# *Chapter 6*

# *Control Charts for Variables*

## LEARNING OBJECTIVES

After completing this chapter you should be able to:

1. Understand the statistical basis of Shewhart control charts for variables
2. Know how to design variables control charts
3. Know how to set up and use $\bar{x}$ and $R$ control charts
4. Know how to estimate process capability from the control chart information
5. Know how to interpret patterns on $\bar{x}$ and $R$ control charts
6. Know how to set up and use $\bar{x}$ and $s$ or $s^2$ control charts
7. Know how to set up and use control charts for individual measurements
8. Understand the importance of the normality assumption for individuals control charts and know how to check this information
9. Understand the rational subgroup concept for variables control charts
10. Determine the average run length for variables control charts

## IMPORTANT TERMS AND CONCEPTS

Average run length
Control chart for individuals units
Control limits
Interpretation of control charts
Moving-range control chart
Natural tolerance limits for a process
Normality and control charts
Operating characteristic (OC) curve for the $\bar{x}$ control chart
Patterns on control charts
Phase I control chart usage
Phase II control chart usage
Probability limits for control charts
Process capability
Process capability ratio (PCR) $C_p$
$R$ control chart
Rational subgroups
$s$ control chart
$s^2$ control chart
Shewhart control charts
Specification limits
Three-sigma control limits
Tier chart or tolerance diagram
Trial control limits
Variable sample size on control charts
Variables control charts
$\bar{x}$ control chart

## EXERCISES

New exercises are marked with ☺

Minitab® Notes:

1. The Minitab convention for determining whether a point is out of control is: (1) if a plot point is within the control limits, it is in control, or (2) if a plot point is on or beyond the limits, it is out of control.
2. Minitab uses pooled standard deviation to estimate standard deviation for control chart limits and capability estimates. This can be changed in dialog boxes or under Tools>Options>Control Charts and Quality Tools>Estimating Standard Deviation.
3. Minitab defines some sensitizing rules for control charts differently than the standard rules. In particular, a run of *n* consecutive points on one side of the center line is defined as 9 points, not 8. This can be changed in dialog boxes, or under Tools > Options > Control Charts and Quality Tools > Tests.

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**6.1.** ☺

A manufacturer of component for automobile transmissions wants to use control charts to monitor a process producing a shaft. The resulting data from 20 samples of 4 shaft diameters that have been measured are:

$$\sum_{i=1}^{20} \bar{x}_i = 10.275, \quad \sum_{i=1}^{20} R_i = 1.012$$

(a) Find the control limits that should be used on the $\bar{x}$ and $R$ control charts.

For $n$ = 4, $A_2$ = 0.729, $D_4$ = 2.282, $D_3$ = 0

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{10.275}{20} = 0.5138; \quad \bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{1.012}{20} = 0.0506$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 0.5138 + 0.729(0.0506) = 0.5507$$

$$\text{CL}_{\bar{x}} = \bar{\bar{x}} = 0.5138$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 0.5138 - 0.729(0.0506) = 0.4769$$

(Equations 6.2, 6.3, 6.4, 6.5)

$$\text{UCL}_{\text{R}} = D_4\bar{R} = 2.282(0.0506) = 0.1155$$

$$\text{CL}_{\text{R}} = \bar{R} = 0.0506$$

$$\text{LCL}_{\text{R}} = D_3\bar{R} = 0(0.0506) = 0.00$$

(b) Assume that the 20 preliminary samples plot in control on both charts. Estimate the process mean and standard deviation.

Process mean = $\bar{\bar{x}} = 0.5138$

For $n$ = 4, $d_2$ = 2.059

Process standard deviation = $\hat{\sigma} = \bar{R}/d_2 = 0.0506 / 2.059 = 0.0246$ (Equation 6.6)

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**6.2.** ☺

A company manufacturing oil seals wants to establish $\bar{x}$ and $R$ control charts on the process. There are 25 preliminary samples of size 4 on the internal diameter of the seal. The summary data (in mm) are as follows:

$$\sum_{i=1}^{25} \bar{x}_i = 1,253.75, \quad \sum_{i=1}^{20} R_i = 14.08$$

(a) Find the control limits that should be used on the $\bar{x}$ and $R$ control charts.

For $n$ = 4, $A_2$ = 0.729, $D_4$ = 2.282, $D_3$ = 0

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} x_i}{m} = \frac{1253.75}{25} = 50.15; \quad \bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{14.08}{25} = 0.563$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 50.15 + 0.729(0.563) = 50.56$$

$$\text{CL}_{\bar{x}} = \bar{\bar{x}} = 50.15$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 50.15 - 0.729(0.563) = 49.74$$ (Equations 6.2, 6.3, 6.4, 6.5)

$$\text{UCL}_R = D_4\bar{R} = 2.282(0.563) = 1.285$$

$$\text{CL}_R = \bar{R} = 0.563$$

$$\text{LCL}_R = D_3\bar{R} = 0(0.563) = 0.00$$

(b) Assume that the 25 preliminary samples plot in control on both charts. Estimate the process mean and standard deviation.

Process mean = $\bar{\bar{x}} = 50.15$

For $n$ = 4, $d_2$ = 2.059

Process standard deviation = $\hat{\sigma} = \bar{R}/d_2 = 0.563/2.059 = 0.273$ (Equation 6.6)

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**6.3.** ☺

Reconsider the situation described in Exercise 6.1. Suppose that several of the preliminary 20 samples plot out of control on the *R* chart. Does this have any impact on the reliability of the control limits on the $\bar{x}$ chart?

Yes. Out-of-control samples on the *R* chart signal a potential problem with process variability, resulting in an unreliable estimate of the process standard deviation, and consequently impacting the accuracy of the upper and lower control limits on the $\bar{x}$ chart.

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**6.4.** ☺

Discuss why it is important to establish control on the *R* chart first when using $\bar{x}$ and $R$ control charts to bring a process into statistical control

If the *R* chart is out of control, the process variability is unstable, and the control limits on the $\bar{x}$ chart (which requires an estimate of process variability) are unstable. Without valid control limits, it is difficult to judge whether the process average is actually in control, and improvement attempts could actually result in a waste of resources and time.

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## 6.5. ☺

A hospital emergency department is monitoring the time required to admit a patient using $\bar{x}$ and $R$ charts. Table 6E.1 presents summary data for 20 subgroups of two patients each (time is in minutes).

**TABLE 6E.1**
Hospital Admission Time Data for Exercise 6.5

| Subgroup | $\bar{x}$ | $R$ | Subgroup | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|
| 1 | 8.3 | 2 | 11 | 8.8 | 3 |
| 2 | 8.1 | 3 | 12 | 9.1 | 5 |
| 3 | 7.9 | 1 | 13 | 5.9 | 3 |
| 4 | 6.3 | 5 | 14 | 9.0 | 6 |
| 5 | 8.5 | 3 | 15 | 6.4 | 3 |
| 6 | 7.5 | 4 | 16 | 7.3 | 3 |
| 7 | 8.0 | 3 | 17 | 5.3 | 2 |
| 8 | 7.4 | 2 | 18 | 7.6 | 4 |
| 9 | 6.4 | 2 | 19 | 8.1 | 3 |
| 10 | 7.5 | 4 | 20 | 8.0 | 2 |

(a) Use these data to determine the control limits for the $\bar{x}$ and $R$ control charts for this patient admitting process.

For $n = 2$, $A_2 = 1.880$, $D_4 = 3.267$, $D_3 = 0$

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{151.4}{20} = 7.6; \quad \bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{63}{20} = 3.2$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 7.6 + 1.880(3.2) = 13.6$$

$$\text{CL}_{\bar{x}} = \bar{\bar{x}} = 7.6$$

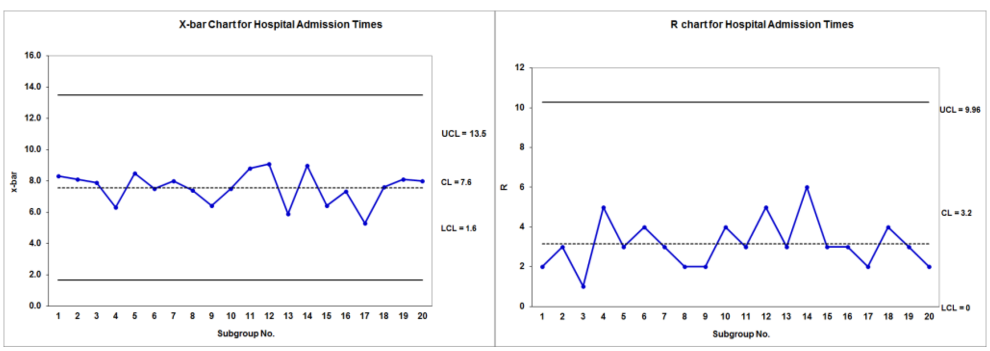
$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 7.6 - 1.880(3.2) = 1.6 \qquad \text{(Equations 6.2, 6.3, 6.4, 6.5)}$$

$$\text{UCL}_R = D_4\bar{R} = 3.267(3.2) = 10.5$$

$$\text{CL}_R = \bar{R} = 3.2$$

$$\text{LCL}_R = D_3\bar{R} = 0(3.2) = 0.00$$

(b) Plot the preliminary data from the first 20 samples on the control charts that you set up in part (a). Is this process in statistical control?




There is no indication of an out-of-control condition on the $R$ chart, indicating that process variability is in control. However points on the $\bar{x}$ chart tend to cluster around the center line, warranting further investigation. This may be sign that the limits were calculated incorrectly, or that the *within* sample variability poorly estimates process variability perhaps due to mixed underlying distributions.

**6.6.** ☺

Components used in a digital SLR camera are manufactured with nominal dimension of 0.3 mm and lower and upper specification limits of 0.295 mm and 0.305 mm respectively. The $\bar{x}$ and $R$ control charts for this process are based on subgroups of size 5 and they exhibit statistical control, with the center line on the $\bar{x}$ at 0.3020 mm and the center line on the $R$ chart at 0.00144 mm.

(a) Estimate the mean and standard deviation of this process.

Process mean = $\bar{\bar{x}} = \text{CL}_{\bar{x}} = 0.3020$

For $n$ = 5, $d_2$ = 2.326

Process standard deviation = $\hat{\sigma} = \bar{R}/d_2 = \text{CL}_{\bar{R}}/d_2 = 0.00154/2.326 = 0.00066$ (Equation 6.6)

(b) Suppose that parts below the lower specification limits can be reworked, but parts above the upper specification limit must be scrapped. Estimate the proportion of scrap and rework produced by this process.

$$p_{rework} = P\{x > 0.305\} = 1 - \Phi\left(\frac{\text{USL} - \text{CL}_{\bar{x}}}{\hat{\sigma}}\right) = 1 - \Phi\left(\frac{0.305 - 0.3020}{0.00066}\right) = 1 - \Phi(4.545) = 1 - 0.999997 = 0.000003$$

$$p_{scrap} = P\{x < 0.295\} = \Phi\left(\frac{\text{LSL} - \text{CL}_{\bar{x}}}{\hat{\sigma}}\right) = \Phi\left(\frac{0.295 - 0.3020}{0.00066}\right) = \Phi(-10.606) = 0$$

(p. 242)

(c) Suppose that the mean of this process can be reset by fairly simple adjustments. What value of the process mean would you recommend?

Recommend centering at the midpoint of the specification, Target = (USL + LSL) / 2 = 0.300

**6.7.**

The data shown in Table 6E.2 are $\bar{x}$ and $R$ values for 24 samples of size $n$ = 5 taken from a process producing bearings. The measurements are made on the inside diameter of the bearing, with only the last three decimals recorded (i.e., 34.5 should be 0.50345).

**TABLE 6E.2**
**Bearing Diameter Data**

| Sample Number | $\bar{x}$ | $R$ | Sample Number | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|
| 1 | 34.5 | 3 | 13 | 35.4 | 8 |
| 2 | 34.2 | 4 | 14 | 34.0 | 6 |
| 3 | 31.6 | 4 | 15 | 37.1 | 5 |
| 4 | 31.5 | 4 | 16 | 34.9 | 7 |
| 5 | 35.0 | 5 | 17 | 33.5 | 4 |
| 6 | 34.1 | 6 | 18 | 31.7 | 3 |
| 7 | 32.6 | 4 | 19 | 34.0 | 8 |
| 8 | 33.8 | 3 | 20 | 35.1 | 4 |
| 9 | 34.8 | 7 | 21 | 33.7 | 2 |
| 10 | 33.6 | 8 | 22 | 32.8 | 1 |
| 11 | 31.9 | 3 | 23 | 33.5 | 3 |
| 12 | 38.6 | 9 | 24 | 34.2 | 2 |

(a) Set up $\bar{x}$ and $R$ charts on this process. Does the process seem to be in statistical control? If necessary, revise the trial control limits.

For $n$ = 5, $A_2$ = 0.577, $D_4$ = 2.114, $D_3$ = 0

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m} = \frac{34.5 + 34.2 + \cdots + 34.2}{24} = 34.00$$

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m} = \frac{3 + 4 + \cdots + 2}{24} = 4.71$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 34.00 + 0.577(4.71) = 36.72$$

$$\text{CL}_{\bar{x}} = \bar{\bar{x}} = 34.00$$

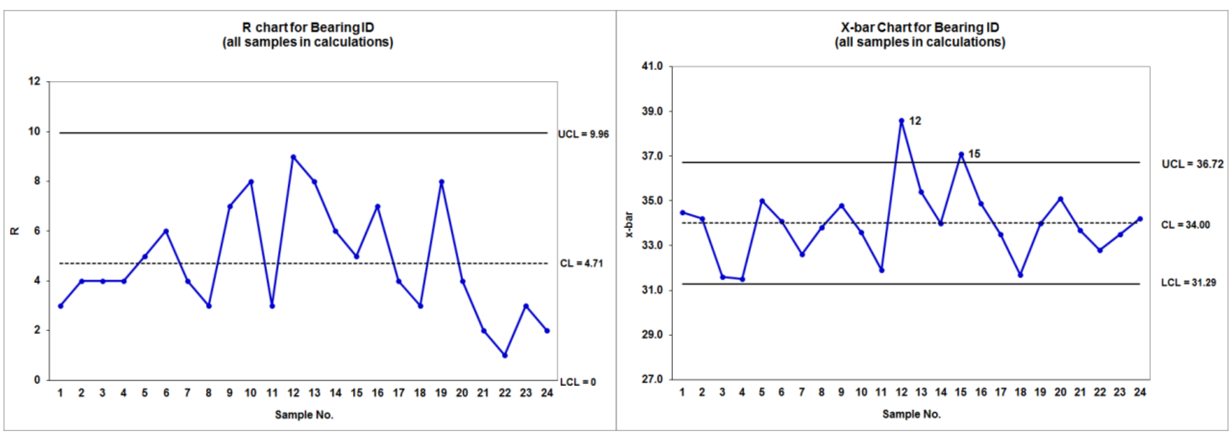
$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 34.00 - 0.577(4.71) = 31.29$$

$$\text{UCL}_R = D_4\bar{R} = 2.114(4.71) = 9.96$$

$$\text{CL}_R = \bar{R} = 4.71$$

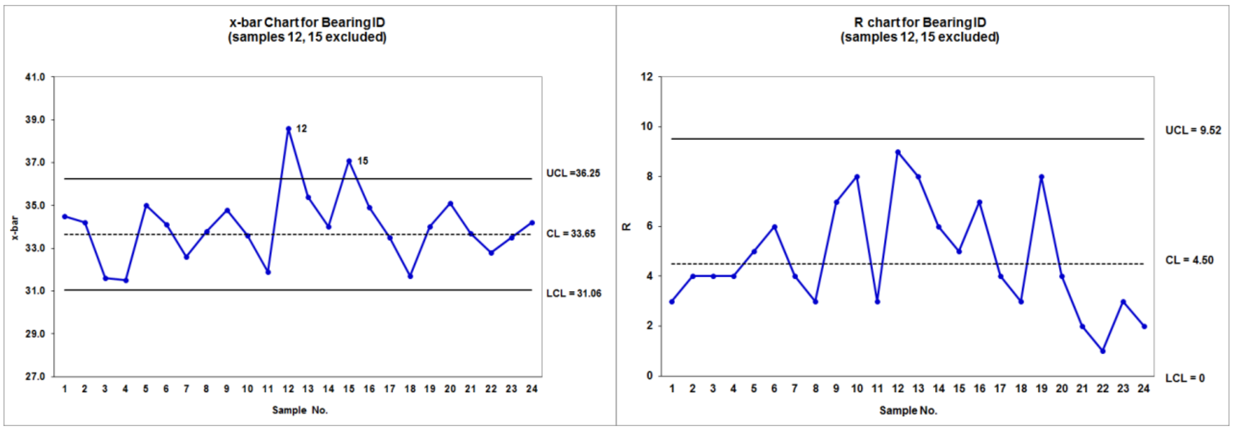
$$\text{LCL}_R = D_3\bar{R} = 0(4.71) = 0.00$$

(Equations 6.2, 6.3, 6.4, 6.5)



The *R* chart is in control, so the $\bar{x}$ chart can be evaluated. The process is not in statistical control; $\bar{x}$ is beyond the upper control limit for both Sample No. 12 and Sample No. 15. Assuming an assignable cause is found for these two out-of-control points, the two samples can be excluded from the control limit calculations. The new process parameter estimates are:

$$\bar{\bar{x}} = 33.65;\ \ \bar{R} = 4.5;\ \ \hat{\sigma}_x = \bar{R}/d_2 = 4.5/2.326 = 1.93$$
$$\text{UCL}_{\bar{x}} = 36.25; \text{CL}_{\bar{x}} = 33.65; \text{LCL}_{\bar{x}} = 31.06$$
$$\text{UCL}_R = 9.52; \text{CL}_R = 4.5; \text{LCL}_R = 0.00$$




(b) If specifications on this diameter are $0.5030 \pm 0.0010$, find the percentage of nonconforming bearings produced by this process. Assume that diameter is normally distributed.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Pr\{x < 20\} + \Pr\{x > 40\} = \Pr\{x < 20\} + \left[1 - \Pr\{x < 40\}\right]$$
$$= \Phi\left(\frac{20 - 33.65}{1.93}\right) + \left[1 - \Phi\left(\frac{40 - 33.65}{1.93}\right)\right] \qquad \text{(p. 242)}$$
$$= \Phi(-7.07) + 1 - \Phi(3.29) = 0 + 1 - 0.99950 = 0.00050$$

**6.8.**

A high-level voltage power supply should have a nominal output voltage of 350 V. A sample of four units is selected each day and tested for process-control purposes. The data shown in Table 6E.3 give the difference between the observed reading on each unit and the nominal voltage times ten; that is,

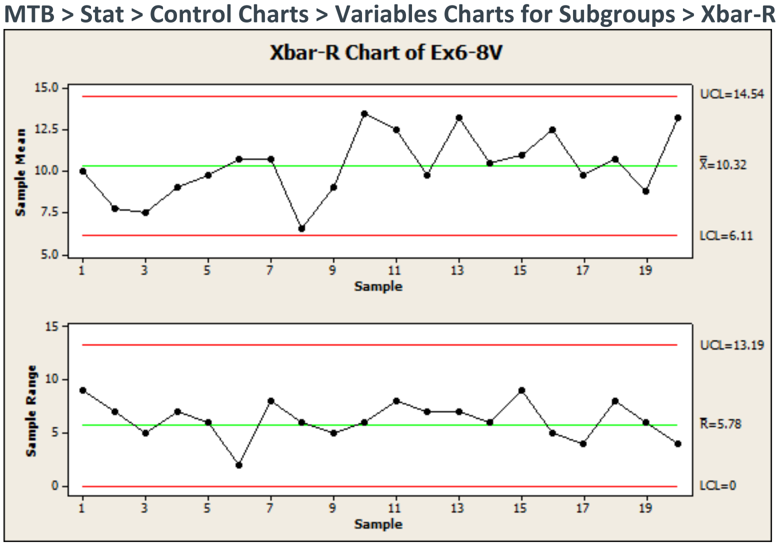
$x_i = (\text{observed voltage on unit } i - 350)10$

**TABLE 6E.3**
Voltage Data for Exercise 6.8

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| 1 | 6 | 9 | 10 | 15 |
| 2 | 10 | 4 | 6 | 11 |
| 3 | 7 | 8 | 10 | 5 |
| 4 | 8 | 9 | 6 | 13 |
| 5 | 9 | 10 | 7 | 13 |
| 6 | 12 | 11 | 10 | 10 |
| 7 | 16 | 10 | 8 | 9 |
| 8 | 7 | 5 | 10 | 4 |
| 9 | 9 | 7 | 8 | 12 |
| 10 | 15 | 16 | 10 | 13 |
| 11 | 8 | 12 | 14 | 16 |
| 12 | 6 | 13 | 9 | 11 |
| 13 | 16 | 9 | 13 | 15 |
| 14 | 7 | 13 | 10 | 12 |
| 15 | 11 | 7 | 10 | 16 |
| 16 | 15 | 10 | 11 | 14 |
| 17 | 9 | 8 | 12 | 10 |
| 18 | 15 | 7 | 10 | 11 |
| 19 | 8 | 6 | 9 | 12 |
| 20 | 13 | 14 | 11 | 15 |

(a) Set up $\bar{x}$ and $R$ charts on this process. Is the process in statistical control?

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R




The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

**6.8. continued**

(b) If specifications are at 350 V $\pm$ 5 V, what can you say about process capability?

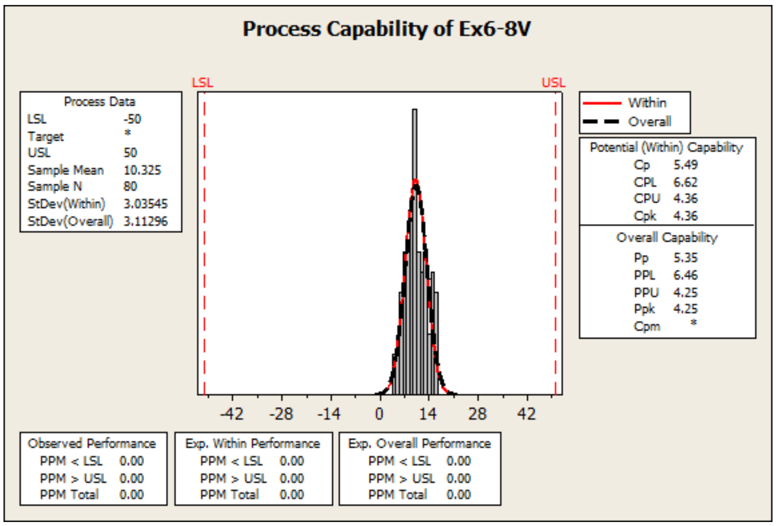
$n = 4, \bar{\bar{x}} = 10.33, \bar{R} = 6.25, \hat{\sigma}_x = \bar{R}/d_2 = 6.25/2.059 = 3.035$. Actual specs are 350 $\pm$ 5 V.

With $x_i$ = (observed voltage on unit $i$ – 350) $\times$ 10: $USL_T$ = +50, $LSL_T$ = –50

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{+50 - (-50)}{6(3.035)} = 5.49$$, so the process is capable.

**MTB > Stat > Quality Tools > Capability Analysis > Normal**

**In the Estimate dialog box, select Rbar to estimate standard deviation**

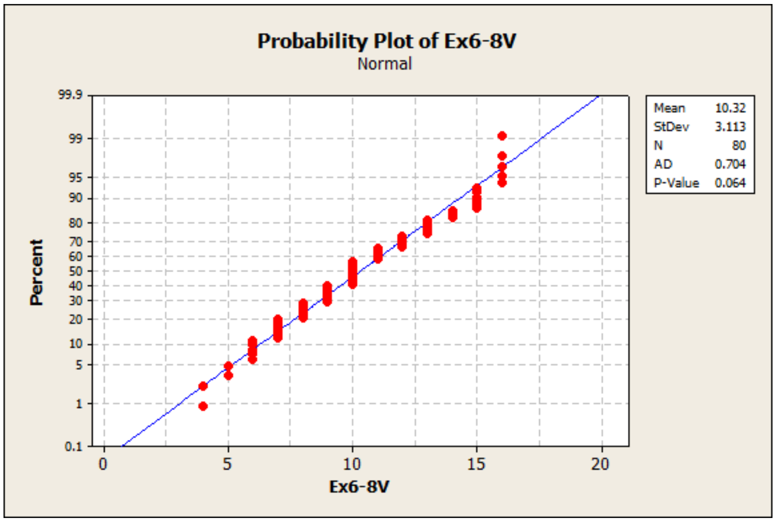



The process is capable, with a Process Capability Ratio (PCR, or $C_p$) of 5.49, and an estimate of the fraction nonconforming less than 0.00 PPM Total.

**6.8. continued**

(c) Is here evidence to support the claim that voltage is normally distributed?

**MTB > Stat > Basic Statistics > Normality Test**




A normal probability plot of the transformed output voltage shows the distribution is close to normal.

**6.9.**

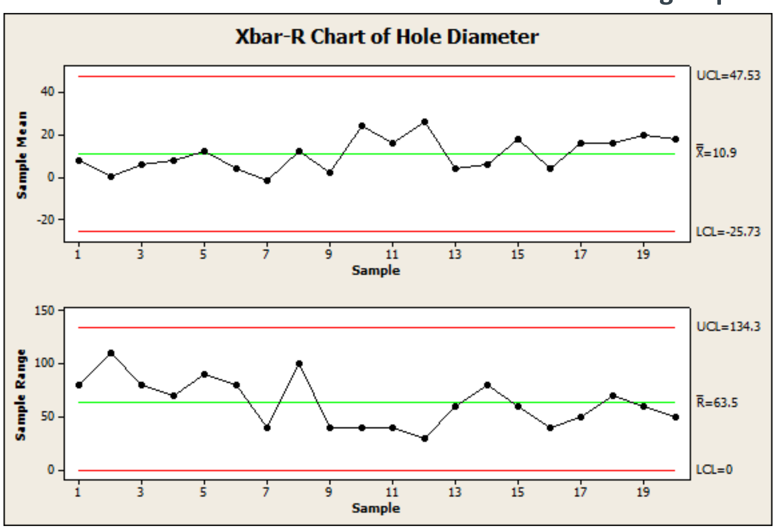
The data shown in Table 6E.4 are the deviations from nominal diameter for holes drilled din a carbon-fiber composite material used in aerospace manufacturing. The values reported are deviations from nominal in ten-thousandths of an inch.

**TABLE 6E.4**
Hole Diameter Data for Exercise 6.9

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| 1 | −30 | +50 | −20 | +10 | +30 |
| 2 | 0 | +50 | −60 | −20 | +30 |
| 3 | −50 | +10 | +20 | +30 | +20 |
| 4 | −10 | −10 | +30 | −20 | +50 |
| 5 | +20 | −40 | +50 | +20 | +10 |
| 6 | 0 | 0 | +40 | −40 | +20 |
| 7 | 0 | 0 | +20 | −20 | −10 |
| 8 | +70 | −30 | +30 | −10 | 0 |
| 9 | 0 | 0 | +20 | −20 | +10 |
| 10 | +10 | +20 | +30 | +10 | +50 |
| 11 | +40 | 0 | +20 | 0 | +20 |
| 12 | +30 | +20 | +30 | +10 | +40 |
| 13 | +30 | −30 | 0 | +10 | +10 |
| 14 | +30 | −10 | +50 | −10 | −30 |
| 15 | +10 | −10 | +50 | +40 | 0 |
| 16 | 0 | 0 | +30 | −10 | 0 |
| 17 | +20 | +20 | +30 | +30 | −20 |
| 18 | +10 | −20 | +50 | +30 | +10 |
| 19 | +50 | −10 | +40 | +20 | 0 |
| 20 | +50 | 0 | 0 | +30 | +10 |

(a) Set up $\bar{x}$ and $R$ charts on the process. Is the process in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

**6.9. continued**

(b) Estimate the process standard deviation using the range method.

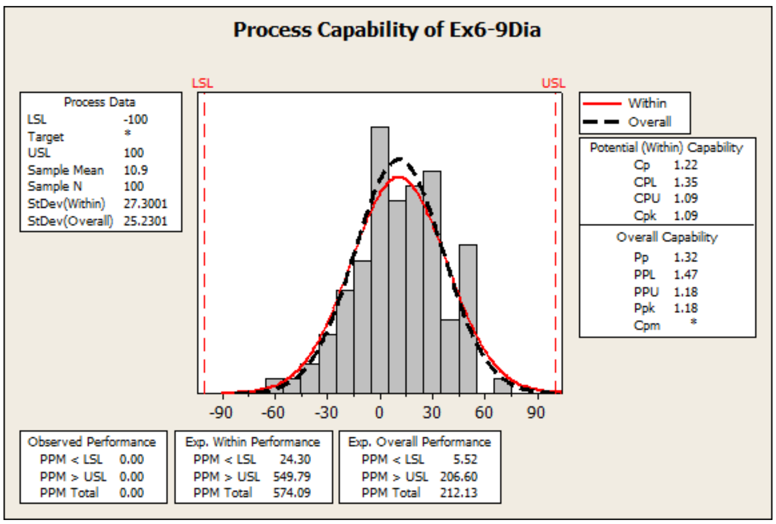
$$\hat{\sigma}_x = \overline{R} / d_2 = 63.5 / 2.326 = 27.3$$

(c) If specifications are at nominal ± 100, what can you say about the capability of this process? Calculate the PCR $C_p$.

USL = +100, LSL = –100

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+100 - (-100)}{6(27.3)} = 1.22$$, so the process is capable.

**MTB > Stat > Quality Tools > Capability Analysis > Normal**



**6.10.**

The thickness of a printed circuit board is an important quality parameter. Data on board thickness (in inches) are given in Table 6E.5 for 25 samples of three boards each.

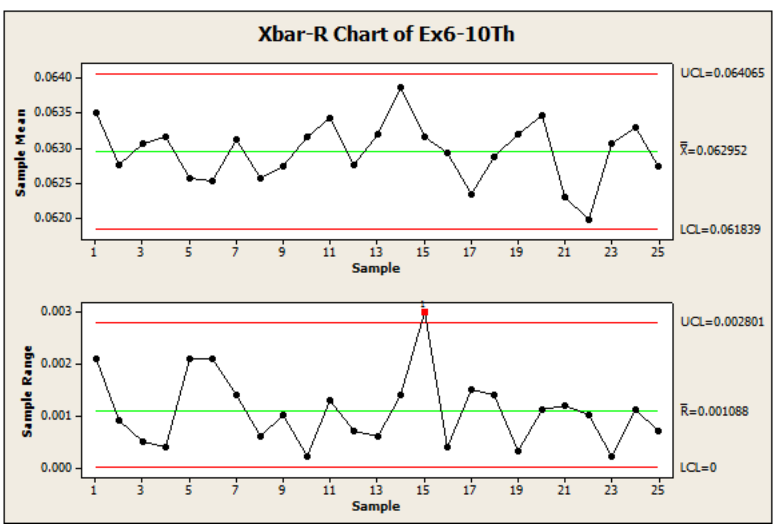
**TABLE 6E.5**
**Printed Circuit Board Thickness for Exercise 6.10**

| Sample Number | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| 1 | 0.0624 | 0.0636 | 0.0645 |
| 2 | 0.0630 | 0.0631 | 0.0622 |
| 3 | 0.0628 | 0.0631 | 0.0633 |
| 4 | 0.0634 | 0.0630 | 0.0631 |
| 5 | 0.0614 | 0.0628 | 0.0635 |
| 6 | 0.0613 | 0.0629 | 0.0634 |
| 7 | 0.0630 | 0.0639 | 0.0625 |
| 8 | 0.0628 | 0.0627 | 0.0622 |
| 9 | 0.0623 | 0.0626 | 0.0633 |
| 10 | 0.0631 | 0.0631 | 0.0633 |
| 11 | 0.0630 | 0.0630 | 0.0643 |
| 12 | 0.0623 | 0.0630 | 0.0630 |
| 13 | 0.0630 | 0.0636 | 0.0630 |
| 14 | 0.0645 | 0.0640 | 0.0631 |
| 15 | 0.0614 | 0.0644 | 0.0637 |
| 16 | 0.0631 | 0.0627 | 0.0630 |
| 17 | 0.0616 | 0.0623 | 0.0631 |
| 18 | 0.0630 | 0.0635 | 0.0621 |
| 19 | 0.0631 | 0.0631 | 0.0634 |
| 20 | 0.0640 | 0.0635 | 0.0629 |
| 21 | 0.0628 | 0.0625 | 0.0616 |
| 22 | 0.0615 | 0.0625 | 0.0619 |
| 23 | 0.0630 | 0.0632 | 0.0630 |
| 24 | 0.0630 | 0.0629 | 0.0640 |
| 25 | 0.0623 | 0.0629 | 0.0630 |

(a) Set up $\bar{x}$ and $R$ control charts. Is the process in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**

### Xbar-R Chart of Ex6-10Th

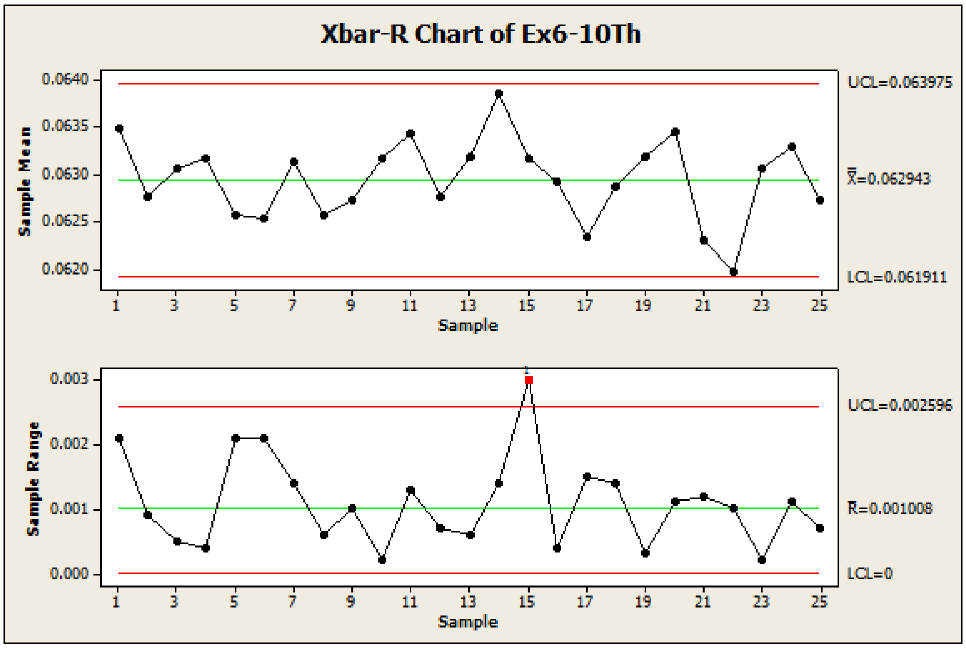



### Test Results for R Chart of Ex6-10Th

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
```

**6.10. continued**

The process is out-of-control, with one point exceeding the UCL on the *R* chart. Assuming an assignable cause is found, remove the out-of-control point (sample 15) and re-calculate control limits. Removing the sample from calculation (in Minitab, **X-bar-R Chart options > Estimate > Omit the following subgroups**), the new control limits are:




(b) Estimate the process standard deviation.

$\hat{\sigma}_x = \overline{R} / d_2 = 0.0010 / 1.693 = 0.0006$

(c) What are the limits that you would expect to contain nearly all the process measurements?

Natural tolerance limits are: $\overline{\overline{x}} \pm 3\hat{\sigma}_x = 0.0630 \pm 3(0.0006) = [0.0612, 0.0648]$

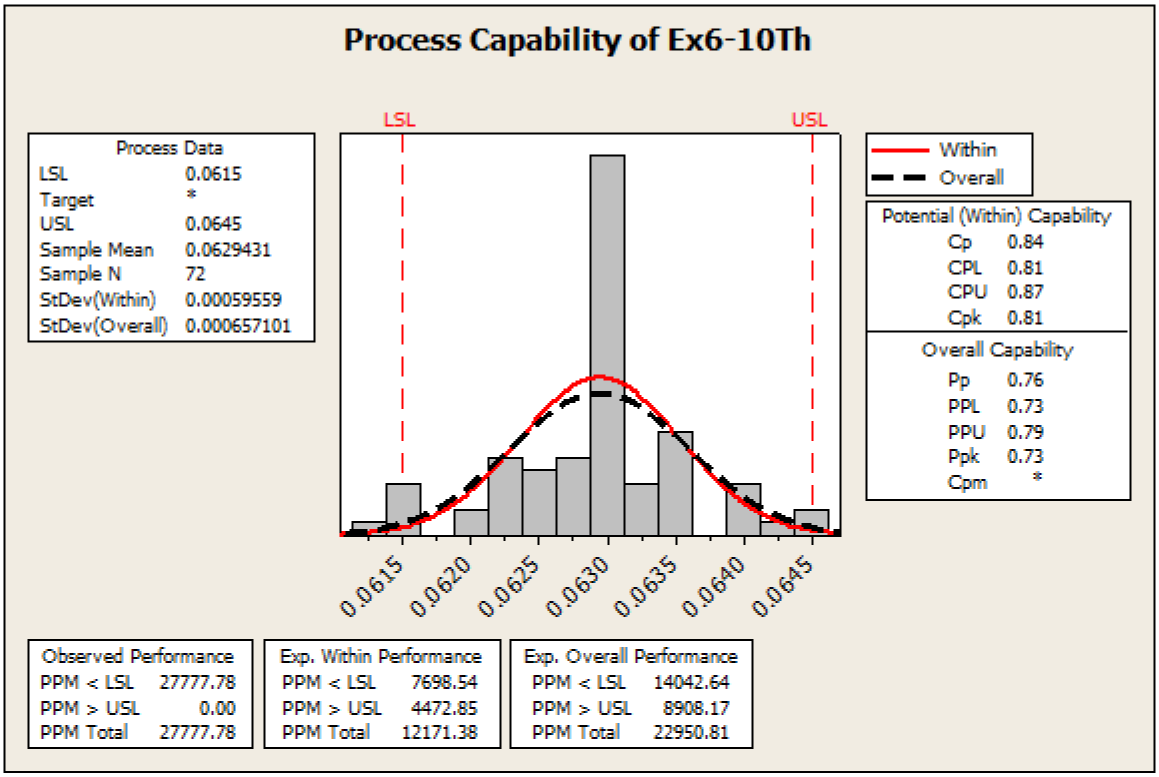
**6.10. continued**

(d) If the specifications are at 0.0630 in. ± 0.0015 in., what is the value of the PCR $C_p$?

Assuming that printed circuit board thickness is normally distributed, and excluding sample 15 from the process capability estimation:

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+0.0015 - (-0.0015)}{6(0.0006)} = 0.8333$$

**MTB > Stat > Quality Tools > Capability Analysis > Normal**



**6.11.**

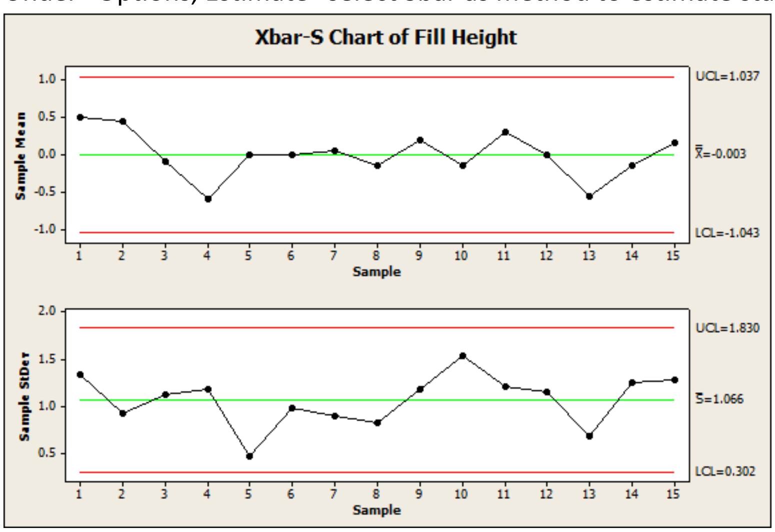
The fill volume of soft-drink beverage bottles is an important quality characteristic. The colume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale a reading fo zero corresponds to the correct fill height. Fifteen samples of size *n* = 10 have been analyzed, and the fill heights are shown in Table 6E.6.

**TABLE 6E.6**
**Fill Height Data for Exercise 6.11**

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5 | 0.5 | 2.0 | −1.0 | 1.0 | −1.0 | 0.5 | 1.5 | 0.5 | −1.5 |
| 2 | 0.0 | 0.0 | 0.5 | 1.0 | 1.5 | 1.0 | −1.0 | 1.0 | 1.5 | −1.0 |
| 3 | 1.5 | 1.0 | 1.0 | −1.0 | 0.0 | −1.5 | −1.0 | −1.0 | 1.0 | −1.0 |
| 4 | 0.0 | 0.5 | −2.0 | 0.0 | −1.0 | 1.5 | −1.5 | 0.0 | −2.0 | −1.5 |
| 5 | 0.0 | 0.0 | 0.0 | −0.5 | 0.5 | 1.0 | −0.5 | −0.5 | 0.0 | 0.0 |
| 6 | 1.0 | −0.5 | 0.0 | 0.0 | 0.0 | 0.5 | −1.0 | 1.0 | −2.0 | 1.0 |
| 7 | 1.0 | −1.0 | −1.0 | −1.0 | 0.0 | 1.5 | 0.0 | 1.0 | 0.0 | 0.0 |
| 8 | 0.0 | −1.5 | −0.5 | 1.5 | 0.0 | 0.0 | 0.0 | −1.0 | 0.5 | −0.5 |
| 9 | −2.0 | −1.5 | 1.5 | 1.5 | 0.0 | 0.0 | 0.5 | 1.0 | 0.0 | 1.0 |
| 10 | −0.5 | 3.5 | 0.0 | −1.0 | −1.5 | −1.5 | −1.0 | −1.0 | 1.0 | 0.5 |
| 11 | 0.0 | 1.5 | 0.0 | 0.0 | 2.0 | −1.5 | 0.5 | −0.5 | 2.0 | −1.0 |
| 12 | 0.0 | −2.0 | −0.5 | 0.0 | −0.5 | 2.0 | 1.5 | 0.0 | 0.5 | −1.0 |
| 13 | −1.0 | −0.5 | −0.5 | −1.0 | 0.0 | 0.5 | 0.5 | −1.5 | −1.0 | −1.0 |
| 14 | 0.5 | 1.0 | −1.0 | −0.5 | −2.0 | −1.0 | −1.5 | 0.0 | 1.5 | 1.5 |
| 15 | 1.0 | 0.0 | 1.5 | 1.5 | 1.0 | −1.0 | 0.0 | 1.0 | −2.0 | −1.5 |

(a) Set up $\bar{x}$ and $S$ control charts on this process. Does the process exhibit statistical control? If necessary construct revised control limits.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S**
Under "Options, Estimate" select Sbar as method to estimate standard deviation.

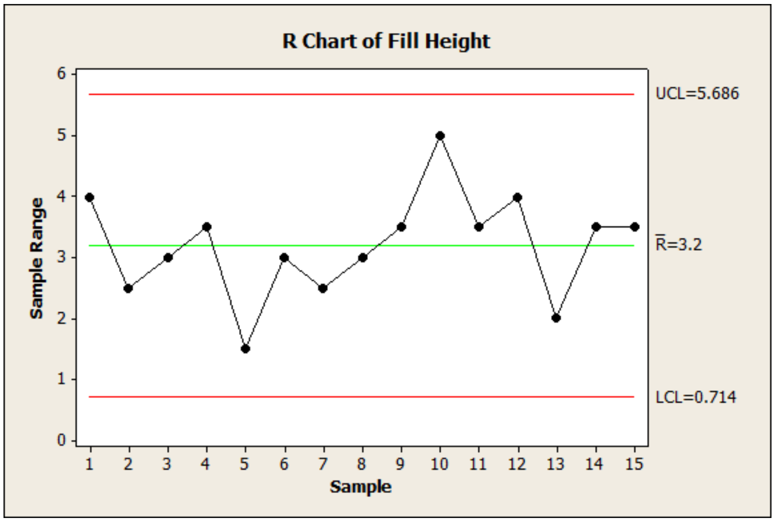



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles.

**6.11. continued**

(b) Set up an *R* chart, and compare it with the *s* chart in part (a).

**MTB > Stat > Control Charts > Variables Charts for Subgroups > R**




The process is in statistical control, with no out-of-control signals, runs, trends, or cycles. There is no difference in interpretation from the $\bar{x}$ and $S$ chart.

(c) Set up an $s^2$ chart and compare it with the *s* chart in part (a).

Let $\alpha$ = 0.010. $n$ = 15, $\bar{s}$ = 1.066.

$$\text{CL} = \bar{s}^2 = 1.066^2 = 1.136$$

$$\text{UCL} = \bar{s}^2/(n-1)\,\chi^2_{\alpha/2,n-1} = 1.066^2/(15-1)\left(\chi^2_{0.010/2,15-1}\right) = 1.066^2/(15-1)\left(31.32\right) = 2.542$$

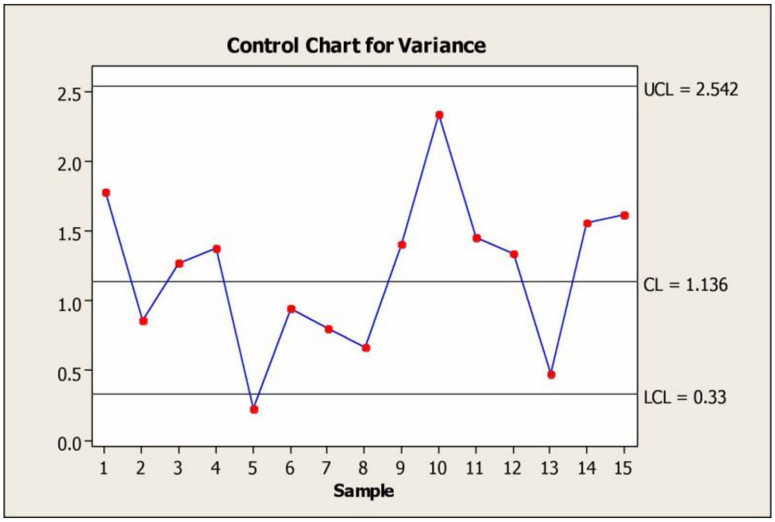
$$\text{LCL} = \bar{s}^2/(n-1)\,\chi^2_{1-(\alpha/2),n-1} = 1.066^2/(15-1)\left(\chi^2_{1-(0.010/2),15-1}\right) = 1.066^2/(15-1)\left(4.07\right) = 0.330$$

Minitab's control chart options do not include an $s^2$ or variance chart. To construct an $s^2$ control chart, first calculate the sample standard deviations and then create a time series plot. To obtain sample standard deviations: **Stat > Basic Statistics > Store Descriptive Statistics**. "Variables" is column with sample data (Ex6.5Vol), and "By Variables" is the sample ID column (Ex6.5Sample). In "Statistics" select "Variance". Results are displayed in the session window. Copy results from the session window by holding down the keyboard "Alt" key, selecting only the variance column, and then copying & pasting to an empty worksheet column (results in Ex6.5Variance).

**6.11. continued**

**Graph > Time Series Plot > Simple**

Control limits can be added using: **Time/Scale > Reference Lines > Y positions**




Sample 5 signals out of control below the lower control limit. Otherwise there are no runs, trends, or cycles. If the limits had been calculated using $\alpha$ = 0.0027 (not tabulated in textbook), sample 5 would be within the limits, and there would be no difference in interpretation from either the $\bar{x} - s$ or the $x$–$R$ chart.

**6.12.**

The net weight (in oz) of a fertilizer product is to be monitored by $\bar{x}$ and $R$ control charts using a sample size of $n$ = 5. Data for 20 preliminary samples are shown in Table 6E.7.

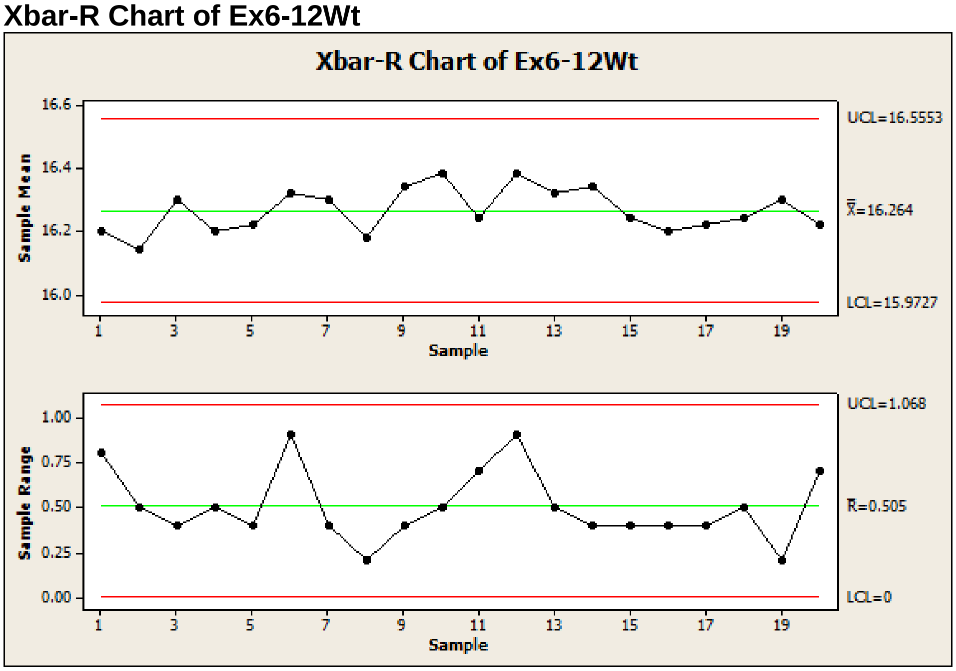
**TABLE 6E.7**
**Data for exercise 6.12**

| Sample Number | x1 | x2 | x3 | x4 | x5 |
|---|---|---|---|---|---|
| 1 | 15.8 | 16.3 | 16.2 | 16.1 | 16.6 |
| 2 | 16.3 | 15.9 | 15.9 | 16.2 | 16.4 |
| 3 | 16.1 | 16.2 | 16.5 | 16.4 | 16.3 |
| 4 | 16.3 | 16.2 | 15.9 | 16.4 | 16.2 |
| 5 | 16.1 | 16.3 | 16.4 | 16.3 | 16.0 |
| 6 | 16.1 | 15.8 | 16.7 | 16.6 | 16.4 |
| 7 | 16.1 | 16.3 | 16.5 | 16.1 | 16.5 |
| 8 | 16.2 | 16.1 | 16.2 | 16.1 | 16.3 |
| 9 | 16.3 | 16.4 | 16.4 | 16.1 | 16.5 |
| 10 | 16.6 | 16.3 | 16.4 | 16.1 | 16.5 |
| 11 | 16.2 | 16.6 | 15.9 | 16.1 | 16.4 |
| 12 | 15.9 | 16.8 | 16.7 | 16.0 | 16.5 |
| 13 | 16.4 | 16.3 | 16.6 | 16.2 | 16.1 |
| 14 | 16.5 | 16.5 | 16.2 | 16.1 | 16.4 |
| 15 | 16.4 | 16.3 | 16.3 | 16.0 | 16.2 |
| 16 | 16.0 | 16.4 | 16.3 | 16.1 | 16.2 |
| 17 | 16.4 | 16.0 | 16.4 | 16.1 | 16.2 |
| 18 | 16.0 | 16.2 | 16.4 | 16.5 | 16.1 |
| 19 | 16.4 | 16.2 | 16.3 | 16.2 | 16.4 |
| 20 | 16.4 | 16.4 | 16.5 | 16.0 | 15.8 |

(a) Set up $\bar{x}$ and $R$ control charts using these data. Does the process exhibit statistical control?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**

**Xbar-R Chart of Ex6-12Wt**




While points on the $\bar{x}$ appear to hug the centerline, the test for 15 points within 1 standard deviation does not signal. The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

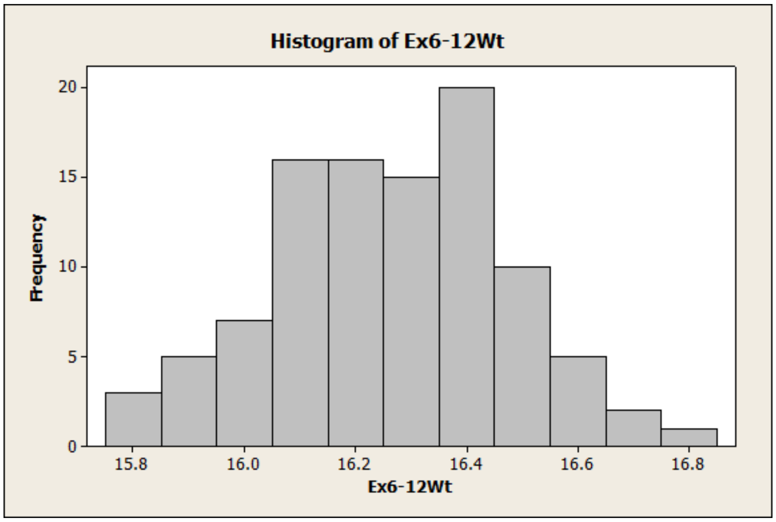
**6.12. continued**

(b) Estimate the process mean and standard deviation.

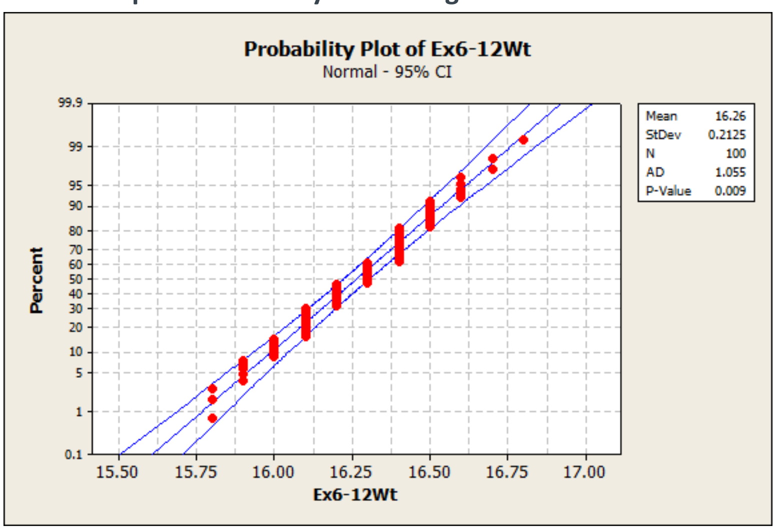
$n = 5;\ \bar{\bar{x}} = 16.264;\ \bar{R} = 0.475;\ \hat{\sigma}_x = \bar{R}/d_2 = 0.505/2.326 = 0.217$

(c) Does the fill weight seem to follow a normal distribution?

**MTB > Graph > Histogram > Simple**




**MTB > Graph > Probability Plot > Single**




Visual examination indicates that fill weights approximate a normal distribution – the histogram has one mode, and is approximately symmetrical with a bell shape. Points on the normal probability plot generally fall along a straight line.

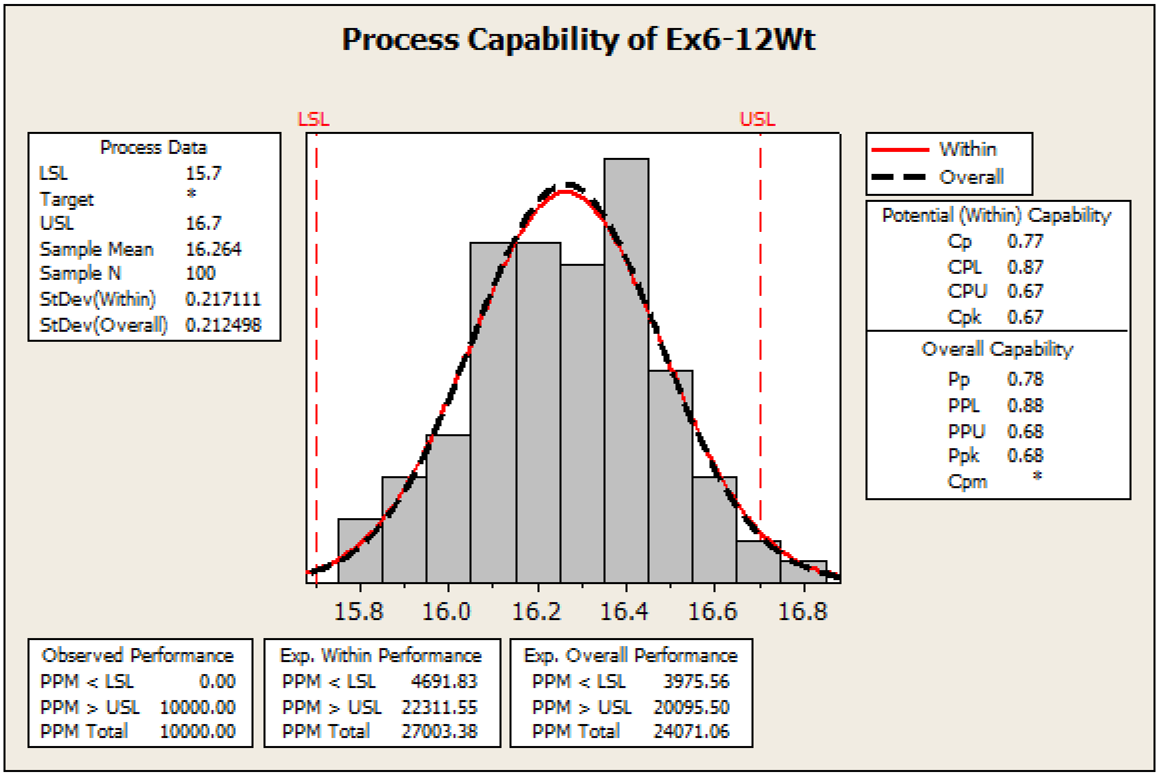
**6.12. continued**

(d) If the specifications are at $16.2 \pm 0.5$, what conclusions would you draw about process capability?

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+0.5 - (-0.5)}{6(0.217)} = 0.77$$, so the process is not capable of meeting specifications.

**MTB > Stat > Quality Tools > Capability Analysis > Normal**

**Under "Estimate" select Rbar as method to estimate standard deviation.**




(e) What fraction of containers produced by this process is likely to be below the lower specification limit of 15.7 oz?

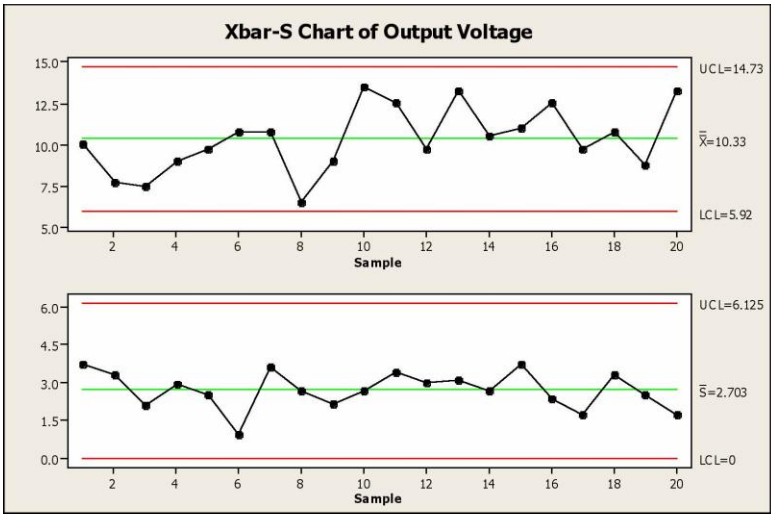
$$\hat{p}_{\text{lower}} = \Pr\{x < \text{LSL}\} = \Pr\{x < 15.7\} = \Phi\left(\frac{15.7 - 16.264}{0.217}\right) = \Phi(-2.60) = 0.0047$$

The Minitab process capability analysis also reports

```
Exp. "Within" Performance
PPM < LSL          4691.83
...
```

**6.13.**
Rework Exercise 6.8 using the *s* chart.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S**

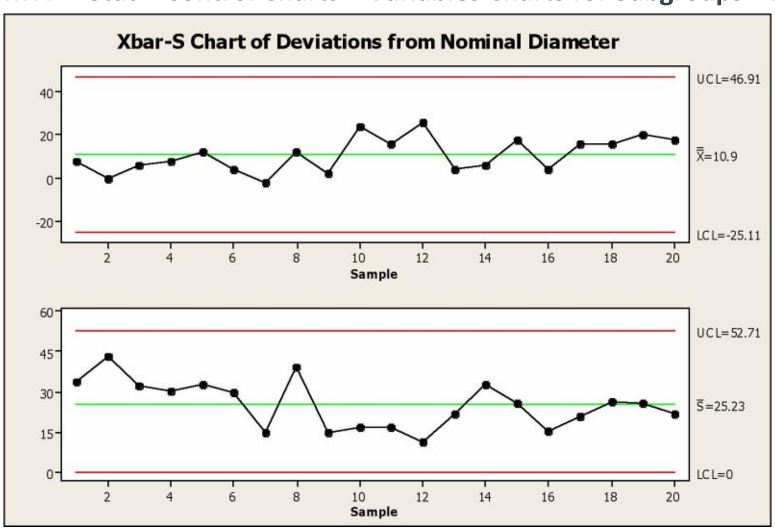



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

---

**6.14.**
Rework Exercise 6.9 using the *s* chart.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S**




The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

---

**6.15.**

Consider the piston ring data shown in Table 6.3. Assume that the specifications on this component are $74.000 \pm 0.05$ mm.

**TABLE 6.3**

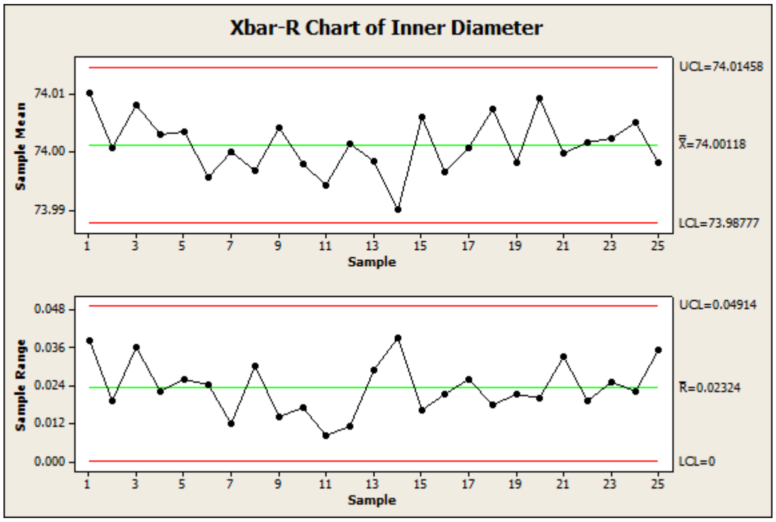
Inside Diameter Measurements (mm) for Automobile Engine Piston Rings

| Sample Number | Observations | | | | | $\bar{x}_i$ | $s_i$ |
|---|---|---|---|---|---|---|---|
| 1 | 74.030 | 74.002 | 74.019 | 73.992 | 74.008 | 74.010 | 0.0148 |
| 2 | 73.995 | 73.992 | 74.001 | 74.011 | 74.004 | 74.001 | 0.0075 |
| 3 | 73.988 | 74.024 | 74.021 | 74.005 | 74.002 | 74.008 | 0.0147 |
| 4 | 74.002 | 73.996 | 73.993 | 74.015 | 74.009 | 74.003 | 0.0091 |
| 5 | 73.992 | 74.007 | 74.015 | 73.989 | 74.014 | 74.003 | 0.0122 |
| 6 | 74.009 | 73.994 | 73.997 | 73.985 | 73.993 | 73.996 | 0.0087 |
| 7 | 73.995 | 74.006 | 73.994 | 74.000 | 74.005 | 74.000 | 0.0055 |
| 8 | 73.985 | 74.003 | 73.993 | 74.015 | 73.988 | 73.997 | 0.0123 |
| 9 | 74.008 | 73.995 | 74.009 | 74.005 | 74.004 | 74.004 | 0.0055 |
| 10 | 73.998 | 74.000 | 73.990 | 74.007 | 73.995 | 73.998 | 0.0063 |
| 11 | 73.994 | 73.998 | 73.994 | 73.995 | 73.990 | 73.994 | 0.0029 |
| 12 | 74.004 | 74.000 | 74.007 | 74.000 | 73.996 | 74.001 | 0.0042 |
| 13 | 73.983 | 74.002 | 73.998 | 73.997 | 74.012 | 73.998 | 0.0105 |
| 14 | 74.006 | 73.967 | 73.994 | 74.000 | 73.984 | 73.990 | 0.0153 |
| 15 | 74.012 | 74.014 | 73.998 | 73.999 | 74.007 | 74.006 | 0.0073 |
| 16 | 74.000 | 73.984 | 74.005 | 73.998 | 73.996 | 73.997 | 0.0078 |
| 17 | 73.994 | 74.012 | 73.986 | 74.005 | 74.007 | 74.001 | 0.0106 |
| 18 | 74.006 | 74.010 | 74.018 | 74.003 | 74.000 | 74.007 | 0.0070 |
| 19 | 73.984 | 74.002 | 74.003 | 74.005 | 73.997 | 73.998 | 0.0085 |
| 20 | 74.000 | 74.010 | 74.013 | 74.020 | 74.003 | 74.009 | 0.0080 |
| 21 | 73.982 | 74.001 | 74.015 | 74.005 | 73.996 | 74.000 | 0.0122 |
| 22 | 74.004 | 73.999 | 73.990 | 74.006 | 74.009 | 74.002 | 0.0074 |
| 23 | 74.010 | 73.989 | 73.990 | 74.009 | 74.014 | 74.002 | 0.0119 |
| 24 | 74.015 | 74.008 | 73.993 | 74.000 | 74.010 | 74.005 | 0.0087 |
| 25 | 73.982 | 73.984 | 73.995 | 74.017 | 74.013 | 73.998 | 0.0162 |
| | | | | | | $\Sigma = 1{,}850.028$ | 0.2351 |
| | | | | | | $\bar{\bar{x}} = 74.001$ | $\bar{s} = 0.0094$ |

**6.15. continued**

(a) Set up $\bar{x}$ and $R$ control charts on this process. Is the process in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b) Note that the control limits on the $\bar{x}$ chart in part (a) are identical to the control limits on the $\bar{x}$ chart in Example 6.3, where the limits were based on *s*. Will this always happen?

The control limits on the $\bar{x}$ charts in Example 6.3 were calculated using $\bar{S}$ to estimate $\sigma$, in this exercise $\bar{R}$ was used to estimate $\sigma$. They will not always be the same, and in general, the $\bar{x}$ control limits based on $\bar{S}$ will be slightly different than limits based on $\bar{R}$ .

(c) Estimate process capability for the piston-ring process. Estimate the percentage of piston rings produced that will be outside of the specification.

$$\hat{\sigma}_x = \bar{R} / d_2 = 0.02324 / 2.326 = 0.009991$$

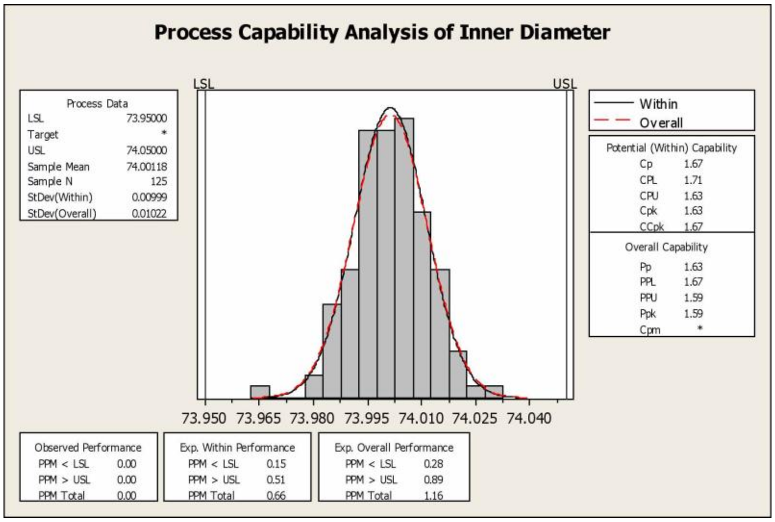
$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{74.05 - 73.95}{6(0.009991)} = 1.668$$

The process is capable of meeting specifications.

**6.15. (c) continued**

**MTB > Stat > Quality Tools > Capability Analysis > Normal**

Under “Estimate” select Rbar as method to estimate standard deviation.

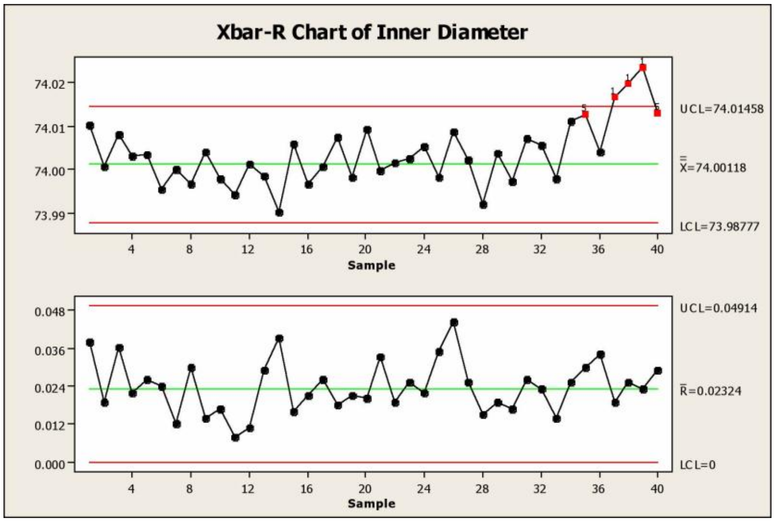



$$
\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < 73.95\} + \Pr\{x > 74.05\} \\
&= \Pr\{x < 73.95\} + \left[1 - \Pr\{x < 74.05\}\right] \\
&= \Phi\left(\frac{73.95 - 74.00118}{0.009991}\right) + \left[1 - \Phi\left(\frac{74.05 - 74.00118}{0.009991}\right)\right] \\
&= \Phi(-5.123) + 1 - \Phi(4.886) \\
&= 0 + 1 - 1 \\
&= 0
\end{aligned}
$$

**6.16.**

Table 6E.8 shows 15 additional samples for the piston ring process (Table 6.3), taken after the initial control charts were established. Plot these data on the $\bar{x}$ and $R$ chart developed in Exercise 6.15. Is the process in control?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




**Test Results for Xbar Chart of Ex6.10ID**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  37, 38, 39
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  38, 39, 40
```

The control charts indicate that the process is in control, until the $\bar{x}$-value from the 37th sample is plotted. Since this point and the three subsequent points plot above the upper control limit, an assignable cause has likely occurred, increasing the process mean.

**6.17.**

Control charts on $\bar{x}$ and $s$ are to be maintained on the torque readings of a earing used in a wingflap actuator assembly. Samples of size $n$ = 10 are to be used, and we know from past experience that when the process is in control, bearing torque has a normal distribution with mean $\mu$ = 80 inch-pounds and standard deviation of $\sigma$ = 10 inch pounds. Find the center line and control limits for these control charts.

$n=10;\ \mu=80$ in-lb; $\sigma_x=10$ in-lb; and $A=0.949;\ B_6=1.669;\ B_5=0.276$

$\text{centerline}_{\bar{x}}=\mu=80$

$\text{UCL}_{\bar{x}}=\mu+A\sigma_x=80+0.949(10)=89.49$

$\text{LCL}_{\bar{x}}=\mu-A\sigma_x=80-0.949(10)=70.51$

$\text{centerline}_S=c_4\sigma_x=0.9727(10)=9.727$

$\text{UCL}_S=B_6\sigma_x=1.669(10)=16.69$

$\text{LCL}_S=B_5\sigma_x=0.276(10)=2.76$

---

**6.18.**

Samples of $n$ = 4 items each are taken from a process at regular intervals. A quality characteristic is measured, and $\bar{x}$ and $R$ values are calculated for each sample. After 50 samples, we have

$\sum_{i=1}^{50}\bar{x}_i=2100;\ \sum_{i=1}^{50}R_i=175;\ m=50$ samples

Assume that the quality characteristic is normally distributed.

(a) Compute control limits for the $\bar{x}$ and $R$ control charts.

$$\bar{\bar{x}}=\frac{\sum_{i=1}^{50}\bar{x}_i}{m}=\frac{2100}{50}=42;\quad \bar{R}=\frac{\sum_{i=1}^{50}R_i}{m}=\frac{175}{50}=3.5$$

$\text{UCL}_{\bar{x}}=\bar{\bar{x}}+A_2\bar{R}=42+0.729(3.5)=44.6$

$\text{LCL}_{\bar{x}}=\bar{\bar{x}}-A_2\bar{R}=42-0.729(3.5)=39.4$

$\text{UCL}_R=D_4\bar{R}=2.282(3.5)=8.0$

$\text{LCL}_R=D_3\bar{R}=0(3.5)=0$

(b) All points on both control charts fall between the control limits computed in part (a). What are the natural tolerance limits of the process?

natural tolerance limits: $\bar{\bar{x}}\pm 3\hat{\sigma}_x=\bar{\bar{x}}\pm 3\left(\bar{R}/d_2\right)=42\pm 3(3.5/2.059)=42\pm 3(1.7)=[36.9,47.1]$

(c) If the specification limits are $41\pm 4.5$, what are your conclusions regarding the ability of the process to produce items within these specifications?

$\hat{C}_P=\frac{\text{USL - LSL}}{6\hat{\sigma}_x}=\frac{+4.5-(-4.5)}{6(1.7)}=0.88$ , so the process is not capable.

**6.18. continued**

(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process producing?

$$\hat{p}_{\text{scrap}} = \Pr\{x < \text{LSL}\} = \Pr\{x < 36\} = \Phi\left(\frac{36-42}{1.7}\right) = \Phi(-3.53) = 0.0002 \text{, or } 0.02\%.$$

$$\hat{p}_{\text{rework}} = \Pr\{x > \text{USL}\} = 1 - \Pr\{x < \text{USL}\} = 1 - \Phi\left(\frac{47-42}{1.7}\right) = 1 - \Phi(2.94) = 1 - 0.9984 = 0.0016 \text{ or } 0.16\%.$$

(e) Make suggestions as to how the process performance could be improved.

First, center the process at 41, not 42, to reduce scrap and rework costs. Second, reduce variability such that the natural process tolerance limits are closer to, say, $\hat{\sigma}_x \approx 1.253$.

---

**6.19.**

Samples of $n$ = 4 items are taken from a process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and $s$ values are calculated for each sample. After 50 subgroups have been analyzed, we have

$$\sum_{i=1}^{50} \bar{x}_i = 1000; \ \sum_{i=1}^{50} S_i = 72; \ m = 50 \text{ subgroups}$$

(a) Compute the control limit for the $\bar{x}$ and $s$ control charts

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{50} \bar{x}_i}{m} = \frac{1000}{50} = 20; \quad \bar{S} = \frac{\sum_{i=1}^{50} S_i}{m} = \frac{72}{50} = 1.44$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S} = 20 + 1.628(1.44) = 22.34$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S} = 20 - 1.628(1.44) = 17.66$$

$$\text{UCL}_S = B_4\bar{S} = 2.266(1.44) = 3.26$$

$$\text{LCL}_S = B_3\bar{S} = 0(1.44) = 0$$

(b) Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process?

natural process tolerance limits: $\bar{\bar{x}} \pm 3\hat{\sigma}_x = \bar{\bar{x}} \pm 3\left(\frac{\bar{S}}{c_4}\right) = 20 \pm 3\left(\frac{1.44}{0.9213}\right) = [15.3, 24.7]$

(c) If the specification limits are $19 \pm 4.0$, what are your conclusions regarding the ability of the process to produce items conforming to specifications?

$$\hat{C}_P = \frac{\text{USL - LSL}}{6\hat{\sigma}_x} = \frac{+4.0-(-4.0)}{6(1.44/0.9213)} = 0.85 \text{, so the process is not capable.}$$

**6.19. continued**

(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process now producing?

$$\hat{p}_{\text{rework}} = \Pr\{x > \text{USL}\} = 1 - \Pr\{x \le \text{USL}\} = 1 - \Phi\left(\frac{23-20}{1.44/0.9213}\right) = 1 - \Phi(1.919) = 1 - 0.9725 = 0.0275 \text{ or } 2.75\%.$$

$$\hat{p}_{\text{scrap}} = \Pr\{x < \text{LSL}\} = \Phi\left(\frac{15-20}{1.44/0.9213}\right) = \Phi(-3.199) = 0.00069\text{ , or } 0.069\%$$

Total = 2.88% + 0.069% = 2.949%

(e) If the process were centered at $\mu$ = 19.0, what would be the effect on percent scrap and rework?

$$\hat{p}_{\text{rework}} = 1 - \Phi\left(\frac{23-19}{1.44/0.9213}\right) = 1 - \Phi(2.56) = 1 - 0.99477 = 0.00523\text{ , or } 0.523\%$$

$$\hat{p}_{\text{scrap}} = \Phi\left(\frac{15-19}{1.44/0.9213}\right) = \Phi(-2.56) = 0.00523\text{ , or } 0.523\%$$

Total = 0.523% + 0.523% = 1.046%

Centering the process would reduce rework, but increase scrap. A cost analysis is needed to make the final decision. An alternative would be to work to improve the process by reducing variability.

**6.20.**

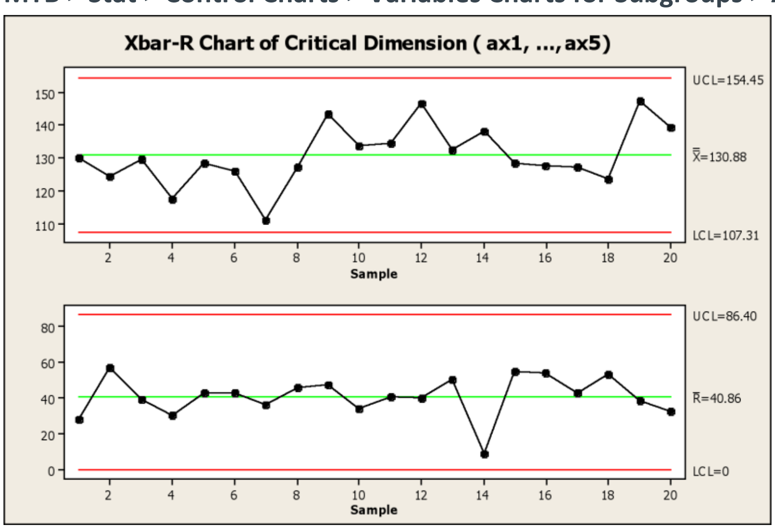
Table 6E.9 presents 20 subgroups of five measurements on the critical dimension of a part produced by a welding process.

**TABLE 6E.9**
Data for Exercise 6.20

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|---|---|
| 1 | 138.1 | 110.8 | 138.7 | 137.4 | 125.4 | 130.1 | 27.9 |
| 2 | 149.3 | 142.1 | 105.0 | 134.0 | 92.3 | 124.5 | 57.0 |
| 3 | 115.9 | 135.6 | 124.2 | 155.0 | 117.4 | 129.6 | 39.1 |
| 4 | 118.5 | 116.5 | 130.2 | 122.6 | 100.2 | 117.6 | 30.0 |
| 5 | 108.2 | 123.8 | 117.1 | 142.4 | 150.9 | 128.5 | 42.7 |
| 6 | 102.8 | 112.0 | 135.0 | 135.0 | 145.8 | 126.1 | 43.0 |
| 7 | 120.4 | 84.3 | 112.8 | 118.5 | 119.3 | 111.0 | 36.1 |
| 8 | 132.7 | 151.1 | 124.0 | 123.9 | 105.1 | 127.4 | 46.0 |
| 9 | 136.4 | 126.2 | 154.7 | 127.1 | 173.2 | 143.5 | 46.9 |
| 10 | 135.0 | 115.4 | 149.1 | 138.3 | 130.4 | 133.6 | 33.7 |
| 11 | 139.6 | 127.9 | 151.1 | 143.7 | 110.5 | 134.6 | 40.6 |
| 12 | 125.3 | 160.2 | 130.4 | 152.4 | 165.1 | 146.7 | 39.8 |
| 13 | 145.7 | 101.8 | 149.5 | 113.3 | 151.8 | 132.4 | 50.0 |
| 14 | 138.6 | 139.0 | 131.9 | 140.2 | 141.1 | 138.1 | 9.2 |
| 15 | 110.1 | 114.6 | 165.1 | 113.8 | 139.6 | 128.7 | 54.8 |
| 16 | 145.2 | 101.0 | 154.6 | 120.2 | 117.3 | 127.6 | 53.3 |
| 17 | 125.9 | 135.3 | 121.5 | 147.9 | 105.0 | 127.1 | 42.9 |
| 18 | 129.7 | 97.3 | 130.5 | 109.0 | 150.5 | 123.4 | 53.2 |
| 19 | 123.4 | 150.0 | 161.6 | 148.4 | 154.2 | 147.5 | 38.3 |
| 20 | 144.8 | 138.3 | 119.6 | 151.8 | 142.7 | 139.4 | 32.2 |

(a) Set up $\bar{x}$ and $R$ control charts on this process. Verify that the process is in statistical control.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

**6.20. continued**

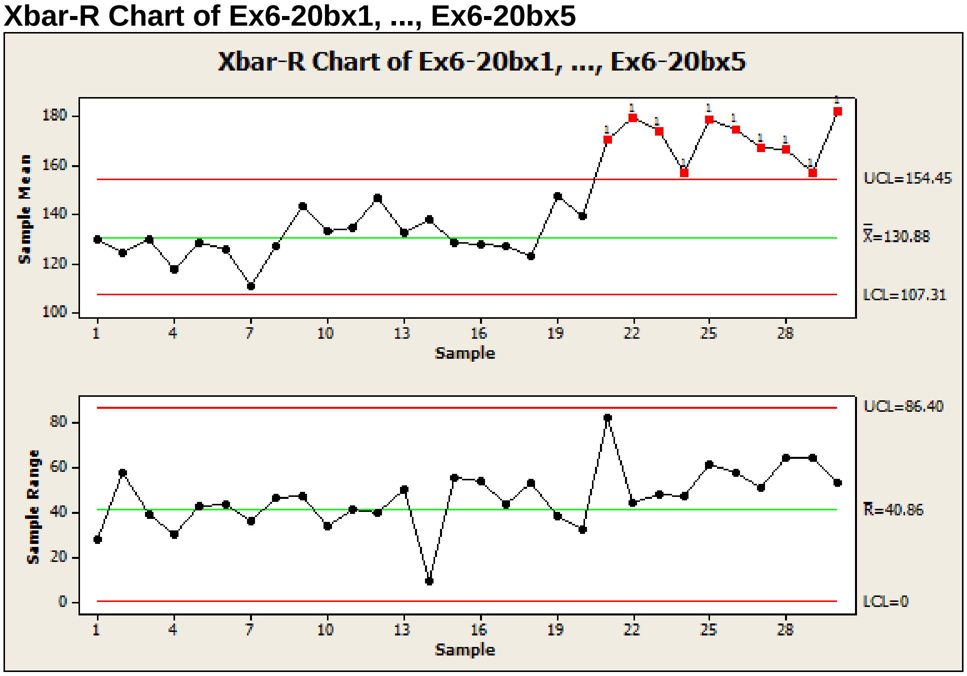
(b) Following the establishment of control charts in part (a) above, 10 new samples in Table 6E.10 were collected Plot the $\bar{x}$ and $R$ values on the control chart you established in part (a) and draw conclusions.

**TABLE 6E.10**
**Additional Data for Exercise 6.20, part (b)**

| Sample Number | *x1* | *x2* | *x3* | *x4* | *x5* | $\bar{x}$ | *R* |
|---|---|---|---|---|---|---|---|
| 21 | 131.0 | 184.8 | 182.2 | 143.3 | 212.8 | 170.8 | 81.8 |
| 22 | 181.3 | 203.2 | 180.7 | 159.1 | 174.3 | 179.7 | 44.1 |
| 23 | 154.8 | 170.2 | 168.4 | 202.7 | 174.4 | 174.1 | 48.0 |
| 24 | 157.5 | 154.2 | 179.1 | 132.2 | 161.9 | 157.0 | 46.9 |
| 25 | 216.3 | 174.3 | 166.2 | 155.5 | 184.3 | 179.3 | 60.8 |
| 26 | 196.9 | 180.2 | 139.2 | 175.2 | 185.0 | 175.3 | 57.7 |
| 27 | 167.8 | 143.9 | 157.5 | 171.8 | 194.9 | 167.2 | 51.0 |
| 28 | 178.2 | 196.7 | 132.4 | 159.4 | 167.6 | 166.9 | 64.3 |
| 29 | 162.6 | 143.6 | 122.8 | 168.9 | 187.2 | 157.0 | 64.4 |
| 30 | 172.1 | 191.7 | 203.4 | 150.4 | 196.3 | 182.8 | 53.0 |

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**
**Under "Options, Estimate" use subgroups 1:20 to calculate control limits.**

**Xbar-R Chart of Ex6-20bx1, ..., Ex6-20bx5**




**Test Results for Xbar Chart of Ex6-20bx1, ..., Ex6-20bx5**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  21, 22, 23, 24, 25, 26, 27, 28, 29, 30
```

Starting at Sample #21, the process average has shifted to above the UCL = 154.45.

**6.20. continued**

(c) Suppose that the assignable cause responsible for the action signals generated in part (b) has been identified and adjustments made to the process to correct its performance. Plot the $\bar{x}$ and $R$ values from the new subgroups shown in Table 6E.11 which were taken following the adjustment, against the control chart limits established in part (a). What are your conclusions?

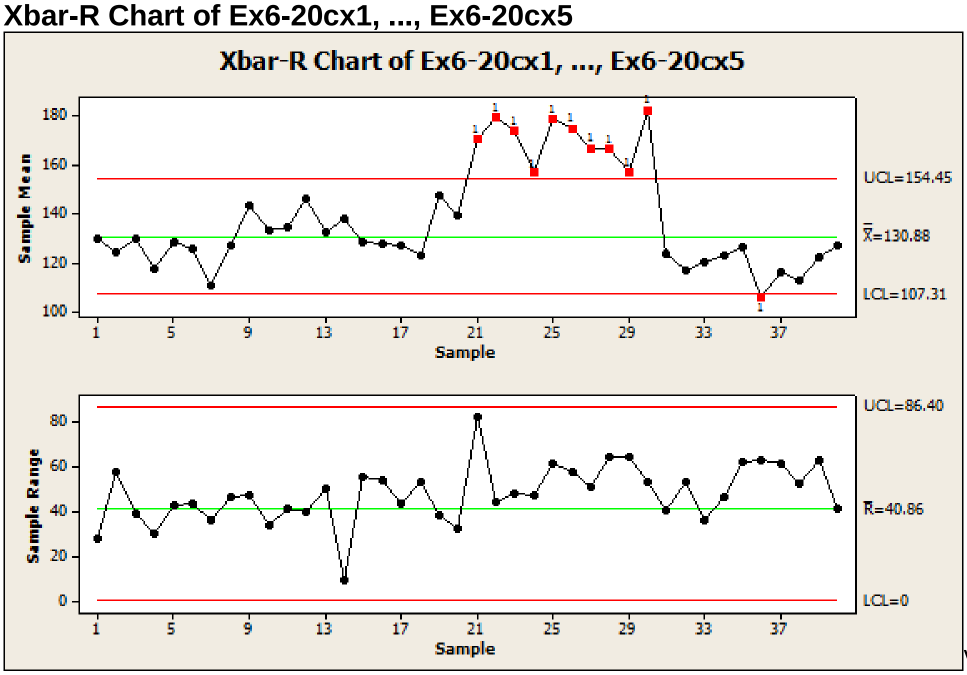
**TABLE 6E.10**

**Additional Data for Exercise 6.20, part (c)**

| Sample Number | *x1* | *x2* | *x3* | *x4* | *x5* | $\bar{x}$ | *R* |
|---|---|---|---|---|---|---|---|
| 31 | 131.5 | 143.1 | 118.5 | 103.2 | 121.6 | 123.6 | 39.8 |
| 32 | 111.0 | 127.3 | 110.4 | 91.0 | 143.9 | 116.7 | 52.8 |
| 33 | 129.8 | 98.3 | 134.0 | 105.1 | 133.1 | 120.1 | 35.7 |
| 34 | 145.2 | 132.8 | 106.1 | 131.0 | 99.2 | 122.8 | 46 |
| 35 | 114.6 | 111.0 | 113.8 | 172.5 | 121.6 | 126.7 | 61.5 |
| 36 | 125.2 | 86.4 | 69.4 | 132.1 | 117.5 | 106.1 | 62.7 |
| 37 | 145.9 | 109.5 | 84.9 | 129.8 | 110.6 | 116.1 | 61 |
| 38 | 123.6 | 114.0 | 135.4 | 83.2 | 107.6 | 112.8 | 52.2 |
| 39 | 90.8 | 151.3 | 119.7 | 96.2 | 153.0 | 122.2 | 62.2 |
| 40 | 107.4 | 148.7 | 127.4 | 125.0 | 127.5 | 127.2 | 41.3 |

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**
**Under "Options, Estimate" use subgroups 1:20 to calculate control limits.**

**Xbar-R Chart of Ex6-20cx1, ..., Ex6-20cx5**




**Test Results for Xbar Chart of Ex6-20cx1, ..., Ex6-20cx5**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36
```

The adjustment overcompensated for the upward shift. The process average is now between $\bar{\bar{x}}$ and the LCL, with a run of ten points below the centerline, and one sample (#36) below the LCL.

**6.21.**

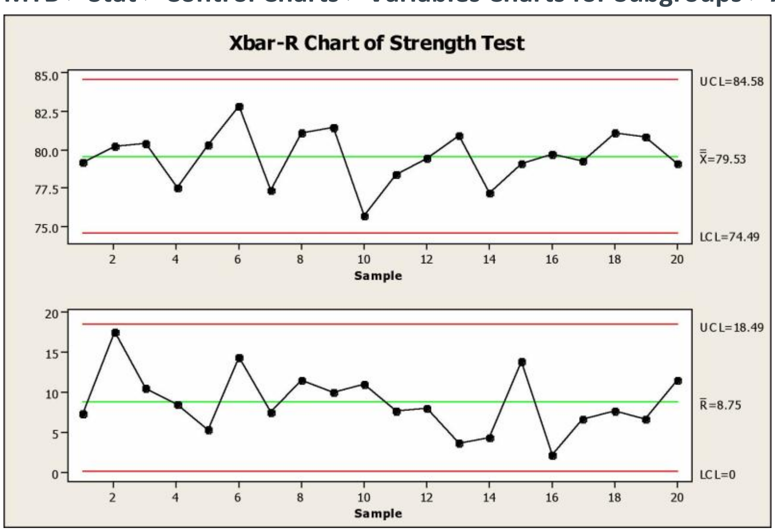
Parts manufactured by an injection molding process are subjected to a compressive strength test. Twenty samples of five parts each are collected, and the compressive strengths (in psi) are shown in Table 6E.12.

**TABLE 6E.12**
Strength Data for Exercise 6.21

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|---|---|
| 1 | 83.0 | 81.2 | 78.7 | 75.7 | 77.0 | 79.1 | 7.3 |
| 2 | 88.6 | 78.3 | 78.8 | 71.0 | 84.2 | 80.2 | 17.6 |
| 3 | 85.7 | 75.8 | 84.3 | 75.2 | 81.0 | 80.4 | 10.4 |
| 4 | 80.8 | 74.4 | 82.5 | 74.1 | 75.7 | 77.5 | 8.4 |
| 5 | 83.4 | 78.4 | 82.6 | 78.2 | 78.9 | 80.3 | 5.2 |
| 6 | 75.3 | 79.9 | 87.3 | 89.7 | 81.8 | 82.8 | 14.5 |
| 7 | 74.5 | 78.0 | 80.8 | 73.4 | 79.7 | 77.3 | 7.4 |
| 8 | 79.2 | 84.4 | 81.5 | 86.0 | 74.5 | 81.1 | 11.4 |
| 9 | 80.5 | 86.2 | 76.2 | 64.1 | 80.2 | 81.4 | 9.9 |
| 10 | 75.7 | 75.2 | 71.1 | 82.1 | 74.3 | 75.7 | 10.9 |
| 11 | 80.0 | 81.5 | 78.4 | 73.8 | 78.1 | 78.4 | 7.7 |
| 12 | 80.6 | 81.8 | 79.3 | 73.8 | 81.7 | 79.4 | 8.0 |
| 13 | 82.7 | 81.3 | 79.1 | 82.0 | 79.5 | 80.9 | 3.6 |
| 14 | 79.2 | 74.9 | 78.6 | 77.7 | 75.3 | 77.1 | 4.3 |
| 15 | 85.5 | 82.1 | 82.8 | 73.4 | 71.7 | 79.1 | 13.8 |
| 16 | 78.8 | 79.6 | 80.2 | 79.1 | 80.8 | 79.7 | 2.0 |
| 17 | 82.1 | 78.2 | 75.5 | 78.2 | 82.1 | 79.2 | 6.6 |
| 18 | 84.5 | 76.9 | 83.5 | 81.2 | 79.2 | 81.1 | 7.6 |
| 19 | 79.0 | 77.8 | 81.2 | 84.4 | 81.6 | 80.8 | 6.6 |
| 20 | 84.5 | 73.1 | 78.6 | 78.7 | 80.6 | 79.1 | 11.4 |

(a) Establish $\bar{x}$ and $R$ control charts for compressive strength using these data. Is the process in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




Yes, the process is in control—though we should watch for a possible cyclic pattern in the averages.

**6.21. continued**

(b) After establishing the control charts in part (a), 15 new subgroups were collected and the compressive strengths are shown in Table 6E.13. Plot the $\overline{x}$ and $R$ values against the control units from part (a) and draw conclusions.

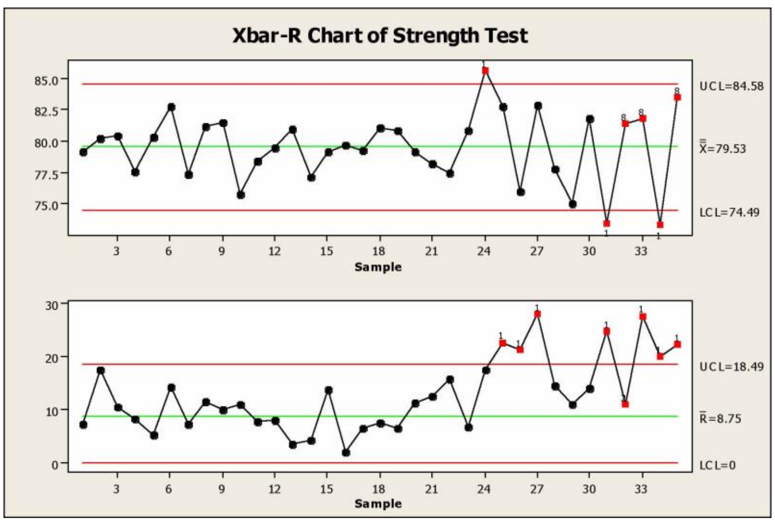
**TABLE 6E.13**

New Data for Exercise 6.21, part (b)

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|---|---|
| 1 | 68.9 | 81.5 | 78.2 | 80.8 | 81.5 | 78.2 | 12.6 |
| 2 | 69.8 | 68.6 | 80.4 | 84.3 | 83.9 | 77.4 | 15.7 |
| 3 | 78.5 | 85.2 | 78.4 | 80.3 | 81.7 | 80.8 | 6.8 |
| 4 | 76.9 | 86.1 | 86.9 | 94.4 | 83.9 | 85.6 | 17.5 |
| 5 | 93.6 | 81.6 | 87.8 | 79.6 | 71.0 | 82.7 | 22.5 |
| 6 | 65.5 | 86.8 | 72.4 | 82.6 | 71.4 | 75.9 | 21.3 |
| 7 | 78.1 | 65.7 | 83.7 | 93.7 | 93.4 | 82.9 | 27.9 |
| 8 | 74.9 | 72.6 | 81.6 | 87.2 | 72.7 | 77.8 | 14.6 |
| 9 | 78.1 | 77.1 | 67.0 | 75.7 | 76.8 | 74.9 | 11.0 |
| 10 | 78.7 | 85.4 | 77.7 | 90.7 | 76.7 | 81.9 | 14.0 |
| 11 | 85.0 | 60.2 | 68.5 | 71.1 | 82.4 | 73.4 | 24.9 |
| 12 | 86.4 | 79.2 | 79.8 | 86.0 | 75.4 | 81.3 | 10.9 |
| 13 | 78.5 | 99.0 | 78.3 | 71.4 | 81.8 | 81.7 | 27.6 |
| 14 | 68.8 | 62.0 | 82.0 | 77.5 | 76.1 | 73.3 | 19.9 |
| 15 | 83.0 | 83.7 | 73.1 | 82.2 | 95.3 | 83.5 | 22.2 |

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**

**Under "Options, Estimate" use subgroups 1:20 to calculate control limits.**




**Test Results for R Chart of Ex6.15bSt**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  25, 26, 27, 31, 33, 34, 35
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  32, 33, 34, 35
```

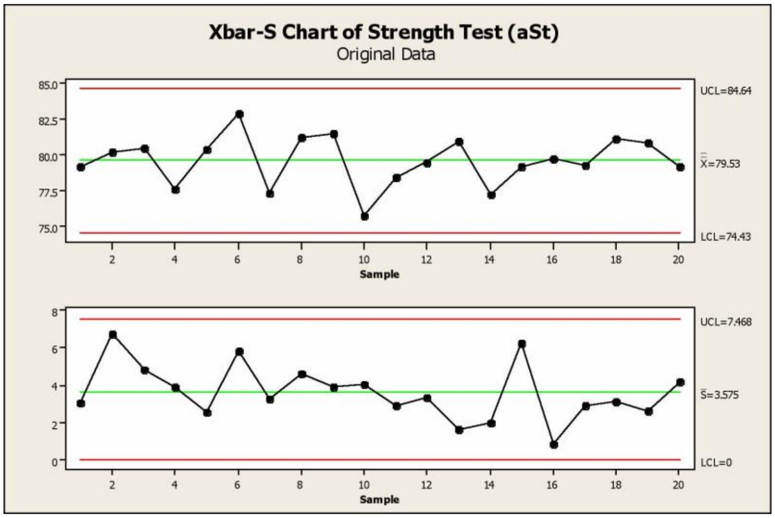
A strongly cyclic pattern in the averages is now evident, but more importantly, there are several out-of-control points on the range chart.

**6.22.**

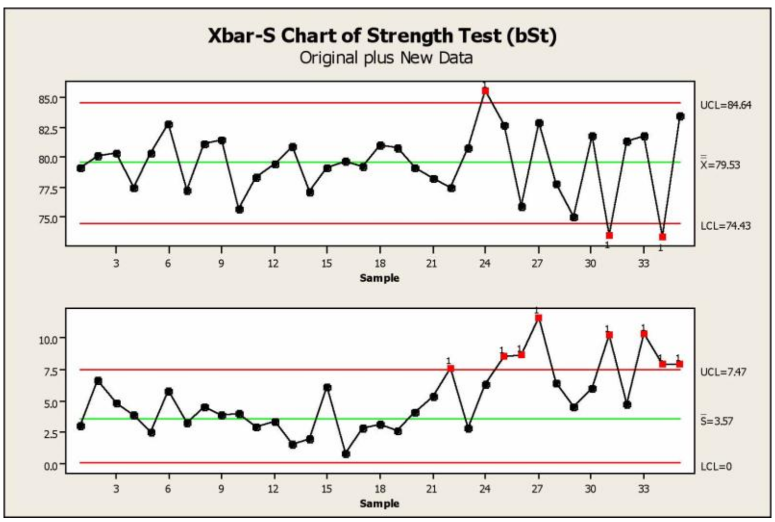
Reconsider the data presented in Exercise 6.21.

(a) Rework both parts (a) and (b) of Exercise 6.21 using the $\bar{x}$ and $s$ charts.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S**




**Under "Options, Estimate" use subgroups 1:20 to calculate control limits.**




**Test Results for Xbar Chart of Ex6.15bSt**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  24, 31, 34
```

**Test Results for S Chart of Ex6.15bSt**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  22, 25, 26, 27, 31, 33, 34, 35
```

**6.22. continued**

(b) Does the *s* chart detect the shift in process variability more quickly than the *R* chart did originally in part (b) of Exercise 6.21?

Yes, the *s* chart detects the change in process variability more quickly than the *R* chart did, at sample #22 versus sample #24.

---

**6.23.**

Consider the $\bar{x}$ and $R$ charts you established in Exercise 6.7 using $n$ = 5.

$n_{\text{old}} = 5;\ \bar{\bar{x}}_{\text{old}} = 34.00;\ \bar{R}_{\text{old}} = 4.7$

(a) Suppose that you wished to continue charting this quality characteristics using $\bar{x}$ and $R$ charts based on a sample size of $n$ =3. What limits would be used on the $\bar{x}$ and $R$ charts?

for $n_{\text{new}}$ = 3

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 + 1.023\left[\frac{1.693}{2.326}\right](4.7) = 37.50$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 - 1.023\left[\frac{1.693}{2.326}\right](4.7) = 30.50$$

$$\text{UCL}_R = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 2.574\left[\frac{1.693}{2.326}\right](4.7) = 8.81$$

$$\text{CL}_R = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{1.693}{2.326}\right](4.7) = 3.42$$

$$\text{LCL}_R = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{1.693}{2.326}\right](4.7) = 0$$

(b) What would be the impact of the decision you made in part (a) on the ability of the $\bar{x}$ chart to detect a $2\sigma$ shift in the mean?

The $\bar{x}$ control limits for $n$ = 5 are “tighter” (31.29, 36.72) than those for $n$ = 3 (30.50, 37.50). This means a $2\sigma$ shift in the mean would be detected more quickly with a sample size of $n$ = 5.

**6.23. continued**

(c) Suppose you wished to continue charting this quality characteristic using $\bar{x}$ and $R$ charts based on a sample size of $n$ = 8. What limits would be used on the $\bar{x}$ and $R$ charts?

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 + 0.373\left[\frac{2.847}{2.326}\right](4.7) = 36.15$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 - 0.373\left[\frac{2.847}{2.326}\right](4.7) = 31.85$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 1.864\left[\frac{2.847}{2.326}\right](4.7) = 10.72$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{2.847}{2.326}\right](4.7) = 5.75$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0.136\left[\frac{2.847}{2.326}\right](4.7) = 0.78$$

(d) What is the impact of using $n$ = 8 on the ability of the $\bar{x}$ chart to detect a $2\sigma$ shift in the mean?

The $\bar{x}$ control limits for $n$ = 8 are even "tighter" (31.85, 36.15), increasing the ability of the chart to quickly detect the $2\sigma$ shift in process mean.

---

**6.24.**

Consider the $\bar{x}$ and $R$ chart that you established in Exercise 6.15 for the piston ring process. Suppose that you want to continue control charting piston ring diameter using $n$ = 4. What limits would be used on the $\bar{x}$ and $R$ chart?

$n_{\text{old}} = 5$, $\bar{\bar{x}}_{\text{old}} = 74.001$, $\bar{R}_{\text{old}} = 0.023$, $n_{\text{new}} = 4$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 74.001 + 1.023\left[\frac{2.059}{2.326}\right](0.023) = 74.022$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 74.001 - 1.023\left[\frac{2.059}{2.326}\right](0.023) = 73.980$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 2.574\left[\frac{2.059}{2.326}\right](0.023) = 0.052$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{2.059}{2.326}\right](0.023) = 0.020$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{2.059}{2.326}\right](0.023) = 0$$

---

**6.25.**

Control charts for $\bar{x}$ and $R$ are maintained for an important quality characteristic. The sample size is $n$= 7; $\bar{x}$ and $R$ are computed for each sample. After 35 samples we have found that

$$\sum_{i=1}^{35} \bar{x}_i = 7{,}805 \text{ and } \sum_{i=1}^{35} R_i = 1{,}200$$

(a) Set up $\bar{x}$ and $R$ charts using these data.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{35} \bar{x}_i}{m} = \frac{7805}{35} = 223; \quad \bar{R} = \frac{\sum_{i=1}^{35} R_i}{m} = \frac{1200}{35} = 34.29$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 223 + 0.419(34.29) = 237.37$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 223 - 0.419(34.29) = 208.63$$

$$\text{UCL}_R = D_4\bar{R} = 1.924(34.29) = 65.97$$

$$\text{LCL}_R = D_3\bar{R} = 0.076(34.29) = 2.61$$

(b) Assuming that both charts exhibit control, estimate the process mean and standard deviation.

$$\hat{\mu} = \bar{\bar{x}} = 223; \quad \hat{\sigma}_x = \bar{R}/d_2 = 34.29/2.704 = 12.68$$

(c) If the quality characteristic is normally distributed and if the specifications are 220 $\pm$ 35, can the process meet the specifications? Estimate the fraction nonconforming.

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+35 - (-35)}{6(12.68)} = 0.92$$ , the process is not capable of meeting specifications.

$$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x < \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le 255\} + \Pr\{x \le 185\}$$

$$= 1 - \Phi\left(\frac{255 - 223}{12.68}\right) + \Phi\left(\frac{185 - 223}{12.68}\right) = 1 - \Phi(2.52) + \Phi(-3.00) = 1 - 0.99413 + 0.00135 = 0.0072$$

(d) Assuming the variance to remain constant, state where the process mean should be located to minimize the fraction nonconforming. What would be the value of the fraction nonconforming under these conditions?

The process mean should be located at the nominal dimension, 220, to minimize non-conforming units.

$$\hat{p} = 1 - \Phi\left(\frac{255 - 220}{12.68}\right) + \Phi\left(\frac{185 - 220}{12.68}\right) = 1 - \Phi(2.76) + \Phi(-2.76) = 1 - 0.99711 + 0.00289 = 0.00578$$

**6.26.**

Samples of size $n$ = 4 are taken from a transactional process every hour. A quality characteristic is measured, and $\bar{x}$ and $R$ are computed for each sample. After 2 samples have been analyzed, we have

$\sum_{i=1}^{25} \bar{x}_i = 657.50$ and $\sum_{i=1}^{25} R_i = 9.00$

The quality characteristic is normally distributed.

(a) Find the control limits for the $\bar{x}$ and $R$ charts.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{25} \bar{x}_i}{m} = \frac{657.50}{25} = 26.30; \quad \bar{R} = \frac{\sum_{i=1}^{25} R_i}{m} = \frac{9.00}{25} = 0.36$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 26.30 + 0.729(0.36) = 26.56$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 26.30 - 0.729(0.36) = 26.04$$

$$\text{UCL}_R = D_4\bar{R} = 2.282(0.36) = 0.82$$

$$\text{LCL}_R = D_3\bar{R} = 0(0.36) = 0$$

(b) Assume that both charts exhibit control. If the specifications are 26.40 $\pm$ 0.50, estimate the fraction nonconforming.

$$\hat{\sigma}_x = \bar{R} / d_2 = 0.36 / 2.039 = 0.177$$

$$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le \text{USL}\} + \Pr\{x < \text{LSL}\}$$

$$= 1 - \Phi\left(\frac{26.90 - 26.30}{0.177}\right) + \Phi\left(\frac{25.90 - 26.30}{0.177}\right) = 1 - \Phi(3.39) + \Phi(-2.26) = 1 - 0.9997 + 0.0119$$

$$= 0.0122$$

(c) If the mean of the process were 26.40, what fraction nonconforming would result?

$$\hat{p} = 1 - \Phi\left(\frac{26.90 - 26.40}{0.177}\right) + \Phi\left(\frac{25.90 - 26.40}{0.177}\right) = 1 - \Phi(2.82) + \Phi(-2.82)$$

$$= 1 - 0.9976 + 0.0024 = 0.0048$$

**6.27.**

Samples of size $n$ = 5 are collected from a process every half hour. After 50 samples have been collected, we calculate $\overline{x} = 20.0$ and $s = 1.5$. Assume that both charts exhibit control and that the quality characteristic is normally distributed.

(a) Estimate the process standard deviation.

$\hat{\sigma}_x = \overline{S} / c_4 = 1.5 / 0.9400 = 1.60$

(b) Find the control limits on the charts.

$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + A_3\overline{S} = 20.0 + 1.427(1.5) = 22.14$

$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - A_3\overline{S} = 20.0 - 1.427(1.5) = 17.86$

$\text{UCL}_S = B_4\overline{S} = 2.089(1.5) = 3.13$

$\text{LCL}_S = B_3\overline{S} = 0(1.5) = 0$

(c) If the process mean shifts to 22, what is the probability of concluding that the process is still in control?

$$\Pr\{\text{in control}\} = \Pr\{\text{LCL} \le \overline{x} \le \text{UCL}\} = \Pr\{\overline{x} \le \text{UCL}\} - \Pr\{\overline{x} \le \text{LCL}\}$$

$$= \Phi\left(\frac{22.14 - 22}{1.6/\sqrt{5}}\right) - \Phi\left(\frac{17.86 - 22}{1.6/\sqrt{5}}\right) = \Phi(0.20) - \Phi(-5.79)$$

$$= 0.57926 - 0 = 0.57926$$

**6.28.** ☺

Control chart for $\bar{x}$ and $R$ are maintained on a process. After 20 preliminary subgroups each of size 5 are evaluated, you have the following data:

$\sum_{i=1}^{20} \bar{x}_i = 5{,}498$ and $\sum_{i=1}^{20} R_i = 60$

(a) Set up control charts using these data.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{20} \bar{x}_i}{m} = \frac{5{,}498}{20} = 274.9; \quad \bar{R} = \frac{\sum_{i=1}^{20} R_i}{m} = \frac{60}{20} = 3$$

$$\mathrm{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 274.9 + 0.577(3) = 276.6$$

$$\mathrm{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 274.9 - 0.577(3) = 273.2$$

$$\mathrm{UCL}_R = D_4\bar{R} = 2.114(3) = 6.3$$

$$\mathrm{LCL}_R = D_3\bar{R} = 0(3) = 0$$

(b) Assume that the process exhibits statistical control. Estimate the process mean and standard deviation.

$$\hat{\mu} = \bar{\bar{x}} = 274.9; \quad \hat{\sigma}_x = \bar{R}/d_2 = 3/2.326 = 1.3$$

(c) Suppose that the quality characteristic is normally distributed with specifications at $275 \pm 6$. Estimate the fraction nonconforming produced by this process.

$$\hat{p} = \Pr\{x > \mathrm{USL}\} + \Pr\{x < \mathrm{LSL}\} = 1 - \Pr\{x \le \mathrm{USL}\} + \Pr\{x < \mathrm{LSL}\}$$

$$= 1 - \Phi\left(\frac{281 - 274.9}{1.3}\right) + \Phi\left(\frac{269 - 274.9}{1.3}\right) = 1 - \Phi(4.69) + \Phi(-4.54) = 1 - 0.999999 + 0.000003$$

$$= 0.000004$$

(d) How much reduction in process variability would be required to make this a Six Sigma process?

The Motorola Six Sigma concept is to reduce the variability in the process so that the specification limits are at least six standard deviations from the mean (text p. 29). If the process mean was centered at the specification midpoint, this would require a standard deviation of

$$\hat{\sigma} = \frac{\mathrm{USL} - \mathrm{midpoint}}{6} = \frac{(275 + 6) - 275}{6} = \frac{6}{6} = 1$$

Or a reduction in process variability of almost 23%, from 1.3 to 1.

**6.29.** ☺

Control chart for $\bar{x}$ and $s$ are maintained on a process. After 25 preliminary subgroups each of size 3 are evaluated, you have the following data:

$$\sum_{i=1}^{25} \bar{x}_i = 55.45 \text{ and } \sum_{i=1}^{25} s_i = 28.67$$

(a) Set up control charts using these data.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{25} \bar{x}_i}{m} = \frac{55.45}{25} = 2.218; \quad \bar{s} = \frac{\sum_{i=1}^{25} s_i}{m} = \frac{28.67}{25} = 1.147$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_3\bar{s} = 2.218 + 1.954(1.147) = 4.459$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_3\bar{s} = 2.218 - 1.954(1.147) = -0.023$$

$$\text{UCL}_s = B_4\bar{s} = 2.568(1.147) = 2.945$$

$$\text{LCL}_s = B_3\bar{s} = 0(1.147) = 0$$

(b) Assume that the process exhibits statistical control. Estimate the process mean and standard deviation.

$$\hat{\mu} = \bar{\bar{x}} = 2.218; \quad \hat{\sigma}_x = \bar{s} / c_4 = 1.147 / 0.8862 = 1.294$$

(c) Suppose that the quality characteristic is normally distributed with specifications at $2.25 \pm 4$. Estimate the fraction nonconforming produced by this process.

$$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le \text{USL}\} + \Pr\{x < \text{LSL}\}$$

$$= 1 - \Phi\left(\frac{6.25 - 2.218}{1.147}\right) + \Phi\left(\frac{-1.75 - 2.218}{1.147}\right) = 1 - \Phi(3.52) + \Phi(-3.46) = 1 - 0.99978 + (1 - 0.99973)$$

$$= 0.00049$$

(d) How much reduction in process variability would be required to make this a Six Sigma process?

The Motorola Six Sigma concept is to reduce the variability in the process so that the specification limits are at least six standard deviations from the mean (text p. 29). If the process mean was centered at the specification midpoint, this would require a standard deviation of

$$\hat{\sigma} = \frac{\text{USL} - \text{midpoint}}{6} = \frac{(2.25 + 4) - 2.25}{6} = \frac{4}{6} = 0.67$$

Or a reduction in process variability of about 40%%, from 1.147 to 0.67.

**6.30.**

An $\bar{x}$ chart is used to control the mean of a normally distributed quality characteristic. It is known that $\sigma$ = 6.0 and $n$ = 4. The center line = 200, UCL = 209, and LCL = 191. If the process mean shifts to 188, find the probability that this shift is detect on the first subsequent sample.

$$\begin{aligned}\Pr\{\text{detect}\} &= 1-\Pr\{\text{not detect}\} = 1-[\Pr\{\text{LCL} \le \bar{x} \le \text{UCL}\}] = 1-[\Pr\{\bar{x} \le \text{UCL}\} - \Pr\{\bar{x} \le \text{LCL}\}] \\ &= 1-\left[\Phi\left(\frac{\text{UCL}_{\bar{x}} - \mu_{\text{new}}}{\sigma_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL}_{\bar{x}} - \mu_{\text{new}}}{\sigma_x/\sqrt{n}}\right)\right] = 1-\left[\Phi\left(\frac{209-188}{6/\sqrt{4}}\right) - \Phi\left(\frac{191-188}{6/\sqrt{4}}\right)\right] \\ &= 1-\Phi(7)+\Phi(1) = 1-1+0.84134 = 0.84134\end{aligned}$$

---

**6.31.**

A critical dimension of a machined part has specifications 100 $\pm$ 10. Control chart analysis indicates tht the process is in control with $\bar{x} = 104$ and $\bar{R} = 9.30$. The control charts use samples of size $n$ = 5. If we assume that the characteristic is normally distributed, can the mean be located (by adjusting the tool position) so that all output meets specifications?

$\hat{\sigma}_x = \bar{R}/d_2 = 9.30/2.326 = 3.998$ and $6\hat{\sigma}_x = 6(3.998) = 23.99$ is larger than the width of the tolerance band, 2(10) = 20. So, even if the mean is located at the nominal dimension, 100, not all of the output will meet specification.

$$\hat{C}_P = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x} = \frac{+10-(-10)}{6(3.998)} = 0.8338$$

---

**6.32.**

A process is to be monitored with standard values $\mu$ = 10 and $\sigma$ = 1.5. The sample size is $n$ = 3.

(a) Find the center line and control limits for the $\bar{x}$ chart.

$\text{centerline}_{\bar{x}} = \mu = 10$

$\text{UCL}_{\bar{x}} = \mu + A\sigma_x = 10 + 1.732(1.5) = 12.60$

$\text{LCL}_{\bar{x}} = \mu - A\sigma_x = 10 - 1.732(1.5) = 7.40$

(b) Find the center line and control limits for the $R$ chart.

$\text{centerline}_R = d_2\sigma_x = 1.693(1.5) = 2.54$

$\text{UCL}_R = D_2\sigma = 4.358(1.5) = 6.54$

$\text{LCL}_R = D_1\sigma = 0(1.5) = 0$

(c) Find the center line and control limits for the $s$ chart.

$\text{centerline}_S = c_4\sigma_x = 0.8862(1.5) = 1.33$

$\text{UCL}_S = B_6\sigma = 2.276(1.5) = 3.41$

$\text{LCL}_S = B_5\sigma = 0(1.5) = 0$

---

**6.33.**

Samples f $n$= 5 units are taken from a process every hour. The $\overline{x}$ and $R$ values for a particular quality characteristic are determined. After 25 sample have been collected, we calculate $\overline{x} = 20$ and $\overline{R} = 4.56$.

(a) What are the three-sigma control limits for the $\overline{x}$ and $R$?

$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R} = 20 + 0.577(4.56) = 22.63$

$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R} = 20 - 0.577(4.56) = 17.37$

$\text{UCL}_R = D_4\overline{R} = 2.114(4.56) = 9.64$

$\text{LCL}_R = D_3\overline{R} = 0(4.56) = 0$

(b) Both charts exhibit control. Estimate the process standard deviation.

$\hat{\sigma}_x = \overline{R} / d_2 = 4.56 / 2.326 = 1.96$

(c) Assume that the process output is normally distributed. If he specifications are 19 $\pm$ 5, what are your conclusions regarding the process capability?

$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{+5-(-5)}{6(1.96)} = 0.85$, so the process is not capable of meeting specifications.

(d) If the process mean shifts to 24, what is the probability of not detecting this shift on the first subsequent sample?

$$\Pr\{\text{not detect}\} = \Pr\{\text{LCL} \le \overline{x} \le \text{UCL}\} = \Pr\{\overline{x} \le \text{UCL}\} - \Pr\{\overline{x} \le \text{LCL}\}$$

$$= \Phi\left(\frac{\text{UCL}_{\overline{x}} - \mu_{\text{new}}}{\hat{\sigma}_x / \sqrt{n}}\right) - \Phi\left(\frac{\text{LCL}_{\overline{x}} - \mu_{\text{new}}}{\hat{\sigma}_x / \sqrt{n}}\right) = \Phi\left(\frac{22.63 - 24}{1.96 / \sqrt{5}}\right) - \Phi\left(\frac{17.37 - 24}{1.96 / \sqrt{5}}\right)$$

$$= \Phi(-1.56) + \Phi(-7.56) = 0.05938 - 0 = 0.05938$$

**6.34.**

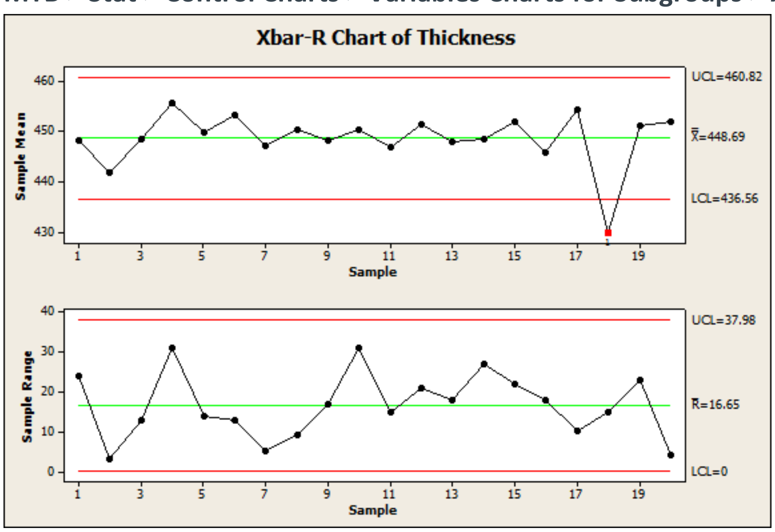
A TiW layer is deposited on a substrate using a sputtering tool. Table 6E.14 contains layer thickness measurements (in angstroms) on 20 subgroups of four substrates.

**TABLE 6E.14**
Layer Thickness Data for Exercise 6.34

| Subgroup | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| 1 | 459 | 449 | 435 | 450 |
| 2 | 443 | 440 | 442 | 442 |
| 3 | 457 | 444 | 449 | 444 |
| 4 | 469 | 463 | 453 | 438 |
| 5 | 443 | 457 | 445 | 454 |
| 6 | 444 | 456 | 456 | 457 |
| 7 | 445 | 449 | 450 | 445 |
| 8 | 446 | 455 | 449 | 452 |
| 9 | 444 | 452 | 457 | 440 |
| 10 | 432 | 463 | 463 | 443 |
| 11 | 445 | 452 | 453 | 438 |
| 12 | 456 | 457 | 436 | 457 |
| 13 | 459 | 445 | 441 | 447 |
| 14 | 441 | 465 | 438 | 450 |
| 15 | 460 | 453 | 457 | 438 |
| 16 | 453 | 444 | 451 | 435 |
| 17 | 451 | 460 | 450 | 457 |
| 18 | 422 | 431 | 437 | 429 |
| 19 | 444 | 446 | 448 | 467 |
| 20 | 450 | 450 | 454 | 454 |

(a) Setup $\bar{x}$ and $R$ control charts on this process. Is the process in control? Revise the control limits as necessary.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**

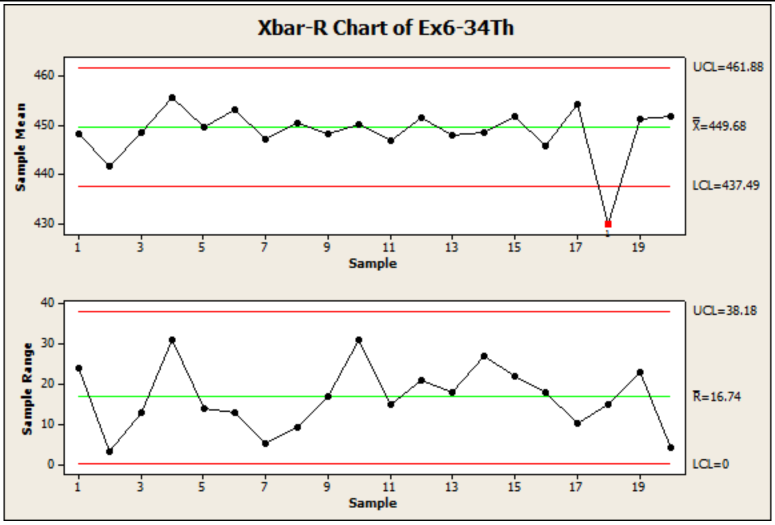



**Test Results for Xbar Chart of Ex6-34Th**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

The process is out of control on the $\bar{x}$ chart at subgroup 18. Excluding subgroup 18 from control limits calculations:

**6.34.(a) continued**




**Test Results for Xbar Chart of Ex6-34Th**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

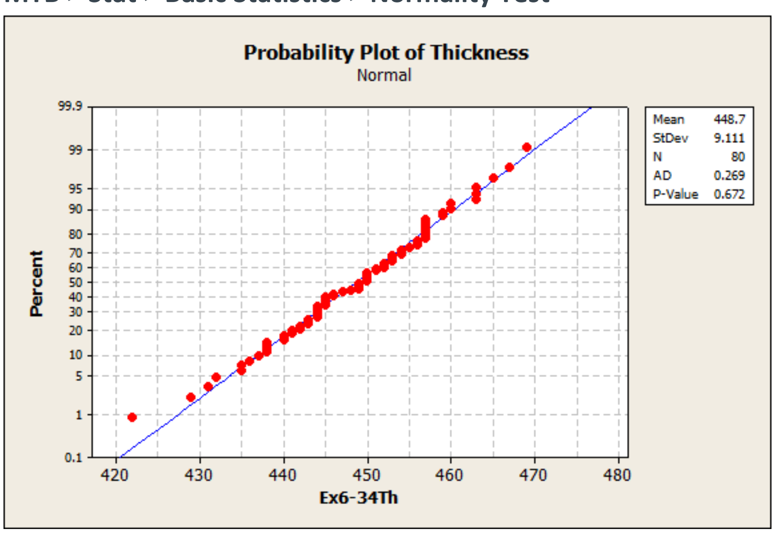
No additional subgroups are beyond the control limits, so these limits can be used for future production.

(b) Estimate the mean and standard deviation of the process.

Excluding subgroup 18: $\bar{\bar{x}} = 449.68$; $\hat{\sigma}_x = \bar{R}/d_2 = 16.74/2.059 = 8.13$

(c) Is the layer thickness normally distributed?

**MTB > Stat > Basic Statistics > Normality Test**




A normal probability plot of the TiW thickness measurements shows the distribution is close to normal.

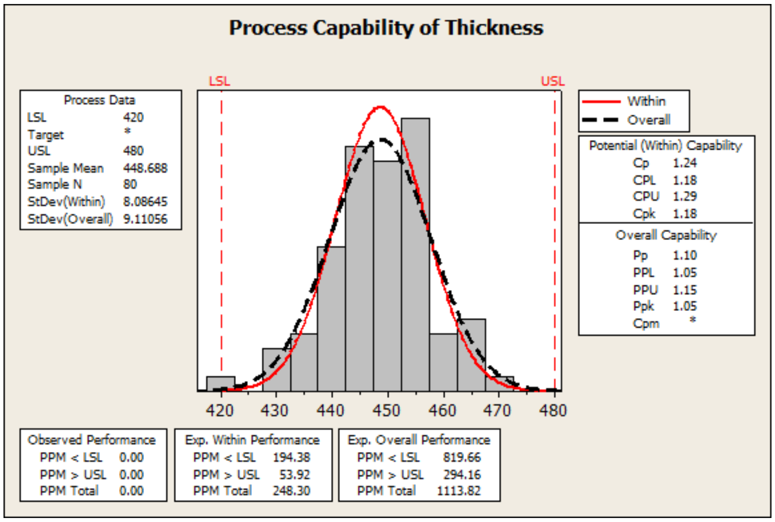
**6.34. continued**

(d) f the specifications are at 450 $\pm$ 30, estimate the process capability.

USL = +30, LSL = –30

$$\hat{C}_P = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x} = \frac{+30-(-30)}{6(8.13)} = 1.23 \text{ , so the process is capable.}$$

**MTB > Stat > Quality Tools > Capability Analysis > Normal**




The Potential (Within) Capability, Cp = 1.24, is estimated from the within-subgroup variation, or in other words, $\sigma_x$ is estimated using $\bar{R}$ . This is the same result as the manual calculation.

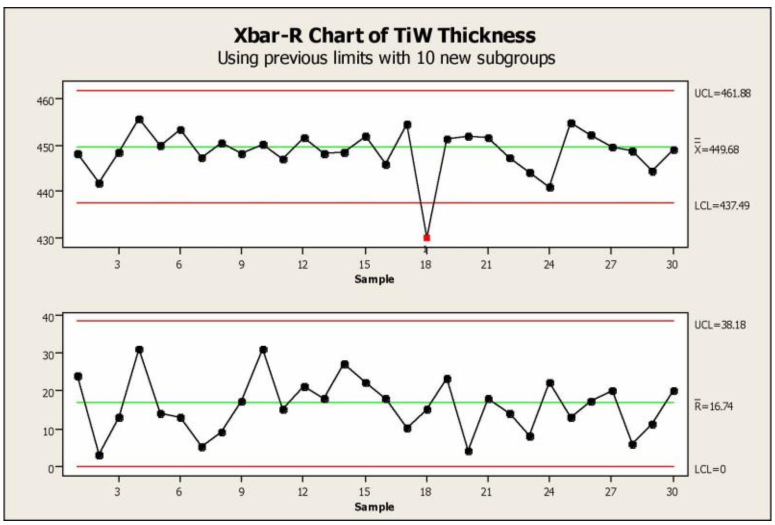
**6.35.**

**Continuation of Exercise 6.34.** Table 6E.15 contains 10 new subgroups of thickness data. Plot this data on the control charts constructed in Exercise 6.26(a). Is the process in statistical control?

**TABLE 6E.15**
Additional Thickness Data for Exercise 6.35.

| Subgroup | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| 21 | 454 | 449 | 443 | 461 |
| 22 | 449 | 441 | 444 | 455 |
| 23 | 442 | 442 | 442 | 450 |
| 24 | 443 | 452 | 438 | 430 |
| 25 | 446 | 459 | 457 | 457 |
| 26 | 454 | 448 | 445 | 462 |
| 27 | 458 | 449 | 453 | 438 |
| 28 | 450 | 449 | 445 | 451 |
| 29 | 443 | 440 | 443 | 451 |
| 30 | 457 | 450 | 452 | 437 |

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




**Test Results for Xbar Chart of Ex6.27Th**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

The process continues to be in a state of statistical control.

**6.36.**

**Continuation of Exercise 6.34.** Suppose that following the construction of the $\bar{x}$ and $R$ control charts in Exercise 6.34, the process engineers decided to change the subgroup size to $n$ = 2. Table 6E.16 contains 10 new subgroups of thickness data. Plot this data on the control charts from Exercise 6.34(a) based on the new subgroup size. Is the process in statistical control?

**TABLE 6E.16**
**Additional Thickness Data for Exercise 6.36**

| Subgroup | $x_1$ | $x_2$ |
|---|---|---|
| 21 | 454 | 449 |
| 22 | 449 | 441 |
| 23 | 442 | 442 |
| 24 | 443 | 452 |
| 25 | 446 | 459 |
| 26 | 454 | 448 |
| 27 | 458 | 449 |
| 28 | 450 | 449 |
| 29 | 443 | 440 |
| 30 | 457 | 450 |

$n_{\text{old}} = 4;\quad \bar{\bar{x}}_{\text{old}} 449.68;\quad \bar{R}_{\text{old}} = 16.74;\quad n_{\text{new}} = 2$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 449.68 + 1.880\left[\frac{1.128}{2.059}\right](16.74) = 466.92$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 449.68 - 1.880\left[\frac{1.128}{2.059}\right](16.74) = 432.44$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 3.267\left[\frac{1.128}{2.059}\right](16.74) = 29.96$$

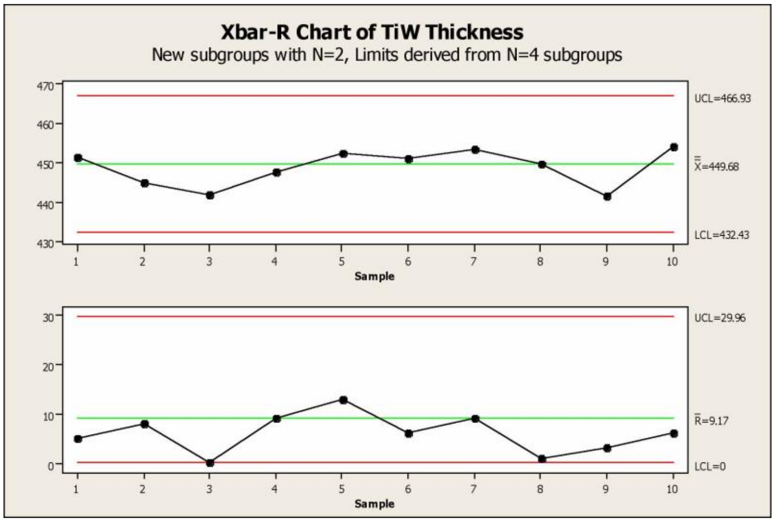
$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{1.128}{2.059}\right](16.74) = 9.17$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{1.128}{2.059}\right](16.74) = 0$$

$$\hat{\sigma}_{\text{new}} = \bar{R}_{\text{new}} / d_{2(\text{new})} = 9.17/1.128 = 8.13$$

**6.36. continued**

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**
**Select Xbar-R options, Parameters, and enter new parameter values.**



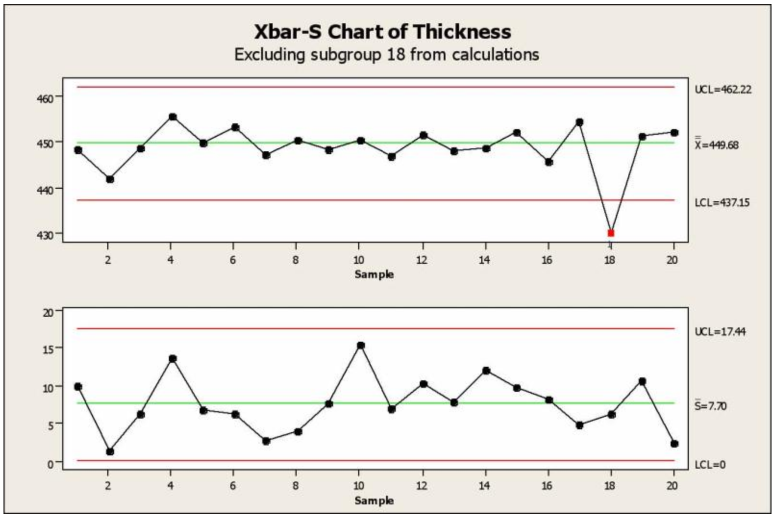

The process remains in statistical control.

**6.37.**

Rework Exercises 6.34 and 6.35 using $\overline{x}$ and $s$ control charts.

The process is out of control on the $\overline{x}$ chart at subgroup 18. After finding assignable cause, exclude subgroup 18 from control limits calculations:

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S**

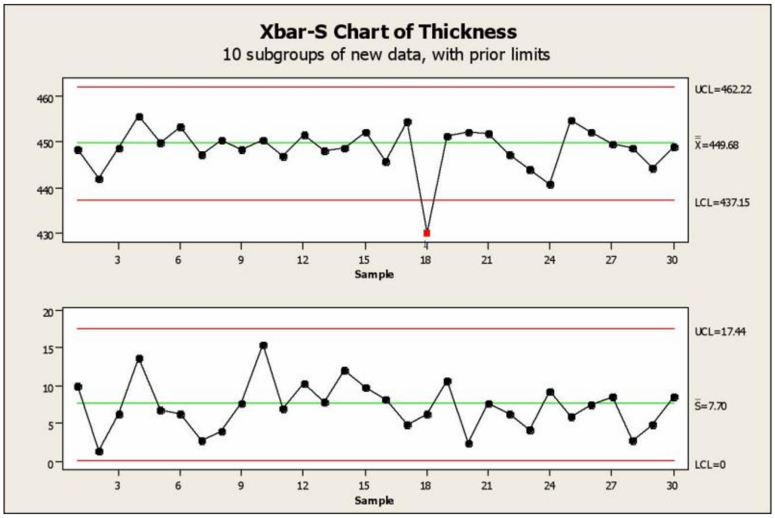



**Xbar-S Chart of Ex6.26Th**

**Test Results for Xbar Chart of Ex6.26Th**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

No additional subgroups are beyond the control limits, so these limits can be used for future production.




The process remains in statistical control.

**6.38.**

Control charts for $\bar{x}$ and $R$ are to be established to control the surface finish of a metal part. Assume that surface finish is normally distributed. Thirty samples of size $n$= 4 parts are collected over a period of time with the following results:

$$\sum_{i=1}^{30} \bar{x}_i = 6000; \quad \sum_{i=1}^{30} R_i = 120$$

(a) Calculate the control limits for $\bar{x}$ and $R$.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{30} \bar{x}_i}{m} = \frac{6000}{30} = 200; \quad \bar{R} = \frac{\sum_{i=1}^{30} R_i}{m} = \frac{120}{30} = 4$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 200 + 0.729(4) = 202.92$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 200 - 0.729(4) = 197.08$$

$$\text{UCL}_R = D_4\bar{R} = 2.282(4) = 9.13$$

$$\text{LCL}_R = D_3\bar{R} = 0(4) = 0$$

(b) Both charts exhibit control. The specifications on surface finish are $200 \pm 5$. What are your conclusions regarding process capability?

$$\hat{\sigma}_x = \bar{R} / d_2 = 4 / 2.059 = 1.9$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+5 - (-5)}{6(1.9)} = 0.88$$

The process is not capable of meeting specification. Even though the process is centered at nominal, the variation is large relative to the tolerance.

(c) For the above $\bar{x}$ chart, find the $\beta$-risk when the true process mean is 199.

$$\beta - \text{risk} = \Pr\{\text{not detect}\} = \Phi\left(\frac{202.92 - 199}{1.9/\sqrt{4}}\right) - \Phi\left(\frac{197.08 - 199}{1.9/\sqrt{4}}\right)$$

$$= \Phi(4.13) - \Phi(-2.02) = 0.99998 - 0.02164 = 0.97834$$

**6.39.**

An $\bar{x}$ chart has a center line of 100, uses three-sigma control limits, and is based on a sample size of four. The process standard deviation is known to be six. If the process mean shifts from 100 to 92, what is the probability of detecting this shift on the first sample following the shift?

$\mu_0 = 100;\quad L = 3;\quad n = 4;\quad \sigma = 6;\quad \mu_1 = 92$

$k = (\mu_1 - \mu_0)/\sigma = (92 - 100)/6 = -1.33$

$$\begin{aligned}
\Pr\{\text{detecting shift on 1st sample}\} &= 1 - \Pr\{\text{not detecting shift on 1st sample}\} \\
&= 1 - \beta \\
&= 1 - \left[\Phi\left(L - k\sqrt{n}\right) - \Phi\left(-L - k\sqrt{n}\right)\right] \\
&= 1 - \left[\Phi\left(3 - (-1.33)\sqrt{4}\right) - \Phi\left(-3 - (-1.33)\sqrt{4}\right)\right] \\
&= 1 - \left[\Phi(5.66) - \Phi(-0.34)\right] \\
&= 1 - \left[1 - 0.37\right] \\
&= 0.37
\end{aligned}$$

---

**6.40.**

The data in Table 6E.17 were collected form a process manufacturing power supplies. The variable of interest is output voltage, and $n = 5$.

**TABLE 6E.17**

Voltage Data for Exercise 6.40

| Sample Number | $\bar{x}$ | $R$ | Sample Number | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|
| 1 | 103 | 4 | 11 | 105 | 4 |
| 2 | 102 | 5 | 12 | 103 | 2 |
| 3 | 104 | 2 | 13 | 102 | 3 |
| 4 | 105 | 11 | 14 | 105 | 4 |
| 5 | 104 | 4 | 15 | 104 | 5 |
| 6 | 106 | 3 | 16 | 105 | 3 |
| 7 | 102 | 7 | 17 | 106 | 5 |
| 8 | 105 | 2 | 18 | 102 | 2 |
| 9 | 106 | 4 | 19 | 105 | 4 |
| 10 | 104 | 3 | 20 | 103 | 2 |

(a) Compute center lines and control limits suitable for controlling future production.

$\bar{\bar{x}} = 104.05;\quad \bar{R} = 3.95$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104.05 + 0.577(3.95) = 106.329$

$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104.05 - 0.577(3.95) = 101.771$

$\text{UCL}_R = D_4\bar{R} = 2.114(3.95) = 8.350$

$\text{LCL}_R = D_3\bar{R} = 0(3.95) = 0$

**6.40.(a) continued**

Sample #4 is out of control on the Range chart. So, excluding #4 and recalculating:

$\bar{\bar{x}} = 104$; $\bar{R} = 3.579$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104 + 0.577(3.579) = 106.065$

$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104 - 0.577(3.579) = 101.935$

$\text{UCL}_R = D_4\bar{R} = 2.114(3.579) = 7.566$

$\text{LCL}_R = D_3\bar{R} = 0(3.579) = 0$

(b) Assume that the quality characteristic is normally distributed. Estimate the process standard deviation.

Without sample #4, $\hat{\sigma}_x = \bar{R} / d_2 = 3.579 / 2.326 = 1.539$

(c) What are the apparent three-sigma natural tolerance limits of the process?

$\text{UNTL} = \bar{\bar{x}} + 3\hat{\sigma}_x = 104 + 3(1.539) = 108.62$

$\text{LNTL} = \bar{\bar{x}} - 3\hat{\sigma}_x = 104 - 3(1.539) = 99.38$

(d) What would be your estimate of the process fraction nonconforming if the specifications on the characteristic were 103 $\pm$4?

$$\hat{p} = 1 - \Phi\left(\frac{107-104}{1.539}\right) + \Phi\left(\frac{99-104}{1.539}\right) = 1 - \Phi(1.95) + \Phi(-3.25) = 1 - 0.9744 + 0.0006 = 0.0262$$

(e) What approaches to reducing the fraction nonconforming can you suggest?

To reduce the fraction nonconforming, first center the process at nominal.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{1.539}\right) + \Phi\left(\frac{99-103}{1.539}\right) = 1 - \Phi(2.60) + \Phi(-2.60) = 1 - 0.9953 + 0.0047 = 0.0094$$

Next work on reducing the variability; if $\hat{\sigma}_x = 0.667$, then almost 100% of parts will be within specification.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{0.667}\right) + \Phi\left(\frac{99-103}{0.667}\right) = 1 - \Phi(5.997) + \Phi(-5.997) = 1 - 1.0000 + 0.0000 = 0.0000$$

**6.41.**

Control charts on $\bar{x}$ and $R$ for samples of size $n$- 5 are to be maintained on the tensile strength in pounds of a yarn. To start the charts, 30 samples were selected, and the mean and range of each computed. This yields $\sum_{i=1}^{30} \bar{x}_i = 607.8$ and $\sum_{i=1}^{30} R_i = 144$

(a) Compute the center line and control limits for the $\bar{x}$ and $R$ control charts.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{607.8}{30} = 20.26; \quad \bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{144}{30} = 4.8$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 20.26 + 0.577(4.8) = 23.03$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 20.26 - 0.577(4.8) = 17.49$$

$$\text{UCL}_R = D_4\bar{R} = 2.114(4.8) = 10.147$$

$$\text{LCL}_R = D_3\bar{R} = 0(4.8) = 0$$

(b) Suppose both charts exhibit control. There is a single lower specification limit of 16 lb. If strength is normally distributed, what fraction of yarn would fail to meet specifications?

$$\hat{\sigma}_x = \bar{R} / d_2 = 4.8 / 2.326 = 2.064$$

$$\hat{p} = \Pr\{x < \text{LSL}\} = \Phi\left(\frac{16 - 20.26}{2.064}\right) = \Phi(-2.064) = 0.0195$$

**6.42.**

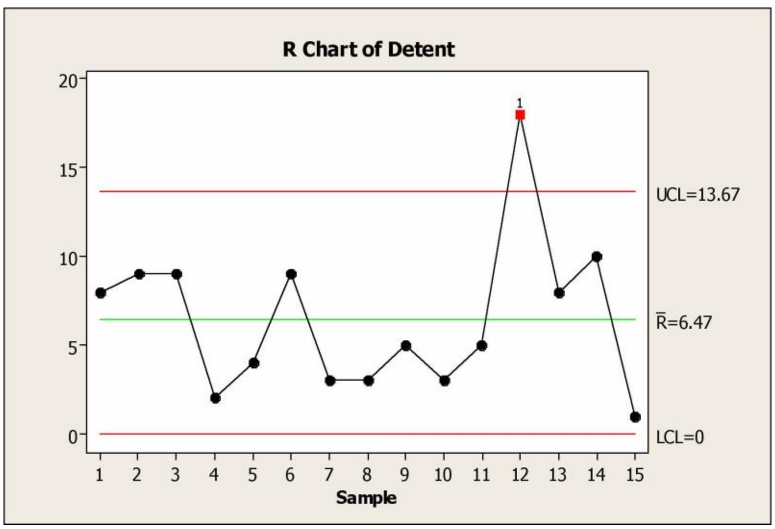
Specifications on a copper flush bushing are 0.3220 and 0.3200 in. Samples of size 5 are taken every 45 minutes with the results shown in Table 6E.18 (measured as deviations from 0.3210 in 0.0001 in.).

**TABLE 6E.18**
Data for Exercise 6.42

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| 1 | 1 | 9 | 6 | 9 | 6 |
| 2 | 9 | 4 | 3 | 0 | 3 |
| 3 | 0 | 9 | 0 | 3 | 2 |
| 4 | 1 | 1 | 0 | 2 | 1 |
| 5 | −3 | 0 | −1 | 0 | −4 |
| 6 | −7 | 2 | 0 | 0 | 2 |
| 7 | −3 | −1 | −1 | 0 | −2 |
| 8 | 0 | −2 | −3 | −3 | −2 |
| 9 | 2 | 0 | −1 | −3 | −1 |
| 10 | 0 | 2 | −1 | −1 | 2 |
| 11 | −3 | −2 | −1 | −1 | 2 |
| 12 | −16 | 2 | 0 | −4 | −1 |
| 13 | −6 | −3 | 0 | 0 | −8 |
| 14 | −3 | −5 | 5 | 0 | 5 |
| 15 | −1 | −1 | −1 | −2 | −1 |

(a) Set up an *R* chart and examine the process for statistical control.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > R**
**Under "Options, Estimate" select Rbar as method to estimate standard deviation.**




**Test Results for R Chart of Ex6.34Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

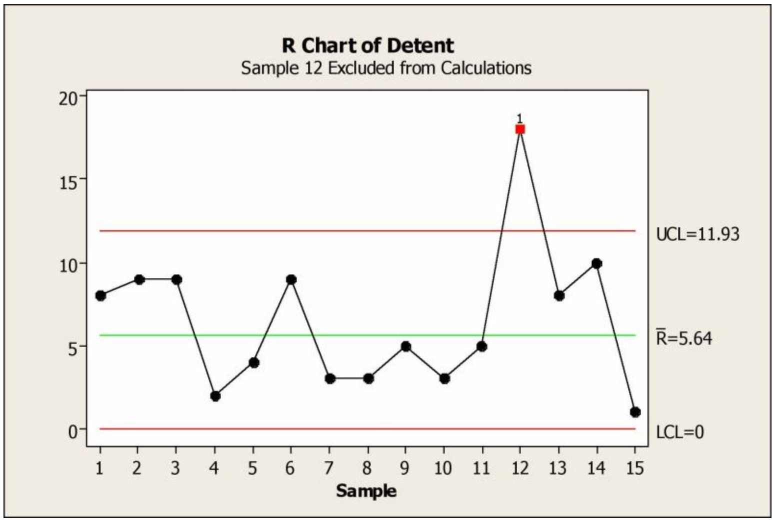
The process is not in statistical control -- sample #12 exceeds the upper control limit on the Range chart.

**6.42. continued**

(b) What parameters would you recommend for an *R* chart for on-line control?

Excluding Sample Number 12:

**MTB > Stat > Control Charts > Variables Charts for Subgroups > R**
**Under "Options, Estimate" omit subgroup 12 and select Rbar.**




### Test Results for R Chart of Ex6.34Det

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

(c) Estimate the standard deviation of the process.

Without sample #12: $\hat{\sigma}_x = \bar{R} / d_2 = 5.64 / 2.326 = 2.42$

(d) What is the process capability?

Assume the cigar lighter detent is normally distributed. Without sample #12:

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{0.3220 - 0.3200}{6(2.42 \times 0.0001)} = 1.38$$

**6.43.**

**Continuation of Exercise 6.42.** Reconsider the data from Exercise 6.42 and establish $\bar{x}$ and $R$ charts with appropriate trial control limits. Revise these trial limits as necessary to produce a set of control charts for monitoring future production. Suppose that the new data in Table 6E.19 are observed.

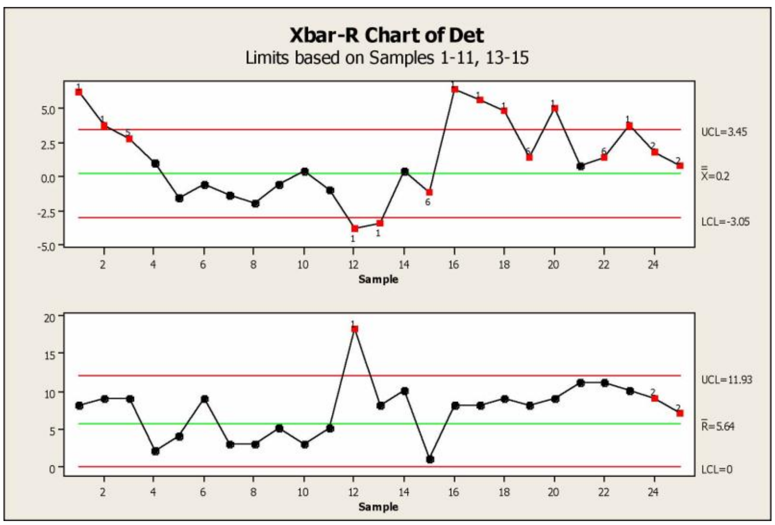
**TABLE 6E.19**
New Data for Exercise 6.43

| Sample Number | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| 16 | 2 | 10 | 9 | 6 | 5 |
| 17 | 1 | 9 | 5 | 9 | 4 |
| 18 | 0 | 9 | 8 | 2 | 5 |
| 19 | –3 | 0 | 5 | 1 | 4 |
| 20 | 2 | 10 | 9 | 3 | 1 |
| 21 | –5 | 4 | 0 | 6 | –1 |
| 22 | 0 | 2 | –5 | 4 | 6 |
| 23 | 10 | 0 | 3 | 1 | 5 |
| 24 | –1 | 2 | 5 | 6 | –3 |
| 25 | 0 | –1 | 2 | 5 | –2 |

(a) Plot these new observations on the control chart. What conclusions can you draw about process stability?

**MTB > Stat > Control Charts > Variables Charts for Subgroups > R**

Under "Options, Estimate" use subgroups 1:11 and 13:15, and select Rbar.




### Test Results for Xbar Chart of Ex6.35Det

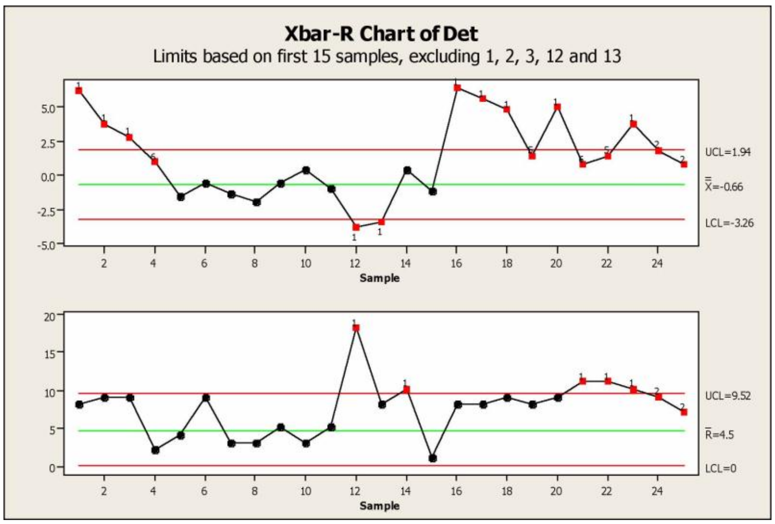
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 12, 13, 16, 17, 18, 20, 23
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  24, 25
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  2, 3, 13, 17, 18, 20
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  15, 19, 20, 22, 23, 24
```

**6.43.(a) continued**

**Test Results for R Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  24, 25
```

We are trying to establish trial control limits from the first 15 samples to monitor future production. Note that samples 1, 2, 12, and 13 are out of control on the $\bar{x}$ chart. If these samples are removed and the limits recalculated, sample 3 is also out of control on the $\bar{x}$ chart. Removing sample 3 gives

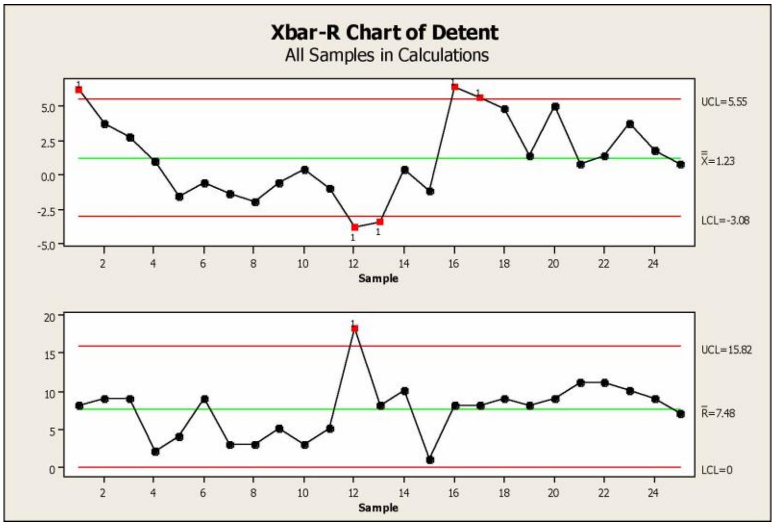



Sample 14 is now out of control on the $R$ chart. No additional samples are out of control on the $\bar{x}$ chart. While the limits on the above charts may be used to monitor future production, the fact that 6 of 15 samples were out of control and eliminated from calculations is an early indication of process instability.

Given the large number of points after sample 15 beyond both the $\bar{x}$ and $R$ control limits on the charts above, the process appears to be unstable.

**6.43. continued**

(b) Use all 25 observations to revise the control limits for the $\overline{x}$ and $R$ charts. What conclusions can you draw now about the process?




With Test 1 only:

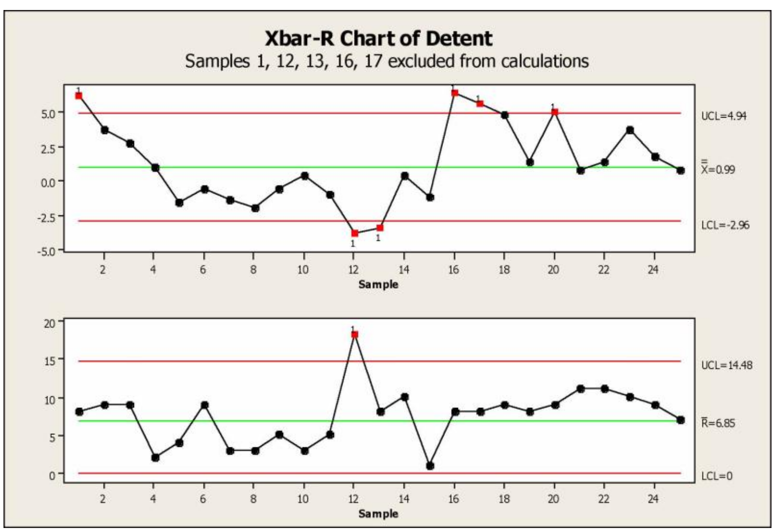
**Test Results for Xbar Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 12, 13, 16, 17
```

**Test Results for R Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

Removing samples 1, 12, 13, 16, and 17 from calculations:



**6.43.(b) continued**

With Test 1 only:

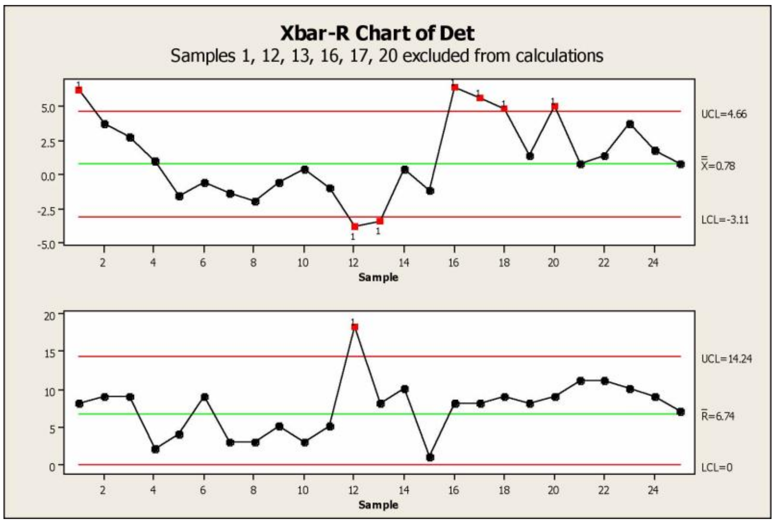
**Test Results for Xbar Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 12, 13, 16, 17, 20
```

**Test Results for R Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

Sample 20 is now also out of control. Removing sample 20 from calculations,




With Test 1 only:

**Test Results for Xbar Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 12, 13, 16, 17, 18, 20
```

**Test Results for R Chart of Ex6.35Det**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

Sample 18 is now out-of-control, for a total 7 of the 25 samples, with runs of points both above and below the centerline. This suggests that the process is inherently unstable, and that the sources of variation need to be identified and removed.

**6.44.**

Two parts are assembled as shown in Figure 6.28. Assume that the dimensions $x$ and $y$ are normally distributed with means $\mu_x$ and $\mu_y$ and standard deviations $\sigma_x$ and $\sigma_y$, respectively. The parts are produced on different machines and are assembled at random. Control charts are maintained on each dimension for the range of each sample ($n$ = 5). Both range charts are in control.

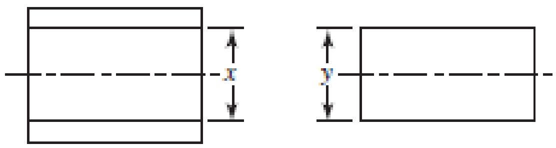



**FIGURE 6.28** Parts for Exercise 6.44.

(a) Given that for 20 samples on the range chart controlling $x$ and 10 samples on the range chart controlling $y$, we have

$$\sum_{i=1}^{20} R_{x,i} = 18.608 \text{ and } \sum_{i=1}^{10} R_{y,i} = 6.978$$

Estimate $\sigma_x$ and $\sigma_y$.

$$\hat{\sigma}_x = \bar{R}_x / d_2 = \left( \sum_{i=1}^{20} R_{x,i} / m_x \right) \Big/ d_2 = (18.608/20)/2.326 = 0.400$$

$$\hat{\sigma}_y = \bar{R}_y / d_2 = \left( \sum_{i=1}^{10} R_{y,i} / m_y \right) \Big/ d_2 = (6.978/10)/2.326 = 0.300$$

(b) If it is desired that the probability of a smaller clearance (i.e., $x - y$) than 0.09 should be 0.006, what distance between the average dimensions (i.e., $\mu_x - \mu_y$) should be specified?

Want $\Pr\{(x - y) < 0.09\} = 0.006$. Let $z = x - y$. Then $\hat{\sigma}_z = \sqrt{\hat{\sigma}_x^2 + \hat{\sigma}_y^2} = \sqrt{0.4^2 + 0.3^2} = 0.500$

$$\Phi\left( \frac{0.09 - z}{\hat{\sigma}_z} \right) = 0.006$$

$$\Phi^{-1}\left( \frac{0.09 - z}{0.500} \right) = \Phi(0.006)$$

$$\left( \frac{0.09 - z}{0.500} \right) = -2.5121$$

$$z = +2.5121(0.500) + 0.09 = 1.346$$

**6.45.**

Control charts for $\bar{x}$ and $R$ are maintained on the tensile strength of a metal fastener. Atfer 30 samples of size $n$= 6 are analyzed, we find that

$$\sum_{i=1}^{30} \bar{x}_i = 12{,}870 \text{ and } \sum_{i=1}^{30} R_i = 1350$$

(a) Compute control limits on the $R$ chart.

$$\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{1350}{30} = 45.0$$

$$\text{UCL}_R = D_4\bar{R} = 2.004(45.0) = 90.18$$

$$\text{LCL}_R = D_3\bar{R} = 0(45.0) = 0$$

(b) Assuming that the $R$ chart exhibits control, estimate the parameters $\mu$ and $\sigma$.

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{12{,}870}{30} = 429.0$$

$$\hat{\sigma}_x = \bar{R} / d_2 = 45.0 / 2.534 = 17.758$$

(c) If the process output is normally distributed, and if the specifications are 440 $\pm$ 40, can the process meet the specifications? Estimate the fraction nonconforming.

USL = 440 + 40 = 480; LSL = 440 - 40 = 400

$$\hat{C}_p \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{480 - 400}{6(17.758)} = 0.751$$

$$\hat{p} = 1 - \Phi\left(\frac{480 - 429}{17.758}\right) + \Phi\left(\frac{400 - 429}{17.758}\right) = 1 - \Phi(2.87) + \Phi(-1.63) = 1 - 0.9979 + 0.0516 = 0.0537$$

(d) If the variance remains constant, where should the mean be located to minimize the fraction nonconforming?

To minimize fraction nonconforming the mean should be located at the nominal dimension (440) for a constant variance.

**6.46.**

Control charts for $\bar{x}$ and $s$ are maintained on a quality characteristic. The sample size is $n$ = 9. After 30 samples, we obtain

$$\sum_{i=1}^{30} \bar{x}_i = 12{,}870 \text{ and } \sum_{i=1}^{30} S_i = 410$$

(a) Find the three-sigma limits for the $s$ chart.

$$\bar{S} = \frac{\sum_{i=1}^{m} S_i}{m} = \frac{410}{30} = 13.667$$

$$\text{UCL}_S = B_4 \bar{S} = 1.761(13.667) = 24.07$$

$$\text{LCL}_S = B_3 \bar{S} = 0.239(13.667) = 3.27$$

(b) Assuming that both charts exhibit control, estimate the parameters $\mu$ and $\sigma$.

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{12{,}870}{30} = 429.0$$

$$\hat{\sigma}_x = \bar{S} / c_4 = 13.667 / 0.9693 = 14.10$$

---

**6.47.**

An $\bar{x}$ chart on a normally distributed quality characteristic is to be established with the standard values $\mu$ = 100, $\sigma$ = 8, and $n$ = 4. Find the following:

(a) The two-sigma control limits.

$$n = 4; \quad \mu = 100; \quad \sigma_x = 8$$

$$\text{UCL}_{\bar{x}} = \mu + 2\sigma_{\bar{x}} = \mu + 2\left(\sigma_x / \sqrt{n}\right) = 100 + 2\left(8 / \sqrt{4}\right) = 108$$

$$\text{LCL}_{\bar{x}} = \mu - 2\sigma_{\bar{x}} = \mu - 2\left(\sigma_x / \sqrt{n}\right) = 100 - 2\left(8 / \sqrt{4}\right) = 92$$

(b) The 0.005 probability limits.

$$k = Z_{\alpha/2} = Z_{0.005/2} = Z_{0.0025} = 2.807$$

$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x / \sqrt{n}\right) = 100 + 2.807\left(8 / \sqrt{4}\right) = 111.228$$

$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x / \sqrt{n}\right) = 100 - 2.807\left(8 / \sqrt{4}\right) = 88.772$$

---

**6.48.**

An $\bar{x}$ chart with three-sigma limits has parameters as follows:

UCL = 104

Center line = 100

LCL = 96

$n$ = 5

Suppose the process quality characteristic being controlled is normally distributed with a true mean of 98 and a standard deviation of8. What is the probability that the control chart would exhibit lack of control by at least the third point plotted?

$$n=5;\ \text{UCL}_{\bar{x}}=104;\ \text{centerline}_{\bar{x}}=100;\ \text{LCL}_{\bar{x}}=96;\ k=3;\ \mu=98;\ \sigma_x=8$$

$$\Pr\{\text{out-of-control signal by at least 3rd plot point}\}=1-\Pr\{\text{not detected by 3rd sample}\}=1-[\Pr\{\text{not detected}\}]^3$$

$$\begin{aligned}\Pr\{\text{not detected}\}&=\Pr\{\text{LCL}_{\bar{x}}\le\bar{x}\le\text{UCL}_{\bar{x}}\}=\Pr\{\bar{x}\le\text{UCL}_{\bar{x}}\}-\Pr\{\bar{x}\le\text{LCL}_{\bar{x}}\}\\&=\Phi\left(\frac{\text{UCL}_{\bar{x}}-\mu}{\sigma_{\bar{x}}}\right)-\Phi\left(\frac{\text{LCL}_{\bar{x}}-\mu}{\sigma_{\bar{x}}}\right)=\Phi\left(\frac{104-98}{8/\sqrt{5}}\right)-\Phi\left(\frac{96-98}{8/\sqrt{5}}\right)=\Phi(1.68)-\Phi(-0.56)\\&=0.9535-0.2877=0.6658\end{aligned}$$

$$1-[\Pr\{\text{not detected}\}]^3=1-(0.6658)^3=0.7049$$

---

**6.49.**

Consider the $\bar{x}$ chart defined in Exercise 6.48. Find the $\text{ARL}_1$ for the chart.

$$\text{ARL}_1=\frac{1}{1-\beta}=\frac{1}{1-\Pr\{\text{not detect}\}}=\frac{1}{1-0.6658}=2.992$$

---

**6.50.**

Control charts for $\bar{x}$ and $s$ with $n$ = 9 are maintained on a quality characteristic. The parameters of these charts are as follows:

| $\bar{x}$ Chart | | | $s$ Chart | |
|---|---|---|---|---|
| UCL = | 201.88 | | UCL = | 3.209 |
| Center line = | 200.00 | | Center line = | 1.822 |
| LCL = | 198.12 | | LCL = | 0.435 |

Both charts exhibit control. Specifications on the quality characteristic are 197.50 and 202.50. What can be said about the ability of the process to produce product that conforms to specification?

$$\hat{C}_p=\frac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x}=\frac{\text{USL}-\text{LSL}}{6\left(\bar{S}/c_4\right)}=\frac{202.50\text{-}197.50}{6\left(1.822/0.9693\right)}=0.44$$

The process is not capable of meeting specifications.

---

**6.51.**

Statistical monitoring of a quality characteristic uses both an $\bar{x}$ and *s* chart. The charts are to be based on the standard values $\mu = 200$ and $\sigma_x = 10$, with $n = 4$.

(a) Find three-sigma control limits for the *s* chart.

$$\text{centerline}_S = c_4\sigma = 0.9213(10) = 9.213$$
$$\text{UCL}_S = B_6\sigma_x = 2.088(10) = 20.88$$
$$\text{LCL}_S = B_5\sigma_x = 0(10) = 0$$

(b) Find a center line and control limits for the $\bar{x}$ chart such that the probability of a type I error is 0.05.

$$k = Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$$
$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x/\sqrt{n}\right) = 200 + 1.96\left(10/\sqrt{4}\right) = 209.8$$
$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x/\sqrt{n}\right) = 200 - 1.96\left(10/\sqrt{4}\right) = 190.2$$

---

**6.52.**

Specifications on a normally distributed dimension are $600 \pm 20$. $\bar{x}$ and *R* charts are maintained on this dimension and have been in control over a long period of time. The parameters of these control charts are as follows ($n = 9$).

| $\bar{x}$ Chart | *s* Chart |
|---|---|
| UCL = 616 | UCL = 32.36 |
| Center line = 610 | Center line = 17.82 |
| LCL = 604 | LCL = 3.28 |

(a) What are your conclusions regarding the capability of the process to produce items within specifications?

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{\text{USL} - \text{LSL}}{6\left(\bar{R}/d_2\right)} = \frac{620 - 580}{6(17.82/2.970)} = 1.111$$

Process is capable of meeting specifications.

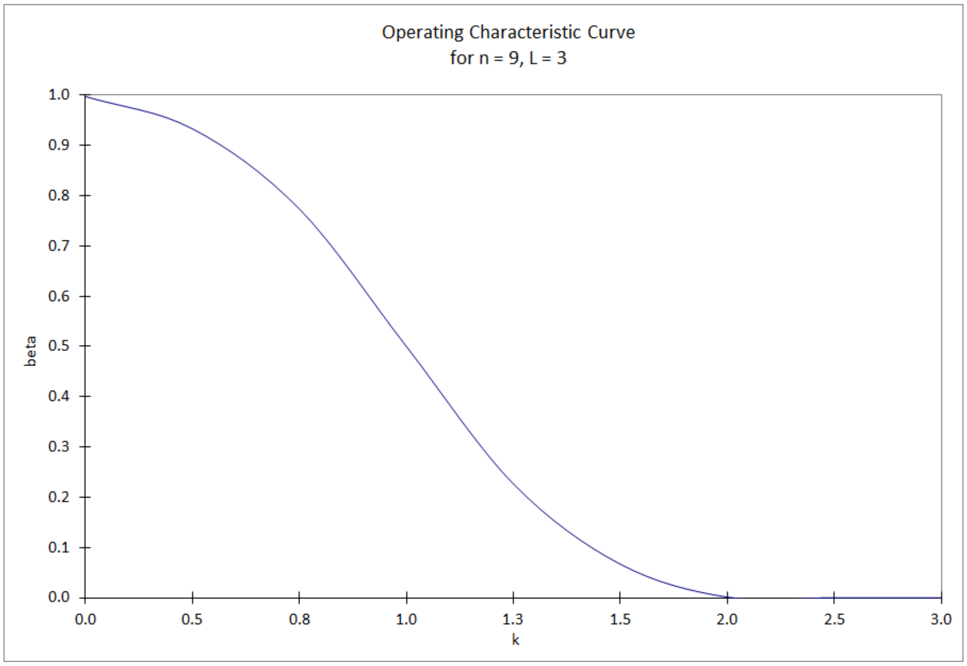
**6.52. continued**

(b) Construct an OC curve for the $\bar{x}$ chart assuming that σ is constant.

$n=9;\ L=3;\ \beta=\Phi\left(L-k\sqrt{n}\right)-\Phi\left(-L-k\sqrt{n}\right)$

for $k$ = {0, 0.5, 0.75, 1.0, 1.25, 1.5, 2.0, 2.5, 3.0},

β = {0.9974, 0.9332, 0.7734, 0.5, 0.2266, 0.0668, 0.0013, 0.0000, 0.0000}




**6.53.**

Thirty samples each of size 7 have been collected to establish control over a process. The following data were collected:

$\sum_{i=1}^{30}\bar{x}_i = 2700$ and $\sum_{i=1}^{30} R_i = 120$

(a) Calculate trial control limits for the two charts.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m}\bar{x}_i}{m} = \frac{2700}{30} = 90;\ \ \bar{R} = \frac{\sum_{i=1}^{m}R_i}{m} = \frac{120}{30} = 4$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 90 + 0.419(4) = 91.676$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 90 - 0.419(4) = 88.324$$

$$\text{UCL}_R = D_4\bar{R} = 1.924(4) = 7.696$$

$$\text{LCL}_R = D_3\bar{R} = 0.076(4) = 0.304$$

(b) On the assumption that the $R$ chart is in control, estimate the process standard deviation.

$$\hat{\sigma}_x = \bar{R}/d_2 = 4/2.704 = 1.479$$

**6.53. continued**

(c) Suppose an *s* chart were desired. What would be the appropriate control limits and center line?

$$\overline{S} = c_4 \hat{\sigma}_x = 0.9594(1.479) = 1.419$$
$$\text{UCL}_S = 1.882(1.419) = 2.671$$
$$\text{LCL}_S = 0.118(1.419) = 0.167$$

---

**6.54.**

An $\bar{x}$ chart is to be established based on the standard values $\mu$ = 600 and $\sigma$ = 15, with *n* = 4. The control limits are to be based on an $\alpha$-risk of 0.01. What are the appropriate control limits?

$$n = 4; \quad \mu = 600; \quad \sigma_x = 15; \quad \alpha = 0.01$$
$$k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$$
$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x/\sqrt{n}\right) = 600 + 2.576\left(15/\sqrt{4}\right) = 619.32$$
$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x/\sqrt{n}\right) = 600 - 2.576\left(15/\sqrt{4}\right) = 580.68$$

---

**6.55.**

$\bar{x}$ and $R$ charts with *n* = 4 are used to monitor a normally distributed quality characteristic. The control chart parameters are

| $\bar{x}$ Chart | *R* Chart |
|---|---|
| UCL = 815 | UCL = 46.98 |
| Center line = 800 | Center line = 20.59 |
| LCL = 785 | LCL = 0 |

Both charts exhibit control. What is the probability that a shift in the process mean to 790 will be detected on the first sample following the shift?

$$\hat{\sigma}_x = \overline{R}/d_2 = 20.59/2.059 = 10$$
$$\Pr\{\text{detect shift on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Pr\{\bar{x} < \text{LCL}\} + 1 - \Pr\{\bar{x} \le \text{UCL}\}$$
$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{785 - 790}{10/\sqrt{4}}\right) + 1 - \Phi\left(\frac{815 - 790}{10/\sqrt{4}}\right)$$
$$= \Phi(-1) + 1 - \Phi(5) = 0.1587 + 1 - 1.0000 = 0.1587$$

---

**6.56.**

Consider the $\bar{x}$ chart in Exercise 6.55. Find the average run length for the chart.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.1587} = 6.30$$

---

**6.57.**

Control charts for $\bar{x}$ and $R$ are in use with the following parameters:

| $\bar{x}$ Chart | $R$ Chart |
|---|---|
| UCL = 363.0 | UCL = 16.18 |
| Center line = 360.0 | Center line = 8.91 |
| LCL = 357.0 | LCL = 1.64 |

The sample size is $n$ = 9. Both charts exhibit control. The quality characteristic is normally distributed.

(a) What is the $\alpha$-risk associated with the $\bar{x}$ chart?

$$\hat{\sigma}_x = \bar{R}/d_2 = 8.91/2.970 = 3.000$$

$$\alpha = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{357 - 360}{3/\sqrt{9}}\right) + 1 - \Phi\left(\frac{363 - 360}{3/\sqrt{9}}\right)$$

$$= \Phi(-3) + 1 - \Phi(3) = 0.0013 + 1 - 0.9987 = 0.0026$$

(b) Specifications on this quality characteristic are $358 \pm 6$. What are your conclusions regarding the ability of the process to produce items within specifications?

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+6 - (-6)}{6(3)} = 0.667$$

The process is not capable of producing all items within specification.

(c) Suppose the mean shifts to 357. What is the probability that the shift will not be detected on the first sample following the shift?

$\mu_{\text{new}} = 357$

$$\Pr\{\text{not detect on 1st sample}\} = \Pr\{\text{LCL} \le \bar{x} \le \text{UCL}\} = \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right)$$

$$= \Phi\left(\frac{363 - 357}{3/\sqrt{9}}\right) - \Phi\left(\frac{357 - 357}{3/\sqrt{9}}\right) = \Phi(6) - \Phi(0) = 1.0000 - 0.5000 = 0.5000$$

(d) What would be the appropriate control limits for the $\bar{x}$ chart if the type I error probability were to be 0.01?

$$\alpha = 0.01; \quad k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + k\sigma_{\bar{x}} = \bar{\bar{x}} + k\left(\hat{\sigma}_x/\sqrt{n}\right) = 360 + 2.576\left(3/\sqrt{9}\right) = 362.576$$

$$\text{LCL}_{\bar{x}} = 360 - 2.576\left(3/\sqrt{9}\right) = 357.424$$

**6.58.**

A normally distributed quality characteristic is monitored through use of an $\bar{x}$ and $R$ chart. These charts have the following parameters ($n$ = 5):

| $\bar{x}$ Chart | | R Chart | |
|---|---|---|---|
| UCL = | 626.0 | UCL = | 21.983 |
| Center line = | 620.0 | Center line = | 10.399 |
| LCL = | 614.0 | LCL = | 0 |

Both charts exhibit control.

(a) What is the estimated standard deviation of the process?

$$\hat{\sigma}_x = \bar{R} / d_2 = 10.399 / 2.326 = 4.471$$

(b) Suppose an $s$ chart were to be substituted for the $R$ chart. What would be the appropriate parameters of the $s$ chart?

$$\bar{S} = c_4\hat{\sigma}_x = 0.9400(4.471) = 4.203; \quad \text{UCL}_S = B_4\bar{S} = 2.089(4.203) = 8.780; \quad \text{LCL}_S = B_3\bar{S} = 0(4.203) = 0$$

(c) If specifications on the product were 610 $\pm$ 10, what would be your estimate of the process fraction nonconforming?

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) = \Phi\left(\frac{600 - 620}{4.471}\right) + 1 - \Phi\left(\frac{620 - 620}{4.471}\right)$$

$$= \Phi(-4.473) + 1 - \Phi(0) = 0.0000 + 1 - 0.5 = 0.5$$

(d) What could be done to reduce this fraction nonconforming?

To reduce the fraction nonconforming, try moving the center of the process from its current mean of 620 closer to the nominal dimension of 610. Also consider reducing the process variability.

(e) What is the probability of detecting a shift in the process mean to 610 on the first sample following the shift ($\sigma$ remains constant)?

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right)$$

$$= \Phi\left(\frac{614 - 610}{4.471/\sqrt{4}}\right) + 1 - \Phi\left(\frac{626 - 610}{4.471/\sqrt{4}}\right) = \Phi(1.79) + 1 - \Phi(7.16) = 0.9632 + 1 - 1.0000 = 0.9632$$

(f) What is the probability of detecting the shift in part (e) by at least the third sample after the shift occurs.

$$\Pr\{\text{detect by 3rd sample}\} = 1 - \Pr\{\text{not detect by 3rd sample}\}$$

$$= 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9632)^3 = 0.99995$$

**6.59.**

Control charts for $\bar{x}$ and $s$ have been maintained on a process and have exhibited statistical control. The sample size is $n$ = 6. The control chart parameters are as follows:

| $\bar{x}$ Chart | $s$ Chart |
|---|---|
| UCL = 708.20 | UCL = 3.420 |
| Center line = 706.00 | Center line = 1.738 |
| LCL = 703.80 | LCL = 0.052 |

(a) Estimate the mean and standard deviation of the process.

$$\hat{\mu} = \bar{\bar{x}} = 706.00; \quad \hat{\sigma}_x = \bar{S} / c_4 = 1.738 / 0.9515 = 1.827$$

(b) Estimate the natural tolerance limits for the process.

$$\text{UNTL} = \bar{\bar{x}} + 3\hat{\sigma}_x = 706 + 3(1.827) = 711.48$$
$$\text{LNTL} = 706 - 3(1.827) = 700.52$$

(c) Assume that the process output is well modeled by a normal distribution. If specifications are 703 and 709, estimate the fraction nonconforming.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) = \Phi\left(\frac{703 - 706}{1.827}\right) + 1 - \Phi\left(\frac{709 - 706}{1.827}\right)$$
$$= \Phi(-1.642) + 1 - \Phi(1.642) = 0.0503 + 1 - 0.9497 = 0.1006$$

(d) Suppose the process mean shifts to 702.00 while the standard deviation remains constant. What is the probability of an out-of-control signal occurring on the first sample following the shift?

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\}$$
$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{703.8 - 702}{1.827/\sqrt{6}}\right) + 1 - \Phi\left(\frac{708.2 - 702}{1.827/\sqrt{6}}\right)$$
$$= \Phi(2.41) + 1 - \Phi(8.31) = 0.9920 + 1 - 1.0000 = 0.9920$$

(e) For the shift in part (d), what is the probability of detecting the shift by at least the third subsequent sample?

$$\Pr\{\text{detect by 3rd sample}\} = 1 - \Pr\{\text{not detect by 3rd sample}\}$$
$$= 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9920)^3 = 1.0000$$

**6.60.**

The following $\bar{x}$ and $s$ charts based on $n$ = 4 have shown statistical control:

| $\bar{x}$ Chart | $s$ Chart |
|---|---|
| UCL = 710 | UCL = 18.08 |
| Center line = 700 | Center line = 7.979 |
| LCL = 690 | LCL = 0 |

(a) Estimate the process parameters $\mu$ and $\sigma$.

$$\hat{\mu} = \bar{\bar{x}} = 700; \quad \hat{\sigma}_x = \bar{S} / c_4 = 7.979 / 0.9213 = 8.661$$

(b) If the specifications are at 705 ± 15, and the process output is normally distributed, estimate the fraction nonconforming.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) = \Phi\left(\frac{690 - 700}{8.661}\right) + 1 - \Phi\left(\frac{720 - 700}{8.661}\right)$$
$$= \Phi(-1.15) + 1 - \Phi(2.31) = 0.1251 + 1 - 0.9896 = 0.1355$$

(c) For the $\bar{x}$ chart, find the probability of a type I error, assuming $\sigma$ is constant.

$$\alpha = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{690 - 700}{8.661/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 700}{8.661/\sqrt{4}}\right)$$
$$= \Phi(-2.31) + 1 - \Phi(2.31) = 0.0104 + 1 - 0.9896 = 0.0208$$

(d) Suppose the process mean shifts to 693 and the standard deviation simultaneously shifts to 12. Find the probability of detecting this shift on the $\bar{x}$ chart on the first subsequent sample.

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\}$$
$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x},\text{new}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x},\text{new}}}\right) = \Phi\left(\frac{690 - 693}{12/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 693}{12/\sqrt{4}}\right)$$
$$= \Phi(-0.5) + 1 - \Phi(2.83) = 0.3085 + 1 - 0.9977 = 0.3108$$

(e) For the shift of part (d), find the average run length.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.3108} = 3.22$$

**6.61.**

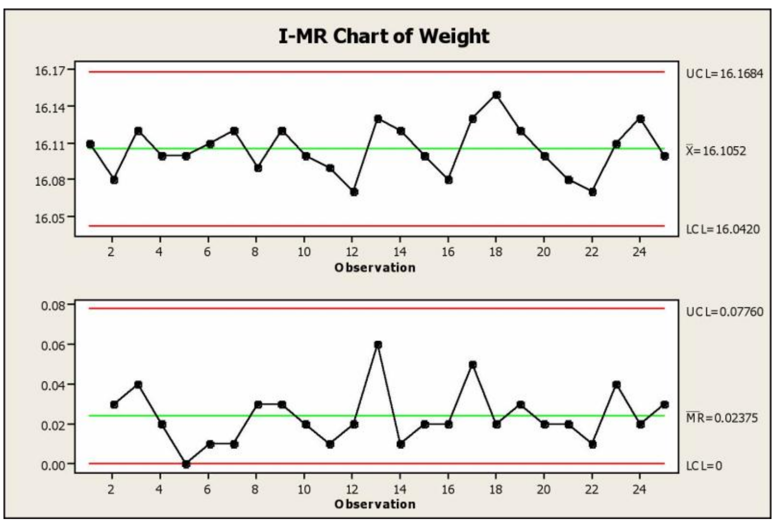
One-pound coffee cans are filled by a machine, sealed, and then weighted automatically. After adjusting for the weight of the can, any package that weighs less than 16 oz is cut out of the conveyor. The weights of 25 successive cans are shown in Table 6E.20. Set up a moving range control chart and a control chart for individuals. Estimate the mean and standard deviation of the amount of coffee packed in each can. Is it reasonable to assume that can weight is normally distributed? If the process remains in control at this level, what percentage of cans will be underfilled?

■ **TABLE 6E.20**

**Can Weight Data for Exercise 6.61**

| Can Number | Weight | Can Number | Weight |
|---|---|---|---|
| 1 | 16.11 | 14 | 16.12 |
| 2 | 16.08 | 15 | 16.10 |
| 3 | 16.12 | 16 | 16.08 |
| 4 | 16.10 | 17 | 16.13 |
| 5 | 16.10 | 18 | 16.15 |
| 6 | 16.11 | 19 | 16.12 |
| 7 | 16.12 | 20 | 16.10 |
| 8 | 16.09 | 21 | 16.08 |
| 9 | 16.12 | 22 | 16.07 |
| 10 | 16.10 | 23 | 16.11 |
| 11 | 16.09 | 24 | 16.13 |
| 12 | 16.07 | 25 | 16.10 |
| 13 | 16.13 | | |

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**

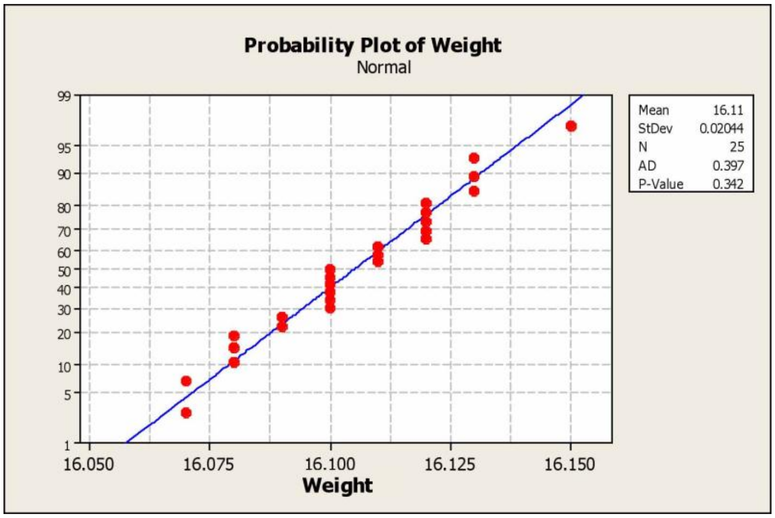



There may be a "sawtooth" pattern developing on the Individuals chart.

$\bar{\bar{x}} = 16.1052$; $\hat{\sigma}_x = 0.021055$; $\overline{MR2} = 0.02375$

**6.61. continued**

**MTB > Stat > Basic Statistics > Normality Test**




Visual examination of the normal probability indicates that the assumption of normally distributed coffee can weights is valid.

$$\begin{aligned}\%\text{underfilled} &= 100\% \times \Pr\{x < 16 \text{ oz}\} \\ &= 100\% \times \Phi\left(\frac{16 - 16.1052}{0.021055}\right) = 100\% \times \Phi(-4.9964) = 0.00003\%\end{aligned}$$

**6.62.**

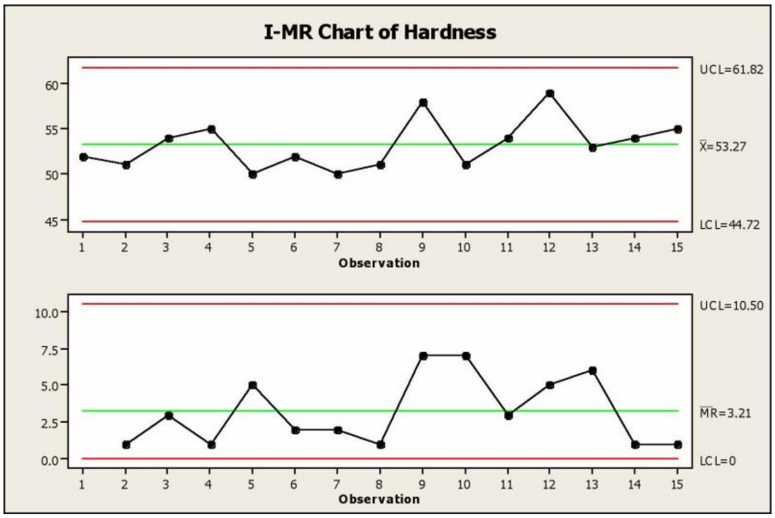
Fifteen successive heats of a steel alloy are tested for hardness. The resulting data are shown in Table 6E.21. Set up a control chart for the moving range and a control chart for individual hardness measurements. Is it reasonable to assume that hardness is normally distributed?

**TABLE 6E.21**

Hardness Data for Exercise 6.62

| Heat | Hardness (coded) | Heat | Hardness (coded) |
|---|---|---|---|
| 1 | 52 | 9 | 58 |
| 2 | 51 | 10 | 51 |
| 3 | 54 | 11 | 54 |
| 4 | 55 | 12 | 59 |
| 5 | 50 | 13 | 53 |
| 6 | 52 | 14 | 54 |
| 7 | 50 | 15 | 55 |
| 8 | 51 | | |

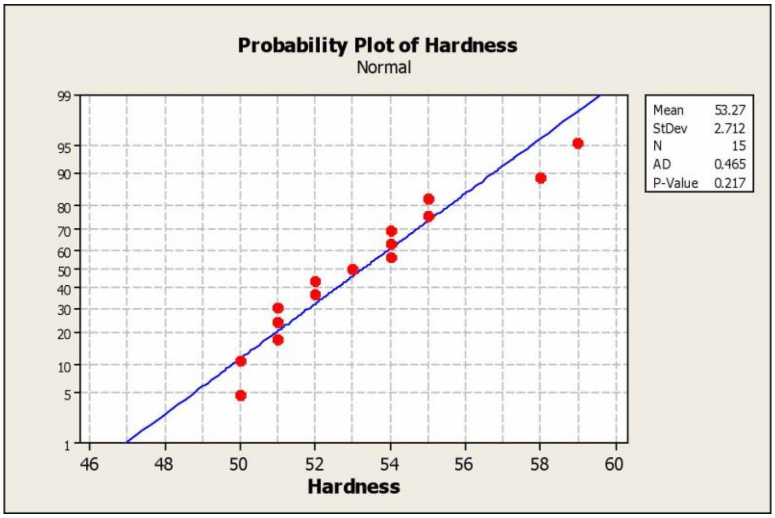
**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




$\bar{\bar{x}} = 53.2667$; $\hat{\sigma}_x = 2.84954$; $\overline{MR2} = 3.21429$

**6.62. continued**

**MTB > Stat > Basic Statistics > Normality Test**




Although the observations at the tails are not very close to the straight line, the *p*-value is greater than 0.05, indicating that it may be reasonable to assume that hardness is normally distributed.

**6.63.**

The viscosity of a polymer is measured hourly. Measurements for the last 20 hours are shown in table 6E.22.

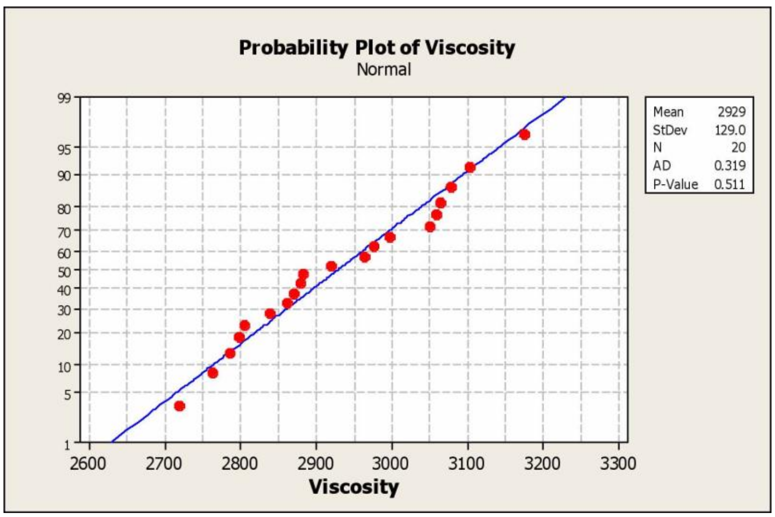
**TABLE 6E.22**

Viscosity Data for Exercise 6.63

| Test | Viscosity | Test | Viscosity |
|---|---|---|---|
| 1 | 2838 | 11 | 3174 |
| 2 | 2785 | 12 | 3102 |
| 3 | 3058 | 13 | 2762 |
| 4 | 3064 | 14 | 2975 |
| 5 | 2996 | 15 | 2719 |
| 6 | 2882 | 16 | 2861 |
| 7 | 2878 | 17 | 2797 |
| 8 | 2920 | 18 | 3078 |
| 9 | 3050 | 19 | 2964 |
| 10 | 2870 | 20 | 2805 |

(a) Does viscosity follow a normal distribution?

**MTB > Stat > Basic Statistics > Normality Test**

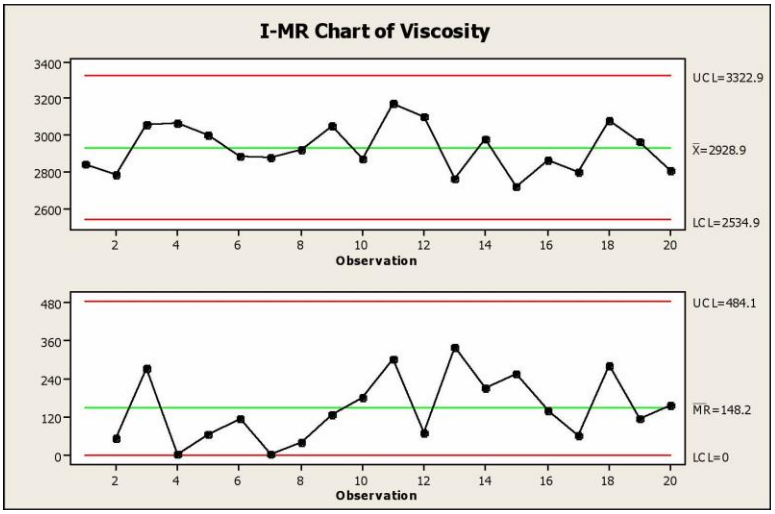



Viscosity measurements do appear to follow a normal distribution.

**6.63. continued**

(b) Set up a control chart on viscosity and a moving range chart. Does the process exhibit statistical control?

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




The process appears to be in statistical control, with no out-of-control points, runs, trends, or other patterns.

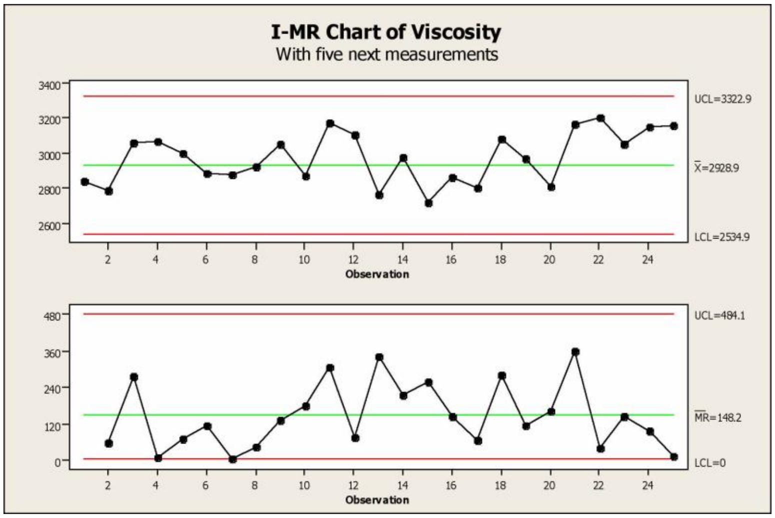
(c) Estimate the process mean and standard deviation.

$\hat{\mu} = \bar{\bar{x}} = 2928.9$; $\hat{\sigma}_x = 131.346$; $\overline{\text{MR2}} = 148.158$

**6.64.**

**Continuation of Exercise 6.63.** The next five measurements on viscosity are 3,163; 3,199; 3,054; 3,147; and 3,156. Do these measurements indicate that the process is in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




All points are inside the control limits. However all of the new points on the *I* chart are above the center line, indicating that a shift in the mean may have occurred.

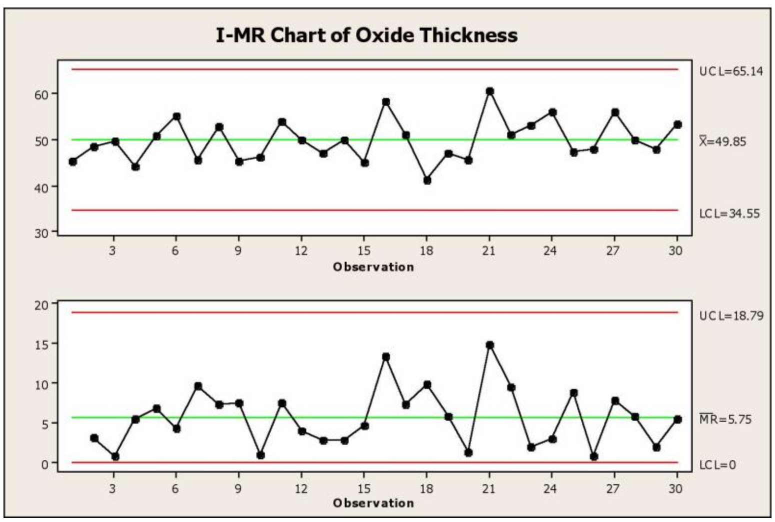
**6.65.**

(a) Thirty observations on the oxide thickness of individual silicon wafers are shown in table 6E.23. Use these data to set up a control chart on oxide thickness and a moving range chart. Does the process exhibit statistical control? Does oxide thickness follow a normal distribution?

**TABLE 6E.23**
**Data. for Exercise 6.65**

| Wafer | Oxide Thickness | Wafer | Oxide Thickness |
|---|---|---|---|
| 1 | 45.4 | 16 | 58.4 |
| 2 | 48.6 | 17 | 51.0 |
| 3 | 49.5 | 18 | 41.2 |
| 4 | 44.0 | 19 | 47.1 |
| 5 | 50.9 | 20 | 45.7 |
| 6 | 55.2 | 21 | 60.6 |
| 7 | 45.5 | 22 | 51.0 |
| 8 | 52.8 | 23 | 53.0 |
| 9 | 45.3 | 24 | 56.0 |
| 10 | 46.3 | 25 | 47.2 |
| 11 | 53.9 | 26 | 48.0 |
| 12 | 49.8 | 27 | 55.9 |
| 13 | 46.9 | 28 | 50.0 |
| 14 | 49.8 | 29 | 47.9 |
| 15 | 45.1 | 30 | 53.4 |

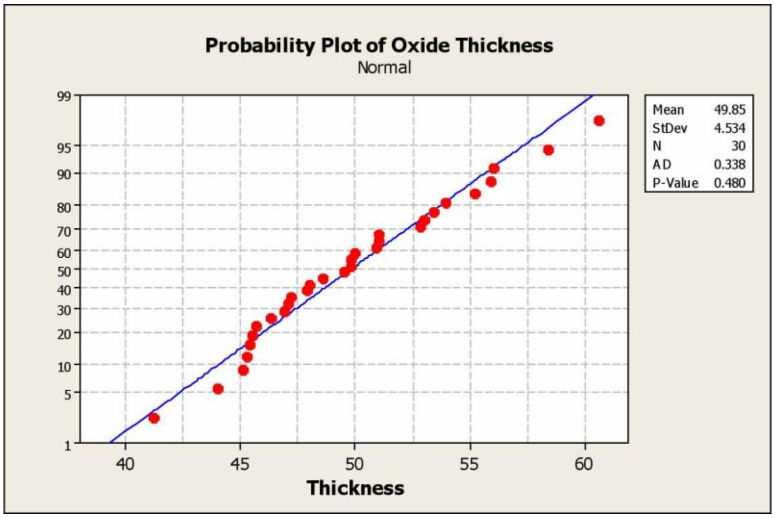
**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




The process is in statistical control.

**6.65.(a) continued**

**MTB > Stat > Basic Statistics > Normality Test**




The normality assumption is reasonable.

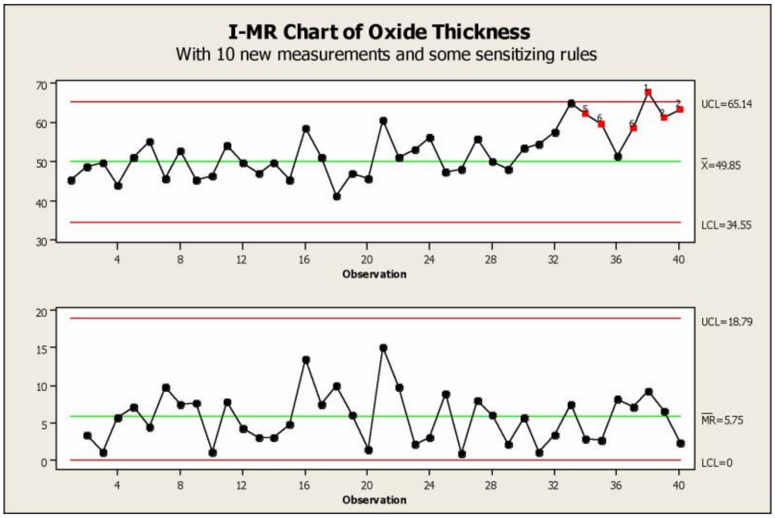
(b) Following the establishment of the control charts in part (a), 10 new wafers were observed. The oxide thickness measurements are as follows:

| Wafer | Oxide Thickness | Wafer | Oxide Thickness |
|---|---|---|---|
| 1 | 54.3 | 6 | 51.5 |
| 2 | 57.5 | 7 | 58.4 |
| 3 | 64.8 | 8 | 67.5 |
| 4 | 62.1 | 9 | 61.1 |
| 5 | 59.6 | 10 | 63.3 |

Plot these observations against the control limits determined in part (a). Is the process in control?

**6.65. continued**

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




## Test Results for I Chart of Ex6.57bTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

We have turned on some of the sensitizing rules in Minitab to illustrate their use. There is a run above the centerline, several 4 of 5 beyond 1 sigma, and several 2 of 3 beyond 2 sigma on the *x* chart. However, even without use of the sensitizing rules, it is clear that the process is out of control during this period of operation.

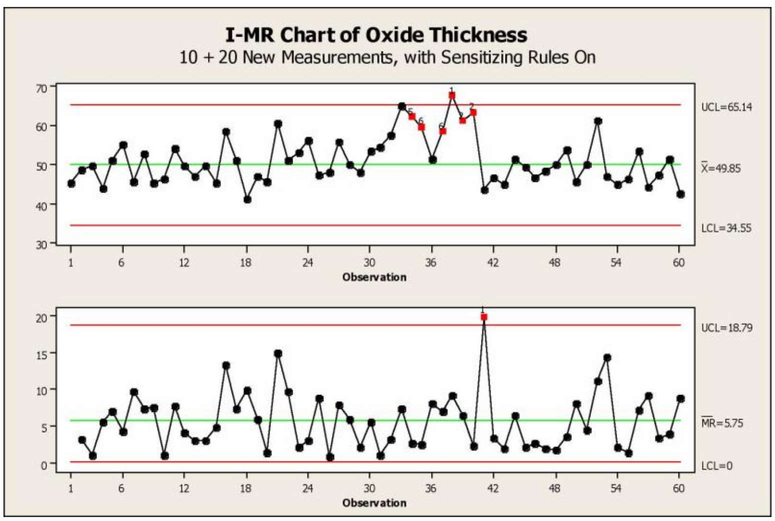
**6.65. continued**

(c) Suppose the assignable cause responsible for the out-of-control signal in part (b) is discovered and removed from the process. Twenty additional wafers are subsequently sampled. Plot the oxide thickness against the part (a) control limits. What conclusions can you draw? The new data are shown in Table 6E.25.

**TABLE 6E.25**
Additional Data for Exercise 6.65, part (c)

| Wafer | Oxide Thickness | Wafer | Oxide Thickness |
|---|---|---|---|
| 1 | 43.4 | 11 | 50.0 |
| 2 | 46.7 | 12 | 61.2 |
| 3 | 44.8 | 13 | 46.9 |
| 4 | 51.3 | 14 | 44.9 |
| 5 | 49.2 | 15 | 46.2 |
| 6 | 46.5 | 16 | 53.3 |
| 7 | 48.4 | 17 | 44.1 |
| 8 | 50.1 | 18 | 47.4 |
| 9 | 53.7 | 19 | 51.3 |
| 10 | 45.6 | 20 | 42.5 |

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




The process has been returned to a state of statistical control.

**6.66.** ☺

The waiting time for treatment in a “minute-clinic” located in a drugstore is monitored using control charts for individuals and the moving range. Table 6E.24 contains 30 successive measurements on waiting time.

■ **TABLE 6E.24**
**Clinic Waiting Time for Exercise 6.66**

| Observation | Waiting Time | Observation | Waiting Time | Observation | Waiting Time |
|---|---|---|---|---|---|
| 1 | 2.49 | 11 | 1.34 | 21 | 1.14 |
| 2 | 3.39 | 12 | 0.50 | 22 | 2.66 |
| 3 | 7.41 | 13 | 4.35 | 23 | 4.67 |
| 4 | 2.88 | 14 | 1.67 | 24 | 1.54 |
| 5 | 0.76 | 15 | 1.63 | 25 | 5.06 |
| 6 | 1.32 | 16 | 4.88 | 26 | 3.40 |
| 7 | 7.05 | 17 | 15.19 | 27 | 1.39 |
| 8 | 1.37 | 18 | 0.67 | 28 | 1.11 |
| 9 | 6.17 | 18 | 4.14 | 29 | 6.92 |
| 10 | 5.12 | 20 | 2.16 | 30 | 36.99 |

(a) Set up individual and moving range control charts using this data.

$$\hat{\mu} = \overline{x} = 4.65; \quad \overline{\text{MR2}} = 4.32$$

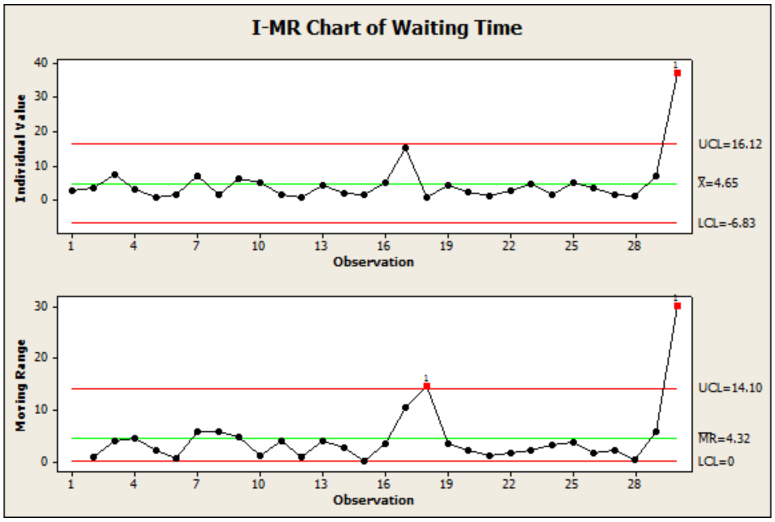
$$\text{UCL}_x = \overline{x} + 3\frac{\overline{\text{MR2}}}{d_2} = 4.65 + 3\left(\frac{4.32}{1.128}\right) = 16.14; \quad \text{LCL}_x = \overline{x} - 3\frac{\overline{\text{MR2}}}{d_2} = 4.65 - 3\left(\frac{4.32}{1.128}\right) = -6.84$$

$$\text{UCL}_{\text{MR}} = D_4\overline{\text{MR2}} = 3.267(4.32) = 14.11; \quad \text{LCL}_{\text{MR}} = D_3\overline{\text{MR2}} = 0(4.32) = 0$$

**6.66. continued**

(b) Plot these observations on the charts constructed in part (a). Interpret the results. Does the process seem to be in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




**Test Results for MR Chart of Ex6-66Wait**

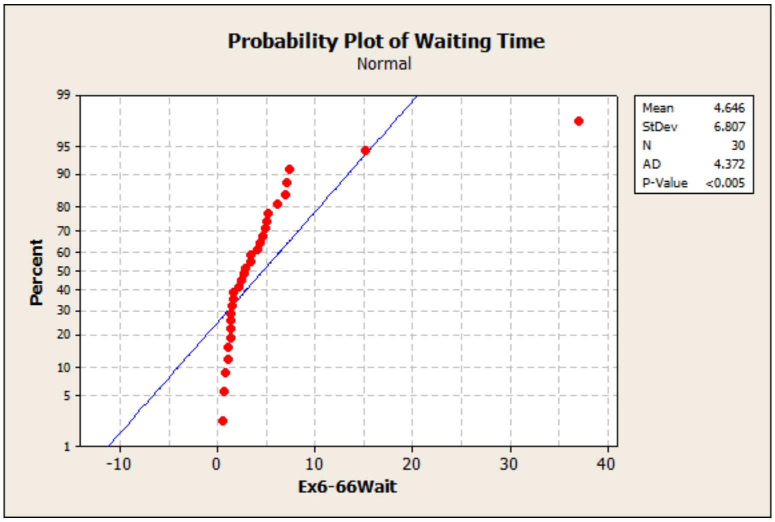
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18, 30
```

No, the process does not seem to be in statistical control, with out-of-control signals at observations 18 and 30.

**6.66. continued**

(c) Plot the waiting time data on a normal probability plot. Is it reasonable to assume normality for these data? Wouldn't a variable like waiting time often tend to have a distribution with a long tail (skewed) to the right? Why?

**MTB > Stat > Basic Statistics > Normality Test**



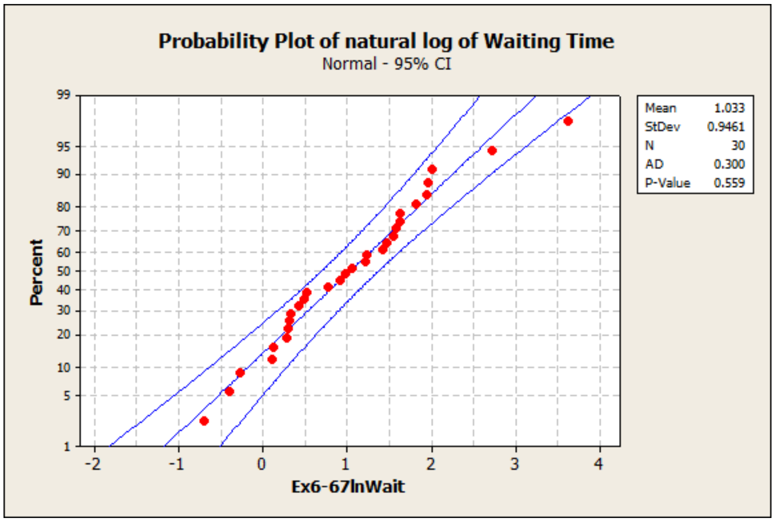

The assumption of normality for waiting time data is not reasonable; the plotted points do not fall along a straight line.

Variables like waiting time often tend to have long tails for two reasons: (1) there is a boundary on the lowest value that can be experienced (perhaps one second if that is the resolution of the measuring device); and (2) the nature of service processes can occasionally result in lengthy waiting times as customers with particularly complicated transactions are served.

**6.67.** ☺

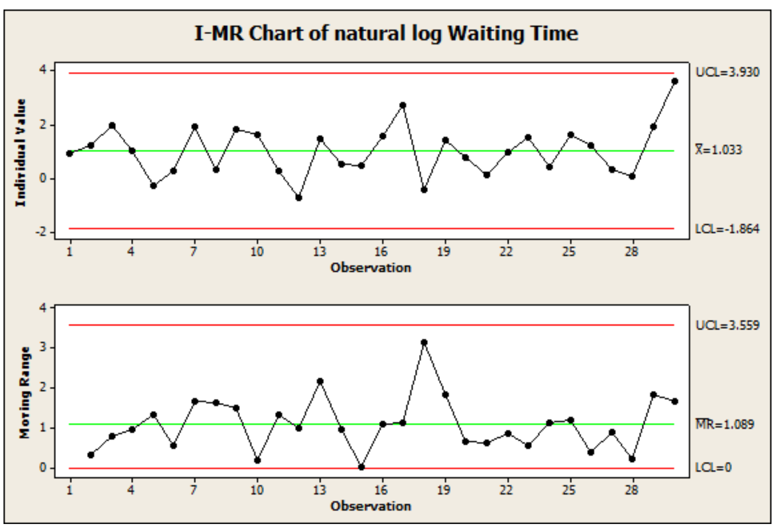
**Continuation of Exercise 6.66.** The waiting time data in Exercise 6.66 may not be normally distributed. Transform these data using a natural log transformation. Plot the transformed data on a normal probability plot and discuss your findings. Set up individual and moving range control charts using the transformed data. Plot the natural log of the waiting time data on these control charts. Compare your results with those from Exercise 6.66.

**MTB > Stat > Basic Statistics > Normality Test**




Applying the natural log transformation to the waiting time data has resulted in data that is approximately normally distributed.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




The process appears to be in statistical control, with no trends, patterns or out of control points.

**6.68.**

Thirty observations on concentration (in g/l) of the active ingredient in a liquid cleaner produced in a continuous chemical process are shown in Table 6E.26.

**TABLE 6E.26**
Data for Exercise 6.68

| Observation | Concentration | Observation | Concentration |
|---|---|---|---|
| 1 | 60.4 | 16 | 99.9 |
| 2 | 69.5 | 17 | 59.3 |
| 3 | 78.4 | 18 | 60.0 |
| 4 | 72.8 | 19 | 74.7 |
| 5 | 78.2 | 20 | 75.8 |
| 6 | 78.7 | 21 | 76.6 |
| 7 | 56.9 | 22 | 68.4 |
| 8 | 78.4 | 23 | 83.1 |
| 9 | 79.6 | 24 | 61.1 |
| 10 | 100.8 | 25 | 54.9 |
| 11 | 99.6 | 26 | 69.1 |
| 12 | 64.9 | 27 | 67.5 |
| 13 | 75.5 | 28 | 69.2 |
| 14 | 70.4 | 29 | 87.2 |
| 15 | 68.1 | 30 | 73.0 |

(a) A normal probability plot of the concentration data is shown in Figure 6.29. The straight line was fit by eye to pass approximately through the 20th and 80th percentiles. Does the normality assumption seem reasonable here?

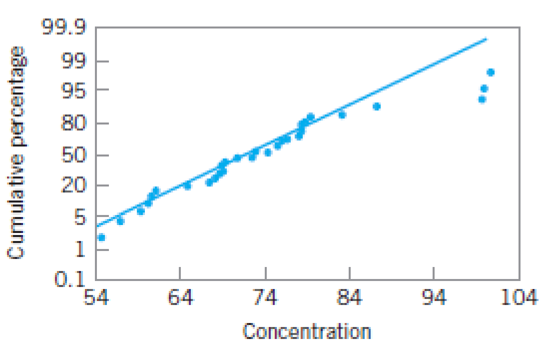


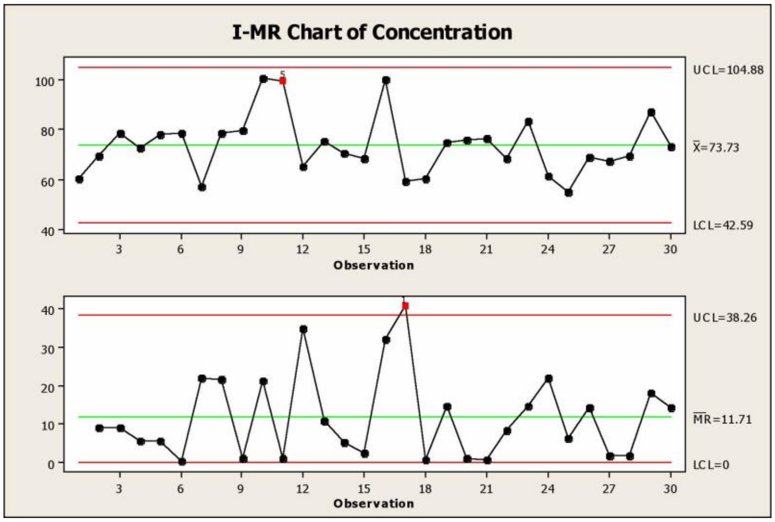

**FIGURE 6.29** Normal Probability Plot of the Concentration Data for Exercise 6.68

The normality assumption is a little bothersome for the concentration data, in particular due to the curve of the larger values and three distant values.

**6.68. continued**

(b) Set up individuals and moving range control charts for the concentration data. Interpret the charts.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




### Test Results for I Chart of Ex6.58C

```
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  11
```

### Test Results for MR Chart of Ex6.58C

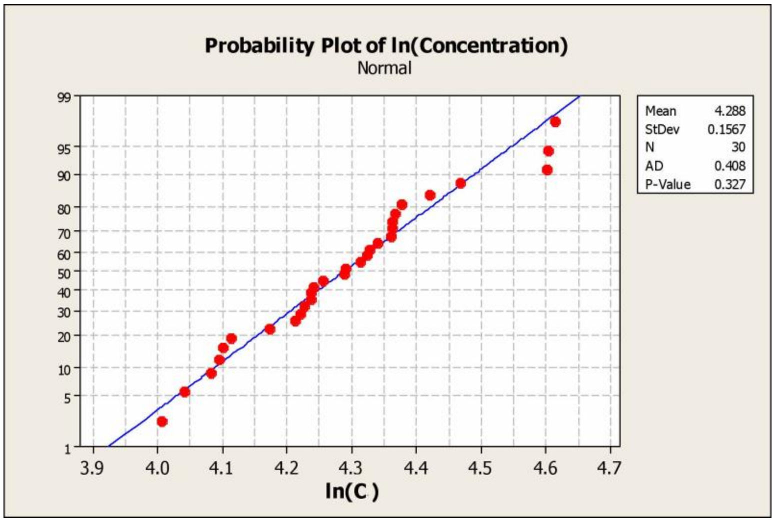
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  17
```

The process is not in control, with two Western Electric rule violations.

**6.68. continued**

(c) Construct a normal probability plot for the natural log of concentration. Is the transformed variable normally distributed?

**MTB > Stat > Basic Statistics > Normality Test**

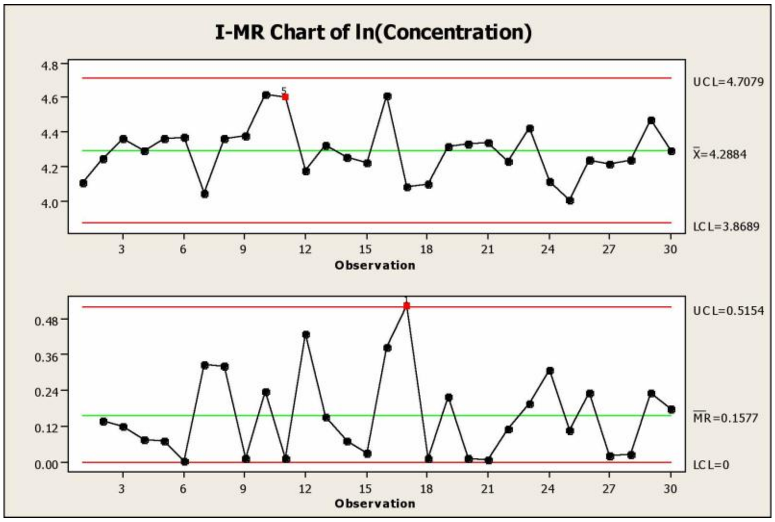



The normality assumption is still troubling for the natural log of concentration, again due to the curve of the larger values and three distant values.

**6.68. continued**

(d) Repeat part (b), using the natural log of concentration as the cahrted variable. Comment on any differences in the charts you note in comparison to those constructed in part (b).

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




### Test Results for I Chart of Ex6.58lnC

```
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  11
```

### Test Results for MR Chart of Ex6.58lnC

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  17
```

The process is still not in control, with the same to Western Electric Rules violations. There does not appear to be much difference between the two control charts (actual and natural log).

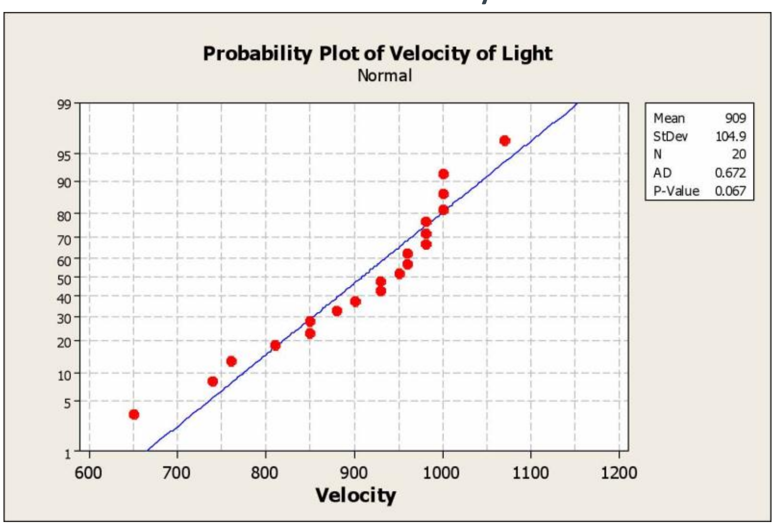
**6.69.**

In 1879, A.A. Michelson measured the velocity of light in air using a modification of a method proposed by the French physicist Foucault. Twenty of these measurements are in table 6E.27 (the value reported is in kilometers per second and has 299,000 subtracted from it). Use these data to set up individuals and moving range control charts. Is there some evidence that the measurements of the velocity of light are normally distributed? O the measurements exhibit statistical control? Revise the control limits if necessary.

**TABLE 6E.27**
Velocity of Light Data for Exercise 6.69

| Measurement | Velocity | Measurement | Velocity |
|---|---|---|---|
| 1 | 850 | 11 | 850 |
| 2 | 1000 | 12 | 810 |
| 3 | 740 | 13 | 950 |
| 4 | 980 | 14 | 1000 |
| 5 | 900 | 15 | 980 |
| 6 | 930 | 16 | 1000 |
| 7 | 1070 | 17 | 980 |
| 8 | 650 | 18 | 960 |
| 9 | 930 | 19 | 880 |
| 10 | 760 | 20 | 960 |

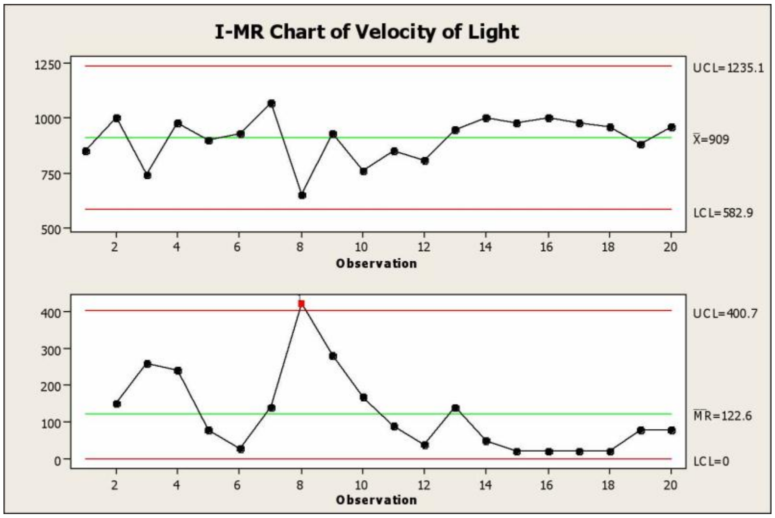
**MTB > Stat > Basic Statistics > Normality Test**




Velocity of light measurements are approximately normally distributed.

**6.69. continued**

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




**I-MR Chart of Ex6.59Vel**

**Test Results for MR Chart of Ex6.59Vel**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8
```

The out-of-control signal on the moving range chart indicates a significantly large difference between successive measurements (7 and 8). Since neither of these measurements seems unusual, use all data for control limits calculations.

There may also be an early indication of less variability in the later measurements. For now, consider the process to be in a state of statistical process control.

**6.70.**

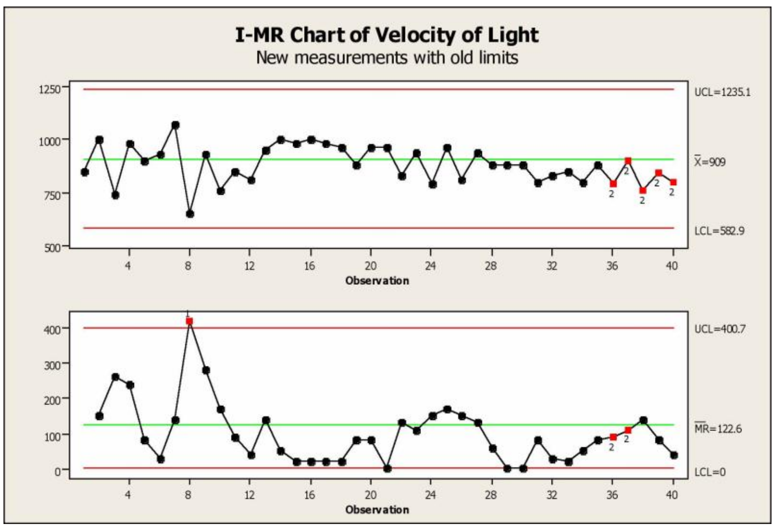
**Continuation of Exercise 6.69.** Michelson actually made 100 measurements on the velocity of light in five trials of 20 observations each. The second set of 20 measurements is shown in Table 6E.28.

**TABLE 6E.28**
Additional Velocity of Light Data for Exercise 6.70

| Measurement | Velocity | Measurement | Velocity |
|---|---|---|---|
| 21 | 960 | 31 | 800 |
| 22 | 830 | 32 | 830 |
| 23 | 940 | 33 | 850 |
| 24 | 790 | 34 | 800 |
| 25 | 960 | 35 | 880 |
| 26 | 810 | 36 | 790 |
| 27 | 940 | 37 | 900 |
| 28 | 880 | 38 | 760 |
| 29 | 880 | 39 | 840 |
| 30 | 880 | 40 | 800 |

(a) Plot these new measurements on the control charts constructed in Exercise 6.69. Are these new measurements in statistical control? Give a practical interpretation of the control charts.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**
**Select I-MR Options, Estimate to specify which subgroups to use in calculations**




### I-MR Chart of Ex6.60Vel

### Test Results for I Chart of Ex6.60Vel

```
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  36, 37, 38, 39, 40
```

### Test Results for MR Chart of Ex6.60Vel

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  36, 37
```

**6.70. continued**

The velocity of light in air is not changing, however the method of measuring is producing varying results—this is a chart of the measurement process. There is a distinct downward trend in measurements, meaning the method is producing gradually smaller measurements.

(b) Is there evidence that the variability in the measurements has decreased between trial 1 and trial 2?

Early measurements exhibit more variability than the later measurements, which is reflected in the number of observations below the centerline of the moving range chart.

**6.71.**

The uniformity of a silicon wafer following an etching process is determined by measuring the layer thickness at several locations and expressing uniformity as the range of the thicknesses. Table 6E.29 presents uniformity determinations for 30 consecutive wafers processed through the etching tool.

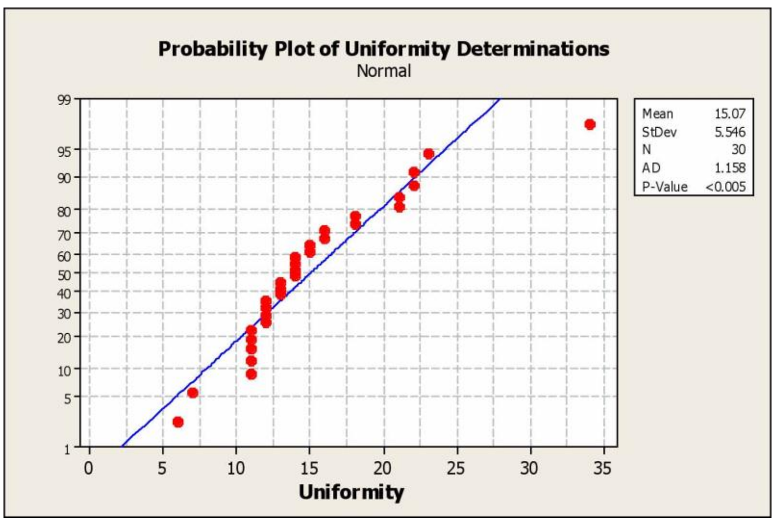
■ **TABLE 6E.29**

**Uniformity Data for Exercise 6.71**

| Wafer | Uniformity | Wafer | Uniformity |
|---|---|---|---|
| 1 | 11 | 16 | 15 |
| 2 | 16 | 17 | 16 |
| 3 | 22 | 18 | 12 |
| 4 | 14 | 19 | 11 |
| 5 | 34 | 20 | 18 |
| 6 | 22 | 21 | 14 |
| 7 | 13 | 22 | 13 |
| 8 | 11 | 23 | 18 |
| 9 | 6 | 24 | 12 |
| 10 | 11 | 25 | 13 |
| 11 | 11 | 26 | 12 |
| 12 | 23 | 27 | 15 |
| 13 | 14 | 28 | 21 |
| 14 | 12 | 29 | 21 |
| 15 | 7 | 30 | 14 |

(a) Is there evidence that uniformity is normally distributed? If not, find a suitable transformation for the data.

**MTB > Stat > Basic Statistics > Normality Test**

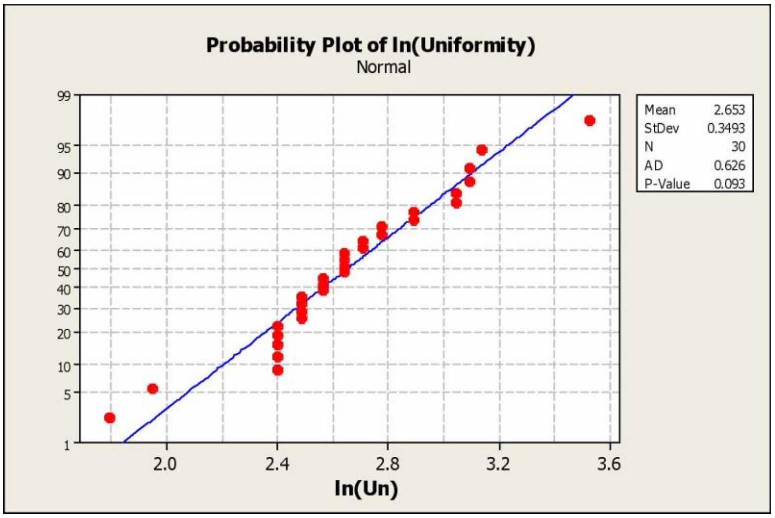



The data are not normally distributed, as evidenced by the "S"- shaped curve to the plot points on a normal probability plot, as well as the Anderson-Darling test p-value.

**6.71. continued**

The data are skewed right, so a compressive transform such as natural log or square-root may be appropriate.

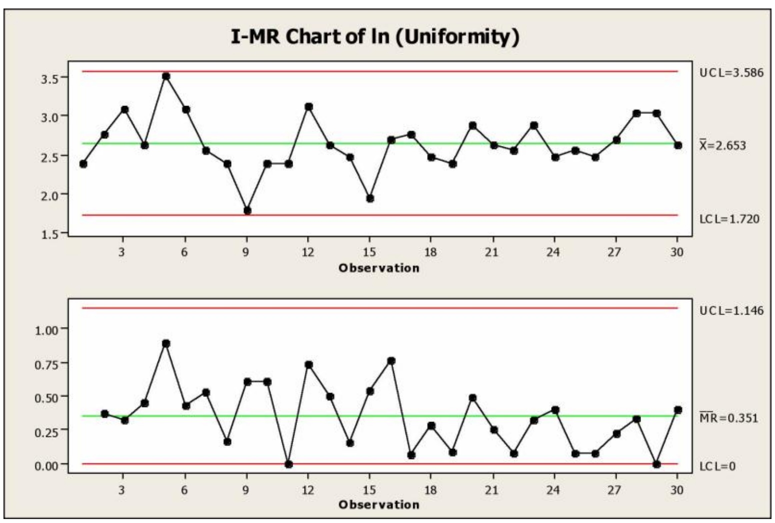
**MTB > Stat > Basic Statistics > Normality Test**




The distribution of the natural-log transformed uniformity measurements is approximately normally distributed.

(b) Construct a control chart for individuals and a moving range control chart for uniformity for the etching process. Is the process in statistical control?

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




The etching process appears to be in statistical control.

**6.72.**

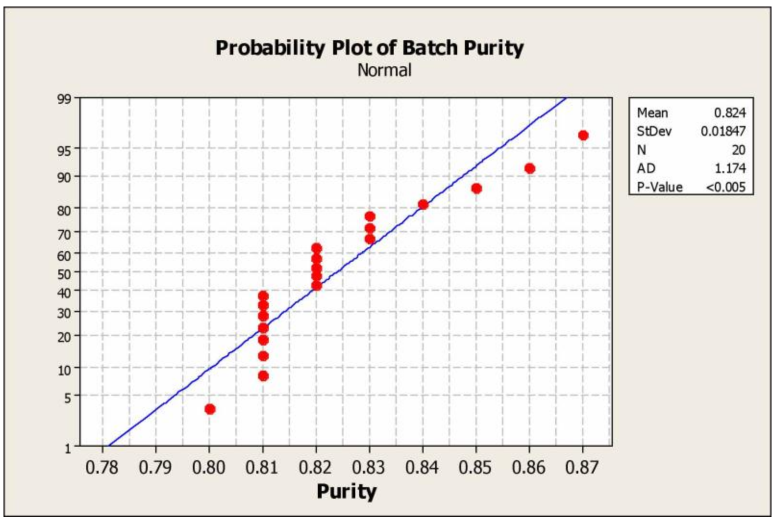
The purity of a chemical product is measured on each batch. Purity determination for 20 successive batches are shown in Table 6E.30.

**TABLE 6E.30**
Purity Data for Exercise 6.72

| Batch | Purity | Batch | Purity |
|---|---|---|---|
| 1 | 0.81 | 11 | 0.81 |
| 2 | 0.82 | 12 | 0.83 |
| 3 | 0.81 | 13 | 0.81 |
| 4 | 0.82 | 14 | 0.82 |
| 5 | 0.82 | 15 | 0.81 |
| 6 | 0.83 | 16 | 0.85 |
| 7 | 0.81 | 17 | 0.83 |
| 8 | 0.80 | 18 | 0.87 |
| 9 | 0.81 | 19 | 0.86 |
| 10 | 0.82 | 20 | 0.84 |

(a) Is purity normally distributed?

**MTB > Stat > Basic Statistics > Normality Test**



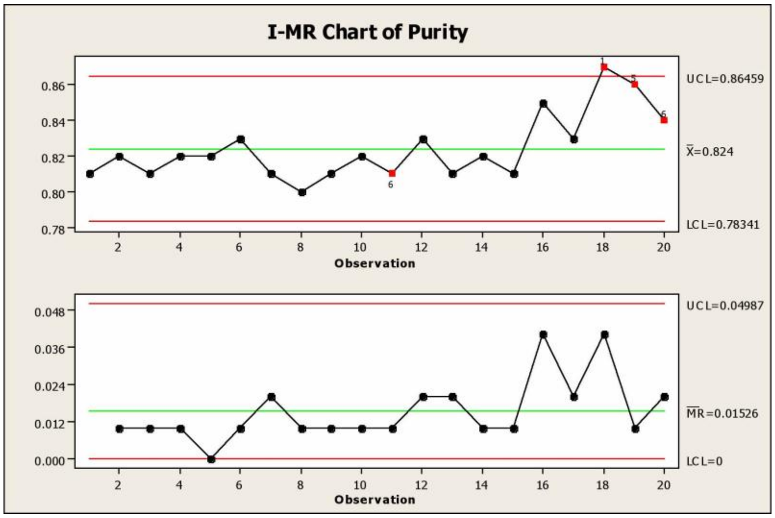

Purity is not normally distributed.

Note that transforming the purity data using natural logarithm, square root, and reciprocal transformations do not result in normally distributed data.

**6.72. continued**

(b) Is the process in statistical process control?

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




**Test Results for I Chart of Ex6.62Pur**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  19
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  11, 20
```

The process is not in statistical control.

(c) Estimate the process mean and standard deviation.

all data: $\hat{\mu} = 0.824$, $\hat{\sigma}_x = 0.0135$

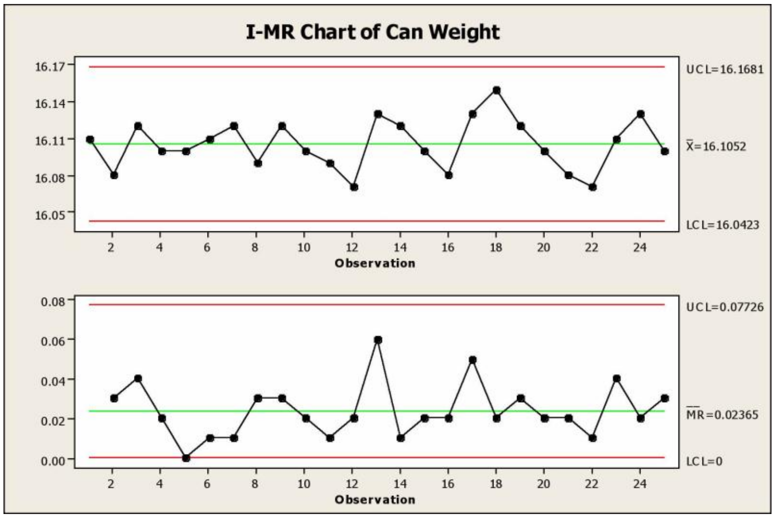
without sample 18: $\hat{\mu} = 0.8216$, $\hat{\sigma}_x = 0.0133$

**6.73.**

Reconsider the situation in Exercise 6.61. Construct an individuals control chart using the median of the span-two moving ranges to estimate variability. Compare this control chart to the one constructed in Exercise 6.61 and discuss.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**

**Select "Estimate" to change the method of estimating sigma**



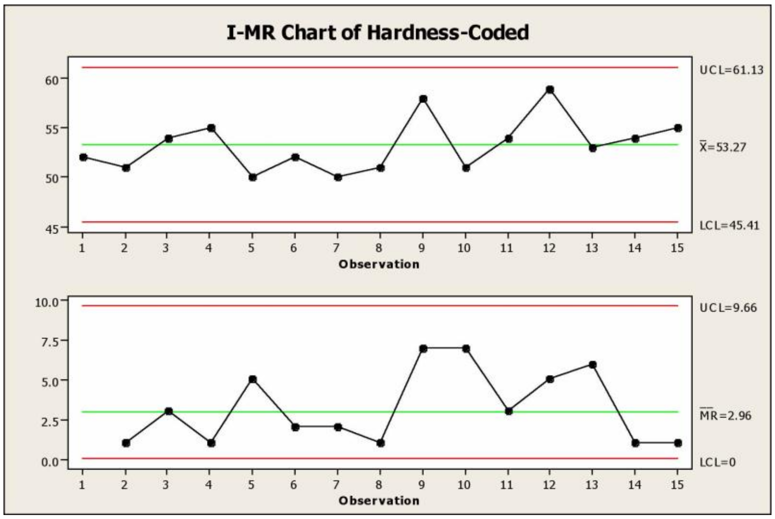

There is no difference between this chart and the one in Exercise 6.53; control limits for both are essentially the same.

**6.74.**

Reconsider the hardness measurements in Exercise 6.62. Construct an individuals control chart using the median of the span-two moving ranges to estimate variability. Compare this control chart to the one constructed in Exercise 6.62 and discuss.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**

**Select "Estimate" to change the method of estimating sigma**

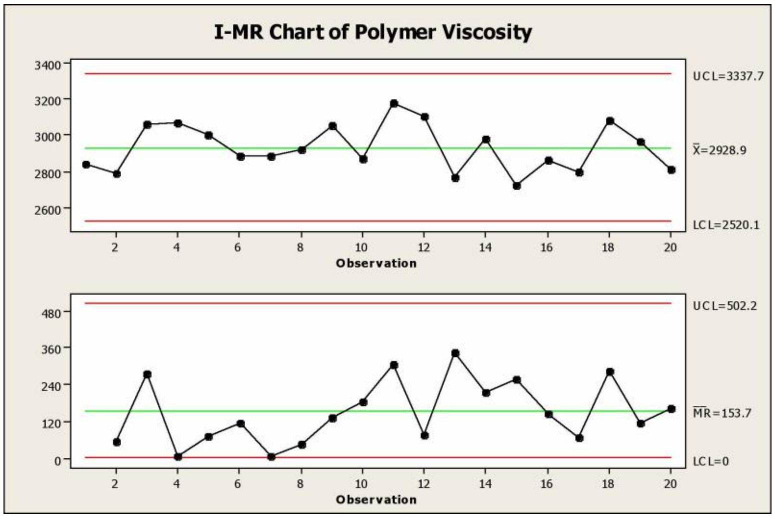



The median moving range method gives slightly tighter control limits for both the Individual and Moving Range charts, with no practical difference for this set of observations.

**6.75.**

Reconsider the polymer viscosity data in Exercise 6.63. Use the median of the span-two moving ranges to estimate σ and set up the individuals control chart. Compare this chart to the one originally constructed using the average moving range method to estimate σ.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**
**Select "Estimate" to change the method of estimating sigma**



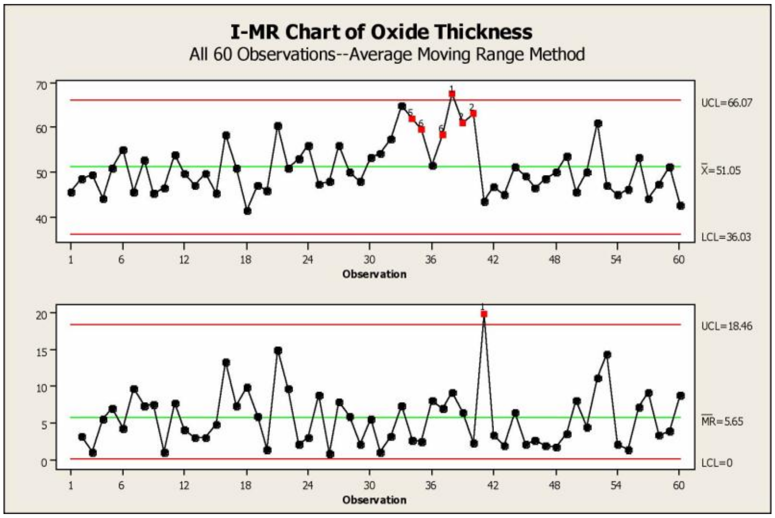

The median moving range method gives slightly wider control limits for both the Individual and Moving Range charts, with no practical meaning for this set of observations.

**6.76.**

**Continuation of Exercise 6.65.** Use all 60 observations on oxide thickness.

(a) Set up an individuals control chart with σ estimated by the average moving range method.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




**Test Results for I Chart of Ex6.57cTh**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

**Test Results for MR Chart of Ex6.57cTh**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

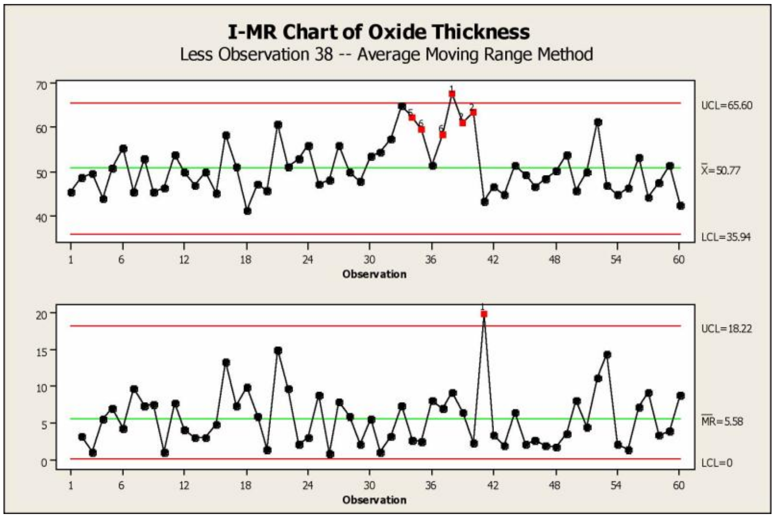
Recall that observations on the Moving Range chart are correlated with those on the Individuals chart—that is, the out-of-control signal on the MR chart for observation 41 is reflected by the shift between observations 40 and 41 on the Individuals chart.

Remove observation 38 and recalculate control limits.

**6.76. (a) continued**

Excluding observation 38 from calculations:

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




### Test Results for I Chart of Ex6.57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

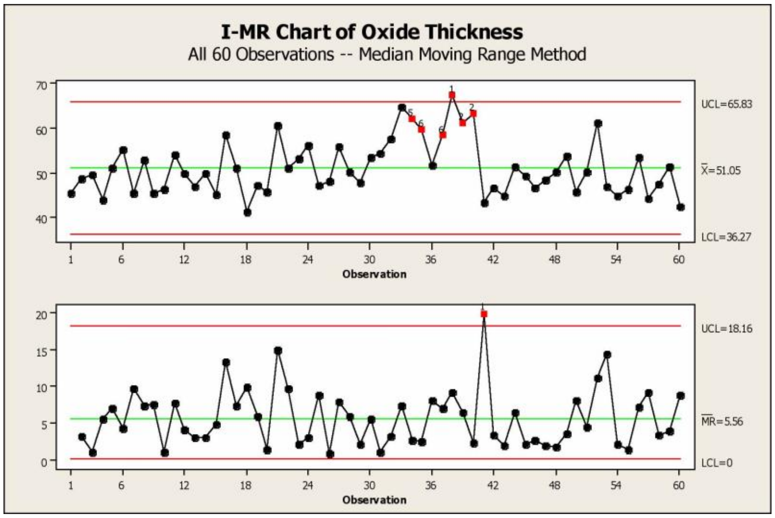
### Test Results for MR Chart of Ex6.57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

**6.76. continued**

(b) Set up an individuals control chart with σ estimated by the median moving range method.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




### Test Results for I Chart of Ex6.57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

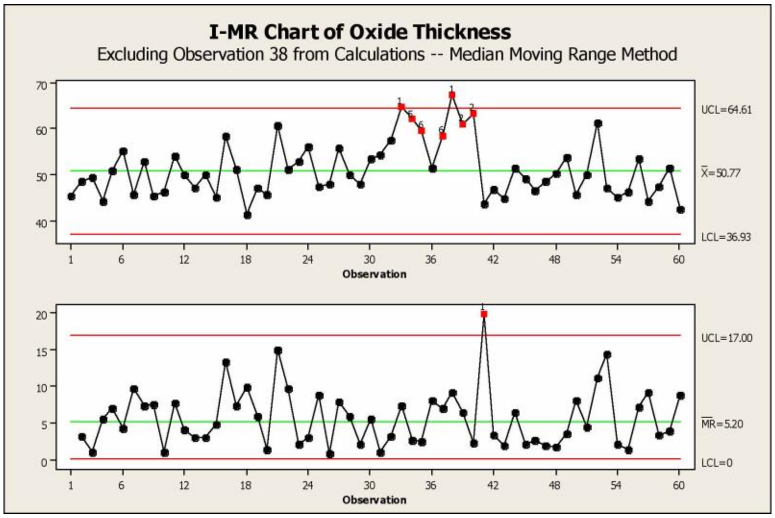
### Test Results for MR Chart of Ex6.57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

**6.76. (b) continued**

Excluding observation 38 from calculations:

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




### Test Results for I Chart of Ex6.57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  33, 38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

### Test Results for MR Chart of Ex6.57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

(c) Compare and discuss the two control charts.

The control limits estimated by the median moving range are tighter and detect the shift in process level at an earlier sample, 33.

**6.77.**

Consider the individuals measurement data shown inTable6E.31.

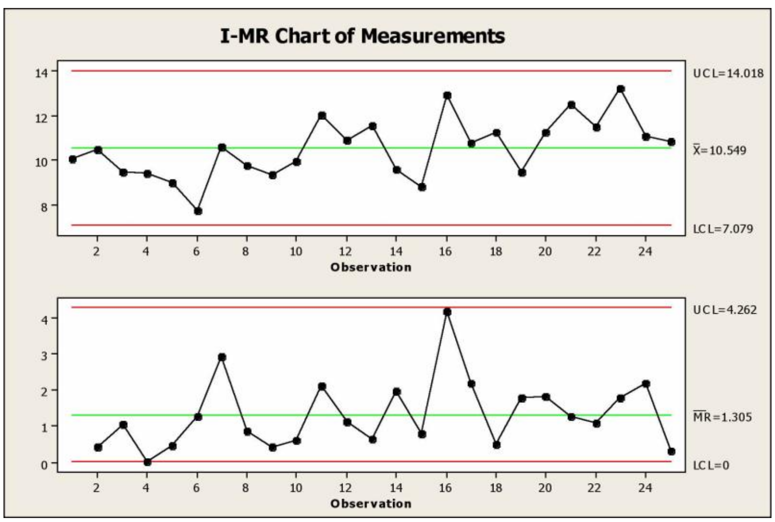
**TABLE 6E.31**

Data for Exercise 6.77

| Observation | $x$ | Observation | $x$ |
|---|---|---|---|
| 1 | 10.07 | 14 | 9.58 |
| 2 | 10.47 | 15 | 8.80 |
| 3 | 9.45 | 16 | 12.94 |
| 4 | 9.44 | 17 | 10.78 |
| 5 | 8.99 | 18 | 11.26 |
| 6 | 7.74 | 19 | 9.48 |
| 7 | 10.63 | 20 | 11.28 |
| 8 | 9.78 | 21 | 12.54 |
| 9 | 9.37 | 22 | 11.48 |
| 10 | 9.95 | 23 | 13.26 |
| 11 | 12.04 | 24 | 11.10 |
| 12 | 10.93 | 25 | 10.82 |
| 13 | 11.54 | | |

(a) Estimate σ using the average of the moving ranges of span two.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




$\hat{\sigma}_x = \overline{R} / d_2 = 1.305 / 1.128 = 1.157$

**6.77. continued**

(b) Estimate $\sigma$ using $s/c_4$.

**MTB > Stat > Basic Statistics > Descriptive Statistics**

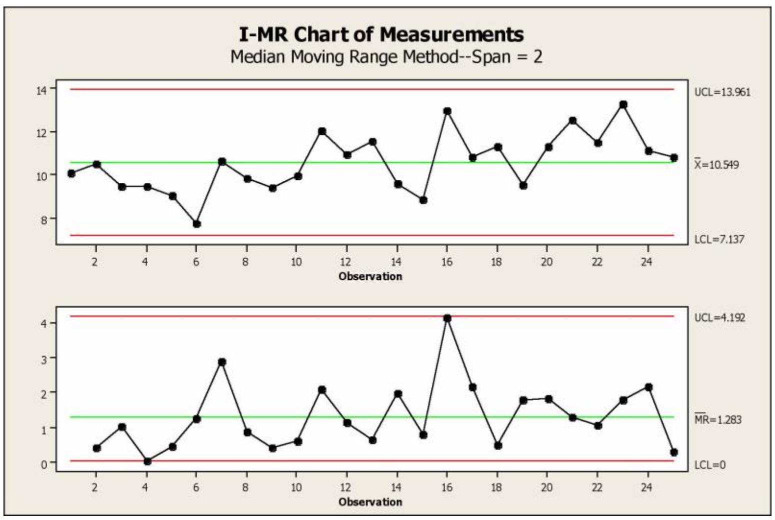
**Descriptive Statistics: Ex6.67Meas**

```
            Total
Variable    Count    Mean  StDev  Median
Ex6.67Meas     25  10.549  1.342  10.630
```

$\hat{\sigma}_x = S / c_4 = 1.342 / 0.7979 = 1.682$

(c) Estimate $\sigma$ using the median of the span-two moving ranges.

**MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR**




$\hat{\sigma}_x = \bar{R} / d_2 = 1.283 / 1.128 = 1.137$

**6.77. continued**

(d) Estimate σ using the average of the moving ranges of span 3, 4, ..., 20.

Average MR3 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 2.049 / 1.693 = 1.210$

Average MR4 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 2.598 / 2.059 = 1.262$

Average MR19 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 5.186 / 3.689 = 1.406$

Average MR20 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 5.36 / 3.735 = 1.435$

(e) Discuss the results you have obtained.

As the span of the moving range is increased, there are fewer observations to estimate the standard deviation, and the estimate becomes less reliable. For this example, $\sigma$ gets larger as the span increases. This tends to be true for unstable processes.

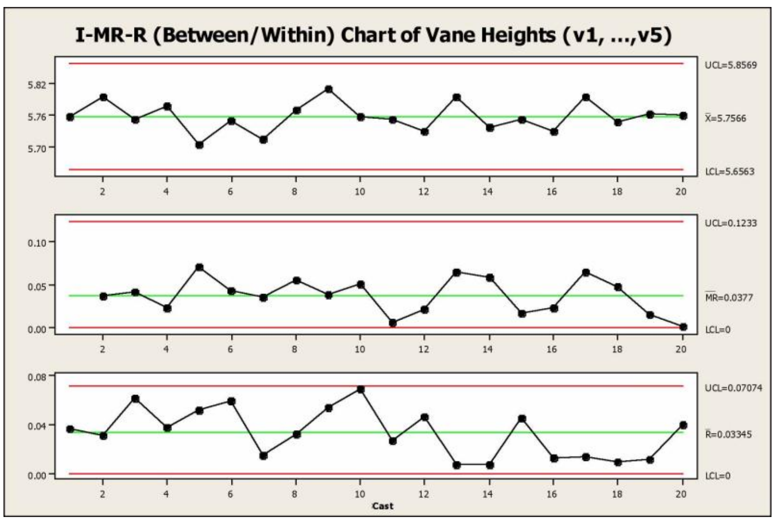
**6.78.**

The vane heights for 20 of the castings from Figure 6.25 are shown in table 6E.32. Construct the "between/within" control charts for these process data using a range chart to monitor the within-castings vane height. Compare these to the control charts shown in Figure 6.27.

**TABLE 6E.32**
Vane Heights for Exercise 6.78

| Casting | Vane 1 | Vane 2 | Vane 3 | Vane 4 | Vane 5 |
|---|---|---|---|---|---|
| 1 | 5.77799 | 5.74907 | 5.76672 | 5.74836 | 5.74122 |
| 2 | 5.79090 | 5.78043 | 5.79163 | 5.79393 | 5.81158 |
| 3 | 5.77314 | 5.71216 | 5.74810 | 5.77292 | 5.75591 |
| 4 | 5.77030 | 5.75903 | 5.77157 | 5.79687 | 5.78063 |
| 5 | 5.72047 | 5.68587 | 5.73302 | 5.70472 | 5.68116 |
| 6 | 5.77265 | 5.76426 | 5.74373 | 5.71338 | 5.74765 |
| 7 | 5.70581 | 5.70835 | 5.71866 | 5.71252 | 5.72089 |
| 8 | 5.76466 | 5.78766 | 5.76115 | 5.77523 | 5.75590 |
| 9 | 5.79397 | 5.83308 | 5.77902 | 5.81122 | 5.82335 |
| 10 | 5.78671 | 5.76411 | 5.75941 | 5.75619 | 5.71787 |
| 11 | 5.75352 | 5.74144 | 5.74109 | 5.76817 | 5.75019 |
| 12 | 5.72787 | 5.70716 | 5.75349 | 5.72389 | 5.73488 |
| 13 | 5.79707 | 5.79231 | 5.79022 | 5.79694 | 5.79805 |
| 14 | 5.73765 | 5.73615 | 5.73249 | 5.74006 | 5.73265 |
| 15 | 5.72477 | 5.76565 | 5.76963 | 5.74993 | 5.75196 |
| 16 | 5.73199 | 5.72926 | 5.72963 | 5.72259 | 5.73513 |
| 17 | 5.79166 | 5.79516 | 5.79903 | 5.78548 | 5.79826 |
| 18 | 5.74973 | 5.74863 | 5.73994 | 5.74405 | 5.74682 |
| 19 | 5.76449 | 5.75632 | 5.76197 | 5.76684 | 5.75474 |
| 20 | 5.75168 | 5.75579 | 5.73979 | 5.77963 | 5.76933 |

**MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)**
**Select "I-MR-R/S Options, Estimate" and choose R-bar method to estimate standard deviation**




**I-MR-R/S Standard Deviations of Ex6.68v1, ..., Ex6.68v5**

```
Standard Deviations
Between         0.0328230
Within          0.0143831
Between/Within  0.0358361
```

The Individuals and Moving Range charts for the subgroup means are identical. When compared to the *s* chart for all data, the *R* chart tells the same story—same data pattern and no out-of-control points. For this example, the control schemes are identical.

**6.79.**

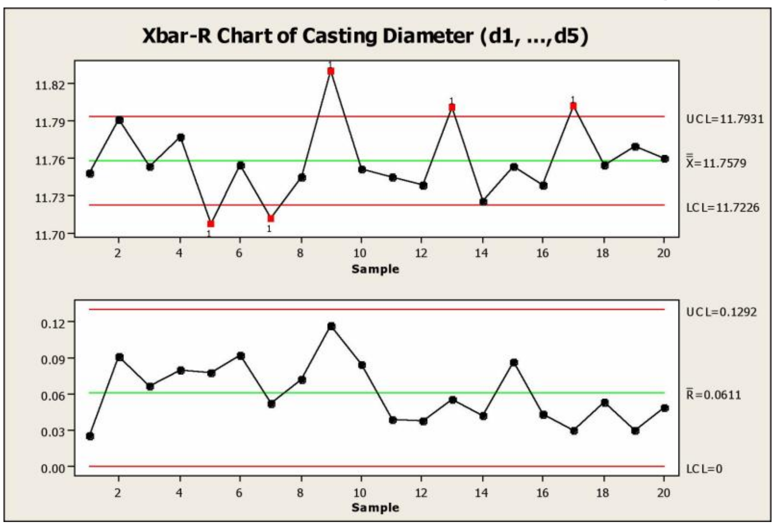
The diameter of the casting in Figure 6.25 is also an important quality characteristic. A coordinate measuring machine is used to measure the diameter of each casting at five different locations. Data for 20 casting are shown in the Table 6E.33.

**TABLE 6E.33**
Diameter Data for Exercise 6.79

| Casting | Diameter 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 11.7629 | 11.7403 | 11.7511 | 11.7474 | 11.7374 |
| 2 | 11.8122 | 11.7506 | 11.7787 | 11.7736 | 11.8412 |
| 3 | 11.7742 | 11.7114 | 11.7530 | 11.7532 | 11.7773 |
| 4 | 11.7833 | 11.7311 | 11.7777 | 11.8108 | 11.7804 |
| 5 | 11.7134 | 11.6870 | 11.7305 | 11.7419 | 11.6642 |
| 6 | 11.7925 | 11.7611 | 11.7588 | 11.7012 | 11.7611 |
| 7 | 11.6916 | 11.7205 | 11.6958 | 11.7440 | 11.7062 |
| 8 | 11.7109 | 11.7832 | 11.7496 | 11.7496 | 11.7318 |
| 9 | 11.7984 | 11.8887 | 11.7729 | 11.8485 | 11.8416 |
| 10 | 11.7914 | 11.7613 | 11.7356 | 11.7628 | 11.7070 |
| 11 | 11.7260 | 11.7329 | 11.7424 | 11.7645 | 11.7571 |
| 12 | 11.7202 | 11.7537 | 11.7328 | 11.7582 | 11.7265 |
| 13 | 11.8356 | 11.7971 | 11.8023 | 11.7802 | 11.7903 |
| 14 | 11.7069 | 11.7112 | 11.7492 | 11.7329 | 11.7289 |
| 15 | 11.7116 | 11.7978 | 11.7982 | 11.7429 | 11.7154 |
| 16 | 11.7165 | 11.7284 | 11.7571 | 11.7597 | 11.7317 |
| 17 | 11.8022 | 11.8127 | 11.7864 | 11.7917 | 11.8167 |
| 18 | 11.7775 | 11.7372 | 11.7241 | 11.7773 | 11.7543 |
| 19 | 11.7753 | 11.7870 | 11.7574 | 11.7620 | 11.7673 |
| 20 | 11.7572 | 11.7626 | 11.7523 | 11.7395 | 11.7884 |

(a) Set up $\bar{x}$ and $R$ charts for this process, assuming the measurements on each casting form a rational subgroup.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**




**Xbar-R Chart of Ex6.69d1, ..., Ex6.69d5**

**Test Results for Xbar Chart of Ex6.69d1, ..., Ex6.69d5**

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  5, 7, 9, 13, 17
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  7
```

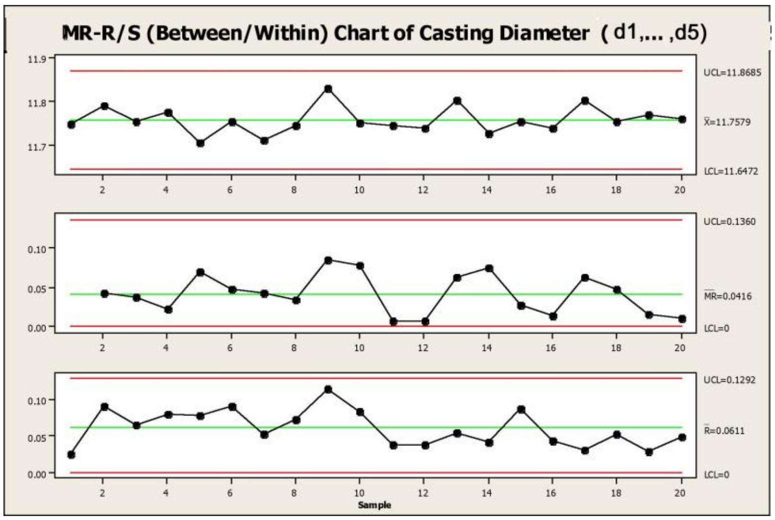
**6.79. continued**

(b) Discuss the charts you have constructed in part (a).

Though the *R* chart is in control, plot points on the $\bar{x}$ chart bounce below and above the control limits. Since these are high precision castings, we might expect that the diameter of a single casting will not change much with location. If no assignable cause can be found for these out-of-control points, we may want to consider treating the averages as an Individual value and graphing "between/within" range charts. This will lead to a understanding of the greatest source of variability, between castings or within a casting.

(c) Construct "between/within" charts for this process.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)**
**Select "I-MR-R/S Options, Estimate" and choose R-bar method to estimate standard deviation**




**I-MR-R/S (Between/Within) Chart of Ex6.69d1, ..., Ex6.69d5**
**I-MR-R/S Standard Deviations of Ex6.69d1, ..., Ex6.69d5**

```
Standard Deviations
Between         0.0349679
Within          0.0262640
Between/Within  0.0437327
```

(d) Do you believe that the charts in part (c) are more informative than those in part (a)? Discuss why.

Yes, the charts in (c) are more informative than those in (a). The numerous out-of-control points on the chart in (a) result from using the wrong source of variability to estimate sample variance. Recall that this is a cast part, and that multiple diameter measurements of any single part are likely to be very similar (text p. 246). With the three charts in (c), the correct variance estimate is used for each chart. In addition, more clear direction is provided by the within and between charts for trouble-shooting out-of-control signals.

**6.79. continued**

(e) Provide a practical interpretation of the "within" chart.

The "within" chart is the usual *R* chart ($n > 1$). It describes the measurement variability within a sample (variability in diameter of a single casting). Though the nature of this process leads us to believe that the diameter at any location on a single casting does not change much, we should continue to monitor "within" to look for wear, damage, etc., in the wax mold.

---

**6.80.**

In the semiconductor industry, the production of microcircuits involves many steps. The wafer fabrication process typically builds these microcircuits on silicon wafers, and there are many microcircuits per wafer. Each production lot consists of between 16 and 48 wafers. Some processing steps treat each wafer separately, so that the batch size for that step is one wafer. It is usually necessary to estimate several components of variation: within-wafer, between-wafer, between-lot, and the total variation.

(a) Suppose that one wafer is randomly selected from each lot and that a single measurement on a critical dimension of interest is taken. Which components of variation could be estimated with these data? What type of control charts would you recommend?

Both total process variability and the overall process average could be estimated from a single measurement on one wafer from each lot. Individuals *X* and Moving Range charts should be used for process monitoring.

(b) Suppose that each wafer is tested at five fixed locations (say, the center and four points at the circumference). The average and range of these within-wafer measurements are $\bar{x}_{WW}$ and $R_{WW}$, respectively. What components of variability are estimated using control charts based on these data?

Assuming that each wafer is processed separately, within-wafer variability could be monitored with a standard $\bar{X}-R$ control chart. The data from each wafer could also be used to monitor between-wafer variability by maintaining an individuals *X* and moving range chart for each of the five fixed positions. The Minitab "between/within" control charts do this in three graphs: (1) wafer mean $(\bar{x}_{ww})$ is an "individual value", (2) moving range is the difference <u>between</u> successive wafers, and (3) sample range is the difference <u>within</u> a wafer $(R_{ww})$. Alternatively, a multivariate process control technique could be used.

(c) Suppose that one measurement point on each wafer is selected and that this measurement is recorded for five consecutive wafers. The average and range of these between-wafer measurements are $\bar{x}_{BW}$ and $R_{BW}$, respectively. What components of variability are estimated using control charts based on these data? Would it be necessary to run separate $\bar{x}$ and $R$ charts for all five locations on the wafer?

Both between-wafer and total process variability could be estimated from measurements at one point on five consecutive wafers. If it is necessary to separately monitor the variation at each location, then either five $\bar{X}-R$ charts or some multivariate technique is needed. If the positions are essentially identical, then only one location, with one $\bar{X}-R$ chart, needs to be monitored.

**6.80. continued**

(d) Consider the question in part (c). How would your answer change if the test sites on each wafer were randomly selected and varied from wafer to wafer?

Within-wafer variability can still be monitored with randomly selected test sites. However, no information will be obtained about the pattern of variability within a wafer.

(e) What type of control charts and rational subgroup scheme would you recommend to control the batch-to-batch variability?

The simplest scheme would be to randomly select one wafer from each lot and treat the average of all measurements on that wafer as one observation. Then a chart for individual *x* and moving range would provide information on lot-to-lot variability.

---

**6.81.**

Consider the situation described in Exercise 6.80. A critical dimension (measured in μm) is of interest to the process engineer. Suppose that five fixed positions are used on each wafer (position 1 is the center) and that two consecutive wafers are selected from each batch. The data that result from several batches are shown in Table 6E.34.

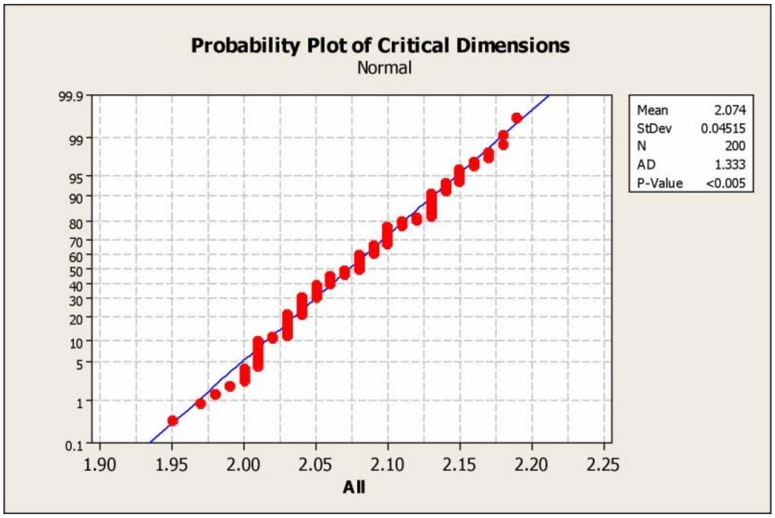
**TABLE 6E.34**
Data for Exercise 6.81

| Lot Number | Wafer Number | Position 1 | 2 | 3 | 4 | 5 | Lot Number | Wafer Number | Position 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2.15 | 2.13 | 2.08 | 2.12 | 2.10 | 11 | 1 | 2.15 | 2.13 | 2.14 | 2.09 | 2.08 |
| | 2 | 2.13 | 2.10 | 2.04 | 2.08 | 2.05 | | 2 | 2.11 | 2.13 | 2.10 | 2.14 | 2.10 |
| 2 | 1 | 2.02 | 2.01 | 2.06 | 2.05 | 2.08 | 12 | 1 | 2.03 | 2.06 | 2.05 | 2.01 | 2.00 |
| | 2 | 2.03 | 2.09 | 2.07 | 2.06 | 2.04 | | 2 | 2.04 | 2.08 | 2.03 | 2.10 | 2.07 |
| 3 | 1 | 2.13 | 2.12 | 2.10 | 2.11 | 2.08 | 13 | 1 | 2.05 | 2.03 | 2.05 | 2.09 | 2.08 |
| | 2 | 2.03 | 2.08 | 2.03 | 2.09 | 2.07 | | 2 | 2.08 | 2.01 | 2.03 | 2.04 | 2.10 |
| 4 | 1 | 2.04 | 2.01 | 2.10 | 2.11 | 2.09 | 14 | 1 | 2.08 | 2.04 | 2.05 | 2.01 | 2.08 |
| | 2 | 2.07 | 2.14 | 2.12 | 2.08 | 2.09 | | 2 | 2.09 | 2.11 | 2.06 | 2.04 | 2.05 |
| 5 | 1 | 2.16 | 2.17 | 2.13 | 2.18 | 2.10 | 15 | 1 | 2.14 | 2.13 | 2.10 | 2.10 | 2.08 |
| | 2 | 2.17 | 2.13 | 2.10 | 2.09 | 2.13 | | 2 | 2.13 | 2.10 | 2.09 | 2.13 | 2.15 |
| 6 | 1 | 2.04 | 2.06 | 1.97 | 2.10 | 2.08 | 16 | 1 | 2.06 | 2.08 | 2.05 | 2.03 | 2.09 |
| | 2 | 2.03 | 2.10 | 2.05 | 2.07 | 2.04 | | 2 | 2.03 | 2.01 | 1.99 | 2.06 | 2.05 |
| 7 | 1 | 2.04 | 2.02 | 2.01 | 2.00 | 2.05 | 17 | 1 | 2.05 | 2.03 | 2.08 | 2.01 | 2.04 |
| | 2 | 2.06 | 2.04 | 2.03 | 2.08 | 2.10 | | 2 | 2.06 | 2.05 | 2.03 | 2.05 | 2.00 |
| 8 | 1 | 2.13 | 2.10 | 2.10 | 2.15 | 2.13 | 18 | 1 | 2.03 | 2.08 | 2.04 | 2.00 | 2.03 |
| | 2 | 2.10 | 2.09 | 2.13 | 2.14 | 2.11 | | 2 | 2.04 | 2.03 | 2.05 | 2.01 | 2.04 |
| 9 | 1 | 1.95 | 2.03 | 2.08 | 2.07 | 2.08 | 19 | 1 | 2.16 | 2.13 | 2.10 | 2.13 | 2.12 |
| | 2 | 2.01 | 2.03 | 2.06 | 2.05 | 2.04 | | 2 | 2.13 | 2.15 | 2.18 | 2.19 | 2.13 |
| 10 | 1 | 2.04 | 2.08 | 2.09 | 2.10 | 2.01 | 20 | 1 | 2.06 | 2.03 | 2.04 | 2.09 | 2.10 |
| | 2 | 2.06 | 2.04 | 2.07 | 2.04 | 2.01 | | 2 | 2.01 | 1.98 | 2.05 | 2.08 | 2.06 |

**6.81. continued**

(a) What can you say about overall process capability?

**MTB > Stat > Basic Statistics > Normality Test**




Although the p-value is very small, the plot points do fall along a straight line, with many repeated values. The wafer critical dimension is approximately normally distributed. The natural tolerance limits ($\pm$ 3 sigma above and below mean) are:

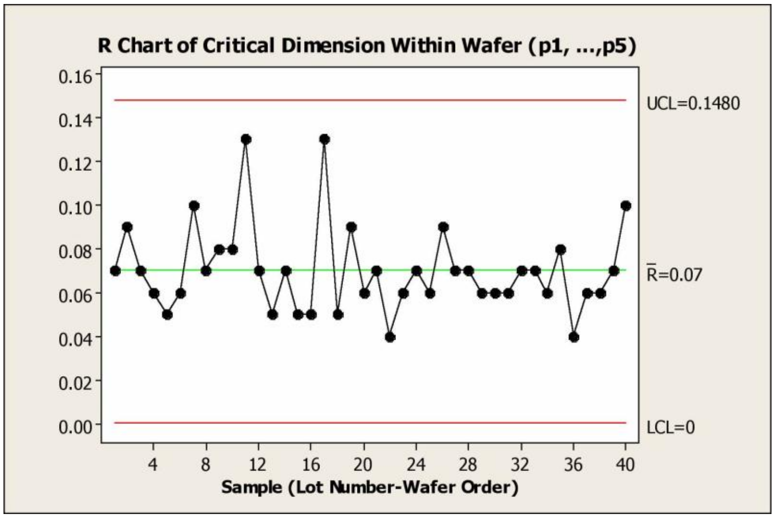
$\bar{x} = 2.074, s = 0.04515$

$\text{UNTL} = \bar{x} + 3s = 2.074 + 3(0.04515) = 2.209; \quad \text{LNTL} = \bar{x} - 3s = 2.074 - 3(0.04515) = 1.939$

**6.81. continued**

(b) Can you construct control charts that allow within-wafer variability to be evaluated?
To evaluate within-wafer variability, construct an *R* chart for each sample of 5 wafer positions (two wafers per lot number), for a total of 40 subgroups.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > R**



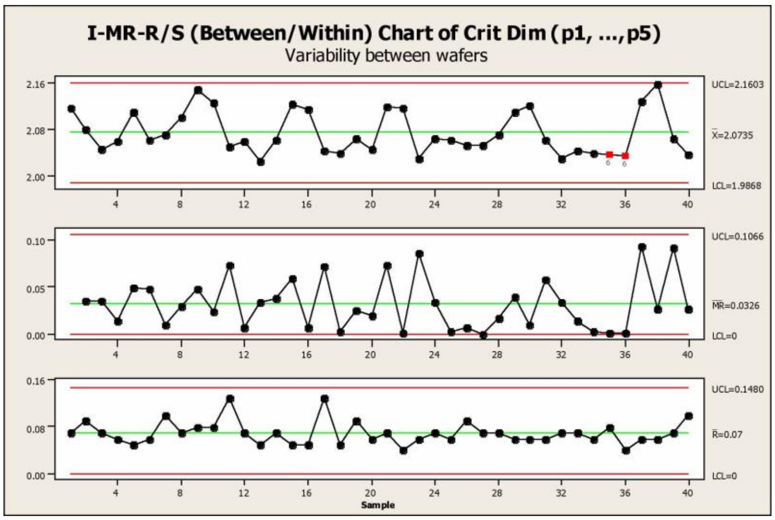

The Range chart is in control, indicating that within-wafer variability is also in control.

**6.81. continued**

(c) What control charts would you establish to evaluate variability between wafers? Set up these charts and use them to draw conclusions about the process.

To evaluate variability between wafers, set up Individuals and Moving Range charts where the *x* statistic is the average wafer measurement and the moving range is calculated between two wafer averages.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)**
**Select "I-MR-R/S Options, Estimate" and choose R-bar method to estimate standard deviation**




### I-MR-R/S Standard Deviations of Ex6.71p1, ..., Ex6.71p5

```
Standard Deviations
Between         0.0255911
Within          0.0300946
Between/Within  0.0395043
```

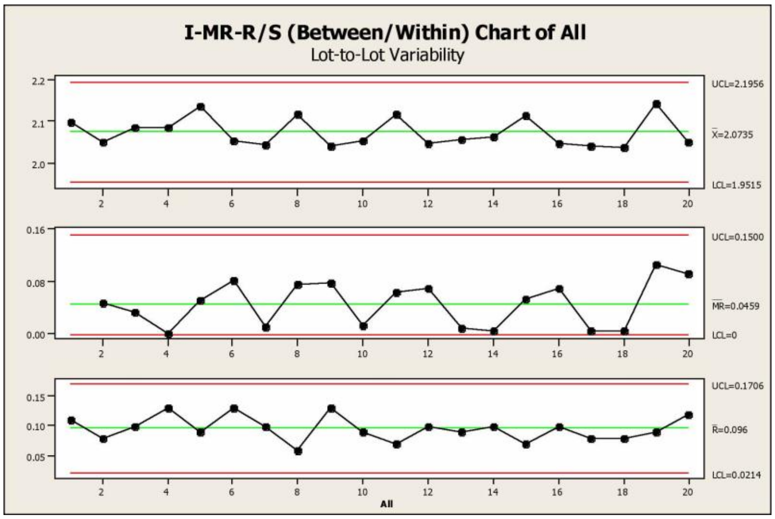
Both "between" control charts (Individuals and Moving Range) are in control, indicating that between-wafer variability is also in-control. The "within" chart (Range) is not required to evaluate variability between wafers.

**6.81. continued**

(d) What control charts would you use to evaluate lot-to-lot variability? Set up these charts and use them to draw conclusions about lot-to-lot variability.

To evaluate lot-to-lot variability, three charts are needed: (1) lot average, (2) moving range between lot averages, and (3) range within a lot—the Minitab "between/within" control charts.

**MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)**




**I-MR-R/S Standard Deviations of Ex6.71All**

```
Standard Deviations
Between          0.0394733
Within           0.0311891
Between/Within   0.0503081
```

All three control charts are in control, indicating that the lot-to-lot variability is also in-control.