Lecture 2 Load and Stress Analysis

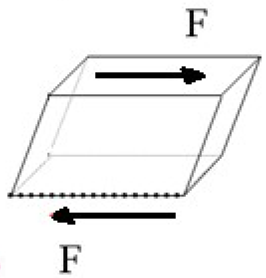
Chapter 3

TYPES OF LOAD

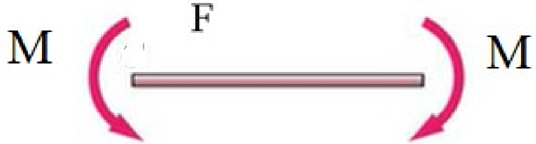
Structures and Machine elements carry different types of loads:

-Normal Force

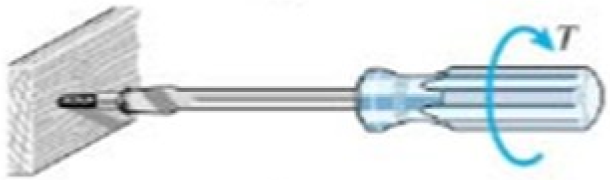
-Shear Force



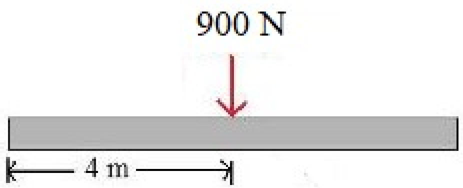
- Bending Moment



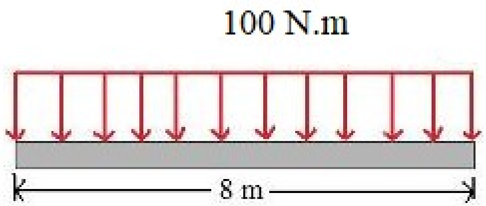
Torsion



(these loads can be concentrated or distributed)



concentrated load



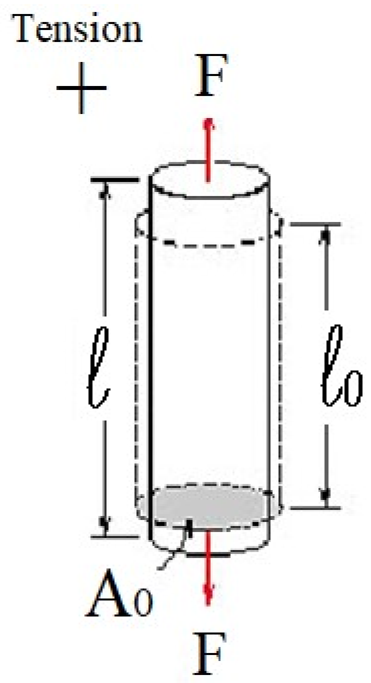
distributed load

Stress analysis of Normal Loads and shear loads

Static load changes relatively slowly with time

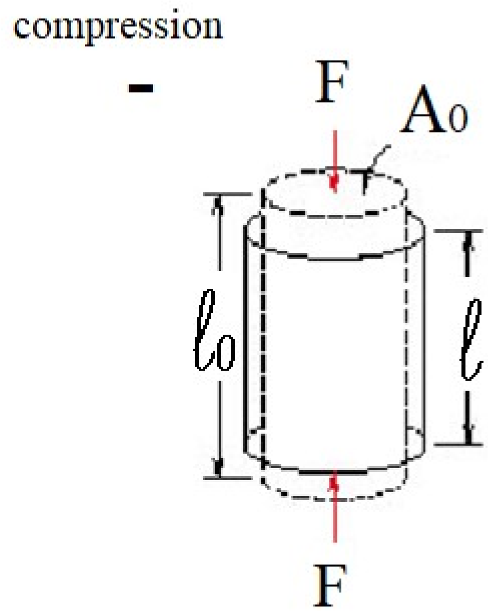
Applied uniformly over a cross section or surface of a member.

Tension
Compression
Shear



$$\sigma = \frac{F_t}{A_o}$$

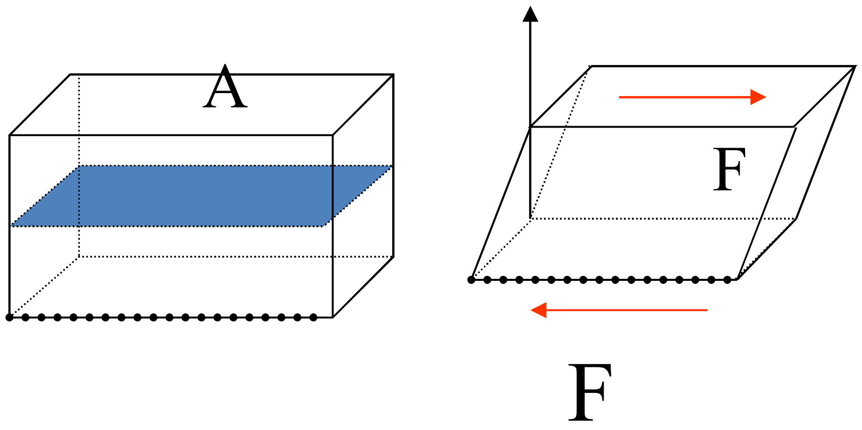
original area before loading



$$\sigma = \frac{-F_t}{A_o}$$

original area before loading

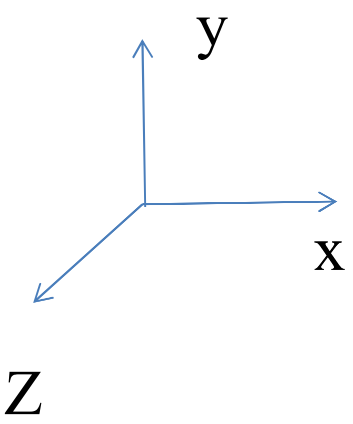
Shear stress = τ = F tangential to the area / A

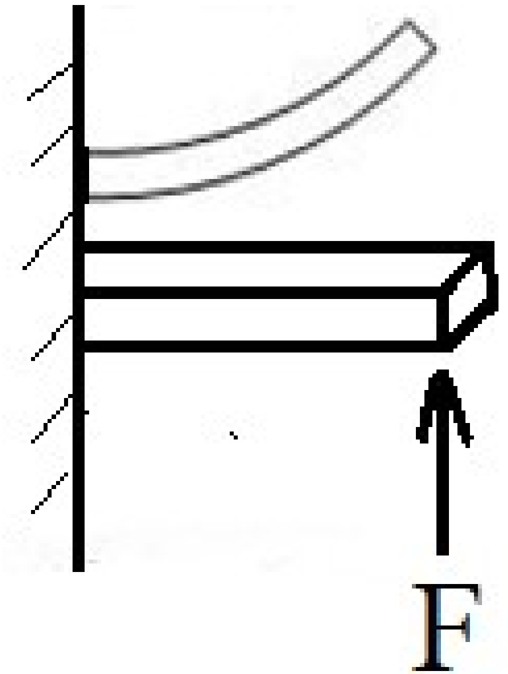


Stress has units:
N/m^2 or lb/in^2

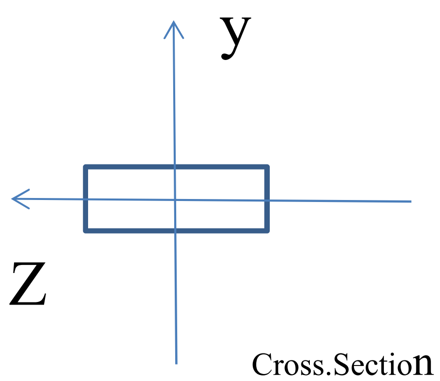
Shear and bending moments in Beams

loads acting transversely to the longitudinal axis the loads create shear forces and bending moments





$M = r \times F$



$M = r \times F$

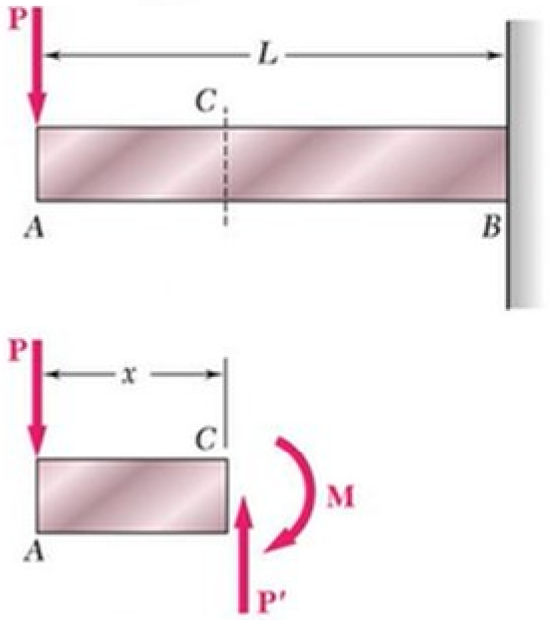
$i \times j = k$

M_Z

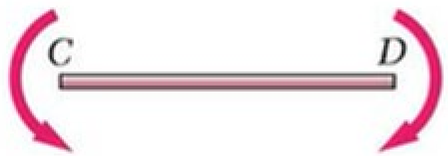
Pure Bending and Non uiform Bending

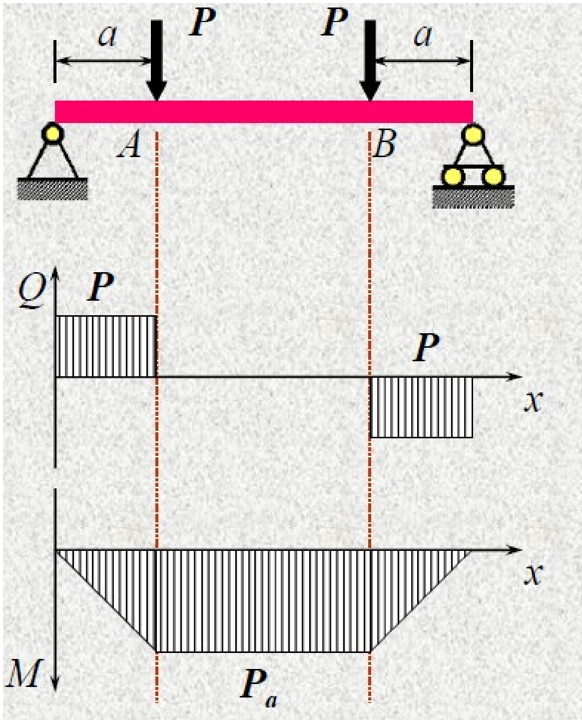
- Pure bending: Shear force (V) = 0 over the section
- Non-uniform bending: V ≠ 0 over the section

Non-uniform bending
- Moment → normal stresses
- Shear force → shear stresses



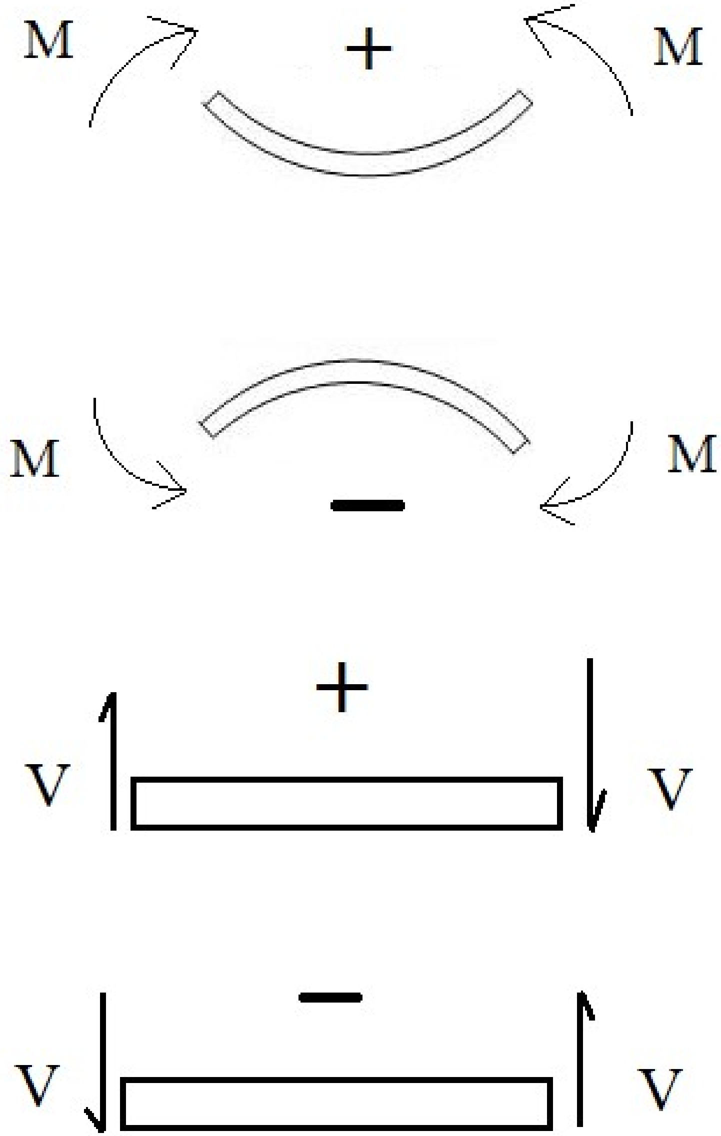
Pure bending
- Moment → normal stresses





there are only bending moment in portion AB and no shearing force . This is called pure bending

The sign convection for shear force and bending moment

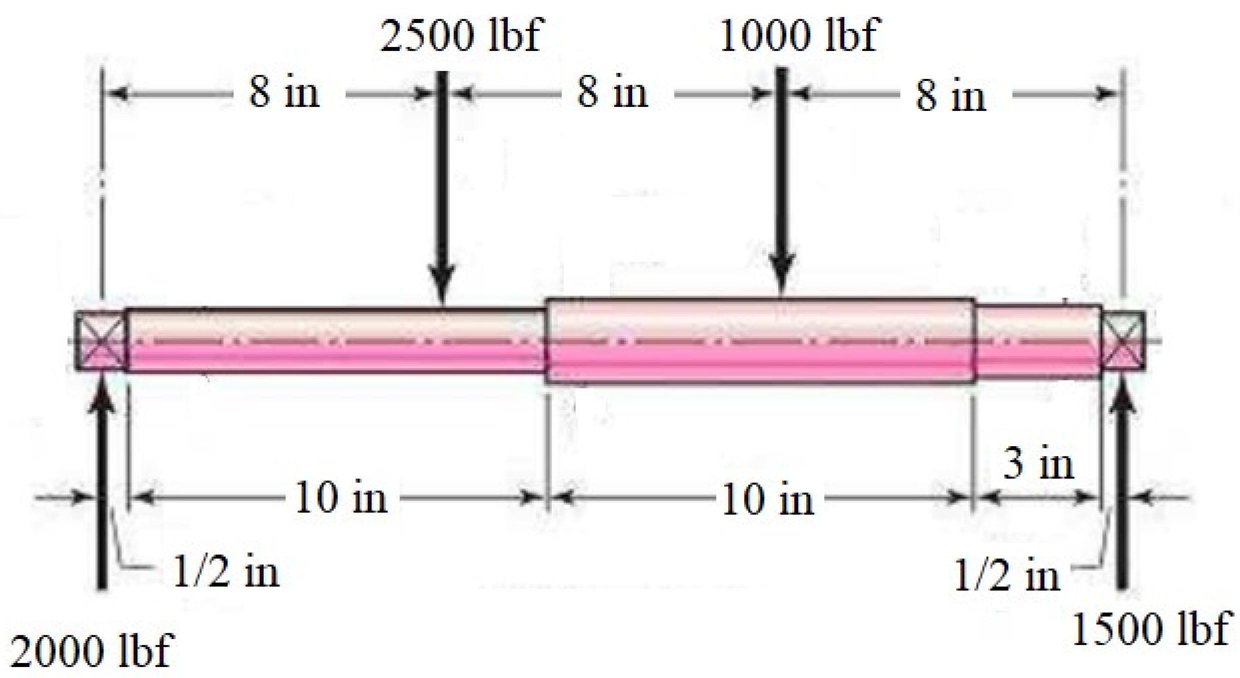


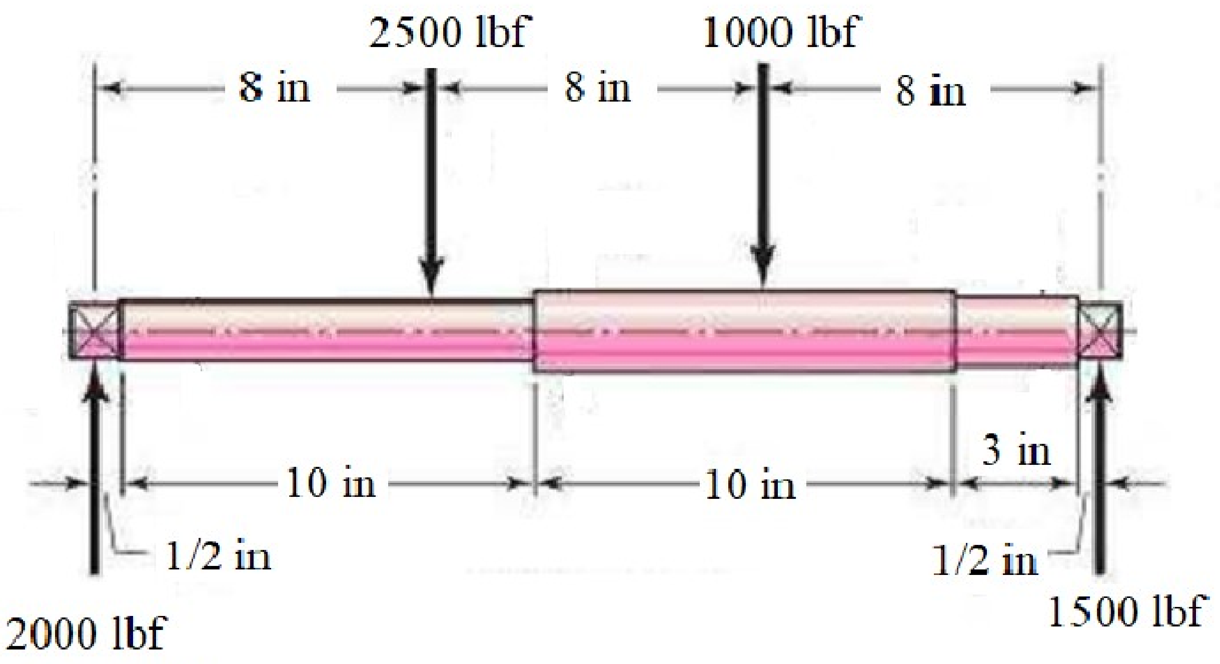
Shear force and bending moment are related by the equation

$$V = \frac{dM}{dx}$$

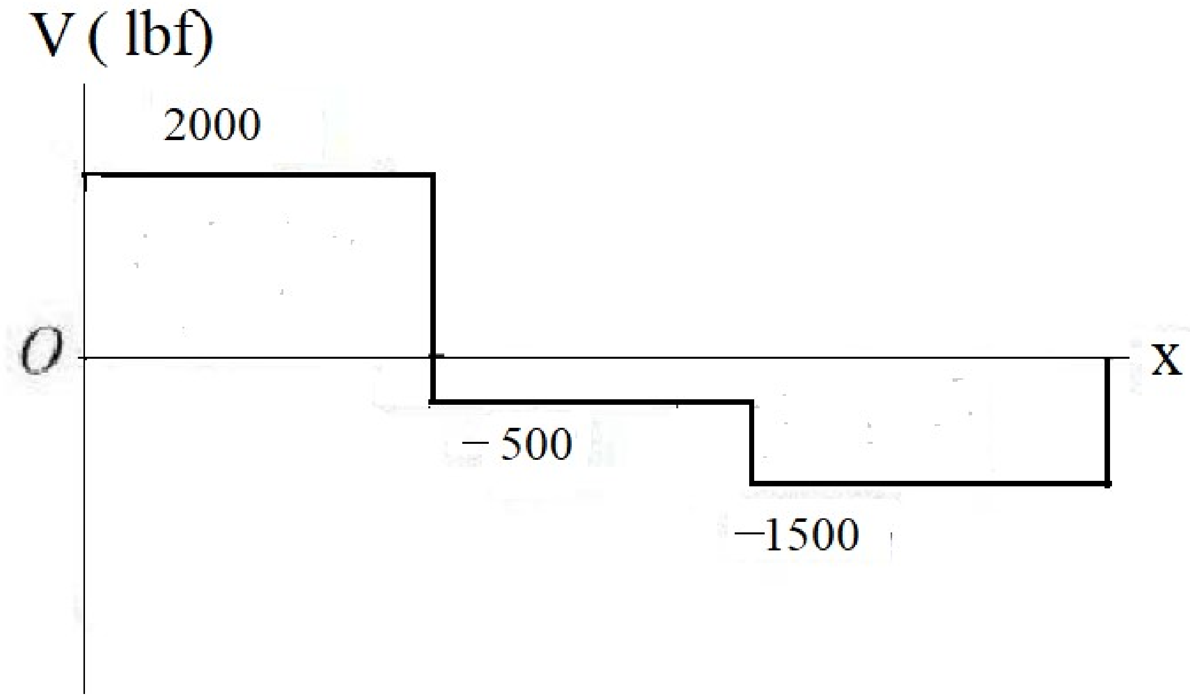
(Shear is the slope of the moment diagram)

Ex: The shaft shown is machined from a AISI 1040 CD steel . Draw the shear force and bending-moment diagrams . What is the maximum shear force V. What is the maximum shear stress developed in the shaft

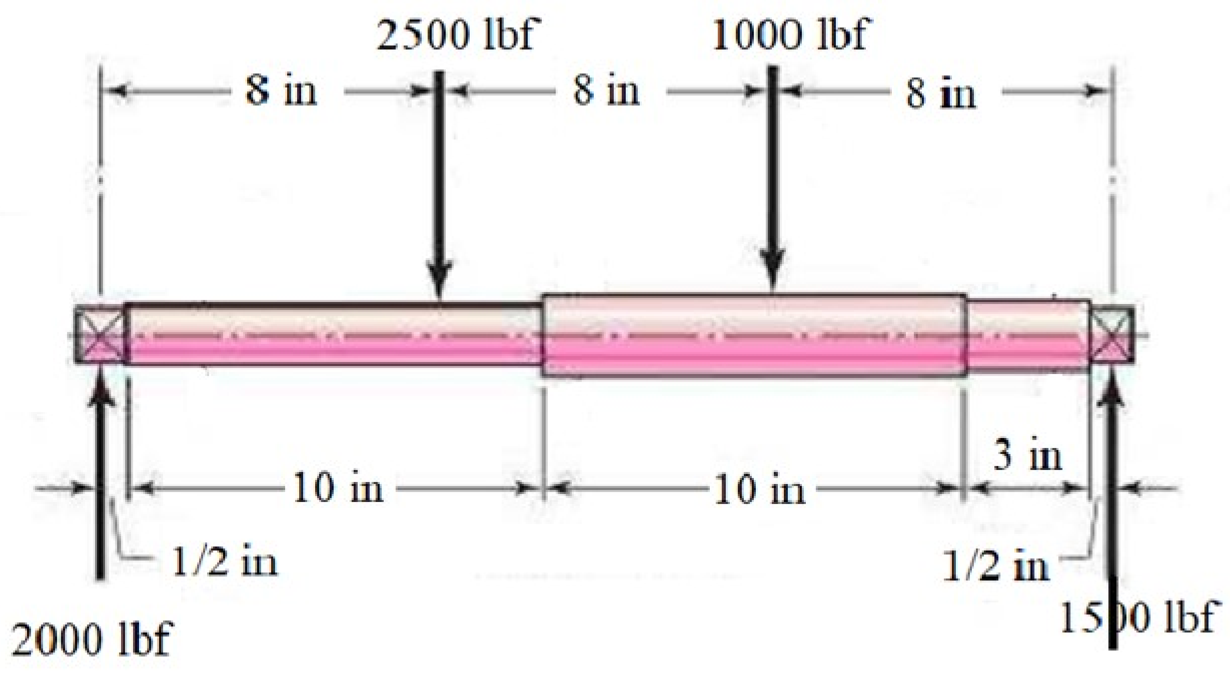




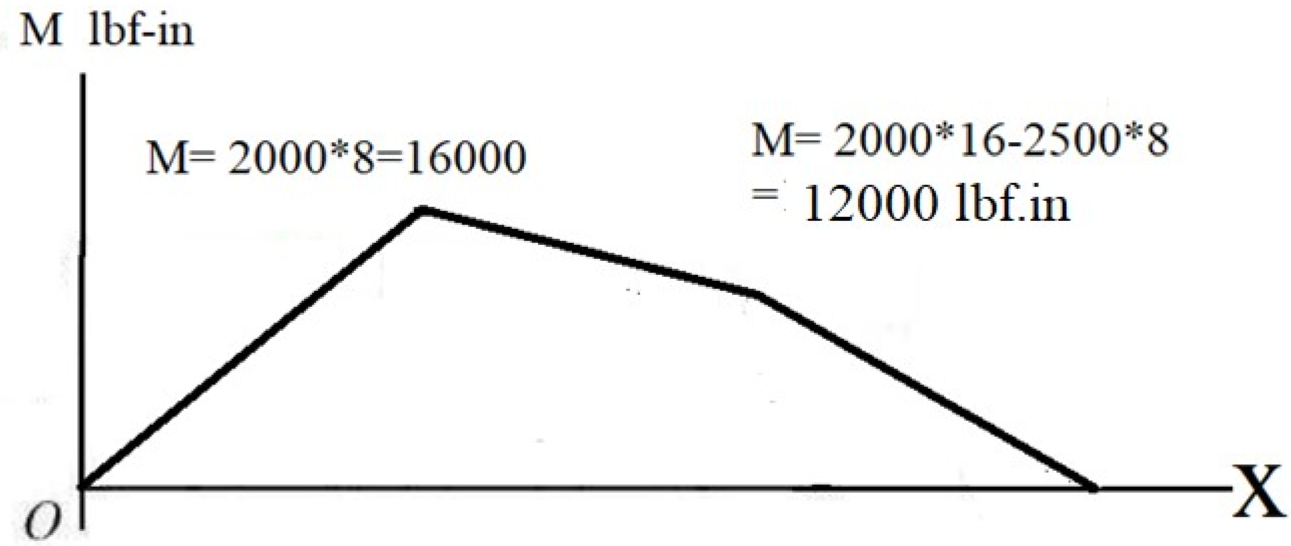
2500 lbf
1000 lbf
8 in
8 in
8 in
3 in
10 in
10 in
1/2 in
1/2 in
2000 lbf
1500 lbf



V (lbf)
2000
O
X
− 500
−1500



2500 lbf
1000 lbf
8 in
8 in
8 in
10 in
10 in
3 in
1/2 in
1/2 in
2000 lbf
1500 lbf



M lbf-in
M= 2000*8=16000
M= 2000*16-2500*8
= 12000 lbf.in
O
X

Stress analysis of Bending and Shear loads

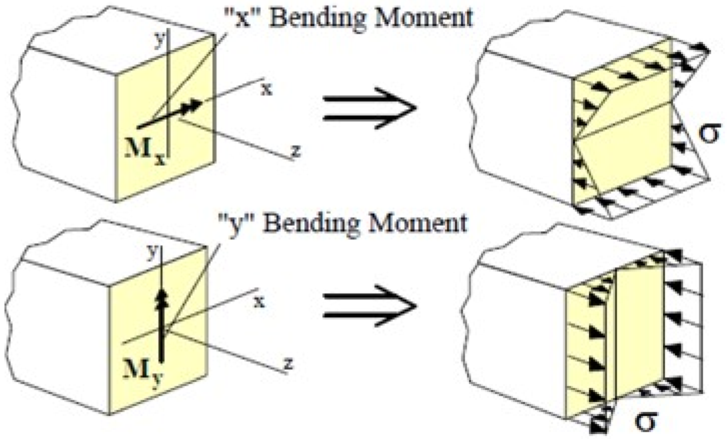
(Internal forces) stresses on the cross section of the bending member

Internal forces:
- Shearing force **V** ⟶ Shearing stress τ
- Bending moment M ⟶ Normal stress σ

3-10 Normal stress for beams in bending

The bending stress in beams subjected to bending moment is found as

$$\sigma_{xx} = \frac{M_z \cdot y}{I_z} \qquad \sigma_{xx} = \frac{M_y \cdot z}{I_y}$$



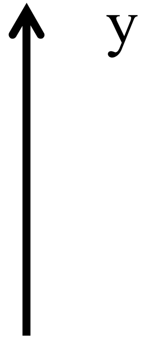
Two Plane Bending

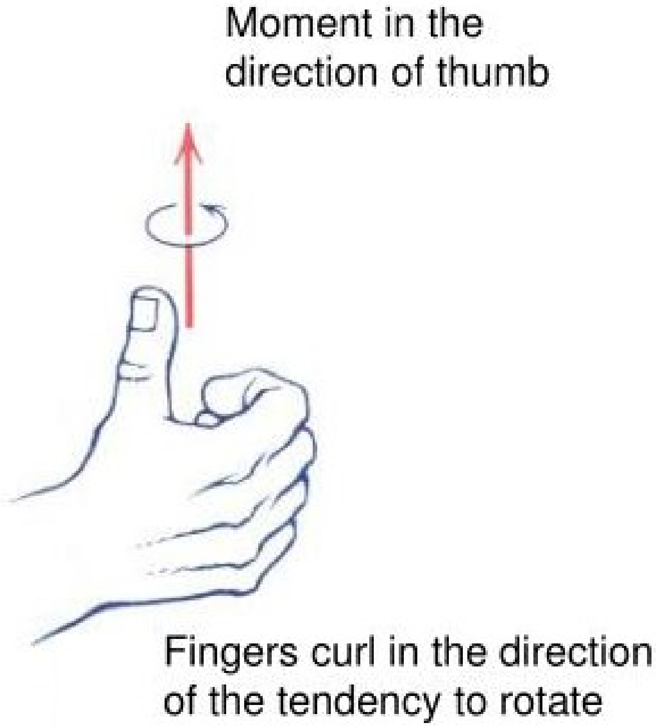
$$\sigma_z = \frac{M_x \cdot y}{I_x} \quad \text{and} \quad \sigma = -\frac{M_y \cdot x}{I_y}$$

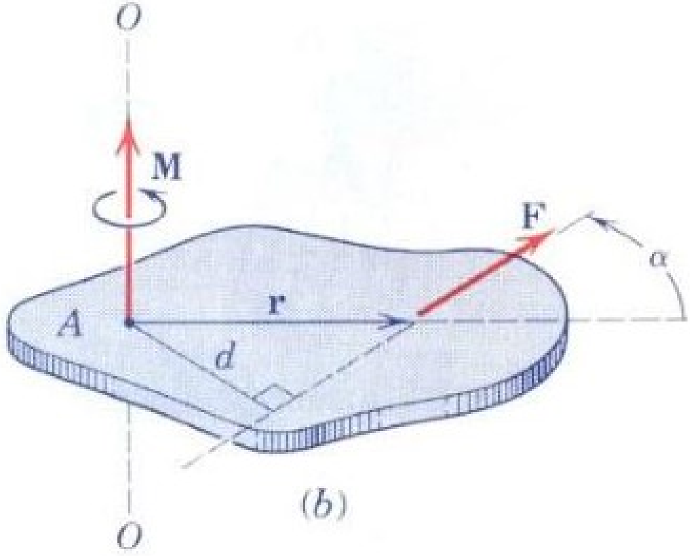
where: $\mathbf{M_x}$ and $\mathbf{M_y}$ are moments about indicated axes
y and **x** are perpendicular from indicated axes
$\mathbf{I_x}$ and $\mathbf{I_y}$ are moments of inertia about indicated axes

$$\sigma_z = \frac{M_x \cdot y}{I_x} - \frac{M_y \cdot x}{I_y}$$

Right hand rule





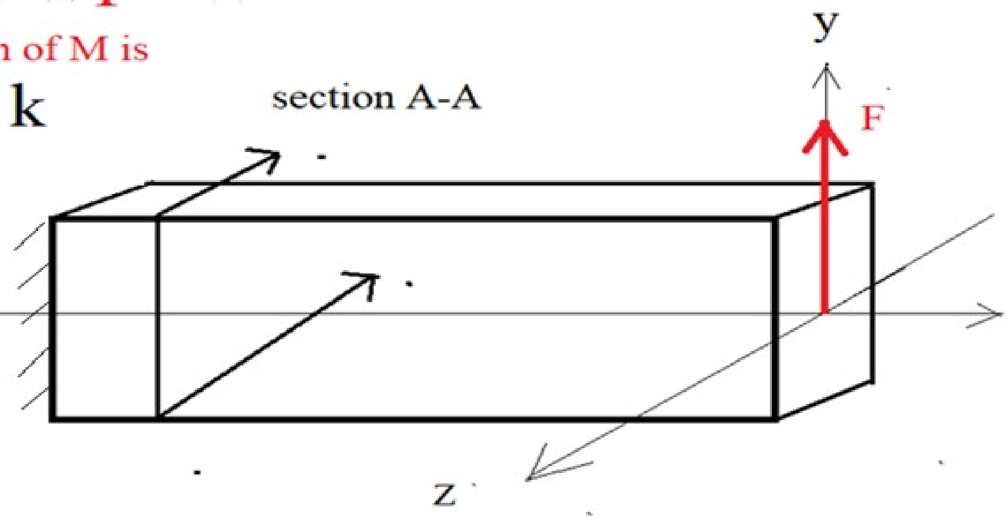


(b)

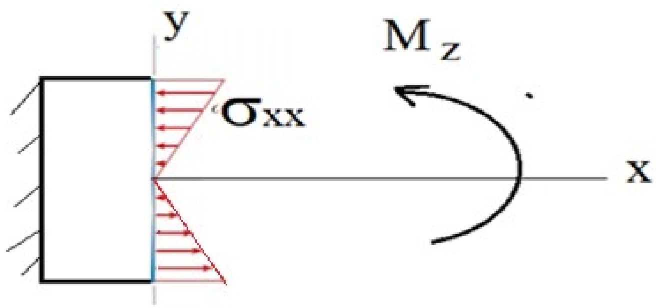
$\vec{M} = \vec{r} \times \vec{F}$

direction of M is

$i \times j = k$



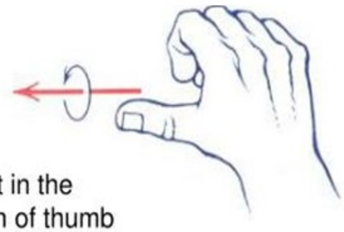
$$\sigma_{xx} = -\frac{M_z \cdot y}{I_z}$$



section A-A

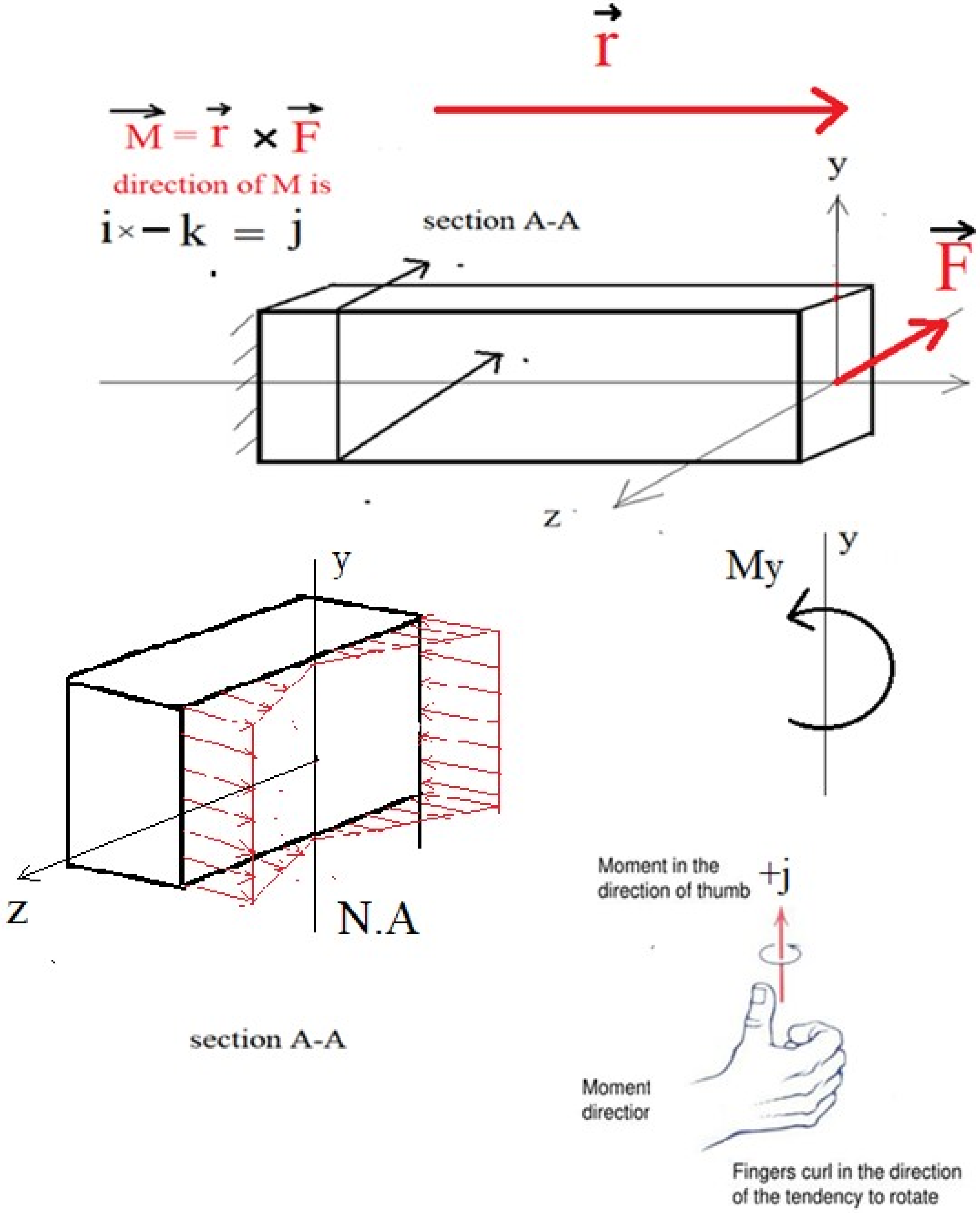
Fingers curl in the direction of the tendency to rotate

+k



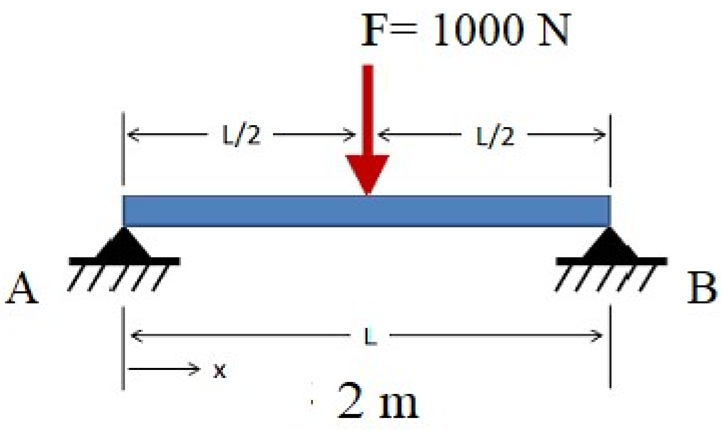
Moment in the direction of thumb

$$\sigma_{xx} = \frac{M_y \cdot Z}{I_y}$$

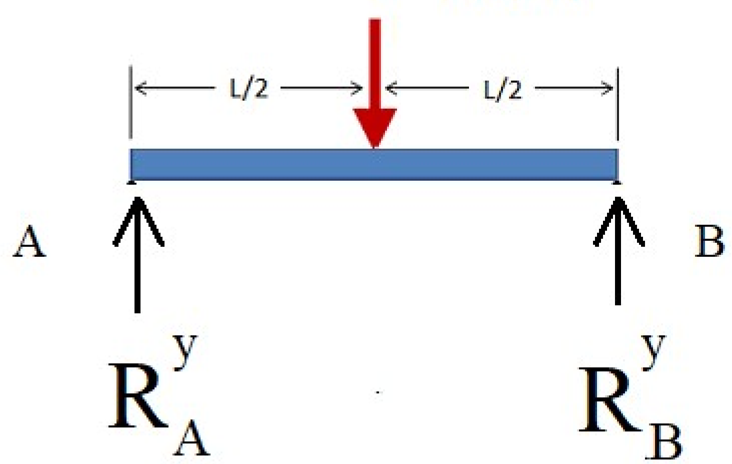


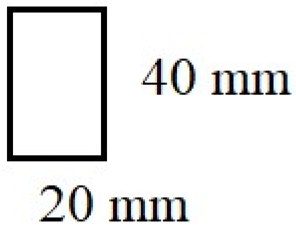
Ex:A simply supported 2 m. beam with a load of 1000 N. acting downward at the center of the beam. The beam used is a rectangular 20 mm by 40 mm steel beam. We would like to determine the maximum

bending (axial) stress which develops in the beam due to the loading



FBD





$$\Sigma M_A^Z = 0$$

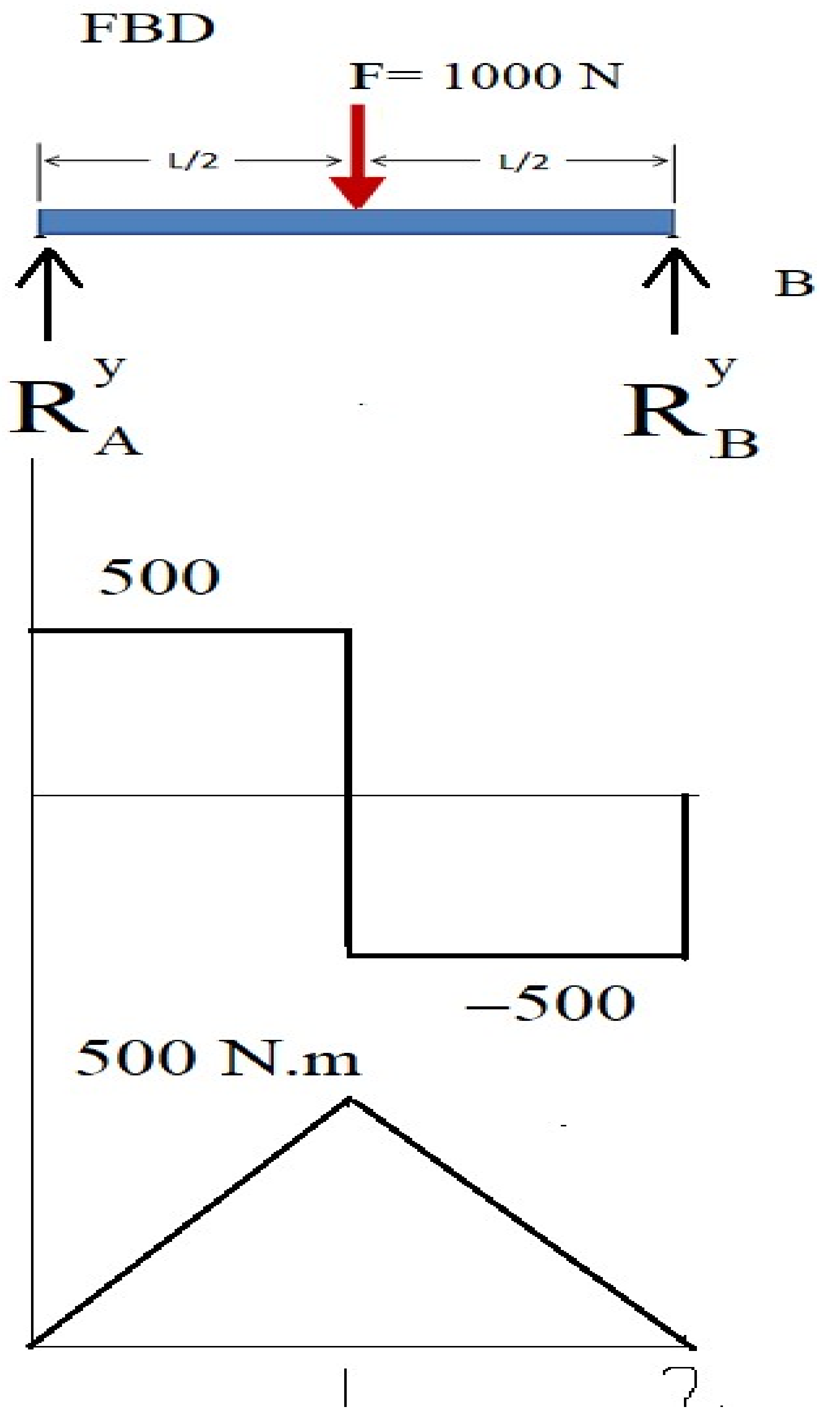
$$2 * R_B^y - 1000*1 = 0$$

$$R_B^y = 500$$

$$\Sigma F_y = 0$$

$$R_A^y + R_B^y - 1000 = 0$$

$$R_A^y = 500$$

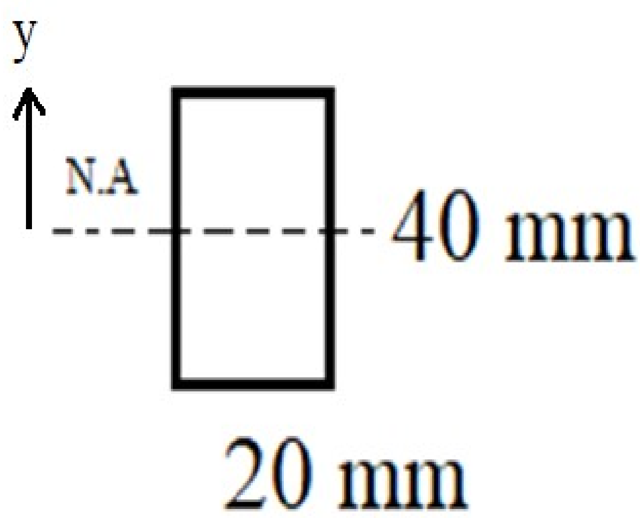


Bending Stress = M y / I

We wish to find the maximum bending stress, which occurs at the outer edge of the beam so

maximum bending moment = 500 N.m (from bending moment diagram)

y = distance from the neutral axis of the cross section to outer edge of beam = 20 mm

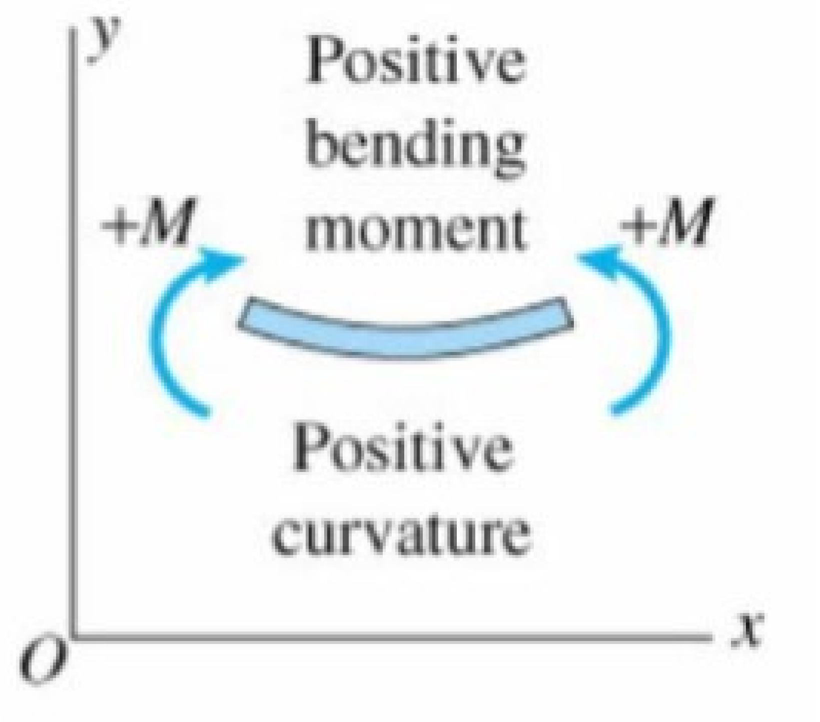


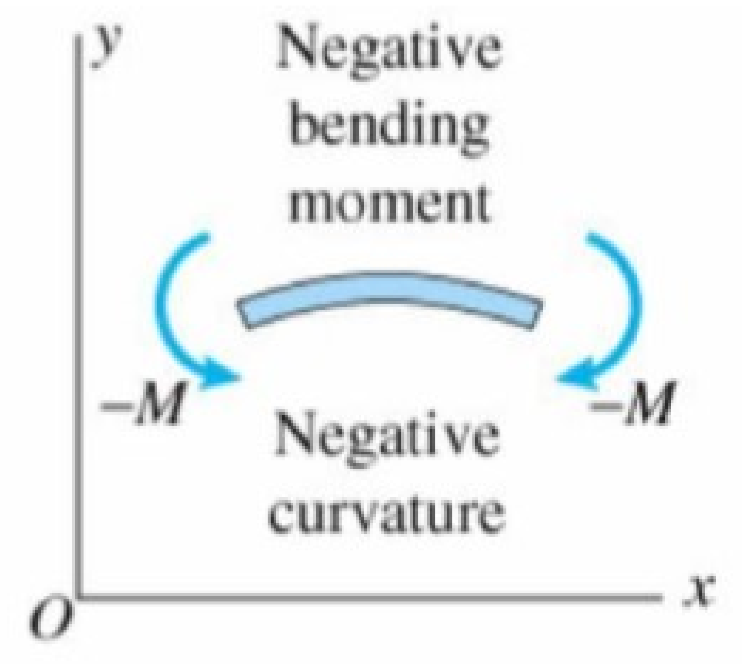
for rectangle:

$I = (1/12)\ bh^3 = 1/12\ (20\times 40^3) = 640000\ mm^4$

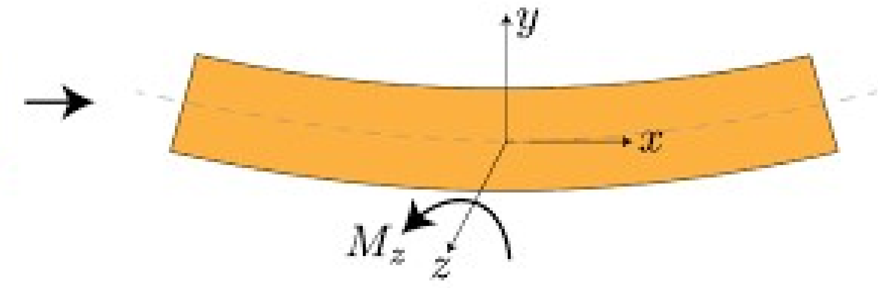
Then, Maximum Bending Stress

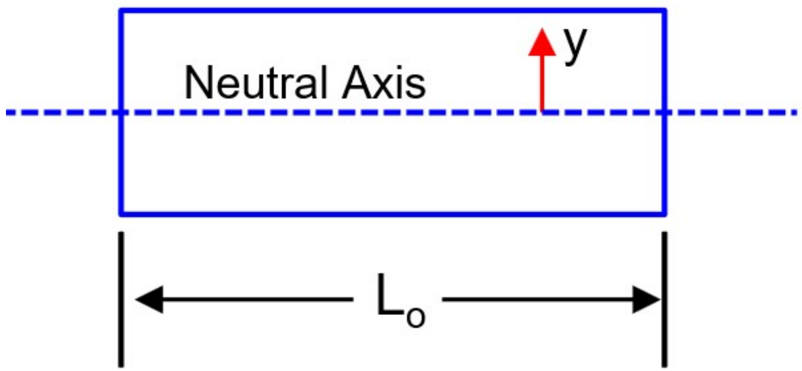
$\sigma_{xx} = M\ y / I = (500\ N.m)\times(0.02\ m)/(64\times10^{-8}\ m^4) = 15625000\ N/m^2$

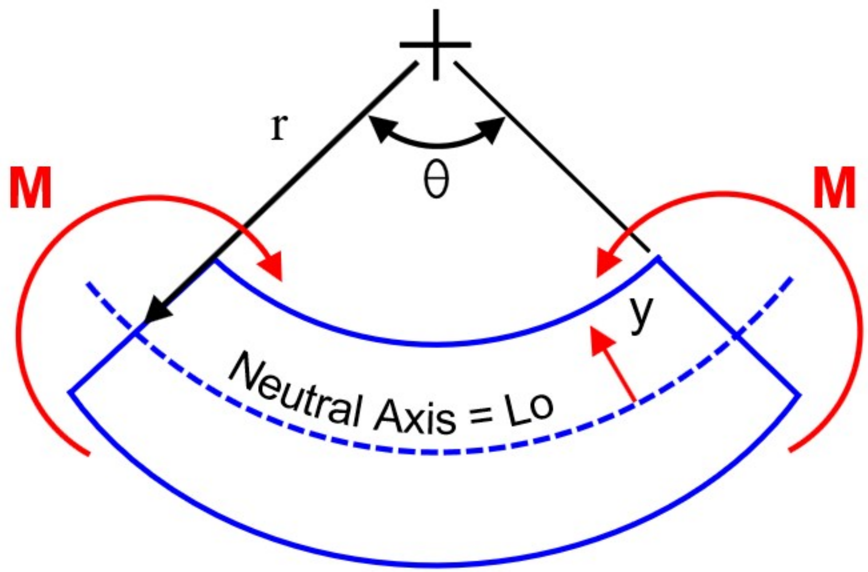

y
Positive bending moment
+M
+M
Positive curvature
O
x

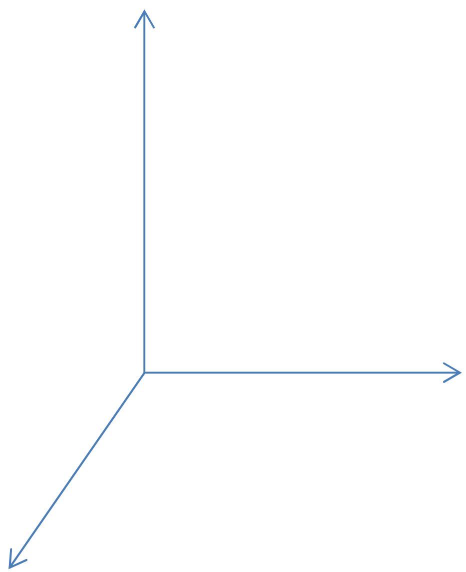

y
Negative bending moment
−M
−M
Negative curvature
O
x

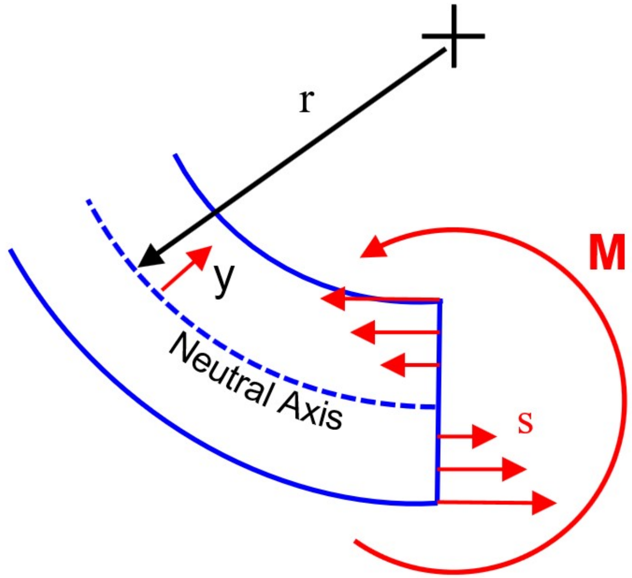

h
L


y
x
M_z
z


Neutral Axis
y
Lo


r
θ
M
M
y
Neutral Axis = Lo

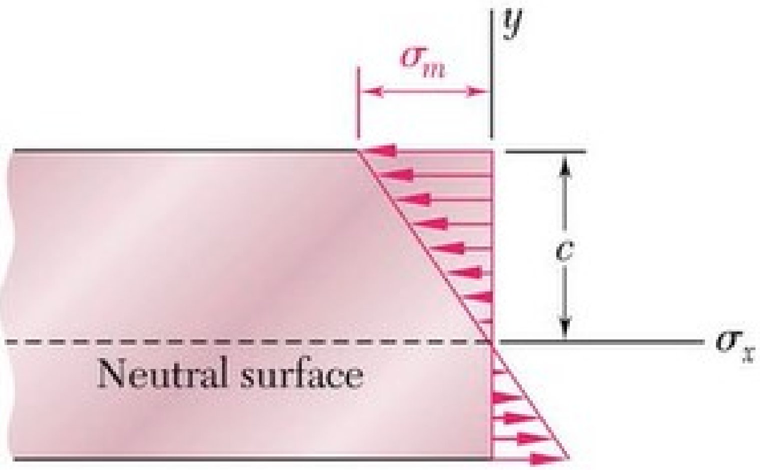



r
M
y
Neutral Axis
s

- The resultant moment about the z-axis must be equal to the applied moment M.

$$\int_A -y\sigma_x dA = \int_A -yE\varepsilon_x dA = \frac{E}{\rho}\int_A y^2 dA = M$$

Definition of the second moment of inertia, I



$$M = \frac{EI}{\rho}$$

$$\sigma_x = -\frac{My}{I}$$

$$\sigma = \frac{Mc}{I}$$

where $c = y_{max}$

:

$$\sigma = \frac{M}{Z}$$

where $Z = \frac{I}{c}$ is called the Section Modulus

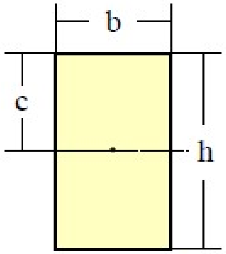
Centroid and Moments of Inertia

Centroid $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A}$$

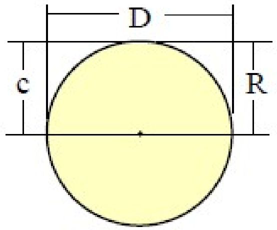
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

Moments of Inertia:



$$\bar{I} = \frac{b \cdot h^3}{12}$$ h is perpendicular to axis

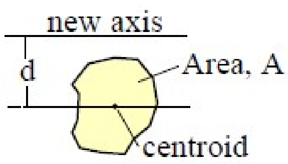
$$Z = \frac{\bar{I}}{c} = \frac{b \cdot h^2}{6}$$



$$\bar{I} = \frac{\pi \cdot D^4}{64} = \frac{\pi \cdot R^4}{4}$$

$$Z = \frac{\bar{I}}{c} = \frac{\pi \cdot D^3}{32} = \frac{\pi \cdot R^3}{4}$$

Parallel Axis Theorem:



$$I = \bar{I} + A \cdot d^2$$

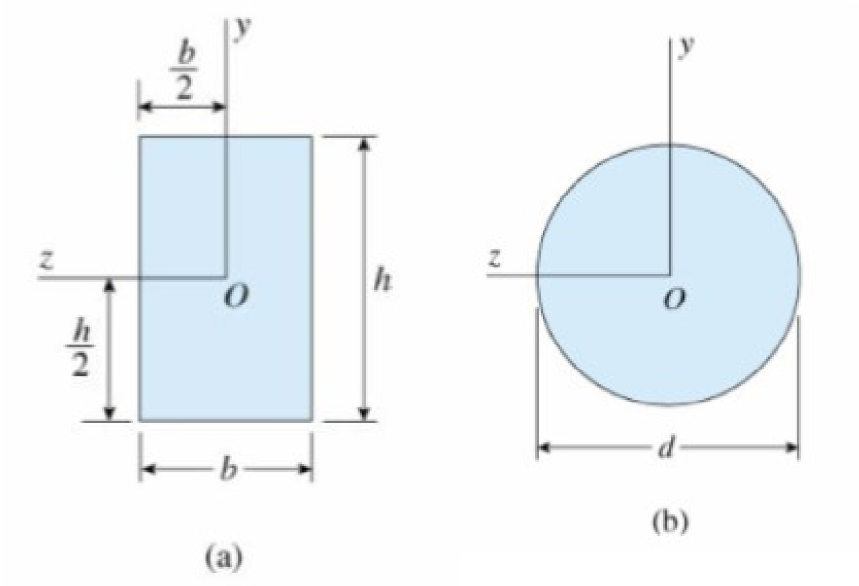
I = Moment of inertia about new axis
$\bar{I}$ = Moment of inertia about the centroidal axis
A = Area of the region
d = perpendicular distance between the two axes.

for rectangular cross section

$$I = \frac{b\,h^3}{12} \qquad S = \frac{b\,h^2}{6}$$

for circular cross section

$$I = \frac{\pi\,d^4}{64} \qquad S = \frac{\pi\,d^3}{32}$$

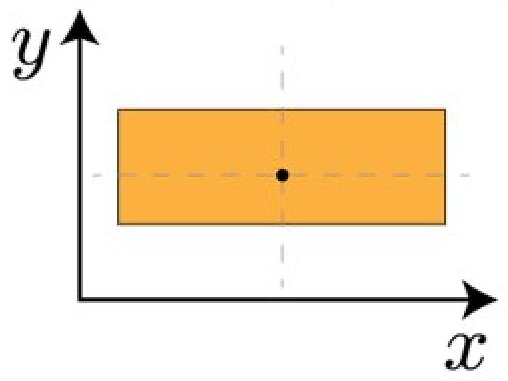


Moment of inertia:

second moment of area.

$$I_x = \int_A y^2 \, dA$$

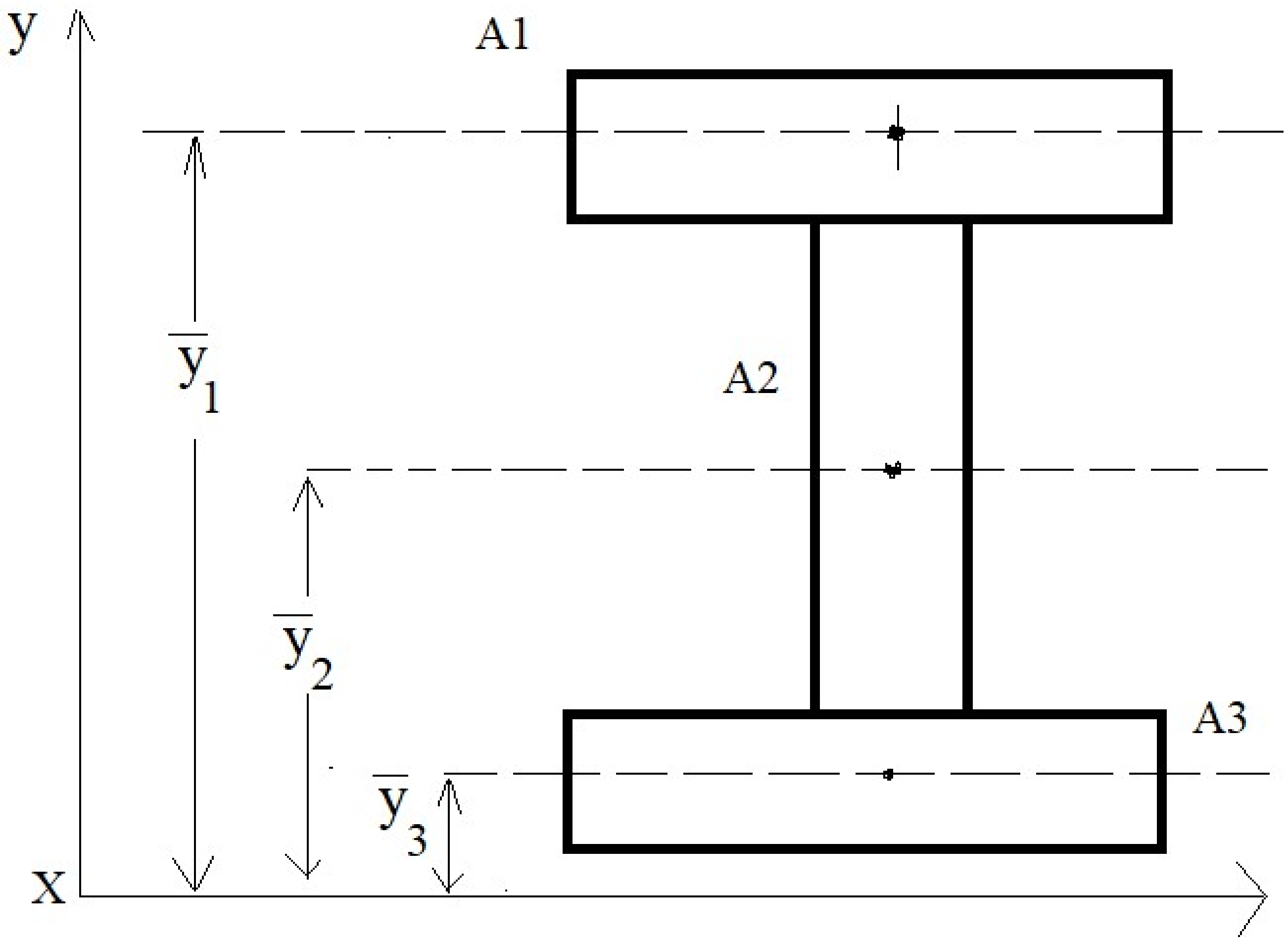
$$I_y = \int_A x^2 \, dA$$



When the axis is not passing through the centroid of an area, we use the parallel axis theorem

$$I_x = \bar{I}_x + A\bar{y}^2$$

Locating the neutral axis for composite areas (cross-sections)

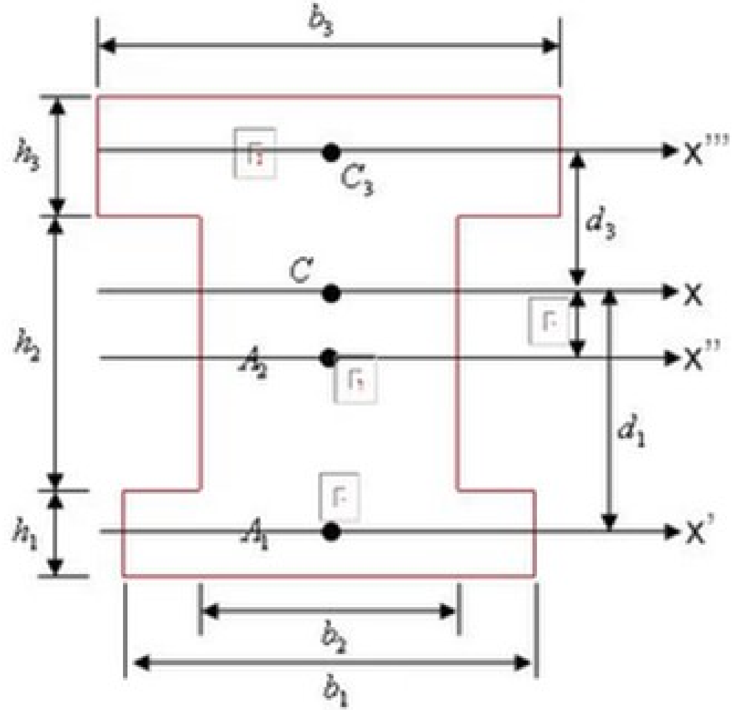


- <u>Centroid</u>

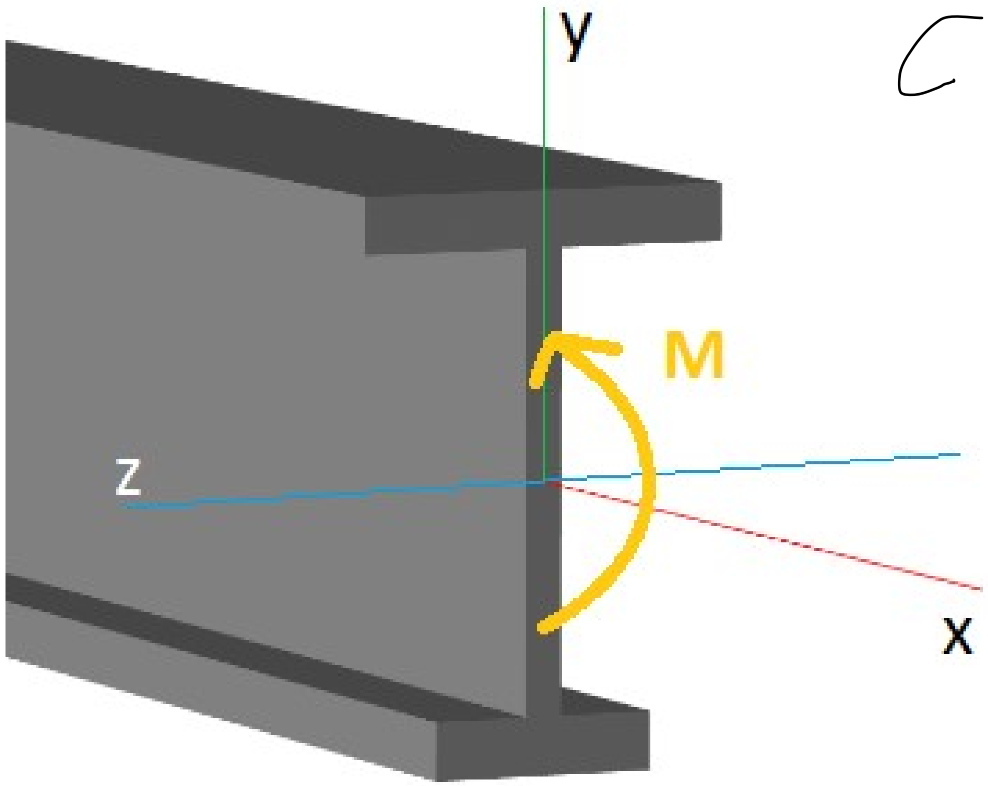
$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + \cdots}{A_1 + A_2 + \cdots}$$

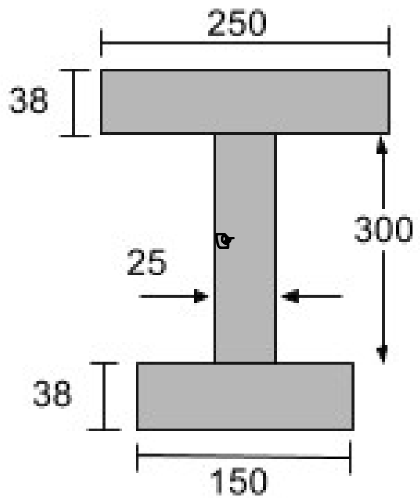
- Divide the composite shape into individual shapes.
- Calculate I of each shape about its own centroid.
- Calculate I of each shape about the centroid of the entire composite shape using the parallel axis theorem: $(I_x)_i = (\overline{I}_{x'})_i + A_i d_i^2$

	$\overline{I}_{x'}$	$A_i(d_i)^2$	$\overline{I}_x + Ad^2$
A1	$b_1h_1^3/12$	$b_1h_1d_1^2$	$b_1h_1(d_1^2+h_1^2/12)$
A2	$b_2h_2^3/12$	$b_2h_2d_2^2$	$b_2h_2(d_2^2+h_2^2/12)$
A3	$b_3h_3^3/12$	$b_3h_3d_3^2$	$b_3h_3(d_3^2+h_3^2/12)$

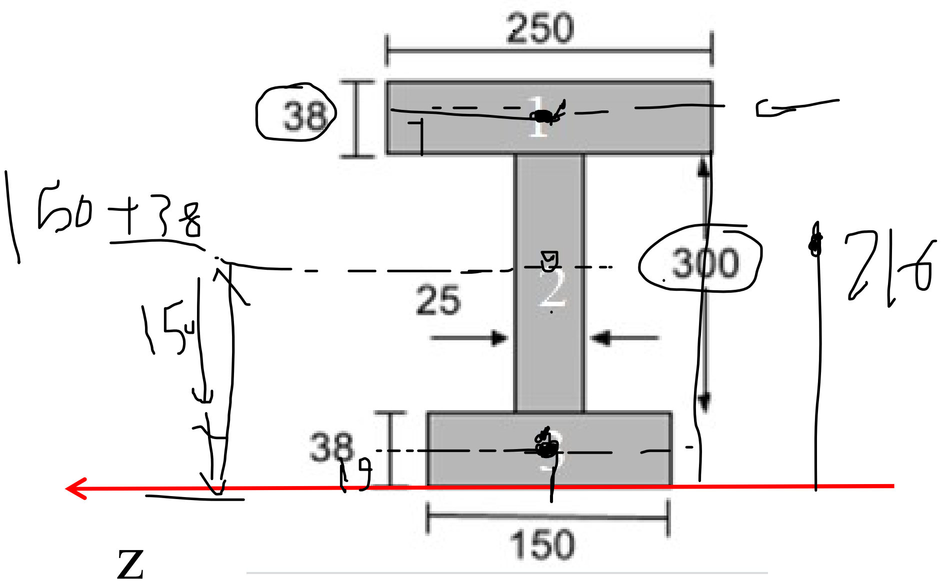


Ex:The I beam is made from steel . Find the moment of inertia for the cross section around the centroidal axis parallel to the z-direction





300 + 38 + 19



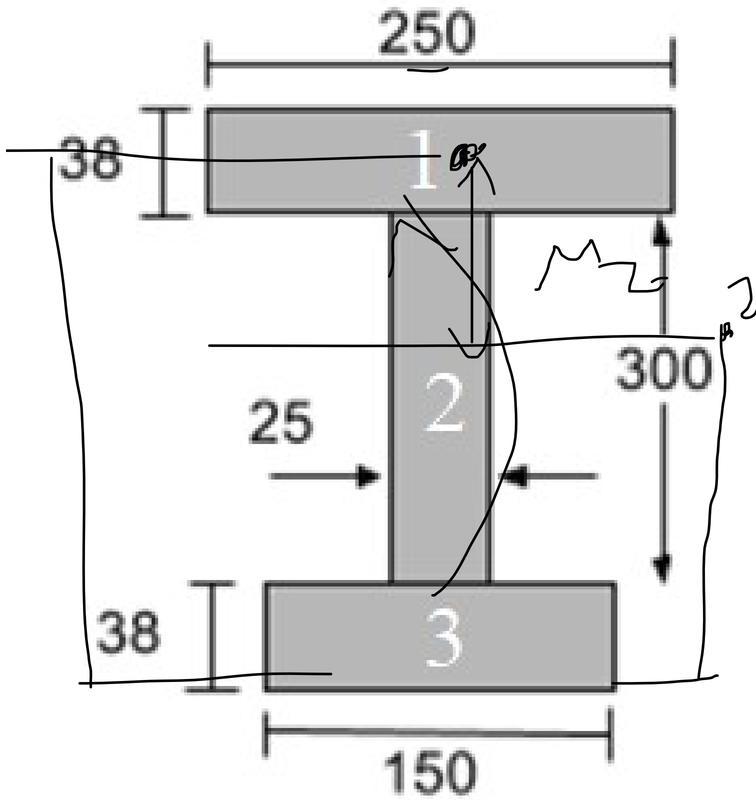
$$A_1 = 250 \times 38 = 9500\ \text{mm}^2$$
$$y_1 = 38 + 300 + \frac{38}{2} = 357\ \text{mm}$$

$$A_2 = 300 \times 25 = 7500\ \text{mm}^2$$
$$y_2 = 38 + \frac{300}{2} = 188\ \text{mm}$$

$$A_3 = 38 \times 150 = 5700\ \text{mm}^2$$
$$y_3 = \frac{38}{2} = 19\ \text{mm}$$

$$\begin{aligned}\bar{y} &= \frac{\sum y_i A_i}{\sum A_i} \\ &= \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} \\ &= \frac{(9500 \times 357) + (7500 \times 188) + (5700 \times 19)}{9500 + 7500 + 5700}\end{aligned}$$

$$\bar{y} = 216.29\ \text{mm}$$



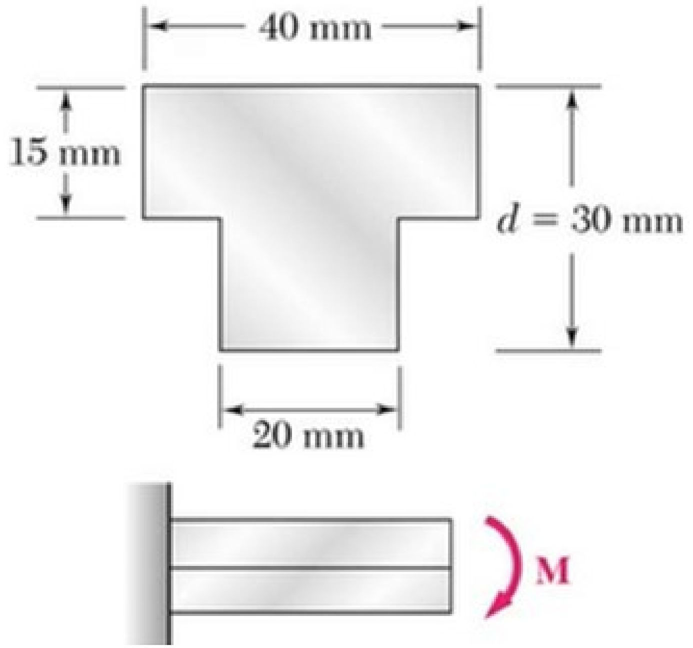
$$\bar{I}_1 = \frac{1}{12}(250)(38)^3 = 1,143,166.667\ \text{mm}^4$$
$$A_1 = 250 \times 38 = 9500\ \text{mm}^2$$
$$d_1 = |y_1 - \bar{y}| = \left|(38 + 300 + \frac{38}{2}) - 216.29\right| = 140.71\ \text{mm}$$

$$\bar{I}_2 = \frac{1}{12}(25)(300)^3 = 56,250,000\ \text{mm}^4$$
$$A_2 = 300 \times 25 = 7500\ \text{mm}^2$$
$$d_2 = |y_2 - \bar{y}| = \left|(38 + \frac{300}{2}) - 216.29\right| = 28.29\ \text{mm}$$

$$\bar{I}_3 = \frac{1}{12}(150)(38)^3 = 685,900\ \text{mm}^4$$
$$A_3 = 150 \times 38 = 5700\ \text{mm}^2$$
$$d_3 = |y_3 - \bar{y}| = \left|\frac{38}{2} - 216.29\right| = 197.29\ \text{mm}$$

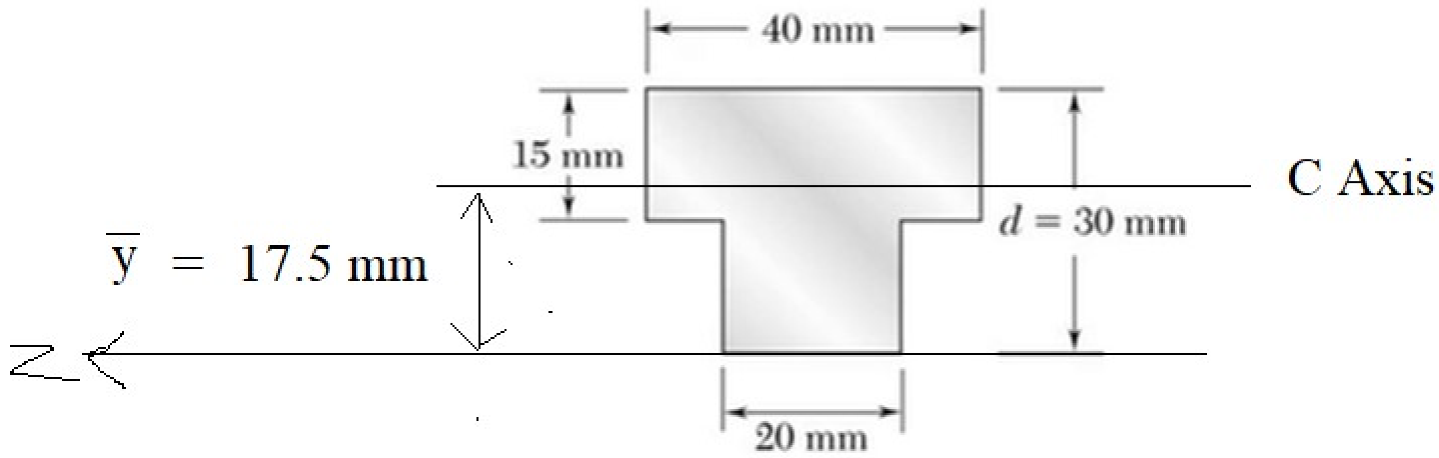
$$\begin{aligned} I_{total} &= \sum (\bar{I}_i + A_i {d_i}^2) \\ &= (\bar{I}_1 + A_1 {d_1}^2) + (\bar{I}_2 + A_2 {d_2}^2) + (\bar{I}_3 + A_3 {d_3}^2) \\ &= (1,143,166.667 + 9500 \times 140.71^2) + (56,250,000 + 7500 \times 28.29^2) + (685,900 + 5700 \times 197.29^2) \\ &= 474,037,947.7\ \text{mm}^4 \end{aligned}$$
$$I_{total} = 4.74 \times 10^8\ \text{mm}^4$$

EX. The beam shown is made of nylon with an allowable stress of 24 MPa in tension and 30 Mpa in compression. Determine the largest **M** that can be applied to the beam.



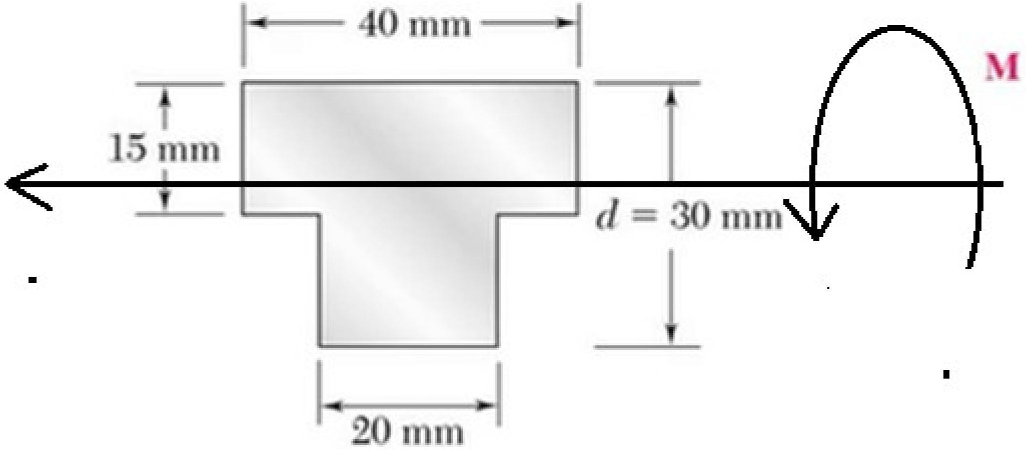
Second moment of area $[I_{zz}] = 61870 \text{ mm}^4$

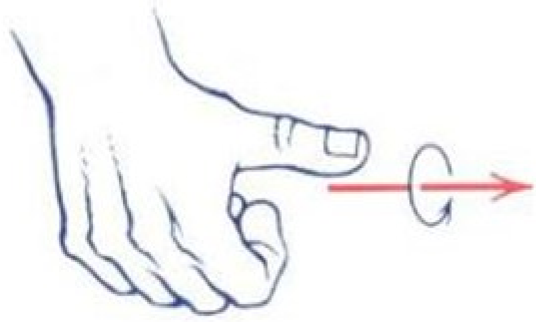
$$\sigma_{xx} = -\frac{M_Z \cdot y}{I_z}$$



in compression

$$-30 \times 10^6 = -\frac{M_Z \cdot -0.0175}{61870 \times 10^{-12}}$$





in Tension

$$24 \times 10^{6} = -\frac{M_z \cdot +0.0125}{61870 \times 10^{-12}}$$