Student Resource Manual

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to accompany

Introduction to Statistical Quality Control

Fourth Edition

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PREFACE

The purpose of this Resource Manual is to provide the student with an in-depth understanding of how to apply the concepts presented in the *Introduction to Statistical Quality Control*, 4th ed., by Douglas C. Montgomery. Along with detailed instructions on how to solve the chapter exercises, insights from practical application are also shared.

In general, solutions have been provided for "Answers to Selected Exercises" listed at the end of the text. However, occasionally a group of "continued" exercises is presented and provide the student with a full solution for a specific data set.

The exercises were solved using statistical software applications widely available for the personal computer:

- Design-Expert© Version 6.0.3 from Stat-Ease, www.statease.com
- JMP® Version 3.2.6 from SAS Institute, www.JMPdiscovery.com
- Excel 2000 from Microsoft®, www.microsoft.com
- MINITAB™ Release 13.1, www.minitab.com

Files with the relevant data sets for each chapter are available for download at www.wiley.com/college/montgomery, under the Online Student Resources for the SQC text.

CONTENTS

CHAPTER 1

Quality Improvement in the Modern Business Environment

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Define quality improvement and the role of statistical methods in variability reduction.

The modern definition of quality, "**Quality** is inversely proportional to variability" (text p. 4), implies that product quality increases as variability in important product characteristics decreases. **Quality improvement** can then be defined as "... the reduction of variability in processes and products" (text p. 6). Since the early 1900's, statistical methods have been used to control and improve quality. In the *Introduction to Statistical Quality Control*, 4th ed., by Douglas C. Montgomery, methods applicable in the key areas of *process control*, *design of experiments*, and *acceptance sampling* are presented.

To understand the potential for application of statistical methods, it may help to envision the system that creates a product as a "black box" (text Figure 1-3). The output of this black box is a product whose quality is defined by one or more quality characteristics that represent dimensions such as conformance to standards, performance, or reliability. Product quality can be evaluated with *acceptance sampling plans*. These plans are typically applied to either the output of a process or the input raw materials and components used in manufacturing. Application of process control techniques (such as control charts) or statistically designed experiments can achieve significant reduction in variability.

Black box inputs are categorized as "incoming raw materials and parts," "controllable inputs," and "uncontrollable inputs."

The quality of incoming raw materials and parts is often assessed with *acceptance sampling plans*. As material is received from suppliers, incoming lots are inspected then dispositioned as acceptable or unacceptable. Once a history of high quality material is established, a customer may accept the supplier's *process control* data in lieu of incoming inspection results.

"Controllable" and "uncontrollable" inputs apply to incoming materials, process variables, and environmental factors. For example, it may be difficult to control the temperature in a heat-treating oven in the sense that some areas of the oven may be cooler while some areas may be warmer. Properties of incoming materials may be very difficult to control. For example, the moisture content and proportion of hardwood in trees used for papermaking have a significant impact on the quality characteristics of the finished paper. Environmental variables such as temperature and relative humidity are often hard to control precisely.

Whether or not controllable and uncontrollable inputs are significant can be determined through process characterization. *Statistically designed experiments* are extremely helpful in characterizing processes and optimizing the relationship between incoming materials, process variables, and product characteristics

Although the initial tendency is to think of manufacturing processes and products, the statistical methods presented in this text can also be applied to *business* processes and products, such as financial transactions and services. In some organizations the opportunity to improve quality in these areas is even greater than it is in manufacturing.

Various quality philosophies and management systems are briefly described in the text; a common thread is the necessity for continuous improvement to increase productivity and reduce cost. The technical tools described in the text are essential for successful quality improvement. Quality management systems alone do not reduce variability and improve quality.

CHAPTER 2

Modeling Process Quality

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Describe variation using the following graphical methods: Stem and leaf plot; Histogram or frequency distribution; Box Plot; Probability Distributions
2. Describe data using the following numerical estimates: Average; Variance; Standard Deviation
3. Understand the difference between continuous and discrete distributions
4. Recognize and use the following discrete distributions: Hypergeometric; Binomial; Poisson; Pascal
5. Recognize and use the following continuous distributions: Normal; Exponential; Gamma; Weibull
6. Apply the Central Limit Theorem
7. Use approximation methods for probability distributions

Exercises

2-1. The fill volume of a soft-drink beverage is being analyzed for variability. Ten bottles, randomly selected from the process, are measured, and the results are as follows (in fluid ounces): 10.05, 10.03, 10.02, 10.04, 10.05, 10.01, 10.02, 10.02, 10.03, 10.01.

(a) Calculate the sample average.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{10.05+10.03+\cdots+10.01}{10} = 10.028 \text{ oz}$$

(b) Calculate the sample standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(10.05^2+\cdots+10.01^2) - \frac{(10.05+\cdots+10.01)^2}{10}}{10-1}} = 0.015 \text{ oz}$$

The Minitab basic statistics output for this data set is:

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Ex2-1	10	10.028	10.025	10.028	0.015	0.005
Variable	Minimum	Maximum	Q1	Q3		
Ex2-1	10.010	10.050	10.018	10.043		

2-3. The nine measurements that follow are furnace temperatures recorded on successive batches in a semiconductor manufacturing process (units are F):

953	955	948
951	957	949
954	950	959

(a) Calculate the sample average.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{953+951+\cdots+959}{9} = 952.9 \text{ °F}$$

(b) Calculate the sample standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(953^2+\cdots+959^2) - \frac{(953+\cdots+959)^2}{9}}{9-1}} = 3.7 \text{ °F}$$

The Minitab basic statistics output for this data set is:

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Ex2-3	9	952.89	953.00	952.89	3.72	1.24
Variable	Minimum	Maximum	Q1	Q3		
Ex2-3	948.00	959.00	949.50	956.00		

2-5. Yield strengths of circular tubes with end caps are measured. The first yields (in kN) are as follows:

96	102	104	108
126	128	150	156

(a) Calculate the sample average.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{96+102+\cdots+156}{8} = 121.25kN$$

(b) Calculate the sample standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(96^2+\cdots+156^2) - \frac{(96+\cdots+156)^2}{8}}{8-1}} = 22.63kN$$

The Minitab basic statistics output for this data set is:

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Ex2-5	8	121.25	117.00	121.25	22.63	8.00
Variable	Minimum	Maximum	Q1	Q3		
Ex2-5	96.00	156.00	102.50	144.50		

2-11. Consider the chemical process yield data in Exercise 2-7. Calculate the sample average and standard deviation.

Exercise 2-7 Data

94.1	87.3	94.1	92.4	84.6	84.4
93.2	84.1	92.1	90.6	83.6	86.6
90.6	90.1	96.4	89.1	85.4	91.7
91.4	95.2	88.2	88.8	89.7	87.5
88.2	86.1	86.4	86.4	87.6	84.2
86.1	94.3	85.0	85.1	85.1	85.1
95.1	93.2	84.9	84.0	89.6	90.5
90.0	86.7	87.3	93.7	90.0	95.6
92.4	83.0	89.6	87.7	90.1	88.3
87.3	95.3	90.3	90.6	94.3	84.1
86.6	94.1	93.1	89.4	97.3	83.7
91.2	97.8	94.6	88.6	96.8	82.9
86.1	93.1	96.3	84.1	94.4	87.3
90.4	86.4	94.7	82.6	96.1	86.4
89.1	87.6	91.1	83.1	98.0	84.5

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{94.1 + 93.2 + \cdots + 84.5}{90} = 89.476$$

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(94.1^2 + \cdots + 84.5^2) - \frac{(94.1 + \cdots + 84.5)^2}{90}}{90-1}} = 4.158$$

The Minitab basic statistics output for this data set is:

```
Variable          N       Mean     Median     TrMean      StDev    SE Mean
Ex2-7            90     89.476     89.250     89.391      4.158      0.438
Variable    Minimum    Maximum         Q1         Q3
Ex2-7        82.600     98.000     86.100     93.125
```

2-15. Suppose that two fair dice are tossed and the random variable observed - say, x -is the sum of the two up faces. Describe the sample space of this experiment, and determine the probability distribution of x.

x: {the sum of two up dice faces}
sample space: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
Calculate the probability of rolling a 2, obtained by rolling a 1 on each die:

$$\Pr\{x=2\} = \Pr\{1,1\} = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Calculate the probability of rolling a 3, 1 and 2 or 2 and 1:

$$\Pr\{x=3\} = \Pr\{1,2\} + \Pr\{2,1\} = \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{2}{36}$$

Calculate the probability of rolling a 4:

$$\Pr\{x=4\} = \Pr\{1,3\} + \Pr\{2,2\} + \Pr\{3,1\} = \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{3}{36}$$

The entire sample space is defined as:

$$p(x) = \begin{cases} 1/36; x=2 \\ 2/36; x=3 \\ 3/36; x=4 \\ 4/36; x=5 \\ 5/36; x=6 \\ 6/36; x=7 \\ 5/36; x=8 \\ 4/36; x=9 \\ 3/36; x=10 \\ 2/36; x=11 \\ 1/36; x=12 \\ 0; \text{ otherwise} \end{cases}$$

2-17. A mechatronic assembly is subjected to a final functional test. Suppose that defects occur at random in these assemblies, and that defects occur according to a Poisson distribution with parameter $\lambda = 0.02$.

(a) What is the probability that an assembly will have exactly one defect?

This is a Poisson distribution with parameter $\lambda = 0.02$, $x \sim \text{POI}(0.02)$.

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.02}(0.02)^1}{1!} = 0.0196$$

(b) What is the probability that an assembly will have one or more defects?

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.02}(0.02)^0}{0!} = 1 - 0.9802 = 0.0198$$

(c) Suppose that you improve the process so that the occurrence rate is cut in half to $\lambda = 0.01$. What effect does this have on the probability that an assembly will have one or more defects?

This is a Poisson distribution with parameter $\lambda = 0.01$, $x \sim \text{POI}(0.01)$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.01}(0.01)^0}{0!} = 1 - 0.9900 = 0.0100$$

Cutting the rate at which defects occur reduces the probability of one or more defects approximately half, from 0.0198 to 0.0100.

2-19. The random variable x takes on the values 1, 2, or 3 with probabilities $(1+3k)/3$, $(1+2k)/3$, and $(0.5+5k)/3$ respectively.

(a) Find the appropriate value of k.

$$p(x)=\begin{cases}(1+3k)/3; & x=1\\ (1+2k)/3; & x=2\\ (0.5+5k)/3; & x=3\\ 0; & \text{otherwise}\end{cases}$$

To solve for k, use $F(x)=\sum_{i=1}^{\infty} p(x_i)=1$

$$\frac{(1+3k)+(1+2k)+(0.5+5k)}{3}=1$$
$$\frac{2.5+10k}{3}=1$$
$$10k=0.5$$
$$k=0.05$$

(b) Find the mean and variance of x.

$$\mu=\sum_{i=1}^{3} x_i p(x_i)=1\times\left[\frac{1+3(0.05)}{3}\right]+2\times\left[\frac{1+2(0.05)}{3}\right]+3\times\left[\frac{0.5+5(0.05)}{3}\right]=1.867$$

$$\sigma^2=\sum_{i=1}^{3} x_i^2 p(x_i)-\mu^2=1^2(0.383)+2^2(0.367)+3^2(0.250)-1.867^2=0.615$$

(c) Find the cumulative distribution function.

Apply $k=0.05$ to earlier equations-*:

$$F(x)=\begin{cases}\frac{1.15}{3}=0.383; x=1\\ \frac{1.15+1.1}{3}=0.750; x=2\\ \frac{1.15+1.1+0.75}{3}=1.000; x=3\end{cases}$$

2-21. A manufacturer of electronic calculators offers a 1-year warranty. If the calculator fails for any reason during this period, it is replaced. The time to failure is well modeled by the following probability distribution:

$f(x)=0.125e^{-0.125x}$ $\quad x>0$

(a) What percentage of the calculators will fail within the warranty period?

This is an exponential distribution with parameter $\lambda=0.125$:
$\Pr\{x\le 1\}=F(1)=1-e^{-0.125(1)}=0.118$
Approximately 11.8% will fail during the first year.

(b) The manufacturing cost of a calculator is \$50, and the profit per sale is \$25. What is the effect of warranty replacement on profit?

Mfg. cost = \$50/calculator Sale profit = \$25/calculator
Net profit = \$[-50(1 + 0.118) + 75]/calculator = \$19.10/calculator.
The replacement warranty effect decreases profit by 25 – 19.10 = \$5.90/calculator.

2-23. A production process operates with 2% nonconforming output. Every hour a sample of 50 units of product is taken, and the number of nonconforming units is counted. If one or more nonconforming units are found, the process is stopped and the quality control technician must search for the cause of nonconforming production. Evaluate the performance of this decision rule.

This is a binomial distribution with parameter $p=0.02$ and $n=50$. The process is stopped if $x\ge 1$.

$$\Pr\{x\ge 1\}=1-\Pr\{x<1\}=1-\Pr\{x=0\}=1-\binom{50}{0}(0.02)^0(1-0.02)^{50}=1-0.364=0.636$$

This decision rule means that 63.6% of the samples will have one or more nonconforming units, and the process will be stopped to look for a cause. This is a somewhat difficult operating situation.

2-25. A random sample of 100 units is drawn from a production process every half hour. The fraction of nonconforming product manufactured is 0.03. What is the probability that $\hat{p}\le 0.04$ if the fraction nonconforming really is 0.03?

This is a binomial distribution with parameter $p=0.03$ and $n=100$.

$$\Pr\{\hat{p}\le 0.04\}=\Pr\{x\le 4\}=\sum_{x=0}^{4}\binom{100}{x}(0.03)^x(1-0.03)^{(100-x)}$$

$$=\binom{100}{0}(0.03)^0(1-0.03)^{100}+\binom{100}{1}(0.03)^1(1-0.03)^{99}+\cdots+\binom{100}{4}(0.03)^4(1-0.03)^{96}=0.818$$

Therefore, the probability is 0.818 that sample fraction nonconforming can be $\hat{p}\le 0.04$ if the population fraction nonconforming really is $p=0.03$.

2-27. An electronic component for a laser range-finder is produced in lots of size N=25. An acceptance testing procedure is used by the purchaser to protect against lots that contain too many nonconforming components. The procedure consists of selecting five components at random from the lot (without replacement) and testing them. If none of the components is nonconforming, the lot is accepted.

(a) If the lot contains three nonconforming components, what is the probability of acceptance?

This is a hypergeometric distribution with N = 25 and n = 5, without replacement.

$$\Pr\{\text{acceptance}\} = p(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

Given D = 3 and x = 0:

$$\Pr\{\text{acceptance}\} = p(0) = \frac{\binom{3}{0}\binom{25-3}{5-0}}{\binom{25}{5}} = \frac{(1)(26,334)}{(53,130)} = 0.496$$

(b) Calculate the desired probability in (a) using the binomial approximation. Is this approximation satisfactory? Why or why not?

For the binomial approximation to the hypergeometric,

$$\Pr\{\text{acceptance}\} = p(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$$

$$p = \frac{D}{N} = \frac{3}{25} = 0.120 \text{ and } n = 5.$$

$$\Pr\{\text{acceptance}\} = p(0) = \binom{5}{0}(0.120)^0(1-0.120)^5 = (1)(1)(0.528) = 0.528$$

This approximation, though close to the exact for x = 0, violates the rule-of-thumb that n/N = 5/25 = 0.20 be less than the suggested 0.1. The binomial approximation is not satisfactory in this case.

(c) Suppose the lot size N=150. Would the binomial approximation be satisfactory in this case?

For N = 150, n/N = 5/150 = 0.033 ≤ 0.1, so the binomial approximation would be a satisfactory approximation the hypergeometric in this case.

(d) Suppose that the purchaser will reject the lot with the decision rule of finding one or more nonconforming components in a sample of size n, and wants the lot to be rejected with probability at least 0.95 if the lot contains five or more nonconforming components. How large should the sample size n be?

Find n to satisfy $\Pr\{x \geq 1 \mid D \geq 5\} \geq 0.95$, or equivalently $\Pr\{x = 0 \mid D = 5\} < 0.05$.

$$p(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$p(0) = \frac{\binom{5}{0}\binom{25-5}{n-0}}{\binom{25}{n}} = \frac{\binom{5}{0}\binom{20}{n}}{\binom{25}{n}}$$

try $n = 10$

$$p(0) = \frac{\binom{5}{0}\binom{20}{10}}{\binom{25}{10}} = \frac{(1)(184,156)}{(3,268,760)} = 0.057$$

try $n = 11$

$$p(0) = \frac{\binom{5}{0}\binom{20}{11}}{\binom{25}{11}} = \frac{(1)(167,960)}{(4,457,400)} = 0.038$$

Let sample size n = 11.

2-29. A textbook has 500 pages on which typographical errors could occur. Suppose that there are exactly 10 such errors randomly located on those pages. Find the probability that a random selection of 50 pages will contain no errors. Find the probability that 50 randomly selected pages will contain at least two errors.

This is a hypergeometric distribution with N = 500 pages, n = 50 pages, and D = 10 errors. Checking n/N = 50/500 = 0.1 ≤ 0.1, the binomial distribution can be used to approximate the hypergeometric, with $p = \frac{D}{N} = \frac{10}{500} = 0.020$.

$$\Pr\{\text{acceptance}\} = p(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$$

$$\Pr\{x = 0\} = p(0) = \binom{50}{0}(0.020)^0(1-0.020)^{50-0} = (1)(1)(0.364) = 0.364$$

$$\Pr\{x \geq 2\} = 1 - \Pr\{x \leq 1\} = 1 - [\Pr\{x = 0\} + \Pr\{x = 1\}] = 1 - p(0) - p(1)$$

$$= 1 - 0.364 - \binom{50}{1}(0.020)^1(1-0.020)^{50-1} = 1 - 0.364 - 0.372 = 0.264$$

2-31. Glass bottles are formed by pouring molten glass into a mold. The molten glass is prepared in a furnace lined with firebrick. As the firebrick wears, small pieces of brick are mixed into the molten glass and finally appear as defects (called "stones") in the bottle. If we can assume that stones occur randomly at the rate of 0.00001 per bottle, what is the probability that a bottle selected at random will contain at least one such defect?

This is a Poisson distribution with $\lambda = 0.00001$ stones/bottle. The Poisson distribution is $\Pr(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x = 0\} = 1 - \frac{e^{-0.00001}(0.00001)^0}{0!} = 1 - 0.99999 = 0.00001$$

2-33. A production process operates in one of two states: in the in-control state, in which most of the units produced conform to specifications, and an out-of-control state, in which most of the units produced are defective. The process will shift from the in-control to the out-of-control state at random. Every hour, a quality control technician checks the process, and if it is in the out-of-control state, the technician detects this with a probability p. Assume that when the process shifts out of control it does so immediately following a check by the inspector, and once a shift has occurred, the process cannot automatically correct itself. If t denotes the number of periods the process remains out of control following a shift before detection, find the probability distribution of t. Find the mean number of periods the process will remain in the out-of-control state.

The distribution for t is based on the number of periods for which the process shifted from in control to out of control, 1, with probability $(p)^1$, and the periods from which the process remains in the out of control state without a shift, $t - 1$, with probability $(1-p)^{t-1}$. There is only one permutation for this situation. Hence, the distribution is the combination of the two states as follows:

$\Pr(t) = p(1-p)^{t-1}; \; t = 1,2,3,\ldots$

The mean is calculated per equation 2-5b: $\mu = \sum_{i=1}^{\infty} x_i p(x_i)$ as follows:

$$\mu = \sum_{t=1}^{\infty} t\left[p(1-p)^{t-1}\right] = p\frac{d}{dq}\left[\sum_{t=1}^{\infty} q^t\right] = \frac{1}{p}$$

2-35. The tensile strength of a metal part is normally distributed with a mean of 40 lb and standard deviation 8 lb. If 50,000 parts are produced, how many would fail to meet a minimum specification of 34-lb tensile strength? How many would have a tensile strength in excel of 48 lb?

$x \sim N(40, 8^2)$; $n = 50{,}000$

How many fail the minimum specification, LSL = 34 lb.? Utilizing the standard normal distribution, Appendix II:

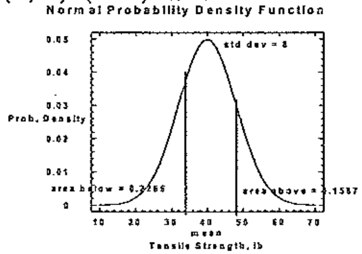
$$\Pr\{x \leq 34\} = \Pr\left\{z \leq \frac{34-40}{8}\right\} = \Pr\{z \leq -0.75\} = \Phi(-0.75) = 0.2266$$

Therefore, the number failing the minimum specification is $(50{,}000) \times (0.2266) = 11{,}330$.

How many exceed 48 lb.? Again, utilizing the standard normal distribution, Appendix II:

$$\Pr\{x > 48\} = 1 - \Pr\{x \leq 48\} = 1 - \Pr\left\{z \leq \frac{48-40}{8}\right\} = 1 - \Pr\{z \leq 1.00\} = 1 - \Phi(1.00) = 1 - 0.8413 = 0.1587$$

The number that exceed 48 lb. is $(50{,}000) \times (0.1587) = 7935$.



2-37. Continuation of Exercise 2-36. Reconsider the power supply manufacturing process in Exercise 2-36. Suppose we wanted to improve the process. Can shifting the means reduce the number of nonconforming units produced? How much would the process variability need to be reduced in order to have all but one of 1000 units conform to the specifications?

Exercise 2-36. The output voltage of a power supply is normally distributed with mean 12 V and standard deviation 0.05 V. If the lower and upper specifications for voltage are 11.90 V and 12.10 V, respectively, what is the probability that a power supply selected at random will conform to the specifications on voltage?

The process, with mean 12V, is currently centered between the specification limits (target = 12V). Shifting the process mean in either direction would increase the number of nonconformities produced. Desire Pr{conformance} = 1 / 1000 = 0.001. Assume that the process remains centered between the specification limits at 12V. Need $\Pr\{x \leq \text{LSL}\} = 0.001 / 2 = 0.0005$.

$\Phi(z) = 0.0005$

$z = \Phi^{-1}(0.0005) = -3.29$

$$z = \frac{\text{LSL} - \bar{x}}{s}, \quad \text{so } s = \frac{\text{LSL} - \bar{x}}{z} = \frac{11.90 - 12}{-3.29} = 0.03$$

Process variance must be reduced to 0.03^2 to have at least 999 of 1000 conform to specification.

2-39. The life of an automotive battery is normally distributed with a mean 900 days and standard deviation 35 day. What fraction of these batteries would be expected to survive beyond 1000 days?

$x \sim N(900, 35^2)$

$$\Pr\{x > 1000\} = 1 - \Pr\{x \leq 1000\} = 1 - \Pr\left\{x \leq \frac{1000-900}{35}\right\} = 1 - \Phi(2.8571) = 1 - 0.9979 = 0.0021$$

The percentage expected to survive more than 1000 days is 0.21%.

2-41. The specifications of an electronic component in a target-acquisition systems are that its life must be between 5000 and 10,000 h. The life is normally distributed with mean 7500 h. The manufacturer realizes a price of $10 per unit produced; however, defective units must be replaced at a cost of $5 to the manufacturer. Two different manufacturing processes can be used, both of which have the same mean life. However, the standard deviation of life for process 1 is 1000h, whereas for process 2 it is only 500 h. Production costs for process 2 are twice those for process 1. What value of production costs will determine the selection between process 1 and 2?

The information required for this exercise is:

$x_1 \sim N(7500, \sigma_1^2 = 1000^2)$; $x_2 \sim N(7500, \sigma_2^2 = 500^2)$; LSL=5,000 h; USL=10,000 h

sales = \$10/unit, defect = \$5/unit, profit = \$10 × Pr{good} + \$5 × Pr{bad} − c

For Process 1

$$\text{proportion defective} = p_1 = 1 - \Pr\{\text{LSL} \le x_1 \le \text{USL}\} = 1 - \Pr\{x_1 \le \text{USL}\} + \Pr\{x_1 \le \text{LSL}\}$$

$$= 1 - \Pr\left\{z_1 \le \frac{10,000 - 7,500}{1,000}\right\} + \Pr\left\{z_1 \le \frac{5,000 - 7,500}{1,000}\right\}$$

$$= 1 - \Phi(2.5) + \Phi(-2.5) = 1 - 0.9938 + 0.0062 = 0.0124$$

profit for process 1 = 10 (1 − 0.0124) + 5 (0.0124) − c_1 = 9.9380 − c_1

For Process 2

$$\text{proportion defective} = p_2 = 1 - \Pr\{\text{LSL} \le x_2 \le \text{USL}\} = 1 - \Pr\{x_2 \le \text{USL}\} + \Pr\{x_2 \le \text{LSL}\}$$

$$= 1 - \Pr\left\{z_2 \le \frac{10,000 - 7,500}{500}\right\} + \Pr\left\{z_2 \le \frac{5,000 - 7,500}{500}\right\}$$

$$= 1 - \Phi(5) + \Phi(-5) = 1 - 1.0000 + 0.0000 = 0.0000$$

profit for process 2 = 10 (1 − 0.0000) + 5 (0.0000) − c_2 = 10 − c_2

If $c_2 > c_1 + 0.0620$, then choose process 1

CHAPTER 3

Inferences about Process Quality

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Sample from Normal, Bernoulli, Poisson distributions
2. Understand and compute point estimates from data sets
3. Make decisions on process data by using statistical inference:
   - Define null and alternate hypothesis
   - Choose and compute test statistics
   - Define critical or reject regions
   - Compare test statistics to critical or reject regions
   - Estimate confidence intervals
4. Generate normal probability plots of data
5. Compare two samples of data
6. Compare multiple samples with analysis of variance

Exercises

3-1. The inside diameter of bearings used in an aircraft landing gear assembly are known to have a standard deviation of $\sigma = 0.002$ cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm.

(a) Test the hypothesis that the mean inside bearing diameter is 8.25 cm. Use a two-side alternative and $\alpha = 0.05$.

$n = 15$; $\bar{x} = 8.2535$ cm; $\sigma = 0.002$ cm
$\mu_0 = 8.25$, $\alpha = 0.05$
Test: H_0: $\mu = 8.25$ vs. H_1: $\mu \neq 8.25$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
First calculate the test statistic Z_0 by

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{8.2535 - 8.25}{0.002/\sqrt{15}} = 6.78$$

From Appendix Table II, identify
$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$
Reject H_0: $\mu = 8.25$, and conclude that the mean bearing ID is not equal to 8.25 cm.

(b) Find the P-value for this test.

From Appendix Table II, find the corresponding cumulative Standard Normal, $\Phi(Z_0)$ for $Z_0 = 6.78$ and calculate:
$P = 2[1 - \Phi(Z_0)] = 2[1 - \Phi(6.78)] = 2[1 - 1.00000] = 0$

(c) Construct a 95% two-sided confidence interval on mean bearing diameter.

By using equation 3-29, the confidence interval on the mean bearing diameter is calculated as follows:

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$8.2535 - 1.96\left(\frac{0.002}{\sqrt{15}}\right) \leq \mu \leq 8.2535 + 1.96\left(\frac{0.002}{\sqrt{15}}\right)$$

$$8.2525 \leq \mu \leq 8.2545$$

3-3. The life of a battery used in a cardiac pacemaker is assumed to be normally distributed. A random sample of 10 batteries is subjected to an accelerated life test by running them continuously at an elevated temperature until failure, and the following lives are obtained.

25.5 h	26.1 h
26.8	23.2
24.2	28.4
25.0	27.8
27.3	25.7

(a) The manufacturer wants to be certain that the mean battery life exceeds 25 h. What conclusions can be drawn from these data (use $\alpha = 0.05$)?

Because σ is unknown, and estimate, S, is calculated and the t-distribution used, one-sided, as follows:
$x \sim N(\mu, \sigma)$, $\bar{x} = 26.0$, $S = 1.62$, $\mu_0 = 25$, $\alpha = 0.05$
Test H_0: $\mu = 25$ vs. H_1: $\mu > 25$. Reject H_0 if $t_0 > t_\alpha$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{26.0 - 25}{1.62/\sqrt{10}} = 1.952$$

$t_{\alpha,n-1} = t_{0.05,10-1} = 1.833$
Reject H_0: $\mu = 25$, and conclude that the mean life exceeds 25 h.

These calculations are validated with Minitab as follows:

```
Test of mu = 25 vs mu > 25
Variable          N      Mean     StDev   SE Mean
Ex3-3            10    26.000     1.625     0.514

Variable         95.0% Lower Bound        T       P
Ex3-3                      25.058     1.95   0.042
```

(b) Construct a 90% two-sided confidence interval on mean life in the accelerated test.

With $\alpha = 0.10$, and equation 3-34, the confidence interval on mean life in the accelerated test is:

$$\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

$$26.0 - 1.833\left(\frac{1.62}{\sqrt{10}}\right) \leq \mu \leq 26.0 + 1.833\left(\frac{1.62}{\sqrt{10}}\right)$$

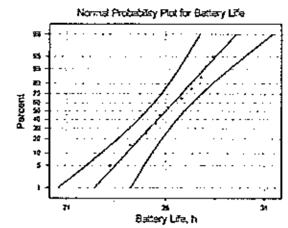
$$25.06 \leq \mu \leq 26.94$$

Again, Minitab validates the calculations as follows:

```
Variable          N      Mean     StDev   SE Mean           90.0% CI
Ex3-3            10    26.000     1.625     0.514   (  25.058,   26.942)
```

(c) Construct a normal probability plot of the battery life data. What conclusions can you draw?

In Minitab, select "Options," then "Obtain Plot Points Using - Modified Kaplan-Meier Method." Using Minitab, the normal probability plot is:



The plotted points fall approximately along a straight line, so the assumption that battery life is normally distributed is appropriate.

3-5. A new process has been developed for applying photoresist to 125-mm silicon wafers used in manufacturing integrated circuits. Ten wafers were tested, and the photoresist thickness measurements shown here were observed:

13.3946 (x 1000 angstroms)	13.4002 (x 1000 angstroms)
13.3987	13.3957
13.3902	13.4015
13.4001	13.3918
13.3965	13.3925

(a) Test the hypothesis that mean thickness is 13.4 x 1000 Å. Use $\alpha = 0.05$ and assume a two-sided alternative.

First, estimate the mean and standard deviation: $x \sim N(\mu,\sigma)$, $n = 10$, $\bar{x} = 13.39618(\times 1000\ \text{Å})$, $S = 0.00391$

$\mu_0 = 13.4\times1000$ Å, $\alpha = 0.05$

Test: H_0: $\mu = 13.4$ vs. H_1: $\mu \neq 13.4$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

Compute the test statistic as follows:

$$t_0 = \frac{\bar{x}-\mu_0}{S/\sqrt{n}} = \frac{13.39618-13.4}{0.00391/\sqrt{10}} = -3.089$$

Identify $t_{\alpha/2,n-1}$ from Appendix Table IV: $t_{\alpha/2,n-1} = t_{0.025,9} = 2.262$

Reject H_0: $\mu = 13.4$, and conclude that the mean thickness differs from 13.4×1000 Å.

This test can also be performed in Minitab as follows:

```
Test of mu = 13.4 vs mu not = 13.4
Variable          N       Mean      StDev    SE Mean
Ex3-5            10    13.3962     0.0039     0.0012
Variable               95.0% CI              T        P
Ex3-5           ( 13.3934, 13.3990)      -3.09    0.013
```

(b) Find a 99% tow-sided confidence interval on mean photoresist thickness. Assume that the thickness is normally distributed.

With $\alpha = 0.01$, identify $t_{\alpha/2,n-1}$ from Appendix Table IV and calculate the confidence interval on mean photoresist thickness as:

$$\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

$$13.39618 - 3.2498\left(\frac{0.00391}{\sqrt{10}}\right) \le \mu \le 13.39618 + 3.2498\left(\frac{0.00391}{\sqrt{10}}\right)$$

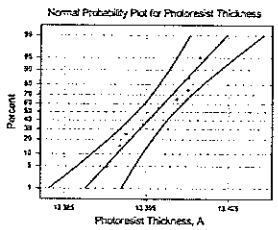
$$13.39216 \le \mu \le 13.40020$$

Using Minitab, the confidence interval calculations are verified:

```
Variable          N       Mean      StDev    SE Mean            99.0% CI
Ex3-5            10    13.3962     0.0039     0.0012   ( 13.3922, 13.4002)
```

(c) Does the normality assumption seem reasonable for these data?

Note: In Minitab, select Options, then "Obtain Plot Points Using - Modified Kaplan-Meier Method"



The plotted points form a reverse "S" shape, instead of a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-7. Ferric chloride is used as a flux in some types of extraction metallurgy processes. This material is shipped in containers, and the container weight varies. It is important to obtain an accurate estimate of mean container weight. Suppose that from long experience a reliable value for the standard deviation of flux container weight is determined to be 4 lb. How large a sample would be required to construct a 95% two-sided confidence interval on the mean that has a total width of 1lb?

$\sigma = 4$ lb, $\alpha = 0.05$, $Z_{\alpha/2} = Z_{0.025} = 1.9600$, total confidence interval width = 1 lb, find n

$$2\left[Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] = \text{total width}$$

$$2\left[1.9600\frac{4}{\sqrt{n}}\right] = 1$$

$$n = 246$$

A sample size of 246 would be required.

3-9. The output voltage of a power supply is assumed to be normally distributed. Sixteen observations taken at random on voltage are shown here.

10.35	9.30	10.00	9.96
11.65	12.00	11.25	9.58
11.54	9.95	10.28	8.37
10.44	9.25	9.38	10.85

$x \sim N(\mu, \sigma)$, $n = 16$; $\bar{x} = 10.259$ V; $S = 0.999$ V

(a) Test the hypothesis that the mean voltage equals 12 V against a two-sided alternative using $\alpha = 0.05$.

$\mu_0 = 12$, $\alpha = 0.05$
Test H_0: $\mu = 12$ vs. H_1: $\mu \neq 12$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{10.259 - 12}{0.999/\sqrt{16}} = -6.971$$

$t_{\alpha/2,n-1} = t_{0.025,16-1} = 2.131$
Reject H_0: $\mu = 12$, and conclude that the mean output voltage differs from 12V.

The Minitab output for this exercise is shown below. The t value calculated by Minitab agrees with the calculations above. Also note that the hypothesis test value of 12 is not included in the 95% confidence interval of 9.727 to 10.792.

```
Test of mu = 12 vs mu not = 12
Variable          N       Mean     StDev   SE Mean
Ex3-9            16     10.259     0.999     0.250
Variable              95.0% CI          T        P
Ex3-9           (   9.727,   10.792)   -6.97   0.000
```

(b) Construct a 95% two-sided confidence interval on μ.

With $\alpha = 0.05$, $t_{\alpha/2,n-1} = t_{0.025,16-1} = 2.131$, the confidence interval is calculated as:

$$\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

$$10.259 - 2.131\left(\frac{0.999}{\sqrt{16}}\right) \le \mu \le 10.259 + 2.131\left(\frac{0.999}{\sqrt{16}}\right)$$

$$9.727 \le \mu \le 10.791$$

The Minitab output for this exercise is:

```
Variable          N       Mean     StDev   SE Mean         95.0% CI
Ex3-9            16     10.259     0.999     0.250   (   9.727,   10.792)
```

(c) Test the hypothesis that $\sigma^2 = 1$ using $\alpha = 0.05$.

$\sigma^2_0 = 1$, $\alpha = 0.05$
Test H_0: $\sigma^2 = 1$ vs. H_1: $\sigma^2 \neq 1$. Reject H_0 if $\chi^2_0 > \chi^2_{\alpha/2,n-1}$ or $\chi^2_0 < \chi^2_{1-\alpha/2,n-1}$.

$$\chi^2_0 = \frac{(n-1)S^2}{\sigma^2_0} = \frac{(16-1)0.999^2}{1} = 14.970$$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,16-1} = 27.488$$

$$\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,16-1} = 6.262$$

Do not reject H_0: $\sigma^2 = 1$, and conclude that there is insufficient evidence that the variance differs from 1.

(d) Construct a 95% two-sided confidence interval on σ.

$\alpha = 0.05$; $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,15} = 27.488$; $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,15} = 6.262$

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(16-1)0.999^2}{27.488} \le \sigma^2 \le \frac{(16-1)0.999^2}{6.262}$$

$$0.545 \le \sigma^2 \le 2.391$$

$$0.738 \le \sigma \le 1.546$$

(e) Construct a 95% upper confidence interval on σ.

$\alpha = 0.05$; $\chi^2_{1-\alpha,n-1} = \chi^2_{0.95,15} = 7.2609$

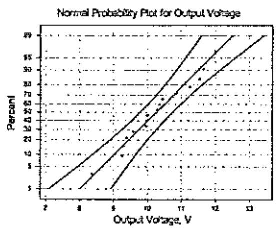
$$\sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}$$

$$\sigma^2 \le \frac{(16-1)0.999^2}{7.2609}$$

$$\sigma^2 \le 2.062$$

$$\sigma \le 1.436$$

(f) Does the assumption of normality seem reasonable for the output voltage?



From visual examination of the plot, the assumption of a normal distribution for output voltage seems appropriate.

3-11. Two quality control technicians measured the surface finish of a metal part, obtaining the data shown. Assume that the measurements are normally distributed.

Technician 1	Technician 2
1.45	1.54
1.37	1.41
1.21	1.56
1.54	1.37
1.48	1.20
1.29	1.31
1.34	1.27
	1.35

(a) Test the hypothesis that the mean surface finish measurements made by the two technicians are equal. Use $\alpha = 0.05$ and assume equal variances.

```
Two-sample T for Ex3-11T1 vs Ex3-11T2
           N      Mean     StDev   SE Mean
Ex3-11T1   7     1.383     0.115     0.043
Ex3-11T2   8     1.376     0.125     0.044
Difference = mu Ex3-11T1 - mu Ex3-11T2
Estimate for difference:  0.0066
95% CI for difference: (-0.1283, 0.1415)
T-Test of difference = 0 (vs not =): T-Value = 0.11  P-Value = 0.917  DF = 12
```

Do not reject H_0: $\mu_1 - \mu_2 = 0$, and conclude that there is not sufficient evidence of a difference between measurements obtained by the two technicians.

(b) What are the practical implications of the test in part (a)? Discuss what practical conclusions you would draw if the null hypothesis were rejected?

The practical implication of this test is that it does not matter which technician measures parts; the readings will be the same. If the null hypothesis had been rejected, we would have been concerned that the technicians obtained different measurements, and an investigation should be undertaken to understand why.

(c) Assuming that the variances are equal, construct a 95% confidence interval on the mean difference in the surface-finish measurements.

$n_1 = 7;\ \bar{x}_1 = 1.383;\ S_1 = 0.115;\ n_2 = 8;\ \bar{x}_2 = 1.376;\ S_2 = 0.125$

$\alpha = 0.05; t_{\alpha/2, n_1+n_2-2} = t_{0.025,13} = 2.1604$

Calculate the pooled standard deviation:

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)0.115^2 + (8-1)0.125^2}{7+8-2}} = 0.120$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(1.383 - 1.376) - 2.1604\sqrt{\frac{1}{7} + \frac{1}{8}} \le (\mu_1 - \mu_2) \le (1.383 - 1.376) + 2.1604\sqrt{\frac{1}{7} + \frac{1}{8}}$$

$$-0.127 \le (\mu_1 - \mu_2) \le 0.141$$

The confidence interval on the mean difference includes zero; therefore, there is no difference in the means. The Minitab output for this data is:

```
Two-sample T for Ex3-11T1 vs Ex3-11T2
           N      Mean     StDev   SE Mean
Ex3-11T1   7     1.383     0.115     0.043
Ex3-11T2   8     1.376     0.125     0.044
Difference = mu Ex3-11T1 - mu Ex3-11T2
Estimate for difference:  0.0066
95% CI for difference: (-0.1280, 0.1412)
T-Test of difference = 0 (vs not =): T-Value = 0.11  P-Value = 0.917  DF = 13
Both use Pooled StDev = 0.120
```

The confidence interval for the difference contains zero. We can conclude that there is no difference in measurements obtained by the two technicians.

(d) Test the hypothesis that the variances of the measurements made by the two technicians are equal. Use $\alpha = 0.05$. What are the practical implications if the null hypothesis is rejected?

$\alpha = 0.05$

Test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_1: \sigma_1^2 \ne \sigma_2^2$. Reject H_0 if $F_0 > F_{\alpha/2, n_1-1, n_2-1}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$.

$$F_0 = \frac{S_1^2}{S_2^2} = \frac{0.115^2}{0.125^2} = 0.8464$$

$F_{\alpha/2, n_1-1, n_2-1} = F_{0.05/2, 7-1, 8-1} = F_{0.025,6,7} = 5.119$

$F_{1-\alpha/2, n_1-1, n_2-1} = F_{1-0.05/2, 7-1, 8-1} = F_{0.975,6,7} = 0.176$

```
Level1     Ex3-11T1
Level2     Ex3-11T2
ConfLvl    95.0000

F-Test (normal distribution)
Test Statistic: 0.846
P-Value       : 0.854
```

Do not reject H_0, and conclude that there is no difference in variability of measurements obtained by the two technicians. If the null hypothesis is rejected, we would have been concerned about the difference in measurement variability between the technicians, and an investigation should be undertaken to understand why.

(e) Construct a 95% confidence interval estimate of the ratio of the variances of technician measurement error.

$\alpha = 0.05;\ F_{1-\alpha/2,n_2-1,n_1-1} = F_{0.975,7,6} = 0.1954;\ F_{\alpha/2,n_2-1,n_1-1} = F_{0.025,7,6} = 5.6955$

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{0.115^2}{0.125^2}(0.1954) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{0.115^2}{0.125^2}(5.6955)$$

$$0.165 \le \frac{\sigma_1^2}{\sigma_2^2} \le 4.821$$

The confidence interval includes one; therefore, there is not difference in the variances.

(f) Construct a 95% confidence interval on the variance of measurement error for technician 2.

$n_2 = 8;\ \bar{x}_2 = 1.376;\ S_2 = 0.125$

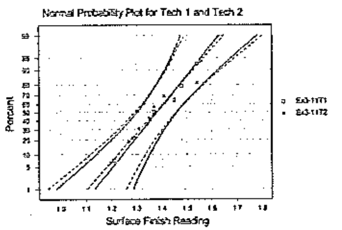
$\alpha = 0.05;\ \chi^2_{\alpha/2,n_2-1} = \chi^2_{0.025,7} = 16.0128;\ \chi^2_{1-\alpha/2,n_2-1} = \chi^2_{0.975,7} = 1.6899$

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(8-1)0.125^2}{16.0128} \le \sigma^2 \le \frac{(8-1)0.125^2}{1.6899}$$

$$0.007 \le \sigma^2 \le 0.065$$

(g) Does the normality assumption seem reasonable for the data?



The normality assumption seems reasonable for these readings.

3-13. Two different hardening processes, (1) saltwater quenching and (2) oil quenching, are used on samples of a particular type of metal alloy. The results are shown here. Assume that hardness is normally distributed.

Saltwater Quench	Oil Quench
145	152
150	150
153	147
148	155
141	140
152	146
146	158
154	152
139	151
148	143

Saltwater quench: $n_1 = 10;\ \bar{x}_1 = 147.6;\ S_1 = 4.97$

Oil quench: $n_2 = 10;\ \bar{x}_2 = 149.4;\ S_2 = 5.46$

(a) Test the hypothesis that the mean hardness for the saltwater quenching process equals the mean hardness for the oil quenching process. Use $\alpha = 0.05$ and assume equal variances.

Assume $\sigma_1^2 = \sigma_2^2$

```
Two-sample T for Ex3-13SQ vs Ex3-13OQ
           N      Mean     StDev   SE Mean
Ex3-13SQ  10    147.60      4.97       1.6
Ex3-13OQ  10    149.40      5.46       1.7
Difference = mu Ex3-13SQ - mu Ex3-13OQ
Estimate for difference:  -1.80
95% CI for difference: (-6.73, 3.13)
T-Test of difference = 0 (vs not =): T-Value = -0.77  P-Value = 0.451  DF = 17
```

Do not reject H_0, and conclude that there is no difference between the quenching processes.

(b) Assuming that the variances σ_1^2 and σ_2^2 are equal, construct a 95% confidence interval on the difference in mean hardness.

From the Minitab output in part (a), $S_1 = 4.97$ and $S_2 = 5.46$.

$\alpha = 0.05;\ t_{\alpha/2,n_1+n_2-2} = t_{0.025,18} = 2.1009$

The pooled standard deviation is

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)4.97^2 + (10-1)5.46^2}{10+10-2}} = 5.22$$

$$(\bar{x}_1-\bar{x}_2)-t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\le(\mu_1-\mu_2)\le(\bar{x}_1-\bar{x}_2)+t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(147.6-149.4)-2.1009\sqrt{\frac{1}{10}+\frac{1}{10}}\le(\mu_1-\mu_2)\le(147.6-149.4)+2.1009\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$-6.7\le(\mu_1-\mu_2)\le 3.1$$

Because the confidence interval of the difference in means includes zero, there is no difference in means.

(c) Construct a 95% confidence interval on the ratio σ_1^2/σ_2^2. Does the assumption made earlier of equal variances seem reasonable?

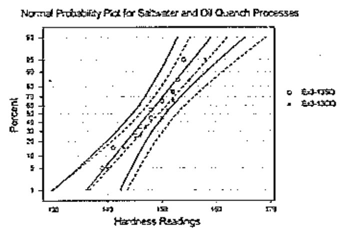
$\alpha=0.05$; $F_{1-\alpha/2,n_2-1,n_1-1}=F_{0.975,9,9}=0.2484$; $F_{\alpha/2,n_2-1,n_1-1}=F_{0.025,9,9}=4.0260$

$$\frac{S_1^2}{S_2^2}F_{1-\alpha/2,n_2-1,n_1-1}\le\frac{\sigma_1^2}{\sigma_2^2}\le\frac{S_1^2}{S_2^2}F_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{4.97^2}{5.46^2}(0.2484)\le\frac{\sigma_1^2}{\sigma_2^2}\le\frac{4.97^2}{5.46^2}(4.0260)$$

$$0.21\le\frac{\sigma_1^2}{\sigma_2^2}\le 3.34$$

(d) Does the assumption of normality seem appropriate for these data?



The normal distribution assumptions for both the saltwater and oil quench methods seem reasonable. However, the slopes on the normal probability plots do not appear to be the same, so the equal variance assumptions do not seem reasonable.

3-15. A random sample of 500 connecting rod pins contains 65 nonconforming units. Estimate the process fraction nonconforming.

(a) Test the hypothesis that the true fraction defective in this process is 0.08. Use $\alpha=0.05$.

$n=500$; $x=65$; $\hat{p}=x/n=65/500=0.130$

$p_0=0.08$, $\alpha=0.05$. Test $H_0: p=0.08$ versus $H_1: p\neq 0.08$. Reject H_0 if $|Z_0|>Z_{\alpha/2}$.

$np_0=500(0.08)=40$

Since $(x=65)>(np_0=40)$,

$$Z_0=\frac{(x-0.5)-np_0}{\sqrt{np_0(1-p_0)}}=\frac{(65-0.5)-40}{\sqrt{40(1-0.08)}}=4.0387$$

$Z_{\alpha/2}=Z_{0.05/2}=Z_{0.025}=1.96$

Reject H_0, and conclude that the sample process fraction nonconforming does differ from 0.08.

(b) Find the P-value for this test.

$P=2\left[1-\Phi|Z_0|\right]=2\left[1-\Phi|4.0387|\right]=2\left[1-0.99997\right]=0.00006$

(c) Construct a 95% upper confidence interval on the true process fraction nonconforming.

$\alpha=0.05$; $Z_\alpha=Z_{0.05}=1.645$

$$p\le\hat{p}+Z_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p\le 0.13+1.645\sqrt{\frac{0.13(1-0.13)}{500}}$$

$p\le 0.155$

The 95% upper confidence interval for the process fraction nonconforming is 0.155.

3-17. A new purification unit is installed in a chemical process. Before its installation, a random sample yielded the following data about the percentage of impurity: $\bar{x}_1=9.85$, $S_1^2=81.73$ and $n_1=10$. After installation, a random sample resulted in $\bar{x}_2=8.08$, $S_2^2=78.46$ and $n_2=8$.

before: $n_1=10$; $\bar{x}_1=9.85$; $S_1^2=81.73$

after: $n_2=8$; $\bar{x}_2=8.08$; $S_2^2=78.46$

(a) Can you conclude that the two variances are equal? Use $\alpha=0.05$.

Test $H_0:\sigma_1^2=\sigma_2^2$ versus $H_1:\sigma_1^2\neq\sigma_2^2$, at $\alpha=0.05$

Reject H_0 if $F_0>F_{\alpha/2,n_1-1,n_2-1}$ or $F_0<F_{1-\alpha/2,n_1-1,n_1-1}$

$F_{\alpha/2,n_1-1,n_2-1}=F_{0.025,9,7}=4.8232$; $F_{1-\alpha/2,n_1-1,n_1-1}=F_{0.975,9,7}=0.2383$

$$F_0=\frac{S_1^2}{S_2^2}=\frac{81.73}{78.46}=1.0417$$

$F_0=1.0417<4.8232$ and >0.2383, so do not reject H_0

The impurity variances before and after are the same.

(b) Can you conclude that the new purification device has reduced the mean percentage of impurity? Use $\alpha = 0.05$.

Test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq$ [illegible] $\alpha = 0.05$
Reject H_0 if $t_0 > t_{\alpha,v}$
The degrees of freedom are first calculated followed by the test on the sample averages.

$$v = \frac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1+1}+\frac{(S_2^2/n_2)^2}{n_2+1}} - 2 = \frac{\left(\frac{81.73}{10}+\frac{78.46}{10}\right)^2}{\frac{(81.73/10)^2}{10+1}+\frac{(78.46/8)^2}{8+1}} - 2 = 19.3 - 2 = 18$$

$t_{\alpha,v} = t_{0.05,18} = 1.7341$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} = \frac{9.85-8.08}{\sqrt{\frac{81.73}{10}+\frac{78.46}{8}}} = 0.4174$$

$t_0 = 0.4174 < 1.7341$, so do not reject H_0
The mean impurities before and after are the same.

3-19. The diameter of a metal rod is measured by 12 inspectors, each using both a micrometer caliper and a vernier caliper. The results are shown here. Is there a difference between the mean measurements produced by the two types of caliper? Use $\alpha = 0.01$.

Inspector	Micrometer Caliper	Vernier Caliper	Inspector	Micrometer Caliper	Vernier Caliper
1	0.150	0.151	7	0.151	0.153
2	0.151	0.150	8	0.153	0.155
3	0.151	0.151	9	0.152	0.154
4	0.152	0.150	10	0.151	0.151
5	0.151	0.151	11	0.151	0.150
6	0.150	0.151	12	0.151	0.152

Calculate the descriptive statistics:
micrometer: $n_1 = 12$; $\bar{x}_1 = 0.1512$; $S_1 = 0.0008$
vernier: $n_2 = 12$; $\bar{x}_2 = 0.1516$; $S_2 = 0.0016$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(12-1)0.0008^2+(12-1)0.0016^2}{12+12-2}} = 0.0013$$

Test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$, at $\alpha = 0.01$
Reject H_0 if $|t_0| > t_{\alpha/2, n_1+n_2-2}$
$t_{\alpha/2,n_1+n_2-2} = t_{0.005,22} = 2.8188$

$$t_0 = \frac{\bar{x}_1-\bar{x}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{0.1512-0.1516}{0.0013\sqrt{\frac{1}{12}+\frac{1}{12}}} = -0.7537$$

$|t_0| = 0.7537 < 2.8188$, so do not reject H_0
There is not a difference in the mean measurements.

3-21. An experiment was conducted to investigate the filling capability of packaging equipment at a winery in Newberg, Oregon. Twenty bottles of Pinot Gris were randomly selected and the fill volume (in ml) measured. Assume that fill volume has a normal distribution. The data are as follows:

753	751	752	753	753
753	752	753	754	754
752	751	752	750	753
755	753	756	751	750

(a) Do the data support the claim that the standard deviation of fill volume is less that 1 ml? Use $\alpha = 0.05$.

Calculate the descriptive statistics:
$n = 20$; $\bar{x} = 752.6$ ml; $S = 1.5$ ml
Test $H_0 : \sigma^2 = 1$ versus $H_1 : \sigma^2 < 1$, at $\alpha = 0.05$
Reject H_0 if $\chi_0^2 < \chi_{1-\alpha,n-1}^2$
$\chi_{1-\alpha,n-1}^2 = \chi_{0.95,19}^2 = 10.1170$

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(20-1)1.5^2}{1} = 42.75$$

$\chi_0^2 = 42.75 > 10.1170$, so do not reject H_0
The standard deviation of the fill volume is not less than 1ml.

(b) Find a 95% two-sided confidence interval on the standard deviation of fill volume.

$\alpha = 0.10$; $\chi_{\alpha/2,n-1}^2 = \chi_{0.05,19}^2 = 30.1435$; $\chi_{1-\alpha/2,n-1}^2 = \chi_{0.95,19}^2 = 10.1170$

$$\frac{(n-1)S^2}{\chi_{\alpha/2,n-1}^2} \le \sigma^2 \le \frac{(n-1)S^2}{\chi_{1-\alpha/2,n-1}^2}$$

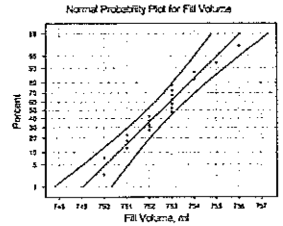
$$\frac{(20-1)1.5^2}{30.1435} \le \sigma^2 \le \frac{(20-1)1.5^2}{10.1170}$$

$$1.4182 \le \sigma^2 \le 4.2256$$

$$1.1909 \le \sigma \le 2.0556$$

The confidence interval does not include unity; therefore, we cannot conclude that the standard deviation of fill volume is less than 1 ml.

(c) Does it seem reasonable to assume that fill volume has a normal distribution?



The plotted points do not fall approximately along a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-23. Consider the hypothesis

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

where σ^2 is known. Derive a general expression for determining the sample size for detecting a true mean of $\mu_1 \neq \mu_0$ with probability $1-\beta$ if the type I error is α.

Let $\mu_1 = \mu_0 + \delta$. From Eqn. 3-68, $\beta = \Phi\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$

If $\delta > 0$, then $\Phi\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$ is likely to be small compared to β. So,

$$\beta \approx \Phi\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$\Phi(\beta) \approx \Phi^{-1}\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$-Z_\beta \approx Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}$$

$$n \approx \left[(Z_{\alpha/2} + Z_\beta)\sigma/\delta\right]^2$$

3-25. Develop a test for the hypotheses

$H_0: \mu_1 = 2\mu_2$

$H_1: \mu_1 \neq 2\mu_2$

where σ_1^2 and σ_2^2 are known.

Given $x \sim N$, n_1, $\bar{x}_1$, n_2, $\bar{x}_2$, x_1 independent of x_2.

Assume $\mu_1 = 2\mu_2$ and let $Q = (\bar{x}_1 - \bar{x}_2)$.

$E(Q) = E(\bar{x}_1 - 2\bar{x}_2) = \mu_1 - 2\mu_2 = 0$

$$\text{var}(Q) = \text{var}(\bar{x}_1 - 2\bar{x}_2) = \text{var}(\bar{x}_1) + \text{var}(2\bar{x}_2) = \text{var}(\bar{x}_1) + 2^2\,\text{var}(\bar{x}_2) = \frac{\text{var}(x_1)}{n_1} + 4\frac{\text{var}(x_2)}{n_2}$$

$$Z_0 = \frac{Q-0}{SD(Q)} = \frac{\bar{x}_1 - 2\bar{x}_2}{\sqrt{\sigma_1^2/n_1 + 4\sigma_2^2/n_2}}$$

And, reject H_0 if $|Z_0| > Z_{\alpha/2}$

3-27. An inspector counts the surface-finish defects in dishwashers. A random sample of five dishwashers contains three such defects. Is there reason to conclude that the mean occurrence rate of surface-finish defects per dishwasher exceeds 0.5? Use the results of part (a) of Exercise 3-26 and assume that $\alpha = 0.05$.

$x \sim \text{Poi}(\lambda)$; $n = 5$; $x = 3$; $\bar{x} = x/N = 3/5 = 0.6$

Test $H_0: \lambda = 0.5$ versus $H_1: \lambda > 0.5$, at $\alpha = 0.05$

Reject H_0 if $Z_0 > Z_\alpha$

$Z_\alpha = Z_{0.05} = 1.645$

$$Z_0 = \frac{\bar{x} - \lambda_0}{\sqrt{\lambda_0/n}} = \frac{0.6 - 0.5}{\sqrt{0.5/5}} = 0.3162$$

$Z_0 = 0.3162 < 1.645$, so do not reject H_0

3-29. An article in *Solid State Technology* (May 1987) describes an experiment to determine the effect of C_2F_6 flow rate on etch uniformity on a silicon wafer used in integrated circuit manufacturing. Three flow rates are tested, and the resulting uniformity (in percent) is observed for six test units at each flow rate. The data are shown in the following table.

	Observation					
C_2F_6 Flow (SCCM)	1	2	3	4	5	6
125	2.7	2.6	4.6	3.2	3.0	3.8
160	4.6	4.9	5.0	4.2	3.6	4.2
200	4.6	2.9	3.4	3.5	4.1	5.2

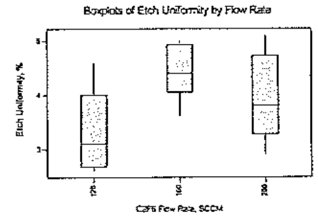
(a) Does C_2F_6 flow rate affect etch uniformity? Answer the question by using an analysis of variance with $\alpha = 0.05$.

The Minitab output is:

Source	DF	SS	MS	F	P
Ex3-29Fl	2	3.648	1.824	3.59	0.053
Error	15	7.630	0.509		
Total	17	11.278			

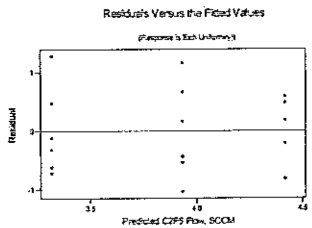
$F_{0.05,2,15} = 3.6823 > F_0$, so flow rate does not affect etch uniformity with significance level $\alpha = 0.05$. However, the P-value is just slightly greater than 0.05, so there is some evidence that gas flow rate affects the etch uniformity.

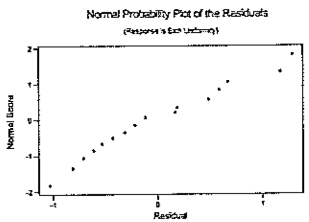
(b) Construct a box plot of etch uniformity data. Use this plot, together with the analysis of variance results, to determine which gas flow rate would be best in terms of etch uniformity (a small percentage is best).



Gas flow rate of 125 SCCM gives smallest percentage uniformity.

(c) Plot the residuals versus predicted C_2F_6 flow. Interpret this plot.





Residuals are satisfactory.

(d) Does the normality assumption seem reasonable in this problem?

Points fall along an approximately straight line, so the normality assumption is reasonable.

3-31. An article in the *ACI Materials Journal* (Vol. 84, 1987, pp. 213-216) describes several experiments investigating the rodding of concrete to remove entrapped air. A 3-in. diameter cylinder was used, and the number of times this rod was used is the design variable. The resulting compressive strength of the concrete specimen is the response. The data are shown in the following table.

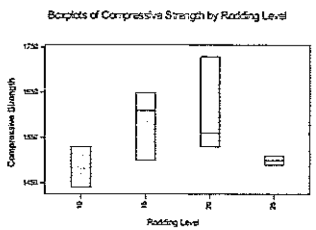
Rodding Level	Compressive Strength		
10	1530	1530	1440
15	1610	1650	1500
20	1560	1730	1530
25	1500	1490	1510

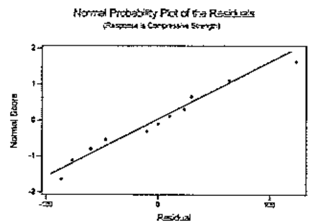
(a) Is there any difference in compressive strength due to the rodding level? Answer this question by using the analysis of variance with $\alpha = 0.05$.

Source	DF	SS	MS	F	P
Ex3-30Ro	3	28633	9544	1.87	0.214
Error	8	40933	5117		
Total	11	69567			

No difference due to rodding level at $\alpha = 0.05$ because the p-value is $0.214 > 0.005$.

(b) Construct box plots of compressive strength by rodding level. Provide a practical interpretation of these plots.





Level 25 exhibits considerably less variability than the other three levels.

(c) Construct a normal probability plot of the residuals from this experiment. Does the assumption of a normal distribution for compressive strength seem reasonable?

The normal distribution assumption for compressive strength is reasonable.

3-33. An aluminum producer manufactures carbon anodes and bakes them in a ring furnace prior to use in the smelting operation. The baked density of the anode is an important quality characteristic, as it may affect anode life. One of the process engineers suspects that firing temperature in the ring furnace affects baked anode density. An experiment was run at four different temperature levels, and six anodes were baked at each temperature level. The data from the experiment follow:

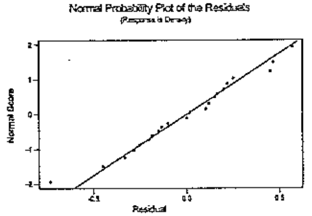
Temperature (°C)	Density					
500	41.8	41.9	41.7	41.6	41.5	41.7
525	41.4	41.3	41.7	41.6	41.7	41.8
550	41.2	41.0	41.6	41.9	41.7	41.3
575	41.0	40.6	41.8	41.2	41.9	41.5

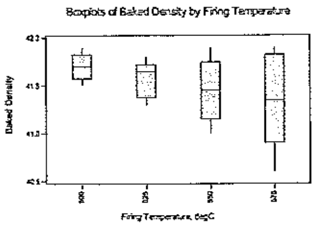
(a) Does firing temperature in the ring furnace affect mean baked anode density?

```
Source     DF       SS        MS        F       P
Ex3-33T     3    0.457     0.152     1.45   0.258
Error      20    2.097     0.105
Total      23    2.553
```

Temperature level does not significantly affect mean baked anode density. The p-value is 0.258 > 0.05.

(b) Find the residuals for this experiment and plot them on a normal probability scale. Comment on the plot.





Normality assumption is reasonable.

(c) What firing temperature would you recommend?

Since statistically there is no evidence to indicate that the means are different, select the temperature with the smallest variance, 500°C (see Boxplot), which probably also incurs the smallest cost (lowest temperature).

3-35. An article in *Environmental International* (Vol. 18, No. 4, 1992) describes an experiment in which the amount of radon released in showers was investigated. Radon-enriched water was used in the experiment, and six different orifice diameters were tested in shower heads. The data from the experiment are shown in the following table.

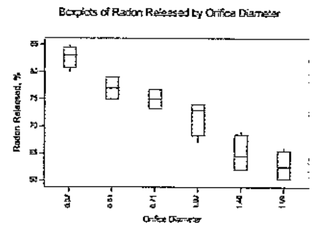
Orifice Diameter	Radon Released (%)			
0.37	80	83	83	85
0.51	75	75	79	79
0.71	74	73	76	77
1.02	67	72	74	74
1.40	62	62	67	69
1.99	60	61	64	66

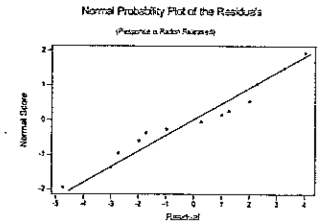
(a) Does the size of the orifice affect the mean percentage of radon released? Use the analysis of variance and $\alpha = 0.05$.

```
Source      DF       SS        MS        F       P
Ex3-35Di     5   1133.38    226.68    30.85   0.000
Error       18    132.25      7.35
Total       23   1265.63
```

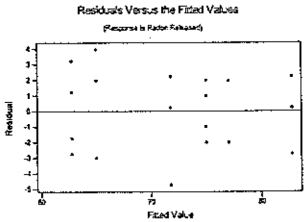
Orifice size does affect mean % radon release, at $\alpha = 0.05$.

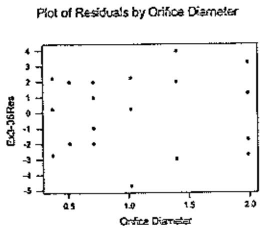
(b) Analyze the results from this experiment.





Smallest % radon released at 1.99 and 1.4 orifice diameters.
Residuals violate the normality assumption; there is a reverse S-shape in the probability plot.





CHAPTER 4

Methods and Philosophy of Statistical Process Control

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Describe and apply "the magnificent seven" fundamental problem-solving tools:
   - Histogram or stem-and-leaf display
   - Check sheet
   - Pareto chart
   - Cause-and-effect diagram
   - Defect concentration diagram
   - Scatter diagram
   - Control chart
2. Explain the difference between chance and assignable causes of process variation and their connection to statistical process control, sample size, sample frequency, and rational subgroups.
3. Construct a Shewhart control chart.
4. Discuss benefits from the use of control charts:
   - Improve productivity
   - Prevent defects
   - Prevent unnecessary process adjustment
   - Provide diagnostic information
   - Provide information on process capability
5. Calculate the ARL_0, ARL_1, and ATS that characterize an SPC scheme.
6. Detect nonrandom patterns of behavior on control charts.

7. Describe the use and misuse of sensitizing rules, including the Western Electric Company decision rules.

Exercises

4-1. What are chance and assignable causes of variability? What part do they play in the operation and interpretation of a Shewhart control chart?

"Chance" or "common" causes of variability represent the inherent, natural variability of a process - its background noise. Variation resulting from "assignable" or "special" causes represents generally large, unsatisfactory disturbances to the usual process performance. Assignable cause variation can usually be traced, perhaps to a change in material, equipment, or operator method.

A Shewhart control chart can be used to monitor a process and to identify occurrences of assignable causes. There is a high probability that an assignable cause has occurred when a plot point is outside the chart's control limits. By promptly identifying these occurrences and acting to permanently remove their causes from the process, we can reduce process variability in the long run.

4-3. Discuss type I and type II errors relative to the control chart. What practical implication in terms of process operation do these two types of errors have?

Relative to the control chart, the type I error represents the probability of concluding the process is out of control when it isn't, meaning a plot point is outside the control limits when in fact the process is still in control. In process operation, high frequencies of false alarms could lead could to excessive investigation costs, unnecessary process adjustment (and increased variability), and lack of credibility for SPC methods.

The type II error represents the probability of concluding the process is in control, when actually it is not; this results from a plot point within the control limits even though the process mean has shifted out of control. The effect on operations of failing to detect an out-of-control shift would be an increase in non-conforming product and associated costs.

4-5. If a process is in a state of statistical control, does it necessarily follow that all or nearly all of the units of product produced will be within specification limits?

No. The fact that a process operates in a state of statistical control does not mean that nearly all product meets specifications. It simply means that process behavior (mean and variation) is statistically predictable. We may very well predict that, say, 50% of the product will not meet specification limits! *Capability* is the term that refers to the ability to meet product specifications, and a process must be in control in order to calculate capability.

This is a key concept! There is no relationship between product specification limits and statistically determined process control limits. Specification limits are used to characterize product as either acceptable ("good") or unacceptable ("bad") for intended use. Control limits are derived from process data.

4-7. What are warning limits on a control chart? How can they be used?

Warning limits on control charts are limits that are inside the control limits. When warning limits are used, control limits are referred to as action limits. Warning limits, say at 2-sigma, can be used to increase chart sensitivity and to potentially signal process changes more quickly than the 3-sigma action limits. The Western Electric rule which addresses this type of shift is to consider a process to be out of control if 2 of 3 plot points (on the same side of the center line) are between 2-sigma and 3-sigma of the chart center line.

4-9. When taking samples or subgroups from a process, do you want assignable causes occurring within the subgroups or between them? Fully explain your answer.

We would want assignable causes to occur between subgroups and would prefer to select samples as close to consecutive as possible. In most SPC applications, process changes will not be self-correcting, but will require action to return the process to its usual performance level. The probability of detecting a change (and initiating a corrective action) is maximized by taking observations within a sample as close together as possible.

4-11. A manufacturing process produces 500 parts per hour. A sample part is selected about every half-hour, and after five parts are obtained, the average of these five measurements is plotted on an $\bar{x}$ control chart.

(a) Is this an appropriate sampling scheme if the assignable cause in the process results in an instantaneous upward shift in the mean that is of very short duration?

No. If the "very short duration" is less than a half hour, it is unlikely that a part will be selected from the out-of-control condition.

(b) If your answer is no, propose an alternative procedure.

The problem is that the process may shift to an out-of-control state and back to an in-control state in less than one-half hour. Each subgroup should be a random sample of all parts produced in the last 2½ hours.

4-13. If the time order of production has not been recorded in a set of data from a process, is it possible to detect the presence of assignable causes?

No. If time order of the data is not preserved, it will be impossible to separate the presence of assignable causes from underlying process variability.

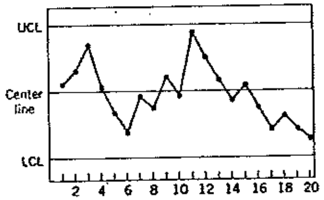
4-15. How do the costs of sampling, the costs of producing an excessive number of defective units, and the costs of searching for assignable causes impact on the choice of parameters of a control chart?

The costs of sampling, excessive defective units, and searches for assignable causes impact selection of the control chart parameters of sample size n, sampling frequency h, and control limit width. The larger n and h, the larger will be the cost of sampling. This sampling cost must be weighed against the cost of producing non-conforming product.

4-17. Consider the control chart shown here. Does the pattern appear random?

4-20. Consider the control chart shown in Exercise 4-17. Would the use of warning limits reveal any potential out-of-control conditions?

4-21. Apply the Western Electric rules to the control chart in Exercise 4-17. Are any of the criteria for declaring the process out of control satisfied?



Evaluate the plot points to the Western Electric rules listed on text p. 174:

- Does any 1 point plot outside the three-sigma control limits? No, all points are within the UCL and LCL.
- Do any 2 of 3 consecutive points plot beyond the two-sigma warning limits? No, points 3, 11, and 20 appear to be beyond two-sigma, but do not violate the decision rule. If warning limits were used, these points would indicate potential out-of-control conditions.
- Do any 4 of 5 consecutive points plot at a distance of one-sigma or beyond from the center line? Point 18 appears to be beyond one-sigma; if so, points 17, 18, 19, and 20 violate this decision rule and form a nonrandom pattern.
- Do any 8 consecutive points plot on one side of the center line? No, there is a good mix of points above and below the center line.

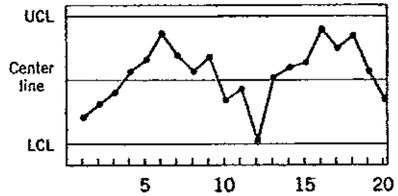
The control chart could also be evaluated to the sensitizing rules listed in Table 4-1 (text p.176), but the danger in applying many rules is a significant increase in the rate of false alarms.

Although there appears to be a decreasing trend from point 11 to point 20, rule 5 (6 points in a row steadily increasing or decreasing) is not violated. In practice, a steadily increasing or decreasing trend is hardly ever observed. If a decreasing trend does exist in this process, rule 2 will likely be violated on the next plot point.

4-19. Consider the control chart shown here. Does the pattern appear random?

4-22. Sketch warning limits on the control chart in Exercise 4-19. Do these limits indicate any potential out-of-control conditions?

4-23. Apply the Western Electric rules to the control chart presented in Exercise 4-19. Would these rules result in any out-of-control signals?



There is a "low—high—low—high—low" pattern, which may be indicative of cyclic behavior. If this pattern continues, further investigation may find a problem with the process such as shift operations, raw material, or time-based effects such as heat or stress build-up. The appearance of up/down runs may be part of the cyclic behavior.

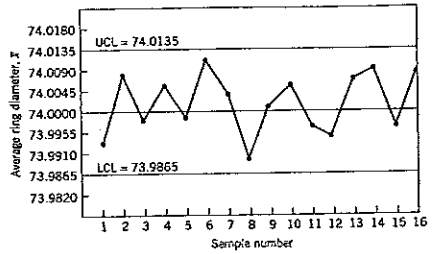
Several single points appear to violate two-sigma warning limits: points 6, 12, 16, and 18.

Evaluate the plot points to the Western Electric rules listed on text p. 174:

- Does any 1 point plot outside the three-sigma control limits? No, though point 12 is close, it is still above the lower control limit.
- Do any 2 of 3 consecutive points plot beyond the two-sigma warning limits? Points 16 and 18 appear to be beyond an upper two-sigma line, so points 16, 17, and 18 violate the decision rule.
- Do any 4 of 5 consecutive points plot at a distance of one-sigma or beyond from the center line? No, the points appear to be randomly distributed relative to the one-sigma distance.
- Do any 8 consecutive points plot on one side of the center line? No, there is a good mix of points above and below the center line.

The control chart could also be evaluated to the sensitizing rules listed in Table 4-1 (text. p. 176), but recall the danger in applying many rules is the increase in the rate of false alarms. Although there appears to be an increasing trend from point 1 to point 6, rule 5 (6 points in a row steadily increasing or decreasing) is not violated—there are only 5 points in the run.

4-25. Consider the X chart for the piston-ring example in this chapter, Figure 4-3. Let ring diameter be normally distributed, and the sample size is $n = 5$.



(a) Find the 2-sigma control limits for this chart.

UCL = 74.0135, LCL = 73.9865, and $\bar{\bar{x}} = 74.0000$

$$\sigma_{\bar{x}} = \frac{\text{UCL} - \bar{\bar{x}}}{3} = \frac{74.0135 - 74.0000}{3} = 0.0045$$

$$\text{UCL}_{2-S} = \bar{\bar{x}} + 2\sigma_{\bar{x}} = 74.0000 + 2(0.0045) = 74.0090$$

$$\text{LCL}_{2-S} = \bar{\bar{x}} - 2\sigma_{\bar{x}} = 74.0000 - 2(0.0045) = 73.9910$$

(b) Suppose it was suggested that the 2-sigma limits be used instead of the typical 3-sigma limits. What effect would this have on the occurrence of false alarms?

Visual examination of the $\bar{x}$ chart in Figure 4-3 shows no point exceeds the 3-sigma control limits, but three points (6, 8, and 14) exceed the 2-sigma warning limits. Decreasing the width of the control limits generally leads to an increase in the number of false alarms.

(c) What effect would the use of 2-sigma limits have on the in-control ARL of the chart?

Use eqn. 4-2 (text p. 167) to evaluate the in-control ARL.

For an $\bar{x}$ chart with three-sigma limits, the probability that a single point falls outside the limits when the process is in control is $p = 2 \times (1 - \Phi(3)) = 2 \times (1 - 0.99865) = 0.0027$:
$\text{ARL}_{0,3\text{-}S} = 1/p = 1/0.0027 \approx 370$

For an $\bar{x}$ chart with two-sigma limits, the probability that a single point falls outside the limits when the process is in control is $p = 2 \times (1 - \Phi(2)) = 2 \times (1 - 0.97725) = 0.0455$:
$\text{ARL}_{0,2\text{-}S} = 1/p = 1/0.0455 \approx 22$.

Using 2-sigma limits shortens the in-control average run length from 370 to 22 samples.

(d) Discuss the meaning of your findings in parts (b) and (c).

Figure 4-3 illustrates the effect of using narrower limits than the 3-sigma control limits— the number of false alarms will increase, the in-control ARL will shorten, and there will be more investigations for assignable causes. Whether this is acceptable depends on the penalty cost for non-conforming product. Are the costs of extra investigations and additional equipment downtime less than the costs of increased nonconformities?

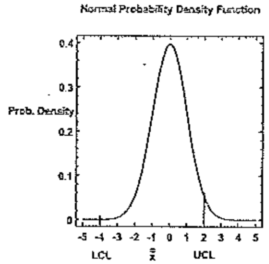
Also, as you will learn in Chapter 11, over-adjustment of a process can actually increase overall process variability. If the reaction plan for out-of-control signals includes adjusting the process, the unintended result may be increased variation.

Another consideration is the *psychological* impact of searching for assignable special causes, when none exist. Repeated false alarms tend to de-sensitize the personnel responsible for detecting and repairing process issues.

4-27. Consider the two decision rules given in Exercise 4-26:
"**Rule 1:** If *one or more* of the next seven samples yield values of the sample average that fall outside the control limits, conclude that the process is out of control.
Rule 2: If *all* of the next seven sample averages fall on the same side of the center line, conclude that the process is out of control."

If the mean of the quality characteristic shifts one standard deviation—that is, goes out of control by one-sigma—and remains there during the collection of the next seven samples, what is the β-risk associated with each decision rule?

Assume the decision rules apply to a normally distributed quality characteristic, the control chart has 3-sigma control limits, and the sample size is $n = 5$.



$\bar{\bar{x}}_{new} = \bar{\bar{x}} + \sigma_{\bar{x}}$

Rule 1:

$\Pr\{\text{inside control limits}\} = \Pr\{LCL \le \bar{x} \le UCL\}$

$= \Phi(+2) - \Phi(-4) = 0.97725 - 0.00003 = 0.97695$

$\beta_1 = \Pr\{\text{all 7 inside control limits}\} = (0.97695)^7 = \underline{0.84939}$

Rule 2:

$\Pr\{1 \text{ above centerline}\} = \Pr\{\bar{\bar{x}} \le \bar{x} \le UCL\}$

$= \Phi(+2) - \Phi(-1) = 0.97725 - 0.15866 = 0.81859$

$\Pr\{1 \text{ below centerline}\} = \Pr\{LCL \le \bar{x} \le \bar{\bar{x}}\}$

$= \Phi(-1) - \Phi(-4) = 0.15866 - 0.00003 = 0.15863$

$\beta_2 = 1 - \Pr\{\text{all above}\} - \Pr\{\text{all below}\} = 1 - (0.81859)^7 - (0.15863)^7 = 0.75370$

4-29. A quality characteristic is monitored by a control chart designed so the probability that a certain out-of-control condition will be detected on the first sample following the shift to that state is $1 - \beta$. Find the following:

(a) The probability that the out-of-control condition will be detected on the second sample following the shift.

$1 - \beta = \Pr\{\text{shift detected on } 1^{st} \text{ sample}\}$, so

$\Pr\{\text{shift detected on } 2^{nd} \text{ sample}\} = \Pr\{\text{not detected on } 1^{st}\} \times \Pr\{\text{detected on } 2^{nd}\}$

$= [1 - (1 - \beta)] \times (1 - \beta) = \beta(1 - \beta)$

(b) The probability that the out-of-control condition will be detected on the m^{th} sample following the shift.

$\Pr\{\text{shift detected on } m^{th} \text{ sample}\} = \Pr\{\text{not detected on } (m-1)\} \times \Pr\{\text{detected on } m^{th}\}$

$= \beta^{m-1}(1 - \beta)$

(c) The expected number of subgroups analyzed before the shift is detected.

$p = \Pr\{\text{any point exceeds control limits}\}$, so $ARL_0 = 1/p = 1/(1-\beta)$

(d) The probability that the first sample following the shift produces a statistic that plots inside the control limits.

$\Pr\{1^{st} \text{ sample inside control limits}\} = \Pr\{\text{shift not detected on } 1^{st} \text{ sample}\} = 1 - (1 - \beta) = \beta$

(e) The probability that the shift is not detected on the mth subsequent sample.

$\Pr\{\text{shift not detected on } m^{th} \text{ sample}\} = [1 - (1 - \beta)]^m = \beta^m$

(f) The probability that two consecutive samples produce statistics that plot outside of the control limits.

$\Pr\{2 \text{ consecutive samples outside control limits}\} = (1 - \beta)(1 - \beta) = (1 - \beta)^2$

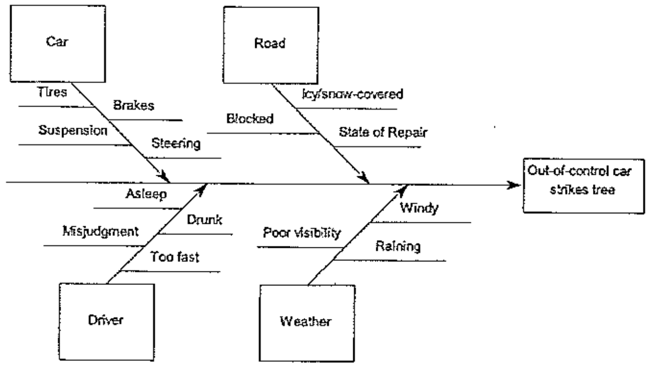
(g) The probability that at least one of the next m samples yields values of the test statistics that plot outside the control limits.

$\Pr\{\text{at least 1 of next } m \text{ samples outside control limits}\}$

$= 1 - \Pr\{\text{none of the next } m \text{ outside control limits}\} = 1 - \beta^m$

4-31. A car has gone out of control during a snowstorm and struck a tree. Construct a cause-and-effect diagram that identifies and outlines the possible causes of the accident.

In construct a cause-and-effect diagram, it is preferable to enlist the opinions and experience of a knowledgeable, cross-functional team. Cause-and-effect diagrams are often constructed with consideration for the 4-M's and 1-E: Machine, Method, Material, Manpower, and Environment. Any logical grouping of causes is appropriate. Many solutions are possible for this problem; one is given below:



CHAPTER 5

Control Charts for Variables

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Design, apply and interpret $\bar{x}$ and R control charts, computing control limits and applying out of control rules
2. Design, apply and interpret $\bar{x}$ and S control charts, computing control limits and applying out of control rules
3. Design, apply, and interpret and x and MR or Shewhart control charts for individual measurements

Exercises

5-1. The data shown here are $\bar{x}$ and R values for 24 samples of size n=5 taken from a process producing bearings. The measurements are made on the inside diameter of the bearing, with only the last three decimals recorded (i.e., 34.5 should be 0.50345).

Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
1	34.5	3	13	35.4	8
2	34.2	4	14	34.0	6
3	31.6	4	15	37.1	5
4	31.5	4	16	34.9	7
5	350.	5	17	33.5	4
6	34.1	6	18	31.7	3
7	32.6	4	19	34.0	8
8	33.8	3	20	35.1	4
9	34.8	7	21	33.7	2

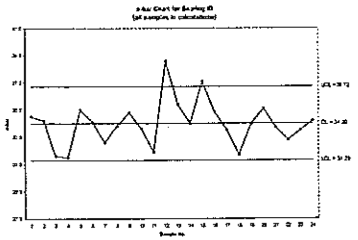
Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
10	33.6	8	22	32.8	1
11	31.9	3	23	33.5	3
12	38.6	9	24	34.2	2

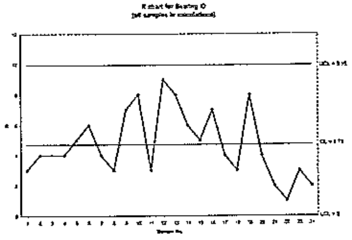
(a) Set up $\bar{x}$ and R charts on this process. Does it seem to be in statistical control? If necessary, revise the trial control limits.

$n=5;\ \bar{\bar{x}}=34.00;\ \bar{R}=4.71$

$UCL_{\bar{x}}=\bar{\bar{x}}+A_2\bar{R}=34.00+0.577(4.71)=36.72$ $\quad LCL_{\bar{x}}=\bar{\bar{x}}-A_2\bar{R}=34.00-0.577(4.71)=31.28$

$UCL_R=D_4\bar{R}=2.115(4.71)=9.96$ $\quad LCL_R=D_3\bar{R}=0(4.71)=0$



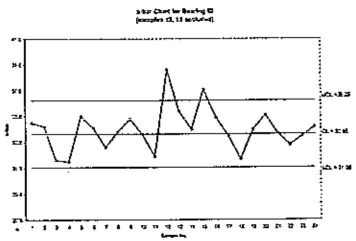


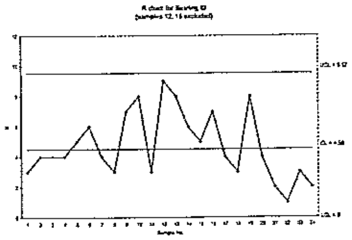
The process is not in statistical control; $\bar{x}$ is beyond the upper control limit for both Sample No. 12 and Sample No. 15. With these two samples excluded from the control limit calculations:

$\bar{\bar{x}}=33.65;\ \bar{R}=4.5;\ \hat{\sigma}_x=\bar{R}/d_2=4.5/2.326=1.93$

$UCL_{\bar{x}}=\bar{\bar{x}}+A_2\bar{R}=33.65+0.577(4.50)=36.25$ $\quad LCL_{\bar{x}}=\bar{\bar{x}}-A_2\bar{R}=33.65-0.577(4.50)=31.05$

$UCL_R=D_4\bar{R}=2.115(4.50)=9.52$ $\quad LCL_R=D_3\bar{R}=0(4.50)=0$





(b) If specifications on this diameter are 0.5030 ± 0.0010, find the percentage of nonconforming bearings produced by this process. Assume that diameter is normally distributed.

$\hat{p}=\Pr\{x<LSL\}+\Pr\{x>USL\}=\Pr\{x<20\}+\Pr\{x>40\}$

$=\Phi\left(\frac{20-33.65}{1.93}\right)+1-\Phi\left(\frac{40-33.65}{1.93}\right)$

$=\Phi(-7.07)+1-\Phi(3.29)=0+1-0.99950=0.00050$

5-3. The data shown here are the deviations from nominal diameter for holes drilled in a carbon-fiber composite material used in aerospace manufacturing. The values reported are deviations from nominal in ten-thousandths of an inch.

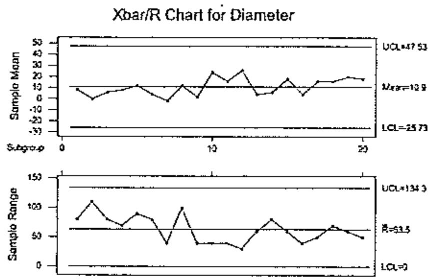
Sample Number	x_1	x_2	x_3	x_4	x_5	Sample Number	x_1	x_2	x_3	x_4	x_5
1	-30	+50	-20	+10	+30	11	+40	0	+20	0	+20
2	0	+50	-60	-20	+30	12	+30	+20	+30	+10	+40
3	-50	+10	+20	+30	+20	13	+30	-30	0	+10	+10
4	-10	-10	+30	-20	+50	14	+30	-10	+50	-10	-30
5	+20	-40	+50	+20	+10	15	+10	-10	+50	+40	0
6	0	0	+40	-40	+20	16	0	0	+30	-10	0
7	0	0	+20	-20	-10	17	+20	+20	+30	+30	-20
8	+70	-30	+30	-10	0	18	+10	-20	+50	+30	+10
9	0	0	+20	-20	+10	19	+50	-10	+40	+20	0
10	+10	+20	+30	+10	+50	20	+50	0	0	+30	+10

(a) Set up $\bar{x}$ and R charts on the process. Is the process in statistical control?

$n=5;\ \bar{\bar{x}}=10.9;\ \bar{R}=63.5$

$UCL_{\bar{x}}=\bar{\bar{x}}+A_2\bar{R}=10.9+0.577(63.5)=47.5$ $\quad LCL_{\bar{x}}=\bar{\bar{x}}-A_2\bar{R}=10.9-0.577(63.5)=-25.7$

$UCL_R=D_4\bar{R}=2.115(63.5)=134.3$ $\quad LCL_R=D_3\bar{R}=0(63.5)=0$



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b) Estimate the process standard deviation using the range method.

$\hat{\sigma}_x = \bar{R} / d_2 = 63.5 / 2.326 = 27.3$

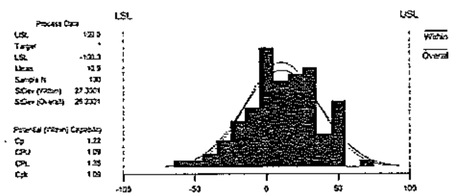
(c) If specifications are at nominal ± 100, what can you say about the capability of this process? Calculate the PCR C_p.

USL = +100, LSL = −100

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+100-(-100)}{6(27.3)} = 1.22 \text{, so the process is capable.}$$

Also, a histogram of the data is shown below with summary statistics as calculated by Minitab:

Process Capability Analysis for Diameter



5-5. The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size n=10 have been analyzed, and the fill heights are shown next.

Sample Number	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	2.5	0.5	2.0	-1.0	1.0	-1.0	0.5	1.5	0.5	-1.5
2	0.0	0.0	0.5	1.0	1.5	1.0	-1.0	1.0	1.5	-1.0
3	1.5	1.0	1.0	-1.0	0.0	-1.5	-1.0	-1.0	1.0	-1.0
4	0.0	0.5	-2.0	0.0	-1.0	1.5	-1.5	0.0	-2.0	-1.5
5	0.0	0.0	0.0	-0.5	0.5	1.0	-0.5	-0.5	0.0	0.0
6	1.0	-0.5	0.0	0.0	0.0	0.5	-1.0	1.0	-2.0	1.0
7	1.0	-1.0	-1.0	-1.0	0.0	1.5	0.0	1.0	0.0	0.0
8	0.0	-1.5	-0.5	1.5	0.0	0.0	0.0	-1.0	0.5	-0.5
9	-2.0	-1.5	1.5	1.5	0.0	0.0	0.5	1.0	0.0	1.0
10	-0.5	3.5	0.0	-1.0	-1.5	-1.5	-1.0	-1.0	1.0	0.5
11	0.0	1.5	0.0	0.0	2.0	-1.5	0.5	-0.5	2.0	-1.0
12	0.0	-2.0	-0.5	0.0	-0.5	2.0	1.5	0.0	0.5	-1.0
13	-1.0	-0.5	-0.5	-1.0	0.0	0.5	0.5	-1.5	-1.0	-1.0
14	0.5	1.0	-1.0	-0.5	-2.0	-1.0	-1.5	0.0	1.5	1.5
15	1.0	0.0	1.5	1.5	1.0	-1.0	0.0	1.0	-2.0	-1.5

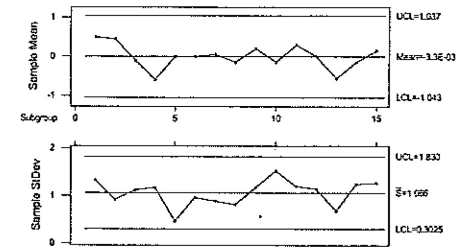
(a) Set up $\bar{x}$ and S charts on this process. Does the process exhibit statistical control? If necessary, construct revised control limits.

$n = 10$; $\bar{\bar{x}} = -0.0033$; $\bar{S} = 1.066$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S} = -0.0033 + 0.975(1.066) = 1.036$ $\quad$ $\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S} = -0.0033 - 0.975(1.066) = -1.043$

$\text{UCL}_R = B_4\bar{S} = 1.716(1.066) = 1.829$ $\quad$ $\text{LCL}_R = B_3\bar{S} = 0.284(1.066) = 0.303$

Xbar/S Chart for Fill Volume



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles.

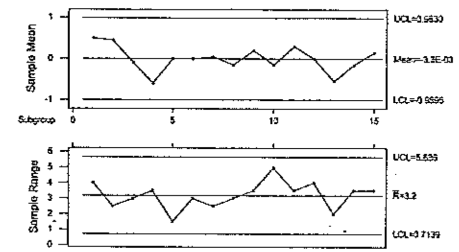
(b) Set up an R chart, and compare with the S chart in part (a).

$n = 10$; $\bar{\bar{x}} = -0.0033$; $\bar{R} = 3.200$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = -0.0033 + 0.308(3.200) = 0.982$ $\quad$ $\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = -0.0033 - 0.308(3.200) = -0.989$

$\text{UCL}_R = D_4\bar{R} = 1.777(3.200) = 5.686$ $\quad$ $\text{LCL}_R = D_3\bar{R} = 0.223(3.200) = 0.714$

Xbar/R Chart for Fill Volume



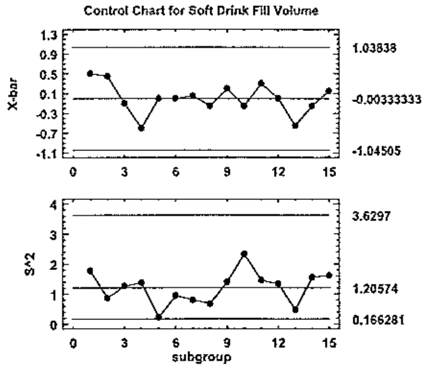
The process is in statistical control, with no out-of-control signals, runs, trends, or cycles. There is no difference in interpretation from the $\bar{x} - S$ chart.

(c) Set up an S^2chart, and compare with the S chart in part (a).

$n=10;\ \bar{\bar{x}}=-0.0033;\ \bar{S}^2=1.066$

$\text{UCL}_{\bar{x}}=\bar{\bar{x}}+A_3\bar{S}=-0.0033+0.975(1.066)=1.036 \qquad \text{LCL}_{\bar{x}}=\bar{\bar{x}}-A_3\bar{S}=-0.0033-0.975(1.066)=-1.043$

$\text{UCL}_R=\dfrac{\bar{S}^2}{n-1}\chi^2_{\alpha/2,n-1}=\dfrac{1.206}{10-1}27.093=3.630 \qquad \text{LCL}_R=\dfrac{\bar{S}^2}{n-1}\chi^2_{1-(\alpha/2),n-1}=\dfrac{1.206}{10-1}1.241=0.166$



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles. There is no difference in interpretation from either the $\bar{x}-R$ or the $\bar{x}-S$ chart.

5-9. Control charts on $\bar{x}$ and S are to be maintained on the torque readings of a bearing used in a wingflap actuator assembly. Samples of size $n=10$ are to be used, and we know from past experience that when the process is in control, bearing torque has a normal distribution with a mean $\mu=80$ inch-pounds and standard deviation $\sigma=10$ inch-pounds. Find the center line and control limits for these control charts.

$n=10;\ \mu=80$ in-lb; $\sigma_x=10$ in-lb; and $A=0.949;\ B_6=1.669;\ B_5=0.276$

$\text{centerline}_{\bar{x}}=\mu=80$

$\text{UCL}_{\bar{x}}=\mu+A\sigma_x=80+0.949(10)=89.49 \qquad \text{LCL}_{\bar{x}}=\mu-A\sigma_x=80-0.949(10)=70.51$

$\text{centerline}_S=c_4\sigma_x=0.9727(10)=9.727$

$\text{UCL}_S=B_6\sigma_x=1.669(10)=16.69 \qquad \text{LCL}_S=B_5\sigma_x=0.276(10)=2.76$

5-11. Samples of $n=6$ items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After 50 subgroups have been analyzed, we have

$$\sum_{i=1}^{50}\bar{x}_i=1000 \quad\text{and}\quad \sum_{i=1}^{50}S_i=75$$

(a) Compute control limits for the $\bar{x}$ and S control charts.

Assume that the quality characteristic is normally distributed.

$n=6$ items/subgroup; $\sum_{i=1}^{50}\bar{x}_i=1000;\ \sum_{i=1}^{50}S_i=75;\ m=50$ subgroups

$$\bar{\bar{x}}=\frac{\sum_{i=1}^{50}\bar{x}_i}{m}=\frac{1000}{50}=20;\ \bar{S}=\frac{\sum_{i=1}^{50}S_i}{m}=\frac{75}{50}=1.50$$

$\text{UCL}_{\bar{x}}=\bar{\bar{x}}+A_3\bar{S}=20+1.287(1.50)=21.93 \qquad \text{LCL}_{\bar{x}}=\bar{\bar{x}}-A_3\bar{S}=20-1.287(1.50)=18.07$

$\text{UCL}_S=B_4\bar{S}=1.970(1.50)=2.955 \qquad \text{LCL}_S=B_3\bar{S}=0.030(1.50)=0.045$

(b) Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process?

The natural process tolerance limits are $\bar{\bar{x}}\pm3\hat{\sigma}_x=\bar{\bar{x}}\pm3\left(\bar{S}/c_4\right)=20\pm3(1.50/0.9515)=[15.27,24.73]$.

(c) If the specification limits are 19 ± 4.0, what are your conclusions regarding the ability of the process to produce items conforming to specifications?

First calculate $\hat{\sigma}_x$, $\hat{\sigma}_x=\bar{S}/c_4=1.50/0.9515=1.58$. Then $\hat{C}_P=\dfrac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x}=\dfrac{+4.0-(-4.0)}{6(1.58)}=0.84$.

The process is not capable.

(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it falls below the lower specification limit it must be scrapped, what percentage scrap and rework is the process now producing?

$$\hat{p}_{\text{rework}}=\Pr\{x>\text{USL}\}=1-\Pr\{x\le\text{USL}\}=1-\Phi\left(\frac{USL-\bar{\bar{x}}}{\hat{\sigma}_x}\right)=1-\Phi\left(\frac{23-20}{1.58}\right)$$

$$=1-\Phi(1.90)=1-0.97120=0.02880=2.88\%$$

$$\hat{p}_{\text{scrap}}=\Pr\{x<\text{LSL}\}=\Phi\left(\frac{USL-\bar{\bar{x}}}{\hat{\sigma}_x}\right)=\Phi\left(\frac{15-20}{1.58}\right)=\Phi(-3.16)=0.0.00078=0.078\%$$

Total = 2.88% + 0.078% = 2.958%

(e) If the process were centered at $\mu = 19.0$, what would be the effect on scrap and rework?

$$\hat{p}_{\text{rework}} = 1 - \Phi\left(\frac{23-19}{1.58}\right) = 1 - \Phi(2.53) = 1 - 0.99432 = 0.00568\text{, or } 0.568\%$$

$$\hat{p}_{\text{scrap}} = \Phi\left(\frac{15-19}{1.58}\right) = \Phi(-2.53) = 0.00568\text{, or } 0.568\%$$

Total = 0.568% + 0.568% = 1.136%

Centering the process would reduce rework, but increase scrap. A cost analysis is needed to make the final decision. An alternative would be to apply resources to improve the process by reducing variability.

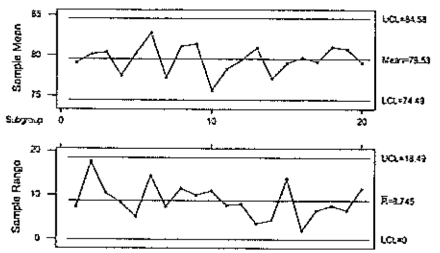
5-13. Parts manufactured by an injection molding process are subjected to a compressive strength test. Twenty samples of five parts each are collected, and the compressive strengths (in psi) are shown in the following table.

Sample Number	x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
1	83.0	81.2	78.7	75.7	77.0	79.1	7.3
2	88.6	78.3	78.8	71.0	84.2	80.2	17.6
3	85.7	75.8	84.3	75.2	81.0	80.4	10.4
4	80.8	74.4	82.5	74.1	75.7	77.5	8.4
5	83.4	78.4	82.6	78.2	78.9	80.3	5.2
6	75.3	79.9	87.3	89.7	81.8	82.8	14.5
7	74.5	78.0	80.8	73.4	79.7	77.3	7.4
8	79.2	84.4	81.5	86.0	74.5	81.1	11.4
9	80.5	86.2	76.2	64.1	80.2	81.4	9.9
10	75.7	75.2	71.1	82.1	74.3	75.7	10.9
11	80.0	81.5	78.4	73.8	78.1	78.4	7.7
12	80.6	81.8	79.3	73.8	81.7	79.4	8.0
13	82.7	81.3	79.1	82.0	79.5	80.9	3.6
14	79.2	74.9	78.6	77.7	75.3	77.1	4.3
15	85.5	82.1	82.8	73.4	71.7	79.1	13.8
16	78.8	79.6	80.2	79.1	80.8	79.7	2.0
17	82.1	78.2	75.5	78.2	82.1	79.2	6.6
18	84.5	76.9	83.5	81.2	79.2	81.1	7.6
19	79.0	77.8	81.2	84.4	81.6	80.8	6.6
20	84.5	73.1	78.6	78.7	80.6	79.1	11.4

(a) Establish $\bar{x}$ and R control charts for compressive strength using these data. Is the process in statistical control?

Original Data

Xbar/R Chart for Compressive Strength



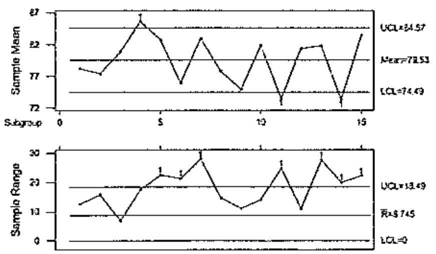
Yes, the process is in control—though we should watch for a possible cyclic pattern in the averages.

(b) After establishing the control charts in part (a), 15 new subgroups were collected and the compressive strengths are shown next. Plot the $\bar{x}$ and R values against the control units from part (a) and draw conclusions.

Sample Number	x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
1	68.9	81.5	78.2	80.8	81.5	78.2	12.6
2	69.8	68.6	80.4	84.3	83.9	77.4	15.7
3	78.5	85.2	78.4	80.3	81.7	80.8	6.8
4	76.9	86.1	86.9	94.4	83.9	85.6	17.5
5	93.6	81.6	87.8	79.6	71.0	82.7	22.5
6	65.5	86.8	72.4	82.6	71.4	75.9	21.3
7	78.1	65.7	83.7	93.7	93.4	82.9	27.9
8	74.9	72.6	81.6	87.2	72.7	77.8	14.6
9	78.1	77.1	67.0	75.7	76.8	74.9	11.0
10	78.7	85.4	77.7	90.7	76.7	81.9	14.0
11	85.0	60.2	68.5	71.1	82.4	73.4	24.9
12	86.4	79.2	79.8	96.0	75.4	81.3	10.9
13	78.5	99.0	78.3	71.4	81.8	81.7	27.6
14	68.8	62.0	82.0	77.5	76.1	73.3	19.9
15	83.0	83.7	73.1	82.2	95.3	83.5	22.2

New Data

Xbar/R Chart for Compressive Strength



A strongly cyclic pattern in the averages is now evident, but more importantly, there are several out-of-control points on the $\bar{x}$ chart (4, 11, 14) and the range chart (points 5, 6, 7, 11, 13, 14, 15).

5-15. Consider the $\bar{x}$ and R charts you established in Exercise 5-1 using $n = 5$.

(a) Suppose that you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of $n = 3$. What limits would be used on the $\bar{x}$ and R charts?

$n = 5$; $\bar{\bar{x}} = 34.00$; $\bar{R} = 4.7$

for $n = 3$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 34 + 1.023(4.7) = 38.81$ $\quad$ $\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 34 - 1.023(4.7) = 29.19$

$\text{UCL}_R = D_4\bar{R} = 2.574(4.7) = 12.1$ $\quad$ $\text{UCL}_R = D_3\bar{R} = 0(4.7) = 0$

(b) What would be the impact of the decision you made in part (a) on the ability of the $\bar{x}$ chart to detect a 2σ shift on the mean?

The $\bar{x}$ control limits for $n = 5$ are "tighter" (31.29, 36.72) than those for $n = 3$ (29.19, 38.81). This means a 2σ shift in the mean would be detected more quickly with a sample size of $n = 5$.

(c) Supposed you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of $n = 8$. What limits would be used on the $\bar{x}$ and R charts?

for $n = 8$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 34 + 0.373(4.7) = 35.75$

$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 34 - 0.373(4.7) = 32.25$

$\text{UCL}_R = D_4\bar{R} = 1.864(4.7) = 8.76$

$\text{UCL}_R = D_3\bar{R} = 0.136(4.7) = 0.64$

(d) What is the impact of using $n = 8$ on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

The $\bar{x}$ control limits for $n = 8$ are even "tighter" (32.25, 35.75), increasing the ability of the chart to quickly detect the 2σ shift in process mean.

5-17. Samples of size $n = 5$ are taken from a manufacturing process every hour. A quality characteristic is measured, and $\bar{x}$ and R are computed for each sample. After 25 samples have been analyzed, we have

$$\sum_{i=1}^{25} \bar{x}_i = 662.50 \text{ and } \sum_{i=1}^{50} R_i = 9.00$$

(a) Find the control limits for the $\bar{x}$ and R charts.

The quality characteristic is normally distributed.

$$n = 5,\ \sum_{i=1}^{25}\bar{x}_i = 662.50,\ \sum_{i=1}^{25} R_i = 9.00,\ m = 25 \text{ samples},\ \bar{\bar{x}} = \frac{\sum_{i=1}^{25}\bar{x}_i}{m} = \frac{662.50}{25} = 26.50\ \ \bar{R} = \frac{\sum_{i=1}^{25} R_i}{m} = \frac{9.00}{25} = 0.36$$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 26.50 + 0.577(0.36) = 26.71$ $\quad$ $\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 26.50 - 0.577(0.36) = 26.29$

$\text{UCL}_R = D_4\bar{R} = 2.115(0.36) = 0.76$ $\quad$ $\text{UCL}_R = D_3\bar{R} = 0(0.36) = 0$

(b) Assume that both charts exhibit control. If the specifications are 26.40 ± 0.50, estimate the fraction nonconforming.

$\hat{\sigma}_x = \bar{R}/d_2 = 0.36/2.326 = 0.155$

$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le \text{USL}\} + \Pr\{x < \text{LSL}\}$

$$= 1 - \Phi\left(\frac{USL - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + \Phi\left(\frac{LSL - \bar{\bar{x}}}{\hat{\sigma}_x}\right) = 1 - \Phi\left(\frac{26.90 - 26.50}{0.155}\right) + \Phi\left(\frac{25.90 - 26.50}{0.155}\right)$$

$$= 1 - \Phi(2.58) + \Phi(-3.87) = 1 - 0.99506 + 0.00005 = 0.00499$$

(c) If the mean of the process were 26.40, what fraction nonconforming would result?

$$\hat{p} = 1 - \Phi\left(\frac{26.90 - 26.40}{0.155}\right) + \Phi\left(\frac{25.90 - 26.40}{0.155}\right) = 1 - \Phi(3.23) + \Phi(-3.23) = 1 - 0.99938 + 0.00062 = 0.00124$$

5-19. An $\bar{x}$ chart is used to control the mean of a normally distributed quality characteristic. It is known that $\sigma = 6.0$ and $n = 4$. The center line = 200, UCL = 209, and LCL = 191. If the process mean shifts to 188, find the probability that this shift is detected on the first subsequent sample.

$$\Pr\{\text{detect}\} = 1 - \Pr\{\text{not detect}\} = 1 - [\Pr\{\text{LCL} \le \bar{x} \le \text{UCL}\}] = 1 - [\Pr\{\bar{x} \le \text{UCL}\} - \Pr\{\bar{x} \le \text{LCL}\}]$$

$$= 1 - \left[\Phi\left(\frac{\text{UCL}_{\bar{x}} - \mu_{new}}{\sigma_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL}_{\bar{x}} - \mu_{new}}{\sigma_x/\sqrt{n}}\right)\right] = 1 - \left[\Phi\left(\frac{209-188}{6/\sqrt{4}}\right) - \Phi\left(\frac{191-188}{6/\sqrt{4}}\right)\right]$$

$$= 1 - \Phi(7) + \Phi(1) = 1 - 1 + 0.84134 = 0.84134$$

5-21. A process is to be monitored with standard values $\mu = 10$ and $\sigma = 2.5$. The sample size is three.

(a) Find the center line and the control limits for the $\bar{x}$ chart.

$n = 3$; $\mu = 10$; $\sigma_x = 2.5$. These are standard values.

$\text{centerline}_{\bar{x}} = \mu = 10$

$\text{UCL}_{\bar{x}} = \mu + A\sigma_x = 10 + 0.949(2.5) = 12.373$ $\quad$ $\text{LCL}_{\bar{x}} = \mu - A\sigma_x = 10 - 0.949(2.5) = 7.628$

(b) Find the center line and control limits for the R chart.

$\text{centerline}_R = d_2\sigma_x = 3.078(2.5) = 7.695$

$\text{UCL}_R = D_2\sigma = 5.469(2.5) = 13.67$ $\quad$ $\text{LCL}_R = D_1\sigma = 0.687(2.5) = 1.72$

(c) Find the center line and control limits for the S chart.

$\text{centerline}_S = c_4\sigma_x = 0.9727(2.5) = 2.432$

$\text{UCL}_S = B_6\sigma = 1.669(2.5) = 4.17$ $\quad$ $\text{LCL}_S = B_5\sigma = 0.276(2.5) = 0.69$

5-23. Control charts for $\bar{x}$ and R are to be established to control the tensile strength of a metal part. Assume that tensile strength is normally distributed. Thirty samples of size $n = 6$ parts are collected over a period of time with the following results:

$$\sum_{i=1}^{30} \bar{x}_i = 6000 \quad \text{and} \quad \sum_{i=1}^{30} R_i = 150$$

(a) Calculate limits for $\bar{x}$ and R.

$$n = 6,\ \sum_{i=1}^{30} \bar{x}_i = 6000,\ \sum_{i=1}^{30} R_i = 150,\ m = 30 \text{ samples},\ \bar{\bar{x}} = \frac{\sum_{i=1}^{30} \bar{x}_i}{m} = \frac{6000}{30} = 200,\ \bar{R} = \frac{\sum_{i=1}^{30} R_i}{m} = \frac{150}{30} = 5$$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 200 + 0.483(5) = 202.42$ $\quad$ $\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 200 - 0.483(5) = 197.59$

$\text{UCL}_R = D_4\bar{R} = 2.004(5) = 10.02$ $\quad$ $\text{UCL}_R = D_3\bar{R} = 0(5) = 0$

(b) Both charts exhibit control. The specifications on tensile strength are 200 ± 5. What are your conclusions regarding process capability?

$\hat{\sigma}_x = \bar{R}/d_2 = 5/2.534 = 1.97$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+5-(-5)}{6(1.97)} = 0.85$$

So, the process is not capable of meeting specification. Even though the process is centered at nominal, the variation is large relative to the tolerance.

(c) For the above $\bar{x}$, find the β-risk when the true process mean is 199.

$$\beta\text{-risk} = \Pr\{\text{not detect}\} = \Phi\left(\frac{UCL - \mu}{\hat{\sigma}_x/\sqrt{n}}\right) - \Phi\left(\frac{LCL - \mu}{\hat{\sigma}_x/\sqrt{n}}\right) = \Phi\left(\frac{202.42 - 199}{1.97/\sqrt{6}}\right) - \Phi\left(\frac{197.59 - 199}{1.97/\sqrt{6}}\right)$$

$$= \Phi(4.25) - \Phi(-1.75) = 1 - 0.04006 = 0.95994$$

5-25. The following data were collected from a process manufacturing power supplies. The variable of interest is output voltage, and $n = 5$.

Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
1	103	4	11	105	4
2	102	5	12	103	2
3	104	2	13	102	3
4	105	11	14	105	4
5	104	4	15	104	5
6	106	3	16	105	3
7	102	7	17	106	5
8	105	2	18	102	2
9	106	4	19	105	4
10	104	3	20	103	2

(a) Computer center lines and control limits suitable for controlling future production.

$\bar{\bar{x}} = 104.05$, $\bar{R} = 3.95$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104.05 + 0.577(3.95) = 106.329$ $\quad$ $\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104.05 - 0.577(3.95) = 101.771$

$\text{UCL}_R = D_4\bar{R} = 2.115(3.95) = 8.354$ $\quad$ $\text{LCL}_R = D_3\bar{R} = 0(3.95) = 0$

By comparing the UCL_R to the data set, sample #4 is out of control on the Range chart. So, excluding #4 and recalculating:

$\bar{\bar{x}} = 104$; $\bar{R} = 3.579$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104 + 0.577(3.579) = 106.064$ $\quad$ $LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104 - 0.577(3.579) = 101.936$

$UCL_R = D_4\bar{R} = 2.115(3.579) = 7.567$ $\quad$ $LCL_R = D_3\bar{R} = 0(3.579) = 0$

(b) Assume that the quality characteristic is normally distributed. Estimate the process standard deviation.

Without sample #4, $\hat{\sigma}_x = \bar{R}/d_2 = 3.579/2.326 = 1.539$

(c) What are the apparent three-sigma natural tolerance limits of the process?

$UNTL = \bar{\bar{x}} + 3\hat{\sigma}_x = 104 + 3(1.539) = 108.62$ $\quad$ $LNTL = \bar{\bar{x}} - 3\hat{\sigma}_x = 104 - 3(1.539) = 99.38$

(d) What would be your estimate of the process faction nonconforming if the specifications on the characteristic were 103 ± 4?

$$\hat{p} = 1 - \Phi\left(\frac{107-104}{1.539}\right) + \Phi\left(\frac{99-104}{1.539}\right) = 1 - \Phi(1.95) + \Phi(-3.25) = 1 - 0.9744 + 0.0006 = 0.0262$$

(e) What approaches to reducing the fraction nonconforming can you suggest?

To reduce the fraction nonconforming, first center the process at nominal from 104 to 103.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{1.539}\right) + \Phi\left(\frac{99-103}{1.539}\right) = 1 - \Phi(2.60) + \Phi(-2.60) = 1 - 0.9953 + 0.0047 = 0.0094.$$

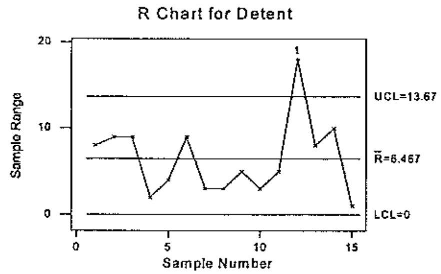
Next work on reducing the variability; if $\hat{\sigma}_x = 0.667$, then almost 100% of parts will be within specification.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{0.667}\right) + \Phi\left(\frac{99-103}{0.667}\right) = 1 - \Phi(5.997) + \Phi(-5.997) = 1 - 1.0000 + 0.0000 = 0.0000$$

5-27. Specifications on a cigar lighter detent are 0.3220 and 0.3200 in. Samples of size five are taken every 45 min with the following results (measured as deviations from 0.3210 in 0.0001 in.).

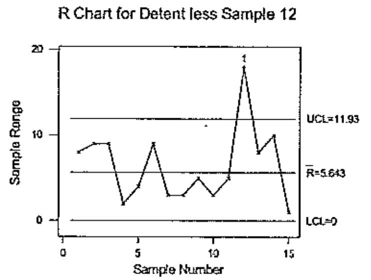
Sample Number	x_1	x_2	x_3	x_4	x_5
1	1	9	6	9	6
2	9	4	3	0	3
3	0	9	0	3	2
4	1	1	0	2	1
5	-3	0	-1	0	-4
6	-7	2	0	0	2
7	-3	-1	-1	0	-2
8	0	-2	-3	-3	-2
9	2	0	-1	-3	-1
10	0	2	-1	-1	2
11	-3	-2	-1	-1	2
12	-16	2	0	-4	-1
13	-6	-3	0	0	-8
14	-3	-5	5	0	5
15	-1	-1	-1	-2	-1

(a) Set up an R chart and examine the process for statistical control.



The process is not in statistical control -- sample #12 exceeds the upper control limit on the Range chart.

(b) What parameters would you recommend for an R chart for on-line control?



Excluding sample #12 and recalculating the parameters, $\bar{R} = 5.643$ and $UCL = 11.93$.

(c) Estimate the standard deviation of the process.

Without sample #12, $\hat{\sigma}_x = \bar{R}/d_2 = 5.64286/2.326 = 2.426$.

(d) What is the process capability?

Without sample #12, $\hat{PCR} = \dfrac{USL - LSL}{6\hat{\sigma}_x} = \dfrac{0.3220 - 0.3200}{6(2.426 \times 0.0001)} = 1.374$

Remember to convert back to the original units by multiplying by 0.0001.

5-29. Two parts are assembled as shown in the figure. Assume that the dimensions x and y are normally distributed with means μ_x and μ_y and standard deviations σ_x and σ_y respectively. The parts are produced on different machines and are assembled at random. Control charts are maintained on each dimension for the range of each sample ($n = 5$). Both range charts are in control.

(a) Given that for 20 samples on the range chart controlling x and 10 samples on the range chart, controlling y, we have

$$\sum_{i=1}^{20} R_{x_i} = 18.608 \quad \text{and} \quad \sum_{i=1}^{10} R_{y_i} = 6.978$$

Estimate σ_x and σ_y.

$n = 5;\ \ m_x = 20;\ \ m_y = 10;\ \ \sum_{i=1}^{20} R_{x,i} = 18.608;\ \ \sum_{i=1}^{10} R_{y,i} = 6.978$

$$\hat{\sigma}_x = \bar{R}_x / d_2 = \left(\sum_{i=1}^{20} R_{x,i} / m_x\right) \Big/ d_2 = (18.086/20)/2.326 = 0.400$$

$$\hat{\sigma}_y = \bar{R}_y / d_2 = \left(\sum_{i=1}^{10} R_{y,i} / m_y\right) \Big/ d_2 = (6.978/10)/2.326 = 0.300$$

(b) If it is desired that the probability of a smaller clearance (i.e., $x - y$) than 0.09 should be 0.006, what distance between the average dimensions (i.e., $\mu_x - \mu_y$) should be specified?

Want $\Pr\{(x-y) < 0.09\} = 0.006$.

Let $z = x - y$. Then $\hat{\sigma}_z = \sqrt{\hat{\sigma}_x^2 + \hat{\sigma}_y^2} = \sqrt{0.4^2 + 0.3^2} = 0.500$.

$$\Phi\left(\frac{0.09 - z}{\hat{\sigma}_z}\right) = 0.006$$

$$\Phi^{-1}\left(\frac{0.09 - z}{0.500}\right) = \Phi(0.006)$$

$$\left(\frac{0.09 - z}{0.500}\right) = -2.5121$$

$$z = +2.5121(0.500) + 0.09 = 1.346$$

5-31. Control charts for $\bar{x}$ and S are maintained on a quality characteristic. The sample size is $n = 4$. After 30 samples, we obtain

$$\sum_{i=1}^{30} \bar{x}_i = 12{,}870 \quad \text{and} \quad \sum_{i=1}^{30} R_i = 410$$

(a) Find the three-sigma limits for the S chart.

$n = 4,\ \ \sum_{i=1}^{30} \bar{x}_i = 12{,}870,\ \ \sum_{i=1}^{30} S_i = 410,\ \ m = 30,\ \ \bar{S} = \dfrac{\sum_{i=1}^{m} S_i}{m} = \dfrac{410}{30} = 13.667$

$UCL_S = B_4 \bar{S} = 2.266(13.667) = 30.969 \qquad LCL_S = B_3 \bar{S} = 0(13.667) = 0$

(b) Assuming that both charts exhibit control, estimate the parameters, μ and σ.

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{12{,}870}{30} = 429.0$$

$$\hat{\sigma}_x = \bar{S} / c_4 = 13.667 / 0.9213 = 14.834$$

5-33. An $\bar{x}$ chart with three-sigma limits has parameters as follows:

UCL =	104
Center Line =	100
LCL =	96
n =	5

Suppose the process quality characteristic being controlled is normally distributed with a true mean of 98 and a standard deviation of 8. What is the probability that the control chart would exhibit lack of control by at least the third point plotted?

$n = 5;\ \ UCL_{\bar{x}} = 104;\ \ \text{centerline}_{\bar{x}} = 100;\ \ LCL_{\bar{x}} = 96;\ \ k = 3;\ \ \mu = 98;\ \ \sigma_x = 8$

Pr{out-of-control signal by at least 3rd plot point} = 1 − Pr{not detected by 3rd sample} = 1 − [Pr{not detected}]^3

$\Pr\{\text{not detected}\} = \Pr\{LCL_{\bar{x}} \le \bar{x} \le UCL_{\bar{x}}\} = \Pr\{\bar{x} \le UCL_{\bar{x}}\} - \Pr\{\bar{x} \le LCL_{\bar{x}}\}$

$$= \Phi\left(\frac{UCL_{\bar{x}} - \mu}{\sigma_{\bar{x}}}\right) - \Phi\left(\frac{LCL_{\bar{x}} - \mu}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{104 - 98}{8/\sqrt{5}}\right) - \Phi\left(\frac{96 - 98}{8/\sqrt{5}}\right) = \Phi(1.68) - \Phi(-0.56)$$

$$= 0.9532 - 0.2881 = 0.6651$$

Pr{out-of-control signal by at least 3rd plot point} = 1 − [Pr{not detected}]^3 = 1 − $(0.6651)^3$ = 0.7058

5-35 Control charts for $\bar{x}$ and S with $n = 4$ are maintained on a quality characteristic. The parameter of these charts are as follows:

$\bar{x}$	Chart	S	Chart
UCL =	210.88	UCL =	2.266
Center Line =	200.00	Center Line =	1.000
LCL =	198.12	LCL =	0

Both charts exhibit control. Specifications on the quality characteristic are 197.50 and 202.50. What can be said about the ability of the process to produce product that conforms to specifications?

$$\hat{C}_P = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x} = \frac{\text{USL}-\text{LSL}}{6\left(\bar{S}/c_4\right)} = \frac{202.50\text{-}197.50}{6(1.000/0.9213)} = 0.7678 .$$

The process is not capable of meeting specifications.

5-37 Specifications on a normally distributed dimension are 600 ± 20. $\bar{x}$ and R charts are maintained on this dimension and have been in control over a long period of time. The parameters of these control charts are as follows ($n = 9$).

$\bar{x}$	Chart	S	Chart
UCL =	616	UCL =	32.36
Center Line =	610	Center Line =	17.82
LCL =	604	LCL =	3.28

(a) What are your conclusions regarding the capability of the process to produce items within specifications?

$n = 9$; USL = 600 + 20 = 620; LSL = 600 - 20 = 580

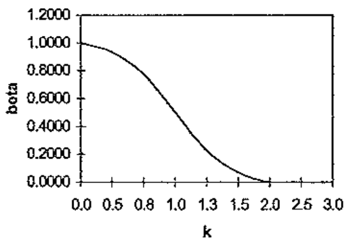
$$\hat{C}_p = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x} = \frac{\text{USL}-\text{LSL}}{6\left(\bar{R}/d_2\right)} = \frac{620-580}{6(17.82/2.970)} = 1.111$$

(b) Construct an OC curve for the $\bar{x}$ chart assuming that σ is constant.

$$n=9;\ L=3;\ \beta = \Phi\left(L-k\sqrt{n}\right)-\Phi\left(-L-k\sqrt{n}\right)$$

for $k =$ {0, 0.5, 0.75, 1.0, 1.25, 1.5, 2.0, 2.5, 3.0}
$\beta =$ {0.9974, 0.9332, 0.7734, 0.5, 0.2266, 0.0668, 0.0013, 0.0000, 0.0000}

Operating Characteristic Curve for n = 9, L = 3



5-39. An $\bar{x}$ chart is to be established based on the standard values $\mu = 600$ and $\sigma = 12$, with $n = 9$. The control limits are to be based on an α-risk of 0.01. What are the appropriate control limits?

$n=9$, $\mu = 600$, $\sigma_x = 12$, α=0.01, $k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$

$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\frac{\sigma_x}{\sqrt{n}}\right) = 600 + 2.576\left(\frac{12}{\sqrt{9}}\right) = 610.3$$

$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\frac{\sigma_x}{\sqrt{n}}\right) = 600 - 2.576\left(\frac{12}{\sqrt{9}}\right) = 589.7$$

5-41. Consider the $\bar{x}$ chart in Exercise 5-40. Find the average run length for the chart.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-\Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.1587} = 6.30$$

5-43. A normally distributed quality characteristic is monitored through the use of an $\bar{x}$ and R chart. These charts have the following parameters ($n = 4$)

$\bar{x}$	Chart	R	Chart
UCL =	626.0	UCL =	18.795
Center Line =	620.0	Center Line =	8.236
LCL =	614.0	LCL =	0

Both charts exhibit control.

(a) What is the estimated standard deviation of the process?

$\hat{\sigma}_x = \bar{R}/d_2 = 8.236/2.059 = 4.000$

(b) Suppose an S chart were to be substituted for the R chart. What would be the appropriate parameters of the S chart.

$\bar{S} = c_4\hat{\sigma}_x = 0.9213(4) = 3.865$

$\text{UCL}_S = B_4\bar{S} = 2.266(3.685) = 8.351 \qquad \text{LCL}_S = B_3\bar{S} = 0(3.685) = 0$

(c) If specifications on the product were 610 ± 15, what would be your estimate of the process fraction nonconforming.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right)$$

$$= \Phi\left(\frac{595 - 620}{4}\right) + 1 - \Phi\left(\frac{625 - 620}{4}\right) = \Phi(-6.25) + 1 - \Phi(1.25) = 0.0000 + 1 - 0.8944 = 0.1056$$

(d) What could be done to reduce this fraction nonconforming?

To reduce the fraction nonconforming, try moving the center of the process from its current mean of 620 closer to the nominal dimension of 610. Also consider reducing the process variability.

(e) What is the probability of detecting a shift in the process mean to 610 on the first sample following the shift (σ remains constant)?

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \mu_{new}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{new}}{\sigma_{\bar{x}}}\right)$$

$$= \Phi\left(\frac{614 - 610}{4/\sqrt{4}}\right) + 1 - \Phi\left(\frac{626 - 610}{4/\sqrt{4}}\right) = \Phi(2) + 1 - \Phi(8) = 0.9772 + 1 - 1.0000 = 0.9772$$

(f) What is the probability of detecting the shift in part (e) by at least the third sample after the shift occurs?

$$\Pr\{\text{detect by 3rd sample}\} = 1 - \Pr\{\text{not detect by 3rd sample}\} = 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9772)^3 = 1.0000$$

5-45. The following $\bar{x}$ and S charts based on $n = 4$ have shown statistical control:

$\bar{x}$	Chart	S	Chart
UCL =	710	UCL =	18.08
Center Line =	700	Center Line =	7.979
LCL =	690	LCL =	0

(a) Estimate the process parameters

$\hat{\mu} = \bar{\bar{x}} = 700; \quad \hat{\sigma}_x = \bar{S}/c_4 = 7.979/0.9213 = 8.661$

(b) If the specifications are at 705 ± 15, and the process output is normally distributed, estimate the fraction nonconforming.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right)$$

$$= \Phi\left(\frac{690 - 700}{8.661}\right) + 1 - \Phi\left(\frac{720 - 700}{8.661}\right) = \Phi(-1.15) + 1 - \Phi(2.31) = 0.1251 + 1 - 0.9896 = 0.1355$$

(c) For the $\bar{x}$ chart, find the probability of a type I error, assuming σ is constant.

$$\alpha = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= \Phi\left(\frac{690 - 700}{8.661/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 700}{8.661/\sqrt{4}}\right) = \Phi(-2.31) + 1 - \Phi(2.31) = 0.0104 + 1 - 0.9896 = 0.0208$$

(d) Suppose the process mean shifts to 693 and the standard deviation simultaneously shifts to 12. Find the probability of detecting this shift on the $\bar{x}$ chart on the first subsequent sample.

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\}$$

$$= \Phi\left(\frac{\text{LCL} - \mu_{new}}{\sigma_{\bar{x},new}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{new}}{\sigma_{\bar{x},new}}\right)$$

$$= \Phi\left(\frac{690 - 693}{12/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 693}{12/\sqrt{4}}\right)$$

$$= \Phi(-0.5) + 1 - \Phi(2.83) = 0.3085 + 1 - 0.9977 = 0.3108$$

(e) For the shift in part (d), find the average run length.

$$\text{ARL}_1 = \frac{1}{1 - \beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.3108} = 3.22$$

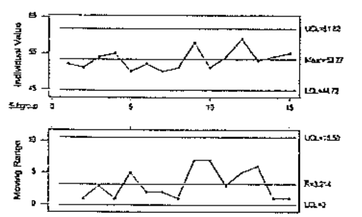
5-47. Fifteen successive heats of a steel alloy are tested for hardness. The resulting data are shown here. Set up a control chart for the moving range and a control chart for individual hardness measurements. Is it reasonable to assume that hardness in normally distributed?

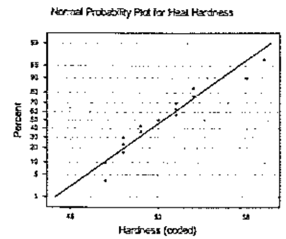
Heat	Hardness (coded)	Heat	Hardness (coded)
1	52	9	58
2	51	10	51
3	54	11	54
4	55	12	59
5	50	13	53
6	52	14	54
7	50	15	55
8	51		

$\bar{\bar{x}} = 53.2667$; $\hat{\sigma}_x = 2.84954$; $\overline{\mathrm{MR}} = 3.21429$

$\mathrm{UCL}_{\bar{x}} = \bar{\bar{x}} + 3\frac{\overline{MR}}{d_2} = 53.2667 + 3\frac{3.21429}{1.128} = 61.8153$ $\quad \mathrm{LCL}_{\bar{x}} = \bar{\bar{x}} - 3\frac{\overline{MR}}{d_2} = 53.2667 - 3\frac{3.21429}{1.128} = 44.7181$

$\mathrm{UCL}_R = D_4\overline{MR} = 3.267(3.21429) = 10.5011$ $\quad \mathrm{LCL}_R = D_3\overline{MR} = 0(3.21429) = 0$





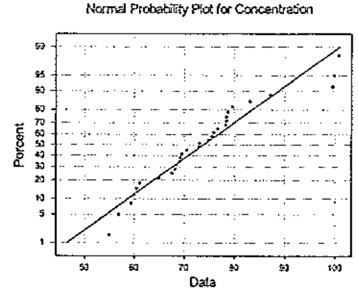
Visual examination of the normal probability plot indicates that it may not be reasonable to assume that hardness is normally distributed. The observations on the tails are not very close to the straight line.

5-51. Thirty observations on concentration (in g/l) of the active ingredient in a liquid cleaner produced in a continuous chemical process are shown here.

Observation	Concentration	Observation	Concentration
1	60.4	16	99.9
2	69.5	17	59.3
3	78.4	18	60.0
4	72.8	19	74.7
5	78.2	20	75.8
6	78.7	21	76.6
7	56.9	22	68.4
8	78.4	23	83.1
9	79.6	24	61.1
10	100.8	25	54.9

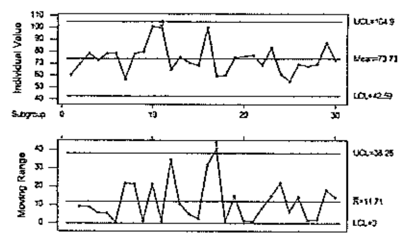
Observation	Concentration	Observation	Concentration
11	99.6	26	69.1
12	64.9	27	67.5
13	75.5	28	69.2
14	70.4	29	87.2
15	68.1	30	73.0

(a) A normal probability plot of the concentration is shown next. The straight line was fit by eye to pass approximately through the twentieth and eightieth percentiles. Does the normality assumption seem reasonable here?



The normality assumption is a little bothersome for the concentration data, in particular due to the curve of the larger values and three distant values in the upper right corner of the plot.

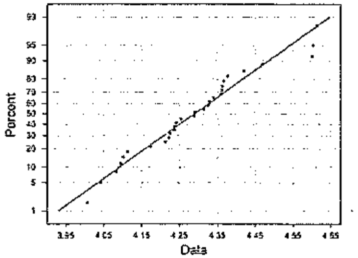
(b) Set up an individuals and moving range control chart for the concentration data. Interpret the charts.



The process is not in control, with two Western Electric rule violations. Point 11 fails "2 of 3" beyond 2 sigma from the center line on the I Chart, and point 17 fails one point greater than 3 sigma from the center line on the MR Chart.

(c) Construct a normal probability plot of the natural log of concentration. Is the transformed variable normally distributed?

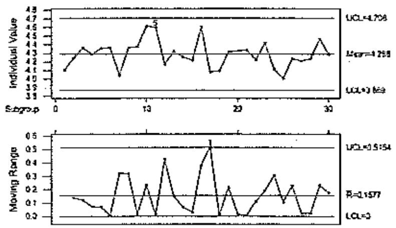
Normal Probability Plot for ln(Concentration)



The normality assumption is still troubling for the natural log of concentration, again due to the curve of the larger values and three distant values.

(d) Repeat part (b), using the natural log of concentration as the charted variable. Comment on any differences in the charts you note in comparison to those constructed in part (b).

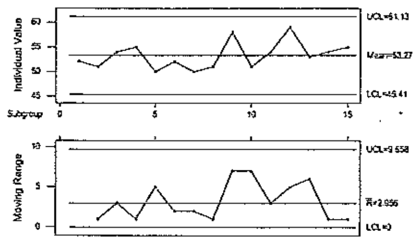
I and MR Chart for ln(Concentration)



The process is still not in control, with the same to Western Electric Rules violations. There does not appear to be much difference between the two control charts (actual and natural log).

5-54. Reconsider the hardness measurements in Exercise 5-47. Construct an individuals control chart using the median of the span-two moving ranges to estimate variability. Compare this control chart to the one constructed in Exercise 5-47 and discuss.

I and MR Chart for Hardness



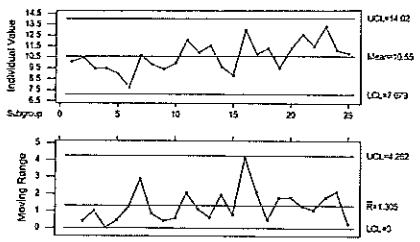
The median moving range method gives slightly tighter control limits for both the Individual and Moving Range charts, with no practical difference for this set of observations.

5-57. Consider the individuals measurement data shown next.

Observation	x	Observation	x
1	10.07	14	9.58
2	10.47	15	8.80
3	9.45	16	12.94
4	9.44	17	10.78
5	8.99	18	11.26
6	7.74	19	9.48
7	10.63	20	11.28
8	9.78	21	12.54
9	9.37	22	11.48
10	9.95	23	13.26
11	12.04	24	11.10
12	10.93	25	10.82
13	11.54		

(a) Estimate σ using the average of the moving ranges of span two.

I and MR Chart for X, avg MR2 method



$\hat{\sigma}_x = \bar{R}/d_2 = 1.305/1.128 = 1.157$

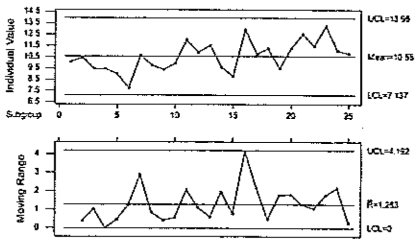
(b) Estimate σ using S/C_4.

Variable	N	Mean	Median	TrMean	StDev
Ex5-57	25	10.549	10.630	10.553	1.342

$\hat{\sigma}_x = S/c_4 = 1.342/0.7979 = 1.682$

(c) Estimate σ using the median of the span-two moving ranges.

I and MR Chart for X, median MR2 method

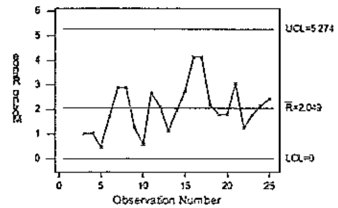


$\hat{\sigma}_x = \bar{R}/d_2 = 1.283/1.128 = 1.137$

(d) Estimate σ using the average of the moving ranges of span 3, 4, ..., 20.

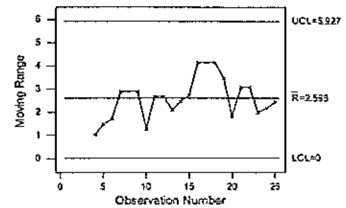
Average MR3 Chart

MR3 Chart for X



Average MR4 Chart

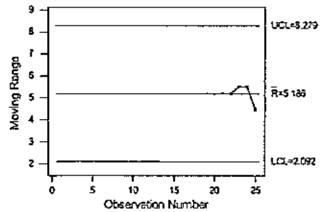
MR4 Chart For X



$\hat{\sigma}_x = \bar{R}/d_2 = 2.049/1.693 = 1.210$ $\hat{\sigma}_x = \bar{R}/d_2 = 2.598/2.059 = 1.262$

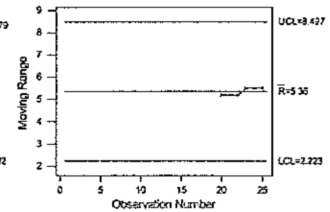
Average MR19 Chart:

MR19 Chart for X



Average MR20 Chart:

MR20 Chart for X



$\hat{\sigma}_x = \bar{R}/d_2 = 5.186/3.689 = 1.406$ $\hat{\sigma}_x = \bar{R}/d_2 = 5.36/3.735 = 1.435$

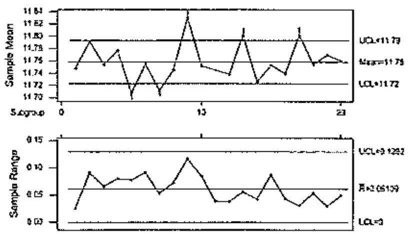
(e) Discuss the results you have obtained.

As the span of the moving range is increased, there are fewer observations to estimate the standard deviation, and the estimate becomes less reliable. For this example, σ gets larger as the span increases. This tends to be true for unstable processes.

5-59. The diameter of the casting in Figure 5-27 is also an important quality characteristic. A coordinate measuring machine is used to measure the diameter of each casting at five different locations. Data for 20 castings are shown in the following table.

Casting	Diameter 1	Diameter 2	Diameter 3	Diameter 4	Diameter 5
1	11.7629	11.7403	11.7511	11.7474	11.7374
2	11.8122	11.7506	11.7787	11.7736	11.8412
3	11.7742	11.7114	11.7530	11.7532	11.7773
4	11.7833	11.7311	11.7777	11.8108	11.7804
5	11.7134	11.6870	11.7305	11.7419	11.6642
6	11.7925	11.7611	11.7588	11.7012	11.7611
7	11.6916	11.7205	11.6958	11.7440	11.7062
8	11.7109	11.7832	11.7496	11.7496	11.7318
9	11.7984	11.8887	11.7729	11.8485	11.8416
10	11.7914	11.7613	11.7356	11.7628	11.7070
11	11.7260	11.7329	11.7424	11.7645	11.7571
12	11.7202	11.7537	11.7328	11.7582	11.7265
13	11.8356	11.7971	11.8023	11.7802	11.7903
14	11.7069	11.7112	11.7492	11.7329	11.7289
15	11.7116	11.7978	11.7982	11.7429	11.7154
16	11.7165	11.7284	11.7571	11.7597	11.7317
17	11.8022	11.8127	11.7864	11.7917	11.8167
18	11.7775	11.7372	11.7241	11.7773	11.7543
19	11.7753	11.7870	11.7574	11.7620	11.7673
20	11.7572	11.7626	11.7523	11.7395	11.7884

(a) Set up $\bar{x}$ and R charts for this process, assuming the measurements on each casting form a rational subgroup.

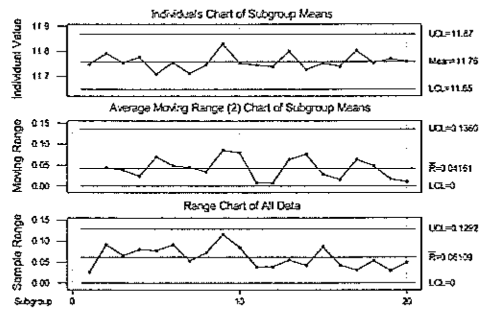


The $\bar{x}$ chart has several out-of-control points. Points 5, 7, 9, 13, and 17 are beyond the upper control limit.

(b) Discuss the chart you have constructed in part (a).

Though the R chart is in control, plot points on the $\bar{x}$ chart bounce below and above the control limits. Since these are high precision castings, we might expect that the diameter of a single casting will not change much with location. If no assignable cause can be found for these out-of-control points, we may want to consider treating the averages as an individual value and graphing "between/within" range charts. This will lead to a understanding of the greatest source of variability, between castings or within a casting.

(c) Construct "between/within" charts for this process.



Between standard deviation = 3.50E-02
Within standard deviation = 2.63E-02
Total standard deviation = 4.37E-02

(d) Do you believe that the charts in part (c) are more informative than those in part (a)? Discuss why.

We are taking several diameter measurements on a single precision casting.

(e) Provide a practical interpretation of the "within" chart.

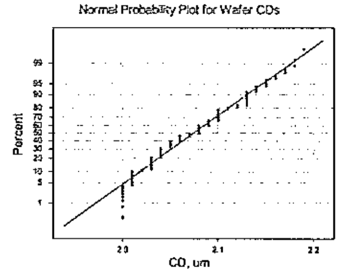
The "within" chart is the usual R chart ($n > 1$). It describes the measurement variability within a sample (variability in diameter of a single casting). Though the nature of this process leads us to believe that the diameter at any location on a single casting does not change much, we should continue to monitor "within" to look for wear, damage, etc., in the wax mold.

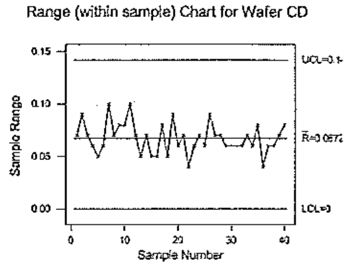
5-61. Consider the situation described in Exercise 5-60. A critical dimension (measured in m) is of interest to the process engineer. Suppose that five fixed positions are used on each wafer (position 1 is the center) and that two consecutive wafers are selected from each batch. The data that result from several batches are shown here.

Lot	Wafer	Position 1	Position 2	Position 3	Position 4	Position 5
1	1	2.15	2.13	2.08	2.12	2.10
1	2	2.13	2.10	2.04	2.08	2.05
2	1	2.02	2.01	2.06	2.05	2.08
2	2	2.03	2.09	2.07	2.06	2.04
3	1	2.13	2.12	2.10	2.11	2.08
3	2	2.03	2.08	2.03	2.09	2.07
4	1	2.04	2.01	2.10	2.11	2.09
4	2	2.07	2.14	2.12	2.08	2.09
5	1	2.16	2.17	2.13	2.18	2.10
5	2	2.17	2.13	2.10	2.09	2.13
6	1	2.04	2.06	2.00	2.10	2.08
6	2	2.03	2.10	2.05	2.07	2.04
7	1	2.04	2.02	2.01	2.00	2.05
7	2	2.06	2.04	2.03	2.08	2.10
8	1	2.13	2.10	2.10	2.15	2.13
8	2	2.10	2.09	2.13	2.14	2.11
9	1	2.00	2.03	2.08	2.07	2.08
9	2	2.01	2.03	2.06	2.05	2.04
10	1	2.04	2.08	2.09	2.10	2.01
10	2	2.06	2.04	2.07	2.04	2.01
11	1	2.15	2.13	2.14	2.09	2.08
11	2	2.11	2.13	2.10	2.14	2.10
12	1	2.03	2.06	2.05	2.01	2.00
12	2	2.04	2.08	2.03	2.10	2.07
13	1	2.05	2.03	2.05	2.09	2.08
13	2	2.08	2.01	2.03	2.04	2.10
14	1	2.08	2.04	2.05	2.01	2.08
14	2	2.09	2.11	2.06	2.04	2.05
15	1	2.14	2.13	2.10	2.10	2.08
15	2	2.13	2.10	2.09	2.13	2.15
16	1	2.06	2.08	2.05	2.03	2.09
16	2	2.03	2.01	2.00	2.06	2.05
17	1	2.05	2.03	2.08	2.01	2.04
17	2	2.06	2.05	2.03	2.05	2.00
18	1	2.03	2.08	2.04	2.00	2.03
18	2	2.04	2.03	2.05	2.01	2.04
19	1	2.16	2.13	2.10	2.13	2.12
19	2	2.13	2.15	2.18	2.19	2.13
20	1	2.06	2.03	2.04	2.09	2.10
20	2	2.01	2.00	2.05	2.08	2.06

(a) What can you say about overall process capability?

```
Variable          N       Mean     Median     TrMean     StDev   SE Mean
Ex5-61A1        200     2.0741     2.0800     2.0730    0.0440    0.0031
Variable    Minimum    Maximum         Q1         Q3
Ex5-61A1     2.0000     2.1900     2.0400     2.1000
```





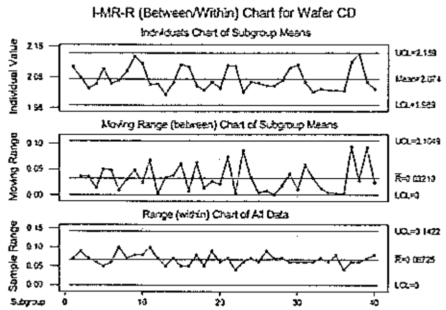
The wafer critical dimension is not normally distributed.

(b) Can you construct control charts that allow within-wafer variability to be evaluated?

To evaluate within-wafer variability, construct a R chart for each sample of 5 positions, for a total of 40 subgroups. The Range chart is in control, indicating that within-wafer variability is also in control.

(c) What control charts would you establish to evaluate variability between wafers? Set up these charts and use them to draw conclusions about the process.

To evaluate variability between wafers, set up Individuals and Moving Range charts where the x statistic is the average wafer measurement and the moving range is calculated between two wafer averages.

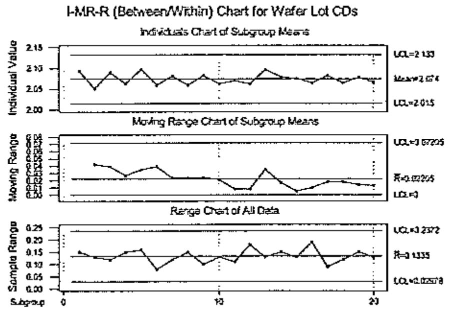


Between standard deviation = 2.54E-02
Within standard deviation = 2.89E-02
Total standard deviation = 3.85E-02

Both "between" control charts (Individuals and Moving Range) are in control, indicating that between-wafer variability is also in-control.

(d) What control charts would you use to evaluate lot-to-lot variability? Set up these charts and use them to draw conclusions about lot-to-lot variability.

To evaluate lot-to-lot variability, three charts are needed: (1) lot average, (2) moving range between lot averages, and (3) range within a lot—the Minitab "between/within" control charts.



Between standard deviation = 1.39E-02
Within standard deviation = 4.34E-02
Total standard deviation = 4.56E-02

All three control charts are in control, also indicating the lot-to-lot variability is in-control.

CHAPTER 6

Control Charts for Attributes

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Design, apply and interpret the control charts for fraction nonconforming data and number nonconforming data
   - The *p* chart
   - The *np* chart
2. Design, apply and interpret the control charts for defects
   - The *c* chart
   - The *u* chart
3. Understand the appropriate control chart for a given process scenario

Exercises

6-1. The data that follow give the number of nonconforming bearing and seal assemblies in samples of size 100. Construct a fraction nonconforming control chart for these data. If any points plot out of control, assume that assignable causes can be found and determine the revised limits.

Sample No.	No. Nonconforming Assemblies	Sample No.	No. Nonconforming Assemblies
1	7	11	6
2	4	12	15
3	1	13	0
4	3	14	9
5	6	15	5

Sample No.	No. Nonconforming Assemblies	Sample No.	No. Nonconforming Assemblies
6	8	16	1
7	10	17	4
8	5	18	5
9	2	19	7
10	7	20	12

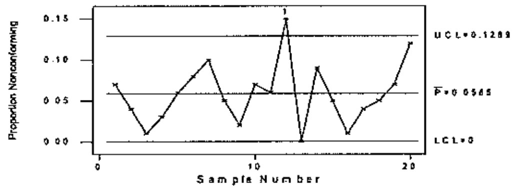
Summary statistics are first calculated: $n=100,\ m=20,\ \sum_{i=1}^{m} D_i = 117,\ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{117}{20(100)} = 0.0585$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 + 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.1289$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 - 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.0585 - 0.0704 \Rightarrow 0$$

The lower control limit as calculated is a negative value, therefore, zero is chosen for the lower limit.

P Chart for Bearing & Seal Assemblies



Sample 12 is out-of-control, so remove from control limit calculation:

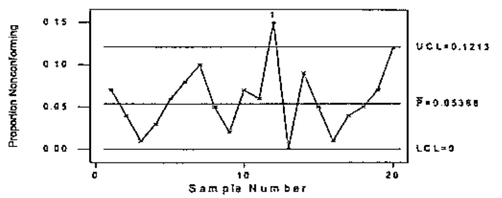
$$n=100,\ m=19,\ \sum_{i=1}^{m} D_i = 102,\ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{102}{19(100)} = 0.0537$$

$$\text{UCL}_p = 0.0537 + 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.1213$$

$$\text{LCL}_p = 0.0537 - 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.0537 - 0.0676 \Rightarrow 0$$

Again, zero is chosen for the lower control limit.

P Chart for Bearing & Seal Assemblies, less sample 12



6-3. The following data represent the results of inspecting all units of a personal computer produced for the last 10 days. Does the process appear to be in control?

Day	Units Inspected	Nonconforming Units	Fraction Nonconforming
1	80	4	0.050
2	110	7	0.064
3	90	5	0.056
4	75	8	0.107
5	130	6	0.038
6	120	6	0.050
7	70	4	0.057
8	125	5	0.040
9	105	8	0.076
10	95	7	0.074

The summary statistics are:

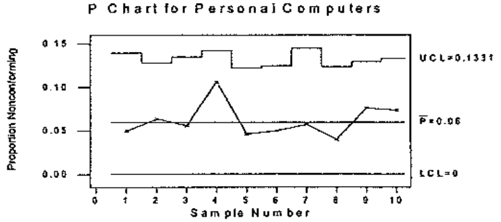
$$m=10,\ \sum_{i=1}^{m} n_i = 1000,\ \sum_{i=1}^{m} D_i = 60,\ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{\sum_{i=1}^{m} n_i} = \frac{60}{1000} = 0.06$$

$$\text{UCL}_i = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \text{ and } \text{LCL}_i = \max\{0, \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_i}\}$$

As an example, for $n = 80$:

$$\text{UCL}_1 = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1}} = 0.06 + 3\sqrt{\frac{0.06(1-0.06)}{80}} = 0.1397$$

$$\text{LCL}_1 = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1}} = 0.06 - 3\sqrt{\frac{0.06(1-0.06)}{80}} = 0.06 - 0.0797 \Rightarrow 0$$



The process appears to be in statistical control.

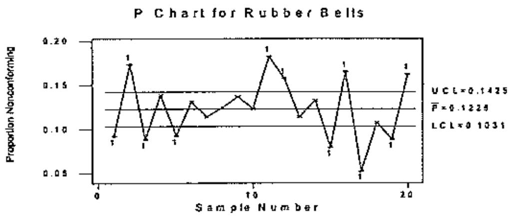
6-5. A process produces rubber belts in lots of size 2500. Inspection records on the last 20 lots reveal the following data.

Lot Number	No. of Nonconforming Belts	Lot Number	No. of Nonconforming Belts
1	230	11	456
2	435	12	394
3	221	13	285
4	346	14	331
5	230	15	198
6	327	16	414
7	285	17	131
8	311	18	269
9	342	19	221
10	308	20	407

(a) Compute trial control limits for a fraction nonconforming control chart.

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1228 + 3\sqrt{\frac{0.1228(1-0.1228)}{2500}} = 0.1425$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1228 - 3\sqrt{\frac{0.1228(1-0.1228)}{2500}} = 0.1031$$



The process is out of control at several points (1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20).

(b) If you wanted to set up a control chart for controlling future production, how would you use these data to obtain the center line and control limits for the chart?

So many subgroups are out of control (11 of 20) that the data should not be used to establish control limits for future production. Instead, the process should be investigated for causes of the wild swings in p.

6-7. A control chart indicates that the current process fraction nonconforming is 0.02. If 50 items are inspected each day, what is the probability of detecting a shift in the fraction nonconforming to 0.04 on the first day after the shift? By the end of the third day following the shift?

$\bar{p} = 0.02; n = 50$

$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.02 + 3\sqrt{0.02(1-0.02)/50} = 0.0794$

$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.02 - 3\sqrt{0.02(1-0.02)/50} = 0.02 - 0.0594 \Rightarrow 0$

Since $p_{new} = 0.04 < 0.1$ and $n = 50$ is "large", use the Poisson approximation to the binomial with $\lambda = np_{new} = 50(0.04) = 2.00$.

$\Pr\{\text{detect}|\text{shift}\} = 1 - \Pr\{\text{not detect}|\text{shift}\} = 1 - \beta$

$= 1 - [\Pr\{D < n\text{UCL} \mid \lambda\} - \Pr\{D \le n\text{LCL} \mid \lambda\}] = 1 - \Pr\{D < 50(0.0794) \mid 2\} + \Pr\{D \le 50(0) \mid 2\}$

$= 1 - \text{POI}(3,2) + \text{POI}(0,2) = 1 - 0.857 + 0.135 = 0.278$

where POI(·) is the cumulative Poisson distribution.

$\Pr\{\text{detected by 3rd sample}\} = 1 - \Pr\{\text{detected after 3rd}\} = 1 - (1 - 0.278)^3 = 0.624$

6-9. Diodes used on printed circuit boards are produced in lots of size 1000. We wish to control the process producing these diodes by taking samples of size 64 from each lot. If the nominal value of the fraction nonconforming is $p = 0.10$ determine the parameters of the appropriate control chart. To what level must the fraction nonconforming increase to make the β-risk equal to 0.50? What is the minimum sample size that would give a positive lower control limit for this chart?

The parameters of the control chart are:

$\bar{p} = 0.10, \quad n = 64$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.10 + 3\sqrt{\frac{0.10(1-0.10)}{64}} = 0.2125$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.10 - 3\sqrt{\frac{0.10(1-0.10)}{64}} = 0.10 - 0.1125 \Rightarrow 0$$

The beta risk is determined with the following function:

$\beta = \Pr\{D < n\text{UCL} \mid p\} - \Pr\{D \le n\text{LCL} \mid p\}$

$= \Pr\{D < 64(0.2125) \mid p\} - \Pr\{D \le 64(0) \mid p\} = \Pr\{D < 13.6) \mid p\} - \Pr\{D \le 0 \mid p\}$

Applying the function iteratively,

p	$Pr\{D \le 13 \mid p\}$	$Pr\{D \le 0 \mid p\}$	β
0.05	0.999999	0.037524	0.962475
0.10	0.996172	0.001179	0.994993
0.20	0.598077	0.000000	0.598077
0.21	0.519279	0.000000	0.519279

p	$Pr\{D \le 13 \mid p\}$	$Pr\{D \le 0 \mid p\}$	β
0.22	0.44154	0.000000	0.44154
0.215	0.480098	0.000000	0.480098
0.212	0.503553	0.000000	0.503553

The fraction of non-conforming would need to increase to 0.212. Assuming $L = 3$ sigma control limits, the minimum sample size to achieve a positive lower control limit is:

$$n > \frac{(1-p)}{p}L^2$$
$$> \frac{(1-0.10)}{0.10}(3)^2$$
$$> 81$$

6-11. A control chart for the fraction nonconforming is to be established using a center line of $p = 0.10$. What sample size is required if we wish to detect a shift in the process fraction nonconforming to 0.20 with probability 0.50?

$p = 0.10,\ p_{new} = 0.20,$ Desire Pr{detect} = 0.50. Assume $k = 3$ sigma control limits.

$\delta = p_{new} - p = 0.20 - 0.10 = 0.10$

$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.10}\right)^2 (0.10)(1-0.10) = 81$$

The required sample size is 81.

6-13. A process is being controlled with a fraction nonconforming control chart. The process average has been shown to be 0.07. Three-sigma control limits are used, and the procedure calls for taking daily samples of 400 items.

(a) Calculate the upper and lower control limits.

$\bar{p} = 0.07;\ k = 3$ sigma control limits; $n = 400$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{p(1-p)}{n}} = 0.07 + 3\sqrt{\frac{0.07(1-0.07)}{400}} = 0.108$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{p(1-p)}{n}} = 0.07 - 3\sqrt{\frac{0.07(1-0.07)}{400}} = 0.032$$

(b) If the process average should suddenly shift to 0.10, what is the probability that the shift would be detected on the first subsequent sample?

$np_{new} = 400(0.10) => 40$, so use the normal approximation to the binomial.

Pr{detect on 1st sample} = 1 − Pr{not detect on 1st sample} = $1 - \beta$

$$= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}]$$
$$= 1 - \Phi\left(\frac{\text{UCL} - p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL} - p}{\sqrt{p(1-p)/n}}\right)$$
$$= 1 - \Phi\left(\frac{0.108 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) + \Phi\left(\frac{0.032 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right)$$
$$= 1 - \Phi(0.533) + \Phi(-4.533) = 1 - 0.703 + 0.000 = 0.297$$

(c) What is the probability that the shift in part (b) would be detected on the first or second sample taken after the shift?

Pr{detect on 1st or 2nd sample} = Pr{detect on 1st} + Pr{not on 1st}×Pr{detect on 2nd}
= 0.297 + (1 − 0.297)(0.297) = 0.506

6-15. A control chart is used to control the fraction nonconforming for a plastic part manufactured in an injection molding process. Ten subgroups yield the following data:

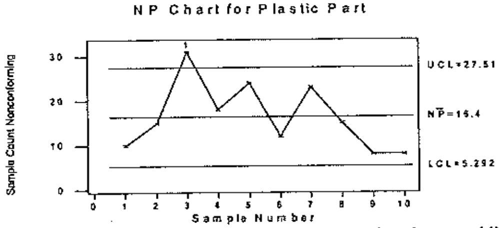
Sample Number	Sample Size	Number Nonconforming
1	100	10
2	100	15
3	100	31
4	100	18
5	100	24
6	100	12
7	100	23
8	100	15
9	100	8
10	100	8

(a) Set up a control chart for the number nonconforming in samples of $n = 100$.

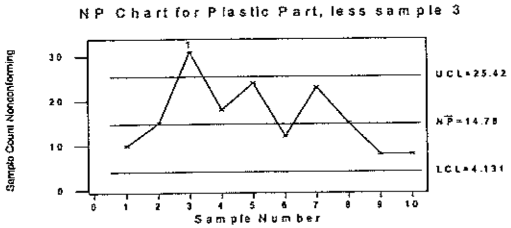
$$m = 10,\ n = 100,\ \sum_{i=1}^{10} D_i = 164,\ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{164}{10(100)} = 0.164,\ n\bar{p} = 16.4$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 16.4 + 3\sqrt{16.4(1-0.164)} = 27.51$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 16.4 - 3\sqrt{16.4(1-0.164)} = 5.292$$



The process is out of control, with point 3 beyond the upper limit. Recalculate the control limits without sample 3:



The control limits are slightly narrower. Of course, point 3 is above the upper limit.

(b) For the chart established in part (a), what is the probability of detecting a shift in the process fraction nonconforming to 0.30 on the first sample after the shift has occurred?

$p_{new} = 0.30$. Since $p = 0.30$ is not too far from 0.50, and $n = 100 > 10$, the normal approximation to the binomial can be used.

$\Pr\{\text{detect on } 1^{st}\} = 1 - \Pr\{\text{not detect on } 1^{st}\} = 1 - \beta$

$$= 1 - [\Pr\{D < \text{UCL} \mid p\} - \Pr\{D \le \text{LCL} \mid p\}] = 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right)$$

$$= 1 - \Phi\left(\frac{25.42 + 0.5 - 30}{\sqrt{30(1-0.3)}}\right) + \Phi\left(\frac{4.13 - 0.5 - 30}{\sqrt{30(1-0.3)}}\right) = 1 - \Phi(-0.8903) + \Phi(-5.7544)$$

$$= 1 - (0.187) + (0.000) = 0.813$$

6-17 (a) A control chart for the number nonconforming is to be established, based on samples of size 400. To start the control chart, 30 samples were selected and the number nonconforming in each sample determined, yielding $\sum_{i=1}^{30} D_i = 1200$. What are the parameters of the np chart?

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{1200}{30(400)} = 0.10, \quad n\bar{p} = 400(0.10) = 40$$

$$\text{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 40 + 3\sqrt{40(1-0.10)} = 58 \qquad \text{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 40 - 3\sqrt{40(1-0.10)} = 22$$

(b) Suppose the process average fraction nonconforming shifted to 0.15. What is the probability that the shift would be detected on the first subsequent sample?

Because $np_{new} = 400\,(0.15) = 60 > 15$, use the normal approximation to the binomial.

$\Pr\{\text{detect on } 1^{st} \text{ sample} \mid p\} = 1 - \Pr\{\text{not detect on } 1^{st} \text{ sample} \mid p\} = 1 - \beta$

$$= 1 - [\Pr\{D < \text{UCL} \mid np\} - \Pr\{D \le \text{LCL} \mid np\}] = 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right)$$

$$= 1 - \Phi\left(\frac{58 + 0.5 - 60}{\sqrt{60(1-0.15)}}\right) + \Phi\left(\frac{22 - 0.5 - 60}{\sqrt{60(1-0.15)}}\right) = 1 - \Phi(-0.210) + \Phi(-5.39)$$

$$= 1 - 0.417 + 0.000 = 0.583$$

6-19. Consider the control chart designed in Exercise 6-18. Find the average run length to detect a shift to a fraction nonconforming of 0.15.

From 6-17(b), $1 - \beta = 0.583$, so $\text{ARL}_1 = \dfrac{1}{1-\beta} = \dfrac{1}{0.583} = 1.715 \approx 2$

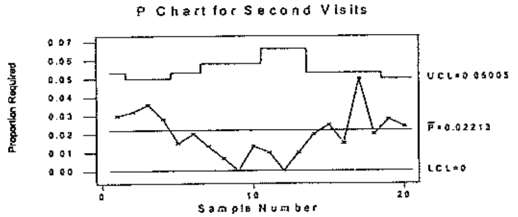
6-21. A maintenance group improves the effectiveness of its repair work by monitoring the number of maintenance requests that require a second call to complete the repair. Twenty weeks of data are available.

Week	Total Requests	Second Visit Req'd	Week	Total Requests	Second Visit Req'd
1	200	6	11	100	1
2	250	8	12	100	0
3	250	9	13	100	1
4	250	7	14	200	4
5	200	3	15	200	5
6	200	4	16	200	3
7	150	2	17	200	10
8	150	1	18	200	4
9	150	0	19	250	7
10	150	2	20	250	6

(a) Find trial control limits for this process.

For a p chart with variable sample size, $\bar{p}=\sum_i D_i / \sum_i n_i = 83/3750 = 0.0221$ and control limits are at $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$

n_i	[LCL_i, UCL_i]
100	[0, 0.0662]
150	[0, 0.0581]
200	[0, 0.0533]
250	[0, 0.0500]

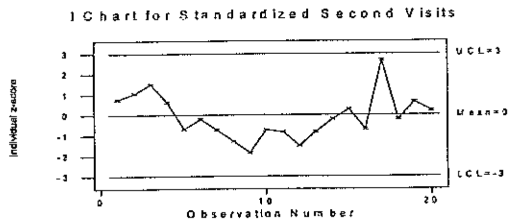


(b) Design a control chart for controlling future production.

There are two approaches for controlling future production. The first approach would be to plot $\hat{p}_i$ and use constant limits unless there is a different size sample or a plot point near a control limit. In those cases, calculate the exact control limits by $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i} = 0.0221 \pm 3\sqrt{0.0216/n_i}$. The second approach, preferred in many cases, would be to construct standardized control limits with control limits at ± 3, and to plot $Z_i = (\hat{p}_i - 0.0221)/\sqrt{0.0221(1-0.0221)/n_i}$.

6-23. Construct a standardized control chart for the data in Exercise 6-21.

$z_i = (\hat{p}_i - \bar{p})/\sqrt{\bar{p}(1-\bar{p})/n_i} = (\hat{p}_i - 0.0221)/\sqrt{0.0216/n_i}$



Process is in statistical control.

6-25. A fraction nonconforming control chart has center line 0.01, UCL = 0.0399, LCL = 0, and n = 100. If three-sigma limits are used, find the smallest sample size that would yield a positive lower control limit.

UCL = 0.0399, $\bar{p}$ = CL = 0.01, LCL = 0, n = 100

$$n > \left(\frac{1-p}{p}\right)L^2$$

$$n > \left(\frac{1-0.01}{0.01}\right)3^2$$

$n > 891$

$n \geq 892$

6-27. A fraction nonconforming control chart with n = 400 has the following parameters:

UCL =	0.0809
Center Line =	0.0500
LCL =	0.0191

(a) Find the width of the control limits in standard deviation units.

$n = 400$, UCL = 0.0809, p = CL = 0.0500, LCL = 0.0191

$0.0809 = 0.05 + L\sqrt{0.05(1-0.05)/400} = 0.05 + L(0.0109)$

The width is $L = 2.8349$

(b) What would be the corresponding parameters for an equivalent control chart based on the number nonconforming?

$CL = np = 400(0.05) = 20$

$UCL = np + 2.8349\sqrt{np(1-p)} = 20 + 2.8349\sqrt{20(1-0.05)} = 32.36$

$LCL = np - 2.8349\sqrt{np(1-p)} = 20 - 2.8349\sqrt{20(1-0.05)} = 7.64$

(c) What is the probability that a shift in the process faction nonconforming to 0.0300 will be detected on the first sample following the shift?

$n = 400$ is large and $p = 0.05 < 0.1$, use Poisson approximation to binomial.

$\Pr\{\text{detect shift to 0.03 on first sample}\} = 1 - \Pr\{\text{not detect}\} = 1 - \beta$

$= 1 - [\Pr\{D < UCL \mid \lambda\} - \Pr\{D \leq LCL \mid \lambda\}] = 1 - \Pr\{D < 32.36 \mid 12\} + \Pr\{D \leq 7.64 \mid 12\}$

$= 1 - POI(32, 12) + POI(7, 12) = 1 - 0.999 + 0.139 = 0.140$

where POI(·) is the cumulative Poisson distribution.

6-29. A fraction nonconforming control chart is to be established with a center line of 0.01 and two-sigma control limits.

(a) How large should the sample size be if the lower control limit is to be nonzero?

$p = 0.01, L = 2$

$n > \left(\frac{1-p}{p}\right)L^2$

$n > \left(\frac{1-0.01}{0.01}\right)2^2$

$n > 396$

$n \geq 397$

(b) How large should the sample size be if we wish the probability of detecting a shift to 0.04 to be 0.50?

$\delta = 0.04 - 0.01 = 0.03$

$n = \left(\frac{L}{\delta}\right)^2 p(1-p) = \left(\frac{2}{0.03}\right)^2 (0.01)(1-0.01) = 44$

6-31. A process that produces bearing housings is controlled with a fraction nonconforming control chart, using sample size $n = 100$ and a center line $= 0.02$.

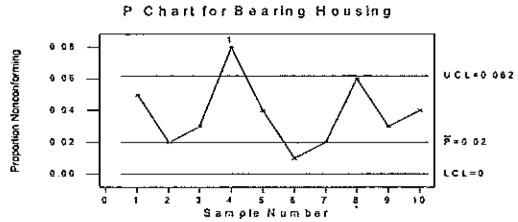
(a) Find the three-sigma limits for this chart.

$n = 100,\ \bar{p} = 0.02$

$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.02 + 3\sqrt{\frac{0.02(1-0.02)}{100}} = 0.062 \qquad \text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.02 - 3\sqrt{\frac{0.02(1-0.02)}{100}} \Rightarrow 0$

(b) Analyze the ten new samples ($n = 100$) shown here for statistical control. What conclusions can you draw about the process now?

Sample Number	Number Noncomforming
1	5
2	2
3	3
4	8
5	4
6	1
7	2
8	6
9	3
10	4



The process is out of control. Sample 4 exceeds the upper control limit. $\bar{p} = 0.038$ and $\hat{\sigma}_p = 0.0191$

6-34. Consider the fraction nonconforming control chart in exercise 6-5. Find the equivalent np chart.

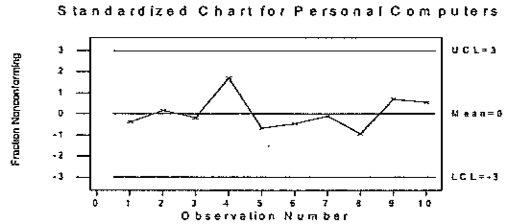
$\text{CL} = n\bar{p} = 2500(0.1228) = 307$

$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 307 + 3\sqrt{307(1-0.1228)} = 356.23$

$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 307 - 3\sqrt{307(1-0.1228)} = 257.77$

6-35. Construct a standardized control chart for the data in Exercise 6-3.

$\bar{p} = 0.06$ and $z_i = (\hat{p}_i - 0.06)\big/\sqrt{0.06(1-0.06)/n_i} = (\hat{p}_i - 0.06)\big/\sqrt{0.0564/n_i}$

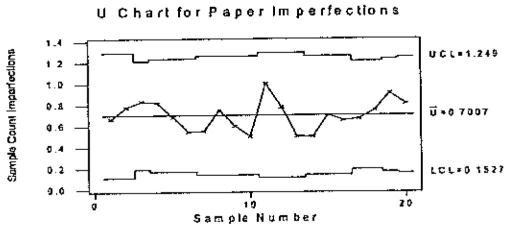


6-37. A paper mill uses a control chart to monitor the imperfection in finished rolls of paper. Production output is inspected for 20 days, and the resulting data are shown here. Use these data to set up a control chart for nonconformities per roll of paper. Does the process appear to be in statistical control? What center line and control limits would you recommend for controlling current production?

Day	No. Of Rolls Produced	Total No. of Imperfections	Day	No. Of Rolls Produced	Total No. of Imperfections
1	18	12	11	18	18
2	18	14	12	18	14
3	24	20	13	18	9
4	22	18	14	20	10
5	22	15	15	20	14
6	22	12	16	20	13
7	20	11	17	24	16
8	20	15	18	24	18
9	20	12	19	22	20
10	20	10	20	21	17

$CL = \bar{u} = 0.7007$ $\quad UCL_i = \bar{u} + 3\sqrt{\bar{u}/n_i} = 0.7007 + 3\sqrt{0.7007/n_i}$ $\quad LCL_i = \bar{u} - 3\sqrt{\bar{u}/n_i} = 0.7007 - 3\sqrt{0.7007/n_i}$

n_i	$[LCL_i, UCL_i]$
18	[0.1088, 1.2926]
20	[0.1392, 1.2622]
21	[0.1527, 1.2487]
22	[0.1653, 1.2361]
24	[0.1881, 1.2133]

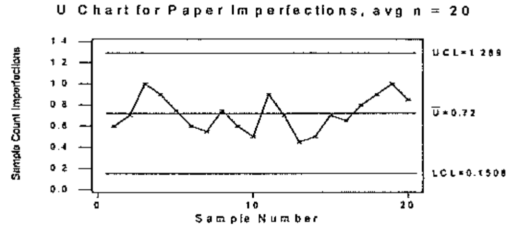


The process appears to be in statistical control. However, it would be recommend to use a standardized control chart for the process.

6-38. Continuation of Exercise 6-37. Consider the papermaking process in Exercise 6-37. Set up a u chart based on an average sample size to control this process.

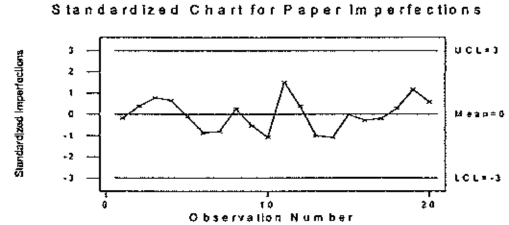
$CL = \bar{u} = 0.7007, \quad \bar{n} = 20.55$

$UCL = \bar{u} + 3\sqrt{\bar{u}/\bar{n}} = 0.7007 + 3\sqrt{0.7007/20.55} = 1.2547$ $\quad LCL = \bar{u} - 3\sqrt{\bar{u}/\bar{n}} = 0.7007 - 3\sqrt{0.7007/20.55} = 0.1468$



6-39. Continuation of Exercise 6-37. Consider the papermaking process in Exercise 6-37. Set up a standardized u chart for this process.

$z_i = (u_i - \bar{u})\big/\sqrt{\bar{u}/n_i} = (u_i - 0.7007)\big/\sqrt{0.7007/n_i}$

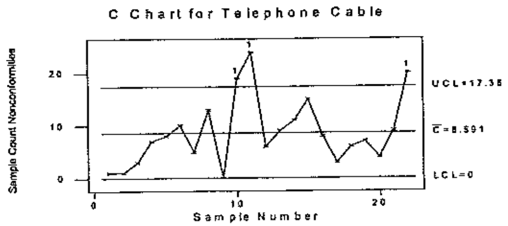


6-41. The following data represent the number of nonconformities per 1000 meters in telephone cable. From analysis of these data, would you conclude that the process is in statistical control? What control procedure would you recommend for future production?

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	1	12	6
2	1	13	9
3	3	14	11
4	7	15	15
5	8	16	8
6	10	17	3
7	5	18	6
8	13	19	7
9	0	20	4
10	19	21	9
11	24	22	20

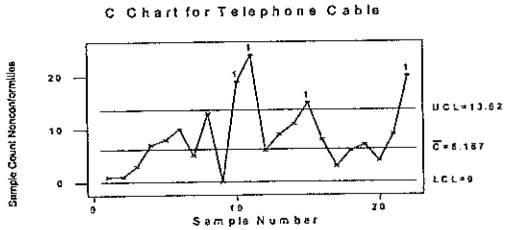
c chart based on 1000 m/inspection unit:

$\text{CL} = \bar{c} = 8.59 \qquad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 8.59 + 3\sqrt{8.59} = 17.384 \qquad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 8.59 - 3\sqrt{8.59} \Rightarrow 0$



Process is not in statistical control; three subgroups exceed the UCL. Exclude subgroups 10, 11 and 22, then re-calculate the control limits. Subgroup 15 will then be out of control and should also be excluded.

$\text{CL} = \bar{c} = 6.17 \qquad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 6.17 + 3\sqrt{6.17} = 13.62 \qquad \text{LCL} \Rightarrow 0$



6-43. Consider the data in Exercise 6-41. Suppose a new inspection unit is defined as 2500 m of wire.

(a) What are the center line and control limits for a control chart for monitoring future production based on the total number of nonconformities in the new inspection unit?

The new inspection unit is $n = 2500/1000 = 2.5$ of the old unit. A c chart of the total number of nonconformities per inspection unit is appropriate.

$\text{CL} = n\bar{c} = 2.5(6.17) = 15.43$

$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 15.43 + 3\sqrt{15.43} = 27.21$

$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 15.43 - 3\sqrt{15.43} = 3.65$

The plot point, $\hat{c}$, is the total number of nonconformities found while inspecting a sample 2500m in length.

(b) What are the center line and control limits for a control chart for average nonconformities in the new inspection unit?

The sample is $n = 1$ new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{111}{(18 \times 1000)/2500} = 15.42$$

$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 15.42 + 3\sqrt{15.42/1} = 27.20 \qquad \text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 15.42 - 3\sqrt{15.42/1} = 3.64$

The plot point, $\hat{u}$, is the average number of nonconformities found in 2500m, and since $n = 1$, this is the same as the total number of nonconformities.

6-45. Find the three-sigma control limits for

(a) A c chart with process average equal to four nonconformities.

$\text{CL} = \bar{c} = 4 \qquad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10 \qquad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$

(b) A u chart with $c = 4$ and $n = 4$.

$c = 4 \quad n = 4$

$\text{CL} = \bar{u} = c/n = 4/4 = 1 \qquad \text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 1 + 3\sqrt{1/4} = 2.5 \qquad \text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 1 - 3\sqrt{1/4} \Rightarrow 0$

5 6-47. Find the three-sigma control limits for

(a) A c chart with process average equal to nine nonconformities.

$\text{CL} = \bar{c} = 9 \qquad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 9 + 3\sqrt{9} = 18 \qquad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 9 - 3\sqrt{9} = 0$

(b) A u chart with $c = 16$ and $n = 4$.

$c = 16 \quad n = 4$

$\text{CL} = \bar{u} = c/n = 16/4 = 4 \qquad \text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 4 + 3\sqrt{4/4} = 7 \qquad \text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 4 - 3\sqrt{4/4} = 1$

6-49. Find 0.975 and 0.025 probability limits for a control chart for nonconformities when $c = 7.6$.

Using the cumulative Poisson distribution

x	$\Pr\{D \le x \mid c = 7.6\}$
2	0.019
3	0.055
12	0.954
13	0.976

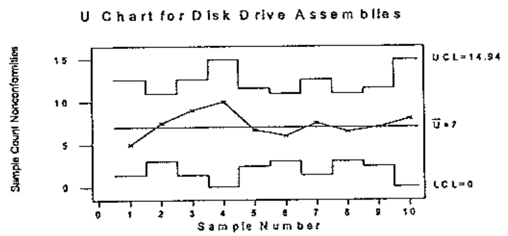
for the c chart, UCL = 13 and LCL = 2. As a comparison, the normal distribution gives

$\text{UCL} = \bar{c} + z_{0.975}\sqrt{\bar{c}} = 7.6 + 1.96\sqrt{1.6} = 13.00$ $\quad \text{LCL} = \bar{c} - z_{0.025}\sqrt{\bar{c}} = 7.6 - 1.96\sqrt{1.6} = 2.20$

6-51. The number of workmanship nonconformities observed in the final inspection of disk-drive assemblies has been tabulated as shown here. Does the process appear to be in control?

Day	No. of Assemblies Inspected	Total No. of Nonconformities
1	2	10
2	4	30
3	2	18
4	1	10
5	3	20
6	4	24
7	2	15
8	4	26
9	3	21
10	1	8

u chart with control limits based on each sample size: $\bar{u} = 7$; $\text{UCL}_i = 7 + 3\sqrt{7/n_i}$; $\text{LCL}_i = 7 - 3\sqrt{7/n_i}$



The process is in statistical control.

6-53. A textile mill wishes to establish a control procedure on flaws in towels it manufactures. Using an inspection unit of 50 units, past inspection data show that 100 previous inspection units had 850 total flaws. What type of control chart is appropriate? Design the control chart such that it has two-sided probability control limits of $\alpha = 0.06$, approximately. Give the center line and control limits.

A c chart with one inspection unit equal to 50 manufacturing units is appropriate. $\bar{c} = 850/100 = 8.5$. From the cumulative Poisson distribution,

x	$\Pr\{D \le x \mid c = 8.5\}$
3	0.030
13	0.949
14	0.973

LCL = 3 and UCL = 13.
For comparison, the normal distribution gives

$\text{UCL} = \bar{c} + z_{0.97}\sqrt{\bar{c}} = 8.5 + 1.88\sqrt{8.5} = 13.98$ $\quad \text{LCL} = \bar{c} + z_{0.03}\sqrt{\bar{c}} = 8.5 - 1.88\sqrt{8.5} = 3.02$

6-55. Assembled portable television sets are subjected to a final inspection for surface defects. A total procedure is established based on the requirement that if the average number on nonconformities per unit is 4.0, the probability of concluding that the process is in control will be 0.99. There is to be no lower control limit. What is the appropriate type of control chart and what is the required upper control limit?

$\bar{u} = 4.0$ average number of nonconformities/unit. Desire $\alpha = 0.99$. Use the cumulative Poisson distribution to determine the UCL.

x	$\Pr\{\text{CL} \le x\}$	x	$\Pr\{\text{CL} \le x\}$
0	0.018	6	0.889
1	0.092	7	0.949
2	0.238	8	0.979
3	0.433	9	0.992
4	0.629	10	0.997
5	0.785	11	0.999

An UCL = 9 will give a probability of 0.992 of concluding the process is in control, when in fact it is.

6-59. A control chart for nonconformities is maintained on a process producing desk calculators. The inspection unit is defined as two calculators. The average number of nonconformities per machine when the process is in control is estimated to be two.

(a) Find the appropriate three-sigma control limits for this size inspection unit.

$\bar{u}$ = average # nonconformities/calculator = 2

Construct a c chart with $\bar{c} = \bar{u} \times n = 2(2) = 4$ nonconformities/inspection unit.

$\text{CL} = \bar{c} = 4 \qquad \text{UCL} = \bar{c} + k\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10 \qquad \text{LCL} = \bar{c} - k\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$

(b) What is the probability of type I error for this control chart?

Type I error =

$\alpha = \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\} = \Pr\{D < 0 \mid 4\} + 1 - \Pr\{D \le 10 \mid 4\}$

$= 0 + 1 - \text{POI}(10, 4) = 1 - 0.997 = 0.003$

where POI(·) is the cumulative Poisson distribution.

6-61. Suppose that we wish to design a control chart for nonconformities per unit with L-sigma limits. Find the minimum sample size that would result in a positive lower control limit for this chart.

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c is the number of nonconformities per unit. Desire L sigma control limits.

$n\bar{c} - L\sqrt{n\bar{c}} > 0$

$n\bar{c} > L\sqrt{n\bar{c}}$

$n > L^2/\bar{c}$

6-63. Suggest at least two non manufacturing scenarios in which attributes control charts could be used for process monitoring?

There are endless possibilities for collection of attributes data from nonmanufacturing processes. Consider a product distribution center (or any warehouse) with processes for filling and shipping orders. One could track the number of orders filled incorrectly (wrong parts, too few/many parts, wrong part labeling,), packaged incorrectly (wrong material, wrong package labeling), invoiced incorrectly, etc. Or consider an accounting firm where attribute control charts are applied to errors in statements, tax preparation, etc. (hopefully caught internally with a verification step).

CHAPTER 7

Process and Measurement System Capability Analysis

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Estimate process capability using a histogram or probability plot
2. Use and interpret process capability ratios: C_p, C_{pk}, C_{pl}, C_{pu}, and C_{pm}
3. Compute confidence intervals on process capability ratios
4. Compute process capability ratios from control charts
5. Determine gage and measurement capability from control charts, tabular data and analysis of variance
6. Set specification limits on discrete components for assemblies and components that interact such as electrical components

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Exercises

7-1. Consider the piston-ring data in Table 5-1. Estimate the process capability assuming that specifications are 74.00 ± 0.035 mm.

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d_2 is found in Appendix VI, Page 761 for $n = 5$

$\hat{\mu} = \bar{\bar{x}} = 74.001$; $\bar{R} = 0.023$; $\hat{\sigma} = \bar{R}/d_2 = 0.023/2.326 = 0.010$

$\text{SL} = 74.000 \pm 0.035 = [73.965, 74.035]$

$$\hat{C}_p = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}} = \frac{74.035-73.965}{6(0.010)} = 1.17$$

$$\hat{C}_{pl} = \frac{\hat{\mu}-\text{LSL}}{3\hat{\sigma}} = \frac{74.001-73.965}{3(0.010)} = 1.20$$

$$\hat{C}_{pu} = \frac{\text{USL}-\hat{\mu}}{3\hat{\sigma}} = \frac{74.035-74.001}{3(0.010)} = 1.13$$

$$\hat{C}_{pk} = \min\left(\hat{C}_{pl}, \hat{C}_{pu}\right) = 1.13$$

7-3. Estimate process capability using $\bar{x}$ and R charts for the power supply voltage data in Exercise 5-2. If specifications are at 350 ± 5 V, calculate C_p, C_{pk} and C_{pkm}. Interpret these capability ratios.

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d_2 is found in Appendix VI, Page 761 for $n = 4$

$\hat{\mu} = \bar{\bar{x}} = 10.375$; $\bar{R}_x = 6.25$; $\hat{\sigma}_x = \bar{R}/d_2 = 6.25/2.059 = 3.04$

$\text{USL}_x = [(350+5)-350]\times 10 = 50$; $\text{LSL}_x = [(350-5)-350]\times 10 = -50$

From Exercise 5-2, $x_i = (\text{obs}_i - 350)\times 10$

$$\hat{C}_p = \frac{\text{USL}_x-\text{LSL}_x}{6\hat{\sigma}_x} = \frac{50-(-50)}{6(3.04)} = 5.48$$

The process produces product that uses approximately 18% (1/5.48 = 0.182) of the total specification band.

$$\hat{C}_{pu} = \frac{\text{USL}_x-\hat{\mu}}{3\hat{\sigma}_x} = \frac{50-10.375}{3(3.04)} = 4.34$$

$$\hat{C}_{pl} = \frac{\hat{\mu}-\text{LSL}_x}{3\hat{\sigma}_x} = \frac{10.375-(-50)}{3(3.04)} = 6.62$$

$$\hat{C}_{pk} = \min(\hat{C}_{pu}, \hat{C}_{pl}) = 4.34$$

This is an extremely capable process, with an estimated percent defective much less than 1ppb, because $z = \frac{USL-\hat{\mu}}{\hat{\sigma}} = 3*\min(\hat{C}_{pu}, \hat{C}_{pl}) = 3(4.34) = 13.02$. Note that the C_{pk} is less than C_p, indicating that the process is not centered and is not achieving potential capability. However, this process capability ratio does not tell *where* the mean is located within the specification band. To estimate the location of the mean we use C_{pm}.

$$V = \frac{T-\bar{x}}{S} = \frac{0-10.375}{3.04} = -3.4128$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{5.48}{\sqrt{1+(-3.4128)^2}} = 1.54$$

Since C_{pm} is greater than 4/3, the mean μ lies within approximately the middle fourth of the specification band.

$$\xi = \frac{\hat{\mu}-T}{\hat{\sigma}} = \frac{10.375-0}{3.04} = 3.41$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1+\xi^2}} = \frac{1.54}{\sqrt{1+3.41^2}} = 0.43$$

7-5. A process is in control with $\bar{\bar{x}} = 100$, $\bar{S} = 1.05$ and $n = 5$. The process specifications are at 95 ± 10. The quality characteristic has a normal distribution.

(a) Estimate the potential capability.

Given: $\hat{\mu} = \bar{\bar{x}} = 100$; $\bar{S} = 1.05$; $\hat{\sigma}_x = \bar{S}/c_4 = 1.05/0.9400 = 1.117$, c_4 is from Appendix VI, page 761.

The potential: $\hat{C}_p = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}} = \frac{(95+10)-(95-10)}{6(1.117)} = 2.98$

(b) Estimate the actual capability.

The actual: $\hat{C}_{pl} = \frac{\hat{\mu}-\text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(95-10)}{3(1.117)} = 4.48$

$$\hat{C}_{pu} = \frac{\text{USL}_x-\hat{\mu}}{3\hat{\sigma}_x} = \frac{(95+10)-100}{3(1.117)} = 1.49$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.49$$

(c) How much could the fallout in the process be reduced if the process were corrected to operate at the nominal specifications?

$$\hat{p}_{\text{potential}} = \Pr\left\{z < \frac{85-95}{1.117}\right\} + 1 - \Pr\left\{z \le \frac{105-95}{1.117}\right\} = \Phi(-8.953) + 1 - \Phi(8.953)$$

$$= 0.000000 + 1 - 1.000000 = 0.000000$$

By moving the process to the center of the specification limits the fallout is reduced from 4 ppm to 0 ppm.

7-7. Consider the two processes shown here (the sample size is $n = 5$):

Process A	Process B
$\bar{\bar{x}}_A = 100$	$\bar{\bar{x}}_B = 105$
$\bar{S}_A = 3$	$\bar{S}_B = 1$

Specifications are at 100 ± 10. Calculate C_p, C_{pk} and C_{pm} and interpret these ratios. Which process would you prefer to use?

Given that $n = 5$,

Process A: $\hat{\mu} = \bar{\bar{x}}_A = 100;\ \bar{S}_A = 3;\ \hat{\sigma}_A = \bar{S}_A / c_4 = 3/0.9400 = 3.191$, c_4 is from Appendix VI, page 761.

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(100+10)-(100-10)}{6(3.191)} = 1.045$$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \frac{(100+10)-100}{3(3.191)} = 1.045$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(100-10)}{3(3.191)} = 1.045$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.045$$

$$V = \frac{T - \bar{\bar{x}}}{S} = \frac{100-100}{3.191} = 0$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{1.045}{\sqrt{1+(0)^2}} = 1.045$$

$C_{pm} = C_{pk}$ for a centered process.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Pr\{x < \text{LSL}\} + 1 - \Pr\{x \le \text{USL}\}$$

$$= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + 1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} = \Pr\left\{z < \frac{90-100}{3.191}\right\} + 1 - \Pr\left\{z \le \frac{110-100}{3.191}\right\}$$

$$= \Phi(-3.134) + 1 - \Phi(3.134) = 0.000863 + 1 - 0.999137 = 0.001726$$

Process B: $\hat{\mu} = \bar{\bar{x}}_B = 105;\ \bar{S}_B = 1;\ \hat{\sigma}_B = \bar{S}_B / c_4 = 1/0.9400 = 1.064$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(100+10)-(100-10)}{6(1.064)} = 3.133$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{105-(100-10)}{3(1.064)} = 4.699$$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{(100+10)-105}{3(1.064)} = 1.566$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.566$$

$$V = \frac{T - \bar{\bar{x}}}{S} = \frac{100-105}{1.064} = -4.699$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{3.133}{\sqrt{1+(-4.699)^2}} = 0.652$$

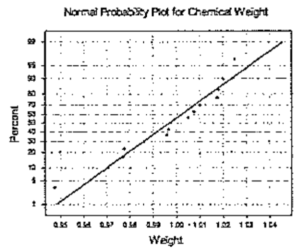
$$\hat{p} = \Pr\left\{z < \frac{90-105}{1.064}\right\} + 1 - \Pr\left\{z \le \frac{110-105}{1.064}\right\} = \Phi(-14.098) + 1 - \Phi(4.699)$$

$$= 0.000000 + 1 - 0.999999 = 0.000001$$

Prefer to use Process B with estimated process fallout of 0.000001 instead of Process A with estimated fallout 0.001726. Process B is not centered but has a much tighter distribution than that of Process A, producing much less fallout.

7-9. The weights of nominal 1-lb containers of a concentrated chemical ingredient are shown here. Prepare a normal probability plot of the data and estimate process capability.

0.9475	0.9775	0.9965	1.0075	1.0180
0.9705	0.9860	0.9975	1.0100	1.0200
0.9770	0.9960	1.0050	1.0175	1.0250



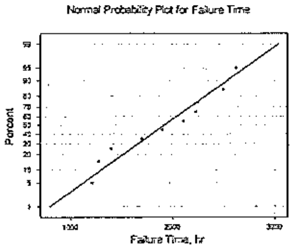
Remember that the standard deviation is the slope of the line and is conveniently estimated as the distance between the 84th and the 50th percentiles.

$\bar{x} \approx x_{50} = 0.9975;\ x_{84} = 1.0200;\ \hat{\sigma} = x_{84} - x_{50} = 1.0200 - 0.9975 = 0.0225$

$6\hat{\sigma} = 6(0.0225) = 0.1350$

7-11. The failure time in hours of 10 LSI memory devices is shown here. Plot the data on normal probability paper and, if appropriate, estimate process capability. Is it safe to estimate the proportion of circuits that fail below 1200h?

1210	2105
1275	2230
1400	2250
1695	2500
1900	2625



The plot shows that the data is not normally distributed; so it is not safe to make any estimations.

7-13. A company has been asked by an important customer to demonstrate that its process capability ratio C_p exceeds 1.33. Its has taken a sample of 50 parts and obtained the point estimate $\hat{C}_p = 1.52$. Assume that the quality characteristic follows a normal distribution. Can the company demonstrate that C_p exceeds 1.33 at the 95% level of confidence? What level of confidence would give a one-sided lower confidence limit on C_p that exceeds 1.33?

Given: $n = 50; \hat{C}_p = 1.52; 1-\alpha = 0.95; \chi^2_{1-\alpha,n-1} = \chi^2_{0.95,49} = 33.9303$

$$\hat{C}_p \sqrt{\frac{\chi^2_{1-\alpha,n-1}}{n-1}} \le C_p$$

$$1.52\sqrt{\frac{33.9303}{49}} = 1.26 \le C_p$$

The company cannot demonstrate that the PCR exceeds 1.33 at a 95% confidence level.

$$1.52\sqrt{\frac{\chi^2_{1-\alpha,49}}{49}} = 1.33$$

$$\chi^2_{1-\alpha,49} = 49\left(\frac{1.33}{1.52}\right)^2 = 37.52$$

$$1-\alpha = 0.88$$

$$\alpha = 0.12$$

The confidence level that can be established is 88%.

7-15. The molecular weight of a particular polymer should fall between 2100 and 2350. Fifty samples of this material were analyzed with the results $\bar{x} = 2275$ and $S = 60$. Assume that molecular weight is normally distributed.

(a) Calculate a point estimate of C_{pk}.

USL = 2350; LSL = 2100; $\bar{x} = 2275; S = 60; n = 50$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{2350-2275}{3(60)} = 0.42$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{2275-2100}{3(60)} = 0.97$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.42$$

(b) Find a 95% confidence interval on C_{pk}.

$\alpha = 0.05; z_{\alpha/2} = z_{0.025} = 1.960$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}^2_{pk}} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}^2_{pk}} + \frac{1}{2(n-1)}}\right]$$

$$0.42\left[1 - 1.96\sqrt{\frac{1}{9(50)(0.42)^2} + \frac{1}{2(50-1)}}\right] \le C_{pk} \le 0.42\left[1 + 1.96\sqrt{\frac{1}{9(50)(0.42)^2} + \frac{1}{2(50-1)}}\right]$$

$$0.2957 \le C_{pk} \le 0.5443$$

7-17. An operator-instrument combination is known to test parts with an average error of zero; however, the standard deviation of measurement error is estimated to be 3. Samples from a controlled process were analyzed, and the total variability was estimated to be $\hat{\sigma} = 5$. What is the true process standard deviation?

$\sigma_{\text{oper*inst}} = 0; \hat{\sigma}_{\text{inst}} = \hat{\sigma}_{\text{meas}} = 3; \hat{\sigma}_{\text{total}} = 5$

$$\hat{\sigma}^2_{\text{total}} = \hat{\sigma}^2_{\text{meas}} + \hat{\sigma}^2_{\text{process}}$$

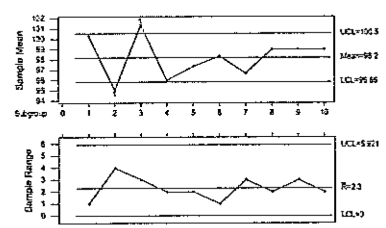
$$\hat{\sigma}_{\text{process}} = \sqrt{\hat{\sigma}^2_{\text{total}} - \hat{\sigma}^2_{\text{meas}}} = \sqrt{5^2 - 3^2} = 4$$

7-19. Ten parts are measured three times by the same operator in a gage capability study. The data are shown here.

	Measurements		
Part Number	1	2	3
1	100	101	100
2	95	93	97
3	101	103	100
4	96	95	97
5	98	98	96
6	99	98	98
7	95	97	98
8	100	99	98
9	100	100	97
10	100	98	99

(a) Describe the measurement error that results from the use of this gage.

Xbar/R Chart for Measurements



Test Results for Xbar Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 2 3

The $\bar{x}$ chart has two out-of-control points, and the R chart is in control. This indicates that the operator is not having difficulty making consistent measurements.

(b) Estimate total variability and product variability.

$\bar{\bar{x}} = 98.2;\ \bar{R} = 2.3;\ \hat{\sigma}_{gage} = \bar{R}/d_2 = 2.3/1.693 = 1.359$

$\hat{\sigma}^2_{total} = 4.717$

$\hat{\sigma}^2_{product} = \hat{\sigma}^2_{total} - \hat{\sigma}^2_{gage} = 4.717 - 1.359^2 = 2.872$

$\hat{\sigma}_{product} = 1.695$

(c) What percentage of total variability is due to the gage?

$$\frac{\hat{\sigma}_{gage}}{\hat{\sigma}_{total}} \times 100 = \frac{1.359}{\sqrt{4.717}} \times 100 = 62.5\%$$

(d) If specifications on the part are at 100 ± 15, find the P/T ratio for this gage. Comment on the adequacy of the gage.

USL = 100 + 15 = 115; LSL = 100 − 15 = 85

$$\frac{P}{T} = \frac{6\hat{\sigma}_{gage}}{USL - LSL} = \frac{6(1.359)}{115-85} = 0.272$$

7-21 The following data were taken by one operator during a gage capability study.

S

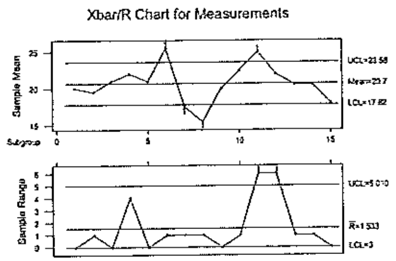
	Measurements		
Part Number	1	2	Abs(Meas 1 – Meas 2)
1	20	20	0
2	19	20	1
3	21	21	0
4	24	20	4
5	21	21	0
6	25	26	1
7	18	17	1
8	16	15	1
9	20	20	0
10	23	22	1
11	28	22	6
12	19	25	6
13	21	20	1
14	20	21	1
15	18	18	0

(a) Estimate the gage capability.

Calculate the range of each pair of measurements. See additional column in data set. Calculate the average range, $\bar{R}$, then find d_2 from Appendix VI, page 761.

$\hat{\sigma}_{gage} = \bar{R}/d_2 = 1.533/1.128 = 1.359$

(b) Does the control chart analysis of these data indicate any potential problem using the gage?



Test Results for Xbar Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 6 7 8 11

Test Results for R Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 11 12

Out-of-control points on R chart indicate operator difficulty with using gage.

7-23. Two parts are $\bar{\bar{x}} = 100$ assembled as shown in the figure. The distributions of x_1 and x_2 are normal, with $\mu_1 = 20$, $\sigma_1 = 0.3$, $\mu_2 = 19.6$, and $\sigma_2 = 0.4$. The specifications of the clearance between the mating parts are 0.5 ± 0.4. What fraction of assemblies will fail to meet specifications if assembly is at random?

Given the distributions: $x_1 \sim N(20, 0.3^2)$; $x_2 \sim N(19.6, 0.4^2)$
Nonconformities will occur if $y = x_1 - x_2 < 0.1$ or $y = x_1 - x_2 > 0.9$
$\mu_y = \mu_1 - \mu_2 = 20 - 19.6 = 0.4$
$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 = 0.3^2 + 0.4^2 = 0.25;\ \sigma_y = 0.50$
$\Pr\{\text{nonconformities}\} = \Pr\{y < \text{LSL}\} + \Pr\{y > \text{USL}\} = \Pr\{y < 0.1\} + \Pr\{y > 0.9\}$
$$= \Pr\{y < 0.1\} + 1 - \Pr\{y \le 0.9\} = \Phi\left(\frac{0.1 - 0.4}{\sqrt{0.25}}\right) + 1 - \Phi\left(\frac{0.9 - 0.4}{\sqrt{0.25}}\right)$$
$$= \Phi(-0.6) + 1 - \Phi(1.00) = 0.2743 + 1 - 0.8413 = 0.4330$$
43.3% of the assemblies will be nonconforming.

7-25. A rectangular piece of metal of width W and length L is cut from a plate of thickness T. If W, L and T are independent random variables with means and standard deviations given here and the density of the metal is 0.08 g/cm^3, what would be the estimated mean and standard deviation of the weights of pieces produced by this process?

Variable	Mean	Standard Deviation
W	10 cm	0.2 cm
L	20 cm	0.3 cm
T	3 cm	0.1 cm

The weight of the part is calculated as the density times the volume.
$\text{weight} = d \times W \times L \times T$
By application of equation 7-29:
$$\text{weight} \cong d\left[\mu_W \mu_L \mu_T + (W - \mu_W)\mu_L \mu_T + (L - \mu_L)\mu_W \mu_T + (T - \mu_T)\mu_W \mu_L\right]$$
By application of equation 7-30:
$$\hat{\mu}_{\text{weight}} \cong d[\mu_W \mu_L \mu_T] = 0.08(10)(20)(3) = 48$$
By application of equation 7-30:
$$\hat{\sigma}^2_{\text{weight}} \cong d^2[\hat{\mu}_W^2 \hat{\sigma}_L^2 \hat{\sigma}_T^2 + \hat{\mu}_L^2 \hat{\sigma}_W^2 \hat{\sigma}_T^2 + \hat{\mu}_T^2 \hat{\sigma}_W^2 \hat{\sigma}_L^2]$$
$$= 0.08^2[10^2(0.3^2)(0.1^2) + 20^2(0.2^2)(0.1^2) + 3^2(0.2^2)(0.3^2)] = 0.00181$$
$$\hat{\sigma}_{\text{weight}} \cong 0.04252$$

7-27. Two resistors are connected to a battery as shown in the figure. Find the approximate expressions for the mean and variance of the resulting current (I). E, R_1 and R_2 are random variables with means μ_E, μ_{R_1}, μ_{R_2} and variances σ_E^2, $\sigma_{R_1}^2$, $\sigma_{R_1}^2$ respectively.

$I = E/(R_1 + R_2)$
$\mu_I \cong \mu_E/(\mu_{R_1} + \mu_{R_2})$ By application of equations 7-30 and 7-31.
$$\sigma_I^2 \cong \frac{\sigma_E^2}{(\mu_{R_1} + \mu_{R_2})} + \frac{\mu_E}{(\mu_{R_1} + \mu_{R_2})^2}(\sigma_{R_1}^2 + \sigma_{R_2}^2)$$

7-29. An assembly of two parts is formed by fitting a shaft into a bearing. It is known that the inside diameters of bearings are normally distributed with mean 2.010 cm and standard deviation 0.002 cm, and that the outside diameters of the shafts are normally distributed with mean 2.004 cm and standard deviation 0.001 cm. Determine the distribution of clearance between the parts if random assembly is used. What is the probability that the clearance is positive?

$\text{ID} \sim N(2.010, 0.002^2)$ and $\text{OD} \sim N(2.004, 0.001^2)$
Interference occurs if $y = \text{ID} - \text{OD} < 0$
$\mu_y = \mu_{\text{ID}} - \mu_{\text{OD}} = 2.010 - 2.004 = 0.006$
$\sigma_y^2 = \sigma_{\text{ID}}^2 + \sigma_{\text{OD}}^2 = 0.002^2 + 0.001^2 = 0.000005$
$\sigma_y = 0.002236$

$\Pr\{\text{positive clearance}\} = 1 - \Pr\{\text{interference}\} = 1 - \Pr\{y < 0\}$

$$= 1 - \Phi\left(\frac{0 - 0.006}{\sqrt{0.000005}}\right) = 1 - \Phi(-2.683) = 1 - 0.0036 = 0.9964$$

99.94% of the assemblies will have a positive clearance.

7-31. A sample of 10 items from a normal population had a mean of 300 and a standard deviation of 10. Using these data, estimate a value for the random variable such that the probability is 0.95 that 90% of the measurements on this random variable will lie below the value.

$x \sim N(300, 10^2)$; $\alpha = 1 - 0.90 = 0.10$; $\gamma = 0.95$; and one-sided

From Appendix VIII: $K = 2.355$

$\text{UTL} = \bar{x} + KS = 300 + 2.355(10) = 323.55$

7-33. A sample of 20 measurements on a normally distributed quality characteristic had $\bar{x} = 350$ and $S = 10$. Find an upper natural tolerance limit that has a probability of 0.90 of containing 95% of the distribution of this quality characteristic.

$x \sim N(350, 10^2)$; $\alpha = 1 - 0.95 = 0.05$; $\gamma = 0.90$; From Appendix VII: $K = 2.208$

$\text{UTL} = \bar{x} + KS = 350 + 2.208(10) = 372.08$

7-35. A random sample of $n = 40$ pipe sections resulted in a mean wall thickness of 0.1264 and a standard deviation of 0.0003 in. We assume that wall thickness is normally distributed.

(a) Between what limits can we say with 95% confidence that 95% of the wall thicknesses should fall?

$x \sim N(0.1264, 0.0003^2)$

$\alpha = 0.05$; $\gamma = 0.95$; from Appendix VII: $K = 2.445$

TI on $x: \bar{x} \pm KS = 0.1264 \pm 2.445(0.003) = [0.1257, 0.1271]$

(b) Construct a 95% confidence interval on the true mean thickness. Explain the difference between this interval and the one constructed in part (a).

$\alpha = 0.05$; $t_{\alpha/2,n-1} = t_{0.025,39} = 2.023$

CI on $\bar{x}: \bar{x} \pm t_{\alpha/2,n-1} S/\sqrt{n} = 0.1264 \pm 2.023\left(0.0003/\sqrt{40}\right) = [0.1263, 0.1265]$

Part (a) is a tolerance interval on individual thickness observations; part (b) is a confidence interval on mean thickness. In part (a), the interval relates to individual observations (random variables), while in part (b) the interval refers to a parameter of a distribution (an unknown constant).

CHAPTER 8

Cumulative Sum and Exponentially Weighted Moving Average Control Charts

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Discuss performance improvements, such as the ability to detect small process shifts, of the cusum and EWMA control charts relative to Shewhart charts.
2. Construct and interpret tabular cusum control charts for process mean and variability, including:
   - One-sided, two-sided, and standardized schemes
   - Selection of the reference value K and the decision interval H
   - Use of a fast initial response (FIR) feature
3. Construct and interpret an EWMA control chart, including:
   - Selection of the weight λ and the control limit width L to achieve desired ARL performance
4. Construct and interpret a MA control chart.

Exercises

8-1. The following data represent individual observations on molecular weight taken hourly from a chemical process. The target value of molecular weight is 1050 and the process standard deviation is thought to be about $\sigma = 25$.

Obs. No.	x	Obs. No.	x	Obs. No.	x	Obs. No.	x
1	1045	6	1008	11	1139	16	1125
2	1055	7	1050	12	1169	17	1163
3	1037	8	1087	13	1151	18	1188
4	1064	9	1125	14	1128	19	1146
5	1095	10	1146	15	1238	20	1167

(a) Set up a tabular cusum for the mean of this process. Design the cusum to quickly detect a shift of about 1.0σ in the process mean.

We are given the target value for the process mean, $\mu_0 = 1050$, as well as the in-control process standard deviation, $\sigma = 25$. The subgroup size for each observation is $n = 1$. The magnitude of the shift we are interested in detecting, in standard deviation units, is $\delta = 1.0$. Therefore the out-of-control value of the process mean is $\mu_1 = \mu_0 + \delta\sigma = 1050 + 1.0(25) = 1075$. The reference value is one-half the magnitude of the shift to be detected, $K = (\delta/2)\sigma = (1.0/2)25 = 12.5$. For the decision interval we use the recommended five times the process standard deviation (text p. 411), $H = 5\sigma = 5(25) = 125$.

The table below presents the tabular cusum scheme. To illustrate the calculations consider the first period, $i = 1$:

$$C_1^- = \max\left[0, (\mu_0 - K) - x_1 + C_{1-1}^-\right] = \max[0, (1050 - 12.5) - 1045 + 0] = \max[0, -7.5] = 0$$

$$C_1^+ = \max\left[0, x_1 - (\mu_0 + K) + C_{1-1}^+\right] = \max[0, 1045 - (1050 + 12.5) + 0] = \max[0, -17.5] = 0$$

Since C_1^+ is zero, N^+ is zero, and since C_1^- is zero, N^- is also zero.

i	x_i	$x_i - (\mu_0 + K)$	C_i^+	N^+	$(\mu_0 - K) - x_i$	C_i^-	N^-
1	1045	-17.5	0	0	-7.5	0	0
2	1055	-7.5	0	0	-17.5	0	0
3	1037	-25.5	0	0	0.5	0.5	1
4	1064	1.5	1.5	1	-26.5	0	0
5	1095	32.5	34	2	-57.5	0	0
6	1008	-54.5	0	0	29.5	29.5	1
7	1050	-12.5	0	0	-12.5	17	2
8	1087	24.5	24.5	1	-49.5	0	0
9	1125	62.5	87	2	-87.5	0	0
10	1146	83.5	170.5	3	-108.5	0	0
11	1139	76.5	247	4	-101.5	0	0
12	1169	106.5	353.5	5	-131.5	0	0
13	1151	88.5	442	6	-113.5	0	0
14	1128	65.5	507.5	7	-90.5	0	0

i	x_i	$x_i - (\mu_0 + K)$	C_i^+	N^+	$(\mu_0 - K) - x_i$	C_i^-	N^-
15	1238	175.5	683	8	-200.5	0	0
...							

The cusum calculations show that the upper side cusum at period 10 is $C_{10}^+ = 170.5$. Since this is the first period at which C_i^+ exceeds $H = 125$, we detect that the process is out of control at observation 10. When the assignable cause occurred—the last time the process was in control—is found by subtracting the counter, N^+, from the detection point, observation $- N^+ = 10 - 3 = 7$.

(b) Is the estimate of σ used in part (a) of this problem reasonable?

We can estimate the process standard deviation from the average moving range of the observations (text p. 257):

$$\overline{MR} = \frac{\sum_{i=2}^{m} MR_i}{(m-1)} = \frac{|1055-1045| + |1037-1055| + \cdots + |1167-1146|}{20-1} = 38.8421$$

$$\hat{\sigma} = \overline{MR}/d_2 = 38.8421/1.128 = 34.4345$$

The estimate used for σ is much smaller than that obtained from the data.

8-2. Rework Exercise 8-1 using a standardized cusum.

A standardized cusum simply uses standardized values of x for the calculations. We select $k = 0.5$ ($K = k\sigma = 0.5(1.0) = 0.5$) and $h = 5$ ($H = h\sigma = 5(1.0) = 5$). For the 1st period (text p. 417):

$$y_1 = \frac{x_1 - \mu_0}{\sigma} = \frac{1045 - 1050}{25} = -0.2$$

$$C_1^+ = \max\left[0, y_1 - k + C_{1-1}^+\right] = \max[0, -0.2 - 0.5 + 0] = \max[0, -0.7] = 0$$

$$C_1^- = \max\left[0, -k - y_1 + C_{1-1}^-\right] = \max[0, -0.5 - (-0.2) + 0] = \max[0, -0.3] = 0$$

i	x_i	y_i	$y_i - k$	C_i^+	N^+	$-y_i - k$	C_i^-	N^-
1	1045	-0.2	-0.7	0	0	-0.3	0	0
2	1055	0.2	-0.3	0	0	-0.7	0	0
3	1037	-0.52	-1.02	0	0	0.02	0.02	1
4	1064	0.56	0.06	0.06	1	-1.06	0	0
5	1095	1.8	1.3	1.36	2	-2.3	0	0
6	1008	-1.68	-2.18	0	0	1.18	1.18	1
7	1050	0	-0.5	0	0	-0.5	0.68	2
8	1087	1.48	0.98	0.98	1	-1.98	0	0
9	1125	3	2.5	3.48	2	-3.5	0	0
10	1146	3.84	3.34	6.82	3	-4.34	0	0
11	1139	3.56	3.06	9.88	4	-4.06	0	0
12	1169	4.76	4.26	14.14	5	-5.26	0	0
13	1151	4.04	3.54	17.68	6	-4.54	0	0
14	1128	3.12	2.62	20.3	7	-3.62	0	0
15	1238	7.52	7.02	27.32	8	-8.02	0	0

i	x_i	y_i	$y_i - k$	C_i^+	N^+	$-y_i - k$	C_i^-	N^-
...								

The process signals out of control at observation 10. The assignable cause occurred after observation 10 − 3 = 7.

8-3. (a) Add a headstart feature to the cusum in Exercise 8-1.

The headstart, or fast initial response (FIR), feature sets the starting values of C_0^+ and C_0^- to some nonzero value, typically $H/2$ for a 50% headstart.

$C_0^+ = C_0^- = H/2 = 125/2 = 62.5$

$$C_1^- = \max\left[0, (\mu_0 - K) - x_1 + C_0^-\right] = \max\left[0, (1050-12.5) - 1045 + 62.5\right] = \max\left[0, 55\right] = 55$$

$$C_1^+ = \max\left[0, x_1 - (\mu_0 + K) + C_0^+\right] = \max\left[0, 1045 - (1050+12.5) + 62.5\right] = \max\left[0, 45\right] = 45$$

i	x_i	$x_i - (\mu_0 + K)$	C_i^+	N^+	$(\mu_0 - K) - x_i$	C_i^-	N^-
FIR			62.5			62.5	
1	1045	-17.5	45	1	-7.5	55	1
2	1055	-7.5	37.5	2	-17.5	37.5	2
3	1037	-25.5	12	3	0.5	38	3
4	1064	1.5	13.5	4	-26.5	11.5	4
5	1095	32.5	46	5	-57.5	0	0
6	1008	-54.5	0	0	29.5	29.5	1
7	1050	-12.5	0	0	-12.5	17	2
8	1087	24.5	24.5	1	-49.5	0	0
9	1125	62.5	87	2	-87.5	0	0
10	1146	83.5	170.5	3	-108.5	0	0
11	1139	76.5	247	4	-101.5	0	0
12	1169	106.5	353.5	5	-131.5	0	0
13	1151	88.5	442	6	-113.5	0	0
14	1128	65.5	507.5	7	-90.5	0	0
15	1238	175.5	683	8	-200.5	0	0
...							

With the headstart, the cusum calculations again show that the upper side cusum at period 10 is $C_{10}^+ = 170.5$. Since this is the first period at which C_i^+ exceeds $H = 125$, we detect that the process is out of control at observation 10. Similarly, the last time the process was in control is found by subtracting the counter, N^+, from the detection point, observation $- N^+ = 10 - 3 = 7$.

(b) Use a combined Shewhart-cusum scheme on the data in Exercise 8-1. Interpret the results of both charts.

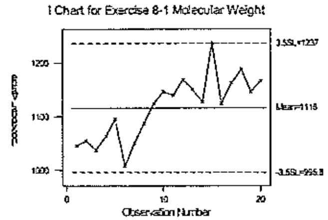
The cusum portion of the procedure is explained above in Exercise 8-1. Construction of the Shewhart Individuals control chart is similar to Chapter 5 (text p. 249), *except* that it is recommended that the control limits should be located approximately 3.5 standard deviations from the center line (text p. 419):

$\bar{x} = 1116$; $\overline{MR} = 38.8421$

$UCL_x = \bar{x} + 3.5\overline{MR}/d_2 = 1116 + 3.5(38.84)/1.128 = 1237$

$CL_x = \bar{x} = 1116$

$LCL_x = \bar{x} - 3.5\overline{MR}/d_2 = 1116 - 3.5(38.84)/1.128 = 995.8$



The Individuals control chart signals out-of-control on the upper side at observation 15 ((x_{15} = 1238) > (UCL = 1237)). Visually we can see that this observation is much larger than the others; however the cusum control chart did not detect it. Use of the combined cusum-Shewhart procedure enabled detection of this event. (Note that observation 6 would have been out of control with 3-sigma limits, LCL = 1013.)

8-5. Rework Exercise 8-4 using the standardized cusum parameters of h = 8.01 and k = 0.25. Compare the results with those obtained previously in Exercise 8-4. What can you say about the theoretical performance of those two cusum schemes?

Theoretical performance of the two schemes is determined from in-control and out-of-control ARLs. From Exercise 8-4,

$\mu_0 = 8.02; \sigma = 0.05; k = 1/2; h = 4.77$

$b = h + 1.166 = 4.77 + 1.166 = 5.936; \delta^* = 0$

$\Delta^+ = \delta^* - k = 0 - 0.5 = -0.5; \Delta^- = -\delta^* - k = -0 - 0.5 = -0.5$

$$ARL_0^+ = ARL_0^- \cong \frac{\exp[-2(-0.5)(5.936)] + 2(-0.5)(5.936) - 1}{2(-0.5)^2} = 742.964$$

$$\frac{1}{ARL_0} = \frac{1}{ARL_0^+} + \frac{1}{ARL_0^-} = \frac{2}{742.964} = 0.0027; ARL_0 = 1/0.0027 = 371.48$$

For Exercise 8-5,

$\mu_0 = 8.02; \sigma = 0.05; k = 0.25; h = 8.01$

$b = h + 1.166 = 8.01 + 1.166 = 9.176; \delta^* = 0$

$\Delta^+ = \delta^* - k = 0 - 0.25 = -0.25; \Delta^- = -\delta^* - k = -0 - 0.25 = -0.25$

$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.25)(9.176)] + 2(-0.25)(9.176) - 1}{2(-0.25)^2} = 741.6771$$

$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{741.6771} = 0.0027; \ \text{ARL}_0 = 1/0.0027 = 370.84$$

The theoretical performance of these two cusum schemes is virtually the same.

8-7. The data that follow are temperature readings from a chemical process in °C, taken every 2 minutes (read the observations down, from left.) The target value for the mean is $\mu_0 = 950$.

953	985	949	937	959	948	958	952
945	973	941	946	939	937	955	931
972	955	966	954	948	955	947	928
945	950	966	935	958	927	941	937
975	948	934	941	963	940	938	950
970	957	937	933	973	962	945	970
959	940	946	960	949	963	963	933
973	933	952	968	942	943	967	960
940	965	935	959	965	950	969	934
936	973	941	956	962	938	981	927

(a) Estimate the process standard deviation.

We can estimate the process standard deviation from the average moving range of the observations (text p. 257):

$$\overline{\text{MR}} = \frac{\sum_{i=2}^{m} \text{MR}_i}{(m-1)} = \frac{|945-953| + |972-945| + \cdots + |927-934|}{80-1} = 13.7215$$

$$\hat{\sigma} = \overline{\text{MR}}/d_2 = 13.7215/1.128 = 12.16$$

(b) Set up and apply a tabular cusum for this process using standardized values $h = 5$ and $k = ½$. Interpret this chart.

Given $\mu_0 = 950$, $k = 0.5$ ($K = k\sigma = 0.5(1.0) = 0.5$), and $h = 5$ ($H = h\sigma = 5(1.0) = 5$). Use the estimated standard deviation, $\sigma = 12.16$. For the 1st period (text p. 417):

$$y_1 = \frac{x_1 - \mu_0}{\sigma} = \frac{953 - 950}{12.16} = 0.2466$$

$$C_1^+ = \max\left[0, y_1 - k + C_{1-1}^+\right] = \max[0, 0.2466 - 0.5 + 0] = \max[0, -0.2534] = 0$$

$$C_1^- = \max\left[0, -k - y_1 + C_{1-1}^-\right] = \max[0, -0.5 - (0.2466) + 0] = \max[0, -0.7466] = 0$$

i	x_i	y_i	C_i^+	N^+	C_i^-	N^-	i	x_i	y_i	C_i^+	N^+	C_i^-	N^-
1	953	0.2466	0.00	0	0.00	0	51	948	-0.1644	0.93	3	0.00	0
2	945	-0.4110	0.00	0	0.00	0	52	937	-1.0687	0.00	0	0.57	1
3	972	1.8085	1.31	1	0.00	0	53	955	0.4110	0.00	0	0.00	0
4	945	-0.4110	0.40	2	0.00	0	54	927	-1.8907	0.00	0	1.39	1
5	975	2.0552	1.95	3	0.00	0	55	940	-0.8221	0.00	0	1.71	2
6	970	1.6441	3.10	4	0.00	0	56	962	0.9865	0.49	1	0.23	3
7	959	0.7399	3.34	5	0.00	0	57	963	1.0687	1.06	2	0.00	0
8	973	1.8907	4.73	6	0.00	0	58	943	-0.5754	0.00	0	0.08	1
9	940	-0.8221	3.41	7	0.32	1	59	950	0.0000	0.00	0	0.00	0
10	936	-1.1509	1.75	8	0.97	2	60	938	-0.9865	0.00	0	0.49	1
11	985	2.8772	4.13	9	0.00	0	61	958	0.6577	0.16	1	0.00	0
12	973	1.8907	5.52	10	0.00	0	62	955	0.4110	0.07	0	0.00	0
13	955	0.4110	5.43	11	0.00	0	63	947	-0.2466	0.00	0	0.00	0
14	950	0.0000	4.93	12	0.00	0	64	941	-0.7399	0.00	0	0.24	1
15	948	-0.1644	4.27	13	0.00	0	65	938	-0.9865	0.00	0	0.73	2
…													

The cusum calculations show that the upper side cusum at period 12 is $C_{12}^+ = 5.52$. Since this is the first period at which C_i^+ exceeds $h = 5$, we detect that the process is out of control at observation 12. When the assignable cause occurred—the last time the process was in control—is found by subtracting the counter, N^+, from the detection point, observation $- N^+ = 12 - 10 = 2$.

8-9. Viscosity measurements on a polymer are made every 10 minutes by an on-line viscometer. Thirty-six observations are shown here (read the observations down, from left). The target viscosity for this process is $\mu_0 = 3200$.

3169	3205	3185	3188
3173	3203	3187	3183
3162	3209	3192	3175
3154	3208	3199	3174
3139	3211	3197	3171
3145	3214	3193	3180
3160	3215	3190	3179
3172	3209	3183	3175
3175	3203	3197	3174

(a) Estimate the process standard deviation.

The process standard deviation is estimated from the average moving range of the observations (text p. 257):

$$\overline{\text{MR}} = \frac{\sum_{i=2}^{m} \text{MR}_i}{(m-1)} = \frac{|3173-3169| + |3162-3173| + \cdots + |3174-3175|}{36-1} = 6.71429$$

$$\hat{\sigma} = \overline{\text{MR}}/d_2 = 6.71429/1.128 = 5.95$$

(b) Construct a tabular cusum for this process using standardized values of $h = 8.01$ and $k = 0.25$.

Given $\mu_0 = 3200$, $k = 0.25$ ($K = k\sigma = 0.25(5.95) = 1.49$), and $h = 5$ ($H = h\sigma = 8.01(5.95) = 47.66$). Use the estimated standard deviation, $\sigma = 5.95$. For the 1st period (text p. 417):

$$y_1 = \frac{x_1 - \mu_0}{\sigma} = \frac{3169 - 3200}{5.95} = -5.21$$

$$C_1^+ = \max\left[0, y_1 - k + C_{1-1}^+\right] = \max\left[0, -5.21 - 0.25 + 0\right] = \max\left[0, -5.46\right] = 0$$

$$C_1^- = \max\left[0, -k - y_1 + C_{1-1}^-\right] = \max\left[0, -0.25 - (-5.21) + 0\right] = \max\left[0, 4.96\right] = 4.96$$

i	x_i	y_i	C_i^+	N^+	C_i^-	N^-	i	x_i	y_i	C_i^+	N^+	C_i^-	N^-
1	3169	-5.21	0.00	0	4.96	1	19	3185	-2.52	7.92	10	43.80	19
2	3173	-4.54	0.00	0	9.24	2	20	3187	-2.18	5.48	11	45.74	20
3	3162	-6.38	0.00	0	15.38	3	21	3192	-1.34	3.89	12	46.83	21
4	3154	-7.73	0.00	0	22.86	4	22	3199	-0.17	3.47	13	46.75	22
5	3139	-10.25	0.00	0	32.85	5	23	3197	-0.50	2.72	14	47.00	23
...													
13	3208	1.34	3.20	4	51.52	13	31	3174	-4.37	0.00	0	64.66	31
14	3211	1.85	4.80	5	49.42	14	32	3171	-4.87	0.00	0	69.28	32
15	3214	2.35	6.90	6	46.82	15	33	3180	-3.36	0.00	0	72.39	33
16	3215	2.52	9.17	7	44.05	16	34	3179	-3.53	0.00	0	75.67	34
17	3209	1.51	10.43	8	42.29	17	35	3175	-4.20	0.00	0	79.62	35
18	3203	0.50	10.69	9	41.53	18	36	3174	-4.37	0.00	0	83.74	36

The process signals out of control when $h = 8.01$ is exceed. This occurs on the lower side at observation 2 and on the upper side at observation 16. Assignable causes occurred after start-up (observation 2 – 2) and after observation 9 (16 – 7).

(c) Discuss the choice of h and k in part (b) of this problem on cusum performance.

Selecting a smaller sized shift to detect, $k = 0.25$, should be balanced by a larger control limit, $h = 8.01$, to give longer in-control ARLs with shorter out-of-control ARLs.

8-11. Apply the scale cusum discussed in Section 8-1.9 to the data in Exercise 8-1.

Let $k = 0.5$, $h = 5$, $\mu_0 = 1050$, and $\sigma = 25$. For the 1st period, the standardized quantity, or scale, for monitoring process variability is found from:

$$y_1 = \frac{x_1 - \mu_0}{\sigma} = \frac{1045 - 1050}{25} = -0.2$$

$$v_1 = \frac{\sqrt{|y_1|} - 0.822}{0.349} = \frac{\sqrt{|-0.2|} - 0.822}{0.349} = -1.07$$

$$S_1^+ = \max\left[0, v_1 - k + S_{1-1}^+\right] = \max\left[0, -1.07 - 0.5 + 0\right] = \max\left[0, -1.57\right] = 0$$

$$S_1^- = \max\left[0, -k - v_1 + S_{1-1}^-\right] = \max\left[0, -0.5 - (-1.07) + 0\right] = \max\left[0, 0.57\right] = 0.57$$

i	x_i	y_i	v_i	S_i^+	N^+	S_i^-	N^-
1	1045	-0.2	-1.07	0.00	0	0.57	1
2	1055	0.2	-1.07	0.00	0	1.15	2
3	1037	-0.52	-0.29	0.00	0	0.94	3
4	1064	0.56	-0.21	0.00	0	0.65	4

i	x_i	y_i	v_i	S_i^+	N^+	S_i^-	N^-
5	1095	1.8	1.49	0.99	1	0.00	0
6	1008	-1.68	1.36	1.85	2	0.00	0
7	1050	0	-2.36	0.00	0	1.86	1
8	1087	1.48	1.13	0.63	1	0.22	2
9	1125	3	2.61	2.74	2	0.00	0
10	1146	3.84	3.26	5.50	3	0.00	0
11	1139	3.56	3.05	8.05	4	0.00	0
12	1169	4.76	3.90	11.44	5	0.00	0
13	1151	4.04	3.40	14.35	6	0.00	0
14	1128	3.12	2.71	16.55	7	0.00	0
15	1238	7.52	5.50	21.56	8	0.00	0
...							

The out-of-control state is detected on the upper side at observation 10, ($S_{10}^+ = 5.50$) > ($h = 5$). The process change occurred at observation – $N^+ = 10 - 3 = 7$; process variability is increasing.

8-13. Consider a standardized two-sided cusum with $k = 0.2$ and $h = 0.8$. Use Siegmund's procedure to evaluate the in-control ARL performance of this scheme. Find ARL_1 for $\delta^* = 0.5$.

Evaluate two-sided cusum with $k = 0.2$ and $h = 8$. Reference text p. 416.

In-control performance:

$\delta^* = 0$, the shift in mean for which ARL is to be calculated

$\Delta^+ = \delta^* - k = 0 - 0.2 = -0.2$

$\Delta^- = -\delta^* - k = -0 - 0.2 = -0.2$

$b = h + 1.166 = 8 + 1.166 = 9.166$

$$ARL_0^+ = ARL_0^- \cong \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2} = \frac{\exp[-2(-0.2)(9.166)] + 2(-0.2)(9.166) - 1}{2(-0.2)^2} = 430.556$$

$$\frac{1}{ARL_0} = \frac{1}{ARL_0^+} + \frac{1}{ARL_0^-} = \frac{2}{430.556} = 0.005$$

$ARL_0 = 1/0.005 = 215.23$

Out-of-control performance:

$\delta^* = 0.5$, shift in mean for which ARL is to be calculated

$\Delta^+ = \delta^* - k = 0.5 - 0.2 = 0.3$

$\Delta^- = -\delta^* - k = -0.5 - 0.2 = -0.7$

$b = h + 1.166 = 8 + 1.166 = 9.166$

$$\text{ARL}_1^+ = \frac{\exp[-2(0.3)(9.166)]+2(0.3)(9.166)-1}{2(0.3)^2} = 25.023$$

$$\text{ARL}_1^- = \frac{\exp[-2(-0.7)(9.166)]+2(-0.7)(9.166)-1}{2(-0.7)^2} = 381,767$$

$$\frac{1}{\text{ARL}_1} = \frac{1}{\text{ARL}_1^+} + \frac{1}{\text{ARL}_1^-} = \frac{1}{25.023} + \frac{1}{381,767} = 0.040$$

$$\text{ARL}_1 = 1/0.040 = 25.02$$

8-15. Rework Exercise 8-1 using an EWMA control chart with $\lambda = 0.1$ and $L = 2.7$. Compare your results to those obtained with the cusum.

$\lambda = 0.1$

$L = 2.7$

$\text{CL} = \mu_0 = 1050$

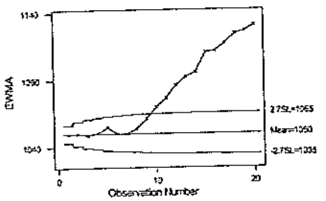
$$\text{UCL}_1 = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 1050 + 2.7(25)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 1050 + 6.75 = 1056.75$$

$$\text{LCL}_1 = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 1050 - 2.7(25)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 1050 - 6.75 = 1043.25$$

$\text{UCL}_{20} = 1065$

$\text{LCL}_{20} = 1035$

EWMA Chart for Ex 8-1 Molecular Weight



The EWMA control chart signals the process is out of control at observation 10. The cusum control scheme in Exercise 8-1 also signaled out-of-control on the upper side at observation 10.

8-19. Reconsider the data in Exercise 8-7. Apply an EWMA control chart to these data using $\lambda = 0.1$ and $L = 2.7$.

Given: $\lambda = 0.1$; $L = 2.7$. From Exercise 8-7: $\hat{\sigma} = 12.16$.

$\text{CL} = \mu_0 = 950$

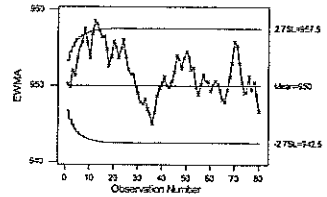
$$\text{UCL}_1 = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 950 + 2.7(12.16)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 950 + 3.28 = 953.28$$

$$\text{LCL}_1 = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 950 - 2.7(12.16)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 950 - 3.28 = 946.72$$

$\text{UCL}_{80} = 957.535$

$\text{LCL}_{80} = 942.465$

EWMA Chart for Ex 8-7, Temperature



The process signals out of control at observations 8, 12, and 13. This contrasts with the cusum control scheme that signals out of control on the upper side cusum at period 12.

8-20. Reconstruct the control chart in Exercise 8-19 using $\lambda = 0.4$ and $L = 3$. Compare this chart to the one constructed in Exercise 8-19.

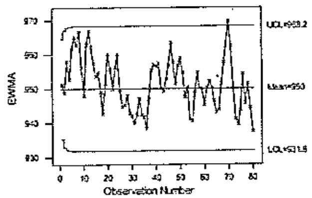
Given: $\lambda = 0.4$; $L = 3$. From Exercise 8-7: $\hat{\sigma} = 12.16$.
$CL = \mu_0 = 950$

$$UCL_1 = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 950 + 3(12.16)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 950 + 3.64 = 953.64$$

$$LCL_1 = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 950 - 3(12.16)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 950 - 3.64 = 946.36$$

$UCL_{80} = 968.24$
$LCL_{80} = 931.76931.76$

EWMA Chart for Ex 8-7 Temperature



With the larger λ, the process is out of control at observation 70, as compared to the in-control chart with the smaller λ.

8-22. Reconsider the data in Exercise 8-9. Set up and apply an EWMA control chart to these data using $\lambda = 0.1$ and $L = 2.7$.

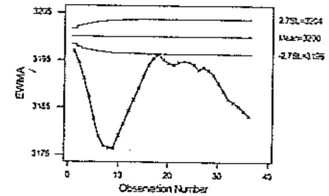
Given: $\lambda = 0.1$; $L = 2.7$. From Exercise 8-9: $\hat{\sigma} = 5.95$
$CL = \mu_0 = 3200$

$$UCL_1 = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 3200 + 2.7(5.95)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 3200 + 1.61 = 3201.61$$

$$LCL_1 = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} = 3200 - 2.7(5.95)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]} = 3200 - 1.61 = 3198.39$$

$UCL_{36} = 3203.69$
$LCL_{36} = 3196.31$

EWMA Chart for Ex 8-9 Viscosity



The EWMA control chart signals that the process is out of control from the first observation. This agrees with the result of applying the cusum control scheme, which signaled on observation 2 and indicated occurrence at start-up.

8-23. Analyze the data in Exercise 8-1 using a moving average control chart with $w = 6$. Compare the results obtained with the cusum control chart in Exercise 8-1.

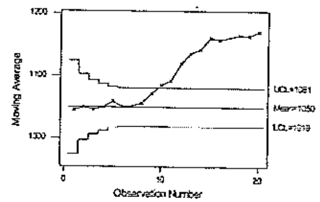
$w = 6$, $\mu_0 = 1050$, $\sigma = 25$
$CL = \mu_0 = 1050$

$$UCL_1 = \mu_0 + 3\left(\frac{\sigma}{\sqrt{1}}\right) = 1050 + 3\left(\frac{25}{\sqrt{1}}\right) = 1125$$

$$LCL_1 = \mu_0 - 3\left(\frac{\sigma}{\sqrt{1}}\right) = 1050 - 3\left(\frac{25}{\sqrt{1}}\right) = 975$$

$$M_6 = \frac{x_6 + x_{6-1} + x_{6-2} + x_{6-3} + x_{6-4} + x_{6-5}}{6} = \frac{1008 + 1095 + 1064 + 1037 + 1055 + 1045}{6} = 1050.67$$

Moving Average Chart for Ex 8-1 Molecular Weight



Process is out of control at observation 10, the same result as for exercise 8-1.

8-31. An EWMA control chart uses λ = 0.4. How wide will the limits be on the Shewhart control chart, expressed as multiple of the width of the steady-state EWMA limits?

For EWMA: steady-state limits are $\pm L\sigma\sqrt{\lambda/(2-\lambda)}$

For Shewhart: steady-state limits are $\pm k\sigma$

$$k\sigma = L\sigma\sqrt{\lambda/(2-\lambda)}$$

$$k = L\sqrt{0.4/(2-0.4)}$$

$$k = 0.5L$$

8-33. Consider the valve failure data in Example 6-6. Set up a one-sided cusum chart for monitoring and detecting an increase in failure rate of the valve. Assume that the target value of the mean time between failures is 700 hr.

Given: $\mu_Y = 700$, so $\mu_Y = 700^{0.2777} = 6.1671$. From Exercise 7-32, $\sigma = 2.0915$. Let $\delta = 1\sigma$, $k = 0.5$, $h = 5$, then $K = 1.0457$, $H = 10.4575$. A one-sided lower cusum is needed to detect an increase in failure rate, or equivalently a decrease in the time-between-failures.

Failure, i	x_i	$x_i = y_i^{0.2777}$	C_i^-	N^-
1	286	4.809865	0.3115	0
2	948	6.709029	0.0000	0
3	536	5.726497	0.0000	0
4	124	3.813671	1.3077	0
5	816	6.435412	0.0000	0
6	729	6.237053	0.0000	0
7	4	1.469576	3.6518	1
8	143	3.967682	4.8055	0
9	431	5.390069	4.5368	0
10	8	1.781509	7.8767	1
11	2837	9.09618	3.9019	0
12	596	5.897744	3.1256	0
13	81	3.388335	4.8586	0
14	227	4.510954	5.4691	0
15	603	5.916898	4.6736	0
16	492	5.591891	4.2031	0
17	1199	7.161238	2.1632	0
18	1214	7.186005	0.0986	0
19	2831	9.090833	0.0000	0
20	96	3.552031	1.5694	0

The rate of occurrence of valve failures on the lower side is in control.

CHAPTER 9

Other Univariate Statistical Process Monitoring and Control Techniques

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Construct and interpret $\bar{x}$ and R DNOM control charts, and apply to short production runs.
2. Construct and interpret both modified and acceptance control charts, and apply to highly capable processes.
3. Understand the monitoring issue for multiple-stream processes (detecting whether one or all streams have shifted), and construct and interpret group control charts.
4. Understand the conditions under which autocorrelated process data occur, and the impact on control schemes such as Shewhart charts.
5. Evaluate the degree of autocorrelation in a data set.
6. Set-up an EWMA procedure to monitor the mean of an autocorrelated process, utilizing residuals from an ARIMA time series model.
7. Construct and apply a moving center-line EWMA control chart to autocorrelated data.
8. Explain adaptive sampling procedures that permit either the sampling interval or the sample size or both to be changed.
9. Design and assess $\bar{x}$ control charts that include economic considerations, such as:
    - Fixed and variable sampling costs
    - Cost of finding an assignable cause
    - Cost of investigating a false alarm
    - Cost of operating in an out-of-control state

Exercises

9-1. Use the following data to set up short run $\bar{x}$ and R charts using the DNOM approach. The nominal dimensions for each part are $T_A = 100$, $T_B = 60$, $T_C = 75$, and $T_D = 50$.

Sample	Part Type	M1	M2	M3	x1	x2	x3	xbar	R
1	A	105	102	103	5	2	3	3.33	3
2	A	101	98	100	1	-2	0	-0.33	3
3	A	103	100	99	3	0	-1	0.67	4
4	A	101	104	97	1	4	-3	0.67	7
5	A	106	102	100	6	2	0	2.67	6
6	B	57	60	59	-3	0	-1	-1.33	3
7	B	61	64	63	1	4	3	2.67	3
8	B	60	58	62	0	-2	2	0.00	4
9	C	73	75	77	-2	0	2	0.00	4
10	C	78	75	76	3	0	1	1.33	3
...									
20	D	53	51	50	3	1	0	1.33	3

$\hat{\sigma}_A = 2.530, n_A = 15, \hat{\mu}_A = 101.40;\quad \hat{\sigma}_B = 2.297, n_B = 9, \hat{\mu}_B = 60.444$

$\hat{\sigma}_C = 1.815, n_C = 18, \hat{\mu}_C = 75.333;\quad \hat{\sigma}_D = 1.875, n_D = 18, \hat{\mu}_D = 50.111$

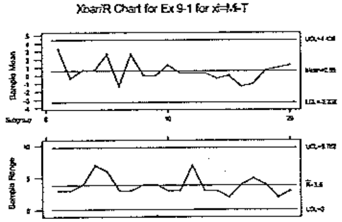
Since the standard deviations and sample sizes are approximately the same, the DNOM chart can be used.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{3.33 + (-0.33) + \cdots + 1.33}{20} = 0.55 \qquad \bar{R} = \frac{\sum_{i=1}^{m} R}{m} = \frac{3 + 3 + \cdots + 3}{20} = 3.8$$

$$\mathrm{CL}_{\bar{x}} = \bar{\bar{x}} = 0.55 \qquad \mathrm{CL}_R = \bar{R} = 3.8$$

$$\mathrm{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 0.55 + 1.023(3.8) = 4.44 \qquad \mathrm{UCL}_R = D_4\bar{R} = 2.574(3.8) = 9.78$$

$$\mathrm{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 0.55 - 1.023(3.8) = -3.34 \qquad \mathrm{LCL}_R = D_3\bar{R} = 0.000(3.8) = 0$$



The process is in control across all part types.

9-5. A machine has four heads. Samples of $n = 3$ units are selected from each head, and the $\bar{x}$ and R values for an important quality characteristic are computed. The data are shown here. Set up group control charts for this process.

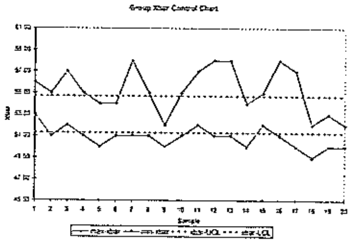
	Head 1		Head 2		Head 3		Head 4		max	min	max
Sample	xbar	R	xbar	R	xbar	R	xbar	R	xbar	xbar	range
1	53	2	54	1	56	2	55	3	56	53	3
2	51	1	55	2	54	4	54	4	55	51	4
3	54	2	52	5	53	3	57	2	57	52	5
4	55	3	54	3	52	1	51	5	55	51	5
5	54	1	50	2	51	2	53	1	54	50	2
6	53	2	51	1	54	2	52	2	54	51	2
7	51	1	53	2	58	5	54	1	58	51	5
8	52	2	54	4	51	2	55	2	55	51	4
9	50	2	52	3	52	1	51	3	52	50	3
10	51	1	55	1	53	3	53	5	55	51	5
...											
20	52	4	52	2	50	3	52	2	52	50	4

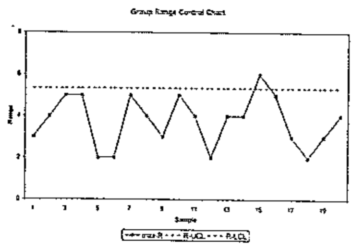
Refer to Section 9-3.2 (text p. 455) for the discussion of group control charts. In any single time period, only the smallest and largest of the four means and the largest range are plotted on the control charts. These are shown in the above table.

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{s}\sum_{i=1}^{m} \bar{x}_{i,j}}{m \times s} = \frac{\sum_{j=1}^{20}\sum_{i=1}^{4} \bar{x}_{i,j}}{20 \times 4} = \frac{53 + 51 + \cdots + 52}{80} = 52.988 \qquad \bar{R} = \frac{\sum_{j=1}^{s}\sum_{i=1}^{m} R_{i,j}}{m \times s} = \frac{\sum_{j=1}^{20}\sum_{i=1}^{4} R_{i,j}}{20 \times 4} = \frac{2 + 1 + \cdots + 2}{80} = 2.338$$

$$\mathrm{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 52.988 + 0.729(2.338) = 54.692 \qquad \mathrm{UCL}_R = D_4\bar{R} = 2.282(2.338) = 5.335$$

$$\mathrm{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 52.988 - 0.729(2.338) = 51.284 \qquad \mathrm{LCL}_R = D_3\bar{R} = 0.000(2.338) = 0$$





There is no situation where one single head gives the maximum or minimum value of $\bar{x}$ six times in a row. There are many values of $\bar{x}_{max}$ and $\bar{x}_{min}$ that are outside the control limits, so the process is out-of-control. The assignable cause affects all heads, not just a specific one.

9-7. Reconsider the data in Exercises 9-5 and 9-6. Suppose the process measurements are individual data values, not subgroup averages.

Sample	X1	MR1	X2	MR2	X3	MR3	X4	MR4	maxX	minX	maxMR
1	53		54		56		55		56	53	
2	51	2	55	1	54	2	54	1	55	51	2
3	54	3	52	3	53	1	57	3	57	52	3
4	55	1	54	2	52	1	51	6	55	51	6
5	54	1	50	4	51	1	53	2	54	50	4
6	53	1	51	1	54	3	52	1	54	51	3
7	51	2	53	2	58	4	54	2	58	51	4
8	52	1	54	1	51	7	55	1	55	51	7
9	50	2	52	2	52	1	51	4	52	50	4
10	51	1	55	3	53	1	53	2	55	51	3
11	52	1	57	2	52	1	55	2	57	52	2
...											
20	52	1	52	1	50	1	52	2	52	50	2

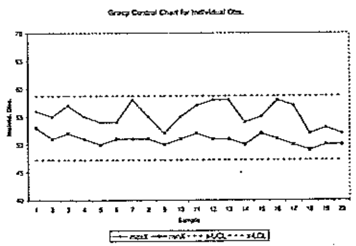
(a) Use observations 1–20 in Exercise 9-5 to construct appropriate group control charts.

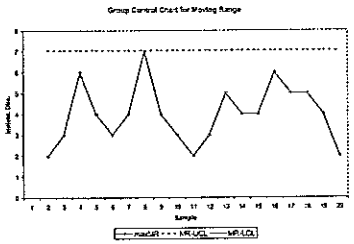
We need to calculate the moving ranges for each stream. From any single sample, only the smallest and largest readings and the largest moving range are plotted on the control charts. These are shown in the above table.

$$\mathrm{UCL}_x = \bar{\bar{x}} + 3\frac{\overline{\mathrm{MR}}}{d_2} = 52.988 + 3\left(\frac{2.158}{1.128}\right) = 58.727 \qquad \mathrm{UCL}_{\mathrm{MR}} = D_4\overline{\mathrm{MR}} = 3.267(2.158) = 7.050$$

$$\mathrm{LCL}_x = \bar{\bar{x}} - 3\frac{\overline{\mathrm{MR}}}{d_2} = 52.988 - 3\left(\frac{2.158}{1.128}\right) = 47.248 \qquad \mathrm{LCL}_{\mathrm{MR}} = D_3\overline{\mathrm{MR}} = 0.000(2.158) = 0$$

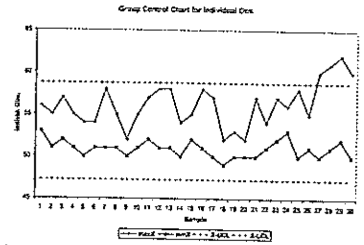
$$\bar{\bar{x}} = \frac{\sum_{j=1}^{s}\sum_{i=1}^{m} x_{i,j}}{m \times s} = \frac{\sum_{j=1}^{4}\sum_{i=1}^{20} x_{i,j}}{20 \times 4} = \frac{53+51+\cdots+52}{80} = 52.988 \qquad \overline{\mathrm{MR}} = \frac{\sum_{j=1}^{s}\sum_{i=2}^{m} \mathrm{MR}_{i,j}}{(m-1)\times s} = \frac{\sum_{j=1}^{4}\sum_{i=2}^{20} \mathrm{MR}_{i,j}}{(20-1)\times 4} = \frac{2+3+\cdots+2}{76} = 2.158$$

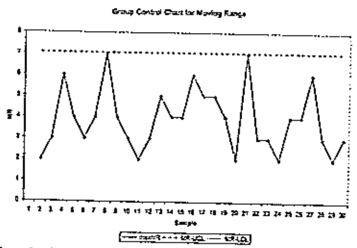




(b) Plot observations 21–30 from Exercise 9-6 on the charts from part (a). Discuss your findings.

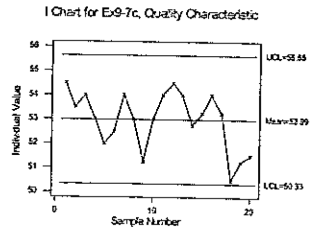
Sample	X1	MR1	X2	MR2	X3	MR3	X4	MR4	maxX	minX	maxMR
21	50	2	54	2	57	7	55	3	57	50	7
22	51	1	53	1	54	3	54	1	54	51	3
23	53	2	52	1	55	1	57	3	57	52	3
24	54	1	54	2	53	2	56	1	56	53	2
25	50	4	51	3	52	1	58	2	58	50	4
26	51	1	55	4	54	2	54	4	55	51	4
27	53	2	50	5	51	3	60	6	60	50	6
28	54	1	51	1	54	3	61	1	61	51	3
29	52	2	52	1	53	1	62	1	62	52	2
30	52	0	53	1	50	3	60	2	60	50	3

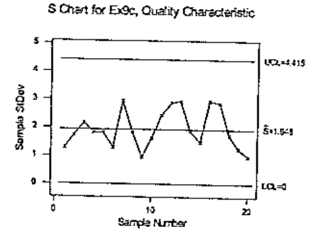




The last four samples from Head 4 are the maximum of all heads, indicating a potential process change.

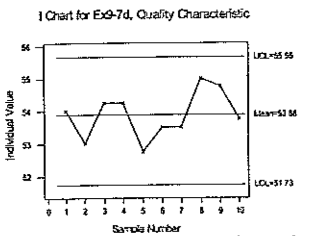
(c) Using observations 1–20, construct an individuals chart using the average of the readings on all four heads as an individual measurement and an S control chart using the individual measurements on each head. Discuss how these charts function relative to the group control chart.

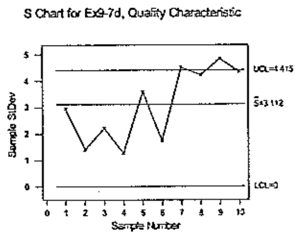




Failure to recognize the multiple stream nature of the process leads to control charts that fail to identify the out-of-control conditions in this process.

(d) Plot observations 21–30 on the control charts from part (c). Discuss your findings.





Only the S chart gives any indication of out-of-control process.

9-9. A sample of five units is taken from a process every half hour. It is known that the process standard deviation is in control with $\sigma = 2.0$. The $\bar{x}$ values for the last 20 samples are below. Specifications on the product are 40 ± 8.

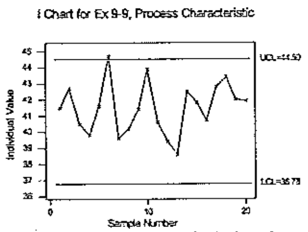
Sample No.	x-bar	Sample No.	x-bar	Sample No.	x-bar	Sample No.	x-bar
1	41.5	6	44.7	11	40.6	16	40.7
2	42.7	7	39.6	12	39.4	17	42.8
3	40.5	8	40.2	13	38.6	18	43.4
4	39.8	9	41.4	14	42.5	19	42.0
5	41.6	10	43.9	15	41.8	20	41.9

(a) Set up a modified control chart on this process. Use 3-sigma limits on the chart and assume that the largest faction nonconforming that is tolerable 0.1%.

Design a modified control chart (text p. 449): $\delta = 0.001$, $z_\delta = z0.001 = 3.0090$, $z_\alpha = k = 3$.

$$\text{UCL} = \text{USL} - \left(z_\delta - 3/\sqrt{n}\right)\sigma = 48 - \left(3.090 - 3/\sqrt{5}\right)(2.0) = 44.503$$

$$\text{LCL} = \text{LSL} + \left(z_\delta - 3/\sqrt{n}\right)\sigma = 32 + \left(3.090 - 3/\sqrt{5}\right)(2.0) = 35.497$$



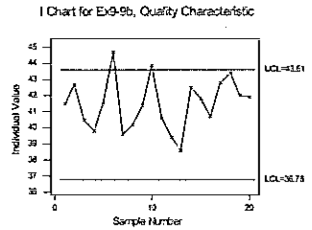
Note that there is no centerline on this control chart. Recall that the modified x control chart is designed to permit the process mean to vary over an interval, under the assumption that the overall process performance is not appreciably affected (text p. 449).

(b) Reconstruct the chart in part (a) using 2-sigma limits. Is there any difference in the analysis of the data?

$\delta = 0.001$; $z_\delta = z_{0.001} = 3.090$; $z_\alpha = k = 2$

$$\text{UCL} = \text{USL} - \left(z_\delta - k/\sqrt{n}\right)\sigma = 48 - \left(3.090 - 2/\sqrt{5}\right)(2.0) = 43.609$$

$$\text{LCL} = \text{LSL} + \left(z_\delta - k/\sqrt{n}\right)\sigma = 32 + \left(3.090 - 2/\sqrt{5}\right)(2.0) = 36.391$$



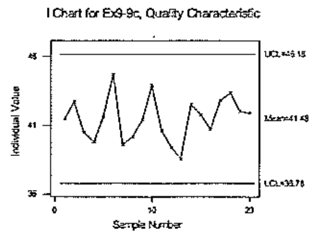
With 3-sigma limits sample #6 exceeds the UCL, while with 2-sigma limits both samples #6 and #10 exceed the UCL.

(c) Suppose that if the true process fraction nonconforming is 5%, we would like to detect this condition with probability 0.95. Construct the corresponding acceptance control chart.

Design acceptance control chart (text p. 453): $\sigma = 2.0$, $n = 5$, USL = 48, LSL = 32, $\gamma = 0.05$, $z_\gamma = z_{0.05} = 1.645$, $1 - \beta = 0.95$, $z_\beta = z_{0.05}$, 1.645.

$$\text{UCL} = \text{USL} - \left(z_\gamma - z_\beta/\sqrt{n}\right)\sigma = 48 - \left(1.645 - 1.645/\sqrt{5}\right)(2.0) = 46.181$$

$$\text{LCL} = \text{LSL} + \left(z_\gamma + -z_\beta/\sqrt{n}\right)\sigma = 32 + \left(1.645 - 1.645/\sqrt{5}\right)(2.0) = 33.819$$



None of the samples exceeds the control limits, reflective of the risks in designing an acceptance control chart.

9-13. An $\bar{x}$ chart is to be designed for a quality characteristic assumed to be normal with a standard deviation of 4. Specifications on the product quality characteristics are 50 ± 20. The control chart is to be designed so that if the fraction nonconforming is1%, the probability of a point falling inside the control limits will be 0.995. The sample size is $n = 4$. What are the control limits and centerline for the chart?

Design a modified control chart (text p. 449): $X \sim N(\mu, 4)$, $n = 4$, USL = 820, LSL = 780, $\delta = 0.01$, $z_\delta = z_{0.01} =$ 2.326, $\alpha = 0.005$, $z_\alpha = z_{0.005} = 2.576$

$$\text{UCL} = \text{USL} - \left(z_\delta - z_\alpha/\sqrt{n}\right)\sigma = (50+20) - \left(2.326 - 2.576/\sqrt{4}\right)(4) = 65.304$$

$$\text{LCL} = \text{LSL} + \left(z_\delta - z_\alpha/\sqrt{n}\right)\sigma = (50-20) + \left(2.326 - 2.576/\sqrt{4}\right)(4) = 34.696$$

Modified control charts generally do not have centerlines, since the mean is allowed to vary over an interval (the control limits) without significantly affecting overall process output.

9-15. A normally distributed quality characteristic is controlled by $\bar{x}$ and R charts having the following parameters ($n = 4$, both charts are in control):

R Chart	$\bar{x}$ Chart
UCL = 18.795	UCL = 626.00
Center line = 8.236	Center line = 620.00
LCL = 0	LCL = 614.00

(a) What is the estimated standard deviation of the quality characteristic x?

Standard deviation is estimated as usual, from the average range (text eqn. 5-6):

$n = 4$, $\bar{R} = 8.236$, $\bar{\bar{x}} = 620.00$

$\hat{\sigma} = \bar{R}/d_2 = 8.236/2.059 = 4.000$

(b) If specifications are 610 ± 15, what is your estimate of the fraction of nonconforming material produced by this process when it is in control at the given level?

Transform variables and use the standard normal distribution to estimate the probability that material exceeds the specification limits (text p. 64):

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Pr\left\{z \le \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right]$$

$$= \Phi\left(\frac{595-620}{4.000}\right) + \left[1 - \Phi\left(\frac{625-620}{4.000}\right)\right] = 0.0000 + [1 - 0.8944] = 0.1056$$

(c) Suppose you wish to establish a modified $\bar{x}$ chart to substitute for the original $\bar{x}$ chart. The process mean is to be controlled so that the fraction nonconforming is less than 0.005. The probability of type I error is to be 0.01. What control limits do you recommend?

Design a modified control chart (text p. 449): $X \sim N(\mu, 4)$, $n = 4$, USL = 625, LSL = 595, $\delta = 0.005$, $z_\delta = z_{0.005} =$ 2.576, $\alpha = 0.01$, $z_\alpha = z_{0.01} = 2.326$.

$$\text{UCL} = \text{USL} - \left(z_\delta - z_\alpha/\sqrt{n}\right)\sigma = 625 - \left(2.576 - 2.326/\sqrt{4}\right)4 = 618.86$$

$$\text{LCL} = \text{LSL} + \left(z_\delta - z_\alpha/\sqrt{n}\right)\sigma = 595 + \left(2.576 - 2.326/\sqrt{4}\right)4 = 601.14$$

9-16. The data that follow are molecular weight measurements made every 2 hours on a polymer (read down, then across from left to right).

2048	2039	2051	2002	2029
2025	2015	2056	1967	2019
2017	2021	2018	1994	2016
1995	2010	2030	2001	2010
1983	2012	2023	2013	2000
1943	2003	2036	2016	2009
1940	1979	2019	2019	1990
1947	2006	2000	2036	1986
1972	2042	1986	2015	1947
1983	2000	1952	2032	1958
...				
1970	2021	2018	2015	2032

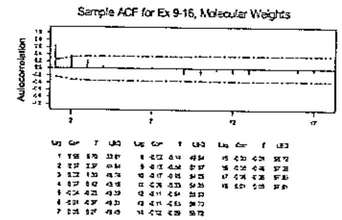
(a) Calculate the sample autocorrelation function and provide an interpretation.

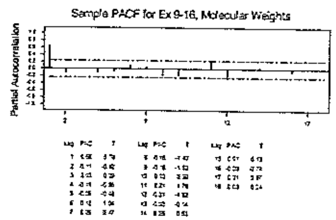
$\bar{x} = 2000.9$. For observations that are $k = 1$ time period apart:

$$r_1 = \frac{\sum_{t=1}^{n-1}(x_t - \bar{x})(x_{t+1} - \bar{x})}{\sum_{t=1}^{n}(x_t - \bar{x})^2}$$

$$= \frac{[(2048-2000.9)(2025-2000.9) + (2025-2000.9)(2017-2000.9) + \cdots + (2015-2000.9)(2032-2000.9)]}{\left[(2048-2000.9)^2 + (2025-2000.9)^2 + \cdots + (2032-2000.9)^2\right]}$$

r_1 0.658253, $r_2 = 0.373245$, $r_3 = 0.220536$, $r_4 = 0.072562$



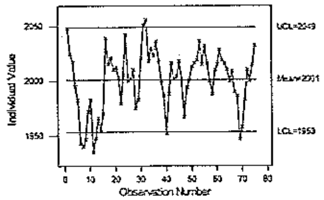


The decaying sine wave of the ACFs combined with a spike at lag 1 for the PACFs suggests an autoregressive process of order 1, AR(1).

(b) Construct an individuals control chart with the standard deviation estimated using the moving range method. How would you interpret the chart? Are you comfortable with this interpretation?

Using the 1st estimator based on a span of $n = 2$ (text p. 258): $\hat{\sigma}_1 = 0.8865\overline{\text{MR}} = 0.8865(17.97) = 15.93$

I Chart for Ex 9-16, Molecular Weight



The process is out of control on the x chart, with big swings and very few observations actually near the mean. This is likely due to the large correlation at low lag. In practice, this would generate many false alarms.

(c) Fit a first-order autoregressive model $x_t = \xi + \phi x_{t-1} + \varepsilon_t$ to the molecular weight data. Set up an individuals control chart on the residuals from this model. Interpret this control chart.

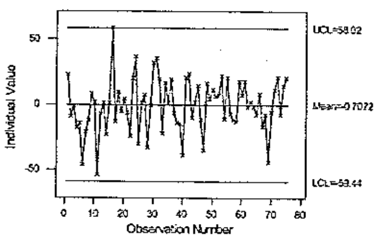
```
ARIMA model for Ex9-16mole
Estimates at each iteration
Iteration      SSE       Parameters
   0         50173.7     0.100 1800.942
   1         41717.0     0.250 1500.843
   2         35687.3     0.400 1200.756
   3         32083.6     0.550  900.693
   4         30929.9     0.675  650.197
   5         30898.4     0.693  613.998
   6         30897.1     0.697  606.956
   7         30897.1     0.698  605.494
   8         30897.1     0.698  605.196
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type          Coef      SE Coef        T         P
AR   1      0.6979      0.0852       8.19     0.000
Constant   605.196       2.364     256.02     0.000
Mean       2003.21        7.82
```


Residuals from this model are:

22.9744	58.9664	35.3741	-11.5322	17.5614
-9.4688	-13.1878	19.4375	-35.3660	-2.2090
-1.4174	9.5614	-22.0519	16.0600	1.7699
-17.8343	-5.6259	16.4678	4.2171	-2.1364
-14.4808	4.0509	1.0931	11.3319	-7.9491
-46.1062	-6.3449	18.9783	5.9572	8.0298
-21.1907	-24.0639	-7.0942	6.8636	-17.2512
-12.0970	19.6854	-14.2301	21.7699	-7.9914
8.0178	36.8424	-14.9702	-11.0942	-44.1998
1.5706	-30.2815	-39.1998	20.5614	-5.9822
...				
1.1326	31.8938	14.2382	7.0509	20.5614

I Chart for Ex 9-16, Residuals from AR(1) Model



A single point, #16, exceeds the upper control limit; otherwise the process seems to be in a reasonable state of statistical control.

9-17. Consider the molecular weight data in Exercise 9-16. Construct a cusum control chart on the residuals from the model you fit to the data in part (c) of that exercise.

Let $\mu_0 = 0$ and estimate σ from: $\hat{\sigma} = \overline{\text{MR}}/d_2 = 22.08/1.128 = 19.57$

Let $\delta = 1\sigma$, $k = 0.5$ and $h = 5$, so $K = k\sigma = 0.5(19.57) = 9.79$, $H = h\sigma = 5(19.57) = 97.87$.

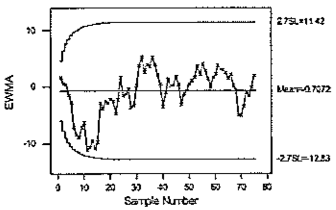
	Residuals from 9-16c	one-sided upper cusum			one-sided lower cusum		
Obs, i	x_i	$x_i - (\mu_0 + K)$	C_i^+	OOC?	$(\mu_0 - K) - x_i$	C_i^-	OOC?
No FIR			0.00			0.00	
1	22.9744	13.19	13.19	no	-32.76	0.00	no
2	-9.4688	-19.26	0.00	no	-0.32	0.00	no
3	-1.4174	-11.20	0.00	no	-8.37	0.00	no
4	-17.8343	-27.62	0.00	no	8.05	8.05	no
5	-14.4808	-24.27	0.00	no	4.69	12.74	no
6	-46.1062	-55.89	0.00	no	36.32	49.06	no
7	-21.1907	-30.98	0.00	no	11.40	60.46	no
8	-12.0970	-21.88	0.00	no	2.31	62.77	no
9	8.0178	-1.77	0.00	no	-17.81	44.97	no
10	1.5706	-8.22	0.00	no	-11.36	33.61	no
...							
75	20.5614	10.77	15.02	no	-30.35	0.00	no

No observations exceed the control limits. Residuals from the AR(1) model indicate that the process is in a reasonable state of statistical control.

9-18. Consider the molecular weight data in Exercise 9-16. Construct an EWMA control chart on the residuals from the model you fit to the data in part(c) of that exercise.

Select $\lambda = 0.100$ and $L = 2.7$ (from text Table 8-10).

EWMA Chart for Ex 9-16 Residuals



The process is in control, with no observations beyond the control limits.

9-19. Set up a moving center-line EWMA control chart for the molecular weight data in Exercise 9-16. Compare it to the residual control chart in Exercise 9-16 (c).

Select $\lambda = 0.15$ (λ could also be selected as the value that minimizes residual sum of squares). From Exercise 9-17 $\hat{\sigma} = \overline{MR}/d_2 = 22.08/1.128 = 19.57$.

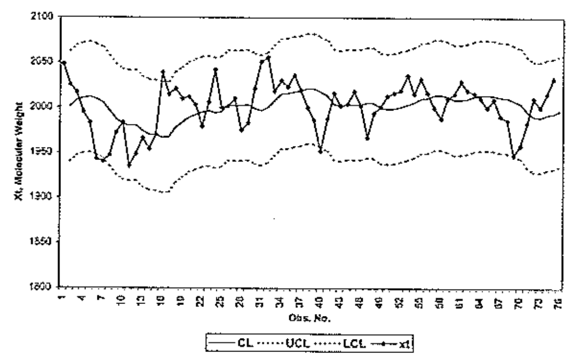
Plot point is x_{t+1} $\quad CL_{t+1} = z_t \quad UCL_{t+1} = z_t + 3\sigma \quad LCL_{t+1} = z_t - 3\sigma$

$z_i = \lambda x_i + (1-\lambda)z_{i-1}$; for $i = 1, \ldots, n$

$$z_0 = \frac{\sum_{i=1}^{n} z_i}{n} = \frac{2008.005 + 2010.554 + \cdots + 2001.973}{75} = 2000.947$$

t	x_t	z_t	CL	UCL	LCL	OOC?
0		2000.947				
1	2048	2008.005	2000.947	2059.657	1942.237	0
2	2025	2010.554	2008.005	2066.715	1949.295	0
3	2017	2011.521	2010.554	2069.264	1951.844	0
4	1995	2009.043	2011.521	2070.231	1952.811	0
5	1983	2005.136	2009.043	2067.753	1950.333	0
6	1943	1995.816	2005.136	2063.846	1946.426	below LCL
...						
16	2039	1978.127	1967.385	2026.095	1908.675	above UCL
...						
40	1952	2003.805	2012.947	2071.657	1954.237	below LCL
...						
75	2032	2001.973	1996.674	2055.384	1937.964	0

EWMA Control Chart for Molecular Weight



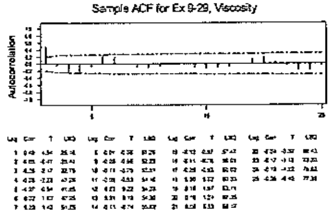
Observations 6, 16 and 40 exceed control limits, compared to no out-of-control observations on the residuals X and MR control charts.

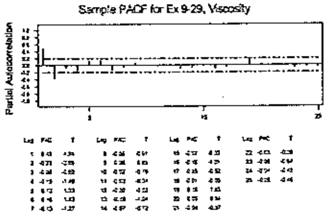
9-29. The viscosity of a chemical product is read every 2 minutes. Some data from this process are shown here (read down, from left to right).

29.33	33.22	27.99	24.28
19.98	30.15	24.13	22.69
25.76	27.08	29.20	26.60
29.00	33.66	34.30	28.86
31.03	36.58	26.41	28.27
32.68	29.04	28.78	28.17
33.56	28.08	21.28	28.58
27.50	30.28	21.71	30.76
26.75	29.35	21.47	30.62
...			
30.83	31.58	31.92	32.44

(a) Is there a serious problem with autocorrelation in these data?

Similar to Exercise 9-16, calculate the sample autocorrelation and partial autocorrelation functions: $r_1 = 0.49$, $r_2 = -0.05$, $r_3 = -0.26$, $r_4 = -0.28$. The $r_1 = 0.49$ indicates a strong positive correlation at lag 1:

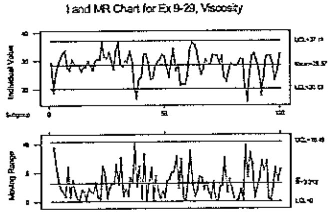




The decaying sine wave of the ACFs combined with a spike at lag 1 for the PACFs suggests a lower order autoregressive process of order 1, either AR(1) or AR(2) (most likely AR(2)).

(b) Set up a control chart for individuals with a moving range used to estimate process variability. What conclusion can you draw from this chart?

$\hat{\sigma} = \overline{MR}/d_2 = 3.212/1.128 = 2.848$



Visual examination of the chart, the sequences of similar points indicate that the mean may wander over time. The numerous points (2, 38, 86, 92) below the lower control limit support the suspicion of positively correlated readings.

(c) Design a cusum control scheme for this process, assuming that the observations are uncorrelated. How does the cusum perform?

$\mu_0 = \bar{x} = 28.57; \hat{\sigma} = 2.848; \delta = 1\sigma; k = 0.5; h = 5$

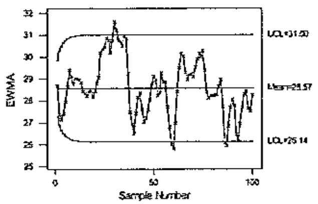
$K = k\sigma = 0.5(2.848) = 1.42; H = h\sigma = 5(2.848) = 14.24$

		one-sided upper cusum				one-sided lower cusum			
Obs, i	x_i	$x_i-(\mu_0+K)$	C_i^+	OOC?	N^+	$(\mu_0-K)-x_i$	C_i^-	OOC?	N^-
No FIR			0.00				0.00		
1	29.33	-0.66	0.00	no		-2.18	0.00	no	
2	19.98	-10.01	0.00	no		7.17	7.17	no	1
3	25.76	-4.23	0.00	no		1.39	8.55	no	2
4	29.00	-0.99	0.00	no		-1.85	6.70	no	3
5	31.03	1.04	1.04	no	1	-3.88	2.81	no	4
...									
29	33.66	3.67	14.27	OOC	10	-6.51	0.00	no	
30	36.58	6.59	20.86	OOC	11	-9.43	0.00	no	
31	29.04	-0.95	19.90	OOC	12	-1.89	0.00	no	
32	28.08	-1.91	17.99	OOC	13	-0.93	0.00	no	
33	30.28	0.29	18.28	OOC	14	-3.13	0.00	no	
34	29.35	-0.64	17.63	OOC	15	-2.20	0.00	no	
35	33.60	3.61	21.24	OOC	16	-6.45	0.00	no	
36	30.29	0.30	21.54	OOC	17	-3.14	0.00	no	
37	20.11	-9.88	11.65	no	18	7.04	7.04	no	1
38	17.51	-12.48	0.00	no		9.64	16.67	OOC	2
39	23.71	-6.28	0.00	no		3.44	20.11	OOC	3
40	24.22	-5.77	0.00	no		2.93	23.03	OOC	4
41	32.43	2.44	2.44	no	1	-5.28	17.75	OOC	5
...									
45	23.62	-6.37	0.00	no		3.53	17.44	OOC	9
46	28.12	-1.87	0.00	no		-0.97	16.46	OOC	10
...									
59	21.47	-8.52	0.00	no		5.68	16.98	OOC	3
60	24.71	-5.28	0.00	no		2.44	19.41	OOC	4
...									
100	32.44	2.45	2.45	no	1	-5.29	1.02	no	16

There are several runs of out-of-control observations on both sides of the cusum control scheme, suggesting a possible problem with autocorrelation in the viscosity readings.

(d) Set up an EWMA control chart with $\lambda = 0.15$ for the process. How does this chart perform?

EWMA Chart for Ex 9-29, Viscosity



This is not an in-control process. There are wide swings in the plot points and several are beyond the control limits. It could also be the case that the autocorrelated structure is not handled well by the EWMA with a small λ.

(e) Set up a moving center-line EWMA scheme for these data.

$\lambda = 0.15$, $\hat{\sigma} = 2.848$, $Z_0 = \bar{x} = 28.479$

i	X_i	Z_i	CL	UCL	LCL	OOC?
0		28.479				
1	29.330	28.607	28.479	37.022	19.937	no
2	19.980	27.313	28.607	37.149	20.064	below LCL
3	25.760	27.080	27.313	35.855	18.770	no
4	29.000	27.368	27.080	35.622	18.537	no
5	31.030	27.917	27.368	35.911	18.825	no
6	32.680	28.632	27.917	36.460	19.375	no
7	33.560	29.371	28.632	37.174	20.089	no
8	27.500	29.090	29.371	37.913	20.828	no
9	26.750	28.739	29.090	37.633	20.548	no
10	30.550	29.011	28.739	37.282	20.197	no
...						
37	20.110	29.234	30.845	39.387	22.302	below LCL
38	17.510	27.476	29.234	37.777	20.692	below LCL
...						
62	36.540	28.409	26.974	35.516	18.431	above UCL
...						
100	23.440	26.886	27.494	36.037	18.951	no

A few observations exceed the lower control limit.

(f) Suppose that a reasonable model for the viscosity data is an AR(2) model. How could this model be used to assist in the development of a statistical process control procedure for viscosity? Set up an appropriate control chart and use it to assess the current state of statistical process control.

An effective method for monitoring a process modeled by a 2nd order autoregressive model is to fit the AR(2) model, then monitor the residuals from that model with conventional control charts. These residuals should be approximately normally and independently distributed, with mean zero and constant variance.

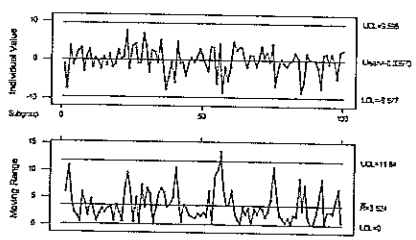
```
ARIMA model for Ex9-29Vis
Estimates at each iteration
Iteration      SSE       Parameters
   0         1590.91     0.100     0.100    22.935
   1         1366.65     0.250    -0.020    22.072
   2         1201.73     0.400    -0.144    21.298
   3         1097.08     0.550    -0.271    20.652
   4         1054.45     0.698    -0.406    20.258
   5         1053.42     0.716    -0.429    20.441
   6         1053.38     0.719    -0.434    20.489
   7         1053.38     0.719    -0.435    20.500
   8         1053.38     0.719    -0.435    20.502
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type          Coef      SE Coef        T        P
AR   1       0.7193     0.0923       7.79    0.000
AR   2      -0.4349     0.0922      -4.72    0.000
Constant    20.5017     0.3278      62.54    0.000
Mean        28.6514     0.4581
```

Residuals from the model are:

-1.18095	4.19682	-1.18129	-6.53172
-7.30751	-0.84016	-2.77212	-1.39570
3.64100	-0.66264	3.51323	0.33572
-1.34237	6.79060	3.28789	-0.90813
0.87056	3.64266	-6.06580	-1.42346
2.46931	-3.13635	4.19716	-0.11632
3.04523	2.59688	-8.43854	0.10905
-2.93024	2.20862	-1.58325	1.95066
1.06134	-0.72128	-5.39393	0.42089
...			
3.66239	1.13418	4.28754	2.77120

I and MR Chart for Ex 9-29 AR(2) Model Residuals



There is a single out-of-control residual on the MR(2) chart at observation 57. Otherwise the process appears to be in control, with a good distribution of points between the control limits and no patterns.

9-31. An $\bar{x}$ chart is used to maintain current control of a process. The cost parameters are a_1 = \$0.50, a_2 = \$0.10, a_3 = \$25, a'_3 = \$50, and a_4 = \$100. A single assignable cause of magnitude $\delta = 2$ occurs, and the duration of the process in control is an exponential random variable with mean 100 hr. Sampling and testing require 0.05 hr, and it takes 2 hr to locate the assignable cause. Assume that the equation 9-31 is the appropriate process model.

(a) Evaluate the cost of the arbitrary control chart design $n = 5, k = 3, h = 1$.

$\lambda = 0.01$/hr or $1/\lambda = 100$hr, $\delta = 2.0$, a_1 = \$0.50/sample, a_2 = \$0.10/unit, a'_3 = \$50, a_3 = \$25, a_4 = \$100/hr, g = 0.05hr/sample, D = 2hr; $n = 5, k = 3, h = 1, \alpha = 0.0027$

$$\beta = \Phi\left(\frac{(\mu_0 + k\sigma/\sqrt{n}) - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{(\mu_0 - k\sigma/\sqrt{n}) - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right) = \Phi\left(k - \delta\sqrt{n}\right) - \Phi\left(-k - \delta\sqrt{n}\right)$$
$$= \Phi\left(3 - 2\sqrt{5}\right) - \Phi\left(-3 - 2\sqrt{5}\right) = \Phi(-1.472) - \Phi(-7.472) = 0.0705 - 0.0000 = 0.0705$$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{1}{2} - \frac{0.01(1^2)}{12} 0.4992$$
$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(1)} = 0.27$$

$E(L)$ = \$4.12/hr

(b) Evaluate the cost of the arbitrary control chart design $n = 5, k = 3, h = 0.5$.

$n = 5, k = 3, h = 1, \alpha = 0.0027, \beta = 0.0705$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{0.5}{2} - \frac{0.01(0.5^2)}{12} 0.2498$$
$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(0.5)} = 0.54$$

$E(L)$ = \$4.98/hr

(c) Determine the economically optimum design.

Solution of this problem requires minimization of $E(L)$ through use of an unconstrained optimization or search technique for repeated evaluation of the cost function (text p. 487). Montgomery, D.C. (1982), "Economic Design of an $\bar{x}$ Control Chart", *Journal of Quality Technology*, Vol. 14, provides a FORTRAN program.

The economically optimum design is $n = 5$, $k_{opt} = 3.080$, $h_{opt} = 1.368$, $\alpha = 0.00207$, $1 - \beta = 0.918$, with an $E(L)$ = \$4.01392/hr.

9-33. An $\bar{x}$ chart is used to maintain current control of a process. The cost parameters are a_1 = \$2, a_2 = \$0.50, a_3 = \$50, a'_3 = \$75, and a_4 = \$200. A single assignable occurs, with magnitude $\delta = 1$, and the run length of the process in control is exponentially distributed with mean 100 hr. It requires 0.05 hr to sample and test, and 1 hr to locate the assignable cause.
(a) Evaluate the cost of the arbitrary $\bar{x}$ chart design $n = 5, k = 3, h = 0.5$.

$\lambda = 0.01$/hr or $1/\lambda = 100$hr, $\delta = 2.0$, a_1 = \$2/sample, a_2 = \$0.50/unit, a'_3 = \$75, a_3 = \$50, a_4 = \$200/hr, g = 0.05 hr/sample, D = 1 hr, $n = 5, k = 3, h = 0.5, \alpha = 0.0027$

$$\beta = \Phi\left(k - \delta\sqrt{n}\right) - \Phi\left(-k - \delta\sqrt{n}\right) = \Phi\left(3 - 1\sqrt{5}\right) - \Phi\left(-3 - 1\sqrt{5}\right) = \Phi(-1.472) - \Phi(-7.472) = 0.775 - 0.0000 = 0.775$$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{0.5}{2} - \frac{0.01(0.5^2)}{12} 0.2498$$
$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(0.5)} = 0.54$$

$E(L)$ = \$16.17/hr

(b) Find the economically optimum design.

Please see comments for solution to Exercise 9-31 (c). The economically optimum design is $n = 10$, $k_{opt} = 2.240$, $h_{opt} = 2.489018$, $\alpha = 0.025091$, $1 - \beta = 0.8218083$, with $E(L)$ = \$10.39762/hr

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CHAPTER 10

Multivariate Process Monitoring and Control

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Describe the nature of the multivariate problem with simultaneous monitoring of two or more related quality characteristics.
2. Construct and interpret a Hotelling T^2 control chart, and calculate phase 1, phase 2, and approximate chi-square control limits.
3. Design and interpret an MEWMA control chart.
4. Build a multiple linear regression model, find residuals, and apply a Regression Adjustment procedure to the residuals.
5. Perform a PCA to minimize the number of variables monitored for significant changes.

Exercises

10-1. The data shown here come from a production process with two observable quality characteristics, x_1 and x_2. The data are sample means of each quality characteristic, based on samples of size $n = 25$. Assume that mean values of the quality characteristics and the covariance matrix were computed from 50 preliminary samples:

$$\bar{\bar{\mathbf{x}}} = \begin{bmatrix} 55 \\ 30 \end{bmatrix}; \quad \mathbf{S} = \begin{bmatrix} 200 & 130 \\ 130 & 120 \end{bmatrix}$$

Sample	$\bar{x}_1$	$\bar{x}_2$	T^2	Sample	$\bar{x}_1$	$\bar{x}_2$	T^2	Sample	$\bar{x}_1$	$\bar{x}_2$	T^2
1	58	32	1.1268	6	53	30	1.6901	11	49	27	5.0704
2	60	33	3.1690	7	42	20	22.8169	12	57	30	1.6901
3	50	27	3.1690	8	55	31	0.7042	13	58	33	1.9014
4	54	31	2.0423	9	46	25	10.6338	14	75	45	52.8169
5	63	38	13.5211	10	50	29	6.6901	15	55	27	6.3380

Construct a T^2 control chart using these data. Use the phase 2 limits.

Phase 2 charts are used for monitoring future production, as compared to phase 1 limits that are used for retrospective analysis of data. For univariate charts such as the $\bar{x}$ chart, the distinction between phase 1 and phase 2 control limits is usually unnecessary for more than $m = 20$ to 25 samples. More care should be exercised for multivariate charts.

Design phase 2 T^2 control charts with $m = 50$ preliminary samples, $n = 25$ sample size, $p = 2$ characteristics. Let $\alpha = 0.001$. Using text eqn. 10-21:

$$\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1} = \frac{2(50+1)(25-1)}{50(25)-50-2+1} F_{0.001,2,1199} = \left(\frac{2448}{1199}\right)(6.948) = 14.186$$

LCL = 0

$$n = 25, \bar{\bar{x}}_1 = 55, \bar{\bar{x}}_2 = 30, \bar{S}_1^2 = 200, \bar{S}_2^2 = 120, \bar{S}_{12} = 130$$

Sample 1: $\bar{x}_1 = 58, \bar{x}_2 = 32$ so $\bar{\mathbf{x}}'_1 = [58, 32]$

$$T_1^2 = n\left(\bar{\mathbf{x}}_1 - \bar{\bar{\mathbf{x}}}\right)'\mathbf{S}^{-1}\left(\bar{\mathbf{x}}_1 - \bar{\bar{\mathbf{x}}}\right) = n\frac{\left[\bar{S}_2^2\left(\bar{x}_1 - \bar{\bar{x}}_1\right)^2 + \bar{S}_1^2\left(\bar{x}_2 - \bar{\bar{x}}_2\right)^2 - 2\left(\bar{S}_{12}\right)\left(\bar{x}_1 - \bar{\bar{x}}_1\right)\left(\bar{x}_2 - \bar{\bar{x}}_2\right)\right]}{\left[\left(\bar{S}_1^2 \times \bar{S}_2^2\right) - \left(\bar{S}_{12}\right)^2\right]}$$

$$= 25\frac{\left[120(58-55)^2 + 200(32-30)^2 - 2(130)(58-55)(32-30)\right]}{\left[(200\times 120) - (130)^2\right]} = 1.1268$$

The process signals out of control samples 7 and 14.

10-3. Reconsider the situation in Exercise 10-1. Suppose that the sample mean vector and sample covariance matrix provided were the actual population parameters. What control limit would be appropriate for phase 2 of the control chart? Apply this limit to the data and discuss any differences in results that you find in comparison to the original choice of control limit.

We need to obtain Phase 2 T^2 control limits for $p = 2$ characteristics. Let $\alpha = 0.001$. Since population parameters are known, the chi-square formula will be used for the upper control limit: $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.001,2} = 13.816$

Sample	$\bar{x}_1$	$\bar{x}_2$	T^2	Sample	$\bar{x}_1$	$\bar{x}_2$	T^2	Sample	$\bar{x}_1$	$\bar{x}_2$	T^2
1	58	32	1.1268	6	53	30	1.6901	11	49	27	5.0704
2	60	33	3.1690	7	42	20	22.8169	12	57	30	1.6901
3	50	27	3.1690	8	55	31	0.7042	13	58	33	1.9014
4	54	31	2.0423	9	46	25	10.6338	14	75	45	52.8169
5	63	38	13.5211	10	50	29	6.6901	15	55	27	6.3380

The process signals out of control at samples 7 and 14. These are the same results as for parameters estimated from samples.

10-5. Consider a T^2 control chart for monitoring $p = 6$ quality characteristics. Suppose that the subgroup size is $n = 3$ and there are 30 preliminary samples available to estimate the sample covariance matrix.

(a) Find the phase 2 control limits assuming that $\alpha = 0.005$.

For $m = 30$ preliminary samples, $n = 3$ sample size, $p = 6$ characteristics, and $\alpha = 0.005$ the Phase 2 limits are:

$$\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1} = \frac{6(30+1)(3-1)}{30(3)-30-6+1} F_{0.005,6,55} = \left(\frac{372}{55}\right)(3.531) = 23.882$$

LCL = 0

(b) Compare the control limits from part (a) to the chi-square control limit. What is the magnitude of the difference in the two control limits?

The chi-square limit is $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.005,6} = 18.548$.

The phase 2 UCL is almost 30% larger than the chi-square limit.

(c) How many preliminary samples would have to be taken to ensure that the exact phase 2 control limit is within 1% of the chi-square control limit?

Given $p = 6$ characteristics, $n = 3$ sample size, $\alpha = 0.005$. Find "m" such that exact phase 2 limit is within 1% of chi-square limit, 1.01(18.548) = 18.733. Use the UCL formula and increase m until it is within 1% of the chi-square limit. The spreadsheet solution using the Excel FINV function is below.

	A	B	C	D	E
1	p =	6			
2	n =	3			
3	alpha =	0.005			
4	m	num	denom	F	UCL
5	30	=$B$1*(A5+1)*($B$2-1)	=A5*$B$2-A5-$B$1+1	=FINV($B$3,$B$1,C5)	=B5/C5*D5

m	num	denom	F	UCL
30	372	55	3.531	23.8820
40	492	75	3.407	22.3527
50	612	95	3.338	21.5042
...	...	...	...	...
718	8628	1431	3.107	18.7332
719	8640	1433	3.107	18.7331
720	8652	1435	3.107	18.7328
721	8664	1437	3.107	18.7325

720 preliminary samples must be taken to ensure that the exact Phase 2 limit is within 1% if the chi-square limit.

10-6. Rework Exercise 10-5, assuming that the subgroup size is $n = 5$.

(a) Phase 2 $\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} = \frac{6(30+1)(5-1)}{30(5)-30-6+1}F_{0.005,6,115} = \left(\frac{744}{115}\right)(3.294) = 21.309$

(b) Chi-square $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.005,6} = 18.548$. The phase 2 UCL is almost 15% larger than the chi-square limit.

(c) $p = 6$ quality characteristics, $n = 5$ sample size, $\alpha = 0.005$. Find m such that the exact phase 2 limit is within 1% of the chi-square limit, 1.01(18.548) = 18.733.

m	num	denom	F	UCL
30	744	115	3.294	21.3087
40	984	155	3.240	20.5692
50	1224	195	3.209	20.1422
...	...	...	...	...
400	9624	1595	3.105	18.7376
410	9864	1635	3.105	18.7330
411	9888	1639	3.105	18.7324
412	9912	1643	3.105	18.7318

411 preliminary samples must be taken to ensure that the exact phase 2 limit is within 1% of the chi-square limit.

10-9. Consider a T^2 control chart for monitoring $p = 10$ quality characteristics. Suppose $n = 3$ and there were 25 preliminary samples available to estimate the sample covariance matrix. Calculate both the phase 1 and the phase 2 control limits (use $\alpha = 0.01$).

Given: $p = 10$ quality characteristics, $n = 3$ sample size, $m = 25$ preliminary samples. Assume: $\alpha = 0.01$.

Phase 1 $\text{UCL} = \frac{p(m-1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} = \frac{10(25-1)(3-1)}{25(3)-25-10+1}F_{0.01,10,41} = \left(\frac{480}{41}\right)(2.788) = 32.638$

Phase 2 $\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} = \frac{10(25+1)(3-1)}{25(3)-25-10+1}F_{0.01,10,41} = \left(\frac{520}{41}\right)(2.788) = 35.360$

10-11. Suppose that we have $p = 3$ quality characteristics, and in correlation form all three variables have variance unity, and all pairwise correlation coefficients are 0.8. The in-control value of the process mean vector is $\mu' = [0, 0, 0]$.

(a) Write out the covariance matrix Σ.

$\bar{S}_1^2 = \bar{S}_2^2 = \bar{S}_3^2 = 1$

$\bar{S}_{12} = \bar{S}_{13} = \bar{S}_{23} = 0.8$

$$\Sigma = \begin{bmatrix} \bar{S}_1^2 & \bar{S}_{12} & \bar{S}_{13} \\ \bar{S}_{12} & \bar{S}_2^2 & \bar{S}_{23} \\ \bar{S}_{13} & \bar{S}_{23} & \bar{S}_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{bmatrix}$$

(b) What is the chi-square control limit for the chart, assuming that $\alpha = 0.05$?

$\alpha = 0.05$, $p = 3$, $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.05,3} = 7.815$

(c) Suppose that a sample of observations results in the standardized observation vector $\mathbf{y}' = [1, 2, 0]$. Calculate the value of the T^2 statistic. Is an out-of-control signal generated?

The sample size is $n = 1$; use equation for individual observations (text p. 522).

$$T^2 = (\mathbf{y}-\boldsymbol{\mu})'\mathbf{S}^{-1}(\mathbf{y}-\boldsymbol{\mu}) = \left(\begin{bmatrix}1\\2\\0\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix}\right)'\begin{bmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{bmatrix}\left(\begin{bmatrix}1\\2\\0\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix}\right) = 11.154$$

Yes. Since ($T^2 = 11.154$) > (UCL = 7.815), an out-of-control signal is generated.

(d) Calculate the diagnostic quantities d_i, $i = 1, 2, 3$ from equation 10-22. Does this information assist in identifying which process variables have shifted?

For Sample 1, $\mathbf{y}' = [1, 2, 0]$, found to be out of control (text p. 521). Attention should be focused on the variables for which d_i are relatively large; for this sample, variables 2 and 3 should be investigated:

Obs., i	y_i	T^2 (from (c))	$T^2_{(i)}$	$d_i = T^2 - T^2_{(i)}$
1	1	11.154	11.111	0.043
2	2	11.154	2.778	8.376
3	0	11.154	5.000	6.154

(e) Suppose that a sample of observations results in the standardized observation vector $\mathbf{y}' = [2, 2, 1]$. Calculate the value of the T^2 statistic. Is an out-of-control signal generated?

The sample size is $n = 1$; use equation for individual observations (text p. 522).

$$T^2 = (\mathbf{y}-\boldsymbol{\mu})'\mathbf{S}^{-1}(\mathbf{y}-\boldsymbol{\mu}) = \left(\begin{bmatrix}2\\2\\1\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix}\right)'\begin{bmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{bmatrix}\left(\begin{bmatrix}2\\2\\1\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix}\right) = 21.800$$

Since ($T^2 = 21.800$) > (UCL = 7.815), an out-of-control signal is generated.

(f) For the case in (e), calculate the diagnostic quantities d_i, $i = 1, 2, 3$ from the equation 10-22. Does this information assist in identifying which process variables have shifted?

Obs., i	y_i	T^2 (from (e))	$T^2_{(i)}$	$d_i = T^2 - T^2_{(i)}$
1	2	21.800	5.000	16.800
2	2	21.800	5.000	16.800
3	1	21.800	4.444	17.356

Since d_i is relatively large for all three variables, each variable should be investigated for contribution to the out-of-control signal.

10-13. Consider the first three process variables in Table 10-5. Calculate an estimate of the sample covariance matrix using both estimators $\mathbf{S}_1$ and $\mathbf{S}_2$ discussed in Section 10-3.2.

The data in Table 10-5 are for $n = 1$ sample size; the estimators in Section 10-3.2 address this problem. $\mathbf{S}_1$ simply pools all m sample observations; $\mathbf{S}_2$ uses differences between successive pairs of observations. There are $n = 3$ variables, $m = 40$ observations for each variable.

$$\bar{\mathbf{x}}' = [\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_3] = [15.339 \quad 0.104 \quad 88.125]$$

$$\mathbf{S}_1 = \frac{1}{(m-1)}\sum_{i=1}^{m}(\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})' = \frac{1}{(40-1)}\sum_{i=1}^{40}\left(\begin{bmatrix} x_{i,1} \\ x_{i,2} \\ x_{i,3} \end{bmatrix} - \begin{bmatrix} 15.339 \\ 0.104 \\ 88.125 \end{bmatrix}\right)\left(\begin{bmatrix} x_{i,1} \\ x_{i,2} \\ x_{i,3} \end{bmatrix} - \begin{bmatrix} 15.339 \\ 0.104 \\ 88.125 \end{bmatrix}\right)' = \begin{bmatrix} 4.440 & -0.016 & 5.395 \\ -0.016 & 0.001 & -0.014 \\ 5.395 & -0.014 & 27.599 \end{bmatrix}$$

Minitab Solution:

```
           Tab10-5x   Tab10-5x   Tab10-5x
Tab10-5x   4.439743
Tab10-5x  -0.016280   0.000964
Tab10-5x   5.394744  -0.014135  27.599359
```

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}'_1 \\ \mathbf{v}'_2 \\ \vdots \\ \mathbf{v}'_{m-1} \end{bmatrix} = \begin{bmatrix} [\mathbf{x}_{1+1} - \mathbf{x}_1]' \\ [\mathbf{x}_{2+1} - \mathbf{x}_2]' \\ \vdots \\ [\mathbf{x}_{m-1+1} - \mathbf{x}_{m-1}]' \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} x_{1,2} - x_{1,1} \\ x_{2,2} - x_{2,1} \\ x_{3,2} - x_{3,1} \end{bmatrix}' \\ \begin{bmatrix} x_{1,3} - x_{1,2} \\ x_{2,3} - x_{2,2} \\ x_{3,3} - x_{3,2} \end{bmatrix}' \\ \vdots \\ \begin{bmatrix} x_{1,40} - x_{1,39} \\ x_{2,40} - x_{2,39} \\ x_{3,40} - x_{3,39} \end{bmatrix}' \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 14.97 - 12.78 \\ 0.1 - 0.15 \\ 90 - 91 \end{bmatrix}' \\ \begin{bmatrix} 15.43 - 14.97 \\ 0.07 - 0.1 \\ 90 - 90 \end{bmatrix}' \\ \vdots \\ \begin{bmatrix} 16.28 - 14.74 \\ 0.13 - 0.07 \\ 86 - 84 \end{bmatrix}' \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 2.19 \\ -0.05 \\ -1 \end{bmatrix}' \\ \begin{bmatrix} 0.46 \\ -0.03 \\ 0 \end{bmatrix}' \\ \vdots \\ \begin{bmatrix} 1.54 \\ 0.06 \\ 2 \end{bmatrix}' \end{bmatrix}$$

$$\mathbf{V}'\mathbf{V} = \begin{bmatrix} 121.101 & -0.256 & 43.720 \\ -0.256 & 0.071 & 0.950 \\ 43.720 & 0.950 & 587.000 \end{bmatrix} \qquad \mathbf{S}_2 = \frac{1}{2}\frac{\mathbf{V}'\mathbf{V}}{(m-1)} = \begin{bmatrix} 1.553 & -0.003 & -0.561 \\ -0.003 & 0.001 & 0.012 \\ -0.561 & 0.012 & 7.526 \end{bmatrix}$$

10-15. Suppose that there are $p = 4$ quality characteristics, and in correlation form all four variables have variance unity and all pairwise correlation coefficients are 0.75. The in-control value of the process mean vector is $\mu' = [0, 0, 0, 0]$, and we want to design an MEWMA control chart to provide good protection against a shift to a new mean vector of $\mathbf{y}' = [1, 1, 1, 1]$. If an in-control ARL_0 of 200 is satisfactory, what value of λ and what upper control limit should be used? Approximately, what is the ARL_1 for detecting the shift in the mean vector?

p =	4			
mu' =	0	0	0	0
Sigma =	1	0.75	0.75	0.75
	0.75	1	0.75	0.75
	0.75	0.75	1	0.75
	0.75	0.75	0.75	1
Sigma-1 =	3.0769	-0.9231	-0.9231	-0.9231
	-0.9231	3.0769	-0.9231	-0.9231
	-0.9231	-0.9231	3.0769	-0.9231
	-0.9231	-0.9231	-0.9231	3.0769
y' =	1	1	1	1
y =	1			
	1			
	1			
	1			
y' Sigma-1 =	0.308	0.308	0.308	0.308
y' Sigma-1 y =	1.231			
delta =	1.109			
ARL0 =	200			

From Table 10-3, select (lambda, H) pair that closely minimizes ARL1.

delta =	1	1.5
lambda =	0.1	0.2
UCL = H =	12.73	13.87
ARL1 =	12.17	6.53

Select $\lambda = 0.1$ with an $UCL = H = 12.73$. This gives an ARL_1 between 7.22 and 12.17.

10-17. Suppose that there are $p = 2$ quality characteristics, and in correlation form all four variables have variance unity and all pairwise correlation coefficients are 0.8. The in-control value of the process mean vector is $\mu' = [0, 0]$, and we want to design an MEWMA control chart to provide good protection against a shift to a new mean vector of $y' = [1, 1]$. If an in-control ARL_0 of 200 is satisfactory, what value of λ and what upper control limit should be used? Approximately, what is the ARL_1 for detecting the shift in the mean vector?

p =	2	
mu' =	0	0
Sigma =	1	0.8
	0.8	1
Sigma-1 =	2.7778	-2.2222
	-2.2222	2.7778
y' =	1	1
y =	1	
	1	
y' Sigma-1 =	0.556	0.556
y' Sigma-1 y =	1.111	
delta =	1.054	
ARL0 =	200	

From Table 10-3, select (lambda, H) pair that closely minimizes ARL1

delta =	1	1	1.5	1.5
lambda =	0.1	0.2	0.2	0.3
UCL = H =	8.64	9.65	9.65	10.08
ARL1 =	10.15	10.20	5.49	5.48

Select $\lambda = 0.2$ with an $UCL = H = 9.65$. This gives an ARL_1 between 5.49 and 10.20.

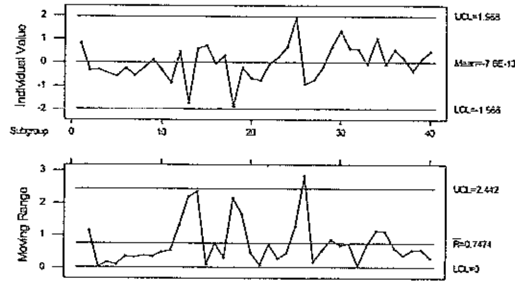
10-19. Consider the cascade process data in Table 10-5. In fitting regression models to both y_1 and y_2 you will find that not all of the process variables are required to obtain a satisfactory regression model for the output variables. Remove the nonsignificant variables from these equations and obtain subset regression models for both y_1 and y_2. Then construct individuals control charts for both sets of residuals. Compare them to the residual charts in the text (Fig. 10-11) and from Exercise 10-18. Are there any substantial differences between the charts from the two different approaches to fitting the regression models?

With this many process variables, a multiple regression analysis package must be used. MINITAB™, JMP® and STATGRAPHICS® Plus are three available software packages. The results below were obtained with MINITAB™ Release 13.1.

For Response y_1:

```
The regression equation is
Tab10-5y1 = 826 + 0.474 Tab10-5x1 + 1.41 Tab10-5x2 - 0.117 Tab10-5x3
          - 0.0824 Tab10-5x4 - 2.39 Tab10-5x5 - 1.30 Tab10-5x6
          + 2.18 Tab10-5x7 + 2.98 Tab10-5x8 + 113 Tab10-5x9

Predictor        Coef     SE Coef        T        P
Constant       825.89       35.14    23.50    0.000
Tab10-5x1     0.47410     0.09038     5.25    0.000
Tab10-5x2       1.413       6.250     0.23    0.823
Tab10-5x3    -0.11684     0.03911    -2.99    0.006
Tab10-5x4    -0.08237     0.02838    -2.90    0.007
Tab10-5x5      -2.392       2.471    -0.97    0.341
Tab10-5x6      -1.298       1.519    -0.85    0.400
Tab10-5x7       2.176       3.087     0.70    0.486
Tab10-5x8      2.9805      0.8497     3.51    0.001
Tab10-5x9      113.22       26.12     4.33    0.000

S = 0.8556      R-Sq = 80.8%      R-Sq(adj) = 75.0%

Analysis of Variance
Source            DF          SS          MS         F        P
Regression         9      92.462      10.274     14.03    0.000
Residual Error    30      21.962       0.732
Total             39     114.424
```

I and MR Chart for Ex 10-19, Residuals from Full y1 Model

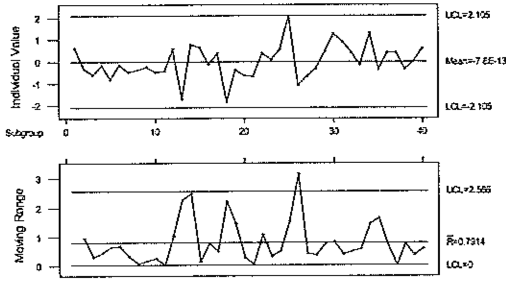


Observation 25 signals out of control on the Individuals chart, and subsequently observation 26 signals out of control on the Moving Range chart.

Remove insignificant variables, x_2, x_5, x_6, and x_7 from consideration and re-fit the regression model in variables x_1, x_3, x_4, x_8, and x_9.

```
The regression equation is
Tab10-5y1 = 819 + 0.431 Tab10-5x1 - 0.124 Tab10-5x3 - 0.0915 Tab10-5x4
          + 2.64 Tab10-5x8 + 115 Tab10-5x9

Predictor          Coef      SE Coef         T        P
Constant         818.80        29.14     28.10    0.000
Tab10-5x1       0.43080      0.08113      5.31    0.000
Tab10-5x3      -0.12396      0.03530     -3.51    0.001
Tab10-5x4      -0.09146      0.02438     -3.75    0.001
Tab10-5x8        2.6367       0.7604      3.47    0.001
Tab10-5x9        114.81        23.65      4.85    0.000

S = 0.8302      R-Sq = 79.5%      R-Sq(adj) = 76.5%

Analysis of Variance

Source            DF         SS          MS          F        P
Regression         5     90.990      18.198      26.40    0.000
Residual Error    34     23.434       0.689
Total             39    114.424
```

I and MR Chart for Ex 10-19, Residuals from Subset y1 Model

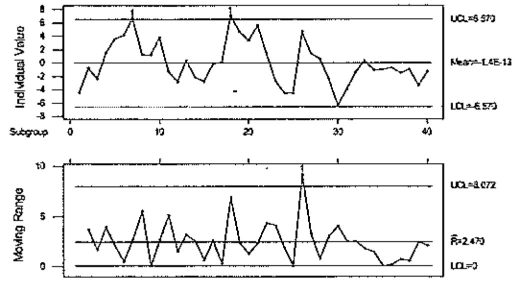


Observation 25 signals out of control on the Individuals chart, and subsequently observation 26 signals out of control on the Moving Range chart.

For Response y_2:

```
The regression equation is
Tab10-5y2 = 215 - 0.666 Tab10-5x1 - 11.6 Tab10-5x2 + 0.435 Tab10-5x3
          + 0.192 Tab10-5x4 - 3.2 Tab10-5x5 + 0.73 Tab10-5x6
          + 6.1 Tab10-5x7 + 10.9 Tab10-5x8 - 215 Tab10-5x9

Predictor          Coef     SE Coef         T        P
Constant          214.6       153.1      1.40    0.171
Tab10-5x1       -0.6659      0.3936     -1.69    0.101
Tab10-5x2        -11.63       27.22     -0.43    0.672
Tab10-5x3        0.4346      0.1704      2.55    0.016
Tab10-5x4        0.1922      0.1236      1.55    0.130
Tab10-5x5         -3.24       10.76     -0.30    0.766
Tab10-5x6         0.728       6.615      0.11    0.913
Tab10-5x7          6.12       13.45      0.46    0.652
Tab10-5x8        10.922       3.701      2.95    0.006
Tab10-5x9        -215.1       113.8     -1.89    0.068

S = 3.727        R-Sq = 68.0%       R-Sq(adj) = 58.4%

Analysis of Variance
Source            DF          SS          MS          F        P
Regression         9      886.87       98.54       7.10    0.000
Residual Error    30      416.63       13.89
Total             39     1303.50
```

I and MR Chart for Ex 10-19, Residuals from Full y2 Model

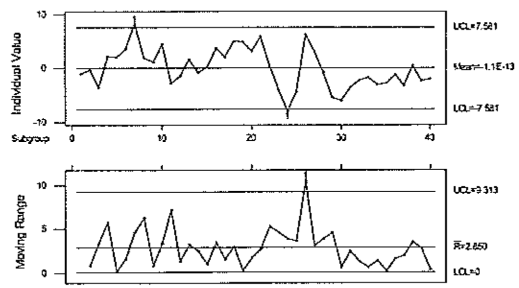


Observations 7 and 24 out of control on the Individuals chart, and observation 26 signals out of control on the Moving Range chart.

Remove insignificant variables, x_1, x_2, x_4, x_5, x_6, and x_7 from consideration and re-fit the regression model in variables x_3, x_8, and x_9.

```
The regression equation is
Tab10-5y2 = 260 + 0.311 Tab10-5x3 + 12.9 Tab10-5x8 - 249 Tab10-5x9

Predictor        Coef    SE Coef        T        P
Constant        260.0      106.2     2.45    0.019
Tab10-5x3      0.3109     0.1526     2.04    0.049
Tab10-5x8      12.902      3.433     3.76    0.001
Tab10-5x9     -248.87      87.40    -2.85    0.007

S = 3.816     R-Sq = 59.8%     R-Sq(adj) = 56.4%

Analysis of Variance

Source            DF        SS        MS        F        P
Regression         3    779.27    259.76    17.84    0.000
Residual Error    36    524.23     14.56
Total             39   1303.50
```

I and MR Chart for Ex10-19 Residuals from Subset y2 Model



Observations 7 and 24 out of control on the Individuals chart, and observation 26 signals out of control on the Moving Range chart.

For response y_1, there is not a significant difference between control charts for residuals from either the full (no out-of-control observations) or subset (observations 25 and 26 are OOC) regression model.

For response y_2, there is not a significant difference between control charts for residuals from either the full (observations 7, 18, and 26 are OOC) or subset (observations 7, 24, and 26 are OOC) regression model.

10-21. Consider the p = 4 process variables in Table 10-6. After applying the PCA procedure to the first 20 observations data (see Table 10-7), suppose that the first three principal components are retained.

(a) Obtain the principal component scores. (Hint: Remember that you must work in standardized variables.)

The results below were obtained with JMP®, Version 4. To work in standardized variables in JMP, use the principal components on correlations.

	x1	x2	x3	x4
x1	1.0000	0.9302	0.2060	0.3595
x2	0.9302	1.0000	0.1669	0.4502
x3	0.2060	0.1669	1.0000	0.3439
x4	0.3595	0.4502	0.3439	1.0000

EigenValue	Percent	Cum Percent
2.3181	57.952	57.952
1.0118	25.295	83.247
0.6088	15.221	98.467
0.0613	1.533	100.000

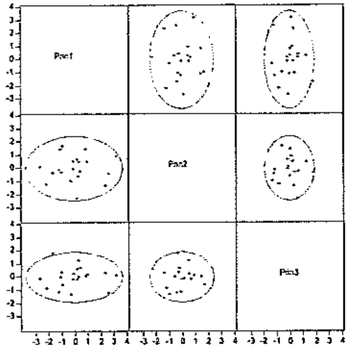
Eigenvectors				
x1	0.59410	-0.33393	0.25699	0.68519
x2	0.60704	-0.32960	0.08341	-0.71826
x3	0.28553	0.79369	0.53368	-0.06092
x4	0.44386	0.38717	-0.80137	0.10440

The first three eigenvalues sum to more than 90% of the variability, so use three principal components.

PRIN1	PRIN2	PRIN3	PRIN1	PRIN2	PRIN3
0.291681	-0.6034	0.024961	0.944697	0.504711	0.179755
0.294281	0.491533	1.23823	-1.2195	-0.09129	-1.11787
0.197337	0.640937	-0.20787	2.608666	-0.42176	-1.19166
0.839022	1.469579	0.039289	-0.12378	-0.08767	-0.19592
3.204876	0.879172	0.124203	-1.10423	1.472593	0.012988
0.203271	-2.29514	0.625447	-0.27825	-0.94763	-1.31445
-0.99211	1.670464	-0.58815	-2.65608	0.135288	-0.11243
-1.70241	-0.36089	1.821569	2.36528	-1.30494	0.322855
-0.14246	0.560808	0.231003	0.411311	-0.21893	0.644795
-0.99498	-0.31493	0.331641	-2.14662	-1.17849	-0.86838

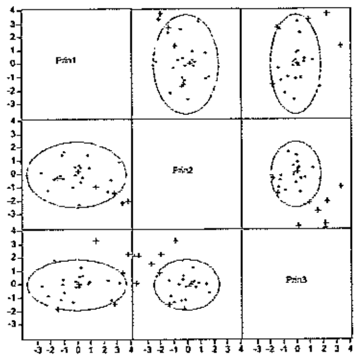
(b) Construct an appropriate set of pairwise plots of the principal component scores.

Perform a multivariate analysis on the principal components scores and examine the scatterplot.



(c) Calculate the principal component scores for the last 10 observations. Plot the scores on the charts from part (b) and interpret the results.

PRIN1	PRIN2	PRIN3
...	...	...
0.074196	0.239359	0.039691
-1.51756	-0.21121	-1.78971
1.408476	-0.87591	3.300787
6.298001	-3.67398	2.172434
3.802025	-1.99584	2.268171
6.490673	-2.73143	1.540329
2.738829	-1.37617	-1.41462
4.958747	-3.94851	2.166644
5.678092	-3.85838	0.128901
3.369657	-2.10878	0.891978



Several of the new observations (21—30) fall outside the confidence ellipse based on the first 20 observations.

CHAPTER 11

Engineering Process Control and SPC

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Describe the EPC approach to variability reduction.
2. Explain the components of an integral control process adjustment scheme:
   - Weighting factor, λ
   - Process gain, g
   - Target, T
   - Set point X_t
   - Disturbance, N_{t+1}
3. Construct and apply a bounded adjustment chart, and understand the meaning of the boundary value L.
4. Discuss the integration of EPC with SPC.

Exercises

11-1. If y_t are the observations and z_t is the EWMA, show that the following relationships are true.

(a) $z_t - z_{t-1} = \lambda(y_t - z_{t-1})$

y_t: observation, z_t = EWMA statistic

$$z_t = \lambda y_t + (1-\lambda)z_{t-1}$$
$$z_t = \lambda y_t + z_{t-1} - \lambda z_{t-1}$$
$$z_t - z_{t-1} = \lambda y_t + z_{t-1} - z_{t-1} - \lambda z_{t-1}$$
$$z_t - z_{t-1} = \lambda y_t - \lambda z_{t-1}$$
$$z_t - z_{t-1} = \lambda(y_t - z_{t-1})$$

(b) $e_t - (1-\lambda)e_{t-1} = y_t - y_{t-1}$

$$z_{t-1} - z_{t-2} = \lambda e_{t-1} \quad \text{(as a result of part (a))}$$
$$z_{t-1} - z_{t-2} + (e_t - e_{t-1}) = \lambda e_{t-1} + (e_t - e_{t-1})$$
$$z_{t-1} + e_t - z_{t-2} - e_{t-1} = e_t - (1-\lambda)e_{t-1}$$
$$y_t - y_{t-1} = e_t - (1-\lambda)e_{t-1}$$

11-3. Consider the data in Table 11-1. Construct a bounded adjustment chart using $\lambda = 0.4$ and $L = 10$. Compare the performance of this chart to the one in Table 11-1 and Fig. 11-12.

Given the weighting factor is $\lambda = 0.4$ and the boundary value is $L = 10$. Assume the target for the process variable is $T = 0$ and the process gain is $g = 0.8$ (from Section 11-2.3).

$\bar{x}_{\text{Orig Output}} = 17.24$, $SS_{\text{Orig Output}} = 21{,}468$

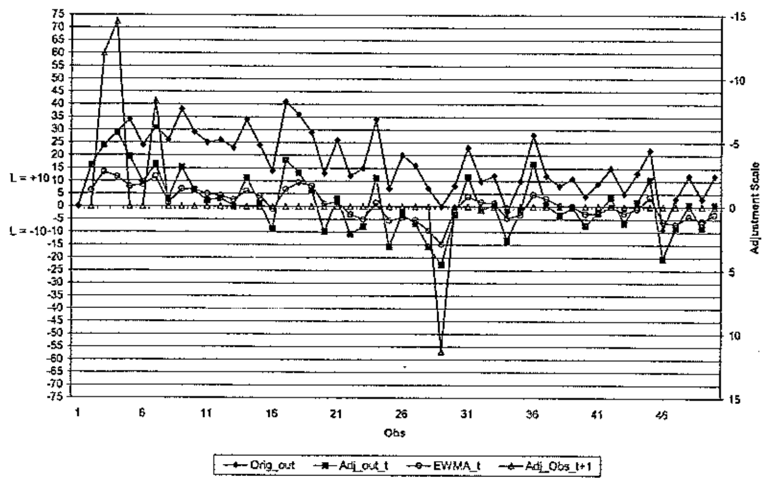
The method for determining the Adjusted Process Output is a little tricky. N_t represents the disturbance in the Original Process Output, $N_t = x_t - x_{t-1}$. Once the process is re-set, the Adjusted Process Output at a single observation (Adj Out_t) is the previous output (Adj Out_t at $t-1$), plus the disturbance (N_t), plus the adjustment made (Adj Obs_t+1 at $t-1$). Under real-time monitoring, it would not be necessary to make this type of correction to the observed Process Output.

At each observation, predict N_t using an EWMA. If the forecast is less than or equal to L, no adjustment to the manipulatable variable is made. If the forecast is greater than L, then an adjustment is made in the usual way. Thus, process adjustments are only made in the periods for which the EWMA forecast is outside the $\pm L$ bounds.

Obs	Orig Out	Orig Nt	Adj Out_t	EWMA_t	|EWMA_t| >L?	Adj Obs_t+1	Cum Adj
1	0	0					
2	16	16	16	6.400	no	0.0	0.0
3	24	8	24	13.440	yes	-12.0	-12.0
*4	29	5	29	11.600	yes	-14.5	-26.5*
5	34	5	19.500	7.800	no	0.000	-26.500
...							
48	12	9	0.625	-3.897	no	0.000	-23.375
49	3	-9	-8.375	-5.688	no	0.000	-23.375
50	12	9	0.625	-3.163	no	0.000	-23.375

* Process is re-set after observation 4. Use original disturbance N_t to determine adjusted process output.

Bounded Adjustment Chart for Ex 11-3



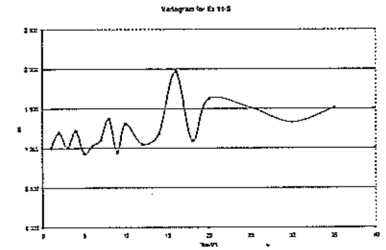
$\bar{x}_{\text{Adj Output}} = 1.615$, $SS_{\text{Adj Output}} = 6344$

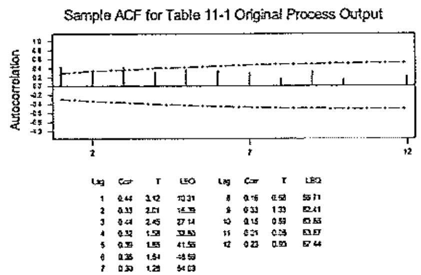
11-5. **The Variogram.** Consider the variance of observations that are m periods apart; that is, $V_m = V(y_{t+m} - y_t)$. A graph of V_m/V_1 versus m is called a variogram. It is a nice way to check a data series for nonstationary (drifting mean) behavior. If a data series is completely uncorrelated (white noise) the variogram will always produce a plot that stays near unity. If the data series is autocorrelated but stationary, the plot of the variogram will increase for a while but as m increases the plot of V_m/V_1 will gradually stabilize and not increase any further. The plot of V_m/V_1 versus m will increase without bound for nonstationary data. Apply this technique to the data in Table 11-1. Is there an indication of nonstationary behavior? Calculate the sample autocorrelation function for the data. Compare the interpretation of both graphs.

$$V_1 = \text{var}\left[(y_{t+1} - y_t) \text{ for all } t = 1, \ldots, 49\right]$$
$$= \text{var}\left[(y_2 - y_1), (y_3 - y_2), \ldots, (y_{50} - y_{49})\right]$$
$$= \text{var}\left[(16-0), (24-16), \ldots, (12-3)\right]$$
$$= 147.11$$

Let period $m = 1$,

t	Yt	m → 1	2	3	4	5	6	7	8	9	10	20	25	30	35
1	0														
2	16	16													
3	24	8	24												
4	29	5	13	29											
5	34	5	10	18	34										
6	24	-10	-5	0	8	24									
7	31	7	-3	2	7	15	31								
8	26	-5	2	-8	-3	2	10	26							
9	38	12	7	14	4	9	14	22	38						
10	29	-9	3	-2	5	-5	0	5	13	29					
	...														
46	-9	-31	-22	-14	-24	-18	-13	-20	-17	-21	-37	-29	-35	-23	-34
47	3	12	-19	-10	-2	-12	-6	-1	-8	-5	-9	-13	-9	-38	-23
48	12	9	21	-10	-1	7	-3	3	8	1	4	5	-3	-24	-11
49	3	-9	0	12	-19	-10	-2	-12	-6	-1	-8	3	-31	-26	-31
50	12	9	0	9	21	-10	-1	7	-3	3	8	4	5	-1	-12
Var_m =		147.11	175.72	147.47	179.02	136.60	151.39	162.43	201.53	138.70	192.18	238.95	222.39	195.36	221.81
Var_m/Var_1 =		1.000	1.195	1.002	1.217	0.929	1.029	1.104	1.370	0.943	1.306	1.624	1.512	1.328	1.508





The variogram increases then stabilizes near 1.5, indicating that the data series is autocorrelated but stationary (mean does not drift). The slow decline in the sample ACF also indicates the data are correlated.

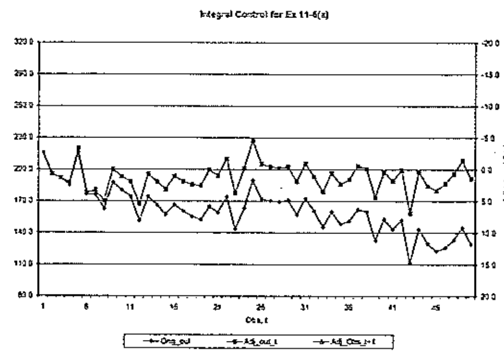
11-6. Consider the observations shown in the following table. The target value for this process is 200.

(a) Set up an integral controller for this process. Assume that the gain for the adjustment variable is $g = 1.2$ and assume that $\lambda = 0.2$ in the EWMA forecasting procedure will provide adequate one-step-ahead predictions.

"Orig_out" is the reading y_t, "Orig_Nt" is the disturbance N_t, "Adj_out_t" is the adjusted output (previous adjusted output, plus disturbance, plus adjustment), "Adj_Obs_t+1" is the adjustment to subsequent output), "Cum_Adj" is the cumulative adjustment to the process.

Obs, t	Orig_out	Orig_Nt	Adj_out_t	Adj_Obs_t+1	Cum_Adj
1	215.8	0.0			
*2	195.8	-20.0	195.8	0.7	0.7*
3	191.3	-4.5	192.0	1.3	2.0
4	185.3	-6.0	187.3	2.1	4.1
5	216.0	30.7	220.1	-3.4	0.8
...					
48	133.2	7.0	195.9	0.7	63.4
49	145.0	11.8	208.4	-1.4	62.0
50	129.5	-15.5	191.5	1.4	63.4

* Process is reset. Use original disturbance N_t to determine adjusted process output.



(b) How much reduction in variability around the target does the integral controller achieve?

	Unadjusted	Adjusted
SS =	1,323,871.8	1,818,510.3
Average =	161.3	192.2
Variance =	467.8	160.9

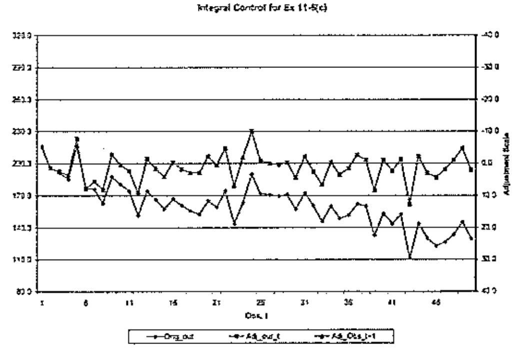
There is a significant reduction in variability with use of an integral control scheme.

(c) Rework parts (a) and (b) assuming that $\lambda = 0.4$. What change does this make in the variability around the target in comparison to that achieved with $\lambda = 0.2$?

$T = 200$, $\lambda = 0.4$, $g = 1.2$.

Obs, t	Orig_out	Orig_Nt	Adj_out_t	Adj_Obs_t+1	Cum_Adj
1	215.8	0.0			
*2	195.8	-20.0	195.8	1.4	1.4*
3	191.3	-4.5	192.7	2.4	3.8
4	185.3	-6.0	189.1	3.6	7.5
5	216.0	30.7	223.5	-7.8	-0.4
...					
48	133.2	7.0	203.4	-1.1	69.0
49	145.0	11.8	214.0	-4.7	64.4
50	129.5	-15.5	193.9	2.0	66.4

* Process is reset. Use original disturbance N_t to determine adjusted process output.



	Unadjusted	Adjusted
SS =	1,323,871.8	1,888,995.0
Average =	161.3	195.9
Variance =	467.8	164.0

The variances are approximately the same for each integral adjustment control scheme.

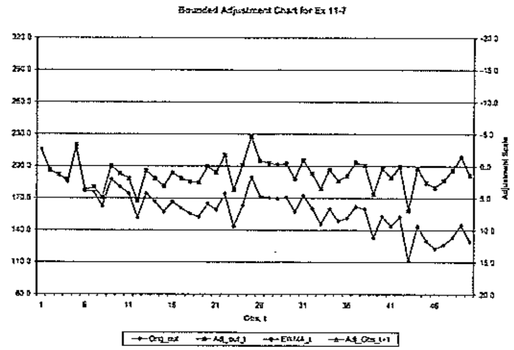
11-7. Use the data in Exercise 11-6 to construct a bounded adjustment chart. Use $\lambda = 0.2$ and set $L = 12$. How does the bounded adjustment chart perform relative to the integral control adjustment procedure in part

(a) of Exercise 11-6?

The solution is similar to that for bounded adjustment chart in Exercise 11-3, with $T = 200$, $\lambda = 0.2$, $L = 12$, $g = 1.2$.

Obs	Orig_out	Orig_Nt	Adj_out_t	EWMA_t	|EWMA_t|>L?	Adj_Obs_t+1	Cum_Adj
1	215.8	0					
*2	195.8	-20	196	39.160	yes	0.7	0.7*
3	191.3	-4.5	192.000	38.400	yes	1.3	2.033
4	185.3	-6	187.333	37.467	yes	2.111	4.144
5	216.0	30.7	220.144	44.029	yes	-3.357	0.787
...							
48	133.2	7	195.894	39.179	yes	0.684	63.379
49	145.0	11.8	208.379	41.676	yes	-1.396	61.982
50	129.5	-15.5	191.482	38.296	yes	1.420	63.402

* Process is reset. Use original disturbance N_t to determine adjusted process output.



	Original	Adjusted
SS =	1,323,872	1,818,510
Average =	161.304	192.237
Variance =	467.8	160.9

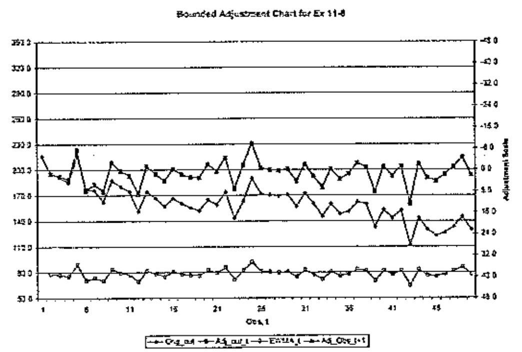
The behavior of the bounded adjustment control scheme is similar to both integral control schemes.

11-8. Rework Exercise 11-7 using $\lambda = 0.4$ and $L = 15$. What differences in the results are obtained?

The solution is similar to that for bounded adjustment chart in Exercise 11-3 with $T = 200$, $\lambda = 0.4$, $L = 15$, $g = 1.2$

Obs, t	Orig_out	Orig_Nt	Adj_out_t	EWMA_t	|EWMA_t|>L?	Adj_Obs_t+1	Cum_Adj
1	215.8	0					
*2	195.8	-20	196	78.320	yes	1.4	1.4*
3	191.3	-4.5	192.700	77.080	yes	2.4	3.833
4	185.3	-6	189.133	75.653	yes	3.622	7.456
5	216.0	30.7	223.456	89.382	yes	-7.819	-0.363
...							
48	133.2	7	203.360	81.344	yes	-1.120	69.040
49	145.0	11.8	214.040	85.616	yes	-4.680	64.360
50	129.5	-15.5	193.860	77.544	yes	2.047	66.407

* Process is reset. Use original disturbance N_t to determine adjusted process output.



	Original	Adjusted
SS =	1,323,872	1,888,995
Average =	161.304	195.934
Variance =	467.81	164.02

Behavior of the bounded adjustment control scheme is similar to both integral control schemes.

CHAPTER 12

Factorial and Fractional Factorial Experiments for Process Design and Improvement

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Understand guidelines for defining, performing and analyzing designed experiments
2. Design and analyze one factor, factorial experiments
3. Design and analyze two factor, factorial experiments, including an interaction
4. Perform residual analysis for checking assumptions and model adequacy
5. Design and analyze 2^k factorial designs
6. Use centerpoints to augment experimental designs to check for model curvature
7. Design and analyze fractional 2^{k-p} factorial designs

Exercises

12-1. An article in *Industrial Quality Control* (1956, pp. 5-8) describes an experiment to investigate the effect of glass type and phosphor type on the brightness of a television tube. The response measured is the current necessary (in microamps) to obtain a specified brightness level. The data are shown here. Analyze the data and draw conclusions.

	Glass Type		
Glass Type	1	2	3
1	28	300	29
	0		0
	29	310	28
	0		5
	28	295	29
	5		0
2	23	260	22
	0		0
	23	240	22
	5		5
	24	235	23
	0		0

Response: Current
ANOVA for Selected Factorial Model
Analysis of Variance table [Partial sum of squares]

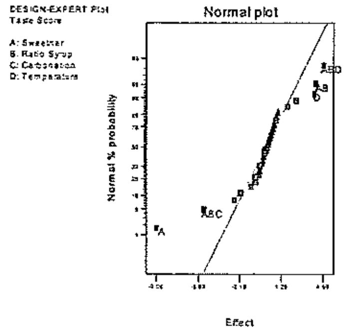
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*15516.67*	*5*	*3103.33*	*58.80*	*< 0.0001*
A	*14450.00*	*1*	*14450.00*	*273.79*	*< 0.0001*
B	*933.33*	*2*	*466.67*	*8.84*	*0.0044*
AB	*133.33*	*2*	*66.67*	*1.26*	*0.3178*
Pure Error	*633.33*	*12*	*52.78*		
Cor Total	*16150.00*	*17*			

Std. Dev.	7.26	R-Squared	0.9608
Mean	263.33	Adj R-Squared	0.9444
C.V.	2.76	Pred R-Squared	0.9118
PRESS	1425.00	Adeq Precision	18.279

No indication of interaction. Glass type (A) and phosphor type (B) significantly affect television tube brightness. The Prob>F statistic for A and B are less than 0.05, or an $\alpha = 0.5$.

12-4. Four factors are thought to possible influence the taste of a soft-drink beverage: type of sweetener (A), ratio of syrup to water (B), carbonation level (C), and the temperature (D). Each factor can be run at two levels producing a 2^4 design. At each run in the design, samples of the beverage are given to test a panel consisting of 20 people. Each tester assigns point score from 1 to 10 to the beverage. Total score is the response variable, and the objective is to find a formulation that maximizes total score. Two replicates of this design are run, and the results are shown here. Analyze the data and draw conclusions.

	Replicate			Replicate	
Treatment Combination	I	II	Treatment Combination	I	II
(1)	190	193	*d*	198	195
a	174	178	*ad*	172	176
b	181	185	*bd*	187	183
ab	183	180	*abd*	185	186
c	177	178	*cd*	199	190
ac	181	180	*acd*	179	175
bc	188	182	*bcd*	187	184
abc	173	170	*abcd*	180	180



From magnitude of effects, A (type of sweetner) is dominant, along with interactions involving ratio of syrup to water (ABC, ABD, AB), and D (temperature). A hierarchical model should be built to include all main factors in the significant interactions.

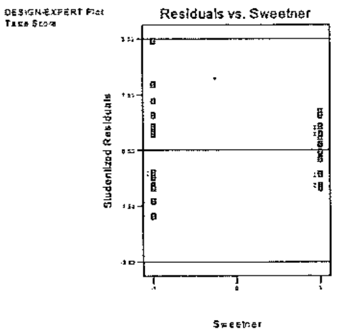
Response: Taste Score
Hierarchical Terms Added after Manual Regression
B C AC AD BC BD
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

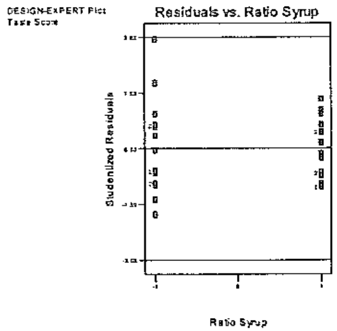
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*1420.59*	*11*	*129.14*	*12.49*	*< 0.0001*
A	*657.03*	*1*	*657.03*	*63.52*	*< 0.0001*
B	*13.78*	*1*	*13.78*	*1.33*	*0.2620*
C	*57.78*	*1*	*57.78*	*5.59*	*0.0283*
D	*124.03*	*1*	*124.03*	*11.99*	*0.0025*
AB	*132.03*	*1*	*132.03*	*12.76*	*0.0019*
AC	*3.78*	*1*	*3.78*	*0.37*	*0.5522*
AD	*38.28*	*1*	*38.28*	*3.70*	*0.0687*
BC	*2.53*	*1*	*2.53*	*0.24*	*0.6262*
BD	*0.28*	*1*	*0.28*	*0.027*	*0.8707*

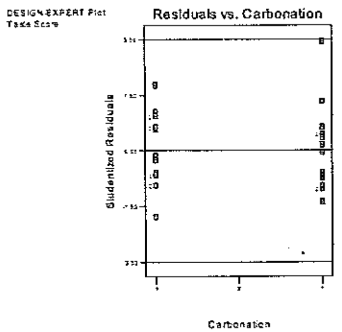
ABC	*215.28*	*1*	*215.28*	*20.81*	*0.0002*
ABD	*175.78*	*1*	*175.78*	*16.99*	*0.0005*
Residual	*206.88*	*20*	*10.34*		
Lack of Fit	*84.38*	*4*	*21.09*	*2.76*	*0.0644*
Pure Error	*122.50*	*16*	*7.66*		
Cor Total	*1627.47*	*31*			

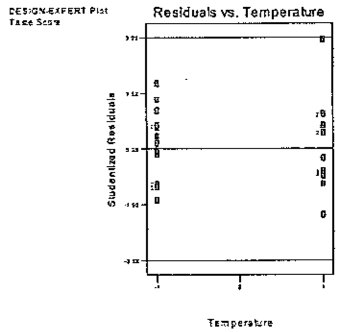
Std. Dev.	3.22	R-Squared	0.8729
Mean	182.78	Adj R-Squared	0.8030
C.V.	1.76	Pred R-Squared	0.6746
PRESS	529.60	Adeq Precision	13.646

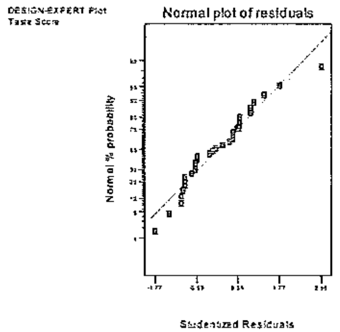
12-5. Consider the experiment in Exercise 12-4. Plot the residuals against the factor levels *A*, *B*, *C* and *D*. Also construct a normal probability plot of the residuals. Comment on these plots.

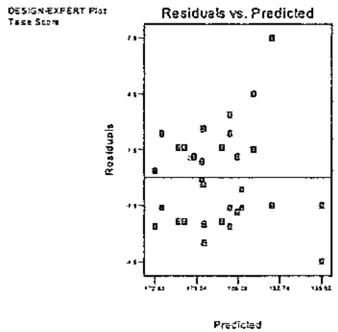












There appears to be a slight indication of inequality of variance for sweetener and syrup ratio, as well as a slight indication of an outlier. This is not serious enough to warrant concern.

12-6. Find the standard error of the effects for the experiment in Exercise 12-4. Using the standard errors as a guide, what factors appear significant?

Factor	Coefficient Estimate	DF	Std Error	95% CI Low	95% CI High
Intercept	*182.78*	*1*	*0.57*	*181.60*	*183.97*
A-Sweetener	*-4.53*	*1*	*0.57*	*-5.72*	*-3.35**
B-Syrup ratio	*-0.66*	*1*	*0.57*	*-1.84*	*0.53*
C-Carbonation	*-1.34*	*1*	*0.57*	*-2.53*	*-0.16**
D-Temperature	*1.97*	*1*	*0.57*	*0.78*	*3.15**
AB	*2.03*	*1*	*0.57*	*0.85*	*3.22**
AC	*0.34*	*1*	*0.57*	*-0.84*	*1.53*
AD	*-1.09*	*1*	*0.57*	*-2.28*	*0.092*
BC	*-0.28*	*1*	*0.57*	*-1.47*	*0.90*
BD	*-0.094*	*1*	*0.57*	*-1.28*	*1.09*
ABC	*-2.59*	*1*	*0.57*	*-3.78*	*-1.41**
ABD	*2.34*	*1*	*0.57*	*1.16*	*3.53**

The ratio of the coefficient estimate to the standard error is distributed as *t* statistic, and a value greater than approximately |2| would be considered significant. Also, if the confidence interval includes zero, the factor is not significant. From examination of the above table, factors *A*, *C*, *D*, *AB*, *ABC* and *ABD* appear to be significant.

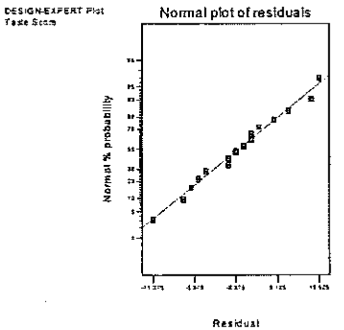
12-7. Suppose that only the data from replicate I in Exercise 12-4 were available. Analyze the data and draw appropriate conclusions.

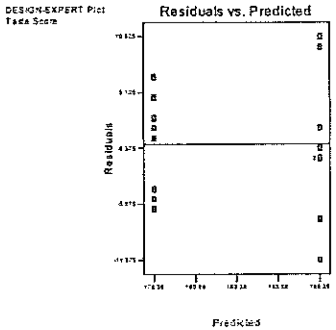
Response: **Taste Score**
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*400.00*	*1*	*400.00*	*10.00*	*0.0069*
A	400.00	1	400.00	10.00	0.0069
Residual	*559.75*	*14*	*39.98*		
Cor Total	*959.75*	*15*			

Std. Dev.	6.32	R-Squared	0.4168
Mean	183.38	Adj R-Squared	0.3751
C.V.	3.45	Pred R-Squared	0.2382
PRESS	731.10	Adeq Precision	4.473

Factor	Coefficient Estimate	DF	Std Error	95% CI Low	95% CI High
Intercept	*183.38*	*1*	*1.58*	*179.98*	*186.77*
A-Sweetener	*-5.00*	*1*	*1.58*	*-8.39*	*-1.61*





12-9. Show how a 2^5 experiment could be set up in two blocks of 16 runs each. Specifically, which runs would be made in each block.

A 2^5 design in two blocks will lose the *ABCDE* interaction to blocks. Compute the contrast for the *ABCDE* interaction. Where the interaction is at the low level, assign those treatment combinations to Block 1. Where the interaction is at the high level, assign those treatment combinations to Block 2.

A	*B*	*C*	*D*	*E*	*ABCDE* Interactio n	Treatment Combinatio n	Block Assigment
-1	-1	-1	-1	-1	-1	(1)	1
1	-1	-1	-1	-1	1	*a*	2
-1	1	-1	-1	-1	1	*c*	2
1	1	-1	-1	-1	-1	*ab*	1
-1	-1	1	-1	-1	1	*c*	2
1	-1	1	-1	-1	-1	*ac*	1
-1	1	1	-1	-1	-1	*bc*	1
1	1	1	-1	-1	1	*abc*	2
-1	-1	-1	1	-1	1	*d*	2
1	-1	-1	1	-1	-1	*ad*	1
-1	1	-1	1	-1	-1	*bd*	1
1	1	-1	1	-1	1	*abd*	2
-1	-1	1	1	-1	-1	*cd*	1
1	-1	1	1	-1	1	*acd*	2
-1	1	1	1	-1	1	*bcd*	2
1	1	1	1	-1	-1	*abcd*	1
-1	-1	-1	-1	1	1	*e*	2
1	-1	-1	-1	1	-1	*ae*	1
-1	1	-1	-1	1	-1	*be*	1
1	1	-1	-1	1	1	*abe*	2
-1	-1	1	-1	1	-1	*ce*	1
1	-1	1	-1	1	1	*ace*	2
-1	1	1	-1	1	1	*bce*	2
1	1	1	-1	1	-1	*abce*	1
-1	-1	-1	1	1	-1	*de*	1
1	-1	-1	1	1	1	*ade*	2
-1	1	-1	1	1	1	*bde*	2
1	1	-1	1	1	-1	*abde*	1
-1	-1	1	1	1	1	*cde*	2
1	-1	1	1	1	-1	*acde*	1
-1	1	1	1	1	-1	*bcde*	1
1	1	1	1	1	1	*abcde*	2

Block 1		Block 2	
(1)	*ae*	*a*	*e*
ab	*be*	*b*	*abe*
ac	*ce*	*c*	*ace*
bc	*abce*	*abc*	*bce*
ad	*de*	*d*	*ade*
bd	*abde*	*abd*	*bde*
cd	*acde*	*acd*	*cde*
abcd	*bcde*	*bcd*	*abcde*

12-11. An article in *Industrial and Engineering Chemistry* ("More on Planning Experiments to Increase Research Efficiency, " 1970, pp. 60-65) uses a 2^{5-2} design to investigate the effect of A = condensation temperature, B = amount of material 1, C = solvent volume, D = condensation time, and E = amount of material 2, on yield. The results obtained are as follows:

$e = 23.2$	$ad = 16.9$	$cd = 23.8$	$bde = 16.8$
$ab = 15.5$	$bc = 16.2$	$ace = 23.4$	$abcde = 18.1$

(a) Verify that the design generators used were $I = ACE$ and $I = BDE$.

From *Design Expert*, Design, Evaluation:

Factorial Effects Defining Contrast
$I = ACE = BDE = ABCD$

(b) Write down a complete defining relation and the aliases from this design.

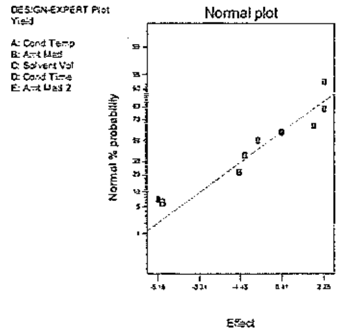
Factorial Effects Aliases
[Est. Terms] Aliased Terms
[Intercept] = Intercept + ACE + BDE
[A] = A + CE + BCD
[B] = B + DE + ACD
[C] = C + AE + ABD
[D] = D + BE + ABC
[E] = E + AC + BD
[BC] = BC + AD + ABE + CDE

(c) Estimate the main effects.

	Term	Effect	SumSqr	% Contribtn
Model	*Intercept*			
Error	*A*	-1.525	4.65125	5.1831
Model	*B*	-5.175	53.5613	59.6858
Error	*C*	2.275	10.3513	11.5349
Error	*D*	-0.675	0.91125	1.01545
Error	*E*	2.275	10.3513	11.5349
Error	*AB*	1.825	6.66125	7.42294
Error	*AD*	-1.275	3.25125	3.62302

(d) Prepare an analysis of variance table. Verify that the AB and AD interactions are available to use as error.

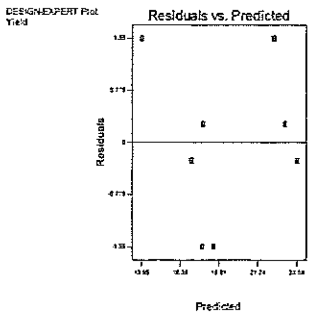
From the normal probability plot of effects below, the main effect B is the only significant effect; hence AB and AD can be used as error.

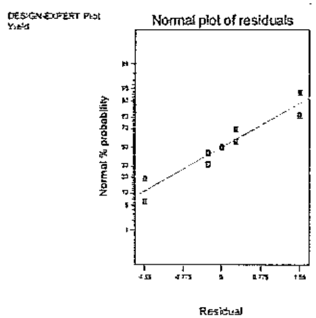


Response: Yield
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	*79.83*	*5*	*15.97*	*3.22*	*0.2537*	*not significant*
A	*4.65*	*1*	*4.65*	*0.94*	*0.4349*	
B	*53.56*	*1*	*53.56*	*10.81*	*0.0814*	
C	*10.35*	*1*	*10.35*	*2.09*	*0.2853*	
D	*0.91*	*1*	*0.91*	*0.18*	*0.7098*	
E	*10.35*	*1*	*10.35*	*2.09*	*0.2853*	
Residual	*9.91*	*2*	*4.96*			
Cor Total	*89.74*	*7*				

(e) Plot the residuals versus the fitted values. Also construct a normal probability plot of the residuals. Comment on the results.





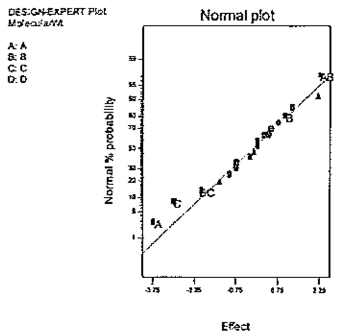
The residuals appear to be satisfactory.

12-13. Reconsider the data in Exercise 12-12. Suppose that four center points were added to this experiment. The molecular weights at the center point are 90, 87, 86, and 93.

Problem 12-12 A 2^4 factorial design has been run in a pilot plant to investigate the effect of four factors on the molecular weight of a polymer. The data from this experiment are as follows (values are coded by dividing by 10).

(1) = 88	d = 88
a = 80	ad = 81
b = 89	bd = 85
ab = 87	abd = 86
c = 86	cd = 85
ac = 81	acd = 79
bc = 82	bcd = 84
abc = 80	$abcd$ = 81

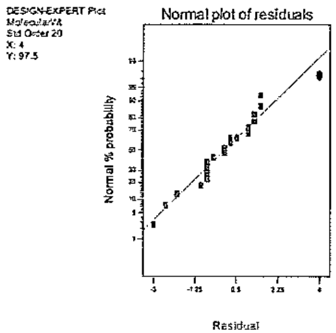
(a) Analyze the data as you did in Exercise 12-12, but include a test for curvature.

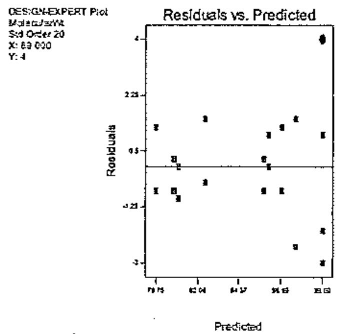


Effects A, C, AB, and BC are significant. Effect B is included due to hierarchy.

Response: MolecularWt
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	132.50	5	26.50	7.10	0.0021	significant
A	56.25	1	56.25	15.08	0.0019	
B	4.00	1	4.00	1.07	0.3193	
C	36.00	1	36.00	9.65	0.0083	
AB	20.25	1	20.25	5.43	0.0366	
BC	16.00	1	16.00	4.29	0.0588	
Curvature	88.20	1	88.20	23.64	0.0003	significant
Residual	48.50	13	3.73			
Lack of Fit	18.50	10	1.85	0.19	0.9820	not significant
Pure Error	30.00	3	10.00			
Cor Total	269.20	19				





Run 8, standard order 20, appears to be an outlier. This is one of the four center points, and could be the cause for the significant curvature effect. It may be of interest to re-analyze the experiment with this run removed.

(b) If curvature is significant in an experiment such as this one, describe what strategy you would pursue next to improve your model of the process.

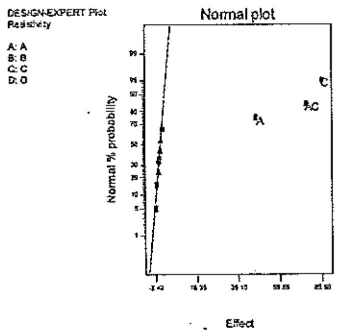
Augment the experiment with additional blocked runs resulting in a central composite experiment as described in Chapter 13.

12-15. A 2^{4-1} design has been used to investigate the effect of four factors on the resistivity of a silicon wafer. The data from this experiment are shown here.

Run	*A*	*B*	*C*	*D*	Resistivity
1	-	-	-	-	33.2
2	+	-	-	0	4.6
3	-	+	-	0	31.2
4	+	+	-	-	9.6
5	-	-	+	0	40.6
6	+	-	+	-	162.4
7	-	+	+	-	39.4
8	+	+	+	0	158.6
9	0	0	0	0	63.4
10	0	0	0	0	62.6
11	0	0	0	0	58.7
12	0	0	0	0	60.9

(a) Estimate the factor effects. Plot the effect estimates on a normal probability scale.

	Term	Effect	SumSqr	% Contribtn
Model	*Intercept*			
Model	*A*	47.7	4550.58	16.146
Error	*B*	-0.5	0.5	0.00177406
Model	*C*	80.6	12992.7	46.0996
Error	*D*	-2.4	11.52	0.0408742
Error	*AB*	1.1	2.42	0.00858643
Model	*AC*	72.8	10599.7	37.6088
Error	*AD*	-2	8	0.0283849
Error	*Curvature*	1.67432	5.60667	0.0198931



(b) Identify a tentative model for this process. Fit the model and test for curvature.

From the normal probability plot of effects, the tentative model includes the intercept, factors *A*, *C*, and the *AC* interaction.

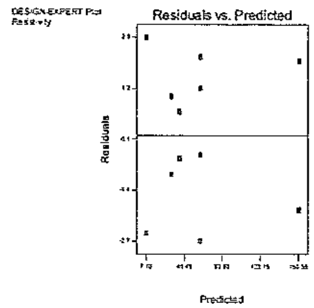
Response: Resistivity
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

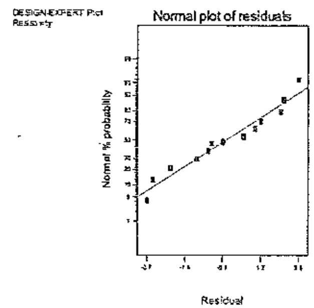
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	28142.98	3	9380.99	1853.95	< 0.0001	*significant*
A	4550.58	1	4550.58	899.32	< 0.0001	
C	12992.72	1	12992.72	2567.73	< 0.0001	
AC	10599.68	1	10599.68	2094.80	< 0.0001	
Curvature	5.61	1	5.61	1.11	0.3275	*not significant*
Residual	35.42	7	5.06			
Lack of Fit	22.44	4	5.61	1.30	0.4327	*not significant*
Pure Error	12.98	3	4.33			
Cor Total	28184.01	11				

The model is as follows:

Resistivity =
+59.95
+23.85 * A
+40.30 * C
+36.40 * A * C

(c) Plot the residuals from the model in part (b) versus the predicted resistivity. Is there any indication on this plot of model inadequacy?





There is no indication of model inadequacy due to the consistency of variance.

(d) Construct a normal probability plot of the residuals. Is there any reason to doubt the validity of the normality assumption?

Although the normal probability plot suggests that the tails of the distribution may be slightly "thicker", it is not enough to doubt the validity of the normality assumption.

CHAPTER 13

Process Optimization with Designed Experiments

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Use the information of the factorial design to find a path of steepest ascent for process optimization
2. Design and analyze response surface designs for estimating second order models
3. Apply evolutionary operation, EVOP, to optimize process output characteristics through a series of small, systematic changes

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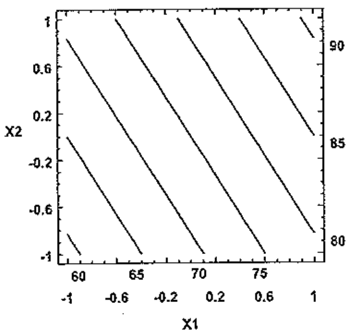
Exercises

13-1. Consider the first-order model

$\hat{y} = 75 + 10x_1 + 6x_2$

(a) Sketch the contours of constant predicted response over the range $-1 \le x_i \le +1, i = 1, 2$.

Contour Plot of First-Order Model



(b) Find the direction of steepest ascent.

Given the equation, find the ratio of coefficients for the step size.

$\hat{y} = 75 + 10x_1 + 6x_2; -1 \le x_1 \le 1; -1 \le x_2 \le 1$

$\frac{x_2}{x_1} = \frac{6}{10} = 0.6$

$\Delta x_1 = 1; \Delta x_2 = 0.6$

13-3. An experiment was run to study the effect of two factors, time and temperature, on the inorganic impurity levels in paper pulp. The results of this experiment are shown here:

x_1	x_2	y
-1	-1	210
1	-1	95
-1	1	218
1	1	100
-1.5	0	225
1.5	0	50
0	-1.5	175
0	1.5	180
0	0	145
0	0	175
0	0	158
0	0	166

(a) What type of experimental design has been used in this study? Is the design rotatable?

This design is a CCD with $k = 2$ and $\alpha = 1.5$. The design is not rotatable because

$1.5 \ne \sqrt[4]{n_F}; \sqrt[4]{n_F} = \sqrt[4]{4} = 1.414$

(b) Fit a quadratic model to the response, using the method of least squares.

Design-Expert® 6.0.0, Analysis (partial output)

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*30583.44*	*5*	*6116.69*	*73.18*	*< 0.0001*
Residual	*501.48*	*6*	*83.58*		
Lack of Fit	*15.48*	*3*	*5.16*	*0.032*	*0.9909*
Pure Error	*486.00*	*3*	*162.00*		
Cor Total	*31084.92*	*11*			

Factor	Coefficient Estimate	DF	Std Error	95% CI Low	95% CI High	VIF
Intercept	*160.87*	*1*	*4.56*	*149.72*	*172.01*	
A-x1	*-58.29*	*1*	*3.14*	*-65.97*	*-50.62*	*1.00*
B-x2	*2.41*	*1*	*3.14*	*-5.26*	*10.08*	*1.00*
A2	*-10.85*	*1*	*3.32*	*-18.97*	*-2.74*	*1.07*
B2	*6.92*	*1*	*3.32*	*-1.19*	*15.04*	*1.07*
AB	*-0.75*	*1*	*4.57*	*-11.94*	*10.44*	*1.00*

Final Equation in Terms of Actual Factors:

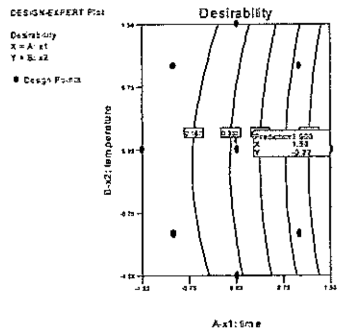
y =
+160.86815
-58.29412 * x1
+2.41176 * x2
-10.85464 * x12
+6.92314 * x22
-0.75000 * x1 * x2

(c) Construct the fitted impurity response surface. What values of x_1 and x_2 would you recommend if you want to minimize the impurity level?

Design-Expert® 6.0.0, Numerical Optimization (partial output)

Name	Goal	Lower Limit	Upper Limit	Lower Wt	Upper Wt	Importance
x1	is in range	-1.5	1.5	1	1	3
x2	is in range	-1.5	1.5	1	1	3
y	minimize	50	225	1	1	3

Number	x1	x2	y	Desirability	
1	1.50	-0.22	49.263	1.000	Selected



At (1.50,-0.22), $y = 49.263$

(d) Suppose that $x_1 = \frac{temp - 750}{50}$ $x_2 = \frac{time - 3}{15}$ where temperature is in °C and time is in hours. Find the optimum operating conditions in terms of the natural variables temperature and time.

Plug in (1.50, -0.22) into the coding equations for the x's to obtain the natural variables:
$Temp = 50x_1 + 750 = 50(+1.50) + 750 = 825$, $Time = 15x_2 + 30 = 15(-0.22) + 30 = 26.7$

13-5. An article in Rubber *Chemistry and Technology* (Vol. 47, 1974, pp. 825-836) describes an experiment that studies the relationship of the Mooney viscosity of rubber to several variables, including silica filler (parts per hundred) and oil filler (parts per hundred). Some of the data form this experiment are shown here, where

$x_1 = \frac{silica - 60}{15}$ $x_2 = \frac{oil - 21}{1.5}$

Coded	Levels	
x_1	x_2	y
-1	-1	13.71
1	-1	14.15
-1	1	12.87
1	1	13.53
-1.4	0	12.99
1.4	0	13.89
0	-1.4	14.16
0	1.4	12.90
0	0	13.75
0	0	13.66
0	0	13.86
0	0	13.63
0	0	13.74

(a) What type of experimental design has been used? Is it rotatable?

The design is a CCD with $k = 2$ and $\alpha = 1.4$. The design is rotatable, $\alpha = \sqrt[4]{n_F} = \sqrt[4]{4} = 1.4$.

(b) Fit a quadratic model to these data. What values of x_1 and x_2 will maximize the Mooney viscosity?

Design-Expert® 6.0.0, Analysis (partial output)

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*2.16*	*5*	*0.43*	*46.56*	*< 0.0001*
Residual	*0.065*	*7*	*9.285E-003*		
Lack of Fit	*0.033*	*3*	*0.011*	*1.35*	*0.3769*
Pure Error	*0.032*	*4*	*8.070E-003*		
Cor Total	*2.23*	*12*			

Factor	Coefficient Estimate	DF	Std Error	95% CI Low	95% CI High	VIF
Intercept	*13.73*	*1*	*0.043*	*13.63*	*13.83*	
A-x1	*0.30*	*1*	*0.034*	*0.22*	*0.38*	*1.00*
B-x2	*-0.41*	*1*	*0.034*	*-0.49*	*-0.33*	*1.00*
A2	*-0.12*	*1*	*0.037*	*-0.21*	*-0.037*	*1.01*
B2	*-0.079*	*1*	*0.037*	*-0.17*	*8.634E-003*	*1.01*
AB	*0.055*	*1*	*0.048*	*-0.059*	*0.17*	*1.00*

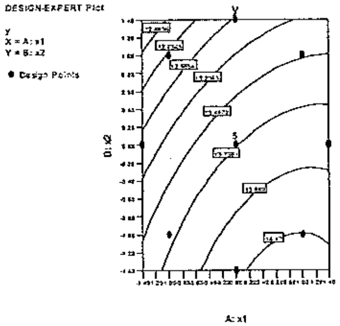
Final Equation in Terms of Actual Factors:

y =
+13.72732
+0.29798 * x1
-0.40707 * x2
-0.12493 * x12
-0.079008 * x22
+0.055000 * x1 * x2

Design-Expert® 6.0.0, Numerical Optimization (partial output)

Name	Goal	Lower Limit	Upper Limit	Lower Wt	Upper Wt	Importance
x1	is in range	-1.4	1.4	1	1	3
x2	is in range	-1.4	1.4	1	1	3
y	maximize	12.87	14.16	1	1	3

Number	x1	x2	y	Desirability	
1	0.95	-1.08	14.1888	1.000	Selected



The maximum can be found at (0.95-1.08) for a viscosity of 14.1888.

13-7. An article by J. J. Pignatiello, Jr. and J. S. Ramberg in the *Journal of Quality Technology* (Vol. 17, 1985, pp. 198-206) describes the use of a replicated fractional factorial to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The data are shown here.

A	*B*	*C*	*D*	*E*			
-	-	-	-	-	7.78	7.78	7.81
+	-	-	+	-	8.15	8.18	7.88
-	+	-	+	-	7.50	7.56	7.50
+	+	-	-	-	7.59	7.56	7.75
-	-	+	+	-	7.54	8.00	7.88
+	-	+	-	-	7.69	8.09	8.06
-	+	+	-	-	7.56	7.52	7.44
+	+	+	+	-	7.56	7.81	7.69
-	-	-	-	+	7.50	7.25	7.12
+	-	-	+	+	7.88	7.88	7.44
-	+	-	+	+	7.50	7.56	7.50
+	+	-	-	+	7.63	7.75	7.56
-	-	+	+	+	7.32	7.44	7.44
+	-	+	-	+	7.56	7.69	7.62
-	+	+	-	+	7.18	7.18	7.25
+	+	+	+	+	7.81	7.50	7.59

(a) Write out the alias structure for this design. What is the resolution of this design?

A+BCD	*AB+CD*	*CE+ABDE*
B+ACD	*AC+BD*	*DE+ABCE*
C+ABD	*AD+BC*	*ABE+CDE*
D+ABC	*AE+BCDE*	*ACE+BDE*
E	*BE+ACDE*	*ADE+BCE*

This is a resolution IV design. All main effects are clear of 2-factor interactions, but some 2-factor interactions are aliased with each other.

(b) Analyze the data. What factors influence mean free height?

Design-Expert® 6.0.0, Analysis (partial output)

Response: free height
ANOVA for Selected Factorial Model
Analysis of Variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	2.08	5	0.42	21.67	< 0.0001
Residual	0.81	42	0.019		
Lack of Fit	0.18	10	0.018	0.91	0.5393
Pure Error	0.63	32	0.020		
Cor Total	2.89	47			

Factor	Coeff Est	DF	Std Error	95% CI Low	95% CI High	VIF
Intercept	7.63	1	0.020	7.59	7.67	
A-furn temp	0.12	1	0.020	0.080	0.16	1.00
B-heat time	-0.081	1	0.020	-0.12	-0.041	1.00
D-hold time	0.046	1	0.020	5.66E-003	0.086	1.00
E-oil temp	-0.12	1	0.020	-0.16	-0.079	1.00
BE	0.077	1	0.020	0.036	0.12	1.00

Final Equation in Terms of Actual Factors:

free height =
+7.62604
+0.12063 * furn temp
-0.081458 * heat time
+0.046042 * hold time
-0.11896 * oil temp
+0.076875 * heat time * oil temp

(c) Calculate the range and standard deviation of free height for each run. Is there any indication that any of these factors affects variability in the free height?

Design-Expert® 6.0.0, Analysis (partial output)

Response: Range
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	0.26	6	0.044	8.22	0.0030
A	0.050	1	0.050	9.22	0.0141
B	0.061	1	0.061	11.41	0.0081
C	2.26E-003	1	2.56E-003	0.42	0.5329
E	5.06E-004	1	5.06E-004	0.094	0.7657

CE	*0.077*	*1*	*0.077*	*14.35*	*0.0043*
ADE(=BCE)	*0.074*	*1*	*0.074*	*13.83*	*0.0048*
Residual	*0.048*	*9*	*5.37E-003*		
Cor Total	*0.31*	*15*			

Factor	Coefficient Estimate	DF	Std Error	95% CI Low	95% CI High	VIF
Intercept	*0.22*	*1*	*0.018*	*0.18*	*0.26*	
A-furn temp	*0.056*	*1*	*0.018*	*0.014*	*0.097*	*1.00*
B-heat time	*-0.062*	*1*	*0.018*	*-0.10*	*-0.020*	*1.00*
C-trans time	*0.012*	*1*	*0.018*	*-0.030*	*0.053*	*1.00*
E-oil temp	*-5.62E-003*	*1*	*0.018*	*-0.047*	*0.036*	*1.00*
CE	*-0.069*	*1*	*0.018*	*-0.11*	*-0.028*	*1.00*
ADE(=BCE)	*0.068*	*1*	*0.018*	*0.027*	*0.11*	*1.00*

Response: StdDev
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*0.076*	*6*	*0.013*	*8.93*	*0.0023*
A	*0.016*	*1*	*0.016*	*10.96*	*0.0091*
B	*0.020*	*1*	*0.020*	*13.75*	*0.0049*
C	*4.00E-004*	*1*	*4.00E-004*	*0.28*	*0.6091*
E	*1.00E-004*	*1*	*1.00E-004*	*0.070*	*0.7970*
CE	*0.021*	*1*	*0.021*	*14.75*	*0.0040*
ADE(=BCE)	*0.020*	*1*	*0.020*	*13.75*	*0.0049*
Residual	*0.013*	*9*	*1.425E-003*		
Cor Total	*0.089*	*15*			

Factor	Coeff Estim	DF	Std Err	95% CI Low	95% CI High	VIF
Intercept	*0.12*	*1*	*9.437E-003*	*0.097*	*0.14*	
A-furn temp	*0.031*	*1*	*9.437E-003*	*9.901E-003*	*0.053*	*1.00*
B-heat time	*-0.035*	*1*	*9.437E-003*	*-0.056*	*-0.014*	*1.00*
C-trans time	*5.00E-003*	*1*	*9.437E-003*	*-0.016*	*0.026*	*1.00*
E-oil temp	*-2.50E-003*	*1*	*9.437E-003*	*-0.024*	*0.019*	*1.00*
CE	*-0.036*	*1*	*9.437E-003*	*-0.058*	*-0.015*	*1.00*
ADE(=BCE)	*0.035*	*1*	*9.437E-003*	*0.014*	*0.056*	*1.00*

Interactions *CE* (transfer time × quench oil temperature) and *ADE=BCE*, along with factors *B* (heating time) and *A* (furnace temperature) are significant for both models of variability. Factors *C* and *E* are included to keep the models hierarchical.

(d) Analyze the residuals from this experiment and comment on your findings.

Mean height: Plot of residuals versus predicted indicates constant variance assumption is reasonable. Normal probability plot of residuals support the normality assumption. Plots of residuals versus each factor show that variance is less at low level of factor *E*.

Range: Plot of residuals versus predicted shows that variance is approximately constant over range of predicted values. Residuals normal probability plot indicate normality assumption is reasonable Plots of residuals versus each factor indicate that the variance may be different at different levels of factor *D*.

Standard Deviation: Residuals versus predicted plot and residuals normal probability plot support constant variance and normality assumptions. Plots of residuals versus each factor indicate that the variance may be different at different levels of factor *D*.

(e) Is this the best possible design for five factors in 16 runs? Specifically, can you find a fractional design for factors in 16 runs with higher resolution than this one?

This is not the best 16-run design for five factors. A resolution V design can be generated with $E = \pm ABCD$, then none of the 2-factor interactions will be aliased with each other.

13-9. Consider the leaf spring experiment in Exercise 13-7. Rework this problem, assuming that factors, *A*, *B*, and *C* are easy to control but factors *D* and *E* are hard to control.

Factors *D* and *E* are noise variables. Assume that $\sigma_D^2 = \sigma_E^2 = 1$. Using equations (13-6) and (13-7), the mean and variance are:

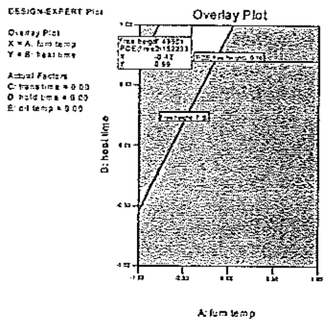
$$\text{Mean Free Height} = 7.63 + 0.12A - 0.081B$$

$$\text{Variance of Free Height} = \sigma^2_D\,(+0.046)^2 + \sigma^2_E\,(-0.12 + 0.077B)^2 + \sigma^2$$

Using $\hat{\sigma}^2 = MS_E = 0.02$:

$$\text{Variance of Free Height} = (0.046)^2 + (-0.12 + 0.077B)^2 + 0.02$$

The plot shows a solution with mean Free Height ≅ 7.50 and minimum standard deviation of Free Height to be ($A = -0.42$, $B = 0.99$).



13-11. The following data were collected by a chemical engineer. The response y is filtration time, x_1 is temperature, and x_2 is pressure. Fit a second-order model.

x_1	x_2	y
-1	-1	54
1	-1	45
-1	1	32
1	1	47
-1.414	0	50
1.414	0	53
0	-1.414	47
0	1.414	51
0	0	41
0	0	39
0	0	44
0	0	42
0	0	40

Design-Expert® 6.0.0, Analysis (partial output)

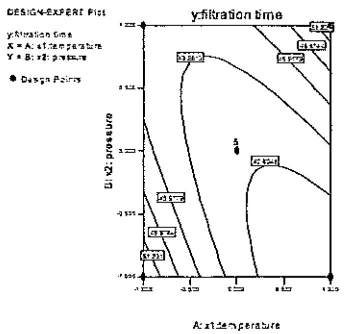
Response: y:filtration time
ANOVA for Response Surface Quadratic Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	*315.60*	*5*	*63.12*	*2.86*	*0.1015*
Residual	*154.40*	*7*	*22.06*		
Lack of Fit	139.60	3	46.53	12.58	0.0167
Pure Error	14.80	4	3.70		
Cor Total	*470.00*	*12*			

Factor	Coefficient Estimate	DF	Std Error	95% CI Low	95% CI High	VIF
Intercept	*41.20*	*1*	*2.10*	*36.23*	*46.17*	
A-x1	*-1.97*	*1*	*1.66*	*-5.90*	*1.96*	*1.00*
B-x2	*1.46*	*1*	*1.66*	*-2.47*	*5.38*	*1.00*
A^2	*3.71*	*1*	*1.78*	*-0.50*	*7.92*	*1.02*
B^2	*2.46*	*1*	*1.78*	*-1.75*	*6.67*	*1.02*
AB	*6.00*	*1*	*2.35*	*0.45*	*11.55*	*1.00*

Final Equation in Terms of Actual Factors:

$$
\begin{aligned}
y = &+41.20000 \\
&-1.96967 * x1 \\
&+1.45711 * x2 \\
&+3.71250 * x1^2 \\
&+2.46250 * x2^2 \\
&+6.00000 * x1 * x2
\end{aligned}
$$



(a) What operating conditions would you recommend if the objective is to minimize the filtration time?

Design-Expert® 6.0.0, Numerical Optimization (partial output)

Name	Goal	Lower Limit	Upper Limit	Lower Wt	Upper Wt	Importance
x1	is in range	-1	1	1	1	3
x2	is in range	-1	1	1	1	3
y	minimize	32	54	1	1	3

Solutions

Number	x1	x2	y	Desirability	
1	.000	-1.000	37.9482	0.730	Selected

(b) What operating conditions would you recommend if the objective is to operate the process at a mean filtration rate very close to 46?

Design-Expert® 6.0.0, Numerical Optimization (partial output)

Name	Goal	Lower Limit	Upper Limit	Lower Wt	Upper Wt	Importance
x1	is in range	-1	1	1	1	3
x2	is in range	-1	1	1	1	3
y	is target = 46	32	54	1	1	3

Solutions Number	x1	x2	y	Desirability	
1	0.825	0.509	45.9999	1.000	Selected
2	-0.734	-0.355	46.0013	1.000	
3	-0.870	-0.070	46.0009	1.000	
4	-0.801	-0.217	46.0006	1.000	
5	-0.576	-0.668	45.9999	1.000	
6	-0.614	-0.595	45.9995	1.000	
7	0.352	0.878	45.9985	1.000	
8	0.302	0.916	46.0012	1.000	
9	-0.508	-0.798	45.998	1.000	
10	-0.564	-0.691	45.9997	1.000	

13-13. Consider the response model in equation 13-5 and the transmission of error approach to finding the variance model (equation 13-7). Suppose that in the response model we use

$$h(\mathbf{x},\mathbf{z})=\sum_{i=1}^{r}\gamma_i z_i+\sum_{i=1}^{k}\sum_{j=1}^{r}\delta_{ij}x_i z_j+\sum_{i<j=2}\sum^{r}\lambda_{ij}z_i z_j$$

What effect does including the interaction terms between the noise variables have on the variance model?

If $h(\mathbf{x},\mathbf{z})=\sum_{i=1}^{r}\gamma_i z_i+\sum_{i=1}^{k}\sum_{j=1}^{r}\delta_{ij}x_i z_j$, then $\frac{\partial h(\mathbf{x},\mathbf{z})}{\partial z_i}=\gamma_i+\sum_{u=1}^{k}\delta_{ui}x_u$, and

$$V\left[y(\mathbf{x},\mathbf{z})\right]=\sigma_z^2\sum_{i=1}^{r}\left(\gamma_i+\sum_{u=1}^{k}\delta_{ui}x_u\right)^2+\sigma^2$$

If $h(\mathbf{x},\mathbf{z})=\sum_{i=1}^{r}\gamma_i z_i+\sum_{i=1}^{k}\sum_{j=1}^{r}\delta_{ij}x_i z_j+\sum_{i<j=2}\sum^{r}\lambda_{ij}z_i z_j$,

then $\sum_{i=1}^{r}\frac{\partial h(\mathbf{x},\mathbf{z})}{\partial z_i}=\sum_{i=1}^{r}\gamma_i+\sum_{i=1}^{r}\sum_{u=1}^{k}\delta_{ui}x_u+\sum_{i<j=2}\sum^{r}\lambda_{ij}(z_i+z_j)$, and

$$V\left[y(\mathbf{x},\mathbf{z})\right]=V\sum_{i=1}^{r}\left[\gamma_i+\sum_{u=1}^{k}\delta_{ui}x_u+\sum_{j>i}^{r}\lambda_{ij}(z_i+z_j)\right]z_i+\sigma^2$$

There will be additional terms in the variance expression arising from the third term inside the square brackets.

CHAPTER 14

Lot-by-Lot Acceptance Sampling for Attributes

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Discuss three key concepts to application of acceptance sampling: used to sentence lots (not estimate lot quality), used to accept/reject lots (not control quality), and used to audit process output (not inspect quality into the product).
2. Describe typical situations in which acceptance sampling is used, and explain the advantages and disadvantages of acceptance sampling.
3. Understand the considerations involved in forming lots for inspection and the importance of random sampling.
4. For single-sampling plans for attributes with given lot size N, sample size n, and acceptance number c, calculate P_a and construct operating-characteristic curves.

Note: Many of the exercises in this chapter are easily solved with spreadsheet application software, such as the BINOMDIST and HYPGEOMDIST functions in Excel.

Exercises

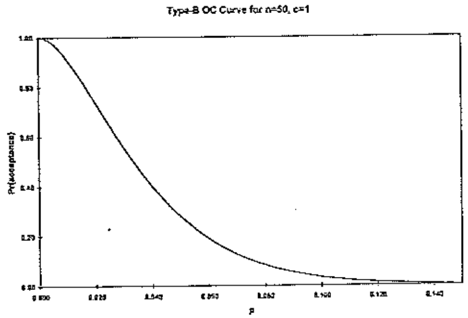
14-1. Draw the type-B OC curve for the single-sampling plan $n = 50$, $c = 1$.

This is a single-sampling plan for attributes. To construct operating characteristic (OC) curve we plot the probability of accepting a lot, P_a, versus various values of the lot fraction defective, p.

The probability of acceptance is the probability that the number of defectives, d, in a sample of size n is less than or equal to acceptance number c. The lot size, N, is relatively large, so the distribution of defectives in a sample is a binomial distribution with parameters n and p. We need to find the probability of observing exactly $d = 0$ defective and the probability of observing exactly $d = 1$ defective. The formulas are given on text p. 683 (eqn. 14-1 and 14-2); the spreadsheet solution is below:

	A	B	C	D
1	n =	50		
2	c =	1		
3	d =	0	1	
4	p	f(d=0)	f(d=1)	Pr{d<=c}
5	0.001	=FACT($B$1)/ (FACT(B$3)*FACT($B$1-B$3))* ($A5^B$3)*((1-$A5)^($B$1-B$3))	=FACT($B$1)/ (FACT(C$3)*FACT($B$1-C$3))* ($A5^C$3)*((1-$A5)^($B$1-C$3))	=+B5+C5

p	*f(d=0)*	*f(d=1)*	*Pr{d<=c}*
0.001	0.95121	0.04761	0.99881
0.002	0.90475	0.09066	0.99540
0.003	0.86051	0.12947	0.98998
0.004	0.81840	0.16434	0.98274
0.005	0.77831	0.19556	0.97387
0.006	0.74015	0.22339	0.96353
0.007	0.70382	0.24807	0.95190
0.008	0.66924	0.26986	0.93910
0.009	0.63633	0.28895	0.92528
0.010	0.60501	0.30556	0.91056
0.020	0.36417	0.37160	0.73577
...			
0.100	0.00515	0.02863	0.03379
...			



The BINOMDIST function could also be used with various values of p and d.

14-3. Suppose that a product is shipped in lots of size $N = 5000$. The receiving inspection procedure used is single sampling with $n = 50$ and $c = 1$.

(a) Draw the type-A OC curve for the plan.

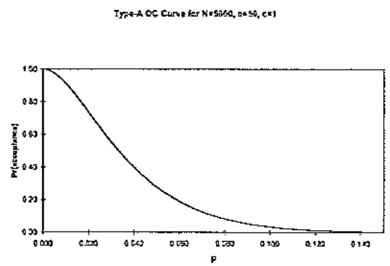
A type-A OC curve is used for an isolated lot, of finite size N. The appropriate sampling distribution is the hypergeometric, with lot size N, sample size n, and acceptance number c. The sample items are selected at random without replacement. The hypergeometric probability distribution is given on text p. 57 (eqn. 2-8) as:

$$p(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

where $p(x)$ is the probability of selecting x defectives from D, the number of defective items in a lot of N items. To plot P_a versus p, find the number of defectives, $D = N * p$. The spreadsheet solution using the Excel HYPGEOMDIST function is below.

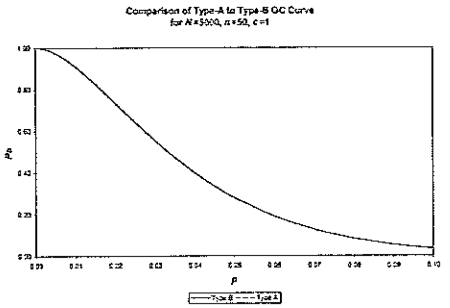
	A	B	C	D	E	F
1	N =	5000				
2	n =	50				
3	c =	1				
4	d =	0	1			
5	p	N*p	D	f(d=0)	f(d=1)	Pr{d<=c}
6	0.001	=$B$1*A6	=INT(B6)	=HYPGEOMDIST ($B$4,$B$2,$C6,$B$1)	=HYPGEOMDIST ($C$4,$B$2,$C6,$B$1)	=D6+E6

p	*N*p*	*D*	*f(d=0)*	*f(d=1)*	*Pr{d<=c}*
0.001	5	5	0.95097	0.04807	0.99904
0.002	10	10	0.90430	0.09151	0.99581
0.003	15	15	0.85988	0.13065	0.99053
0.004	20	20	0.81759	0.16581	0.98340
0.005	25	25	0.77735	0.19726	0.97461
0.006	30	30	0.73905	0.22527	0.96432
0.007	35	35	0.70260	0.25011	0.95271
0.008	37.5	37	0.68852	0.25921	0.94773
0.008	40	40	0.66791	0.27201	0.93992
0.009	45	45	0.63491	0.29118	0.92609
0.010	50	50	0.60350	0.30785	0.91135
0.020	100	100	0.36234	0.37347	0.73581
0.030	150	150	0.21641	0.33807	0.55448
0.040	200	200	0.12856	0.27059	0.39915
0.050	250	250	0.07595	0.20196	0.27791
0.060	300	300	0.04462	0.14391	0.18854
0.070	350	350	0.02607	0.09914	0.12521



(b) Draw the type-B OC curve for this plan and compare it to the type-A OC curve found in part (a).

The type-B OC curve was constructed in exercise 14-1. Both curves are shown in the graph below:



(c) Which curve is appropriate for this situation?

The difference between the two curves is small; either is appropriate.

14-5. Find a single-sampling plan for which $p_1 = 0.05$, $\alpha = 0.05$, $p_2 = 0.15$, and $\beta = 0.10$.

A lot is accepted if c or fewer defective items are found in the sample of size n.

To specify a sampling plan, it does not matter which two points on the operating characteristic (OC) curve are specified. If $p_1 = 0.05$ is considered an "acceptable" lot and $p_2 = 0.15$ is considered an "unacceptable" lot, we can use the producer's risk α and the consumer's risk β to determine a sampling plan. The probability of accepting lots with $p_1 = 0.05$ fraction defective is desired to be $1 - \alpha = 1 - 0.05 = 0.95$. The probability of accepting lots with $p_2 = 0.15$ fraction defective is desired to be $\beta = 0.10$.

Use the binomial nomograph, Figure 14-9 on text p. 690. Draw one line connecting $p_1 = 0.05$ on the left with 0.95 on the right. Draw the second line connecting $p_2 = 0.15$ on the left with 0.10 on the right. The sampling plan is $n = 80$ and $c = 7$.

14-7. A company uses the following acceptance-sampling procedure. A sample equal to 10% of the lot is taken. If 2% or less of the items in the sample are defective, the lot is accepted; otherwise it is rejected. If submitted lots vary in size from 5000 to 10,000 units, what can you say about the protection by this plan? If 0.05 is the desired LTPD, does this scheme offer reasonable protection to the consumer?

The poorest quality level or percent defective that a consumer is will tolerate (accept) in a single lot is the LTPD.

The minimum lot size is $N_1 = 5000$ units, the maximum is $N_2 = 10,000$ units. Sample size is proportionate to the lot size, $n = 0.10 * N$. The consumer desires an LTPD = 2% = 0.05 fraction defective.

The LTPD along with the lot size N establishes the maximum number of defective units, c, that can exist. For various values of lot fraction defective p, we can evaluate the single-sampling plan and the probability of rejecting

the lot. The formulas on text p. 683 (eqn. 14-1 and 14-2) apply. The spreadsheet solution using the Excel BINOMDIST function is below:

	A	B	C	D	E	F
1	LTPD=	0.05				
2						
3		N1 =	5000		N2 =	10000
4		n1 =	=0.1*C3		n1 =	=0.1*F3
5		pmax =	0.02		pmax =	0.02
6		cmax =	=C5*C4		cmax =	=F5*F4
7		binomial			binomial	
8	p	Pr{d<=10}	Pr{reject}		Pr{d<=20}	Pr{reject}
9	0.001	=BINOMDIST($C$6,$C$4,A9,TRUE)	=1-B9		=BINOMDIST($F$6,$F$4,A9,TRUE)	=1-E9

p	*Pr{d<=10}*	*Pr{reject}*	*Pr{d<=20}*	*Pr{reject}*	*difference*
0.0010	1.00000	0.0000	1.00000	0.0000	0.00000
0.0020	1.00000	0.0000	1.00000	0.0000	0.00000
0.0030	1.00000	0.0000	1.00000	0.0000	0.00000
0.0040	0.99999	0.0000	1.00000	0.0000	-0.00001
0.0050	0.99994	0.0001	1.00000	0.0000	-0.00006
0.0060	0.99972	0.0003	1.00000	0.0000	-0.00027
0.0070	0.99903	0.0010	0.99999	0.0000	-0.00095
0.0080	0.99729	0.0027	0.99991	0.0001	-0.00263
0.0090	0.99359	0.0064	0.99959	0.0004	-0.00600
0.0100	0.98676	0.0132	0.99850	0.0015	-0.01175
0.0200	0.58304	0.4170	0.55910	0.4409	0.02395
0.0250	0.29404	0.7060	0.18221	0.8178	0.11183
0.0300	0.11479	0.8852	0.03328	0.9667	0.08151
0.0400	0.00967	0.9903	0.00030	0.9997	0.00938
0.0500	0.00046	0.9995	0.00000	1.0000	0.00046
0.0600	0.00001	1.0000	0.00000	1.0000	0.00001
0.0700	0.00000	1.0000	0.00000	1.0000	0.00000

Different sample sizes offer different levels of protection. For $N = 5,000$, $P_a(p = 0.025) = 0.294$; while for $N = 10,000$, $P_a(p = 0.025) = 0.182$. Also, the consumer is protected from a LTPD = 0.05 by $P_a(N = 5,000) = 0.00046$ and $P_a(N = 10,000) = 0.00000$, but pays for the high probability of rejecting acceptable lots like those with $p = 0.025$.

14-9. Consider the single-sampling plan found in Exercise 14-4. Suppose that lots of $N = 2000$ are submitted. Draw the ATI curve for this plan. Draw the AOQ curve and find the AOQL.

From Exercise 14-4: $p_1 = 0.01$, $1 - \alpha = 1 - 0.05 = 0.95$, $p_2 = 0.10$, $\beta = 0.10$. From the binomial nomograph, select $n = 35$ and $c = 1$, resulting in actual $\alpha = 0.04786$ and $\beta = 0.12238$. Single-sampling plan for $N = 2,000$ is $n = 35$ and $c = 1$. The formulas on text p. 683 (eqn. 14-1 and 14-2) apply.

$$\text{AOQ} = \frac{P_a p(N-n)}{N} = \frac{P_a p(2000-35)}{2000} = (1965/2000)P_a p$$

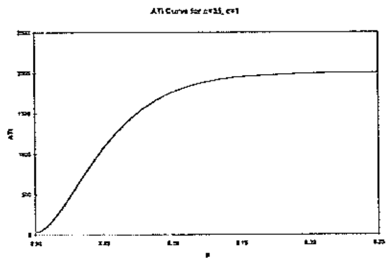
$$\text{AOQL} = 0.0234$$

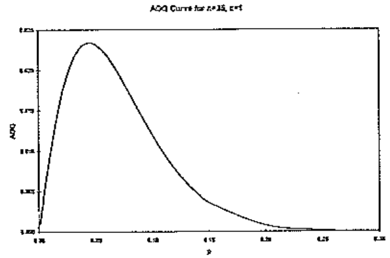
$$\text{ATI} = n + (1 - P_a)(N - n) = 35 + (1 - P_a)(2000 - 35) = 2000 - 1965P_a$$

The spreadsheet solution using the Excel BINOMDIST function is below:

	A	B	C	D	E	F
	N =	2000				
1	n =	35				
2	c =	1				
3	p	Pa=Pr{d<=1}		ATI		AOQ
4	0.001	=BINOMDIST ($B$3,$B$2,A4,TRUE)		=$B$1-($B$1-$B$2)*B4		=B4*A4*($B$1-$B$2)/$B$1

p	*Pa=Pr{d<=1}*	*ATI*	*AOQ*
0.0010	0.99942	36	0.0010
0.0020	0.99772	39	0.0020
0.0030	0.99499	45	0.0029
0.0040	0.99128	52	0.0039
0.0050	0.98667	61	0.0048
0.0060	0.98121	72	0.0058
0.0070	0.97498	84	0.0067
0.0080	0.96802	98	0.0076
0.0090	0.96039	113	0.0085
0.0100	0.95214	129	0.0094
...			
0.0400	0.58903	843	0.0231
0.0450	0.52873	961	0.0234 AOQL
0.0500	0.47203	1072	0.0232
...			
0.3000	0.00006	2000	0.0000





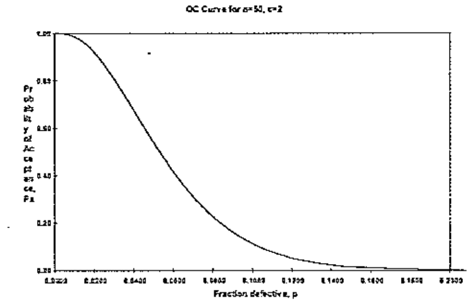
14-11. Suppose that a vendor ships components in lots of size 5000. A single-sampling plan with $n = 50$ and $c = 2$ is being used for receiving inspection. Rejected lots are screened, and all defective items are reworked and returned to the lot.

(a) Draw the OC curve for this plan

Draw a type-B OC curve; the formulas on text p. 683 (eqn. 14-1 and 14-2) apply. The spreadsheet solution using the Excel BINOMDIST function is below:

	A	B	C
1	N = 5000	n =	50
2		c =	2
3		binomial	
4	p	Pa=Pr(d<=1)	Pr(reject)
5	0.001	=BINOMDIST($C$2,$C$1,A5,TRUE)	=1-B5

p	*Pa=Pr{d<=1}*	*Pr{reject}*
0.0010	0.99998	0.00002
0.0020	0.99985	0.00015
0.0030	0.99952	0.00048
0.0040	0.99891	0.00109
0.0050	0.99794	0.00206
0.0060	0.99657	0.00343
0.0070	0.99474	0.00526
0.0080	0.99242	0.00758
0.0090	0.98957	0.01043
0.0100	0.98618	0.01382
0.1020	0.10368	0.89632
0.1030	0.09985	0.90015
0.1040	0.09614	0.90386
0.2500	0.00009	0.99991
0.3000	0.00000	1.00000



(b) Find the level of lot quality that will be rejected 90% of the time.

From reading the spreadsheet table, $p = 0.1030$ will be rejected about 90% of the time. From visual examination of the OC curve, at $P_a = 0.10$, $p \approx 0.10$.

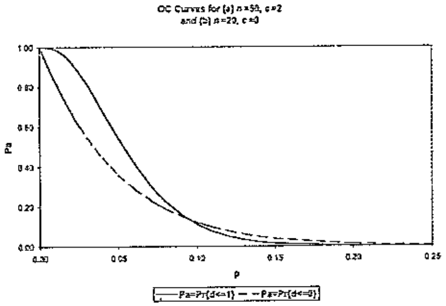
(c) Management has objected to the use of the above sampling procedure and wants to use a plan with an acceptance number $c = 0$, arguing that this is more consistent with their zero-defects program. What do you think of this?

A zero-defects sampling plan, with acceptance number $c = 0$, will be extremely hard on the vendor because the P_a is low even if the lot fraction defective is extremely low. Generally, quality improvement begins with the manufacturing process control, not the sampling plan.

(d) Design a single-sampling plan with $c = 0$ that will give a 0.90 probability of rejection of lots having the quality level found in part (b). Note that the two plans are now matched at the LTPD point. Draw the OC curve for this plan and compare it to the one for $n = 50$, $c = 2$ in part (a).

From the nomograph in Figure 14-9 (text p. 690), select $n = 20$, yielding $P_a = 1 - 0.11372 = 0.88638 \approx 0.90$. The OC curve for this zero-defects plan is much steeper.

p	Pa=Pr{d<=0}	Pr{reject}
0.0010	0.98019	0.01981
0.0020	0.96075	0.03925
0.0030	0.94168	0.05832
0.0040	0.92297	0.07703
0.0050	0.90461	0.09539
0.0060	0.88660	0.11340
0.0070	0.86893	0.13107
0.0080	0.85160	0.14840
0.0090	0.83459	0.16541
0.0100	0.81791	0.18209
0.0200	0.66761	0.33239



(e) Suppose that incoming lots are 0.5% nonconforming. What is the probability of rejecting these lots under both plans? Calculate the ATI at this point for both plans. Which plan do you prefer? Why?

$\Pr\{\text{reject} \mid p = 0.005, c = 0\} = 0.09539$

$\Pr\{\text{reject} \mid p = 0.005, c = 2\} = 0.00206$

$\text{ATI}_{c=0} = n + (1 - P_a)(N - n) = 20 + (0.09539)(5000 - 20) = 495$

$\text{ATI}_{c=2} = 50 + (0.00206)(5000 - 50) = 60$

The $c = 2$ plan is preferred because the $c = 0$ plan will reject good lots 10% of the time. In addition, the average total inspection required by the $c = 2$ sampling plan is much less at 60 inspections per lot versus 495.

14-13. (a) Derive an item-by-item sequential-sampling plan for which $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.10$, and $\beta = 0.10$.

$p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.10$, and $\beta = 0.10$

$$k = \log\frac{p_2(1-p_1)}{p_1(1-p_2)} = \log\frac{0.10(1-0.01)}{0.01(1-0.10)} = 1.0414$$

$$h_1 = \left(\log\frac{1-\alpha}{\beta}\right)\Big/k = \left(\log\frac{1-0.05}{0.10}\right)\Big/1.0414 = 0.9389 \qquad h_2 = \left(\log\frac{1-\beta}{\alpha}\right)\Big/k = \left(\log\frac{1-0.10}{0.05}\right)\Big/1.0414 = 1.2054$$

$$s = \left(\log\frac{1-p_1}{1-p_2}\right)\Big/k = \left(\log\frac{1-0.01}{1-0.10}\right)\Big/1.0414 = 0.0397$$

$$X_A = -h_1 + sn = -0.9389 + 0.0397n$$

$$X_R = h_2 + sn = 1.2054 + 0.0397n$$

	A	B	C	D	E
1	p1 =	0.01		k =	=LOG((B2*(1-B1))/(B1*(1-B2)))
2	p2 =	0.1		h1 =	=(LOG((1-B3)/B4))/E1
3	alpha =	0.05		h2 =	=(LOG((1-B4)/B3))/E1
4	beta =	0.1		s =	=LOG((1-B1)/(1-B2))/E1
5	n	XA	XR	Acc	Rej
6	1	=-$E$2+$E$4*A6	=$E$3+$E$4*A6	=IF(B6<0,"can't accept",FLOOR(B6,1))	=IF(C6<0,"can't reject",CEILING(C6,1))

n	XA	XR	Acc	Rej
1	-0.899	1.245	can't accept	2
2	-0.859	1.285	can't accept	2
3	-0.820	1.325	can't accept	2
4	-0.780	1.364	can't accept	2
5	-0.740	1.404	can't accept	2
6	-0.700	1.444	can't accept	2
7	-0.661	1.484	can't accept	2
8	-0.621	1.523	can't accept	2
9	-0.581	1.563	can't accept	2
10	-0.541	1.603	can't accept	2
...				
23	-0.025	2.120	can't accept	3
24	0.015	2.159	0	3
...				
45	0.850	2.994	0	3
46	0.890	3.034	0	4
47	0.929	3.074	0	4
48	0.969	3.113	0	4
49	1.009	3.153	1	4
50	1.049	3.193	1	4

The sampling plan is $n = 49$; Acc = 1; Rej = 4.

(b) Draw the OC curve for this plan.

Three points on the OC curve are:

$p_1 = 0.01$; $P_a = 1 - \alpha = 0.95$

$$p = s = 0.0397;\ P_a = \frac{h_2}{h_1 + h_2} = \frac{1.2054}{0.9389 + 1.2054} = 0.5621$$

$p_2 = 0.10$; $P_a = \beta = 0.10$

14-15. Consider rectifying inspection for single sampling. Develop an AOQ equation assuming that all defective items are removed but *not* replaced with good ones.

Incoming lots of size N are of quality p. A sample of size n is drawn; the defective units are not replaced. Incoming lots have an expected number of defective units equal to Np. The expected number of defective units in the sample is $P_a \times np$. After sampling, inspection, and removal of defective units, outgoing lots have an expected number of defective units equal to $P_a \times p \times (N - n)$.

$$\text{AOQ} = \frac{\text{Defective units in uninspected portion of lot}}{N - [\text{Defective units removed}|\text{Acceptance} + \text{Defective units removed}|\text{Rejection}]}$$

$$= \frac{P_a \times p \times (N - n)}{N - [P_a \times (np) + (1 - P_a) \times (Np)]}$$

14-16. A vendor ships a component in lots of size $N = 3000$. The AQL has been established for this product at 1%. Find the normal, tightened, and reduced single-sampling plans for this situation from MIL STD 105E, assuming that general inspection level II is appropriate.

Given: $N = 3000$, AQL = 1%, General Inspection Level II, Single Sampling
From Table 14-4: Sample size code letter = K
From Table 14-5, Normal sampling plan: $n = 125$, Ac = 3, Re = 4
From Table 14-6, Tightened sampling plan: $n = 125$, Ac = 2, Re = 3
From Table 14-7, Reduced sampling plan: $n = 50$, Ac = 1, Re = 4 (meaning the lot would be accepted if two defectives were found, but the next lot would be inspected under normal inspection, text p. 707, 4b)

14-17. Repeat Exercise 14-16, using general inspection level I. Discuss the differences in the various sampling plans.

Given: $N = 3000$, AQL = 1%, General Inspection Level I, Single Sampling
From Table 14-4: Sample size code letter = H
From Table 14-5, Normal sampling plan: $n = 50$, Ac = 1, Re = 2
From Table 14-6, Tightened sampling plan: use sample size code letter = J, $n = 80$, Ac = 1, Re = 2
From Table 14-7, Reduced sampling plan: $n = 20$, Ac = 0, Re = 2 (meaning the lot would be accepted if one defective was found, but the next lot would be inspected under normal inspection, text p. 707, 4b)
The primary difference between general inspection levels I and II is the amount of inspection required. For example, Normal sampling requires 50 items for I versus 125 for II. Level II is the usual plan; Level I is generally selected when less discrimination is needed.

14-19. MIL STD 105E is being used to inspect incoming lots of size $N = 5000$. Single sampling, general inspection level II, and an AQL of 0.65% are being used.

(a) Find the normal, tightened, and reduced inspection plans.

Given: $N = 5000$, AQL = 0.65%, General Inspection Level II, Single Sampling
From Table 14-4: Sample size code letter = L
From Table 14-5, Normal sampling plan: $n = 200$, Ac = 3, Re = 4
From Table 14-6, Tightened sampling plan: $n = 200$, Ac = 2, Re = 3
From Table 14-7, Reduced sampling plan: $n = 80$, Ac = 1, Re = 4 (meaning the lot would be accepted if two defectives were found, but the next lot would be inspected under normal inspection, text p. 707, 4b)

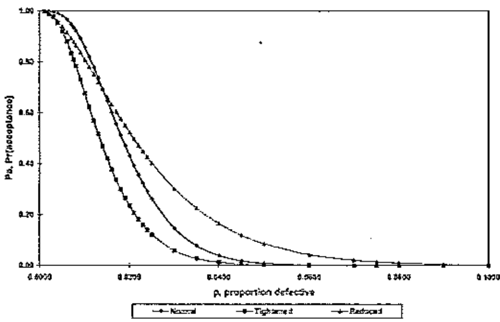
(b) Draw the OC curves of the normal, tightened, and reduced inspection plans on the same graph.

Draw a type-B OC curve; the formulas on text p. 683 (eqn. 14-1 and 14-2) apply. The spreadsheet solution using the Excel BINOMDIST function is below:

	A	B	C	D
1	N = 5000	normal	tightened	reduced
2	n =	200	200	80
3	c =	3	2	1
4	p	Pa=Pr{d<=3}	Pa=Pr{d<=2}	Pa=Pr{d<=1}
5	0.001	=BINOMDIST($B$3,$B$2,A5,TRUE)	=BINOMDIST($C$3,$C$2,A5,TRUE)	=BINOMDIST($D$3,$D$2,A5,TRUE)

p	Pa=Pr{d<=3}	Pa=Pr{d<=2}	Pa=Pr{d<=1}
0.0010	0.9999	0.9989	0.9970
0.0020	0.9992	0.9922	0.9885
0.0030	0.9957	0.9771	0.9755
0.0040	0.9911	0.9529	0.9588
0.0050	0.9813	0.9202	0.9389
0.0050	0.9557	0.8800	0.9153
0.0070	0.9459	0.8340	0.8915
0.0080	0.9220	0.7838	0.8553
0.0090	0.8922	0.7309	0.8377
0.0100	0.8580	0.5757	0.8092
0.0200	0.4315	0.2351	0.5230
0.0400	0.0395	0.0125	0.1554
0.0500	0.0018	0.0004	0.0433
0.0800	0.0001	0.0000	0.0101

OC Curves for N=2000, II, AQL=0.65%



14-21. We wish to find a single-sampling plan for a situation where lots are shipped from a vendor. The vendor's process operates at a fallout level of 0.50% defective. We want the AOQL from the inspection activity to be 3%

(a) Find the appropriate Dodge-Romig plan.

We desire a Dodge-Romig, single sampling, AOQL plan with an AOQL = 3% for an average process fallout of p = 0.50% defective. Refer to Table 14-8 for AOQL = 3.0%, and use the second column "0.07–0.50%". The minimum sampling plan (requiring the least amount of inspection) that meets the quality requirements is to form lots of size $50{,}001 \le N \le 100{,}000$; $n = 55$; $c = 3$.

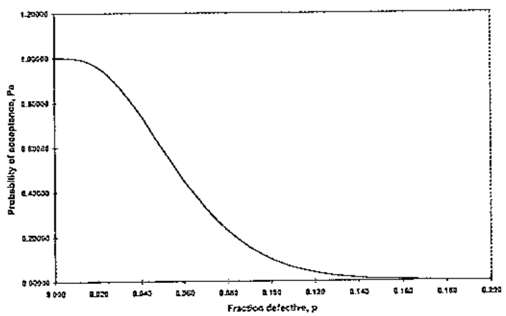
(b) Draw the OC curve and the ATI curve for this plan. How much inspection will be necessary, on the average, if the vendor's process operates close to the average fallout level?

For the type-B OC curve; the formulas on text p. 583 (eqn. 14-1 and 14-2) apply. For the ATI curve, the formula on text p. 593 (eqn. 14-5) applies. The spreadsheet solutions using the Excel BINOMDIST function are below:

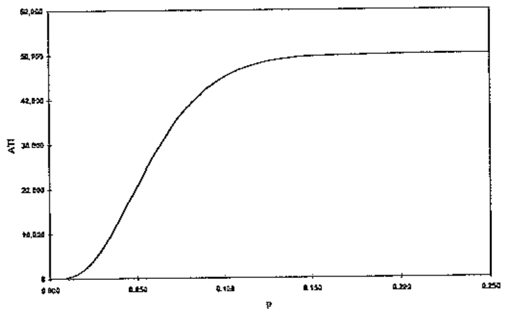
	A	B	C
1	N =	50001	
2	n =	55	
3	c =	3	
4			
5	p	Pa	ATI
5	0.001	=BINOMDIST($B$3,$B$2,A5,TRUE)	=$B$2+(1-B5)*($B$1-$B$2)

p	Pa	ATI
0.001	1.00000	55
0.002	0.99999	55
0.003	0.99995	57
0.004	0.99985	72
0.005	0.99957	82
0.006	0.99934	98
0.007	0.99884	123
0.008	0.99812	159
0.009	0.99713	208
0.010	0.99583	273
0.020	0.95852	2131
0.040	0.73783	13157
0.060	0.44755	27547
0.080	0.22527	38702
0.100	0.09955	45030
0.101	0.09525	45244
0.102	0.09112	45451
0.103	0.08714	45550
0.104	0.08331	45841
0.105	0.07953	45025
0.110	0.05327	45841
0.120	0.03923	48042
0.130	0.02377	48814
0.140	0.01410	49297
0.150	0.00820	49592
0.200	0.00042	49980

OC Curve for Dodge-Romig, n=55, c=3



ATI Curve for N=50,001, n=55, c=3



let $N = 50{,}001$

$P_a = \text{Binom}(3, 65, 0.005) = 0.99967$

$\text{ATI} = n + (1 - P_a)(N - n) = 65 + (1 - 0.99967)(50{,}001 - 65) = 82$

On average, if the vendor's process operates close to process average, the average inspection required will be 82 units.

(c) What is the LTPD protection for this plan?

Table 14-8 also gives the LTPD for an AOQL plan. The LTPD for this plan is 10.3%, which is also the point on the OC curve for which $P_a = 0.10$. This provides assurance that 90% of incoming lots that are as bad as 10.3% defective will be rejected.

CHAPTER 15

Other Acceptance-Sampling Techniques

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Discuss the advantages and disadvantages of variables sampling plans:
   - Advantages include smaller sample sizes with more information
   - Disadvantages include need to know distribution of characteristics, separate plans for each characteristic, and can reject a lot without a defect
2. Construct and apply MIL STD 414 variables-sampling plans
3. Construct and apply other variables sampling procedures:
   - Chain sampling for situations in which testing is destructive or very expensive
   - Continuous sampling, especially CSP-1, for continuous production where lots are not naturally formed
   - Skip-lot sampling for product from a supplier of consistently good quality

Exercises

15-1. The density of a plastic part used in a cellular telephone is required to be at least 0.70 g/cm3. The parts are supplied in large lots, and a variables sampling plan is to be used to sentence the lots. It is desired to have p1 = 0.02, p2 = 0.10, α = 0.10, and β = 0.05. The process variability of the manufacturing process is unknown but will be estimated by the sample standard deviation.

(a) Find an appropriate variables sampling plan, using Procedure 1.

We can use Procedure 1 (text p. 724), where a lot is rejected if the estimate of the critical distance k is greater than Z_{LSL} (indicating the mean is to close to the lower specification limit). The lower specification limit is LSL = 0.070 g/cm^3.

To specify a sampling plan, it does not matter which two points on the operating characteristic (OC) curve are specified. If $p_1 = 0.02$ is considered an "acceptable" lot and $p_2 = 0.10$ is considered an "unacceptable" lot, we can use the producer's risk α and the consumer's risk β to determine a sampling plan. The probability of accepting lots with $p_1 = 0.02$ fraction defective is desired to be $1-\alpha = 1-0.10 = 0.90$. The probability of accepting lots with $p_2 = 0.10$ fraction defective is desired to be $\beta = 0.05$.

Use the variables nomograph, Figure 15-2 on text p. 726. To find the sampling plan, draw one line connecting $p_1 = 0.02$ on the left with 0.90 on the right. Draw the second line connecting $p_2 = 0.10$ on the left with 0.05 on the right. At the intersection of the lines we read $k = 1.7$. Follow the curved line from the intersection point to the upper sample size scale (for σ unknown) and interpolate $n = 35$.

The procedure is to select a random sample of $n = 35$ parts, record the density, calculate the sample $\bar{x}$ and S, and accept the lot if:

$$Z_{LSL} = \frac{\bar{x} - \text{LSL}}{S} \geq 1.7$$

(b) Suppose that a sample of the appropriate size was taken, and $\bar{x} = 0.73$, $S = 1.05 \times 10\text{-}2$. Should the lot be accepted or rejected?

$\bar{x} = 0.73$; $S = 1.05 \times 10^{-2}$

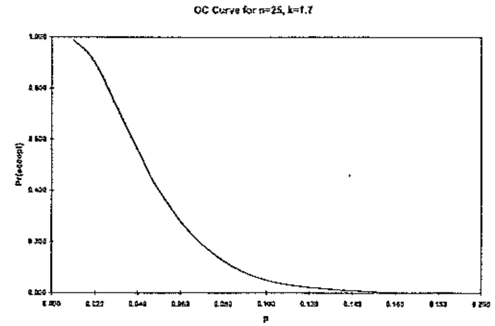
$$Z_{LSL} = \frac{0.73 - 0.70}{1.05 \times 10^{-2}} = 2.8571 \geq 1.7$$

So accept the lot.

(c) Sketch the OC curve for this sampling plan. Find the probability of accepting lots that are 5% defective.

From the variables nomograph at $n = 35$ and $k = 1.7$, select values of p and find P_a.

	p	Pr{accept}	
	0.010	0.988	
	0.016	0.945	
p1	0.020	0.900	1-alpha
	0.025	0.820	
	0.030	0.730	
	0.040	0.560	
	0.050	0.400	
	0.070	0.190	
p2	0.100	0.050	beta
	0.150	0.005	
	0.190	0.001	



$P_a\{p = 0.05\} \approx 0.38$ (from nomograph)

15-3. Describe how rectifying inspection can be used with variables sampling. What are the appropriate equations for the AOQ and the ATI, assuming single sampling and requiring that all defective items found in either sampling or 100% inspection are replaced by good ones?

Recall that rectifying inspection programs require that discovered defective items are either removed for further disposition or are replaced with known good items. Rejected lots are screened and have zero defective items. Accepted lots have some outgoing fraction defective p_1 less than the incoming fraction defective p_0.

The equations do not change:
$\text{AOQ} = P_a p\,(N-n)/N$ (eqn. 14-4, text p. 691)
$\text{ATI} = n + (1-P_a)(N-n)$ (eqn. 14-6, text p. 693)

The design of a variables plan in rectifying inspection is somewhat different from the attribute plan design, and generally involves some trial-and-error search. For example, for a given $\text{AOQL} = P_a p_m (N-n)/N$ (where p_m is the value of p that maximizes AOQ), we know n and k are related, because both P_a and p_m are functions of n and k. Suppose n is arbitrarily specified. Then a k can be found to satisfy the AOQL equation. No convenient mathematical method exists to do this, and special Romig tables are usually employed. Now, for a specified process average, n and k will define P_a.

Finally, ATI is found from the above equation. Repeat until the n and k that minimize ATI are found.

15-4. An inspector for a military agency desires a variables sampling plan for use with an AQL of 1.5%, assuming that lots are of size 7000. If the standard deviation of the lot or process is unknown, derive a sampling plan using Procedure 1 from MIL STD 414.

Given: AQL = 1.5%, N = 7000, standard deviation unknown
Assume: Single-Specification limit—Form 1, Inspection Level IV ("normal")
From Table 15-1 (Table A-2): Sample Size Code Letter = M
From Table 15-2 (Table B-1): n = 50, k_{normal} = 1.80 (read from AQL headings at top of table), $k_{tightened}$ = 1.93 (read from AQL headings at bottom of table)

Consult a copy of MIL STD 414 for complete tables containing the reduced sampling plan ($n_{reduced}$ = 20, $k_{reduced}$ = 1.51).

15-5. How does the sample size found in Exercise 15-4 compare with what would have been used under MIL STD 105E?

Under MIL STD 105E with Inspection level II, Sample size code letter = L, use Tables 14-4 through 14-7:

	Normal	Tightened	Reduced
n	200	200	80
Ac	7	5	3
Re	8	6	6

The MIL STD 414 sample sizes are considerably smaller than those for MIL STD 105E.

15-7. A soft-drink bottler purchases nonreturnable glass bottles from a vendor. The lower specification on bursting strength in the bottles is 225 psi. The bottler wishes to use variables sampling to sentence the lots, and has decided to use an AQL of 1%. Find an appropriate set of normal and tightened sampling plans from the standard. Suppose that a lot is submitted, and the sample results yield $\overline{x}$ = 255 and S = 10. Determine the disposition of the lot using Procedure 1. The lot size is N = 100,000.

Given: AQL = 1%, N = 100,000, Standard Deviation unknown, Single-Specification limit—Form 1, LSL = 225 psi
Assume: Inspection Level IV ("normal")
From Table 15-1 (Table A-2): Sample Size Code Letter = O
From Table 15-2 (Table B-1), Normal Sampling: n = 100, k_{normal} = 2.00 (read from AQL headings at top of table)
From Table 15-2 (Table B-1), Tightened Sampling: n = 100, $k_{tightened}$ = 2.14 (read from AQL headings at bottom of table)

Assume Normal Sampling is currently in effect.
$\overline{x} = 255; S = 10$

$$Z_{LSL} = \frac{\overline{x} - \text{LSL}}{S} = \frac{255 - 225}{10} = 3.000 > 2.00$$

So accept the lot.

15-9. A standard of 0.3 ppm has been established for formaldehyde emission levels in wood products. Suppose that the standard deviation of emissions in an individual board is σ = 0.10 ppm. Any lot that contains 1% of its items above 0.3 ppm is considered acceptable. Any lot that has 8% or more of its items above 0.3 ppm is considered unacceptable. Good lots are to be accepted with probability 0.95, and bad lots are to be rejected with probability 0.90.

(a) Derive a variables sampling plan for this situation.

Use the variables nomograph, Figure 15-2 on text p. 726. To find the sampling plan, draw one line connecting p_1 = 0.01 on the left with $1 - \alpha$ = 0.95 on the right. Draw the second line connecting p_2 = 0.08 on the left with $\beta = 1 - 0.90 = 0.10$ on the right.

At the intersection of the lines we read k = 1.8. Follow the curved line from the intersection point to the lower sample size scale (for σ known) and interpolate n = 30. The sampling plan is n = 30 and k = 1.8.

(b) Using the 1% nonconformance level as an AQL, and assuming that lots consist of 5000 panels, find an appropriate set of sampling plans from MIL STD 414, assuming σ unknown. Compare the sample sizes and the protection that both producer and consumer obtain from this plan with the plan derived in part (a).

Given: AQL = 1%, N = 5000, σ unknown, single specification limit
Assume: Inspection Level IV
From Table 15-1 (MIL STD 414, Table A-2): Sample Size Code Letter = M
From Table 15-2 (MIL STD 414, Table B-1), Normal Sampling: n = 50, k = 1.93
From Table 15-2 (MIL STD 414, Table B-1), Tightened Sampling: n = 50, k = 2.08

A sampling plan constructed with σ known (n = 30) requires smaller sample sizes than σ unknown n = 50).

The plan in part (a) is designed with p_1 = 1%, representing the level of quality for which the producer desires a high probability (0.95) of acceptance, and with p_2 = 8%, the level of quality that a consumer wants a low probability (0.10) of accepting. In part (b) an AQL of 1% represents the poorest level of quality that the consumer considers acceptable—focusing on the producer's risk end of the OC curve, and the consumer's protection varies depending on the choice of inspection level.

(c) Find an attributes sampling plan that has the same OC curve as the variables sampling plan derived in part (a). Compare the sample sizes required for equivalent protection. Under what circumstances would variables sampling be more economically efficient?

Use the binomial nomograph, Figure 14-9 on text p. 690. To find the sampling plan, draw one line connecting p_1 = 0.01 on the left with $1 - \alpha$ = 0.95 on the right. Draw the second line connecting p_2 = 0.08 on the left with $\beta = 1 - 0.90 = 0.10$ on the right. The sampling plan is n = 60 and c = 2.

The sample size is slightly larger than required for the variables plan (a). Note that the variables sampling would be more efficient if σ were known.

(d) Using the 1% nonconforming as an AQL, find an attributes sampling plan from MIL STD 105E. Compare the sample sizes and the protection obtained from this plan with the plans derived in parts (a), (b), and (c).

Given: $N = 5000$, AQL = 1%, Single Sampling
Assume: General Inspection Level II
From Table 14-4: Sample size code letter = L
From Table 14-5, Normal sampling plan: $n = 200$, Ac = 5, Re = 6
From Table 14-6, Tightened sampling plan: $n = 200$, Ac = 3, Re = 4
From Table 14-7, Reduced sampling plan: $n = 80$, Ac = 2, Re = 5

The sample size of the MIL STD 105E normal sampling plan, $n = 200$, is much larger than for the other plans (30, 50, and 60 respectively). The comparison of the protection offered by plan (a) versus plan (b), is the same for plan (c) and plan (d).

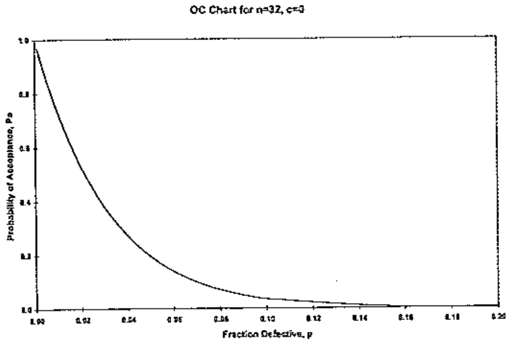
15-11. An electronics manufacturer buys memory devices in lots of 30,000 from a vendor. The vendor has a long record of good quality performance, with an average fraction defective of approximately 0.10%. The quality engineering department has suggested using a conventional acceptance-sampling plan with $n = 32$, $c = 0$.

(a) Draw the OC curve of this sampling plan.

For the type-B OC curve; the formulas on text p. 683 (eqn. 14-1 and 14-2) apply. The spreadsheet solutions using the Excel BINOMDIST function are:

	A	B	C
1	N = 30,000	n =	32
2		c =	0
3		binomial	
4	p	Pa	Pr{reject}
5	0.001	=BINOMDIST($C$2,$C$1,A5,TRUE)	=1-B5

p	*Pa*	*Pr{reject}*
0.0010	0.9685	0.0315
0.0015	0.9531	0.0469
0.0020	0.9379	0.0621
0.0030	0.9083	0.0917
0.0040	0.8796	0.1204
0.0050	0.8518	0.1482
0.0060	0.8248	0.1752
0.0070	0.7987	0.2013
0.0080	0.7733	0.2267
0.0090	0.7488	0.2512
0.0100	0.7250	0.2750
0.0200	0.5239	0.4761
0.0400	0.2708	0.7292
0.0600	0.1381	0.8619
0.0800	0.0694	0.9306
0.1000	0.0343	0.9657
0.2000	0.0008	0.9992



(b) If lots are of a quality that is near the vendor's long-term process average, what is the average total inspection at that level of quality?

Equation 14-6 (text p. 693) still applies: $\text{ATI} = n + (1 - P_a)(N - n) = 32 + (1 - 0.9685)(30000 - 32) = 976$

(c) Consider a chain-sampling plan with $n = 32$, $c = 0$, and $i = 3$. Contrast the performance of this plan with the conventional sampling plan $n = 32$, $c = 0$.

Using eqn. 15-3 (text p. 37), where $P(0, n)$ is the probability of obtaining 0 defective and $P(1, n)$ is the probability of obtaining 1 defective.:

$P_a = P(0,n) + P(1,n)[P(0,n)]^i$

$P(0,n) = P(0,32) = 0.9685$

$P(1,n) = P(1,32) = 0.0310$

$P_a = 0.9685 + (0.0310)(0.9685)^3 = 0.9968$

Using eqn. 14-6 on text p. 693, ATI = $n + (1 - Pa)(N - n) = 32 + (1 - 0.9967)(30{,}000 - 32) = 131$. Compared to conventional sampling, the P_a for chain sampling is slightly larger, but the average number inspected is much smaller.

(d) How would the performance of this chain-sampling plan change if we substituted $i = 4$ in part (c)?

The formula on text p. 737 (eqn. 15-3) applies. The spreadsheet solution using the Excel BINOMDIST function is below:

	E	F	G	H
20	P(0,32)	P(1,32)	Pa part(c)	Pa part(d)
21	=BINOMDIST(0,32,0.001,FALSE)	=BINOMDIST(1,32,0.001,FALSE)	=E21+F21*E21^3	=E21+F21*E21^4

P(0,32)	*P(1,32)*	*Pa part(c)*	*Pa part(d)*
0.9685	0.0310	0.9967	0.9958

$P_a = 0.9958$, there is little change in performance by increasing i. Using eqn. 14-6 on text p. 693, ATI = $n + (1 - Pa)(N - n) = 32 + (1 - 0.9958)(30{,}000 - 32) = 158$

15-13. A chain-sampling plan is used for the inspection of lots of size $N = 500$. The sample size is $n = 6$. If the sample contains no defectives, the lot is accepted. If one defective is found, the lot is accepted provided that the samples from the four previous lots are free of defectives. Determine the probability of acceptance of a lot that is 2% defective.

$N = 500$, $n = 6$
The sampling plan is to accept the lot if $c = 0$. If $c = 1$, accept the lot if $i = 4$ previous lots were free of defectives.

To find $P_a\{p = 0.02\}$, use eqn. 15-3 (text p. 737):

$P_a = P(0,n) + P(1,n)[P(0,n)]^i = P(0,6) + P(1,6)[P(0,6)]^4$

$= 0.88584 + 0.10847(0.88584)^4 = 0.95264$

15-14. Suppose that a manufacturing process operates in continuous production, such that continuous sampling plans could be applied. Determine three different CSP-1 sampling plans that could be used for an AOQL of 0.198%.

We need to find f, the fraction of units to be inspected and i, the clearance number. From Table 15-3, for AOQL = 0.198%, select any three values of f and find the value for i. Three different CSP-1 plans would be:

1. $f = ½$ and $i = 140$
2. $f = 1/10$ and $i = 550$
3. $f = 1/100$ and $i = 1302$

The final choice of i and f is based on practical manufacturing considerations.

To use one of the plans, begin by inspecting all units 100%. As soon as i consecutive units are found to be defect-free, discontinue 100% inspection and inspection fraction f. Sample units are randomly selected, one at a time, from the production flow. If a unit is defective, 100% inspection is resumed. Defective units are either reworked or replaced with good units.

15-15. For the sampling plans developed in Exercise 15-14, compare the plans' performance in terms of average fraction inspected, given that the process is in control at an average fallout level 0.15%. Compare the plans in terms of their operating characteristic curves.

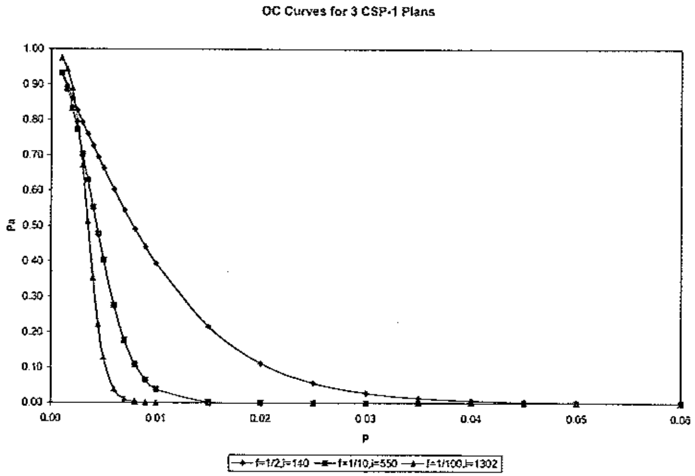
The average process fallout is $p = 0.15\% = 0.0015$, so $q = 1 - p = 0.9985$. Use eqns. 15-4 – 15-7 to evaluate OC curves (text pp. 738 – 739). The Excel spreadsheet solutions are:

	A	B	C	D
1	AOQL = 0.198%			
2	p =	0.0015		
3	q =	=1-B2		
4				
5	f =	0.5		
6	i =	140		
7				
8	u =	=(1-$B$3^B6)/($B$2*$B$3^B6)		
9	v =	=1/(B5*$B$2)		
10	AFI =	=(B8+B5*B9)/(B8+B9)		
11	Pa(p=.0015) =	=B9/(B8+B9)		
12				
13	p	u	v	Pa
14	0.001	=(1-(1-A14)^$B$6)/(A14*(1-A14)^$B$6)	=1/($B$5*A14)	=C14/(B14+C14)

	f = 1/2 and i = 140			f = 1/10 and i = 550			f = 1/100 and i = 1302		
p	u	v	Pa	u	v	Pa	u	v	Pa
0.0010	1.5035E+02	2000.00	0.9301	7.3373E+02	10000.00	0.9316	2.6790E+03	100000.00	0.9739
0.0015	1.5592E+02	1333.33	0.8953	8.5553E+02	6666.67	0.8863	4.0401E+03	66666.67	0.9429
0.0020	1.6175E+02	1000.00	0.8608	1.0037E+03	5000.00	0.8328	6.2765E+03	50000.00	0.8885
0.0025	1.6788E+02	800.00	0.8266	1.1848E+03	4000.00	0.7715	1.0010E+04	40000.00	0.7998
0.0030	1.7431E+02	666.67	0.7927	1.4066E+03	3333.33	0.7032	1.6331E+04	33333.33	0.6712
0.0035	1.8106E+02	571.43	0.7594	1.6795E+03	2857.14	0.6298	2.7161E+04	28571.43	0.5127
0.0040	1.8816E+02	500.00	0.7266	2.0162E+03	2500.00	0.5536	4.5912E+04	25000.00	0.3526
0.0045	1.9562E+02	444.44	0.6944	2.4329E+03	2222.22	0.4774	7.8875E+04	22222.22	0.2202
0.0050	2.0346E+02	400.00	0.6628	2.9502E+03	2000.00	0.4040	1.3638E+05	20000.00	0.1279
0.0060	2.2037E+02	333.33	0.6020	4.3972E+03	1666.67	0.2749	4.2131E+05	16666.67	0.0381
0.0070	2.3909E+02	285.71	0.5444	6.6619E+03	1428.57	0.1766	1.3395E+06	14285.71	0.0106
0.0080	2.5984E+02	250.00	0.4904	1.0238E+04	1250.00	0.1088	4.3521E+06	12500.00	0.0029
0.0090	2.8284E+02	222.22	0.4400	1.5930E+04	1111.11	0.0652	1.4383E+07	11111.11	0.0008
0.0100	3.0839E+02	200.00	0.3934	2.5056E+04	1000.00	0.0384	4.8192E+07	10000.00	0.0002
0.0200	7.9590E+02	100.00	0.1116	3.3467E+06	500.00	0.0001	1.3262E+13	5000.00	0.0000

	f = 1/2 and i = 140			f = 1/10 and i = 550			f = 1/100 and i = 1302		
p	u	v	Pa	u	v	Pa	u	v	Pa
0.0300	2.3371E+03	66.67	0.0277	6.2867E+08	333.33	0.0000	5.5729E+18	3333.33	0.0000
0.0400	7.5602E+03	50.00	0.0066	1.4085E+11	250.00	0.0000	3.0255E+24	2500.00	0.0000
0.0500	2.6266E+04	40.00	0.0015	3.5731E+13	200.00	0.0000	2.0179E+30	2000.00	0.0000
0.0600	9.6355E+04	33.33	0.0003	1.0035E+16	166.67	0.0000	1.6195E+36	1666.67	0.0000

1. $f = ½$ and $i = 140$: $u = 155.915$, $v = 1333.3$, AFI = 0.5523, $P_a = 0.8953$
2. $f = 1/10$ and $i = 550$: $u = 855.530$, $v = 6666.7$, AFI = 0.2024, $P_a = 0.8863$
3. $f = 1/100$ and $i = 1302$: $u = 4040.000$, $v = 66{,}666.7$, AFI = 0.0666, $P_a = 0.9429$



The OC curves for the 1st and 2nd plans ($f = ½$ and $i = 140$, $f = 1/10$ and $i = 550$) are much steeper, indicating more discriminating plans.