CHAPTER 15

Section 15-3

15-1 a) $\bar{\bar{x}} = \dfrac{7805}{36} = 216.81 \qquad \bar{r} = \dfrac{1200}{36} = 33.33$

$\bar{X}$ *chart*

$$UCL = CL + A_2\bar{r} = 216.81 + 0.483(33.33) = 232.91$$
$$CL = 216.81$$
$$LCL = CL - A_2\bar{r} = 216.81 - 0.483(33.33) = 200.71$$

R *chart*

$$UCL = D_4\bar{r} = 2.004(33.33) = 66.79$$
$$CL = 33.33$$
$$LCL = D_3\bar{r} = 0(33.33) = 0$$

b) $\hat{\mu} = \bar{\bar{x}} = 216.81$

$$\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{33.33}{2.534} = 13.15$$

15-2 a) $\bar{\bar{x}} = \dfrac{362.75}{36} = 10.076 \qquad \bar{r} = \dfrac{8.60}{36} = 0.239 \qquad \bar{s} = \dfrac{3.64}{36} = 0.1011$

$\bar{X}$ *Chart*

$$UCL = CL + A_2\bar{r} = 10.076 + 0.483 * 0.239 = 10.191$$
$$CL = 10.076$$
$$UCL = CL - A_2\bar{r} = 10.076 - 0.483 * 0.239 = 9.961$$

R *Chart*

$$UCL = D_4\bar{r} = 2.004 * 0.239 = 0.479$$
$$CL = 0.239$$
$$UCL = D_3\bar{r} = 0 * 0.239 = 0$$

b) $c_4 = 0.9515$

$\bar{X}$ *chart*

$$UCL = CL + 3\frac{\bar{s}}{c_4\sqrt{n}} = 10.076 + 3\frac{0.1011}{0.9515\sqrt{6}} = 10.206$$
$$CL = 10.076$$
$$LCL = CL - 3\frac{\bar{s}}{c_4\sqrt{n}} = 10.076 - 3\frac{0.1011}{0.9515\sqrt{6}} = 9.946$$

S chart

$$UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2} = 0.1011 + 3\left(\frac{0.1011}{0.9515}\right)\sqrt{1-0.9515^2} = 0.1992$$

$$CL = 0.1456$$

$$LCL = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2} = 0.1011 - 3\left(\frac{0.1011}{0.9515}\right)\sqrt{1-0.9515^2} = 0.003 \rightarrow 0$$

15-3 a) $\bar{\bar{x}} = \frac{4460}{25} = 178.4 \qquad \bar{s} = \frac{271.6}{25} = 10.864$

$\bar{X}$ *chart*

$$UCL = CL + 3\frac{\bar{s}}{c_4\sqrt{n}} = 178.4 + 3\left(\frac{10.864}{0.94\sqrt{5}}\right) = 193.91$$

$$CL = 178.4$$

$$LCL = CL - 3\frac{\bar{s}}{c_4\sqrt{n}} = 178.4 - 3\left(\frac{10.864}{0.94\sqrt{5}}\right) = 162.89$$

S chart

$$UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2} = 10.864 + 3\left(\frac{10.864}{0.94}\right)\sqrt{0.1164} = 22.693$$

$$CL = 13.58$$

$$LCL = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2} = 10.864 - 3\left(\frac{10.864}{0.94}\right)\sqrt{0.1164} = -0.965 \rightarrow 0$$

b) Process mean and standard deviation

$$\hat{\mu} = \bar{\bar{x}} = 178.4 \qquad \hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{10.864}{0.94} = 11.557$$

15-4 a) $\bar{\bar{x}} = 20.0 \qquad \frac{\bar{r}}{d_2} = 1.4 \qquad d_2 = 2.326 \quad \bar{r} = 1.4(2.326) = 3.2564$

$\bar{X}$ *chart*

$$UCL = CL + A_2\bar{r} = 20.0 + 0.577(3.2564) = 21.88$$

$$CL = 20.0$$

$$LCL = CL - A_2\bar{r} = 20.0 - 0.577(3.2564) = 18.12$$

R chart

$$UCL = D_4\bar{r} = 2.115(3.2564) = 6.89$$

$$CL = 3.2564$$

$$LCL = D_3\bar{r} = 0(3.2564) = 0$$

$\bar{X}$ *chart* where $\bar{s}/c_4 = 1.5$

$$UCL = CL + 3\frac{\bar{s}}{c_4\sqrt{n}} = 20.0 + 3(1.5)/\sqrt{5} = 22.0125$$

$$CL = 20.0$$

$$LCL = CL - 3\frac{\bar{s}}{c_4\sqrt{n}} = 20.0 - 3(1.5)/\sqrt{5} = 17.9875$$

$$S \quad Chart \quad c_4 = 0.94 \quad so \quad \bar{s} = 1.41$$

$$UCL = \bar{s} + \frac{3\bar{s}}{c_4}\sqrt{1-c_4^2} = 1.41 + 3(1.5)\sqrt{1-0.94^2} = 2.945$$

$$CL = 1.41$$

$$UCL = \bar{s} - \frac{3\bar{s}}{c_4}\sqrt{1-c_4^2} = 1.41 - 3(1.5)\sqrt{1-0.94^2} = 0$$

15-5 a) $\bar{\bar{x}} = \frac{140.03}{35} = 4.0009 \qquad \bar{r} = \frac{13.63}{35} = 0.3894$

For the $\bar{X}$ chart:

$$UCL = \bar{\bar{x}} + A_2\bar{r} = 4.0009 + (0.577)(0.3894) = 4.226$$
$$LCL = \bar{\bar{x}} - A_2\bar{r} = 4.0009 - (0.577)(0.3894) = 3.776$$

For the *r* chart:

$$UCL = D_4\bar{r} = (2.115)(0.3894) = 0.8236$$
$$LCL = D_2\bar{r} = (0)(0.3894) = 0$$

b) $\bar{s} = \frac{5.10}{35} = 0.1457$

For the $\bar{X}$ chart:

$$UCL = \bar{\bar{x}} + \frac{3\bar{s}}{c_4\sqrt{n}} = 4.0009 + \frac{3(0.1457)}{0.94\sqrt{5}} = 4.2087$$

$$LCL = \bar{\bar{x}} - \frac{3\bar{s}}{c_4\sqrt{n}} = 4.0009 - \frac{3(0.1457)}{0.94\sqrt{5}} = 3.7931$$

For the *S* chart:

$$UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2} = 0.1457 + 3\left(\frac{0.1457}{0.94}\right)\sqrt{1-0.94^2} = 0.3043$$

$$LCL = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2} = 0.1457 - 3\left(\frac{0.1457}{0.94}\right)\sqrt{1-0.94^2} = -0.0129$$

Because the LCL is negative it is set to zero.

15-6 For the $\bar{X}$ chart:

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}} = 49$$

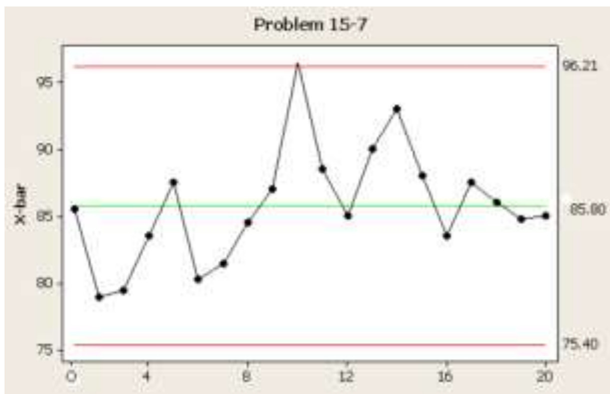
$$LCL = \mu - \frac{3\sigma}{\sqrt{n}} = 42$$

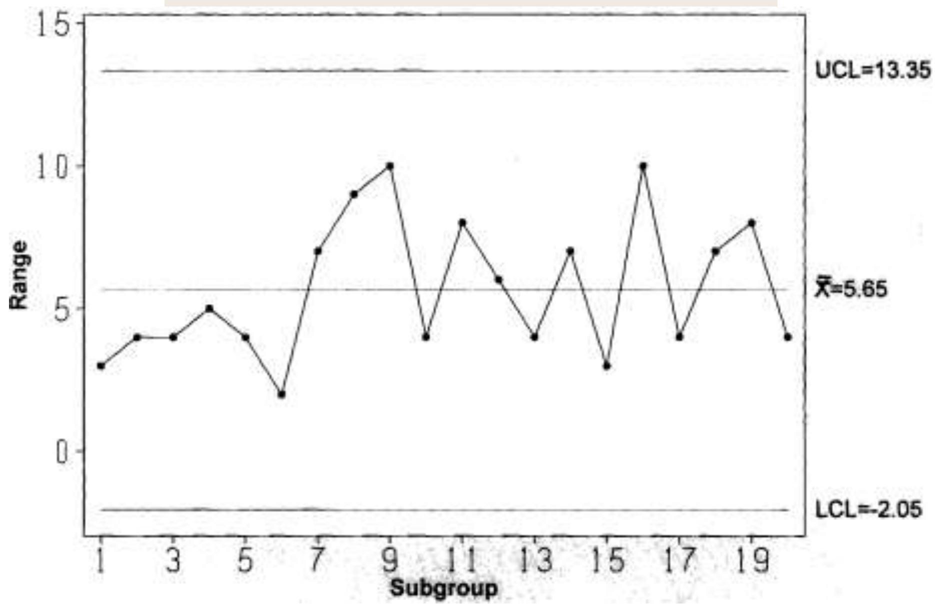
The difference $UCL - LCL = \frac{6\sigma}{\sqrt{n}} = 49 - 42 = 7$

$$n = \left(\frac{6(2.25)}{7}\right)^2 = 3.72 \approx 4$$

15-7 a)

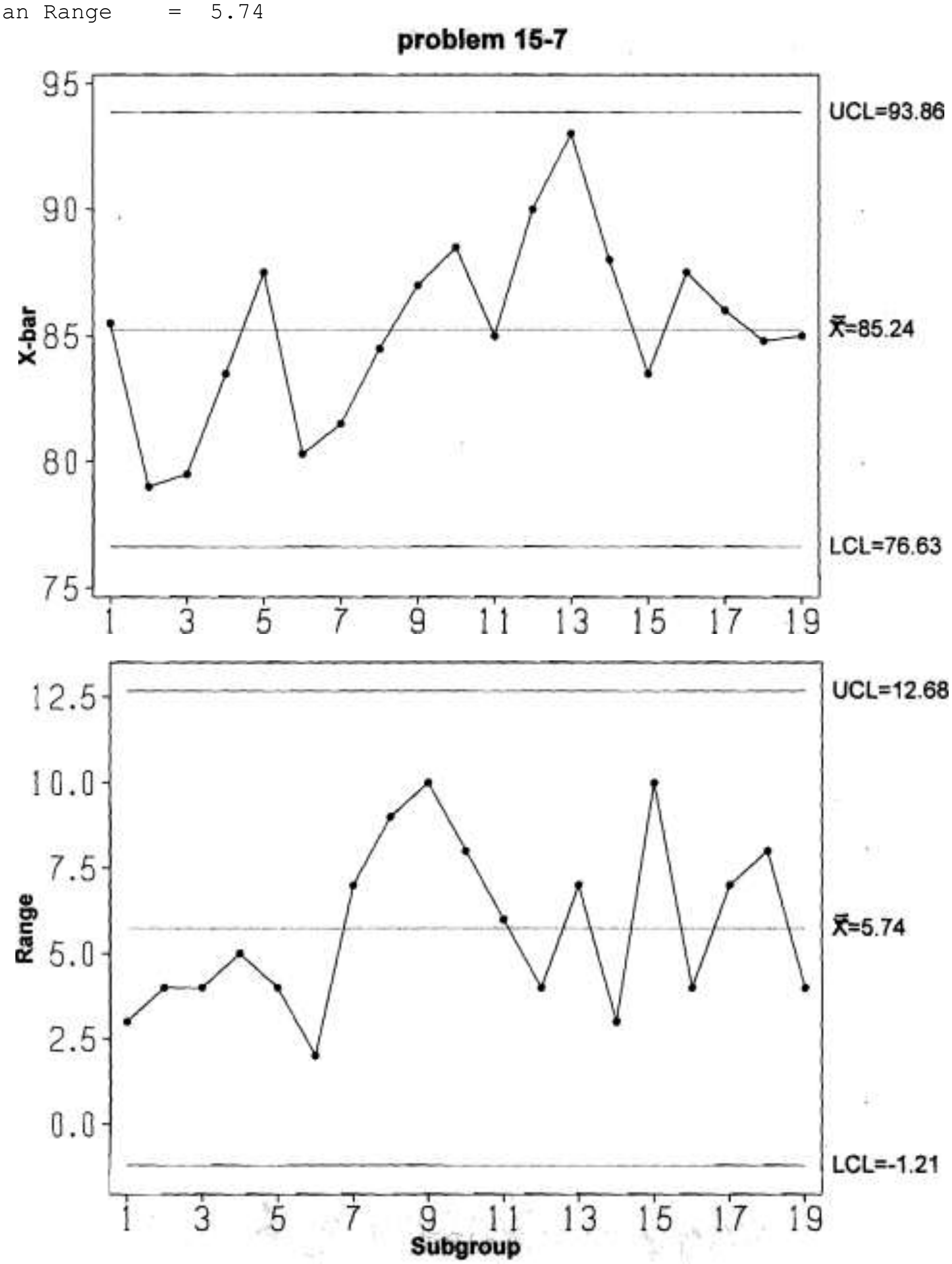
```
                    X-bar and Range - Initial Study
                             Problem 15-7

X-bar                                | Range
-----                                | -----
UCL:  +    3.0 sigma = 96.21         | UCL:  +    3.0 sigma = 13.35
Centerline           = 85.80         | Centerline           =  5.65
LCL:  -    3.0 sigma = 75.40         | LCL:  -    3.0 sigma =  -2.05
                                     |
out of limits = 1                    | out of limits = 0
-----------------------------------------------------------------
Chart: Both      Normalize: No

   20 subgroups, size 5                      0 subgroups excluded

Estimated
process mean  = 85.80
process sigma =  3.46678
mean Range    =  5.65
```





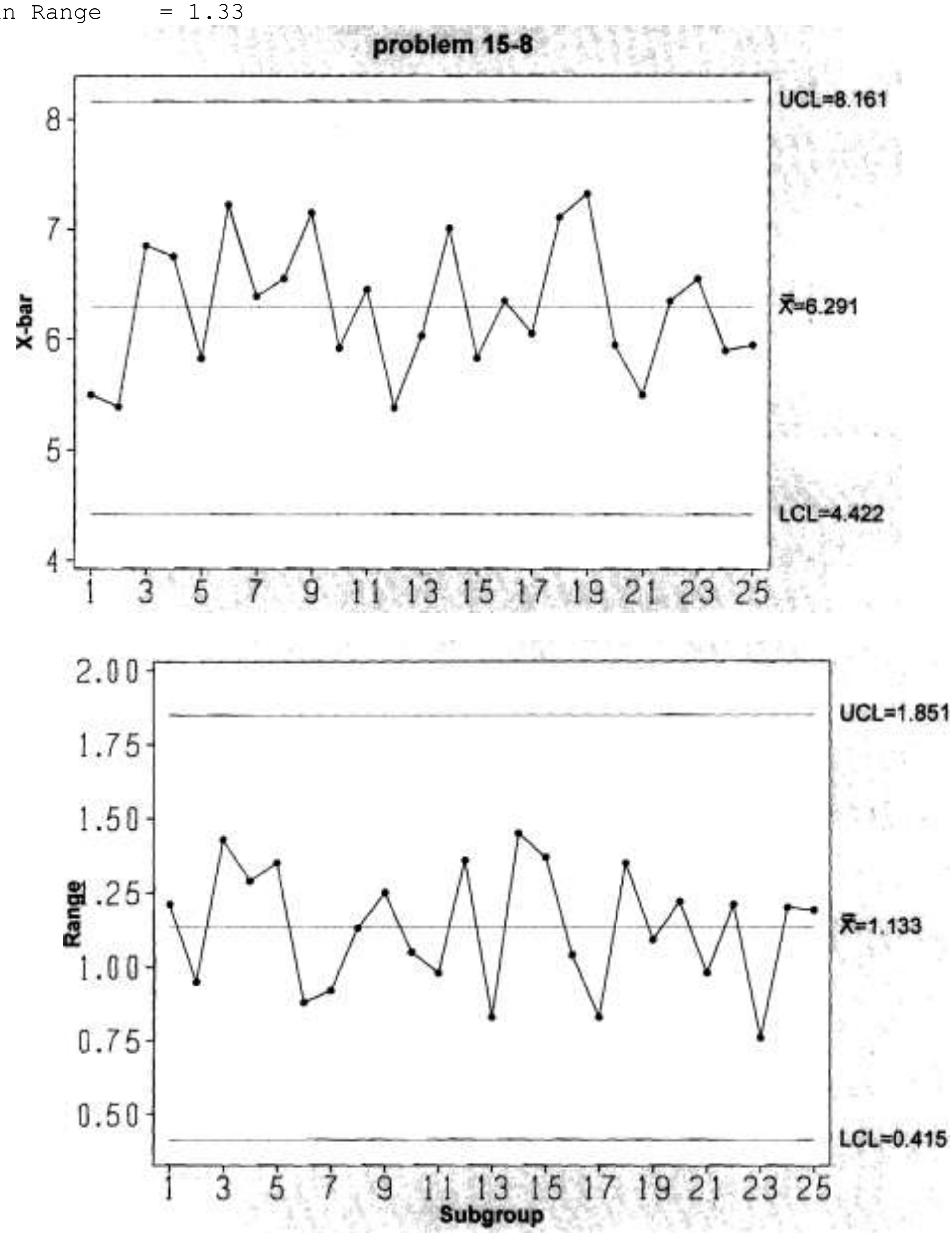
b)

```
                                   Charting xbar
X-bar                               | Range
-----                               | -----
UCL:  +    3.0 sigma = 93.86        | UCL:  +    3.0 sigma = 12.68
Centerline           = 85.24        | Centerline           =  5.74
LCL:  -    3.0 sigma = 76.63        | LCL:  -    3.0 sigma =  0

out of limits = 0                   | out of limits = 0
------------------------------------------------------------------
20 subgroups, size 5                   1 subgroups excluded
Estimated
process mean  = 85.24
process sigma =  2.87136
mean Range    =  5.74
```



15-8 a)

```
                              X-bar and Range - Initial Study
Charting Problem 15-8
X-bar                                  | Range
-----                                  | -----
UCL:  +    3.0 sigma = 8.161           | UCL:  +    3.0 sigma = 1.851
Centerline           = 6.291           | Centerline           = 1.133
LCL:  -    3.0 sigma = 4.422           | LCL:  -    3.0 sigma = 0.415
                                       |
out of limits = 0                      | out of limits = 0

Estimated
process mean  = 6.2912
process sigma = 0.623153
mean Range    = 1.33
```

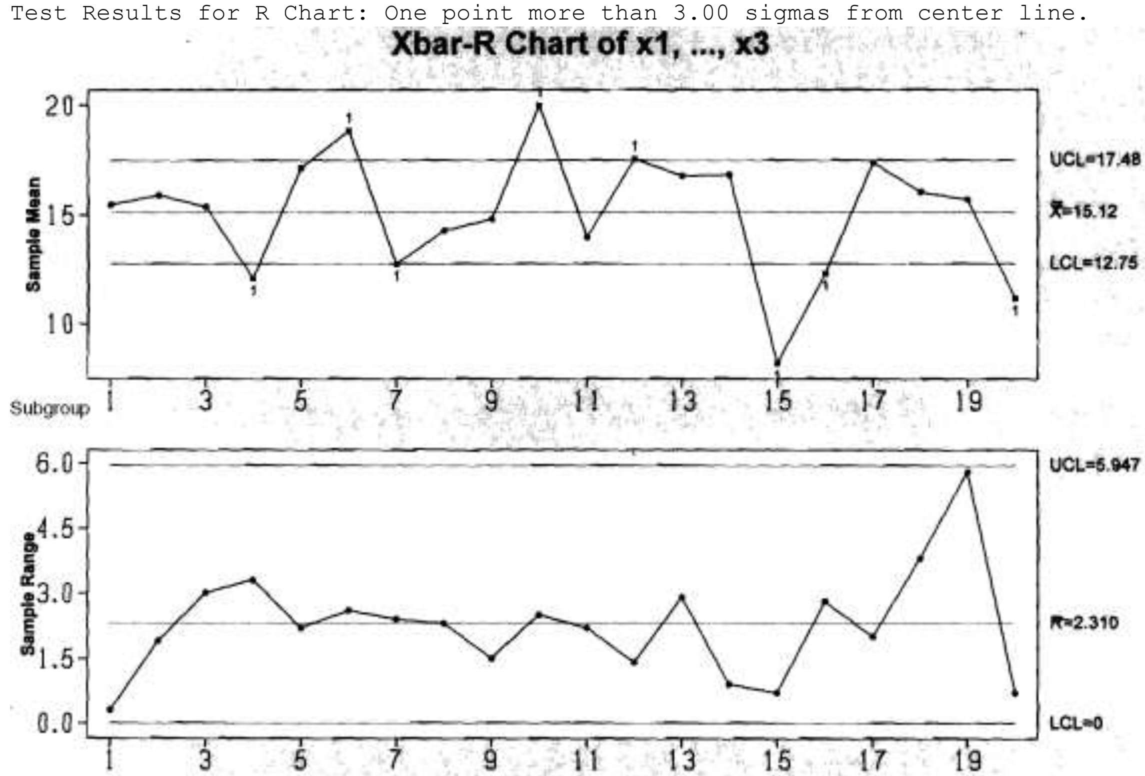


There are no points beyond the control limits. The process appears to be in control.

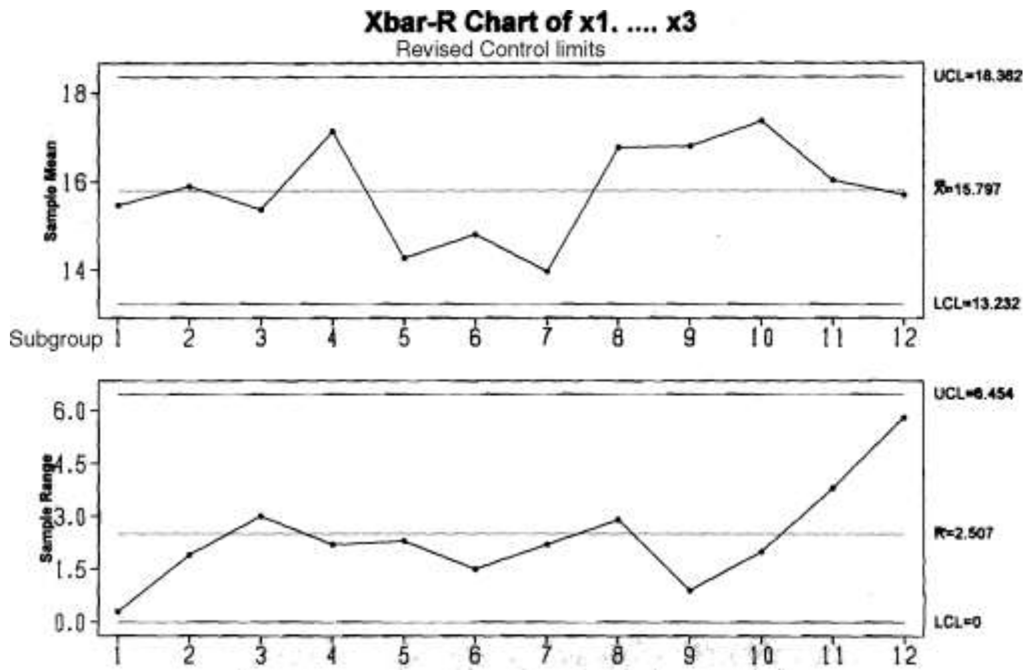
b) No points fell beyond the control limits. The limits do not need to be revised.

15-9 a)

```
                         X-bar and Range - Initial Study
                            Charting Problem 15-9
X-bar                                     | Range
-----                                     | -----
UCL:  +    3.0 sigma = 17.48              | UCL:  +    3.0 sigma =  5.947
Centerline           = 15.12              | Centerline           =  2.31
LCL:  -    3.0 sigma = 12.75              | LCL:  -    3.0 sigma =  0
                                          |

Test Results: X-bar One point more than 3.00 sigmas from center line.
Test Failed at points: 4 6 7 10 12 15 16 20

Test Results for R Chart: One point more than 3.00 sigmas from center line.
```

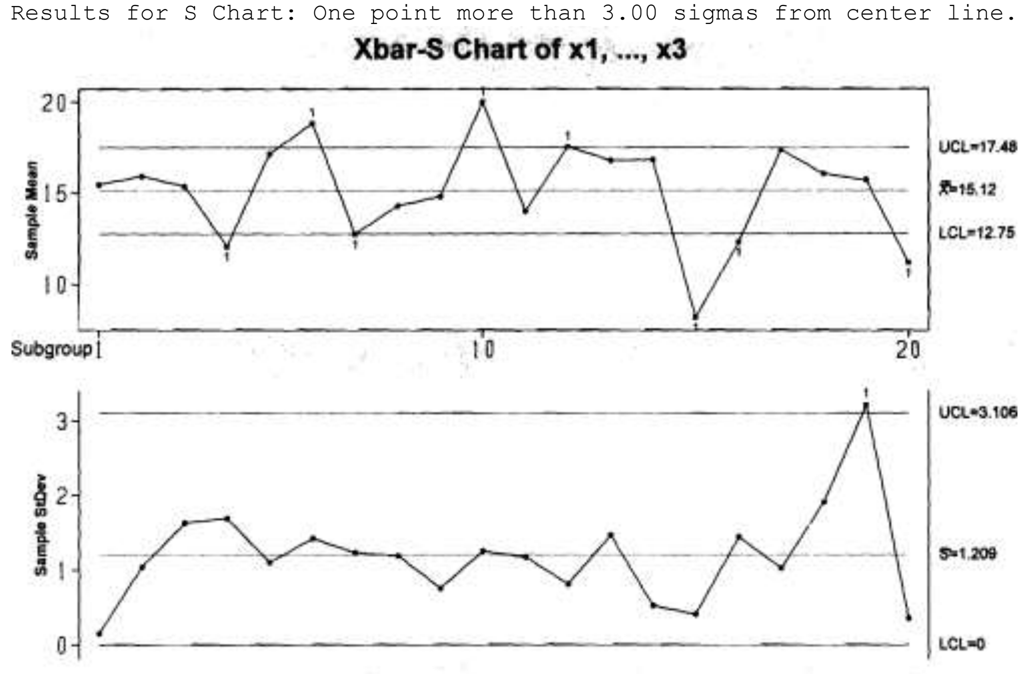


b) Removed points 4, 6, 7, 10, 12, 15, 16, and 20 and revised the control limits. The control limits are not as wide after being revised: *X*-bar UCL = 18.362, CL = 15.797, LCL = 13.232 and R UCL = 6.454, *R*-bar = 2.507, LCL = 0.

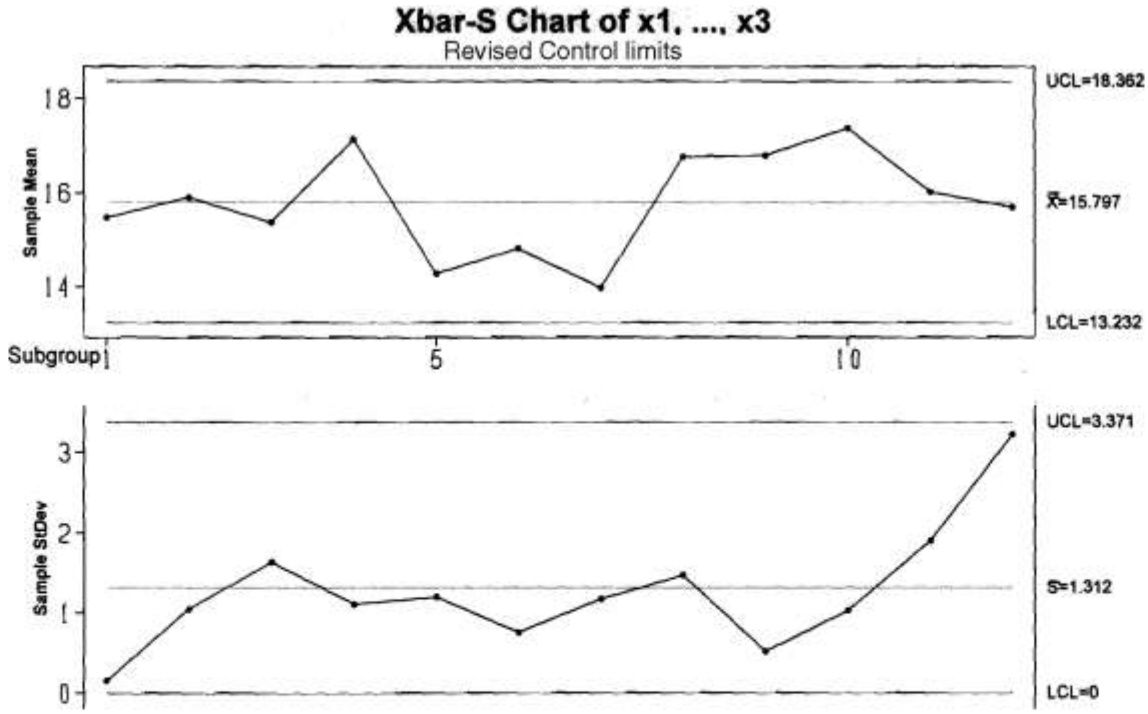


c)

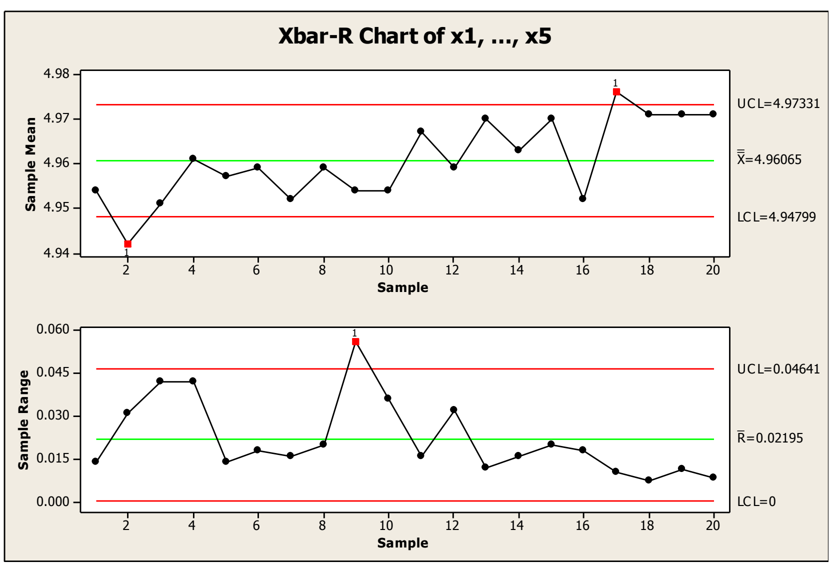
```
                        X-bar and StDev - Initial Study
                           Charting Problem 15-7
X-bar                                    | StDev
-----                                    | -----
UCL:  +    3.0 sigma = 17.48             | UCL:  +    3.0 sigma =  3.106
Centerline           = 15.12             | Centerline           =  1.209
LCL:  -    3.0 sigma = 12.75             | LCL:  -    3.0 sigma =  0
                                         |

Test Results: X-bar One point more than 3.00 sigmas from center line.
Test Failed at points: 4 6 7 10 12 15 16 20
Test Results for S Chart: One point more than 3.00 sigmas from center line.
```



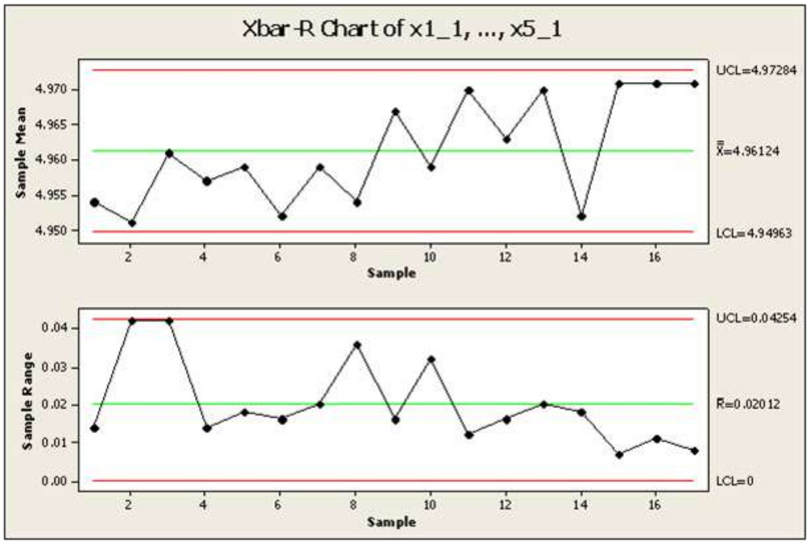
Removed points 4, 6, 7, 10, 12, 15, 16, and 20 and revised the control limits. The control limits are not as wide after being revised: *X*-bar UCL = 18.362, CL = 15.797, LCL = 13.232 and *S* UCL = 3.371, *S*-bar = 1.372, LCL = 0.



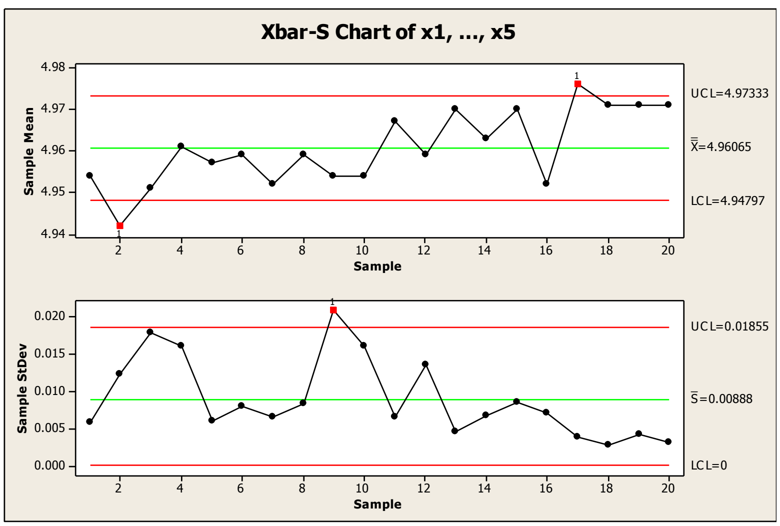
15-10 a) The average range is used to estimate the standard deviation. Samples 2, 9, and 17 are out-of-control.

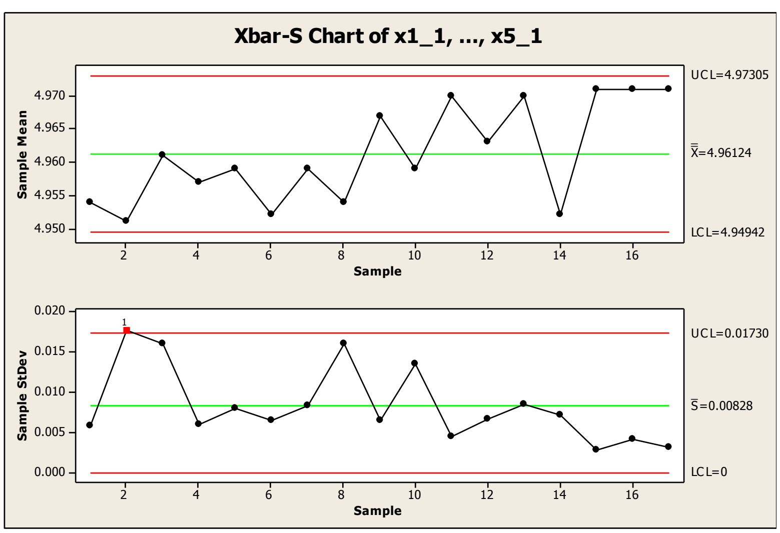


b)

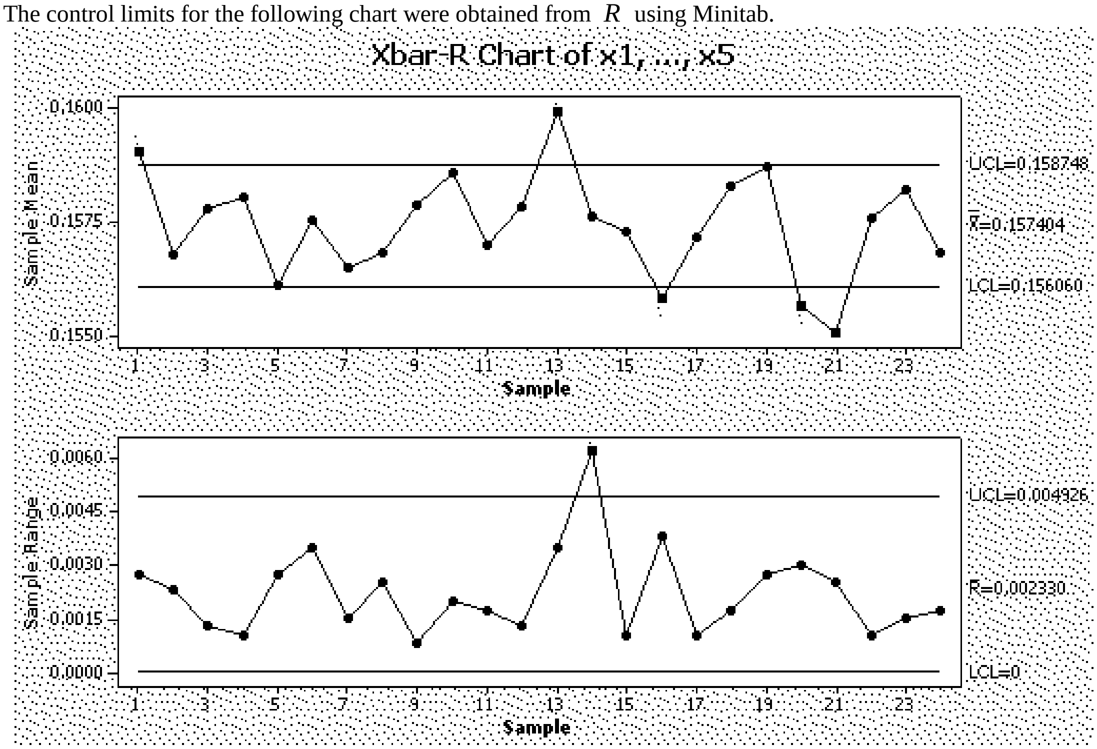


c) For $\overline{X}$ -S chart

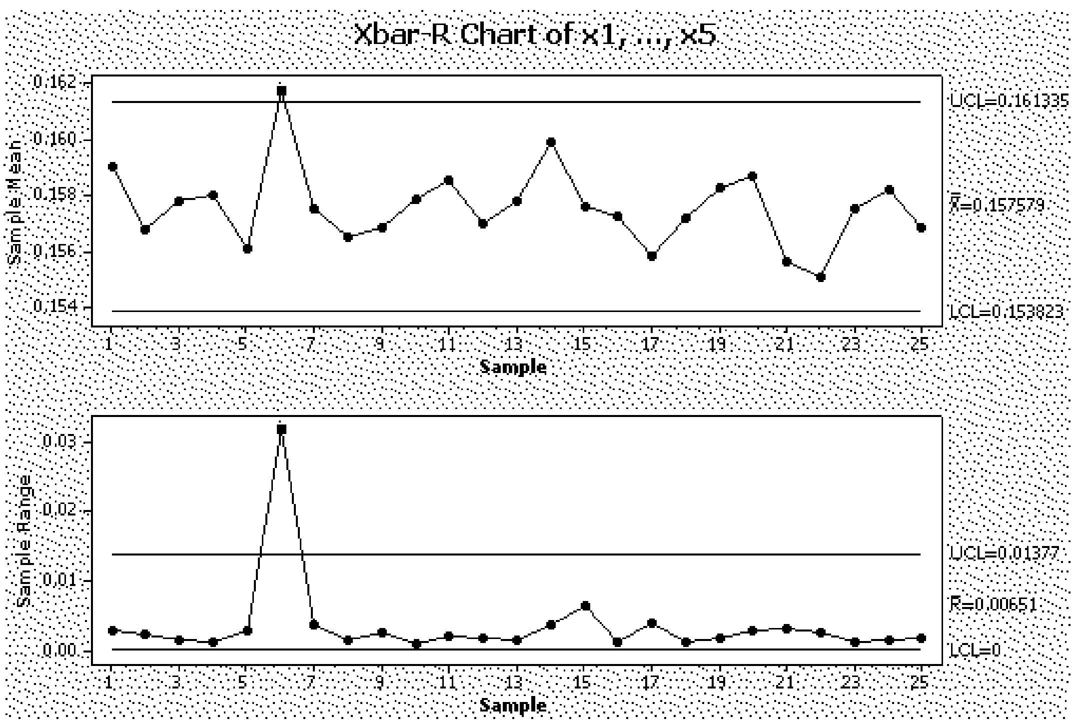




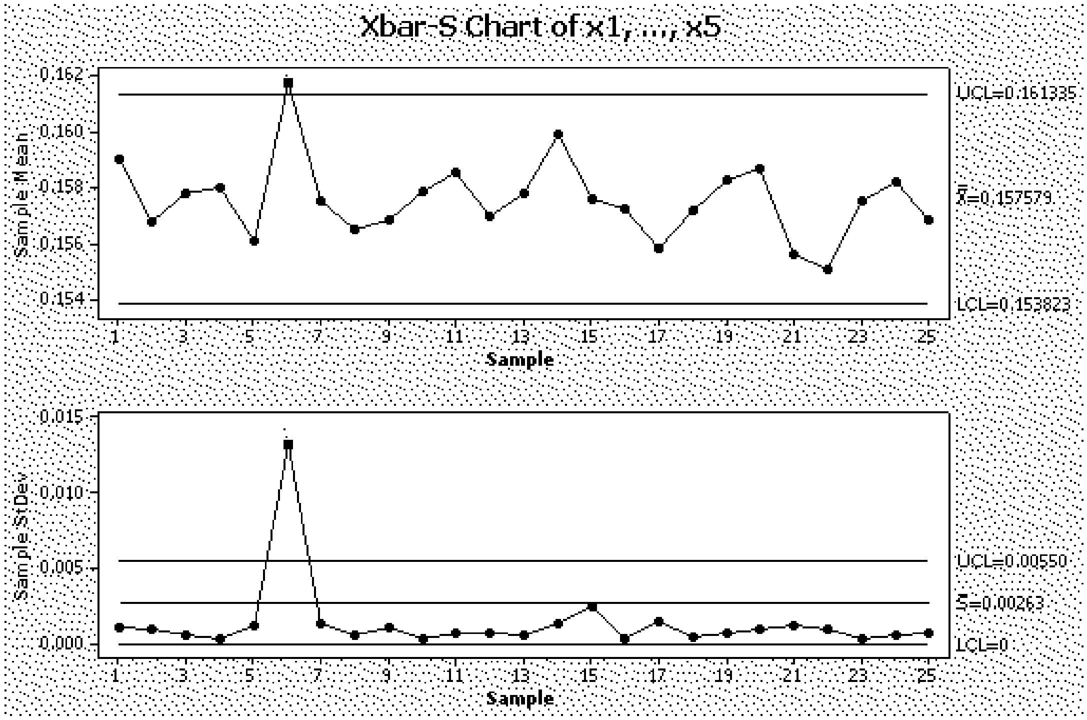
15-11 a) The control limits for the following chart were obtained from $\bar{R}$ using Minitab.



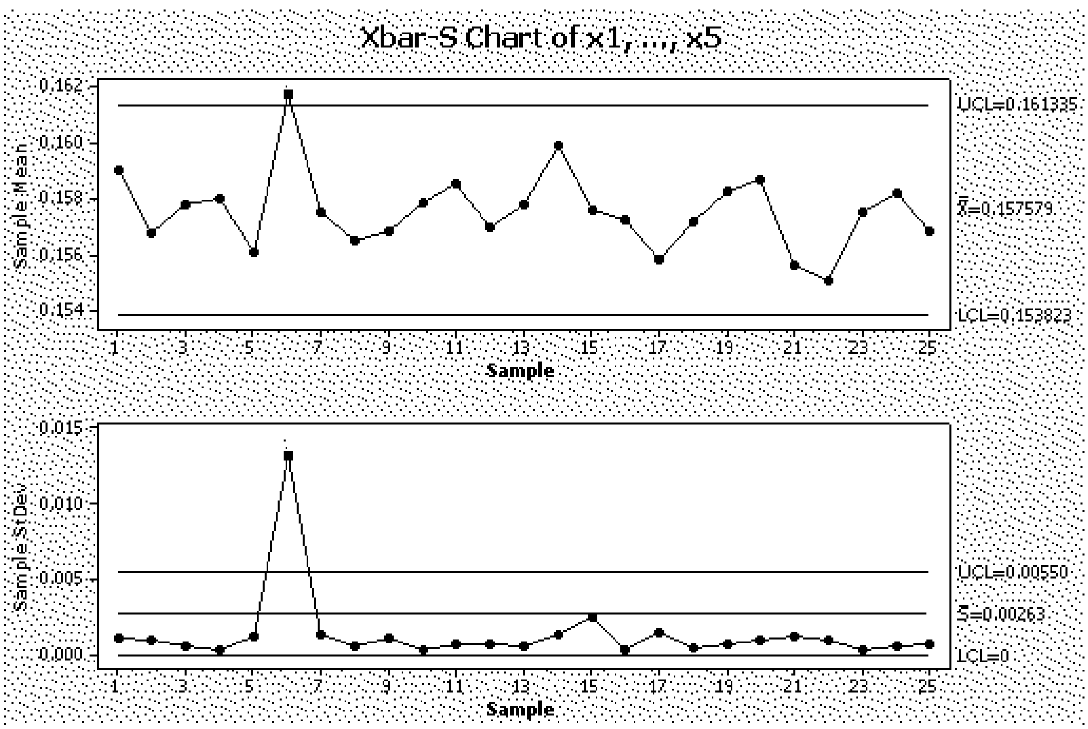
b) The test failed at point 6. The control limits are revised one time by omitting the out-of-control points. However, the charts still show additional out-of-control signals.



c) The control limits for the following charts were obtained from $\bar{S}$ using Minitab



d) The test failed at point 6. The control limits are revised one time by omitting the out-of-control point. However, the charts still show an out-of-control signal.



15-12

(a) $\bar{\bar{x}} = \dfrac{114.35}{24} = 4.76 \quad \bar{r} = \dfrac{8.59}{24} = 0.36$

The value of A_2 for samples of size 6 is $A_2 = 0.483$ from Appendix Table XI.

$\bar{X}$ *Chart*

$$UCL = CL + A_2\bar{r} = 4.76 + 0.483 * 0.36 = 4.93$$

$$UCL = CL - A_2\bar{r} = 4.76 - 0.483 * 0.36 = 4.59$$

R *Chart*

$$UCL = D_4\bar{r} = 2.004 * 0.36 = 0.72$$

$$UCL = D_3\bar{r} = 0 * 0.36 = 0$$

(b) $\bar{s} = \dfrac{3.65}{24} = 0.15$

$\bar{X}$ *Chart*

$$UCL = CL + \frac{3\bar{s}}{c_4\sqrt{n}} = 4.76 + \frac{3 * 0.15}{0.9515\sqrt{6}} = 4.95$$

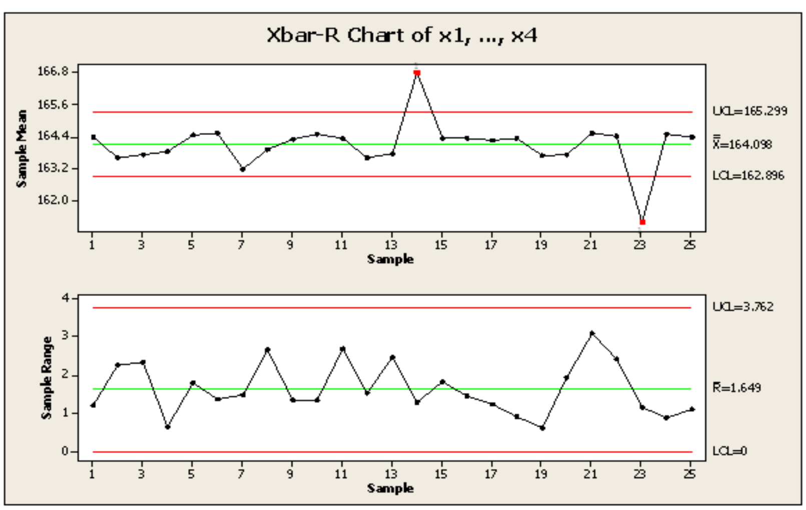
$$UCL = CL - \frac{3\bar{s}}{c_4\sqrt{n}} = 4.76 - \frac{3 * 0.15}{0.9515\sqrt{6}} = 4.57$$

S *Chart*

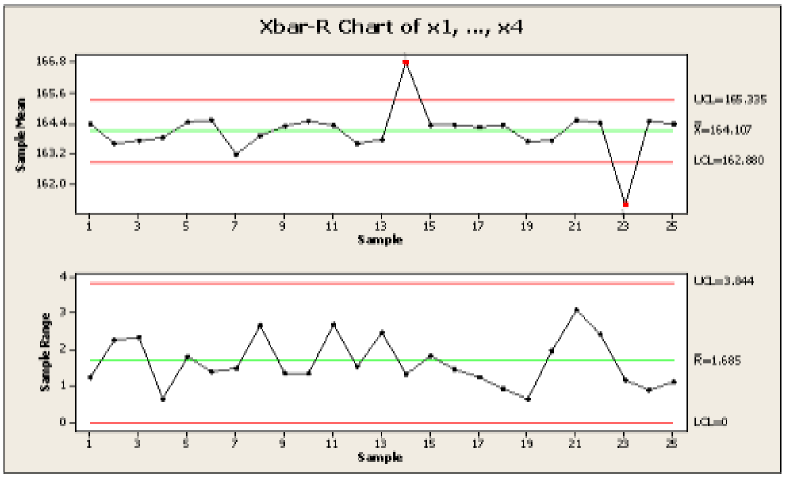
$$UCL = \bar{s} + \frac{3\bar{s}}{c_4}\sqrt{1 - c_4^2} = 0.15 + \frac{3 * 0.15}{0.9515}\sqrt{1 - 0.9515^2} = 0.295$$

$$UCL = \bar{s} - \frac{3\bar{s}}{c_4}\sqrt{1 - c_4^2} = 0.15 - \frac{3 * 0.15}{0.9515}\sqrt{1 - 0.9515^2} = 0.005$$

15-13 a) The control limits for the following chart were obtained from $\bar{R}$.



b) The test failed at points 14 and 23. The control limits are revised by omitting the out-of-control points from the control limit calculations.



An additional point is out-of-control and limits might be estimated again with this point eliminated.

15-14 For the $\overline{X}$ chart:

$\overline{X}$ *Chart*

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}} = 67.83$$

$$UCL = \mu - \frac{3\sigma}{\sqrt{n}} = 62.34$$

The difference $UCL - LCL = \dfrac{6\sigma}{\sqrt{n}} = 67.83 - 62.34 = 5.49$

$$n = (\frac{6*2.05}{5.49})^2 = 5$$

15-15 a)

$$\bar{\bar{x}} = 30.0 \quad \frac{\bar{r}}{d_2} = 1.5 \quad d_2 = 2.704 \quad \bar{r} = 1.5*2.704 = 4.056$$

$\overline{X}$ *Chart*

$$UCL = CL + A_2\bar{r} = 30 + 0.419*4.056 = 31.70$$

$$CL = 30$$

$$UCL = CL - A_2\bar{r} = 30 - 0.419*4.056 = 28.30$$

R *Chart*

$$UCL = D_4\bar{r} = 1.924*4.056 = 7.80$$

$$CL = 4.056$$

$$UCL = D_3\bar{r} = 0.076*4.056 = 0.31$$

b)

$$\bar{X} \quad Chart \quad where \quad \bar{s}/c_4 = 1.8$$

$$UCL = CL + \frac{3\bar{s}}{c_4\sqrt{n}} = 30 + \frac{3*1.8}{\sqrt{7}} = 32.04$$

$$CL = 30$$

$$UCL = CL - \frac{3\bar{s}}{c_4\sqrt{n}} = 30 - \frac{3*1.8}{\sqrt{7}} = 28.96$$

$$S \quad Chart \quad c_4 = 0.9594 \quad so \quad \bar{s} = 1.73$$

$$UCL = \bar{s} + \frac{3\bar{s}}{c_4}\sqrt{1-c_4^2} = 3.25$$

$$CL = 1.73$$

$$UCL = \bar{s} - \frac{3\bar{s}}{c_4}\sqrt{1-c_4^2} = 0.21$$

15-16 a) The difference $UCL - LCL = 6\hat{\sigma}_{\bar{X}} = 49.0 - 40.00 = 9.0$

Therefore, $\hat{\sigma}_{\bar{X}} = \frac{9.0}{6} = 1.5$ because the limits are six standard errors wide. Then $\hat{\sigma}_{\bar{X}} = \frac{\hat{\sigma}}{\sqrt{4}}$. Therefore, $\hat{\sigma} = 1.5 \times 2 = 3.0$.

b) No, the calculation in part (a) is valid regardless of the method used to construct the control chart.

15-17 a) The difference $UCL - LCL = 6\hat{\sigma}_X = 74.98 - 64.68 = 10.3$

Therefore, $\hat{\sigma}_X = \frac{10.3}{6} = 1.72$, because the limits are six standard errors wide. Then $\hat{\sigma}_X = \frac{\hat{\sigma}}{\sqrt{4}}$.

Therefore , $\hat{\sigma} = 1.72 * 2 = 3.44$.

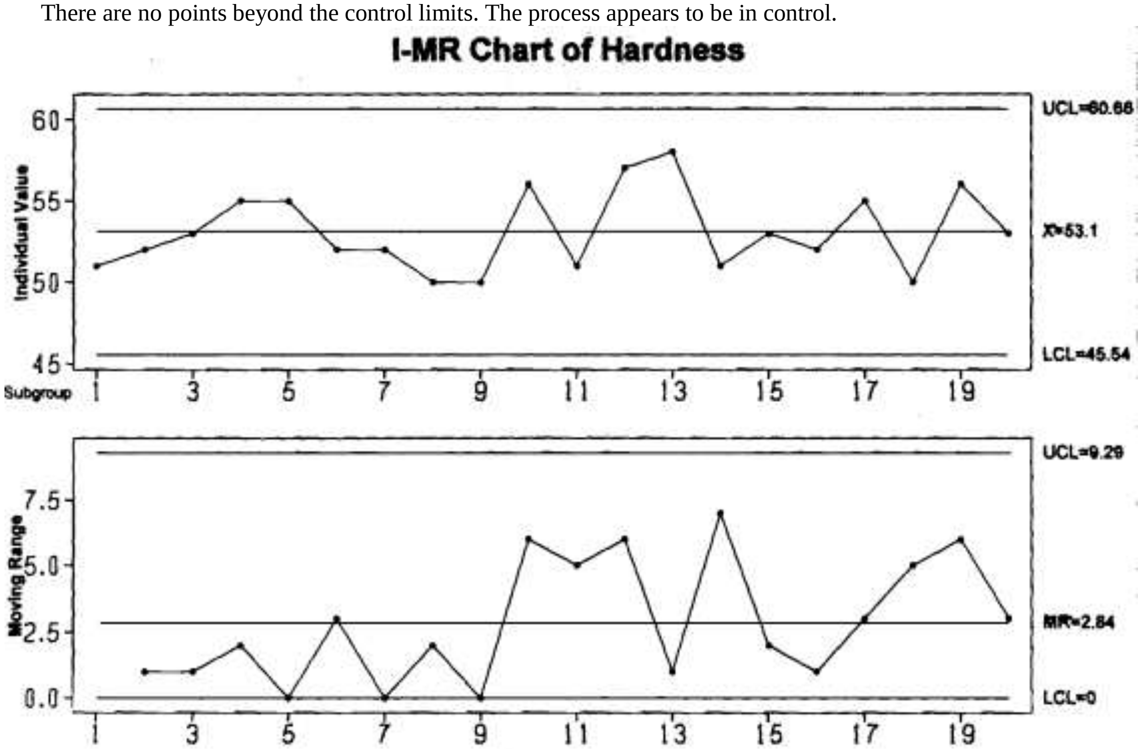
b) No, the calculation in part (a) is valid regardless of the method used to construct the control chart.

Section 15-4

15-18 a)

```
                        Individuals and MR(2) - Initial Study
-----------------------------------------------------------------
                             Charting Problem 15-18

Ind.x                                 | MR(2)
-----                                 | -----
UCL:  +    3.0 sigma = 60.66          | UCL:  +    3.0 sigma = 9.29
Centerline           = 53.1           | Centerline           = 2.84
LCL:  -    3.0 sigma = 45.54          | LCL:  -    3.0 sigma = 0
                                      |
out of limits = 0                     | out of limits = 0
-----------------------------------------------------------------
Chart: Both      Normalize: No

   20 subgroups, size 1                        0 subgroups excluded

Estimated
process mean  = 53.1
process sigma =  2.51960
mean MR       =  2.84
```

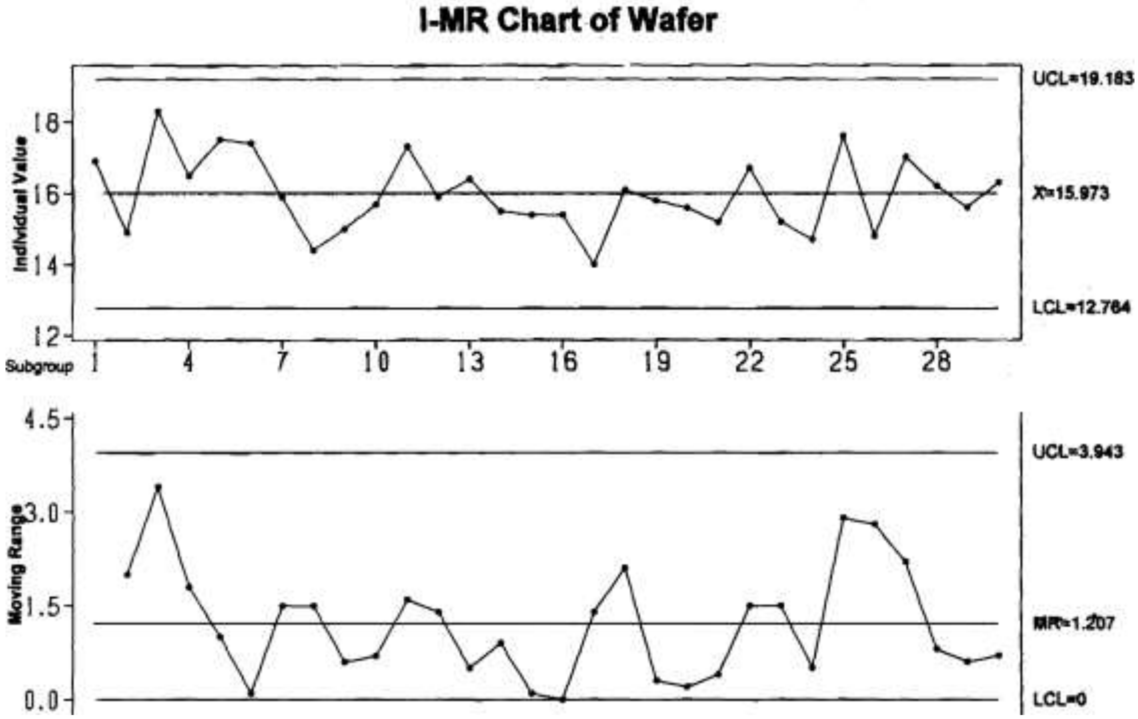
There are no points beyond the control limits. The process appears to be in control.



b) Estimated process mean and standard deviation

$$\hat{\mu} = \bar{x} = 53.1 \qquad \hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{2.84}{1.128} = 2.158$$

15-19 a) The process appears to be in statistical control. There are no points beyond the control limits.

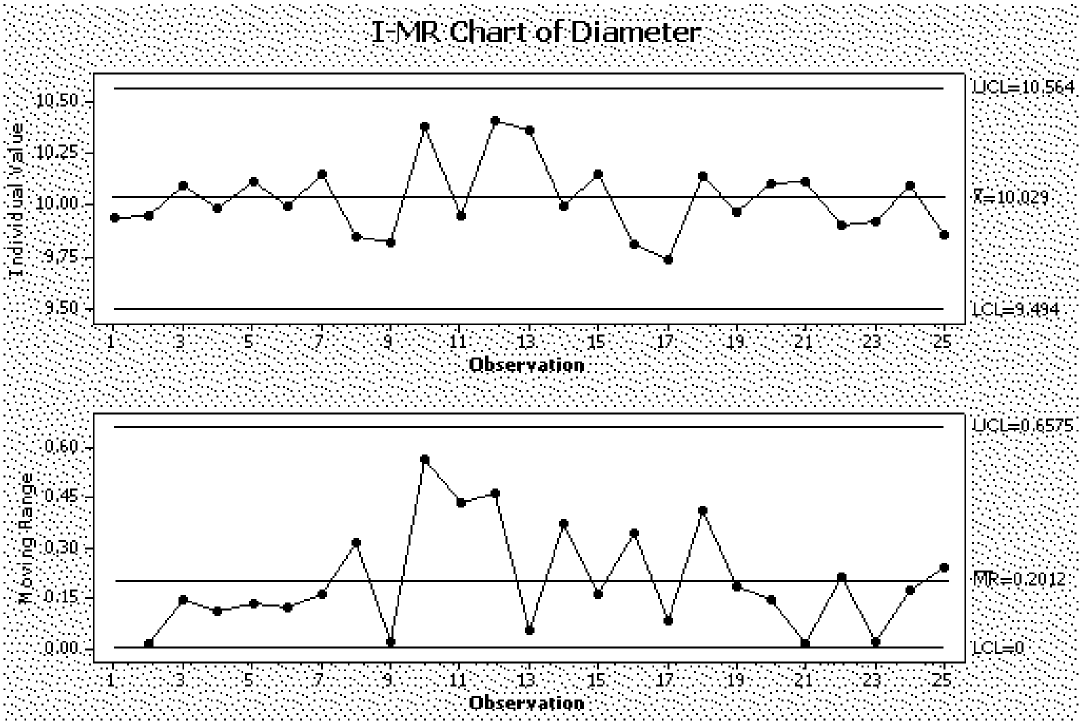


b) Estimated process mean and standard deviation

$$\hat{\mu} = \bar{x} = 15.973 \qquad \hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{1.207}{1.128} = 1.07$$

15-20 a) Ind.x and MR(2) - Initial Study

```
-----------------------------------------------------------------
                             Charting diameter

Ind.x                                | MR(2)
-----                                | -----
UCL:  +    3.0 sigma = 10.564        | UCL:  +    3.0 sigma = 0.6575
Centerline           = 10.029        | Centerline           = 0.2012
LCL:  -    3.0 sigma =  9.494        | LCL:  -    3.0 sigma = 0
                                     |
out of limits = 0                    | out of limits = 0
-----------------------------------------------------------------


   Chart: Both      Normalize: No

   25 subgroups, size 1                    0 subgroups excluded

Estimated
process mean  = 10.0288
process sigma =  0.178413
mean MR(2)    =  0.2012
```



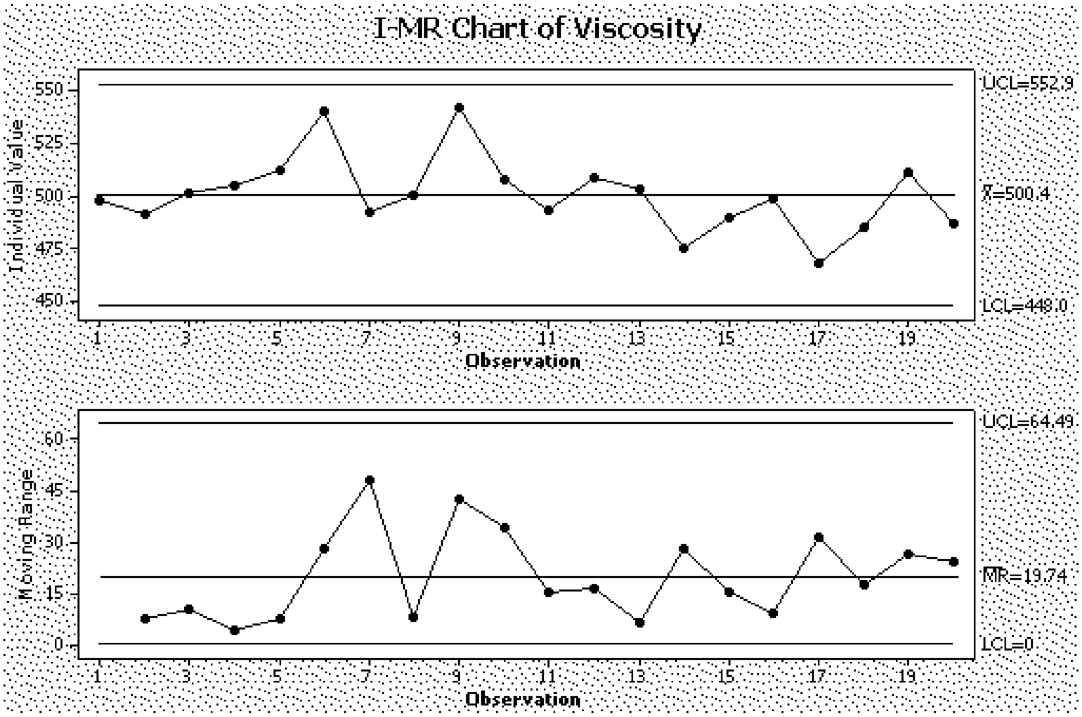
There are no points beyond the control limits. The process appears to be in control.

b) Estimated process mean and standard deviation

$$\hat{\mu} = \bar{x} = 10.029 \qquad \hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{0.2012}{1.128} = 0.17837$$

15-21 a)

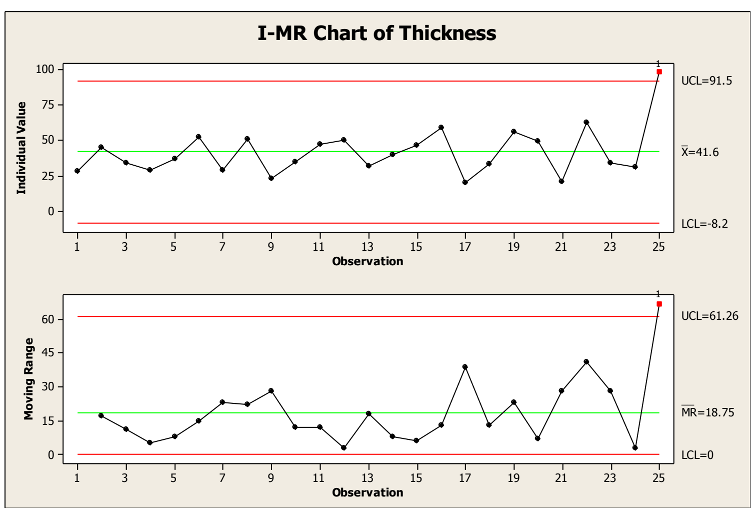
```
                        Ind.x and MR(2) - Initial Study
-----------------------------------------------------------------
                            Charting Problem 15-21

Ind.x                                | MR(2)
-----                                | -----
UCL:  +    3.0 sigma = 552.9         | UCL:  +    3.0 sigma = 64.49
Centerline           = 500.4         | Centerline           = 19.74
LCL:  -    3.0 sigma = 448           | LCL:  -    3.0 sigma =  0
                                     |
out of limits = 0                    | out of limits = 0
-----------------------------------------------------------------

Chart: Both      Normalize: No

   20 subgroups, size 1                        0 subgroups excluded
Estimated
process mean  = 500.45
process sigma =  17.4972
mean MR(2)    =  19.74
```



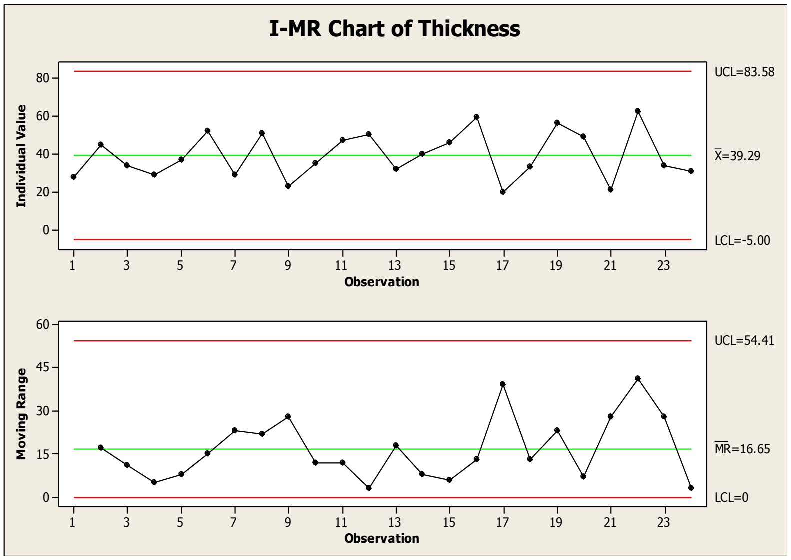
b) Estimated process mean and standard deviation

$$\hat{\mu} = \bar{x} = 500.45 \qquad \hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{19.74}{1.128} = 17.5$$

15-22 a) The process is out of control. The control charts follow.

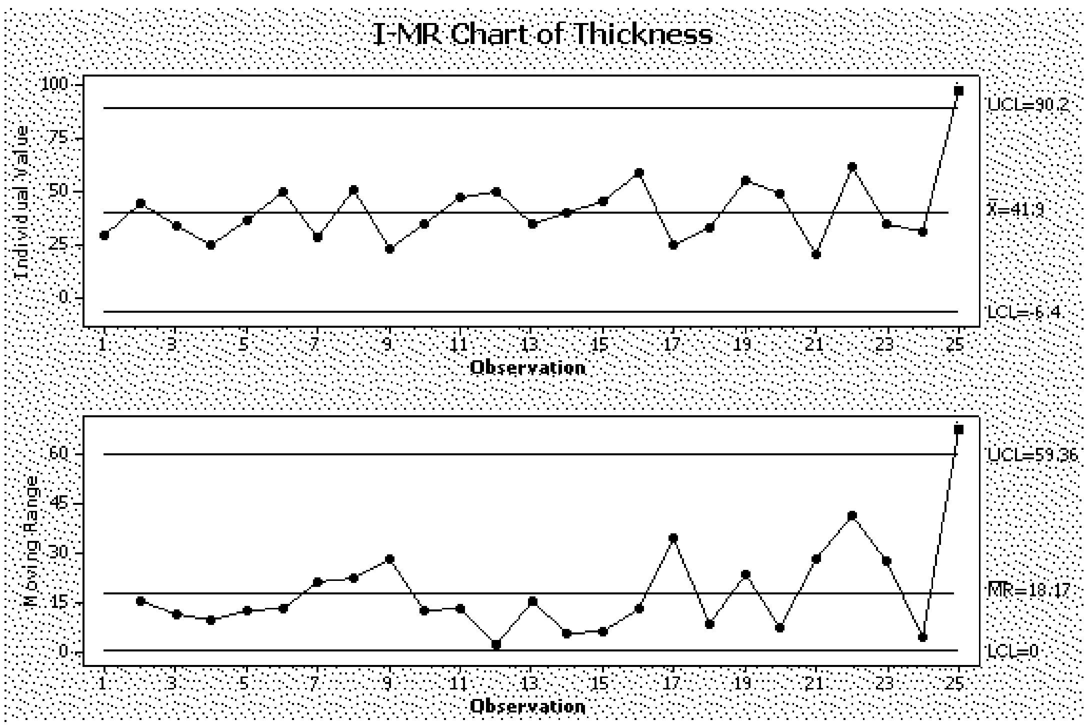


Remove the out-of-control observation 25:

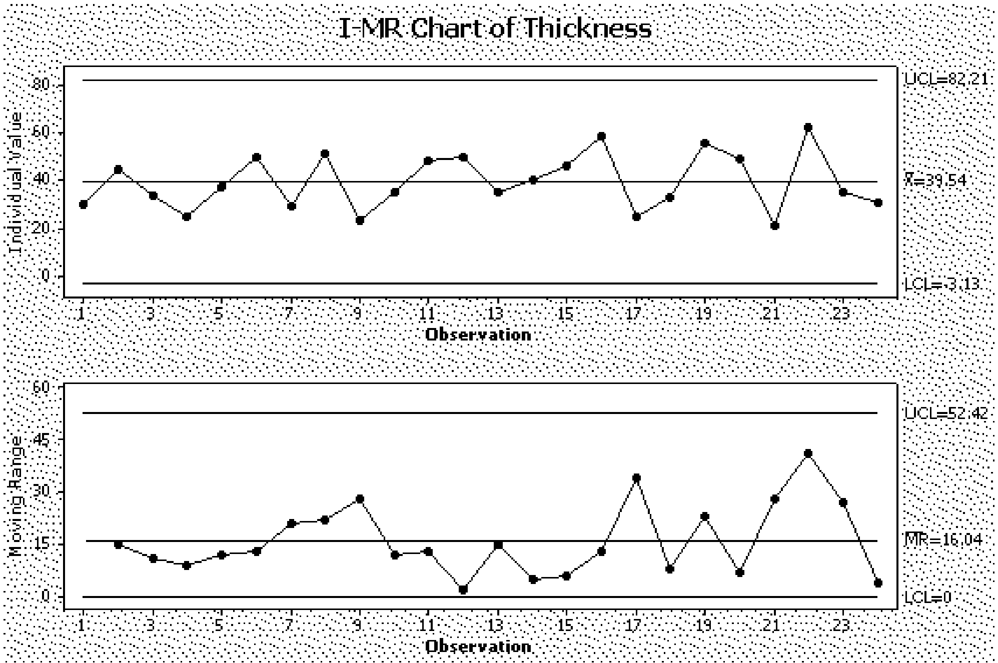


b) From the centerline of the x chart, $\hat{\mu} = 39.29$. Also, $\hat{\sigma} = \overline{MR}/d_2$ = 16.65/1.128 = 14.76

15-23 a) The process is out of control. The control charts from Minitab follow.



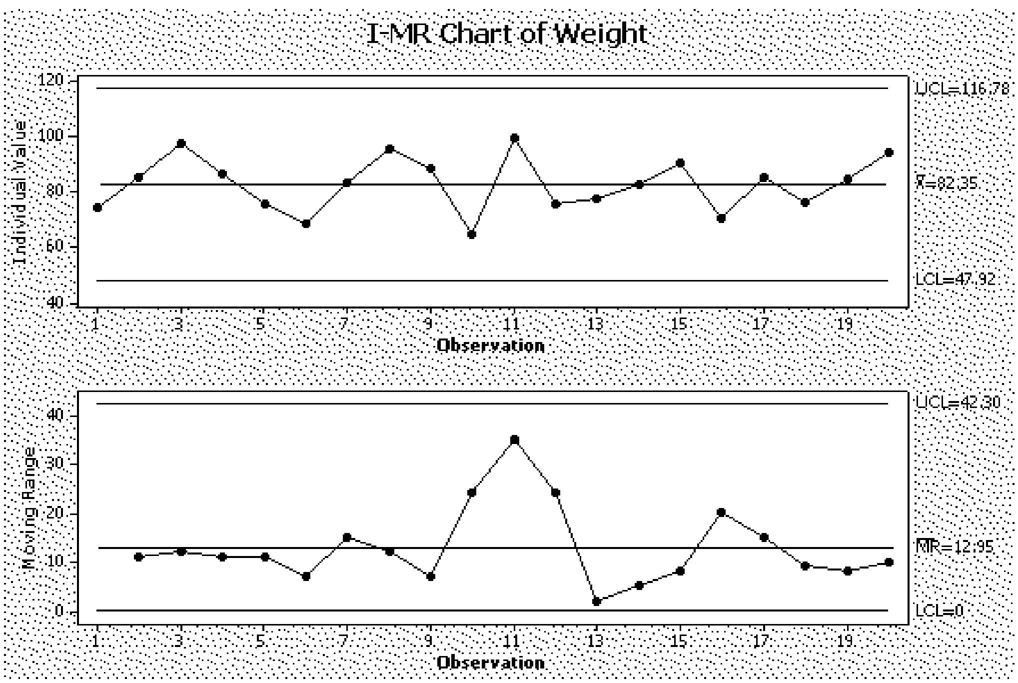
Remove the out-of-control observation 25:



b) From the centerline of the x chart, $\hat{\mu} = 39.59$.

Also, $\hat{\sigma} = \overline{MR}/d_2 = 16.04/1.128 = 14.22$

15-24 a) The process is in control. The control charts from Minitab follow.



b) From the centerline of the x chart, $\hat{\mu} = 82.$

From x chart the difference $UCL - LCL$ = 116.78 – 47.92 = 68.86 = 6 $\hat{\sigma}$

Therefore, $\hat{\sigma}$ = 68.86/6 = 11.48

Section 15-5

15-25 a) The natural tolerance limits are 90 ± 18.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{40}{36} = 1.1111$$

Since the mean of the process is not centered at the nominal dimension,

$$PCR_k = \min\left[\frac{10}{18}, \frac{30}{18}\right]$$

The small PCR_k indicates that the process is likely to produce units outside the specification limits.
The fraction defective is

$$\begin{aligned} P(X < LSL) + P(X > USL) &= P\left(Z < \frac{-10}{6}\right) + P\left(Z > \frac{30}{6}\right) \\ &= P(Z < -1.67) + P(Z > 5) \\ &= 0.04806 + 0 \\ &= 0.04806 \end{aligned}$$

b) The natural tolerance limits are 100 ± 18.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{40}{36} = 1.1111$$

Since the mean of the process is centered at the nominal dimension,
PCR = 1.1111
Since the process natural tolerance limits lie inside the specifications, very few defective units will be produced.
The fraction defective is $2\Phi(-20/6) = 0.0858\%$

15-26 a) $PCR = \frac{USL - LSL}{6\sigma} = \frac{15}{6\sigma} = 1.5$ so $\sigma = 1.6667$

b) (20 + 35)/2 = 27.5 When the process is centered at the nominal dimension, the fraction defective is minimized for any σ.

15-27 a) The natural tolerance limits are 25 ± 6.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{30}{12} = 2.50$$

Since the mean of the process is not centered at the nominal dimension,

$$PCR_k = \min\left[\frac{21}{6}, \frac{9}{6}\right] = 1.5$$

Since the process natural tolerance limits lie inside the specifications, very few defective units will be produced.
The fraction defective is $2\Phi(-15/2) = 0$

b) The natural tolerance limits are 28 ± 6.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{30}{12} = 2.5$$

Since the mean of the process is not centered at the nominal dimension,

$$PCR_k = \min\left[\frac{18}{6}, \frac{12}{6}\right] = 2$$

The fraction defective is $P(X < LSL) + P(X > USL) = P(Z < -18/2) + P(Z > 12/2) = 0 + 0 = 0$

c) The measure of actual capability decreases and the fraction defective remains the same when the process mean is shifted from the center of the specification limits.

15-28 a) If the process uses 69% of the specification band, then 6σ = 0.69(USL − LSL). Because the process is centered

$$3\sigma = 0.69(USL - \mu) = 0.69(\mu - LSL) = 0.69(USL - \mu)$$

$$4.35\sigma = USL - \mu = \mu - LSU$$

$$PC = PCR_k \quad = \min\left[\frac{4.35\sigma}{3\sigma}, \frac{4.35\sigma}{3\sigma}\right] = 1.45$$

Because *PCR* and PCR_k exceed unity, the natural tolerance limits are inside the specification limits and few defective units should be produced.

b) Assuming a normal distribution with 6σ = 0.69(USL − LSL) and a centered process, then 3σ = 0.69(USL − μ). Consequently, USL − μ = 4.35σ and μ − LSL = 4.35σ

$$P(X > USL) = P\left(Z > \frac{4.35\sigma}{\sigma}\right) = P(Z > 4.35) = 1 - P(Z < 4.35)$$
$$= 1 - 1 = 0$$

By symmetry, the fraction defective is 2[P(X > USL)] = 0.

15-29 a) If the process uses 90% of the spec band then 6σ = 0.9(USL − LSL) and $PCR = \dfrac{USL - LSL}{0.9(USL - LSL)} = \dfrac{1}{0.9} = 1.11$

Then 3σ = 0.9(USL − μ) = 0.9(μ − LSL)
Therefore,

$$PCR_k = \min\left[\frac{3.33\sigma}{3\sigma}, \frac{3.33\sigma}{3\sigma}\right] = 1.11$$

Because *PCR* and PCR_k exceed unity, the natural tolerance limits are inside the specification limits and few defective units should be produced.

b) Assuming a normal distribution with 6σ = 0.9(USL − LSL) and a centered process, then 3σ = 0.9(USL − μ). Consequently, USL − μ = 3.33σ and μ − LSL = 3.33σ

$$P(X > USL) = P\left(Z > \frac{3.33\sigma}{\sigma}\right) = P(Z > 3.33) = 1 - P(Z < 3.33)$$
$$= 1 - 0.999566 = 0.000434$$

By symmetry, the fraction defective is 2[P(X > USL)] = 0.000868

15-30 Assume a normal distribution with $\hat{\mu} = 216.81$ and $\hat{\sigma} = \dfrac{33.33}{2.534} = 13.15$

$$P\left(X < LSL\right) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z < \frac{170 - 216.81}{13.15}\right) = P\left(Z < -3.56\right)$$
$$= 0.000185$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z > \frac{270 - 216.81}{13.15}\right) = P(Z > 4.04)$$
$$= 0$$

Therefore, the proportion nonconforming is given by
P(X < LSL) + P(X > USL) = 0.000185 + 0 = 0.000185

$$PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{270 - 170}{6(13.15)} = 1.267$$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \ \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{270 - 216.81}{3(13.15)}, \ \frac{216.81 - 170}{3(13.15)}\right] \\ &= \min[1.348, \ 1.187] \\ &= 1.187 \end{aligned}$$

The process capability is marginal.

15-31 a) Assume a normal distribution with $\hat{\mu} = 10.076$ and $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{0.239}{2.534} = 0.094$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z < \frac{9.5 - 10.076}{0.094}\right) = P(Z < -6.13)$$
$$= 0$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z > \frac{10.5 - 10.076}{0.094}\right) = P(Z > 4.51)$$
$$= 0$$

Therefore, the proportion nonconforming is given by P(X < LSL) + P(X > USL) = 0 + 0 = 0

b) $PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{10.5 - 9.5}{6(0.094)} = 1.77$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \ \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{10.5 - 10.076}{3(0.094)}, \ \frac{10.076 - 9.5}{3(0.094)}\right] \\ &= \min[1.50, \ 2.04] \\ &= 1.5 \end{aligned}$$

Because PCR and PCR_k exceed unity, the natural tolerance limits are inside the specification limits and few defective units should be produced.
Because $PCR_k \cong PCR$ the process appears to be centered.

15-32 a) Assume a normal distribution with $\hat{\mu} = 178.4$ and $\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{10.864}{0.94} = 11.56$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z < \frac{140 - 178.4}{11.56}\right) = P(Z < -3.32)$$
$$= 0.00045$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z > \frac{260 - 178.4}{11.56}\right) = P(Z > 7.05)$$
$$= 0$$

Probability of producing a part outside the specification limits is 0.00045 + 0 = 0.00045

$$PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{260 - 140}{6(11.56)} = 1.73$$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}},\ \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{260 - 178.4}{3(11.56)},\ \frac{178.4 - 140}{3(11.56)}\right] \\ &= \min\left[2.35,\ 1.11\right] \\ &= 1.11 \end{aligned}$$

Because *PCR* and PCR_k exceed unity, the natural tolerance limits are inside the specification limits and few defective units should be produced. The estimated proportion nonconforming is given by P(X < LSL) + P(X > USL) = 0.00045 + 0 = 0.00045

15-33 Assuming a normal distribution with $\hat{\mu} = 20.0$ and $\hat{\sigma} = 1.4$

$$PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{25 - 15}{6(1.4)} = 1.19$$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \hat{\mu}}{3\hat{\sigma}},\ \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{25 - 20}{3(1.4)},\ \frac{20 - 15}{3(1.4)}\right] \\ &= \min\left[1.19,\ 1.19\right] \\ &= 1.19 \end{aligned}$$

The process is capable.

15-34 a) $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{5.74}{2.326} = 2.468 \quad \hat{\sigma} = 0.0002468$

$$PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{1.2611 - 1.2561}{6(0.0002468)} = 3.38$$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}},\ \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{1.2611 - 1.2586}{3(0.0002468)},\ \frac{1.2586 - 1.2561}{3(0.0002468)}\right] \\ &= \min\left[3.38,\ 3.38\right] \\ &= 3.38 \end{aligned}$$

Because *PCR* and PCR_k exceed unity, the natural tolerance limits are inside the specification limits and few defective units should be produced.
Because $PCR_k = PCR$ the process is centered.

b) Assume a normal distribution with $\hat{\mu} = 1.2586$ and $\hat{\sigma} = 0.0002468$

$$P\left(X < LSL\right) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right) = P\left(Z < -10.13\right) = 0$$

$$\begin{aligned} P(X > USL) &= P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right) = P(Z > 10.13) = 1 - P(Z < 4.46) \\ &= 0 \end{aligned}$$

Therefore, the proportion nonconforming is given by P(X < LSL) + P(X > USL) = 0 + 0 = 0

15-35 Assuming a normal distribution with $\hat{\mu} = 6.291$ and $\hat{\sigma} = \dfrac{1.33}{1.693} = 0.786$

$$PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{8-4}{6(0.786)} = 0.85$$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \overline{\overline{x}}}{3\hat{\sigma}}, \frac{\overline{\overline{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{8 - 6.291}{3(0.786)}, \frac{6.291 - 4}{3(0.786)}\right] \\ &= \min\left[0.725,\ 0.972\right] \\ &= 0.725 \end{aligned}$$

The process capability is poor.

15-36

$$\hat{\sigma} = \frac{\overline{r}}{d_2} = \frac{2.31}{1.693} = 1.364$$

$$\overline{x} = 15.12$$

$$P(X > 13) + P(X < 7)$$

$$= P\left(Z > \frac{15 - 15.12}{1.693}\right) + P\left(Z < \frac{5 - 15.12}{1.693}\right)$$

$$= P(Z > -1.25) + P(Z < -4.796)$$

$$= 0.8944 + 0.0 = 0.8944$$

$$PCR = \frac{13-7}{6(1.693)} = 0.591$$

Because the estimated *PCR* is less than unity, the process capability is not good.

15-37 Assuming a normal distribution with $\hat{\mu} = 500.45$ and $\hat{\sigma} = 17.497$

$$PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{530 - 470}{6(17.497)} = 0.572$$

$$\begin{aligned} PCR_k &= \min\left[\frac{USL - \overline{\overline{x}}}{3\hat{\sigma}}, \frac{\overline{\overline{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{530 - 500.45}{3(17.497)}, \frac{500.45 - 470}{3(17.497)}\right] \\ &= \min\left[0.563,\ 0.58\right] \\ &= 0.563 \end{aligned}$$

Because the process capability ratios are less than unity, the process capability appears to be poor.

15-38 a) The natural tolerance limits are 120 ± 3(6.5) = (100.5, 139.5)
The fraction conforming is
P(110 < X < 130) = P[(110 – 120)/6.5 < Z < (130 – 120)/6.5] = P[–1.5385 < Z < 1.5385] = 0.87644
Therefore the fraction defective = 1 – 0.87644 = 0.12356
PCR = 20/(6 × 6.5) = 0.513
PCR_k = 0.513 because the process is centered within the specifications.

b) The shift is 1.5 × 6.5 = 9.75. The natural tolerance limits are 129.75 ± 3(6.5) = (110.25, 149.25)
The fraction conforming is
P(110 < X < 130) = P[(110 – 129.75)/6.5 < Z < (130 – 129.75)/6.5] = P[–3.04 < Z < 0.04] = 0.51477
Therefore, the fraction defective = 1 – 0.51477 = 0.48523
PCR remains the same = 20/(6 × 6.5) = 0.513.
The nearest specification to the process mean is 130. Therefore,
PCR_k = (130 – 129.75)/(3 × 6.5) = 0.0128

c) The fraction defective increases in part (b) when the process mean shifts from the center of the specifications. This change is reflected in the decreased PCR_k.

15-39 a) The natural tolerance limits are 150 ± 3(σ) = 150 ± 18
The standard deviation = 6.

b) PCR = 40/ (6 × 6) = 1.11
PCR_k = 1.11 because the process is centered within the specifications.
The process width = 18 and the specification width = 20. The percentage of the specification width used by the process = 18/20 = 90%
c) The fraction conforming is
P (130 < X < 170) = P [(130 – 150)/6 < Z < (170 – 150)/6] = P [-3.33 < Z < 3.33] = 0.9991
The fraction defective = 1 – the fraction conforming = 1- 0.9991 = 0.0009

15-40 a) For the $\bar{x}$ chart: The difference $UCL - LCL = 6\hat{\sigma}_{\overline{X}} = 28.8 - 24.6 = 4.2$

Therefore, $\hat{\sigma}_{\overline{X}} = \dfrac{4.2}{6} = 0.7$ and $\hat{\sigma} = 0.7\sqrt{4} = 1.4$

b) PCR = (32 – 24)/(6 × 1.4) = 0.9524
The control charts are centered at (28.8 + 24.6)/2 = 26.7 so this value estimates the process mean.
PCR_k = (26.7 – 24)/(3 × 1.4) = 0.6429

15-41 a) The difference $UCL - LCL = 6\hat{\sigma} = 1.80 - 1.62 = 0.18$

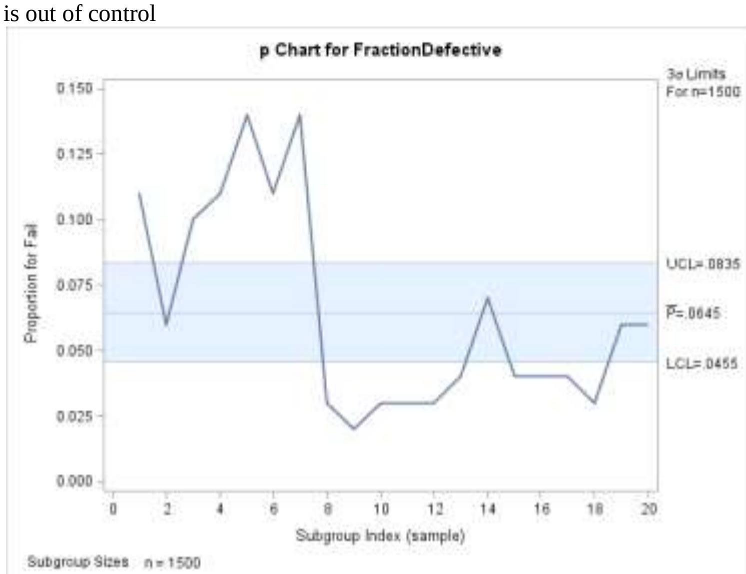
Therefore, $\hat{\sigma} = \dfrac{0.18}{6} = 0.03$

b) PCR = (1.84 – 1.64)/(6 × 0.03) = 1.11

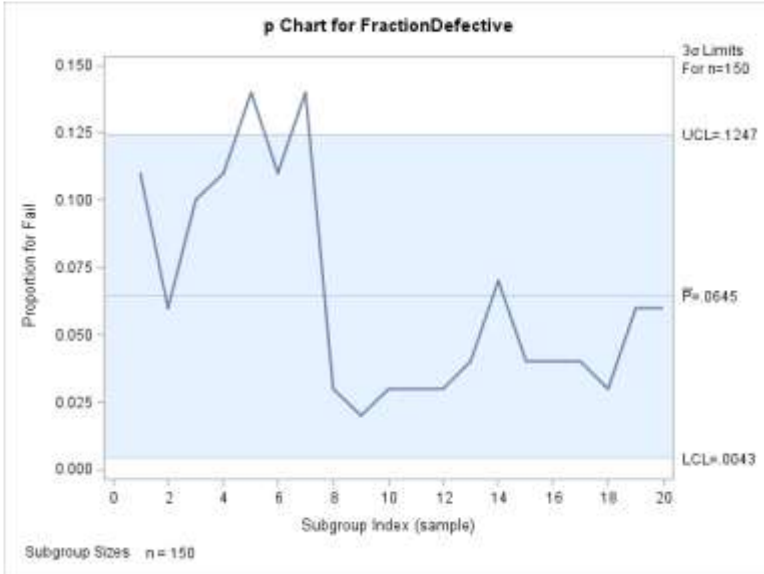
The control charts are centered at (1.80 + 1.62)/2 = 1.71 so this value estimates the process mean.
PCR_k = (1.71 – 1.64)/(3 × 0.03) = 0.778

<u>Section 15-6</u>

15-42 a) This process is out of control

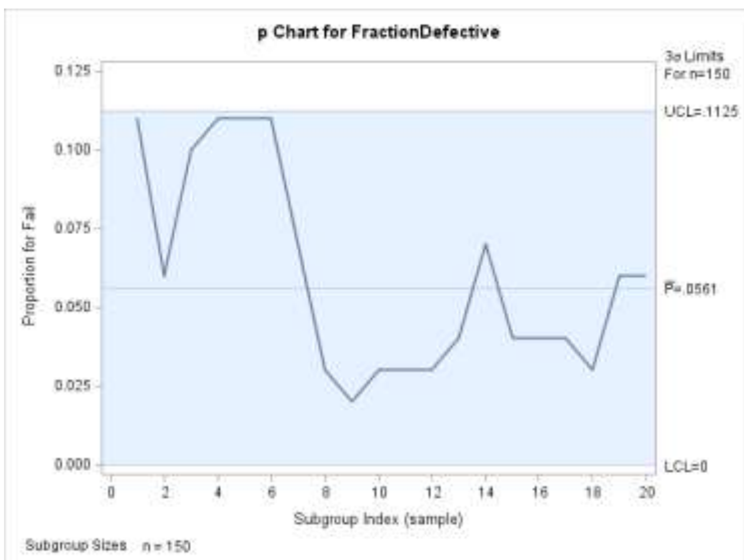


b)



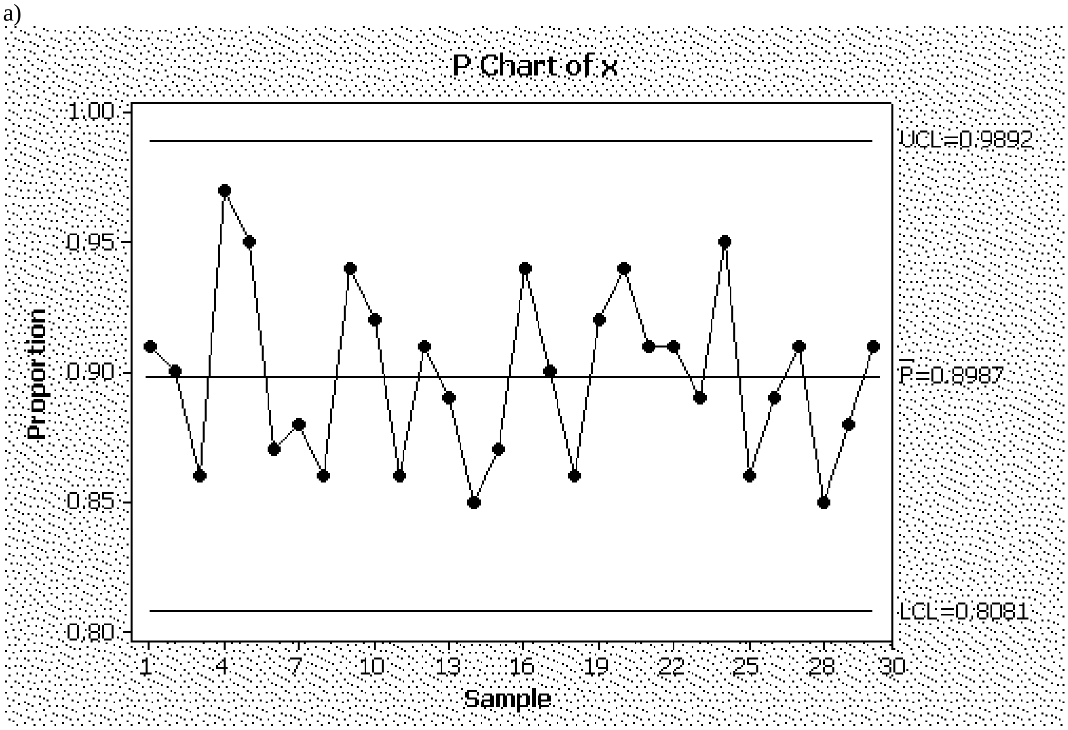
The process is still out of control, but not as many points fall outside of the control limits. The control limits are wider for smaller values of *n*.
Test Failed at points 5 and 7.

The following chart eliminates points 5 and 7.



c) The larger sample size leads to a smaller standard deviation for the proportions and. Thus. narrower control limits.

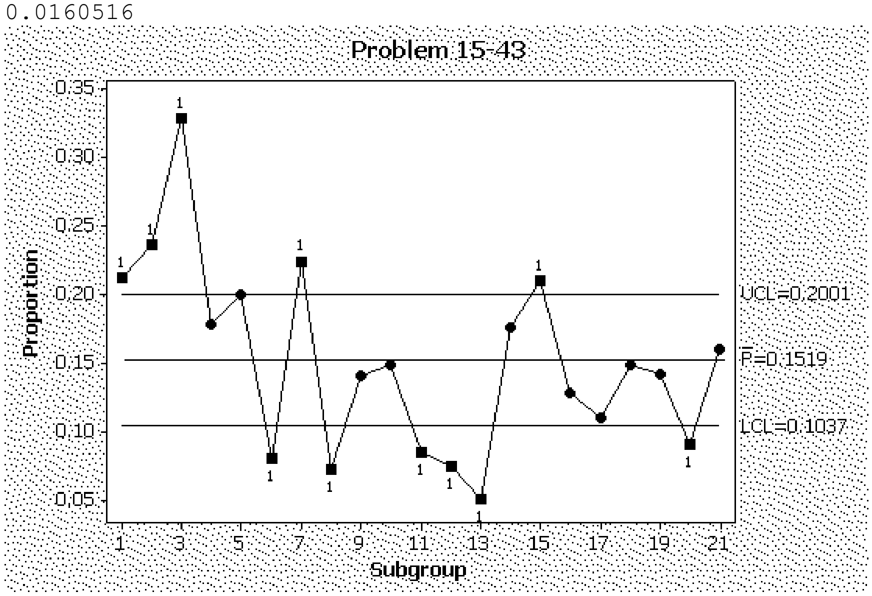
15-43 a)



b) The process appears to be in statistical control.

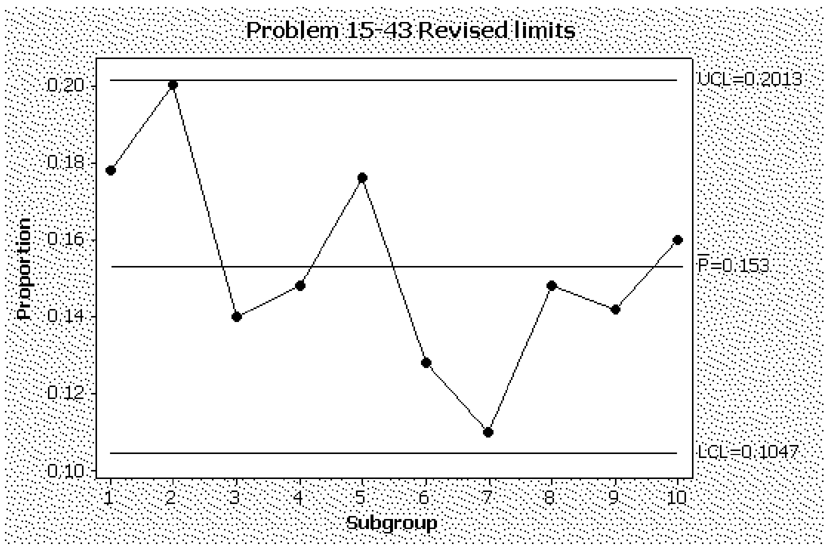
15-44

```
                         P Chart - Initial Study
                           Charting Problem 15-44
P Chart
-----
UCL:  +    3.0 sigma = 0.2001
Centerline           = 0.1519
LCL:  -    3.0 sigma = 0.1037

out of limits = 12
Estimated
mean P  = 0.1519
sigma   = 0.0160516
```



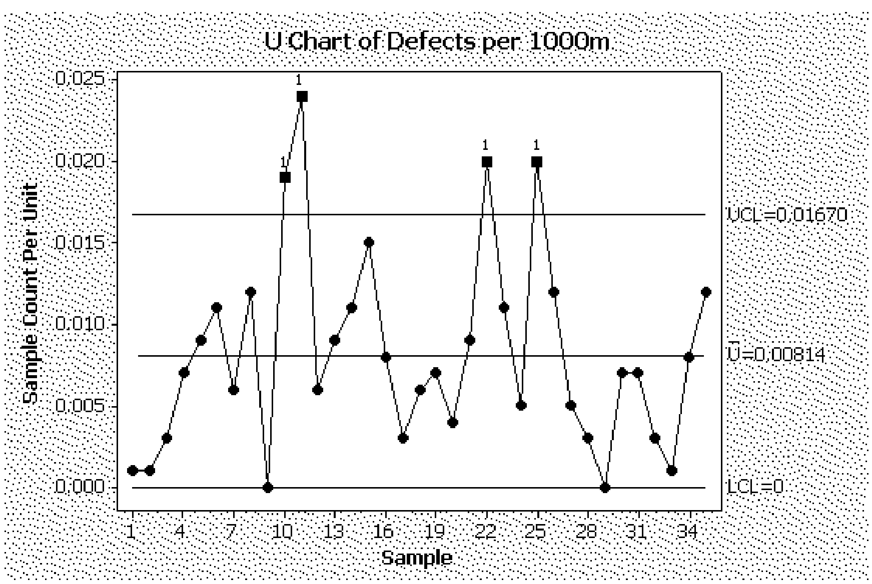
The samples with out-of-control points are 1, 2, 3, 6, 7, 8, 11, 12, 13, 15, and 20. The control limits need to be revised.

```
                      P Chart - Revised Limits
                       Charting Problem 15-44
P Chart
-----
UCL:  +    3.0 sigma = 0.2013
Centerline           = 0.153
LCL:  -    3.0 sigma = 0.1047
out of limits = 0
Estimated
mean P  = 0.153
sigma   = 0.0160991
```

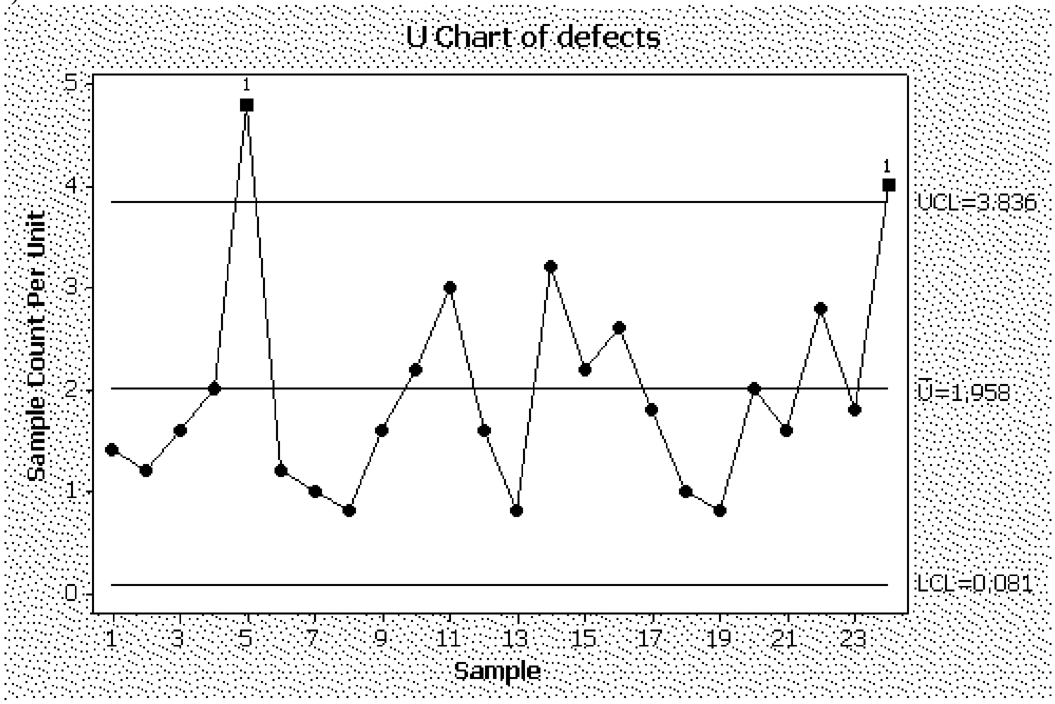


There are no further points out of control for the revised limits.

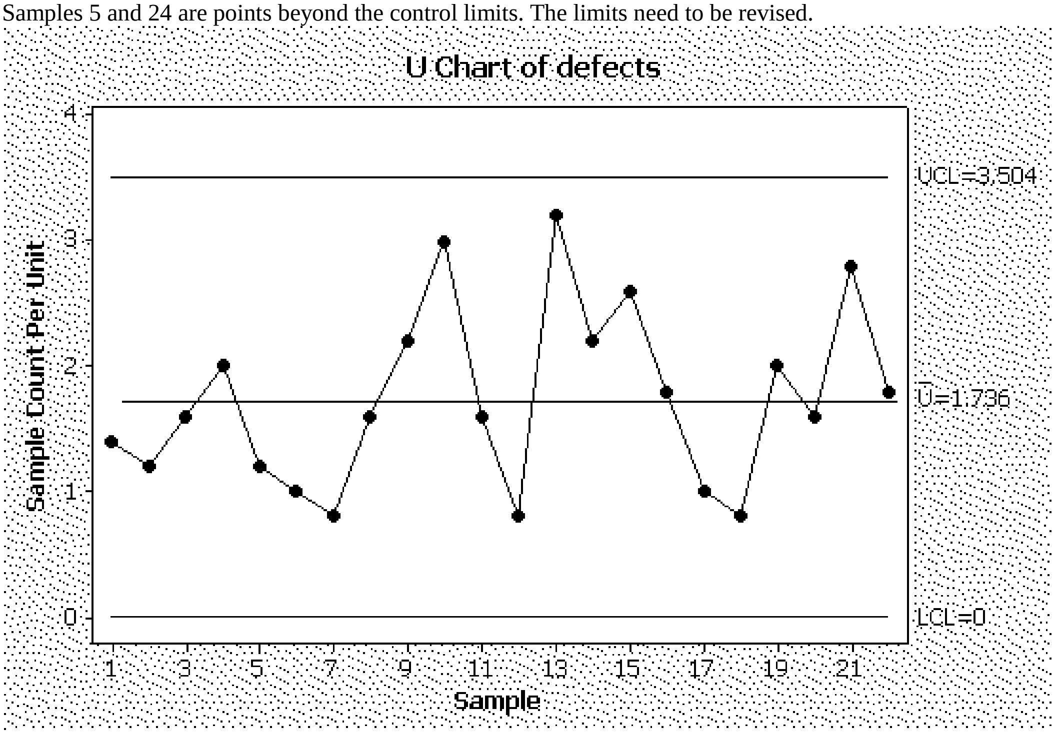
15-45 The process does not appear to be in control.



15-46 a)

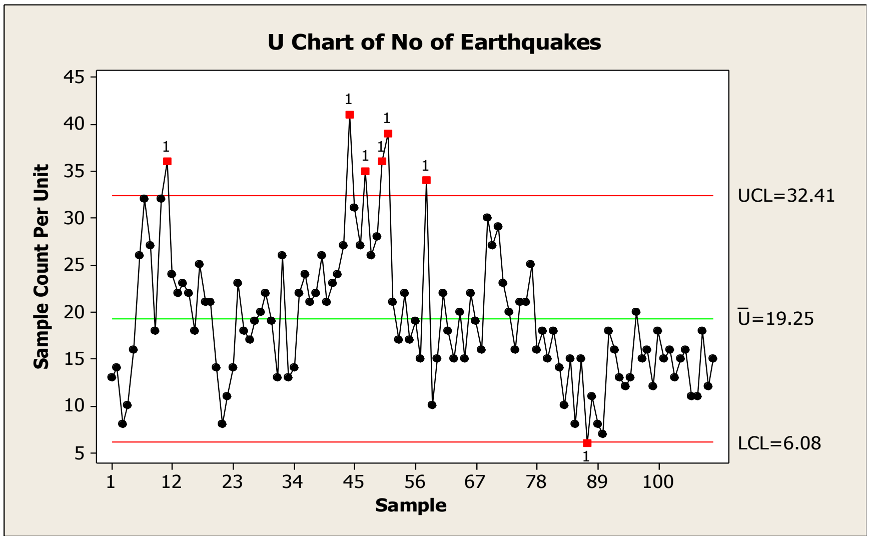


Samples 5 and 24 are points beyond the control limits. The limits need to be revised.

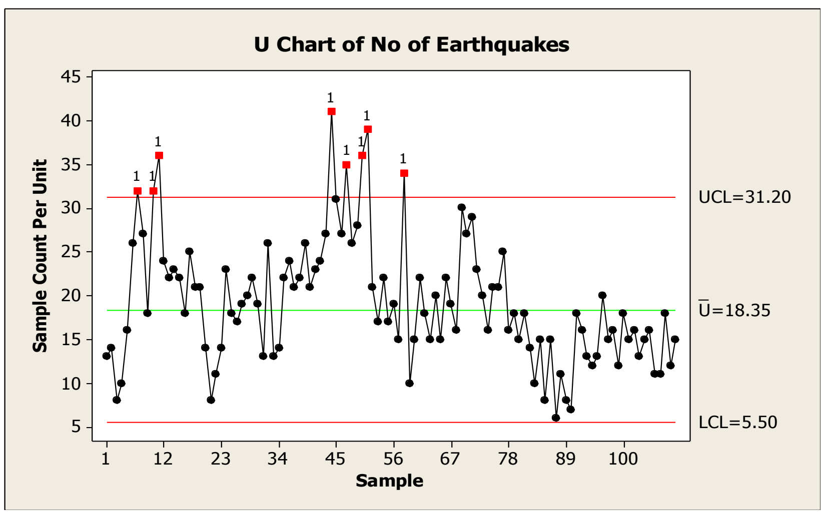


b) The control limits are calculated without the out-of-control points. There are no further points out of control for the revised limits.

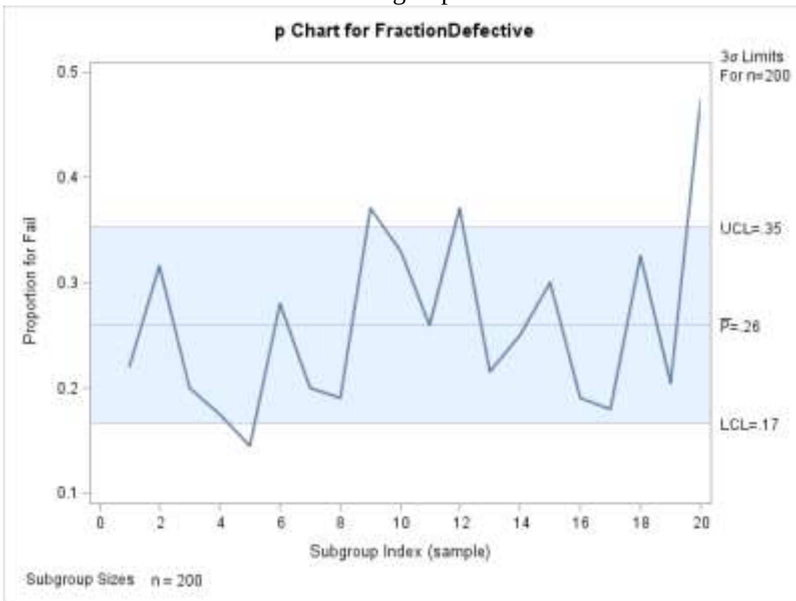
15-47 a)



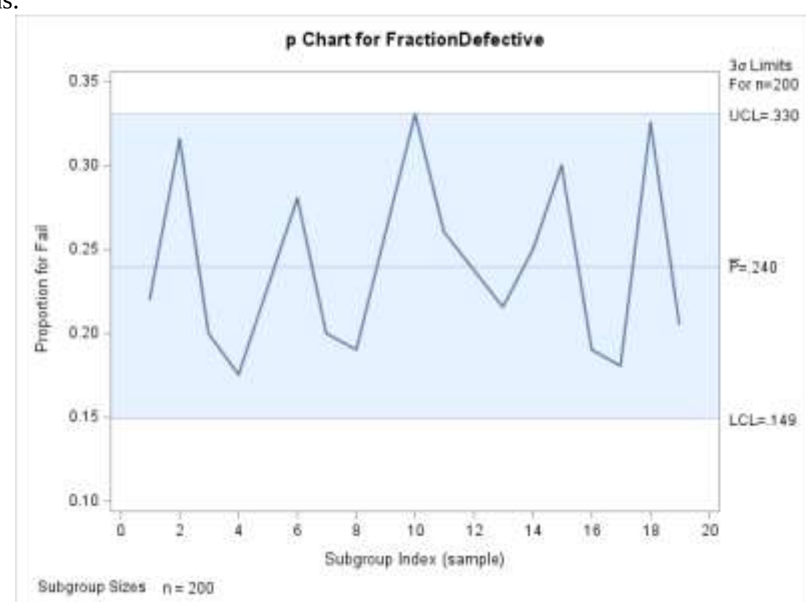
b) No. The process is out-of-control at observations 11, 44, 47, 50, 51, 58, and 87. The control limits are revised one time by omitting the out-of-control points. However, the chart shows additional out-of-control signals.



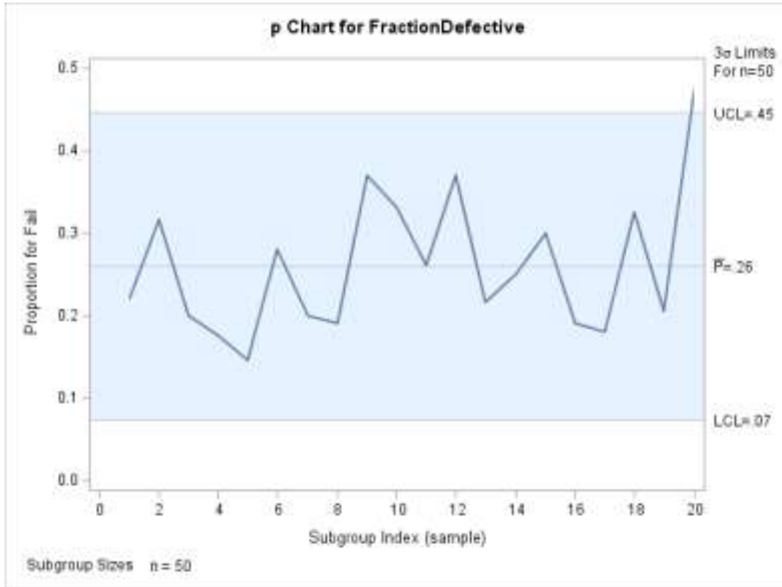
15-48 a) The process is NOT in control. The P chart for subgroups of size 200 follows.



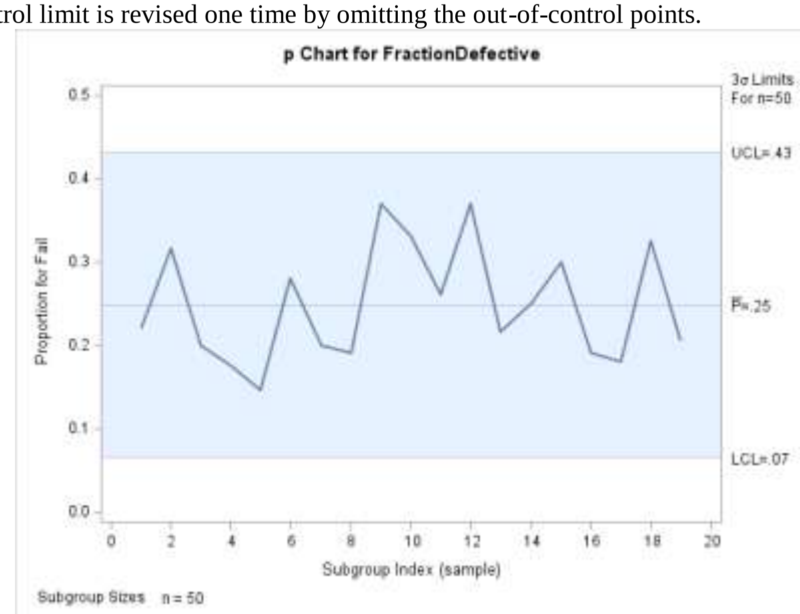
The P chart for subgroups of size 200 follows with points 5, 9, 12, 20 removed from the control limits calculations.



b) The P chart for subgroups of size 50 follows.



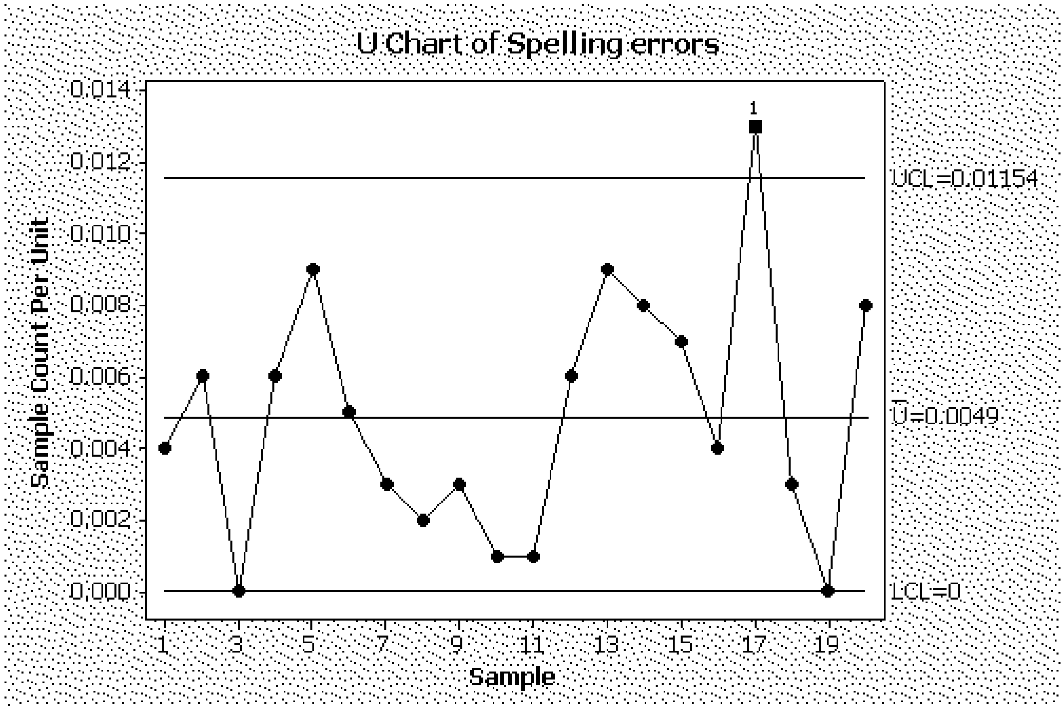
The P chart for subgroups of size 100 follows with point 20 removed from the calculations for the control limits. The control limit is revised one time by omitting the out-of-control points.



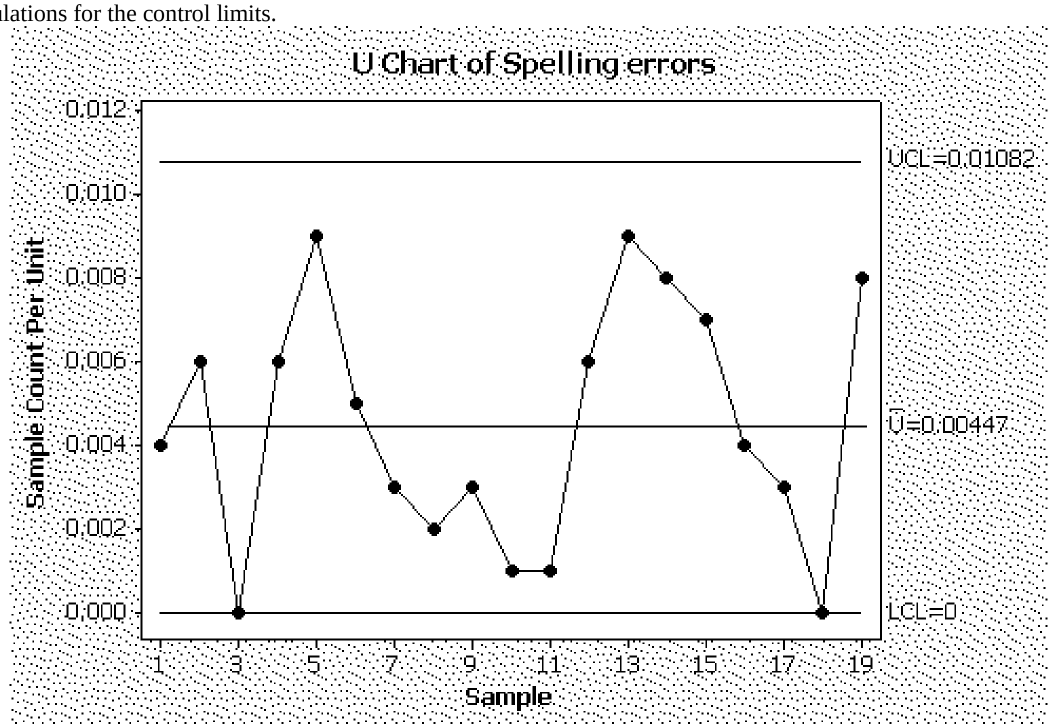
c) The control limits in parts (a) and (b) differ because the subgroup size has changed. The number of defectives is from 200 wafers in part (a) and the same number of defectives is from 50 wafers in part (b). This changes the centerline of the charts and the chart in part (a) also has tighter limits because it uses a larger sample size.

15-49 a) The U chart is appropriate for these data.

b) The U chart from Minitab follows.



c) The process is out-of-control at point 17. The U chart from Minitab follows with point 17 removed from the calculations for the control limits.



Section 15-7

15-50 a) (109 – 100) /3 = 3

b)

$$P(91 < X < 109) = P(\frac{91-96}{3} < \frac{X-\mu}{\sigma_X} < \frac{109-96}{3}) = P(-1.67 < Z < 4.33)$$
$$= P(Z < 4.33) - P(Z < -1.67) = 0.953$$

The probability of detecting is 1- 0.953 = 0.047.

c) 1/0.047 = 21.28 ARL to detect the shift is about 21.

15-51 a) $\mu + 3\frac{\sigma}{\sqrt{n}} = UCL$

$$100 + 3\frac{\sigma}{\sqrt{9}} = 106$$

$$\sigma = \frac{3}{3}(106-100) = 6$$

b) $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{6}{3} = 2, \qquad \mu = 96$

$$P(94 < X < 106) = P\left(\frac{94-96}{2} < \frac{\overline{X}-\mu}{\sigma_{\bar{x}}} < \frac{106-96}{2}\right) = P(-1 < Z < 5)$$
$$= P(Z < 5) - P(Z < -1) = 1 - 0.1587 = 0.8413$$

The probability that this shift will be detected on the next sample is p = 1– 0.8413 = 0.1587.

c) $ARL = \frac{1}{p} = \frac{1}{0.1587} = 6.301$

15-52 a) $\sigma_{\bar{x}}$ = 0.0045 μ = 73.990

$$P(73.9865 < \overline{X} < 74.0135)$$
$$= P\left(\frac{73.9865-73.99}{0.0045} < \frac{X-\mu}{\hat{\sigma}_{\bar{x}}} < \frac{74.0135-73.99}{0.0045}\right)$$
$$= P(-0.78 < Z < 5.22) = P(Z < 5.22) - P(Z < -0.78)$$
$$= 0.7823$$

The probability that this shift will be detected on the next sample is p = 1–0.7823 = 0.2177.

b) $ARL = \frac{1}{p} = \frac{1}{0.2177} = 4.6$

15-53 a) $\hat{\sigma} = \dfrac{\bar{R}}{d_2} = \dfrac{0.39}{2.326} = 0.168$ $\quad \hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{0.168}{\sqrt{5}} = 0.075$, $\quad \mu = 14.6$

$$P(14.315 < X < 14.705) = P\left(\frac{14.315-14.6}{0.075} < \frac{X-\mu}{\sigma_x} < \frac{14.705-14.6}{0.075}\right)$$
$$= P(-3.8 < Z < 1.4) = P(Z < 1.4) - P(Z < -3.8)$$
$$= 0.919243 - 0.000072 = 0.919171$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.919171 = 0.0808$.

b) $ARL = \dfrac{1}{p} = \dfrac{1}{0.0808} = 12.4$

15-54 a) $\hat{\sigma} = \dfrac{\bar{R}}{d_2} = 16.91,$ $\quad \hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{16.91}{\sqrt{5}} = 7.562,$ $\quad \mu = 210$

$$P(203.250 < \bar{X} < 242.58) = P\left(\frac{203.250-210}{7.562} < \frac{\bar{X}-\mu}{\sigma_{\bar{x}}} < \frac{242.58-210}{7.562}\right)$$
$$= P(-0.89 < Z < 4.33) = P(Z < 4.33) - P(Z < -0.89)$$
$$= 1 - 0.1867 = 0.8133$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.8133 = 0.1867$.

b) $ARL = \dfrac{1}{p} = \dfrac{1}{0.1867} = 5.4$

15-55 a) $\hat{\sigma} = \dfrac{\bar{R}}{d_2} = 1.34$ $\quad \hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{1.34}{\sqrt{6}} = 0.5471$, $\quad \mu = 17$

$$P(18.25 < X < 21.65) = P\left(\frac{18.25-17}{0.5471} < \frac{\bar{X}-\mu}{\sigma_{\bar{x}}} < \frac{21.65-17}{0.5471}\right)$$
$$= P(2.28 < Z < 8.5) = P(Z < 8.5) - P(Z < 2.28)$$
$$= 1 - 0.9887 = 0.0113$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.0113 = 0.9887$.

b) $ARL = \dfrac{1}{p} = \dfrac{1}{0.9887} = 1.011$

15-56 a) $\hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{2.4664}{\sqrt{6}} = 1.007$, $\mu = 36$

$$P(30.78 < \bar{X} < 37.404) = P\left(\frac{30.78-36}{1.007} < \frac{\bar{X}-\mu}{\hat{\sigma}_{\bar{x}}} < \frac{37.404-36}{1.007}\right)$$
$$= P(-5.18 < Z < 1.39) = P(Z < 1.39) - P(Z < -5.18)$$
$$= 0.9177 - 0 = 0.9177$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.9177 = 0.0823$.

b) $ARL = \dfrac{1}{p} = \dfrac{1}{0.0823} = 12.15$

15-57 a) $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{2.25}{1.693} = 1.329$ $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{1.329}{\sqrt{3}} = 0.767,$ $\mu = 13$

$$P(12.70 < \bar{X} < 17.5) = P\left(\frac{12.70-13}{0.767} < \frac{\bar{X}-\mu}{\sigma_{\bar{X}}} < \frac{17.5-13}{0.767}\right)$$
$$= P(-0.3 < Z < 4.5) = P(Z < 4.5) - P(Z < -0.3)$$
$$= 1 - 0.3821 = 0.6179$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.6179 = 0.3821$.

b) $ARL = \frac{1}{p} = \frac{1}{0.3821} = 2.62$

15-58 a) $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.000924}{2.970} = 0.000311$ $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{0.000311}{\sqrt{9}} = 0.000104$, $\mu = 0.0625$

$$P(0.0624 < \bar{X} < 0.0635) = P\left(\frac{0.0624-0.0625}{0.000104} < \frac{\bar{X}-\mu}{\sigma_{\bar{x}}} < \frac{0.0635-0.0625}{0.000104}\right)$$
$$= P(-0.96 < Z < 9.62) = P(Z < 9.62) - P(Z < -0.96)$$
$$= 1 - 0.1685 = 0.8315$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.8315 = 0.1685$.

b) $ARL = \frac{1}{p} = \frac{1}{0.1685} = 5.93$

15-59 a) $\hat{\sigma} = \frac{\bar{R}}{d_2} = 0.669$ $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{0.669}{\sqrt{3}} = 0.386$, $\mu = 6.5$

$$P(5.125 < \bar{X} < 7.443 \mid \mu = 6.5) = P\left(\frac{5.125-6.5}{0.386} < \frac{\bar{X}-\mu}{\sigma_{\bar{x}}} < \frac{7.443-6.5}{0.386}\right)$$
$$= P(-3.56 < Z < 2.44) = P(Z < 2.44) - P(Z < -3.56)$$
$$= 0.9927 - 0.0002 = 0.9925$$

The probability that this shift will be detected on the next sample is $p = 1 - 0.9925 = 0.0075$.

b) $ARL = \frac{1}{p} = \frac{1}{0.0075} = 133.3$

15-60 a) The difference $UCL - LCL = 6\hat{\sigma}_{\bar{X}} = 220 - 180 = 40$
Therefore, $\hat{\sigma}_{\bar{X}} = \frac{40}{6} = 6.67$ and $\hat{\sigma}_{\bar{X}} = \frac{\hat{\sigma}}{\sqrt{3}}$. Therefore, $\hat{\sigma} = 6.67\sqrt{3} = 11.55$

b) $P(180 < \bar{X} < 220 \mid \mu = 195) = P[(180 - 195)/6.67 < Z < (220 - 195)/6.67] = P(-2.25 < Z < 3.75) = 0.9877$.
Therefore, the probability the shift is detected = 1 – 0.9877 = 0.0123

c) ARL = $1/p$, where p is the probability a point exceeds a control limit. From part (b) $p = 0.0123$. Therefore ARL = 1/0.0123 = 81.3.

15-61 a) The difference $UCL - LCL = 6\hat{\sigma}_{\bar{X}} = 24.81 - 23.75 = 1.06$

Therefore, $\hat{\sigma}_{\bar{X}} = \dfrac{1.06}{6} = 0.1767$ and $\hat{\sigma} = \dfrac{1.06\sqrt{3}}{6} = 0.306$

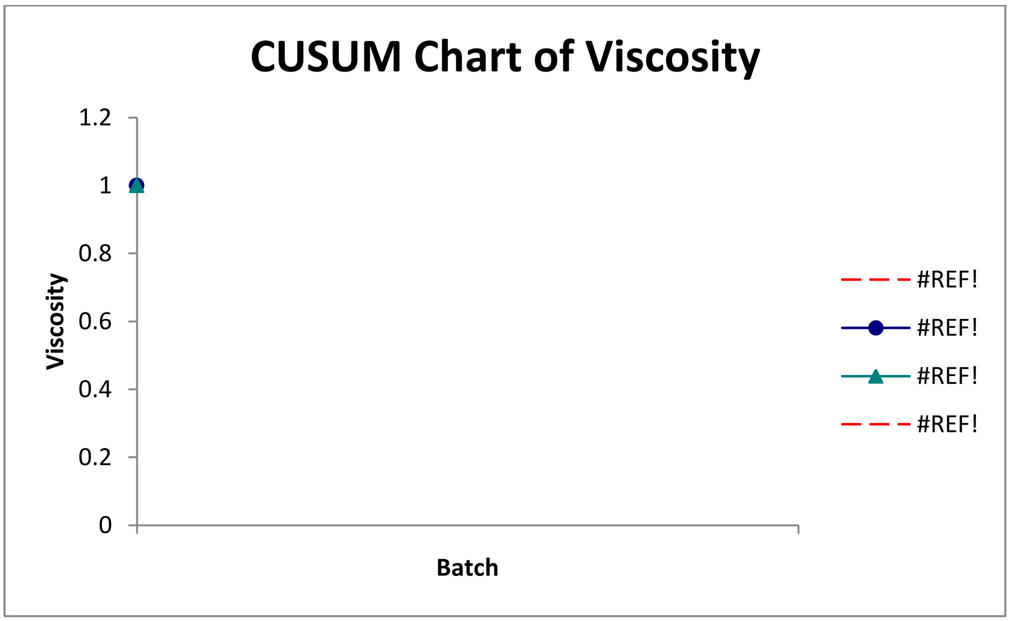
$$P(23.75 < \bar{X} < 24.81 \mid \mu = 24.2) = P\left(\frac{23.75 - 24.2}{0.1767} < \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} < \frac{24.81 - 24.2}{0.1767}\right)$$
$$= P(-2.5467 < Z < 3.4522) = P(Z < 3.4522) - P(Z < -2.5467)$$
$$= 0.9997 - 0.0054 = 0.9943$$

Therefore the probability the shift is detected = 1 – 0.9943 = 0.0057 ≈ 0.006

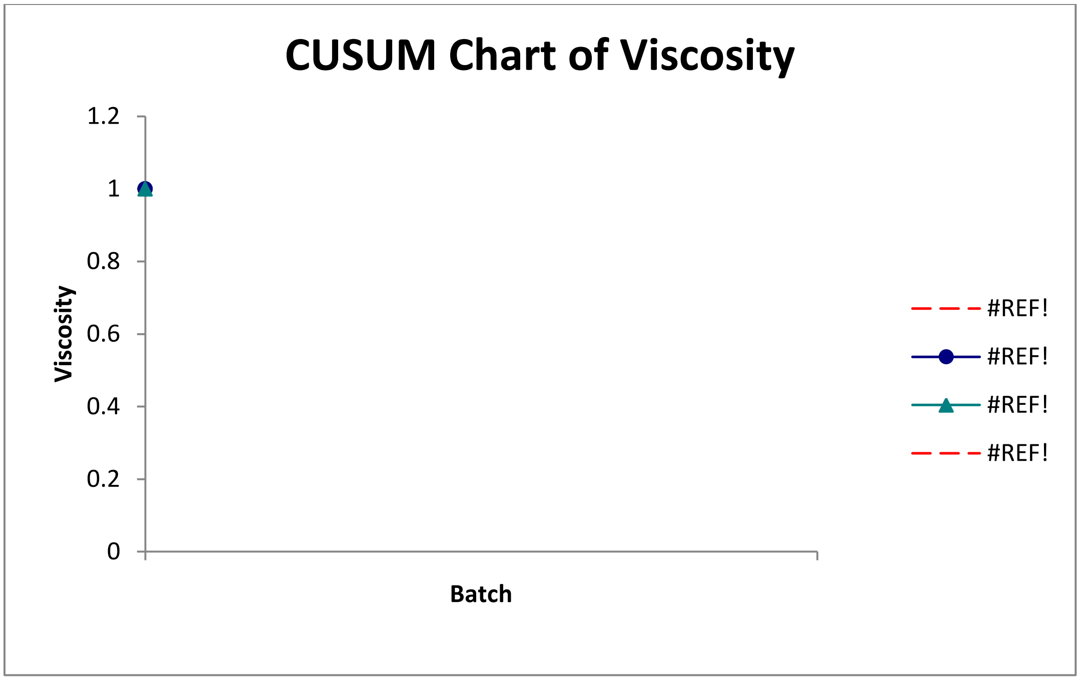
c) ARL = 1/*p*, where *p* is the probability a point exceeds a control limit. From part (b) *p* = 0.0079. Therefore ARL = 1/0.0057 = 175.44

Section 15-8

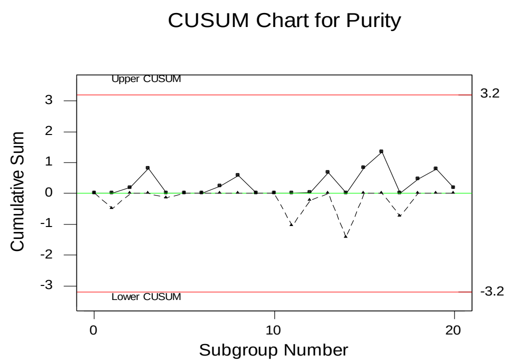
15-62 a) Yes, this process is in-control. CUSUM chart with *h* = 4 and *k* = 0.5 is shown.



b) Yes, this process has shifted out-of-control. For the CUSUM estimated from all the data (with *k* = 0.5 and *h* = 4) observation 20 exceeds the upper limit.

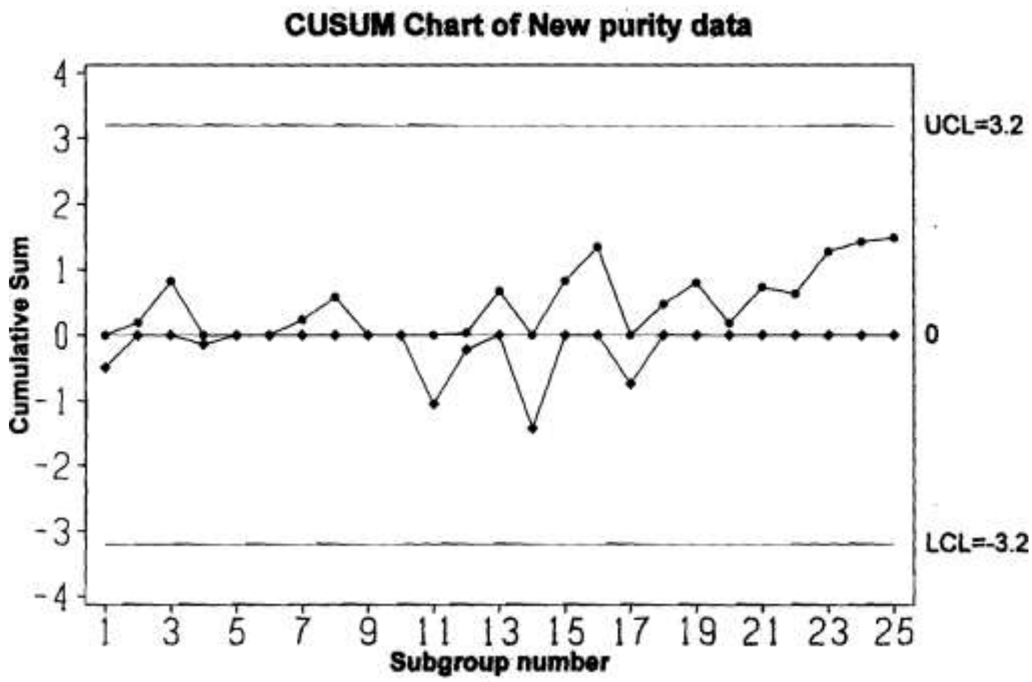


15-63 a) CUSUM Control chart with $k = 0.5$ and $h = 4$



The CUSUM control chart for purity does not indicate an out-of-control situation. The S_H values do not plot beyond the values of $-H$ and H.

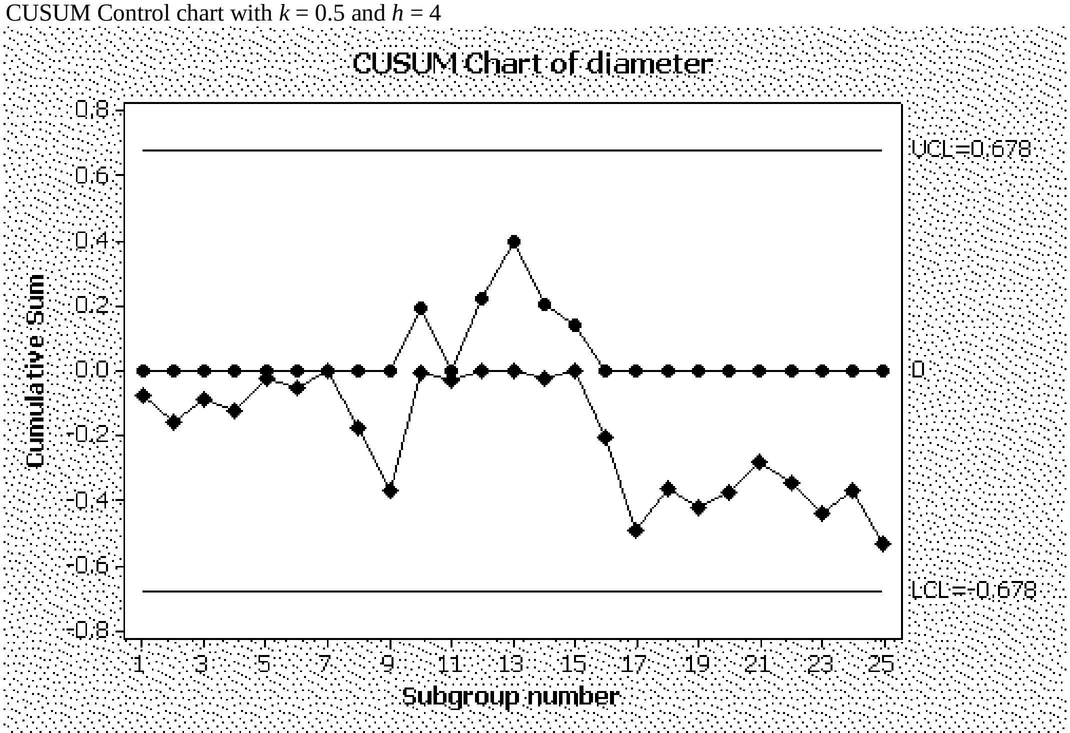
b) CUSUM Control chart with $k = 0.5$ and $h = 4$



The process appears to be moving out of statistical control.

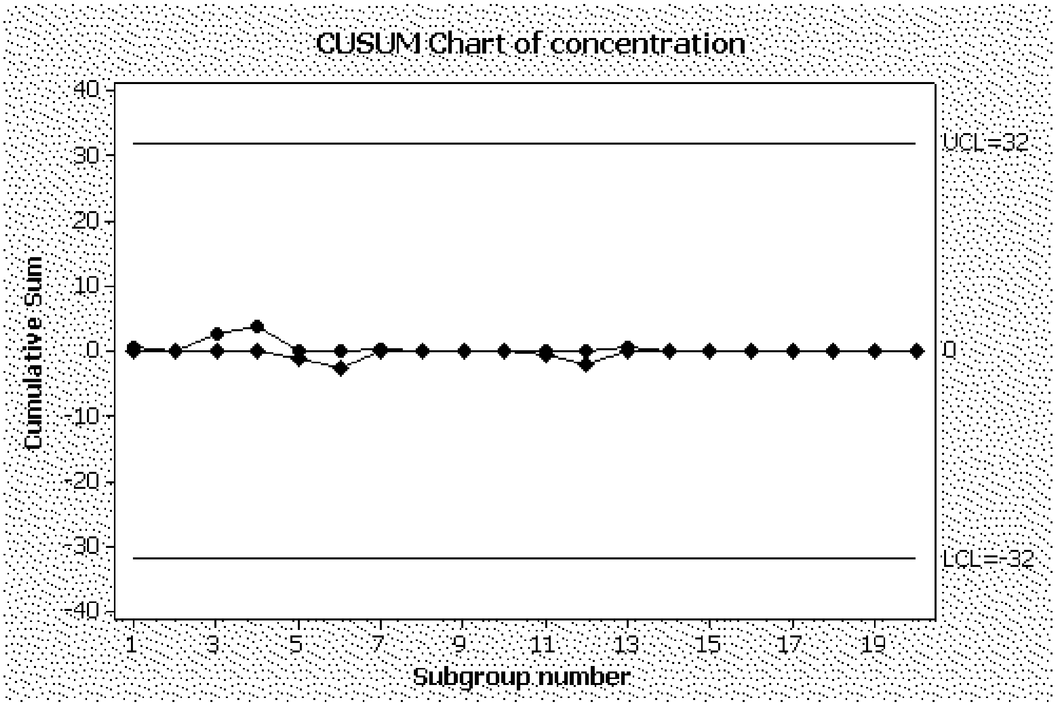
15-64 a) $\hat{\sigma}$ = 0.1736

b) CUSUM Control chart with k = 0.5 and h = 4



The process appears to be in control at the specified target level.

15-65 a) CUSUM Control chart with $k = 0.5$ and $h = 4$



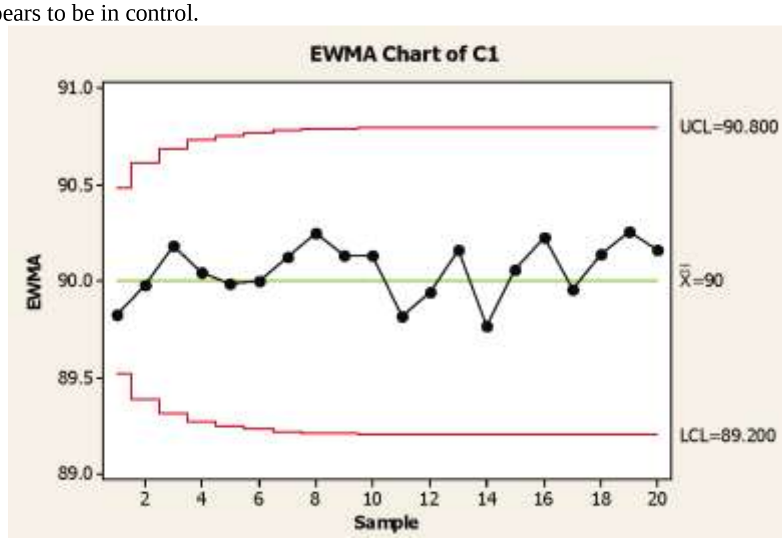
The process appears to be in statistical control.

b) With the target = 100 a shift to 104 is a shift of 104 – 100 = 4 = 0.5σ. From Table 15-9 with $h = 4$ and a shift of 0.5, ARL = 26.6

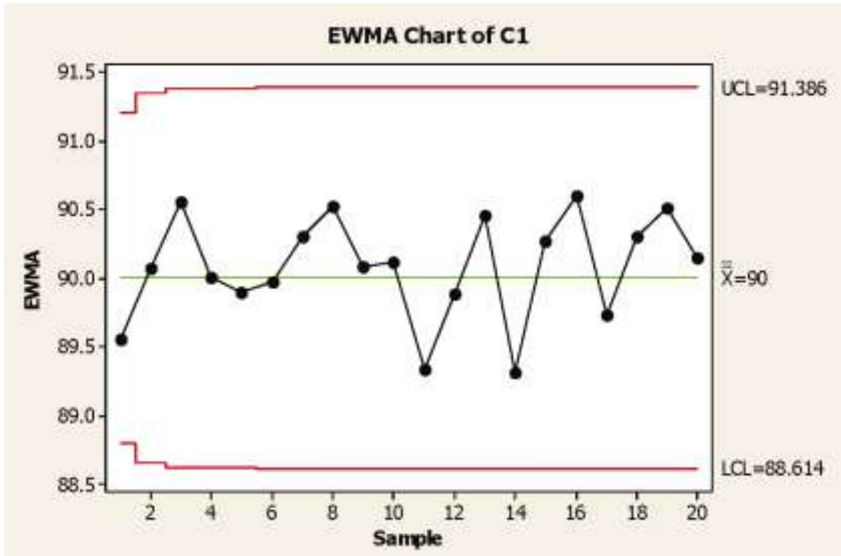
15-66 a) A shift to 52 is a shift of $\dfrac{\mu - \mu_0}{\sigma} = \dfrac{52-50}{4}$ = 0.5 standard deviations. From Table 15-10, ARL = 38.0

b) If $n = 4$, the shift to 52 is a shift of $\dfrac{\mu - \mu_0}{\sigma/\sqrt{n}} = \dfrac{52-50}{4/\sqrt{4}}$ = 1 standard deviation. From Table 15-10, ARL = 10.4

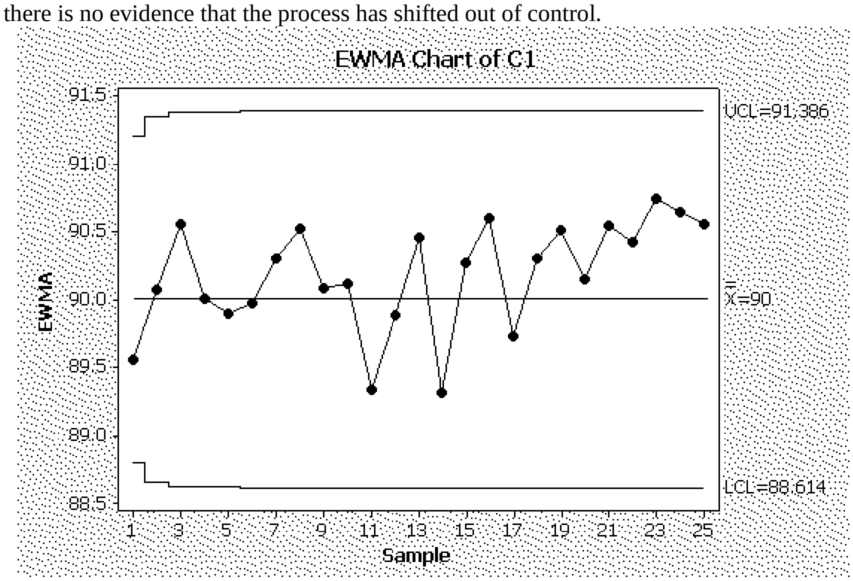
15-67 a) The process appears to be in control.



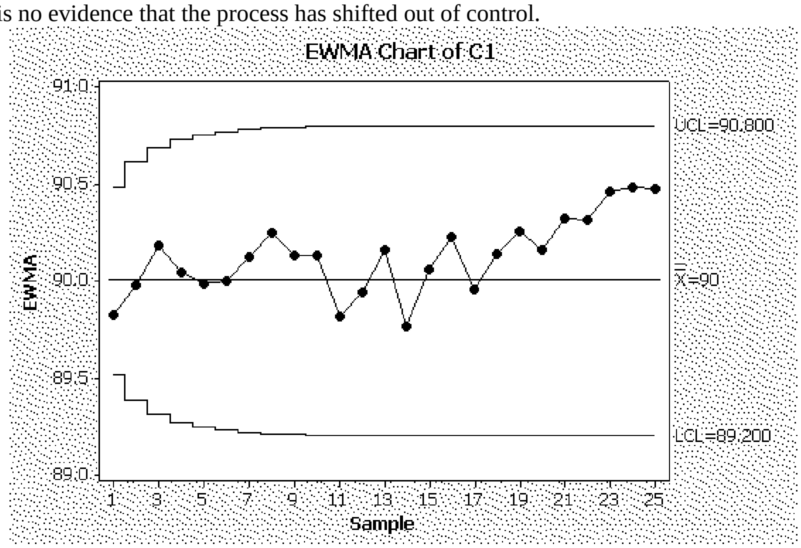
b) The process appears to be in control.



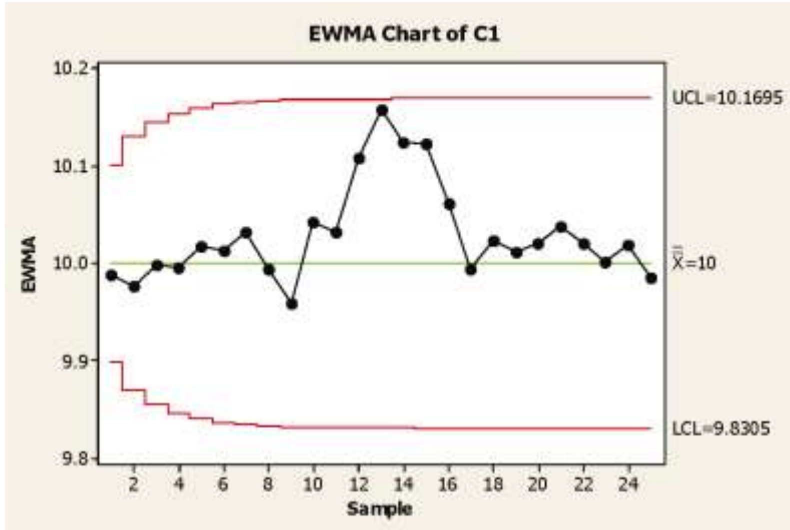
c) For part (a), there is no evidence that the process has shifted out of control.



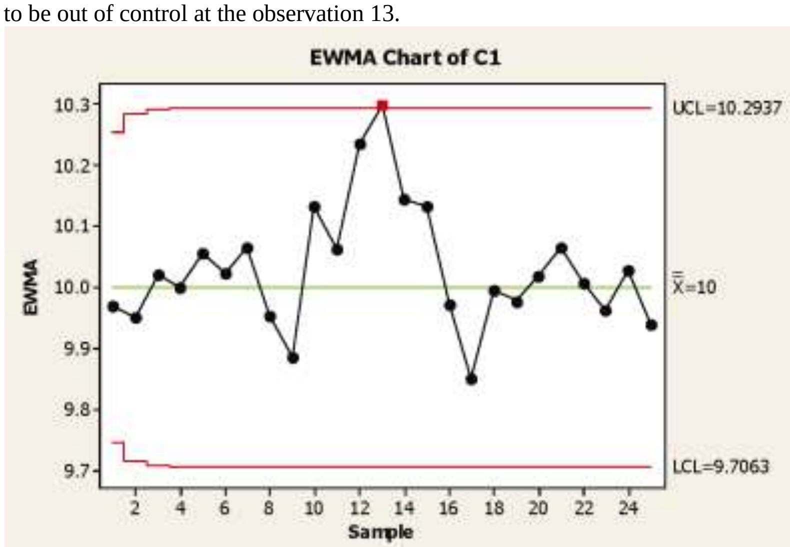
For part b), there is no evidence that the process has shifted out of control.



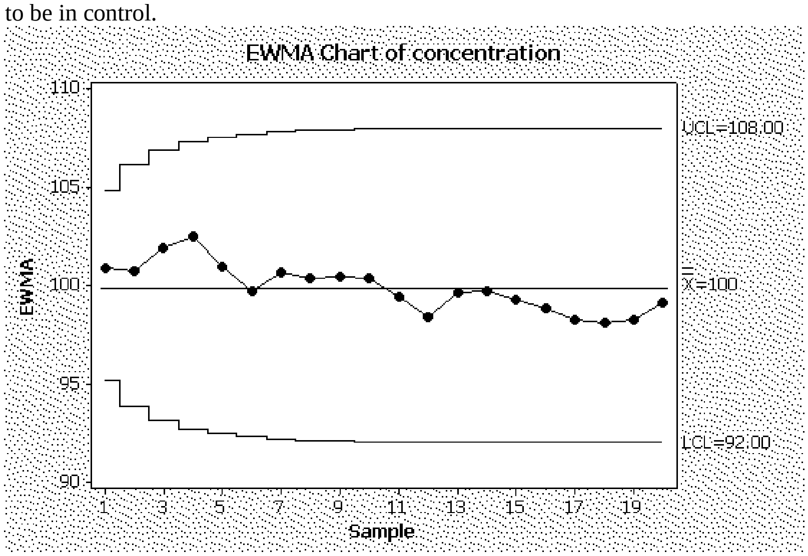
15-68 a) The estimated standard deviation is 0.169548.
b) The process appears to be in control.



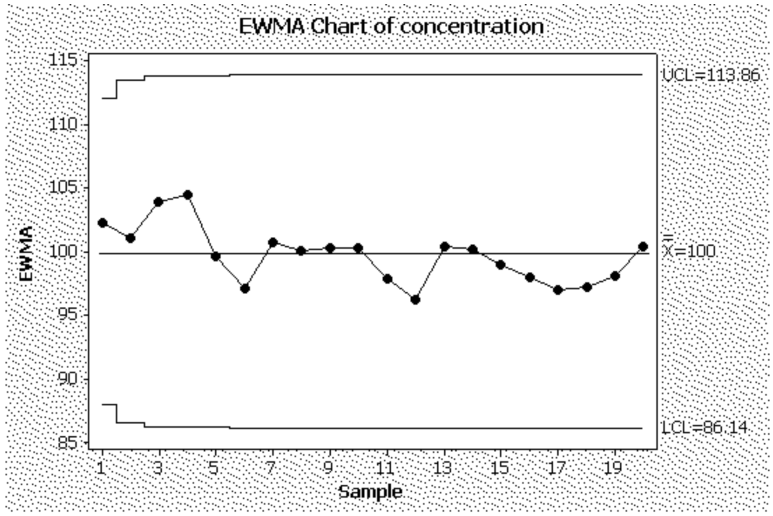
c) The process appears to be out of control at the observation 13.



15-69 a) The process appears to be in control.



b) The process appears to be in control.



c) Because the shift is 0.5σ, a smaller λ is preferred to detect the shift quickly. Therefore, the chart in part (a) is preferred.

15-70 a) The shift of the mean is 0.5σ. So we prefer $\lambda = 0.1$ and L = 2.81 because this setting has the smaller ARL = 31.3.

b) The shift of the mean is $1\sigma_{\overline{X}}$. So we prefer $\lambda = 0.1$ and L= 2.81 because this setting has the smaller ARL = 10.3

c) The shift of the mean is $3\sigma_{\overline{X}}$. Solving $1/(\frac{2}{\sqrt{n}}) = 3$ for n gives us the required sample size of 36.

15-71 a) With a target = 100 and a shift to 101 results in a shift of $\frac{101-100}{4}$ = 0.25 standard deviations.

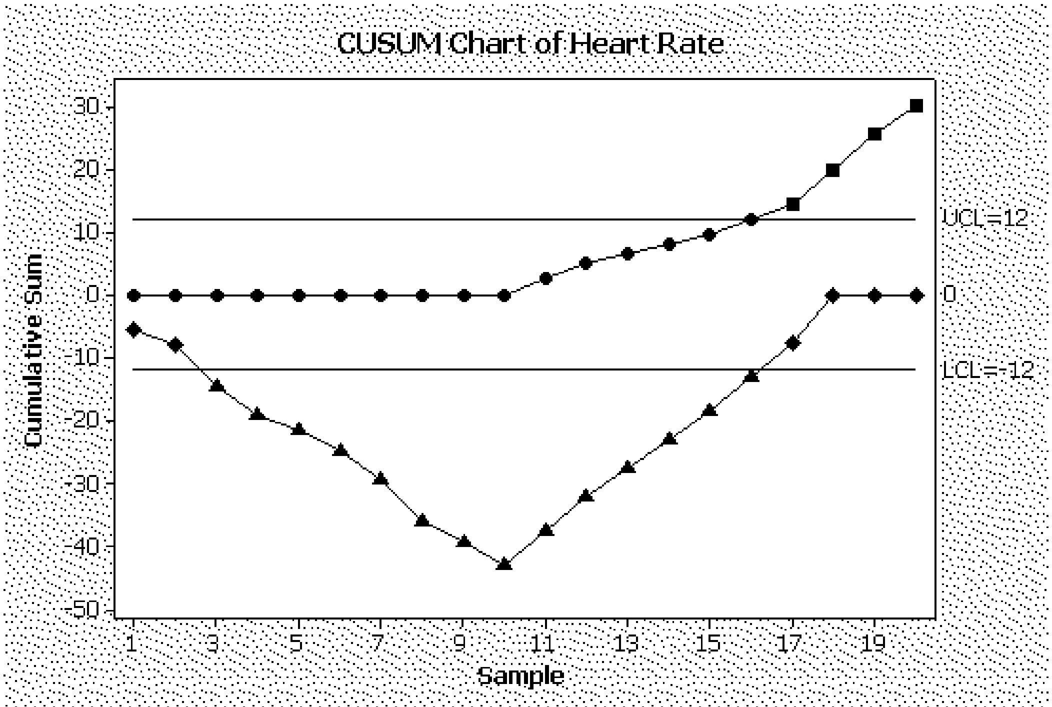
From Table 15-10, ARL = 139. The hours of production are 2(139) = 278.

b) The ARL = 139. However, the time to obtain 139 samples is now 0.5(139) = 69.5.

c) From Table 15-10, the ARL when there is no shift is 465. Consequently, the time between false alarms is 0.5(465) = 232.5 hours. Under the old interval, false alarms occurred every 930 hours.

d) If the process shifts to 101, the shift is $\frac{\mu - \mu_0}{\sigma/\sqrt{n}} = \frac{101-100}{4/\sqrt{4}} = 0.5$ standard deviation. From Table 15-10, the ARL for this shift is 38. Therefore, the time to detect the shift is 2(38) = 76 hours. Although this time is slightly longer than the result in part (b), the time between false alarms is 2(465) = 930 hours, which is better than the result in part (c).

15-72 a) The process is not in control. The control chart from Minitab follows with $h = 4$ and $k = 0.5$.

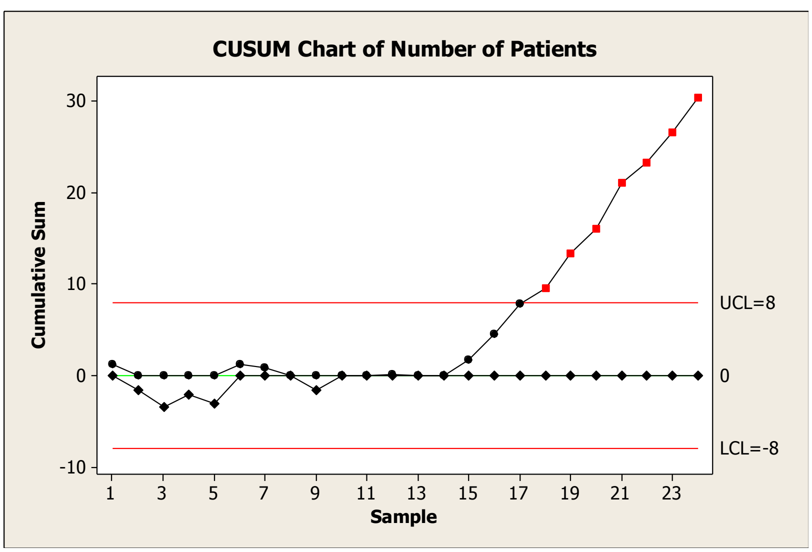


b) The shift from 75 to 80 is a shift of 5/3 = 1.67 standard deviation units.

If $h = 4$ and $k = 0.5$ then Table 15-9 can be used. The shift of 1.67 is between the table values 3 and 4. Therefore, 3.34 < ARL < 4.75.

If $h = 5$ and $k = 0.5$ then Table 15-9 can be used. The shift of 1.67 is between the table values 3 and 4. Therefore, 4.01 < ARL < 5.75.

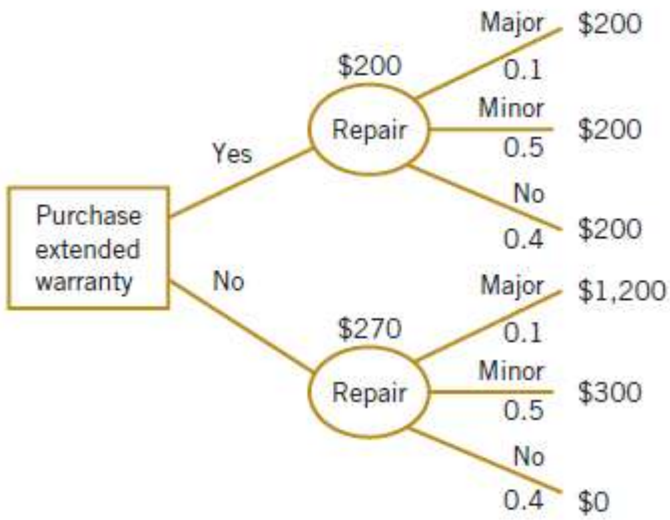
15-73 a) The process is not in control. The control chart follows.



b) The shift from 160 to 164 is a shift of 4/2 = 2 standard deviation units. If $h = 4$ and $k = 0.5$ then Table 15-10 can be used. The shift of 2 leads to ARL = 3.34.

Section 15-10

15-74



a) Minimax criteria: purchase cost = 200, max cost if not purchased = 1000, therefore the minimum cost decision is to purchase

b) Most probable criteria: purchase cost = 200, most probable cost if not purchased = 300, so purchase

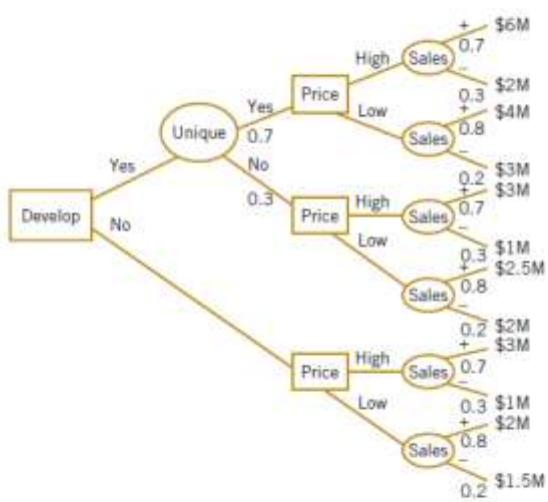
c) Expected cost: purchase cost = 200, expected cost if not purchased = 0.1(1000) + 0.5(300) + 0.4(0) = 250, so purchase

15-75 a) Minimax criteria: purchase cost = 200, max cost if not purchased = 1200, therefore the minimum cost decision is to purchase

b) Most probable criteria: purchase cost = 200, most probable cost if not purchased is either 300 or 0 because they both have the same probability. Because 300 is greater than 200 and 0 is less than 200, the solution from the most probable criterion is not defined in this case.

c) Expected cost: purchase cost = 200, expected cost if not purchased = 0.2(1000) + 0.4(300) + 0.4(0) = 320, so purchase

15-76



Decisions:
1. When a new product is developed and a unique product is achieved, the most probable outcomes for the high and low prices are 6M and 4M, respectively. Therefore, the price is set high.

2. When a new product is developed and a unique product is not achieved, the most probable outcomes for the high and low prices are 3M and 2.5M, respectively. Therefore, the price is set high.

3. When a new product is not developed, the most probable outcomes for the high and low prices are 3M and 2M, respectively. Therefore, the price is set to high.

4. When a new product is developed, the most probably outcome is that it is unique. The price decision based on the most probable outcome is to price high with the most probable outcome 6M.

When a new product is not developed, the price decision based on the most probable outcome is to price high with the most probable outcome 3M.

Therefore, the decision is to develop a new product.

The choice is different from pessimistic approach in the example.

15-77 Decisions:
1. When a new product is developed and a unique product is achieved, the expected outcomes for the high and low prices are 0.7(6M) + 0.3(2M) = 4.8M and 0.8(4M) + 0.2(3M) = 3.8, respectively. Therefore, the price is set high.

2. When a new product is developed and a unique product is not achieved, the expected outcomes for the high and low prices are 0.7(3M) + 0.3(1M) = 2.4M and 0.8(2.5M) + 0.2(2M) = 2.4, respectively. Therefore, there is no difference in expected value between the price high and low decisions.

3. When a new product is not developed, the expected outcomes for the high and low prices are 0.7(3M) + 0.3(1M) = 2.4M and 0.8(2M) + 0.2(1.5M) = 1.9M, respectively. Therefore, the price is set high.

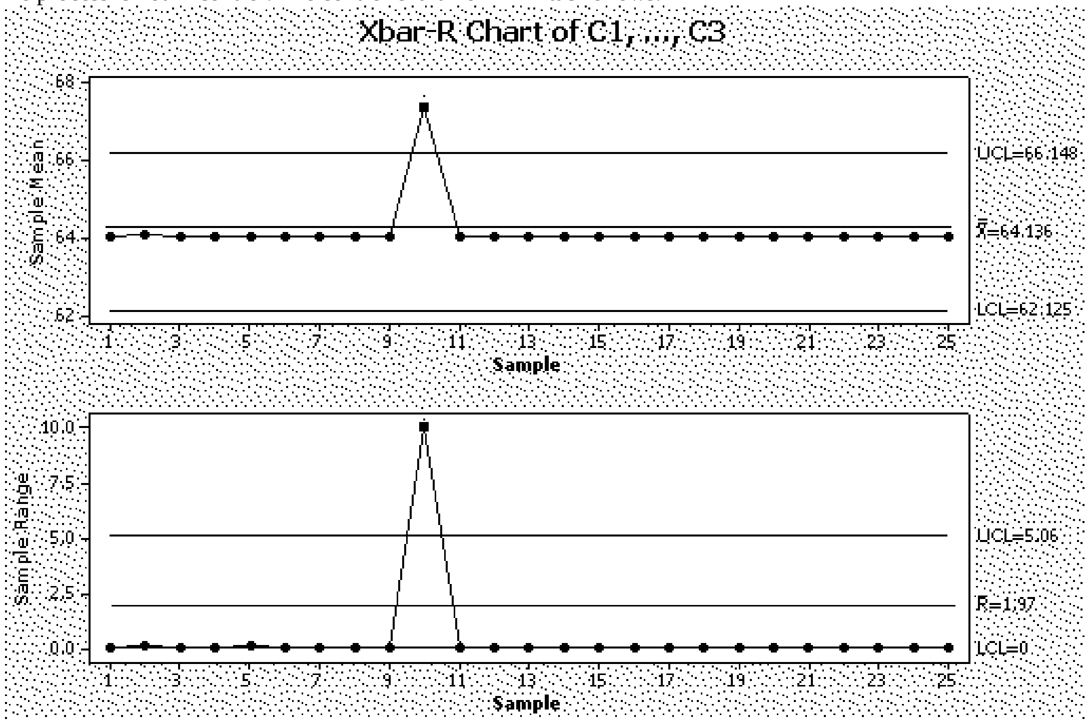
4. When a new product is developed, the expected outcome at the unique node is 0.7(4.8M) + 0.3(2.4M) = 4.08M, where the expected outcomes at the price decision nodes are used in this calculation. When a new product is not developed, the price decision is to price high with an expected outcome of 2.4M.

Therefore, the choice from the expected value criterion is to develop a new product.

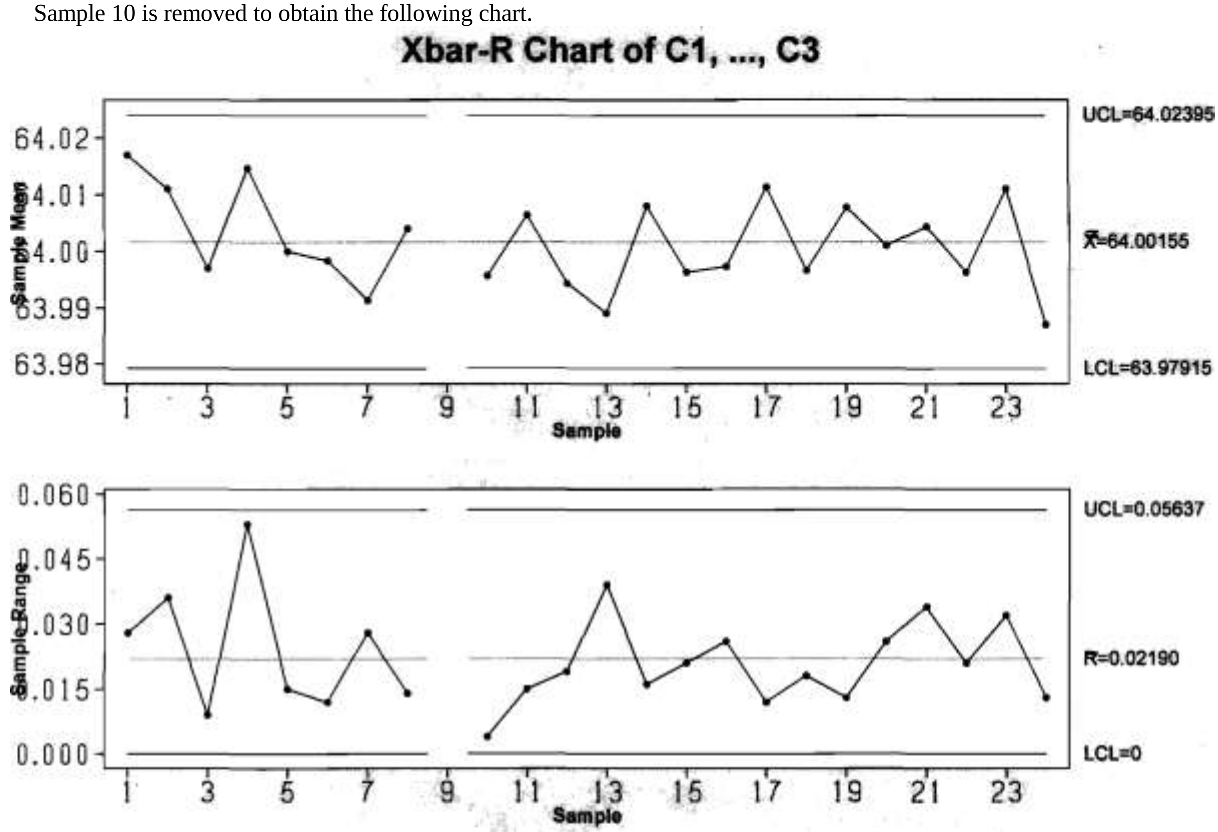
The choice is different from pessimistic approach in the example.

Supplementary Exercises

15-78 a) The process is not in control. The control chart from Minitab follows.



Sample 10 is removed to obtain the following chart.



b) estimates: mean = 64.002, stdev = 0.0314/1.693 = 0.01294

c) $PCR = 0.359$
d) $PCR_k = 0.576$

e) The value of the variance is found by solving $PCR_k = \dfrac{\bar{\bar{x}} - LSL}{3\sigma} = 2.0$ for σ. This yields $\dfrac{64.002 - 63.98}{3\sigma} = 2.0$,

and $\sigma = 0.0037$

f) P(63.9792 < Z < 64.024)
= P[(63.9792 – 64.01)/(0.01294/sqrt(3))] < Z < (64.024 – 64.01)/(0.01294/sqrt(3))]
= P(–4.12 < Z < 1.87) = 0.969. Therefore, the probability of detection = 1 – 0.969 = 0.031
ARL = 1/0.031 = 32.26

15-79 (a) $\bar{\bar{x}} = \dfrac{6508}{20} = 325.4 \quad \bar{r} = \dfrac{1000}{20} = 50$

The value of A_2 for samples of size 5 is $A_2 = 0.577$ from Appendix Table XI.

$$\bar{X} \quad Chart$$

$$UCL = CL + A_2\bar{r} = 325.4 + 0.577 * 50 = 354.25$$

$$CL = 325.4$$

$$UCL = CL - A_2\bar{r} = 325.4 - 0.577 * 50 = 296.55$$

$$R \quad Chart$$

$$UCL = D_4\bar{r} = 2.115 * 50 = 105.75$$

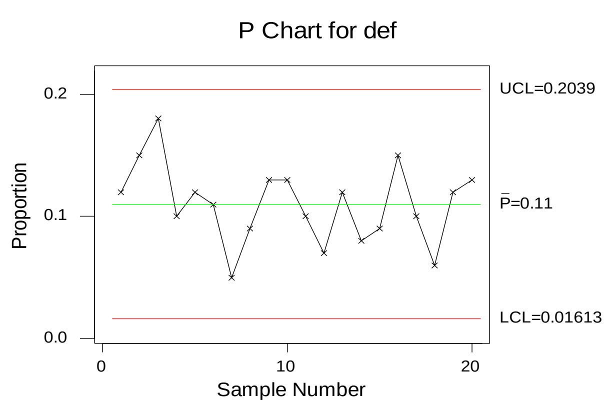
$$CL = 50$$

$$UCL = D_3\bar{r} = 0 * 50 = 0$$

(b) $\hat{\mu} = \bar{\bar{x}} = 325.4$

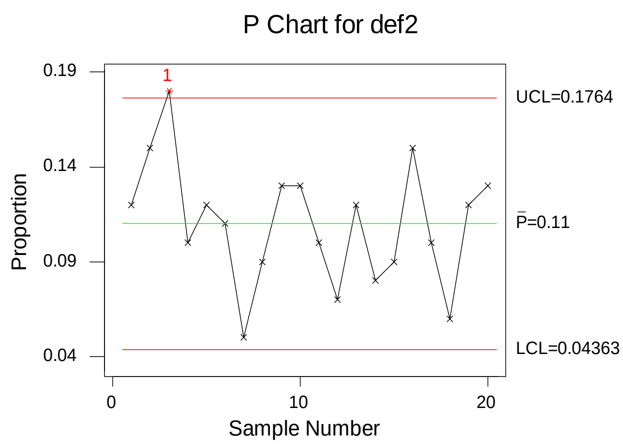
$$\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{50}{2.326} = 21.50$$

15-80 a)



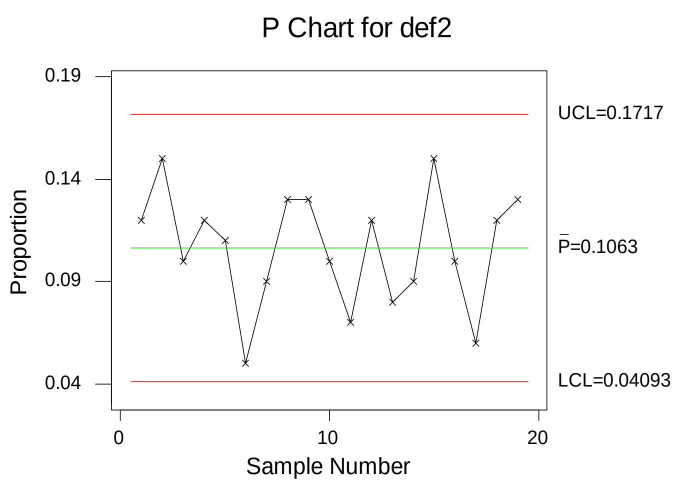
There are no points beyond the control limits. The process is in control.

b)



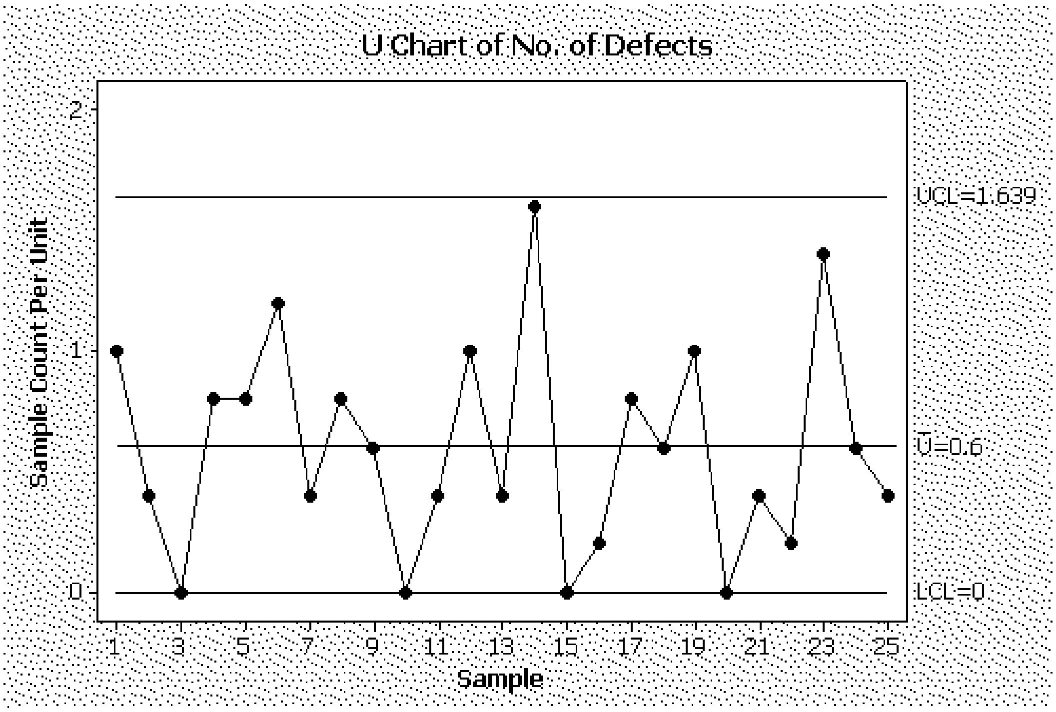
There is one point beyond the upper control limit. The process is out of control. The revised limits are:

There are no further points beyond the control limits.



c) A larger sample size with the same percentage of defective items will result in more narrow control limits. The control limits corresponding to the larger sample size are more sensitive to process shifts.

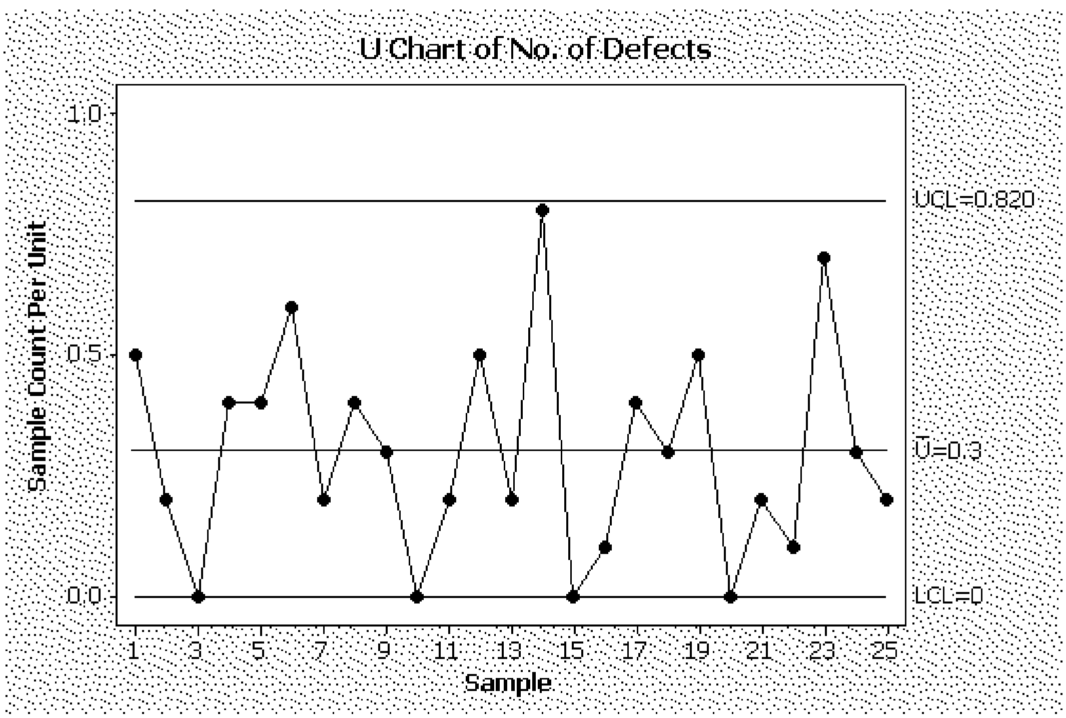
15-81 a)



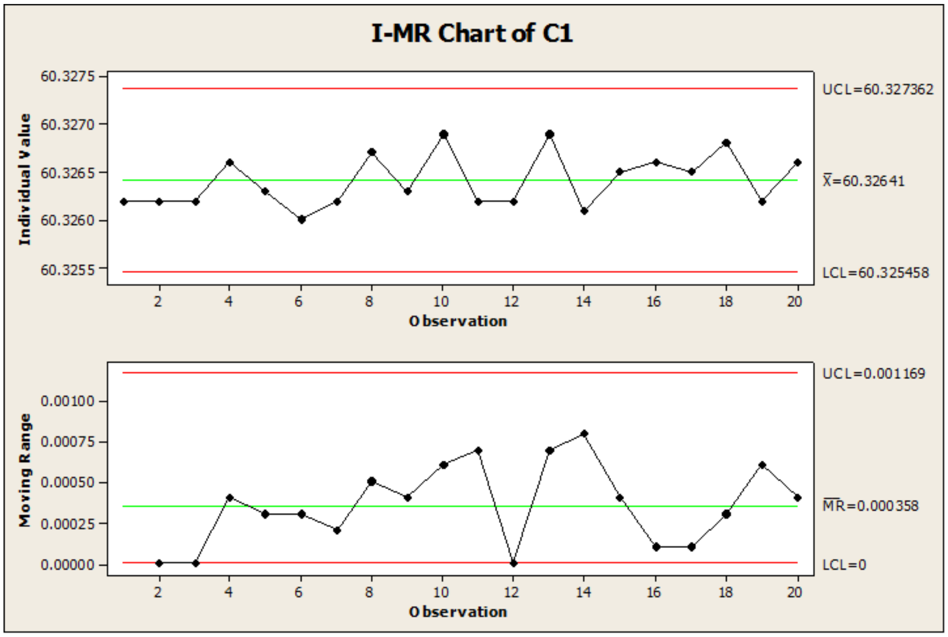
All the points are within the control limits. The process is under control.

b) There are no points beyond the control limits. So the control limits were not revised.

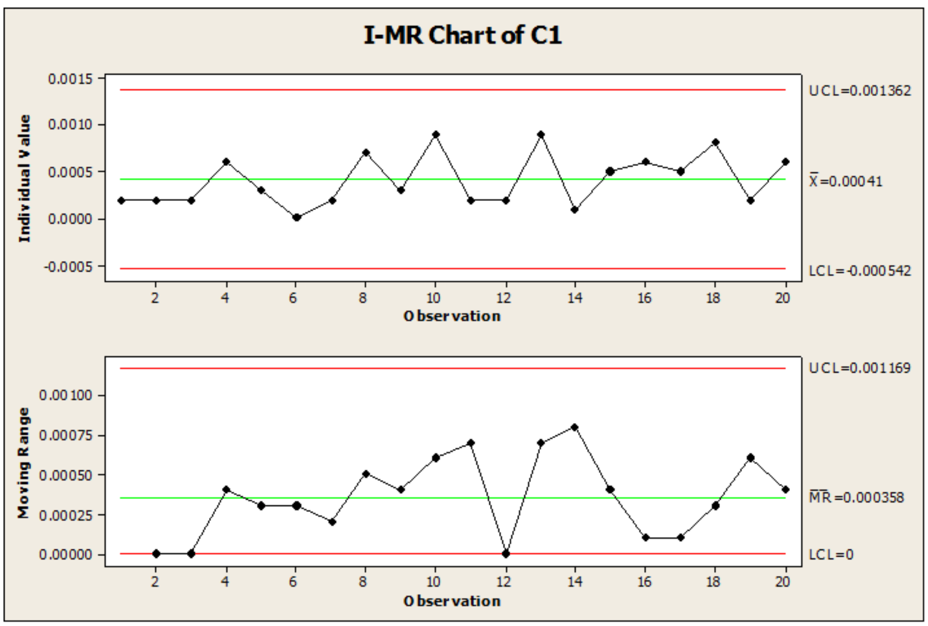
c) The control limits are narrower for a sample size of 10



15-82 a) Using I-MR chart.



b) The chart is identical to the chart in part (a) except for the scale of the individuals chart.

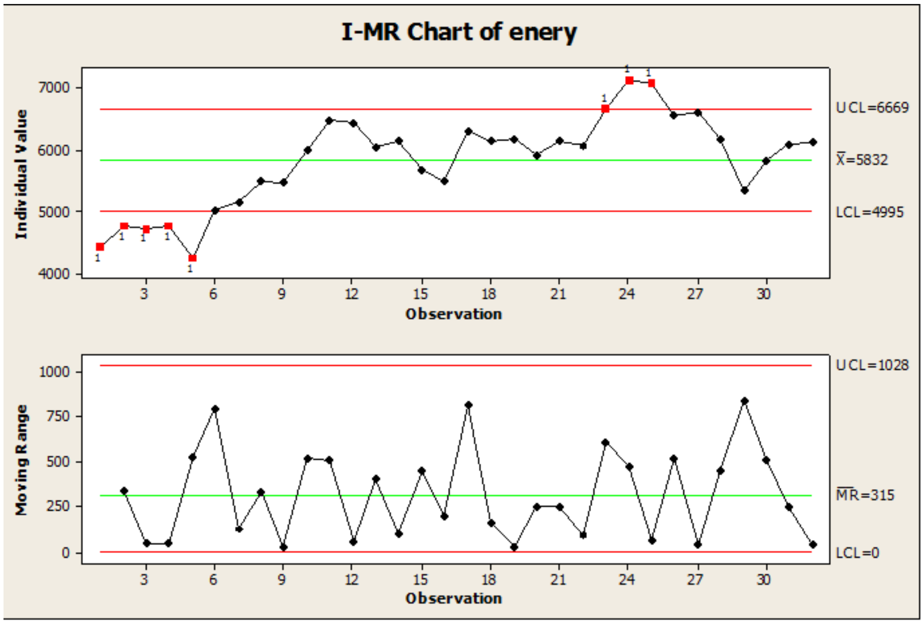


c)The estimated mean is 60.3264. The estimated standard deviation is 0.0003173.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{0.002}{6(0.0003173)} = 1.0505$$

$$PCR_k = \min\left[\frac{0.0009}{3\sigma}, \frac{0.0011}{3\sigma}\right] = 0.9455$$

15-83 a)



b) The data does not appear to be generated from an in-control process. The average tends to drift to larger values and then drop back off over the last 5 values.

15-84 (a) $\bar{\bar{x}} = 558.766 \quad \bar{s} = 86.4208$

Trial control limits :

S chart

UCL= 170.2482

CL = 86.4208

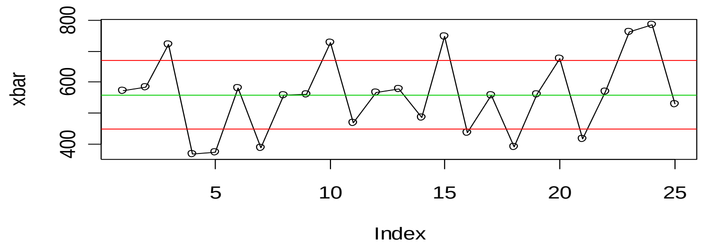
LCL = 2.59342

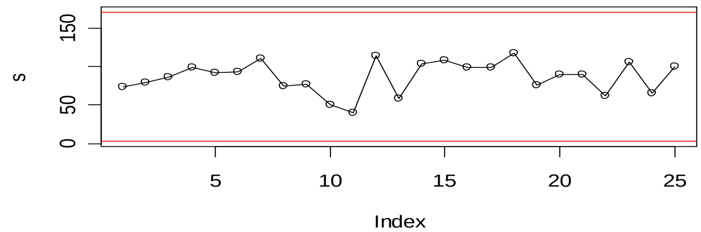
X bar chart

UCL= 670.0045

CL = 558.766

LCL = 447.5275





b) An estimate of σ is given by $\overline{S}/c_4 = 86.4208/0.9515 = 90.8259$

PCR=500/(6*90.8259) = 0.9175 and PCR$_k$ = $\min\left[\dfrac{830-558.77}{3(90.8259)}, \dfrac{558.77-330}{3(90.8259)}\right]$ = 0.8396

Based on the capability ratios above (both < 1), the process is operating off-center and will result in a large number of non-conforming units.

c) To determine the new variance, solve $PCR_k = 2$ for σ.

Because PCR$_k$= $\dfrac{558.77-330}{3\sigma}$, we find σ = 38.128 or σ^2=1453.77.

d) The probability that $\overline{X}$ falls within the control limits is

$$P\left(447.5275 < \overline{X} < 670.0045\right) = P\left(\frac{447.5275-580}{\frac{90.8259}{\sqrt{6}}} < Z < \frac{670.0045-580}{\frac{90.8259}{\sqrt{6}}}\right) =$$

$$P(-3.57 < Z < 2.43) = 0.9923$$

Thus, p = 0.0077 and ARL=1/p=129.87. The probability that the shift will be detected in the next sample is 0.0077.

15-85 (a) $\bar{\bar{x}} = \dfrac{685.38}{30} = 22.846 \quad \bar{r} = \dfrac{6.47}{30} = 0.216 \quad \bar{s} = \dfrac{3.75}{30} = 0.125$

The value of A_2 for samples of size 5 is A_2 = 0.577 from Appendix Table XI.

$$\overline{X} \quad Chart$$
$$UCL = CL + A_2\bar{r} = 22.846 + 0.577 * 0.216 = 22.971$$
$$CL = 22.846$$
$$UCL = CL - A_2\bar{r} = 22.846 - 0.577 * 0.216 = 22.721$$

$$R \quad Chart$$
$$UCL = D_4\bar{r} = 2.115 * 0.216 = 0.457$$
$$CL = 0.216$$
$$UCL = D_3\bar{r} = 0 * 0.216 = 0$$

(b) $c_4 = 0.9400$

$\overline{X}$ *Chart*

$$UCL = CL + \frac{3\overline{s}}{c_4\sqrt{n}} = 22.846 + \frac{3*0.125}{0.94\sqrt{5}} = 23.024$$

$$CL = 22.846$$

$$UCL = CL - \frac{3\overline{s}}{c_4\sqrt{n}} = 22.846 - \frac{3*0.125}{0.94\sqrt{5}} = 22.668$$

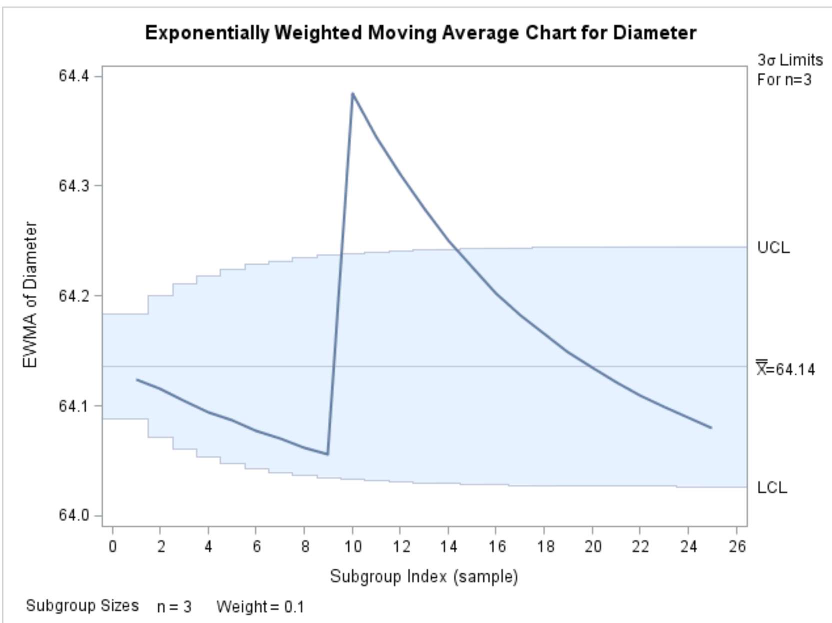
S *Chart*

$$UCL = \overline{s} + \frac{3\overline{s}}{c_4}\sqrt{1-c_4^2} = 0.261$$

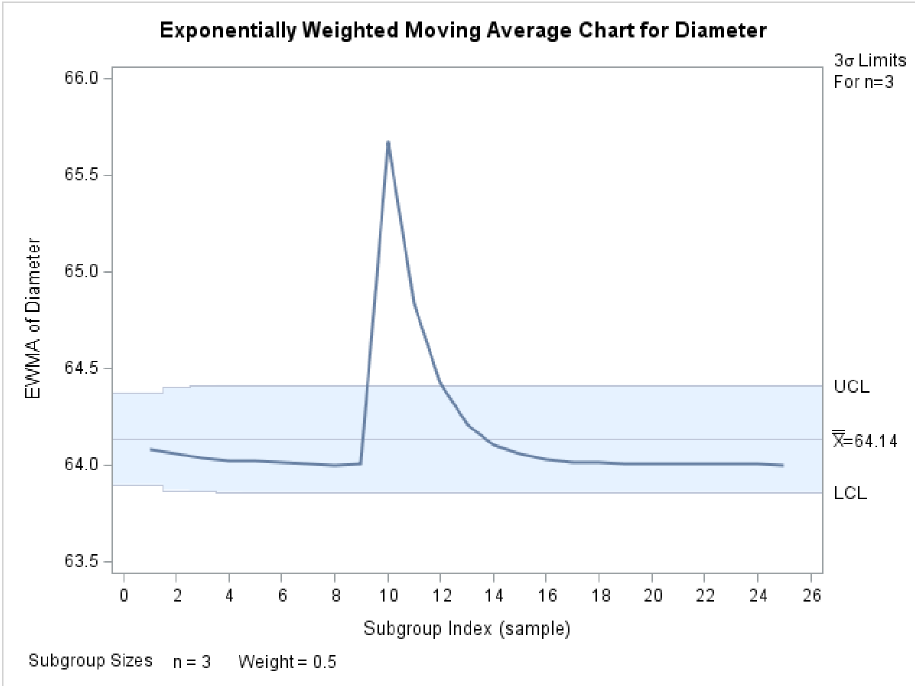
$$CL = 0.216$$

$$UCL = \overline{s} - \frac{3\overline{s}}{c_4}\sqrt{1-c_4^2} = 0$$

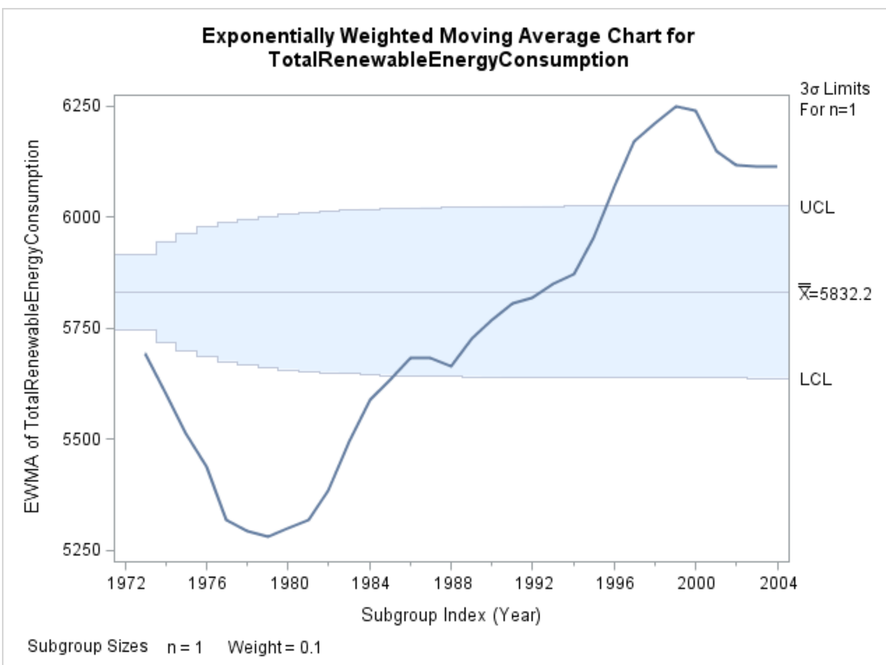
15-86 a) The following control chart use the average range from 25 subgroups of size 3 to estimate the process standard deviation. The software uses a pooled estimate of variance as the default method for an EWMA control chart so that the range method was selected from the options. Points are clearly out of control.



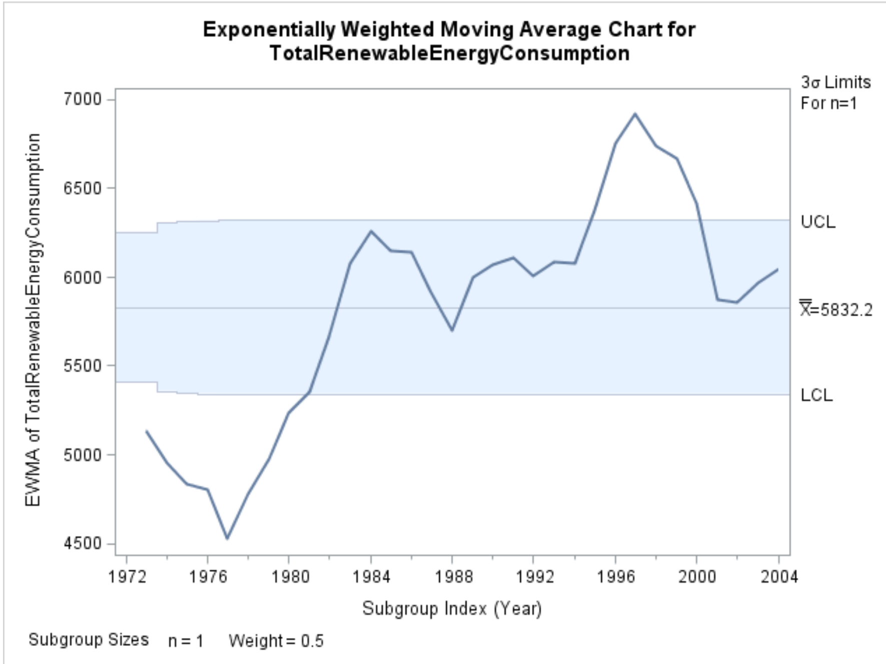
b) The following control chart use the average range from 25 subgroups of size 3 to estimate the process standard deviation. There is a large shift in the mean at sample 10 and the process is out of control at this point.



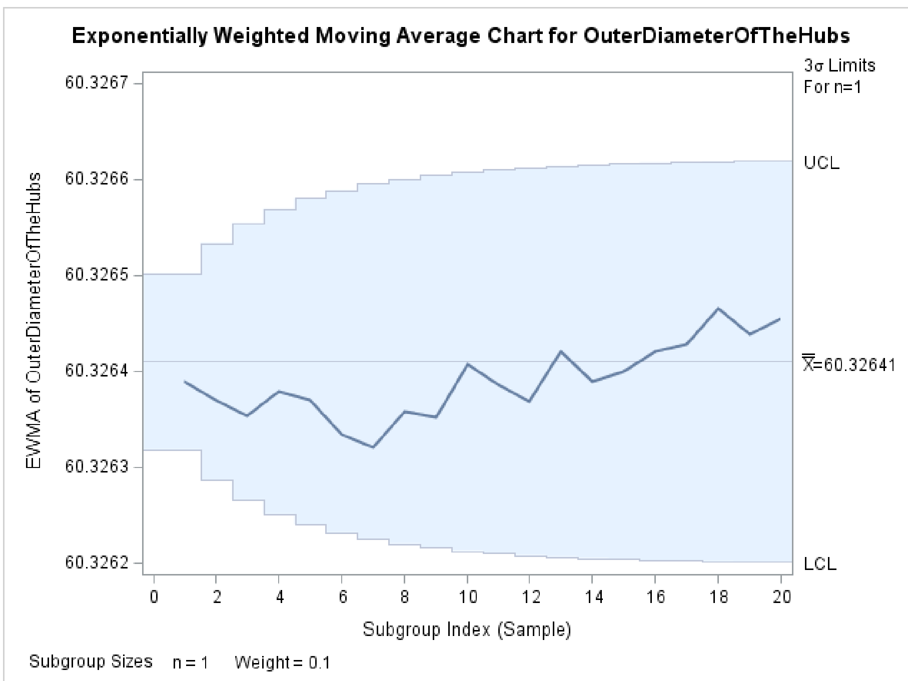
15-87 a) The data appear to be generated from an out-of-control process.



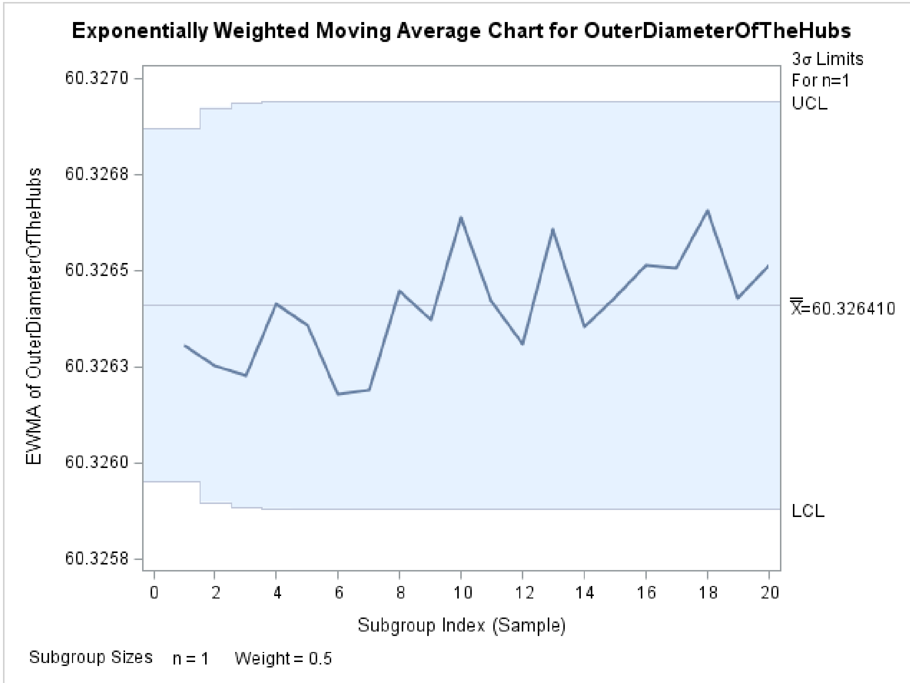
b) The data appear to be generated from an out-of-control process.



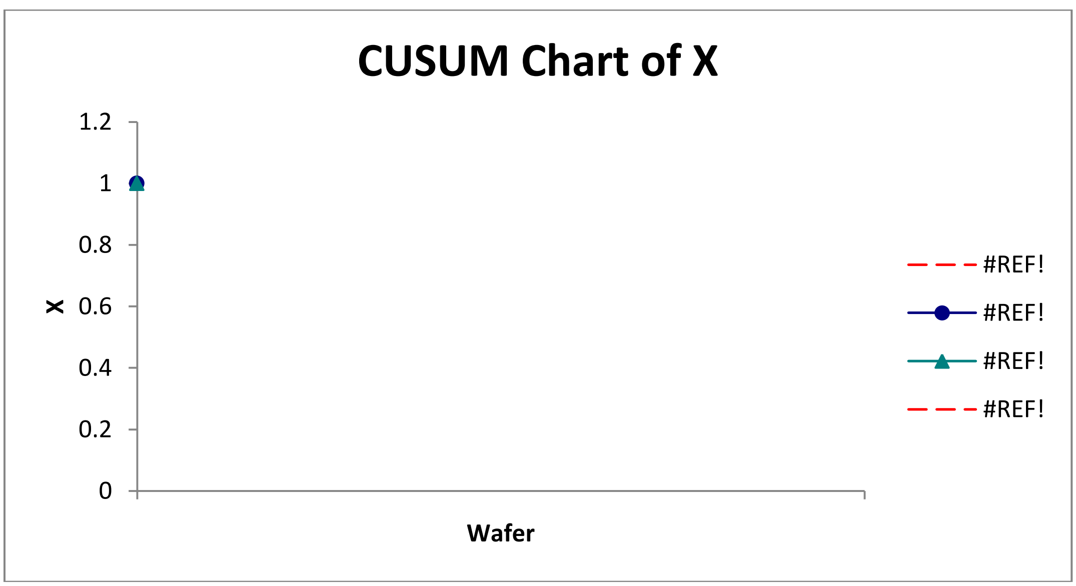
15-88 a) The process appears to be in control.



b) The process appears to be in control.

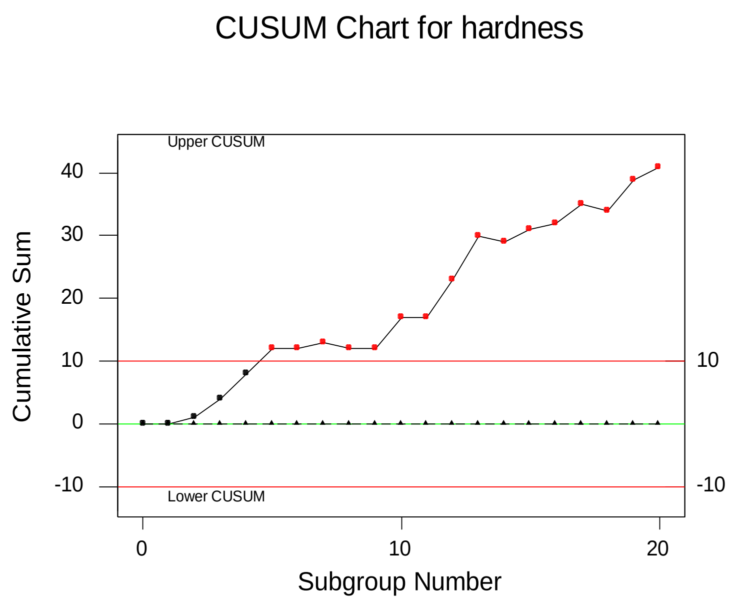


15-89

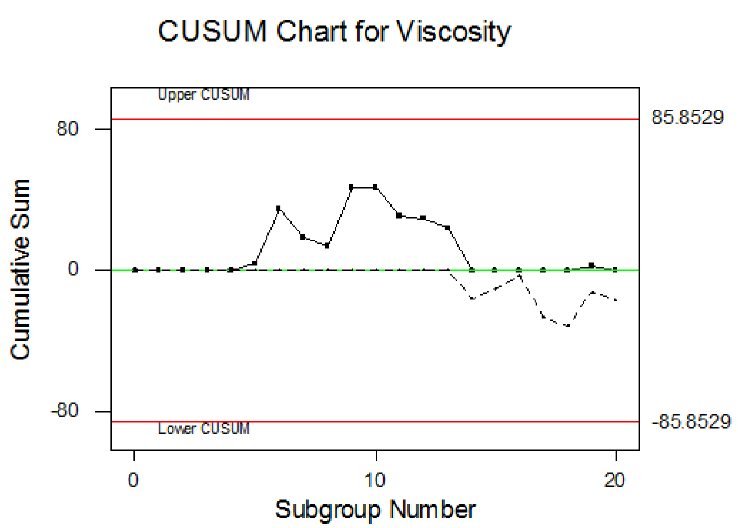


The process standard deviation is estimated using the average moving range of size 2 with MR/d_2, where d_2 = 1.128. The estimate is 1.05. Recommendation for k and h are 0.5 and 4 or 5, respectively for n = 1. For this chart h = 5 was used.

15-90 The process is not in control.
$K = k\sigma = 1$, so that $k = 0.5$
$H = h\sigma = 10$, so that $h = 5$

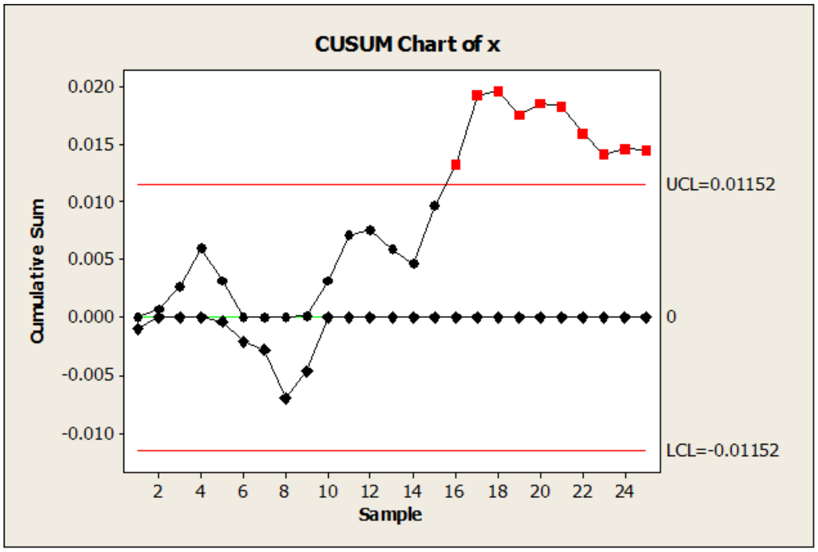


15-91



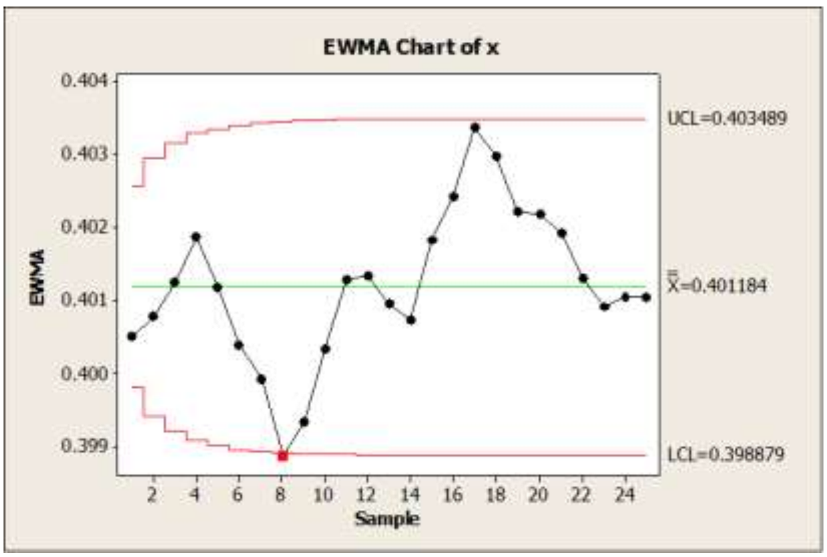
Process standard deviation is estimated using the average moving range of size 2 with MR/d_2, where d_2 = 1.128 for a moving range of 2. The estimate is 17.17. Recommendation for k and h are 0.5 and 4 or 5, respectively, for n = 1.

15-92 a)



Here σ is estimated using the moving range: 0.0026/1.128=0.0023. H and K were computed using k = 0.5 and h = 5. The process is not in control.

b) EWMA gives similar results.



15-93 a) Let p denote the probability that a point plots outside of the control limits when the mean has shifted from μ_0 to $\mu = \mu_0 + 1.5\sigma$. Then,

$$P(LCL < \overline{X} < UCL) = P\left(\mu_0 - \frac{3\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + \frac{3\sigma}{\sqrt{n}}\right)$$

$$= P\left(\frac{-1.5\sigma}{\sigma/\sqrt{n}} - 3 < \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} < \frac{-1.5\sigma}{\sigma/\sqrt{n}} + 3\right)$$

$$= P(-6.67 < Z < -0.67) = P(Z < -0.67) - P(Z < -6.67)$$

$$= 0.251$$

Therefore, the probability the shift is undetected for three consecutive samples is $(1 - p)^3 = (0.251)^3 = 0.0158$.

b) If 2-sigma control limits were used, then

$$1-p = P(LCL < \overline{X} < UCL) = P\left(\mu_0 - \frac{2\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + \frac{2\sigma}{\sqrt{n}}\right)$$

$$= P\left(\frac{-1.5\sigma}{\sigma/\sqrt{n}} - 2 < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{-1.5\sigma}{\sigma/\sqrt{n}} + 2\right)$$

$$= P(-5.67 < Z < -1.67) = P(Z < -1.67) - P(Z < -5.67)$$

$$= 0.047$$

Therefore, the probability the shift is undetected for three consecutive samples is $(1-p)^3 = (0.047)^3 =$ 0.0001.

c) The 2-sigma limits are narrower than the 3-sigma limits. Because the 2-sigma limits have a smaller probability of a shift being undetected, the 2-sigma limits would be better than the 3-sigma limits for a mean shift of 1.5σ. However, the 2-sigma limits would result in more signals when the process has not shifted (false alarms).

15-94 a)

$$\overline{\overline{x}} = \frac{5345}{25} = 213.8 \quad \overline{s} = \frac{326.52}{25} = 13.06 \qquad c_4 = 0.9213$$

$$\overline{X} \quad Chart$$

$$UCL = CL + \frac{3\overline{s}}{c_4\sqrt{n}} = 213.8 + \frac{3*13.06}{0.9213\sqrt{4}} = 235.06$$

$$CL = 213.8$$

$$UCL = CL - \frac{3\overline{s}}{c_4\sqrt{n}} = 213.8 - \frac{3*13.06}{0.9213\sqrt{4}} = 192.54$$

$$S \quad Chart$$

$$UCL = \overline{s} + \frac{3\overline{s}}{c_4}\sqrt{1-c_4^2} = 29.60$$

$$CL = 13.06$$

$$UCL = \overline{s} - \frac{3\overline{s}}{c_4}\sqrt{1-c_4^2} = 0$$

b) Process mean and standard deviation are

$$\hat{\mu} = \overline{\overline{x}} = 213.8$$

$$\hat{\sigma} = \frac{\overline{s}}{c_4} = \frac{13.06}{0.9213} = 14.18$$

15-95 a) Because ARL = 370, on the average we expect there to be one false alarm every 370 hours. Each 30-day month contains 30 × 24 = 720 hours of operation. Consequently, we expect 720/370 = 1.9 false alarms each month

$$P(X > \overline{X} + 3\hat{\sigma}) + P(X < \overline{X} - 3\hat{\sigma}) = P(z > 3) + P(z < -3) = 2(0.00135) = 0.0027$$

ARL=1/p=1/0.0027=370.37

b) With 2-sigma limits the probability of a point plotting out of control is determined as follows, when $\mu = \mu_0 + \sigma$

$P(X > UCL) + P(X < LCL)$

$$= P\left(\frac{X - \mu_0 - \sigma}{\sigma} > \frac{\mu_0 + 2\sigma - \mu_0 - \sigma}{\sigma}\right) + P\left(\frac{X - \mu_0 - \sigma}{\sigma} < \frac{\mu_0 - 2\sigma - \mu_0 - \sigma}{\sigma}\right)$$

$$= P(Z > 1) + P(Z < -3)$$

$$= 1 - P(Z < 1) + [1 - P(Z < 3)]$$

$$= 1 - 0.84134 + 1 - 0.99865$$

$$= 0.160$$

Therefore, ARL=1/p = 1/0.160 = 6.25. The 2-sigma limits reduce the ARL for detecting a shift in the mean of magnitude σ. However, the next part of this solution shows that the number of false alarms increases with 2-sigma limits.

c) 2σ limits

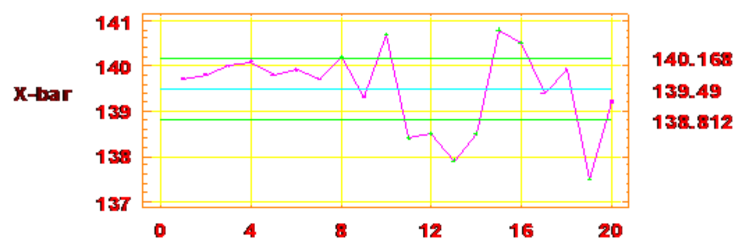
$$P(X > \overline{X} + 2\hat{\sigma}) + P(X < \overline{X} - 2\hat{\sigma}) = P(z > 2) + P(z < -2) = 2(0.02275) = 0.0455$$

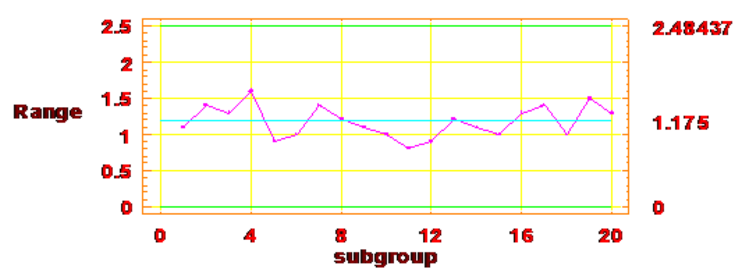
AR L= 1/ p= 1/0.0455 = 21.98. This ARL is not satisfactory. There would be too many false alarms. We would expect 32.76 false alarms per month.

15-96 a) X-bar and Range - Initial Study

Charting xbar

X-bar	Range
UCL: + 3.0 sigma = 140.168	UCL: + 3.0 sigma = 2.48437
Centerline = 139.49	Centerline = 1.175
LCL: - 3.0 sigma = 138.812	LCL: - 3.0 sigma = 0
out of limits = 9	out of limits = 0
Estimated
process mean = 139.49
process sigma = 0.505159
mean Range = 1.175



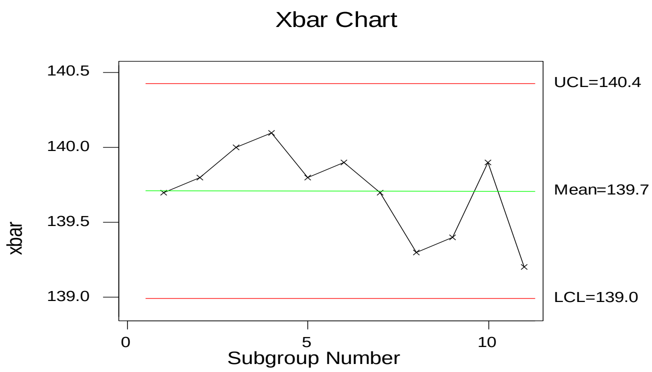


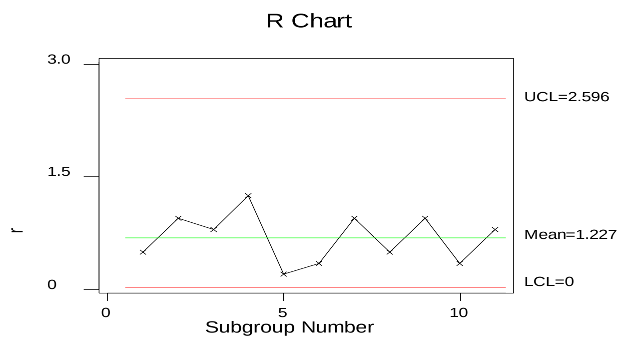
There are points beyond the control limits. The process is out of control. The points are 8, 10, 11, 12, 13, 14, 15, 16, and 19.

b) Revised control limits are given in the table below:

X-bar and Range - Initial Study

```
                    Charting Xbar
X-bar                          | Range
-----                          | -----
UCL: +    3.0 sigma = 140.417       | UCL: +    3.0 sigma = 2.595682
Centerline         = 139.709        | Centerline         = 1.227273
LCL: -    3.0 sigma = 139.001       | LCL: -    3.0 sigma = 0
out of limits = 0                   | out of limits = 0
```





There are no further points beyond the control limits.

The process standard deviation estimate is given by $\hat{\sigma} = \dfrac{\overline{R}}{d_2} = \dfrac{1.227273}{2.326} = 0.5276$

c) $PCR = \dfrac{USL - LSL}{6\hat{\sigma}} = \dfrac{142 - 138}{6(0.528)} = 1.26$

$$PCR_k = \min\left[\frac{USL - \overline{\overline{x}}}{3\hat{\sigma}}, \frac{\overline{\overline{x}} - LSL}{3\hat{\sigma}}\right] = \min\left[\frac{142 - 139.709}{3(0.528)}, \frac{139.709 - 138}{3(0.528)}\right]$$

$$= \min[1.45, 1.08] = 1.08$$

Because the process capability ratios are less than unity, the process capability appears to be poor. PCR is slightly larger than PCR_k indicating that the process is somewhat off center.

d) In order to make this process a "six-sigma process", the variance σ^2 would have to be decreased such that PCR_k = 2.0. The value of the variance is found by solving $PCR_k = \dfrac{\overline{\overline{x}} - LSL}{3\sigma} = 2.0$ for σ:

$$\frac{139.709-138}{3\sigma}=2.0$$
$$6\sigma=139.709-138$$
$$\sigma=\frac{139.709-138}{6}$$
$$\sigma=0.2848$$

Therefore, the process variance would have to be decreased to $\sigma^2 = (0.2848)^2 = 0.081$.

e) $\hat{\sigma}_{\overline{X}} = 0.528$

$$p=P(139.001<X<140.417 \mid \mu=139.7)$$
$$=P\left(\frac{139.001-139.7}{0.528}<\frac{X-\mu}{\sigma_x}<\frac{140.417-139.7}{0.528}\right)$$
$$=P(-1.32<Z<1.35)$$
$$=P(Z<1.36)-P(Z<-1.32)$$
$$=0.913085-0.093418$$
$$=0.8197$$

The probability that this shift will be detected on the next sample is $1 - p = 1–0.8197 = 0.1803$.

$$ARL=\frac{1}{1-p}=\frac{1}{0.1803}=5.55$$

15-97 a) The probability of having no signal is $P(-3<X<3)=0.9973$

P(No signal in 3 samples)=$(0.9973)^3 = 0.9919$
P(No signal in 5 samples)=$(0.9973)^5 = 0.9866$
P(No signal in 15 samples)=$(0.9973)^{15} = 0.9603$

15-98 PCR = 2 but $\mu=USL+2\sigma$

$$P(X<USL)=P\left(Z<\frac{(\mu-2\sigma)-\mu}{\sigma}\right)=P(Z<-2)=0.0228$$

15-99 a) The P(LCL < $\hat{P}$ < UCL), when $p = 0.07$, is needed.

$$LCL=\overline{p}-3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}=0.05-3\sqrt{\frac{0.05(1-0.05)}{100}}=-0.015\to 0$$
$$UCL=\overline{p}+3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}=0.05+3\sqrt{\frac{0.05(1-0.05)}{100}}=0.115$$

Therefore, when $p = 0.07$

$$P(0\le\hat{P}\le 0.115)=P(\hat{P}\le 0.115)=P\left(\frac{\hat{P}-0.07}{\sqrt{\frac{0.07(0.93)}{100}}}\le\frac{0.115-0.07}{\sqrt{\frac{0.07(0.93)}{100}}}\right)$$
$$=P(Z\le 1.76)=0.96$$

Using the normal approximation to the distribution of $\hat{P}$. Therefore, the probability of detecting the shift on the first sample following the shift is 1 − 0.96 = 0.04.

b) The probability that the control chart detects a shift to 0.07 on the second sample, but not the first, is (0.96)0.04= 0.0384. This uses the fact that the samples are independent to multiply the probabilities.

c) p = 0.10

$$P(0 \le \hat{P} \le 0.115) = P(\hat{P} \le 0.115) = P\left(\frac{\hat{P} - 0.10}{\sqrt{\frac{0.10(0.90)}{100}}} \le \frac{0.115 - 0.10}{\sqrt{\frac{0.10(0.90)}{100}}} \right)$$

$$= P(Z \le 0.5) = 0.69146$$

from the normal approximation to the distribution of $\hat{P}$. Therefore, the probability of detecting the shift on the first sample following the shift is 1 − 0.69146 = 0.30854.

The probability that the control chart detects a shift to 0.10 on the second sample after the shift, but not the first, is 0.69146(0.30854) = 0.2133.

c) A larger shift is generally easier to detect. Therefore, we should expect a shift to 0.10 to be detected quicker than a shift to 0.07.

15-100 $\bar{u} = 8$

a) n = 4

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 8 + 3\sqrt{\frac{8}{4}} = 12.24$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 8 - 3\sqrt{\frac{8}{4}} = 3.76$$

P($\bar{U}$ > 12.24 when λ = 16) = $P\left(Z > \frac{12.24 - 16}{\sqrt{16/4}} \right)$

= P(Z > −1.88) = 1 − P(Z < −1.88) = 1 − 0.03005= 0.96995

P($\bar{U}$ < 3.78) = $P\left(Z < \frac{3.76 - 16}{\sqrt{16/4}} \right)$

= P(Z < −6.12)
= 0

So the probability is 0.96995.

b) n = 10

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 8 + 3\sqrt{\frac{8}{10}} = 10.68$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 8 - 3\sqrt{\frac{8}{10}} = 5.32$$

P(U > 10.68 when λ = 16) = $P\left(Z > \frac{10.68 - 16}{\sqrt{\frac{16}{10}}} \right)$ = P(Z > −4.22) = 1

So the probability is 1.

15-101 $\bar{u} = 10$

a) $n = 1$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 10 + 3\sqrt{\frac{10}{1}} = 19.49$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 10 - 3\sqrt{\frac{10}{1}} = 0.51$$

P($\bar{U}$ > 19. 94 when λ = 14) = $P\left(Z > \frac{19.94 - 14}{\sqrt{14}}\right)$

= P(Z > 1.47)= 1 – P(Z < 1.47) = 1 – 0.9292 = 0.0708

and

P($\bar{U}$ < 0.51) = $P\left(Z < \frac{0.51 - 14}{\sqrt{14}}\right) = 0$

b) $n = 4$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 10 + 3\sqrt{\frac{10}{4}} = 14.74$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 10 - 3\sqrt{\frac{10}{4}} = 5.26$$

P($\bar{U}$ > 14.74 when λ = 14) = $P\left(Z > \frac{14.74 - 14}{\sqrt{\frac{14}{4}}}\right)$ = P(Z >0.40) = 1 – 0.6554 = 0.3446

P($\bar{U}$ < 5.26 when λ = 14) = $P\left(Z < \frac{5.26 - 14}{\sqrt{\frac{14}{4}}}\right)$ = P(Z < –4.67) = 0

15-102 $\bar{u} = 8$

a) $n = 5$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 8 + 3\sqrt{\frac{8}{5}} = 11.79$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 8 - 3\sqrt{\frac{8}{5}} = 4.21$$

P($\bar{U}$ > 11.79 when λ = 16) = $P\left(Z > \frac{11.79 - 16}{\sqrt{16/5}}\right)$

= P(Z > –2.35) = 1 – P(Z < –2.35)
= 1 – 0.009387 = 0.99061

P($\bar{U}$ < 4.21) = $P\left(Z < \frac{4.21 - 16}{\sqrt{16/5}}\right)$

= P(Z < –6.59)
= 0

So the probability is 0.99061.

b) $n = 15$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 8 + 3\sqrt{\frac{8}{15}} = 10.19$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 8 - 3\sqrt{\frac{8}{15}} = 5.81$$

P(U > 10.19 when λ = 16) = $P\left(Z > \dfrac{10.19 - 16}{\sqrt{\dfrac{16}{15}}}\right)$ = P(Z > –5.63) = 1

So the probability is 1.

15-103 $\bar{u} = 10$

a) n = 2

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 10 + 3\sqrt{\frac{10}{2}} = 16.71$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 10 - 3\sqrt{\frac{10}{2}} = 3.29$$

P($\bar{U}$ > 16.71 when λ = 14) = $P\left(Z > \dfrac{16.71 - 14}{\sqrt{7}}\right)$

= P(Z > 1.02)= 1 – P(Z < 1.02) = 1 – 0.8461 = 0.1339

and

P($\bar{U}$ < 3.29) = $P\left(Z < \dfrac{3.29 - 14}{\sqrt{7}}\right)$ = 0

b) n = 5

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 10 + 3\sqrt{\frac{10}{5}} = 14.24$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 10 - 3\sqrt{\frac{10}{5}} = 5.76$$

P($\bar{U}$ > 14.24 when λ = 14) = $P\left(Z > \dfrac{14.24 - 14}{\sqrt{\dfrac{14}{5}}}\right)$ = P(Z > 0.14) = 1 – 0.5558 = 0.4442

P($\bar{U}$ < 5.76 when λ = 14) = $P\left(Z < \dfrac{5.76 - 14}{\sqrt{\dfrac{14}{5}}}\right)$ = P(Z < –4.92) = 0

15-104 a) According to the table, if there is no shift, ARL = 500.
If the shift in mean is $1\sigma_{\overline{X}}$, ARL = 17.5.

b) According to the table, if there is no shift, ARL = 500.
If the shift in mean is $2\sigma_{\overline{X}}$, ARL = 3.63.

15-105 a) The natural tolerance limits are 200± 15.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{40}{30} = 1.33.$$

Since the mean of the process is centered at the nominal dimension,

$$PCR = 1.33.$$

Since the process natural tolerance limits lie inside the specifications, very few defective units will be produced.

The fraction defective is $2\Phi(-20/5) = 0.00633\%$

b) The natural tolerance limits are 206± 15.

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{40}{30} = 1.33.$$

Since the mean of the process is not centered at the nominal dimension,

$$PCR_k = \min[\frac{10}{15}, \frac{30}{15}].$$

The small PCR_k indicates that the process is likely to produce units outside the specification limits.
The fraction defective is

$$\begin{aligned} P(X < LSL) + P(X > USL) &= P(Z < \frac{-10}{5}) + P(Z > \frac{30}{5}) \\ &= P(Z < -2) + P(Z > 6) \\ &= 0.02275 + 0 \\ &= 0.02275 \end{aligned}$$

15-106 a)

$$PCR = \frac{USL - LSL}{6\sigma} = \frac{12}{6\sigma} = 1.5$$ so $\sigma = 1.33$

b) (55+67)/2=61 when the process is centered at the nominal dimension, the fraction defective is minimized for any σ.

15-107 a) If the process uses 80% of the specification band, then 6σ = 0.8(USL – LSL). Since the process is centered,

$$3\sigma = 0.8(USL - \mu) = 0.8(\mu - LSL)$$

$$3.75\sigma = USL - \mu = \mu - LSL$$

$$PCR = PCR_k = \min[\frac{3.75\sigma}{3\sigma}, \frac{3.75\sigma}{3\sigma}] = 1.25$$

Since PCR and PCR_k exceed unity, the natural tolerance limits are inside the specification limits and few defective units should be produced.

b) Assuming a normal distribution with $6\sigma = 0.8(USL - LSL)$ and a centered process, then $3\sigma = 0.8(USL - \mu)$. Consequently, $USL - \mu = 3.75\sigma$ and $\mu - LSL = 3.75\sigma$.

$$P(X > USL) = P(Z > \frac{3.75\sigma}{\sigma}) = P(Z > 3.75) = 1 - P(Z < 3.75) \approx 0$$

By symmetry, the fraction defective is $2[P(X > USL)] = 0$.

Mind Expanding Exercises

15-108 Let p denote the probability that a point plots outside of the control limits when the mean has shifted from μ_0 to $\mu = \mu_0 + 1.5\sigma$. Then:

$$\begin{aligned}1-p &= P\left(LCL < \overline{X} < UCL \mid \mu = \mu_0 + 1.5\sigma\right)\\ &= P\left(\mu_0 - 3\frac{\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + 3\frac{\sigma}{\sqrt{n}} \mid \mu = \mu_0 + 1.5\sigma\right)\\ &= P\left(\frac{-1.5\sigma}{\sigma/\sqrt{n}} - 3 < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{-1.5\sigma}{\sigma/\sqrt{n}} + 3\right)\\ &= P\left(-3 - 1.5\sqrt{n} < Z < +3 - 1.5\sqrt{n}\right) \text{ when } n = 4\\ &= P(-6 < Z < 0) = 0.5\end{aligned}$$

Therefore, the probability the shift is undetected for three consecutive samples is $(1 - p)^3 = 0.5^3 = 0.125$.

If 2-sigma control limits were used, then

$$\begin{aligned}1-p &= P\left(LCL < \overline{X} < UCL \mid \mu = \mu_0 + 1.5\sigma\right) = P\left(\mu_0 - 2\frac{\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + 2\frac{\sigma}{\sqrt{n}} \mid \mu = \mu_0 + 1.5\sigma\right)\\ &= P\left(\frac{-1.5\sigma}{\sigma/\sqrt{n}} - 2 < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{-1.5\sigma}{\sigma/\sqrt{n}} + 2\right) \text{ when } n = 4\\ &= P(-5 < Z < -1) = 0.1587 - 0 = 0.1587\end{aligned}$$

Therefore, the probability the shift is undetected for three consecutive samples is $(1 - p)^3 = 0.1587^3 = 0.004$.

15-109

$$\begin{aligned}LCL &= \mu_0 - k\sigma/\sqrt{n}\\ CL &= \mu_0\\ UCL &= \mu_0 + k\sigma/\sqrt{n}\end{aligned}$$

a)

$$\begin{aligned}1-p &= 1 - P\left(LCL < \overline{X} < UCL \mid \mu = \mu_0 + \delta\sigma\right)\\ &= 1 - P\left(\mu_0 - k\frac{\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + k\frac{\sigma}{\sqrt{n}} \mid \mu = \mu_0 + \delta\sigma\right)\\ 1 &= 1 - P\left(-k - \frac{\delta\sigma}{\sigma/\sqrt{n}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < k - \frac{\delta\sigma}{\sigma/\sqrt{n}}\right)\\ &= 1 - P(-k - \delta\sqrt{n} < Z < k - \delta\sqrt{n})\\ &= 1 - [\Phi(k - \delta\sqrt{n}) - \Phi(-k\delta\sqrt{n}]\end{aligned}$$

where $\Phi(Z)$ is the standard normal cumulative distribution function.

15-110

$$\begin{aligned}LCL &= \mu_0 - k\sigma/\sqrt{n}\\ CL &= \mu_0\\ UCL &= \mu_0 + k\sigma/\sqrt{n}\end{aligned}$$

a) ARL = $1/p$ where p is the probability a point plots outside of the control limits. Then,

$$1 - p = P\left(LCL < \overline{X} < UCL \mid \mu_0\right) = P\left(-k < \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} < k \mid \mu_0\right)$$
$$= P(-k < Z < k) = \Phi(k) - \Phi(-k) = 2\Phi(k) - 1$$

where $\Phi(Z)$ is the standard normal cumulative distribution function. Therefore, $p = 2 - 2\Phi(k)$ and ARL = $1/[2-2\Phi(k)]$. The mean time until a false alarm is $1/p$ hours.

b) ARL = $1/p$ where

$$1 - p = P\left(LCL < \overline{X} < UCL \mid \mu_1 = \mu_0 + \delta\sigma\right) = P\left(-k - \frac{\delta\sigma}{\sigma/\sqrt{n}} < \overline{X} < k - \frac{\delta\sigma}{\sigma/\sqrt{n}}\right)$$
$$= P\left(-k - \sqrt{n}\delta < Z < k - \sqrt{n}\delta\right)$$
$$= \Phi(k - \sqrt{n}\delta) - \Phi(-k - \sqrt{n}\delta)$$

and

$$p = 1 - \Phi(k - \sqrt{n}\delta) + \Phi(-k - \sqrt{n}\delta)$$

c) ARL = $1/p$ where $1 - p = P(-3<Z<3) = 0.9973$. Thus, ARL = 1/0.0027 = 370.4.

If $k = 2$, $1 - p = P(-2<Z<2) = 0.9545$ and ARL = $1/p$ = 22.0.

The 2-sigma limits result in a false alarm for every 22 points on the average. This is a high number of false alarms for the routine use of a control chart.

d) From part (b), ARL = 1/p, where 1 - p = $P(-3-\sqrt{5} < Z < 3-\sqrt{5}) = 0.7764$. ARL = 4.47 assuming 3- sigma control limits.

15-111 Determine n such that

$$0.5 = P(LCL < \hat{P} < UCL \mid p = p_c)$$
$$= P\left(\frac{\overline{p} - k\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} < \frac{\overline{P} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} < \frac{\overline{p} + k\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} \mid p = p_c\right)$$
$$= P\left(\frac{\overline{p} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} - \frac{k\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{p_c(1-p_c)}} < Z < \frac{\overline{p} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} - \frac{k\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{p_c(1-p_c)}}\right)$$

Use the fact that if $p_c > \overline{p}$ then the probability is approximately equal to the probability that Z is less than the upper limit.

$$\cong P\left(Z < \frac{\overline{p} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} - \frac{k\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{p_c(1-p_c)}}\right)$$

Then, the probability above approximately equals 0.5 if

$$\frac{\overline{p} - p_c}{\sqrt{\frac{p_c(1-p_c)}{n}}} = \frac{k\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{p_c(1-p_c)}}$$

Solving for *n*, $n = \dfrac{k^2 \bar{p}(1-\bar{p})}{(\bar{p}-p_c)^2}$

15-112 The LCL is $\bar{p} - k\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

$\bar{p} - k\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 0$ or $n = \dfrac{k^2(1-\bar{p})}{\bar{p}}$

15-113 The $P(LCL < \hat{P} < UCL | p = 0.08)$ is desired. Now, using the normal approximation:

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.05 - 3\sqrt{\frac{0.05(0.95)}{100}} = -0.015 \to 0$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.05 + 3\sqrt{\frac{0.05(0.95)}{100}} = 0.115$$

$$P(0 < \hat{P} < 0.115 \mid p = 0.08) = P(\hat{P} < 0.115 \mid p = 0.08)$$

$$= P\left(\frac{\hat{P} - 0.08}{\sqrt{\frac{0.08(1-0.08)}{100}}} < \frac{0.115 - 0.08}{\sqrt{\frac{0.08(1-0.08)}{100}}}\right) = P(Z < 1.29) = 0.90$$

Therefore, the probability of detecting shift on the first sample following the shift is 1 – 0.90 = 0.10.

The probability of detecting a shift by at least the third sample following the shift can be determined from the geometric distribution to be $0.10 + 0.90(0.10) + 0.90^2(0.10) = 0.27$

15-114 The process should be centered at the middle of the specifications; that is, at 100.

For an $\bar{x}$ chart:

$$CL = \mu_0 = 100$$

$$LCL = \mu_0 - k\sigma/\sqrt{n} = 100 - 3(5)/2 = 92.5$$

$$UCL = \mu_0 + k\sigma/\sqrt{n} = 100 + 3(5)/2 = 107.5$$

$$P(LCL < \bar{X} < UCL \mid \mu = 105) = P\left(\frac{92.5-105}{5/2} < \frac{\bar{X}-105}{5/2} < \frac{107.5-105}{5/2}\right)$$

$$= P(-5 < Z < 1) = 0.84$$

The requested probability is then 1 – 0.84 = 0.16. The ARL = 1/0.16 = 6.25. With $\mu = 105$, the specifications at 100 ± 15 and $\sigma = 5$, the probability of a defective item is

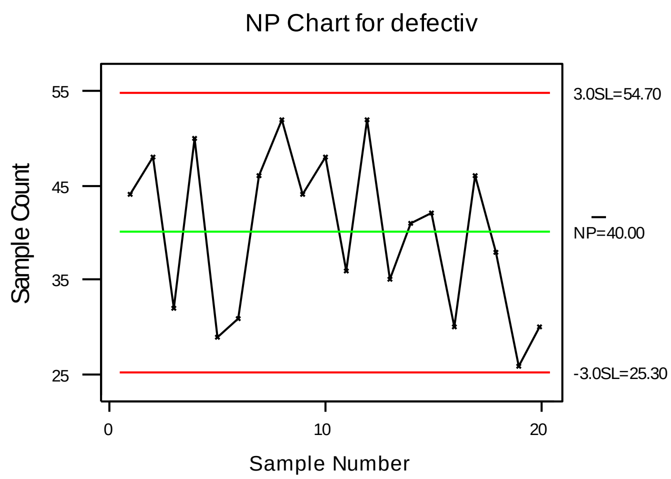
$$P(X < 85) + P(X > 115) = P\left(\frac{X-105}{5} < \frac{85-105}{5}\right) + P\left(\frac{X-105}{5} > \frac{115-105}{5}\right)$$

$$= P(Z < -4) + P(Z > 2) = 0.0228$$

Therefore, the average number of observations until a defective occurs, follows from the geometric distribution to be 1/0.0228 = 43.86. However, the $\bar{x}$ chart only requires 6.25 samples of 4 observations each = 6.25(4) = 25 observations, on average, to detect the shift.

15-115 a) Let X denote the number of defectives in a sample of n. Then X has a binomial distribution with E(X) = np and V(X) = np(1 - p). Therefore, the estimates of the mean and standard deviation of X are $n\bar{p}$ and $\sqrt{n\bar{p}(1-\bar{p})}$, respectively. Using these estimates results in the given control limits.

b) Data from example 16-4



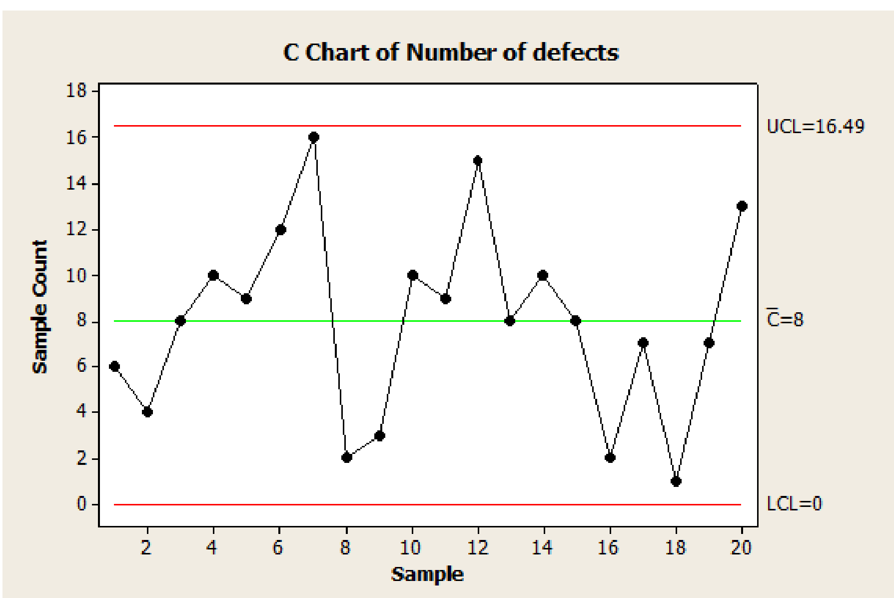
c)

$$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} < n\hat{P} < n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} < \hat{P} < \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Therefore, the *np* control chart always provides results equivalent to the *p* chart.

15-116 a) The center line $\overline{C} = 8$, and UCL=16.49 and LCL=0



b)Yes

15-117 Because $-3 < Z < 3$ if and only if

$$-3 < \frac{\hat{P} - \overline{p}}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}} < 3 \quad or \quad \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} < \hat{P}_i < \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

a point is in control on this chart if and only if the point is in control on the original *p* chart.

15-118 For unequal sample sizes, the *p* control chart can be used with the value of *n* equal to the size of each sample. That is,

$$Z_i = \frac{\hat{P} - \overline{p}}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_i}}}$$ where n_i is the size of the *i*th sample.