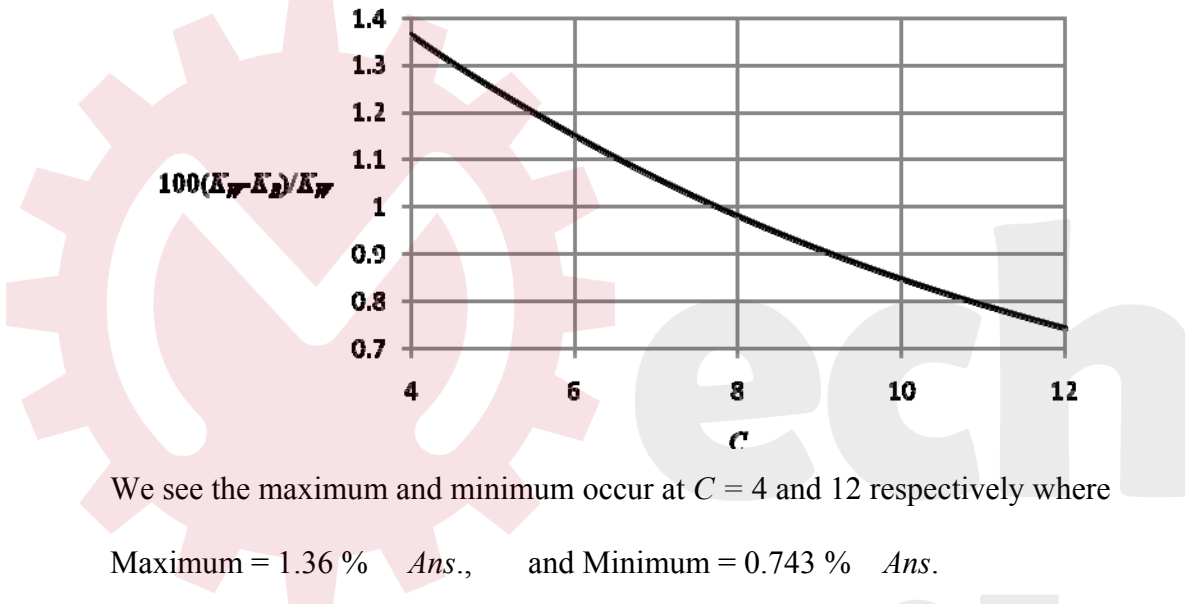


Chapter 10

10-1 From Eqs. (10-4) and (10-5)

$$K_W - K_B = \frac{4C-1}{4C-4} + \frac{0.615}{C} - \frac{4C+2}{4C-3}$$

Plot $100(K_W - K_B)/K_W$ vs. C for $4 \leq C \leq 12$ obtaining



10-2 $A = Sd^m$

$$\dim(A_{\text{uscu}}) = [\dim(S) \dim(d^m)]_{\text{uscu}} = \text{kpsi} \cdot \text{in}^m$$

$$\dim(A_{\text{SI}}) = [\dim(S) \dim(d^m)]_{\text{SI}} = \text{MPa} \cdot \text{mm}^m$$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}} \cdot \frac{\text{mm}^m}{\text{in}^m} A_{\text{uscu}} = 6.894757 (25.4)^m A_{\text{uscu}} \doteq 6.895 (25.4)^m A_{\text{uscu}} \quad \text{Ans.}$$

For music wire, from Table 10-4:

$$A_{\text{uscu}} = 201 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.145; \quad \text{what is } A_{\text{SI}}?$$

$$A_{\text{SI}} = 6.895 (25.4)^{0.145} (201) = 2215 \text{ MPa} \cdot \text{mm}^m \quad \text{Ans.}$$

10-3 Given: Music wire, $d = 2.5 \text{ mm}$, $\text{OD} = 31 \text{ mm}$, plain ground ends, $N_t = 14$ coils.

(a) Table 10-1: $N_a = N_t - 1 = 14 - 1 = 13$ coils

$$L_s = d N_t = 2.5(14) = 35 \text{ mm}$$

Table 10-4: $m = 0.145, \quad A = 2211 \text{ MPa}\cdot\text{mm}^m$

Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{2211}{2.5^{0.145}} = 1936 \text{ MPa}$

Table 10-6: $S_{sy} = 0.45(1936) = 871.2 \text{ MPa}$

$$D = \text{OD} - d = 31 - 2.5 = 28.5 \text{ mm}$$

$$C = D/d = 28.5/2.5 = 11.4$$

Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(11.4)+2}{4(11.4)-3} = 1.117$

Eq. (10-7): $F_s = \frac{\pi d^3 S_{sy}}{8 K_B D} = \frac{\pi (2.5^3) 871.2}{8 (1.117) 28.5} = 167.9 \text{ N}$

Table 10-5: $d = 2.5/25.4 = 0.098 \text{ in} \Rightarrow G = 81.0(10^3) \text{ MPa}$

Eq. (10-9): $k = \frac{d^4 G}{8 D^3 N_a} = \frac{2.5^4 (81) 10^3}{8 (28.5^3) 13} = 1.314 \text{ N/mm}$

$$L_0 = \frac{F_s}{k} + L_s = \frac{167.9}{1.314} + 35 = 162.8 \text{ mm} \quad \text{Ans.}$$

(b) $F_s = 167.9 \text{ N} \quad \text{Ans.}$

(c) $k = 1.314 \text{ N/mm} \quad \text{Ans.}$

(d) $(L_0)_{cr} = \frac{2.63(28.5)}{0.5} = 149.9 \text{ mm} . \text{ Spring needs to be supported.} \quad \text{Ans.}$

10-4 Given: Design load, $F_1 = 130 \text{ N}$.

Referring to Prob. 10-3 solution, $C = 11.4$, $N_a = 13$ coils, $S_{sy} = 871.2 \text{ MPa}$, $F_s = 167.9 \text{ N}$, $L_0 = 162.8 \text{ mm}$ and $(L_0)_{cr} = 149.9 \text{ mm}$.

Eq. (10-18): $4 \leq C \leq 12 \quad C = 11.4 \quad \text{O.K.}$

Eq. (10-19): $3 \leq N_a \leq 15 \quad N_a = 13 \quad \text{O.K.}$

Eq. (10-17): $\xi = \frac{F_s}{F_1} - 1 = \frac{167.9}{130} - 1 = 0.29$

Eq. (10-20): $\xi \geq 0.15, \quad \xi = 0.29 \quad O.K.$

From Eq. (10-7) for static service

$$\tau_1 = K_B \left(\frac{8F_1 D}{\pi d^3} \right) = 1.117 \frac{8(130)(28.5)}{\pi(2.5)^3} = 674 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{871.2}{674} = 1.29$$

Eq. (10-21): $n_s \geq 1.2, \quad n = 1.29 \quad O.K.$

$$\tau_s = \tau_1 \left(\frac{167.9}{130} \right) = 674 \left(\frac{167.9}{130} \right) = 870.5 \text{ MPa}$$

$$S_{sy} / \tau_s = 871.2 / 870.5 \doteq 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe (but was the basis of the design). *Not O.K.*

$L_0 \leq (L_0)_{cr}$: $162.8 \geq 149.9 \quad \text{Not O.K.}$

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Given: Oil-tempered wire, $d = 0.2$ in, $D = 2$ in, $N_t = 12$ coils, $L_0 = 5$ in, squared ends.

(a) Table 10-1: $L_s = d(N_t + 1) = 0.2(12 + 1) = 2.6$ in *Ans.*

(b) Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils
Table 10-5: $G = 11.2$ Mpsi

Eq. (10-9): $k = \frac{d^4 G}{8D^3 N} = \frac{0.2^4 (11.2) 10^6}{8(2^3) 10} = 28 \text{ lbf/in}$

$F_s = k y_s = k (L_0 - L_s) = 28(5 - 2.6) = 67.2 \text{ lbf} \quad \text{Ans.}$

(c) Eq. (10-1): $C = D/d = 2/0.2 = 10$

Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$

Eq. (10-7): $\tau_s = K_B \frac{8FD}{\pi d^3} = 1.135 \frac{8(67.2)2}{\pi(0.2^3)} = 48.56(10^3) \text{ psi}$

Table 10-4: $m = 0.187, A = 147 \text{ kpsi} \cdot \text{in}^m$

Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{147}{0.2^{0.187}} = 198.6 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.50 S_{ut} = 0.50(198.6) = 99.3 \text{ kpsi}$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{99.3}{48.56} = 2.04 \quad \text{Ans.}$$

10-6 Given: Oil-tempered wire, $d = 4 \text{ mm}$, $C = 10$, plain ends, $L_0 = 80 \text{ mm}$, and at $F = 50 \text{ N}$, $y = 15 \text{ mm}$.

(a) $k = F/y = 50/15 = 3.333 \text{ N/mm}$ Ans.

(b) $D = Cd = 10(4) = 40 \text{ mm}$

$$\text{OD} = D + d = 40 + 4 = 44 \text{ mm} \quad \text{Ans.}$$

(c) From Table 10-5, $G = 77.2 \text{ GPa}$

Eq. (10-9): $N_a = \frac{d^4 G}{8kD^3} = \frac{4^4 (77.2) 10^3}{8(3.333) 40^3} = 11.6 \text{ coils}$

Table 10-1: $N_t = N_a = 11.6 \text{ coils}$ Ans.

(d) Table 10-1: $L_s = d(N_t + 1) = 4(11.6 + 1) = 50.4 \text{ mm}$ Ans.

(e) Table 10-4: $m = 0.187, A = 1855 \text{ MPa} \cdot \text{mm}^m$

Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{1855}{4^{0.187}} = 1431 \text{ MPa}$

Table 10-6: $S_{sy} = 0.50 S_{ut} = 0.50(1431) = 715.5 \text{ MPa}$

$$y_s = L_0 - L_s = 80 - 50.4 = 29.6 \text{ mm}$$

$$F_s = k y_s = 3.333(29.6) = 98.66 \text{ N}$$

Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.135 \frac{8(98.66)40}{\pi(4^3)} = 178.2 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{715.5}{178.2} = 4.02 \quad \text{Ans.}$$

10-7 Static service spring with: HD steel wire, $d = 0.080$ in, OD = 0.880 in, $N_t = 8$ coils, plain and ground ends.

Preliminaries

Table 10-5: $A = 140 \text{ kpsi} \cdot \text{in}^m$, $m = 0.190$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{140}{0.080^{0.190}} = 226.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45(226.2) = 101.8 \text{ kpsi}$

Then,

$$D = \text{OD} - d = 0.880 - 0.080 = 0.8 \text{ in}$$

$$\text{Eq. (10-1): } C = D/d = 0.8/0.08 = 10$$

$$\text{Eq. (10-5): } K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135$$

Table 10-1: $N_a = N_t - 1 = 8 - 1 = 7$ coils

$$L_s = dN_t = 0.08(8) = 0.64 \text{ in}$$

Eq. (10-7) For solid-safe, $n_s = 1.2$:

$$F_s = \frac{\pi d^3 S_{sy} / n_s}{8K_B D} = \frac{\pi(0.08^3)[101.8(10^3) / 1.2]}{8(1.135)(0.8)} = 18.78 \text{ lbf}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.08^4 (11.5)10^6}{8(0.8^3)7} = 16.43 \text{ lbf/in}$$

$$y_s = \frac{F_s}{k} = \frac{18.78}{16.43} = 1.14 \text{ in}$$

$$\text{(a) } L_0 = y_s + L_s = 1.14 + 0.64 = 1.78 \text{ in} \quad \text{Ans.}$$

$$\text{(b) Table 10-1: } p = \frac{L_0}{N_t} = \frac{1.78}{8} = 0.223 \text{ in} \quad \text{Ans.}$$

$$\text{(c) From above: } F_s = 18.78 \text{ lbf} \quad \text{Ans.}$$

$$\text{(d) From above: } k = 16.43 \text{ lbf/in} \quad \text{Ans.}$$

$$\text{(e) Table 10-2 and Eq. (10-13): } (L_0)_{cr} = \frac{2.63D}{\alpha} = \frac{2.63(0.8)}{0.5} = 4.21 \text{ in}$$

Since $L_0 < (L_0)_{cr}$, buckling is unlikely *Ans.*

10-8 Given: Design load, $F_1 = 16.5 \text{ lbf}$.

Referring to Prob. 10-7 solution, $C = 10$, $N_a = 7$ coils, $S_{sy} = 101.8 \text{ kpsi}$, $F_s = 18.78 \text{ lbf}$, $y_s = 1.14 \text{ in}$, $L_0 = 1.78 \text{ in}$, and $(L_0)_{cr} = 4.208 \text{ in}$.

Eq. (10-18): $4 \leq C \leq 12$ $C = 10$ *O.K.*

Eq. (10-19): $3 \leq N_a \leq 15$ $N_a = 7$ *O.K.*

Eq. (10-17): $\xi = \frac{F_s}{F_1} - 1 = \frac{18.78}{16.5} - 1 = 0.14$

Eq. (10-20): $\xi \geq 0.15$, $\xi = 0.14$ *not O.K.*, but probably acceptable.

From Eq. (10-7) for static service

$$\tau_1 = K_B \left(\frac{8F_1 D}{\pi d^3} \right) = 1.135 \frac{8(16.5)(0.8)}{\pi(0.080)^3} = 74.5(10^3) \text{ psi} = 74.5 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{101.8}{74.5} = 1.37$$

Eq. (10-21): $n_s \geq 1.2$, $n = 1.37$ *O.K.*

$$\tau_s = \tau_1 \left(\frac{18.78}{16.5} \right) = 74.5 \left(\frac{18.78}{16.5} \right) = 84.8 \text{ kpsi}$$

$$n_s = S_{sy} / \tau_s = 101.8 / 84.8 = 1.20$$

Eq. (10-21): $n_s \geq 1.2$, $n_s = 1.2$ It is solid-safe (basis of design). *O.K.*

Eq. (10-13) and Table 10-2: $L_0 \leq (L_0)_{cr}$ $1.78 \text{ in} \leq 4.208 \text{ in}$ *O.K.*

10-9 Given: A228 music wire, sq. and grd. ends, $d = 0.007 \text{ in}$, $OD = 0.038 \text{ in}$, $L_0 = 0.58 \text{ in}$, $N_t = 38$ coils.

$$D = OD - d = 0.038 - 0.007 = 0.031 \text{ in}$$

Eq. (10-1): $C = D/d = 0.031/0.007 = 4.429$

Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(4.429)+2}{4(4.429)-3} = 1.340$

Table (10-1): $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 12.0 \text{ Mpsi}$

Eq. (10-9): $k = \frac{d^4 G}{8D^3 N_a} = \frac{0.007^4 (12.0) 10^6}{8(0.031^3) 36} = 3.358 \text{ lbf/in}$

Table (10-1): $L_s = dN_t = 0.007(38) = 0.266 \text{ in}$

$$y_s = L_0 - L_s = 0.58 - 0.266 = 0.314 \text{ in}$$

$$F_s = ky_s = 3.358(0.314) = 1.054 \text{ lbf}$$

Eq. (10-7): $\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.340 \frac{8(1.054) 0.031}{\pi(0.007^3)} = 325.1(10^3) \text{ psi} \quad (1)$

Table 10-4: $A = 201 \text{ kpsi} \cdot \text{in}^m$, $m = 0.145$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{201}{0.007^{0.145}} = 412.7 \text{ kpsi}$$

$$\text{Table 10-6: } S_{sy} = 0.45 S_{ut} = 0.45(412.7) = 185.7 \text{ kpsi}$$

$\tau_s > S_{sy}$, that is, $325.1 > 185.7$ kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{[185.7(10^3)/1.2]\pi(0.007^3)}{8(1.340)3.358(0.031)} = 0.149 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.266 + 0.149 = 0.415 \text{ in} \quad \text{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-10 Given: B159 phosphor-bronze, sq. and grd. ends, $d = 0.014$ in, OD = 0.128 in, $L_0 = 0.50$ in, $N_t = 16$ coils.

$$\text{Eq. (10-1): } D = \text{OD} - d = 0.128 - 0.014 = 0.114 \text{ in}$$

$$C = D/d = 0.114/0.014 = 8.143$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(8.143)+2}{4(8.143)-3} = 1.169$$

$$\text{Table (10-1): } N_a = N_t - 2 = 16 - 2 = 14 \text{ coils}$$

$$\text{Table 10-5: } G = 6 \text{ Mpsi}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.014^4 (6)10^6}{8(0.114^3)14} = 1.389 \text{ lbf/in}$$

$$\text{Table (10-1): } L_s = dN_t = 0.014(16) = 0.224 \text{ in}$$

$$y_s = L_0 - L_s = 0.50 - 0.224 = 0.276 \text{ in}$$

$$F_s = ky_s = 1.389(0.276) = 0.3834 \text{ lbf}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.169 \frac{8(0.3834)0.114}{\pi(0.014^3)} = 47.42(10^3) \text{ psi} \quad (1)$$

$$\text{Table 10-4: } A = 145 \text{ kpsi}\cdot\text{in}^m, m = 0$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{145}{0.014^0} = 145 \text{ kpsi}$$

$$\text{Table 10-6: } S_{sy} = 0.35 S_{ut} = 0.35(145) = 47.25 \text{ kpsi}$$

$\tau_s > S_{sy}$, that is, $47.42 > 47.25$ kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{[47.25(10^3)/1.2]\pi(0.014^3)}{8(1.169)1.389(0.114)} = 0.229 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.224 + 0.229 = 0.453 \text{ in} \quad \text{Ans.}$$

10-11 Given: A313 stainless steel, sq. and grd. ends, $d = 0.050$ in, OD = 0.250 in, $L_0 = 0.68$ in, $N_t = 11.2$ coils.

$$D = \text{OD} - d = 0.250 - 0.050 = 0.200 \text{ in}$$

$$\text{Eq. (10-1): } C = D/d = 0.200/0.050 = 4$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(4)+2}{4(4)-3} = 1.385$$

$$\text{Table (10-1): } N_a = N_t - 2 = 11.2 - 2 = 9.2 \text{ coils}$$

$$\text{Table 10-5: } G = 10 \text{ Mpsi}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.050^4 (10) 10^6}{8(0.2^3) 9.2} = 106.1 \text{ lbf/in}$$

$$\text{Table (10-1): } L_s = dN_t = 0.050(11.2) = 0.56 \text{ in}$$

$$y_s = L_0 - L_s = 0.68 - 0.56 = 0.12 \text{ in}$$

$$F_s = ky_s = 106.1(0.12) = 12.73 \text{ lbf}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.385 \frac{8(12.73)0.2}{\pi(0.050^3)} = 71.8(10^3) \text{ psi}$$

$$\text{Table 10-4: } A = 169 \text{ kpsi} \cdot \text{in}^m, m = 0.146$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{169}{0.050^{0.146}} = 261.7 \text{ kpsi}$$

$$\text{Table 10-6: } S_{sy} = 0.35 S_{ut} = 0.35(261.7) = 91.6 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{91.6}{71.8} = 1.28 \quad \text{Spring is solid-safe } (n_s > 1.2) \quad \text{Ans.}$$

10-12 Given: A227 hard-drawn wire, sq. and grd. ends, $d = 0.148$ in, OD = 2.12 in, $L_0 = 2.5$ in, $N_t = 5.75$ coils.

$$D = \text{OD} - d = 2.12 - 0.148 = 1.972 \text{ in}$$

$$\text{Eq. (10-1): } C = D/d = 1.972/0.148 = 13.32 \quad (\text{high})$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(13.32)+2}{4(13.32)-3} = 1.099$$

$$\text{Table (10-1): } N_a = N_t - 2 = 5.75 - 2 = 3.75 \text{ coils}$$

$$\text{Table 10-5: } G = 11.4 \text{ Mpsi}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.148^4 (11.4) 10^6}{8(1.972^3) 3.75} = 23.77 \text{ lbf/in}$$

$$\text{Table (10-1): } L_s = dN_t = 0.148(5.75) = 0.851 \text{ in}$$

$$y_s = L_0 - L_s = 2.5 - 0.851 = 1.649 \text{ in}$$

$$F_s = ky_s = 23.77(1.649) = 39.20 \text{ lbf}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.099 \frac{8(39.20)1.972}{\pi(0.148^3)} = 66.7(10^3) \text{ psi}$$

$$\text{Table 10-4: } A = 140 \text{ kpsi}\cdot\text{in}^m, m = 0.190$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{140}{0.148^{0.190}} = 201.3 \text{ kpsi}$$

$$\text{Table 10-6: } S_{sy} = 0.35 S_{ut} = 0.45(201.3) = 90.6 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{90.6}{66.7} = 1.36 \quad \text{Spring is solid-safe } (n_s > 1.2) \quad \text{Ans.}$$

10-13 Given: A229 OQ&T steel, sq. and grd. ends, $d = 0.138$ in, $OD = 0.92$ in, $L_0 = 2.86$ in, $N_t = 12$ coils.

$$D = OD - d = 0.92 - 0.138 = 0.782 \text{ in}$$

$$\text{Eq. (10-1): } C = D/d = 0.782/0.138 = 5.667$$

$$\text{Eq. (10-5): } K_B = \frac{4C + 2}{4C - 3} = \frac{4(5.667) + 2}{4(5.667) - 3} = 1.254$$

$$\text{Table (10-1): } N_a = N_t - 2 = 12 - 2 = 10 \text{ coils}$$

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 11.5$ Mpsi.

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.138^4 (11.5)10^6}{8(0.782^3)10} = 109.0 \text{ lbf/in}$$

$$\text{Table (10-1): } L_s = dN_t = 0.138(12) = 1.656 \text{ in}$$

$$y_s = L_0 - L_s = 2.86 - 1.656 = 1.204 \text{ in}$$

$$F_s = ky_s = 109.0(1.204) = 131.2 \text{ lbf}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.254 \frac{8(131.2)0.782}{\pi(0.138^3)} = 124.7(10^3) \text{ psi} \quad (1)$$

$$\text{Table 10-4: } A = 147 \text{ kpsi}\cdot\text{in}^m, m = 0.187$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{147}{0.138^{0.187}} = 212.9 \text{ kpsi}$$

$$\text{Table 10-6: } S_{sy} = 0.50 S_{ut} = 0.50(212.9) = 106.5 \text{ kpsi}$$

$\tau_s > S_{sy}$, that is, $124.7 > 106.5$ kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{[106.5(10^3)/1.2]\pi(0.138^3)}{8(1.254)109.0(0.782)} = 0.857 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 1.656 + 0.857 = 2.51 \text{ in} \quad \text{Ans.}$$

10-14 Given: A232 chrome-vanadium steel, sq. and grd. ends, $d = 0.185 \text{ in}$, $\text{OD} = 2.75 \text{ in}$, $L_0 = 7.5 \text{ in}$, $N_t = 8 \text{ coils}$.

$$D = \text{OD} - d = 2.75 - 0.185 = 2.565 \text{ in}$$

$$\text{Eq. (10-1): } C = D/d = 2.565/0.185 = 13.86 \quad (\text{high})$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(13.86)+2}{4(13.86)-3} = 1.095$$

$$\text{Table (10-1): } N_a = N_t - 2 = 8 - 2 = 6 \text{ coils}$$

$$\text{Table 10-5: } G = 11.2 \text{ Mpsi.}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.185^4 (11.2) 10^6}{8(2.565^3) 6} = 16.20 \text{ lbf/in}$$

$$\text{Table (10-1): } L_s = dN_t = 0.185(8) = 1.48 \text{ in}$$

$$y_s = L_0 - L_s = 7.5 - 1.48 = 6.02 \text{ in}$$

$$F_s = ky_s = 16.20(6.02) = 97.5 \text{ lbf}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.095 \frac{8(97.5) 2.565}{\pi (0.185^3)} = 110.1(10^3) \text{ psi} \quad (1)$$

$$\text{Table 10-4: } A = 169 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.168$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{169}{0.185^{0.168}} = 224.4 \text{ kpsi}$$

$$\text{Table 10-6: } S_{sy} = 0.50 S_{ut} = 0.50(224.4) = 112.2 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{112.2}{110.1} = 1.02 \quad \text{Spring is not solid-safe } (n_s < 1.2)$$

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s) \pi d^3}{8K_B k D} = \frac{[112.2(10^3)/1.2] \pi (0.185^3)}{8(1.095) 16.20 (2.565)} = 5.109 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 1.48 + 5.109 = 6.59 \text{ in} \quad \text{Ans.}$$

10-15 Given: A313 stainless steel, sq. and grd. ends, $d = 0.25 \text{ mm}$, $\text{OD} = 0.95 \text{ mm}$, $L_0 = 12.1 \text{ mm}$, $N_t = 38 \text{ coils}$.

$$D = \text{OD} - d = 0.95 - 0.25 = 0.7 \text{ mm}$$

$$\text{Eq. (10-1): } C = D/d = 0.7/0.25 = 2.8 \quad (\text{low})$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(2.8)+2}{4(2.8)-3} = 1.610$$

Table (10-1): $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 69.0(10^3)$ MPa.

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{0.25^4 (69.0) 10^3}{8(0.7^3) 36} = 2.728 \text{ N/mm}$$

Table (10-1): $L_s = dN_t = 0.25(38) = 9.5$ mm
 $y_s = L_0 - L_s = 12.1 - 9.5 = 2.6$ mm
 $F_s = ky_s = 2.728(2.6) = 7.093$ N

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.610 \frac{8(7.093)0.7}{\pi(0.25^3)} = 1303 \text{ MPa} \quad (1)$$

Table 10-4 (dia. less than table): $A = 1867 \text{ MPa} \cdot \text{mm}^m$, $m = 0.146$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{1867}{0.25^{0.146}} = 2286 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35 S_{ut} = 0.35(2286) = 734$ MPa

$\tau_s > S_{sy}$, that is, $1303 > 734$ MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s) \pi d^3}{8K_B k D} = \frac{(734/1.2) \pi (0.25^3)}{8(1.610) 2.728 (0.7)} = 1.22 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 9.5 + 1.22 = 10.72 \text{ mm} \quad \text{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-16 Given: A228 music wire, sq. and grd. ends, $d = 1.2$ mm, OD = 6.5 mm, $L_0 = 15.7$ mm, $N_t = 10.2$ coils.

$$D = \text{OD} - d = 6.5 - 1.2 = 5.3 \text{ mm}$$

$$\text{Eq. (10-1): } C = D/d = 5.3/1.2 = 4.417$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(4.417)+2}{4(4.417)-3} = 1.368$$

Table (10-1): $N_a = N_t - 2 = 10.2 - 2 = 8.2$ coils

Table 10-5 ($d = 1.2/25.4 = 0.0472$ in): $G = 81.7(10^3)$ MPa.

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{1.2^4 (81.7) 10^3}{8(5.3^3) 8.2} = 17.35 \text{ N/mm}$$

Table (10-1): $L_s = dN_t = 1.2(10.2) = 12.24$ mm
 $y_s = L_0 - L_s = 15.7 - 12.24 = 3.46$ mm
 $F_s = ky_s = 17.35(3.46) = 60.03$ N

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.368 \frac{8(60.03)5.3}{\pi(1.2^3)} = 641.4 \text{ MPa} \quad (1)$$

$$\text{Table 10-4: } A = 2211 \text{ MPa}\cdot\text{mm}^m, \quad m = 0.145$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{2211}{1.2^{0.145}} = 2153 \text{ MPa}$$

$$\text{Table 10-6: } S_{sy} = 0.45 S_{ut} = 0.45(2153) = 969 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{969}{641.4} = 1.51 \quad \text{Spring is solid-safe } (n_s > 1.2) \quad \text{Ans.}$$

10-17 Given: A229 OQ&T steel, sq. and grd. ends, $d = 3.5 \text{ mm}$, $\text{OD} = 50.6 \text{ mm}$, $L_0 = 75.5 \text{ mm}$, $N_t = 5.5 \text{ coils}$.

$$\text{Eq. (10-1): } D = \text{OD} - d = 50.6 - 3.5 = 47.1 \text{ mm}$$

$$C = D/d = 47.1/3.5 = 13.46 \quad (\text{high})$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(13.46)+2}{4(13.46)-3} = 1.098$$

$$\text{Table (10-1): } N_a = N_t - 2 = 5.5 - 2 = 3.5 \text{ coils}$$

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 79.3(10^3) \text{ MPa}$.

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{3.5^4 (79.3)10^3}{8(47.1^3)3.5} = 4.067 \text{ N/mm}$$

$$\text{Table (10-1): } L_s = dN_t = 3.5(5.5) = 19.25 \text{ mm}$$

$$y_s = L_0 - L_s = 75.5 - 19.25 = 56.25 \text{ mm}$$

$$F_s = ky_s = 4.067(56.25) = 228.8 \text{ N}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.098 \frac{8(228.8)47.1}{\pi(3.5^3)} = 702.8 \text{ MPa} \quad (1)$$

$$\text{Table 10-4: } A = 1855 \text{ MPa}\cdot\text{mm}^m, \quad m = 0.187$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{1855}{3.5^{0.187}} = 1468 \text{ MPa}$$

$$\text{Table 10-6: } S_{sy} = 0.50 S_{ut} = 0.50(1468) = 734 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{734}{702.8} = 1.04 \quad \text{Spring is not solid-safe } (n_s < 1.2)$$

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B k D} = \frac{(734/1.2)\pi(3.5^3)}{8(1.098)4.067(47.1)} = 48.96 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 19.25 + 48.96 = 68.2 \text{ mm} \quad \text{Ans.}$$

10-18 Given: B159 phosphor-bronze, sq. and grd. ends, $d = 3.8 \text{ mm}$, $\text{OD} = 31.4 \text{ mm}$, $L_0 = 71.4 \text{ mm}$, $N_t = 12.8 \text{ coils}$.

$$D = \text{OD} - d = 31.4 - 3.8 = 27.6 \text{ mm}$$

$$\text{Eq. (10-1): } C = D/d = 27.6/3.8 = 7.263$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(7.263)+2}{4(7.263)-3} = 1.192$$

$$\text{Table (10-1): } N_a = N_t - 2 = 12.8 - 2 = 10.8 \text{ coils}$$

$$\text{Table 10-5: } G = 41.4(10^3) \text{ MPa.}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{3.8^4 (41.4)10^3}{8(27.6^3)10.8} = 4.752 \text{ N/mm}$$

$$\text{Table (10-1): } L_s = dN_t = 3.8(12.8) = 48.64 \text{ mm}$$

$$y_s = L_0 - L_s = 71.4 - 48.64 = 22.76 \text{ mm}$$

$$F_s = ky_s = 4.752(22.76) = 108.2 \text{ N}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.192 \frac{8(108.2)27.6}{\pi(3.8^3)} = 165.2 \text{ MPa} \quad (1)$$

$$\text{Table 10-4 } (d = 3.8/25.4 = 0.150 \text{ in}): A = 932 \text{ MPa}\cdot\text{mm}^m, m = 0.064$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{932}{3.8^{0.064}} = 855.7 \text{ MPa}$$

$$\text{Table 10-6: } S_{sy} = 0.35 S_{ut} = 0.35(855.7) = 299.5 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{299.5}{165.2} = 1.81 \quad \text{Spring is solid-safe } (n_s > 1.2) \quad \text{Ans.}$$

10-19 Given: A232 chrome-vanadium steel, sq. and grd. ends, $d = 4.5 \text{ mm}$, $\text{OD} = 69.2 \text{ mm}$, $L_0 = 215.6 \text{ mm}$, $N_t = 8.2 \text{ coils}$.

$$D = \text{OD} - d = 69.2 - 4.5 = 64.7 \text{ mm}$$

$$\text{Eq. (10-1): } C = D/d = 64.7/4.5 = 14.38 \quad (\text{high})$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(14.38)+2}{4(14.38)-3} = 1.092$$

$$\text{Table (10-1): } N_a = N_t - 2 = 8.2 - 2 = 6.2 \text{ coils}$$

$$\text{Table 10-5: } G = 77.2(10^3) \text{ MPa.}$$

$$\text{Eq. (10-9): } k = \frac{d^4 G}{8D^3 N_a} = \frac{4.5^4 (77.2)10^3}{8(64.7^3)6.2} = 2.357 \text{ N/mm}$$

$$\text{Table (10-1): } L_s = dN_t = 4.5(8.2) = 36.9 \text{ mm}$$

$$y_s = L_0 - L_s = 215.6 - 36.9 = 178.7 \text{ mm}$$

$$F_s = ky_s = 2.357(178.7) = 421.2 \text{ N}$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.092 \frac{8(421.2)64.7}{\pi(4.5^3)} = 832 \text{ MPa} \quad (1)$$

$$\text{Table 10-4: } A = 2005 \text{ MPa}\cdot\text{mm}^m, \quad m = 0.168$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{2005}{4.5^{0.168}} = 1557 \text{ MPa}$$

$$\text{Table 10-6: } S_{sy} = 0.50 S_{ut} = 0.50(1557) = 779 \text{ MPa}$$

$\tau_s > S_{sy}$, that is, $832 > 779$ MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s , giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B k D} = \frac{(779/1.2)\pi(4.5^3)}{8(1.092)2.357(64.7)} = 139.5 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 36.9 + 139.5 = 176.4 \text{ mm} \quad \text{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-20 Given: A227 HD steel.

From the figure: $L_0 = 4.75$ in, OD = 2 in, and $d = 0.135$ in. Thus

$$D = \text{OD} - d = 2 - 0.135 = 1.865 \text{ in}$$

(a) By counting, $N_t = 12.5$ coils. Since the ends are squared along $1/4$ turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \quad \text{Ans.}$$

$$p = 4.75 / 12 = 0.396 \text{ in} \quad \text{Ans.}$$

The solid stack is 13 wire diameters

$$L_s = 13(0.135) = 1.755 \text{ in} \quad \text{Ans.}$$

(b) From Table 10-5, $G = 11.4$ Mpsi

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.135^4 (11.4)(10^6)}{8(1.865^3)(12)} = 6.08 \text{ lbf/in} \quad \text{Ans.}$$

$$(c) F_s = k(L_0 - L_s) = 6.08(4.75 - 1.755)(10^{-3}) = 18.2 \text{ lbf} \quad \text{Ans.}$$

$$(d) C = D/d = 1.865/0.135 = 13.81$$

$$K_B = \frac{4(13.81) + 2}{4(13.81) - 3} = 1.096$$

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.096 \frac{8(18.2)(1.865)}{\pi(0.135^3)} = 38.5(10^3) \text{ psi} = 38.5 \text{ kpsi} \quad \text{Ans.}$$

10-21 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (a) and (b). For N_a , $k = F_{\max}/y = 20/2 = 10$ lbf/in. For τ_s , $F = F_s = 20(1 + \xi) = 20(1 + 0.15) = 23$ lbf.

(a) Spring over a Rod					(b) Spring in a Hole				
Source	Parameter	Values			Source	Parameter	Values		
	d	0.075	0.080	0.085		d	0.075	0.080	0.085
	ID	0.800	0.800	0.800		OD	0.950	0.950	0.950
	D	0.875	0.880	0.885		D	0.875	0.870	0.865
Eq. (10-1)	C	11.667	11.000	10.412	Eq. (10-1)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-5)	K_B	1.115	1.122	1.129	Eq. (10-5)	K_B	1.115	1.123	1.133
Eq. (10-7)	τ_s	135.335	112.948	95.293	Eq. (10-7)	τ_s	135.335	111.787	93.434
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	-0.282	-0.391	-0.536	Eq. (10-22)	fom	-0.282	-0.398	-0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-22 In Prob. 10-21, there is an advantage of first selecting d as one can select from the available sizes (Table A-28). Selecting C first, requires a calculation of d where then a size must be selected from Table A-28. Consider part (a) of the problem. It is required that

$$\text{ID} = D - d = 0.800 \text{ in.} \quad (1)$$

From Eq. (10-1), $D = Cd$. Substituting this into the first equation yields

$$d = \frac{0.800}{C - 1} \quad (2)$$

Starting with $C = 10$, from Eq. (2) we find that $d = 0.089$ in. From Table A-28, the closest diameter is $d = 0.090$ in. Substituting this back into Eq. (1) gives $D = 0.890$ in, with $C = 0.890/0.090 = 9.889$, which are acceptable. From this point the solution is the same as Prob. 10-21. For part (b), use

$$OD = D + d = 0.950 \text{ in.} \quad (3)$$

and,
$$d = \frac{0.800}{C - 1} \quad (4)$$

(a) Spring over a rod				(b) Spring in a Hole				
Source		Parameter	Values	Source		Parameter	Values	
		C	10.000	10.5			C	10.000
Eq. (2)		d	0.089	0.084	Eq. (4)		d	0.086
Table A-28		d	0.090	0.085	Table A-28		d	0.085
Eq. (1)		D	0.890	0.885	Eq. (3)		D	0.865
Eq. (10-1)		C	9.889	10.412	Eq. (10-1)		C	10.176
Eq. (10-9)		N_a	13.669	11.061	Eq. (10-9)		N_a	11.846
Table 10-1		N_t	15.669	13.061	Table 10-1		N_t	13.846
Table 10-1		L_s	1.410	1.110	Table 10-1		L_s	1.177
$1.15y + L_s$		L_0	3.710	3.410	$1.15y + L_s$		L_0	3.477
Eq. (10-13)		$(L_0)_{cr}$	4.681	4.655	Eq. (10-13)		$(L_0)_{cr}$	4.550
Table 10-4		A	201.000	201.000	Table 10-4		A	201.000
Table 10-4		m	0.145	0.145	Table 10-4		m	0.145
Eq. (10-14)		S_{ut}	284.991	287.363	Eq. (10-14)		S_{ut}	287.363
Table 10-6		S_{sy}	128.246	129.313	Table 10-6		S_{sy}	129.313
Eq. (10-5)		K_B	1.135	1.128	Eq. (10-5)		K_B	1.135
Eq. (10-7)		τ_s	81.167	95.223	Eq. (10-7)		τ_s	93.643
$n_s = S_{sy}/\tau_s$		n_s	1.580	1.358	$n_s = S_{sy}/\tau_s$		n_s	1.381
Eq. (10-22)		fom	-0.725	-0.536	Eq. (10-22)		fom	-0.555

Again, for $n_s \geq 1.2$, the optimal size is $d = 0.085$ in.

Although this approach used less iterations than in Prob. 10-21, this was due to the initial values picked and not the approach.

10-23 One approach is to select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm. Starting with $d = 2$ mm,

$$D = ID + d = 11.25 + 2 = 13.25 \text{ mm}$$

$$C = D/d = 13.25/2 = 6.625 \quad (\text{acceptable})$$

Table 10-5 ($d = 2/25.4 = 0.0787$ in): $G = 79.3$ GPa

$$\text{Eq. (10-9): } N_a = \frac{d^4 G}{8kD^3} = \frac{2^4(79.3)10^3}{8(4.286)13.25^3} = 15.9 \text{ coils}$$

Assume squared and closed.

$$\text{Table 10-1: } N_t = N_a + 2 = 15.9 + 2 = 17.9 \text{ coils}$$

$$L_s = dN_t = 2(17.9) = 35.8 \text{ mm}$$

$$y_s = L_0 - L_s = 48 - 35.8 = 12.2 \text{ mm}$$

$$F_s = ky_s = 4.286(12.2) = 52.29 \text{ N}$$

$$\text{Eq. (10-5): } K_B = \frac{4C+2}{4C-3} = \frac{4(6.625)+2}{4(6.625)-3} = 1.213$$

$$\text{Eq. (10-7): } \tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.213 \left[\frac{8(52.29)13.25}{\pi(2^3)} \right] = 267.5 \text{ MPa}$$

$$\text{Table 10-4: } A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190$$

$$\text{Eq. (10-14): } S_{ut} = \frac{A}{d^m} = \frac{1783}{2^{0.190}} = 1563 \text{ MPa}$$

$$\text{Table 10-6: } S_{sy} = 0.45S_{ut} = 0.45(1563) = 703.3 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{703.3}{267.5} = 2.63 > 1.2 \quad O.K.$$

No other diameters in the given range work. So specify

A227-47 HD steel, $d = 2 \text{ mm}$, $D = 13.25 \text{ mm}$, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9 \text{ coils}$, $N_a = 15.9 \text{ coils}$, $k = 4.286 \text{ N/mm}$, $L_s = 35.8 \text{ mm}$, and $L_0 = 48 \text{ mm}$.
Ans.

10-24 Select A227 HD steel for its low cost. Try $L_0 = 48 \text{ mm}$, then for $y = 48 - 37.5 = 10.5 \text{ mm}$ when $F = 45 \text{ N}$. The spring rate is $k = F/y = 45/10.5 = 4.286 \text{ N/mm}$.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm.

$$D - d = 11.25 \quad (1)$$

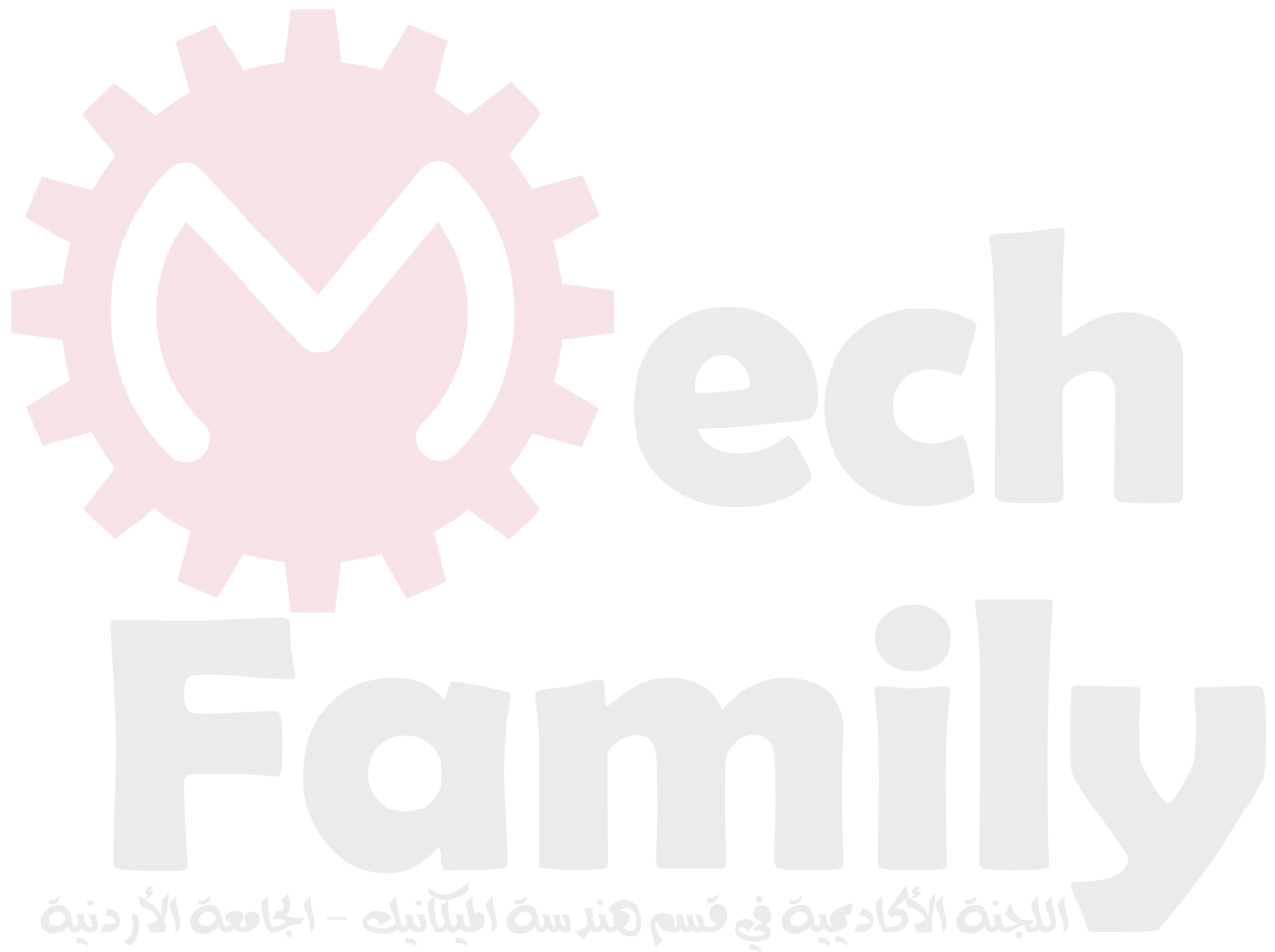
$$\text{and, } D = Cd \quad (2)$$

Starting with $C = 8$, gives $D = 8d$. Substitute into Eq. (1) resulting in $d = 1.607 \text{ mm}$.

Selecting the nearest diameter in the given range, $d = 1.6 \text{ mm}$. From this point, the calculations are shown in the third column of the spreadsheet output shown. We see that for $d = 1.6 \text{ mm}$, the spring is not solid safe. Iterating on C we find that $C = 6.5$ provides acceptable results with the specifications

A227-47 HD steel, $d = 2 \text{ mm}$, $D = 13.25 \text{ mm}$, ID = 11.25 mm, OD = 15.25 mm, squared

and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm.
Ans.



Source		Parameter Values		
	C	8.000	7	6.500
Eq. (2)	d	1.607	1.875	2.045
Table A-28	d	1.600	1.800	2.000
Eq. (1)	D	12.850	13.050	13.250
Eq. (10-1)	C	8.031	7.250	6.625
Eq. (10-9)	N_a	7.206	10.924	15.908
Table 10-1	N_t	9.206	12.924	17.908
Table 10-1	L_s	14.730	23.264	35.815
$L_0 - L_s$	y_s	33.270	24.736	12.185
$F_s = ky_s$	F_s	142.594	106.020	52.224
Table 10-4	A	1783.000	1783.000	1783.000
Table 10-4	m	0.190	0.190	0.190
Eq. (10-14)	S_{ut}	1630.679	1594.592	1562.988
Table 10-6	S_{sy}	733.806	717.566	703.345
Eq. (10-5)	K_B	1.172	1.200	1.217
Eq. (10-7)	τ_s	1335.568	724.943	268.145
$n_s = S_{sy}/\tau_s$	n_s	0.549	0.990	2.623

The only difference between selecting C first rather than d as was done in Prob. 10-23, is that once d is calculated, the closest wire size must be selected. Iterating on d uses available wire sizes from the beginning.

10-25 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be made aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at
1 - (800) - 237 - 5225
www.centuryspring.com
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-26 Given: ID = 0.6 in, $C = 10$, $L_0 = 5$ in, $L_s = 5 - 3 = 2$ in, sq. & grd ends, unpeened, HD A227 wire.

(a) With ID = $D - d = 0.6$ in and $C = D/d = 10 \Rightarrow 10d - d = 0.6 \Rightarrow d = 0.0667$ in *Ans.*, and $D = 0.667$ in.

(b) Table 10-1: $L_s = dN_t = 2$ in $\Rightarrow N_t = 2/0.0667$ 30 coils *Ans.*

(c) Table 10-1: $N_a = N_t - 2 = 30 - 2 = 28$ coils
 Table 10-5: $G = 11.5$ Mpsi
 Eq. (10-9): $k = \frac{d^4 G}{8D^3 N_a} = \frac{0.0667^4 (11.5) 10^6}{8(0.667^3) 28} = 3.424 \text{ lbf/in} \quad \text{Ans.}$

(d) Table 10-4: $A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$
 Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{140}{0.0667^{0.190}} = 234.2 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.45 S_{ut} = 0.45 (234.2) = 105.4 \text{ kpsi}$

$F_s = k y_s = 3.424(3) = 10.27 \text{ lbf}$
 Eq. (10-5): $K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.135 \frac{8(10.27) 0.667}{\pi (0.0667^3)}$$

$$= 66.72(10^3) \text{ psi} = 66.72 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \quad \text{Ans.}$$

(e) $\tau_a = \tau_m = 0.5 \tau_s = 0.5(66.72) = 33.36 \text{ kpsi}, r = \tau_a / \tau_m = 1$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67 S_{ut} = 0.67(234.2) = 156.9 \text{ kpsi}$

The Gerber ordinate intercept for the Zimmerli data is

$$S_e = \frac{S_{sa}}{1 - (S_{sm} / S_{su})^2} = \frac{35}{1 - (55 / 156.9)^2} = 39.9 \text{ kpsi}$$

Table 6-7, p. 307,

$$S_{sa} = \frac{r^2 S_{su}^2}{2 S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2 S_{se}}{r S_{su}} \right)^2} \right]$$

$$= \frac{1^2 (156.9^2)}{2(39.9)} \left\{ -1 + \sqrt{1 + \left[\frac{2(39.9)}{1(156.9)} \right]^2} \right\} = 37.61 \text{ kpsi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{37.61}{33.36} = 1.13 \quad \text{Ans.}$$

10-27 Given: $OD \leq 0.9 \text{ in}, C = 8, L_0 = 3 \text{ in}, L_s = 1 \text{ in}, y_s = 3 - 1 = 2 \text{ in}$, sq. ends, unpeened, music wire.

(a) Try $OD = D + d = 0.9 \text{ in}, C = D/d = 8 \Rightarrow D = 8d \Rightarrow 9d = 0.9 \Rightarrow d = 0.1 \text{ Ans.}$

$$D = 8(0.1) = 0.8 \text{ in}$$

(b) Table 10-1: $L_s = d(N_t + 1) \Rightarrow N_t = L_s / d - 1 = 1/0.1 - 1 = 9 \text{ coils}$ *Ans.*

Table 10-1: $N_a = N_t - 2 = 9 - 2 = 7 \text{ coils}$

(c) Table 10-5: $G = 11.75 \text{ Mpsi}$

Eq. (10-9): $k = \frac{d^4 G}{8D^3 N_a} = \frac{0.1^4 (11.75) 10^6}{8(0.8^3) 7} = 40.98 \text{ lbf/in}$ *Ans.*

(d) $F_s = ky_s = 40.98(2) = 81.96 \text{ lbf}$

Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(8)+2}{4(8)-3} = 1.172$

Eq. (10-7): $\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.172 \frac{8(81.96)0.8}{\pi(0.1^3)} = 195.7(10^3) \text{ psi} = 195.7 \text{ kpsi}$

Table 10-4: $A = 201 \text{ kpsi}\cdot\text{in}^m$, $m = 0.145$

Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{201}{0.1^{0.145}} = 280.7 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.45 S_{ut} = 0.45(280.7) = 126.3 \text{ kpsi}$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{126.3}{195.7} = 0.645 \quad \text{Ans.}$$

(e) $\tau_a = \tau_m = \tau_s / 2 = 195.7/2 = 97.85 \text{ kpsi}$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67 S_{ut} = 0.67(280.7) = 188.1 \text{ kpsi}$

The Gerber ordinate intercept for the Zimmerli data is

$$S_e = \frac{S_{sa}}{1 - (S_{sm} / S_{su})^2} = \frac{35}{1 - (55 / 188.1)^2} = 36.83 \text{ kpsi}$$

Table 6-7, p. 307,

$$S_{sa} = \frac{r^2 S_{su}^2}{2 S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2 S_{se}}{r S_{su}} \right)^2} \right]$$

$$= \frac{1^2 (188.1^2)}{2(38.3)} \left\{ -1 + \sqrt{1 + \left[\frac{2(38.3)}{1(188.1)} \right]^2} \right\} = 36.83 \text{ kpsi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{36.83}{97.85} = 0.376 \quad \text{Ans.}$$

Obviously, the spring is severely under designed and will fail statically and in fatigue. Increasing C would improve matters. Try $C = 12$. This yields $n_s = 1.83$ and $n_f = 1.00$.

10-28 Note to the Instructor: In the first printing of the text, the wire material was incorrectly identified as music wire instead of oil-tempered wire. This will be corrected in subsequent printings. We are sorry for any inconvenience.

Given: $F_{\max} = 300$ lbf, $F_{\min} = 150$ lbf, $\Delta y = 1$ in, OD = 2.1 – 0.2 = 1.9 in, $C = 7$, unpeened, sq. & grd., oil-tempered wire.

(a) $D = \text{OD} - d = 1.9 - d \quad (1)$

$C = D/d = 7 \Rightarrow D = 7d \quad (2)$

Substitute Eq. (2) into (1)

$7d = 1.9 - d \Rightarrow d = 1.9/8 = 0.2375$ in *Ans.*

(b) From Eq. (2): $D = 7d = 7(0.2375) = 1.663$ in *Ans.*

(c) $k = \frac{\Delta F}{\Delta y} = \frac{300 - 150}{1} = 150$ lbf/in *Ans.*

(d) Table 10-5: $G = 11.6$ Mpsi

Eq. (10-9): $N_a = \frac{d^4 G}{8D^3 k} = \frac{0.2375^4 (11.6) 10^6}{8(1.663^3) 150} = 6.69$ coils

Table 10-1: $N_t = N_a + 2 = 8.69$ coils *Ans.*

(e) Table 10-4: $A = 147$ kpsi·in^{*m*}, $m = 0.187$

Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{147}{0.2375^{0.187}} = 192.3$ kpsi

Table 10-6: $S_{sy} = 0.5 S_{ut} = 0.5(192.3) = 96.15$ kpsi

Eq. (10-5): $K_B = \frac{4C + 2}{4C - 3} = \frac{4(7) + 2}{4(7) - 3} = 1.2$

Eq. (10-7): $\tau_s = K_B \frac{8F_s D}{\pi d^3} = S_{sy}$

$$F_s = \frac{\pi d^3 S_{sy}}{8 K_B D} = \frac{\pi (0.2375^3) 96.15 (10^3)}{8 (1.2) 1.663} = 253.5 \text{ lbf}$$

$$y_s = F_s / k = 253.5 / 150 = 1.69 \text{ in}$$

Table 10-1: $L_s = N_t d = 8.46(0.2375) = 2.01 \text{ in}$

$$L_0 = L_s + y_s = 2.01 + 1.69 = 3.70 \text{ in} \quad \text{Ans.}$$

10-29 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N} \theta$$

The torsion of a section is $T = PR$ where $dL = R d\theta$

$$\begin{aligned} \delta_p &= \frac{\partial U}{\partial P} = \frac{1}{GJ} \int T \frac{\partial T}{\partial P} dL = \frac{1}{GJ} \int_0^{2\pi N} PR^3 d\theta \\ &= \frac{P}{GJ} \int_0^{2\pi N} \left(R_1 + \frac{R_2 - R_1}{2\pi N} \theta \right)^3 d\theta \\ &= \frac{P}{GJ} \left(\frac{1}{4} \right) \left(\frac{2\pi N}{R_2 - R_1} \right) \left[\left(R_1 + \frac{R_2 - R_1}{2\pi N} \theta \right)^4 \right]_0^{2\pi N} \\ &= \frac{\pi P N}{2GJ(R_2 - R_1)} (R_2^4 - R_1^4) = \frac{\pi P N}{2GJ} (R_1 + R_2)(R_1^2 + R_2^2) \\ J &= \frac{\pi}{32} d^4 \quad \therefore \delta_p = \frac{16PN}{Gd^4} (R_1 + R_2)(R_1^2 + R_2^2) \\ k &= \frac{P}{\delta_p} = \frac{d^4 G}{16N(R_1 + R_2)(R_1^2 + R_2^2)} \quad \text{Ans.} \end{aligned}$$

10-30 Given: $F_{\min} = 4 \text{ lbf}$, $F_{\max} = 18 \text{ lbf}$, $k = 9.5 \text{ lbf/in}$, $\text{OD} \leq 2.5 \text{ in}$, $n_f = 1.5$.

For a food service machinery application select A313 Stainless wire.

Table 10-5: $G = 10(10^6) \text{ psi}$

Note that for $0.013 \leq d \leq 0.10 \text{ in}$ $A = 169$, $m = 0.146$
 $0.10 < d \leq 0.20 \text{ in}$ $A = 128$, $m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7 \text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11 \text{ lbf}, \quad r = 7 / 11$$

Try, $d = 0.080$ in, $S_{ut} = \frac{169}{(0.08)^{0.146}} = 244.4$ kpsi
 $S_{su} = 0.67S_{ut} = 163.7$ kpsi, $S_{sy} = 0.35S_{ut} = 85.5$ kpsi

Try unpeened using Zimmerli's endurance data: $S_{sa} = 35$ kpsi, $S_{sm} = 55$ kpsi

Gerber: $S_{se} = \frac{S_{sa}}{1 - (S_{sm} / S_{su})^2} = \frac{35}{1 - (55 / 163.7)^2} = 39.5$ kpsi

$$S_{sa} = \frac{(7 / 11)^2 (163.7)^2}{2(39.5)} \left\{ -1 + \sqrt{1 + \left[\frac{2(39.5)}{(7 / 11)(163.7)} \right]^2} \right\} = 35.0 \text{ kpsi}$$

$$\alpha = S_{sa} / n_f = 35.0 / 1.5 = 23.3 \text{ kpsi}$$

$$\beta = \frac{8F_a}{\pi d^2} (10^{-3}) = \left[\frac{8(7)}{\pi (0.08^2)} \right] (10^{-3}) = 2.785 \text{ kpsi}$$

$$C = \frac{2(23.3) - 2.785}{4(2.785)} + \sqrt{\left[\frac{2(23.3) - 2.785}{4(2.785)} \right]^2 - \frac{3(23.3)}{4(2.785)}} = 6.97$$

$$D = Cd = 6.97(0.08) = 0.558 \text{ in}$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4(6.97) + 2}{4(6.97) - 3} = 1.201$$

$$\tau_a = K_B \left(\frac{8F_a D}{\pi d^3} \right) = 1.201 \left[\frac{8(7)(0.558)}{\pi (0.08^3)} (10^{-3}) \right] = 23.3 \text{ kpsi}$$

$$n_f = 35 / 23.3 = 1.50 \text{ checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ coils}$$

$$N_t = 31.02 + 2 = 33 \text{ coils, } L_s = dN_t = 0.08(33) = 2.64 \text{ in}$$

$$y_{\max} = F_{\max} / k = 18 / 9.5 = 1.895 \text{ in}$$

$$y_s = (1 + \xi)y_{\max} = (1 + 0.15)(1.895) = 2.179 \text{ in}$$

$$L_0 = 2.64 + 2.179 = 4.819 \text{ in}$$

$$(L_0)_{cr} = 2.63 \frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}$$

$$\tau_s = 1.15(18 / 7)\tau_a = 1.15(18 / 7)(23.3) = 68.9 \text{ kpsi}$$

$$n_s = S_{sy} / \tau_s = 85.5 / 68.9 = 1.24$$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2 (0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128	128
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{se}	39.452	39.654	40.046	40.469
S_{sa}	35.000	35.000	35.000	35.000
α	23.333	23.333	23.333	23.333
β	2.785	2.129	1.602	1.228
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.993	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_S	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179
L_0	4.818	3.606	3.020	2.764
$(L_0)_{cr}$	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
$f_s(\text{Hz})$	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory. The specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = $0.879 + 0.092 = 0.971$ in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns *Ans.*

10-31 The steps are the same as in Prob. 10-23 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

The problem then proceeds as in Prob. 10-23. The results for the wire sizes are shown below (see solution to Prob. 10-23 for additional details).

Iteration of d for the first trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{se}	52.706	53.239	54.261	55.345	L_s	1.295	0.774	0.517	0.410
S_{sa}	43.513	43.560	43.634	43.691	$y_{\max}$	2.875	2.875	2.875	2.875
α	29.008	29.040	29.090	29.127	L_0	4.170	3.649	3.392	3.285
β	2.785	2.129	1.602	1.228	$(L_0)_{cr}$	3.809	5.924	9.354	14.219
C	9.052	12.309	16.856	22.433	τ_s	85.782	85.876	86.022	86.133
D	0.724	1.126	1.778	2.703	n_s	0.997	0.977	0.941	0.907
					$f(\text{Hz})$	140.040	145.559	149.938	152.966

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line ($n_s < 1$). The table below uses $n_f = 2$.

Iteration of d for the second trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{se}	52.706	53.239	54.261	55.345	L_s	3.379	1.765	1.000	0.667
S_{sa}	43.513	43.560	43.634	43.691	$y_{\max}$	2.875	2.875	2.875	2.875
α	21.756	21.780	21.817	21.845	L_0	6.254	4.640	3.875	3.542
β	2.785	2.129	1.602	1.228	$(L_0)_{cr}$	2.691	4.266	6.821	10.449
C	6.395	8.864	12.292	16.485	τ_s	64.336	64.407	64.517	64.600
D	0.512	0.811	1.297	1.986	n_s	1.329	1.302	1.255	1.210
					$f(\text{Hz})$	98.936	104.827	109.340	112.409

The satisfactory spring has design specifications of: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = $0.811 + 0.092 = 0.903$ in, $L_0 = 4.266$ in, and $N_t = 19.6$ turns. *Ans.*

10-32 This is the same as Prob. 10-30 since $S_{sa} = 35$ kpsi. Therefore, the specifications are:

A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.84$ turns *Ans.*

10-33 For the Gerber fatigue-failure criterion, $S_{su} = 0.67S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm} / S_{su})^2}, \quad S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}} \right)^2} \right]$$

The equation for S_{sa} is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_a	8.915	6.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.630	8.160
S_{sa}	34.658	34.652	K_B	1.111	1.095
α	23.105	23.101	τ_a	23.105	23.101
β	1.732	1.523	n_f	1.500	1.500
C	12.004	13.851	τ_s	70.855	70.844
D	1.260	1.551	n_s	1.770	1.754
ID	1.155	1.439	f_n	105.433	106.922
OD	1.365	1.663	fom	-0.973	-1.022

There are only slight changes in the results.

10-34 As in Prob. 10-35, the basic change is S_{sa} .

For Goodman, $S_{se} = \frac{S_{sa}}{1 - (S_{sm} / S_{su})}$

Recalculate S_{sa} with

$$S_{sa} = \frac{rS_{se}S_{su}}{rS_{su} + S_{se}}$$

Calculations for the last 2 diameters of Ex. 10-5 are given below.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_a	9.153	6.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.572	8.090
S_{sa}	34.386	34.380	K_B	1.112	1.096
α	22.924	22.920	τ_a	22.924	22.920
β	1.732	1.523	n_f	1.500	1.500
C	11.899	13.732	τ_s	70.301	70.289
D	1.249	1.538	n_s	1.784	1.768
ID	1.144	1.426	f_n	104.509	106.000
OD	1.354	1.650	fom	-0.986	-1.034

There are only slight differences in the results.

10-35 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi $\cdot$ in^m, $m = 0.190$, rel cost = 1.

Try $d = 0.067$ in, $S_{ut} = \frac{140}{(0.067)^{0.190}} = 234.0$ kpsi

Table 10-6: $S_{sy} = 0.45S_{ut} = 105.3$ kpsi

Table 10-7: $S_y = 0.75S_{ut} = 175.5$ kpsi

Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2} [(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C - 1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C - 1) \left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 1 \right)$$

$$C^2 - \frac{1}{4} \left(1 + \frac{\pi d^2 S_y}{4n_y F_{\max}} - 1 \right) C + \frac{1}{4} \left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 2 \right) = 0$$

$$C = \frac{1}{2} \left[\frac{\pi d^2 S_y}{16n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16n_y F_{\max}} \right)^2 - \frac{\pi d^2 S_y}{4n_y F_{\max}} + 2} \right] \text{ take positive root}$$

$$= \frac{1}{2} \left\{ \frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} \right.$$

$$\left. + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)} \right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2} \right\} = 4.590$$

$$D = Cd = 4.59(0.067) = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D} \left[\frac{33\,500}{\exp(0.105C)} \pm 1000 \left(4 - \frac{C-3}{6.5} \right) \right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)} \left\{ \frac{33\,500}{\exp[0.105(4.590)]} - 1000 \left(4 - \frac{4.590-3}{6.5} \right) \right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer. Therefore, use $F_i = 7$ lbf

$$k = \frac{18 - 7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4 (11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18 \text{ lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

$$\text{Body: } K_B = \frac{4C + 2}{4C - 3} = \frac{4(4.590) + 2}{4(4.590) - 3} = 1.326$$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3} (10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in, } C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4) - 1}{4(4) - 4} = 1.25$$

$$\tau_B = (K)_B \left[\frac{8F_{\max} D}{\pi d^3} \right] = 1.25 \left[\frac{8(18)(0.3075)}{\pi(0.067)^3} \right] (10^{-3}) = 58.58 \text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1) \frac{\pi^2 d^2 (N_b + 2) D}{4} = - \frac{\pi^2 (0.067)^2 (44.88 + 2) (0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18 \text{ lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	-0.160	-0.144	-0.138	-0.135	-0.133	-0.135	-0.138	-0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-36 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD = 1.5 in, $d = 0.162$ in.

$$D = \text{OD} - d = 1.5 - 0.162 = 1.338 \text{ in}$$

(a) Eq. (10-39):

$$\begin{aligned} L_0 &= 2(D - d) + (N_b + 1)d \\ &= 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12 \text{ in} \quad \text{Ans.} \end{aligned}$$

or

$$2d + L_0 = 2(0.162) + 16.12 = 16.45 \text{ in overall}$$

(b)

$$\begin{aligned} C &= \frac{D}{d} = \frac{1.338}{0.162} = 8.26 \\ K_B &= \frac{4C + 2}{4C - 3} = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166 \end{aligned}$$

$$\tau_i = K_B \left[\frac{8F_i D}{\pi d^3} \right] = 1.166 \frac{8(16)(1.338)}{\pi(0.162)^3} = 14\,950 \text{ psi} \quad \text{Ans.}$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.162)^4 (11.4)(10^6)}{8(1.338)^3 (84.4)} = 4.855 \text{ lbf/in} \quad \text{Ans.}$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m$, $m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$

$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$

$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$

$$= \frac{\pi (0.162)^3 (103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162 / 2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$

$$F = \frac{\pi (0.162)^3 (103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$

$$S_{yt} = \sigma = F \left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2} \right]$$

$$F = \frac{155.3(10^3)}{\left[16(1.099)(1.338) \right] / \left[\pi (0.162)^3 \right] + \left\{ 4 / \left[\pi (0.162)^2 \right] \right\}} = 85.8 \text{ lbf}$$

$$= \min(110.8, 103.9, 85.8) = 85.8 \text{ lbf} \quad \text{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \text{Ans.}$$

10-37 $F_{\min} = 9 \text{ lbf}$, $F_{\max} = 18 \text{ lbf}$

$$F_a = \frac{18 - 9}{2} = 4.5 \text{ lbf}, \quad F_m = \frac{18 + 9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \leq d \leq 0.1$ $A = 169 \text{ kpsi} \cdot \text{in}^m$, $m = 0.146$
 $0.1 \leq d \leq 0.2$ $A = 128 \text{ kpsi} \cdot \text{in}^m$, $m = 0.263$
 $E = 28 \text{ Mpsi}$, $G = 10 \text{ Gpsi}$

Try $d = 0.081 \text{ in}$ and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45 S_{ut} = 109.8 \text{ kpsi}$

$$S_e = \frac{S_r / 2}{1 - [S_r / (2 S_{ut})]^2} = \frac{109.8 / 2}{1 - [(109.8 / 2) / 243.9]^2} = 57.8 \text{ kpsi}$$

$$r = F_a / F_m = 4.5 / 13.5 = 0.333$$

Table 7-10:

$$S_a = \frac{r^2 S_{ut}^2}{2 S_e} \left[-1 + \sqrt{1 + \left(\frac{2 S_e}{r S_{ut}} \right)^2} \right]$$

$$S_a = \frac{(0.333)^2 (243.9^2)}{2(57.8)} \left[-1 + \sqrt{1 + \left[\frac{2(57.8)}{0.333(243.9)} \right]^2} \right] = 42.2 \text{ kpsi}$$

Hook bending

$$(\sigma_a)_A = F_a \left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2} \right] = \frac{S_a}{(n_f)_A} = \frac{S_a}{2}$$

$$\frac{4.5}{\pi d^2} \left[\frac{(4C^2 - C - 1)16C}{4C(C - 1)} + 4 \right] = \frac{S_a}{2}$$

This equation reduces to a quadratic in C (see Prob. 10-35). The useable root for C is

$$\begin{aligned}
 C &= 0.5 \left[\frac{\pi d^2 S_a}{144} + \sqrt{\left(\frac{\pi d^2 S_a}{144} \right)^2 - \frac{\pi d^2 S_a}{36} + 2} \right] \\
 &= 0.5 \left\{ \frac{\pi(0.081)^2(42.2)(10^3)}{144} + \sqrt{\left[\frac{\pi(0.081)^2(42.2)(10^3)}{144} \right]^2 - \frac{\pi(0.081)^2(42.2)(10^3)}{36} + 2} \right\} \\
 &= 4.91
 \end{aligned}$$

$$D = Cd = 0.398 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D} \left[\frac{33\,500}{\exp(0.105C)} \pm 1000 \left(4 - \frac{C-3}{6.5} \right) \right]$$

Use the lowest F_i in the preferred range.

$$\begin{aligned}
 F_i &= \frac{\pi(0.081)^3}{8(0.398)} \left[\frac{33\,500}{\exp[0.105(4.91)]} - 1000 \left(4 - \frac{4.91-3}{6.5} \right) \right] \\
 &= 8.55 \text{ lbf}
 \end{aligned}$$

For simplicity we will round up to next 1/4 integer.

$$F_i = 8.75 \text{ lbf}$$

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i) / k = 2.602 + (18 - 8.75) / 36 = 2.859 \text{ in}$$

$$(\sigma_a)_A = \frac{4.5(4)}{\pi d^2} \left(\frac{4C^2 - C - 1}{C - 1} + 1 \right)$$

$$= \frac{18(10^{-3})}{\pi(0.081)^2} \left[\frac{4(4.91^2) - 4.91 - 1}{4.91 - 1} + 1 \right] = 21.1 \text{ kpsi}$$

$$(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \text{ checks}$$

$$\text{Body: } K_B = \frac{4C + 2}{4C - 3} = \frac{4(4.91) + 2}{4(4.91) - 3} = 1.300$$

$$\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a} \tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}$$

The repeating allowable stress from Table 7-8 is

$$S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is

$$S_{se} = \frac{73.17 / 2}{1 - [(73.17 / 2) / 163.4]^2} = 38.5 \text{ kpsi}$$

From Table 6-7,

$$(n_f)_{\text{body}} = \frac{1}{2} \left(\frac{163.4}{33.47} \right)^2 \left(\frac{11.16}{38.5} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(33.47)(38.5)}{163.4(11.16)} \right]^2} \right\} = 2.53$$

$$\text{Let } r_2 = 2d = 2(0.081) = 0.162$$

$$C_2 = \frac{2r_2}{d} = 4, \quad (K)_B = \frac{4(4) - 1}{4(4) - 4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B} \tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B} \tau_m = \frac{1.25}{1.30}(33.47) = 32.18 \text{ kpsi}$$

$$\text{Table 10-8: } (S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3 \text{ kpsi}$$

$$(S_{se})_B = \frac{68.3 / 2}{1 - [(68.3 / 2) / 163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2} \left(\frac{163.4}{32.18} \right)^2 \left(\frac{10.73}{35.7} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(32.18)(35.7)}{163.4(10.73)} \right]^2} \right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2} \left[\frac{(4C^2 - C - 1)}{C - 1} + 1 \right]$$

$$= \frac{4(18)}{\pi(0.081^2)} \left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1 \right] (10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\begin{aligned}\tau_i &= (F_i / F_a)\tau_a = (8.75 / 4.5)(11.16) = 21.7 \text{ kpsi} \\ r &= \tau_a / (\tau_m - \tau_i) = 11.16 / (33.47 - 21.7) = 0.948 \\ (S_{sa})_y &= \frac{r}{r+1}(S_{sy} - \tau_i) = \frac{0.948}{0.948+1}(85.4 - 21.7) = 31.0 \text{ kpsi} \\ (n_y)_{\text{body}} &= \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78\end{aligned}$$

Hook shear:

$$\begin{aligned}S_{sy} &= 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi} \\ \tau_{\max} &= (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi} \\ (n_y)_B &= \frac{73.2}{42.9} = 1.71 \\ \text{fom} &= -\frac{7.6\pi^2 d^2 (N_b + 2)D}{4} = -\frac{7.6\pi^2 (0.081)^2 (23.3 + 2)(0.398)}{4} = -1.239\end{aligned}$$

A tabulation of several wire sizes follow

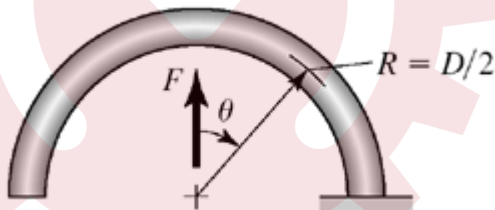
d	0.081	0.085	0.092	0.098	0.105	0.12
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
S_a	42.136	41.841	41.360	40.980	40.570	39.786
C	4.903	5.484	6.547	7.510	8.693	11.451
D	0.397	0.466	0.602	0.736	0.913	1.374
OD	0.478	0.551	0.694	0.834	1.018	1.494
F_i (calc)	8.572	7.874	6.798	5.987	5.141	3.637
F_i (rd)	8.75	9.75	10.75	11.75	12.75	13.75
k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.86	17.90	11.38	8.03	5.55	2.77
N_b	23.50	17.54	11.02	7.68	5.19	2.42
L_0	2.617	2.338	2.127	2.126	2.266	2.918
$L_{18 \text{ lbf}}$	2.874	2.567	2.328	2.300	2.412	3.036
$(\sigma_a)_A$	21.068	20.920	20.680	20.490	20.285	19.893
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
K_B	1.301	1.264	1.216	1.185	1.157	1.117
$(\tau_a)_{\text{body}}$	11.141	10.994	10.775	10.617	10.457	10.177
$(\tau_m)_{\text{body}}$	33.424	32.982	32.326	31.852	31.372	30.532
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.531	2.547	2.569	2.583	2.596	2.616
$(K)_B$	1.250	1.250	1.250	1.250	1.250	1.250
$(\tau_a)_B$	10.705	10.872	11.080	11.200	11.294	11.391
$(\tau_m)_B$	32.114	32.615	33.240	33.601	33.883	34.173
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717

$(n_f)_B$	2.519	2.463	2.388	2.341	2.298	2.235
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\max}$	84.273	83.682	82.720	81.961	81.139	79.573
$(n_y)_A$	1.592	1.592	1.592	1.592	1.592	1.592
τ_i	21.663	23.820	25.741	27.723	29.629	31.097
r	0.945	1.157	1.444	1.942	2.906	4.703
$(S_{sy})_{\text{body}}$	85.372	84.773	83.800	83.030	82.198	80.611
$(S_{sa})_y$	30.958	32.688	34.302	36.507	39.109	40.832
$(n_y)_{\text{body}}$	2.779	2.973	3.183	3.438	3.740	4.012
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\max}$	42.819	43.486	44.321	44.801	45.177	45.564
$(n_y)_B$	1.709	1.671	1.621	1.589	1.560	1.516
fom	-1.246	-1.234	-1.245	-1.283	-1.357	-1.639

↑ optimal fom

The shaded areas show the conditions not satisfied.

10-38 For the hook,



$$M = FR \sin \theta, \quad \partial M / \partial F = R \sin \theta$$

$$\delta_F = \frac{1}{EI} \int_0^{\pi/2} F (R \sin \theta)^2 R d\theta = \frac{\pi}{2} \frac{FR^3}{EI}$$

The total deflection of the body and the two hooks

$$\begin{aligned} \delta &= \frac{8FD^3 N_b}{d^4 G} + 2 \left(\frac{\pi}{2} \frac{FR^3}{EI} \right) = \frac{8FD^3 N_b}{d^4 G} + \frac{\pi F (D/2)^3}{E(\pi/64)(d^4)} \\ &= \frac{8FD^3}{d^4 G} \left(N_b + \frac{G}{E} \right) = \frac{8FD^3 N_a}{d^4 G} \\ \therefore N_a &= N_b + \frac{G}{E} \quad \text{Q.E.D.} \end{aligned}$$

10-39 Table 10-5 ($d = 4 \text{ mm} = 0.1575 \text{ in}$): $E = 196.5 \text{ GPa}$

Table 10-4 for A227:

$$A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190$$

Eq. (10-14):
$$S_{ut} = \frac{A}{d^m} = \frac{1783}{4^{0.190}} = 1370 \text{ MPa}$$

Eq. (10-57):
$$S_y = \sigma_{\text{all}} = 0.78 S_{ut} = 0.78(1370) = 1069 \text{ MPa}$$

$$D = OD - d = 32 - 4 = 28 \text{ mm}$$

$$C = D/d = 28/4 = 7$$

Eq. (10-43):
$$K_i = \frac{4C^2 - C - 1}{4C(C - 1)} = \frac{4(7^2) - 7 - 1}{4(7)(7 - 1)} = 1.119$$

Eq. (10-44):
$$\sigma = K_i \frac{32Fr}{\pi d^3}$$

At yield, $Fr = M_y$, $\sigma = S_y$. Thus,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi(4^3)1069(10^{-3})}{32(1.119)} = 6.00 \text{ N} \cdot \text{m}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{k}$$

where from Eq. (10-51):
$$k = \frac{d^4 E}{10.8DN}$$

Thus,

$$N = 2.5 - \frac{M_y}{d^4 E / (10.8DN)}$$

Solving for N gives

$$\begin{aligned} N &= \frac{2.5}{1 + [10.8DM_y / (d^4 E)]} \\ &= \frac{2.5}{1 + \{[10.8(28)(6.00)] / [4^4(196.5)]\}} = 2.413 \text{ turns} \end{aligned}$$

This means $(2.5 - 2.413)(360^\circ)$ or 31.3° from closed. *Ans.*

Treating the hand force as in the middle of the grip,

$$r = 112.5 - 87.5 + \frac{87.5}{2} = 68.75 \text{ mm}$$

$$F_{\max} = \frac{M_y}{r} = \frac{6.00(10^3)}{68.75} = 87.3 \text{ N} \quad \text{Ans.}$$

10-40 The spring material and condition are unknown. Given $d = 0.081$ in and $OD = 0.500$,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4 E}{10.8 D N} = \frac{(0.081)^4 (28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

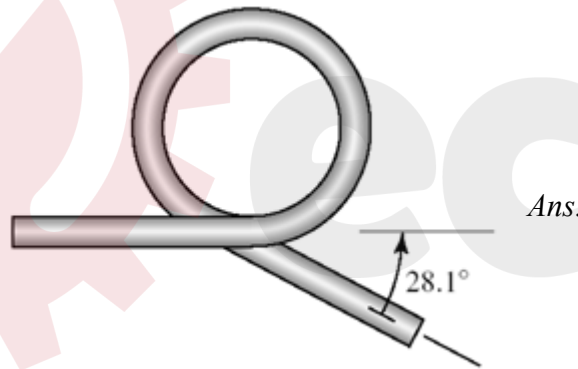
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180° , say 165° . This uses up $165/360$ or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360^\circ) = 28.1^\circ$). The original configuration of the spring was



(b)

$$C = \frac{D}{d} = \frac{0.419}{0.081} = 5.17$$

$$K_i = \frac{4C^2 - C - 1}{4C(C - 1)} = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168$$

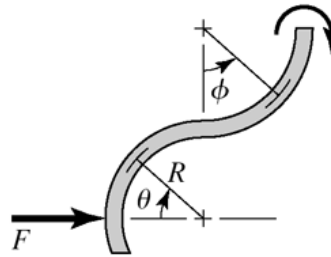
$$\sigma = K_i \frac{32M}{\pi d^3} = 1.168 \left[\frac{32(13.25)}{\pi(0.081)^3} \right] = 297(10^3) \text{ psi} = 297 \text{ kpsi} \quad \text{Ans.}$$

To achieve this stress level, the spring had to have set removed.

10-41 (a) Consider half and double results



Upper 180° section:



$$M = F[R + R(1 - \cos \phi)]$$

$$= FR(2 - \cos \phi), \quad \frac{\partial M}{\partial F} = R(2 - \cos \phi)$$

Lower section: $M = FR \sin \theta, \quad \frac{\partial M}{\partial F} = R \sin \theta$

Considering bending only:

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI} \left[\int_0^{l/2} 9FR^2 dx + \int_0^\pi FR^2(2 - \cos \phi)^2 R d\phi + \int_0^{\pi/2} F(R \sin \theta)^2 R d\theta \right]$$

$$= \frac{2F}{EI} \left[\frac{9}{2} R^2 l + R^3 \left(4\pi - 4 \sin \phi \Big|_0^\pi + \frac{\pi}{2} \right) + R^3 \left(\frac{\pi}{4} \right) \right]$$

$$= \frac{2FR^2}{EI} \left(\frac{19\pi}{4} R + \frac{9}{2} l \right) = \frac{FR^2}{2EI} (19\pi R + 18l)$$

The spring rate is

$$k = \frac{F}{\delta} = \frac{2EI}{R^2(19\pi R + 18l)} \quad \text{Ans.}$$

(b) Given: A227 HD wire, $d = 2$ mm, $R = 6$ mm, and $l = 25$ mm.

Table 10-5 ($d = 2$ mm = 0.0787 in): $E = 197.2$ MPa

$$k = \frac{2(197.2)10^9 \pi (0.002^4) / (64)}{0.006^2 [19\pi(0.006) + 18(0.025)]} = 10.65(10^3) \text{ N/m} = 10.65 \text{ N/mm} \quad \text{Ans.}$$

(c) The maximum stress will occur at the bottom of the top hook where the bending-moment is $3FR$ and the axial force is F . Using curved beam theory for bending,

Eq. (3-65), p. 119: $\sigma_i = \frac{Mc_i}{Aer_i} = \frac{3FRc_i}{(\pi d^2 / 4)e(R - d/2)}$

Axial: $\sigma_a = \frac{F}{A} = \frac{F}{\pi d^2 / 4}$

Combining,
$$\sigma_{\max} = \sigma_i + \sigma_a = \frac{4F}{\pi d^2} \left[\frac{3Rc_i}{e(R-d/2)} + 1 \right] = S_y$$

$$F = \frac{\pi d^2 S_y}{4 \left[\frac{3Rc_i}{e(R-d/2)} + 1 \right]} \quad (1) \quad \text{Ans.}$$

For the clip in part (b),

Eq. (10-14) and Table 10-4: $S_{ut} = A/d^m = 1783/2^{0.190} = 1563 \text{ MPa}$

Eq. (10-57): $S_y = 0.78 S_{ut} = 0.78(1563) = 1219 \text{ MPa}$

Table 3-4, p. 121:

$$r_n = \frac{l^2}{2(6 - \sqrt{6^2 - l^2})} = 5.95804 \text{ mm}$$

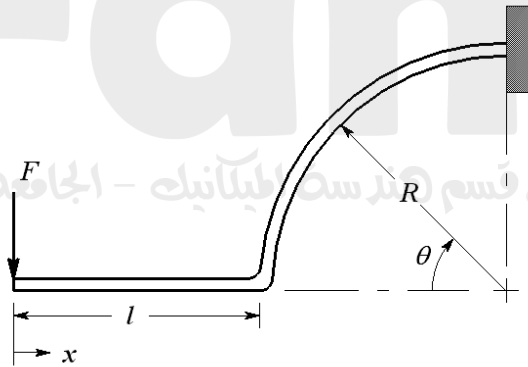
$$e = r_c - r_n = 6 - 5.95804 = 0.04196 \text{ mm}$$

$$c_i = r_n - (R - d/2) = 5.95804 - (6 - 2/2) = 0.95804 \text{ mm}$$

Eq. (1):

$$F = \frac{\pi (0.002^2) 1219 (10^6)}{4 \left[\frac{3(6) 0.95804}{0.04196(6-1)} + 1 \right]} = 46.0 \text{ N} \quad \text{Ans.}$$

10-42 (a)



$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x \quad 0 \leq x \leq l$$

$$M = Fl + FR(1 - \cos \theta), \quad \frac{\partial M}{\partial F} = l + R(1 - \cos \theta) \quad 0 \leq \theta \leq l$$

$$\begin{aligned} \delta_F &= \frac{1}{EI} \int_0^l -Fx(-x)dx + \int_0^{\pi/2} F[l + R(1 - \cos \theta)]^2 R d\theta \\ &= \frac{F}{12EI} \{4l^3 + 3R[2\pi l^2 + 4(\pi - 2)lR + (3\pi - 8)R^2]\} \end{aligned}$$

The spring rate is

$$k = \frac{F}{\delta_F} = \frac{12EI}{4l^3 + 3R[2\pi l^2 + 4(\pi - 2)lR + (3\pi - 8)R^2]} \quad \text{Ans.}$$

(b) Given: A313 stainless wire, $d = 0.063$ in, $R = 0.625$ in, and $l = 0.5$ in.

Table 10-5: $E = 28$ Mpsi

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (0.063^4) = 7.733(10^{-7}) \text{ in}^4$$

$$\begin{aligned} k &= \frac{12(28)10^6 (7.733)10^{-7}}{4(0.5^3) + 3(0.625)[2\pi(0.5^2) + 4(\pi - 2)0.5(0.625) + (3\pi - 8)(0.625^2)]} \\ &= 36.3 \text{ lbf/in} \quad \text{Ans.} \end{aligned}$$

(c) Table 10-4: $A = 169 \text{ kpsi} \cdot \text{in}^m$, $m = 0.146$

$$\text{Eq. (10-14):} \quad S_{ut} = A/d^m = 169/0.063^{0.146} = 253.0 \text{ kpsi}$$

$$\text{Eq. (10-57):} \quad S_y = 0.61 S_{ut} = 0.61(253.0) = 154.4 \text{ kpsi}$$

One can use curved beam theory as in the solution for Prob. 10-41. However, the equations developed in Sec. 10-12 are equally valid.

$$C = D/d = 2(0.625 + 0.063/2)/0.063 = 20.8$$

$$\text{Eq. (10-43):} \quad K_t = \frac{4C^2 - C - 1}{4C(C - 1)} = \frac{4(20.8^2) - 20.8 - 1}{4(20.8)(20.8 - 1)} = 1.037$$

Eq. (10-44), setting $\sigma = S_y$:

$$K_i \frac{32Fr}{\pi d^3} = S_y \Rightarrow 1.037 \frac{32F(0.5+0.625)}{\pi(0.063^3)} = 154.4(10^3)$$

Solving for F yields $F = 3.25 \text{ lbf}$ *Ans.*

Try solving part (c) of this problem using curved beam theory. You should obtain the same answer.

10-43 (a) $M = -Fx$

$$\sigma = \left| \frac{M}{I/c} \right| = \frac{Fx}{I/c} = \frac{Fx}{bh^2/6}$$

Constant stress,

$$\frac{bh^2}{6} = \frac{Fx}{\sigma} \Rightarrow h = \sqrt{\frac{6Fx}{b\sigma}} \quad (1) \quad \text{Ans.}$$

At $x = l$,

$$h_o = \sqrt{\frac{6Fl}{b\sigma}} \Rightarrow h = h_o \sqrt{x/l} \quad \text{Ans.}$$

(b) $M = -Fx, \partial M / \partial F = -x$

$$\begin{aligned} y &= \int_0^l \frac{M(\partial M / \partial F)}{EI} dx = \frac{1}{E} \int_0^l \frac{-Fx(-x)}{\frac{1}{12}bh_o^3(x/l)^{3/2}} dx = \frac{12Fl^{3/2}}{bh_o^3E} \int_0^l x^{1/2} dx \\ &= \frac{2}{3} \frac{12Fl^{3/2}}{bh_o^3E} l^{3/2} = \frac{8Fl^3}{bh_o^3E} \end{aligned}$$

$$k = \frac{F}{y} = \frac{bh_o^3E}{8l^3} \quad \text{Ans.}$$

10-44 Computer programs will vary.

10-45 Computer programs will vary.