The University of Jordan

Faculty of Engineering

Department of Industrial Engineering

Probabilistic Operations Research IE 0916555 — **Paper part of the Final Exam**

1st Semester 2022/2023 — **Date: 29. 1. 2023**

Course Instructor: Prof. Dr. Mohammad D. AL-Tahat. — **Time:**

EXAM INSTRUCTIONS

1. The Final Exam is a **Closed Book** and **closed Notes**.
2. It is forbidden to exchange any other things between you.
3. The instructor(s) proctor(s), and TA(s) will not respond to any question during the exam. If you think that something is wrong, **state** (write) your concern on the final Answering sheet.
4. Mobile phones **are strictly not allowed** to be with you during the exam. All phones must be turned off and put away. Students cannot use the cell phone calculator.
5. Write your name clearly in the **Arabic** language in all assigned place(s)

$$f(t)=\lambda e^{-\lambda t} \qquad P(t_a \le T)=1-e^{-\lambda} \qquad P_n(T)=\frac{(\lambda T)^n}{n!}e^{-\lambda T} \qquad P_n=\left(\frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1}\right)P_0 \qquad \sum_{i=0}^{\infty}x^i=\frac{1}{1-x}$$

$$S_n=\frac{a_1(1-r^n)}{1-r} \qquad L_q=\sum_{n=c+1=5}^{\infty}(n-c)p_n \qquad L_s=\sum_{n=0}^{\infty}np_n \qquad \mu_{ij}=\sum_{n=1}^{\infty}nf_{ij}^{(n)} \qquad \pi P=\pi$$

$$P^n=P^{n-m}P^m$$

$$L_s=\frac{\rho}{1-\rho}, \quad L_q=\frac{\rho^2}{1-\rho}, \quad W_s=\frac{L_s}{\lambda}, \quad W_q=W_s-\frac{1}{\mu}, \quad W_q=W_s-t_s, \quad \bar{c}=L_s-L_q, \quad W_q=\frac{L_q}{\lambda_{eff}},$$

$$L_s=\frac{\rho[1-(N+1)\rho^N+N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})},\rho\ne 1, \quad p_N=\frac{(1-\rho)\rho^N}{1-\rho^{N+1}}, \quad \lambda_{eff}=\lambda(1-p_N), \quad W_s=\frac{L_s}{\lambda_{eff}}, \quad L_q=L_s-\frac{\lambda_{eff}}{\mu},$$

$$L_q=\frac{\rho^{c+1}}{(c-1)!(c-\rho)^2}p_0, \quad \left[p_0=\left\{\left(\sum_{n=0}^{c-1}\frac{\rho^n}{n!}\right)+\frac{\rho^c}{c!}\left(\frac{1}{1-\frac{\rho}{c}}\right)\right\}^{-1}\left(\text{when } \frac{\rho}{c}<1\right)\right], \quad W_q=\frac{L_q}{\lambda} \qquad \|\mu_{ij}\|=(I-N_j)^{-1}1, j\ne i$$

$$L_q=\begin{cases}\dfrac{\rho^{c+1}}{(c-1)!(c-\rho)^2}\left\{1-\left(\dfrac{\rho}{c}\right)^{N-c+1}-(N-c+1)\left(1-\dfrac{\rho}{c}\right)\left(\dfrac{\rho}{c}\right)^{N-c}\right\}p_0 & ,\dfrac{\rho}{c}\ne 1\\[2ex] \dfrac{\rho^c(N-c)(N-c+1)}{2c!}p_0 & ,\dfrac{\rho}{c}=1\end{cases}$$

$$p_n=\begin{cases}\dfrac{\rho^n}{n!}p_0, & 0\le n<c\\[2ex] \dfrac{\rho^n}{c!c^{n-c}}p_0, & c\le n\le N\end{cases} \quad \text{where, } p_0=\begin{cases}\left(\left(\displaystyle\sum_{n=0}^{c-1}\frac{\rho^n}{n!}\right)+\dfrac{\rho^c\left(1-\left(\frac{\rho}{c}\right)^{N-c+1}\right)}{c!\left(1-\frac{\rho}{c}\right)}\right)^{-1} & ,\dfrac{\rho}{c}\ne 1\\[2ex] \left(\left(\displaystyle\sum_{n=0}^{c-1}\frac{\rho^n}{n!}\right)+\dfrac{\rho^c}{c!}(N-c+1)\right)^{-1} & ,\dfrac{\rho}{c}=1\end{cases} \qquad p_N=\frac{\rho^N}{c!c^{N-c}}p_0 \qquad \alpha_i=\sum_{j=0}^{m-1}r_{ij}$$

General Rate Matrix

System State index

$$R=\begin{array}{c} \\ 0\\ 1\\ \\ m-1\end{array}\begin{array}{c}\begin{array}{ccccc}0 & 1 & 2 & \dots & m\text{-}1\end{array}\\ \begin{bmatrix}0 & r_{01} & r_{02} & \dots & r_{0,m-1}\\ r_{10} & r_{11} & r_{12} & \dots & r_{1,m-1}\\ . & . & . & \dots & .\\ r_{m-1,0} & r_{m-1,1} & r_{m-1,2} & \dots & 0\end{bmatrix}\end{array}$$