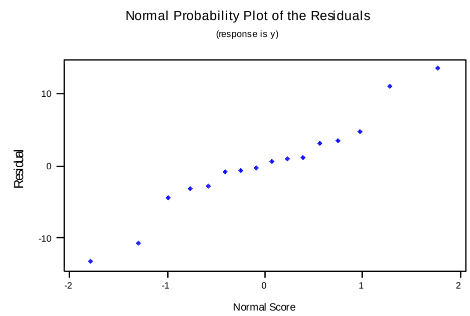
b) There is more variability on the response associated with the low setting of factor C.











c) Some of the information from the experiment is lost because the design is run in 4 blocks. This causes us to lose information on the ABC interaction even though we have replicated the experiment twice. If it is possible to run the experiment in only 2 blocks, there would be information on all interactions.

d) To have data on all interactions, we could run the experiment so that each replicate is a block. In that case, there would be only two blocks.

14-42 a) Because the sum of squares associated with blocks is large relative to sum of squares for residual error, we conclude that blocking is important to reduce nuisance variation in this experiment.

b) The effects for all three-factor interaction terms (ABC, ABD, ACD, and BCD) are used to generate the residual error in ANOVA because these effects do not appear in the ANOVA table. The four-factor interaction effect is confounded with blocks.

c) Coef of AD $= \dfrac{Effect}{2} = \dfrac{30.28}{2} = 15.14$

t test of AD $= \dfrac{Coef}{Se\,Coef} = \dfrac{15.14}{9.928} = 1.525$

The degrees of freedom for blocks are 15 – 4 – 6 – 4 = 1.
Also, we know there are two blocks so the degrees of freedom = 2 – 1 = 1.

Adj MS of 2-way Interactions $= \dfrac{Adj\ SS}{DF} = \dfrac{6992}{6} = 1165.33$

14-43 a) The effect of Fab Temperature (D) is aliased with the three factor interaction Dispatching time (A)**Rework Delay time* (B)**Rework Level* (C) and this alias set is confounded with blocks.

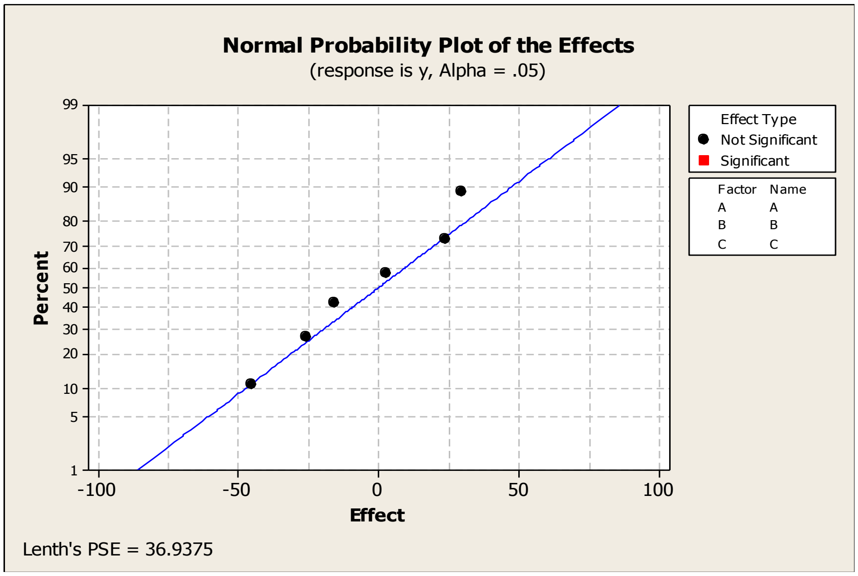
That is, Block = *Fab Temperature = Dispatching time*Rework Delay time*Rework Level*

A concern is that the alias set confounded with blocks contains the main effect of Fab Temperature.

b) Computer software will often not analyze an experiment with a main effect confounded with blocks. Therefore, the experiment is handled as a three-factor experiment in factors A, B, C confounded in two blocks. Information on factor D is lost because it is confounded with blocks.

```
Factorial Fit: y versus Block, A, B, C

Estimated Effects and Coefficients for y (coded units)
```

```
Term       Effect    Coef
Constant           263.81
Block                7.06
A           29.37   14.69
B           23.63   11.81
C          -15.88   -7.94
A*B        -25.63  -12.81
A*C          2.38    1.19
B*C        -45.38  -22.69
```



From the normal probability plot of effects, there does not appear to be any significant effects. However, the effect estimates in the table show that the A*C effect = 0.13 is much smaller than the others. If only this effect represents the magnitude of noise, the following computer output shows that all the other effects are significant. Some additional data is needed here to estimate noise and to choose among the results that all but the A*C effect are significant or no effects are significant.

Factorial Fit: y versus Block, A, B, C

```
Estimated Effects and Coefficients for y (coded units)

Term       Effect    Coef  SE Coef       T      P
Constant           263.81    1.188  222.16  0.003
Block                7.06    1.188    5.95  0.106
A           29.37   14.69    1.188   12.37  0.051
B           23.63   11.81    1.188    9.95  0.064
C          -15.88   -7.94    1.188   -6.68  0.095
A*B        -25.63  -12.81    1.188  -10.79  0.059
B*C        -45.37  -22.69    1.188  -19.11  0.033

S = 3.35876   R-Sq = 99.88%   R-Sq(adj) = 99.14%

Analysis of Variance for y (coded units)

Source              DF   Seq SS   Adj SS   Adj MS       F      P
Blocks               1   399.03   399.03   399.03   35.37  0.106
Main Effects         3  3346.09  3346.09  1115.36   98.87  0.074
2-Way Interactions   2  5431.06  5431.06  2715.53  240.71  0.046
Residual Error       1    11.28    11.28    11.28
Total                7  9187.47
```

Section 14-7

14-44 a) Design 2^{5-1}

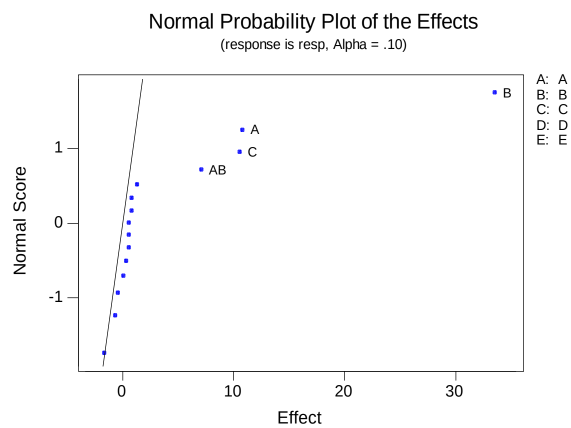
Run No.	A	B	C	D	E	resp
1	-1	-1	-1	-1	1	8
2	1	-1	-1	-1	-1	9
3	-1	1	-1	-1	-1	34
4	1	1	-1	-1	1	52
5	-1	-1	1	-1	-1	16
6	1	-1	1	-1	1	22
7	-1	1	1	-1	1	45
8	1	1	1	-1	-1	60
9	-1	-1	-1	1	-1	8
10	1	-1	-1	1	1	10
11	-1	1	-1	1	1	30
12	1	1	-1	1	-1	50
13	-1	-1	1	1	1	15
14	1	-1	1	1	-1	21
15	-1	1	1	1	-1	44
16	1	1	1	1	1	63

b)
```
Design Generators:  E = ABCD

Alias Structure

I + ABCDE

A + BCDE
B + ACDE
C + ABDE
D + ABCE
E + ABCD
AB + CDE
AC + BDE
AD + BCE
AE + BCD
BC + ADE
BD + ACE
BE + ACD
CD + ABE
CE + ABD
DE + ABC
```

c)
```
Term                   Effect      Coef
Constant                         30.4375
factor_A              10.8750    5.4375
factor_B              33.6250   16.8125
factor_C              10.6250    5.3125
factor_D              -0.6250   -0.3125
factor_E               0.3750    0.1875
factor_A*factor_B      7.1250    3.5625
factor_A*factor_C      0.6250    0.3125
factor_A*factor_D      0.8750    0.4375
factor_A*factor_E      1.3750    0.6875
factor_B*factor_C      0.8750    0.4375
factor_B*factor_D     -0.3750   -0.1875
factor_B*factor_E      0.1250    0.0625
```

```
factor_C*factor_D      0.6250     0.3125
factor_C*factor_E      0.6250     0.3125
factor_D*factor_E     -1.6250    -0.8125
```

d) Factors A, B, and C and interaction AB are significant

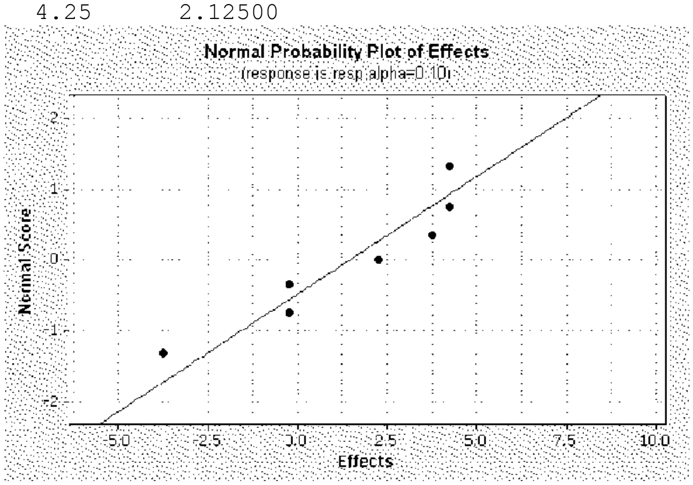


e)

Term	Effect	Coef	StDev Coef	T	P
Constant		30.438	0.4243	71.73	0.000
factor_A	10.875	5.438	0.4243	12.81	0.000
factor_B	33.625	16.812	0.4243	39.62	0.000
factor_C	10.625	5.313	0.4243	12.52	0.000
factor_A*factor_B	7.125	3.562	0.4243	8.40	0.000

14-45 Estimated Effects and Coefficients for resp (coded units)
Estimated Effects and Coefficients for yield (coded units)

Term	Effect	Coef
Constant		17.8750
A	3.75	1.87500
B	-0.25	-0.125000
C	2.25	1.12500
D	4.25	2.12500
A*B	-0.25	-0.125000
A*C	-3.75	-1.87500
A*D	4.25	2.12500



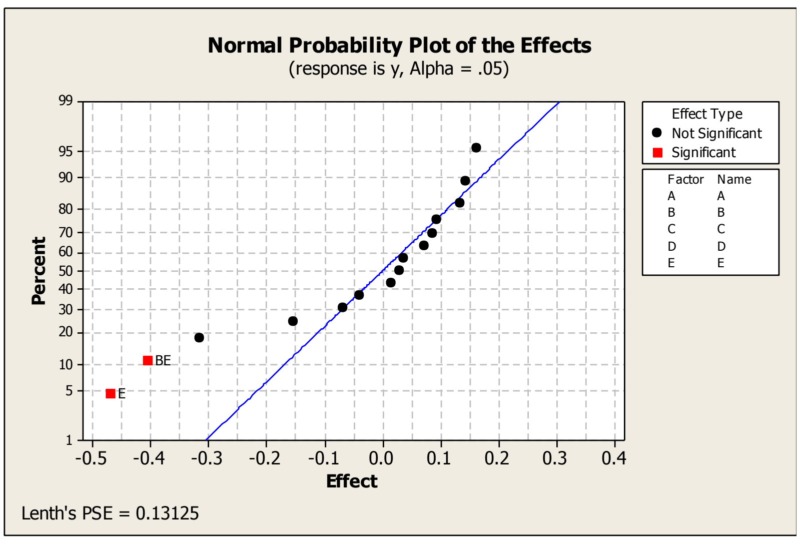
None of the factors appear to be significant in the 2^{4-1} design.

14-46 a) The generator is E= -ABCD

b) The resolution is resolution V

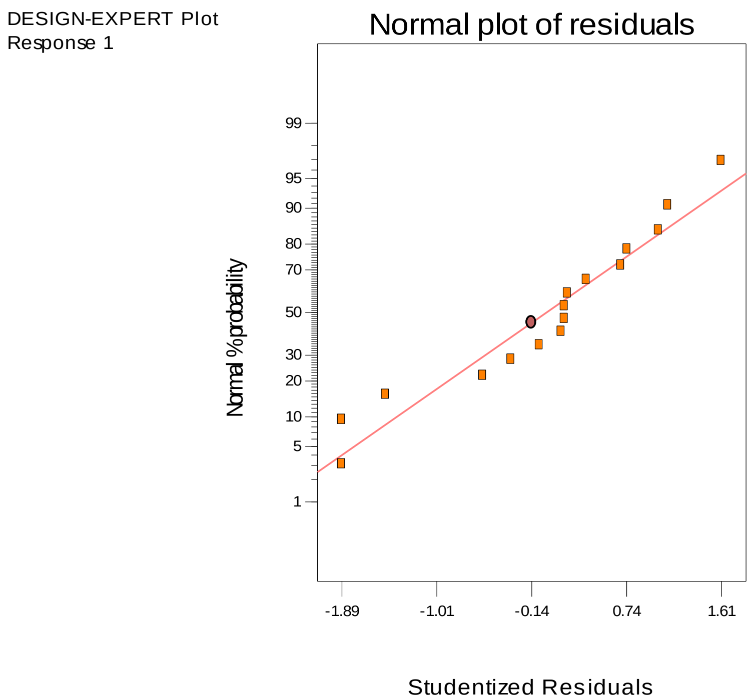
c) Estimated Effects and Coefficients for y (coded units)

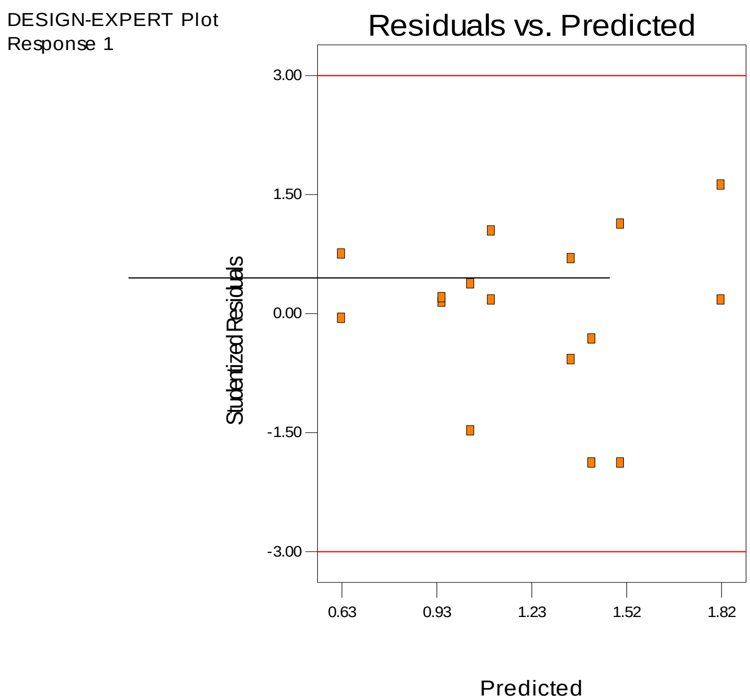
Term	Effect	Coef
Constant		1.2263
A	0.1450	0.0725
B	0.0875	0.0438
C	0.0375	0.0187
D	-0.0375	-0.0187
E	-0.4700	-0.2350
A*B	0.0150	0.0075
A*C	0.0950	0.0475
A*D	0.0300	0.0150
A*E	-0.1525	-0.0762
B*C	-0.0675	-0.0338
B*D	0.1625	0.0813
B*E	-0.4050	-0.2025
C*D	0.0725	0.0363
C*E	0.1350	0.0675
D*E	-0.3150	-0.1575



From the graph shown above, E, BE, and DE are important effects. From the effect estimation from table above these same effects are computed to be the largest effects (in absolute value).

d) For the model with E, BE, and DE the normality assumption and constant variance seem to be reasonable.

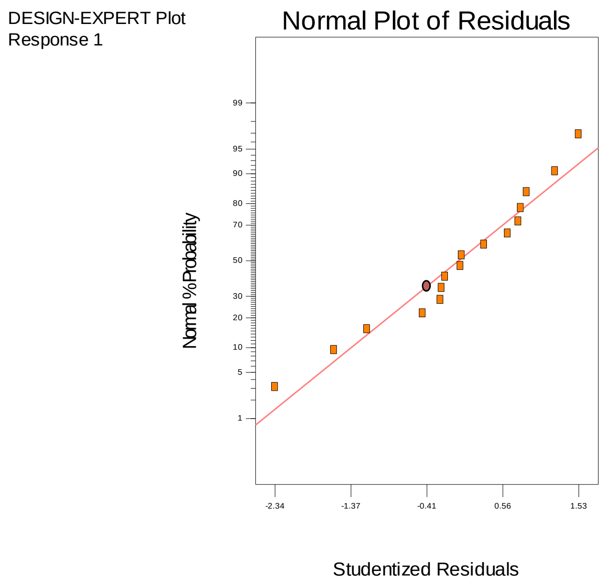


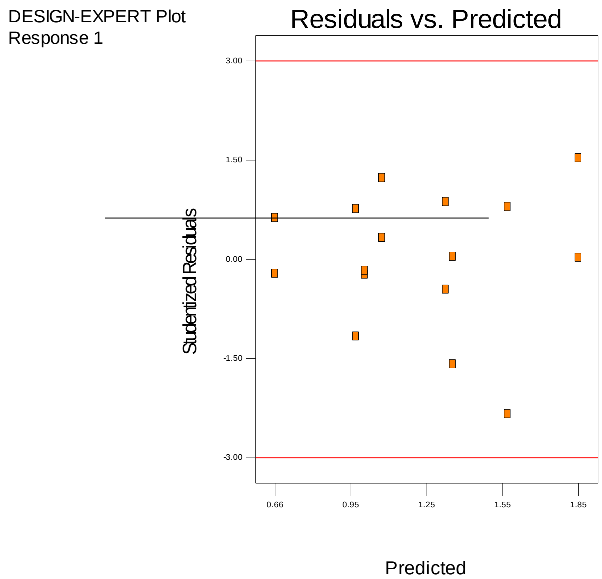


For a hierarchical model, the effects to include are B, D, E, BE, and DE. The ANOVA and residual plot follow:

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	1.97	5	0.39	8.94	0.0019
B	*0.031*	*1*	*0.031*	*0.69*	*0.4242*
D	*5.625E-003*	*1*	*5.625E-003*	*0.13*	*0.7284*
E	*0.88*	*1*	*0.88*	*20.03*	*0.0012*
BE	*0.66*	*1*	*0.66*	*14.87*	*0.0032*
DE	*0.40*	*1*	*0.40*	*9.00*	*0.0134*
Residual	0.44	10	0.044		
Cor Total	2.41	15			





e) For the non-hierarchical model $\hat{y} = 1.23 - 0.24E - 0.20BE - 0.16DE$
This equation could be used for further study of the process.

14-47 a)
```
Design Generators:  D = ABC
Alias Structure
I + ABCD
A + BCD
B + ACD
C + ABD
D + ABC
AB + CD
AC + BD
AD + BC
```

b)

Term	Effect	Coef
Constant		70.7500
A	16.5	8.25000
B	1.5	0.750000
C	14.0	7.00000
D	14.0	7.00000
A*B	-3.5	-1.75000
A*C	-16.0	-8.00000
A*D	24.0	12.0000

A, C, D, AC, and AD have large estimated effects.

c) Estimated Effects and Coefficients for rate

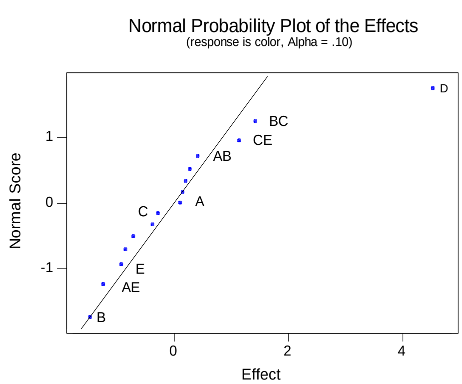
Term	Effect	Coef	SE Coef	T	P
Constant		70.750	1.346	52.55	0.000
A	16.5	8.250	1.346	6.13	0.026
C	14.0	7.000	1.346	5.20	0.035
D	14.0	7.000	1.346	5.20	0.035
A*C	-16.0	-8.000	1.346	-5.94	0.027
A*D	24.0	12.000	1.346	8.91	0.012

Analysis of Variance for rate

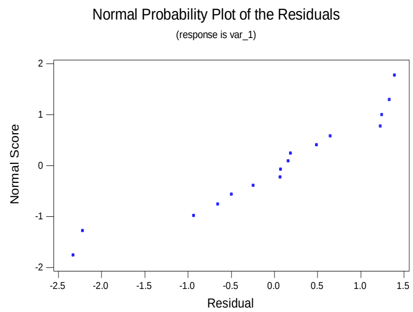
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	1328.50	1328.50	442.83	30.54	0.032
2-way interaction	2	1664.00	1664.00	832.00	57.38	0.020
Error	2	29.00	29.00	14.50		
Total	7	3021.50				

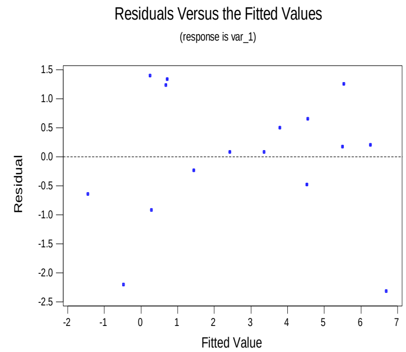
A, C, D, AC, and AD are significant. This appears to be the appropriate model for the data.

14-48 a) Several factors and interactions are potentially significant.

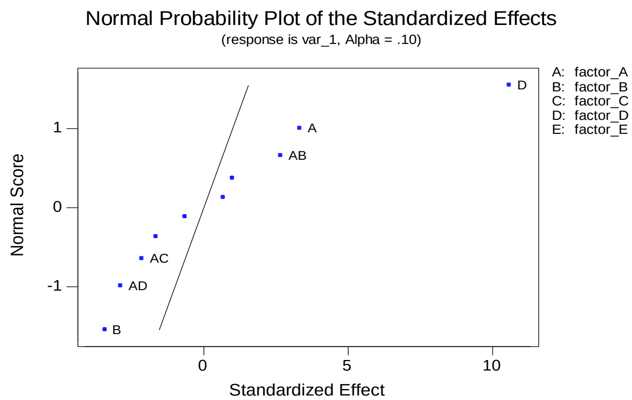


b) There are no serious problems with the residual plots. The normal probability plot has some curvature and there is a little more variability at the lower and higher ends of the fitted values.





c) Normal probability plot shows that we can collapse using only factors A, B, and D



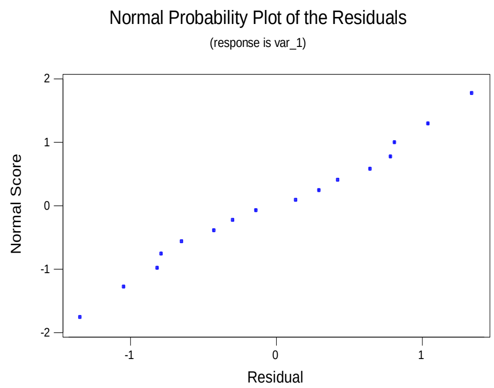
Estimated Effects and Coefficients for var_1

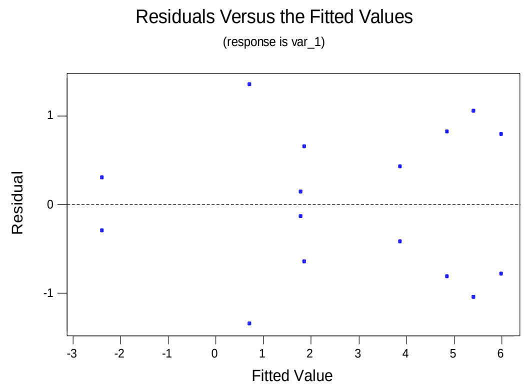
Term	Effect	Coef	StDev Coef	T	P
Constant		2.7700	0.2762	10.03	0.000
factor_A	1.4350	0.7175	0.2762	2.60	0.032
factor_B	-1.4650	-0.7325	0.2762	-2.65	0.029
factor_D	4.5450	2.2725	0.2762	8.23	0.000
factor_A*factor_B	1.1500	0.5750	0.2762	2.08	0.071
factor_A*factor_D	-1.2300	-0.6150	0.2762	-2.23	0.057
factor_B*factor_D	0.1200	0.0600	0.2762	0.22	0.833
factor_A*factor_B*factor_D	-0.3650	-0.1825	0.2762	-0.66	0.527

Analysis of Variance for var_1

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	99.450	99.4499	33.1500	27.15	0.000
2-Way Interactions	3	11.399	11.3992	3.7997	3.11	0.088
3-Way Interactions	1	0.533	0.5329	0.5329	0.44	0.527
Residual Error	8	9.767	9.7668	1.2208		
Pure Error	8	9.767	9.7668	1.2208		
Total	15	121.149				

Factors A, B, D, AB and AD are significant.

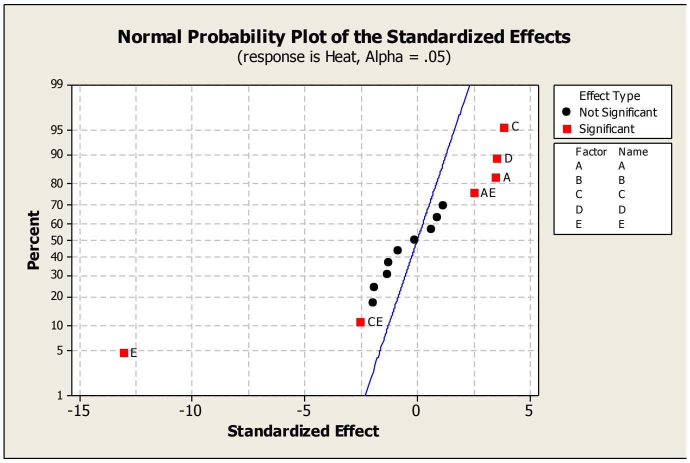




The normal probability plot does not indicate problesm. The reduced model ignores factor C and it is two replicates of a full factorial experiment in factors A, B, and D. There are 8 unique test points with two replicates at each. The model shown has 8 coefficients so that the fitted value is the mean of the replicates at each of the 8 unique test points. Therefore, at each unique test point there are equal positive and negative residuals. Consequently, the plot of residuals versus fitted values has symmetry about zero.

14-49 a) Estimated Effects and Coefficients for Heat (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		14.256	0.2370	60.15	0.000
A	1.659	0.829	0.2370	3.50	0.003
B	-0.041	-0.021	0.2370	-0.09	0.932
C	1.840	0.920	0.2370	3.88	0.001
D	1.679	0.839	0.2370	3.54	0.003
E	-6.178	-3.089	0.2370	-13.03	0.000
A*B	0.301	0.151	0.2370	0.64	0.534
A*C	-0.915	-0.457	0.2370	-1.93	0.071
A*D	-0.391	-0.196	0.2370	-0.83	0.421
A*E	1.195	0.598	0.2370	2.52	0.023
B*C	0.555	0.278	0.2370	1.17	0.259
B*D	-0.609	-0.304	0.2370	-1.28	0.217
B*E	-0.593	-0.296	0.2370	-1.25	0.229
C*D	0.430	0.215	0.2370	0.91	0.378
C*E	-1.199	-0.599	0.2370	-2.53	0.022
D*E	-0.905	-0.453	0.2370	-1.91	0.074



The model is

$$\hat{y} = 14.2546 + 0.829x_1 + 0.920x_3 + 0.839x_4 - 3.809x_5 + 0.598x_1x_5 - 0.599x_3x_5$$

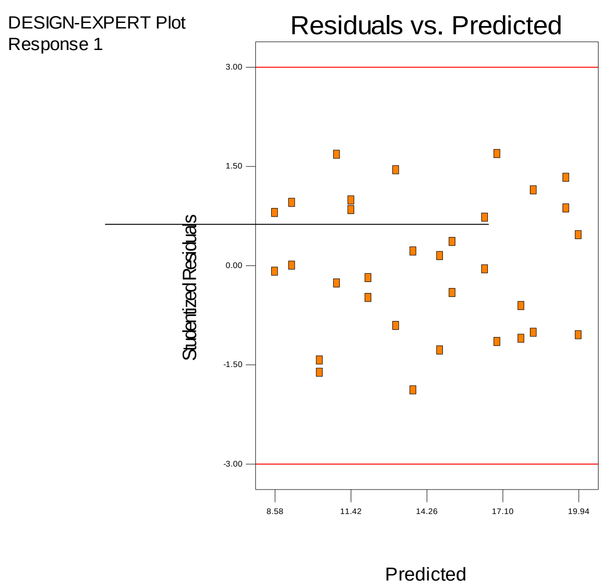
b)

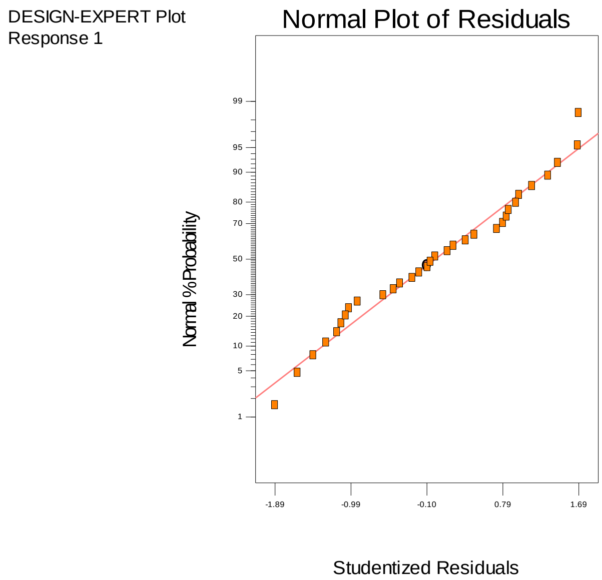
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	399.85	6	66.64	31.03	< 0.0001
A	*22.01*	*1*	*22.01*	*10.25*	*0.0037*
C	*27.08*	*1*	*27.08*	*12.61*	*0.0016*
D	*22.55*	*1*	*22.55*	*10.50*	*0.0034*
E	*305.29*	*1*	*305.29*	*142.16*	*< 0.0001*
AE	*11.42*	*1*	*11.42*	*5.32*	*0.0297*
CE	*11.50*	*1*	*11.50*	*5.35*	*0.0292*
Residual	53.69	25	2.15		
Lack of Fit	*24.93*	*9*	*2.77*	*1.54*	*0.2157*
Pure Error	*28.76*	*16*	*1.80*		
Cor Total	453.54	31			

The model is significant with significant main effects and two-factor interactions.

c) The residual plots do not show any violations of the assumptions.





d) The actual factor levels are not provided so only the model in the coded variables can be presented

$$\hat{y} = 14.2546 - 0.829x_1 + 0.920x_3 + 0.839x_4 - 3.809x_5 + 0.598x_1x_5 - 0.599x_3x_5$$

e) Use the t-test to test individual effect as shown below

Term	Effect	Coef	SE Coef	T	P
Constant		14.256	0.2370	60.15	0.000
A	1.659	0.829	0.2370	3.50	0.003
B	-0.041	-0.021	0.2370	-0.09	0.932
C	1.840	0.920	0.2370	3.88	0.001
D	1.679	0.839	0.2370	3.54	0.003
E	-6.178	-3.089	0.2370	-13.03	0.000
A*B	0.301	0.151	0.2370	0.64	0.534
A*C	-0.915	-0.457	0.2370	-1.93	0.071
A*D	-0.391	-0.196	0.2370	-0.83	0.421
A*E	1.195	0.598	0.2370	2.52	0.023
B*C	0.555	0.278	0.2370	1.17	0.259
B*D	-0.609	-0.304	0.2370	-1.28	0.217
B*E	-0.593	-0.296	0.2370	-1.25	0.229
C*D	0.430	0.215	0.2370	0.91	0.378
C*E	-1.199	-0.599	0.2370	-2.53	0.022
D*E	-0.905	-0.453	0.2370	-1.91	0.074

At $\alpha = 0.05$, the t-test provides the same result as using normal probability plot in part (a).

14-50 a) The design generators are I = ACE and I = BDE. This is verified by looking at the following table. The contrast for E is calculated using E = AC and the contrast for D is calculated using D = BE.

A	B	C	D	E	reponse
-1	-1	-1	-1	1	23.2
1	1	-1	-1	-1	15.5
1	-1	-1	1	-1	16.9
-1	1	1	-1	-1	16.2
-1	-1	1	1	-1	23.8
1	-1	1	-1	1	23.4
-1	1	-1	1	1	16.8
1	1	1	1	1	18.1

b) Design Generator: D = BE, E = AC
Defining Relation: I = ACE = BDE =ABCDE

Aliases
A=CE=BCDE=ABDE
B=DE=ACDE=ABCE
C=AE=ABDE=BCDE
D=BE=ABCE=ACDE
E=AC=BD=ABCD

c) Estimated Effects and Coefficients for response (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		19.238	0.7871	24.44	0.002
A	-1.525	-0.762	0.7871	-0.97	0.435
B	-5.175	-2.587	0.7871	-3.29	0.081
C	2.275	1.138	0.7871	1.45	0.285
D	-0.675	-0.337	0.7871	-0.43	0.710
E	2.275	1.137	0.7871	1.45	0.285

d) Estimated Effects and Coefficients for response (coded units)

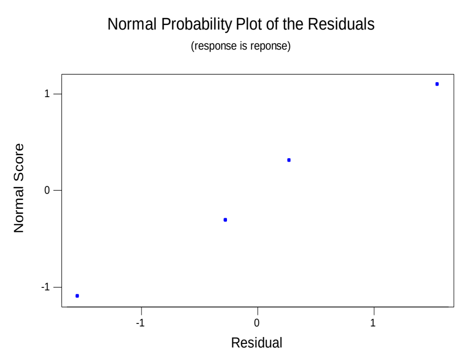
Term	Effect	Coef	SE Coef	T	P
Constant		19.238	1.138	16.91	0.038
A	-1.525	-0.762	1.138	-0.67	0.624
B	-5.175	-2.587	1.138	-2.27	0.264
C	2.275	1.138	1.138	1.00	0.500
D	-0.675	-0.337	1.138	-0.30	0.816
A*B	1.825	0.913	1.138	0.80	0.570
A*D	-1.275	-0.638	1.138	-0.56	0.675

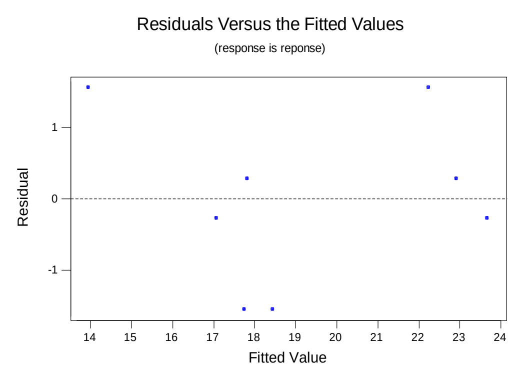
Analysis of Variance for response (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	69.475	69.475	17.369	1.68	0.517
2-Way Interactions	2	9.913	9.913	4.956	0.48	0.715
Residual Error	1	10.351	10.351	10.351		
Total	7	89.739				

Interactions AD and AB are not significant in the model, and therefore may be used as error.

e) The normal probability plot and the plot of the residuals versus fitted values are satisfactory.

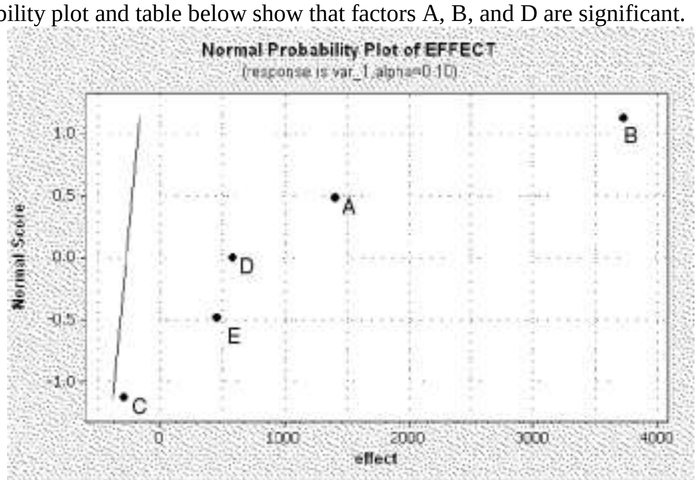




14-51 Generator E = ABCD for 2^{5-1}, Resolution V

A	B	C	D	E	var_1
-1	-1	-1	-1	1	800
1	-1	-1	-1	-1	900
-1	1	-1	-1	-1	3400
1	1	-1	-1	1	6200
-1	-1	1	-1	-1	600
1	-1	1	-1	1	1200
-1	1	1	-1	1	2500
1	1	1	-1	-1	5300
-1	-1	-1	1	-1	1000
1	-1	-1	1	1	1500
-1	1	-1	1	1	4500
1	1	-1	1	-1	6100
-1	-1	1	1	1	1500
1	-1	1	1	-1	800
-1	1	1	1	-1	3300
1	1	1	1	1	6800

The normal probability plot and table below show that factors A, B, and D are significant.




```
Estimated Effects and Coefficients for var_1 (coded units)

Term          Effect       Coef     SE Coef        T       P
Constant                 2900.0      222.6     13.03   0.000
A            1400         700.0      222.6      3.15   0.010
B            3725        1862.5      222.6      8.37   0.000
C            -300        -150.0      222.6     -0.67   0.516
D             575         287.5      222.6      1.29   0.225
E             450         225.0      222.6      1.01   0.336
Analysis of Variance for var_1,
Source           DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects      5    65835000   65835000   13167000  16.61  0.217
Error            10     7925000    7925000     792500
Total            15    73760000
```

Factors A, B and D are significant. In these factors, the design is a 2^2 with two replicates.

14-52 a) 2_{III}^{6-3}

```
Alias Structure

I + ABD + ACE + BCF + DEF + ABEF + ACDF + BCDE

A + BD + CE
B + AD + CF
C + AE + BF
D + AB + EF
E + AC + DF
F + BC + DE
AF + BE + CD
```

b) 2_{IV}^{8-4}

```
Alias Structure

I + ABCG + ABDH + ABEF + ACDF + ACEH + ADEG + AFGH + BCDE + BCFH + BDFG + BEGH
+ CDGH + CEFG + DEFH

A
B
C
D
E
F
G
H
AB + CG + DH + EF
AC + BG + DF + EH
AD + BH + CF + EG
AE + BF + CH + DG
AF + BE + CD + GH
AG + BC + DE + FH
AH + BD + CE + FG
```

14-53 a) Because factors A, B, C, and E form a word in the complete defining relation, it can be verified that the resulting design is two replicates of a 2^{4-1} fractional factorial.

b) Because factors A, B, C, and E form a word in the complete defining relation, it can be verified that the resulting design is two replicates of a 2^{4-1} fractional factorial. This is different than the design that results when C and E are dropped from the 2^{6-2}. When C and E are dropped, the result is a full factorial because the factors ABDF do not form a word in the complete defining relation.

14-54 a) Suppose A = x_1, B = x_2, C = x_3, D = x_4, E = x_5, F = x_6, G = x_7, H = x_8.

Generators are computer software defaults.

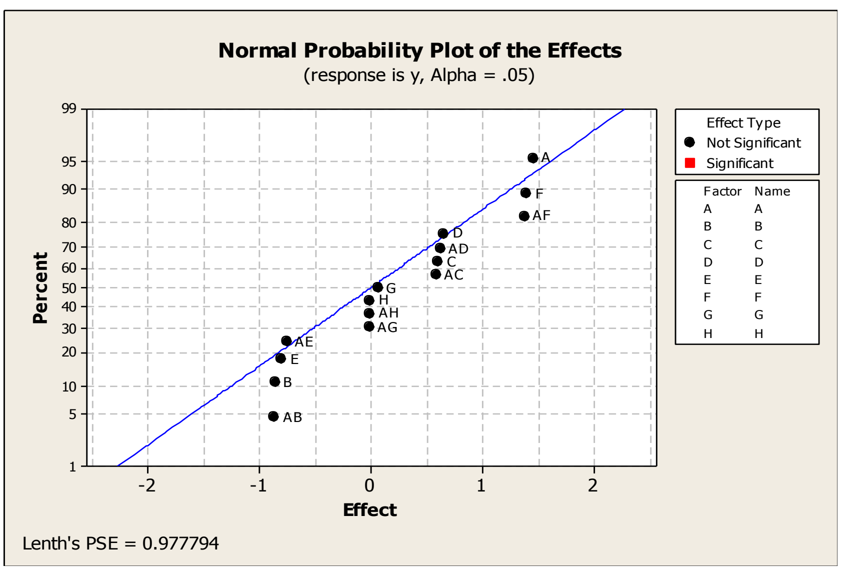
```
Design Generators: E = BCD, F = ACD, G = ABC, H = ABD


Alias Structure (up to order 4)

I + ABCG + ABDH + ABEF + ACDF + ACEH + ADEG + AFGH + BCDE + BCFH + BDFG + BEGH
     + CDGH + CEFG + DEFH

A + BCG + BDH + BEF + CDF + CEH + DEG + FGH
B + ACG + ADH + AEF + CDE + CFH + DFG + EGH
C + ABG + ADF + AEH + BDE + BFH + DGH + EFG
D + ABH + ACF + AEG + BCE + BFG + CGH + EFH
```

```
E + ABF + ACH + ADG + BCD + BGH + CFG + DFH
F + ABE + ACD + AGH + BCH + BDG + CEG + DEH
G + ABC + ADE + AFH + BDF + BEH + CDH + CEF
H + ABD + ACE + AFG + BCF + BEG + CDG + DEF
AB + CG + DH + EF + ACDE + ACFH + ADFG + AEGH + BCDF + BCEH + BDEG + BFGH
AC + BG + DF + EH + ABDE + ABFH + ADGH + AEFG + BCDH + BCEF + CDEG + CFGH
AD + BH + CF + EG + ABCE + ABFG + ACGH + AEFH + BCDG + BDEF + CDEH + DFGH
AE + BF + CH + DG + ABCD + ABGH + ACFG + ADFH + BCEG + BDEH + CDEF + EFGH
AF + BE + CD + GH + ABCH + ABDG + ACEG + ADEH + BCFG + BDFH + CEFH + DEFG
AG + BC + DE + FH + ABDF + ABEH + ACDH + ACEF + BDGH + BEFG + CDFG + CEGH
AH + BD + CE + FG + ABCF + ABEG + ACDG + ADEF + BCGH + BEFH + CDFH + DEGH
```

b) Estimated Effects and Coefficients for y (coded units)

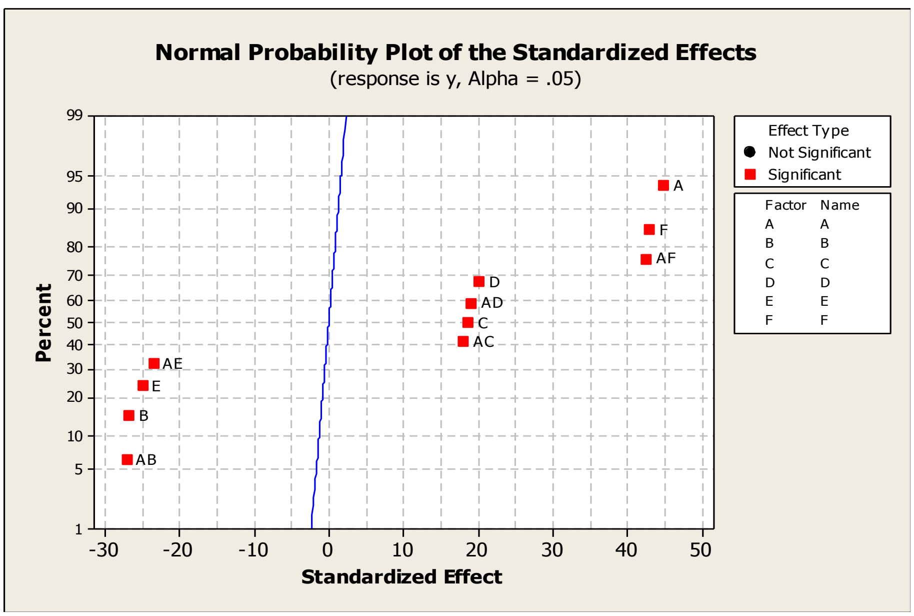
Term	Effect	Coef
Constant		0.7786
A	1.4497	0.7249
B	-0.8624	-0.4312
C	0.6034	0.3017
D	0.6519	0.3259
E	-0.8052	-0.4026
F	1.3864	0.6932
G	0.0591	0.0296
H	-0.0129	-0.0064
A*B	-0.8708	-0.4354
A*C	0.5811	0.2906
A*D	0.6186	0.3093
A*E	-0.7566	-0.3783
A*F	1.3718	0.6859
A*G	-0.0176	-0.0088
A*H	-0.0137	-0.0068

c) The normal probability plot of the effects follows.



From the effects table and the normal probability plot effects G, H, AG, and AH are smaller than the others. If these are used to estimate error the following estimates and normal plot are obtained.

```
Estimated Effects and Coefficients for y (coded units)

Term        Effect     Coef  SE Coef       T      P
Constant             0.7786  0.01612   48.31  0.000
A           1.4497   0.7249  0.01612   44.98  0.000
B          -0.8624  -0.4312  0.01612  -26.75  0.000
C           0.6034   0.3017  0.01612   18.72  0.000
D           0.6519   0.3259  0.01612   20.22  0.000
E          -0.8052  -0.4026  0.01612  -24.98  0.000
F           1.3864   0.6932  0.01612   43.01  0.000
A*B        -0.8708  -0.4354  0.01612  -27.02  0.000
A*C         0.5811   0.2906  0.01612   18.03  0.000
A*D         0.6186   0.3093  0.01612   19.19  0.000
A*E        -0.7566  -0.3783  0.01612  -23.47  0.000
A*F         1.3718   0.6859  0.01612   42.56  0.000


S = 0.0644648   R-Sq = 99.96%   R-Sq(adj) = 99.85%

Analysis of Variance for y (coded units)

Source               DF   Seq SS   Adj SS   Adj MS       F      P
Main Effects          6  24.8193  24.8193  4.13654  995.39  0.000
2-Way Interactions    5  15.7318  15.7318  3.14635  757.11  0.000
Residual Error        4   0.0166   0.0166  0.00416
Total                15  40.5676
```



With this estimate of error, the remaining effects are all significant. Also, any interpretations of these effects need to consider the aliases from the alias structure shown previously.

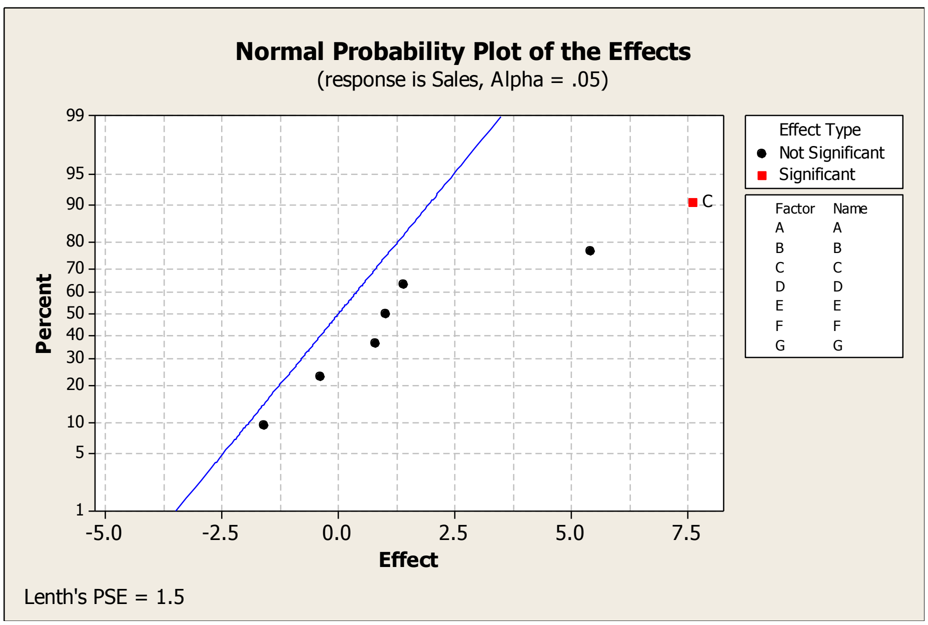
14-55 a)

```
Alias Structure (up to order 3)
I + A*B*D + A*C*E + A*F*G + B*C*F + B*E*G + C*D*G + D*E*F
A + B*D + C*E + F*G + B*C*G + B*E*F + C*D*F + D*E*G
B + A*D + C*F + E*G + A*C*G + A*E*F + C*D*E + D*F*G
C + A*E + B*F + D*G + A*B*G + A*D*F + B*D*E + E*F*G
D + A*B + C*G + E*F + A*C*F + A*E*G + B*C*E + B*F*G
E + A*C + B*G + D*F + A*B*F + A*D*G + B*C*D + C*F*G
F + A*G + B*C + D*E + A*B*E + A*C*D + B*D*G + C*E*G
G + A*F + B*E + C*D + A*B*C + A*D*E + B*D*F + C*E*F
```

b) Factorial Fit: Sales versus A, B, C, D, E, F, G

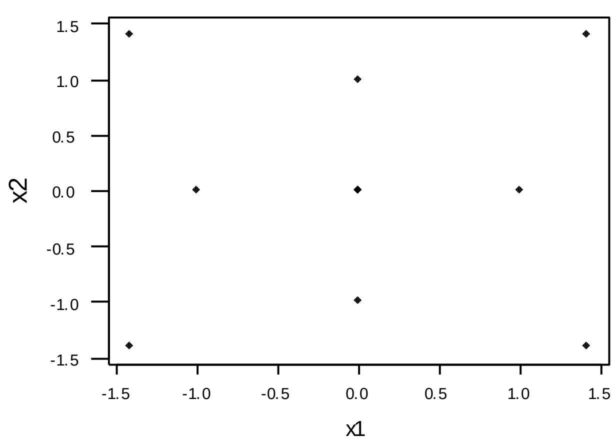
```
Estimated Effects and Coefficients for Sales (coded units)

Term        Effect     Coef
Constant            15.0000
A           5.4000   2.7000
B          -0.4000  -0.2000
C           7.6000   3.8000
D          -1.6000  -0.8000
E           1.4000   0.7000
F           1.0000   0.5000
G           0.8000   0.4000
```

c) The plot indicates that only Factor C is a significant effect, but one might also consider the effect of A as sufficiently distant from the line to be considered significant..



Section 14-8

14-56 a)

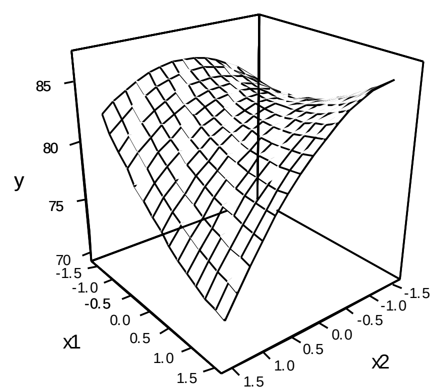


b)

```
Estimated Regression Coefficients for y
Term            Coef       StDev          T       P
Constant       82.024      0.5622    145.905  0.000
x1             -1.115      0.4397     -2.536  0.044
x2             -2.408      0.4397     -5.475  0.002
x1*x1           0.861      0.7343      1.172  0.286
x2*x2          -1.590      0.7342     -2.165  0.074
x1*x2          -1.801      0.3477     -5.178  0.002
S = 1.390       R-Sq = 92.0%     R-Sq(adj) = 85.3%

Analysis of Variance for y
Source          DF    Seq SS    Adj SS    Adj MS     F       P
Regression       5   132.837   132.837   26.5674  13.74  0.003
  Linear         2    70.393    70.391   35.1957  18.21  0.003
  Square         2    10.602    10.610    5.3048   2.74  0.142
  Interaction    1    51.842    51.842   51.8425  26.82  0.002
Residual Error   6    11.600    11.600    1.9333
  Lack-of-Fit    3    10.052    10.052    3.3507   6.50  0.079
  Pure Error     3     1.548     1.548    0.5158
Total           11   144.437
```

The second order model appears to be significant for the interaction term (p = 0.002). However, the square terms are not significant (p = 0.142).

c)

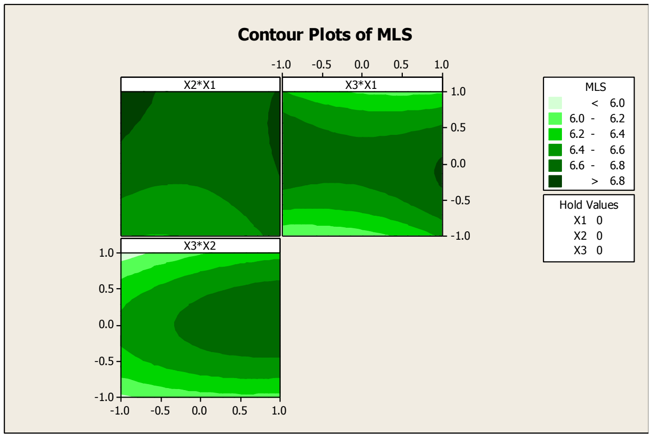


There appears to be a saddle point in the experimental region. The yield increases as x_1 is decreased and x_2 is near the zero level.

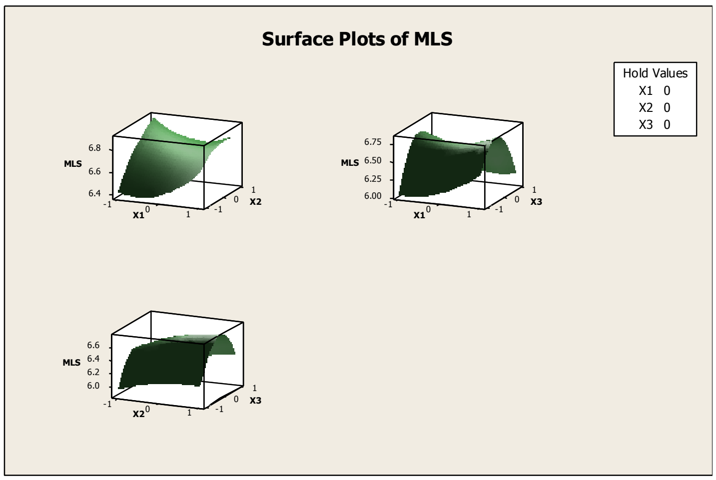
14-57 a)

$$\hat{y} = 6.65821 + 0.04201x_1 + 0.15468x_2 + 0.02895x_3 + 0.11452x_1^2 - 0.07433x_2^2$$
$$- 0.51248x_3^2 - 0.08453x_1x_2 - 0.15555x_1x_3 + 0.06693x_2x_3$$

b) Contour Plots

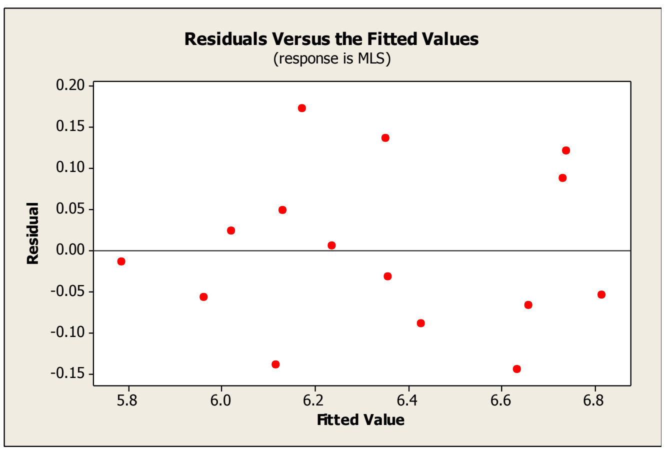


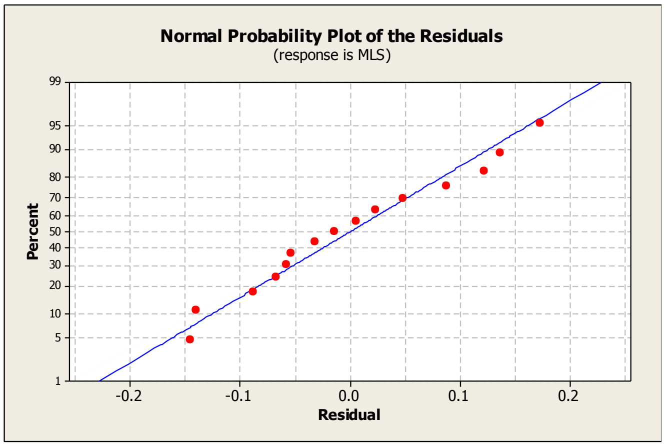
Response surface plots



There is curvature from the second-order effects.

c) The residual plots appear reasonable.





d) Adding additional center points would be a good idea to improve the estimates of the coefficients as well as to allow an independent estimate of error to be obtained.

14-58 Move 1.5 units in the direction of x_1 for every –0.8 unit in the direction of x_2. Thus, the path of steepest ascent passes through the point (0, 0) and has a slope –0.8/1.5 = –0.533.

14-59 a) $20 + 5x_1 + 2x_2 > 12$ $\quad$ $25 + 3x_1 + 4x_2 < 27.50$

$x_2 > -\frac{5}{2}x_1 - 4$ $\quad$ $x_2 < -0.75x_1 + 0.625$

The feasible region is between these two lines, which can be shown graphically on the x_1-x_2 plane.

b) Operating the process with $x_1 = 1.5$ and $x_2 = -1.5$ results in y_1 and y_2 comfortably within the feasible region.

14-60 a) A central composite design has been used but it is not rotatable.

b)

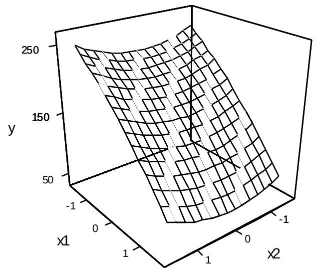
Term	Coef	StDev	T	P
Constant	150.04	7.821	19.184	0.000
x1	-58.47	5.384	-10.861	0.000
x2	3.35	5.384	0.623	0.556
x1*x1	-6.53	5.693	-1.147	0.295
x2*x2	10.58	5.693	1.859	0.112
x1*x2	0.50	7.848	0.064	0.951

S = 15.70 R-Sq = 95.4% R-Sq(adj) = 91.6%

Analysis of Variance for y

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	5	30688.7	30688.7	6137.7	24.91	0.001
Linear	2	29155.4	29155.4	14577.7	59.17	0.000
Square	2	1532.3	1532.3	766.1	3.11	0.118
Interaction	1	1.0	1.0	1.0	0.00	0.951
Residual Error	6	1478.2	1478.2	246.4		
Lack-of-Fit	3	4.2	4.2	1.4	0.00	1.000
Pure Error	3	1474.0	1474.0	491.3		
Total	11	32166.9				

The linear terms appear to be significant ($p = 0.001$) while both the square terms and interaction terms are insignificant ($p = 0.118$ and $p = 0.951$, respectively). Because x_1 is the only significant factor, to minimize *ash* increase the value of x_1.



14-61 a) Response Surface Regression

```
Estimated Regression Coefficients for y

Term        Coef  SE Coef       T      P
Constant  327.62    38.76   8.453  0.000
x3        131.47    17.94   7.328  0.000
x2        109.43    17.94   6.099  0.000
x1        177.00    17.94   9.866  0.000
x3*x3     -29.06    31.08  -0.935  0.363
x2*x2     -22.38    31.08  -0.720  0.481
x1*x1      32.01    31.08   1.030  0.317
x3*x2      43.58    21.97   1.983  0.064
x3*x1      75.47    21.97   3.435  0.003
x2*x1      66.03    21.97   3.005  0.008

S = 76.12   R-Sq = 92.7%   R-Sq(adj) = 88.8%

Analysis of Variance for y

Source          DF   Seq SS   Adj SS  Adj MS      F      P
Regression       9  1248237  1248237  138693  23.94  0.000
  Linear         3  1090558  1090558  363519  62.74  0.000
  Square         3    14219    14219    4740   0.82  0.502
  Interaction    3   143461   143461   47820   8.25  0.001
Residual Error  17    98498    98498    5794
Total           26  1346735
Reduced model:

Term        Coef  SE Coef       T      P
Constant  314.67    15.46  20.354  0.000
x3        131.47    18.93   6.943  0.000
x2        109.43    18.93   5.779  0.000
x1        177.00    18.93   9.348  0.000
x3*x1      75.47    23.19   3.255  0.004
x2*x1      66.03    23.19   2.847  0.010

S = 80.33   R-Sq = 89.9%   R-Sq(adj) = 87.5%
```

The quadratic model for y_1 is

$$y_1 = 314.67 + 177.00x_1 + 109.43x_2 + 131.47x_3 + 66.03x_1x_2 + 75.47x_1x_3$$

b) Response Surface Regression

```
Estimated Regression Coefficients for y2
```

```
Term        Coef  SE Coef      T      P
Constant   48.00    7.808  6.147  0.000
x3         29.19    9.563  3.052  0.006
x2         15.32    9.563  1.602  0.123
x1         11.53    9.563  1.205  0.240

S = 40.57   R-Sq = 36.7%   R-Sq(adj) = 28.4%

Analysis of Variance for y2

Source          DF   Seq SS   Adj SS   Adj MS     F      P
Regression       3  21957.3  21957.3  7319.09  4.45  0.013
  Linear         3  21957.3  21957.3  7319.09  4.45  0.013
Residual Error  23  37863.6  37863.6  1646.24
Total           26  59820.9
```

The linear model for y_2 is given by

$$y_2 = 48.00 + 29.19x_3$$

c) The equations for y_1 and y_2 are used to determine values for the x's. Given values for x_1 and x_2, a value for x_3 can be solved to set y_1 to a target. Each x_i should range from -1 to 1 to stay within the experimental region for the models. The standard deviation is minimized with the smallest feasible value for x_3. When x_3 = -1 at least one of x_1 and x_2 must exceed 1 in order to set y_1 = 500. Therefore, x_3 is greater than -1. To keep a solution within the feasible region, we set x_1 = 1 and x_2 = 1. With these values the value for x_3 that sets y_1 = 500 is x_3 = -0.808 and this minimizes the standard deviation.

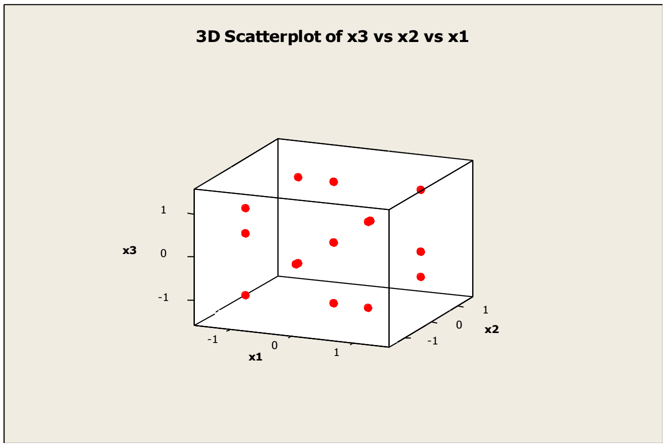
14-62 a) $y = 15 + 1.2x_1 - 2.1x_2 + 1.8x_3 - 0.6x_4$

The direction of steepest ascent is in the direction of the vector (1.2, –2.1, 1.8, –0.6)

b) The point along the path of steepest descent that is 3 units away from (0,0,0,0) is given by:

$$\frac{3\cdot(1.2,\quad -2.1,\quad 1.8,\quad -0.6)}{\sqrt{1.2^2+(-2.1)^2+(1.8)^2+(-0.6)^2}} = \frac{3\cdot(1.2,\quad -2.1,\quad 1.8,\quad -0.6)}{3.074} = (1.17,\quad -2.05,\quad 1.76,\quad -0.59)$$

14-63 a) A plot of the *coded* data follows.



b) Computer results are shown below for the first-order and second-order models for the *coded* data. Note that the coded data are computed after a natural logarithm transform is applied to the original data. That is, the center point for the variable *speed* is not (117+36)/2 = 76.5, but instead [ln(117) + ln(36)]/2 = 4.1728 and exp(4.1728) = 65.

Response Surface Regression: y versus x1, x2, x3

The analysis was done using coded units.

Estimated Regression Coefficients for y

Term	Coef	SE Coef	T	P
Constant	3.2422	0.1120	28.955	0.000
x1	-0.2594	0.1371	-1.891	0.073
x2	1.8931	0.1371	13.804	0.000
x3	0.1963	0.1371	1.431	0.168

S = 0.5485   R-Sq = 90.7%   R-Sq(adj) = 89.4%

Analysis of Variance for y

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	3	59.0253	59.0253	19.6751	65.39	0.000
Linear	3	59.0253	59.0253	19.6751	65.39	0.000
Residual Error	20	6.0180	6.0180	0.3009		
Lack-of-Fit	11	5.7727	5.7727	0.5248	19.26	0.000
Pure Error	9	0.2453	0.2453	0.0273		
Total	23	65.0432				

Note that the lack-of-fit test is significant for the first-order model (P-value near zero) and this indicates that a second-order model should be considered.

Response Surface Regression: Surface Roughness versus x1, x2, x3

The analysis was done using coded units.

Estimated Regression Coefficients for Surface Roughness

Term	Coef	SE Coef	T	P
Constant	2.47142	0.08780	28.147	0.000
x1	-0.25937	0.04809	-5.393	0.000
x2	1.89296	0.04809	39.361	0.000
x3	0.19625	0.04809	4.081	0.001
x1*x1	0.29946	0.05553	5.393	0.000
x2*x2	0.65308	0.05553	11.760	0.000
x3*x3	0.20358	0.05553	3.666	0.003
x1*x2	-0.00612	0.06801	-0.090	0.930
x1*x3	-0.11863	0.06801	-1.744	0.103
x2*x3	0.06038	0.06801	0.888	0.390

S = 0.1924   R-Sq = 99.2%   R-Sq(adj) = 98.7%

Analysis of Variance for Surface Roughness

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	9	64.5252	64.5252	7.1695	193.74	0.000
Linear	3	59.0252	59.0252	19.6751	531.67	0.000
Square	3	5.3579	5.3579	1.7860	48.26	0.000
Interaction	3	0.1420	0.1420	0.0473	1.28	0.320
Residual Error	14	0.5181	0.5181	0.0370		
Lack-of-Fit	5	0.2728	0.2728	0.0546	2.00	0.172
Pure Error	9	0.2453	0.2453	0.0273		
Total	23	65.0432				

The linear and pure quadratic terms appear to be significant (P-value = 0 and P-value = 0) while the interaction terms are insignificant (P-value = 0.32). The lack-of-fit test is not significant and this indicates a better fit for the second-order model.

Reduced model

Response Surface Regression: Roughness versus x1, x2, x3

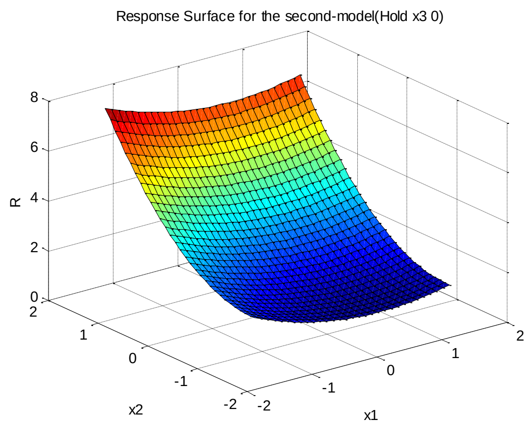
The analysis was done using coded units.

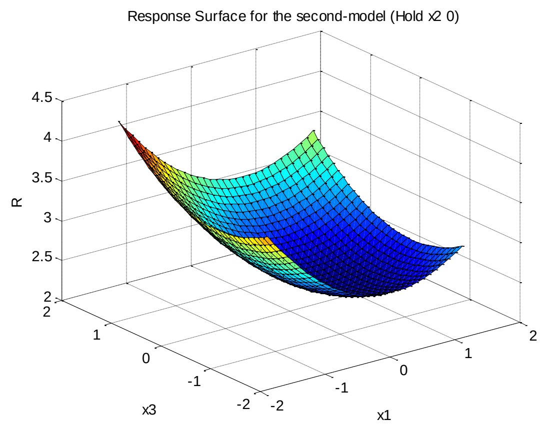
```
Estimated Regression Coefficients for Roughness

Term          Coef  SE Coef       T      P
Constant    2.4714  0.08994  27.478  0.000
x1         -0.2594  0.04926  -5.265  0.000
x2          1.8930  0.04926  38.425  0.000
x3          0.1963  0.04926   3.984  0.001
x1*x1       0.2995  0.05688   5.264  0.000
x2*x2       0.6531  0.05688  11.481  0.000
x3*x3       0.2036  0.05688   3.579  0.002

S = 0.1971   R-Sq = 99.0%   R-Sq(adj) = 98.6%
```

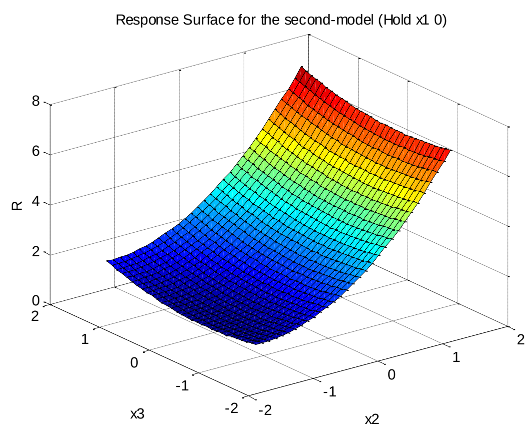
The quadratic model of the coded variable is

$$y_1 = 2.4714 - 0.2594x_1 + 1.8930x_2 + 0.1963x_3 + 0.2995x_1^2 + 0.6531x_2^2 + 0.2036x_3^2$$

c) There is curvature in the fitted surface from the second-order effects.







<u>Supplemental Exercises</u>

14-64 a) Estimated Effects and Coefficients for var_1 (coded units)

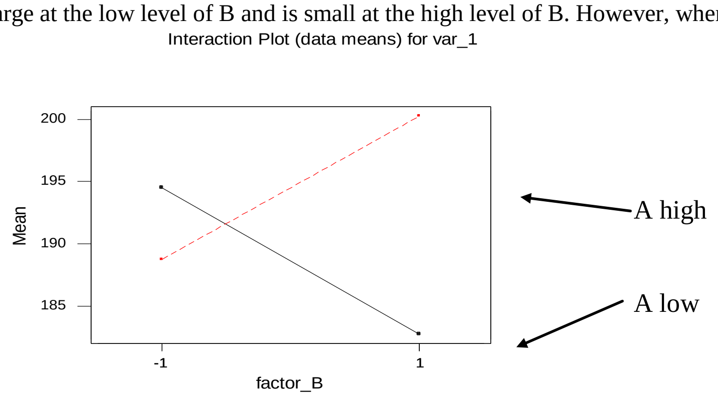
Term	Effect	Coef	SE Coef	T	P
Constant		191.563	1.158	165.49	0.000
factor_A (PH)	5.875	2.937	1.158	2.54	0.026
factor_B (CC)	-0.125	-0.062	1.158	-0.05	0.958
factor_A*factor_B	11.625	5.812	1.158	5.02	0.000

Analysis of Variance for var_1 (coded units)

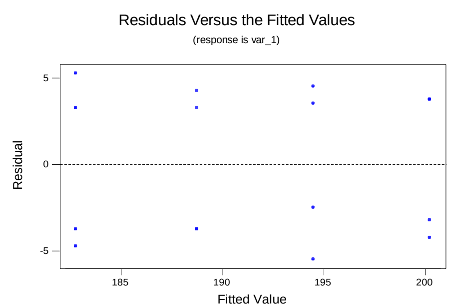
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	138.125	138.125	69.06	3.22	0.076
2-Way Interactions	1	540.562	540.562	540.56	25.22	0.000
Residual Error	12	257.250	257.250	21.44		
Pure Error	12	257.250	257.250	21.44		
Total	15	935.938				

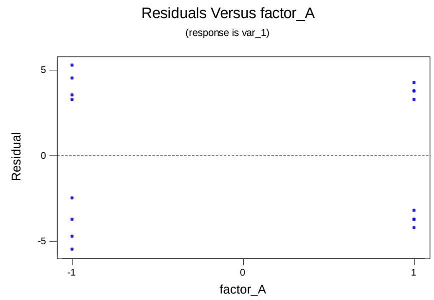
The main effect of *pH* and the interaction of *pH*Catalyst Concentration* (CC) are significant at the 0.05 level of significance. The model used is viscosity = $191.563 + 2.937x_1 - 0.062x_2 + 5.812x_{12}$

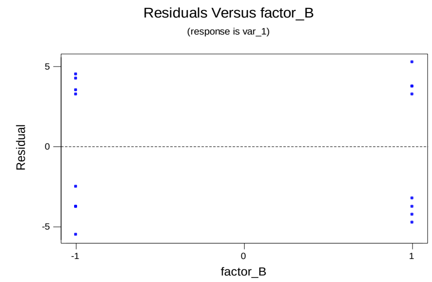
b) The interaction plot shows that there is a strong interaction. When Factor A is at its low level, the mean response is large at the low level of B and is small at the high level of B. However, when A is at its high level, the results reverse.

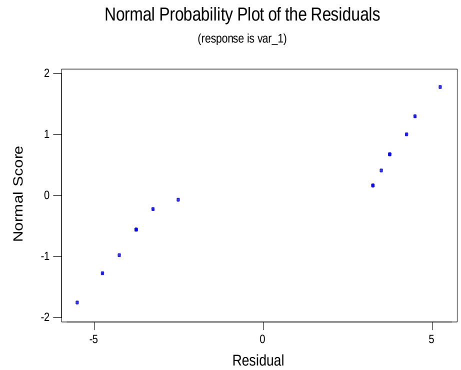


c) The plots of the residuals show that the equality of variance assumption is reasonable. However, there is a large gap in the middle of the normal probability plot. Sometimes, this can indicate that there is another variable that has an effect on the response, but which is not included in the experiment. For example, in this experiment, note that the replicates in each cell have two pairs of values that are very similar, but there is a rather large difference in the mean values of the two pairs. (Cell 1 has 189 and 192 as one pair and 198 and 199 as the other.)









14-65 a)

Factor	Type	Levels	Values
Gear Typ	fixed	3	20 24 28
Time	fixed	2	90 120

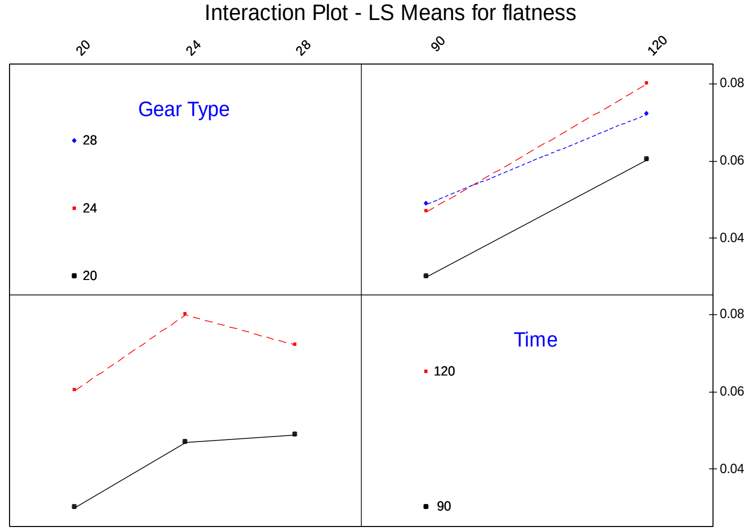
Analysis of Variance for flatness, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Gear Typ	2	0.0007591	0.0007591	0.0003796	3.90	0.082
Time	1	0.0024941	0.0024941	0.0024941	25.66	0.002
Gear Typ*Time	2	0.0000505	0.0000505	0.0000253	0.26	0.779
Error	6	0.0005833	0.0005833	0.0000972		
Total	11	0.0038870				

Term		Coef	SE Coef	T	P
Constant		0.056500	0.002846	19.85	0.000
Gear Typ					
20		-0.011125	0.004025	-2.76	0.033
24		0.007000	0.004025	1.74	0.133
Time					
90		-0.014417	0.002846	-5.07	0.002
Gear Typ*Time					
20	90	-0.000708	0.004025	-0.18	0.866
24	90	-0.002083	0.004025	-0.52	0.623

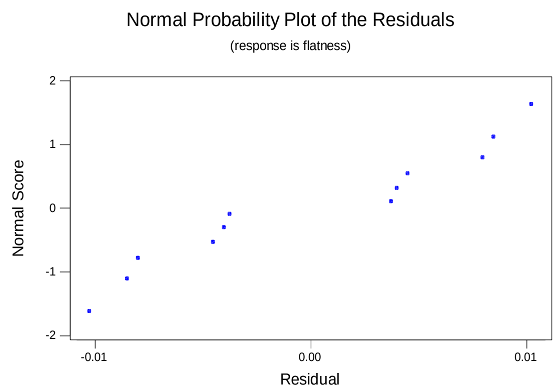
There is weak evidence that flatness distortion is different for the different gear types (p = 0.082). Gear type is significant at α = 0.1, but not at α = 0.05. Also, the gear type 20 coefficient has a p-value = 0.033. Heat-treating time affects the flatness distortion (p = 0.002). There is no evidence that factors interact (p = 0.779).

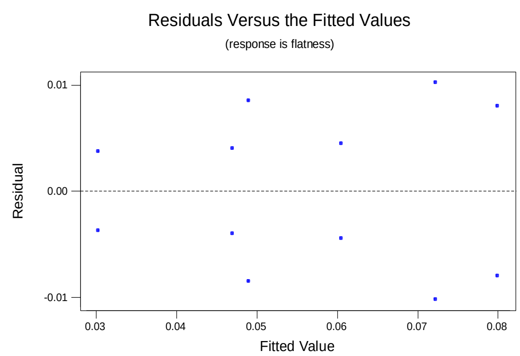
b)

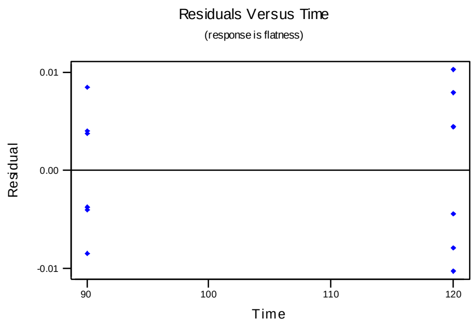


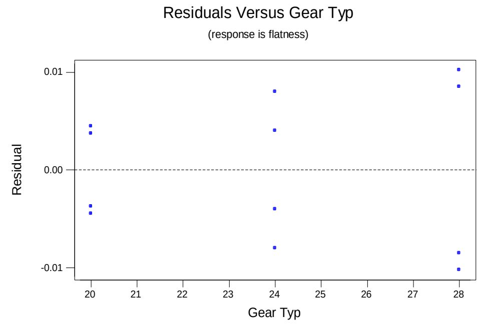
The interaction plot for the effects indicates that there is no interaction between gear type and time. The interaction plot indicates there may be some significant difference between the low and high levels of time. There is also a difference between one of the gear types and the other two.

c) The model used is $\hat{y} = 0.0565 = 0.0111x_1 - 0.01144x_2$









The residual plots are adequate. There does not appear to be any serious departure from normality or violation of the assumption of constant variance.

14-66 a)

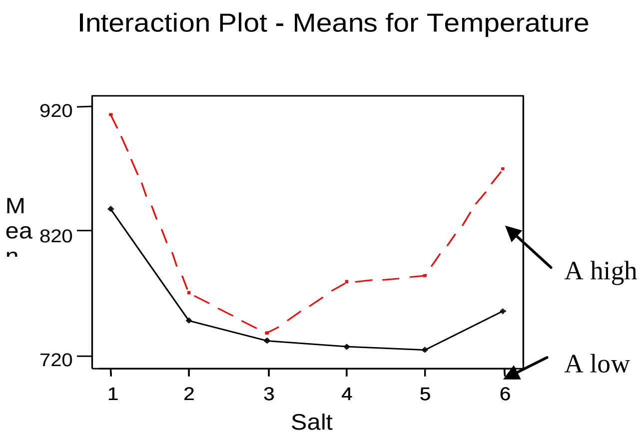
Factor	Type	Levels	Values					
Level	fixed	2	1	2				
Salt	fixed	6	1	2	3	4	5	6

Analysis of Variance for temperat

Source	DF	SS	MS	F	P
level	1	27390	27390	63.24	0.000
salt	5	86087	17217	39.75	0.000
Interaction	5	11459	2292	5.29	0.002
Error	24	10395	433		
Total	35	135332			

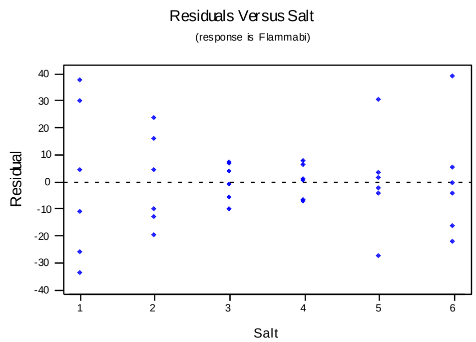
There is a significant difference between the application levels, the salts, and there is a significant interaction between the two factors.

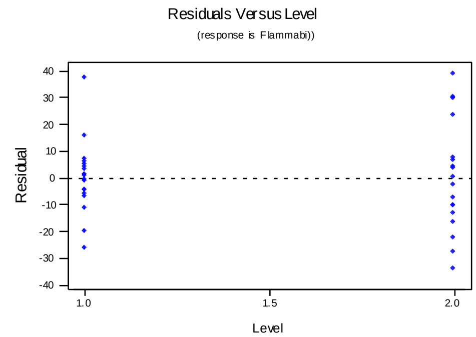
b)

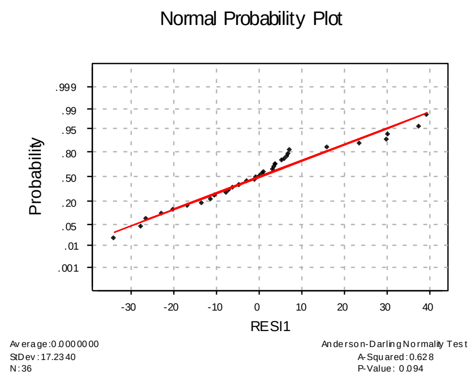


From the interaction plot, we see that the untreated salt 1 has a higher flammability average than any of the other five levels. The remaining five levels ($MgCl_2$, NaCl, $CaCO_3$, $CaCl_2$, Na_2CO_3) also seem to differ. Overall, application level 1 increases the flammability average. Also, the difference between application levels varies with the salt type and this indicates a significant interaction.

c)







The residual plots do not indicate major problems with the assumptions. There is some concern with the constant variability assumption in the plot of residuals versus salts.

14-67 a)

Factor	Type	Levels	Values
Solder B	fixed	3	75 130 260
Align Me	fixed	3	1 2 3

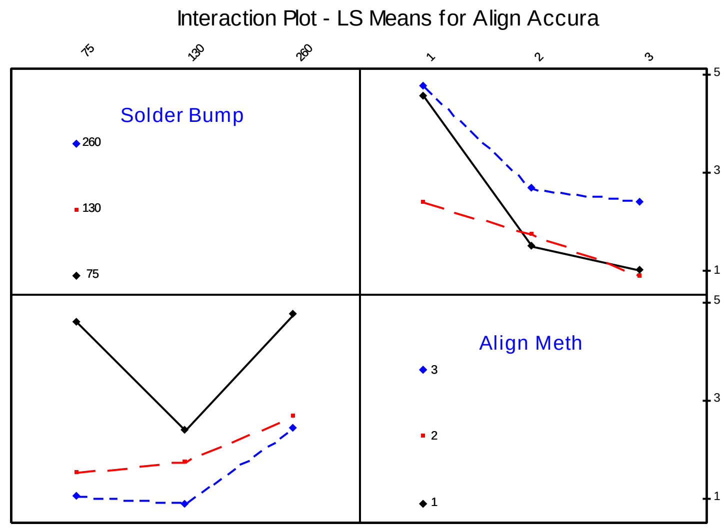
Analysis of Variance for Align Ac, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Solder B	2	7.7757	7.7757	3.8879	297.92	0.000
Align Me	2	20.1241	20.1241	10.0621	771.04	0.000
Solder B*Align Me	4	3.5001	3.5001	0.8750	67.05	0.000
Error	9	0.1174	0.1174	0.0130		
Total	17	31.5174				

Term		Coef	SE Coef	T	P
Constant		2.43611	0.02693	90.47	0.000
Solder B					
75		-0.07278	0.03808	-1.91	0.088
130		-0.76611	0.03808	-20.12	0.000
Align Me					
1		1.46389	0.03808	38.44	0.000
2		-0.46778	0.03808	-12.28	0.000
Solder B*Align Me					
75	1	0.73778	0.05385	13.70	0.000
75	2	-0.39556	0.05385	-7.35	0.000
130	1	-0.74889	0.05385	-13.91	0.000
130	2	0.53778	0.05385	9.99	0.000

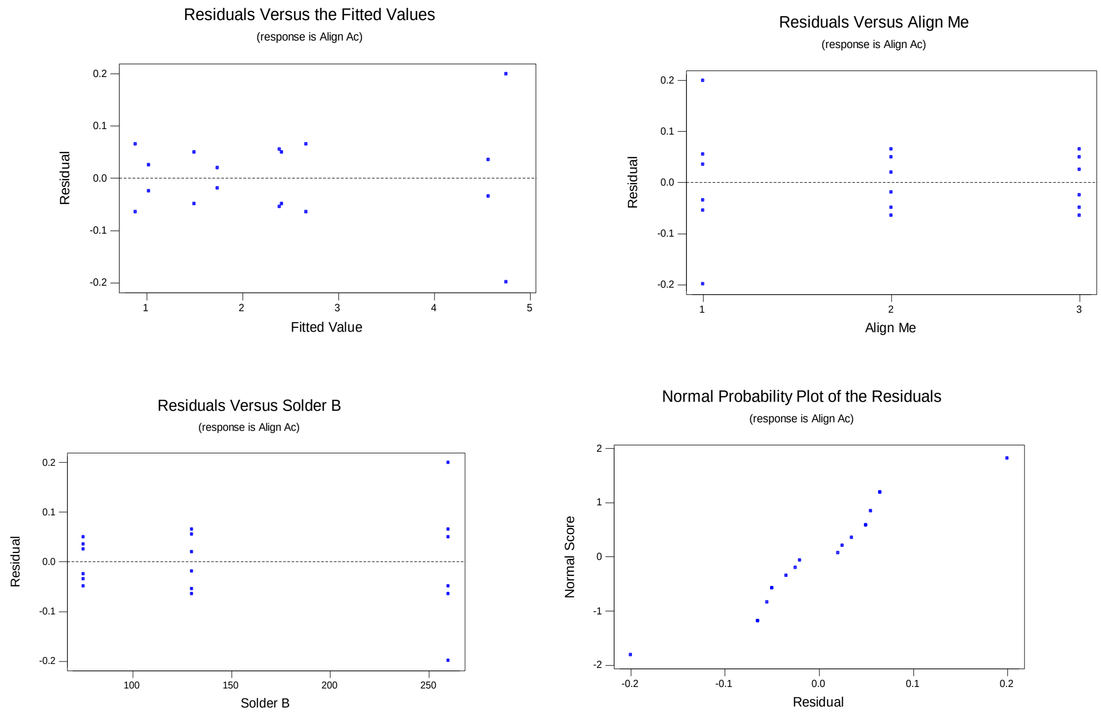
The analysis indicates that both solder size and alignment method significantly affect alignment accuracy. The interaction between solder size and alignment method is also significant in affecting alignment accuracy.

b) The lines for factor A intersect at the lower level of alignment.



Because the smaller value is preferred, to improve alignment accuracy solder size should be set at its middle level (130μm) while the alignment method used should be method 3. There is not much difference between bump size of 130 μm and 75 μm when method 3 is used.

c)



The normal probability plot does not suggest a departure from normality. The assumption of constant variance may be of concern. It appears that the variability is lower for the high level of the factor A.

14-68 a) Analysis of Variance for Tool Life

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	2549.937500	364.276786	3.70	0.0434
Error	8	788.500000	98.562500		
Corrected Total	15	3338.437500			

R-Square	Coeff Var	Root MSE	Y Mean
0.763812	11.51891	9.927865	86.18750

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	5.062500	5.062500	0.05	0.8264
B	1	473.062500	473.062500	4.80	0.0598
A*B	1	76.562500	76.562500	0.78	0.4038
C	1	540.562500	540.562500	5.48	0.0473
A*C	1	1314.062500	1314.062500	13.33	0.0065
B*C	1	14.062500	14.062500	0.14	0.7155
A*B*C	1	126.562500	126.562500	1.28	0.2900

Cutting angle and the speed-angle interaction are significant.

b) Estimated Effects and Coefficients for Tool Life

Parameter Estimates

Variable	Effect	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept		86.1875	2.48197	34.73	<.0001
speed	1.125	0.5625	2.48197	0.23	0.8264
hardness	10.875	5.4375	2.48197	2.19	0.0598
angle	11.625	5.8125	2.48197	2.34	0.0473
speed * hardness	-4.375	-2.1875	2.48197	-0.88	0.4038
speed * angle	-18.125	-9.0625	2.48197	-3.65	0.0065
hardness * angle	-1.875	-0.9375	2.48197	-0.38	0.7155
speed * hardness * angle	-5.625	-2.8125	2.48197	-1.13	0.29

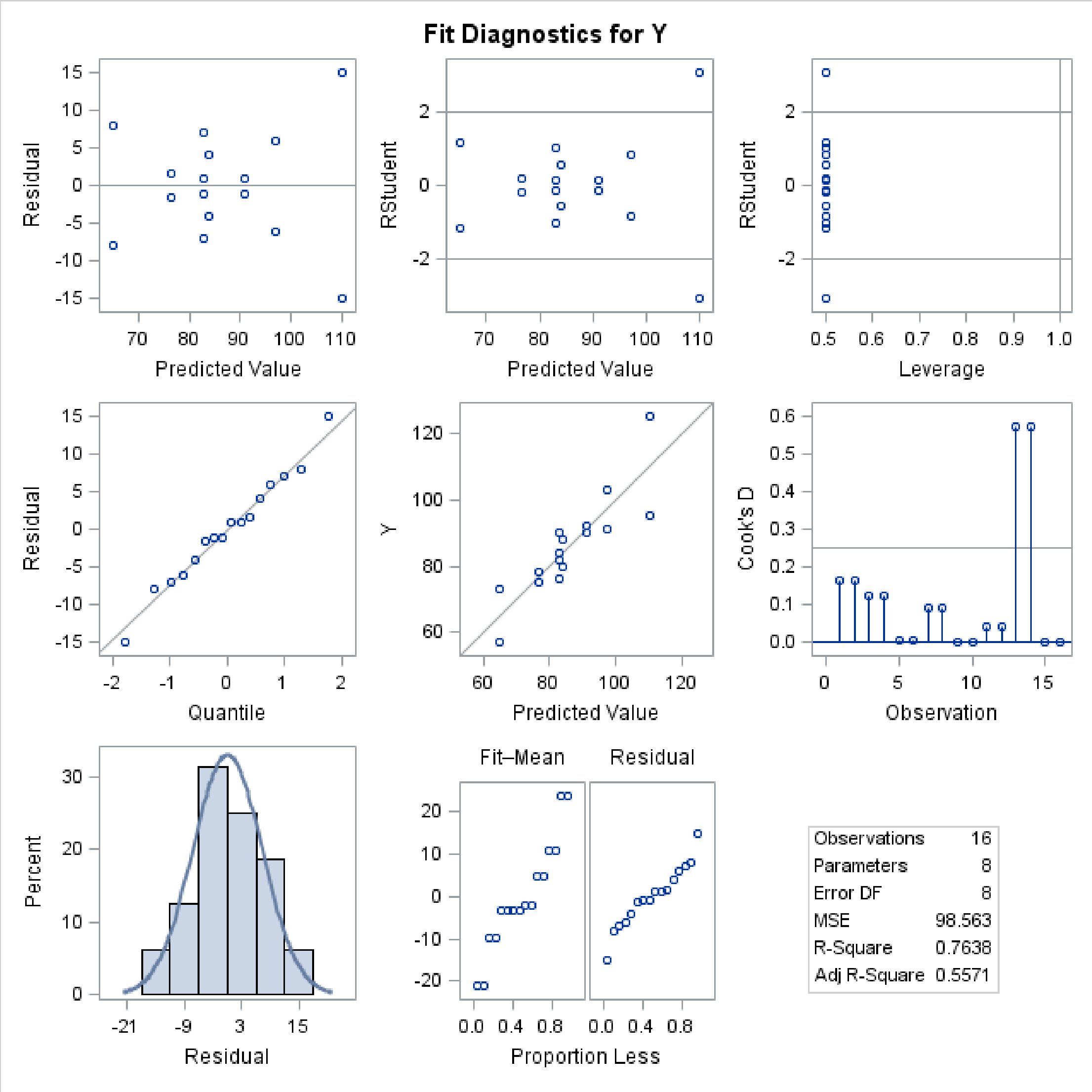
The three largest effects are B, C and AC interaction. The regression model is

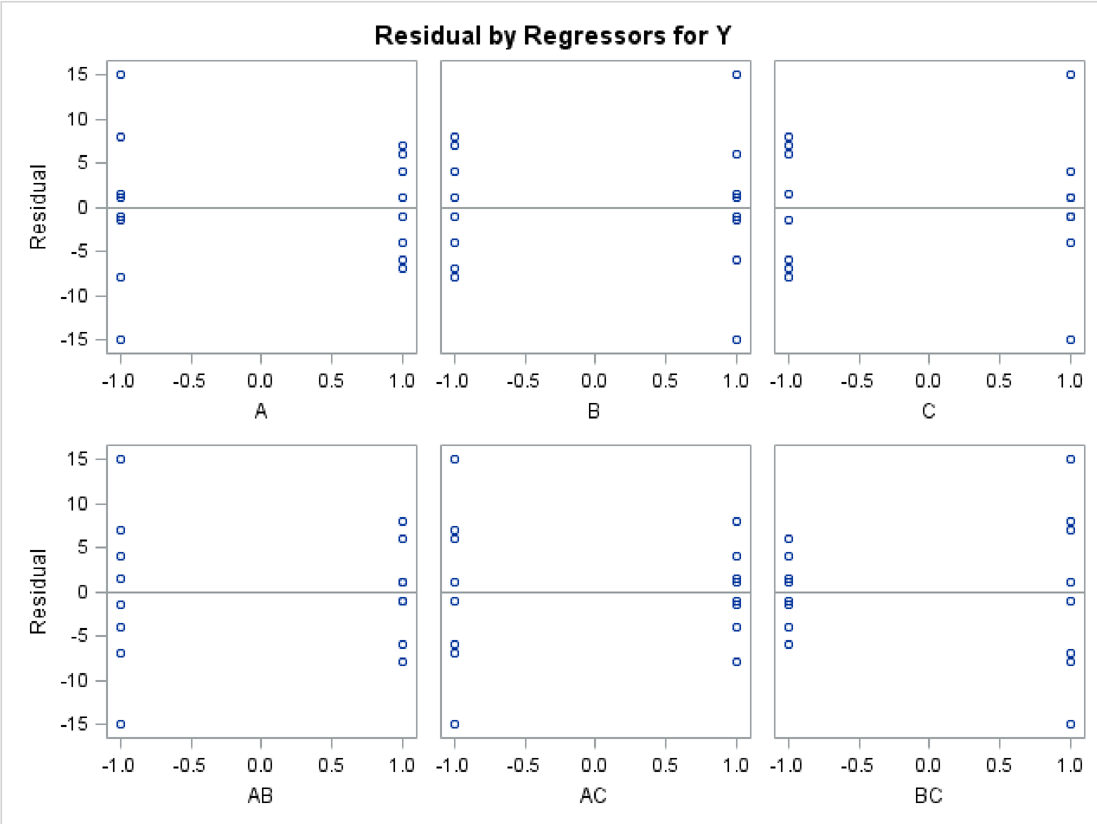
$$\hat{y} = 86.188 + \left(\frac{10.875}{2}\right)x_2 + \left(\frac{11.625}{2}\right)x_3 + \left(\frac{-18.125}{2}\right)x_1x_3$$

where x_2 represents factor B, x_3 represents factor C, and x_1x_3 represents the AC interaction. Thus,

$$\hat{y} = 86.188 + 5.438x_2 + 5.813x_3 - 9.063x_1x_3$$

c) Analysis of the residuals shows that all assumptions are reasonable.





14-69

Parameter Estimates

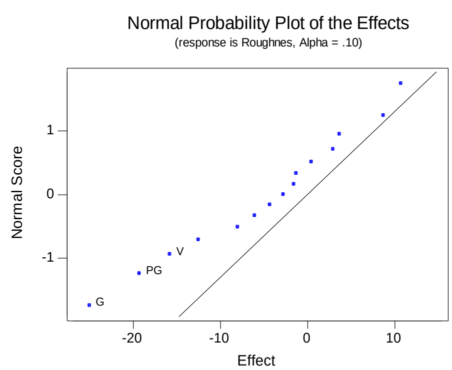
Variable	Effect	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept		154.5	0.55261	279.58	<.0001
A	16.25	8.125	0.55261	14.7	<.0001
B	-0.875	-0.4375	0.55261	-0.79	0.4394
C	10.875	5.4375	0.55261	9.84	<.0001
D	8.25	4.125	0.55261	7.46	<.0001
AB	-0.875	-0.4375	0.55261	-0.79	0.4394
AC	-0.625	-0.3125	0.55261	-0.57	0.5791
AD	9.25	4.625	0.55261	8.37	<.0001
BC	-1	-0.5	0.55261	-0.9	0.3782
BD	1.625	0.8125	0.55261	1.47	0.1597
CD	-2.375	-1.1875	0.55261	-2.15	0.0463
ABC	0.75	0.375	0.55261	0.68	0.5065
ABD	-0.125	-0.0625	0.55261	-0.11	0.9113
BCD	0	0	0.55261	0	1
ABCD	-0.5	-0.25	0.55261	-0.45	0.6567

Factors A, C, and D are significant as well as the interaction AD and CD.

14-70 a)

Term	Effect
V	-15.75
F	8.75
P	10.75
G	-25.00
V*F	3.00
V*P	-8.00
V*G	-2.75
F*P	-6.00
F*G	3.75
P*G	-19.25
V*F*P	-1.25
V*F*G	0.50
V*P*G	-1.50
F*P*G	-12.50
V*F*P*G	-4.25

b)



According to the probability plot, factors V, P, and G and, PG are possibly significant.

Estimated Effects and Coefficients for roughnes (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		102.75	2.986	34.41	0.000
V	-15.75	-7.87	2.986	-2.64	0.046
F	8.75	4.37	2.986	1.46	0.203
P	10.75	5.37	2.986	1.80	0.132
G	-25.00	-12.50	2.986	-4.19	0.009
V*F	3.00	1.50	2.986	0.50	0.637
V*P	-8.00	-4.00	2.986	-1.34	0.238
V*G	-2.75	-1.38	2.986	-0.46	0.665
F*P	-6.00	-3.00	2.986	-1.00	0.361
F*G	3.75	1.88	2.986	0.63	0.558
P*G	-19.25	-9.63	2.986	-3.22	0.023

Analysis of Variance for roughness (coded units)

Analysis of Variance for Roughness (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	4260.7	4260.7	1065.2	7.46	0.024
2-Way Interactions	6	2004.7	2004.7	334.1	2.34	0.184
Residual Error	5	713.5	713.5	142.7		
Total	15	6979.0				

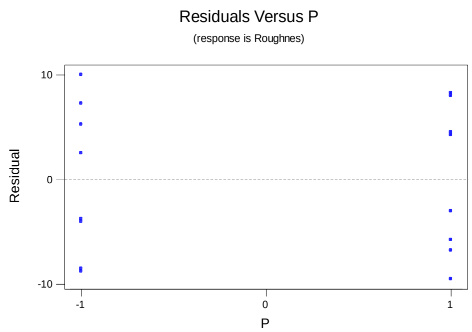
$$\hat{y} = 102.75 - 7.87x_1 + 5.37x_3 - 12.50x_4 - 9.63x_{34}$$

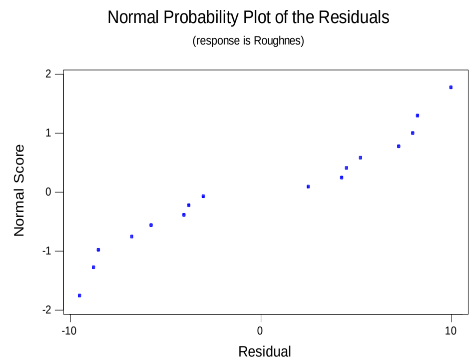
c) From the analysis, we see that water jet pressure (P), abrasive grain size (G), and jet traverse speed (V) are significant along with the interaction of water jet pressure and abrasive grain size. The model without the interaction is a reasonable model. With the interaction, there is a problem with collinearity. Without the interaction, Factor P is likely not required in the model.

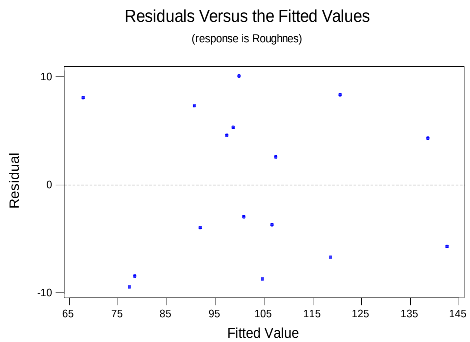
```
Analysis of Variance for Roughnes, using Adjusted SS for Tests

Source      DF     Seq SS     Adj SS     Adj MS       F       P
V            1      992.2      992.2      992.2    3.74   0.077
P            1      306.2      306.2      306.2    1.16   0.304
G            1     2500.0     2500.0     2500.0    9.43   0.010
Error       12     3180.5     3180.5      265.0
Total       15     6979.0
```

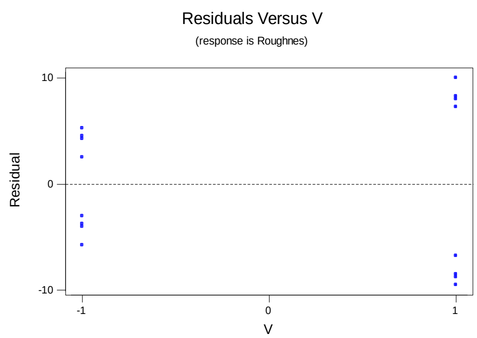
d) To minimize, abrasive grain size should be at the higher level with jet traverse speed at the lower level.

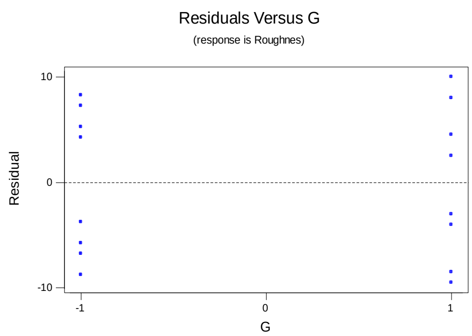
e) The residual plots appear to indicate the assumption of constant variance may not be met. The assumption of normality appears reasonable.

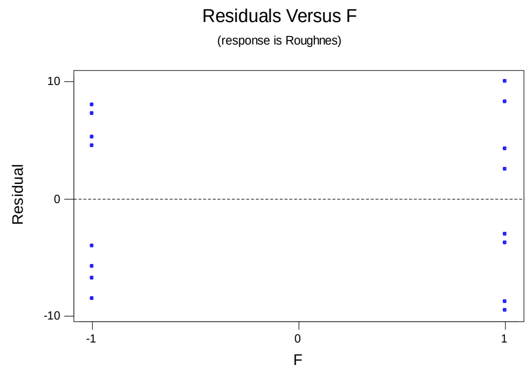












14-71 Move 2 units in the direction of x_1 for every –3.2 units in the direction of x_2. Thus, the path of steepest ascent passes through the point (0, 0) and has a slope –3.2/2 = –1.6.

14-72

(a)

$$9 + x_1 + 3x_2 > 15 \qquad 11 + 4x_1 + 2x_2 < 22$$

$$x_2 > 2 - \frac{1}{3}x_1 \qquad x_2 < 5.5 - 2x_1$$

The feasible region is between these two lines, which can be shown graphically on the x_1-x_2 plane.

(b) Operating the process with $x_1 = 0$ and $x_2 = 3$ results in y_1 and y_2 within the feasible region.

14-73 a) $y = 10 + 2.2x_1 - 1.7x_2 + 1.5x_3 - 0.8x_4$

The direction of steepest ascent is in the direction of the vector (2.2, -1.7, 1.5, -0.8).

b) The point along the path of steepest descent that is 5 units away from (0,0,0,0) is given by:

$$\frac{5*(2.2,-1.7,1.5,-0.8)}{\sqrt{2.2^2 + (-1.7)^2 + (1.5)^2 + (-0.8)^2}} = \frac{5*(2.2,-1.7,1.5,-0.8)}{3.259} = (3.38,-2.61,2.30,-1.23)$$

14-74

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	46289.88	11572.47	14.22	0.0272
Error	3	2441	813.6667		
Corrected Total	7	48730.88			

R-Square	Coeff Var	Root MSE	Y Mean
0.949909	22.26329	28.52484	128.125

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	16256.25	16256.25	19.98	0.0209
B	1	24180.25	24180.25	29.72	0.0121
AB	1	2450.25	2450.25	3.01	0.1811

Curvature	1	3403.125	3403.125	4.18	0.1334

Only main effects are significant. Interaction effects and curvature are not significant at α = 0.05.

For the curvature, since $F_0 = 4.18 < F_{0.05,1,3} = 10.13$, there is no evidence to conclude that curvature is significant at α = 0.05.

14-75 a) The generator for this fraction was I = ABCD

```
I = A*B*C*D
A = B*C*D
B = A*C*D
C = A*B*D
D = A*B*C
E = A*B*C*D*E
A*B = C*D
A*C = B*D
A*D = B*C
A*E = B*C*D*E
B*E = A*C*D*E
C*E = A*B*D*E
D*E = A*B*C*E
A*B*E = C*D*E
A*C*E = B*D*E
A*D*E = B*C*E
```

b) Estimated Effects and Coefficients for freeheig

Term	Effect	Coef	StDev Coef	T	P
Constant		7.6400	0.01901	401.97	0.000
A	0.2133	0.1067	0.01901	5.61	0.000
B	-0.1925	-0.0963	0.01901	-5.06	0.000
C	-0.0783	-0.0392	0.01901	-2.06	0.048
D	0.0625	0.0313	0.01901	1.64	0.110
E	-0.2100	-0.1050	0.01901	-5.52	0.000
A*B	-0.0008	-0.0004	0.01901	-0.02	0.983
A*C	0.0300	0.0150	0.01901	0.79	0.436
A*D	0.0058	0.0029	0.01901	0.15	0.879
A*E	0.0350	0.0175	0.01901	0.92	0.364
B*E	0.1242	0.0621	0.01901	3.27	0.003
C*E	-0.0617	-0.0308	0.01901	-1.62	0.115
D*E	0.0108	0.0054	0.01901	0.28	0.777
A*B*E	0.0308	0.0154	0.01901	0.81	0.423
A*C*E	0.0483	0.0242	0.01901	1.27	0.213
A*D*E	-0.0308	-0.0154	0.01901	-0.81	0.423

Analysis of Variance for freeheig

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	1.64052	1.64052	0.32810	18.92	0.000
2-Way Interactions	7	0.25797	0.25797	0.03685	2.13	0.069
3-Way Interactions	3	0.05085	0.05085	0.01695	0.98	0.416
Residual Error	32	0.55487	0.55487	0.01734		
Pure Error	32	0.55487	0.55487	0.01734		
Total	47	2.50420				

Based on the analysis, factors A, B, C, and E are significant. The interaction of BE is also significant.

c)

A	B	C	D	E	Range
-1	-1	-1	-1	-1	0.03
1	-1	-1	1	-1	0.30
-1	1	-1	1	-1	0.06

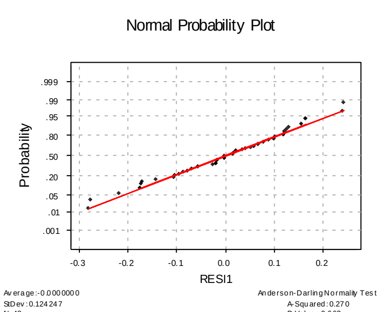
1	1	-1	-1	-1	0.19
-1	-1	1	1	-1	0.46
1	-1	1	-1	-1	0.40
-1	1	1	-1	-1	0.12
1	1	1	1	-1	0.25
-1	-1	-1	-1	1	0.06
1	-1	-1	1	1	0.44
-1	1	-1	1	1	0.06
1	1	-1	-1	1	0.19
-1	-1	1	1	1	0.12
1	-1	1	-1	1	0.13
-1	1	1	-1	1	0.07
1	1	1	1	1	0.31

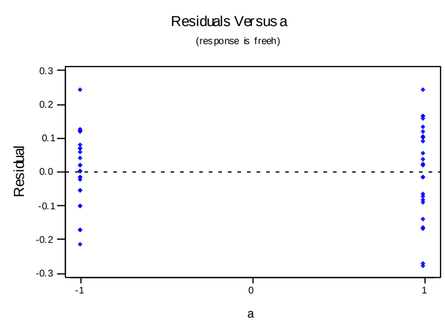
```
Estimated Effects and Coefficients for Range
Term          Effect      Coef  StDev Coef       T      P
Constant               0.19938     0.02714    7.35  0.000
A            0.15375   0.07688     0.02714    2.83  0.018
B           -0.08625  -0.04313     0.02714   -1.59  0.143
C            0.06625   0.03312     0.02714    1.22  0.250
D            0.10125   0.05062     0.02714    1.87  0.092
E           -0.05375  -0.02687     0.02714   -0.99  0.345

Analysis of Variance for Range
Source                DF      Seq SS     Adj SS     Adj MS       F       P
Main Effects           5      0.1944     0.1944    0.03889    3.30   0.051
Residual Error        10      0.1179     0.1179    0.01179
Total                 15      0.3123
```

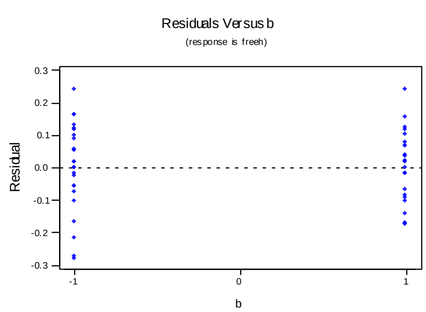
From the analysis, factor A is significant for variability in free height.

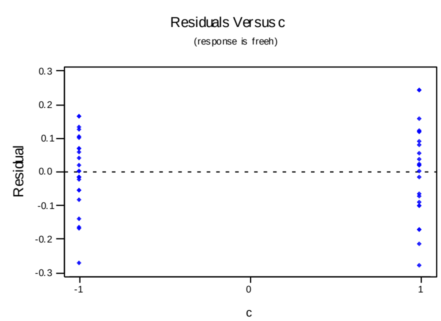
Using the model $\hat{y} = 0.19938 + 0.07688x_1$

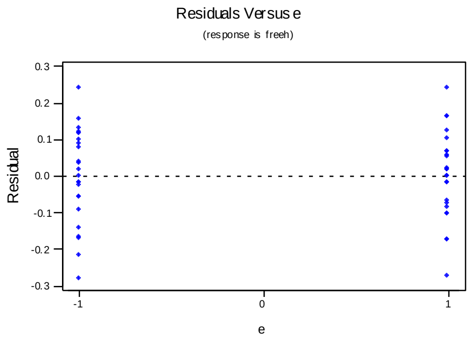
d)











The residual plots appear to be adequate.

14-76 a) The design used is a 2^2 full factorial with 2 replicates.
b) Factors x_1 and x_2 are significant. The interaction between x_1 and x_2 is not significant

Term	Effect	Coef	SE Coef	T	P
Constant		13.5800	0.1241	109.42	0.000
x1	0.7950	0.3975	0.1241	3.20	0.033
x2	-1.1600	-0.5800	0.1241	-4.67	0.009
x1*x2	0.0850	0.0425	0.1241	0.34	0.749

Analysis of Variance for y (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	3.95525	3.95525	1.97763	16.05	0.012
2-Way Interactions	1	0.01445	0.01445	0.01445	0.12	0.749
Residual Error	4	0.49290	0.49290	0.12323		
Pure Error	4	0.49290	0.49290	0.12322		
Total	7	4.46260				

14-77 a)

1. H_0: $\tau_1 = \tau_2 = \tau_3 = 0$

   H_1: at least one $\tau_i \neq 0$

2. H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

   H_1: at least one $\beta_j \neq 0$

3. H_0: $(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{13} = ... = (\tau\beta)_{34} = 0$

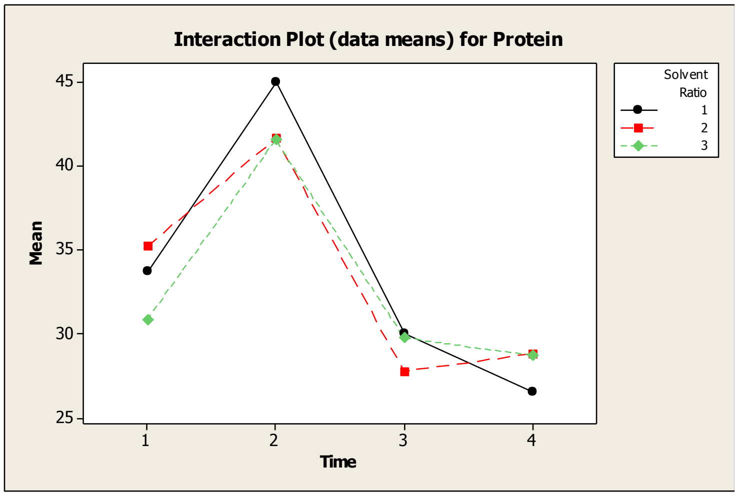
   H_1: at least one $(\tau\beta)_{ij} \neq 0$

Analysis of Variance for Protein, using Adjusted SS for Tests

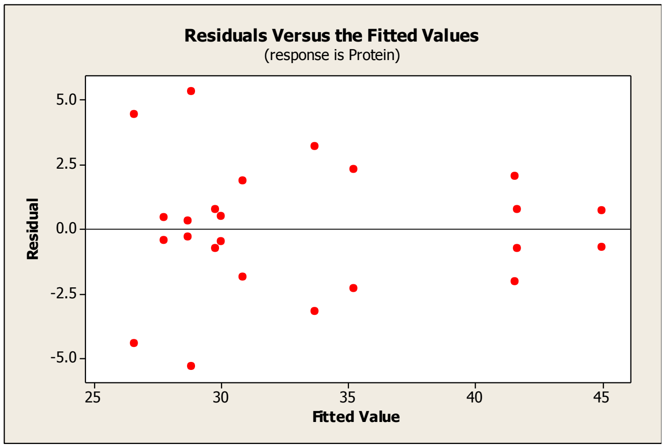
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Solvent Ratio	2	4.88	4.88	2.44	0.20	0.821
Time	3	803.93	803.93	267.98	21.96	0.000
Solvent Ratio*Time	6	42.62	42.62	7.10	0.58	0.738
Error	12	146.41	146.41	12.20		
Total	23	997.84				

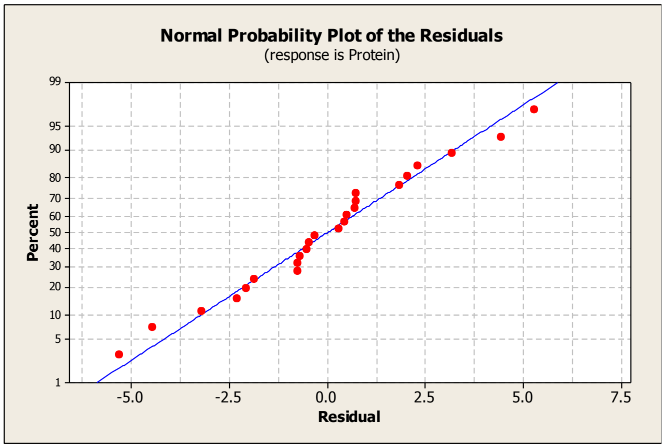
The only the time effect is significant.

b) The mean percentage of protein extracted of solvent 2 highest at time1, solvent 2 is highest at time 2 and 3, but lowest at time 4. The lines cross, but they are approximately parallel. This supports the ANOVA results that the interaction is not significant.



c) The plot of the residuals versus the fitted values shows a concern with the assumption of equal variances. The normality assumption appears reasonable.





14-78 a)

1. H$_0$: $\tau_1 = \tau_2 = \tau_3 = 0$

H$_1$: at least one $\tau_i \neq 0$

2. H$_0$: $\beta_1 = \beta_2 = \beta_3 = 0$

H$_1$: at least one $\beta_j \neq 0$

3. H$_0$: $(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{13} = ... = (\tau\beta)_{33} = 0$

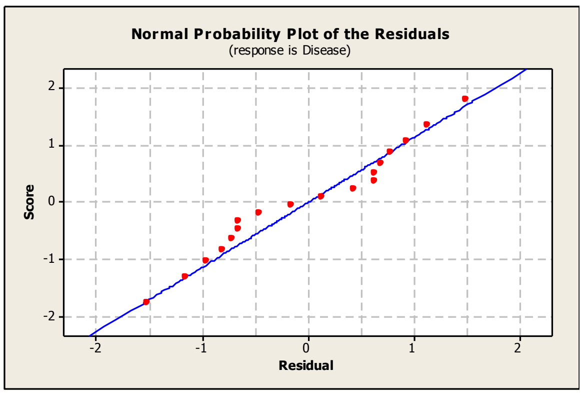
H$_1$: at least one $(\tau\beta)_{ij} \neq 0$

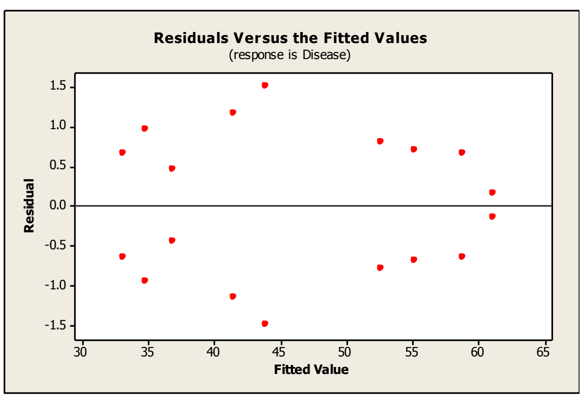
b)

```
Analysis of Variance for Disease

Source              DF       SS      MS       F      P
Nitrogen             2   924.73  462.37  311.71  0.000
Potassium            2   353.52  176.76  119.17  0.000
Nitrogen*Potassium   4   551.46  137.86   92.94  0.000
Error                9    13.35    1.48
Total               17  1843.06

S = 1.21792   R-Sq = 99.28%   R-Sq(adj) = 98.63%
```

All effects in the model are significant.

c) The residual plots do not show any violations of the model assumptions.





d) s = 1.21792 from the ANOVA estimates σ

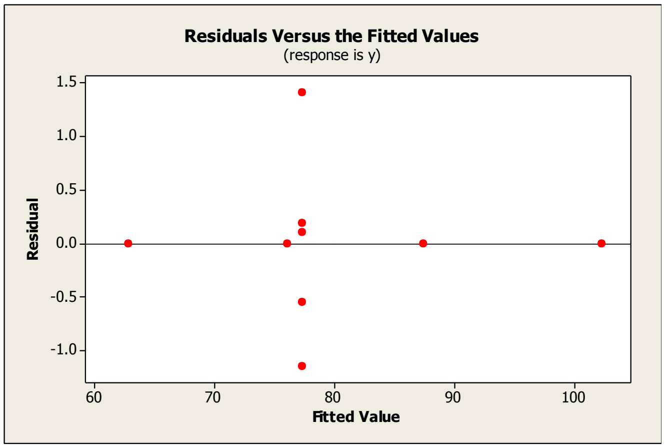
14-79

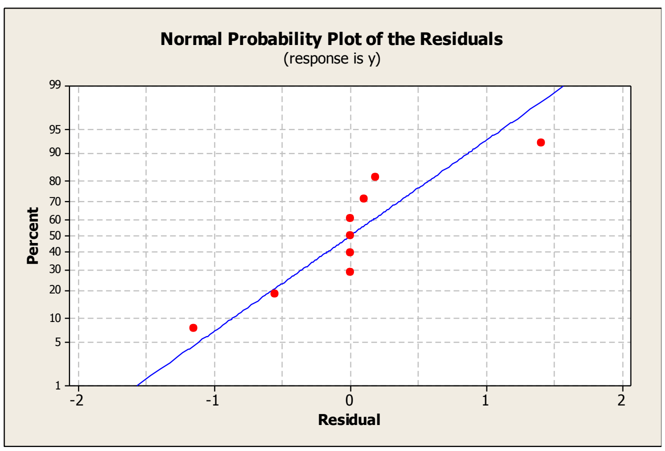
```
Analysis of Variance for y (coded units)

Source              DF   Seq SS   Adj SS   Adj MS       F      P
Main Effects         2  842.381  842.381  421.191  461.67  0.000
2-Way Interactions   1    0.638    0.638    0.638    0.70  0.450
  Curvature          1   52.598   52.598   52.598   57.65  0.002
Residual Error       4    3.649    3.649    0.912
  Pure Error         4    3.649    3.649    0.912
Total                8  899.267
```

Yes, the curvature is important in this region of the factors because the P-value =0.002 is smaller than $\alpha = 0.05$.

b) Residual

Trial	X1	X2	y	Residual
1	0	0	76.187	-1.1474
2	-1	-1	62.874	-0.0000
3	0	0	77.523	0.1886
4	1	-1	76.133	0.0000
5	1	1	102.324	0.0000
6	0	0	76.782	-0.5524
7	0	0	77.438	0.1036
8	-1	1	87.467	0.0000
9	0	0	78.742	1.4076





The linear model does not provide a good fit to this data.

c)

```
Estimated Regression Coefficients for y

Term          Coef  SE Coef        T      P
Constant   108.623   0.2660  408.367  0.000
x1           1.756   0.2103    8.349  0.000
x2           2.269   0.2103   10.790  0.000
x1*x1       -0.587   0.2255   -2.602  0.035
x2*x2       -0.925   0.2255   -4.104  0.005
x1*x2       -0.702   0.2974   -2.360  0.050
```

```
S = 0.5948   R-Sq = 96.8%   R-Sq(adj) = 94.5%

Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS      F      P
Regression       5  75.3066  75.3066  15.0613  42.57  0.000
  Linear         2  65.8426  65.8426  32.9213  93.06  0.000
  Square         2   7.4942   7.4942   3.7471  10.59  0.008
  Interaction    1   1.9698   1.9698   1.9698   5.57  0.050
Residual Error   7   2.4763   2.4763   0.3538
  Lack-of-Fit    3   0.8529   0.8529   0.2843   0.70  0.599
  Pure Error     4   1.6234   1.6234   0.4059
Total           12  77.7829
```

The model is $\hat{y} = 108.623 + 1.756x_1 + 2.269x_2 - 0.587x_1^2 - 0.925x_2^2 - 0.702x_1x_2$
This model is a better fit than the model from part (a).

14-80 a) Generators are E = ABC, F = ABD, and G = ACD

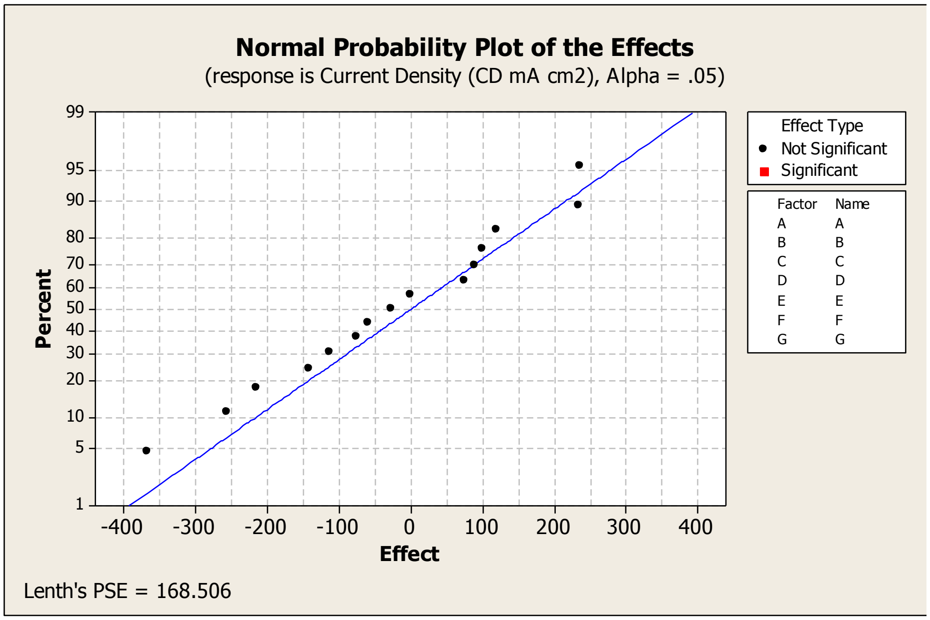
I = ABCE = ABDF = CDEF = ACDG = BDEG = BCFG = AEFG

```
Alias Structure (up to order 3)
I
A + B*C*E + B*D*F + C*D*G + E*F*G
B + A*C*E + A*D*F + C*F*G + D*E*G
C + A*B*E + A*D*G + B*F*G + D*E*F
D + A*B*F + A*C*G + B*E*G + C*E*F
E + A*B*C + A*F*G + B*D*G + C*D*F
F + A*B*D + A*E*G + B*C*G + C*D*E
G + A*C*D + A*E*F + B*C*F + B*D*E
A*B + C*E + D*F
A*C + B*E + D*G
A*D + B*F + C*G
A*E + B*C + F*G
A*F + B*D + E*G
A*G + C*D + E*F
B*G + C*F + D*E
A*B*G + A*C*F + A*D*E + B*C*D + B*E*F + C*E*G + D*F*G
```

b)
Factorial Fit: Current Density (CD mA cm2) versus A, B, C, D, E, F, G

```
Estimated Effects and Coefficients for Current Density (CD mA cm2) (coded
     units)
Term      Effect    Coef
Constant           206.9
A          -74.9   -37.5
B           76.1    38.0
C         -366.4  -183.2
D          236.9   118.5
E         -213.4  -106.7
F          119.9    60.0
G          101.9    51.0
A*B        234.8   117.4
A*C         90.8    45.4
A*D       -112.3   -56.2
A*E        -58.2   -29.1
A*F          0.7     0.3
A*G       -254.8  -127.4
B*G       -139.8   -69.9
A*B*G      -25.1   -12.5
```

c) Although the effects C, D, E, F, G, AD, AG, and BG are large, these effects are not indicated as significant in the normal probability plot of the effects.



d) From part (b), the effect of ABG interaction = -25.1. The contrast of the ABG interactions

= (Effect of ABG interactions x 2^{8-4})/2 = -25.1 x 2^3 = -200.8

The sum of square for the ABG interaction

= Contrast of ABG interactions^2 / 2^{8-4} = 2520 (= 2518 with more precision from computer software)

14-81 a) Generators are E = BCD, F = ACD, and G = ABC

I = BCDE = ACDF = ABEF = ABCG = ADEG = BDFG = CEFG

```
Alias Information for Terms in the Model.

I + A*C*D*F + A*B*E*F + A*B*C*G + A*D*E*G + B*C*D*E + B*D*F*G + C*E*F*G
A + B*E*F + B*C*G + C*D*F + D*E*G + A*B*C*D*E + A*B*D*F*G + A*C*E*F*G
B + A*E*F + A*C*G + C*D*E + D*F*G + A*B*C*D*F + A*B*D*E*G + B*C*E*F*G
C + A*D*F + A*B*G + B*D*E + E*F*G + A*B*C*E*F + A*C*D*E*G + B*C*D*F*G
D + A*C*F + A*E*G + B*C*E + B*F*G + A*B*D*E*F + A*B*C*D*G + C*D*E*F*G
E + A*B*F + A*D*G + B*C*D + C*F*G + A*C*D*E*F + A*B*C*E*G + B*D*E*F*G
F + A*C*D + A*B*E + B*D*G + C*E*G + A*B*C*F*G + A*D*E*F*G + B*C*D*E*F
G + A*B*C + A*D*E + B*D*F + C*E*F + A*C*D*F*G + A*B*E*F*G + B*C*D*E*G
A*B + C*G + E*F + A*C*D*E + A*D*F*G + B*C*D*F + B*D*E*G + A*B*C*E*F*G
A*C + B*G + D*F + A*B*D*E + A*E*F*G + B*C*E*F + C*D*E*G + A*B*C*D*F*G
A*D + C*F + E*G + A*B*C*E + A*B*F*G + B*D*E*F + B*C*D*G + A*C*D*E*F*G
A*E + B*F + D*G + A*B*C*D + A*C*F*G + B*C*E*G + C*D*E*F + A*B*D*E*F*G
A*F + B*E + C*D + A*B*D*G + A*C*E*G + B*C*F*G + D*E*F*G + A*B*C*D*E*F
A*G + B*C + D*E + A*B*D*F + A*C*E*F + B*E*F*G + C*D*F*G + A*B*C*D*E*G
B*D + C*E + F*G + A*B*C*F + A*D*E*F + A*C*D*G + A*B*E*G + B*C*D*E*F*G
A*B*D + A*C*E + A*F*G + B*C*F + B*E*G + C*D*G + D*E*F + A*B*C*D*E*F*G
```

b)
Factorial Fit: Weight versus A, B, C, D, E, F, G

Estimated Effects and Coefficients for Weight (coded units)

Term	Effect	Coef
Constant		16.525
A	1.825	0.913
B	1.225	0.612
C	10.500	5.250
D	-0.325	-0.162
E	1.400	0.700
F	1.550	0.775
G	-5.600	-2.800
A*B	1.200	0.600
A*C	-3.525	-1.762
A*D	0.750	0.375
A*E	0.525	0.263
A*F	6.125	3.063
A*G	-3.975	-1.987
B*D	-0.150	-0.075
A*B*D	-0.775	-0.387

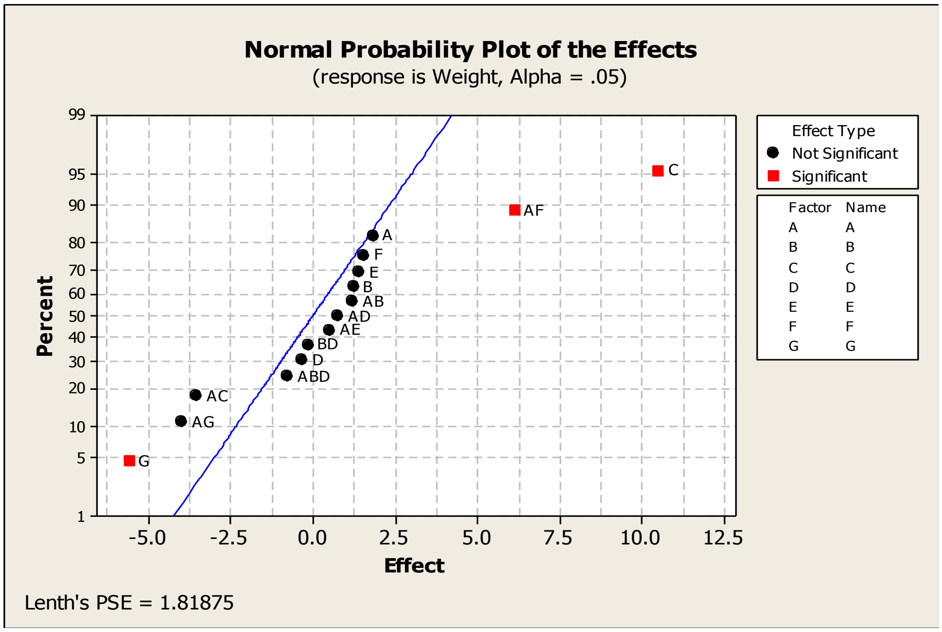
Factorial Fit: Cellular versus A, B, C, D, E, F, G

Estimated Effects and Coefficients for Cellular (coded units)

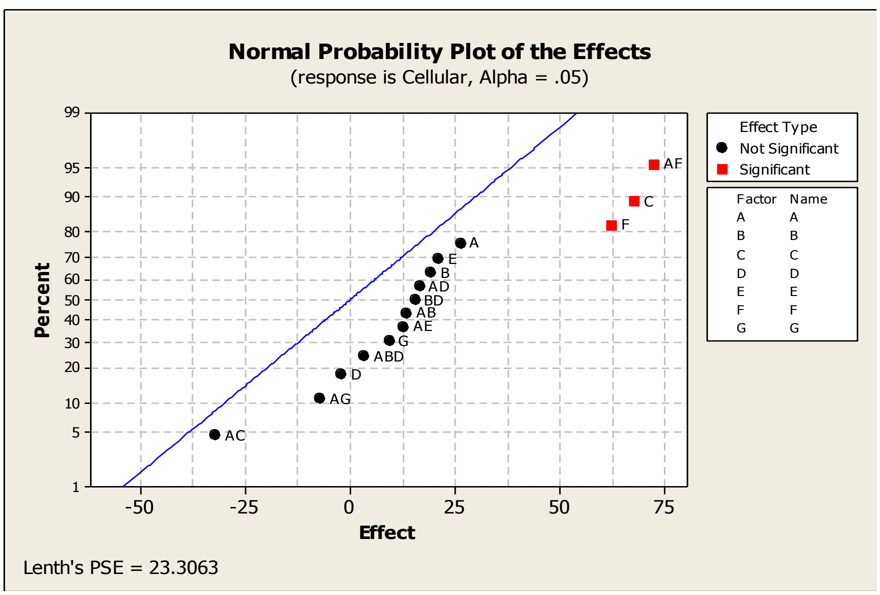
Term	Effect	Coef
Constant		93.24
A	26.41	13.21
B	19.44	9.72
C	67.71	33.86
D	-1.96	-0.98
E	21.09	10.54
F	62.46	31.23
G	9.44	4.72
A*B	13.51	6.76
A*C	-32.06	-16.03
A*D	16.66	8.33
A*E	12.71	6.36
A*F	72.54	36.27
A*G	-6.99	-3.49
B*D	15.54	7.77
A*B*D	3.41	1.71

c)

Effects Plot for WeightContent



For weight, the effects labeled as AF, C, and G are marked as significant. Also, AC and AG might be considered important. These effect labels actually represent alias sets and need to be interpreted with the alias table shown above.



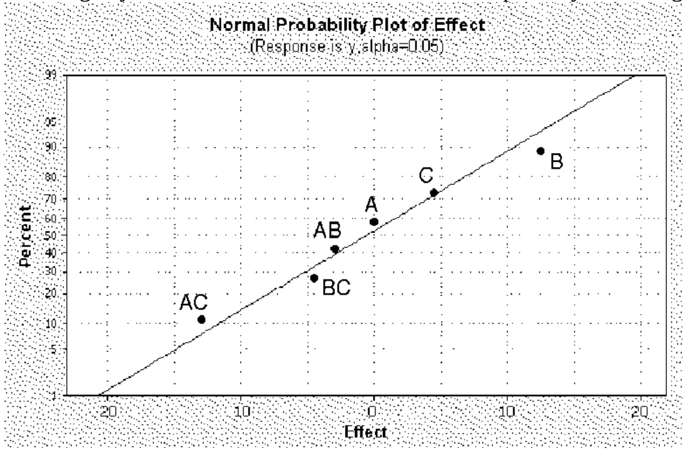
For cellular content, the effects labeled as AF, C, and F are marked as significant and AC are also indicated as potentially important. These effect labels actually represent alias sets and need to be interpreted with the alias table shown above.

14-82 a)

Std Order	Run Order	Center Pt	Blocks	A	B	C	y
1	1	1	1	-1	-1	-1	30
2	2	1	1	1	1	-1	60
3	3	1	1	1	-1	1	42
4	4	1	1	-1	1	1	63
5	5	1	2	1	-1	-1	34

6	6	1	2	-1	1	-1	38
7	7	1	2	-1	-1	1	40
8	8	1	2	1	1	1	35

b) The Minitab results do not flag any effects, but the effects for B, AC, and possibly C are large.



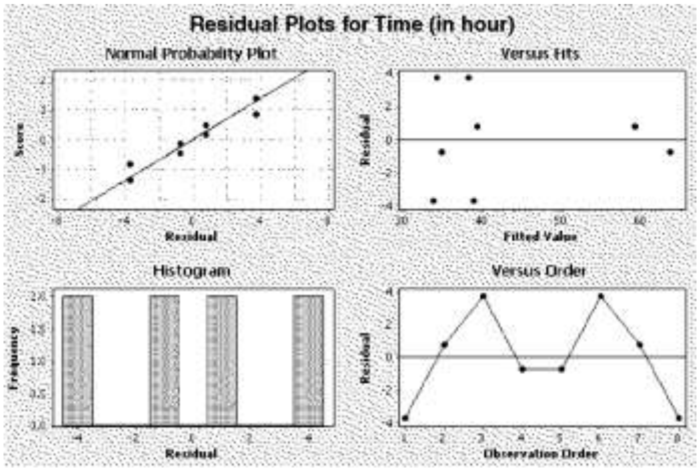
Factorial Fit: Time (in hours) versus Block, A, B, C

```
Estimated Effects and Coefficients for Time (in hours) (coded units)

Term        Effect         Coef
Constant                60.7500
Blocks                 -12.0000
A              0.0   0.00000000
B             12.5      6.25000
C              4.5      2.25000
AB            -3.0     -1.50000
AC           -13.0     -6.50000
BC            -4.5     -2.25000
```

If a hierarchical model is applied, the main effect A is added to the model. The Minitab result shows that no effects are significant at $\alpha = 0.05$. However, effect B and AC are significant at $\alpha = 0.1$. Residuals plots do not indicate any serious model failures. There is some increased variability at the lower fitted values.

Factorial Fit: Time (in hours) versus Block, A, B, C

```
Estimated Effects and Coefficients for Time (in hours) (coded units)

Term        Effects     Coef  SE Coef      T      P
Constant              60.750    6.047  10.05  0.010
Blocks               -12.000    3.824  -3.14  0.088
A             -0.00   -0.000    1.912  -0.00  1.000
B             12.50    6.250    1.912   3.27  0.082
C              4.50    2.250    1.912   1.18  0.360
AC           -13.00   -6.500    1.912  -3.40  0.077

S = 5.40833   R-Sq = 94.4%   R-Sq(adj) = 80.3%

Analysis of Variance for time (in hours) (coded units)
Source              DF   Seq SS  Adj SS  Adj MS      F      P
Blocks               1   288.00  288.00  288.00   9.85  0.088
Main Effects         3   353.00  353.00  117.67   4.02  0.480
2-way interactions   1   338.00  338.00  338.00  11.56  0.077
Error                2    58.50   58.50   29.25
Total                7  1037.50
```



14-83 a)

A	B	Mean	StDev	Sum
1	1	21.3333	6.027714	64
1	2	20	7.549834	60
1	3	32.6667	3.511885	98
2	1	31	6.244998	93
2	2	33	6.557439	99
2	3	23	10	69

	Factor B			
Factor A	1	2	3	$y_{i.}$
1	64	60	98	222
2	93	99	69	261
$y_{.j}$	157	159	167	483 = $y_{..}$

$$SS_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn} = \frac{1}{(3)(3)}[222^2 + 261^2] - \frac{483^2}{18} = 84.5$$

$$SS_B = \frac{1}{an}\sum_{i=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn} = \frac{1}{(2)(3)}[157^2 + 159^2 + 167^2] - \frac{483^2}{18} = 9.3333$$

$$SS_{Interaction} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_A - SS_B$$

$$= \frac{1}{(3)}[64^2 + 60^2 + 98^2 + 93^2 + 99^2 + 69^2] - \frac{483^2}{18} - 84.5 - 9.3333 = 449.3333$$

b) `stDev` = $\sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

For A =1, B = 1, `stDev = 6.027714` = $\sqrt{\dfrac{\sum_{i=1}^{3}(x_i - 21.3333)^2}{2}}$

$$\sum_{i=1}^{3}(x_i - 21.3333)^2 = 6.027714^2 \times 2 = 72.666672$$

$$(x_1 - 21.3333)^2 + (x_2 - 21.3333)^2 + (x_3 - 21.3333)^2 = 72.66672$$

$$(x_1^2 + x_2^2 + x_3^2) - (2 \times 21.3333)(x_1 + x_2 + x_3) + (3 \times 21.3333^2) = 72.66672$$

$$(x_1^2 + x_2^2 + x_3^2) = 72.66672 + (2 \times 21.3333)(64) - (3 \times 21.3333^2) = 1438$$

A	B	Mean	StDev	sum	sum of (x - xbar)^2	$x_1^2+x_2^2+x_3^2$
1	1	21.3333	6.027714	64	72.666672	1438
1	2	20	7.549834	60	113.999987	1314
1	3	32.6667	3.511885	98	24.666673	3226
2	1	31	6.244998	93	78.000000	2961
2	2	33	6.557439	99	86.000012	3353
2	3	23	10	69	200	1787

$SS_{Total} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \dfrac{y_{...}^2}{abn} = [1438 + 1314 + 3226 + 2961 + 3353 + 1787] - \dfrac{483^2}{18} = 1119$

$SS_{Error} = SS_{Total} - SS_A - SS_B - SS_{Interaction}$ = 1119 - 84.5 – 9.3333 – 449.3333 = 575

c) Total trials = 6 treatments * 3 replicates = 18 trials.

The ANOVA table

```
Source        DF    SS           MS           F            P-value
A             1     84.5         84.5         1.7624       0.209
B             2     9.3333       4.6667       0.0973       0.908
AB            2     449.3333     224.6667     4.686        0.031
Error         12    575          47.9446
Total         17    1119
```

14-84 a) Factor A has = 3 + 1 = 4 levels. Factor B has 2 + 1 = 3 levels.

b) The total degrees of freedom =11 which implies the total runs = 12. Therefore, one replicate was used.

c) The two-factor interaction term (AB) is not significant.

d) Degree of freedom of error = 11 – 3 – 2 = 6

$MS(B) = \dfrac{SS_B}{df_B} = \dfrac{SS_B}{2} = 17335441$, then $SS_B = 34670882$

$$MS_E = \frac{SS_E}{df_E} = \frac{1784195}{6} = 297365.83$$

$$F_A = \frac{MS_A}{MS_E} = \frac{404590}{297365.83} = 1.3601$$

14-85 a) Generators are E = ABC, F = BCD, and G = ABD. Note that the generator for factor G differs from the Minitab default.

I = ABCE = ABDG = CDEG = ACFG = BEFG = BCDF =ADEF

```
Alias Structure (up to order 3)
I
A + B*C*E + B*D*G + C*F*G + D*E*F
B + A*C*E + A*D*G + C*D*F + E*F*G
C + A*B*E + A*F*G + B*D*F + D*E*G
D + A*B*G + A*E*F + B*C*F + C*E*G
E + A*B*C + A*D*F + B*F*G + C*D*G
F + A*C*G + A*D*E + B*C*D + B*E*G
G + A*B*D + A*C*F + B*E*F + C*D*E
A*B + C*E + D*G
A*C + B*E + F*G
A*D + B*G + E*F
A*E + B*C + D*F
A*F + C*G + D*E
A*G + B*D + C*F
B*F + C*D + E*G
```

b) Factorial Fit: Yield(%) versus A, B, C, D, E, F, G

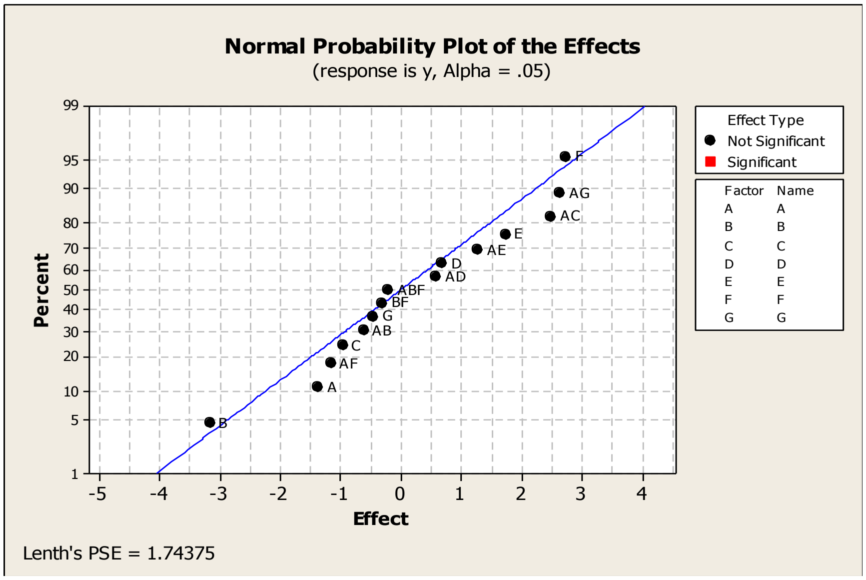
Estimated Effects and Coefficients for Yield(%) (coded units)

Term	Effect	Coef
Constant		96.044
A	-1.362	-0.681
B	-3.162	-1.581
C	-0.962	-0.481
D	0.687	0.344
E	1.738	0.869
F	2.737	1.369
G	-0.462	-0.231

c)
Factorial Fit: y versus A, B, C, D, E, F, G

Estimated Effects and Coefficients for y (coded units)

Term	Effect	Coef
Constant		96.044
A	-1.362	-0.681
B	-3.162	-1.581
C	-0.962	-0.481
D	0.687	0.344
E	1.738	0.869
F	2.737	1.369
G	-0.462	-0.231
A*B	-0.612	-0.306
A*C	2.487	1.244
A*D	0.587	0.294
A*E	1.287	0.644
A*F	-1.163	-0.581
A*G	2.638	1.319
B*F	-0.312	-0.156
A*B*F	-0.213	-0.106



The computer effects plot does not indicate any significant effects. However, effects B, F, AG, AC are large (in absolute value). A model with the smaller effects A, AF, C, AB, G, BF, ABF pooled into error could be used to test the other effects. These effects are labels for the alias sets in the table above and the aliases need to be used to interpret these results.

b) $MS_{\text{Two-way Interaction}} = \dfrac{SS_{Two-way\ Interaction}}{df_{Two-way\ Interaction}} = \dfrac{67.884}{7} = 9.698$

$SS_{\text{Residual Error}} = SS_{\text{Total}} - SS_{\text{Main Effects}} - SS_{\text{Two-way Interactioin}} = 163.999 - 95.934 - 67.884 = 0.181$

$MS_{\text{Residual Error}} = \dfrac{SS_{ResidualError}}{df_{ResidualError}} = \dfrac{0.181}{1} = 0.181$

F-test = $\dfrac{MS_{Main\ Effects}}{MS_{ResidualError}} = \dfrac{13.7049}{0.181} = 75.72$

P-value = 0.088

<u>Mind Expanding Exercises</u>

14-86 The ABCD interaction is

$$\frac{1}{8}\left[(1)+ab+ac+bc+ad+bd+cd+abcd\right]-\left[a+b+c+d+abc+abd+acd+bcd\right]$$

If *ab* is missing, then ABCD interaction will be zero when

[550 + ab + 642 + 601 + 749 + 1052 + 1075 + 729] – [669 + 604 + 633 + 1037 + 635 + 868 + 860 + 1063] = 0

Therefore, *ab* + 5398 - 6369 = 0 or *ab* = 971. After estimating *ab*, only the A and AD effects appear significant.

14-87 Two three-factor interactions could be used to generate the blocks such as ABC and ACD. This would confound these effects and ABC(ACD) = BD with blocks. Therefore, only one two-factor and no main effects are confounded with blocks.

14-88

	A	**B**	**AB**	**block**
(1)	-	-	+	1
a	+	-	-	2
b	-	+	-	2
ab	+	+	+	1

The block effect is estimated by $\frac{a+b}{2}-\frac{(1)-ab}{2}$ which is the same as the estimate of the effect of AB.

14-89 a) A different effect can be confounded in each replicate as follows.

Replicate 1 ABC confounded		Replicate 2 AB confounded		Replicate 3 BC confounded		Replicate 4 AC confounded	
(1)	a	(1)	a	(1)	b	(1)	a
ab	b	c	b	a	c	b	c
ac	c	ab	ac	bc	ab	ac	ab
bc	abc	abc	bc	abc	ac	abc	bc

b)

Source of Variation	Degrees of freedom
Replicates	3
Blocks with replicates	4
[or ABC (rep. 1) + AB (rep. 2) + BC (rep. 3) + AC (rep. 4)]	
A	1
B	1
C	1
AB (from replicates 1, 3, and 4)	1
AC (from replicates 1, 2, and 3)	1
BC (from replicates 1, 2, and 4)	1
ABC (from replicates 2, 3, and 4)	1
Error (by subtraction)	17
Total	31

In calculating an interaction sum of squares, only data from the replicates in which the interaction is un-confounded are used.

14-90

A	B	C	D	E = ABCD	AB = CDE	block
-	-	-	-	+	+	1
+	-	-	-	-	-	2
-	+	-	-	-	-	2
+	+	-	-	+	+	1
-	-	+	-	-	+	1
+	-	+	-	+	-	2
-	+	+	-	+	-	2
+	+	+	-	-	+	1
-	-	-	+	-	+	1
+	-	-	+	+	-	2
-	+	-	+	+	-	2
+	+	-	+	-	+	1
-	-	+	+	+	+	1
+	-	+	+	-	-	2
-	+	+	+	-	-	2
+	+	+	+	+	+	1

This uses AB = CDE as the effect to confound with blocks.

14-91 The generators are F = ABCD and G = ABDE. The complete defining relation is
I = ABCDF = ABDEG = CEFG.
The design can be constructed in four blocks by confounding ACE = AFG and BCE = BFG with blocks. This also confounds AB = CDF = DEG with blocks.
Yes, a two-factor interaction is confounded with blocks. The best blocking scheme confounds only one two-factor interaction with blocks.

14-92 The generators are E = ABC, F = BCD, and G = ACD.
The complete defining relation is
I = ABCE = BCDF = ADEF = ACDG = BDEG = ABFG = CEFG.
The alias set
ABD = CDE = ACF = BEF = BCG = AEG = DFG

can be used to construct the blocks. Then, only three-factor interactions are confounded with blocks.

14-93 a)

A	B	C	D = AB	E = AC	F = BC	G = ABC
-	-	-	+	+	+	-
+	-	-	-	-	+	+
-	+	-	-	+	-	+
+	+	-	+	-	-	-
-	-	+	+	-	-	+
+	-	+	-	+	-	-
-	+	+	-	-	+	-
+	+	+	+	+	+	+

The complete defining relation is
I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF = ABCG = CDG = BEG = ADEG = AFG = BDFG = CEFG = ABCDEFG.

The alias structure follows (including only one- and two-factor effects).
A = BD = CE = FG
B = AD = CF = EG
C = AE = BF = DG
D = AB = EF = CG
E = AC = DF = BG
F = BC = DE = AG
G = CD = BE = AF

b) The complete defining relation is
I = -ABD = -ACE = BCDE = -BCF = ACDF = ABEF = -DEF = ABCG = -CDG = -BEG = ADEG = -AFG = BDFG = CEFG = -ABCDEFG.

The aliases (up to two-factor effects) are:
A = -BD = -CE = -FG
B = -AD = -CF = -EG
C = -AE = -BF = -DG
D = -AB = -EF = -CG
E = -AC = -DF = -BG
F = -BC = -DE = -AG
G = -CD = -BE = -AF

c) All the main effects can be estimated. Use the average response from the alias set that contains the main effect in each fraction. The two-factor effects cancel when this average is computed.

14-94 When the square root of the sum of squares for curvature is divided by the square root of mean squared error, the resulting statistic is

$$\frac{|\bar{y}_F - \bar{y}_C|}{\hat{\sigma}\sqrt{\frac{1}{n_F} + \frac{1}{n_C}}}$$

and this is a t-statistic used to compare two means. If this t-statistic is significant, $\bar{y}_F - \bar{y}_C$ is large meaning curvature is significant. This test is equivalent to the F test from the ANOVA testing for curvature. This statistic is compared to a t distribution with the degrees of freedom associated with the estimate of σ.

If a random variable with a *t* distribution is squared, the resulting random variable has an *F* distribution with one degree of freedom in the numerator and degrees of freedom in the denominator equal to the degrees of freedom of the *t* statistic. Therefore, the reference distribution for the *F* test is the square of the reference distribution for the *t* test.