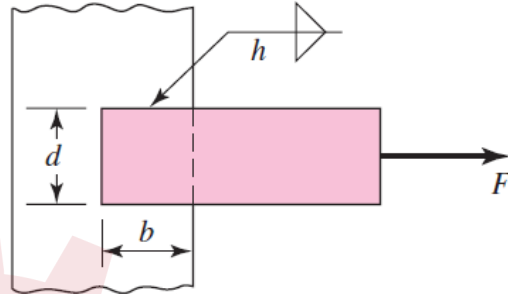


Chapter 9

**Figure for Probs.
9-1 to 9-4**



9-1 Given, $b = 50 \text{ mm}$, $d = 50 \text{ mm}$, $h = 5 \text{ mm}$, $\tau_{\text{allow}} = 140 \text{ MPa}$.

$$F = 0.707 hl \tau_{\text{allow}} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \quad \text{Ans.}$$

9-2 Given, $b = 2 \text{ in}$, $d = 2 \text{ in}$, $h = 5/16 \text{ in}$, $\tau_{\text{allow}} = 25 \text{ kpsi}$.

$$F = 0.707 hl \tau_{\text{allow}} = 0.707(5/16)[2(2)](25) = 22.1 \text{ kip} \quad \text{Ans.}$$

9-3 Given, $b = 50 \text{ mm}$, $d = 30 \text{ mm}$, $h = 5 \text{ mm}$, $\tau_{\text{allow}} = 140 \text{ MPa}$.

$$F = 0.707 hl \tau_{\text{allow}} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \quad \text{Ans.}$$

9-4 Given, $b = 4 \text{ in}$, $d = 2 \text{ in}$, $h = 5/16 \text{ in}$, $\tau_{\text{allow}} = 25 \text{ kpsi}$.

$$F = 0.707 hl \tau_{\text{allow}} = 0.707(5/16)[2(4)](25) = 44.2 \text{ kip} \quad \text{Ans.}$$

9-5 Prob. 9-1 with E7010 Electrode.

Table 9-6: $f = 14.85 h \text{ kip/in} = 14.85 [5 \text{ mm}/(25.4 \text{ mm/in})] = 2.923 \text{ kip/in}$

$$= 2.923(4.45/25.4) = 0.512 \text{ kN/mm}$$

$$F = fl = 0.512[2(50)] = 51.2 \text{ kN} \quad \text{Ans.}$$

9-6 Prob. 9-2 with E6010 Electrode.

Table 9-6: $f = 14.85 h \text{ kip/in} = 14.85(5/16) = 4.64 \text{ kip/in}$

$$F = fl = 4.64[2(2)] = 18.6 \text{ kip} \quad \text{Ans.}$$

9-7 Prob. 9-3 with E7010 Electrode.

Table 9-6: $f = 14.85 h \text{ kip/in} = 14.85 [5 \text{ mm}/(25.4 \text{ mm/in})] = 2.923 \text{ kip/in}$
 $= 2.923(4.45/25.4) = 0.512 \text{ kN/mm}$

$$F = fl = 0.512[2(50)] = 51.2 \text{ kN} \quad \text{Ans.}$$

9-8 Prob. 9-4 with E6010 Electrode.

Table 9-6: $f = 14.85 h \text{ kip/in} = 14.85(5/16) = 4.64 \text{ kip/in}$

$$F = fl = 4.64[2(4)] = 37.1 \text{ kip} \quad \text{Ans.}$$

9-9 Table A-20:

1018 CD: $S_{ut} = 440 \text{ MPa}$, $S_y = 370 \text{ MPa}$

1018 HR: $S_{ut} = 400 \text{ MPa}$, $S_y = 220 \text{ MPa}$

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned} \tau_{all} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(400), 0.40(220)] \\ &= \min(120, 88) = 88 \text{ MPa} \end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{all} = 0.707(5)[2(50)](88)(10^{-3}) = 31.1 \text{ kN} \quad \text{Ans.}$

9-10 Table A-20:

1020 CD: $S_{ut} = 68 \text{ kpsi}$, $S_y = 57 \text{ kpsi}$

1020 HR: $S_{ut} = 55 \text{ kpsi}$, $S_y = 30 \text{ kpsi}$

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned} \tau_{all} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(55), 0.40(30)] \\ &= \min(16.5, 12.0) = 12.0 \text{ kpsi} \end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{all} = 0.707(5/16)[2(2)](12.0) = 10.6 \text{ kip} \quad \text{Ans.}$

9-11 Table A-20:1035 HR: $S_{ut} = 500$ MPa, $S_y = 270$ MPa1035 CD: $S_{ut} = 550$ MPa, $S_y = 460$ MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{all} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(500), 0.40(270)] \\ &= \min(150, 108) = 108 \text{ MPa}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{all} = 0.707(5)[2(50)](108)(10^{-3}) = 38.2 \text{ kN} \quad \text{Ans.}$

9-12 Table A-20:1035 HR: $S_{ut} = 72$ kpsi, $S_y = 39.5$ kpsi1020 CD: $S_{ut} = 68$ kpsi, $S_y = 57$ kpsi, 1020 HR: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{all} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(55), 0.40(30)] \\ &= \min(16.5, 12.0) = 12.0 \text{ kpsi}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{all} = 0.707(5/16)[2(4)](12.0) = 21.2 \text{ kip} \quad \text{Ans.}$

9-13

Eq. (9-3): $\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+50)]} = 141 \text{ MPa} \quad \text{Ans.}$

9-14

Eq. (9-3): $\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(2+2)]} = 22.6 \text{ kpsi} \quad \text{Ans.}$

9-15 Eq. (9-3): $\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+30)]} = 177 \text{ MPa} \quad \text{Ans.}$

9-16 Eq. (9-3): $\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(4+2)]} = 15.1 \text{ kpsi} \quad \text{Ans.}$

9-17 $b = d = 50 \text{ mm}$, $c = 150 \text{ mm}$, $h = 5 \text{ mm}$, and $\tau_{\text{allow}} = 140 \text{ MPa}$.

(a) *Primary shear*, Table 9-1, Case 2 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{50[3(50^2) + 50^2]}{6} = 83.33(10^3) \text{ mm}^3$$

$$J = 0.707 h J_u = 0.707(5)(83.33)(10^3) = 294.6(10^3) \text{ mm}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F(10^3)(25)}{294.6(10^3)} = 14.85F$$

$$\tau_{\text{max}} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{14.85^2 + (2.829 + 14.85)^2} = 23.1F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{140}{23.1} = 6.06 \text{ kN} \quad \text{Ans.}$$

(b) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21 \text{ kpsi} = 21(6.89) = 145 \text{ MPa}$

1020 HR bar: $S_{ut} = 380 \text{ MPa}$, $S_y = 210 \text{ MPa}$

1015 HR support: $S_{ut} = 340 \text{ MPa}$, $S_y = 190 \text{ MPa}$

Table 9-3, E7010 Electrode: $S_{ut} = 482 \text{ MPa}$, $S_y = 393 \text{ MPa}$

The support controls the design.

$$\begin{aligned} \text{Table 9-4: } \tau_{\text{allow}} &= \min(0.30S_{ut}, 0.40S_y) = \min[0.30(340), 0.40(190)] = \min(102, 76) \\ &= 76 \text{ MPa} \end{aligned}$$

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{76}{23.1} = 3.29 \text{ kN} \quad \text{Ans.}$$

9-18 $b = d = 2 \text{ in}$, $c = 6 \text{ in}$, $h = 5/16 \text{ in}$, and $\tau_{\text{allow}} = 25 \text{ kpsi}$.

(a) *Primary shear*, Table 9-1 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.132F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{2[3(2^2) + 2^2]}{6} = 5.333 \text{ in}^3$$

$$J = 0.707 h J_u = 0.707(5/16)(5.333) = 1.178 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.178} = 5.942F$$

$$\tau_{\max} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{5.942^2 + (1.132 + 5.942)^2} = 9.24F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{25}{9.24} = 2.71 \text{ kip} \quad \text{Ans.}$$

(b) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21 \text{ kpsi}$

1020 HR bar: $S_{ut} = 55 \text{ kpsi}$, $S_y = 30 \text{ kpsi}$

1015 HR support: $S_{ut} = 50 \text{ kpsi}$, $S_y = 27.5 \text{ kpsi}$

Table 9-3, E7010 Electrode: $S_{ut} = 70 \text{ kpsi}$, $S_y = 57 \text{ kpsi}$

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(50), 0.40(27.5)] = \min(15, 11) = 11 \text{ kpsi}$

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{11}{9.24} = 1.19 \text{ kip} \quad \text{Ans.}$$

9-19 $b = 50 \text{ mm}$, $c = 150 \text{ mm}$, $d = 30 \text{ mm}$, $h = 5 \text{ mm}$, and $\tau_{\text{allow}} = 140 \text{ MPa}$.

(a) *Primary shear*, Table 9-1, Case 2 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{50[3(30^2) + 50^2]}{6} = 43.33(10^3) \text{ mm}^3$$

$$J = 0.707 h J_u = 0.707(5)(43.33)(10^3) = 153.2(10^3) \text{ mm}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{175F(10^3)(15)}{153.2(10^3)} = 17.13F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{175F(10^3)(25)}{153.2(10^3)} = 28.55F$$

$$\tau_{\max} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{17.13^2 + (2.829 + 28.55)^2} = 35.8F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{140}{35.8} = 3.91 \text{ kN} \quad \text{Ans.}$$

(b) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21 \text{ kpsi} = 21(6.89) = 145 \text{ MPa}$

1020 HR bar: $S_{ut} = 380 \text{ MPa}$, $S_y = 210 \text{ MPa}$

1015 HR support: $S_{ut} = 340 \text{ MPa}$, $S_y = 190 \text{ MPa}$

Table 9-3, E7010 Electrode: $S_{ut} = 482 \text{ MPa}$, $S_y = 393 \text{ MPa}$

The support controls the design.

$$\begin{aligned} \text{Table 9-4: } \tau_{\text{allow}} &= \min(0.30S_{ut}, 0.40S_y) = \min[0.30(340), 0.40(190)] = \min(102, 76) \\ &= 76 \text{ MPa} \end{aligned}$$

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{76}{35.8} = 2.12 \text{ kN} \quad \text{Ans.}$$

9-20 $b = 4 \text{ in}$, $c = 6 \text{ in}$, $d = 2 \text{ in}$, $h = 5/16 \text{ in}$, and $\tau_{\text{allow}} = 25 \text{ kpsi}$.

(a) Primary shear, Table 9-1 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(4)} = 0.5658F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{4[3(2^2) + 4^2]}{6} = 18.67 \text{ in}^3$$

$$J = 0.707 h J_u = 0.707(5/16)(18.67) = 4.125 \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{8F(1)}{4.125} = 1.939F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{4.125} = 3.879F$$

$$\tau_{\max} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{1.939^2 + (0.5658 + 3.879)^2} = 4.85F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{25}{4.85} = 5.15 \text{ kip} \quad \text{Ans.}$$

(b) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi

1020 HR bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

1015 HR support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-3, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(50), 0.40(27.5)] = \min(15, 11)$

$$= 11 \text{ kpsi}$$

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{11}{4.85} = 2.27 \text{ kip} \quad \text{Ans.}$$

9-21 Given, $b = 50 \text{ mm}$, $c = 150 \text{ mm}$, $d = 50 \text{ mm}$, $h = 5 \text{ mm}$, $\tau_{\text{allow}} = 140 \text{ MPa}$.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau'_y = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50+50)} = 1.414F$$

Secondary shear:

Table 9-1: $J_u = \frac{(b+d)^3}{6} = \frac{(50+50)^3}{6} = 166.7(10^3) \text{ mm}^3$
 $J = 0.707 h J_u = 0.707(5)166.7(10^3) = 589.2(10^3) \text{ mm}^4$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F(10^3)(25)}{589.2(10^3)} = 7.425F$$

Maximum shear:

$$\tau_{\text{max}} = \sqrt{\tau_x''^2 + (\tau'_y + \tau_y'')^2} = F\sqrt{7.425^2 + (1.414 + 7.425)^2} = 11.54F$$

$$F = \frac{\tau_{\text{allow}}}{11.54} = \frac{140}{11.54} = 12.1 \text{ kN} \quad \text{Ans.}$$

9-22 Given, $b = 2 \text{ in}$, $c = 6 \text{ in}$, $d = 2 \text{ in}$, $h = 5/16 \text{ in}$, $\tau_{\text{allow}} = 25 \text{ kpsi}$.

Primary shear:

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2+2)} = 0.5658F$$

Secondary shear:

Table 9-1: $J_u = \frac{(b+d)^3}{6} = \frac{(2+2)^3}{6} = 10.67 \text{ in}^3$
 $J = 0.707 h J_u = 0.707(5/16)10.67 = 2.357 \text{ in}^4$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{2.357} = 2.970F$$

Maximum shear:

$$\tau_{\text{max}} = \sqrt{\tau_x''^2 + (\tau'_y + \tau_y'')^2} = F\sqrt{2.970^2 + (0.566 + 2.970)^2} = 4.618F$$

$$F = \frac{\tau_{\text{allow}}}{4.618} = \frac{25}{4.618} = 5.41 \text{ kip} \quad \text{Ans.}$$

9-23 Given, $b = 50 \text{ mm}$, $c = 150 \text{ mm}$, $d = 30 \text{ mm}$, $h = 5 \text{ mm}$, $\tau_{\text{allow}} = 140 \text{ MPa}$.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau'_y = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50+30)} = 1.768F$$

Secondary shear:

Table 9-1: $J_u = \frac{(b+d)^3}{6} = \frac{(50+30)^3}{6} = 85.33(10^3) \text{ mm}^3$
 $J = 0.707 h J_u = 0.707(5)85.33(10^3) = 301.6(10^3) \text{ mm}^4$

$$\tau''_x = \frac{Mr_y}{J} = \frac{175F(10^3)(15)}{301.6(10^3)} = 8.704F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{175F(10^3)(25)}{301.6(10^3)} = 14.51F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{8.704^2 + (1.768 + 14.51)^2} = 18.46F$$

$$F = \frac{\tau_{\text{allow}}}{18.46} = \frac{140}{18.46} = 7.58 \text{ kN} \quad \text{Ans.}$$

9-24 Given, $b = 4$ in, $c = 6$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear:

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(4+2)} = 0.3772F$$

Secondary shear:

Table 9-1: $J_u = \frac{(b+d)^3}{6} = \frac{(4+2)^3}{6} = 36 \text{ in}^3$
 $J = 0.707 h J_u = 0.707(5/16)36 = 7.954 \text{ in}^4$

$$\tau''_x = \frac{Mr_y}{J} = \frac{8F(1)}{7.954} = 1.006F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{7.954} = 2.012F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{1.006^2 + (0.3772 + 2.012)^2} = 2.592F$$

$$F = \frac{\tau_{allow}}{2.592} = \frac{25}{2.592} = 9.65 \text{ kip} \quad Ans.$$

9-25 Given, $b = 50 \text{ mm}$, $d = 50 \text{ mm}$, $h = 5 \text{ mm}$, E6010 electrode.

$$A = 0.707(5)(50 + 50 + 50) = 530.3 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 320 \text{ MPa}$.

$$\text{Eq. 6-19 and Table 6-2, pp. 287, 288:} \quad k_a = 272(320)^{-0.995} = 0.875$$

$$k_b = 1 \text{ (uniform shear)}, k_c = 0.59 \text{ (torsion, shear)}, k_d = 1$$

$$\text{Eqs. (6-8) and (6-18):} \quad S_e = 0.875(1)(0.59)(1)(0.5)(320) = 82.6 \text{ MPa}$$

$$\text{Electrode endurance: E6010, Table 9-3,} \quad S_{ut} = 427 \text{ MPa}$$

$$\text{Eq. 6-19 and Table 6-2, pp. 287, 288:} \quad k_a = 272(427)^{-0.995} = 0.657$$

$$\text{As before, } k_b = 1 \text{ (direct shear)}, k_c = 0.59 \text{ (torsion, shear)}, k_d = 1$$

$$S_e = 0.657(1)(0.59)(1)(0.5)(427) = 82.8 \text{ MPa}$$

The members and electrode are basically of equal strength. We will use $S_e = 82.6 \text{ MPa}$.
For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{allow} A}{K_{fs}} = \frac{82.6(530.3)}{2.7} = 16.2(10^3) \text{ N} = 16.2 \text{ kN} \quad Ans.$$

9-26 Given, $b = 2 \text{ in}$, $d = 2 \text{ in}$, $h = 5/16 \text{ in}$, E6010 electrode.

$$A = 0.707(5/16)(2 + 2 + 2) = 1.326 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 47 \text{ kpsi}$.

$$\text{Eq. 6-19 and Table 6-2, pp. 287, 288:} \quad k_a = 39.9(47)^{-0.995} = 0.865$$

$$k_b = 1 \text{ (uniform shear)}, k_c = 0.59 \text{ (torsion, shear)}, k_d = 1$$

$$\text{Eqs. (6-8) and (6-18):} \quad S_e = 0.865(1)(0.59)(1)(0.5)(47) = 12.0 \text{ kpsi}$$

$$\text{Electrode endurance: E6010, Table 9-3,} \quad S_{ut} = 62 \text{ kpsi}$$

$$\text{Eq. 6-19 and Table 6-2, pp. 287, 288:} \quad k_a = 39.9(62)^{-0.995} = 0.657$$

As before, $k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.657(1)(0.59)(1)(0.5)(62) = 12.0 \text{ kpsi}$$

Thus the members and electrode are of equal strength. For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{12.0(1.326)}{2.7} = 5.89 \text{ kip} \quad \text{Ans.}$$

9-27 Given, $b = 50 \text{ mm}$, $d = 30 \text{ mm}$, $h = 5 \text{ mm}$, E7010 electrode.

$$A = 0.707(5)(50 + 50 + 30) = 459.6 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 320 \text{ MPa}$.

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 272(320)^{-0.995} = 0.875$

$k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

Eqs. (6-8) and (6-18): $S_e = 0.875(1)(0.59)(1)(0.5)(320) = 82.6 \text{ MPa}$

Electrode endurance: E6010, Table 9-3, $S_{ut} = 482 \text{ MPa}$

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 272(482)^{-0.995} = 0.582$

As before, $k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.582(1)(0.59)(1)(0.5)(482) = 82.7 \text{ MPa}$$

The members and electrode are basically of equal strength. We will use $S_e = 82.6 \text{ MPa}$. For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{82.6(459.6)}{2.7} = 14.1(10^3) \text{ N} = 14.1 \text{ kN} \quad \text{Ans.}$$

9-28 Given, $b = 4 \text{ in}$, $d = 2 \text{ in}$, $h = 5/16 \text{ in}$, E7010 electrode.

$$A = 0.707(5/16)(4 + 4 + 2) = 2.209 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 47 \text{ kpsi}$.

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 39.9(47)^{-0.995} = 0.865$

$k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

Eqs. (6-8) and (6-18): $S_e = 0.865(1)(0.59)(1)(0.5)(47) = 12.0 \text{ kpsi}$

Electrode endurance: E7010, Table 9-3, $S_{ut} = 70 \text{ kpsi}$

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 39.9(70)^{-0.995} = 0.582$

As before, $k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.582(1)(0.59)(1)(0.5)(70) = 12.0 \text{ kpsi}$$

Thus the members and electrode are of equal strength. For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{12.0(2.209)}{2.7} = 9.82 \text{ kip} \quad \text{Ans.}$$

9-29 Primary shear: $\tau' = 0$ (why?)

Secondary shear:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1.5)^3 = 21.21 \text{ in}^3$

$$J = 0.707 h J_u = 0.707(1/4)(21.21) = 3.749 \text{ in}^4$$

2 welds: $\tau'' = \frac{Mr}{2J} = \frac{8F(1.5)}{2(3.749)} = 1.600F$

$$\tau'' = \tau_{\text{allow}} \Rightarrow 1.600F = 20 \Rightarrow F = 12.5 \text{ kip} \quad \text{Ans.}$$

9-30 $l = 2 + 4 + 4 = 10 \text{ in}$

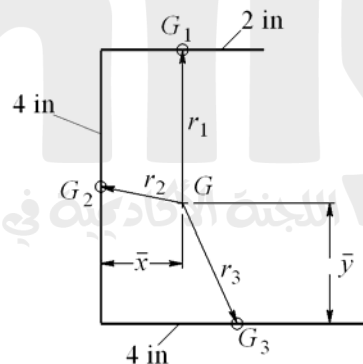
$$\bar{x} = \frac{2(1) + 4(0) + 4(2)}{10} = 1 \text{ in}$$

$$\bar{y} = \frac{2(4) + 4(2) + 4(0)}{10} = 1.6 \text{ in}$$

$$M = FR = F(10 - 1) = 9F$$

$$r_1 = \sqrt{(1-1)^2 + (4-1.6)^2} = 2.4 \text{ in}, \quad r_2 = \sqrt{1^2 + (2-1.6)^2} = 1.077 \text{ in}$$

$$r_3 = \sqrt{(2-1)^2 + 1.6^2} = 1.887 \text{ in}$$



$$J_{G_1} = \frac{1}{12}(0.707)(5/16)(2^3) = 0.1473 \text{ in}^4$$

$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5/16)(4^3) = 1.178 \text{ in}^4$$

$$\begin{aligned} J &= \sum_{i=1}^3 (J_i + A_i r_{G_i}^2) \\ &= 0.1473 + 0.707(5/16)(2)(2.4^2) + 1.178 + 0.707(5/16)(4)(1.077^2) \\ &\quad + 1.178 + 0.707(5/16)(4)(1.887^2) = 9.220 \text{ in}^4 \end{aligned}$$

$$\alpha = \tan^{-1}\left(\frac{1.6}{4-1}\right) = 28.07^\circ$$

$$r = \sqrt{1.6^2 + (4-1)^2} = 3.4 \text{ in}$$

Primary shear (τ in kpsi, F in kip) :

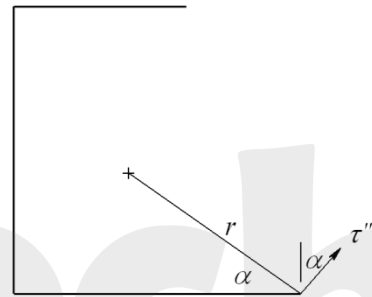
$$\tau' = \frac{V}{A} = \frac{F}{0.707(5/16)(10)} = 0.4526F$$

Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{9F(3.4)}{9.220} = 3.319F$$

$$\begin{aligned} \tau_{\max} &= \sqrt{(3.319F \sin 28.07^\circ)^2 + (3.319F \cos 28.07^\circ + 0.4526F)^2} \\ &= 3.724F \end{aligned}$$

$$\tau_{\max} = \tau_{\text{allow}} \Rightarrow 3.724F = 25 \Rightarrow F = 6.71 \text{ kip} \quad \text{Ans.}$$



9-31 $l = 30 + 50 + 50 = 130 \text{ mm}$

$$\bar{x} = \frac{30(15) + 50(0) + 50(25)}{130} = 13.08 \text{ mm}$$

$$\bar{y} = \frac{30(50) + 50(25) + 50(0)}{130} = 21.15 \text{ mm}$$

$$M = FR = F(200 - 13.08) \\ = 186.92 F \text{ (M in N}\cdot\text{m, } F \text{ in kN)}$$

$$r_1 = \sqrt{(15 - 13.08)^2 + (50 - 21.15)^2} = 28.92 \text{ mm}, \quad r_2 = \sqrt{13.08^2 + (25 - 21.15)^2} = 13.63 \text{ mm}$$

$$r_3 = \sqrt{(25 - 13.08)^2 + 21.15^2} = 24.28 \text{ mm}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5)(30^3) = 7.954(10^3) \text{ mm}^4$$

$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5)(50^3) = 36.82(10^3) \text{ mm}^4$$

$$J = \sum_{i=1}^3 (J_i + A_i r_{G_i}^2) \\ = 7.954(10^3) + 0.707(5)(30)(28.92^2) + 36.82(10^3) + 0.707(5)(50)(13.63^2) \\ + 36.82(10^3) + 0.707(5)(50)(24.28^2) = 307.3(10^3) \text{ mm}^4$$

$$\alpha = \tan^{-1}\left(\frac{21.15}{50 - 13.08}\right) = 29.81^\circ$$

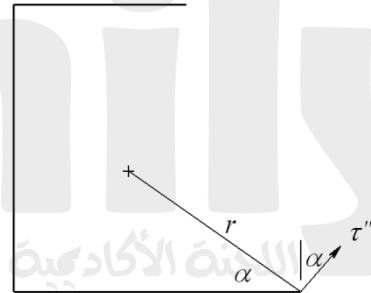
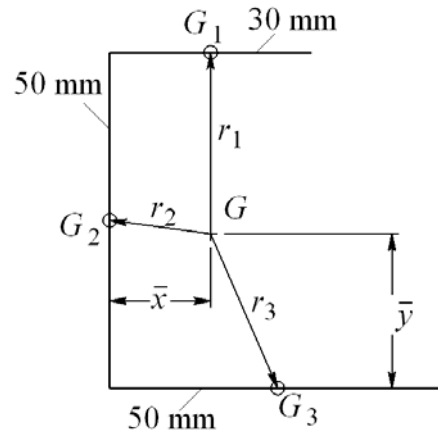
$$r = \sqrt{21.15^2 + (50 - 13.08)^2} = 42.55 \text{ mm}$$

Primary shear (τ in MPa, F in kN) :

$$\tau' = \frac{V}{A} = \frac{F(10^3)}{0.707(5)(130)} = 2.176F$$

Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{186.92F(10^3)(42.55)}{307.3(10^3)} = 25.88F$$



$$\tau_{\max} = \sqrt{(25.88F \sin 29.81^\circ)^2 + (25.88F \cos 29.81^\circ + 2.176F)^2}$$

$$= 27.79F$$

$$\tau_{\max} = \tau_{\text{allow}} \Rightarrow 27.79 F = 140 \Rightarrow F = 5.04 \text{ kN} \quad \text{Ans.}$$

9-32

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833 \left(\frac{a^2}{h} \right)$	5
2.	$\text{fom}' = \frac{a(3a^2 + a^2)}{6(2a)h} = \frac{a^2}{3h} = 0.3333 \left(\frac{a^2}{h} \right)$	1
3.	$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083 \left(\frac{a^2}{h} \right)$	4
4.	$\text{fom}' = \frac{1}{3ah} \left(\frac{8a^3 + 6a^3 + a^3}{12} - \frac{a^4}{2a+a} \right) = 0.3056 \left(\frac{a^2}{h} \right)$	2
5.	$\text{fom}' = \frac{(2a)^3}{6h} \frac{1}{4a} = \frac{8a^3}{24ah} = 0.3333 \left(\frac{a^2}{h} \right)$	1
6.	$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = 0.25 \left(\frac{a^2}{h} \right)$	3

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9-33

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{I_u}{lh} = \frac{(a^3/12)}{ah} = 0.0833 \left(\frac{a^2}{h} \right)$	6
2.	$\text{fom}' = \frac{(a^3/6)}{2ah} = 0.0833 \left(\frac{a^2}{h} \right)$	6
3.	$\text{fom}' = \frac{(aa^2/2)}{2ah} = 0.25 \left(\frac{a^2}{h} \right)$	1
4.*	$\text{fom}' = \frac{(a^2/12)(6a+a)}{3ah} = \frac{7a^2}{36h} = 0.1944 \left(\frac{a^2}{h} \right)$	2
5. & 7.	$\bar{x} = \frac{a}{2}, \quad \bar{y} = \frac{a^2}{a+2a} = \frac{a}{3}$ $I_u = \frac{2a^3}{3} - 2a^2 \frac{a}{3} + (a+2a) \left(\frac{a}{3} \right)^2 = \frac{a^3}{3}$ $\text{fom}' = \frac{I_u}{lh} = \frac{(a^3/3)}{3ah} = \frac{1}{9} \left(\frac{a^2}{h} \right) = 0.1111 \left(\frac{a^2}{h} \right)$	5
6. & 8.	$\text{fom}' = \frac{(a^2/6)(3a+a)}{4ah} = \frac{1}{6} \left(\frac{a^2}{h} \right) = 0.1667 \left(\frac{a^2}{h} \right)$	3
9.	$\text{fom}' = \frac{\pi(a/2)^3}{\pi ah} = \frac{a^2}{8h} = 0.125 \left(\frac{a^2}{h} \right)$	4

*Note. Because this section is not symmetric with the vertical axis, out-of-plane deflection may occur unless special precautions are taken. See the topic of “shear center” in books with more advanced treatments of mechanics of materials.

9-34 Attachment and member (1018 HR), $S_y = 220$ MPa and $S_{ut} = 400$ MPa.

The member and attachment are weak compared to the properties of the lowest electrode.

Decision Specify the E6010 electrode

Controlling property, Table 9-4: $\tau_{all} = \min[0.3(400), 0.4(220)] = \min(120, 88) = 88$ MPa

For a static load, the parallel and transverse fillets are the same. Let the length of a bead be $l = 75$ mm, and n be the number of beads.

$$\tau = \frac{F}{n(0.707)hl} = \tau_{all}$$

$$nh = \frac{F}{0.707l\tau_{all}} = \frac{100(10^3)}{0.707(75)(88)} = 21.43$$

where h is in millimeters. Make a table

Number of beads, n	Leg size, h (mm)
1	21.43
2	10.71
3	7.14
4	5.36 $\rightarrow$ 6 mm

Decision Specify $h = 6$ mm on all four sides.

Weldment specification:

Pattern: All-around square, four beads each side, 75 mm long

Electrode: E6010

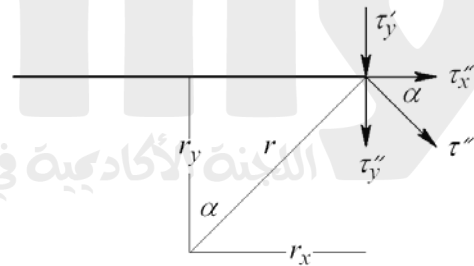
Leg size: $h = 6$ mm

9-35 *Decision:* Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-32) and have thus reduced a synthesis problem to an analysis problem:

Table 9-1, case 2, rotated 90° : $A = 1.414hd = 1.414(h)(75) = 106.05h \text{ mm}^2$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{106.05h} = \frac{113.2}{h}$$



Secondary shear:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{75[3(75^2) + 75^2]}{6} = 281.3(10^3) \text{ mm}^3$$

$$J = 0.707(h)(281.3)(10^3) = 198.8(10^3)h \text{ mm}^4$$

With $\alpha = 45^\circ$,

$$\tau_x'' = \frac{Mr \cos 45^\circ}{J} = \frac{Mr_y}{J} = \frac{12(10^3)(187.5)(37.5)}{198.8(10^3)h} = \frac{424.4}{h} = \tau_y''$$

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = \frac{1}{h} \sqrt{424.4^2 + (113.2 + 424.4)^2} = \frac{684.9}{h}$$

Attachment and member (1018 HR): $S_y = 220$ MPa, $S_{ut} = 400$ MPa

Decision: Use E60XX electrode which is stronger

$$\tau_{\text{all}} = \min[0.3(400), 0.4(220)] = 88 \text{ MPa}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{684.9}{h} = 88 \text{ MPa}$$

$$h = \frac{684.9}{88} = 7.78 \text{ mm}$$

Decision: Specify 8 mm leg size

Weldment Specifications:

Pattern: Parallel horizontal fillet welds

Electrode: E6010

Type: Fillet

Length of each bead: 75 mm

Leg size: 8 mm

9-36 Problem 9-35 solves the problem using parallel horizontal fillet welds, each 75 mm long obtaining a leg size rounded up to 8 mm.

For this problem, since the width of the plate is fixed and the length has not been determined, we will explore reducing the leg size by using two vertical beads 75 mm long and two horizontal beads such that the beads have a leg size of 6 mm.

Decision: Use a rectangular weld bead pattern with a leg size of 6 mm (case 5 of Table 9-1 with b unknown and $d = 75$ mm).

Materials:

Attachment and member (1018 HR): $S_y = 220$ MPa, $S_{ut} = 400$ MPa

From Table 9-4, AISC welding code,

$$\tau_{\text{all}} = \min[0.3(400), 0.4(220)] = \min(120, 88) = 88 \text{ MPa}$$

Select a stronger electrode material from Table 9-3.

Decision: Specify E6010

Solving for b : In Prob. 9-35, every term was linear in the unknown h . This made solving for h relatively easy. In this problem, the terms will not be linear in b , and so we will use an iterative solution with a spreadsheet.

Throat area and other properties from Table 9-1:

$$A = 1.414(6)(b + 75) = 8.484(b + 75) \quad (1)$$

$$J_u = \frac{(b+75)^3}{6}, \quad J = 0.707 (6) J_u = 0.707(b+75)^3 \quad (2)$$

Primary shear (τ in MPa, h in mm):

$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{A} \quad (3)$$

Secondary shear (See Prob. 9-35 solution for the definition of α):

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos \alpha = \frac{Mr}{J} \cos \alpha = \frac{Mr_y}{J} = \frac{12(10^3)(150 + b/2)(37.5)}{0.707(b+75)^3} \quad (4)$$

$$\tau''_y = \tau'' \sin \alpha = \frac{Mr}{J} \sin \alpha = \frac{Mr_x}{J} = \frac{12(10^3)(150 + b/2)(b/2)}{0.707(b+75)^3} \quad (5)$$

$$\tau_{\max} = \sqrt{\tau'^2_y + (\tau''_x + \tau''_y)^2} \quad (6)$$

Enter Eqs. (1) to (6) into a spreadsheet and iterate for various values of b . A portion of the spreadsheet is shown below.

b (mm)	A (mm ²)	J (mm ⁴)	τ'_y (Mpa)	τ''_y (Mpa)	τ''_x (Mpa)	$\tau_{\max}$ (Mpa)	
41	984.144	1103553.5	12.19334	69.5254	38.00722	90.12492	
42	992.628	1132340.4	12.08912	67.9566	38.05569	88.63156	
43	1001.112	1161623.6	11.98667	66.43718	38.09065	87.18485	< 88 Mpa
44	1009.596	1191407.4	11.88594	64.96518	38.11291	85.7828	

We see that $b \geq 43$ mm meets the strength goal.

Weldment Specifications:

Pattern: Horizontal parallel weld tracks 43 mm long, vertical parallel weld tracks 75 mm long

Electrode: E6010

Leg size: 6 mm

9-37 Materials:

Member and attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Table 9-4: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8$ kpsi

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi,

$$\tau_{all} = \min[0.3(62), 0.4(50)] = 20 \text{ kpsi}$$

Decision: Since 1018 HR is weaker than the E6010 electrode, use $\tau_{all} = 12.8$ kpsi

Decision: Use an all-around square weld bead track.

$$l_1 = 6 + a = 6 + 6.25 = 12.25 \text{ in}$$

Throat area and other properties from Table 9-1:

$$A = 1.414h(b + d) = 1.414(h)(6 + 6) = 16.97h$$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{F}{A} = \frac{20(10^3)}{16.97h} = \frac{1179}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b + d)^3}{6} = \frac{(6 + 6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{20(10^3)(6.25 + 3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{max} = \sqrt{\tau_x'^2 + (\tau_y' + \tau_y'')^2} = \frac{1}{h} \sqrt{2726^2 + (1179 + 2726)^2} = \frac{4762}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{max} = \tau_{all} \Rightarrow \frac{4762}{h} = 12.8(10^3) \Rightarrow h = \frac{4762}{12.8(10^3)} = 0.372 \text{ in}$$

Decision:

Specify 3/8 in leg size

Specifications:

Pattern: All-around square weld bead track

Electrode: E6010

Type of weld: Fillet

Weld bead length: 24 in

Leg size: 3/8 in

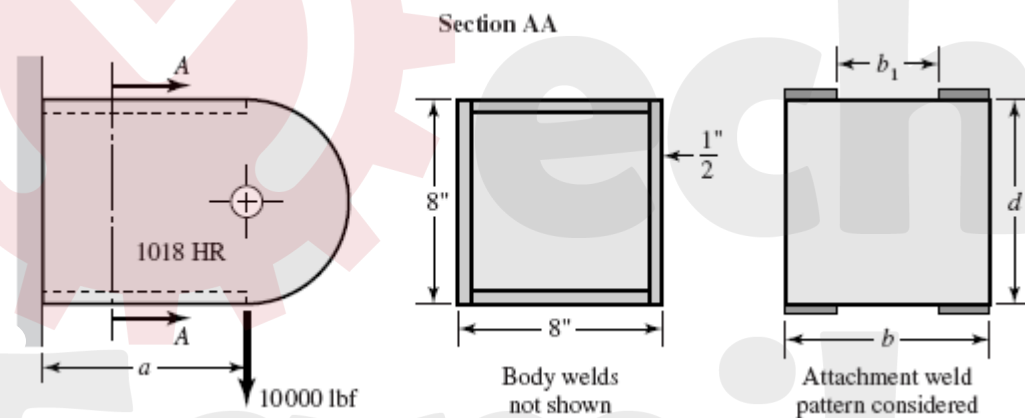
Attachment length: 12.25 in

9-38 This is a good analysis task to test a student's understanding.

- (1) Solicit information related to a priori decisions.
- (2) Solicit design variables b and d .
- (3) Find h and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.
- (4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-39 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-2 can be added or subtracted to obtain the properties of a contemplated weld pattern. The instructor can control the level of complication. We have left the presentation of the drawing to you. Here is *one* possibility. Study the problem's opportunities, and then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1 . From Table 9-3, case 3:

$$A = 1.414h(b - b_1)$$

$$I_u = \frac{bd^2}{2} - \frac{b_1d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{all} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8 - 2) = 8.48h \text{ in}^2$$

$$I_u = (8 - 2)(8^2 / 2) = 192 \text{ in}^3$$

$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$

$$\tau'' = \frac{10\,000(10)(8 / 2)}{135.7h} = \frac{2948}{h} \text{ psi}$$

$$\tau_{\max} = \frac{1}{h} \sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800 \text{ psi}$$

from which $h = 0.248 \text{ in}$. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5 \text{ in}$$

$$A = 8.48(0.248) = 2.10 \text{ in}^2$$

$$I = 135.7(0.248) = 33.65 \text{ in}^4$$

$$\text{vol} = \frac{h^2}{2} l = \frac{0.248^2}{2} 12 = 0.369 \text{ in}^3$$

$$\text{eff} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 \text{ in}$$

$$\tau' = \frac{1179}{0.248} = 4754 \text{ psi}$$

$$\tau'' = \frac{2948}{0.248} = 11\,887 \text{ psi}$$

$$\tau_{\max} = \frac{3175}{0.248} = 12\,800 \text{ psi}$$

Now consider the case of uninterrupted welds,

$$b_1 = 0$$

$$A = 1.414(h)(8 - 0) = 11.31h$$

$$I_u = (8 - 0)(8^2 / 2) = 256 \text{ in}^3$$

$$I = 0.707(256)h = 181h \text{ in}^4$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$

$$\tau'' = \frac{10\,000(10)(8 / 2)}{181h} = \frac{2210}{h}$$

$$\tau_{\max} = \frac{1}{h} \sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$

$$h = \frac{\tau_{\max}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186 \text{ in}$$

Do not round off h .

$$A = 11.31(0.186) = 2.10 \text{ in}^2$$

$$I = 181(0.186) = 33.67 \text{ in}^4$$

$$\tau' = \frac{884}{0.186} = 4753 \text{ psi}, \quad \text{vol} = \frac{0.186^2}{2} 16 = 0.277 \text{ in}^3$$

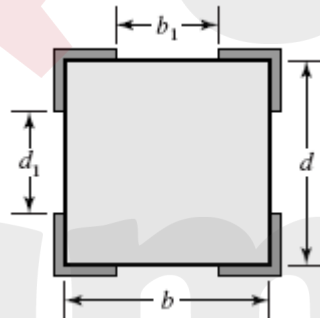
$$\tau'' = \frac{2210}{0.186} = 11882 \text{ psi}$$

$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0 \text{ in}$$

$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7 \text{ in}$$

Conclusions: To meet allowable stress limitations, I and A do not change, nor do τ and σ . To meet the shortened bead length, h is increased proportionately. However, volume of bead laid down increases as h^2 . The uninterrupted bead is superior. In this example, we did not round h and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a follow up task analyzing an alternative weld pattern.



9-40 From Table 9-2

For the box $A = 1.414h(b + d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414h(b - b_1 + d - d_1)$$

$$I_u = \frac{d^2}{6}(3b + d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2} = \frac{1}{2}(b - b_1)d^2 + \frac{1}{6}(d^3 - d_1^3)$$

Length of bead $l = 2(b - b_1 + d - d_1)$

$$\text{fom} = I_u / hl$$

9-41 Computer programs will vary.

9-42 **Note to the Instructor.** In the first printing of the ninth edition, the loading was stated incorrectly. In the fourth line, “bending moment of 100 kip · in in” should read, “10 kip bending load 10 in from”. This will be corrected in the printings that follow. We apologize if this has caused any inconvenience.

$\tau_{\text{all}} = 12$ kpsi. Use Fig. 9-17(a) for general geometry, but employ $\equiv$ beads and then $||$ beads.

Horizontal parallel weld bead pattern
 $b = 3$ in, $d = 6$ in

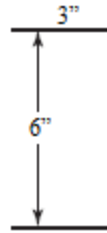


Table 9-2: $A = 1.414hb = 1.414(h)(3) = 4.24h$ in²

$$I_u = \frac{bd^2}{2} = \frac{3(6)^2}{2} = 54$$
 in³

$$I = 0.707hI_u = 0.707(h)(54) = 38.2h$$
 in⁴

$$\tau' = \frac{10}{4.24h} = \frac{2.358}{h}$$
 kpsi

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{38.2h} = \frac{7.853}{h}$$
 kpsi

$$\tau_{\text{max}} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h} \sqrt{2.358^2 + 7.853^2} = \frac{8.199}{h}$$
 kpsi

Equate the maximum and allowable shear stresses.

$$\tau_{\text{max}} = \tau_{\text{all}} = \frac{8.199}{h} = 12$$

from which $h = 0.683$ in. It follows that

$$I = 38.2(0.683) = 26.1$$
 in⁴

The volume of the weld metal is

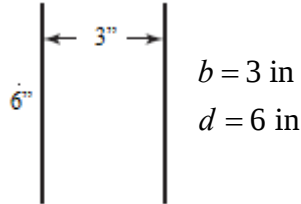
$$\text{vol} = \frac{h^2 l}{2} = \frac{(0.683)^2 (3+3)}{2} = 1.40$$
 in³

The effectiveness, (eff)_H, is

$$(\text{eff})_H = \frac{I}{\text{vol}} = \frac{26.1}{1.4} = 18.6 \text{ in}$$

$$(\text{fom}')_H = \frac{I_u}{hl} = \frac{54}{0.683(3+3)} = 13.2 \text{ in}$$

Vertical parallel weld beads



From Table 9-2, case 2

$$A = 1.414hd = 1.414(h)(6) = 8.48h \text{ in}^2$$

$$I_u = \frac{d^3}{6} = \frac{6^3}{6} = 72 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(72) = 50.9h$$

$$\tau' = \frac{10}{8.48h} = \frac{1.179}{h} \text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{50.9h} = \frac{5.894}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h} \sqrt{1.179^2 + 5.894^2} = \frac{6.011}{h} \text{ kpsi}$$

Equating $\tau_{\max}$ to τ_{all} gives $h = 0.501 \text{ in}$. It follows that

$$I = 50.9(0.501) = 25.5 \text{ in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.501^2}{2} (6+6) = 1.51 \text{ in}^3$$

$$(\text{eff})_V = \frac{I}{\text{vol}} = \frac{25.5}{1.51} = 16.7 \text{ in}$$

$$(\text{fom}')_V = \frac{I_u}{hl} = \frac{72}{0.501(6+6)} = 12.0 \text{ in}$$

The ratio of $(\text{eff})_V / (\text{eff})_H$ is $16.7 / 18.6 = 0.898$. The ratio $(\text{fom}')_V / (\text{fom}')_H$ is $12.0 / 13.2 = 0.909$. This is not surprising since

$$\text{eff} = \frac{I}{\text{vol}} = \frac{I}{(h^2/2)l} = \frac{0.707hI_u}{(h^2/2)l} = 1.414 \frac{I_u}{hl} = 1.414 \text{fom}'$$

The ratios $(\text{eff})_V / (\text{eff})_H$ and $(\text{fom}')_V / (\text{fom}')_H$ give the same information.

9-43 $F = 0$, $T = 15 \text{ kip}\cdot\text{in.}$

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283 \text{ in}^3$, $J = 0.707(1/4) 6.283 = 1.111 \text{ in}^4$

$$\tau_{\max} = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi} \quad \text{Ans.}$$

9-44 $F = 2 \text{ kip}$, $T = 0$.

Table 9-2: $A = 1.414 \pi h r = 1.414 \pi (1/4)(1) = 1.111 \text{ in}^2$

$I_u = \pi r^3 = \pi(1)^3 = 3.142 \text{ in}^3$, $I = 0.707(1/4) 3.142 = 0.5553 \text{ in}^4$

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$\tau'' = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

$$\tau_{\max} = (\tau'^2 + \tau''^2)^{1/2} = (1.80^2 + 21.6^2)^{1/2} = 21.7 \text{ kpsi} \quad \text{Ans.}$$

9-45 $F = 2 \text{ kip}$, $T = 15 \text{ kip}\cdot\text{in.}$

Bending:

Table 9-2: $A = 1.414 \pi h r = 1.414 \pi (1/4)(1) = 1.111 \text{ in}^2$

$I_u = \pi r^3 = \pi(1)^3 = 3.142 \text{ in}^3$, $I = 0.707(1/4) 3.142 = 0.5553 \text{ in}^4$

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283 \text{ in}^3$, $J = 0.707(1/4) 6.283 = 1.111 \text{ in}^4$

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau''_M)^2 + (\tau''_T)^2} = \sqrt{1.80^2 + 21.6^2 + 13.5^2} = 25.5 \text{ kpsi} \quad \text{Ans.}$$

9-46 $F = 2 \text{ kip}$, $T = 15 \text{ kip}\cdot\text{in.}$

Bending:

Table 9-2: $A = 1.414 \pi h r = 1.414 \pi h (1) = 4.442h \text{ in}^2$

$$I_u = \pi r^3 = \pi (1)^3 = 3.142 \text{ in}^3, \quad I = 0.707 h (3.142) = 2.221h \text{ in}^4$$

$$\tau' = \frac{V}{A} = \frac{2}{4.442h} = \frac{0.4502}{h} \text{ kpsi}$$

$$(\tau''_M)_M = \frac{Mr}{I} = \frac{2(6)(1)}{2.221h} = \frac{5.403}{h} \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi (1)^3 = 6.283 \text{ in}^3, \quad J = 0.707 h (6.283) = 4.442 \text{ in}^4$

$$(\tau''_T)_T = \frac{Tr}{J} = \frac{15(1)}{4.442h} = \frac{3.377}{h} \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau''_M)^2 + (\tau''_T)^2} = \sqrt{\left(\frac{0.4502}{h}\right)^2 + \left(\frac{5.403}{h}\right)^2 + \left(\frac{3.377}{h}\right)^2} = \frac{6.387}{h} \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{6.387}{h} = 20 \quad \Rightarrow \quad h = 0.319 \text{ in} \quad \text{Ans.}$$

Should specify a $\frac{3}{8}$ in weld. *Ans.*

9-47 $h = 9 \text{ mm}$, $d = 200 \text{ mm}$, $b = 25 \text{ mm}$

From Table 9-2, case 2:

$$A = 1.414(9)(200) = 2.545(10^3) \text{ mm}^2$$

$$I_u = \frac{d^3}{6} = \frac{200^3}{6} = 1.333(10^6) \text{ mm}^3$$

$$I = 0.707h I_u = 0.707(9)(1.333)(10^6) = 8.484(10^6) \text{ mm}^4$$

$$\tau' = \frac{F}{A} = \frac{25(10^3)}{2.545(10^3)} = 9.82 \text{ MPa}$$

$$M = 25(150) = 3750 \text{ N}\cdot\text{m}$$

$$\tau'' = \frac{Mc}{I} = \frac{3750(100)}{8.484(10^6)}(10^3) = 44.20 \text{ MPa}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{9.82^2 + 44.20^2} = 45.3 \text{ MPa} \quad \text{Ans.}$$

9-48 Note to the Instructor. In the first printing of the ninth edition, the vertical dimension of 5 in should be to the top of the top plate. This will be corrected in the printings that follow. We apologize if this has caused any inconvenience.

$$h = 0.25 \text{ in}, b = 2.5 \text{ in}, d = 5 \text{ in.}$$

$$\text{Table 9-2, case 5: } A = 0.707h(b + 2d) = 0.707(0.25)[2.5 + 2(5)] = 2.209 \text{ in}^2$$

$$\bar{y} = \frac{d^2}{b + 2d} = \frac{5^2}{2.5 + 2(5)} = 2 \text{ in}$$

$$\begin{aligned} I_u &= \frac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2 \\ &= \frac{2(5^3)}{3} - 2(5^2)(2) + [2.5 + 2(5)](2^2) = 33.33 \text{ in}^3 \end{aligned}$$

$$I = 0.707 h I_u = 0.707(1/4)(33.33) = 5.891 \text{ in}^4$$

Primary shear:

$$\tau' = \frac{F}{A} = \frac{2}{2.209} = 0.905 \text{ kpsi}$$

Secondary shear (the critical location is at the bottom of the bracket):

$$y = 5 - 2 = 3 \text{ in}$$

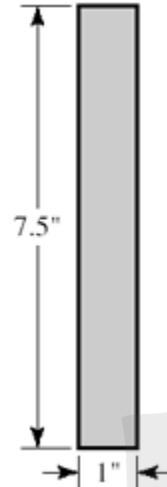
$$\tau'' = \frac{My}{I} = \frac{2(5)(3)}{5.891} = 5.093 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{0.905^2 + 5.093^2} = 5.173 \text{ kpsi}$$

$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{18}{5.173} = 3.48 \quad \text{Ans.}$$

- 9-49** The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment. Use a rectangular, weld-all-around pattern – Table 9-2, case 6:

$$\begin{aligned}
 A &= 1.414 h(b + d) = 1.414(1/16)(1 + 7.5) \\
 &= 0.7512 \text{ in}^2 \\
 \bar{x} &= b / 2 = 0.5 \text{ in} \\
 \bar{y} &= d / 2 = 7.5 / 2 = 3.75 \text{ in} \\
 I_u &= \frac{d^2}{6}(3b + d) = \frac{7.5^2}{6}[3(1) + 7.5] = 98.44 \text{ in}^3 \\
 I &= 0.707 h I_u = 0.707(1/16)(98.44) = 4.350 \text{ in}^4 \\
 M &= (3.75 + 0.5)W = 4.25W \\
 \tau' &= \frac{V}{A} = \frac{W}{0.7512} = 1.331W \\
 \tau'' &= \frac{Mc}{I} = \frac{4.25W(7.5 / 2)}{4.350} = 3.664W \\
 \tau_{\max} &= \sqrt{\tau'^2 + \tau''^2} = W\sqrt{1.331^2 + 3.664^2} = 3.90W
 \end{aligned}$$



Material properties: The allowable stress given is low. Let's demonstrate that. For the 1020 CD bracket, use HR properties of $S_y = 30$ kpsi and $S_{ut} = 55$. The 1030 HR support, $S_y = 37.5$ kpsi and $S_{ut} = 68$. The E6010 electrode has strengths of $S_y = 50$ and $S_{ut} = 62$ kpsi.

Allowable stresses:

1020 HR: $\tau_{\text{all}} = \min[0.3(55), 0.4(30)] = \min(16.5, 12) = 12 \text{ kpsi}$

1020 HR: $\tau_{\text{all}} = \min[0.3(68), 0.4(37.5)] = \min(20.4, 15) = 15 \text{ kpsi}$

E6010: $\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = \min(18.6, 20) = 18.6 \text{ kpsi}$

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value. Therefore, the allowable shear stress is

$$\tau_{\text{all}} = \min(14.4, 12, 18.0) = 12 \text{ kpsi}$$

However, the allowable stress in the problem statement is 1.5 kpsi which is low from the weldment perspective. The load associated with this strength is

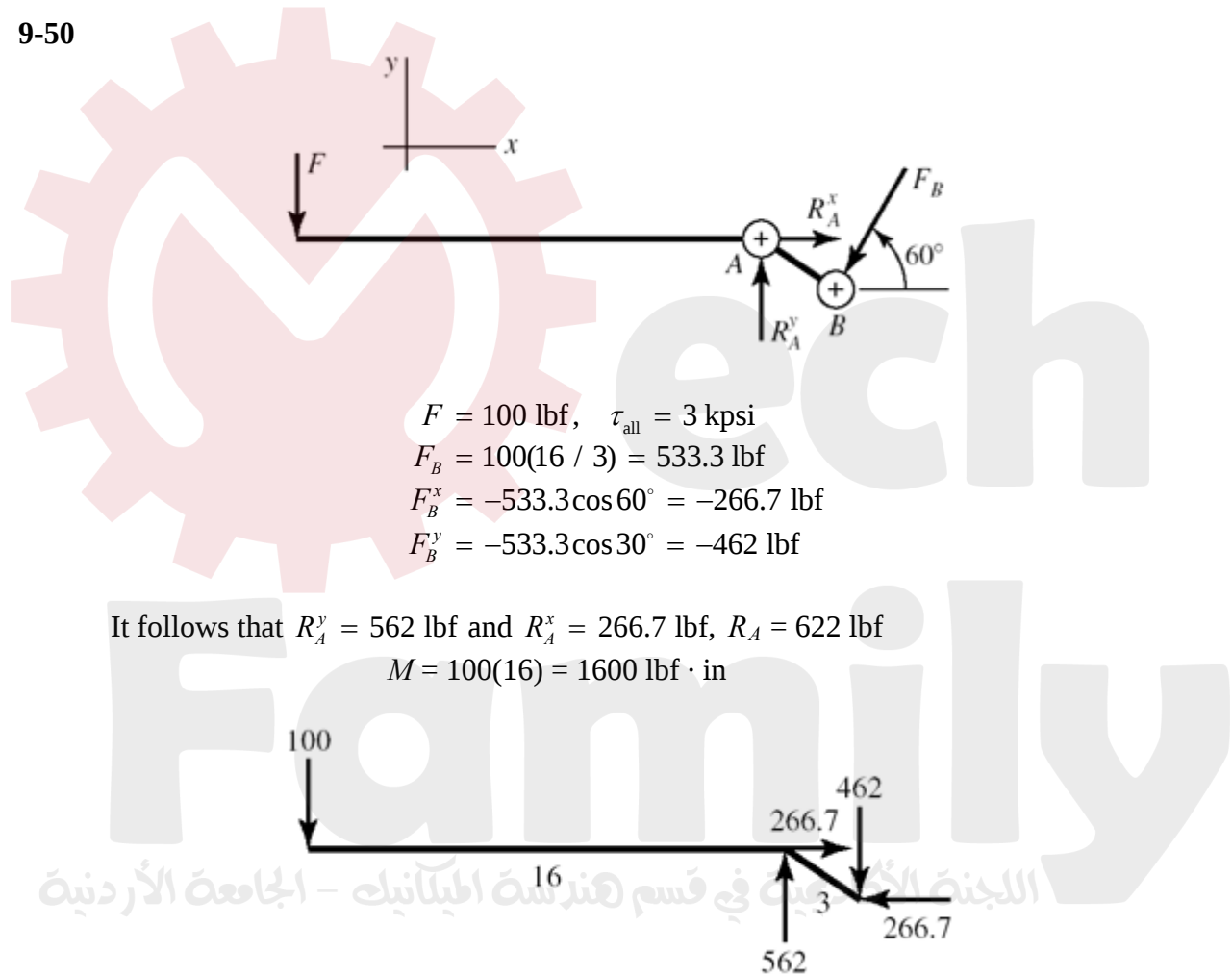
$$\begin{aligned}
 \tau_{\max} &= \tau_{\text{all}} = 3.90W = 1500 \\
 W &= \frac{1500}{3.90} = 385 \text{ lbf}
 \end{aligned}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load associated with this strength is $W = 12\,000/3.90 = 3077$ lbf. Can the bracket carry such a load?

There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

9-50



The OD of the tubes is 1 in. From Table 9-1, case 6:

$$A = 2[1.414(\pi hr)] = 2(1.414)(\pi h)(1 / 2) = 4.442h \text{ in}^2$$

$$J_u = 2\pi r^3 = 2\pi(1 / 2)^3 = 0.7854 \text{ in}^3$$

$$J = 2(0.707)hJ_u = 1.414(0.7854)h = 1.111h \text{ in}^4$$

$$\tau' = \frac{V}{A} = \frac{622}{4.442h} = \frac{140.0}{h}$$

$$\tau'' = \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.111h} = \frac{720.1}{h}$$

The shear stresses, τ' and τ'' , are additive algebraically

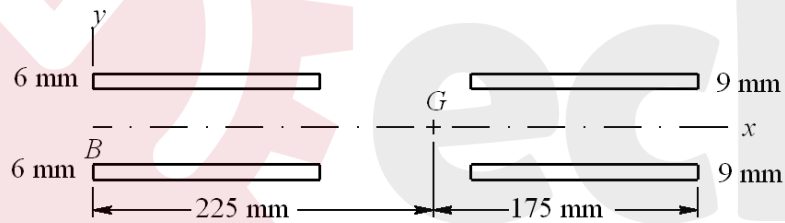
$$\tau_{\max} = \frac{1}{h}(140.0 + 720.1) = \frac{860}{h} \text{ psi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{860}{h} = 3000$$

$$h = \frac{860}{3000} = 0.287 \rightarrow 5 / 16 \text{ in}$$

Decision: Use 5/16 in fillet welds Ans.

9-51



For the pattern in bending shown, find the centroid G of the weld group.

$$\bar{x} = \frac{75(6)(150) + 325(9)(150)}{(6)(150) + (9)(150)} = 225 \text{ mm}$$

$$I_{6\text{mm}} = 2(I_G + A\bar{x}^2)_{6\text{mm}}$$

$$= 2 \left[\frac{0.707(6)(150^3)}{12} + 0.707(6)(150)(225 - 75)^2 \right] = 31.02(10^6) \text{ mm}^4$$

$$I_{9\text{mm}} = 2 \left[\frac{0.707(9)(150^3)}{12} + 0.707(9)(150)(175 - 75)^2 \right] = 22.67(10^6) \text{ mm}^4$$

$$I = I_{6\text{mm}} + I_{9\text{mm}} = (31.02 + 22.67)(10^6) = 53.69(10^6) \text{ mm}^4$$

The critical location is at B . With τ in MPa, and F in kN

$$\tau' = \frac{V}{A} = \frac{F(10^3)}{2[0.707(6+9)(150)]} = 0.3143F$$

$$\tau'' = \frac{Mc}{I} = \frac{200F(10^3)(225)}{53.69(10^6)} = 0.8381F$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.3143^2 + 0.8381^2} = 0.8951F$$

Materials:

1015 HR (Table A-20): $S_y = 190$ MPa, E6010 Electrode (Table 9-3): $S_y = 345$ MPa

Eq. (5-21), p. 225 $\tau_{\text{all}} = 0.577(190) = 109.6$ MPa

$$F = \frac{\tau_{\text{all}} / n}{0.8951} = \frac{109.6 / 2}{0.8951} = 61.2 \text{ kN} \quad \text{Ans.}$$

9-52 In the textbook, Fig. Problem 9-52b is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$

(b) $F_y = 1200 \sin 30^\circ = 600 \text{ lbf} \quad \text{Ans.}$

(c) $F_x = 1200 \cos 30^\circ = 1039 \text{ lbf} \quad \text{Ans.}$

(d) From Table 9-2, case 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through G and parallel to z is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \quad \text{Ans.}$$

(e) Refer to Fig. Problem 9-52b. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \sqrt{\tau_1^2 + \tau_2^2} = \sqrt{617^2 + 1069^2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1234^2 + 916^2} = 1537 \text{ psi} \quad \text{Ans.}$$

(f) Materials:

1018 HR Member: $S_y = 32 \text{ kpsi}$, $S_{ut} = 58 \text{ kpsi}$ (Table A-20)

E6010 Electrode: $S_y = 50 \text{ kpsi}$ (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.577S_y}{\tau_{\max}} = \frac{0.577(32)}{1.537} = 12.0 \quad \text{Ans.}$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to bh .

$$A_1 \doteq bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location A ,

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \sqrt{\sigma_y^2 + 3\tau_{xy}^2} = \sqrt{2648^2 + 3(1662)^2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad \text{Ans.}$$

The clip on the mooring line bears against the side of the 1/2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad \text{Ans.}$$

Further investigation of this situation requires more detail than is included in the task statement.

- (h)** In shear fatigue, the weakest constituent of the weld melt is 1018 HR with $S_{ut} = 58$ kpsi, Eq. (6-8), p. 282, gives

$$S'_e = 0.504S_{ut} = 0.504(58) = 29.2 \text{ kpsi}$$

Eq. (6-19), p. 287: $k_a = 14.4(58)^{-0.718} = 0.780$

For the size factor estimate, we first employ Eq. (6-25), p. 289, for the equivalent diameter

$$d_e = 0.808\sqrt{0.707hb} = 0.808\sqrt{0.707(2.5)(0.25)} = 0.537 \text{ in}$$

Eq. (6-20), p. 288, is used next to find k_b

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.537}{0.30}\right)^{-0.107} = 0.940$$

Eq. (6-26), p. 290: $k_c = 0.59$

From Eq. (6-18), p. 287, the endurance strength in shear is

$$S_{se} = 0.780(0.940)(0.59)(29.2) = 12.6 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied

$$\tau_a = \tau_m = K_{fs} \frac{\tau_{\max}}{2} = 2.7 \frac{1.537}{2} = 2.07 \text{ kpsi}$$

Table 6-7, p. 307: Gerber factor of safety n_f , adjusted for shear, with $S_{su} = 0.67S_{ut}$

$$n_f = \frac{1}{2} \left(\frac{S_{su}}{\tau_m} \right)^2 \frac{\tau_a}{S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su} \tau_a} \right)} \right]$$

$$= \frac{1}{2} \left[\frac{0.67(58)}{2.07} \right]^2 \left(\frac{2.07}{12.6} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(2.07)(12.6)}{0.67(58)(2.07)} \right]^2} \right\} = 5.55 \quad \text{Ans.}$$

Attachment metal should be checked for bending fatigue.

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.2829F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.85 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{14F(2)}{18.85} = 1.485F$$

Thus, the maximum shear and allowable load are:

$$\tau_{\max} = F\sqrt{1.485^2 + (0.2829 + 1.485)^2} = 2.309F$$

$$F = \frac{\tau_{\text{all}}}{2.309} = \frac{25}{2.309} = 10.8 \text{ kip} \quad \text{Ans.}$$

The load for part (a) has increased by a factor of $10.8/2.71 = 3.99$ Ans.

(b) From Prob. 9-18b, $\tau_{\text{all}} = 11 \text{ kpsi}$

$$F_{\text{all}} = \frac{\tau_{\text{all}}}{2.309} = \frac{11}{2.309} = 4.76 \text{ kip}$$

The allowable load in part (b) has increased by a factor of $4.76/1.19 = 4$ Ans.

9-54 Purchase the hook having the design shown in Fig. Problem 9-54b. Referring to text Fig. 9-29a, this design reduces peel stresses.

9-55 (a)

$$\bar{\tau} = \frac{1}{l} \int_{-l/2}^{l/2} \frac{P\omega \cosh(\omega x)}{4b \sinh(\omega l / 2)} dx = \frac{A_1}{l} \int_{-l/2}^{l/2} \cosh(\omega x) dx = \frac{A_1}{\omega} \sinh(\omega x) \Big|_{-l/2}^{l/2}$$

$$= \frac{A_1}{\omega} [\sinh(\omega l / 2) - \sinh(-\omega l / 2)] = \frac{A_1}{\omega} [\sinh(\omega l / 2) - (-\sinh(\omega l / 2))]$$

$$= \frac{2A_1 \sinh(\omega l / 2)}{\omega} = \frac{P\omega}{4bl \sinh(\omega l / 2)} [2 \sinh(\omega l / 2)] = \frac{P}{2bl} \quad \text{Ans.}$$

(b) $\tau(l/2) = \frac{P\omega \cosh(\omega l / 2)}{4b \sinh(\omega l / 2)} = \frac{P\omega}{4b \tanh(\omega l / 2)} \quad \text{Ans.}$

$$(c) \quad K = \frac{\tau(l/2)}{\bar{\tau}} = \frac{P\omega}{4b \tanh(\omega l/2)} \left(\frac{2bl}{P} \right) = \frac{\omega l/2}{\tanh(\omega l/2)} \quad Ans.$$

For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

$$K = \frac{\omega l}{2} \frac{\exp(\omega l/2) - \exp(-\omega l/2)}{\exp(\omega l/2) + \exp(-\omega l/2)} \quad Ans.$$

9-56 This is a computer programming exercise. All programs will vary.

