

Chapter 2

2-1 From Tables A-20, A-21, A-22, and A-24c,

(a) UNS G10200 HR: $S_{ut} = 380$ (55) MPa (kpsi), $S_{yt} = 210$ (30) Mpa (kpsi) *Ans.*

(b) SAE 1050 CD: $S_{ut} = 690$ (100) MPa (kpsi), $S_{yt} = 580$ (84) Mpa (kpsi) *Ans.*

(c) AISI 1141 Q&T at 540°C (1000°F): $S_{ut} = 896$ (130) MPa (kpsi), $S_{yt} = 765$ (111) Mpa (kpsi) *Ans.*

(d) 2024-T4: $S_{ut} = 446$ (64.8) MPa (kpsi), $S_{yt} = 296$ (43.0) Mpa (kpsi) *Ans.*

(e) Ti-6Al-4V annealed: $S_{ut} = 900$ (130) MPa (kpsi), $S_{yt} = 830$ (120) Mpa (kpsi) *Ans.*

2-2 (a) Maximize yield strength: Q&T at 425°C (800°F) *Ans.*

(b) Maximize elongation: Q&T at 650°C (1200°F) *Ans.*

2-3 Conversion of kN/m^3 to kg/m^3 multiply by $1(10^3) / 9.81 = 102$

AISI 1018 CD steel: Tables A-20 and A-5

$$\frac{S_y}{\rho} = \frac{370(10^3)}{76.5(102)} = 47.4 \text{ kN} \cdot \text{m/kg} \quad \text{Ans.}$$

2011-T6 aluminum: Tables A-22 and A-5

$$\frac{S_y}{\rho} = \frac{169(10^3)}{26.6(102)} = 62.3 \text{ kN} \cdot \text{m/kg} \quad \text{Ans.}$$

Ti-6Al-4V titanium: Tables A-24c and A-5

$$\frac{S_y}{\rho} = \frac{830(10^3)}{43.4(102)} = 187 \text{ kN} \cdot \text{m/kg} \quad \text{Ans.}$$

ASTM No. 40 cast iron: Tables A-24a and A-5. Does not have a yield strength. Using the ultimate strength in tension

$$\frac{S_{ut}}{\rho} = \frac{42.5(6.89)(10^3)}{70.6(102)} = 40.7 \text{ kN} \cdot \text{m/kg} \quad \text{Ans}$$

2-4

AISI 1018 CD steel: Table A-5

$$\frac{E}{\gamma} = \frac{30.0(10^6)}{0.282} = 106(10^6) \text{ in} \quad \text{Ans.}$$

2011-T6 aluminum: Table A-5

$$\frac{E}{\gamma} = \frac{10.4(10^6)}{0.098} = 106(10^6) \text{ in} \quad \text{Ans.}$$

Ti-6Al-6V titanium: Table A-5

$$\frac{E}{\gamma} = \frac{16.5(10^6)}{0.160} = 103(10^6) \text{ in } Ans.$$

No. 40 cast iron: Table A-5

$$\frac{E}{\gamma} = \frac{14.5(10^6)}{0.260} = 55.8(10^6) \text{ in } Ans.$$

2-5

$$2G(1+\nu) = E \Rightarrow \nu = \frac{E - 2G}{2G}$$

From Table A-5

$$\text{Steel: } \nu = \frac{30.0 - 2(11.5)}{2(11.5)} = 0.304 \text{ Ans.}$$

$$\text{Aluminum: } \nu = \frac{10.4 - 2(3.90)}{2(3.90)} = 0.333 \text{ Ans.}$$

$$\text{Beryllium copper: } \nu = \frac{18.0 - 2(7.0)}{2(7.0)} = 0.286 \text{ Ans.}$$

$$\text{Gray cast iron: } \nu = \frac{14.5 - 2(6.0)}{2(6.0)} = 0.208 \text{ Ans.}$$

2-6 (a) $A_0 = \pi(0.503)^2/4$, $\sigma = P_i / A_0$

For data in elastic range, $\epsilon = \Delta l / l_0 = \Delta l / 2$

$$\text{For data in plastic range, } \epsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 = \frac{A_0}{A} - 1$$

On the next two pages, the data and plots are presented. Figure (a) shows the linear part of the curve from data points 1-7. Figure (b) shows data points 1-12. Figure (c) shows the complete range. **Note:** The exact value of A_0 is used without rounding off.

(b) From Fig. (a) the slope of the line from a linear regression is $E = 30.5 \text{ Mpsi}$ Ans.

From Fig. (b) the equation for the dotted offset line is found to be

$$\sigma = 30.5(10^6)\epsilon - 61\,000 \quad (1)$$

The equation for the line between data points 8 and 9 is

$$\sigma = 7.60(10^5)\epsilon + 42\,900 \quad (2)$$

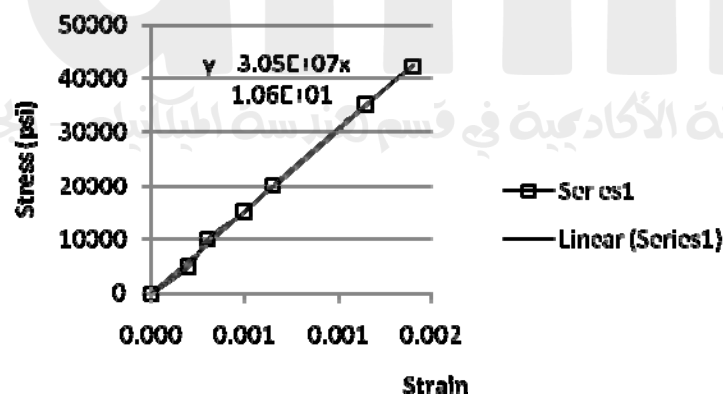
Solving Eqs. (1) and (2) simultaneously yields $\sigma = 45.6$ kpsi which is the 0.2 percent offset yield strength. Thus, $S_y = 45.6$ kpsi Ans.

The ultimate strength from Figure (c) is $S_u = 85.6$ kpsi Ans.

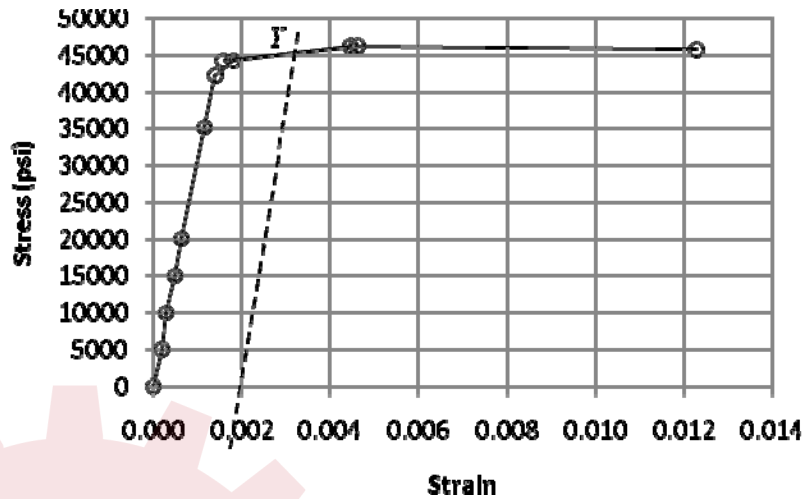
The reduction in area is given by Eq. (2-12) is

$$R = \frac{A_0 - A_f}{A_0} (100) = \frac{0.1987 - 0.1077}{0.1987} (100) = 45.8 \% \quad \text{Ans.}$$

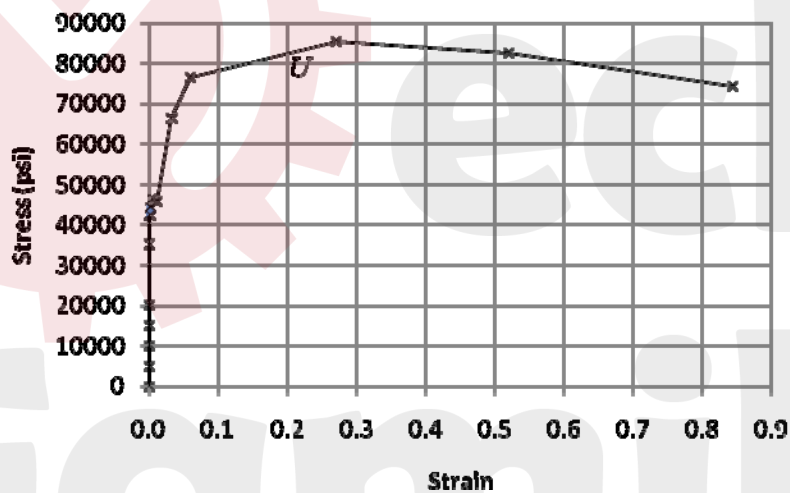
Data Point	P_i	$\Delta l, A_i$	ϵ	σ
1	0	0	0	0
2	1000	0.0004	0.00020	5032
3	2000	0.0006	0.00030	10065
4	3000	0.001	0.00050	15097
5	4000	0.0013	0.00065	20130
6	7000	0.0023	0.00115	35227
7	8400	0.0028	0.00140	42272
8	8800	0.0036	0.00180	44285
9	9200	0.0089	0.00445	46298
10	8800	0.1984	0.00158	44285
11	9200	0.1978	0.00461	46298
12	9100	0.1963	0.01229	45795
13	13200	0.1924	0.03281	66428
14	15200	0.1875	0.05980	76492
15	17000	0.1563	0.27136	85551
16	16400	0.1307	0.52037	82531
17	14800	0.1077	0.84506	74479



(a) Linear range



(b) Offset yield



(c) Complete range

(c) The material is ductile since there is a large amount of deformation beyond yield.

(d) The closest material to the values of S_y , S_{ut} , and R is SAE 1045 HR with $S_y = 45$ kpsi, $S_{ut} = 82$ kpsi, and $R = 40$ %. *Ans.*

2-7 To plot σ_{true} vs. ϵ , the following equations are applied to the data.

$$\sigma_{\text{true}} = \frac{P}{A}$$

Eq. (2-4)

$$\varepsilon = \ln \frac{l}{l_0} \quad \text{for } 0 \leq \Delta l \leq 0.0028 \text{ in}$$

$$\varepsilon = \ln \frac{A_0}{A} \quad \text{for } \Delta l > 0.0028 \text{ in}$$

$$\text{where } A_0 = \frac{\pi(0.503)^2}{4} = 0.1987 \text{ in}^2$$

The results are summarized in the table below and plotted on the next page. The last 5 points of data are used to plot $\log \sigma$ vs $\log \varepsilon$

The curve fit gives $m = 0.2306$
 $\log \sigma_0 = 5.1852 \Rightarrow \sigma_0 = 153.2 \text{ kpsi} \quad \text{Ans.}$

For 20% cold work, Eq. (2-14) and Eq. (2-17) give,

$$A = A_0 (1 - W) = 0.1987 (1 - 0.2) = 0.1590 \text{ in}^2$$

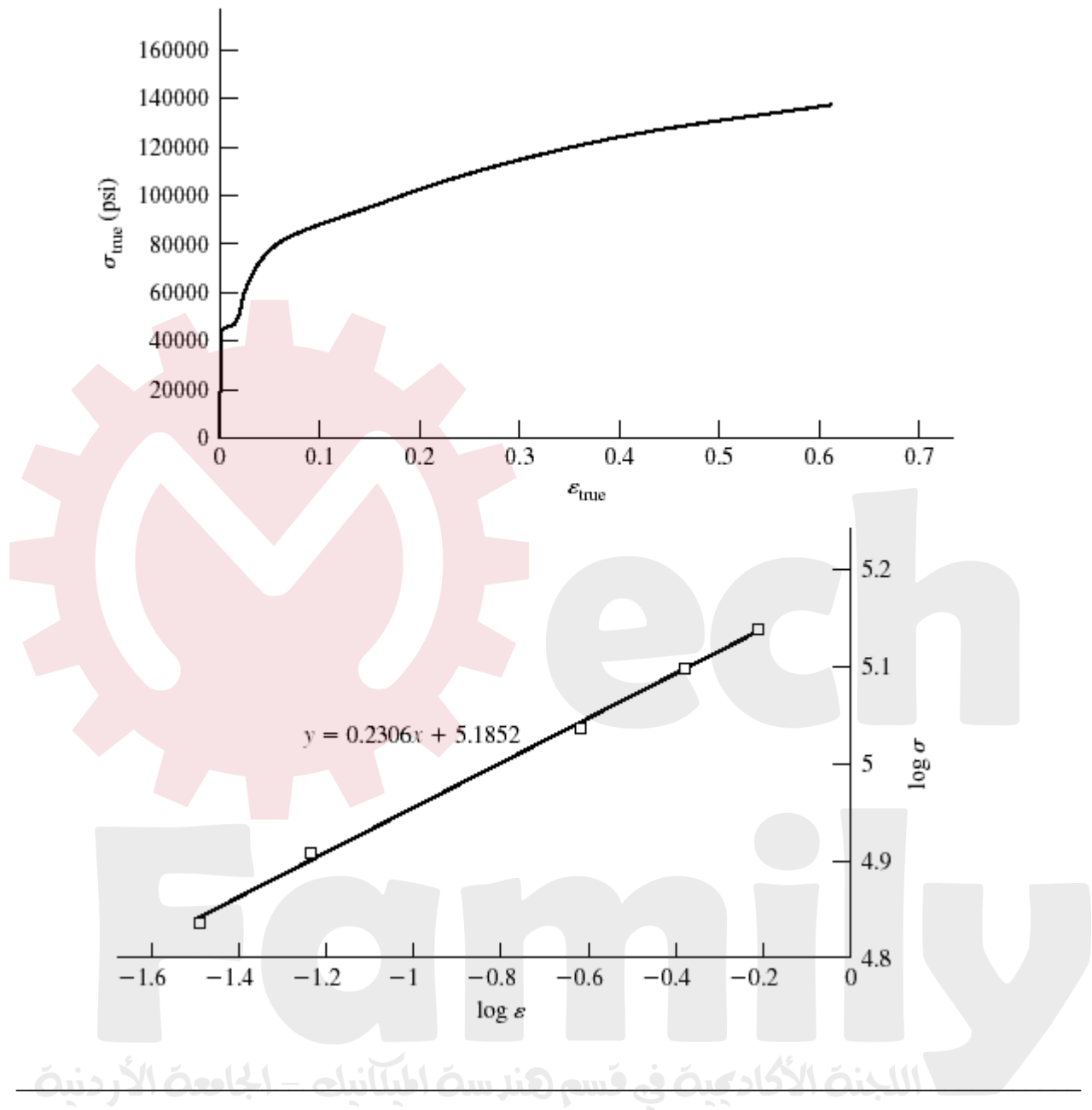
$$\varepsilon = \ln \frac{A_0}{A} = \ln \frac{0.1987}{0.1590} = 0.2231$$

$$\text{Eq. (2-18): } S_y = \sigma_0 \varepsilon^m = 153.2(0.2231)^{0.2306} = 108.4 \text{ kpsi} \quad \text{Ans.}$$

$$\text{Eq. (2-19), with } S_u = 85.6 \text{ from Prob. 2-6,}$$

$$S'_u = \frac{S_u}{1 - W} = \frac{85.6}{1 - 0.2} = 107 \text{ kpsi} \quad \text{Ans.}$$

P	ΔL	A	ε	σ_{true}	$\log \varepsilon$	$\log \sigma_{\text{true}}$
0	0	0.198 713	0	0		
1000	0.0004	0.198 713	0.000 2	5032.388	-3.699 01	3.701 774
2000	0.0006	0.198 713	0.000 3	10 064.78	-3.522 94	4.002 804
3000	0.001	0.198 713	0.000 5	15 097.17	-3.301 14	4.178 895
4000	0.0013	0.198 713	0.000 65	20 129.55	-3.187 23	4.303 834
7000	0.0023	0.198 713	0.001 149	35 226.72	-2.939 55	4.546 872
8400	0.0028	0.198 713	0.001 399	42 272.06	-2.854 18	4.626 053
8800	0.0036	0.198 4	0.001 575	44 354.84	-2.802 61	4.646 941
9200	0.0089	0.197 8	0.004 604	46 511.63	-2.336 85	4.667 562
9100		0.196 3	0.012 216	46 357.62	-1.913 05	4.666 121
13200		0.192 4	0.032 284	68 607.07	-1.491 01	4.836 369
15200		0.187 5	0.058 082	81 066.67	-1.235 96	4.908 842
17000		0.156 3	0.240 083	108 765.20	-0.619 64	5.036 49
16400		0.130 7	0.418 956	125 478.20	-0.377 83	5.098 568
14800		0.107 7	0.612 511	137 418.80	-0.212 89	5.138 046



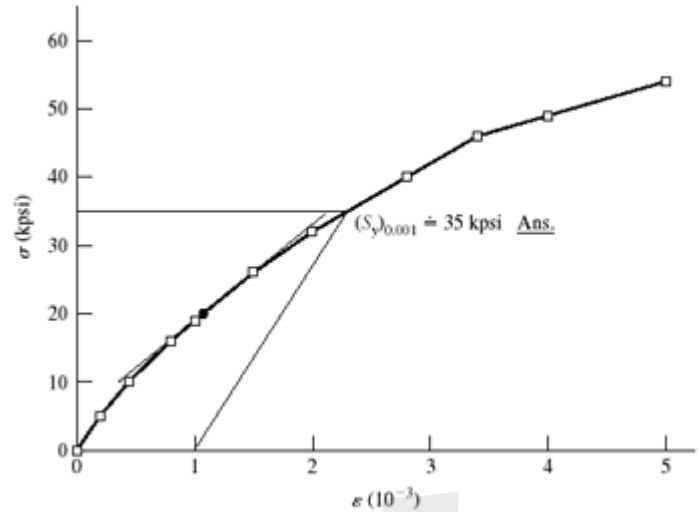
2-8 Tangent modulus at $\sigma = 0$ is

$$E = \frac{\Delta \sigma}{\Delta \epsilon} = \frac{5000 - 0}{0.2(10^{-3}) - 0} = 25(10^6) \text{ psi} \quad \text{Ans.}$$

At $\sigma = 20 \text{ kpsi}$

$$E_{20} = \frac{(26-19)(10^3)}{(1.5-1)(10^{-3})} = 14.0(10^6) \text{ psi} \quad \text{Ans.}$$

$\epsilon (10^{-3})$	$\sigma \text{ (kpsi)}$
0	0
0.20	5
0.44	10
0.80	16
1.0	19
1.5	26
2.0	32
2.8	40
3.4	46
4.0	49
5.0	54



2-9 $W = 0.20$,

(a) Before cold working: Annealed AISI 1018 steel. Table A-22, $S_y = 32 \text{ kpsi}$, $S_u = 49.5 \text{ kpsi}$, $\sigma_0 = 90.0 \text{ kpsi}$, $m = 0.25$, $\epsilon_f = 1.05$

After cold working: Eq. (2-16), $\epsilon_u = m = 0.25$

Eq. (2-14), $\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$

Eq. (2-17), $\epsilon_i = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 < \epsilon_u$

Eq. (2-18), $S_y' = \sigma_0 \epsilon_i^m = 90(0.223)^{0.25} = 61.8 \text{ kpsi} \quad \text{Ans.} \quad 93\% \text{ increase} \quad \text{Ans.}$

Eq. (2-19), $S_u' = \frac{S_u}{1-W} = \frac{49.5}{1-0.20} = 61.9 \text{ kpsi} \quad \text{Ans.} \quad 25\% \text{ increase} \quad \text{Ans.}$

(b) Before: $\frac{S_u}{S_y} = \frac{49.5}{32} = 1.55$ After: $\frac{S_u'}{S_y'} = \frac{61.9}{61.8} = 1.00 \quad \text{Ans.}$

Lost most of its ductility

2-10 $W = 0.20$,

(a) Before cold working: AISI 1212 HR steel. Table A-22, $S_y = 28 \text{ kpsi}$, $S_u = 61.5 \text{ kpsi}$, $\sigma_0 = 110 \text{ kpsi}$, $m = 0.24$, $\epsilon_f = 0.85$

After cold working: Eq. (2-16), $\epsilon_u = m = 0.24$

$$\text{Eq. (2-14), } \frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$$

$$\text{Eq. (2-17), } \epsilon_i = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 < \epsilon_u$$

$$\text{Eq. (2-18), } S_y' = \sigma_0 \epsilon_i^m = 110(0.223)^{0.24} = 76.7 \text{ kpsi} \quad \text{Ans. 174\% increase} \quad \text{Ans.}$$

$$\text{Eq. (2-19), } S_u' = \frac{S_u}{1-W} = \frac{61.5}{1-0.20} = 76.9 \text{ kpsi} \quad \text{Ans. 25\% increase} \quad \text{Ans.}$$

$$\text{(b) Before: } \frac{S_u}{S_y} = \frac{61.5}{28} = 2.20 \quad \text{After: } \frac{S_u'}{S_y'} = \frac{76.9}{76.7} = 1.00 \quad \text{Ans.}$$

Lost most of its ductility

2-11 $W = 0.20$,

(a) Before cold working: 2024-T4 aluminum alloy. Table A-22, $S_y = 43.0$ kpsi, $S_u = 64.8$ kpsi, $\sigma_0 = 100$ kpsi, $m = 0.15$, $\epsilon_f = 0.18$

After cold working: Eq. (2-16), $\epsilon_u = m = 0.15$

$$\text{Eq. (2-14), } \frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$$

$$\text{Eq. (2-17), } \epsilon_i = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 > \epsilon_f \quad \text{Material fractures.} \quad \text{Ans.}$$

2-12 For $H_B = 275$, Eq. (2-21), $S_u = 3.4(275) = 935$ MPa Ans.

2-13 Gray cast iron, $H_B = 200$.

$$\text{Eq. (2-22), } S_u = 0.23(200) - 12.5 = 33.5 \text{ kpsi} \quad \text{Ans.}$$

From Table A-24, this is probably ASTM No. 30 Gray cast iron Ans.

2-14 Eq. (2-21), $0.5H_B = 100 \Rightarrow H_B = 200$ Ans.

2-15 For the data given, converting H_B to S_u using Eq. (2-21)

H_B	S_u (kpsi)	S_u^2 (kpsi)
230	115	13225
232	116	13456
232	116	13456
234	117	13689
235	117.5	13806.25
235	117.5	13806.25
235	117.5	13806.25
236	118	13924
236	118	13924
239	119.5	14280.25
$\Sigma S_u =$	1172	$\Sigma S_u^2 =$ 137373

$$\bar{S}_u = \frac{\Sigma S_u}{N} = \frac{1172}{10} = 117.2 \div 117 \text{ kpsi} \quad \text{Ans.}$$

Eq. (20-8),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N \bar{S}_u^2}{N - 1}} = \sqrt{\frac{137373 - 10(117.2)^2}{9}} = 1.27 \text{ kpsi} \quad \text{Ans.}$$

2-16 For the data given, converting H_B to S_u using Eq. (2-22)

H_B	S_u (kpsi)	S_u^2 (kpsi)
230	40.4	1632.16
232	40.86	1669.54
232	40.86	1669.54
234	41.32	1707.342
235	41.55	1726.403
235	41.55	1726.403
235	41.55	1726.403
236	41.78	1745.568
236	41.78	1745.568
239	42.47	1803.701
$\Sigma S_u =$	414.12	$\Sigma S_u^2 =$ 17152.63

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{414.12}{10} = 41.4 \text{ kpsi} \quad \text{Ans.}$$

Eq. (20-8),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{17152.63 - 10(41.4)^2}{9}} = 1.20 \quad \text{Ans.}$$

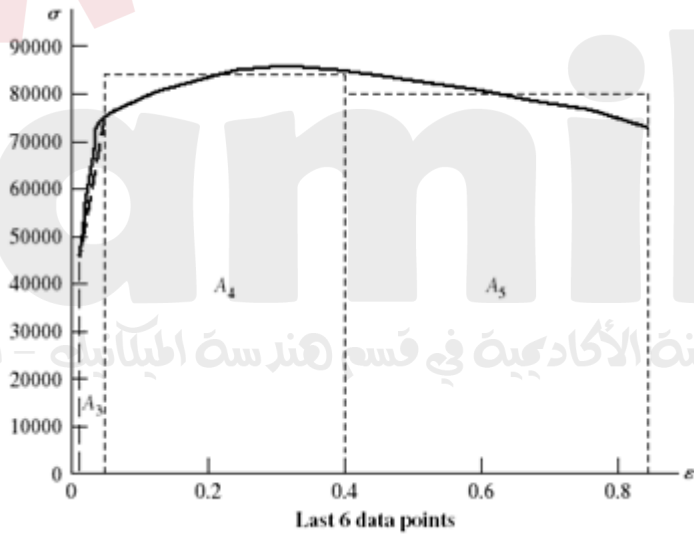
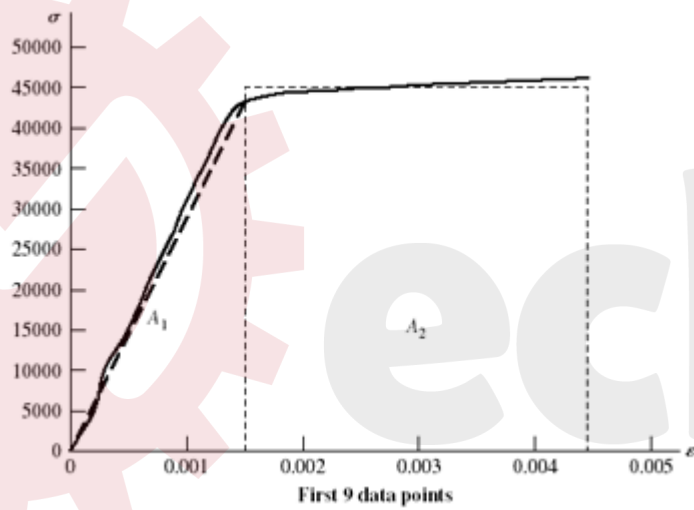
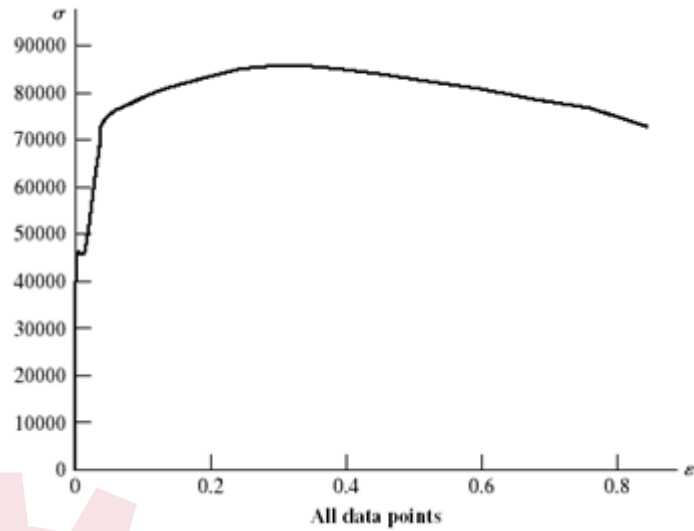
2-17 (a) $u_R \doteq \frac{45.5^2}{2(30)} = 34.5 \text{ in} \cdot \text{lbf} / \text{in}^3 \quad \text{Ans.}$

(b)

P	ΔL	A	$A_0 / A - 1$	ϵ	$\sigma = P/A_0$
0	0			0	0
1000	0.0004			0.0002	5 032.39
2000	0.0006			0.0003	10 064.78
3000	0.0010			0.0005	15 097.17
4000	0.0013			0.000 65	20 129.55
7000	0.0023			0.001 15	35 226.72
8400	0.0028			0.0014	42 272.06
8800	0.0036			0.0018	44 285.02
9200	0.0089			0.004 45	46 297.97
9100		0.1963	0.012 291	0.012 291	45 794.73
13200		0.1924	0.032 811	0.032 811	66 427.53
15200		0.1875	0.059 802	0.059 802	76 492.30
17000		0.1563	0.271 355	0.271 355	85 550.60
16400		0.1307	0.520 373	0.520 373	82 531.17
14800		0.1077	0.845 059	0.845 059	74 479.35

From the figures on the next page,

$$\begin{aligned}
 u_T &\doteq \sum_{i=1}^5 A_i = \frac{1}{2} (43\,000)(0.001\,5) + 45\,000(0.004\,45 - 0.001\,5) \\
 &\quad + \frac{1}{2} (45\,000 + 76\,500) + (0.059\,8 - 0.004\,45) \\
 &\quad + 81\,000(0.4 - 0.059\,8) + 80\,000(0.845 - 0.4) \\
 &\doteq 66.7(10^3) \text{ in} \cdot \text{lbf/in}^3 \quad \text{Ans.}
 \end{aligned}$$



2-18, 2-19 These problems are for student research. No standard solutions are provided.

2-20 Appropriate tables: Young's modulus and Density (Table A-5) 1020 HR and CD (Table A-20), 1040 and 4140 (Table A-21), Aluminum (Table A-24), Titanium (Table A-24c)

Appropriate equations:

$$\text{For diameter, } \sigma = \frac{F}{A} = \frac{F}{(\pi/4)d^2} = S_y \quad \Rightarrow \quad d = \sqrt{\frac{4F}{\pi S_y}}$$

Weight/length = ρA , Cost/length = \$/in = (\$/lbf) Weight/length,

Deflection/length = $\delta/L = F/(AE)$

With $F = 100 \text{ kips} = 100(10^3) \text{ lbf}$,

Material	Young's Modulus	Density	Yield Strength	Cost/lbf	Diameter	Weight/length	Cost/length	Deflection/length
units	Mpsi	lbf/in ³	kpsi	\$/lbf	in	lbf/in	\$/in	in/in
1020 HR	30	0.282	30	\$0.27	2.060	0.9400	\$0.25	1.000E-03
1020 CD	30	0.282	57	\$0.30	1.495	0.4947	\$0.15	1.900E-03
1040	30	0.282	80	\$0.35	1.262	0.3525	\$0.12	2.667E-03
4140	30	0.282	165	\$0.80	0.878	0.1709	\$0.14	5.500E-03
Al	10.4	0.098	50	\$1.10	1.596	0.1960	\$0.22	4.808E-03
Ti	16.5	0.16	120	\$7.00	1.030	0.1333	\$0.93	7.273E-03

The selected materials with minimum values are shaded in the table above.

Ans.

2-21 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three would favor steel, cast iron, or maybe a less common ferrous material. The expectation would likely be hot-rolled steel. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 7.95 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{7.95 \text{ lbf}}{[\pi(1 \text{ in})^2 / 4](36 \text{ in})} = 0.281 \text{ lbf/in}^3 \doteq 0.28 \text{ lbf/in}^3$$

which agrees well with the unit weight of 0.282 lbf/in³ reported in Table A-5 for carbon steel. Nickel steel and stainless steel have similar unit weights, but surface finish and darker coloring do not favor their selection. To select a likely specification from Table

A-20, perform a Brinell hardness test, then use Eq. (2-21) to estimate an ultimate strength of $S_u = 0.5H_B = 0.5(200) = 100$ kpsi. Assuming the material is hot-rolled due to the rough surface finish, appropriate choices from Table A-20 would be one of the higher carbon steels, such as hot-rolled AISI 1050, 1060, or 1080. *Ans.*

- 2-22** First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous material like aluminum. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 2.90 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{2.9 \text{ lbf}}{[\pi(1 \text{ in})^2 / 4](36 \text{ in})} = 0.103 \text{ lbf/in}^3 \doteq 0.10 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.098 lbf/in³ reported in Table A-5 for aluminum. No other materials come close to this unit weight, so the material is likely aluminum. *Ans.*

- 2-23** First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous copper-based material such as copper, brass, or bronze. To further distinguish the material, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 9 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{9.0 \text{ lbf}}{[\pi(1 \text{ in})^2 / 4](36 \text{ in})} = 0.318 \text{ lbf/in}^3 \doteq 0.32 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.322 lbf/in³ reported in Table A-5 for copper. Brass is not far off (0.309 lbf/in³), so the deflection test could be used to gain additional insight. From the measured deflection and utilizing the deflection equation for an end-loaded cantilever beam from Table A-9, Young's modulus is determined to be

$$E = \frac{Fl^3}{3I_y} = \frac{100(24)^3}{3(\pi(1)^4/64)(17/32)} = 17.7 \text{ Mpsi}$$

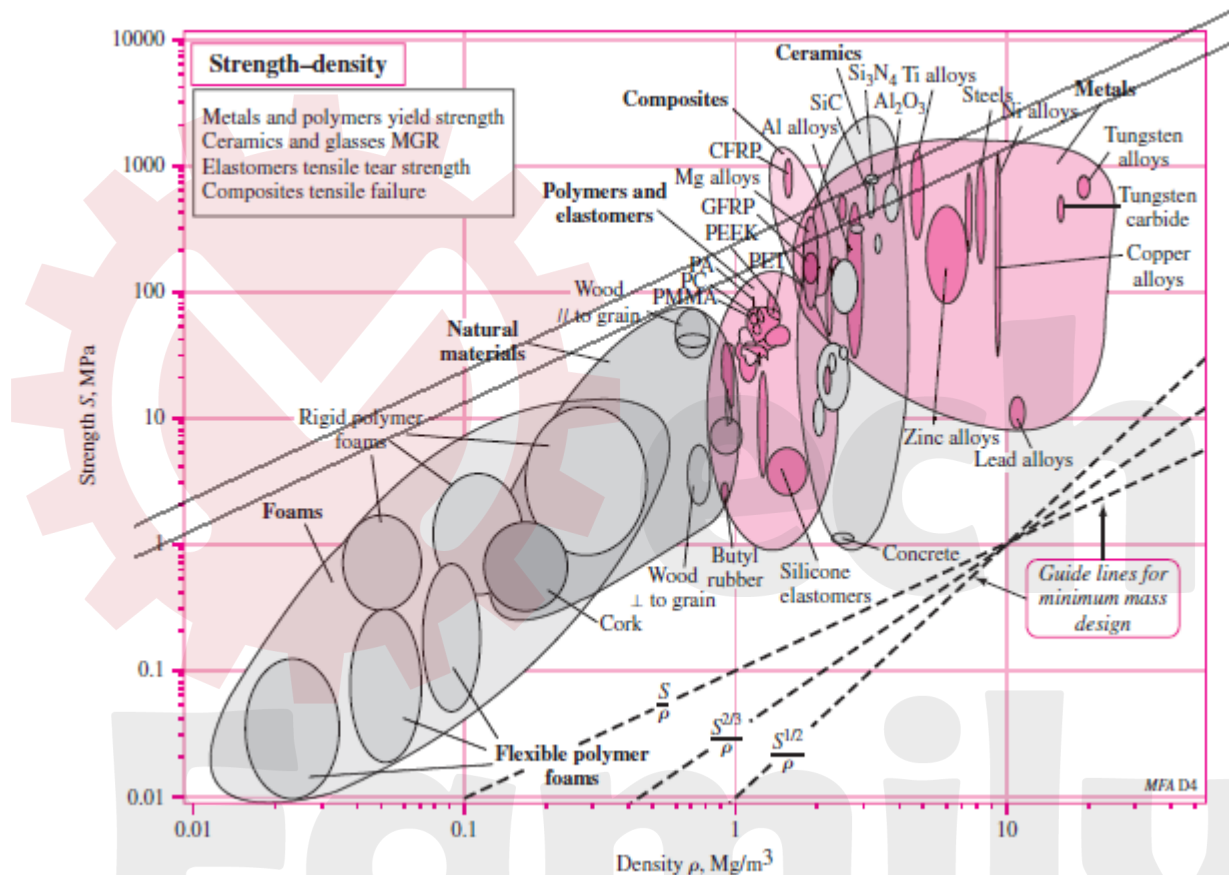
which agrees better with the modulus for copper (17.2 Mpsi) than with brass (15.4 Mpsi). The conclusion is that the material is likely copper. *Ans.*

- 2-24 and 2-25** These problems are for student research. No standard solutions are provided.
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2-26 For strength, $\sigma = F/A = S \Rightarrow A = F/S$
 For mass, $m = A l \rho = (F/S) l \rho$

Thus, $f_3(M) = \rho/S$, and maximize S/ρ ($\beta = 1$)

In Fig. (2-19), draw lines parallel to S/ρ

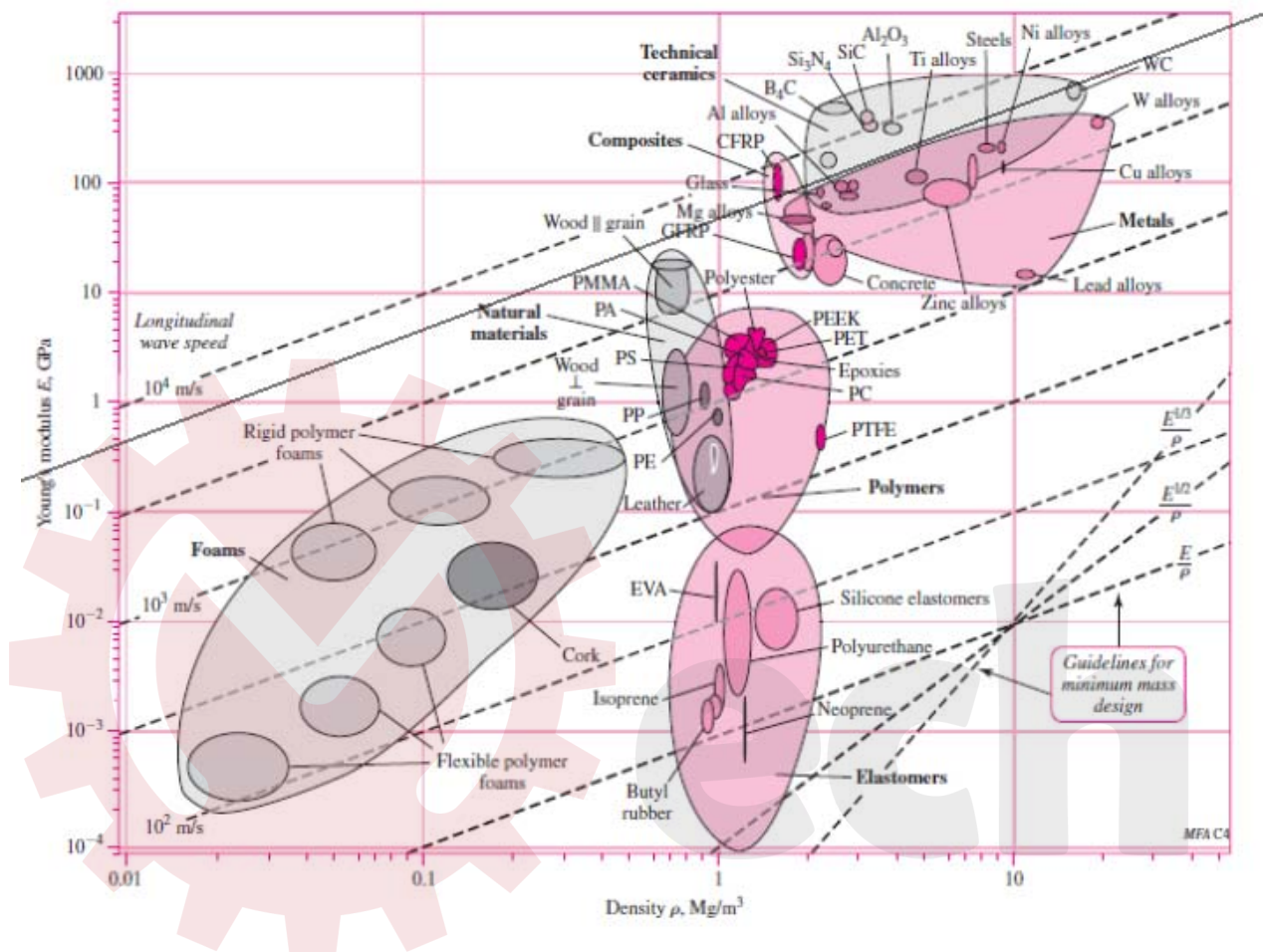


From the list of materials given, both **aluminum alloy** and **high carbon heat treated steel** are good candidates, having greater potential than tungsten carbide or polycarbonate. The higher strength aluminum alloys have a slightly greater potential. Other factors, such as cost or availability, may dictate which to choose. *Ans.*

2-27 For stiffness, $k = AE/l \Rightarrow A = kl/E$
 For mass, $m = A l \rho = (kl/E) l \rho = kl^2 \rho/E$

Thus, $f_3(M) = \rho/E$, and maximize E/ρ ($\beta = 1$)

In Fig. (2-16), draw lines parallel to E/ρ



From the list of materials given, **tungsten carbide (WC)** is best, closely followed by aluminum alloys, and then followed by high carbon heat-treated steel. They are close enough that other factors, like cost or availability, would likely dictate the best choice. Polycarbonate polymer is clearly not a good choice compared to the other candidate materials. Ans.

2-28 For strength,

$$\sigma = Fl/Z = S \quad (1)$$

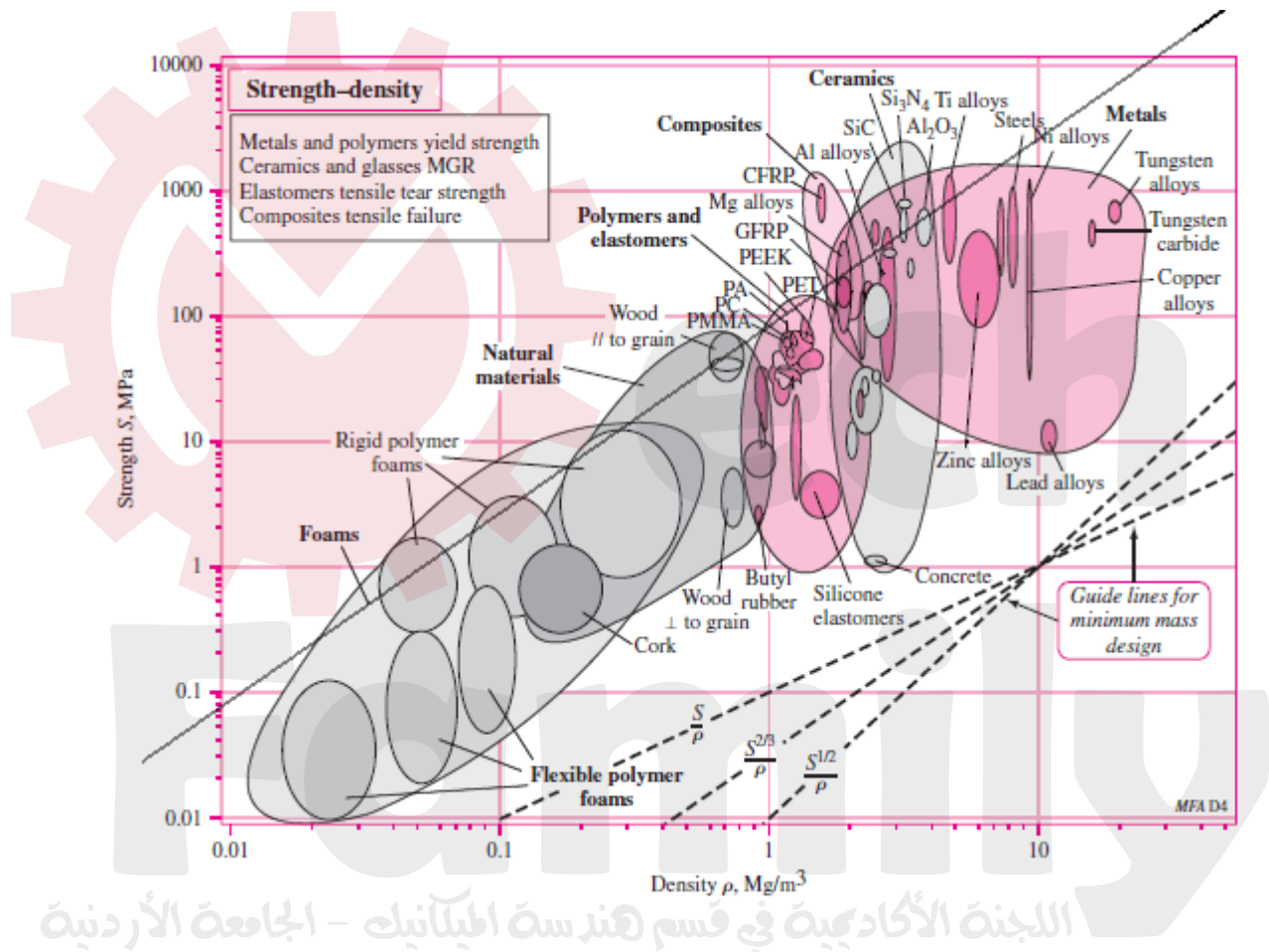
where Fl is the bending moment and Z is the section modulus [see Eq. (3-26b), p. 90]. The section modulus is strictly a function of the dimensions of the cross section and has the units in³ (ips) or m³ (SI). Thus, for a given cross section, $Z = C(A)^{3/2}$, where C is a number. For example, for a circular cross section, $C = (4\sqrt{\pi})^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \quad \Rightarrow \quad A = \left(\frac{Fl}{CS} \right)^{2/3} \quad (2)$$

For mass,
$$m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3} \left(\frac{\rho}{S^{2/3}}\right)$$

Thus, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$ ($\beta = 2/3$)

In Fig. (2-19), draw lines parallel to $S^{2/3}/\rho$



From the list of materials given, a higher strength **aluminum alloy** has the greatest potential, followed closely by high carbon heat-treated steel. Tungsten carbide is clearly not a good choice compared to the other candidate materials. .Ans.

2-29 Eq. (2-26), p. 65, applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where C is a constant. For example, consider a rectangular section of height h and width b , where for a given scaled shape, $h = cb$, where c is a

constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$ (a constant).

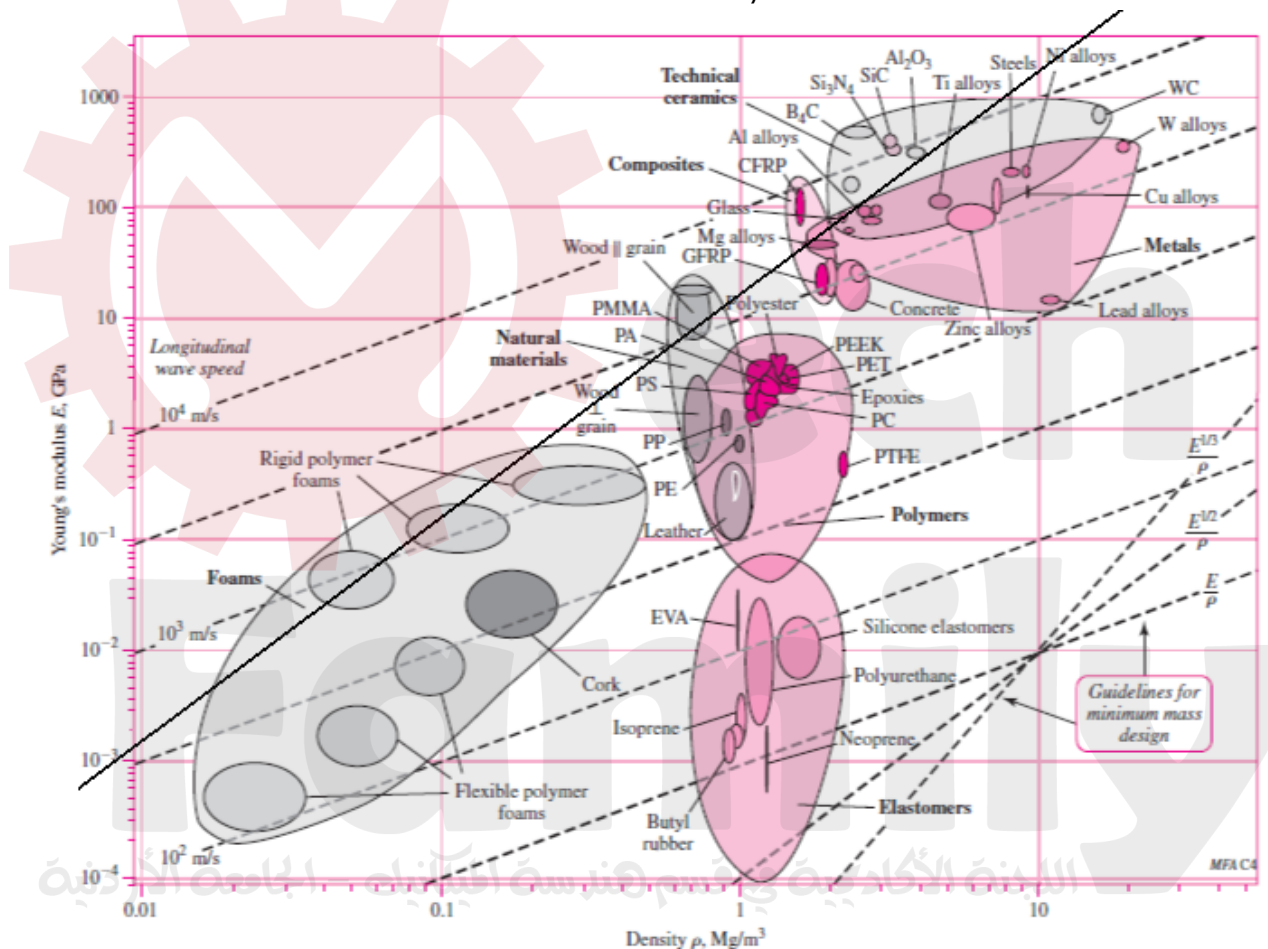
Thus, Eq. (2-27) becomes

$$A = \left(\frac{kl^3}{3CE} \right)^{1/2}$$

and Eq. (2-29) becomes

$$m = Al\rho = \left(\frac{k}{3C} \right)^{1/2} l^{5/2} \left(\frac{\rho}{E^{1/2}} \right)$$

Thus, minimize $f_3(M) = \frac{\rho}{E^{1/2}}$, or maximize $M = \frac{E^{1/2}}{\rho}$. From Fig. (2-16)



From the list of materials given, **aluminum alloys** are clearly the best followed by steels and tungsten carbide. Polycarbonate polymer is not a good choice compared to the other candidate materials. Ans.

- 2-30 For stiffness, $k = AE/l \Rightarrow A = kl/E$
For mass, $m = Al\rho = (kl/E) l\rho = kl^2 \rho/E$

So, $f_3(M) = \rho/E$, and maximize E/ρ . Thus, $\beta = 1$. Ans.

2-31 For strength, $\sigma = F/A = S \Rightarrow A = F/S$

For mass, $m = Al\rho = (F/S)l\rho$

So, $f_3(M) = \rho/S$, and maximize S/ρ . Thus, $\beta = 1$. Ans.

2-32 Eq. (2-26), p. 65, applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where C is a constant. For example, consider a rectangular section of height h and width b , where for a given scaled shape, $h = cb$, where c is a constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$.

Thus, Eq. (2-27) becomes

$$A = \left(\frac{kl^3}{3CE} \right)^{1/2}$$

and Eq. (2-29) becomes

$$m = Al\rho = \left(\frac{k}{3C} \right)^{1/2} l^{5/2} \left(\frac{\rho}{E^{1/2}} \right)$$

So, minimize $f_3(M) = \frac{\rho}{E^{1/2}}$, or maximize $M = \frac{E^{1/2}}{\rho}$. Thus, $\beta = 1/2$. Ans.

2-33 For strength,

$$\sigma = Fl/Z = S \quad (1)$$

where Fl is the bending moment and Z is the section modulus [see Eq. (3-26b), p. 90]. The section modulus is strictly a function of the dimensions of the cross section and has the units in³ (ips) or m³ (SI). Thus, for a given cross section, $Z = C(A)^{3/2}$, where C is a number. For example, for a circular cross section, $C = (4\sqrt{\pi})^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \Rightarrow A = \left(\frac{Fl}{CS} \right)^{2/3} \quad (2)$$

For mass,

$$m = Al\rho = \left(\frac{Fl}{CS} \right)^{2/3} l\rho = \left(\frac{F}{C} \right)^{2/3} l^{5/3} \left(\frac{\rho}{S^{2/3}} \right)$$

So, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$. Thus, $\beta = 2/3$. Ans.

2-34 For stiffness, $k = AE/l$, or, $A = kl/E$.

Thus, $m = \rho Al = \rho (kl/E)l = kl^2 \rho /E$. Then, $M = E / \rho$ and $\beta = 1$.

From Fig. 2-16, lines parallel to E / ρ for ductile materials include steel, titanium, molybdenum, aluminum alloys, and composites.

For strength, $S = F/A$, or, $A = F/S$.

Thus, $m = \rho Al = \rho F/S l = Fl \rho /S$. Then, $M = S/\rho$ and $\beta = 1$.

From Fig. 2-19, lines parallel to S/ρ give for ductile materials, steel, aluminum alloys, nickel alloys, titanium, and composites.

Common to both stiffness and strength are steel, titanium, aluminum alloys, and composites. *Ans.*

