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12–1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \text{m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

SOLUTION

$a = 2t - 6$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t (2t - 6)\,dt$$

$v = t^2 - 6t$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7$ m **Ans.**

Ans:
$v = 0$
$s = 80.7$ m

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12–2.

The acceleration of a particle as it moves along a straight line is given by $a = (4t^3 - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 2\ \text{m}$ and $v = 5\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 5\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_5^v dv = \int_0^t (4\,t^3 - 1)\,dt$$

$$v = t^4 - t + 5$$

$$\int_2^s ds = \int_0^t (t^4 - t + 5)\,dt$$

$$s = \frac{1}{5}t^5 - \frac{1}{2}t^2 + 5\,t + 2$$

When $t = 5$ s,

$$v = 625\ \text{m/s} \qquad \textbf{Ans.}$$

$$s = 639.5\ \text{m} \qquad \textbf{Ans.}$$

Since $v \neq 0$ then

$$d = 639.5 - 2 = 637.5\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v = 625\ \text{m/s}$
$s = 639.5\ \text{m}$
$d = 637.5\ \text{m}$

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12–3.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

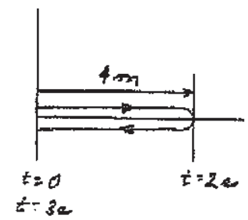
SOLUTION

$v = 6t - 3t^2$

$a = \dfrac{dv}{dt} = 6 - 6t$

At $t = 3$ s

$a = -12 \text{ m/s}^2$ **Ans.**



$ds = v\,dt$

$$\int_0^s ds = \int_0^t (6t - 3t^2)\,dt$$

$s = 3t^2 - t^3$

At $t = 3$ s

$s = 0$ **Ans.**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

when $t = 2$ s, $\quad s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans.**

Ans:
$a = -12 \text{ m/s}^2$
$s = 0$
$s_T = 8$ m
$(v_{sp})_{avg} = 2.67$ m/s

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***12–4.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$s = 10t^2 + 20$

(a) $s|_{1\text{ s}} = 10(1)^2 + 20 = 30\text{ mm}$

$s|_{5\text{ s}} = 10(5)^2 + 20 = 270\text{ mm}$

$\Delta s = 270 - 30 = 240\text{ mm}$ **Ans.**

(b) $\Delta t = 5 - 1 = 4\text{ s}$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{240}{4} = 60\text{ mm/s}$ **Ans.**

(c) $a = \dfrac{d^2 s}{dt^2} = 20\text{ mm/s}^2$ (for all t) **Ans.**

Ans:
$\Delta s = 240\text{ mm}$
$v_{avg} = 60\text{ mm/s}$
$a = 20\text{ mm/s}^2$

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12–5.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

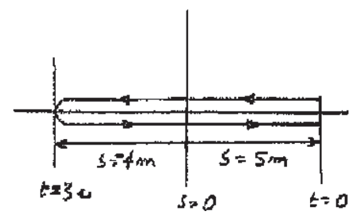
SOLUTION

$s = t^2 - 6t + 5$

$v = \frac{ds}{dt} = 2t - 6$

$a = \frac{dv}{dt} = 2$

$v = 0$ when $t = 3$



$s|_{t=0} = 5$

$s|_{t=3} = -4$

$s|_{t=6} = 5$

$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{0}{6} = 0$ **Ans.**

$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{9 + 9}{6} = 3 \text{ m/s}$ **Ans.**

$a|_{t=6} = 2 \text{ m/s}^2$ **Ans.**

Ans:
$v_{avg} = 0$
$(v_{sp})_{avg} = 3 \text{ m/s}$
$a|_{t=6\text{ s}} = 2 \text{ m/s}^2$

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12–6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$+\downarrow s = s_1 + v_1 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(9.81)(2)^2$$

$$s_A = 19.62 \text{ m}$$

$$s_A = 0 + 0 + \frac{1}{2}(9.81)(1)^2$$

$$s_B = 4.91 \text{ m}$$

$\Delta s = 19.62 - 4.91 = 14.71 \text{ m}$ **Ans.**

Ans:

$\Delta s = 14.71 \text{ m}$

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12–7.

A bus starts from rest with a constant acceleration of 1 m/s^2. Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION

***Kinematics*:**

$v_0 = 0$, $v = 25$ m/s, $s_0 = 0$, and $a_c = 1$ m/s^2.

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

$25 = 0 + (1)t$

$t = 25$ s **Ans.**

$(\overset{+}{\rightarrow})$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$25^2 = 0 + 2(1)(s - 0)$

$s = 312.5$ m **Ans.**

Ans:

$t = 25$ s
$s = 312.5$ m

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***12–8.**

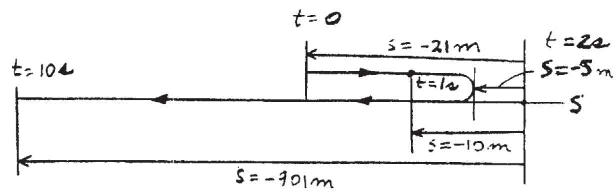
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$$v = 12 - 3t^2 \qquad \textbf{(1)}$$

$$a = \frac{dv}{dt} = -6t\big|_{t=4} = -24 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\int_{-10}^{s} ds = \int_{1}^{t} v\,dt = \int_{1}^{t} (12 - 3t^2)\,dt$$



$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s\big|_{t=0} = -21$$

$$s\big|_{t=10} = -901$$

$$\Delta s = -901 - (-21) = -880 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (1):

$$v = 0 \text{ when } t = 2s$$

$$s\big|_{t=2} = 12(2) - (2)^3 - 21 = -5$$

$$s_T = (21 - 5) + (901 - 5) = 912 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$a = -24 \text{ m/s}^2$
$\Delta s = -880$ m
$s_T = 912$ m

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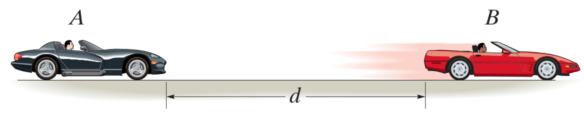
12–9.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



SOLUTION

Motion of car A:

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right| \qquad \textbf{Ans.}$$

Ans:

$$S_{BA} = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$$

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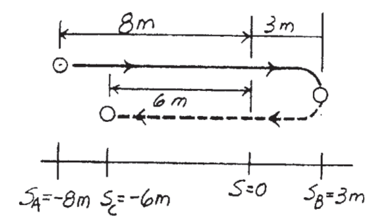
12–10.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

SOLUTION



Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{\text{Tot}} = s_{A \to B} + s_{B \to C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{avg} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{avg} = 0.222$ m/s
$(v_{sp})_{avg} = 2.22$ m/s

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12–11.

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})$ hr $= 30$ s **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792$ km $= 792$ m **Ans.**

Ans:
$t = 30$ s
$s = 792$ m

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***12–12.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$v = 1.29$ m/s **Ans.**

Ans:
$v = 1.29$ m/s

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12–13.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$a = 2t - 1$

$dv = a\,dt$

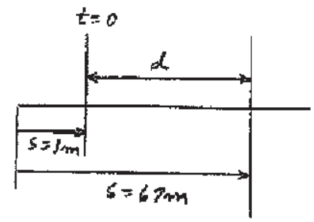
$$\int_2^v dv = \int_0^t (2t - 1)dt$$

$v = t^2 - t + 2$

$dx = v\,dt$

$$\int_t^s ds = \int_0^t (t^2 - t + 2)dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$



When $t = 6$ s

$v = 32$ m/s **Ans.**

$s = 67$ m **Ans.**

Since $v \neq 0$ for $0 \leq t \leq 6$ s, then

$d = 67 - 1 = 66$ m **Ans.**

Ans:
$v = 32$ m/s
$s = 67$ m
$d = 66$ m

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12–14.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$(\xrightarrow{+})\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$(\xrightarrow{+})\qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30$ m/s, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\xrightarrow{+})\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30$ m/s, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1\ \text{m/s}^2$. Thus,

$$(\xrightarrow{+})\qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\xrightarrow{+}\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km}$$ **Ans.**

Ans:
$s = 28.4$ km

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12–15.

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\big|_2^s = -4t\big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = -\frac{16}{(8t + 1)^2} \text{ m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{16}{(8t + 1)^2} \text{ m/s}$$

$$a = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$

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***12–16.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

SOLUTION

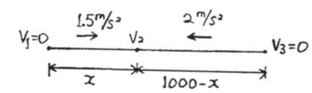
Using formulas of constant acceleration:

$v_2 = 1.5\,t_1$

$x = \frac{1}{2}(1.5)(t_1^2)$

$0 = v_2 - 2\,t_2$

$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$



Combining equations:

$t_1 = 1.33\,t_2; \quad v_2 = 2\,t_2$

$x = 1.33\,t_2^2$

$1000 - 1.33\,t_2^2 = 2\,t_2^2 - t_2^2$

$t_2 = 20.702\text{ s}; \qquad t_1 = 27.603\text{ s}$

$t = t_1 + t_2 = 48.3\text{ s}$ **Ans.**

Ans:
$t = 48.3\text{ s}$

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12–17.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{d\nu}{dt} = -\mathrm{k}\nu^3$$

$$\int_{\nu 0}^{\nu} \nu^{-3}\, d\nu = \int_0^t -k\, dt$$

$$-\frac{1}{2}(\nu^{-2} - \nu_0^{-2}) = -kt$$

$$\nu = \left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{-\frac{1}{2}}$$ **Ans.**

$$ds = \nu\, dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{\nu_0}\right]$$ Ans.

Ans:

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$$

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12–18.

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{6\text{ m/s}}^{v} \frac{v}{-1.5v^{\frac{1}{2}}}\,dv$$

$$s = \int_{6\text{ m/s}}^{v} -0.6667\,v^{\frac{1}{2}}\,dv$$

$$= \left(-0.4444v^{\frac{3}{2}} + 6.532\right)\text{ m}$$

When $v = 0$, $\quad s = -0.4444\left(0^{\frac{3}{2}}\right) + 6.532 = 6.53\text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = -\int_{6\text{ m/s}}^{v} \frac{dv}{1.5v^{\frac{1}{2}}}$$

$$t = -1.333\left(v^{\frac{1}{2}}\right)\Big|_{6\text{ m/s}}^{v} = \left(3.266 - 1.333v^{\frac{1}{2}}\right)\text{s}$$

When $v = 0$, $\quad t = 3.266 - 1.333\left(0^{\frac{1}{2}}\right) = 3.27\text{ s}$ **Ans.**

Ans:
$s = 6.53\text{ m}$
$t = 3.27\text{ s}$

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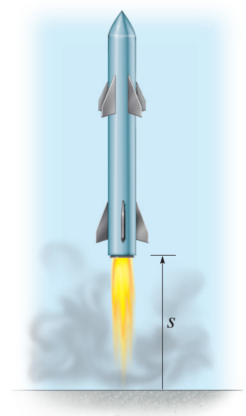
12–19. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this attitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$b = 6\ \mathrm{m/s^2} \qquad c = 0.02\ \mathrm{s^{-2}} \qquad s_{p1} = 2000\ \mathrm{m}$

$$a_p = b + c\,s_p = v_p \frac{dv_p}{ds_p}$$

$$\int_0^{v_p} v_p\, dv_p = \int_0^{s_p} \left(b + c\,s_p\right) ds_p$$

$$\frac{v_p^2}{2} = b\,s_p + \frac{c}{2} s_p^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b\,s_p + c\,s_p^2} \qquad v_{p1} = \sqrt{2b\,s_{p1} + c\,s_{p1}^2} \qquad v_{p1} = 322.49\ \mathrm{m/s}$$ **Ans.**

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b\,s_p + c\,s_p^2}}\, ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b\,s_p + c\,s_p^2}}\, ds_p \qquad t_1 = 19.27\ \mathrm{s}$$ **Ans.**

Ans:
$v_{p1} = 322.49\ \dfrac{\mathrm{m}}{\mathrm{s}}$
$t_1 = 19.27\ \mathrm{s}$

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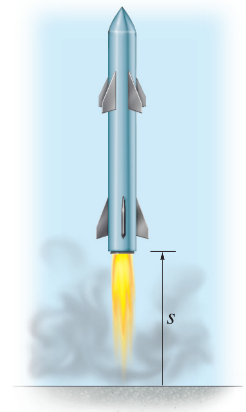
***12–20.**

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a\,ds = v\,dv$$

$$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$$

$$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12\,s + 0.02\,s^2}$$

$$ds = v\,dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}}\ln\left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$$

$t = 5.62$ s **Ans.**

Ans:
$t = 5.62$ s

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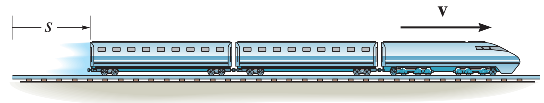
12–21.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})$ m/s^2, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.



SOLUTION

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}(v^5 - 32)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s} \qquad \textbf{Ans.}$$

$$a\,ds = v\,dv$$

$$ds = \frac{v\,dv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$v = 3.93 \text{ m/s}$
$s = 9.98 \text{ m}$

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12–22.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

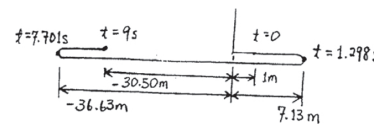
$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$

$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$



Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

Ans:
(a) $s = -30.5$ m
(b) $s_{\text{Tot}} = 56.0$ m
(c) $v = 10$ m/s

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12–23.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \to \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \to \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \to 1$. Then, from Eq. (1)

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
(a) $v = 45.5$ m/s
(b) $v_{max} = 100$ m/s

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***12–24.**

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 0 + (-6)(8) + \frac{1}{2}(9.81)(8)^2$$

$$= 265.92 \text{ m}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

Altitude $= h + h' = 265.92 + 48 = 314$ m **Ans.**

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$v = -6 + 9.81(8) = 72.5$ m/s **Ans.**

Ans:
$h = 314$ m
$v = 72.5$ m/s

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12–25.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20\ \text{m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$a = -2v$

$\dfrac{dv}{dt} = -2v$

$\displaystyle\int_{20}^{v} \frac{dv}{v} = \int_0^t -2\, dt$

$\ln \dfrac{v}{20} = -2t$

$v = \left(20e^{-2t}\right)\ \text{m/s}$ **Ans.**

$a = \dfrac{dv}{dt} = \left(-40e^{-2t}\right)\ \text{m/s}^2$ **Ans.**

$\displaystyle\int_0^s ds = v\, dt = \int_0^t (20e^{-2t})dt$

$s = -10e^{-2t}\big|_0^t = -10\left(e^{-2t} - 1\right)$

$s = 10\left(1 - e^{-2t}\right)\ \text{m}$ **Ans.**

Ans:
$v = \left(20e^{-2t}\right)\ \text{m/s}$
$a = \left(-40e^{-2t}\right)\ \text{m/s}^2$
$s = 10\left(1 - e^{-2t}\right)\ \text{m}$

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12–26.

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\frac{v^2}{2}\bigg|_0^v = \frac{1}{6}s^{3/2}\bigg|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2}\ \text{m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

Ans:
$v = 0.781$ m/s

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12–27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

Ans:
$t = 0.549\left(\frac{v_f}{g}\right)$

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***12–28.**

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)$ m/s^2, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$(+\downarrow)$
$$dv = adt$$
$$\int_{27}^{v} dv = \int_0^t -6tdt$$
$$v = \left(27 - 3t^2\right)\text{ m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\text{ s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$(+\downarrow)$
$$ds = vdt$$
$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$
$$s = \left(27t - t^3\right)\text{ m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0\text{ m} \qquad \textbf{Ans.}$$

Ans:
$s = 54.0$ m

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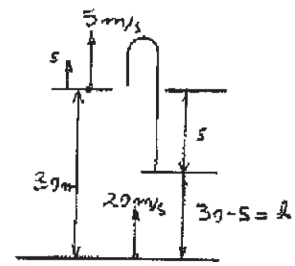
12–29.

A ball A is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

SOLUTION

Origin at roof:

Ball A:



$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball B:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$

Solving,

$t = 2$ s **Ans.**

$s = 9.62$ m

Distance from ground,

$d = (30 - 9.62) = 20.4$ m **Ans.**

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

Require

$$s_A = s_B$$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2$ s **Ans.**

$s_B = 20.4$ m **Ans.**

Ans:
$h = 20.4$ m
$t = 2$ s

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12–30.

A boy throws a ball straight up from the top of a 12-m high tower. If the ball falls past him 0.75 s later, determine the velocity at which it was thrown, the velocity of the ball when it strikes the ground, and the time of flight.

SOLUTION

***Kinematics*:** When the ball passes the boy, the displacement of the ball in equal to zero.

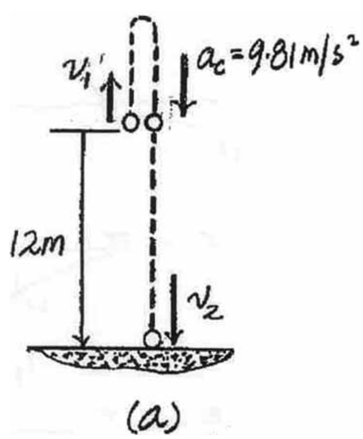
Thus, $s = 0$. Also, $s_0 = 0$, $v_0 = v_1$, $t = 0.75$ s, and $a_c = -9.81 \text{ m/s}^2$.

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v_1(0.75) + \frac{1}{2}(-9.81)(0.75^2)$$

$$v_1 = 3.679 \text{ m/s} = 3.68 \text{ m/s} \qquad \textbf{Ans.}$$

When the ball strikes the ground, its displacement from the roof top is $s = -12$ m. Also, $v_0 = v_1 = 3.679$ m/s, $t = t_2$, $v = v_2$, and $a_c = -9.81 \text{ m/s}^2$.



$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-12 = 0 + 3.679t_2 + \frac{1}{2}(-9.81)t_2^2$$

$$4.905t_2^2 - 3.679t_2 - 12 = 0$$

$$t_2 = \frac{3.679 \pm \sqrt{(-3.679)^2 - 4(4.905)(-12)}}{2(4.905)}$$

Choosing the positive root, we have

$$t_2 = 1.983 \text{ s} = 1.98 \text{ s} \qquad \textbf{Ans.}$$

Using this result,

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$v_2 = 3.679 + (-9.81)(1.983)$$

$$= -15.8 \text{ m/s} = 15.8 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_1 = 3.68$ m/s
$t_2 = 1.98$ s
$v_2 = 15.8 \text{ m/s} \downarrow$

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12–31.

The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

SOLUTION

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t\Big|_0^t = -\frac{1}{k}\ln(v_0 - ks)\Big|_0^s$$

$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - ks}\right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right) \qquad \textbf{Ans.}$$

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}\left(1 - e^{-kt}\right)\right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right)$$

$$a = -kv_0 e^{-kt} \qquad \textbf{Ans.}$$

Ans:

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

$$a = -kv_0 e^{-kt}$$

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***12–32.**

Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

SOLUTION

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h$, $t_A = t'$, and $(a_c)_A = -g$.

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A{}^2$$

$$h = 0 + v_0 t' + \frac{1}{2}(-g)(t')^2$$

$$h = v_0 t' - \frac{g}{2}t'^2 \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad v_A = (v_A)_0 + (a_c)_A t_A$$

$$v_A = v_0 + (-g)(t')$$

$$v_A = v_0 - gt' \qquad \textbf{(2)}$$

The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B{}^2$$

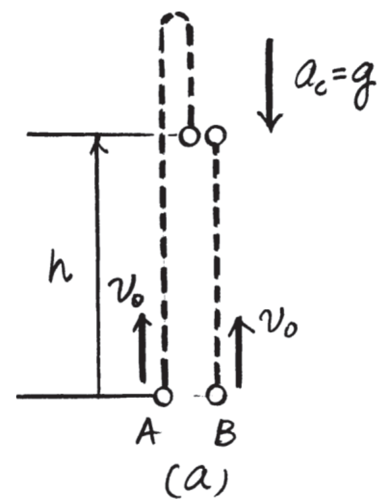
$$h = 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2$$

$$h = v_0(t' - t) - \frac{g}{2}(t' - t)^2 \qquad \textbf{(3)}$$

$$(+\uparrow) \qquad v_B = (v_B)_0 + (a_c)_B t_B$$

$$v_B = v_0 + (-g)(t' - t)$$

$$v_B = v_0 - g(t' - t) \qquad \textbf{(4)}$$



Solving Eqs. (1) and (3),

$$v_0 t' - \frac{g}{2}t'^2 = v_0(t' - t) - \frac{g}{2}(t' - t)^2$$

$$t' = \frac{2v_0 + gt}{2g} \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$v_A = v_0 - g\left(\frac{2v_0 + gt}{2g}\right)$$

$$= -\frac{1}{2}gt = \frac{1}{2}gt\downarrow \qquad \textbf{Ans.}$$

$$v_B = v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right)$$

$$= \frac{1}{2}gt \uparrow \qquad \textbf{Ans.}$$

Ans:
$t' = \dfrac{2v_0 + gt}{2g}$
$v_A = \dfrac{1}{2}gt \downarrow$
$v_B = \dfrac{1}{2}gt \uparrow$

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12–33.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\ \text{m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\left.\frac{v^2}{2}\right|_v^0 = \left.\frac{g_0 R^2}{R + y}\right|_0^\infty$$

$$v = \sqrt{2 g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\ \text{m/s} = 11.2\ \text{km/s}$$ **Ans.**

Ans:
$v = 11.2\ \text{km/s}$

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12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–33), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–33.

SOLUTION

From Prob. 12–33,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R + y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0\,R^2 \int_{y_0}^{y} \frac{dy}{(R + y)^2} = \int_0^v v\,dv$$

$$g_0\,R^2 \left[\frac{1}{R + y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0\,R^2 \left[\frac{1}{R + y} - \frac{1}{R + y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0\,(y_0 - y)}{(R + y)(R + y_0)}} \qquad \textbf{Ans.}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow \qquad \textbf{Ans.}$$

Ans:

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

$$v = 3.02 \text{ km/s}$$

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12–35.

A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the $v-t$ graph and determine the maximum speed of the train.

SOLUTION

For stage (1) motion,

$$(\overset{+}{\rightarrow}) \qquad v_1 = v_0 + (a_c)_1 t$$

$$v_{max} = 0 + (a_c)_1 t_1$$

$$v_{max} = (a_c)_1 t_1 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad v_1^2 = v_0^2 + 2(a_c)_1(s_1 - s_0)$$

$$v_{max}{}^2 = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

$$t_1 = \frac{2000}{v_{max}} \qquad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of v_{max} for $t = (t_2 - t_1)$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2$$

$$1000 + 2000 = 1000 + v_{max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{max}} \qquad \textbf{(4)}$$

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_3 = v_2 + (a_c)_3 t$$

$$0 = v_{max} - (a_c)_3(360 - t_2)$$

$$v_{max} = (a_c)_3(360 - t_2) \qquad \textbf{(5)}$$

$$(\overset{+}{\rightarrow}) \qquad v_3^2 = v_2^2 + 2(a_c)_3(s_3 - s_2)$$

$$0 = v_{max}{}^2 + 2[-(a_c)_3](4000 - 3000)$$

$$(a_c)_3 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(6)}$$

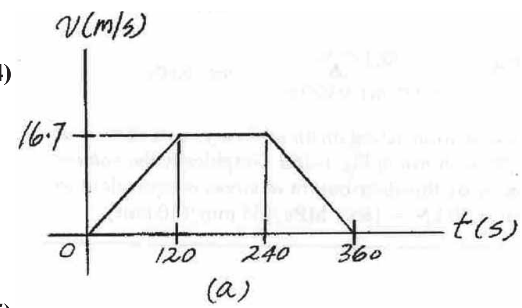
Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

$$360 - t_2 = \frac{2000}{v_{max}} \qquad \textbf{(7)}$$

Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120 \text{ s} \qquad t_2 = 240 \text{ s}$$

$$v_{max} = 16.7 \text{ m/s} \qquad \textbf{Ans.}$$



Based on the above results, the $v-t$ graph is shown in Fig. a.

$v = v_{max}$ for 2 min < t < 4 min.

Ans:
$v_{max} = 16.7$ m/s

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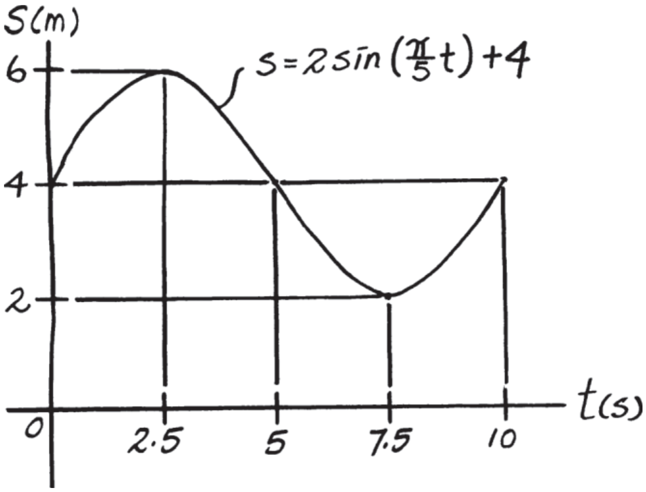
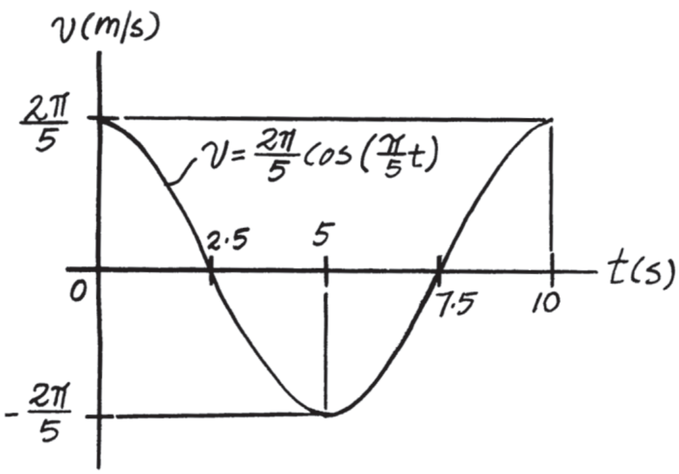
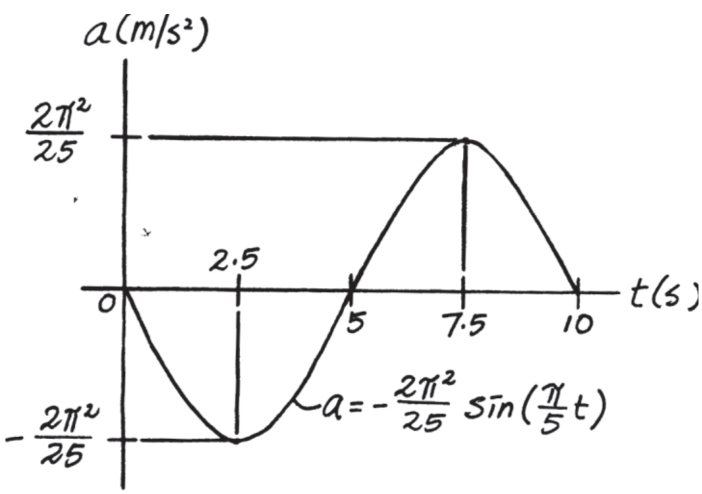
***12–36.**

If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.

SOLUTION







Ans:

$s = 2 \sin\left(\frac{\pi}{5}t\right) + 4$

$v = \frac{2\pi}{5} \cos\left(\frac{\pi}{5}t\right)$

$a = -\frac{2\pi^2}{25} \sin\left(\frac{\pi}{5}t\right)$

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12–37.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \le t \le 4$ s.

SOLUTION

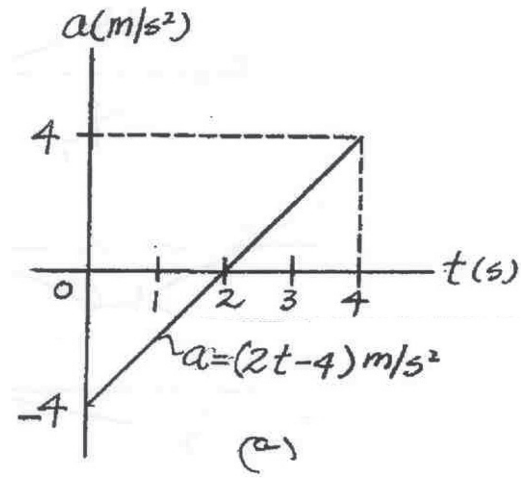
a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$
$$a = (2t - 4)\,\text{m/s}^2$$

Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$
$$a|_{t=2} = 0$$
$$a|_{t=4\,\text{s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. a.



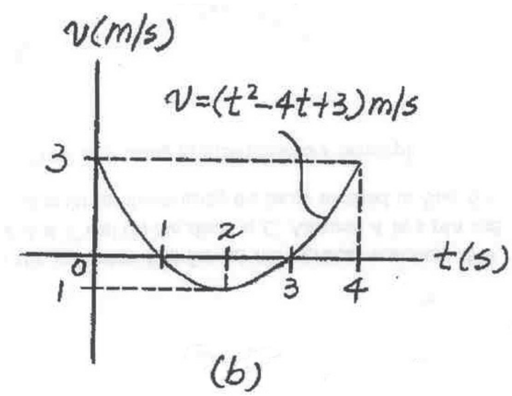
v–t Graph: The slope of the $v-t$ graph is zero when $a = \dfrac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\ \text{s}$$

The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\,\text{s}} = 0^2 - 4(0) + 3 = 3\ \text{m/s}$$
$$v|_{t=2\,\text{s}} = 2^2 - 4(2) + 3 = -1\ \text{m/s}$$
$$v|_{t=4\,\text{s}} = 4^2 - 4(4) + 3 = 3\ \text{m/s}$$

The $v-t$ graph is shown in Fig. b.



Ans:
$a|_{t=0} = -4\ \text{m/s}^2$
$a|_{t=2\,\text{s}} = 0$
$a|_{t=4\,\text{s}} = 4\ \text{m/s}^2$
$v|_{t=0} = 3\ \text{m/s}$
$v|_{t=2\,\text{s}} = -1\ \text{m/s}$
$v|_{t=4\,\text{s}} = 3\ \text{m/s}$

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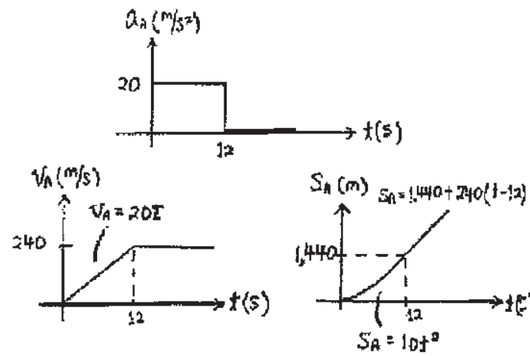
12–38.

Two rockets start from rest at the same elevation. Rocket A accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket B accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s. Construct the a–t, v–t, and s–t graphs for each rocket until $t = 20$ s. What is the distance between the rockets when $t = 20$ s?

SOLUTION

For rocket A

For $t < 12$ s



$$+\uparrow\ v_A = (v_A)_0 + a_A t$$

$$v_A = 0 + 20\,t$$

$$v_A = 20\,t$$

$$+\uparrow\ s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(20)\,t^2$$

$$s_A = 10\,t^2$$

When $t = 12$ s, $\quad v_A = 240$ m/s

$s_A = 1440$ m

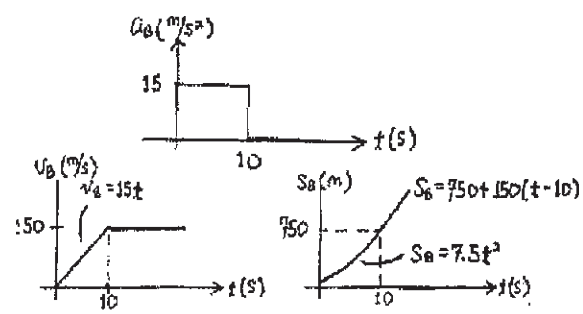
For $t > 12$ s

$v_A = 240$ m/s

$s_A = 1440 + 240(t - 12)$

For rocket B

For $t < 10$ s



$$+\uparrow\ v_B = (v_B)_0 + a_B t$$

$$v_B = 0 + 15\,t$$

$$v_B = 15\,t$$

$$+\uparrow\ s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(15)\,t^2$$

$$s_B = 7.5\,t^2$$

When $t = 10$ s, $\quad v_B = 150$ m/s

$s_B = 750$ m

For $t > 10$ s

$v_B = 150$ m/s

$s_B = 750 + 150(t - 10)$

When $t = 20$ s, $\quad s_A = 3360$ m, $\quad s_B = 2250$ m

$\Delta s = 1110\text{ m} = 1.11\text{ km}$ **Ans.**

Ans:
$\Delta s = 1.11$ km

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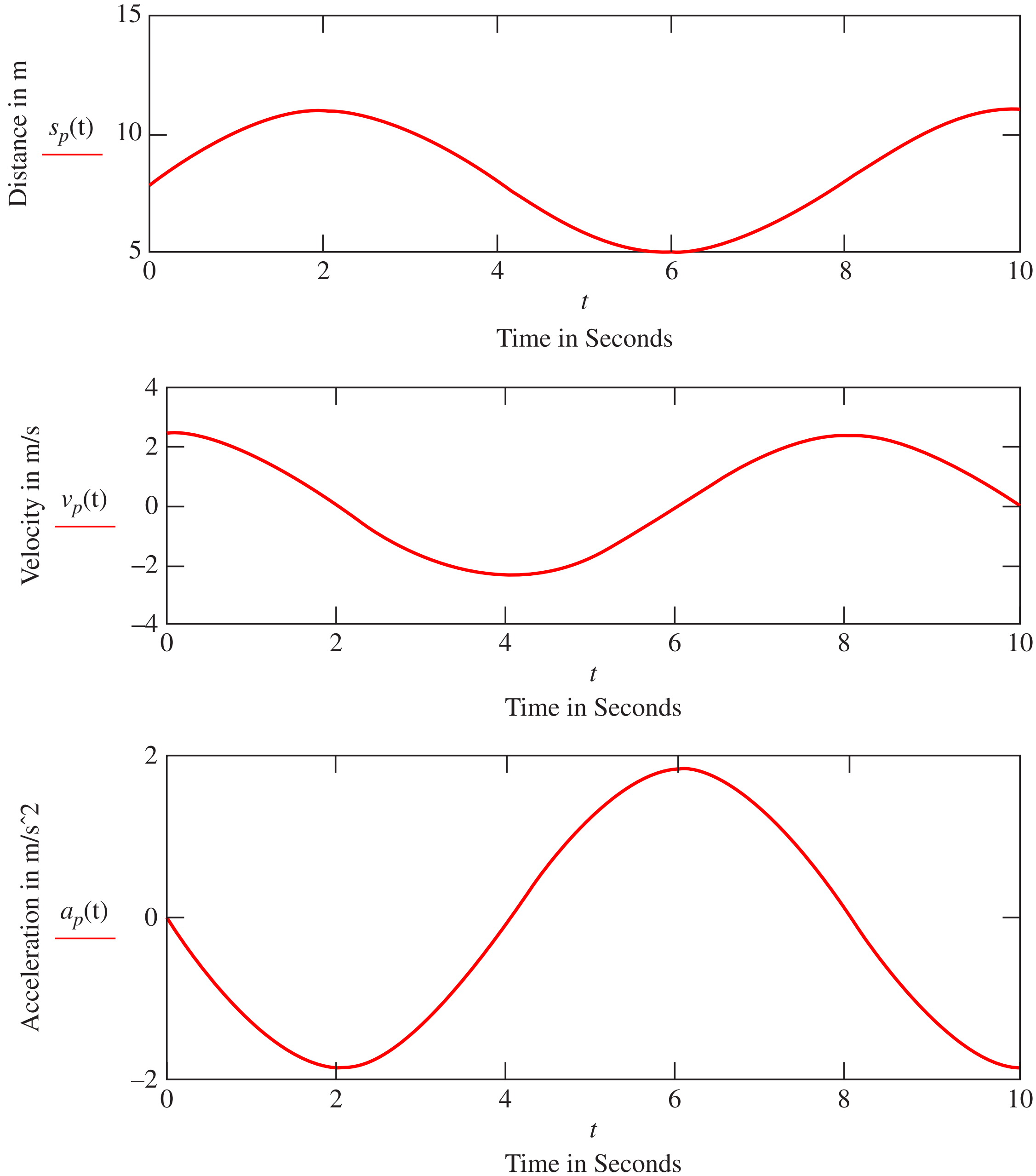
12–39.

If the position of a particle is defined by $s = [3 \sin (\pi/4)t + 8]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.

SOLUTION



Ans:

$$s = 3 \sin\left(\frac{\pi}{4} t\right) + 8$$

$$v = \frac{3\pi}{4} \cos\left(\frac{\pi}{4} t\right)$$

$$a = -\frac{3\pi^2}{16} \sin\left(\frac{\pi}{4} t\right)$$

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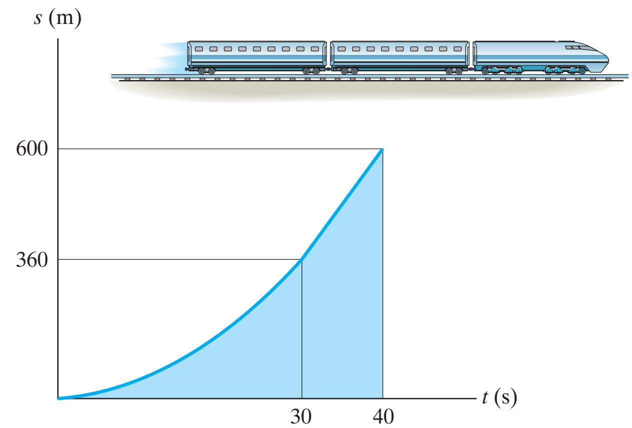
***12–40.**

The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \leq t \leq 40$ s. For $0 \leq t \leq 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \geq 30$ s.



SOLUTION

$0 \leq t \leq 30$:

$$s = 0.4t^2 \qquad \textbf{Ans.}$$

$$v = \frac{ds}{dt} = 0.8t \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = 0.8 \qquad \textbf{Ans.}$$

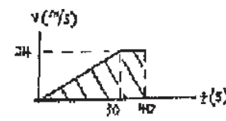
$30 \leq t \leq 40$:

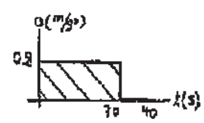
$$s - 360 = \left(\frac{600 - 360}{40 - 30}\right)(t - 30)$$

$$s = 24(t - 30) + 360 \qquad \textbf{Ans.}$$

$$v = \frac{ds}{dt} = 24 \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = 0 \qquad \textbf{Ans.}$$





Ans:

$s = 0.4t^2$

$v = \dfrac{ds}{dt} = 0.8t$

$a = \dfrac{dv}{dt} = 0.8$

$s = 24(t - 30) + 360$

$v = \dfrac{ds}{dt} = 24$

$a = \dfrac{dv}{dt} = 0$

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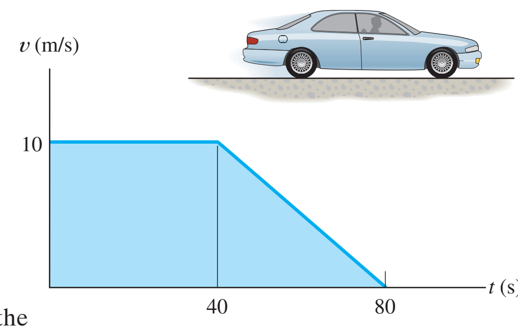
12–41.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

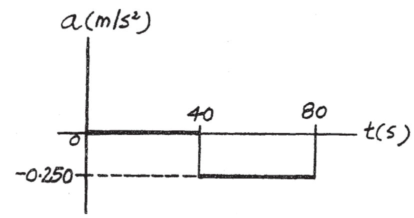
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40$ s,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80$ s, $\dfrac{v - 10}{t - 40} = \dfrac{0 - 10}{80 - 40}$, $v = \left(-\dfrac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$



For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

Ans:
$s = 600$ m.
For $0 \leq t < 40$ s,
$a = 0$.
For $40 \text{ s} < t \leq 80$ s,
$a = -0.250 \text{ m/s}^2$

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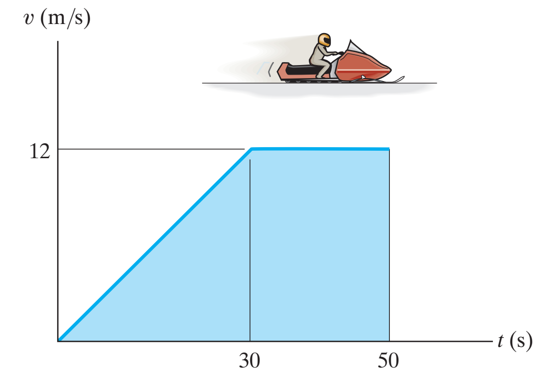
12–42. The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.



SOLUTION

***s* – *t* Graph:** The position function in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0\text{ s} \le t < 30\text{ s}$, $v = \dfrac{12}{30}t = \left(\dfrac{2}{5}t\right)$ m/s.

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{2}{5}t\,dt$$

$$s = \left(\frac{1}{5}t^2\right)\text{ m}$$

At $t = 30$ s, $\qquad s = \dfrac{1}{5}\left(30^2\right) = 180$ m

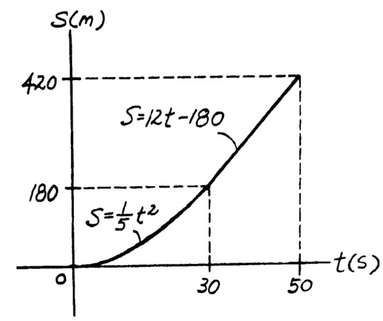
For time interval $\mathbf{30\ s < t \le 50\ s}$,

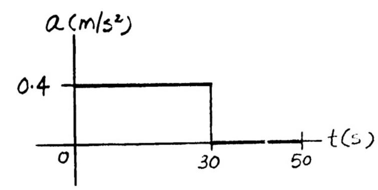
$$ds = v\,dt$$

$$\int_{180\text{ m}}^s ds = \int_{30\text{ s}}^t 12\,dt$$

$$s = (12t - 180)\text{ m}$$

At $t = 50$ s, $\qquad s = 12(50) - 180 = 420$ m





***a* – *t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $\mathbf{0\ s \le t < 30\ s}$ and $\mathbf{30\ s < t \le 50\ s}$, $a = \dfrac{dv}{dt} = \dfrac{2}{5} = 0.4\text{ m/s}^2$ and $a = \dfrac{dv}{dt} = 0$, respectively.

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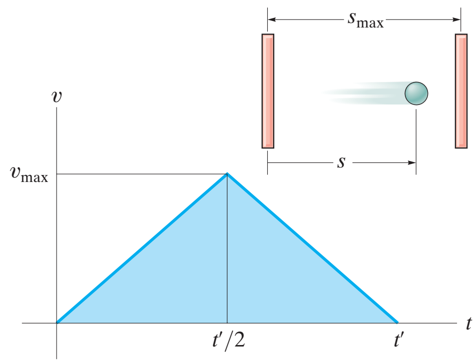
12–43.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

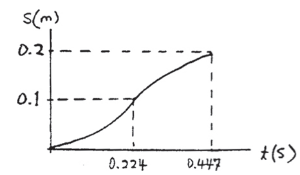
$a_c = 4\text{ m/s}^2$

$\frac{s}{2} = 100\text{ mm} = 0.1\text{ m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\text{ m/s} \qquad = 0.894\text{ m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721\text{ s} \qquad = 0.447\text{ s}$ **Ans.**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224\text{ s},$

$s = 0.1\text{ m}$

$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t{+}1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t{+}1.788)\,dt$

$s = -2\,t^2 + 1.788\,t - 0.2$

When $t = 0.447$ s,

$s = 0.2\text{ m}$ **Ans.**

Ans:
$v_{max} = 0.894\text{ m/s}$
$t' = 0.447\text{ s}$
$s = 0.2\text{ m}$

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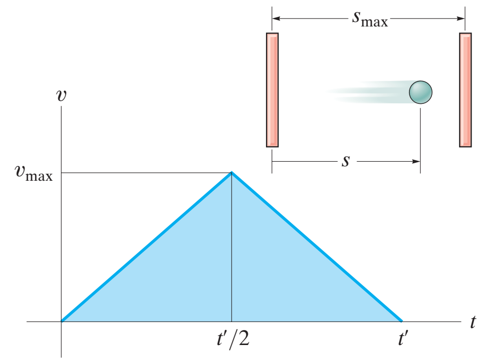
***12–44.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$$v = 100\,t$$

$$a = \frac{dv}{dt} = 100$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 100\,t\,dt$$

$$s = 50\,t^2$$

When $t = 0.1$ s,

$$s = 0.5 \text{ m}$$

For $0.1 \text{ s} < t < 0.2$ s,

$$v = -100\,t + 20$$

$$a = \frac{dv}{dt} = -100$$

$$ds = v\,dt$$

$$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)\text{dt}$$

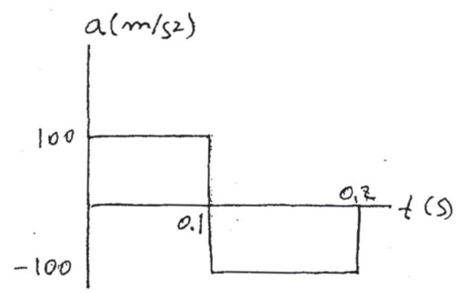
$$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$$

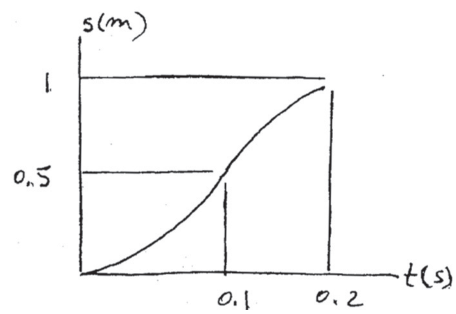
$$s = -50\,t^2 + 20\,t - 1$$

When $t = 0.2$ s,

$$s = 1 \text{ m}$$

When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.





Ans:
When $t = 0.1$ s,
$s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When $t = 0.2$ s, $s = 1$ m.

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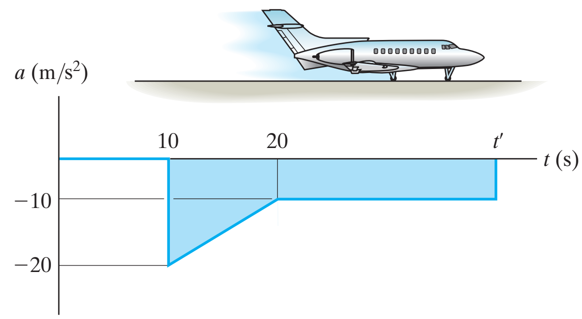
12–45. The motion of a jet plane just after landing on a runway is described by the a–t graph. Determine the time t' when the jet plane stops. Construct the v–t and s–t graphs for the motion. Here $s = 0$, and $v = 150$ m/s when $t = 0$.



SOLUTION

***v–t* Graph.** The v–t function can be determined by integrating $dv = a\,dt$. For $0 \le t < 10$ s, $a = 0$. Using the initial condition $v = 150$ m/s at $t = 0$,

$$\int_{150\text{ m/s}}^{v} dv = \int_0^t 0\,dt$$

$$v - 150 = 0$$

$$v = 150\text{ m/s} \qquad \textbf{Ans.}$$

For $10\text{ s} < t < 20$ s, $\dfrac{a - (-10)}{t - 10} = \dfrac{-5 - (-10)}{20 - 10}$, $a = \left(\dfrac{1}{2}t - 15\right)\text{m/s}^2$. Using the initial condition $v = 150$ m/s at $t = 10$ s,

$$\int_{150\text{ m/s}}^{v} dv = \int_{10\text{ s}}^{t} \left(\frac{1}{2}t - 15\right)dt$$

$$v - 150 = \left(\frac{1}{4}t^2 - 15t\right)\Big|_{10\text{ s}}^{t}$$

$$v = \left\{\frac{1}{4}t^2 - 15t + 275\right\}\text{m/s} \qquad \textbf{Ans.}$$

At $t = 20$ s,

$$v|_{t=20\text{ s}} = \frac{1}{4}(20^2) - 15(20) + 275 = 75\text{ m/s}$$

For $20\text{ s} < t \le t'$, $a = -5$ m/s. Using the initial condition $v = 75$ m/s at $t = 20$ s,

$$\int_{75\text{ m/s}}^{v} dv = \int_{20\text{ s}}^{t} -5\,dt$$

$$v - 75 = (-5t)\Big|_{20\text{ s}}^{t}$$

$$v = \{-5t + 175\}\text{ m/s} \qquad \textbf{Ans.}$$

It is required that at $t = t'$, $v = 0$. Thus

$$0 = -5t' + 175$$

$$t' = 35.0\text{ s} \qquad \textbf{Ans.}$$

Using these results, the v–t graph shown in Fig. a can be plotted.

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12–45. Continued

s-t Graph. The s–t function can be determined by integrating $ds = vdt$. For $0 \leq t < 10$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 150\, dt$$

$$s = \{150\, t\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 10$ s,

$$s|_{t=10\text{ s}} = 150(10) = 1500 \text{ m}$$

For $10 \text{ s} < t < 20$ s, the initial condition is $s = 1500$ m at $t = 10$ s.

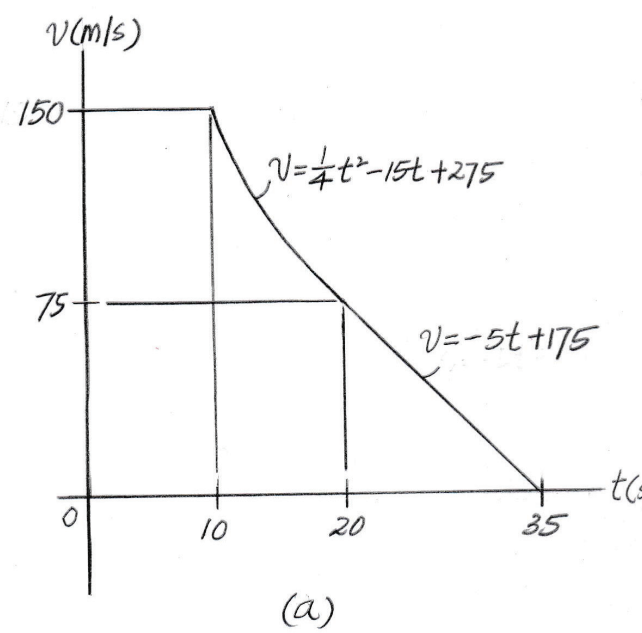
$$\int_{1500\text{ m}}^s ds = \int_{10\text{ s}}^t \left(\frac{1}{4}t^2 - 15t + 275\right)dt$$

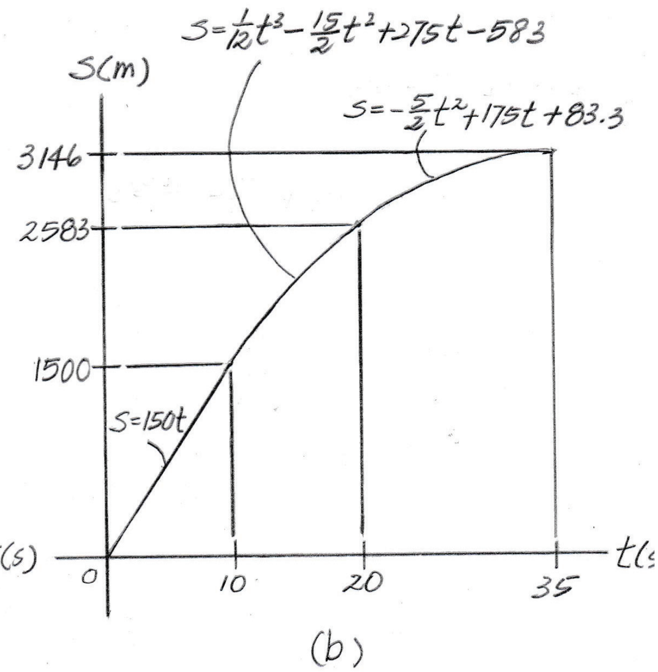
$$s - 1500 = \left(\frac{1}{12}t^3 - \frac{15}{2}t^2 + 275t\right)\Big|_{10\text{ s}}^t$$

$$s = \left\{\frac{1}{12}t^3 - \frac{15}{2}t^2 + 275t - 583\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 20$ s,

$$s|_{t=20\text{ s}} = \frac{1}{12}(20^3) - \frac{15}{2}(20^2) + 275(20) - 583 = 2583 \text{ m}$$





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12–45. Continued

For 20 s < $t \leq 35$ s, the initial condition is $s = 2583$ m at $t = 20$ s.

$$\int_{2583\text{ m}}^{s} ds = \int_{20\text{ s}}^{t} (-5t + 175)\,dt$$

$$s - 2583 = \left(-\frac{5}{2}t^2 + 175t\right)\Big|_{20\text{ s}}^{t}$$

$$s = \left\{-\frac{5}{2}t^2 + 175t + 83.3\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 35$ s,

$$s|_{t=35\text{ s}} = -\frac{5}{2}(35^2) + 175(35) + 83.3 = 3146\text{ m}$$

using these results, the *s*-*t* graph shown in Fig. *b* can be plotted.

Ans:
$v = 150\text{ m/s}$
$v = \left\{\frac{1}{4}t^2 - 15t + 275\right\}\text{ m/s}$
$v = \{-5t + 175\}\text{ m/s}$
$t' = 35.0\text{ s}$
$s = \{150t\}\text{ m}$
$s = \left\{\frac{1}{12}t^3 - \frac{15}{2}t^2 + 275t - 583\right\}\text{ m}$
$s = \left\{\frac{-5}{2}t^2 + 175t + 83.3\right\}\text{ m}$

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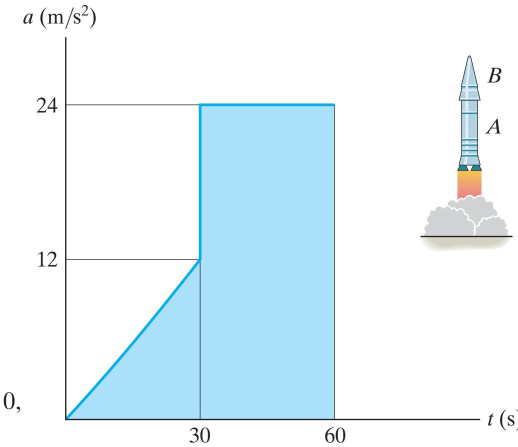
12–46.

A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A, burns out and the second stage, B, ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 60$ s.



SOLUTION

***v*–*t* Graph.** The $v-t$ function can be determined by integrating $dv = a\,dt$.

For $0 \leq t < 30$ s, $a = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s^2. Using the initial condition $v = 0$ at $t = 0$,

$$\int_0^v dv = \int_0^t \frac{2}{5}t\,dt$$

$$v = \left\{\frac{1}{5}t^2\right\} \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$v\Big|_{t=30\text{ s}} = \frac{1}{5}(30^2) = 180 \text{ m/s}$$

For $30 < t \leq 60$ s, $a = 24$ m/s^2. Using the initial condition $v = 180$ m/s at $t = 30$ s,

$$\int_{180\text{ m/s}}^{v} dv = \int_{30\text{ s}}^{t} 24\,dt$$

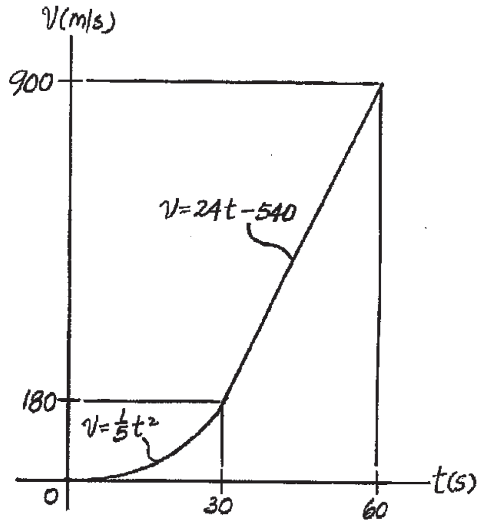
$$v - 180 = 24\,t\Big|_{30\text{ s}}^{t}$$

$$v = \{24t - 540\} \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 60$ s,

$$v\Big|_{t=60\text{ s}} = 24(60) - 540 = 900 \text{ m/s}$$

Using these results, $v-t$ graph shown in Fig. a can be plotted.



(a)

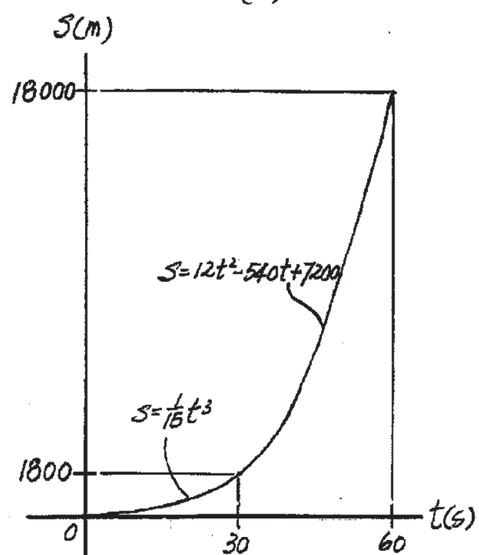
***s*–*t* Graph.** The $s-t$ function can be determined by integrating $ds = v\,dt$. For $0 \leq t < 30$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{1}{5}t^2 dt$$

$$s = \left\{\frac{1}{15}t^3\right\} \text{m} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$s\Big|_{t=30\text{ s}} = \frac{1}{15}(30^3) = 1800 \text{ m}$$



(b)

Ans:
For $0 \leq t < 30$ s, $v = \left\{\frac{1}{5}t^2\right\}$ m/s, $s = \left\{\frac{1}{15}t^3\right\}$ m

For $30 \leq t \leq 60$ s, $v = \{24t - 540\}$ m/s,

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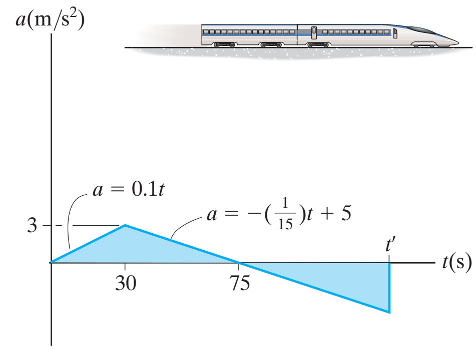
12–47.

The a–t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v–t and s–t graphs.



SOLUTION

$v-t$ Graph: For the time interval $0 \leq t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right) \text{ m/s}$$

When $t = 30$ s,

$$v\big|_{t=30\text{ s}} = 0.05\left(30^2\right) = 45 \text{ m/s}$$

or the time interval $30 \text{ s} < t \leq t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{45\text{ m/s}}^v dv = \int_{30\text{ s}}^t \left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right) \text{ m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09 \text{ s} = 133 \text{ s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

This equation is the same as the one obtained previously.

The slope of the v–$t$graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\big|_{t=75\text{ s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5 \text{ m/s}$$

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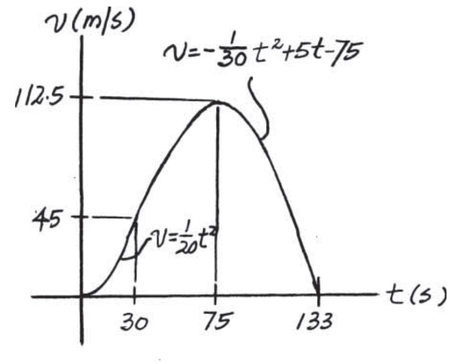
12–47. continued

The v–t graph is shown in Fig. a.



(a)

$s-t$ Graph: Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = vdt$. For the time interval $0 \leq t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$(\overset{+}{\rightarrow})$ $\qquad ds = vdt$

$$\int_0^s ds = \int_0^t 0.05t^2\, dt$$

$$s = \left(\frac{1}{60}t^3\right) \text{ m}$$

When $t = 30$ s,

$$s\big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \leq t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$(\overset{+}{\rightarrow})$ $\qquad ds = vdt$

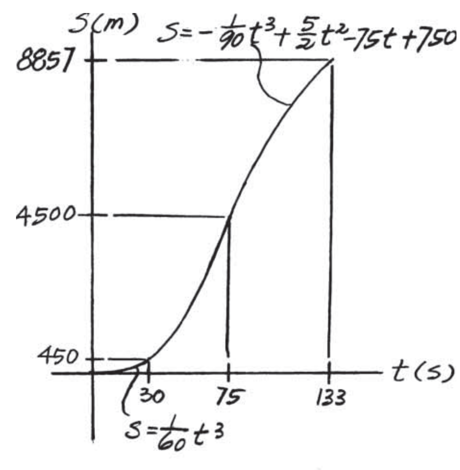
$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right)dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right) \text{ m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s\big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500 \text{ m}$$

$$s\big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857 \text{ m} \qquad \textbf{Ans.}$$



(b)

The s–t graph is shown in Fig. b.

When $t = 30$ s,

$v = 45$ m/s and $s = 450$ m.

When $t = 75$ s,

$v = v_{\max} = 112.5$ m/s and $s = 4500$ m.

When $t = 133$ s,

$v = 0$ and $s = 8857$ m.

Ans:
$t' = 133$ s
$s\big|_{t=133.09\text{ s}} = 8857$ m

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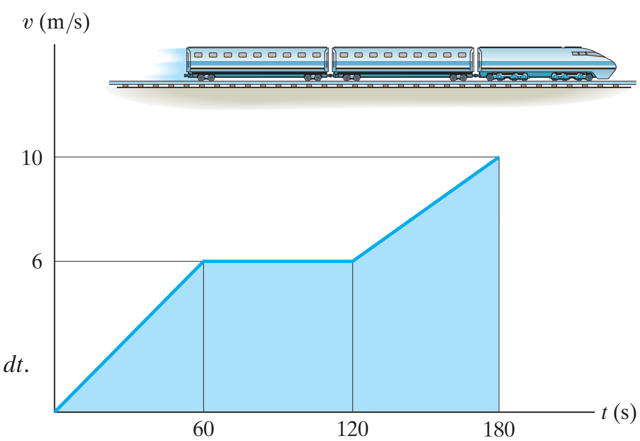
***12–48.**

The v–t graph for a train has been experimentally determined. From the data, construct the s–t and a–t graphs for the motion for $0 \le t \le 180$ s. When $t = 0$, $s = 0$.



SOLUTION

***s–t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 60$ s, $v = \frac{6}{60}t = \left(\frac{1}{10}t\right)$ m/s. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t \left(\frac{1}{10}t\right)dt$$

$$s = \left\{\frac{1}{20}t^2\right\}\text{ m} \qquad \textbf{Ans.}$$

When $t = 60$ s,

$$s\big|_{t-60\text{ s}} = \frac{1}{20}\left(60^2\right) = 180\text{ m}$$

For 60 s $< t < 120$ s, $v = 6$ m/s. Using the initial condition $s = 180$ m at $t = 60$ s,

$$\int_{180\text{ m}}^s ds = \int_{60\text{ s}}^t 6\,dt$$

$$s - 180 = 6t\Big|_{60\text{ s}}^t$$

$$s = \{6t - 180\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 120$ s,

$$s\big|_{t-120\text{ s}} = 6(120) - 180 = 540\text{ m}$$

For 120 s $< t \le 180$ s, $\frac{v - 6}{t - 120} = \frac{10 - 6}{180 - 120}$; $v = \left\{\frac{1}{15}t - 2\right\}$ m/s. Using the initial condition $s = 540$ m at $t = 120$ s,

$$\int_{540\text{ m}}^s ds = \int_{120\text{ s}}^t \left(\frac{1}{15}\text{t} - 2\right)dt$$

$$s - 540 = \left(\frac{1}{30}t^2 - 2t\right)\Big|_{120\text{ s}}^t$$

$$s = \left\{\frac{1}{30}t^2 - 2t + 300\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 180$ s,

$$s\big|_{t=180\text{ s}} = \frac{1}{30}\left(180^2\right) - 2(180) + 300 = 1020\text{ m}$$

Using these results, s–t graph shown in Fig. *a* can be plotted.

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***12–48. Continued**

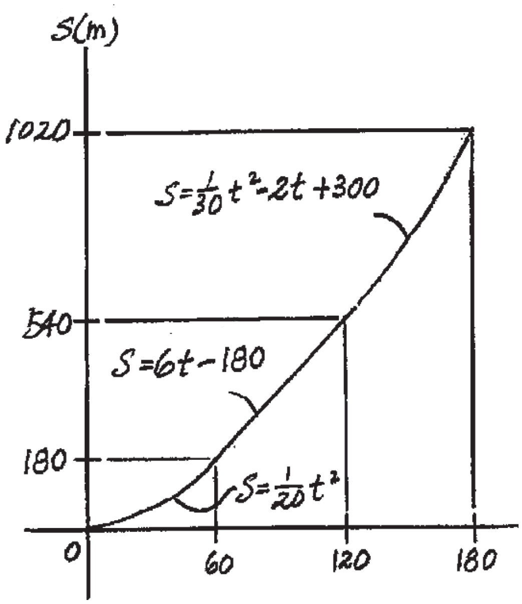
***a–t* Graph.** The a–t function can be determined using $a = \dfrac{dv}{dt}$.

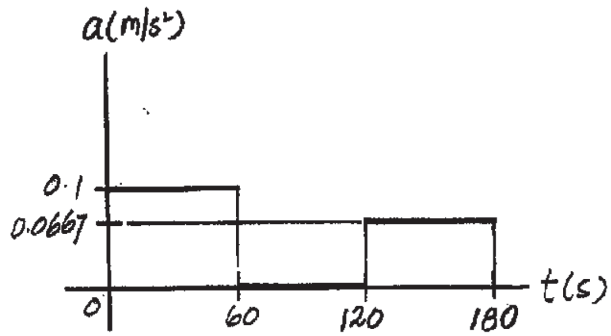
For $0 \leq t < 60$ s, $\quad a = \dfrac{d\left(\frac{1}{10}t\right)}{dt} = 0.1 \text{ m/s}^2$ **Ans.**

For $60 \text{ s} < t < 120$ s, $\quad a = \dfrac{d(6)}{dt} = 0$ **Ans.**

For $120 \text{ s} < t \leq 180$ s, $\quad a = \dfrac{d\left(\frac{1}{15}t - 2\right)}{dt} = 0.0667 \text{ m/s}^2$ **Ans.**

Using these results, a–t graph shown in Fig. b can be plotted.





Ans:
For $0 \leq t < 60$ s,
$s = \left\{\dfrac{1}{20}t^2\right\}$ m,
$a = 0.1 \text{ m/s}^2$.
For $60 \text{ s} < t < 120$ s,
$s = \{6t - 180\}$ m,
$a = 0$. For $120 \text{ s} < t \leq 180$ s,
$s = \left\{\dfrac{1}{30}t^2 - 2t + 300\right\}$ m,
$a = 0.0667 \text{ m/s}^2$.

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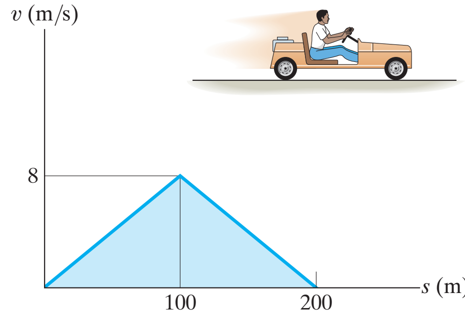
12–49.

The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

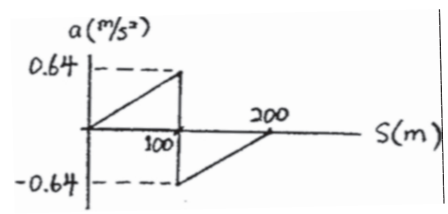
For $0 \leq s < 100$

$v = 0.08\,s, \qquad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32\text{ m/s}^2$ **Ans.**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32\text{ m/s}^2$ **Ans.**

Also,

$v\,dv = a\,ds$

$a = v\left(\frac{dv}{ds}\right)$

At $s = 50$ m,

$a = 4\left(\frac{8}{100}\right) = 0.32\text{ m/s}^2$ **Ans.**

At $s = 150$ m,

$a = 4\left(\frac{-8}{100}\right) = -0.32\text{ m/s}^2$ **Ans.**

At $s = 100$ m, a changes from $a_{max} = 0.64\text{ m/s}^2$

to $a_{min} = -0.64\text{ m/s}^2$.

Ans:
$a = 0.32\text{ m/s}^2$,
$a = -0.32\text{ m/s}^2$

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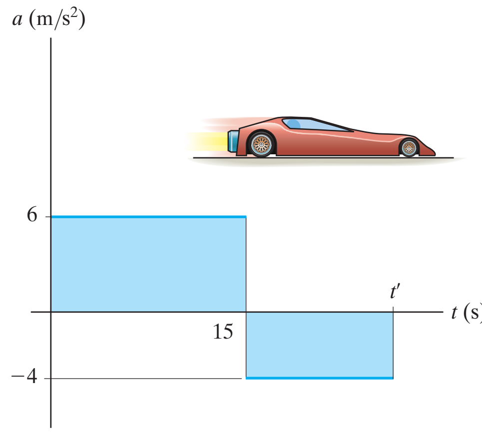
12–50.

The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0, s = 0$.



SOLUTION

***v–t* Function.** The $v-t$ function can be determined by integrating $dv = a\,dt$. For $0 \leq t < 15$ s, $a = 6$ m/s^2. Using the initial condition $v = 10$ m/s at $t = 0$,

$$\int_{10\text{ m/s}}^{v} dv = \int_0^t 6dt$$

$$v - 10 = 6t$$

$$v = \{6t + 10\}\text{ m/s}$$

The maximum velocity occurs when $t = 15$ s. Then

$$v_{max} = 6(15) + 10 = 100\text{ m/s}$$ **Ans.**

For 15 s $< t \leq t'$, $a = -4$ m/s, Using the initial condition $v = 100$ m/s at $t = 15$ s,

$$\int_{100\text{ m/s}}^{v} dv = \int_{15\text{ s}}^{t} -4dt$$

$$v - 100 = (-4t)\Big|_{15\text{ s}}^{t}$$

$$v = \{-4t + 160\}\text{ m/s}$$

It is required that $v = 0$ at $t = t'$. Then

$$0 = -4t' + 160 \qquad t' = 40\text{ s}$$ **Ans.**

Ans:
$v_{max} = 100$ m/s
$t' = 40$ s

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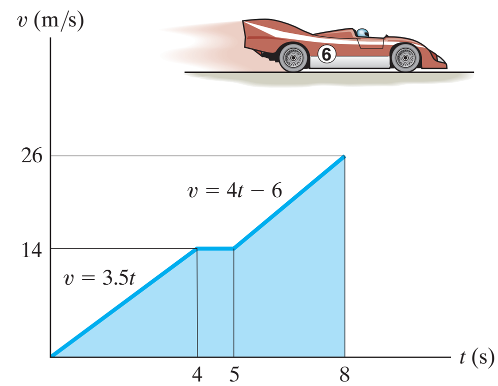
12–51.

The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v–t graph. The flat part of the graph is caused by shifting gears. Draw the a–t graph and determine the maximum acceleration of the car.



SOLUTION

For $0 \leq t < 4$ s

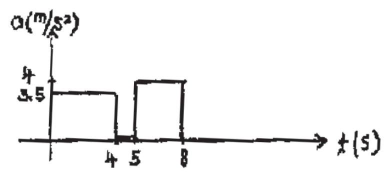
$$a = \frac{\Delta v}{\Delta t} = \frac{14}{4} = 3.5 \text{ m/s}^2$$

For $4 \text{ s} \leq t < 5$ s

$$a = \frac{\Delta v}{\Delta t} = 0$$

For $5 \text{ s} \leq t < 8$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{26 - 14}{8 - 5} = 4 \text{ m/s}^2$$



$a_{\text{max}} = 4.00 \text{ m/s}^2$ **Ans.**

Ans:
$a_{\text{max}} = 4.00 \text{ m/s}^2$

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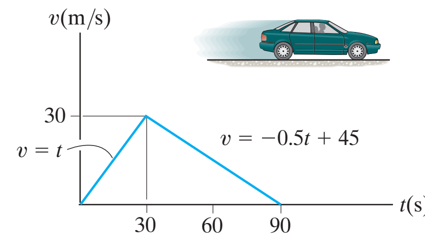
***12–52.** A car starts from rest and travels along a straight road with a velocity described by the graph. Determine the total distance traveled until the car stops. Construct the *s–t* and *a–t* graphs.



SOLUTION

***s − t* Graph:** For the time interval $0 \leq t < 30$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t t\,dt$$

$$s = \left(\frac{t^2}{2}\right)\text{ m}$$

When $t = 30$ s,

$$s = \frac{30^2}{2} = 450\text{ m}$$

For the time interval $30\text{ s} < t \leq 90$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

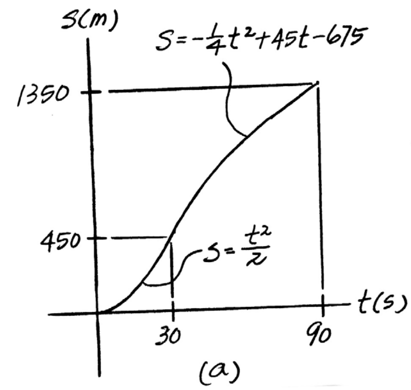
$$\int_{450\text{ m}}^s ds = \int_{30\text{s}}^t (-0.5t + 45)dt$$

$$s = \left(-\frac{1}{4}t^2 + 45t - 675\right)\text{ m}$$

When $t = 90$ s,

$$s\big|_{t=90\text{ s}} = -\frac{1}{4}\left(90^2\right) + 45(90) - 675 = 1350\text{ m}$$ **Ans.**

The $s - t$ graph shown is in Fig. *a*.



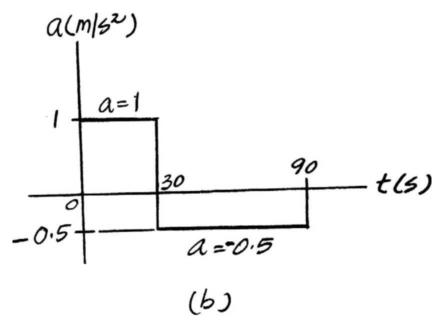
***a − t* Graph:** For the time interval $0 < t < 30$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(t) = 1\text{ m/s}^2$$

For the time interval $30\text{ s} < t \leq 90$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-0.5t + 45) = -0.5\text{ m/s}^2$$

The $a - t$ graph is shown in Fig. *b*.



Note: Since the change in position of the car is equal to the area under the $v - t$ graph, the total distance traveled by the car is

$$\Delta s = \int v\,dt$$

$$s\big|_{t=90\text{ s}} - 0 = \frac{1}{2}(90)(30)$$

$$s\big|_{t=90\text{ s}} = 1350\text{ m}$$

Ans:
$s\big|_{t=90\text{ s}} = 1350$ m

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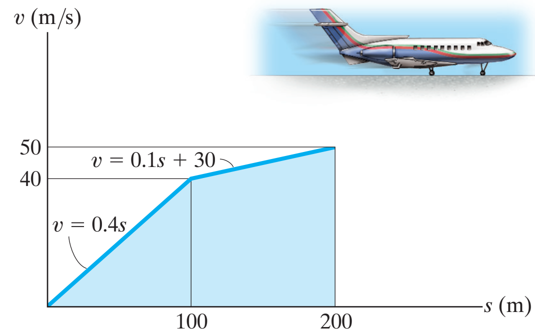
12–53. The v–s graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = 100$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

$$a = v\frac{dv}{ds}$$

$s_1 = 100$ m $\quad s_4 = 150$ m

$s_2 = 200$ m $\quad v_1 = 40$ m/s

$s_3 = 50$ m $\quad v_2 = 50$ m/s

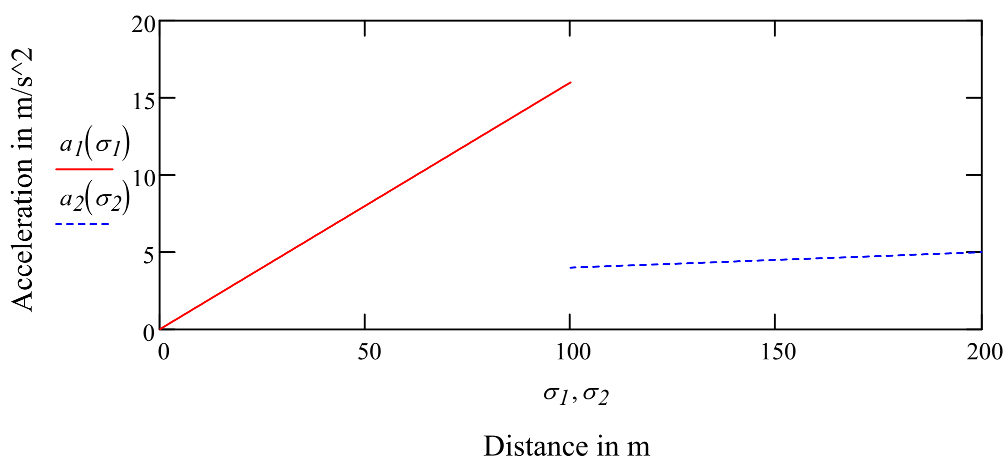
$0 < s_3 < s_1 \qquad a_3 = \left(\frac{s_3}{s_1}\right)v_1\left(\frac{v_1}{s_1}\right) \qquad a_3 = 8.00\text{ m/s}^2$ **Ans.**

$s_1 < s_4 < s_2 \qquad a_4 = \left[v_1 + \frac{s_4 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1} \qquad a_4 = 4.50\text{ m/s}^2$ **Ans.**

The graph

$\sigma_1 = 0, 0.01s_1 .. s_1 \qquad a_1(\sigma_1) = \frac{\sigma_1}{s_1}\frac{v_1^{\,2}}{s_1}\ \text{s}^2/\text{m}$

$\sigma_2 = s_1, 1.01s_1 .. s_2 \qquad a_2(\sigma_2) = \left[v_1 + \frac{\sigma_2 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1}\ \text{s}^2/\text{m}$



Ans:
$a_3 = 8.00\text{ m/s}^2$
$a_4 = 4.50\text{ m/s}^2$

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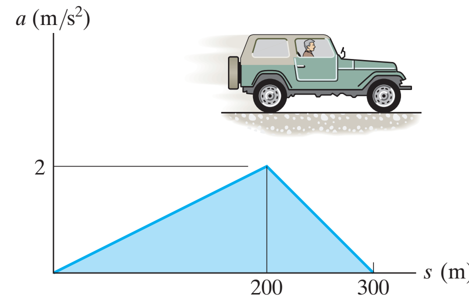
12–54.

The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0, v = 0$.



SOLUTION

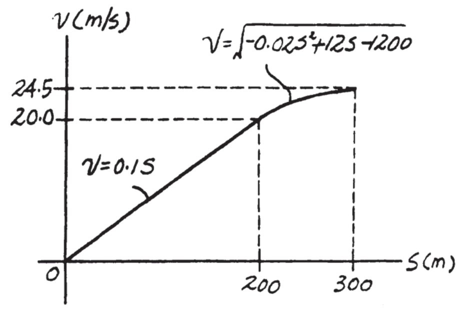
***a*−*s* Graph**: The function of acceleration a in terms of s for the interval $0 \text{ m} \le s < 200 \text{ m}$ is

$$\frac{a - 0}{s - 0} = \frac{2 - 0}{200 - 0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \le 300 \text{ m}$,

$$\frac{a - 2}{s - 200} = \frac{0 - 2}{300 - 200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$

***v*−*s* Graph**: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0 \text{ m} \le s < 200 \text{ m}}$,



$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval $\mathbf{200 \text{ m} < s \le 300 \text{ m}}$,

$$vdv = ads$$

$$\int_{20.0 \text{ m/s}}^{v} vdv = \int_{200 \text{ m}}^{s} (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

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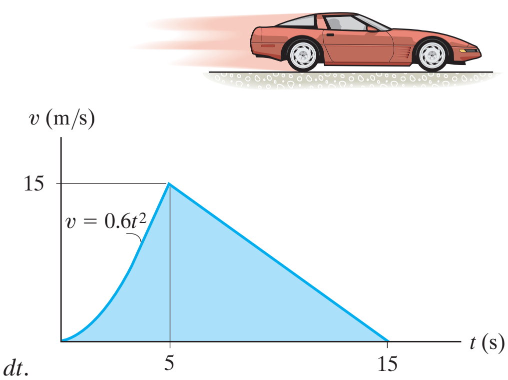
12–55.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t and a–t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0$, $s = 0$.



SOLUTION

s–t Graph. The s–t function can be determined by integrating $ds = v\,dt$. For $0 \leq t < 5$ s, $v = 0.6t^2$. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t 0.6t^2 dt$$

$$s = \left\{0.2t^3\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 5$ s,

$$s\big|_{t=5\text{ s}} = 0.2\left(5^3\right) = 25 \text{ m}$$

For $5 \text{ s} < t \leq 15$ s, $\dfrac{v - 15}{t - 5} = \dfrac{0 - 15}{15 - 5}$; $v = \dfrac{1}{2}(45 - 3t)$. Using the initial condition

$s = 25$ m at $t = 5$ s,

$$\int_{25\text{ m}}^s ds = \int_{5\text{ s}}^t \frac{1}{2}(45 - 3t)dt$$

$$s - 25 = \frac{45}{2}t - \frac{3}{4}t^2 - 93.75$$

$$s = \left\{\frac{1}{4}\left(90t - 3t^2 - 275\right)\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 15$ s,

$$s = \frac{1}{4}\left[90(15) - 3\left(15^2\right) - 275\right] = 100 \text{ m} \qquad \textbf{Ans.}$$

Thus the average speed is

$$v_{\text{avg}} = \frac{s_T}{t} = \frac{100 \text{ m}}{15 \text{ s}} = 6.67 \text{ m/s} \qquad \textbf{Ans.}$$

using these results, the s–t graph shown in Fig. *a* can be plotted.

Ans:

$s = \left\{0.2t^3\right\}$ m

$s = \left\{\dfrac{1}{4}\left(90t - 3t^2 - 275\right)\right\}$ m

$s = 100$ m

$v_{\text{avg}} = 6.67$ m/s

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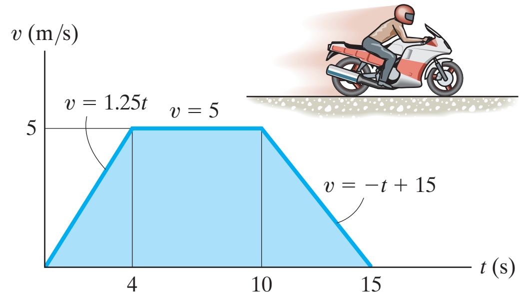
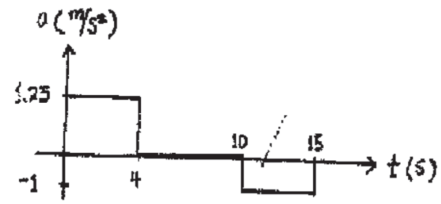
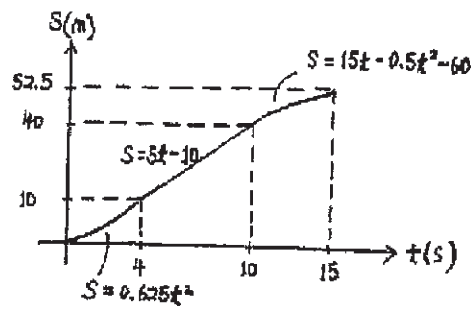
***12–56.**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.



SOLUTION





For $t < 4$ s

$$a = \frac{dv}{dt} = 1.25$$

$$\int_0^s ds = \int_0^t 1.25\,t\,dt$$

$$s = 0.625\,t^2$$

When $t = 4$ s, $\quad s = 10$ m

For $4 \text{ s} < t < 10$ s

$$a = \frac{dv}{dt} = 0$$

$$\int_{10}^s ds = \int_4^t 5\,dt$$

$$s = 5\,t - 10$$

When $t = 10$ s, $\quad s = 40$ m

For $10 \text{ s} < t < 15$ s

$$a = \frac{dv}{dt} = -1$$

$$\int_{40}^s ds = \int_{10}^t (15 - t)\,dt$$

$$s = 15\,t - 0.5\,t^2 - 60$$

When $t = 15$ s, $\quad s = 52.5$ m **Ans.**

Ans:
When $t = 15$ s, $\quad s = 52.5$ m

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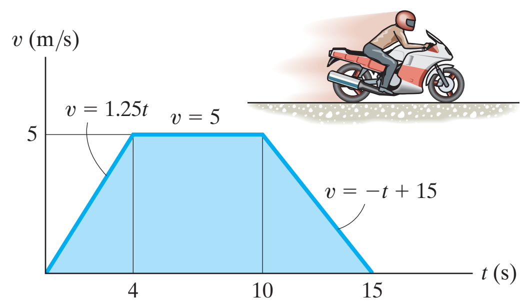
12–57.

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



SOLUTION

At $t = 8$ s

$$a = \frac{dv}{dt} = 0$$ **Ans.**

$$\Delta s = \int v\, dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8 - 4)(5) = 30$$

$s = 30$ m **Ans.**

At $t = 12$ s

$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2$$ **Ans.**

$$\Delta s = \int v\, dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10 - 4)(5) + \frac{1}{2}(15 - 10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

$s = 48$ m **Ans.**

Ans:
At $t = 8$ s,
$a = 0$ and $s = 30$ m.
At $t = 12$ s,
$a = -1 \text{ m/s}^2$
and $s = 48$ m.

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12–58.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

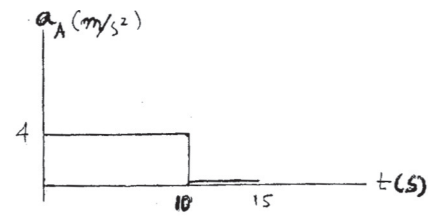
$v = v_0 + a_c t$

$v_A = 0 + 4t$

At $t = 10$ s, $\quad v_A = 40$ m/s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$$

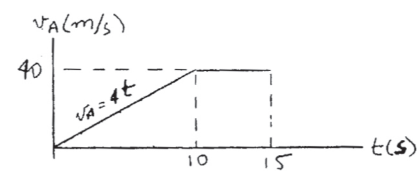


At $t = 10$ s, $\qquad s_A = 200$ m

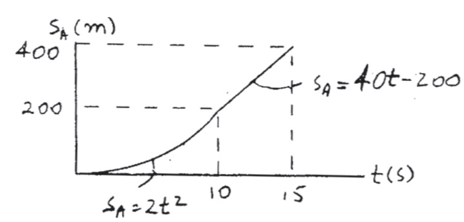
$t > 10$ s, $\qquad ds = v\,dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$



At $t = 15$ s, $\qquad s_A = 400$ m



Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\qquad t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

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12–58. Continued

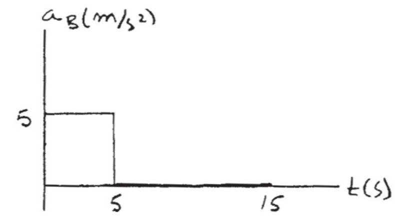
At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

$$s_B = 25t - 62.5$$

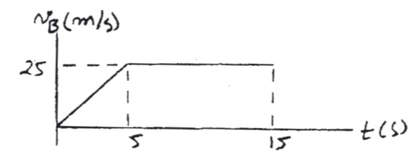


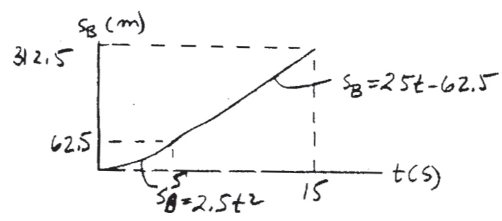
When $t = 15$ s, $\quad s_B = 312.5$

Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.





Ans:
When $t = 5$ s,
$s_B = 62.5$ m.
When $t = 10$ s,
$v_A = (v_A)_{\max} = 40$ m/s and
$s_A = 200$ m.
When $t = 15$ s,
$s_A = 400$ m and $s_B = 312.5$ m.
$\Delta s = s_A - s_B = 87.5$ m

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12–59. A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s - t$, $v - t$, and $a - t$ graphs for the particle for $0 \leq t \leq 3$ s.

SOLUTION

$s = t^3 - 3t^2 + 2t$

$v = \frac{ds}{dt} = 3t^2 - 6t + 2$

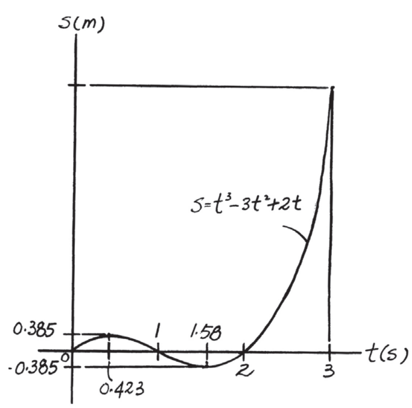
$a = \frac{dv}{dt} = 6t - 6$

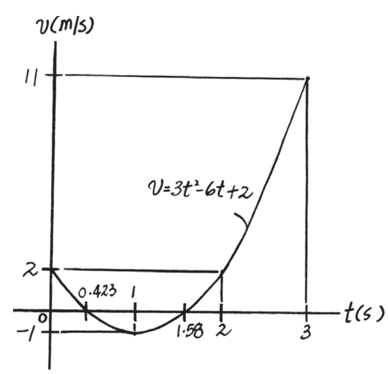
$v = 0$ at $0 = 3t^2 - 6t + 2$

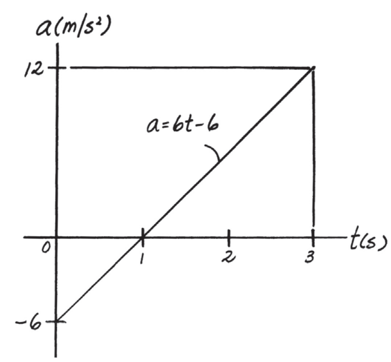
$t = 1.577$ s, and $t = 0.4226$ s,

$s|_{t=1.577} = -0.386$ m

$s|_{t=0.4226} = 0.385$ m







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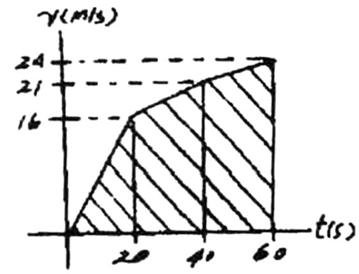
***12–60.**

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.



SOLUTION

The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_T = 980$ m

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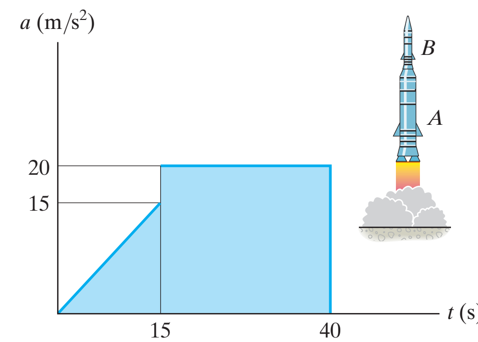
12–61.

A two-stage rocket is fired vertically from rest with the acceleration shown. After 15 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 40$ s.



SOLUTION

For $0 \leq t < 15$

$a = t$

$$\int_0^v dv = \int_0^t t\, dt$$

$$v = \frac{1}{2}t^2$$

$v = 112.5$ when $t = 15$ s

$$\int_0^s ds = \int_0^t \frac{1}{2}t^2\, dt$$

$$s = \frac{1}{6}t^3$$

$s = 562.5$ when $t = 15$ s

For $15 < t < 40$

$a = 20$

$$\int_{112.5}^{v} dv = \int_{1.5}^{t} 20\, dt$$

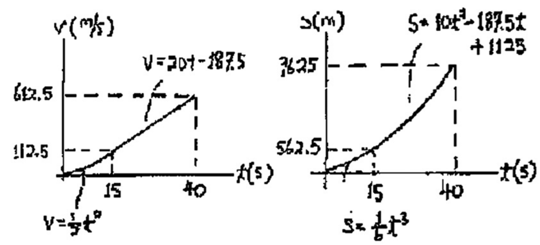
$$v = 20t - 187.5$$

$v = 612.5$ when $t = 40\ s$

$$\int_{562.5}^{s} ds = \int_{15}^{t} (20\,t - 187.5)\, dt$$

$$s = 10\,t^2 - 187.5\,t + 1125$$

$s = 9625$ when $t = 40$ s



Ans:

For $0 \leq t < 15$ s,

$$v = \left\{\frac{1}{2}t^2\right\} \text{m/s}$$

$$s = \left\{\frac{1}{6}t^3\right\} \text{m}$$

For $15\text{ s} < t \leq 40$ s,

$v = \{20t - 187.5\}$ m/s

$s = \{10t^2 - 187.5t + 1125\}$ m

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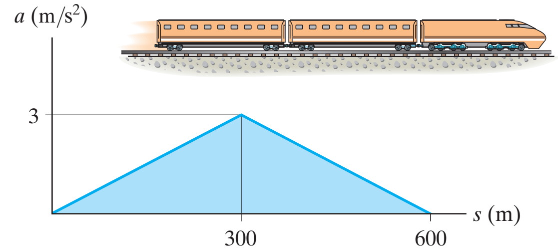
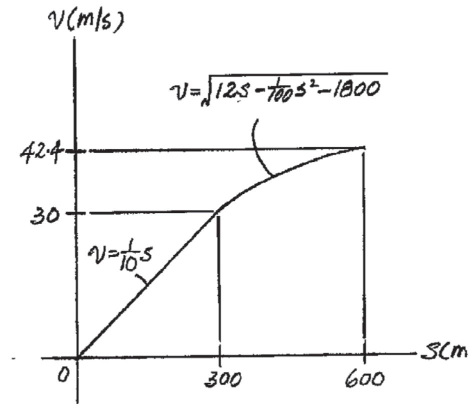
12–62. The motion of a train is described by the a–s graph shown. Draw the v–s graph if $v = 0$ at $s = 0$.



SOLUTION



***v–s* Graph.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 300$ m, $a = \left(\frac{3}{300}\right)s = \left(\frac{1}{100}s\right)$ m/s^2. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(\frac{1}{100}s\right) ds$$

$$\frac{v^2}{2} = \frac{1}{200}s^2$$

$$v = \left\{\frac{1}{10}s\right\} \text{ m/s} \qquad \textbf{Ans.}$$

At $s = 300$ m,

$$v|_{s=300\text{ m}} = \frac{1}{10}(300) = 30 \text{ m/s}$$

For $300 \text{ m} < s \le 600$ m, $\frac{a-3}{s-300} = \frac{0-3}{600-300}$; $a = \left\{-\frac{1}{100}s + 6\right\}$ m/s^2, using the initial condition $v = 30$ m/s at $s = 300$ m,

$$\int_{30\text{ m/s}}^v v\,dv = \int_{300\text{ m}}^s \left(-\frac{1}{100}s + 6\right) ds$$

$$\left.\frac{v^2}{2}\right|_{30\text{ m/s}}^{v} = \left.\left(-\frac{1}{200}s^2 + 6s\right)\right|_{300\text{ m}}^{s}$$

$$\frac{v^2}{2} - 450 = 6s - \frac{1}{200}s^2 - 1350$$

$$v = \left\{\sqrt{12s - \frac{1}{100}s^2 - 1800}\right\} \text{ m/s} \qquad \textbf{Ans.}$$

At $s = 600$ m,

$$v = \sqrt{12\,(600) - \frac{1}{100}(600^2) - 1800} = 42.43 \text{ m/s}$$

Using these results, the $v-s$ graph shown in Fig. a can be plotted.

Ans:
$v = \left\{\frac{1}{10}s\right\}$ m/s

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12–63. If the position of a particle is defined as $s = (5t - 3t^2)$ m, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 2.5$ s.

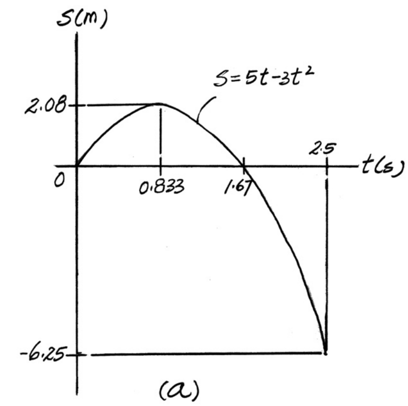
SOLUTION

***s–t* Graph.** When s = 0,

$$0 = 5t - 3t^2 \quad 0 = t(5 - 3t) \quad t = 0 \quad \text{and} \quad t = 1.675$$

At $t = 10$ s,

$$s|_{t=2\text{ s}} = 5(2.5) - 3(2.5^2) = -6.25 \text{ m}$$



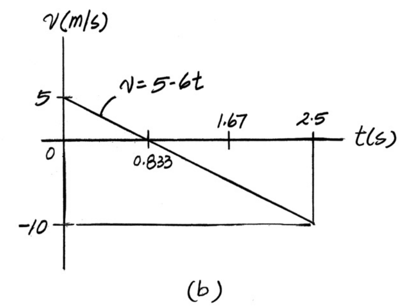
***v–t* Graph.** The velocity function can be determined using $v = \dfrac{ds}{dt}$, Thus $v = \dfrac{ds}{dt} = \{5 - 6t\}$ m/s

At $t = 0$, $v|_{t=0} = 5 - 6(0) = 5$ m/s

At $t = 2.5$ s, $v|_{t=2.5\text{ s}} = 5 - 6(2.5) = -10$ m/s

when $v = 0$,

$$0 = 5 - 6t \quad t = 0.833 \text{ s}$$

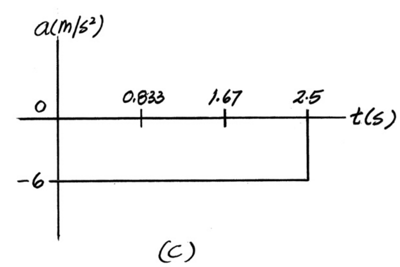


Then, at $t = 0.833$ s,

$$s|_{t=0.833\text{ s}} = 5(0.833) - 3(0.833^2) = 2.08 \text{ m}$$

***a–t* Graph.** The acceleration function can be determined using $a = \dfrac{dv}{dt}$. Thus

$$a = \frac{dv}{dt} = -6 \text{ m/s}^2 \qquad \textbf{Ans.}$$



s–t, v–t and a–t graph shown in Fig. a, b, and c can be plotted using these results.

Ans:
$v = \{5 - 6t\}$ m/s
$a = -6$ m/s^2

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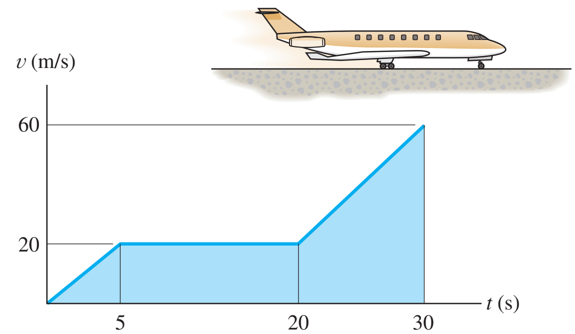
***12–64.** From experimental data, the motion of a jet plane while traveling along a runway is defined by the v–t graph. Construct the s–t and a–t graphs for the motion. When $t = 0, s = 0$.



SOLUTION

***s* − *t* Graph:** The position in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $\mathbf{0\ s \leq t < 5\ s}$, $v = \frac{20}{5}t = (4t)$ m/s.

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4t\,dt$$

$$s = \left(2t^2\right)\ \text{m}$$

When $t = 5$ s, $\quad s = 2\left(5^2\right) = 50$ m,

For time interval $\mathbf{5\ s < t < 20\ s}$,

$$ds = v\,dt$$

$$\int_{50\ \text{m}}^s ds = \int_{5\ \text{a}}^t 20\,dt$$

$$s = (20t - 50)\ \text{m}$$

When $t = 20$ s, $\quad s = 20(20) - 50 = 350$ m

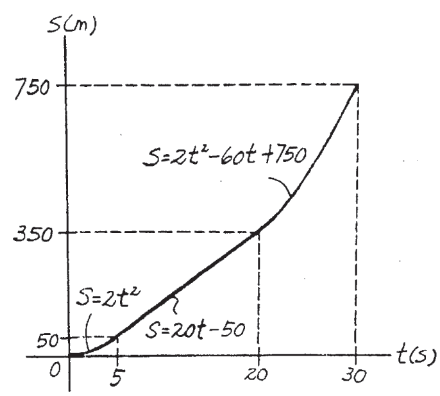
For time interval $\mathbf{20\ s < t \leq 30\ s}$, $\frac{v - 20}{t - 20} = \frac{60 - 20}{30 - 20}$, $v = (4t - 60)$ m/s.

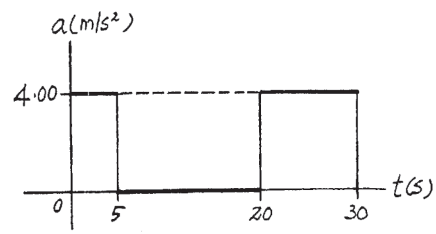
$$ds = v\,dt$$

$$\int_{350\ \text{m}}^s ds = \int_{20\ \text{a}}^t (4t - 60)\,dt$$

$$s = \left(2t^2 - 60t + 750\right)\ \text{m}$$

When $t = 30$ s, $\quad s = 2\left(30^2\right) - 60(30) + 750 = 750$ m





***a* − *t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $\mathbf{0\ s \leq t < 5\ s}$, $\mathbf{5\ s < t < 20\ s}$ and $\mathbf{20\ s < t \leq 30\ s}$, $a = \frac{dv}{dt} = 4.00\ \text{m/s}^2$, $a = \frac{dv}{dt} = 0$ and $a = \frac{dv}{dt} = 4.00\ \text{m/s}^2$, respectively.

Ans:
For $0 \leq t < 5$ s,
$s = \left\{2t^2\right\}$ m and $a = 4\ \text{m/s}^2$.
For $5\ \text{s} < t < 20$ s,
$s = \{20t - 50\}$ m and $a = 0$.
For $20\ \text{s} < t \leq 30$ s,
$s = \left\{2t^2 - 60t + 750\right\}$ m
and $a = 4\ \text{m/s}^2$.

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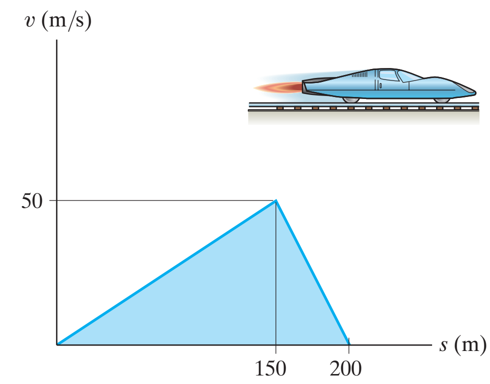
12–65. The v–s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



SOLUTION

$0 \le s \le 150$ m: $\quad v = \frac{1}{3}s$

$$dv = \frac{1}{3}ds$$

$$v\,dv = a\,ds$$

$$\frac{1}{3}s\left(\frac{1}{3}ds\right) = a\,ds$$

$$a = \frac{1}{9}s$$

At $s = 100$ m, $\quad a = \frac{1}{9}(100) = 11.1\ \text{m/s}^2$ **Ans.**

$150\ \text{m} \le s \le 200\ \text{m}$; $\quad v = 200 - s,$

$$dv = -\,ds$$

$$v\,dv = a\,ds$$

$$(200 - s)(-\,ds) = a\,ds$$

$$a = s - 200$$

At $s = 175$ m, $\quad a = 175 - 200 = -25\ \text{m/s}^2$ **Ans.**

Ans:
At $s = 100$ m, $a = 11.1\ \text{m/s}^2$
At $s = 175$ m, $a = -25\ \text{m/s}^2$

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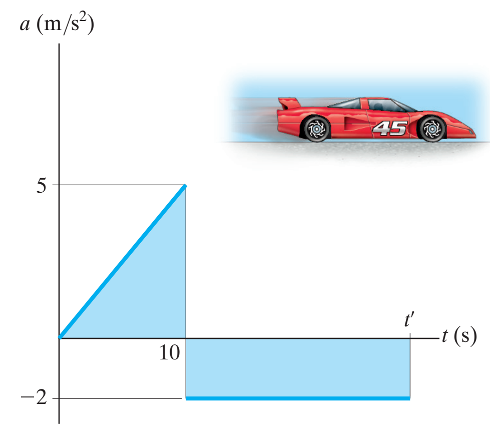
12–66.

The a–t graph for a car is shown. Construct the v–t and s–t graphs if the car starts from rest at $t = 0$. At what time t' does the car stop?



SOLUTION

$a_1 = 5\ \text{m/s}^2$

$a_2 = -2\ \text{m/s}^2$

$t_1 = 10\ \text{s}$

$k = \dfrac{a_1}{t_1}$

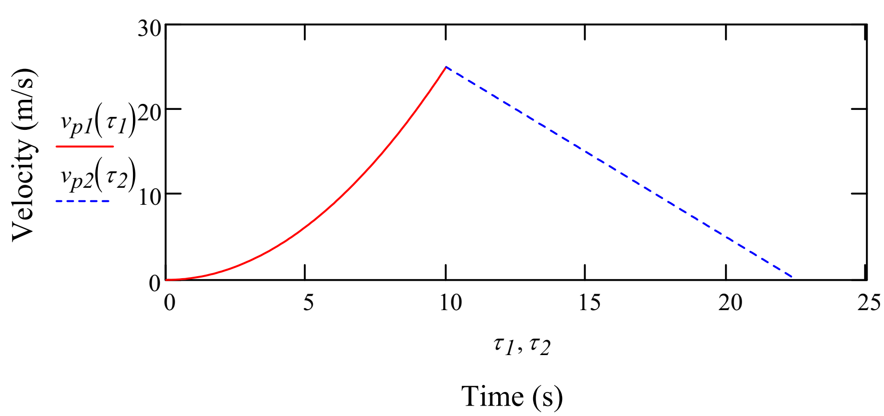
$a_{p1}(t) = kt \qquad v_{p1}(t) = k\dfrac{t^2}{2} \qquad s_{p1}(t) = k\dfrac{t^3}{6}$

$a_{p2}(t) = a_2 \qquad v_{p2}(t) = v_{p1}(t_1) + a_2(t - t_1)$

$$s_{p2}(t) = s_{p1}(t_1) + v_{p1}(t_1)(t - t_1) + \frac{1}{2}a_2(t - t_1)^2$$

Guess $t' = 12\ \text{s}$ Given $v_{p2}(t') = 0$ $t' = \text{Find}(t')$ $t' = 22.5\ \text{s}$ **Ans.**

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t'$



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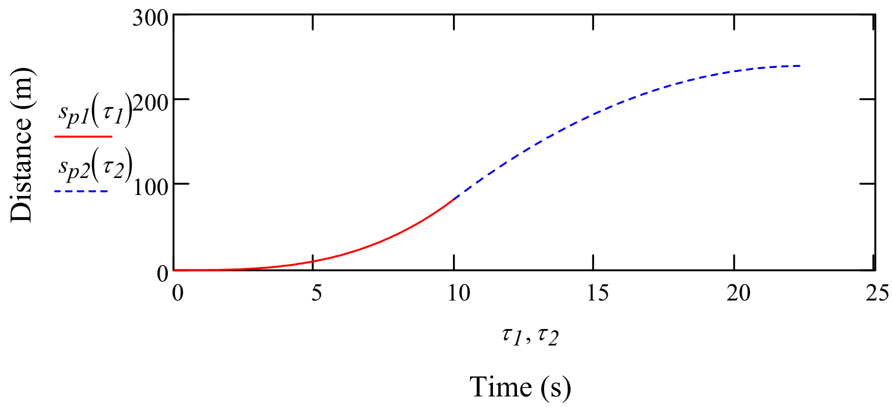
12–66. Continued



Ans:
$t' = 22.5\text{ s}$

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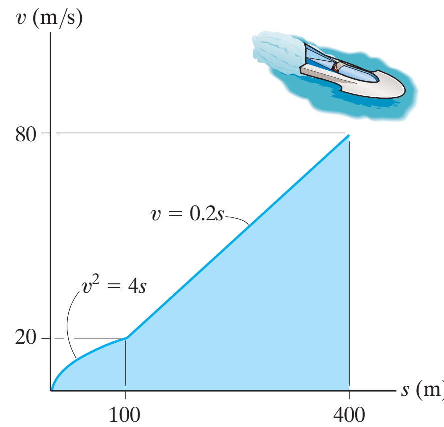
12–67.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and a–s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

***s–t* Graph:** For $0 \le s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

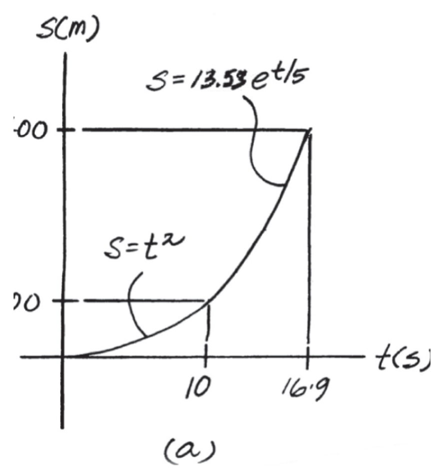
$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right)\text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10\text{ s} \qquad \textbf{Ans.}$$



For $100\text{ m} < s \le 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

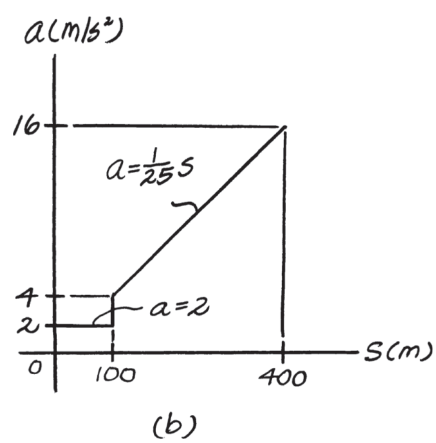
$$s = \left(13.53e^{t/5}\right)\text{ m}$$

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93\text{ s} = 16.9\text{ s} \qquad \textbf{Ans.}$$

The s–t graph is shown in Fig. a.



***a–s* Graph:** For $0\text{ m} \le s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2\text{ m/s}^2$$

For $100\text{ m} < s \le 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4\text{ m/s}^2$$

$$a\big|_{s=400\text{ m}} = 0.04(400) = 16\text{ m/s}^2$$

The a–s graph is shown in Fig. b.

Ans:
When $s = 100$ m,
$t = 10$ s.
When $s = 400$ m,
$t = 16.9$ s.
$a\big|_{s=100\text{ m}} = 4\text{ m/s}^2$
$a\big|_{s=400\text{ m}} = 16\text{ m/s}^2$

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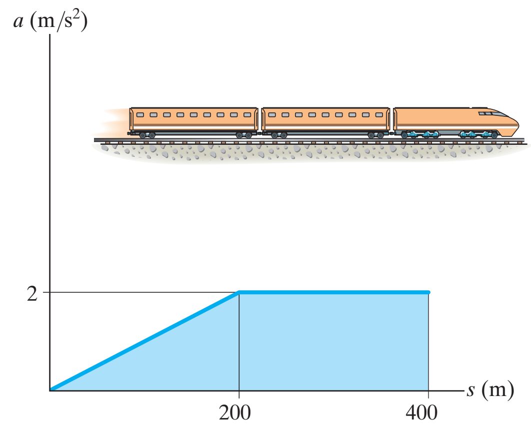
***12–68.** The a–s graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the v–s graph. $v = 0$ at $s = 0$.



SOLUTION

$$s_1 = 200 \text{ m}$$

$$s_2 = 400 \text{ m}$$

$$a_1 = 2 \frac{\text{m}}{\text{s}^2}$$

$$\sigma_1 = 0, 0.01s_1 .. s_1$$

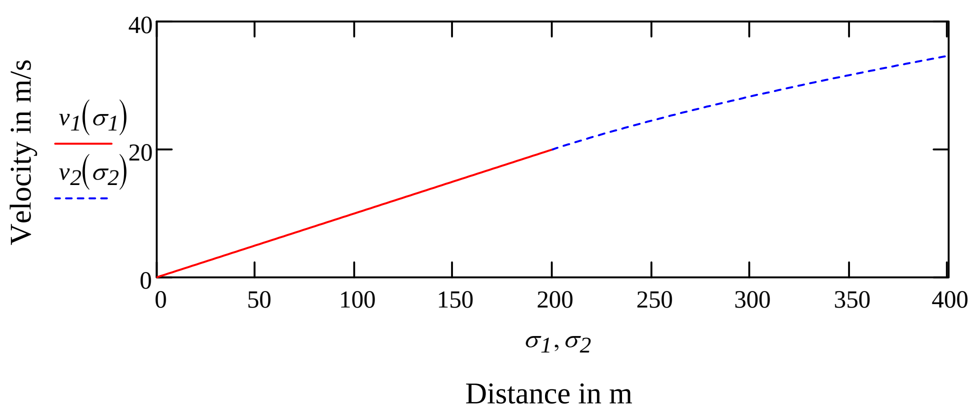
$$\sigma_2 = s_1, 1.01s_1 .. s_2$$

For $0 < s < s_1$ $\quad k = \frac{a_1}{s_1} \quad a_{c1} = ks$

$$a = ks = v\frac{dv}{ds} \qquad \int_0^v v\,dv = \int_0^s ks\,ds \qquad \frac{v^2}{2} = \frac{k}{2}s^2 \qquad v_1(\sigma_1) = \sqrt{k}\,\sigma_1 \frac{\text{s}}{\text{m}}$$

For $s_1 < s < s_2$ $\quad a_{c2} = a_1 \quad a = ks = v\frac{dv}{ds} \quad \int_{v_1}^{v} v\,dv = \int_{s_1}^{s} a_1\,ds$

$$\frac{v^2}{2} - \frac{v_1^2}{2} = a_1(s - s_1) \qquad v_2(\sigma_2) = \sqrt{2a_1(\sigma_2 - s_1) + ks_1^2}\,\frac{\text{s}}{\text{m}}$$



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12–69.

The velocity of a particle is given by $\boldsymbol{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\boldsymbol{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \boldsymbol{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7,\ 16.0,\ 14.0)\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$a = 80.2\ \text{m/s}^2$
$(42.7, 16.0, 14.0)$ m

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12–70. The position of a particle is defined by $r = \{5(\cos 2t)\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2t\mathbf{i} + 8 \cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $v = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$\mathbf{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector from can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2t\mathbf{i} - 16 \sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\mathbf{i} - 16 \sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Traveling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{(1)}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{(2)}$$

Adding Eqs (1) and (2) yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \qquad \textbf{(Equation of an Ellipse) (Q.E.D.)}$$

Ans:
$v = 9.68$ m/s
$a = 169.8$ m/s^2

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12–71.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \boldsymbol{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \boldsymbol{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m}$$ **Ans.**

Ans:
$\Delta\mathbf{r} = \{6\mathbf{i} + 4\mathbf{j}\}$ m

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***12–72.**

If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

SOLUTION

$$\mathbf{v}(t) = 0.8t^2\,\mathbf{i} + 12t^{1/2}\,\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{a} = \frac{dv}{dt} = 1.6t\,\mathbf{i} + 6t^{1/2}\mathbf{j}$$

When $t = 2$ s, $\quad \mathbf{a} = 3.2\mathbf{i} + 4.243\mathbf{j}$

$$a = \sqrt{(3.2)^2 + (4.243)^2} = 5.31 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$u_o = \frac{\mathbf{a}}{a} = 0.6022\mathbf{i} + 0.7984\mathbf{j}$$

$$\alpha = \cos^{-1}(0.6022) = 53.0^\circ \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}(0.7984) = 37.0^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(0) = 90.0^\circ \qquad \textbf{Ans.}$$

Ans:
$a = 5.31 \text{ m/s}^2$
$\alpha = 53.0^\circ$
$\beta = 37.0^\circ$
$\gamma = 90.0^\circ$

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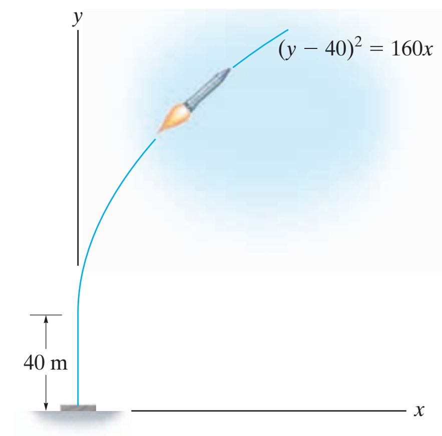
12–73.

When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



SOLUTION

$v_y = 180$ m/s

$(y - 40)^2 = 160\,x$

$2(y - 40)v_y = 160v_x$ **(1)**

$2(80 - 40)(180) = 160v_x$

$v_x = 90$ m/s

$v = \sqrt{90^2 + 180^2} = 201$ m/s **Ans.**

$a_y = \dfrac{d\,v_y}{dt} = 0$

From Eq. 1,

$2\,v_y^2 + 2(y - 40)a_y = 160\,a_x$

$2(180)^2 + 0 = 160\,a_x$

$a_x = 405$ m/s^2

$a = 405$ m/s^2 **Ans.**

Ans:
$v = 201$ m/s
$a = 405$ m/s^2

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12–74. A particle is traveling with a velocity of $\mathbf{v} = \{3\sqrt{t}e^{-0.2t}\mathbf{i} + 4e^{-0.8t^2}\mathbf{j}\}$ m/s, where t is in seconds. Determine the magnitude of the particle's displacement from $t = 0$ to $t = 3$ s. Use Simpson's rule with $n = 100$ to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = 2$ s?

SOLUTION

Given: $a = 3\ \text{m/s}^{3/2}$ $\quad b = -0.2\ \text{s}^{-1}$ $\quad c = 4\ \text{m/s}$ $\quad d = -0.8\ \text{s}^{-2}$ $\quad t_1 = 3\ \text{s}$ $\quad t_2 = 2\ \text{s}$

$n = 100$

Displacement

$$x_1 = \int_0^{t_1} a\sqrt{t}e^{bt}\,dt \qquad x_1 = 7.34\ \text{m} \qquad y_1 = \int_0^{t_1} c e^{dt^2}\,dt \qquad y_1 = 3.96\ \text{m}$$

$$d_1 = \sqrt{x_1^2 + y_1^2} \qquad d_1 = 8.34\ \text{m}$$ **Ans.**

Acceleration

$$a_x = \frac{d}{dt}\left(a\sqrt{t}e^{bt}\right) = \frac{a}{2\sqrt{t}}e^{bt} + ab\sqrt{t}e^{bt} \qquad a_{x2} = \frac{a}{\sqrt{t_2}}e^{bt_2}\left(\frac{1}{2} + bt_2\right)$$

$$a_y = \frac{d}{dt}\left(ce^{dt^2}\right) = 2cdte^{dt^2} \qquad a_{y2} = 2cdt_2e^{dt_2^2}$$

$$a_{x2} = 0.14\ \text{m/s}^2 \qquad a_{y2} = -0.52\ \text{m/s}^2 \qquad a_2 = \sqrt{a_{x2}^2 + a_{y2}^2} \qquad a_2 = 0.541\ \text{m/s}^2$$ **Ans.**

Ans:
$d_1 = 8.34$ m,
$a_2 = 0.541\ \text{m/s}^2$

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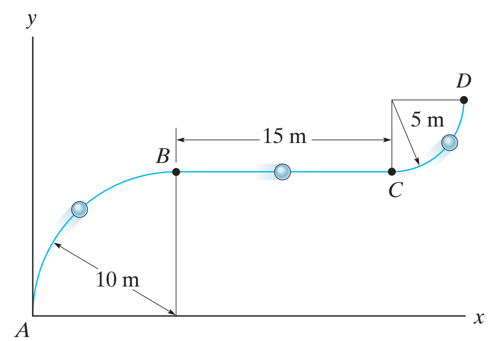
12–75.

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

Ans:
$(v_{\text{sp}})_{\text{avg}} = 4.28 \text{ m/s}$

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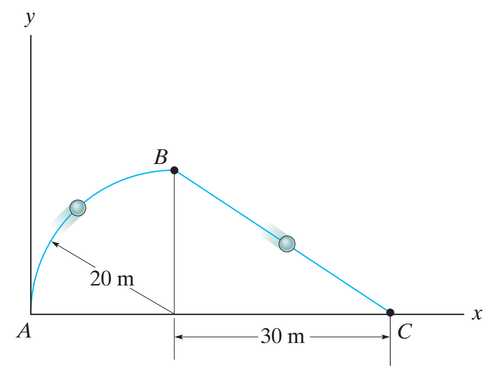
***12–76.**

A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A. Determine its average speed when it goes around the closed path.



SOLUTION

The total distance traveled is

$$S_{\text{Tot}} = S_{AB} + S_{BC} + S_{CA}$$

$$= 20\left(\frac{\pi}{2}\right) + \sqrt{20^2 + 30^2} + (30 + 20)$$

$$= 117.47 \text{ m}$$

The total time taken is

$$t_{\text{Tot}} = t_{AB} + t_{BC} + t_{CA}$$

$$= 5 + 8 + 10$$

$$= 23 \text{ s}$$

Thus, the average speed is

$$(v_{sp})_{avg} = \frac{S_{\text{Tot}}}{t_{\text{Tot}}} = \frac{117.47 \text{ m}}{23 \text{ s}} = 5.107 \text{ m/s} = 5.11 \text{ m/s}$$ **Ans.**

Ans:
$(v_{sp})_{avg} = 5.11 \text{ m/s}$

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12–77.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3 \text{ m/s} \qquad v_y = 3(2) = 6 \text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ m/s} = 8.55 \text{ m} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \qquad a_y = 3 \text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:
$v = 8.55$ m/s
$a = 5.82$ m/s^2

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12–78.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1003$ m/s
$a = 103$ m/s^2

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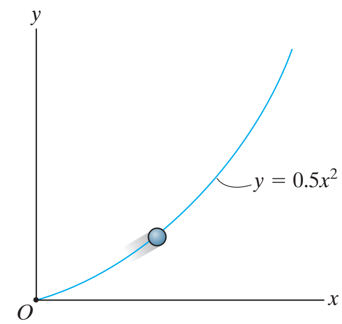
12–79. The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ m/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t\,dt$$

$$x = \left(2.50t^2\right)\text{ m}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ m. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ m and $y = 3.125\left(1^4\right) = 3.125$ m. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00\text{ m} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ m/s $a_x = \dfrac{dv_x}{dt} = 5$ m/s², and $x = 2.50$ m. Then, from Eq. (2)

$$a_y = 5^2 + 2.50(5) = 37.5\text{ m/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 4.00$ m
$a = 37.8$ m/s²

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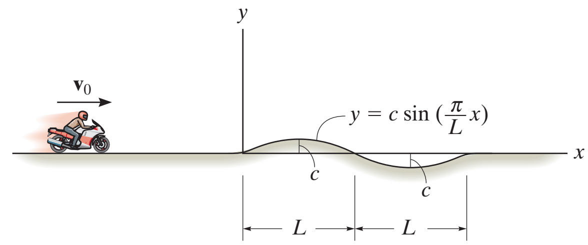
***12–80.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c \sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\,v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

Ans:

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

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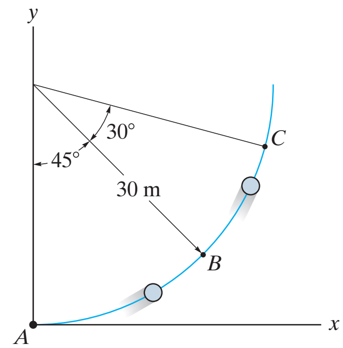
12–81.

A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



SOLUTION

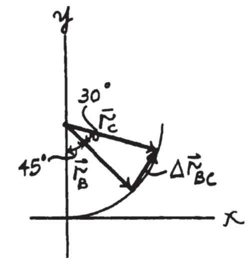
Position: The coordinates for points B and C are [30 sin 45°, 30 − 30 cos 45°] and [30 sin 75°, 30 − 30 cos 75°]. Thus,

$$\mathbf{r}_B = (30 \sin 45° - 0)\mathbf{i} + [(30 - 30 \cos 45°) - 30]\mathbf{j}$$

$$= \{21.21\mathbf{i} - 21.21\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_C = (30 \sin 75° - 0)\mathbf{i} + [(30 - 30 \cos 75°) - 30]\mathbf{j}$$

$$= \{28.98\mathbf{i} - 7.765\mathbf{j}\}\text{ m}$$



Average Velocity: The displacement from point B to C is $\Delta\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\}$ m.

$$(\mathbf{v}_{BC})_{\text{avg}} = \frac{\Delta\mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{BC})_{\text{avg}} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}$ m/s

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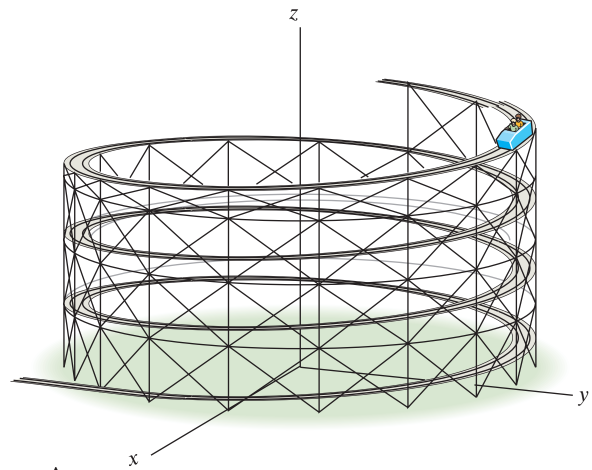
12–82.

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



SOLUTION

$x = c \sin kt \qquad \dot{x} = ck \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -ck \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2k^2 + b^2}$ **Ans.**

$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$ **Ans.**

Ans:
$v = \sqrt{c^2 k^2 + b^2}$
$a = ck^2$

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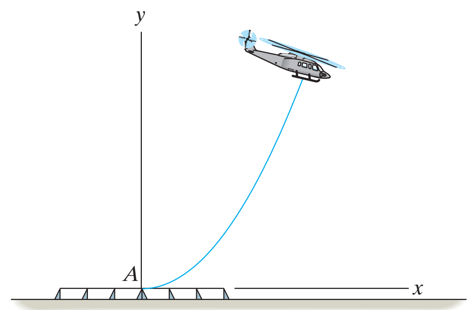
12–83.

The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



SOLUTION

$x = 2t^2 \qquad y = 0.04t^3$

At $t = 10$ s, $\qquad x = 200$ m $\qquad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204$ m **Ans.**

$$v_x = \frac{dx}{dt} = 4t$$

$$a_x = \frac{dv_x}{dt} = 4$$

$$v_y = \frac{dy}{dt} = 0.12t^2$$

$$a_y = \frac{dv_y}{dt} = 0.24t$$

At $t = 10$ s,

$v = \sqrt{(40)^2 + (12)^2} = 41.8$ m/s **Ans.**

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66$ m/s^2 **Ans.**

Ans:
$d = 204$ m
$v = 41.8$ m/s
$a = 4.66$ m/s^2

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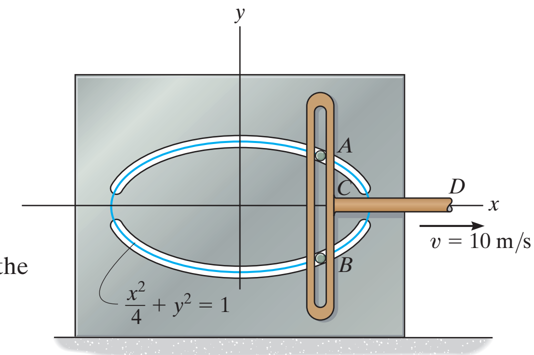
***12–84.**

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s} \downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x{}^2 + xa_x\right) + 2\left(v_y{}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2 \downarrow$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 10.4$ m/s
$a = 38.5$ m/s^2

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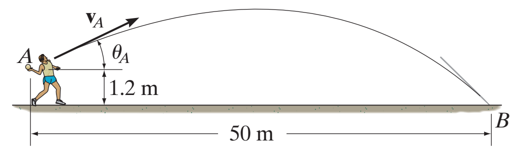
12–85.

It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = v_A \cos\theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$50 = 0 + v_A \cos\theta_A (2.5)$$

$$v_A \cos\theta_A = 20 \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = v_A \sin\theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81\ \text{m/s}^2$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.2 = 0 + v_A \sin\theta_A (2.5) + \frac{1}{2}(-9.81)\left(2.5^2\right)$$

$$v_A \sin\theta_A = 11.7825 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 23.2$ m/s

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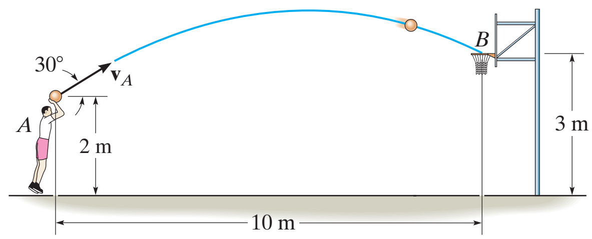
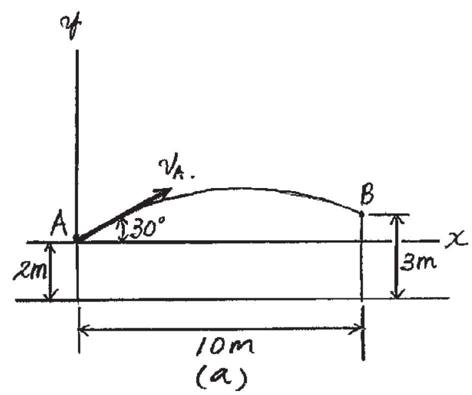
12–86.

Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



SOLUTION



Coordinate System. The origin of the x-y coordinate system will be set to coinside with point A as shown in Fig. a

Horizontal Motion. Here $(v_A)_x = v_A \cos 30° \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10 \text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$10 = 0 + v_A \cos 30° \, t$$

$$t = \frac{10}{v_A \cos 30°} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow}) \quad (v_B)_x = (v_A)_x = v_A \cos 30° \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = v_A \sin 30° \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1 \text{ m} \uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$1 = 0 + v_A \sin 30° \, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5\, v_A \, t + 1 = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow) \quad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = v_A \sin 30° + (-9.81)t$$

$$(v_B)_y = 0.5\, v_A - 9.81t \qquad \textbf{(4)}$$

Solving Eq. (1) and (3)

$$v_A = 11.705 \text{ m/s} = 11.7 \text{ m/s} \qquad \textbf{Ans.}$$

$$t = 0.9865 \text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30° = 10.14 \text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825 \text{ m/s} = 3.825 \text{ m/s} \downarrow$$

Thus, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83 \text{ m/s} = 10.8 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{3.825}{10.14}\right) = 20.67° = 20.7° \qquad \textbf{Ans.}$$

Ans:
$v_A = 11.7 \text{ m/s}$
$v_B = 10.8 \text{ m/s}$
$\theta = 20.7° ⦪$

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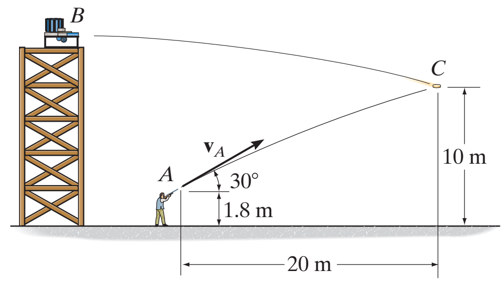
12–87. A projectile is fired from the platform at B. The shooter fires his gun from point A at an angle of 30°. Determine the muzzle speed of the bullet if it hits the projectile at C.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $x_A = 0$ and $x_C = 20$ m. Thus,

$$\left(\xrightarrow{+}\right) \qquad x_C = x_A + (v_A)_x t$$

$$20 = 0 + v_A \cos 30°\, t \qquad \textbf{(1)}$$

***y*-Motion:** Here, $y_A = 1.8$, $(v_A)_y = v_A \sin 30°$, and $a_y = -g = -9.81\ \text{m/s}^2$. Thus,

$$(+\uparrow) \qquad y_C = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$10 = 1.8 + v_A \sin 30°(t) + \frac{1}{2}(-9.81)(t)^2$$

Thus,

$$10 - 1.8 = \left(\frac{20 \sin 30°}{\cos 30°(t)}\right)(t) - 4.905(t)^2$$

$$t = 0.8261\ \text{s}$$

So that

$$v_A = \frac{20}{\cos 30°(0.8261)} = 28.0\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 28.0$ m/s

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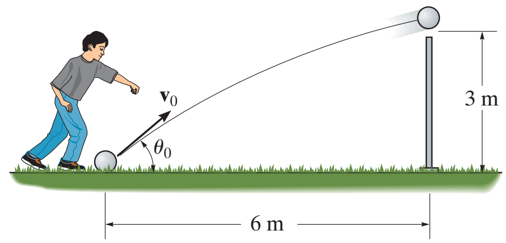
***12–88.**

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad x = x_0 + (v_0)x^t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81\,\text{m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0(\sin\theta)t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0(\sin\theta)t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\dfrac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57^\circ$$

$$\theta = 58.28^\circ = 58.3^\circ \qquad \textbf{Ans.}$$

Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57^\circ - \cos^2 58.28^\circ}} = 9.76\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\theta = 58.3^\circ$
$(v_0)_{\min} = 9.76\text{ m/s}$

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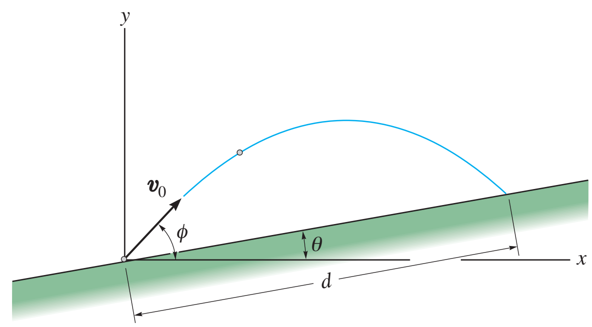
12–89.

A projectile is given a velocity $\mathbf{v}_0$ at an angle ϕ above the horizontal. Determine the distance d to where it strikes the sloped ground. The acceleration due to gravity is g.



SOLUTION

$(\xrightarrow{+})\qquad s = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$ **Ans.**

Ans:

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

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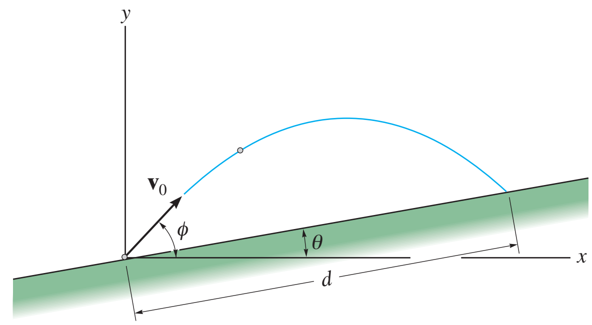
12-90.

A projectile is given a velocity $\mathbf{v}_0$. Determine the angle ϕ at which it should be launched so that d is a maximum. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s_x = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)\qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

Require:

$$\frac{d(d)}{d\phi} = \frac{v_0^2}{g\cos\theta}\left[\cos 2\phi(2) - 2\tan\theta(2\cos\phi)(-\sin\phi)\right] = 0$$

$$\cos 2\phi + \tan\theta\sin 2\phi = 0$$

$$\frac{\sin 2\phi}{\cos 2\phi}\tan\theta + 1 = 0$$

$$\tan 2\phi = -\text{ctn}\,\theta$$

$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta) \qquad \textbf{Ans.}$$

Ans:

$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta)$$

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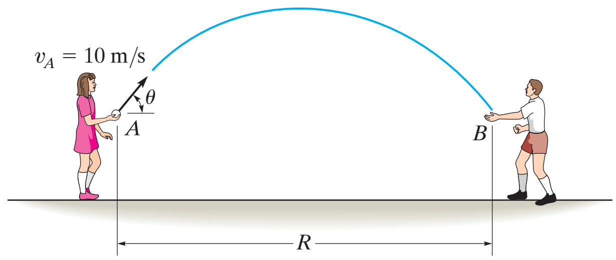
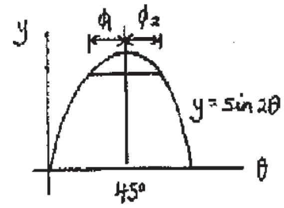
12–91.

The girl at A can throw a ball at $v_A = 10$ m/s. Calculate the maximum possible range $R = R_{max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



SOLUTION



$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$R = 0 + (10 \cos \theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10 \sin \theta = 10 \sin \theta - 9.81t$$

$$t = \frac{20}{9.81} \sin \theta$$

Thus, $\quad R = \dfrac{200}{9.81} \sin \theta \cos \theta$

$$R = \frac{100}{9.81} \sin 2\theta \qquad \textbf{(1)}$$

Require,

$$\frac{dR}{d\theta} = 0$$

$$\frac{100}{9.81} \cos 2\theta(2) = 0$$

$$\cos 2\theta = 0$$

$$\theta = 45^\circ \qquad \textbf{Ans.}$$

$$R = \frac{100}{9.81}(\sin 90^\circ) = 10.2 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$R_{max} = 10.2$ m
$\theta = 45^\circ$

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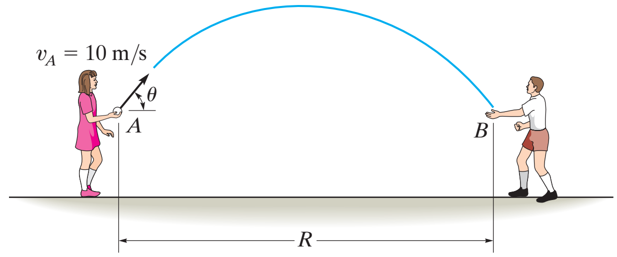
***12–92.**

Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15°, i.e., $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. Assume the ball is caught at the same elevation from which it is thrown.



SOLUTION

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$R = 0 + (10\cos\theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10\sin\theta = 10\sin\theta - 9.81t$$

$$t = \frac{20}{9.81}\sin\theta$$

Thus, $\quad R = \dfrac{200}{9.81}\sin\theta\cos\theta$

$$R = \frac{100}{9.81}\sin 2\theta \qquad \textbf{(1)}$$

Since the function $y = \sin 2\theta$ is symmetric with respect to $\theta = 45°$ as indicated, Eq. (1) will be satisfied if $|\phi_1| = |\phi_2|$

Choosing $\phi = 15°$ or $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$, and substituting into Eq. (1) yields

$R = 8.83$ m **Ans.**

Ans:
$R = 8.83$ m

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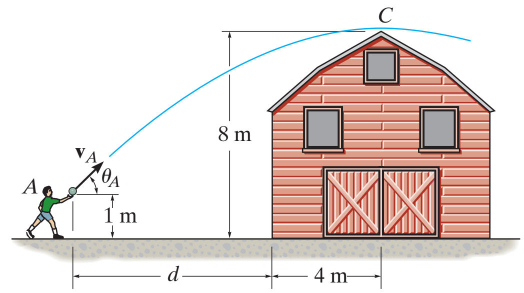
12–93. The boy at A attempts to throw a ball over the roof of a barn with an initial speed of $v_A = 15$ m/s. Determine the angle θ_A at which the ball must be thrown so that it reaches its maximum height at C. Also, find the distance d where the boy should stand to make the throw.



SOLUTION

Vertical Motion: The vertical component, of initial and final velocity are $(v_0)_y = (15 \sin \theta_A)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$(+\uparrow)$ $$v_y = (v_0) + a_c t$$

$$0 = 15 \sin \theta_A + (-9.81)t \qquad \textbf{[1]}$$

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + 15 \sin \theta_A t + \frac{1}{2}(-9.81)t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta_A = 51.38° = 51.4° \qquad \textbf{Ans.}$$

$$t = 1.195 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos \theta_A = 15 \cos 51.38° = 9.363$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d + 4 = 0 + 9.363(1.195)$$

$$d = 7.18 \text{ m} \qquad \textbf{Ans.}$$

Ans:

$\theta_A = 51.4°$
$d = 7.18$ m

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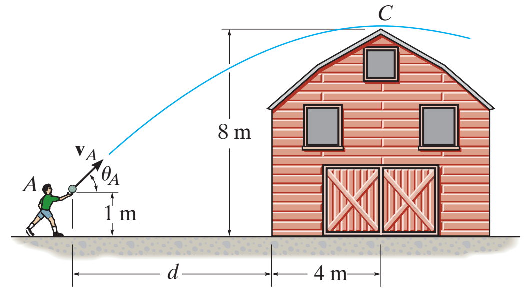
12–94. The boy at A attempts to throw a ball over the roof of a barn such that it is launched at an angle $\theta_A = 40°$. Determine the minimum speed v_A at which he must throw the ball so that it reaches its maximum height at C. Also, find the distance d where the boy must stand so that he can make the throw.



SOLUTION

Vertical Motion: The vertical components of initial and final velocity are $(v_0)_y = (v_A \sin 40°)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$$(+\uparrow) \qquad v_y = (v_0) + a_c t$$

$$0 = v_A \sin 40° + (-9.81)\,t \qquad \textbf{[1]}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + v_A \sin 40° t + \frac{1}{2}(-9.81)\,t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_A = 18.23 \text{ m/s} = 18.2 \text{ m/s}$$

$$t = 1.195 \text{ s} \qquad \textbf{Ans.}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos \theta_A = 18.23 \cos 40° = 13.97$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d + 4 = 0 + 13.97(1.195)$$

$$d = 12.7 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$t = 1.195$ s
$d = 12.7$ m

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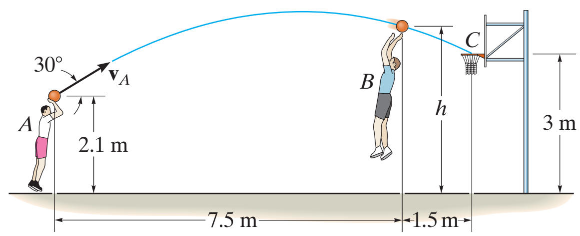
12-95.

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$a = 2.1 \text{ m}$

$b = 7.5 \text{ m}$

$c = 1.5 \text{ m}$

$d = 3 \text{ m}$

$\theta = 30°$

$g = 9.81 \text{ m/s}^2$

Guesses $\quad v_A = 3 \text{ m/s} \quad t_B = 1 \text{ s} \quad t_C = 1 \text{ s} \quad h = 3.5 \text{ m}$

Given $\quad b + c = v_A \cos(\theta) t_C \qquad b = v_A \cos(\theta) t_B$

$$d = \frac{-g}{2} t_C^{\,2} + v_A \sin(\theta) t_C + a \qquad h = \frac{-g}{2} t_B^{\,2} + v_A \sin(\theta) t_B + a$$

$$\begin{pmatrix} v_A \\ t_B \\ t_C \\ h \end{pmatrix} = \text{Find}(v_A, t_B, t_C, h) \qquad \begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.94 \end{pmatrix} \text{s} \qquad v_A = 11.1 \text{ m/s} \qquad h = 3.45 \text{ m} \quad \textbf{Ans.}$$

Ans:
$v_A = 11.1$ m/s
$h = 3.45$ m

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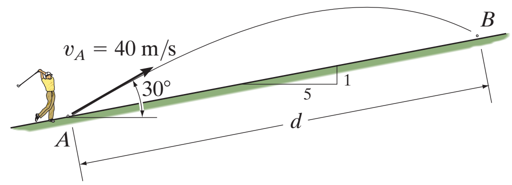
***12–96.** The golf ball is hit at A with a speed of $v_A = 40\,\text{m/s}$ and directed at an angle of 30° with the horizontal as shown. Determine the distance d where the ball strikes the slope at B.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 40\cos 30° = 34.64\text{ m/s}$, $x_A = 0$, and $x_B = d\left(\dfrac{5}{\sqrt{5^2+1}}\right) = 0.9806d$. Thus,

$$\left(\xrightarrow{+}\right) \qquad x_B = x_A + (v_A)_x t$$

$$0.9806d = 0 + 34.64t$$

$$t = 0.02831d \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 40\sin 30° = 20\text{ m/s}$, $y_A = 0$, $y_B = d\left(\sqrt{\dfrac{1}{\sqrt{5^2+1}}}\right) = 0.1961d$, and $a_y = -g = -9.81\text{ m/s}^2$.

Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$0.1961d = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 20t + 0.1961d = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$4.905(0.02831d)^2 - 20(0.02831d) + 0.1961d = 0$$

$$3.9303\left(10^{-3}\right)d^2 - 0.37002d = 0$$

$$d\left[3.9303\left(10^{-3}\right)d - 0.37002\right] = 0$$

Since $d \neq 0$, then

$$3.9303\left(10^{-3}\right)d = 0.37002 = 0$$

$$d = 94.1\text{ m} \qquad \textbf{Ans.}$$

Ans:
$d = 94.1$ m

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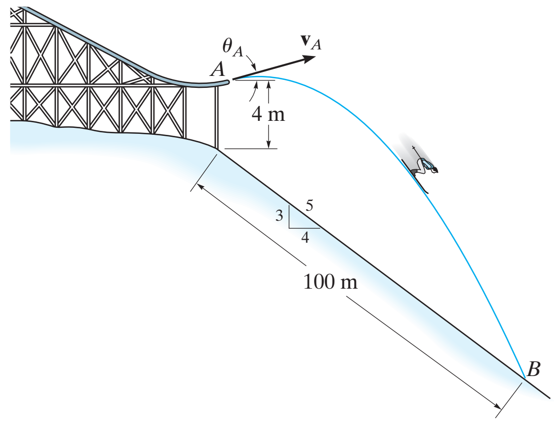
12–97.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



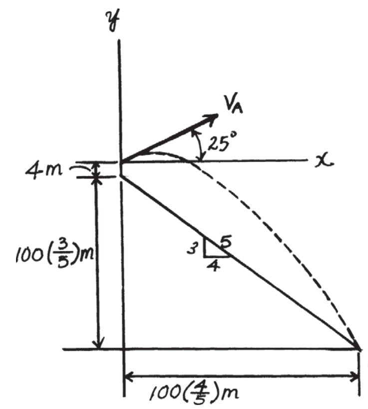
SOLUTION

$(\xrightarrow{+})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$



Solving,

$$v_A = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

$$t_{AB} = 4.54 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 19.4$ m/s
$t_{AB} = 4.54$ s

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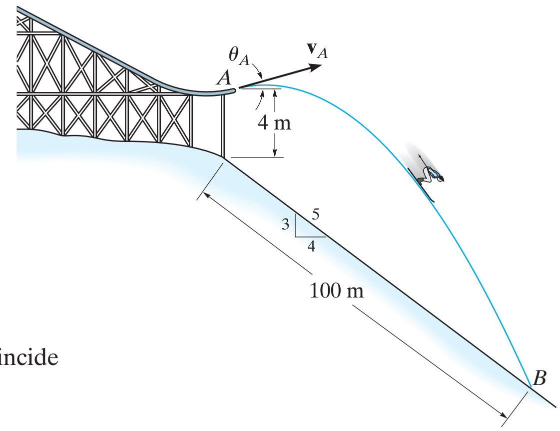
12–98.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

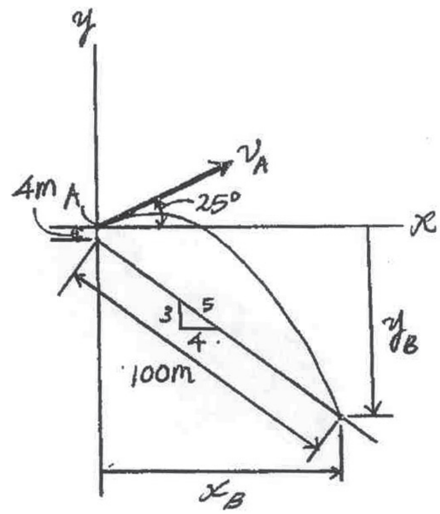
Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. a.

x-motion: Here, $x_A = 0,\ x_B = 100\left(\frac{4}{5}\right) = 80\text{ m}$ and $(v_A)_x = v_A \cos 25°$.

$$\left(\xrightarrow{+}\right) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \qquad \textbf{(1)}$$



(a)

y-motion: Here, $y_A = 0,\ y_B = -[4 + 100\left(\frac{3}{5}\right)] = -64\text{ m}$ and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81\text{ m/s}^2$.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-64 = 0 + v_A \sin 25°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\, t = 64 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yields

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

$$\frac{80}{v_A \cos 25°} = 4.545$$

$$v_A = 19.42\text{ m/s} = 19.4\text{ m/s} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465\text{ s}$$

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12–98. Continued

Using this result,

$$(+\uparrow) \qquad (v_B)_y = (v_A)_y + a_y t$$

$$= 19.42 \sin 25° + (-9.81)(4.5446)$$

$$= -36.37 \text{ m/s} = 36.37 \text{ m/s} \downarrow$$

And

$$(\overset{+}{\rightarrow}) \qquad (v_B)_x = (v_A)_x = v_A \cos 25° = 19.42 \cos 25° = 17.60 \text{ m/s} \rightarrow$$

Thus,

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2}$$

$$= \sqrt{36.37^2 + 17.60^2}$$

$$= 40.4 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 19.4 \text{ m/s}$
$v_B = 40.4 \text{ m/s}$

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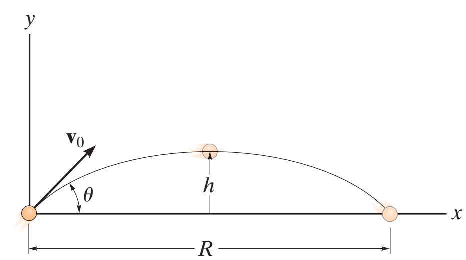
12–99.

The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

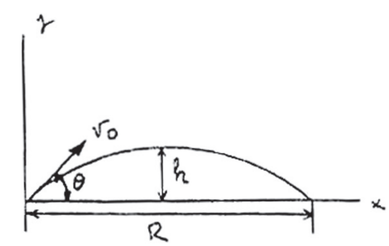
$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 0 + (v_0 \sin\theta)\, t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$$

$$R = \frac{v_0^2}{g}\sin 2\theta \qquad \textbf{Ans.}$$



$$t = \frac{R}{v_0 \cos\theta} = \frac{v_0^2\,(2 \sin\theta \cos\theta)}{v_0\, g \cos\theta}$$

$$= \frac{2v_0}{g}\sin\theta \qquad \textbf{Ans.}$$

$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_0 \sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g}\sin^2\theta \qquad \textbf{Ans.}$$

Ans:
$R = \dfrac{v_0}{g}\sin 2\theta$
$t = \dfrac{2v_0}{g}\sin\theta$
$h = \dfrac{v_0^2}{2g}\sin^2\theta$

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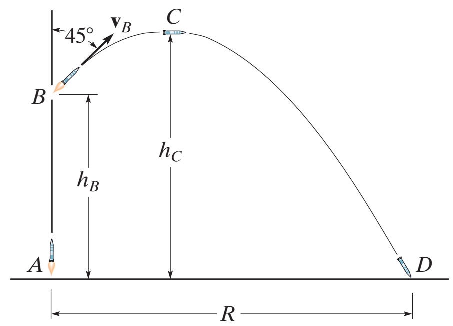
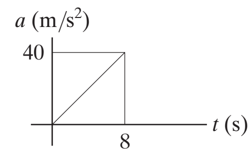
***12–100.**

The missile at A takes off from rest and rises vertically to B, where its fuel runs out in 8 s. If the acceleration varies with time as shown, determine the missile's height h_B and speed v_B. If by internal controls the missile is then suddenly pointed 45° as shown, and allowed to travel in free flight, determine the maximum height attained, h_C, and the range R to where it crashes at D.





SOLUTION

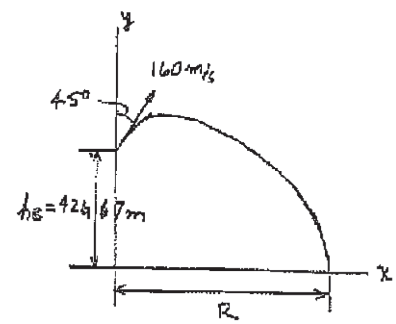
$a = \frac{40}{8}t = 5t$

$dv = a\,dt$

$\int_0^v dv = \int_0^t 5t\,dt$

$v = 2.5t^2$

When $t = 8$ s, $\quad v_B = 2.5(8)^2 = 160$ m/s **Ans.**



$ds = v\,dt$

$\int_0^s ds = \int_0^t 2.5t^2\,dt$

$x = \frac{2.5}{3}t^3$

$h_B = \frac{2.5}{3}(8)^3 = 426.67 = 427$ m **Ans.**

$(v_B)_x = 160 \sin 45° = 113.14$ m/s

$(v_B)_y = 160 \cos 45° = 113.14$ m/s

$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$

$0^2 = (113.14)^2 + 2(-9.81)(s_c - 426.67)$

$h_c = 1079.1$ m $= 1.08$ km **Ans.**

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$R = 0 + 113.14t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0 = 426.67 + 113.14t + \frac{1}{2}(-9.81)t^2$

Solving for the positive root, $t = 26.36$ s

Then,

$R = 113.14\,(26.36) = 2983.0 = 2.98$ km **Ans.**

Ans:
$v_B = 160$ m/s
$h_B = 427$ m
$h_C = 1.08$ km
$R = 2.98$ km

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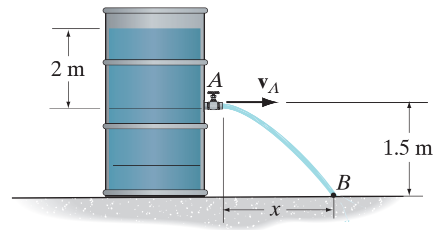
12–101.

The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A. The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

***x*-Motion:** Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$x = 0 + 6.264 t_A \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 0$, $a_y = -g = -9.81 \text{ m/s}^2$, $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.5 = 0 + 0 + \frac{1}{2}(-9.81) t_A{}^2$$

$$t_A = 0.553 \text{ s} \qquad \textbf{Ans.}$$

Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$t_A = 0.553$ s
$x = 3.46$ m

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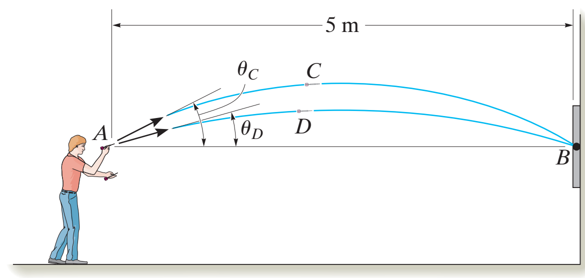
12–102.

The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C $(>\theta_D)$, then the second dart is thrown at θ_D.



SOLUTION

$(\xrightarrow{+})\qquad s = s_0 + v_0 t$

$$5 = 0 + (10\cos\theta)\,t \qquad \textbf{(1)}$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-10\sin\theta = 10\sin\theta - 9.81\,t$$

$$t = \frac{2(10\sin\theta)}{9.81} = 2.039\sin\theta$$

From Eq. (1),

$$5 = 20.39\sin\theta\cos\theta$$

Since $\quad \sin 2\theta = 2\sin\theta\cos\theta$

$$\sin 2\theta = 0.4905$$

The two roots are $\theta_D = 14.7^\circ$ **Ans.**

$\theta_C = 75.3^\circ$ **Ans.**

From Eq. (1): $t_D = 0.517$ s

$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s **Ans.**

Ans:
$\theta_D = 14.7^\circ$
$\theta_C = 75.3^\circ$
$\Delta t = 1.45$ s

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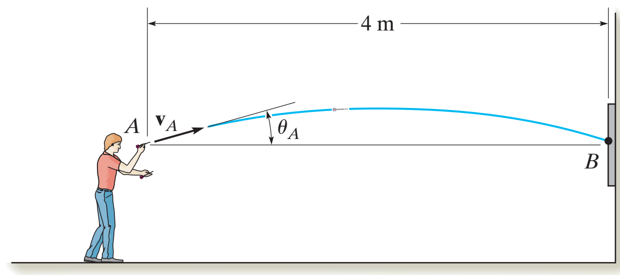
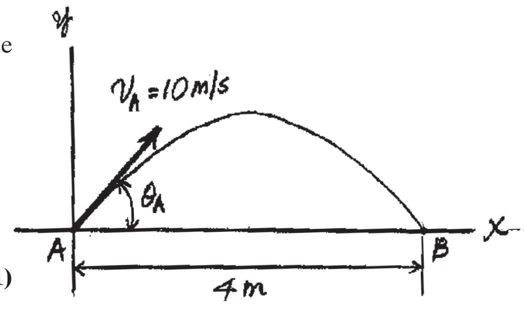
12–103.

If the dart is thrown with a speed of 10 m/s, determine the shortest possible time before it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION

Coordinate System. The origin of the x-y coordinate system will be set to coincide with point A as shown in Fig. *a*.



(a)

Horizontal Motion. Here, $(v_A)_x = 10\cos\theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4 \text{ m} \rightarrow$.

$$(\xrightarrow{+}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$4 = 0 + 10\cos\theta_A\, t$$

$$t = \frac{4}{10\cos\theta_A} \qquad \textbf{(1)}$$

Also,

$$(\xrightarrow{+}) \quad (v_B)_x = (v_A)_x = 10\cos\theta_A \rightarrow \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = 10\sin\theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (10\sin\theta_A)\,t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - (10\sin\theta_A)\,t = 0$$

$$t\,(4.905t - 10\sin\theta_A) = 0$$

Since $t \neq 0$, then

$$4.905t - 10\sin\theta_A = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow) \quad (v_B)_y^2 = (v_A)_y^2 + 2\,a_y\,[(s_B)_y - (s_A)_y]$$

$$(v_B)_y^2 = (10\sin\theta_A)^2 + 2\,(-9.81)\,(0 - 0)$$

$$(v_B)_y = -10\sin\theta_A = 10\sin\theta_A \downarrow \qquad \textbf{(4)}$$

Substitute Eq. (1) into (3)

$$4.905\left(\frac{4}{10\cos\theta_A}\right) - 10\sin\theta_A = 0$$

$$1.962 - 10\sin\theta_A\cos\theta_A = 0$$

Using the trigonometry identity $\sin 2\theta_A = 2\sin\theta_A\cos\theta_A$, this equation becomes

$$1.962 - 5\sin 2\theta_A = 0$$

$$\sin 2\theta_A = 0.3924$$

$$2\theta_A = 23.10° \text{ and } 2\theta_A = 156.90°$$

$$\theta_A = 11.55° \text{ and } \theta_A = 78.45°$$

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12–103. Continued

Since the shorter time is required, Eq. (1) indicates that smaller θ_A must be choosen. Thus

$$\theta_A = 11.55° = 11.6° \qquad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 11.55°} = 0.4083 \text{ s} = 0.408 \text{ s} \qquad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 11.55° = 9.7974 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 11.55° = 2.0026 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{9.7974^2 + 2.0026^2} = 10 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{2.0026}{9.7974}\right) = 11.55° = 11.6° \qquad \textbf{Ans.}$$

Ans:
$\theta_A = 11.6°$
$t = 0\ 408$ s
$v_B = 10$ m/s
$\theta_B = 11.6°$

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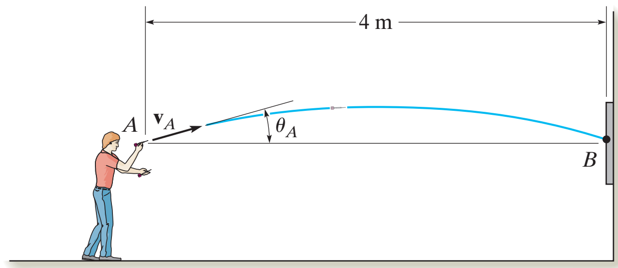
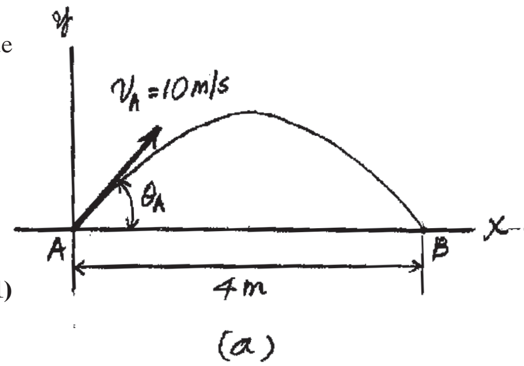
***12–104.**

If the dart is thrown with a speed of 10 m/s, determine the longest possible time when it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION



Coordinate System. The origin of the x-y coordinate system will be set to coincide with point A as shown in Fig. a.

Horizontal Motion. Here, $(v_A)_x = 10\cos\theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4\text{ m} \rightarrow$.

$$(\xrightarrow{+})\quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$4 = 0 + 10\cos\theta_A\, t$$

$$t = \frac{4}{10\cos\theta_A} \qquad \textbf{(1)}$$

Also,

$$(\xrightarrow{+})\quad (v_B)_x = (v_A)_x = 10\cos\theta_A \rightarrow \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = 10\sin\theta_A\uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = -9.81\text{ m/s}^2\downarrow$.

$$(+\uparrow)\quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (10\sin\theta_A)\,t + \frac{1}{2}(-9.81)\,t^2$$

$$4.905t^2 - (10\sin\theta_A)\,t = 0$$

$$t\,(4.905t - 10\sin\theta_A) = 0$$

Since $t \neq 0$, then

$$4.905\,t - 10\sin\theta_A = 0 \qquad \textbf{(3)}$$

Also,

$$(v_B)_y^2 = (v_A)_y^2 + 2\,a_y\,[(s_B)_y - (s_A)_y]$$

$$(v_B)_y^2 = (10\sin\theta_A)^2 + 2\,(-9.81)\,(0 - 0)$$

$$(v_B)_y = -10\sin\theta_A = 10\sin\theta_A\downarrow \qquad \textbf{(4)}$$

Substitute Eq. (1) into (3)

$$4.905\left(\frac{4}{10\cos\theta_A}\right) - 10\sin\theta_A = 0$$

$$1.962 - 10\sin\theta_A\cos\theta_A = 0$$

Using the trigonometry identity $\sin 2\theta_A = 2\sin\theta_A\cos\theta_A$, this equation becomes

$$1.962 - 5\sin 2\theta_A = 0$$

$$\sin 2\theta_A = 0.3924$$

$$2\theta_A = 23.10° \text{ and } 2\theta_A = 156.90°$$

$$\theta_A = 11.55° \text{ and } \theta_A = 78.44°$$

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12–104. Continued

Since the longer time is required, Eq. (1) indicates that larger θ_A must be choosen. Thus,

$$\theta_A = 78.44° = 78.4° \qquad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 78.44°} = 1.9974 \text{ s} = 2.00 \text{ s} \qquad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 78.44° = 2.0026 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 78.44° = 9.7974 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{2.0026^2 + 9.7974^2} = 10 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{9.7974}{2.0026}\right) = 78.44° = 78.4° \qquad \textbf{Ans.}$$

Ans:
$\theta_A = 78.4°$
$t = 2.00$ s
$\theta_B = 78.4°$

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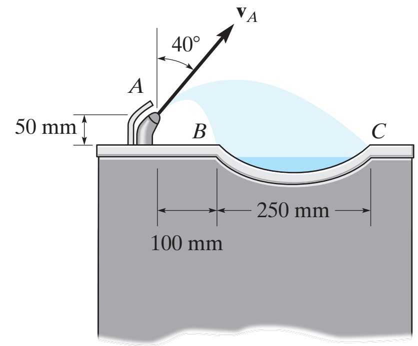
12–105.

The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



SOLUTION

Horizontal Motion:

$$(\xrightarrow{+}) \qquad s = v_0 t$$

$$R = v_A \sin 40^\circ\, t \quad t = \frac{R}{v_A \sin 40^\circ} \tag{1}$$

Vertical Motion:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-\,0.05 = 0 + v_A \cos 40^\circ t + \frac{1}{2}(-9.81)t^2 \tag{2}$$

Substituting Eq.(1) into (2) yields:

$$-0.05 = v_A \cos 40^\circ\left(\frac{R}{v_A \sin 40^\circ}\right) + \frac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40^\circ}\right)^2$$

$$v_A = \sqrt{\frac{4.905 R^2}{\sin 40^\circ\,(R \cos 40^\circ + 0.05 \sin 40^\circ)}}$$

At point B, $\quad R = 0.1$ m.

$$v_{\min} = v_A = \sqrt{\frac{4.905\,(0.1)^2}{\sin 40^\circ\,(0.1 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 0.838 \text{ m/s} \qquad \textbf{Ans.}$$

At point C, $\quad R = 0.35$ m.

$$v_{\max} = v_A = \sqrt{\frac{4.905\,(0.35)^2}{\sin 40^\circ\,(0.35 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 1.76 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{\min} = 0.838$ m/s
$v_{\max} = 1.76$ m/s

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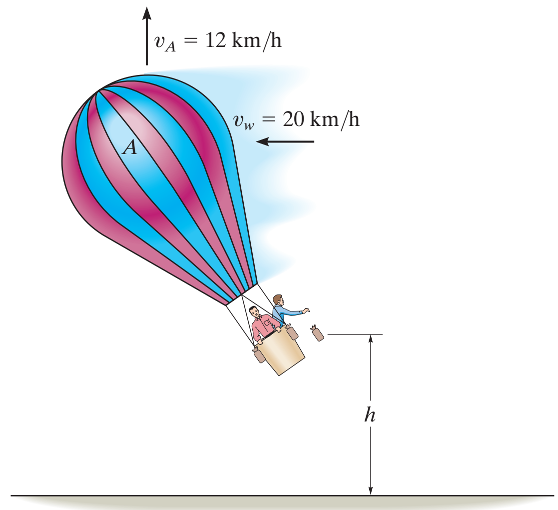
12–106. The balloon A is ascending at the rate $v_A = 12$ km/h and is being carried horizontally by the wind at $v_w = 20$ km/h. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?



SOLUTION

$v_A = 12 \text{ km/h}$

$v_w = 20 \text{ km/h}$

$h = 50 \text{ m}$

$g = 9.81 \text{ m/s}^2$

$a_x = 0 \qquad a_y = -g$

$v_x = v_w \qquad v_y = -gt + v_A$

$s_x = v_w t \qquad s_y = \frac{-1}{2} g t^2 + v_A t + h$

Thus $\quad 0 = \frac{-1}{2} g t^2 + v_A t + h \qquad t = \frac{v_A + \sqrt{v_A^2 + 2gh}}{g} \qquad t = 3.55 \text{ s}$ **Ans.**

$v_x = v_w \qquad v_y = -gt + v_A \qquad v = \sqrt{v_x^2 + v_y^2} \qquad v = 32.0 \text{ m/s}$ **Ans.**

Ans:
$t = 3.55$ s
$v = 32.0$ m/s

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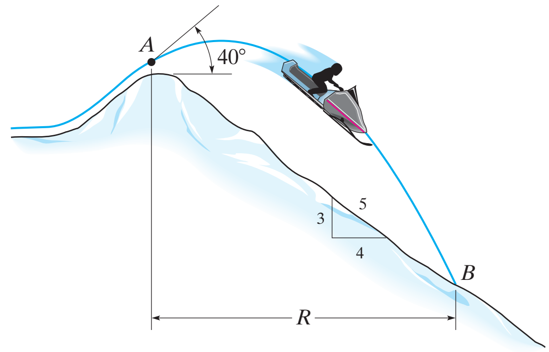
12–107.

The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



SOLUTION

$(\xrightarrow{+})$ $\quad s_B = s_A + v_A t$

$$R = 0 + 10 \cos 40^\circ\, t$$

$(+\uparrow)$ $\quad s_B = s_A + v_A t + \frac{1}{2} a_c t^2$

$$-R\left(\frac{3}{4}\right) = 0 + 10 \sin 40^\circ t - \frac{1}{2}(9.81)\, t^2$$

Solving:

$R = 19.0$ m **Ans.**

$t = 2.48$ s **Ans.**

Ans:
$R = 19.0$ m
$t = 2.48$ s

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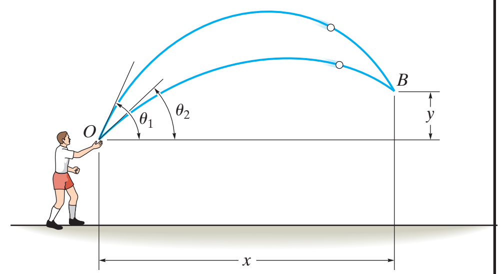
***12–108.**

A boy throws a ball at O in the air with a speed v_0 at an angle θ_1. If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in midair at B.



SOLUTION

Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_1 t_1 + \frac{1}{2}(-g)t_1^2 \qquad \textbf{(1)}$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_2 t_2 + \frac{1}{2}(-g)t_2^2 \qquad \textbf{(2)}$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$\left(\xrightarrow{+}\right) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_1\, t_1 \qquad \textbf{(3)}$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$\left(\xrightarrow{+}\right) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_2\, t_2 \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4), we have

$$t_2 = \frac{\cos\theta_1}{\cos\theta_2} t_1 \qquad \textbf{(5)}$$

Equating Eqs. (1) and (2), we have

$$v_0 t_1 \sin\theta_1 - v_0 t_2 \sin\theta_2 = \frac{1}{2} g\left(t_1^2 - t_2^2\right) \qquad \textbf{(6)}$$

Solving Eq. [5] into [6] yields

$$t_1 = \frac{2v_0 \cos\theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$t_2 = \frac{2v_0 \cos\theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

Thus, the time between the throws is

$$\Delta t = t_1 - t_2 = \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos\theta_2 - \cos\theta_1)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)} \qquad \textbf{Ans.}$$

Ans:

$$\Delta t = \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)}$$

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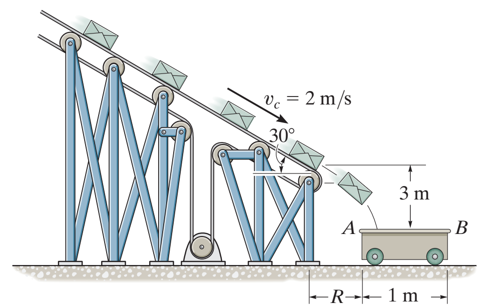
12–109.

Small packages traveling on the conveyor belt fall off into a l-m-long loading car. If the conveyor is running at a constant speed of $v_C = 2$ m/s, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2 \sin 30° = 1.00$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3$ m, respectively.

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$3 = 0 + 1.00(t) + \frac{1}{2}(9.81)\left(t^2\right)$$

Choose the positive root $t = 0.6867$ s

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2 \cos 30° = 1.732$ m/s and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R = 0 + 1.732(0.6867) = 1.19 \text{ m} \qquad \textbf{Ans.}$$

If $s_x = R + 1$, then

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R + 1 = 0 + 1.732(0.6867)$$

$$R = 0.189 \text{ m} \qquad \textbf{Ans.}$$

Thus, $R_{\min} = 0.189$ m, $R_{\max} = 1.19$ m **Ans.**

Ans:

$R_{\min} = 0.189$ m
$R_{\max} = 1.19$ m

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12–110.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

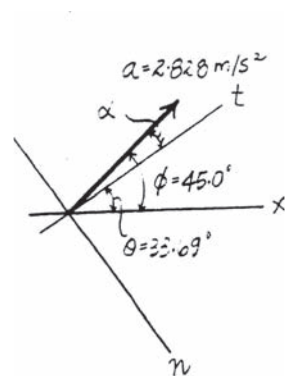
Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity **v**, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\ \text{m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21\ \text{m/s} \qquad \textbf{Ans.}$$

The velocity **v** makes an angle $\theta = \tan^{-1}\frac{4}{6} = 33.69°$ with the x axis.



Acceleration: To determine the acceleration **a**, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\ \text{m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\ \text{m/s}^2$$

The acceleration **a** makes an angle $\phi = \tan^{-1}\frac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a \sin\alpha = 2.828 \sin 11.31° = 0.555\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_t = a \cos\alpha = 2.828 \cos 11.31° = 2.77\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v_n = 0$ and $v_t = 7.21$ m/s

$a_n = 0.555\ \text{m/s}^2$

$a_t = 2.77\ \text{m/s}^2$

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12–111.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$ **Ans.**

Ans:
$v = 38.7$ m/s

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***12–112.**

A particle moves along the curve $y = b\sin(cx)$ with a constant speed v. Determine the normal and tangential components of its velocity and acceleration at any instant.

SOLUTION

$$v = 2\ \frac{\text{m}}{\text{s}} \qquad b = 1\ \text{m} \qquad c = \frac{1}{\text{m}}$$

$$y = b\sin(cx) \qquad y' = bc\cos(cx) \qquad y'' = -bc^2\sin(cx)$$

$$\rho = \frac{\sqrt{\left(1 + y'^2\right)^3}}{y''} = \frac{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}{-bc^2\sin(cx)}$$

$$a_n = \frac{v^2 bc\sin(cx)}{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}} \qquad a_t = 0 \qquad v_t = 0 \qquad v_n = 0 \qquad \textbf{Ans.}$$

Ans:

$$a_n = \frac{v^2 bc\sin(cx)}{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}$$

$a_t = 0$

$v_t = 0$

$v_n = 0$

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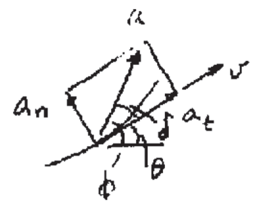
12–113.

The position of a particle is defined by $\mathbf{r} = \{4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 1$ s.

SOLUTION

$$\mathbf{r} = 4(t - \sin t)\,\mathbf{i} + (2\,t^2 - 3)\mathbf{j}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 4(1 - \cos t)\mathbf{i} + (4\,t)\mathbf{j}$$



$$\mathbf{v}|_{t=1} = 1.83879\mathbf{i} + 4\mathbf{j}$$

$$v = \sqrt{(1.83879)^2 + (4)^2} = 4.40 \text{ m/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{4}{1.83879}\right) = 65.312^\circ \measuredangle\theta$$

$$\mathbf{a} = 4 \sin \mathbf{r}\mathbf{i} + 4\mathbf{j}$$

$$\mathbf{a}|_{t=1} = 3.3659\mathbf{i} + 4\mathbf{j}$$

$$a = \sqrt{(3.3659)^2 + (4)^2} = 5.22773 \text{ m/s}^2$$

$$\phi = \tan^{-1}\left(\frac{4}{3.3659}\right) = 49.920^\circ \measuredangle\theta$$

$$\delta = \theta - \phi = 15.392^\circ$$

$$a_2 = 5.22773 \cos 15.392^\circ = 5.04 \text{ m/s}^2$$ **Ans.**

$$a_n = 5.22773 \sin 15.392^\circ = 1.39 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 4.40$ m/s
$a_t = 5.04$ m/s^2
$a_n = 1.39$ m/s^2

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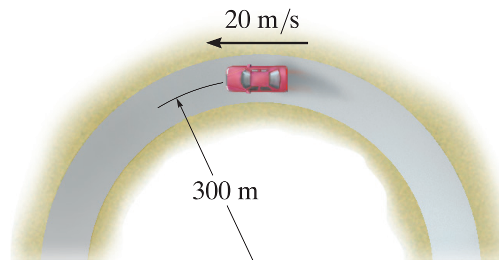
12–114. The car travels along the curve having a radius of 300 m. If its speed is uniformly increased from 15 m/s to 27 m/s in 3 s, determine the magnitude of its acceleration at the instant its speed is 20 m/s.



SOLUTION

$v_1 = 15 \text{ m/s} \qquad t = 3 \text{ s}$

$R = 300 \text{ m}$

$v_2 = 27 \text{ m/s}$

$v_3 = 20 \text{ m/s}$

$$a_t = \frac{v_2 - v_1}{t} \qquad a_n = \frac{v_3^{\,2}}{R} \qquad a = \sqrt{a_t^{\,2} + a_n^{\,2}} \qquad a = 4.22 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:
$a = 4.22 \text{ m/s}^2$

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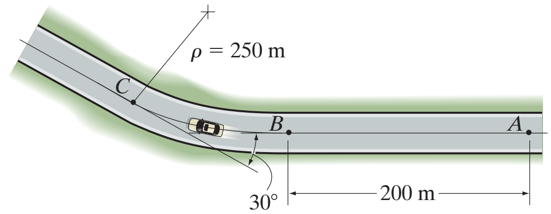
12–115. When the car reaches point A it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (-\frac{1}{4}t^{1/2})$ m/s^2. Determine the magnitude of acceleration of the car just before it reaches point C.



SOLUTION

Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a_t\,dt$$

$$\int_{25\text{ m/s}}^{v} dv = \int_0^t -\frac{1}{4}t^{1/2}\,dt$$

$$v = \left(25 - \frac{1}{6}t^{3/2}\right)\text{ m/s} \qquad \textbf{(1)}$$

Position: Using the initial conditions $s = 0$ when $t = 0$ s,

$$\int ds = \int v\,dt$$

$$\int_0^s ds = \int_0^t \left(25 - \frac{1}{6}t^{3/2}\right)dt$$

$$s = \left(25t - \frac{1}{15}t^{5/2}\right)\text{ m}$$

Acceleration: When the car reaches C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus,

$$330.90 = 25t - \frac{1}{15}t^{5/2}$$

Solving by trial and error,

$$t = 15.942\text{ s}$$

Thus, using Eq. (1).

$$v_C = 25 - \frac{1}{6}(15.942)^{3/2} = 14.391\text{ m/s}$$

$$(a_t)_C = \dot{v} = -\frac{1}{4}\left(15.942^{1/2}\right) = -0.9982\text{ m/s}^2$$

$$(a_n)_C = \frac{v_C^{\,2}}{\rho} = \frac{14.391^2}{250} = 0.8284\text{ m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C^{\,2} + (a_n)_C^{\,2}} = \sqrt{(-0.9982)^2 + 0.8284^2} = 1.30\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.30$ m/s^2

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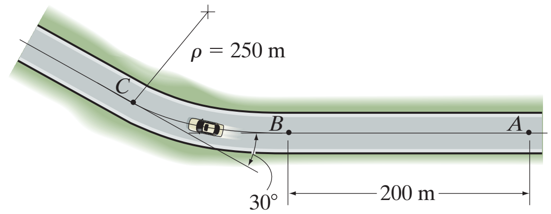
***12–116.** When the car reaches point A, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1)\ \text{m/s}^2$. Determine the magnitude of acceleration of the car just before it reaches point C.



SOLUTION

Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a\,ds$$

$$\int_{25\text{ m/s}}^{v} v\,dv = \int_0^s (0.001s - 1)\,ds$$

$$v = \sqrt{0.001s^2 - 2s + 625}$$

Acceleration: When the car is at point C, $s_C = 200 + 250\left(\dfrac{\pi}{6}\right) = 330.90$ m. Thus, the speed of the car at C is

$$v_C = \sqrt{0.001\left(330.90^2\right) - 2(330.90) + 625} = 8.526\ \text{m/s}^2$$

$$(a_t)_C = \dot{v} = [0.001(330.90) - 1] = -0.6691\ \text{m/s}^2$$

$$(a_n)_C = \frac{{v_C}^2}{\rho} = \frac{8.526^2}{250} = 0.2908\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{{(a_t)_C}^2 + {(a_n)_C}^2} = \sqrt{(-0.6691)^2 + 0.2908^2} = 0.730\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.730\ \text{m/s}^2$

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12–117. At a given instant, a car travels along a circular curved road with a speed of 20 m/s while decreasing its speed at the rate of 3 m/s^2. If the magnitude of the car's acceleration is 5 m/s^2, determine the radius of curvature of the road.

SOLUTION

Acceleration: Here, the car's tangential component of acceleration of $a_t = -3\ m/s^2$. Thus,

$$a = \sqrt{{a_t}^2 + {a_n}^2}$$

$$5 = \sqrt{(-3)^2 + {a_n}^2}$$

$$a_n = 4\ m/s^2$$

$$a_n = \frac{v^2}{\rho}$$

$$4 = \frac{20^2}{\rho}$$

$$\rho = 100\ m$$ **Ans.**

Ans:
$\rho = 100$ m

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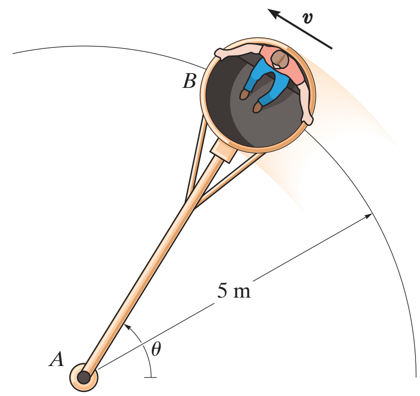
12–118.

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right) \qquad \textbf{(1)}$$

When $\theta = 30°$, the car has traveled a distance of $s = r\theta = 5\left(\dfrac{30°}{180°}\pi\right) = 2.618\ \text{m}$. The time required for the car to travel this distance can be obtained by applying $v = \dfrac{ds}{dt}$.

$$ds = v\,dt$$

$$\int_0^{2.618\ \text{m}} ds = \int_0^t 0.5\left(e^t - 1\right) dt$$

$$2.618 = 0.5\left(e^t - t - 1\right)$$

Solving by trial and error $\quad t = 2.1234\ \text{s}$

Substituting $t = 2.1234\ \text{s}$ into Eq. (1) yields

$$v = 0.5\left(e^{2.1234} - 1\right) = 3.680\ \text{m/s} = 3.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for the car at $t = 2.1234\ \text{s}$ is $a_t = 0.5e^{2.1234} = 4.180\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 3.68\ \text{m/s}$
$a = 4.98\ \text{m/s}^2$

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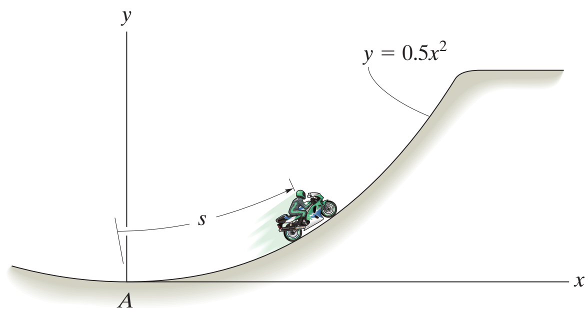
12–119.

The motorcycle is traveling at 1 m/s when it is at A. If the speed is then increased at $\dot{v} = 0.1\text{ m/s}^2$, determine its speed and acceleration at the instant $t = 5$ s.



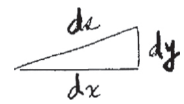
SOLUTION

$a_t = v = 0.1\,\text{m/s}^2$

$$s = s_0 + v_0\,t + \frac{1}{2}a_c t^2$$

$$s = 0 + 1(5) + \frac{1}{2}(0.1)(5)^2 = 6.25\text{ m}$$

$$\int_0^{6.25} ds = \int_0^{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$



$y = 0.5x^2$

$$\frac{dy}{dx} = x$$

$$\frac{d^2y}{dx^2} = 1$$

$$6.25 = \int_0^x \sqrt{1 + x^2}\,dx$$

$$6.25 = \frac{1}{2}\left[x\sqrt{1 + x^2} + 1\text{n}\left(x + \sqrt{1 + x^2}\right)\right]_0^x$$

$$x\sqrt{1 + x^2} + 1\text{n}\left(x + \sqrt{1 + x^2}\right) = 12.5$$

Solving,

$x = 3.184$ m

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{[1 + x^2]^{\frac{3}{2}}}{|1|}\right|_{x=3.184} = 37.17\text{ m}$$

$v = v_0 + a_c t$

$= 1 + 0.1(5) = 1.5\text{ m/s}$ **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{(1.5)^2}{37.17} = 0.0605\text{ m/s}^2$$

$a = \sqrt{(0.1)^2 + (0.0605)^2} = 0.117\text{ m/s}^2$ **Ans.**

Ans:
$v = 1.5\text{ m/s}$
$a = 0.117\text{ m/s}^2$

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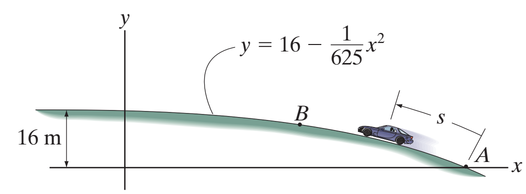
***12–120.**

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m and $x = 50$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B{}^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B $(s = 51.5 \text{ m})$

$$a_t = \left[0.225(51.5) - 3.75\right] = -2.591 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2$$ **Ans.**

Ans:
$a = 2.75 \text{ m/s}^2$

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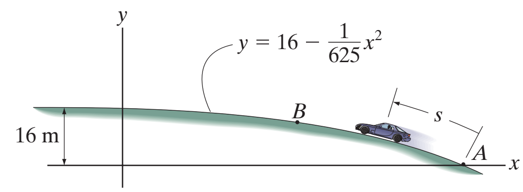
12–121.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 101.68$ m and $x = 0$.



SOLUTION

Velocity: The speed of the car at C is

$$v_C^{\,2} = v_A^{\,2} + 2a_t\,(s_C - s_A)$$

$$v_C^{\,2} = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = 0.5 \text{ m/s}$$

$$a_n = \frac{v_C^{\,2}}{\rho} = \frac{22.361^2}{312.5} = 1.60 \text{ m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.68 \text{ m/s}^2$

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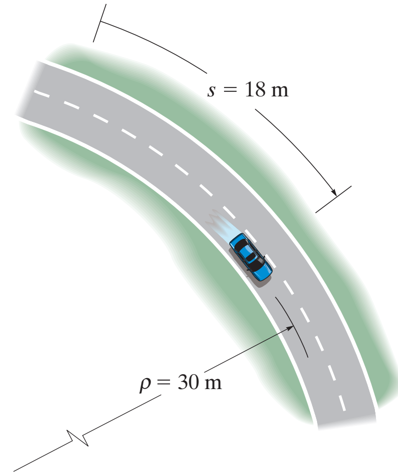
12–122.

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \text{s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85\ \text{m/s} = 19.9\ \text{m/s} \qquad \textbf{Ans.}$$

$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\ \text{s}} = 20.35\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 19.9$ m/s
$a = 24.2$ m/s^2

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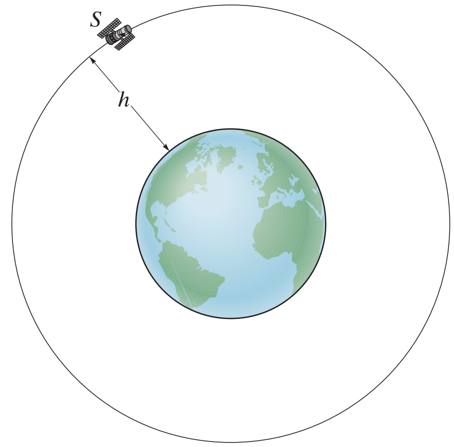
12–123.

The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2, determine the altitude h. Assume the earth's diameter to be 12 713 km.



SOLUTION

$$v = 20 \text{ Mm/h} = \frac{20(10^6)}{3600} = 5.56(10^3) \text{ m/s}$$

Since $a_t = \dfrac{dv}{dt} = 0$, then,

$$a = a_n = 2.5 = \frac{v^2}{\rho}$$

$$\rho = \frac{(5.56(10^3))^2}{2.5} = 12.35(10^6) \text{ m}$$

The radius of the earth is

$$\frac{12\,713(10^3)}{2} = 6.36(10^6) \text{ m}$$

Hence,

$$h = 12.35(10^6) - 6.36(10^6) = 5.99(10^6) \text{ m} = 5.99 \text{ Mm}$$ **Ans.**

Ans:
$h = 5.99$ Mm

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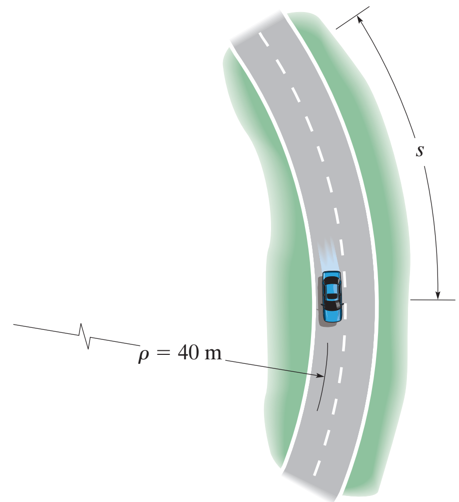
***12–124.**

The car has an initial speed $v_0 = 20$ m/s. If it increases its speed along the circular track at $s = 0$, $a_t = (0.8s)$ m/s^2, where s is in meters, determine the time needed for the car to travel $s = 25$ m.



SOLUTION

The distance traveled by the car along the circular track can be determined by integrating $v\,dv = a_t\,ds$. Using the initial condition $v = 20$ m/s at $s = 0$,

$$\int_{20\text{ m/s}}^{v} v\,dv = \int_0^{s} 0.8\,s\,ds$$

$$\left.\frac{v^2}{2}\right|_{20\text{ m/s}}^{v} = 0.4\,s^2$$

$$v = \left\{\sqrt{0.8\left(s^2 + 500\right)}\right\}\text{ m/s}$$

The time can be determined by integrating $dt = \dfrac{ds}{v}$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^t dt = \int_0^{25\text{ m}} \frac{ds}{\sqrt{0.8\left(s^2 + 500\right)}}$$

$$t = \frac{1}{\sqrt{0.8}}\left[\ln\left(s + \sqrt{s^2 + 500}\right)\right]\Big|_0^{25\text{ m}}$$

$$= 1.076\text{ s} = 1.08\text{ s}$$ **Ans.**

Ans:
$t = 1.08$ s

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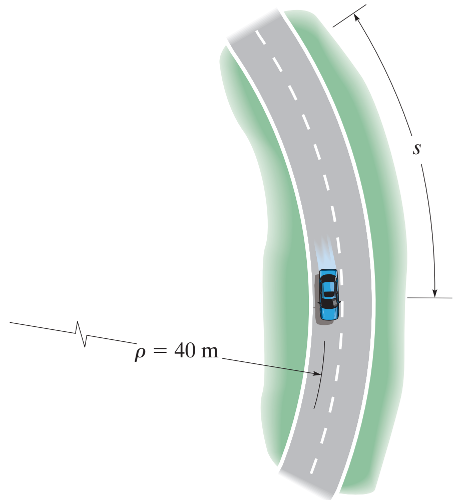
12–125.

The car starts from rest at $s = 0$ and increases its speed at $a_t = 4\ \text{m/s}^2$. Determine the time when the magnitude of acceleration becomes $20\ \text{m/s}^2$. At what position s does this occur?



SOLUTION

Acceleration. The normal component of the acceleration can be determined from

$$\theta_r = \frac{v^2}{\rho}; \qquad a_r = \frac{v^2}{40}$$

From the magnitude of the acceleration

$$a = \sqrt{a_t^2 + a_n^2}; \quad 20 = \sqrt{4^2 + \left(\frac{v^2}{40}\right)^2} \quad v = 28.00\ \text{m/s}$$

Velocity. Since the car has a constant tangential accelaration of $a_t = 4\ \text{m/s}^2$,

$$v = v_0 + a_t t; \quad 28.00 = 0 + 4t$$

$t = 6.999\ \text{s} = 7.00\ \text{s}$ **Ans.**

$$v^2 = v_0^2 + 2a_t s; \;\; 28.00^2 = 0^2 + 2(4)\ s$$

$s = 97.98\ \text{m} = 98.0\ \text{m}$ **Ans.**

Ans:
$t = 7.00\ \text{s}$
$s = 98.0\ \text{m}$

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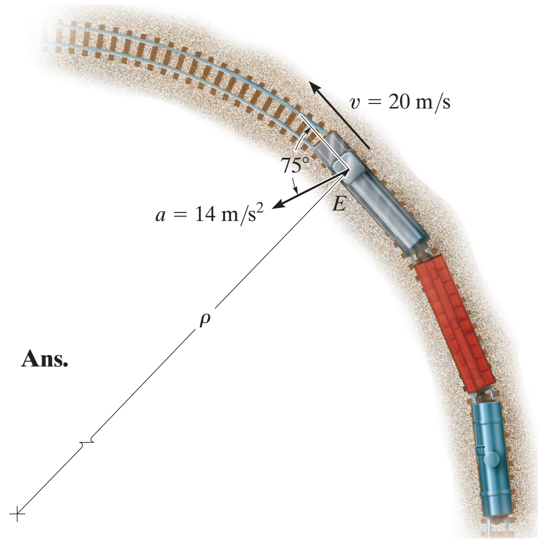
12–126.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$ **Ans.**

$a_n = 14 \sin 75°$

$$a_n = \frac{(20)^2}{\rho}$$

$\rho = 29.6 \text{ m}$ **Ans.**

Ans:
$a_t = 3.62 \text{ m/s}^2$
$\rho = 29.6 \text{ m}$

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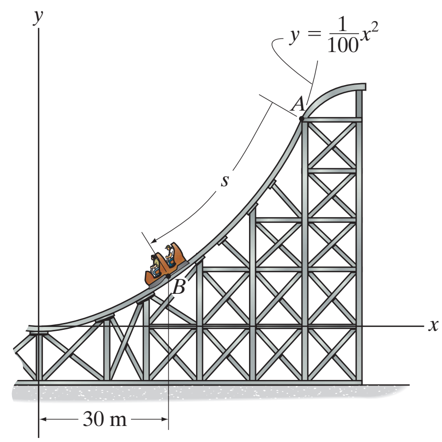
12–127.

When the roller coaster is at B, it has a speed of 25 m/s, which is increasing at $a_t = 3\ \text{m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



SOLUTION

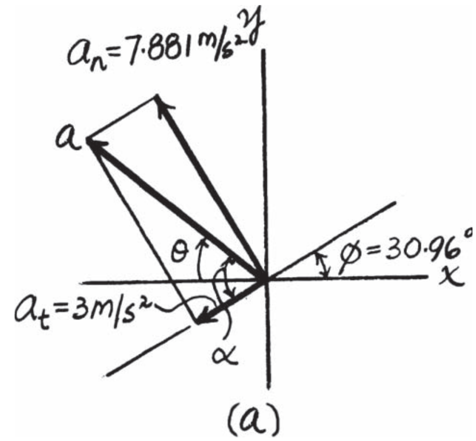
Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\text{ m}$$



Acceleration:

$$a_t = \dot{v} = 3\ \text{m/s}^2$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{25^2}{79.30} = 7.881\ \text{m/s}^2$$

The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{3^2 + 7.881^2} = 8.43\ \text{m/s}^2 \qquad \textbf{Ans.}$$

The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

Ans:

$\theta = 38.2°$

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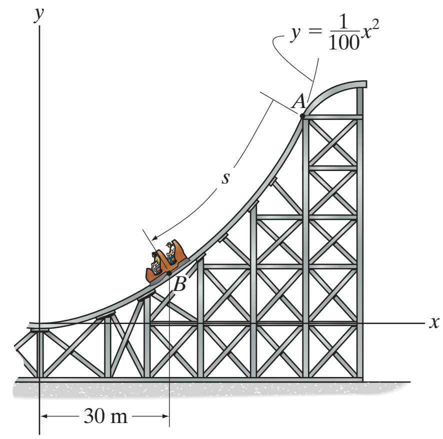
***12–128.**

If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s)\ \mathrm{m/s^2}$, determine the magnitude of its acceleration when it reaches B where $s_B = 40$ m.



SOLUTION

Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^s \left(6 - 0.06s\right)ds$$

$$v = \left(\sqrt{12s - 0.06s^2}\right)\ \mathrm{m/s} \qquad \textbf{(1)}$$

Thus,

$$v_B = \sqrt{12\left(40\right) - 0.06\left(40\right)^2} = 19.60\ \mathrm{m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\dfrac{1}{50}x\right)^2\right]^{3/2}}{\left|\dfrac{1}{50}\right|}\right|_{x=30\ \mathrm{m}} = 79.30\ \mathrm{m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.600\ \mathrm{m/s^2}$$

$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842\ \mathrm{m/s^2}$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{3.600^2 + 4.842^2} = 6.03\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

Ans:
$a = 6.03\ \mathrm{m/s^2}$

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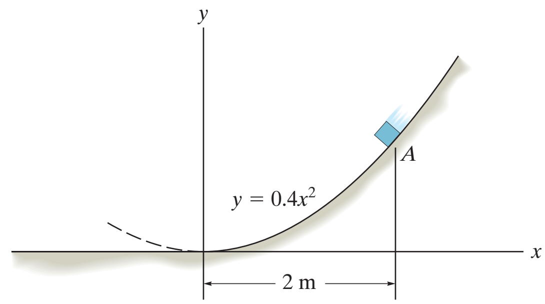
12–129.

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v = 8$ m/s and the increase in speed is $dv/dt = 4$ m/s^2. Determine the magnitude of the acceleration of the box at this instant.



SOLUTION

$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = \left.0.8x\right|_{x=2\text{ m}} = 1.6$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{|0.8|} = 8.396\text{ m}$$

$$a_n = \frac{{v_B}^2}{\rho} = \frac{8^2}{8.396} = 7.622\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\text{ m/s}^2$$ **Ans.**

Ans:
$a = 8.61$ m/s^2

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12–130.

The position of a particle traveling along a curved path is $s = (3t^3 - 4t^2 + 4)$ m, where t is in seconds. When $t = 2$ s, the particle is at a position on the path where the radius of curvature is 25 m. Determine the magnitude of the particle's acceleration at this instant.

SOLUTION

Velocity:

$$v = \frac{d}{dt}\left(3t^3 - 4t^2 + 4\right) = \left(9t^2 - 8t\right)\text{m/s}$$

When $t = 2$ s,

$$v|_{t=2\text{ s}} = 9\left(2^2\right) - 8(2) = 20\text{ m/s}$$

Acceleration:

$$a_t = \frac{dv}{ds} = \frac{d}{dt}\left(9t^2 - 8t\right) = \left(18t - 8\right)\text{m/s}^2$$

$$a_t|_{t=2\text{ s}} = 18(2) - 8 = 28\text{ m/s}^2$$

$$a_n = \frac{\left(v|_{t=2\text{s}}\right)^2}{\rho} = \frac{20^2}{25} = 16\text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t{}^2 + a_n^2} = \sqrt{28^2 + 16^2} = 32.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 32.2\text{ m/s}^2$

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12–131.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)\ \text{m/s}^2$, determine the magnitude of the particle's acceleraton four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\text{ m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20\ln\frac{v}{10}$$

$$v = (10e^{t/20})\ \text{m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\ \text{m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\ \text{m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0.6107^2 + 2.984^2} = 3.05\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 3.05\ \text{m/s}^2$

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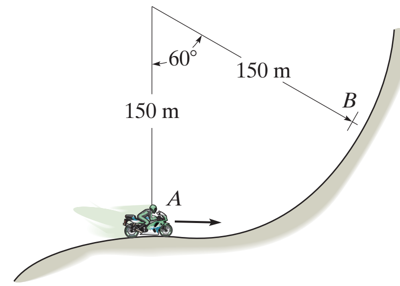
***12–132.**

The motorcycle is traveling at 40 m/s when it is at A. If the speed is then decreased at $\dot{v} = -(0.05\,s)$ m/s^2, where s is in meters measured from A, determine its speed and acceleration when it reaches B.



SOLUTION

Velocity. The velocity of the motorcycle along the circular track can be determined by integrating $vdv = a_t ds$ with the initial condition $v = 40$ m/s at $s = 0$. Here, $a_t = -0.05s$.

$$\int_{40\text{ m/s}}^{v} vdv = \int_0^s -0.05\,sds$$

$$\left.\frac{v^2}{2}\right|_{40\text{ m/s}}^{v} = -0.025\,s^2\Big|_0^s$$

$$v = \left\{\sqrt{1600 - 0.05\,s^2}\right\}\text{ m/s}$$

At B, $s = r\theta = 150\left(\frac{\pi}{3}\right) = 50\pi$ m. Thus

$$v_B = v|_{s=50\pi\text{m}} = \sqrt{1600 - 0.05(50\pi)^2} = 19.14\text{ m/s} = 19.1\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration. At B, the tangential and normal components are

$$a_t = 0.05(50\pi) = 2.5\pi\text{ m/s}^2$$

$$a_n = \frac{v_B^2}{\rho} = \frac{19.14^2}{150} = 2.4420\text{ m/s}^2$$

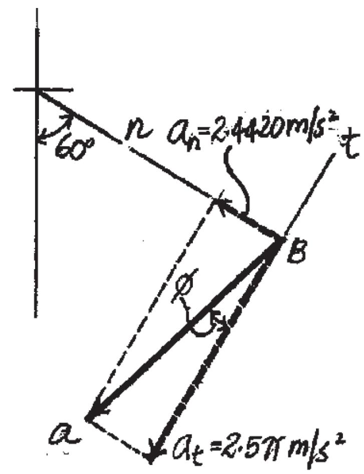
Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2.5\pi)^2 + 2.4420^2} = 8.2249\text{ m/s}^2 = 8.22\text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by angle ϕ measured from the negative t-axis, Fig. a.

$$\phi = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{2.4420}{2.5\pi}\right)$$

$$= 17.27° = 17.3° \qquad \textbf{Ans.}$$



(a)

Ans:
$v_B = 19.1$ m/s
$a = 8.22$ m/s^2
$\phi = 17.3°$
up from negative−t axis

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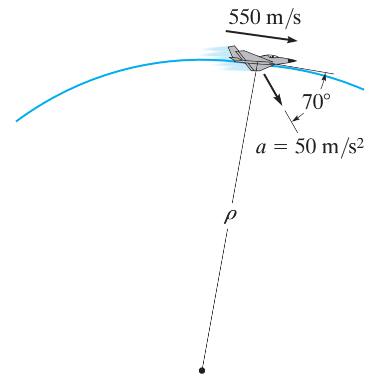
12–133.

At a given instant the jet plane has a speed of 550 m/s and an acceleration of 50 m/s^2 acting in the direction shown. Determine the rate of increase in the plane's speed, and also the radius of curvature ρ of the path.



SOLUTION

Acceleration. With respect to the n–t coordinate established as shown in Fig. a, the tangential and normal components of the acceleration are

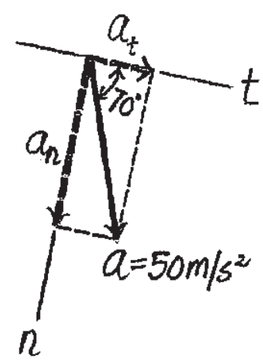
$$a_t = 50 \cos 70° = 17.10 \text{ m/s}^2 = 17.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = 50 \sin 70° = 46.98 \text{ m/s}^2$$

However,

$$a_n = \frac{v^2}{\rho}; \qquad 46.98 = \frac{550^2}{\rho}$$

$$\rho = 6438.28 \text{ m} = 6.44 \text{ km} \qquad \textbf{Ans.}$$



(a)

Ans:
$a_t = 17.1 \text{ m/s}^2$
$a_n = 46.98 \text{ m/s}^2$
$\rho = 6.44 \text{ km}$

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12–134.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s^2.

SOLUTION

$a_t = 2 \text{ m/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36 \text{ m/s}^2$$ **Ans.**

Ans:
$a = 2.36 \text{ m/s}^2$

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12–135.

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s} \qquad \textbf{Ans.}$$

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\bigg|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 1.96$ m/s
$a = 0.930$ m/s^2

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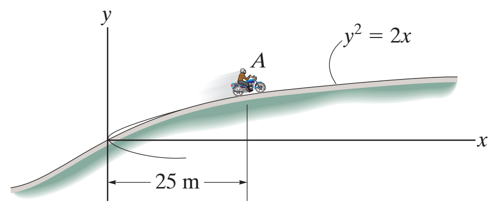
***12–136.**

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21 \text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.763\text{ m/s}^2$

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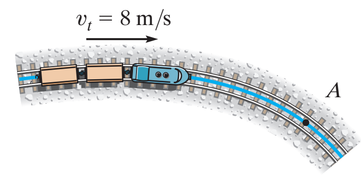
12–137.

When $t = 0$, the train has a speed of 8 m/s, which is increasing at 0.5 m/s^2. Determine the magnitude of the acceleration of the engine when it reaches point A, at $t = 20$ s. Here the radius of curvature of the tracks is $\rho_A = 400$ m.



SOLUTION

Velocity. The velocity of the train along the track can be determined by integrating $dv = a_t\,dt$ with initial condition $v = 8$ m/s at $t = 0$.

$$\int_{8\text{ m/s}}^{v} dv = \int_0^t 0.5\,dt$$

$$v - 8 = 0.5\,t$$

$$v = \{0.5\,t + 8\}\text{ m/s}$$

At $t = 20$ s,

$$v\big|_{t=20\,s} = 0.5(20) + 8 = 18\text{ m/s}$$

Acceleration. Here, the tangential component is $a_t = 0.5\text{ m/s}^2$. The normal component can be determined from

$$a_n = \frac{v^2}{\rho} = \frac{18^2}{400} = 0.81\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{0.5^2 + 0.81^2}$$

$$= 0.9519\text{ m/s}^2 = 0.952\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.952\text{ m/s}^2$

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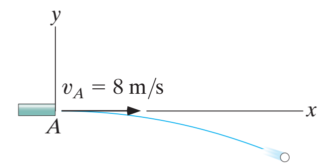
12–138.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



SOLUTION

$v_x = 8$ m/s

$(\xrightarrow{+})$ $\quad s = v_0 t$

$x = 8t$

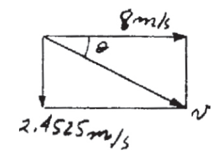
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

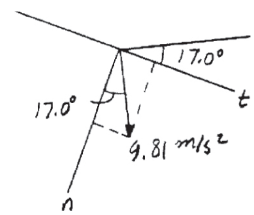
$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$

$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$

$y = -0.0766x^2$ (Parabola) **Ans.**





$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

$a_x = 0 \quad a_y = 9.81$ m/s^2

$a_n = 9.81 \cos 17.04° = 9.38$ m/s^2 **Ans.**

$a_t = 9.81 \sin 17.04° = 2.88$ m/s^2 **Ans.**

Ans:
$y = -0.0766x^2$
$v = 8.37$ m/s
$a_n = 9.38$ m/s^2
$a_t = 2.88$ m/s^2

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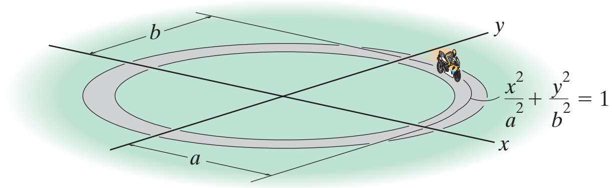
12–139.

The motorcycle travels along the elliptical track at a constant speed v. Determine its greatest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{a}} = \frac{v^2a}{b^2} \qquad \textbf{Ans.}$$

Ans:

$$a_{\max} = \frac{v^2a}{b^2}$$

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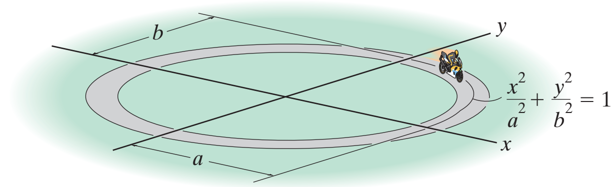
***12–140.**

The motorcycle travels along the elliptical track at a constant speed v. Determine its smallest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = 0$, $y = b$,

$$|\rho| = \frac{a^2}{b}$$

Thus

$$a_t = 0$$

$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^3}{b}} = \frac{v^2b}{a^2}$$ **Ans.**

Ans:

$$a_{\min} = \frac{v^2b}{a^2}$$

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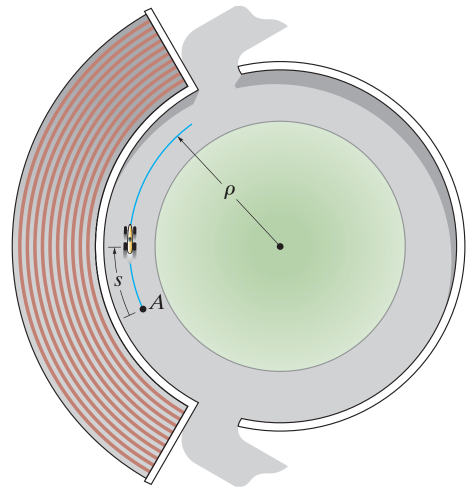
12–141.

The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^{v} v\,dv$$

$$\left.\frac{0.4s^2}{2}\right|_0^s = \left.\frac{v^2}{2}\right|_{15}^{v}$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$\left.1\text{n}\,(s + \sqrt{s^2 + 562.5})\right|_0^s = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5}) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$t = 1.21$ s **Ans.**

Ans:
$t = 1.21$ s

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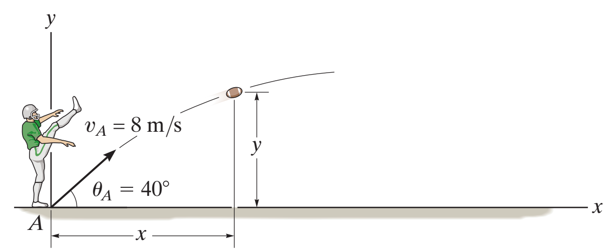
12–142.

The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.



SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8\cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

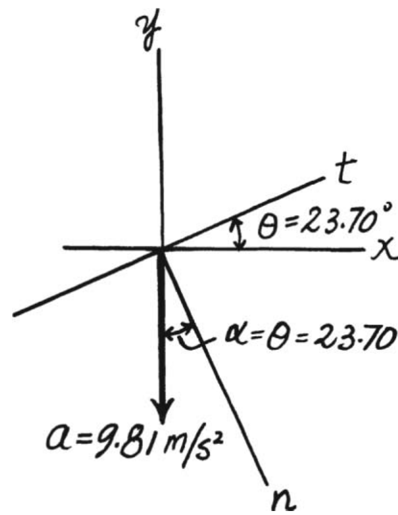
$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad \textbf{(1)}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8\sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad \textbf{(2)}$$



Eliminate t from Eqs (1) and (2),we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans.}$$

Acceleration: When $t = 0.25$ s, from Eq. (1), $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = -a\sin\alpha = -9.81\sin 23.70° = -3.94\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a\cos\alpha = 9.81\cos 23.70° = 8.98\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$y = \left\{0.839x - 0.131x^2\right\}$ m
$a_t = -3.94$ m/s^2
$a_n = 8.98$ m/s^2

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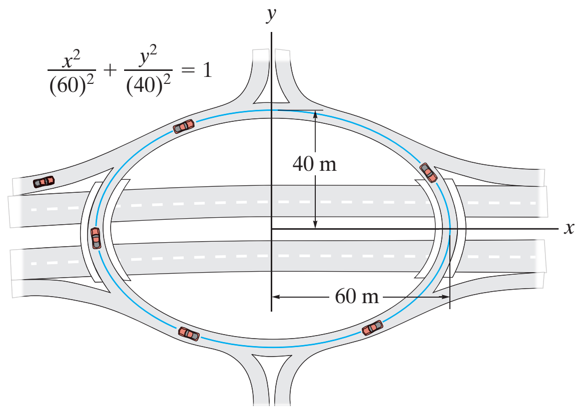
12–143.

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the minimum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{b^4}{a^2y^2}\right)\left(\frac{x^2}{a^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2} + \frac{b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^2}{a^2y^3}\right|}$$

At $x = 0$, $y = h$,

$$\rho = \frac{a^2}{b}$$

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12–143. Continued

Thus

$a_t = 0$

$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^2}{b}} = \frac{v^2 b}{a^2}$$

Set $a = 60$ m, $b = 40$ m,

$$v = \frac{60(10)^3}{3600} = 16.67 \text{ m/s}$$

$$a_{\min} = \frac{(16.67)^2(40)}{(60)^2} = 3.09 \text{ m/s}^2$$ **Ans.**

Ans:
$a_{\min} = 3.09 \text{ m/s}^2$

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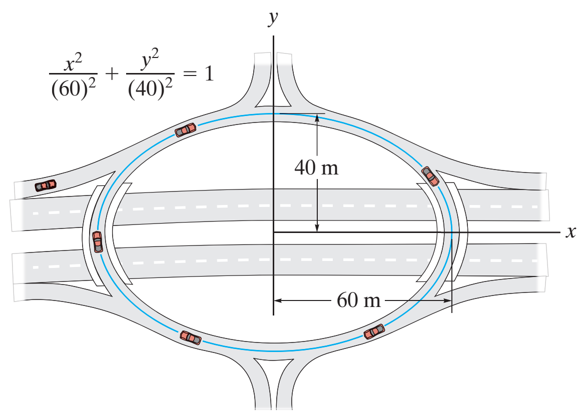
***12–144.**

Cars move around the “traffic circle” which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{n}} = \frac{v^2a}{b^3}$$

Set $a = 60$ m, $b = 40$ m, $v = \frac{60(10^3)}{3600} = 16.67$ m/s

$$a_{\max} = \frac{(16.67)^2(60)}{(40)^2} = 10.4 \text{ m/s}^2$$ **Ans.**

Ans:
$a_{\max} = 10.4 \text{ m/s}^2$

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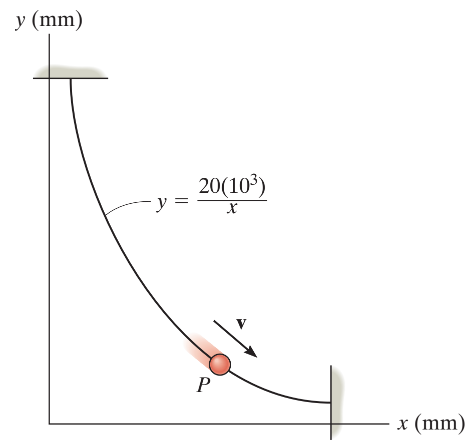
12–145.

The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.



SOLUTION

$v = 300 \text{ mm/s}$

$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{20(10^3)}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=200} = -\frac{20(10^3)}{x^2} = -0.5$$

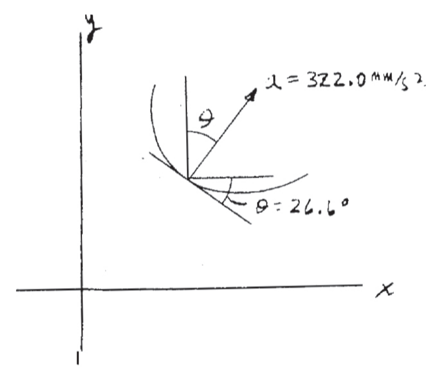
$$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{40(10^3)}{x^3} = 5(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-0.5)^2]^{\frac{3}{2}}}{\left|5(10^{-3})\right|} = 279.5 \text{ mm}$$

$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{(0)^2 + (322)^2} = 322 \text{ mm/s}^2 \qquad \textbf{Ans.}$$



Since $\frac{dy}{dx} = -0.5$,

$$\theta = \tan^{-1}(-0.5) = 26.6° \qquad \textbf{Ans.}$$

Ans:

$a = 322 \text{ mm/s}^2$
$\theta = 26.6°$

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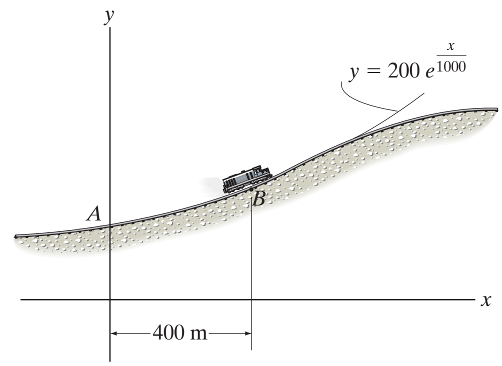
12–146.

The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5\ \text{m/s}^2$. Determine the magnitude of acceleration of the train at this point.



SOLUTION

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}}$$

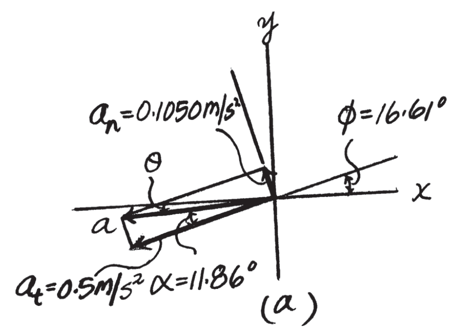
$$\frac{d^2y}{dx^2} = 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}.$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1+\left(0.2\,e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96\text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.5\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050\ \text{m/s}^2$$



The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1050^2} = 0.511\ \text{m/s}^2$$ **Ans.**

Ans:

$a = 0.511\ \text{m/s}^2$

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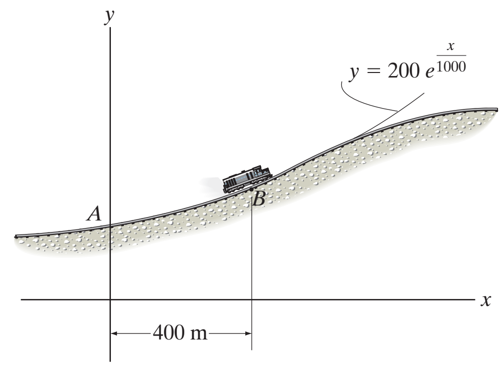
12–147.

The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25\ \text{m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B, where $s_{AB} = 412$ m.



SOLUTION

Velocity: The speed of the train at B can be determined from

$$v_B{}^2 = v_A{}^2 + 2a_t\,(s_B - s_A)$$

$$v_B{}^2 = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34\ \text{m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = -0.25\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822\ \text{m/s}^2$$

The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-0.5\right)^2 + 0.1822^2} = 0.309\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.309\ \text{m/s}^2$

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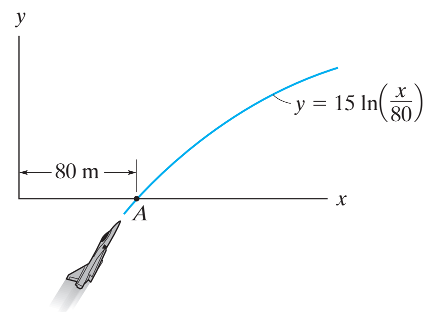
***12–148.**

The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s^2 when it reaches point A. Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.



SOLUTION

$$y = 15\ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x=80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x=80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x=80\text{ m}} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=80\text{ m}}$$

$$= \frac{[1+(0.1875)^2]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(120)^2}{449.4} = 32.04\text{ m/s}^2$$

$$a_n = -40\text{ m/s}^2$$

$$a = \sqrt{(-40)^2 + (32.04)^2} = 51.3\text{ m/s}^2$$ **Ans.**

Since

$$\frac{dy}{dx} = \tan\theta = 0.1875$$

$$\theta = 10.6^\circ$$ **Ans.**

Ans:
$a = 51.3\text{ m/s}^2$
$\theta = 10.6^\circ$

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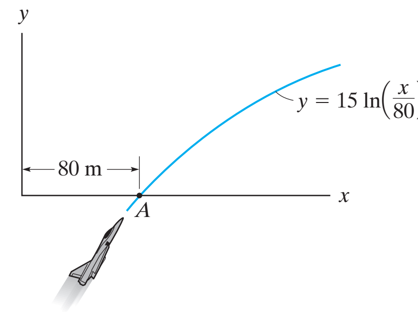
12–149.

The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point $A(y = 0)$.



SOLUTION

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x = 80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x = 80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x = 80\text{ m}}$$

$$= \frac{\left[1 + (0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9\text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 26.9\text{ m/s}^2$$ **Ans.**

Ans:

$a = 26.9\text{ m/s}^2$

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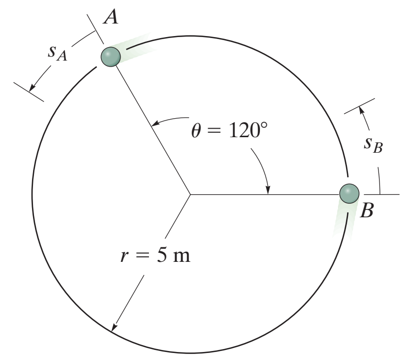
12–150.

Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A)$ m/s^2, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



SOLUTION

Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5\left(\frac{120°}{180°}\right)\pi = 10.47 \text{ m}$$

When $t = 1$ s, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$vdv = ads$$

$$\int_{8\text{ m/s}}^{v} vdv = \int_0^s 0.4\, sds$$

$$v = 0.6325\sqrt{s^2 + 160} \qquad \textbf{(1)}$$

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.6325\sqrt{s^2 + 160}}$$

$$1 = \frac{1}{0.6325}\left(\text{In}\left[\frac{\sqrt{s^2 + 160} + s}{\sqrt{160}}\right]\right)$$

$$s = 8.544 \text{ m}$$

Thus the distance between the two cyclists after $t = 1$ s is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \qquad \textbf{Ans.}$$

Acceleration:

For A, when $t = 1$ s,

$$(a_t)_A = \dot{v}_A = 0.4(8.544) = 3.4176 \text{ m/s}^2$$

$$v_A = 0.6325\sqrt{8.544^2 + 160} = 9.655 \text{ m/s}$$

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2$$

The magnitude of the A's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

For B, when $t = 1$ s,

$$(a_t)_B = \dot{v}_A = 0$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2$$

The magnitude of the B's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 11.0$ m
$a_A = 19.0$ m/s^2
$a_B = 12.8$ m/s^2

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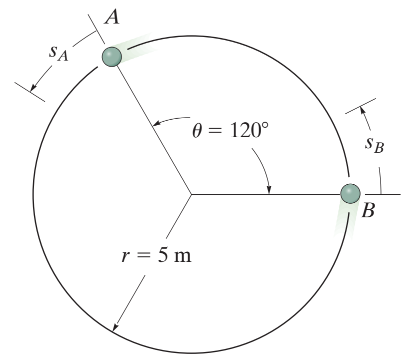
12–151.

Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t\ \text{m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



SOLUTION

Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)t^2 = \left(8t + 2t^2\right)\text{ m} \qquad \textbf{[1]}$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\text{ m/s}}^{v_A} dv_A = \int_0^t 0.8\,t\,dt$$

$$v_A = \left(0.4t^2 + 8\right)\text{ m/s} \qquad \textbf{[2]}$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right)dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\text{ s} = 2.51\text{ s}$ **Ans.**

Note: If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\text{ s} > 2.51\text{ s}$.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8\,(2.5074) = 2.006\ \text{m/s}^2$ and $(a_t)_B = 4\ \text{m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\ \text{m/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\ \text{m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$t = 2.51$ s
$a_A = 22.2\ \text{m/s}^2$
$a_B = 65.1\ \text{m/s}^2$

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***12–152.**

A particle P moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

SOLUTION

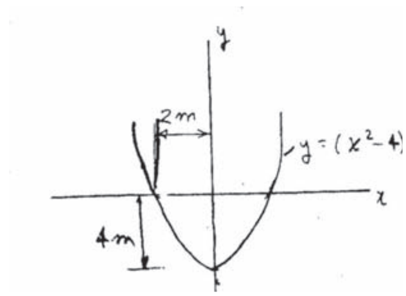
$y = (x^2 - 4)$

$$a_t = \frac{dv}{dt} = 0,$$

To obtain maximum $a = a_n$, ρ must be a minimum.

This occurs at:

$x = 0, \quad y = -4$ m **Ans.**



Hence,

$$\left.\frac{dy}{dx}\right|_{x=0} = 2x = 0; \quad \frac{d^2y}{dx^2} = 2$$

$$\rho_{min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + 0]^{\frac{3}{2}}}{|2|} = \frac{1}{2}$$

$$(a)_{max} = (a_n)_{max} = \frac{v^2}{\rho_{min}} = \frac{5^2}{\frac{1}{2}} = 50 \text{ m/s}^2$$ **Ans.**

Ans:
$x = 0, \quad y = -4$ m
$(a)_{max} = 50$ m/s^2

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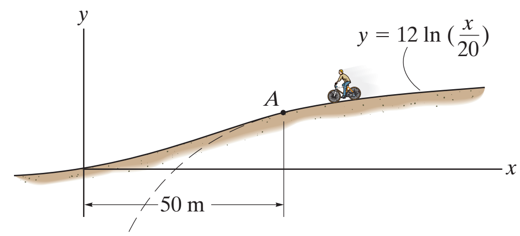
12–153.

When the bicycle passes point A, it has a speed of 6 m/s, which is increasing at the rate of $\dot{v} = (0.5)\ \text{m/s}^2$. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = 12\ln\left(\frac{x}{20}\right)$$

$$\frac{dy}{dx} = 12\left(\frac{1}{x/20}\right)\left(\frac{1}{20}\right) = \frac{12}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{12}{x^2}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1+\left(\frac{12}{x}\right)^2\right]^{3/2}}{\left|-\frac{12}{x^2}\right|}\right|_{x=50\text{ m}} = 226.59\text{ m}$$

Acceleration:

$$a_t = v = 0.5\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{6^2}{226.59} = 0.1589\text{ m/s}^2$$

The magnitude of the bicycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.1589^2} = 0.525\text{ m/s}^2$$ **Ans.**

Ans:
$a = 0.525\text{ m/s}^2$

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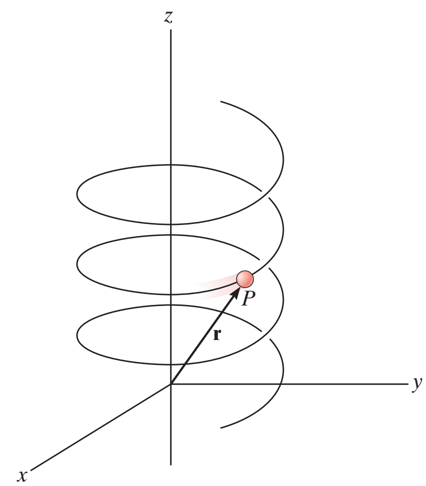
12–154.

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$

$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$

When $t = 8$ s,

$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$

$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

Ans:

$\alpha = 52.5°$
$\beta = 142°$
$\gamma = 85.1°$
$\alpha = 128°, \beta = 37.9°, \gamma = 94.9°$

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12–155.

If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4 \cos t = -1.66459$

$\ddot{r} = -4 \sin t = -3.6372$

$\theta = 2\,e^{-t}$

$\dot{\theta} = -2\,e^{-t} = -0.27067$

$\ddot{\theta} = 2\,e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97$ m/s^2 **Ans.**

Ans:
$v_r = -1.66$ m/s
$v_\theta = -2.07$ m/s
$a_r = -4.20$ m/s
$a_\theta = 2.97$ m/s^2

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***12–156.**

A particle travels around a limaçon, defined by the equation $r = b - a\cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

SOLUTION

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta}$$ **Ans.**

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta}$$ **Ans.**

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta}$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$$ **Ans.**

Ans:
$v_r = \dot{r} = a\sin\theta\dot{\theta}$
$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta}$
$a_r = (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta}$
$a_\theta = (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$

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12–157. A particle moves along a circular path of radius 300 mm. If its angular velocity is $\dot{\theta} = (2t^2)$ rad/s, where t is in seconds, determine the magnitude of the particle's acceleration when $t = 2$ s.

SOLUTION

Time Derivatives:

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 2t^2\Big|_{t=2\text{ s}} = 8 \text{ rad/s} \qquad \ddot{\theta} = 4t\Big|_{t=2\text{ s}} = 8 \text{ rad/s}^2$$

Velocity: The radial and transverse components of the particle's velocity are

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 0.3(8) = 2.4 \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} = \sqrt{0^2 + 2.4^2} = 2.4 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3\left(8^2\right) = -19.2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(8) + 0 = 2.4 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2} = \sqrt{(-19.2)^2 + 2.4^2} = 19.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 2.4$ m/s
$a = 19.3$ m/s^2

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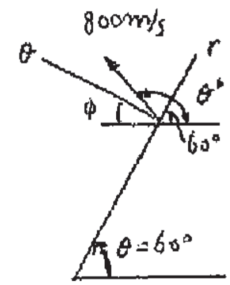
12–158.

For a short time a rocket travels up and to the right at a constant speed of 800 m/s along the parabolic path $y = 600 - 35x^2$. Determine the radial and transverse components of velocity of the rocket at the instant $\theta = 60°$, where θ is measured counterclockwise from the x axis.

SOLUTION



$y = 600 - 35x^2$

$\dot{y} = -70x\dot{x}$

$\dfrac{dy}{dx} = -70x$

$\tan 60° = \dfrac{y}{x}$

$y = 1.732051x$

$1.732051x = 600 - 35x^2$

$x^2 + 0.049487x - 17.142857 = 0$

Solving for the positive root,

$x = 4.1157$ m

$\tan\theta' = \dfrac{dy}{dx} = -288.1$

$\theta' = 89.8011°$

$\phi = 180° - 89.8011° - 60° = 30.1989°$

$v_r = 800 \cos 30.1989° = 691$ m/s **Ans.**

$v_\theta = 800 \sin 30.1989° = 402$ m/s **Ans.**

Ans:

$v_r = 691$ m/s

$v_\theta = 402$ m/s

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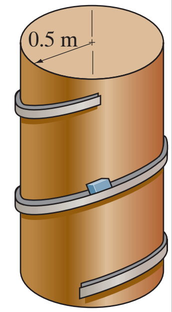
12–159.

The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



SOLUTION

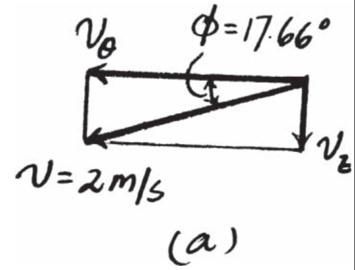
Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\left[\dfrac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. *a*, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$



Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

Since $\mathbf{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2 + a_z{}^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 7.26 \text{ m/s}^2$

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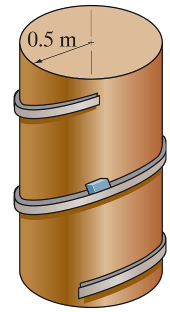
***12–160.** The box slides down the helical ramp such that $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



SOLUTION

Time Derivatives:

$$r = 0.5\text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right)\text{ rad/s} \qquad \ddot{\theta} = (3t)\text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t)\text{ m/s} \qquad \ddot{z} = -0.4\text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325\text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325\text{ s}} = 1.5(2.325)^2 = 8.108\text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325\text{ s}} = 3(2.325) = 6.975\text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325\text{ s}} = -0.4(2.325) = -0.92996\text{ m/s}$$

$$\ddot{z}\big|_{t=2.325\text{ s}} = -0.4\text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385\text{ m/s}$$

$$v_z = \dot{z} = -0.92996\text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487\text{ m/s}^2$$

$$a_z = \ddot{z} = -0.4\text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 4.16$ m/s
$a = 33.1$ m/s^2

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12–161.

If a particle's position is described by the polar coordinates $r = (2\sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$\theta = 4t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2\sin 2\theta = 1.9787$

$\dot{r} = 4\cos 2\theta\,\dot{\theta} = -2.3280$

$\ddot{r} = -8\sin 2\theta(\dot{\theta})^2 + 8\cos 2\theta\,\ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6$ m/s^2 **Ans.**

Ans:
$v_r = -2.33$ m/s
$v_\theta = 7.91$ m/s
$a_r = -158$ m/s^2
$a_\theta = -18.6$ m/s^2

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12–162.

If a particle moves along a path such that $r = (e^{at})$ m and $\theta = t$, where t is in seconds, plot the path $r = f(\theta)$, and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = e^{at} \qquad \dot{r} = ae^{at} \qquad \ddot{r} = a^2e^{et}$

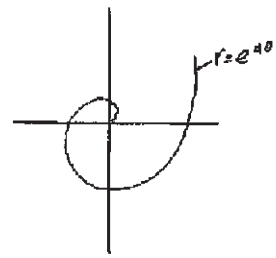
$\theta = t \qquad \dot{\theta} = 1 \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = ae^{at}$ **Ans.**

$v_a = r\dot{\theta} = e^{at}(1) = e^{at}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = a^2e^{at} - e^{at}(1)^2 = e^{at}(a^2 - 1)$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{at}(0) + 2(ae^{at})(1) = 2ae^{at}$ **Ans.**



Ans:

$v_r = ae^{at}$

$v_\theta = e^{at}$

$a_r = e^{at}(a^2 - 1)$

$a_\theta = 2ae^{at}$

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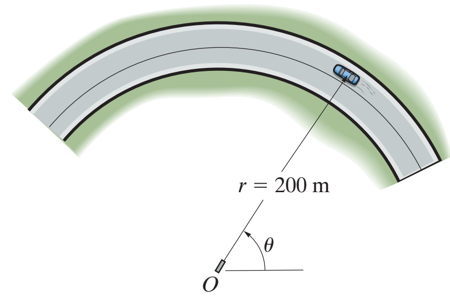
12–163.

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 20^2} = 20 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v = 20$ m/s
$a = 5.39$ m/s^2

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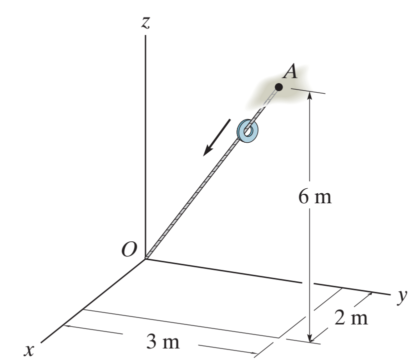
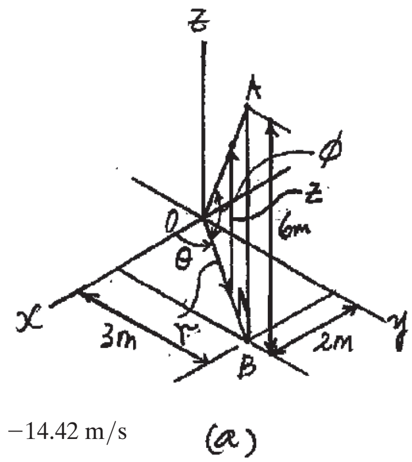
***12–164.**

The small washer is sliding down the cord OA. When it is at the midpoint, its speed is 28 m/s and its acceleration is 7 m/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

The position of the washer can be defined using the cylindrical coordinate system (r, θ and z) as shown in Fig. a. Since θ is constant, there will be no transverse component for **v** and **a**. The velocity and acceleration expressed as Cartesian vectors are



$$\mathbf{v} = v\left(-\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 28\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}\}\ \text{m/s}$$

$$\mathbf{a} = a\left(-\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 7\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}\}\ \text{m}^2/\text{s}$$

$$\mathbf{u}_r = \frac{\mathbf{r}_{OB}}{r_{OB}} = \frac{2\mathbf{i} + 3\mathbf{j}}{\sqrt{2^2 + 3^2}} = \frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$$

$$\mathbf{u}_z = \mathbf{k}$$

Using vector dot product

$$v_r = \mathbf{v}\cdot\mathbf{u}_r = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -8\left(\frac{2}{\sqrt{13}}\right) + \left[-12\left(\frac{3}{\sqrt{13}}\right)\right] = -14.42\ \text{m/s}$$

$$v_z = \mathbf{v}\cdot\mathbf{u}_z = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot(\mathbf{k}) = -24.0\ \text{m/s}$$

$$a_r = \mathbf{a}\cdot\mathbf{u}_r = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -2\left(\frac{2}{\sqrt{13}}\right) + \left[-3\left(\frac{3}{\sqrt{13}}\right)\right] = -3.606\ \text{m/s}^2$$

$$a_z = a\cdot\mathbf{u}_z = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\mathbf{k} = -6.00\ \text{m/s}^2$$

Thus, in vector form

$\mathbf{v} = \{-14.2\,\mathbf{u}_r - 24.0\,\mathbf{u}_z\}$ m/s **Ans.**

$\mathbf{a} = \{-3.61\,\mathbf{u}_r - 6.00\,\mathbf{u}_z\}$ m/s^2 **Ans.**

These components can also be determined using trigonometry by first obtain angle ϕ shown in Fig. a.

$$OA = \sqrt{2^2 + 3^2 + 6^2} = 7\ \text{m} \qquad OB = \sqrt{2^2 + 3^2} = \sqrt{13}$$

Thus,

$\sin\phi = \dfrac{6}{7}$ and $\cos\phi = \dfrac{\sqrt{13}}{7}$. Then

$$v_r = -v\cos\phi = -28\left(\frac{\sqrt{13}}{7}\right) = -14.42\ \text{m/s}$$

$$v_z = -v\sin\phi = -28\left(\frac{6}{7}\right) = -24.0\ \text{m/s}$$

$$a_r = -a\cos\phi = -7\left(\frac{\sqrt{13}}{7}\right) = -3.606\ \text{m/s}^2$$

$$a_z = -a\sin\phi = -7\left(\frac{6}{7}\right) = -6.00\ \text{m/s}^2$$

Ans:

$\mathbf{v} = \{-14.2\mathbf{u}_r - 24.0\mathbf{u}_z\}$ m/s
$\mathbf{a} = \{-3.61\mathbf{u}_r - 6.00\mathbf{u}_z\}$ m/s^2

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12–165.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans.}$$

Ans:

$\dot{\mathbf{a}} = (\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta})\mathbf{u}_r$
$+ (3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3)\mathbf{u}_\theta + (\dddot{z})\mathbf{u}_z$

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12–166. A particle is moving along a circular path having a 400-mm radius. Its position as a function of time is given by $\theta = (2t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

SOLUTION

$$t = \sqrt{\frac{\theta_1}{b}} \qquad t = 0.51\text{ s}$$

$$r = 400\text{ mm} \qquad b = 2\text{ rad/s}^2 \qquad \theta_1 = 30°$$

$$\theta = bt^2 \qquad \theta' = 2bt \qquad \theta'' = 2b$$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta''\right)^2} \qquad a = 2.32\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 2.32\text{ m/s}^2$

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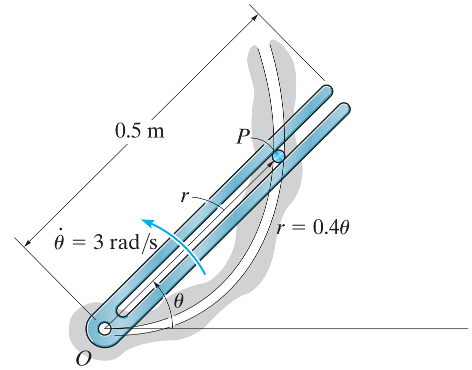
12–167.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$$\dot{r} = 0.4\,\dot{\theta}$$

$$\ddot{r} = 0.4\,\ddot{\theta}$$

At $\theta = \dfrac{\pi}{3}$, $\quad r = 0.4189$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$v = \dot{r} = 1.20$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans.**

Ans:
$v_r = 1.20$ m/s
$v_\theta = 1.26$ m/s
$a_r = -3.77$ m/s^2
$a_\theta = 7.20$ m/s^2

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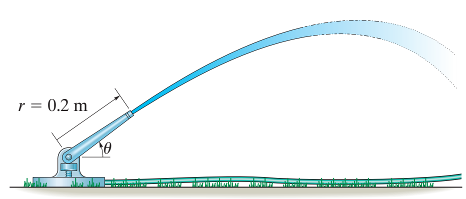
***12–168.** At the instant shown, the watersprinkler is rotating with an angular speed $\dot{\theta} = 2\ \text{rad/s}$ and an angular acceleration $\ddot{\theta} = 3\ \text{rad/s}^2$. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of $3\ \text{m/s}$, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 0.2$ m.



SOLUTION

$$\theta' = 2\ \text{rad/s}$$

$$\theta'' = 3\ \text{rad/s}^2$$

$$r' = 3\ \text{m/s}$$

$$r = 0.2\ \text{m}$$

$$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 3.03\ \text{m/s} \qquad \textbf{Ans.}$$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2} \qquad a = 12.63\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 3.03\ \text{m/s}$
$a = 12.63\ \text{m/s}^2$

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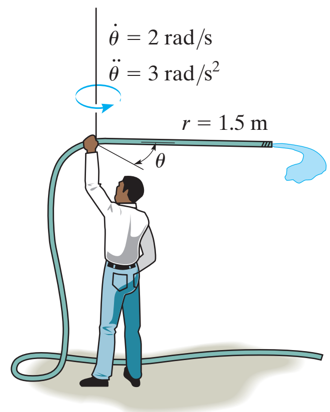
12–169.

At the instant shown, the man is twirling a hose over his head with an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If it is assumed that the hose lies in a horizontal plane, and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 1.5$ m.



SOLUTION

$r = 1.5$

$\dot{r} = 3$

$\ddot{r} = 0$

$\dot{\theta} = 2$

$\ddot{\theta} = 3$

$v_r = \dot{r} = 3$

$v_\theta = r\dot{\theta} = 1.5(2) = 3$

$v = \sqrt{(3)^2 + (3)^2} = 4.24$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(2)^2 = 6$

$a_\theta = r\,\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.5(3) + 2(3)(2) = 16.5$

$a = \sqrt{(6)^2 + (16.5)^2} = 17.6$ m/s^2 **Ans.**

Ans:
$v = 4.24$ m/s
$a = 17.6$ m/s^2

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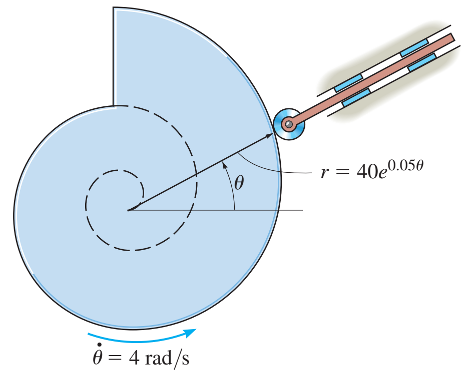
12–170.

The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



SOLUTION

$$r = 40e^{0.05\,\theta}$$

$$\dot{r} = 2e^{0.05\theta}\dot{\theta}$$

$$\ddot{r} = 0.1e^{0.05\,\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\,\theta}\ddot{\theta}$$

$$\theta = \frac{\pi}{6}$$

$$\dot{\theta} = -4$$

$$\ddot{\theta} = 0$$

$$r = 40e^{\,0.05\left(\frac{\pi}{6}\right)} = 41.0610$$

$$\dot{r} = 2e^{\,0.05\left(\frac{\pi}{6}\right)}(-4) = -8.2122$$

$$\ddot{r} = 0.1e^{\,0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 0 = 1.64244$$

$v = \dot{r} = -8.2122 = 8.21$ mm/s **Ans.**

$a = \ddot{r} - r\dot{\theta}^2 = 1.642\,44 - 41.0610(-4)^2 = -665.33 = -665$ mm/s^2 **Ans.**

Ans:
$v_r = 8.21$ mm/s
$a = -665$ mm/s^2

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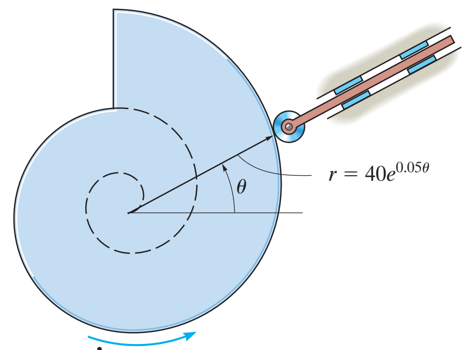
12–171.

Solve Prob. 12–170, if the cam has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when its angular velocity is $\dot{\theta} = 4\ \text{rad/s}$ at $\theta = 30°$.



SOLUTION

$$r = 40e^{0.05\theta}$$

$$\dot{r} = 2e^{0.05\theta}\dot{\theta}$$

$$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$$

$$\theta = \frac{\pi}{6}$$

$$\dot{\theta} = -4$$

$$\ddot{\theta} = -2$$

$$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$$

$$\dot{r} = 2e^{0.05\left(\left(\frac{\pi}{6}\right)\right.}(-4) = -8.2122$$

$$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(-2) = -2.4637$$

$$v = \dot{r} = 8.2122 = 8.21\ \text{mm/s}$$ **Ans.**

$$a = \ddot{r} - r\dot{\theta}^2 = -2.4637 - 41.0610(-4)^2 = -659\ \text{mm/s}^2$$ **Ans.**

Ans:

$v = 8.21$ mm/s
$a = -659\ \text{mm/s}^2$

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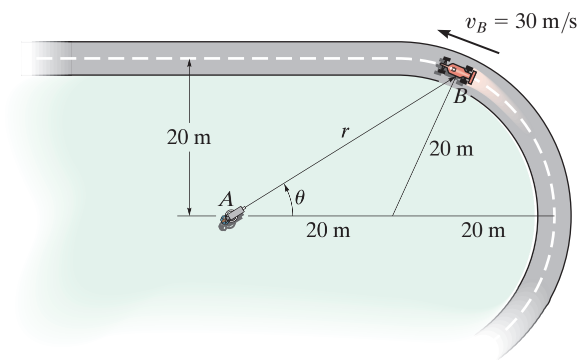
***12–172.**

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.



SOLUTION

$r = 2(20)\cos\theta = 40\cos\theta$

$\dot{r} = -(40\sin\theta)\dot{\theta}$

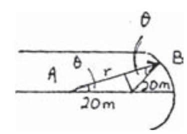
$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\,\dot{\theta}\,\mathbf{u}_\theta$

$v = \sqrt{(\dot{r})^2 + (r\,\dot{\theta})^2}$

$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$

$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$

$\dot{\theta} = \dfrac{30}{40} = 0.75 \text{ rad/s}$ **Ans.**



Ans:
$\dot{\theta} = 0.75$ rad/s

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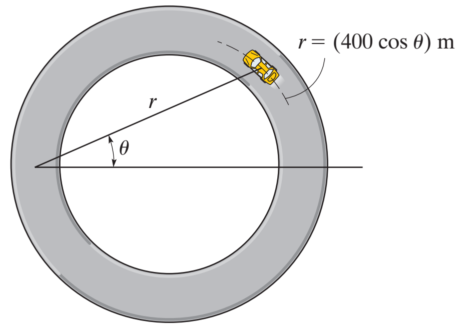
12–173.

The car travels around the circular track with a constant speed of 20 m/s. Determine the car's radial and transverse components of velocity and acceleration at the instant $\theta = \pi/4$ rad.



SOLUTION

$v = 20 \text{ m/s}$

$\theta = \dfrac{\pi}{4} = 45°$

$r = 400 \cos\theta$

$\dot{r} = -400 \sin\theta\,\dot{\theta}$

$\ddot{r} = -400(\cos\theta(\dot{\theta})^2 + \sin\theta\,\ddot{\theta})$

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$0 = \dot{r}\ddot{r} + r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})$

Thus

$r = 282.84$

$(20)^2 = [-400 \sin 45°\,\dot{\theta}]^2 + [282.84\,\dot{\theta}]^2$

$\dot{\theta} = 0.05$

$\dot{r} = -14.14$

$0 = -14.14[-400(\cos 45°)(0.05)^2 + \sin 45°\,\ddot{\theta}] + 282.84(0.05)[-14.14(0.05) + 282.84\ddot{\theta}]$

$\ddot{\theta} = 0$

$\ddot{r} = -0.707$

$v_r = \dot{r} = -14.1 \text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 282.84(0.05) = 14.1 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -0.707 - 282.84(0.05)^2 = -1.41 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \dot{\theta} + 2(-14.14)(0.05) = -1.41 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -14.1 \text{ m/s}$
$v_\theta = 14.1 \text{ m/s}$
$a_r = -1.41 \text{ m/s}^2$
$a_\theta = -1.41 \text{ m/s}^2$

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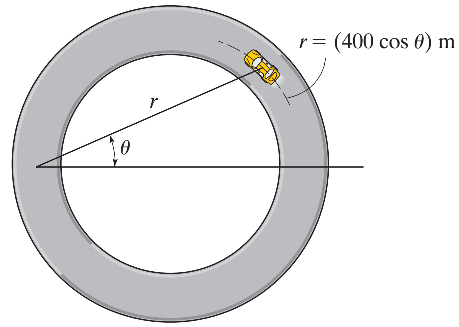
12–174.

The car travels around the circular track such that its transverse component is $\theta = (0.006t^2)$ rad, where t is in seconds. Determine the car's radial and transverse components of velocity and acceleration at the instant $t = 4$ s.



SOLUTION

$\theta = 0.006\, t^2\big|_{t=4} = 0.096 \text{ rad} = 5.50°$

$\dot{\theta} = 0.012\, t\big|_{t=4} = 0.048 \text{ rad/s}$

$\ddot{\theta} = 0.012 \text{ rad/s}^2$

$r = 400 \cos \theta$

$\dot{r} = -400 \sin \theta\, \dot{\theta}$

$\ddot{r} = -400(\cos \theta\, (\dot{\theta})^2 + \sin\theta\, \ddot{\theta})$

At $\theta = 0.096$ rad

$r = 398.158$ m

$\dot{r} = -1.84037$ m/s

$\ddot{r} = -1.377449 \text{ m/s}^2$

$v_r = \dot{r} = -1.84$ m/s **Ans.**

$v_\theta = r\,\dot{\theta} = 398.158(0.048) = 19.1$ m/s **Ans.**

$a_r = \ddot{r} - r\,(\dot{\theta})^2 = -1.377449 - 398.158(0.048)^2 = -2.29 \text{ m/s}^2$ **Ans.**

$a_\theta = r\,\ddot{\theta} = 2\dot{r}\,\dot{\theta} = 398.158\,(0.012) + 2(-1.84037)(0.048) = 4.60 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -1.84$ m/s
$v_\theta = 19.1$ m/s
$a_r = -2.29 \text{ m/s}^2$
$a_\theta = 4.60 \text{ m/s}^2$

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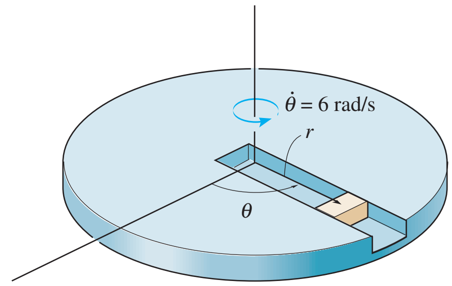
12–175.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$\dot{r} = 4t|_{t=1} = 4 \qquad \ddot{r} = 4$

$\dot{\theta} = 6 \qquad \ddot{\theta} = 0$

$$\int_0^1 dr = \int_0^1 4t\, dt$$

$r = 2t^2\Big]_0^1 = 2\text{ m}$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6\text{ m/s}$ **Ans.**

$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$ **Ans.**

$= 83.2\text{ m/s}^2$

Ans:
$v = 12.6\text{ m/s}$
$a = 83.2\text{ m/s}^2$

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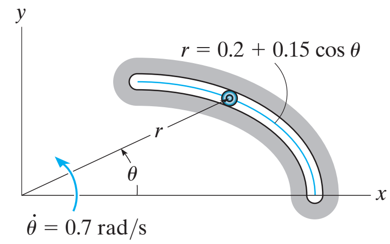
***12–176.** The pin follows the path described by the equation $r = (0.2 + 0.15\cos\theta)$ m. At the instant $\theta = 30°$, $\dot{\theta} = 0.7$ rad/s and $\ddot{\theta} = 0.5$ rad/s^2. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.



SOLUTION

$a = 0.2\ \text{m}$

$b = 0.15\ \text{m}$

$\theta_1 = 30°$

$\theta' = 0.7\ \text{rad/s}$

$\theta'' = 0.5\ \text{rad/s}^2$

$\theta = \theta_1$

$r = a + b\cos(\theta) \qquad r' = -b\sin(\theta)\theta' \qquad r'' = -b\cos(\theta)\theta'^2 - b\sin(\theta)\theta''$

$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 0.237\ \text{m/s}$ **Ans.**

$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + (r\theta'' + 2r'\theta')^2} \qquad a = 0.278\ \text{m/s}^2$ **Ans.**

Ans:
$v = 0.237\ \text{m/s}$
$a = 0.278\ \text{m/s}^2$

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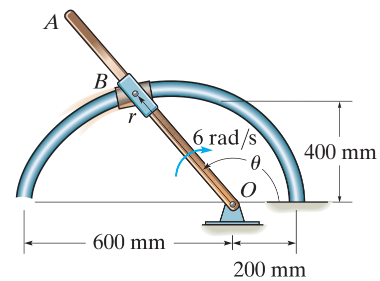
12–177.

The rod OA rotates clockwise with a constant angular velocity of 6 rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 150°$.



SOLUTION

Velocity. Using the chain rule, the first and second time derivatives of r can be determined.

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\dot{\theta} = \{200(\sin\theta)\dot{\theta}\} \text{ mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\} \text{ mm/s}^2$$

The radial and transverse components of the velocity are

$$v_r = \dot{r} = \{200(\sin\theta)\dot{\theta}\} \text{ mm/s}$$

$$v_\theta = r\dot{\theta} = \{200(2 - \cos\theta)\dot{\theta}\} \text{ mm/s}$$

Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$,

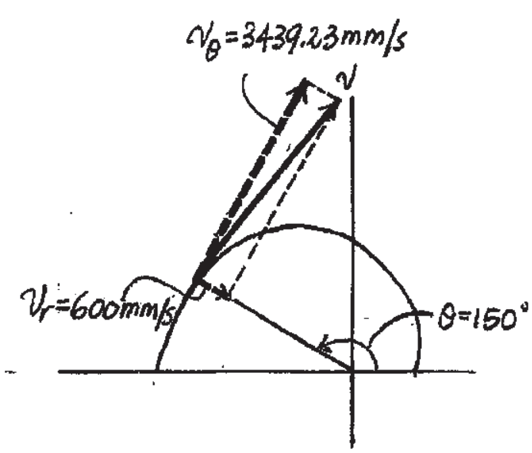
$$v_r = 200(\sin 150°)(-6) = -600 \text{ mm/s}$$

$$v_\theta = 200(2 - \cos 150°)(-6) = -3439.23 \text{ mm/s}$$

Thus, the magnitude of the velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} \quad \sqrt{(-600)^2 + (-3439.23)^2} = 3491 \text{ mm/s} = 3.49 \text{ m/s} \qquad \textbf{Ans.}$$

These components are shown in Fig. a



Ans:
$v = 3.49$ m/s

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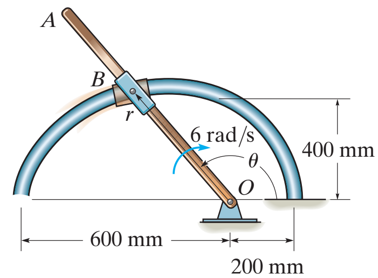
12–178.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–177 when $\theta = 150°$.



SOLUTION

Acceleration. Using the chain rule, the first and second time derivatives of r can be determined

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\dot{\theta} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

Here, since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$

$$r = 200(2 - \cos 150°) = 573.21\ \text{mm}$$

$$\dot{r} = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$\ddot{r} = 200\left[(\cos 150°)(-6)^2 + \sin 150°(0)\right] = -6235.38\ \text{mm/s}^2$$

The radial and transverse components of the acceleration are

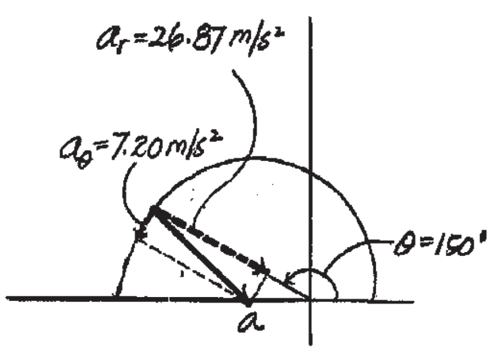
$$a_r = \ddot{r} - r\dot{\theta}^2 = -6235.38 - 573.21\,(-6)^2 = -26870.77\ \text{mm/s}^2 = -26.87\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 573.21(0) + 2(-600)(-6) = 7200\ \text{mm/s}^2 = 7.20\ \text{m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-26.87)^2 + 7.20^2} = 27.82\ \text{m/s}^2 = 27.8\ \text{m/s}^2 \qquad \textbf{Ans.}$$

These components are shown in Fig. *a*.



(a)

Ans:
$a = 27.8\ \text{m/s}^2$

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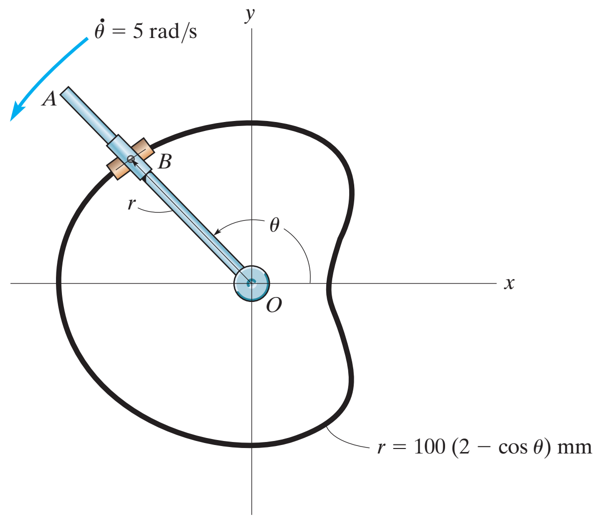
12–179.

The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100 \sin\theta\dot{\theta} = 500 \sin\theta$

$\ddot{r} = 500 \cos\theta\dot{\theta} = 2500 \cos\theta$

At $\theta = 120°$,

$v_r = \dot{r} = 500 \sin 120° = 433.013$

$v_\theta = r\dot{\theta} = 100\,(2 - \cos 120°)(5) = 1250$

$v = \sqrt{(433.013)^2 + (1250)^2} = 1322.9 \text{ mm/s} = 1.32 \text{ m/s}$ **Ans.**

Ans:
$v = 1.32$ m/s

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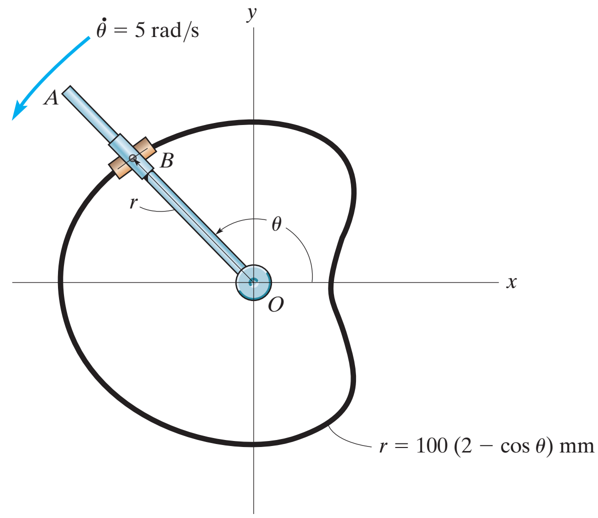
***12–180.**

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–179 when $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$\dot{\theta} = 0$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100 \sin\theta\dot{\theta} = 500 \sin\theta$

$\ddot{r} = 500 \cos\theta\dot{\theta} = 2500 \cos\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 = 2500\cos\theta - 100(2 - \cos\theta)(5)^2 = 5000(\cos 120° - 1) = -7500 \text{ mm/s}^2$

$a_s = r\dot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(500\sin\theta)(5) = 5000\sin 120° = 4330.1 \text{ mm/s}^2$

$a = \sqrt{(-7500)^2 + (4330.1)^2} = 8660.3 \text{ mm/s}^2 = 8.66 \text{ m/s}^2$ **Ans.**

Ans:
$a = 8.66 \text{ m/s}^2$

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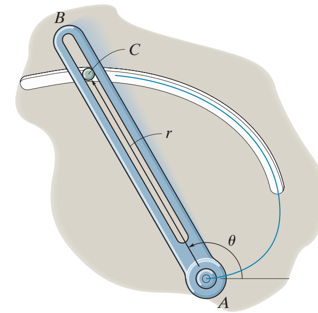
12–181. The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = a\,\dot{\theta}$
$v_\theta = a\,\theta\dot{\theta}$
$a_r = -a\theta\dot{\theta}^2$
$a_\theta = 2a\dot{\theta}^2$

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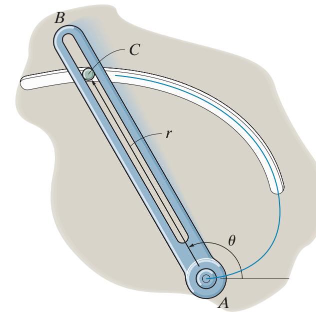
12–182. The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\,\theta)$ m, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



SOLUTION

Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4$ rad/s^2.

$$r = 1.5\theta \qquad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \qquad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6 \text{ m/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{\frac{1}{2}(6t^2 + \pi)\right\}$ m. At $t = 1$ s, $r = \frac{1}{2}\left[6(1^2) + \pi\right] = 4.571$ m, $\dot{r} = 6(1) = 6.00$ m/s and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4.571\,(4) = 18.3 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571\left(4^2\right) = -67.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)\,(4) = 66.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = 6.00$ m/s
$v_\theta = 18.3$ m/s
$a_r = -67.1$ m/s^2
$a_\theta = 66.3$ m/s^2

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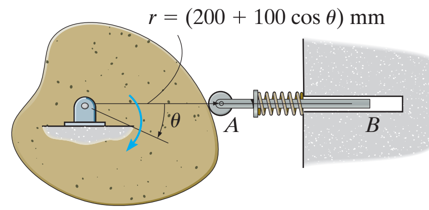
12–183. If the cam rotates clockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB at the instant $\theta = 30°$. The surface of the cam has a shape of limaçon defined by $r = (200 + 100 \cos \theta)$ mm.



SOLUTION

Time Derivatives:

$$r = (200 + 100 \cos \theta) \text{ mm}$$

$$\dot{r} = (-100 \sin \theta\dot{\theta}) \text{ mm/s} \qquad \dot{\theta} = 5 \text{ rad/s}$$

$$\ddot{r} = -100\left[\sin \theta\ddot{\theta} + \cos \theta\dot{\theta}^2\right] \text{ mm/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 30°$,

$$r\big|_{\theta=30°} = 200 + 100 \cos 30° = 286.60 \text{ mm}$$

$$\dot{r}\big|_{\theta=30°} = -100 \sin 30°(5) = -250 \text{ mm/s}$$

$$\dot{r}\big|_{\theta=30°} = -100\left[0 + \cos 30°\left(5^2\right)\right] = -2165.06 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250 \text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2156.06 - 286.60(5^2) = -9330 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v_r = -250$ mm/s
$a_r = -9330$ mm/s^2

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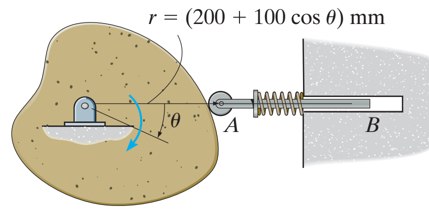
***12–184.** At the instant $\theta = 30°$, the cam rotates with a clockwise angular velocity of $\dot{\theta} = 5$ rad/s and, angular acceleration of $\ddot{\theta} = 6$ rad/s^2. Determine the magnitudes of the velocity and acceleration of the follower rod AB at this instant. The surface of the cam has a shape of a limaçon defined by $r = (200 + 100 \cos\theta)$ mm.



SOLUTION

Time Derivatives:

$$r = (200 - 100\cos\theta)\text{ mm}$$

$$\dot{r} = (-100\sin\theta\dot{\theta})\text{ mm/s}$$

$$\dot{r} = -100\left[\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right]\text{ mm/s}^2$$

When $\theta = 30°$,

$$r\big|_{\theta=30°} = 200 + 100\cos 30° = 286.60\text{ mm}$$

$$\dot{r}\big|_{\theta=30°} = -100\sin 30°(5) = -250\text{ mm/s}$$

$$\ddot{r}\big|_{\theta=30°} = -100\left[\sin 30°(6) + \cos 30°\left(5^2\right)\right] = -2465.06\text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250\text{ mm/s}$$ **Ans.**

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2465.06 - 286.60\left(5^2\right) = -9630\text{ mm/s}^2$$ **Ans.**

Ans:

$v_r = -250$ mm/s
$a_r = -9630$ mm/s^2

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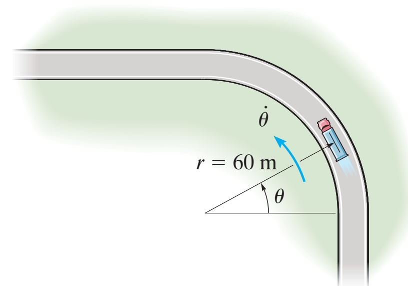
12–185.

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



SOLUTION

$r = 60$

$\dot{r} = 0$

$\ddot{r} = 0$

$v = 20$

$v_r = \dot{r} = 0$

$v_\theta = r\,\dot{\theta} = 60\,\dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$

$20 = 60\,\dot{\theta}$

$\dot{\theta} = 0.333$ rad/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2$

$\quad = 0 - 60(0.333)^2$

$\quad = -6.67$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\quad = 60\ddot{\theta}$

Since

$v = r\dot{\theta}$

$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$

$0 = 0 + 60\,\ddot{\theta}$

$\ddot{\theta} = 0$

Thus,

$a_\theta = 0$

$a = \left|a_r\right| = 6.67$ m/s^2 **Ans.**

Ans:
$\dot{\theta} = 0.333$ rad/s
$a = 6.67$ m/s^2

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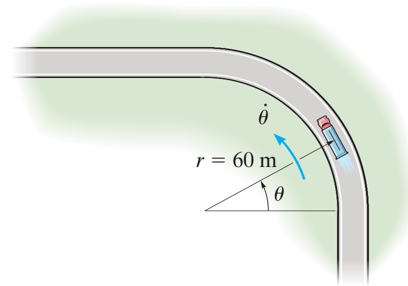
12–186.

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at $3\ \text{m/s}^2$. Determine the truck's radial and transverse components of acceleration.



SOLUTION

$r = 60$

$a_t = 3\ \text{m/s}^2$

$$a_n = \frac{v^2}{r} = \frac{(20)^2}{60} = 6.67\ \text{m/s}^2$$

$a_r = -a_n = -6.67\ \text{m/s}^2$ **Ans.**

$a_\theta = a_t = 3\ \text{m/s}^2$ **Ans.**

Ans:
$a_r = -6.67\ \text{m/s}^2$
$a_\theta = 3\ \text{m/s}^2$

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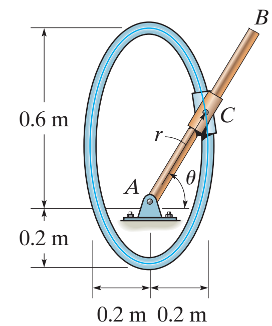
12–187.

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5\,t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |(0.4 \sin\theta + 0.2)|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0$, $\theta = 0$. Use Simpson's rule with $n = 50$ to determine θ at $t = 1$ s.



SOLUTION

$\dot{\theta} = e^{0.5\,t^2}\big|_{t=1} = 1.649 \text{ rad/s}$

$\ddot{\theta} = e^{0.5\,t^2}\,t\big|_{t=1} = 1.649 \text{ rad/s}^2$

$\theta = \displaystyle\int_0^1 e^{0.5\,t^2}dt = 1.195 \text{ rad} = 68.47°$

$r = 0.4 \sin\theta + 0.2$

$\dot{r} = 0.4 \cos\theta\,\dot{\theta}$

$\ddot{r} = -0.4 \sin\theta\,\dot{\theta}^2 + 0.4\cos\theta\,\ddot{\theta}$

At $t = 1$ s,

$r = 0.5721$

$\dot{r} = 0.2421$

$\ddot{r} = -0.7697$

$v_r = \dot{r} = 0.242 \text{ m/s}$ **Ans.**

$v_\theta = r\,\dot{\theta} = 0.5721(1.649) = 0.943 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.7697 - 0.5721(1.649)^2$

$a_r = -2.33 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 0.5721(1.649) + 2(0.2421)(1.649)$

$a_\theta = 1.74 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = 0.242 \text{ m/s}$
$v_\theta = 0.943 \text{ m/s}$
$a_r = -2.33 \text{ m/s}^2$
$a_\theta = 1.74 \text{ m/s}^2$

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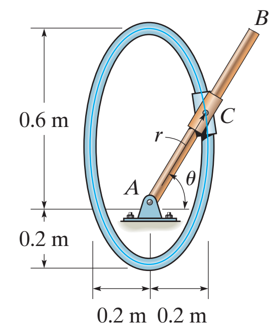
***12–188.**

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the mechanism is to be designed so that the largest speed given to the collar is 6 m/s, determine the required constant angular velocity $\dot{\theta}$ of rod AB. The path defined by the fixed rod is $r = (0.4 \sin \theta + 0.2)$ m.



SOLUTION

$r = 0.4 \sin \theta + 0.2$

$\dot{r} = 0.4 \cos \theta \, \dot{\theta}$

$v_r = \dot{r} = 0.4 \cos \theta \, \dot{\theta}$

$v_\theta = r \, \dot{\theta} = (0.4 \sin \theta + 0.2) \, \dot{\theta}$

$v^2 = v_r^2 + v_\theta^3$

$(6)^2 = [(0.4 \cos \theta)^2 + (0.4 \sin \theta + 0.2)^2](\dot{\theta})^2$

$36 = [0.2 + 0.16 \sin \theta](\dot{\theta})^2$

The greatest speed occurs when $\theta = 90°$.

$\dot{\theta} = 10.0$ rad/s **Ans.**

Ans:
$\dot{\theta} = 10.0$ rad/s

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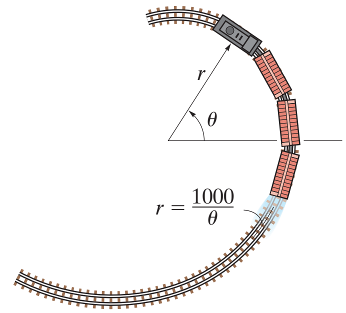
12–189. For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



SOLUTION

$$r = \frac{1000}{\theta}$$

$$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$$

Since

$$v^2 = (\dot{r})^2 + (r\,\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$$

Thus,

$$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$$

At $\theta = \dfrac{9\pi}{4}$

$$\dot{\theta} = 0.140$$

$$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$$

$v_r = \dot{r} = -2.80$ m/s **Ans.**

$v_\theta = r\dot{\theta} = \dfrac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans.**

Ans:
$v_r = -2.80$ m/s
$v_\theta = 19.8$ m/s

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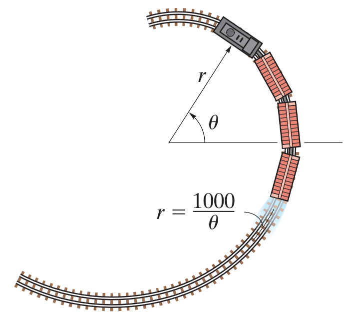
12–190.

For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



SOLUTION

$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$$r = \frac{1000}{\theta}$$

$\dot{r} = -1000(\theta^{-2})\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$

When $\theta = \frac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

$v_r = \dot{r} = -4.00$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans.**

Ans:
$v_r = -4.00$ m/s
$v_\theta = 28.3$ m/s
$a_r = -5.43$ m/s^2
$a_\theta = -1.60$ m/s

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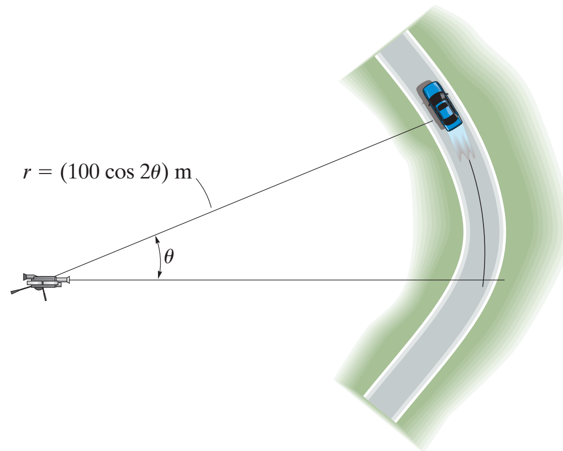
12–191.

The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\dot{\theta}) \text{ m/s}$$

At $\theta = 15°$,

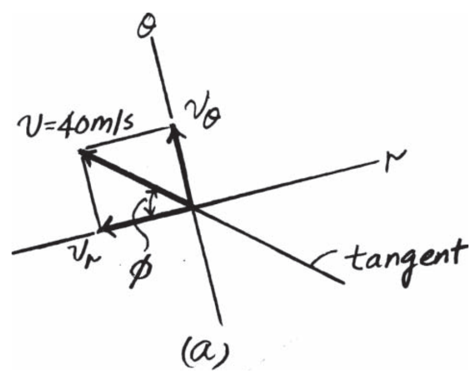
$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. *a*, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$



and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 0.302$ rad/s

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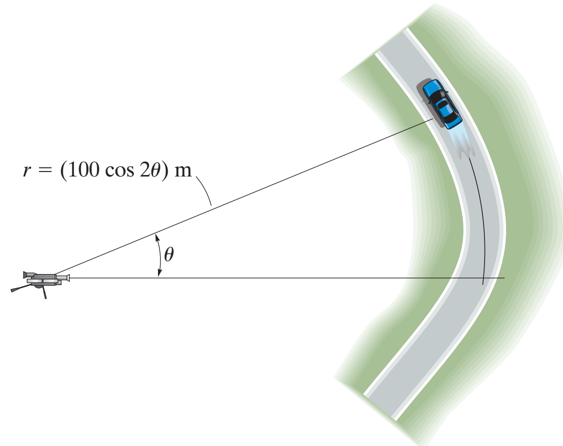
***12–192.**

When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\,\dot{\theta}) \text{ m/s}$$

$$\ddot{r} = -200\left[(\sin\ 2\theta)\,\ddot{\theta} + 2\,(\cos 2\theta)\,\dot{\theta}^2\right] \text{ m/s}^2$$

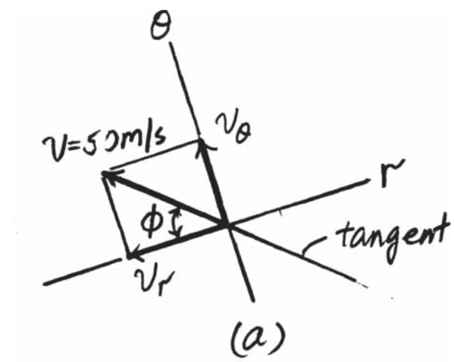
At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

$$\ddot{r}\big|_{\theta=15°} = -200\left[\sin 30°\ddot{\theta} + 2 \cos 30°\dot{\theta}^2\right] = \left(-100\ddot{\theta} - 346.41\dot{\theta}^2\right) \text{ m/s}^2$$

Velocity: Referring to Fig. a, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$. Thus,



$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.378 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 0.378$ rad/s

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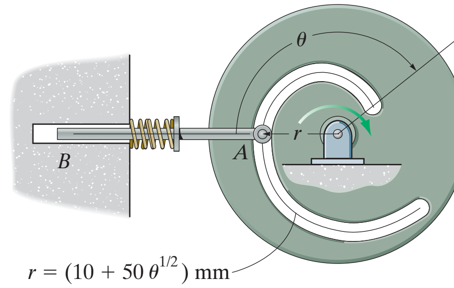
12–193.

If the circular plate rotates clockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB when $\theta = 2/3\pi$ rad.



SOLUTION

Time Derivaties:

$$r = \left(10 + 50\theta^{1/2}\right)\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\dot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[0 - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = -9.279\text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = 25.9\text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -9.279 - 82.36\left(1.5^2\right) = -195\text{ mm/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = 25.9$ mm/s
$a_r = -195$ mm/s^2

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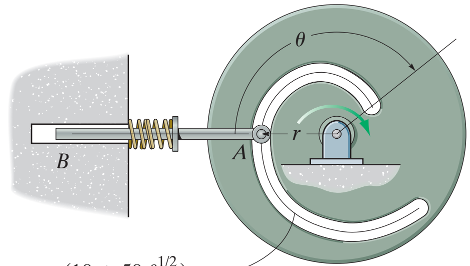
12–194.

When $\theta = 2/3\pi$ rad, the angular velocity and angular acceleration of the circular plate are $\dot{\theta} = 1.5$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the rod AB at this instant.



SOLUTION

Time Derivatives:

$$r = (10 + 50\theta^{1/2})\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{ mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[\left(\frac{2\pi}{3}\right)^{-1/2}(3) - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = 42.55\text{ mm/s}^2$$

For the rod,

$v = \dot{r} = 25.9$ mm/s **Ans.**

$a = \ddot{r} = 42.5$ mm/s^2 **Ans.**

Ans:

$v = 25.9$ mm/s

$a = 42.5$ mm/s^2

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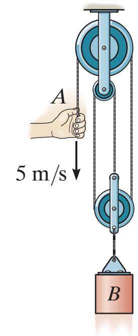
12–195.

If the end of the cable at A is pulled down with a speed of 5 m/s, determine the speed at which block B rises.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A, respectively, as shown in Fig. *a*. This is a single-cord pulley system. Thus,

$$s_B + 2(s_B - a) + s_A = l$$

$$3s_B + s_A = l + 2a \qquad \textbf{(1)}$$

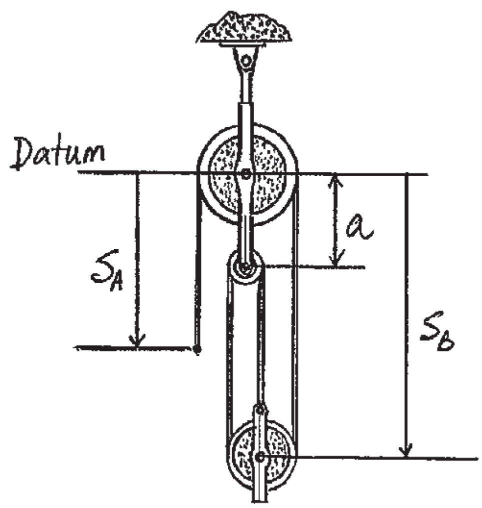
Time Derivative. Taking the time derivative of Eq. (1),

$$3v_B + v_A = 0 \qquad \textbf{(2)}$$

Here $v_A = +5$ m/s, since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 5 = 0 \qquad v_B = -1.667 \text{ m/s} = 1.67 \text{ m/s}\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_B is directed toward the negative sense of s_B.



(a)

Ans:
$v_B = 1.67$ m/s

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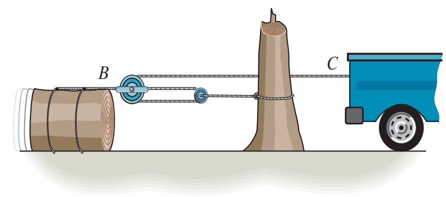
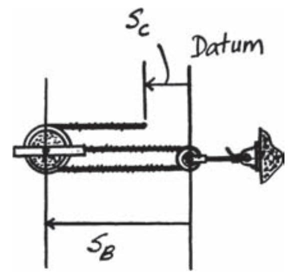
***12–196.** Determine the displacement of the log if the truck at C pulls the cable 1.2 m to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

Since $\Delta s_C = -1.2$, then

$3\Delta s_B = -1.2$

$\Delta s_B = -0.4 \text{ m} = 0.4 \text{ m} \rightarrow$ **Ans.**

Ans:
$\Delta s_B = 0.4 \text{ m} \rightarrow$

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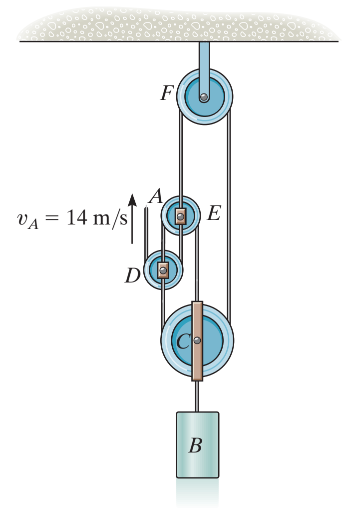
12–197. If the end A of the cable is moving upwards at $v_A = 14$ m/s, determine the speed of block B.



SOLUTION

$v_A = -14$ m/s

$L_1 = (s_D - s_A) + (s_D - s_E)$

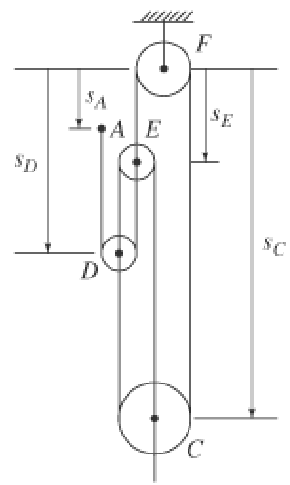
$0 = 2v_D - v_A - v_E$

$L_2 = (s_D - s_E) + (s_C - s_E)$

$0 = v_D + v_C - 2v_E$

$L_3 = (s_C - s_D) + s_C + s_E$

$0 = 2v_C - v_D + v_E$



Guesses $\quad v_C = 1$ m/s $\quad v_D = 1$ m/s $\quad v_E = 1$ m/s

Given $\quad 0 = 2v_D - v_A - v_E$

$0 = v_D + v_C - 2v_E$

$0 = 2v_C - v_D + v_E$

$$\begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \text{Find}(v_C, v_D, v_E) \qquad \begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \\ -6 \end{pmatrix} \text{ m/s}$$

$v_B = v_C \qquad v_B = -2$ m/s $\quad$ Positive means down, Negative means up $\quad$ **Ans.**

Ans:

$v_B = -2$ m/s Positive means down, Negative means up.

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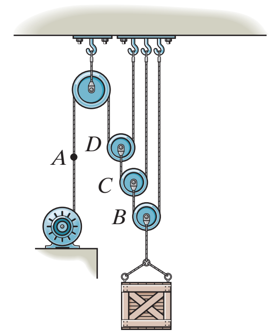
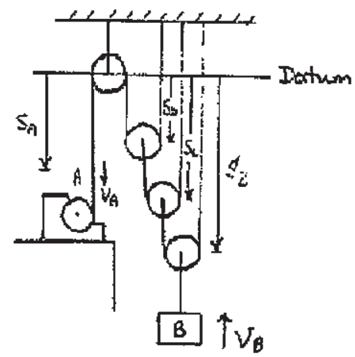
12–198.

Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load 6 m in 1.5 s.



SOLUTION



$v_B = \dfrac{6}{1.5} = 4\text{ m/s}\uparrow$

$s_B + (s_B - s_C) = l_1$

$s_C + (s_C - s_D) = l_2$

$s_A + 2\,s_D = l_3$

Thus,

$2s_B - s_C = l_1$

$2s_C - s_D = l_2$

$s_A + 2s_D = l_3$

$2v_A = v_C$

$2v_C = v_D$

$v_A = -2v_D$

$2(2v_B) = v_D$

$v_A = -2(4v_B)$

$v_A = -8v_B$

$v_A = -8(-4) = 32\text{ m/s}\downarrow$ **Ans.**

Ans:
$v_A = 32\text{ m/s}\downarrow$

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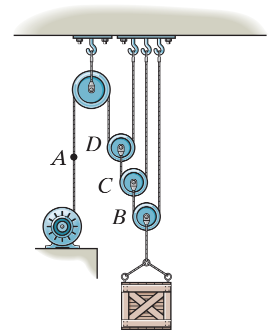
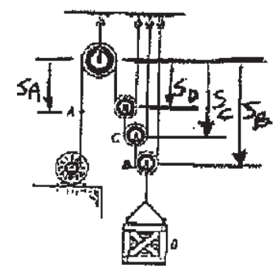
12–199.

Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



SOLUTION



$v_B = \dfrac{6}{1.5} = 4\text{ m/s} \uparrow$

$s_B + (s_B - s_C) = l_1$

$s_C + (s_C - s_D) = l_2$

$s_A + 2s_D) = l_3$

Thus,

$2s_B - s_C = l_1$

$2s_C - s_D = l_2$

$s_A + 2s_D = l_3$

$2v_B = v_C$

$2v_C = v_D$

$v_A = -2v_D$

$v_A = -8v_B$

$3\,t^2 = -8v_B$

$v_B = \dfrac{-3}{8}t^2$

$s_B = \displaystyle\int_0^t \frac{-3}{8}t^2\,dt$

$s_B = \dfrac{-1}{8}t^3$

$-7 = \dfrac{-1}{8}t^3$

$t = 3.83\text{ s}$ **Ans.**

Ans:
$t = 3.83\text{ s}$

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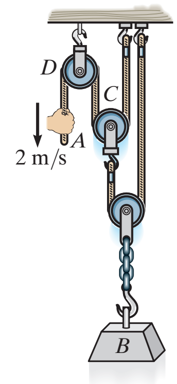
***12–200.**

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

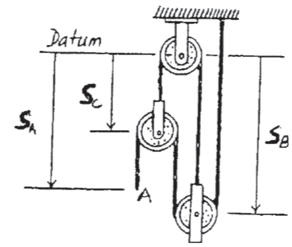


SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$



Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

Since $v_A = 2$ m/s, from Eq. (3)

$(+\downarrow)$
$$2 + 4v_B = 0$$

$$v_B = -0.5\text{ m/s} = 0.5\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

Ans:
$v_B = 0.5$ m/s

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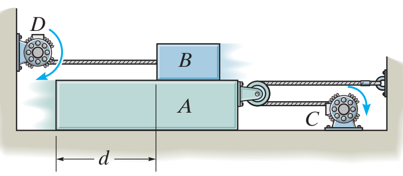
12–201.

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

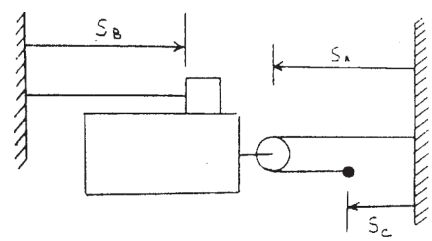
$2v_A = v_C$

$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

$s_A = 0.125\,t^4 \quad \rightarrow$



For B:

$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07$ s **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050$

$v_B = 5(1.0656) = 5.3281$ m/s

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93$ m/s $\quad \rightarrow$ **Ans.**

Ans:
$t = 1.07$ s
$v_{A/B} = 5.93$ ms/s $\rightarrow$

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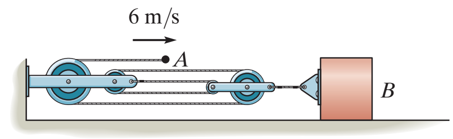
12–202.

Determine the speed of the block at B.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A respectively as shown in Fig. a. This is a single cord pulley system. Thus,

$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b \qquad \textbf{(1)}$$

Time Derivative. Taking the time derivative of Eq. (1),

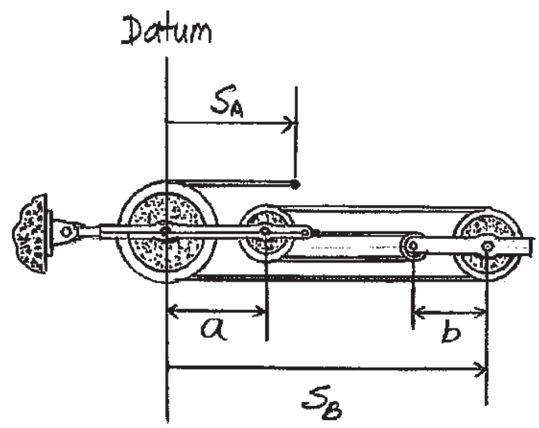
$$4v_B + v_A = 0 \qquad \textbf{(2)}$$

Here, $v_A = +6$ m/s since it is directed toward the positive sense of s_A. Thus,

$$4v_B + 6 = 0$$

$$v_B = -1.50 \text{ m/s} = 1.50 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed towards negative sense of s_B.



(a)

Ans:
$v_B = 1.50$ m/s

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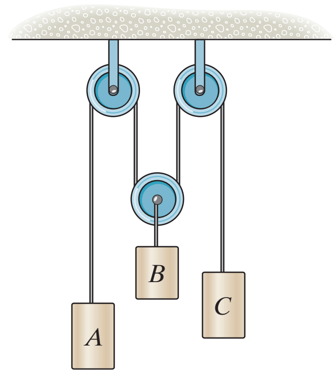
12-203. If block A is moving downward with a speed of 2 m/s while C is moving up at 1 m/s, determine the speed of block B.



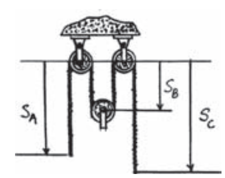
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

$2 + 2v_B - 1 = 0$

$v_B = -0.5\text{ m/s} = 0.5\text{ m/s}\ \uparrow$ **Ans.**



Ans:

$2 + 2v_B - 1 = 0$

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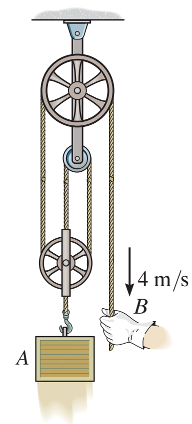
***12–204.**

Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

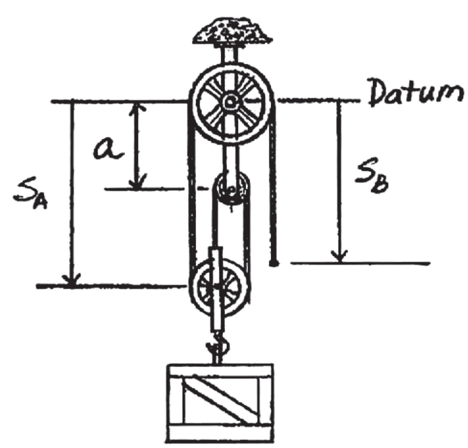
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

$$4 + 3v_A = 0 \qquad v_A = -133 \text{ m/s} = 1.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

Ans:
$v_A = 1.33$ m/s

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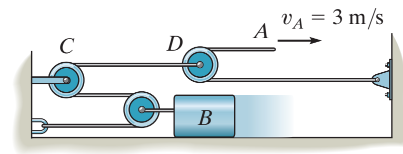
12–205.

If the end A of the cable is moving at $v_A = 3\text{ m/s}$, determine the speed of block B.



SOLUTION

Position Coordinates. The positions of pulley B, D and point A are specified by position coordinates s_B, s_D and s_A respectively as shown in Fig. *a*. The pulley system consists of two cords which give

$$2\,s_B + s_D = l_1 \qquad \textbf{(1)}$$

and

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$2v_B + v_D = 0 \qquad \textbf{(3)}$$

$$v_A - 2v_D = 0 \qquad \textbf{(4)}$$

Eliminate v_0 from Eqs. (3) and (4),

$$v_A + 4v_B = 0 \qquad \textbf{(5)}$$

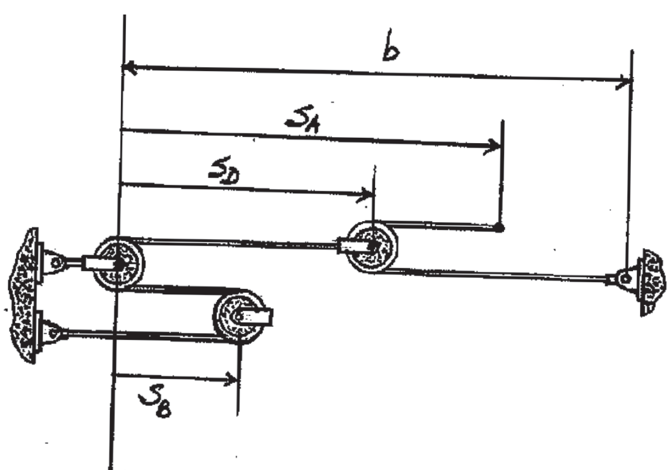
Here $v_A = +3\text{ m/s}$ since it is directed toward the positive sense of s_A.

Thus

$$3 + 4v_B = 0$$

$$v_B = -0.75\text{ m/s} = 0.75\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_D$ is directed toward the negative sense of $\mathbf{s}_B$.



(a)

Ans:
$v_B = 0.75\text{ m/s}$

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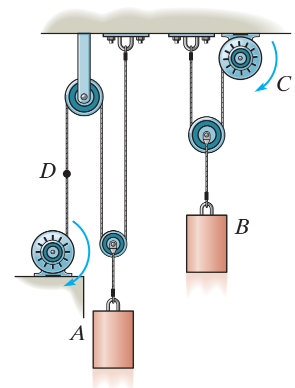
12–206.

The motor draws in the cable at C with a constant velocity of $v_C = 4$ m/s. The motor draws in the cable at D with a constant acceleration of $a_D = 8$ m/s^2. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

(a) $a_D = 8 \text{ m/s}^2$

$v_D = 8\,t$

$s_D = 4\,t^2$

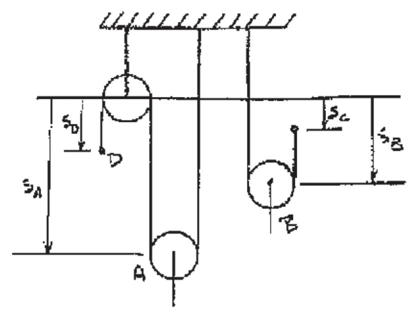
$s_D + 2s_A = l$

$\Delta s_D = -2\Delta s_A$ **(1)**

$\Delta s_A = -2\,t^2$

$-3 = -2\,t^2$

$t = 1.2247 = 1.22 \text{ s}$ **Ans.**



(b) $v_A = s_A = -4\,t = -4(1.2247) = -4.90 \text{ m/s} = 4.90 \text{ m/s}\uparrow$

$s_B + (s_B - s_C) = l$

$2v_B = v_C = -4$

$v_B = -2 \text{ m/s} = 2 \text{ m/s}\uparrow$

$(+\downarrow)$ $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-4.90 = -2 + v_{A/B}$

$v_{A/B} = -2.90 \text{ m/s} = 2.90 \text{ m/s}\uparrow$ **Ans.**

Ans:
$v_{A/B} = 2.90 \text{ m/s}\uparrow$

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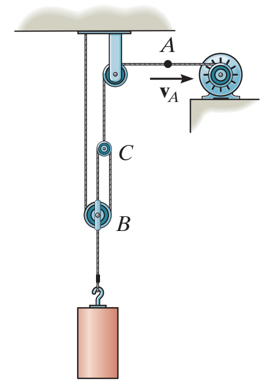
12–207.

Determine the time needed for the load at B to attain a speed of 10 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 3 m/s^2.



SOLUTION

Position Coordinates. The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. a. The pulley system consists of two cords which gives

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \qquad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivative twice of Eqs. (1) and (2),

$$3a_B - 2a_C = 0 \qquad \textbf{(3)}$$

And

$$a_C + a_A = 0 \qquad \textbf{(4)}$$

Eliminate a_C from Eqs. (3) and (4)

$$3a_B + 2a_A = 0$$

Here, $a_A = +3\text{ m/s}^2$ since it is directed toward the positive sense of s_A. Thus,

$$3a_B + 2(3) = 0 \qquad a_B = -2\text{ m/s}^2 = 2\text{ m/s}^2 \uparrow$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of s_B. Applying kinematic equation of constant acceleration,

$+\uparrow \qquad v_B = (v_B)_0 + a_B t$

$$10 = 0 + 2t$$

$$t = 5.00\text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 5.00$ s

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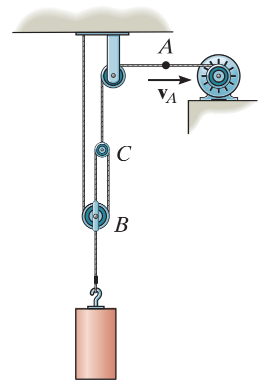
***12–208.**

The cable at A is being drawn toward the motor at $v_A = 8$ m/s. Determine the velocity of the block.



SOLUTION

Position Coordinates. The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. a. The pulley system consists of two cords which give

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \qquad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$3v_B - 2v_C = 0 \qquad \textbf{(3)}$$

And

$$v_C + v_A = 0 \qquad \textbf{(4)}$$

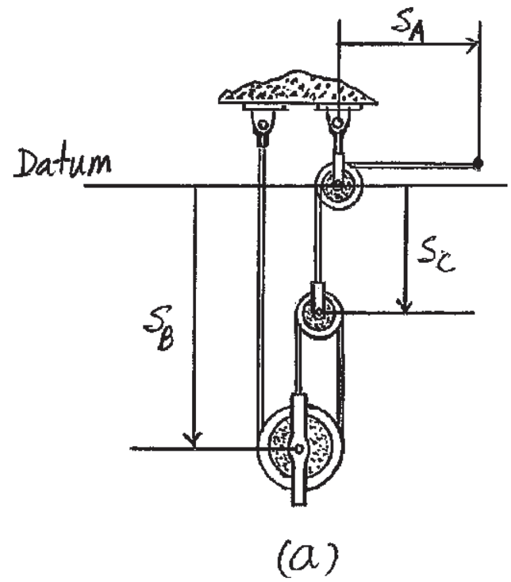
Eliminate v_C from Eqs. (3) and (4),

$$3v_B + 2v_A = 0$$

Here $v_A = +8$ m/s since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 2(8) = 0 \qquad v_B = -5.33 \text{ m/s} = 5.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of s_B.



(a)

Ans:
$v_B = 5.33 \text{ m/s}\uparrow$

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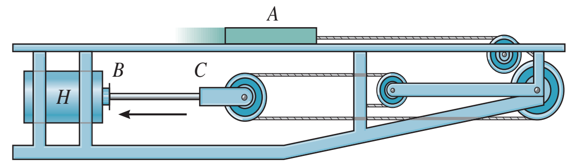
12-209. If the hydraulic cylinder H draws in rod BC at 1 m/s, determine the speed of slider A.



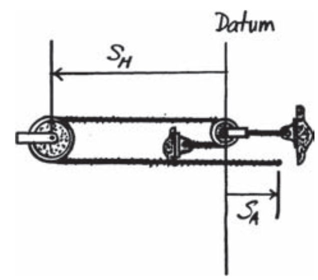
SOLUTION

$2s_H + s_A = l$

$2v_H = -v_A$

$2(1) = -v_A$

$v_A = -2 \text{ m/s} = 2 \text{ m/s} \leftarrow$ **Ans.**



Ans:
$v_A = 2 \text{ m/s} \leftarrow$

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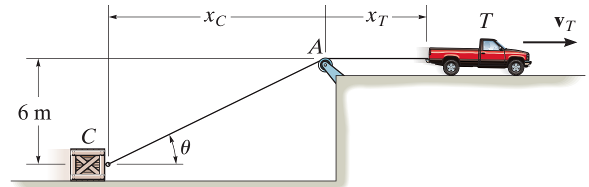
12–210. If the truck travels at a constant speed of $v_T = 1.8\ \text{m/s}$, determine the speed of the crate for any angle θ of the rope. The rope has a length of 30 m and passes over a pulley of negligible size at A. *Hint:* Relate the coordinates x_T and x_C to the length of the rope and take the time derivative. Then substitute the trigonometric relation between x_C and θ.



SOLUTION

$$\sqrt{(6)^2 + x_C^2} + x_T = l = 30$$

$$\frac{1}{2}\left((6)^2 + (x_C)^2\right)^{-\frac{1}{2}}\left(2x_C\dot{x}_C\right) + \dot{x}_T = 0$$

Since $\dot{x}_T = v_T = 1.8\ \text{m/s}$, $v_c = \dot{x}_c$, and

$$x_C = 6 \text{ ctn}\,\theta$$

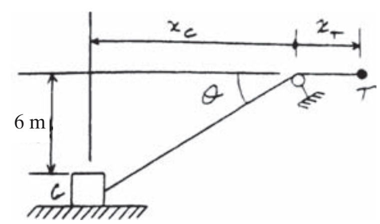
Then,

$$\frac{(6 \text{ ctn}\,\theta)v_C}{(36 + 36 \text{ ctn}^2\,\theta)^{\frac{1}{2}}} = -1.8$$

Since $1 + \text{ctn}^2\,\theta = \csc^2\theta$,

$$\left(\frac{\text{ctn}\,\theta}{\csc\theta}\right)v_C = \cos\theta v_C = -1.8$$

$$v_C = -1.8\sec\theta = (1.8\sec\theta)\ \text{m/s} \rightarrow$$ **Ans.**



Ans:
$v_C = (1.8\sec\theta)\ \text{m/s} \rightarrow$

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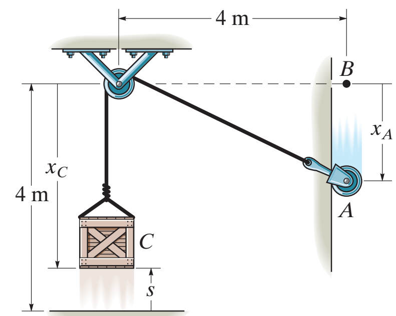
12–211.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2\text{ m/s}$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1\text{ m}$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

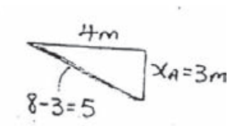


SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$



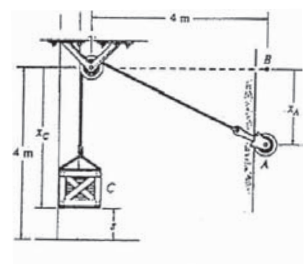
$l = 8\text{ m}$, and when $s = 1\text{ m}$,

$x_C = 3\text{ m}$

$x_A = 3\text{ m}$

$v_A = \dot{x}_A = 2\text{ m/s}$

$a_A = \ddot{x}_A = 0$



Thus,

$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

$v_C = -1.2\text{ m/s} = 1.2\text{ m/s}\uparrow$ **Ans.**

$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

$a_C = -0.512\text{ m/s}^2 = 0.512\text{ m/s}^2\uparrow$ **Ans.**

Ans:

$v_C = 1.2\text{ m/s}\uparrow$
$a_C = 0.512\text{ m/s}^2\uparrow$

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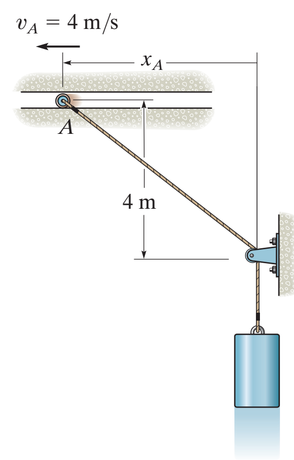
***12–212.**

The roller at A is moving with a velocity of $v_A = 4\text{ m/s}$ and has an acceleration of $a_A = 2\text{ m/s}^2$ when $x_A = 3\text{ m}$. Determine the velocity and acceleration of block B at this instant.



SOLUTION

Position Coordinates. The position of roller A and block B are specified by position coordinates x_A and y_B respectively as shown in Fig. a. We can relate these two position coordinates by considering the length of the cable, which is constant

$$\sqrt{x_A^2 + 4^2} + y_B = l$$

$$y_B = l - \sqrt{x_A^2 + 16} \qquad \textbf{(1)}$$

Velocity. Taking the time derivative of Eq. (1) using the chain rule,

$$\frac{dy_B}{dt} = 0 - \frac{1}{2}\left(x_A^2 + 16\right)^{-\frac{1}{2}}(2x_A)\frac{dx_A}{dt}$$

$$\frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 16}}\frac{dx_A}{dt}$$

However, $\dfrac{dy_B}{dt} = v_B$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 16}}v_A \qquad \textbf{(2)}$$

At $x_A = 3\text{ m}$, $v_A = +4\text{ m/s}$ since $\mathbf{v}_A$ is directed toward the positive sense of x_A. Then Eq. (2) give

$$v_B = -\frac{3}{\sqrt{3^2 + 16}}(4) = -2.40\text{ m/s} = 2.40\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of y_B.

Acceleration. Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[x_A\left(-\frac{1}{2}\right)(x_A^2 + 16)^{-3/2}(2x_A)\frac{dx_A}{dt} + (x_A^2 + 16)^{-1/2}\frac{dx_A}{dt}\right]$$

$$v_A - x_A(x_A^2 + 16)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{dx_A}{dt} = v_A$. Then

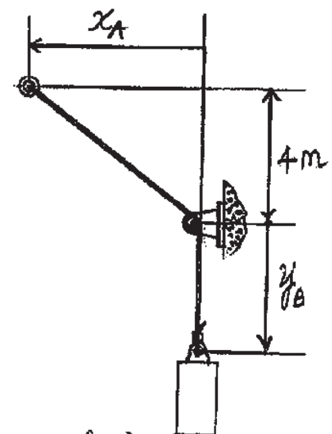
$$a_B = \frac{x_A^2 v_A^2}{\left(x_A^2 + 16\right)^{3/2}} - \frac{v_A^2}{\left(x_A^2 + 16\right)^{1/2}} - \frac{x_A a_A}{\left(x_A^2 + 16\right)^{1/2}}$$

$$a_B = -\frac{16\,v_A^2 + a_A x_A\left(x_A^2 + 16\right)}{\left(x_A^2 + 16\right)^{3/2}}$$

At $x_A = 3\text{ m}$, $v_A = +4\text{ m/s}$, $a_A = +2\text{ m/s}^2$ since $\mathbf{v}_A$ and a_A are directed toward the positive sense of x_A.

$$a_B = -\frac{16\left(4^2\right) + 2(3)\left(3^2 + 16\right)}{\left(3^2 + 16\right)^{3/2}} = -3.248\text{ m/s}^2 = 3.25\text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of y_B.



(a)

Ans:
$v_B = 2.40\text{ m/s}\uparrow$
$a_B = 3.25\text{ m/s}^2\uparrow$

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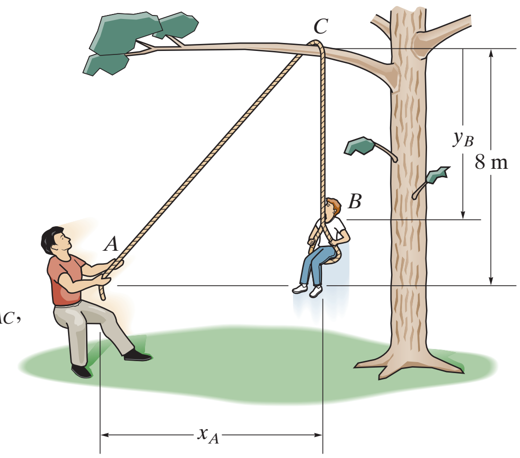
12–213.

The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) and realizing that $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $x_A = 4$ m, from Eq. [2]

$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

Ans:
$v_B = 0.671$ m/s $\uparrow$

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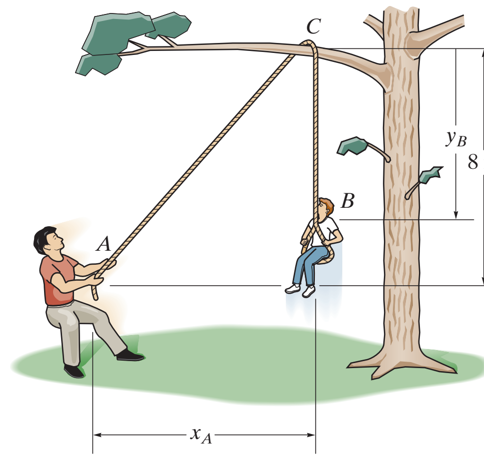
12–214.

The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) Where $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $y_B = 4$ m, from Eq. (1), $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A\left[s_A - (s_0)_A\right]$$

$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$

$$v_A = 1.891\ \text{m/s}$$

Substitute the above results into Eq. (2) yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

Ans:

$v_B = 1.41\ \text{m/s}\uparrow$

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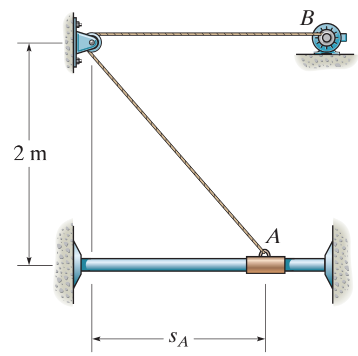
12–215.

The motor draws in the cord at B with an acceleration of $a_B = 2\ \text{m/s}^2$. When $s_A = 1.5$ m, $v_B = 6$ m/s. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

Position Coordinates. The position of collar A and point B are specified by s_A and s_C respectively as shown in Fig. a. We can relate these two position coordinates by considering the length of the cable, which is constant.

$$s_B + \sqrt{s_A^2 + 2^2} = l$$

$$s_B = l - \sqrt{s_A^2 + 4} \tag{1}$$

Velocity. Taking the time derivative of Eq. (1),

$$\frac{ds_B}{dt} = 0 - \frac{1}{2}\left(s_A^2 + 4\right)^{-1/2}\left(2s_A\frac{ds_A}{dt}\right)$$

$$\frac{ds_B}{dt} = -\frac{s_A}{\sqrt{s_A^2 + 4}}\frac{ds_A}{dt}$$

However, $\dfrac{ds_B}{dt} = v_B$ and $\dfrac{ds_A}{dt} = v_A$. Then this equation becomes

$$v_B = -\frac{s_A}{\sqrt{s_A^2 + 4}}v_A \tag{2}$$

At the instant $s_A = 1.5$ m, $v_B = +6$ m/s. v_B is positive since it is directed toward the positive sense of s_B.

$$6 = -\frac{1.5}{\sqrt{1.5^2 + 4}}v_A$$

$$v_A = -10.0\ \text{m/s} = 10.0\ \text{m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_A is directed toward the negative sense of s_A.

Acceleration. Taking the time derivative of Eq. (2),

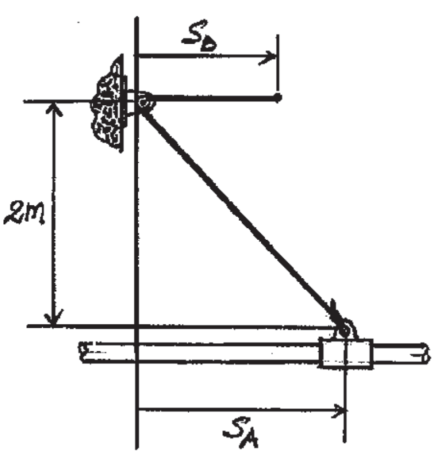
$$\frac{dv_B}{dt} = -\left[s_A\left(-\frac{1}{2}\right)\left(s_A^2 + 4\right)^{-3/2}\left(2s_A\frac{ds_A}{dt}\right) + \left(s_A^2 + 4\right)^{-1/2}\frac{ds_A}{dt}\right]$$

$$v_A - s_A\left(s_A^2 + 4\right)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{ds_A}{dt} = v_A$. Then

$$a_B = \frac{s_A^2 v_A^2}{\left(s_A^2 + 4\right)^{3/2}} - \frac{v_A^2}{\left(s_A^2 + 4\right)^{1/2}} - \frac{a_A s_A}{\left(s_A^2 + 4\right)^{1/2}}$$

$$a_B = -\frac{4v_A^2 + a_A s_A\left(s_A^2 + 4\right)}{\left(s_A^2 + 4\right)^{3/2}}$$



(a)

At the instant $s_A = 1.5$ m, $a_B = +2\ \text{m/s}^2$. $\mathbf{a}_B$ is positive since it is directed toward the positive sense of s_B. Also, $v_A = -10.0$ m/s. Then

$$2 = -\left[\frac{4(-10.0)^2 + a_A(1.5)(1.5^2 + 4)}{(1.5^2 + 4)^{3/2}}\right]$$

$$a_A = -46.0\ \text{m/s}^2 = 46.0\ \text{m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_A$ is directed toward the negative sense of s_A.

Ans:
$v_A = 10.0\ \text{m/s} \leftarrow$
$a_A = 46.0\ \text{m/s}^2 \leftarrow$

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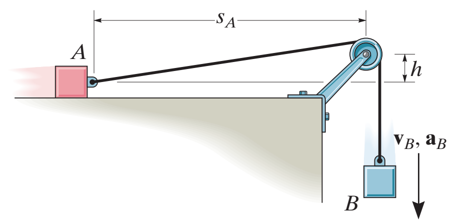
***12–216.**

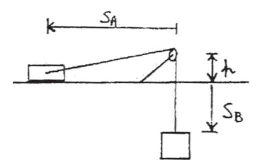
If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_B^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2} 2 s_A \, \dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\,\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\,(1 + \left(\frac{h}{s_A}\right)^2)^{1/2}$$ **Ans.**

$$a_A = \dot{v}_A = \;-\,\dot{v}_B(1 + \left(\frac{h}{s_A}\right)^2)^{1/2} - v_B\left(\frac{1}{2}\right)(1 + \left(\frac{h}{s_A}\right)^2)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -\,a_B(1+\left(\frac{h}{s_B}\right)^2)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\,(1+\left(\frac{h}{s_A}\right)^2)^{-1/2}$$ **Ans.**

Ans:

$$v_A = -v_B\,(1+\left(\frac{h}{s_A}\right)^2)^{1/2}$$

$$a_A = -a_B\,(1+\left(\frac{h}{s_B}\right)^2)^{1/2} + \frac{v_A v_B h^2}{s_A^3}(1+\left(\frac{h}{S_A}\right)^2)^{-1/2}$$

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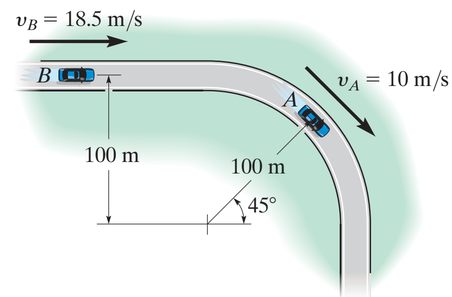
12–217.

At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = 10\cos 45°\mathbf{i} - 10\sin 45°\mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\}\ \text{m/s}$$

$$v_B = \{18.5\mathbf{i}\}\ \text{m/s}$$

$$v_{A/B} = v_A - v_B$$

$$= (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\}\ \text{m/s}$$

$$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4\ \text{m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{7.071}{11.429} = 31.7° \qquad \textbf{Ans.}$$

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1\ \text{m/s}^2 \qquad (a_A)_t = 5\ \text{m/s}^2$$

$$\mathbf{a}_A = (5\cos 45° - 1\cos 45°)\mathbf{i} + (-1\sin 45° - 5\sin 45°)\mathbf{j}$$

$$= \{2.828\mathbf{i} - 4.243\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{a}_B = \{2\mathbf{i}\}\ \text{m/s}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{u}_B$$

$$= (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\}\ \text{m/s}^2$$

$$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{4.243}{0.828} = 79.0° \qquad \textbf{Ans.}$$

Ans:

$v_{A/B} = 13.4$ m/s

$\theta_v = 31.7°$

$a_{A/B} = 4.32$ m/s^2

$\theta_a = 79.0°$

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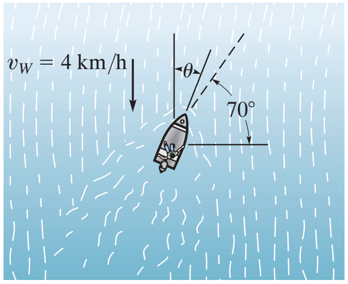
12–218.

The boat can travel with a speed of 16 km/h in still water. The point of destination is located along the dashed line. If the water is moving at 4 km/h, determine the bearing angle θ at which the boat must travel to stay on course.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_W + \mathbf{v}_{B/W}$

$v_B \cos 70°\mathbf{i} + v_B \sin 70°\mathbf{j} = -4\mathbf{j} + 16 \sin \theta\mathbf{i} + 16 \cos \theta\mathbf{j}$

$(\overset{+}{\rightarrow})$ $\quad v_B \cos 70° = 0 + 16 \sin \theta$

$(+\uparrow)$ $\quad v_B \sin 70° = -4 + 16 \cos \theta$

$2.748 \sin \theta - \cos \theta + 0.25 = 0$

Solving,

$\theta = 15.1°$ **Ans.**

Ans:
$\theta = 15.1°$

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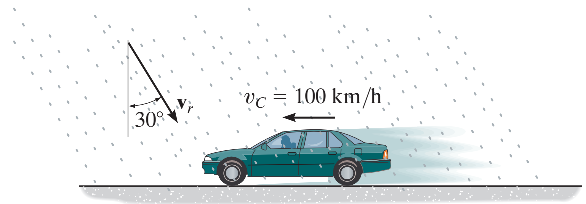
12-219. The car is traveling at a constant speed of 100 km/h. If the rain is falling at 6 m/s in the direction shown, determine the velocity of the rain as seen by the driver.



SOLUTION

Solution I

Vector Analysis: The speed of the car is $v_c = \left(100\,\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000\text{ m}}{1\text{ km}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)$ $= 27.78$ m/s. The velocity of the car and the rain expressed in Cartesian vector form are $\mathbf{v}_c = [-27.78\mathbf{i}]$ m/s and $\mathbf{v}_r = [6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}]$ m/s. Applying the relative velocity equation, we have

$$\mathbf{v}_r = \mathbf{v}_c + \mathbf{v}_{r/c}$$

$$3\mathbf{i} - 5.196\mathbf{j} = -27.78\mathbf{i} + \mathbf{v}_{r/c}$$

$$\mathbf{v}_{r/c} = [30.78\mathbf{i} - 5.196\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{r/c}$ is given by

$$v_{r/c} = \sqrt{30.78^2 + (-5.196)^2} = 31.2\text{ m/s} \qquad \textbf{Ans.}$$

and the angle $x_{r/c}$ makes with the x axis is

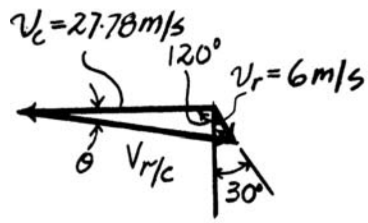
$$\theta = \tan^{-1}\left(\frac{5.196}{30.78}\right) = 9.58° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$v_{r/c} = \sqrt{27.78^2 + 6^2 - 2(27.78)(6)\cos 120°}$$

$$= 19.91\text{ m/s} = 19.9\text{ m/s} \qquad \textbf{Ans.}$$



(a)

Using the result of $v_{r/c}$ and applying the law of sines,

$$\frac{\sin\theta}{6} = \frac{\sin 120°}{31.21}$$

$$\theta = 9.58° \qquad \textbf{Ans.}$$

Ans:
$v_{r/c} = 31.2$ m/s
$\theta = 9.58°$
$v_{r/c} = 19.9$ m/s
$\theta = 9.58°$

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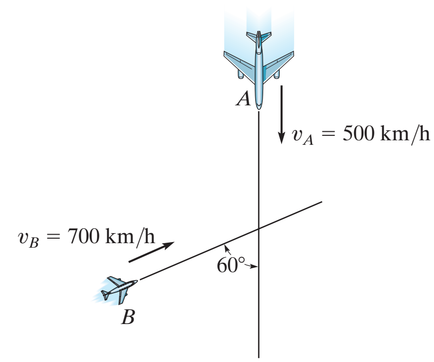
***12–220.**

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 500\text{ km/h}$ and $v_B = 700\text{ km/h}$ such that the angle between their straight-line courses is $\theta = 60°$, determine the velocity of plane B with respect to plane A.



SOLUTION

Relative Velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form,

$$\mathbf{v}_A = \{-500\,\mathbf{j}\}\text{ km/h}$$

$$\mathbf{v}_B = \{700\sin 60°\mathbf{i} + 700\cos 60°\mathbf{j}\}\text{ km/h} = \left\{350\sqrt{3}\mathbf{i} + 350\mathbf{j}\right\}\text{ km/h}$$

Applying the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$350\sqrt{3}\mathbf{i} + 350\mathbf{j} = -500\mathbf{j} + \mathbf{v}_{B/A}$$

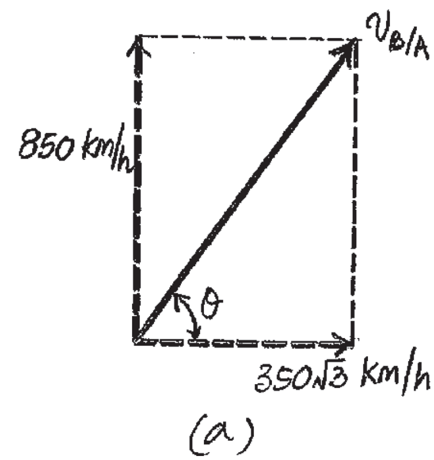
$$\mathbf{v}_{B/A} = \left\{350\sqrt{3}\mathbf{i} + 850\mathbf{j}\right\}\text{km/h}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(350\sqrt{3})^2 + 850^2} = 1044.03\text{ km/h} = 1044\text{ km/h} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. a.

$$\theta = \tan^{-1}\left(\frac{850}{350\sqrt{3}}\right) = 54.50° = 54.5° \measuredangle \qquad \textbf{Ans.}$$



Ans:
$v_{B/A} = 1044\text{ km/h}$
$\theta = 54.5° \measuredangle$

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12–221.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is coming from the east. If the car's speed is 80 km/h, the instrument indicates that the wind is coming from the northeast. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2 \cos 45°\mathbf{i} + (v_{W/C})_2 \sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2 \cos 45°\mathbf{i} + (v_{w/c})_2 \sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2 \cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2 \sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2 \sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \qquad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \qquad \textbf{Ans.}$$

and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \qquad \textbf{Ans.}$$

Ans:
$v_w = 58.3$ km/h
$\theta = 59.0°$

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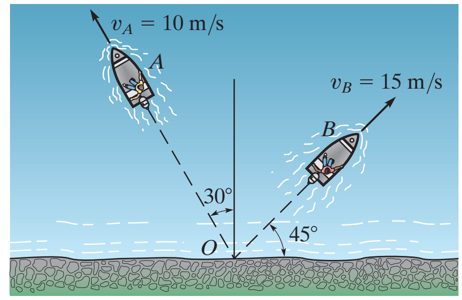
12–222.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10\text{ m/s}$ and $v_B = 15\text{ m/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 600 m apart?



SOLUTION

Relative Velocity. The velocity triangle shown in Fig. *a* is drawn based on the relative velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. Using the cosine law,

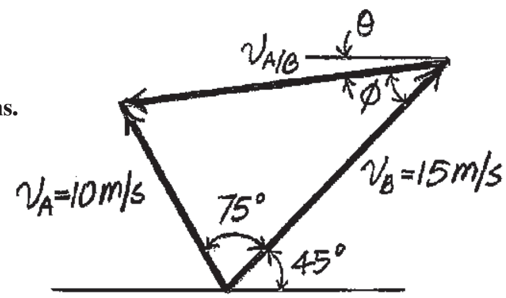
$$v_{A/B} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 75^\circ} = 15.73\text{ m/s} = 15.7\text{ m/s} \qquad \textbf{Ans.}$$

Then, the sine law gives

$$\frac{\sin\phi}{10} = \frac{\sin 75^\circ}{15.73} \qquad \phi = 37.89^\circ$$

The direction of $\mathbf{v}_{A/B}$ is defined by

$$\theta = 45^\circ - \phi = 45^\circ - 37.89^\circ = 7.11^\circ \ \measuredangle \qquad \textbf{Ans.}$$



(a)

Alternatively, we can express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form

$$\mathbf{v}_A = \{-10\sin 30^\circ\mathbf{i} + 10\cos 30^\circ\mathbf{j}\}\text{ m/s} = \{-5.00\mathbf{i} + 5\sqrt{3}\mathbf{j}\}\text{ m/s}$$

$$\mathbf{v}_B = \{15\cos 45^\circ\mathbf{i} + 15\sin 45^\circ\mathbf{j}\}\text{ m/s} = \{7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j}\}\text{ m/s}.$$

Applying the relative velocity equation

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-500\mathbf{i} + 5\sqrt{3}\mathbf{j} = 7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-15.61\mathbf{i} - 1.946\mathbf{j}\}\text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-15.61)^2 + (-1.946)^2} = 15.73\text{ m/s} = 15.7\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *b*,

$$\theta = \tan^{-1}\left(\frac{1.946}{15.61}\right) = 7.1088^\circ = 7.11^\circ \ \measuredangle \qquad \textbf{Ans.}$$

(b)

Here $s_{A/B} = 600\text{ m}$. Thus

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{600}{15.73} = 38.15\text{ s} = 38.1\text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_{A/B} = 15.7\text{ m/s}$
$\theta = 7.11^\circ \ \measuredangle$
$t = 38.1\text{ s}$

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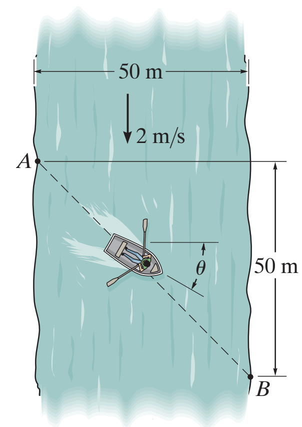
12-223. A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



SOLUTION

Relative Velocity:

$$\mathbf{v}_b = \mathbf{v}_r + \mathbf{v}_{b/r}$$

$$v_b \sin 45°\mathbf{i} - v_b \cos 45°\mathbf{j} = -2\mathbf{j} + 5\cos\theta\mathbf{i} - 5\sin\theta\mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b \sin 45° = 5\cos\theta \qquad \textbf{[1]}$$

$$-v_b \cos 45° = -2 - 5\sin\theta \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta = 28.57°$$

$$v_b = 6.210 \text{ m/s} = 6.21 \text{ m/s} \qquad \textbf{Ans.}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4 \text{ s} \qquad \textbf{Ans.}$$

Ans:

$v_b = 6.21$ m/s
$t = 11.4$ s

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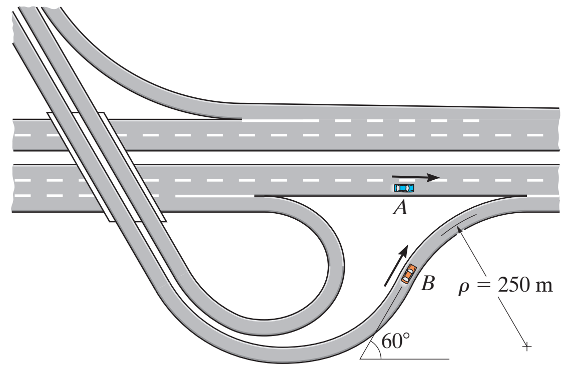
***12–224.** At the instant shown car A is traveling with a velocity of 30 m/s and has an acceleration of 2 m/s^2 along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed of 15 m/s, which is decreasing at 0.8 m/s^2. Determine the relative velocity and relative acceleration of B with respect to A at this instant.



SOLUTION

$$\mathbf{v_{Av}} = \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{a_{Av}} = \begin{pmatrix} a_A \\ 0 \end{pmatrix}$$

$$\mathbf{v_{Bv}} = v_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{a_{Bv}} = v'_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_B^{\,2}}{\rho} \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \end{pmatrix}$$

$v_A = 30\ \text{m/s}$

$v_B = 15\ \text{m/s}$

$a_A = 2\ \text{m/s}^2$

$v'_B = -0.8\ \text{m/s}^2$

$\rho = 250\ \text{m}$

$\theta = 60°$

$$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}} \qquad \mathbf{v_{BA}} = \begin{pmatrix} -22.5 \\ 12.99 \end{pmatrix} \text{m/s} \qquad |\mathbf{v_{BA}}| = 26.0 \qquad \textbf{Ans.}$$

$$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}} \qquad \mathbf{a_{BA}} = \begin{pmatrix} -1.621 \\ -1.143 \end{pmatrix} \text{m/s}^2 \qquad |\mathbf{a_{BA}}| = 1.983\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$|\mathbf{v_{BA}}| = 26.0$

$|\mathbf{a_{BA}}| = 1.983\ \text{m/s}^2$

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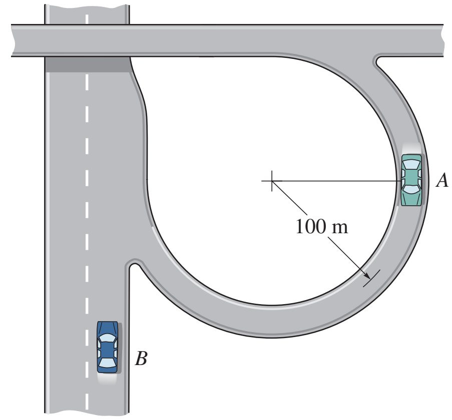
12–225.

At the instant shown, car A has a speed of 20 km/h, which is being increased at the rate of 300 km/h^2 as the car enters an expressway. At the same instant, car B is decelerating at 250 km/h^2 while traveling forward at 100 km/h. Determine the velocity and acceleration of A with respect to B.



SOLUTION

$\mathbf{v}_A = \{-20\mathbf{j}\}$ km/h $\qquad \mathbf{v}_B = \{100\mathbf{j}\}$ km/h

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (-20\mathbf{j} - 100\mathbf{j}) = \{-120\mathbf{j}\} \text{ km/h}$$

$v_{A/B} = 120$ km/h $\downarrow$ **Ans.**

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{20^2}{0.1} = 4000 \text{ km/h}^2 \qquad (a_A)_t = 300 \text{ km/h}^2$$

$$\mathbf{a}_A = -4000\mathbf{i} + (-300\mathbf{j})$$

$$= \{-4000\mathbf{i} - 300\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_B = \{-250\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$

$$= (-4000\mathbf{i} - 300\mathbf{j}) - (-250\mathbf{j}) = \{-4000\mathbf{i} - 50\mathbf{j}\} \text{ km/h}^2$$

$a_{A/B} = \sqrt{(-4000)^2 + (-50)^2} = 4000$ km/h^2 **Ans.**

$\theta = \tan^{-1}\frac{50}{4000} = 0.716°$ **Ans.**

Ans:
$v_{A/B} = 120$ km/h $\downarrow$
$a_{A/B} = 4000$ km/h^2
$\theta = 0.716°$

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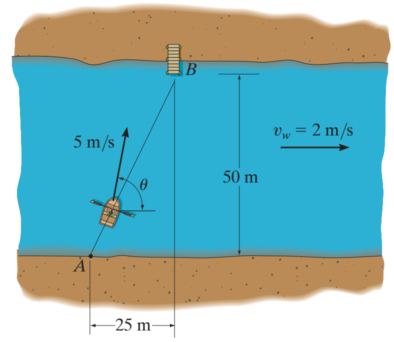
12–226.

The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B.



SOLUTION

Solution I

Vector Analysis: Here, the velocity $\mathbf{v}_b$ of the boat is directed from A to B. Thus, $\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$. The magnitude of the boat's velocity relative to the flowing river is $v_{b/w} = 5$ m/s. Expressing $\mathbf{v}_b$, $\mathbf{v}_w$, and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43\mathbf{i} + v_b \sin 63.43\mathbf{j} = 0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = 2\mathbf{i} + 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = (2 + 5\cos\theta)\mathbf{i} + 5\sin\theta\mathbf{j}$$

Equating the **i** and **j** components, we have

$$0.4472v_b = 2 + 5\cos\theta \qquad \textbf{(1)}$$

$$0.8944v_b = 5\sin\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56 \text{ m/s} \qquad \theta = 84.4° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$5^2 = 2^2 + v_b{}^2 - 2(2)(v_b)\cos 63.43°$$

$$v_b{}^2 - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$

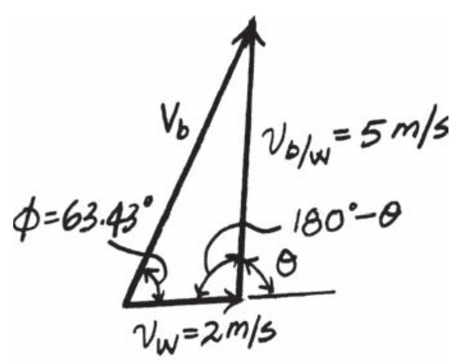
Choosing the positive root,

$$v_b = 5.563 \text{ m/s} = 5.56 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of v_b and applying the law of sines,

$$\frac{\sin(180° - \theta)}{5.563} = \frac{\sin 63.43°}{5}$$

$$\theta = 84.4° \qquad \textbf{Ans.}$$



(a)

Ans:

$v_b = 5.56$ m/s
$\theta = 84.4°$

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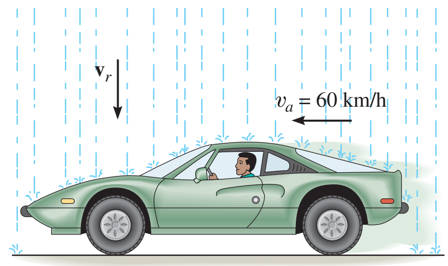
12–227.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION

$v_r = v_a + v_{r/a}$

$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$

$(\overset{+}{\rightarrow})\qquad 0 = -60 + v_{r/a}\cos 30°$

$(+\uparrow)\qquad -v_r = 0 - v_{r/a}\sin 30°$

$v_{r/a} = 69.3$ km/h

$v_r = 34.6$ km/h **Ans.**

Ans:
$v_r = 34.6\ \text{km/h}\downarrow$

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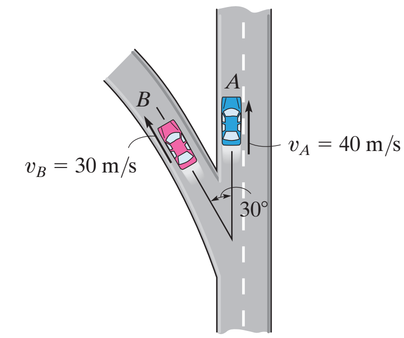
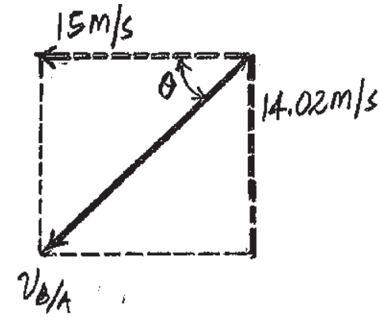
***12–228.** At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If B is increasing its velocity by 2 m/s^2, while A maintains a constant velocity, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ as Cartesian vectors.

$\mathbf{v}_A = \{40\mathbf{j}\}$ m/s $\quad \mathbf{v}_B = \{-30 \sin 30°\mathbf{i} + 30 \cos 30°\mathbf{j}\}$ m/s $= \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}$ m/s



(a)

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\} \text{ m/s}$$

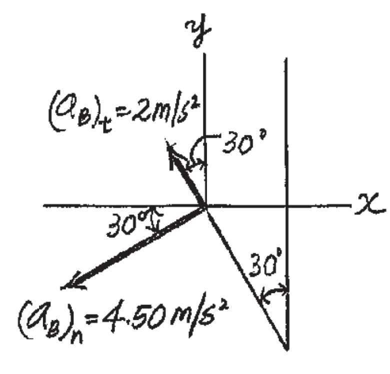
Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53 \text{ m/s} = 20.5 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *a*

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \qquad \textbf{Ans.}$$

Relative Acceleration. Here, $(a_B)_t = 2$ m/s^2 and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.50$ m/s^2 and their directions are shown in Fig. *b*. Then, express $\mathbf{a}_B$ as a Cartesian vector,



(b)

$$\mathbf{a}_B = (-2 \sin 30° - 4.50 \cos 30°)\mathbf{i} + (2 \cos 30° - 4.50 \sin 30°)\mathbf{j}$$

$$= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\} \text{ m/s}^2$$

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

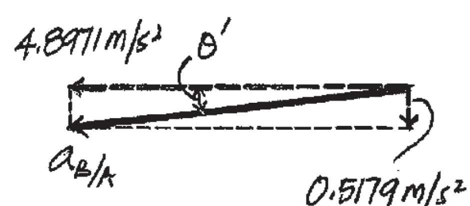
$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244 \text{ m/s}^2 = 4.92 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by angle θ', Fig. *c*,



(c)

$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038° = 6.04° \qquad \textbf{Ans.}$$

Ans:
$v_{B/A} = 20.5$ m/s
$\theta_v = 43.1°$
$a_{B/A} = 4.92$ m/s^2
$\theta_a = 6.04°$

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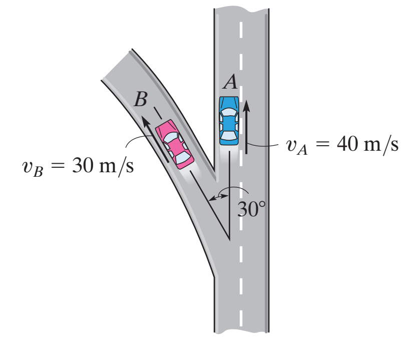
12–229. At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If A is increasing its speed at 4 m/s^2, whereas the speed of B is decreasing at 3 m/s^2, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ as Cartesian vectors.

$$\mathbf{v}_A = \{40\mathbf{j}\}\ \text{m/s} \quad \mathbf{v}_B = \{-30\sin 30°\mathbf{i} + 30\cos 30°\mathbf{j}\}\ \text{m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}\ \text{m/s}$$

Applying the relative velocity equation,

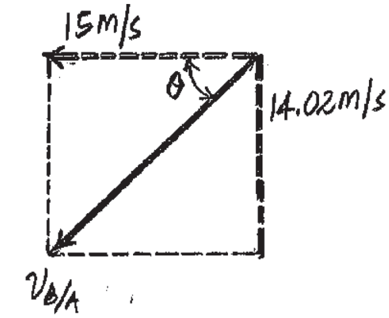
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\}\ \text{m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53\ \text{m/s} = 20.5\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

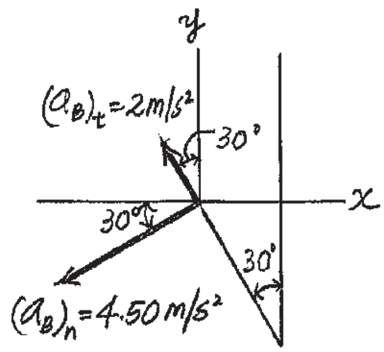
And its direction is defined by angle θ, Fig. a

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \qquad \textbf{Ans.}$$

Relative Acceleration. Here, $(a_B)_t = 2\ \text{m/s}^2$ and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.50\ \text{m/s}^2$ and their directions are shown in Fig. b. Then, express $\mathbf{a}_B$ as a Cartesian vector,

$$\mathbf{a}_B = (-2\sin 30° - 4.50\cos 30°)\mathbf{i} + (2\cos 30° - 4.50\sin 30°)\mathbf{j}$$

$$= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\ \text{m/s}^2$$



(b)

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

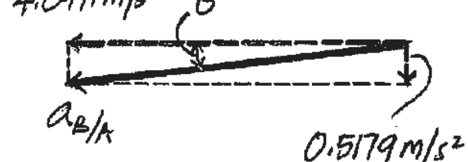
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244\ \text{m/s}^2 = 4.92\ \text{m/s}^2 \qquad \textbf{Ans.}$$



(c)

And its direction is defined by angle θ', Fig. c,

$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038° = 6.04° \qquad \textbf{Ans.}$$

Ans:
$v_{B/A} = 20.5$ m/s
$\theta_v = 43.1°$
$a_{B/A} = 4.92$ m/s^2
$\theta_a = 6.04°$

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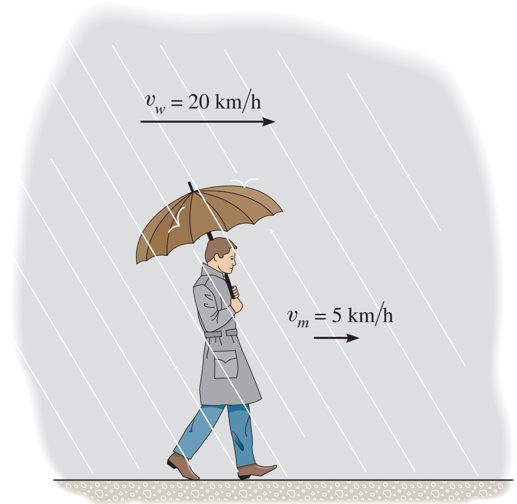
12–230.

A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man.



SOLUTION

Relative Velocity: The velocity of the rain must be determined first. Applying Eq. 12–34 gives

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} = 20\,\mathbf{i} + (-7\,\mathbf{j}) = \{20\,\mathbf{i} - 7\,\mathbf{j}\}\ \text{km/h}$$

Thus, the relatives velocity of the rain with respect to the man is

$$\mathbf{v}_r = \mathbf{v}_m + \mathbf{v}_{r/m}$$

$$20\,\mathbf{i} - 7\,\mathbf{j} = 5\,\mathbf{i} + \mathbf{v}_{r/m}$$

$$\mathbf{v}_{r/m} = \{15\,\mathbf{i} - 7\,\mathbf{j}\}\ \text{km/h}$$

The magnitude of the relative velocity $\mathbf{v}_{r/m}$ is given by

$$v_{r/m} = \sqrt{15^2 + (-7)^2} = 16.6\ \text{km/h} \qquad \textbf{Ans.}$$

And its direction is given by

$$\theta = \tan^{-1}\frac{7}{15} = 25.0° \qquad \textbf{Ans.}$$

Ans:
$v_{r/m} = 16.6$ km/h
$\theta = 25.0°$

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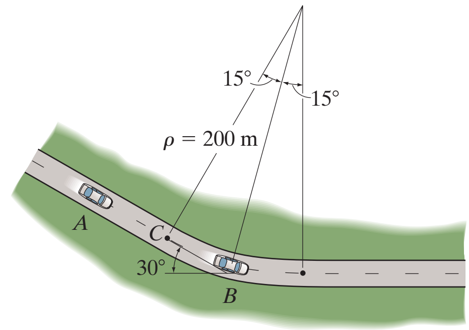
12–231.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.

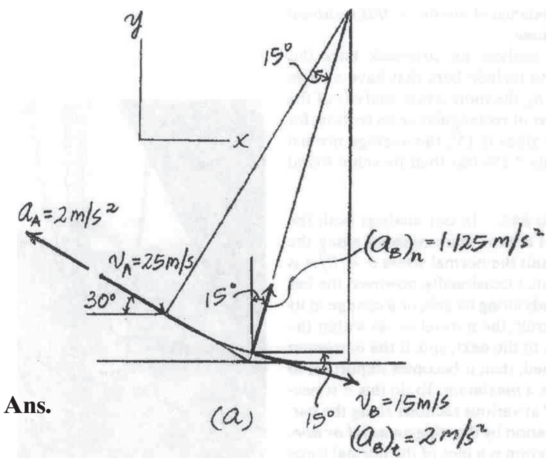


SOLUTION

Velocity: Referring to Fig. *a*, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25\cos 30°\,\mathbf{i} - 25\sin 30°\,\mathbf{j}]\text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_B = [15\cos 15°\,\mathbf{i} - 15\sin 15°\,\mathbf{j}]\text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}]\text{ m/s}$$



Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

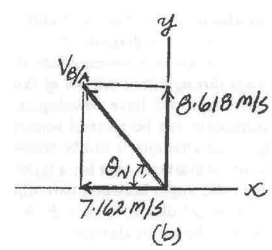
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2\text{ m/s} \qquad \textbf{Ans.}$$

The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3° \qquad \textbf{Ans.}$$



Ans:
$v_{B/A} = 11.2\text{ m/s}$
$\theta = 50.3°$

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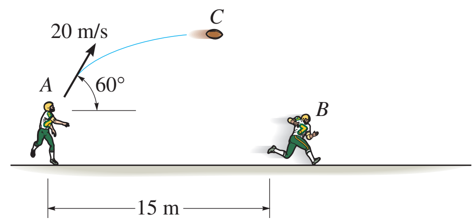
***12–232.**

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$(\overset{+}{\rightarrow})\, s = s_0 + v_0 t$

$s_C = 0 + 20 \cos 60^\circ\, t$

$(+\uparrow) \quad v = v_0 + a_c t$

$-20 \sin 60^\circ = 20 \sin 60^\circ - 9.81\, t$

$t = 3.53$ s

$s_C = 35.31$ m

Player B:

$(\overset{+}{\rightarrow})\ s_B = s_0 + \nu_B\, t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75$ m/s **Ans.**

At the time of the catch

$(v_C)_x = 20 \cos 60^\circ = 10 \text{ m/s} \rightarrow$

$(v_C)_y = 20 \sin 60^\circ = 17.32 \text{ m/s} \downarrow$

$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.751\mathbf{i} + (v_{C/B})_x\mathbf{i} + (v_{C/B})_y\mathbf{j}$

$(\overset{+}{\rightarrow}) \quad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow) \quad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25 \text{ m/s} \rightarrow$

$(v_{C/B})_y = 17.32 \text{ m/s} \downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8$ m/s **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{4.25}\right) = 76.2^\circ$ ⦪ **Ans.**

$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

$-9.81\,\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81 \text{ m/s}^2\downarrow$ **Ans.**

Ans:
$v_B = 5.75$ m/s
$v_{C/B} = 17.8$ m/s
$\theta = 76.2^\circ$ ⦪
$a_{C/B} = 9.81 \text{ m/s}^2\downarrow$

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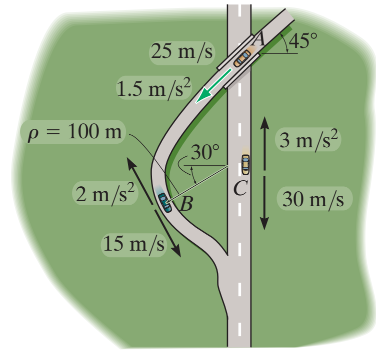
12–233.

Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



SOLUTION

Velocity: The velocity of cars A and C expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s}$$ **Ans.**

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9°$$ **Ans.**

Acceleration: The acceleration of cars A and C expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2$$ **Ans.**

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4°$$ **Ans.**

Ans:
$v_{A/C} = 21.5$ m/s
$\theta_v = 34.9°$
$a_{A/C} = 4.20$ m/s^2
$\theta_a = 75.4°$

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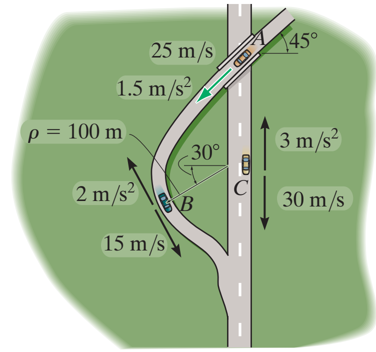
12–234.

Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car B relative to car C.



SOLUTION

Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15\cos 60^\circ\,\mathbf{i} - 15\sin 60^\circ\,\mathbf{j}]\text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2^\circ \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \dfrac{{v_B}^2}{\rho} = \dfrac{15^2}{100} = 2.25\text{ m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2\cos 60^\circ\,\mathbf{i} + 2\sin 60^\circ\mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}]\text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25\cos 30^\circ\,\mathbf{i} + 2.25\sin 30^\circ\,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57^\circ \qquad \textbf{Ans.}$$

Ans:
$v_{B/C} = 18.6$ m/s
$\theta_v = 66.2^\circ$
$a_{B/C} = 0.959\text{ m/s}^2$
$\theta_a = 8.57^\circ$

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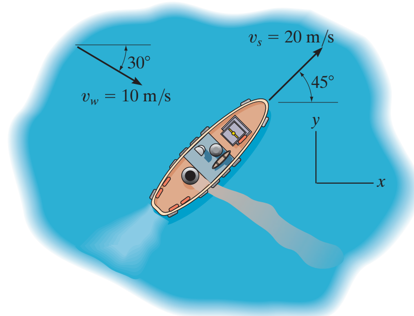
12–235.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10\cos 30°\,\mathbf{i} - 10\sin 30°\,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \qquad \textbf{Ans.}$$

Solution II

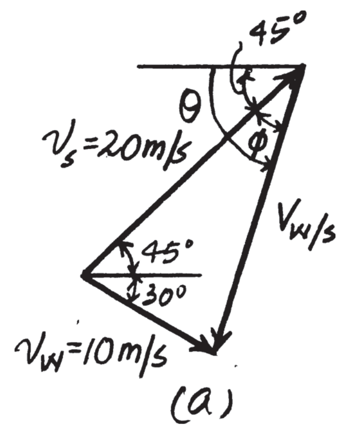
Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,

$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

$$= 19.91 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin\phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$



Thus,

$$\theta = 45° + \phi = 74.0° \qquad \textbf{Ans.}$$

Ans:
$v_{w/s} = 19.9$ m/s
$\theta = 74.0°$

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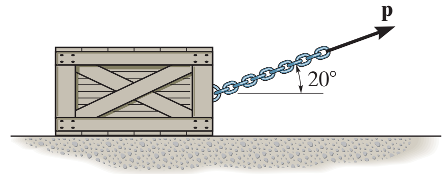
13–1.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



SOLUTION

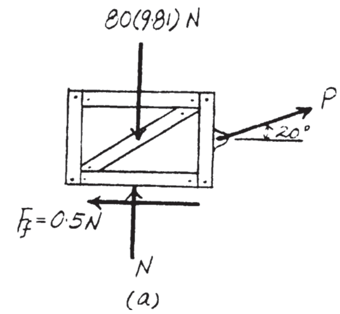
Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N + P\sin 20° - 80(9.81) = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P\cos 20° - 0.5N = 0 \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2) yields

$$P = 353.29 \text{ N} \qquad N = 663.97 \text{ N}$$



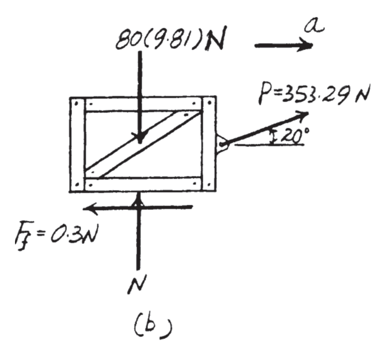
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 353.29\sin 20° = 80(0)$$

$$N = 663.97 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 353.29\cos 20° - 0.3(663.97) = 80a$$

$$a = 1.66 \text{ m/s}^2 \qquad \textbf{Ans.}$$



Ans:
$a = 1.66 \text{ m/s}^2$

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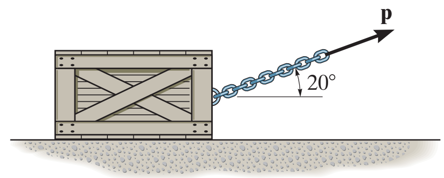
13–2.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



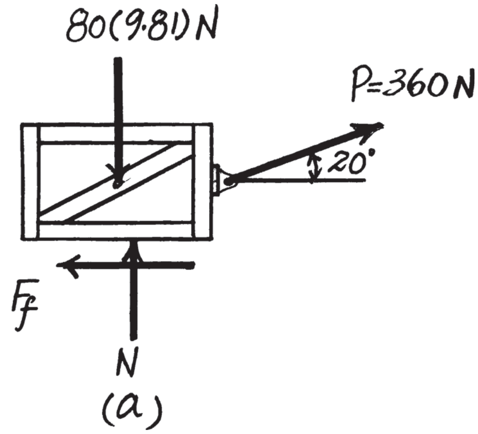
SOLUTION

Equations of Equilibrium: At $t = 2$ s, $P = 90\left(2^2\right) = 360$ N. From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad N + 360 \sin 20° - 80(9.81) = 0 \qquad N = 661.67 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 360 \cos 20° - F_f = 0 \qquad F_f = 338.29 \text{ N}$$

Since $F_f > (F_f)_{\max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.



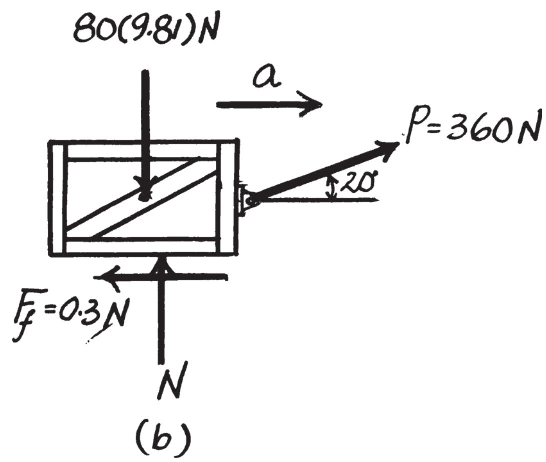
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 360 \sin 20° = 80(0)$$

$$N = 661.67 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 360 \cos 20° - 0.3(661.67) = 80a$$

$$a = 1.75 \text{ m/s}^2$$ **Ans.**



Ans:
$a = 1.75 \text{ m/s}^2$

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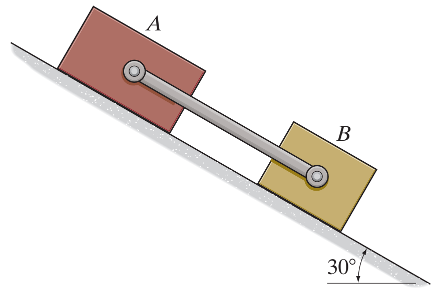
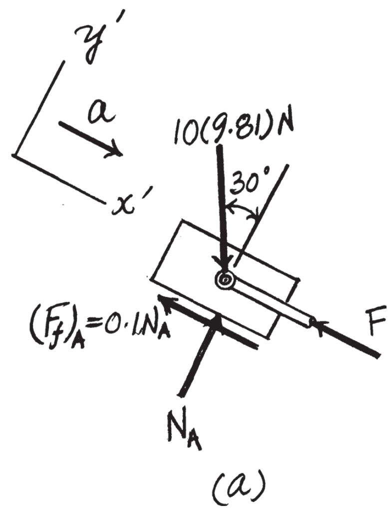
13–3.

If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1N_A$ and $(F_f)_B = \mu_B N_B = 0.3N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration **a**.



Equations of Motion: By referring to Figs. (a) and (b),

$+\nearrow \Sigma F_{y'} = ma_{y'}; \qquad N_A - 10(9.81)\cos 30° = 10(0)$

$N_A = 84.96 \text{ N}$

$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 10(9.81)\sin 30° - 0.1(84.96) - F = 10a$

$40.55 - F = 10a \qquad$ **(1)**

and

$+\nearrow \Sigma F_{y'} = ma_{y'}; \qquad N_B - 6(9.81)\cos 30° = 6(0)$

$N_B = 50.97 \text{ N}$

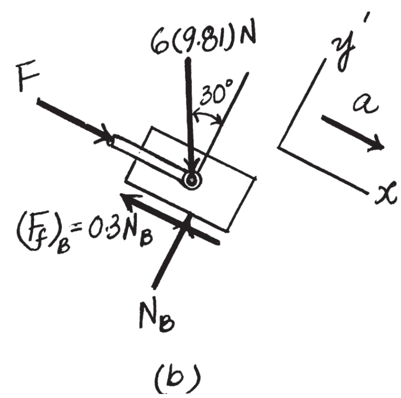
$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad F + 6(9.81)\sin 30° - 0.3(50.97) = 6a$

$F + 14.14 = 6a \qquad$ **(2)**

Solving Eqs. (1) and (2) yields

$a = 3.42 \text{ m/s}^2$

$F = 6.37 \text{ N} \qquad$ **Ans.**



Ans:
$F = 6.37 \text{ N}$

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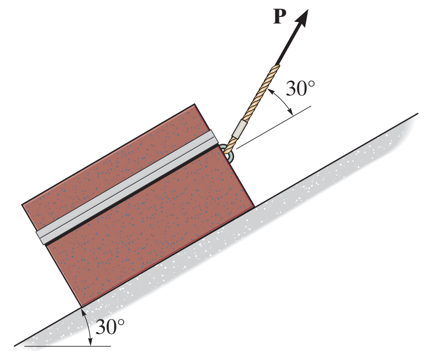
***13–4.**

If $P = 400$ N and the coefficient of kinetic friction between the 50-kg crate and the inclined plane is $\mu_k = 0.25$, determine the velocity of the crate after it travels 6 m up the plane. The crate starts from rest.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is assumed to be directed up the plane. The acceleration **a** of the crate is also assumed to be directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

$$\Sigma F_{y'} = ma_{y'}; \qquad N + 400\sin 30° - 50(9.81)\cos 30° = 50(0)$$

$$N = 224.79 \text{ N}$$

Using the result of **N**,

$$\Sigma F_{x'} = ma_{y'}; \qquad 400\cos 30° - 50(9.81)\sin 30° - 0.25(224.79) = 50a$$

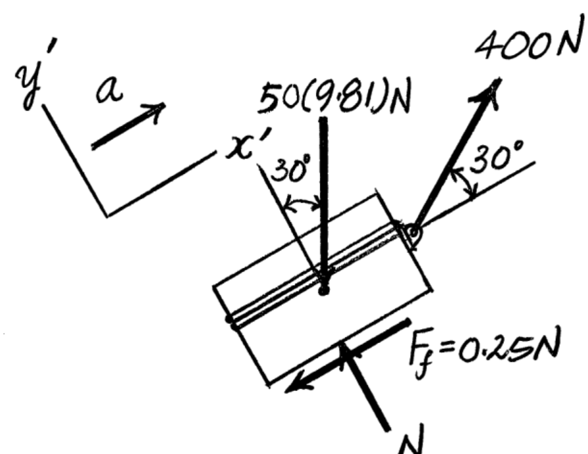
$$a = 0.8993 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(0.8993)(6 - 0)$$

$$v = 3.29 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 3.29$ m/s

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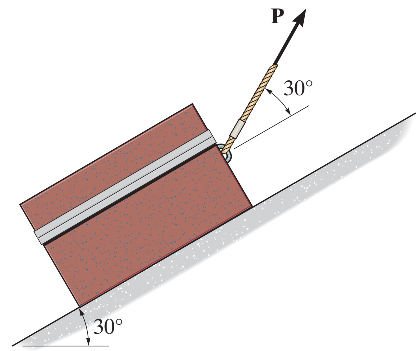
13–5.

If the 50-kg crate starts from rest and travels a distance of 6 m up the plane in 4 s, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION

Kinematics: Here, the acceleration **a** of the crate will be determined first since its motion is known.

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$6 = 0 + 0 + \frac{1}{2}a(4^2)$$

$$a = 0.75\ \text{m/s}^2$$

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

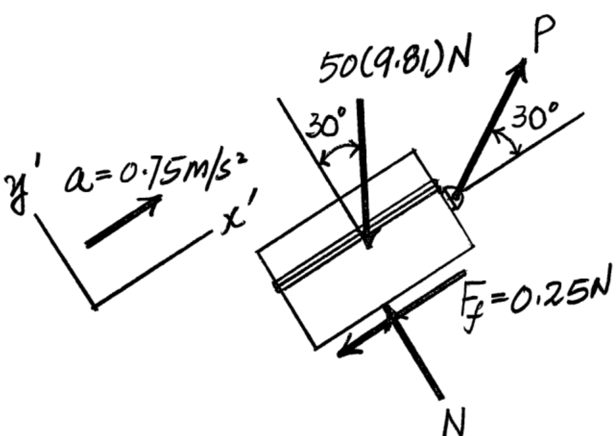
$\Sigma F_{y'} = ma_{y'};\qquad N + P\sin 30° - 50(9.81)\cos 30° = 50(0)$

$$N = 424.79 - 0.5P$$

Using the results of **N** and **a**,

$\Sigma F_{x'} = ma_{x'};\qquad P\cos 30° - 0.25(424.79 - 0.5P) - 50(9.81)\sin 30° = 50(0.75)$

$P = 392\ \text{N}$ **Ans.**



(a)

Ans:
$P = 392$ N

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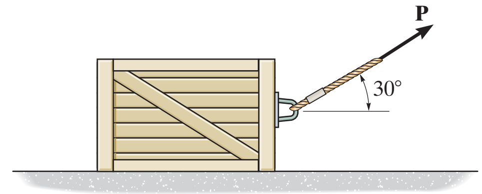
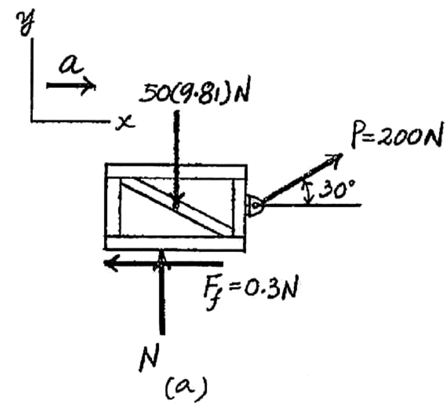
13–6.

If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when $t = 3$ s. The crate starts from rest, and $P = 200$ N.



SOLUTION

Free-Body Diagram: The kinetic friction $F_f = \mu_k N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.



Equations of Motion: Here, $a_y = 0$. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad N - 50(9.81) + 200 \sin 30° = 0$$

$$N = 390.5 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 200 \cos 30° - 0.3(390.5) = 50a$$

$$a = 1.121 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v = 0 + 1.121(3) = 3.36 \text{ m/s} \qquad \textbf{Ans.}$$

and

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.121)(3^2) = 5.04 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$v = 3.36$ m/s
$s = 5.04$ m

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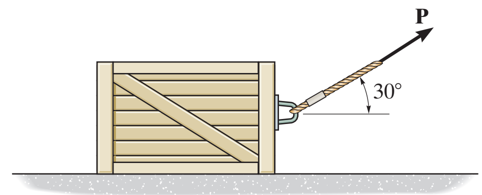
13–7.

If the 50-kg crate starts from rest and achieves a velocity of $v = 4\ \text{m/s}$ when it travels a distance of 5 m to the right, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



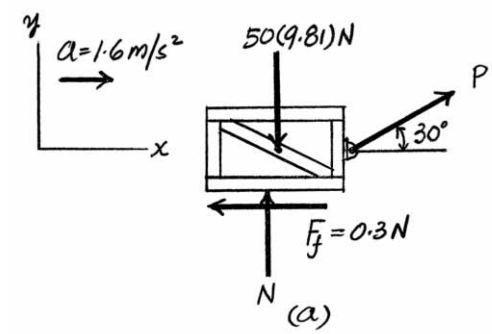
SOLUTION

Kinematics: The acceleration **a** of the crate will be determined first since its motion is known.

$(\overset{+}{\rightarrow})\qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$$4^2 = 0^2 + 2a(5 - 0)$$

$$a = 1.60\ \text{m/s}^2 \rightarrow$$



Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.3N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.

Equations of Motion:

$+\uparrow \Sigma F_y = ma_y;\qquad N + P\sin 30° - 50(9.81) = 50(0)$

$$N = 490.5 - 0.5P$$

Using the results of **N** and **a**,

$\overset{+}{\rightarrow} \Sigma F_x = ma_x;\qquad P\cos 30° - 0.3(490.5 - 0.5P) = 50(1.60)$

$$P = 224\ \text{N}$$ **Ans.**

Ans:
$P = 224\ \text{N}$

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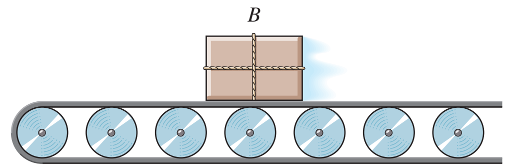
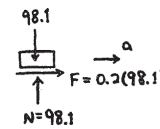
***13–8.**

The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = ma_x;$ $\quad 0.2(98.1) = 10\,a$

$a = 1.962\ \text{m/s}^2$

$(\overset{+}{\rightarrow})\ v = v_0 + a_c t$

$4 = 0 + 1.962\,t$

$t = 2.04\ \text{s}$ **Ans.**

Ans:
$t = 2.04\ \text{s}$

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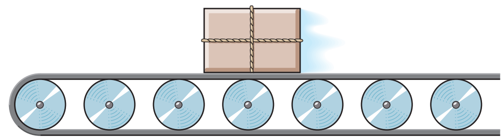
13–9.

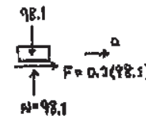
The conveyor belt is designed to transport packages of various weights. Each 10-kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the distance the package will slide on the belt before coming to rest.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.15\,m(9.81) = ma$

$$a = 1.4715 \text{ m/s}^2$$



$(\xrightarrow{+})\; v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (5)^2 + 2(-1.4715)(s - 0)$$

$$s = 8.49 \text{ m}$$

Ans.

Ans:
$s = 8.49$ m

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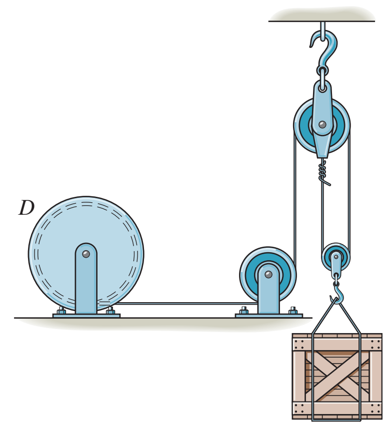
13–10.

The winding drum D is drawing in the cable at an accelerated rate of $5\ \mathrm{m/s^2}$. Determine the cable tension if the suspended crate has a mass of 800 kg.



SOLUTION

$$s_A + 2\,s_B = l$$

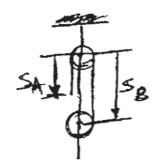
$$a_A = -2\,a_B$$

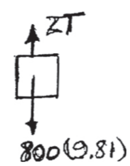
$$5 = -2\,a_B$$

$$a_B = -2.5\ \mathrm{m/s^2} = 2.5\ \mathrm{m/s^2}\uparrow$$

$+\uparrow \Sigma F_y = ma_y;\qquad 2T - 800(9.81) = 800(2.5)$

$T = 4924\ \mathrm{N} = 4.92\ \mathrm{kN}$ **Ans.**





Ans:
$T = 4.92\ \mathrm{kN}$

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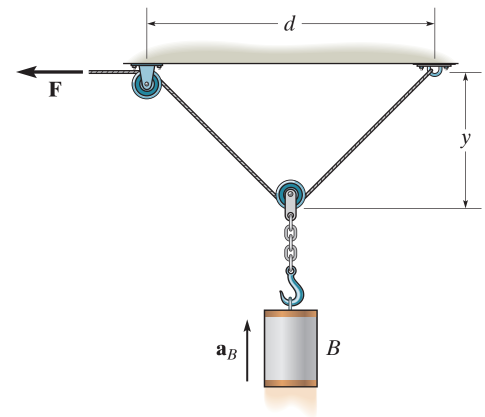
13–11.

Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **F** as a function of the block's vertical position y so that when **F** is applied, the block rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord and pulleys.

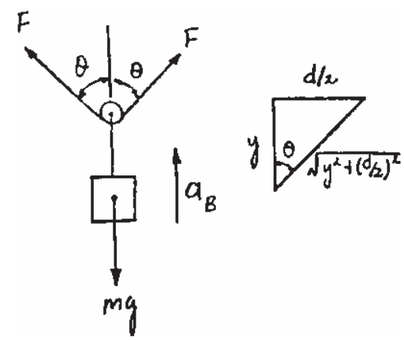


SOLUTION

$+\uparrow \Sigma F_y = ma_y;\qquad 2F\cos\theta - mg = ma_B \qquad \text{where } \cos\theta = \dfrac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$

$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$ **Ans.**



Ans:

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$

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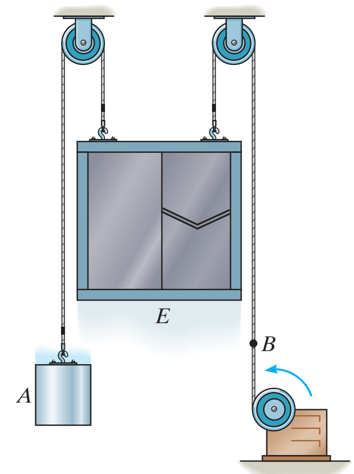
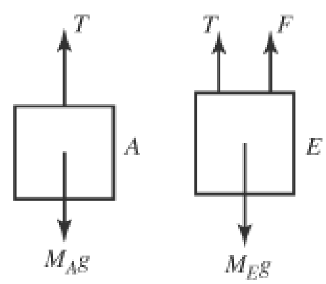
***13–12.** The elevator E has a mass of 500 kg and the counterweight at A has a mass of 150 kg. If the elevator attains a speed of 10 m/s after it rises 40 m, determine the constant force developed in the cable at B. Neglect the mass of the pulleys and cable.



SOLUTION

Guesses $\quad T = 1\text{ kN} \quad F = 1\text{ kN} \quad a = 1\text{ m/s}^2$

Given $\quad T - M_A g = -M_A a \qquad F + T - M_E g = M_E a \qquad v^2 = 2ah$



$M_E = 500\text{ kg}$

$M_A = 150\text{ kg}$

$v = 10\text{ m/s}$

$h = 40\text{ m}$

$$\begin{pmatrix} F \\ T \\ a \end{pmatrix} = \text{Find}(F, T, a) \qquad a = 1.250\text{ m/s}^2 \quad T = 1.28\text{ kN} \qquad F = 4.24\text{ kN} \quad \textbf{Ans.}$$

Ans:

$F = 4.24\text{ kN}$

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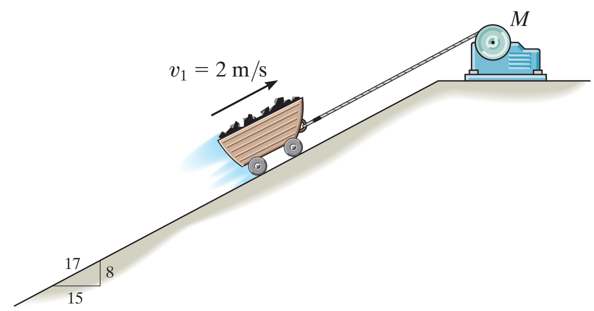
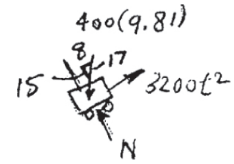
13–13.

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



SOLUTION

$$\nearrow + \Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$



$$dv = adt$$

$$\int_2^v dv = \int_0^2 (8t^2 - 4.616)\, dt$$

$$v = 14.1 \text{ m/s}$$ **Ans.**

Ans:
$v = 14.1$ m/s

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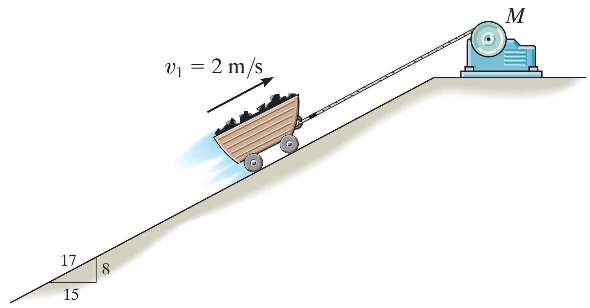
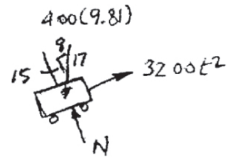
13–14.

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



SOLUTION

$\nearrow + \Sigma F_{x'} = ma_{x'};\qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$



$$dv = a\,dt$$

$$\int_2^v dv = \int_0^t (8t^2 - 4.616)\,dt$$

$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

$$\int_0^s ds = \int_0^2 (2.667\,t^3 - 4.616t + 2)\,dt$$

$$s = 5.43 \text{ m}$$ **Ans.**

Ans:
$s = 5.43$ m

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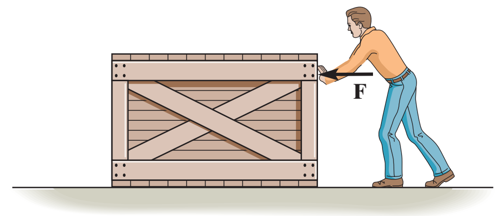
13–15.

The 75-kg man pushes on the 150-kg crate with a horizontal force **F**. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$, show that the man is able to move the crate. What is the greatest acceleration the man can give the crate?



SOLUTION

Equation of Equilibrium. Assuming that the crate is on the verge of sliding $(F_f)_C = \mu_s N_C = 0.3N_C$. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N_C - 150(9.81) = 0 \qquad N_C = 1471.5 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - F = 0 \qquad F = 441.45 \text{ N}$

Referring to the FBD of the man, Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad N_m - 75(9.81) = 0 \qquad N_B = 735.75 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 441.45 - (F_f)_m = 0 \qquad (F_f)_m = 441.45 \text{ N}$

Since $(F_f)_m < \mu'_s N_m = 0.8(735.75) = 588.6$ N, **the man is able to move the crate**.

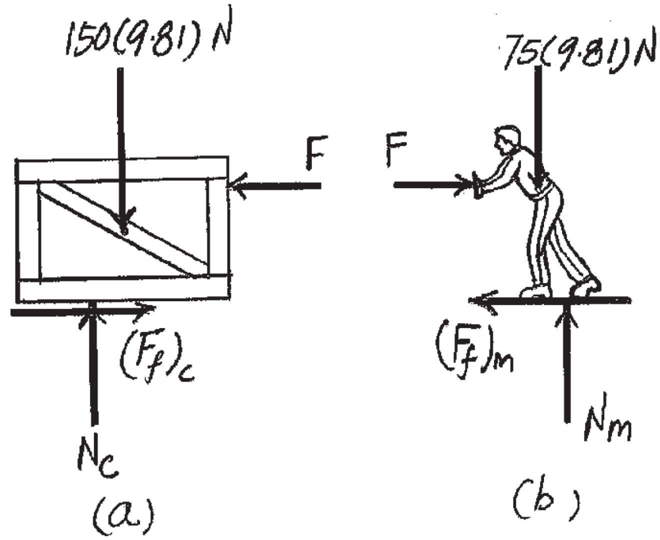
Equation of Motion. The greatest acceleration of the crate can be produced when the man is on the verge of slipping. Thus, $(F_f)_m = \mu'_s N_m = 0.8(735.75) = 588.6$ N.

$\xrightarrow{+} \Sigma F_x = 0; \qquad F - 588.6 = 0 \qquad F = 588.6 \text{ N}$

Since the crate slides, $(F_f)_C = \mu_k N_C = 0.2(1471.5) = 294.3$ N. Thus,

$\xleftarrow{+} \Sigma F_x = ma_x; \qquad 588.6 - 294.3 = 150\,a$

$a = 1.962 \text{ m/s}^2 = 1.96 \text{ m/s}^2$ **Ans.**



Ans:
$a = 1.96 \text{ m/s}^2$

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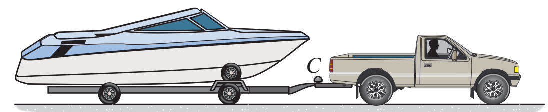
***13–16.**

The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.



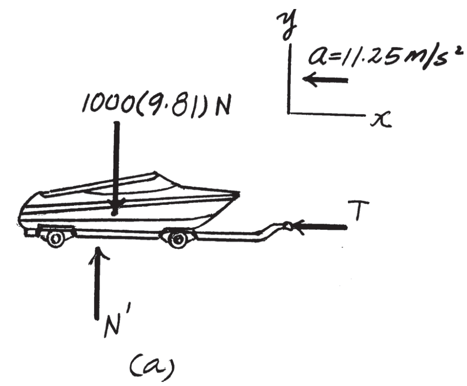
SOLUTION

Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\overset{+}{\rightarrow}\right) \quad v^2 = v_0^{\,2} + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25\text{ m/s}^2 = 11.25\text{ m/s}^2 \leftarrow$$



Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling C is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

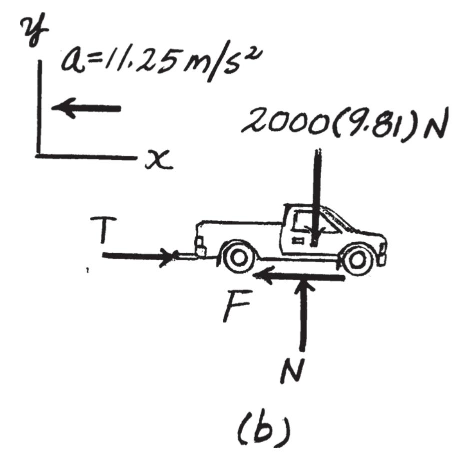
$\overset{+}{\rightarrow}\Sigma F_x = ma_x;$ $\quad -T = 1000(-11.25)$

$T = 11\,250\text{ N} = 11.25\text{ kN}$ **Ans.**

Using the results of **a** and **T** and referring to Fig. (b),

$+\uparrow\Sigma F_x = ma_x;$ $\quad 11\,250 - F = 2000(-11.25)$

$F = 33\,750\text{ N} = 33.75\text{ kN}$ **Ans.**



Ans:
$T = 11.25$ kN
$F = 33.75$ kN

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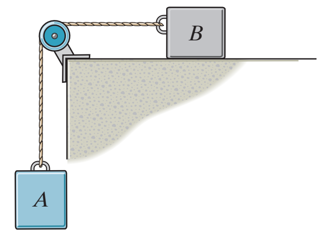
13–17.

Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μ_k, and the mass of each block is m. Neglect the mass of the pulleys and cord.



SOLUTION

Free Body Diagram. Since the pulley is smooth, the tension is constant throughout the entire cord. Since block B is required to slide, $F_f = \mu_k N$. Also, blocks A and B are attached together with inextensible cord, so $a_A = a_B = a$. The FBDs of blocks A and B are shown in Figs. a and b, respectively.

Equations of Motion. For block A, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - mg = m(-a) \qquad \textbf{(1)}$$

For block B, Fig. b,

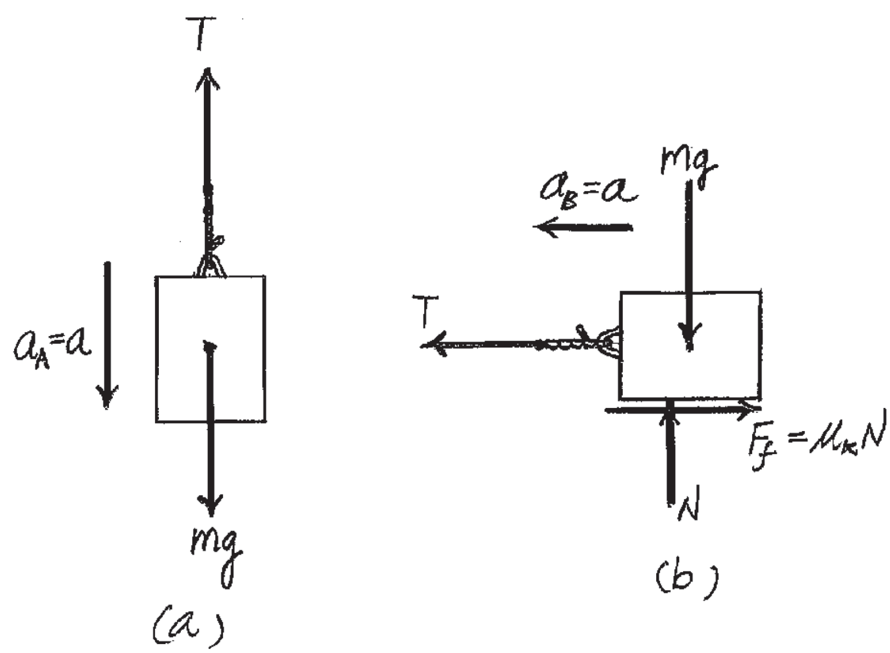
$$+\uparrow \Sigma F_y = ma_y; \qquad N - mg = m(0) \qquad N = mg$$

$$(\overset{+}{\leftarrow}) \Sigma F_x = ma_x; \quad T - \mu_k mg = ma \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$a = \frac{1}{2}(1 - \mu_k)\, g \qquad \textbf{Ans.}$$

$$T = \frac{1}{2}(1 + \mu_k)\, mg$$



Ans:
$a = \frac{1}{2}(1 - \mu_k)\, g$

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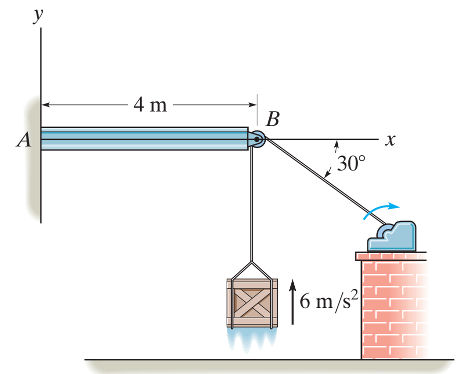
13–18.

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2. Determine the components of force reaction and the couple moment at the fixed support A.



SOLUTION

Equation of Motion. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow\Sigma F_y = ma_y;\qquad T - 50(9.81) = 50(6)\qquad T = 790.5\text{ N}$

Equations of Equilibrium. Since the pulley is smooth, the tension is constant throughout entire cable. Referring to the FBD of the pulley shown in Fig. *b*,

$\xrightarrow{+}\Sigma F_x = 0;\qquad 790.5\cos 30^\circ - B_x = 0\qquad B_x = 684.59\text{ N}$

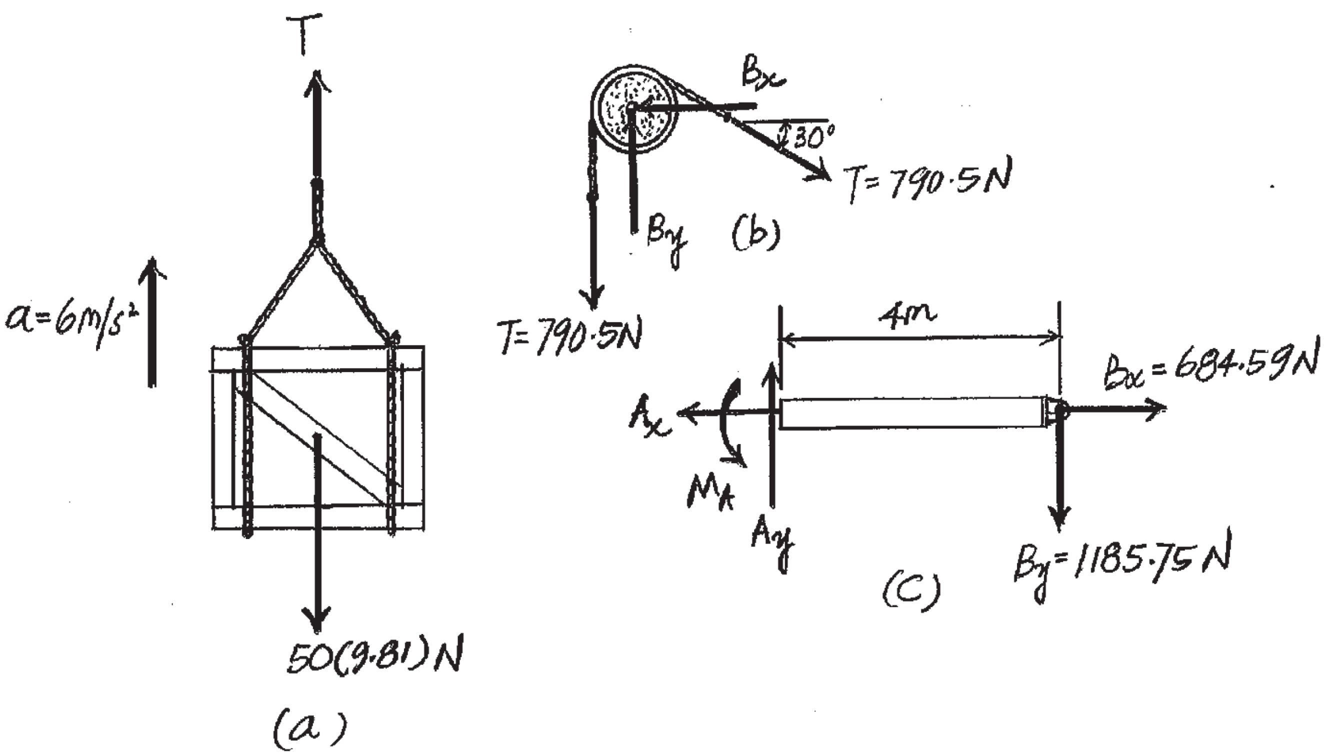
$+\uparrow\Sigma F_y = 0;\qquad B_y - 790.5 - 790.5\sin 30^\circ = 0\qquad B_y = 1185.75\text{ N}$

Consider the FBD of the cantilever beam shown in Fig. *c*,

$\xrightarrow{+}\Sigma F_x = 0;\qquad 684.59 - A_x = 0\qquad A_x = 684.59\text{ N} = 685\text{ N}$ **Ans.**

$+\uparrow\Sigma F_y = 0;\qquad A_y - 1185.75 = 0\qquad A_y = 1185.75\text{ N} = 1.19\text{ kN}$ **Ans.**

$\circlearrowleft+\Sigma M_A = 0;\quad M_A - 1185.75(4) = 0\qquad M_A = 4743\text{ N}\cdot\text{m} = 4.74\text{ kN}\cdot\text{m}$ **Ans.**



Ans:
$A_x = 685\text{ N}$
$A_y = 1.19\text{ kN}$
$M_A = 4.74\text{ kN}\cdot\text{m}$

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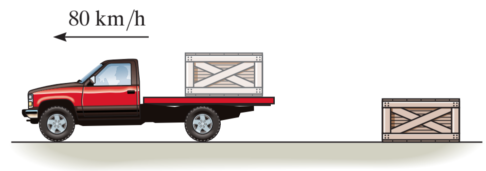
13–19. A crate having a mass of 60 kg falls horizontally off the back of a truck which is traveling at 80 km/h. Determine the coefficient of kinetic friction between the road and the crate if the crate slides 45 m on the ground with no tumbling along the road before coming to rest. Assume that the initial speed of the crate along the road is 80 km/h.

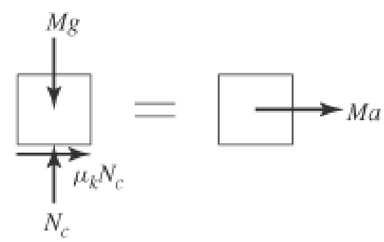


SOLUTION

$N_C - Mg = 0 \qquad N_C = Mg$

$\mu_k N_C = Ma \qquad a = \mu_k g$

$$\frac{v^2}{2} = ad = \mu_k g d$$



Given:

$M = 60 \text{ kg} \qquad d = 45 \text{ m}$

$v = 80 \text{ km/h} \qquad g = 9.81 \text{ m/s}^2$

$$\mu_k = \frac{v^2}{2gd} \qquad \mu_k = 0.559 \quad \textbf{Ans.}$$

Ans:
$\mu_k = 0.559$

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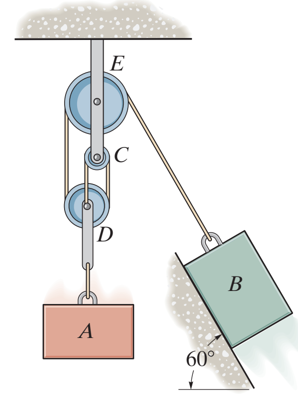
***13–20.** Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.

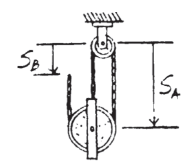


SOLUTION

Kinematic: Applying equation $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, we have

$$(\nwarrow +) \qquad 0.75 = 0 + 0 + \frac{1}{2} a_B \left(2^2\right) \qquad a_B = 0.375 \text{ m/s}^2$$

Establishing the position - coordinate equation, we have



$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$

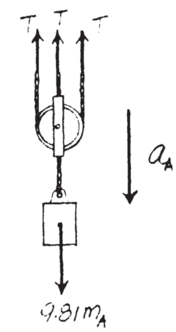
Taking time derivative twice yields

$$3a_A - a_B = 0 \tag{1}$$

From Eq.(1),

$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),



(a)

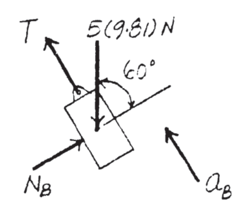
$$\nwarrow + \Sigma F_{y'} = ma_{y'}; \qquad T - 5(9.81)\sin 60° = 5(0.375)$$

$$T = 44.35 \text{ N}$$

From FBD(a),

$$+\uparrow \Sigma F_y = ma_y; \qquad 3(44.35) - 9.81m_A = m_A(-0.125)$$

$$m_A = 13.7 \text{ kg} \qquad \textbf{Ans.}$$



(b)

Ans:
$m_A = 13.7$ kg

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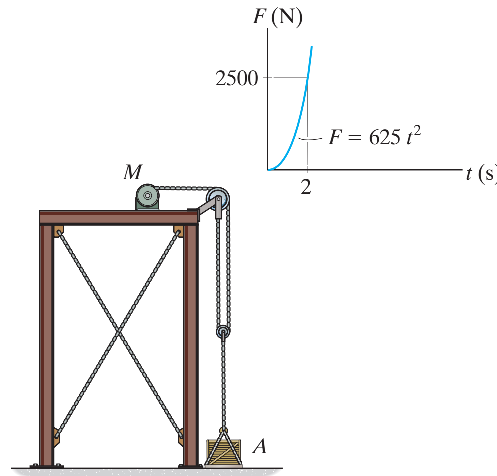
13–21.

The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$+\uparrow \Sigma F_y = 0;\qquad 2(625t^2) - 400(9.81) = 0$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = \left(625t^2\right)$ N. By referring to Fig. *a*,

$+\uparrow \Sigma F_y = ma_y;\qquad 2\left(625t^2\right) - 400(9.81) = 400a$

$$a = (3.125t^2 - 9.81)\text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $1.772\text{ s} \le t < 2$ s, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$(+\uparrow)\qquad \int dv = \int a\,dt$

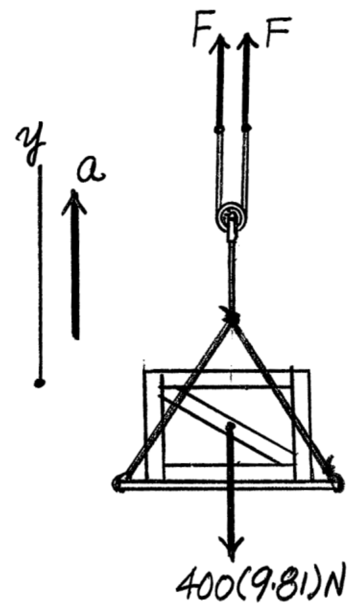
$$\int_0^v dv = \int_{1.772\text{ s}}^t \left(3.125t^2 - 9.81\right)dt$$

$$v = \left(1.0417t^3 - 9.81t\right)\Big|_{1.772\text{ s}}^{t}$$

$$= \left(1.0417t^3 - 9.81t + 11.587\right)\text{ m/s}$$

When $t = 2$ s,

$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 0.301$ m/s

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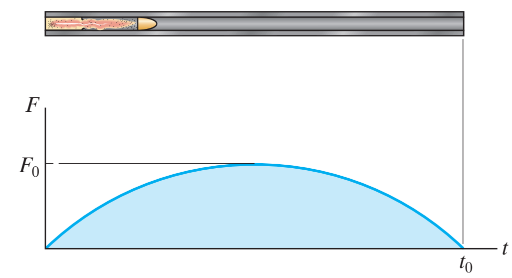
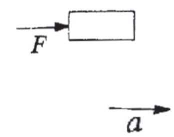
13–22.

The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t/t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \sin\left(\frac{\pi t}{t_0}\right) = ma$$

$$a = \frac{dv}{dt} = \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)$$

$$\int_0^v dv = \int_0^t \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)dt \qquad v = -\left(\frac{F_0 t_0}{\pi m}\right)\cos\left(\frac{\pi t}{t_0}\right)\Bigg]_0^t$$

$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$.

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$ **Ans.**

$$\int_0^s ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)dt$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[\mathrm{t} - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right]_0^t$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left(t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

Ans:

$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left[1 - \cos\left(\frac{\pi t}{t_0}\right)\right]$$

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right]$$

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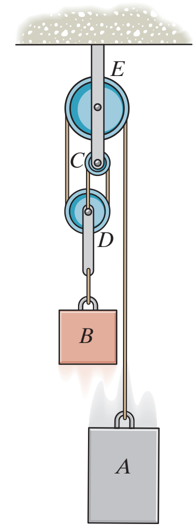
13–23.

The 50-kg block A is released from rest. Determine the velocity of the 15-kg block B in 2 s.



SOLUTION

Kinematics. As shown in Fig. *a*, the position of block B and point A are specified by s_B and s_A respectively. Here the pulley system has only one cable which gives

$$s_A + s_B + 2(s_B - a) = l$$

$$s_A + 3s_B = l + 2a \qquad \textbf{(1)}$$

Taking the time derivative of Eq. (1) twice,

$$a_A + 3a_B = 0 \qquad \textbf{(2)}$$

Equations of Motion. The FBD of blocks B and A are shown in Fig. *b* and *c*. To be consistent to those in Eq. (2), $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed towards the positive sense of their respective position coordinates s_A and s_B. For block B,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3T - 15(9.81) = 15(-a_B) \qquad \textbf{(3)}$$

For block A,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 50(9.81) = 50(-a_A) \qquad \textbf{(4)}$$

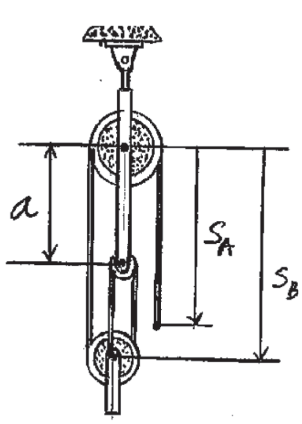
Solving Eqs. (2), (3) and (4),

$$a_B = -2.848 \text{ m/s}^2 = 2.848 \text{ m/s}^2 \uparrow \qquad a_A = 8.554 \text{ m/s}^2 \qquad T = 63.29 \text{ N}$$

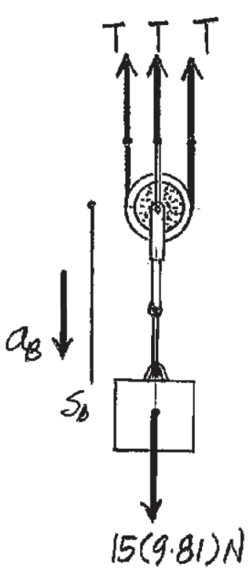
The negative sign indicates that $\mathbf{a}_B$ acts in the sense opposite to that shown in FBD. The velocity of block B can be determined using

$$+\uparrow \quad v_B = (v_A)_0 + a_B t; \qquad v_B = 0 + 2.848(2)$$

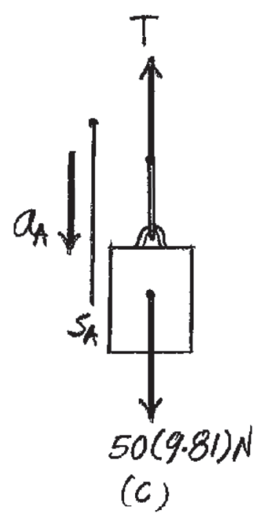
$$v_B = 5.696 \text{ m/s} = 5.70 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$v_B = 5.70 \text{ m/s} \uparrow$

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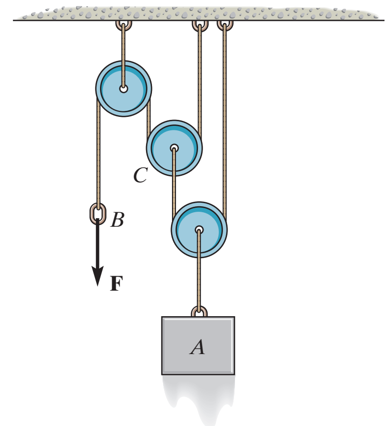
***13–24.**

If the supplied force $F = 150$ N, determine the velocity of the 50-kg block A when it has risen 3 m, starting from rest.



SOLUTION

Equations of Motion. Since the pulleys are smooth, the tension is constant throughout each entire cable. Referring to the FBD of pulley C, Fig. a, of which its mass is negligible.

$+\uparrow \Sigma F_y = 0; \qquad 150 + 150 - T = 0 \qquad T = 300 \text{ N}$

Subsequently, considered the FBD of block A shown in Fig. b,

$+\uparrow \Sigma F_y = ma_y; \qquad 300 + 300 - 50(9.81) = 50a$

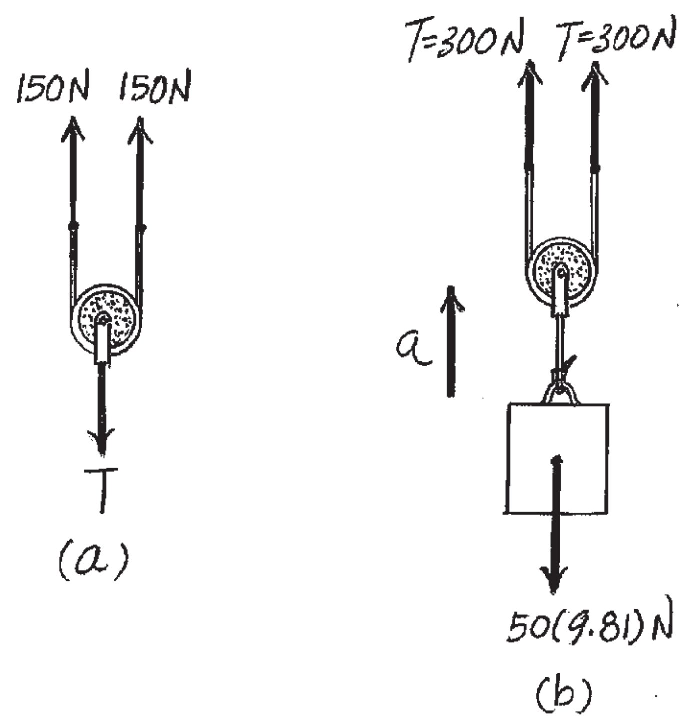
$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of **a**,

$(+\uparrow) \quad v^2 = v_0^2 + 2a_c s;$

$$v^2 = 0^2 + 2(2.19)(3)$$

$$v = 3.6249 \text{ m/s} = 3.62 \text{ m/s}$$ **Ans.**



Ans:
$v = 3.62 \text{ m/s}\uparrow$

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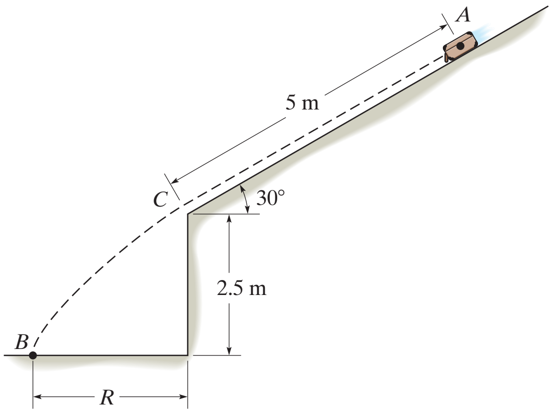
13–25.

A 60-kg suitcase slides from rest 5 m down the smooth ramp. Determine the distance R where it strikes the ground at B. How long does it take to go from A to B?



SOLUTION

Equation of Motion. Referring to the FBD of the suitcase shown in Fig. *a*

$+\swarrow \Sigma F_{x'} = ma_{x'}; \qquad 60(9.81) \sin 30° = 60a \qquad a = 4.905 \text{ m/s}^2$

Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$(+\swarrow)\ v^2 = v_0^2 + 2a_c s; \qquad v^2 = 0^2 + 2(4.905)(5)$

$$v = 7.0036 \text{ m/s}$$

$(+\swarrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2; \qquad 5 = 0 + 0 + \frac{1}{2}(4.905)t_{AC}^2$

$$t_{AC} = 1.4278 \text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. *b*, the vertical motion gives

$(+\downarrow) \quad s_y = (s_0)_y + v_y t + \frac{1}{2} a_y t^2;$

$$2.5 = 0 + 7.0036 \sin 30°\, t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905\, t_{CB}^2 + 3.5018\, t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4412 \text{ s}$$

Then, the horizontal motion gives

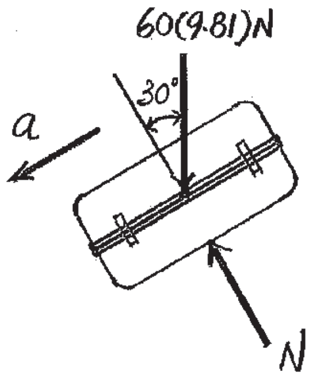
$(\overset{+}{\leftarrow}) \quad s_x = (s_0)_x + v_x t;$

$$R = 0 + 7.0036 \cos 30° (0.4412)$$

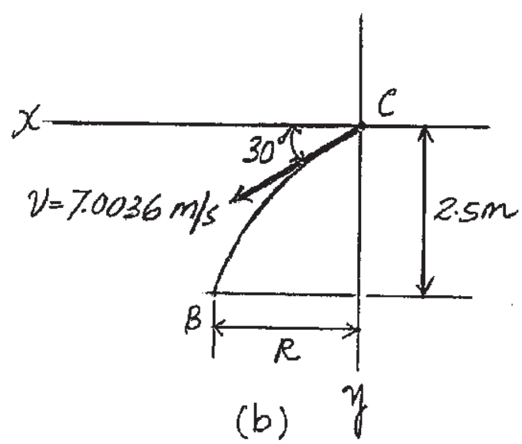
$$= 2.676 \text{ m} = 2.68 \text{ m}$$ **Ans.**

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.4278 + 0.4412 = 1.869 \text{ s} = 1.87 \text{ s}$$ **Ans.**



(a)



(b)

Ans:
$(\overset{+}{\leftarrow}) \quad s_x = 2.68$ m
$t_{AB} = 1.87$ s

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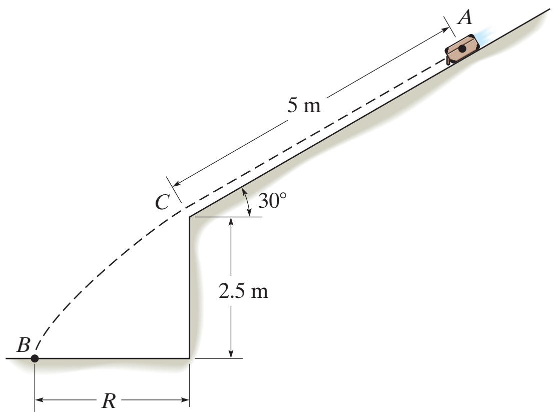
13–26.

Solve Prob. 13–25 if the suitcase has an initial velocity down the ramp of $v_A = 2\text{ m/s}$, and the coefficient of kinetic friction along AC is $\mu_k = 0.2$.



SOLUTION

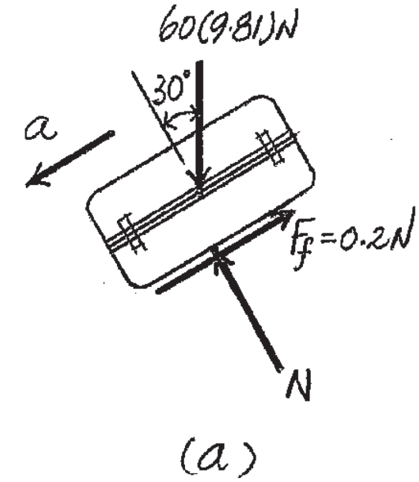
Equations of Motion. The friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the suitcase shown in Fig. a

$$\nwarrow^{+} \Sigma F_{y'} = ma_{y'}; \qquad N - 60(9.81)\cos 30^\circ = 60(0)$$

$$N = 509.74\text{ N}$$

$$+\swarrow \Sigma F_{x'} = ma_{x'}; \qquad 60(9.81)\sin 30^\circ - 0.2(509.74) = 60\,a$$

$$a = 3.2059\text{ m/s}^2 \swarrow$$



Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$$(+\swarrow) \qquad v^2 = v_0^2 + 2a_c s; \qquad v^2 = 2^2 + 2(3.2059)(5)$$

$$v = 6.0049\text{ m/s} \swarrow$$

$$(+\swarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2; \qquad 5 = 0 + 2t_{AC} + \frac{1}{2}(3.2059)t_{AC}^2$$

$$1.6029\,t_{AC}^2 + 2t_{AC} - 5 = 0$$

Solve for positive root,

$$t_{AC} = 1.2492\text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. b, the vertical motion gives

$$(+\downarrow) \qquad s_y = (s_0)_y + v_y t + \frac{1}{2}a_y t^2;$$

$$2.5 = 0 + 6.0049\sin 30^\circ\, t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905\,t_{CB}^2 + 3.0024\,t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4707\text{ s}$$

Then, the horizontal motion gives

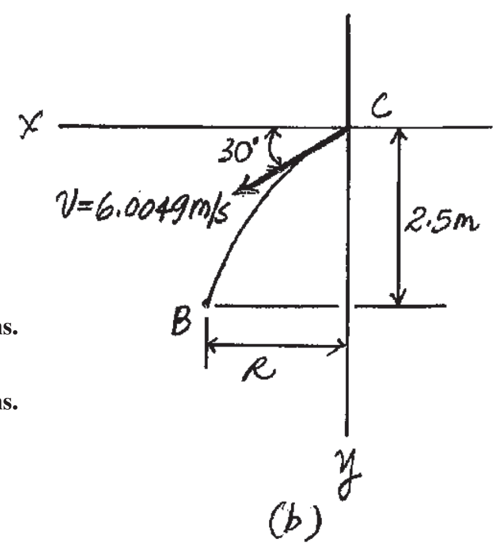
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t;$$

$$R = 0 + 6.0049\cos 30^\circ\,(0.4707)$$

$$= 2.448\text{ m} = 2.45\text{ m} \qquad \textbf{Ans.}$$

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.2492 + 0.4707 = 1.7199\text{ s} = 1.72\text{ s} \qquad \textbf{Ans.}$$



Ans:
$R = 2.45\text{ m}$
$t_{AB} = 1.72\text{ s}$

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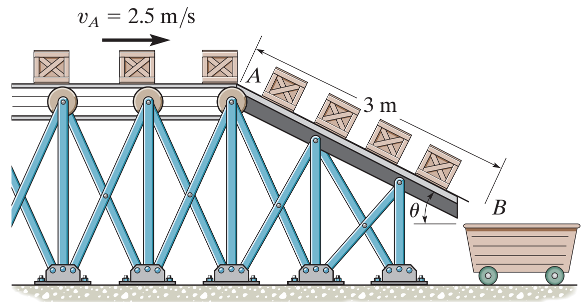
13–27.

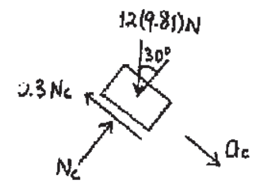
The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5\text{ m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.



SOLUTION

$\nearrow + \Sigma F_y = ma_y;\qquad N_C - 12(9.81)\cos 30° = 0$

$$N_C = 101.95\text{ N}$$



$+\searrow \Sigma F_x = ma_x;\qquad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$

$$a_C = 2.356\text{ m/s}^2$$

$(+\searrow)\qquad v_B^2 = v_A^2 + 2\,a_C(s_B - s_A)$

$$v_B^2 = (2.5)^2 + 2(2.356)(3 - 0)$$

$$v_B = 4.5152 = 4.52\text{ m/s}$$ **Ans.**

Ans:
$v_B = 4.52\text{ m/s}$

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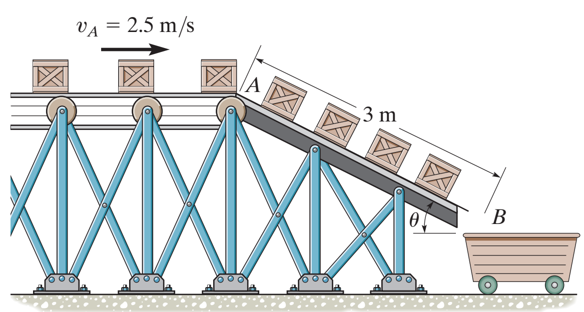
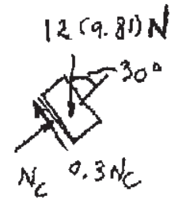
***13–28.**

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5\text{ m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the smallest incline θ of the ramp so that the crates will slide off and fall into the cart.



SOLUTION



$(+\searrow)\quad v_B^2 = v_A^2 + 2a_C(s_B - s_A)$

$$0 = (2.5)^2 + 2(a_C)(3 - 0)$$

$$a_C = 1.0417$$

$\nearrow + \Sigma F_y = ma_y;\qquad N_C - 12(9.81)\cos\theta = 0$

$$N_C = 117.72\cos\theta$$

$+\searrow \Sigma F_x = ma_x;\qquad 12(9.81)\sin\theta - 0.3(N_C) = 12\,(1.0417)$

$$117.72\sin\theta - 35.316\cos\theta - 12.5 = 0$$

Solving,

$\theta = 22.6°$ **Ans.**

Ans:
$\theta = 22.6°$

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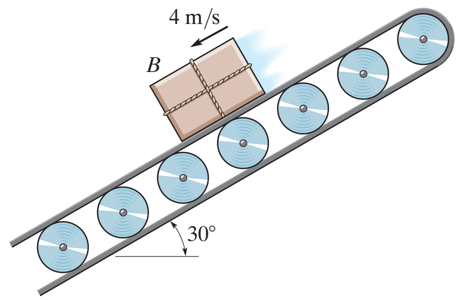
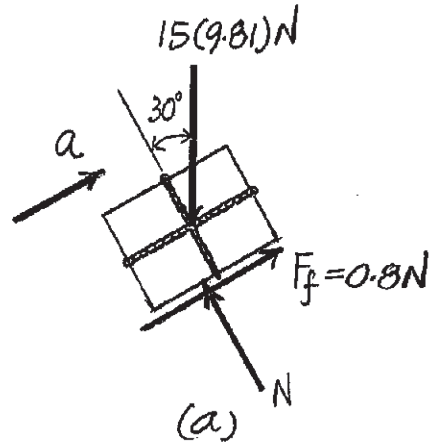
13–29.

The conveyor belt is moving downward at 4 m/s. If the coefficient of static friction between the conveyor and the 15-kg package B is $\mu_s = 0.8$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION

Equations of Motion. It is required that the package is on the verge to slide. Thus, $F_f = \mu_s N = 0.8N$. Referring to the FBD of the package shown in Fig. a,



$+\nwarrow \Sigma F_{y'} = ma_{y'};\quad N - 15(9.81)\cos 30° = 15(0)\quad N = 127.44 \text{ N}$

$+\nearrow \Sigma F_{x'} = ma_{x'};\quad 0.8(127.44) - 15(9.81)\sin 30° = 15\,a$

$$a = 1.8916 \text{ m/s}^2 \nearrow$$

Kinematic. Since the package is required to stop, $v = 0$. Here $v_0 = 4$ m/s.

$(+\swarrow)\quad v = v_0 + a_0 t;$

$$0 = 4 + (-1.8916)\,t$$

$$t = 2.1146 \text{ s} = 2.11 \text{ s}$$ **Ans.**

Ans:
$t = 2.11$ s

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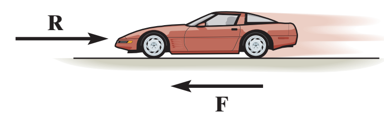
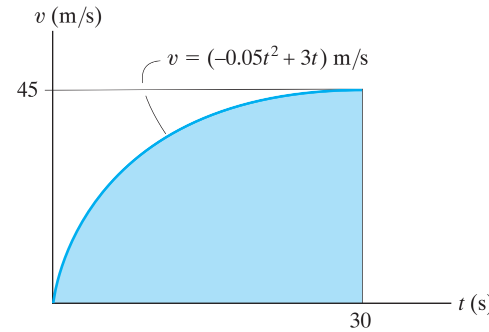
13–30.

The 1.5 Mg sports car has a tractive force of $F = 4.5$ kN. If it produces the velocity described by v-t graph shown, plot the air resistance R versus t for this time period.





SOLUTION

Kinematic. For the v–t graph, the acceleration of the car as a function of t is

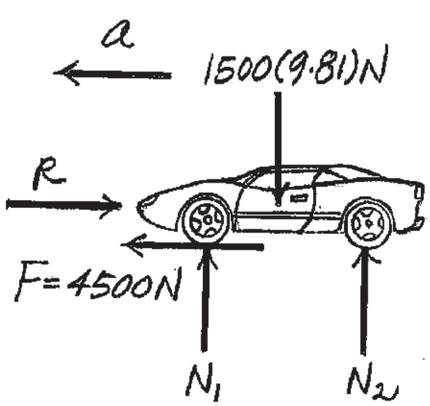
$$a = \frac{dv}{dt} = \{-0.1t + 3\}\ \text{m/s}^2$$

Equation of Motion. Referring to the FBD of the car shown in Fig. a,

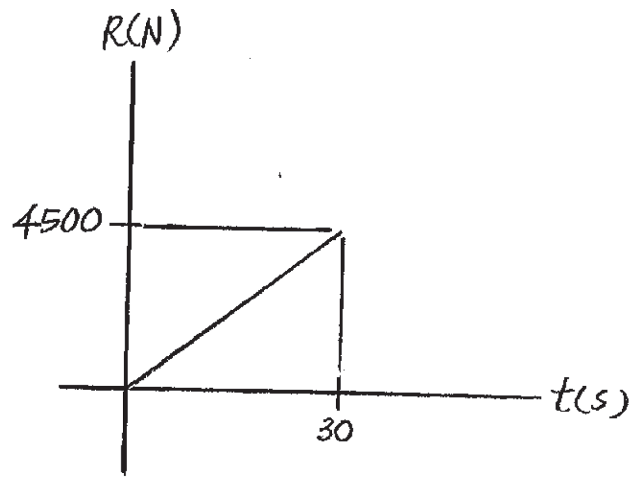
$$(\overset{+}{\leftarrow})\,\Sigma F_x = ma_x; \qquad 4500 - R = 1500(-0.1t + 3)$$

$$R = \{150t\}\ \text{N}$$

The plot of R vs t is shown in Fig. b



(a)



Ans:
$R = \{150t\}$ N

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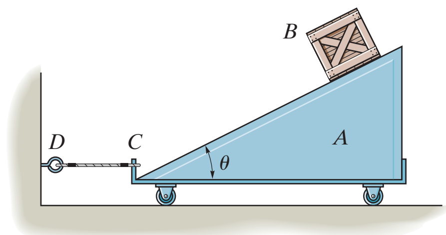
13–31.

Crate B has a mass m and is released from rest when it is on top of cart A, which has a mass $3m$. Determine the tension in cord CD needed to hold the cart from moving while B is sliding down A. Neglect friction.

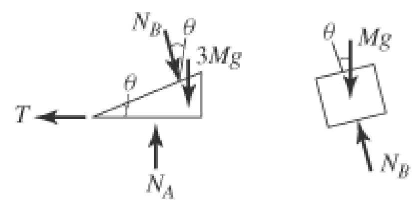


SOLUTION

Block B:

$$N_B - mg\cos(\theta) = 0$$

$$N_B = mg\cos(\theta)$$



Cart :

$$-T + N_B\sin(\theta) = 0$$

$$T = mg\sin(\theta)\cos(\theta)$$

$$T = \left(\frac{mg}{2}\right)\sin(2\theta) \qquad \textbf{Ans.}$$

Ans:

$T = \left(\dfrac{mg}{2}\right)\sin(2\theta)$

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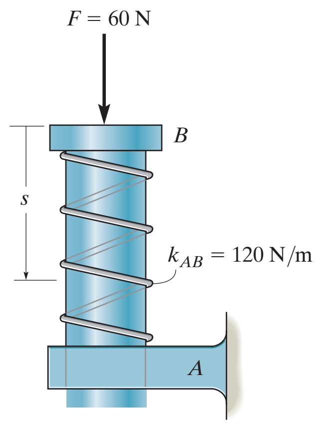
***13–32.**

The 4-kg smooth cylinder is supported by the spring having a stiffness of $k_{AB} = 120$ N/m. Determine the velocity of the cylinder when it moves downward $s = 0.2$ m from its equilibrium position, which is caused by the application of the force $F = 60$ N.



SOLUTION

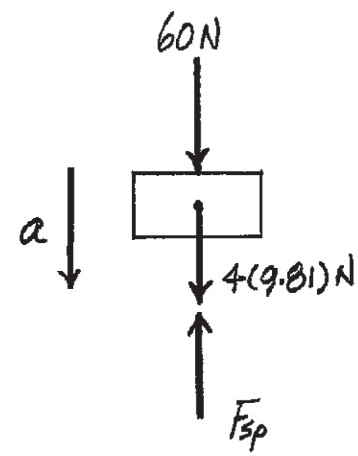
Equation of Motion. At the equilibrium position, realizing that $F_{sp} = kx_0 = 120x_0$ the compression of the spring can be determined from

$+\uparrow \Sigma F_y = 0; \qquad 120x_0 - 4(9.81) = 0 \qquad x_0 = 0.327 \text{ m}$

Thus, when 60 N force is applied, the compression of the spring is $x = s + x_0 = s + 0.327$. Thus, $F_{sp} = kx = 120(s + 0.327)$. Then, referring to the FBD of the collar shown in Fig. *a*,

$+\uparrow \Sigma F_y = ma_y; \qquad 120(s + 0.327) - 60 - 4(9.81) = 4(-a)$

$$a = \{15 - 30\,s\} \text{ m/s}^2$$



(a)

Kinematics. Using the result of **a** and integrate $\int v\,dv = a\,ds$ with the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s (15 - 30\,s)ds$$

$$\frac{v^2}{2} = 15\,s - 15\,s^2$$

$$v = \left\{\sqrt{30(s - s^2)}\right\} \text{ m/s}$$

At $s = 0.2$ m,

$$v = \sqrt{30(0.2 - 0.2^2)} = 2.191 \text{ m/s} = 2.19 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 2.19$ m/s

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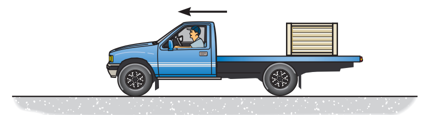
13–33.

The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.



SOLUTION

Free-Body Diagram: When the crate accelerates with the truck, the frictional force F_f develops. Since the crate is required to be on the verge of slipping, $F_f = \mu_s N = 0.3N$.

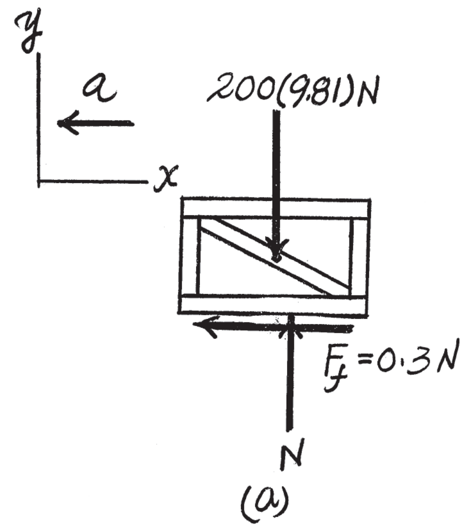
Equations of Motion: Here, $a_y = 0$. By referring to Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200(9.81) = 200(0)$$

$$N = 1962 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.3(1962) = 200(-a)$$

$$a = 2.943 \text{ m/s}^2 \leftarrow$$



Kinematics: The final velocity of the truck is $v = \left(60\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right) =$ 16.67 m/s. Since the acceleration of the truck is constant,

$$(\overset{+}{\leftarrow}) \qquad v = v_0 + a_c t$$

$$16.67 = 0 + 2.943t$$

$$t = 5.66 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 5.66$ s

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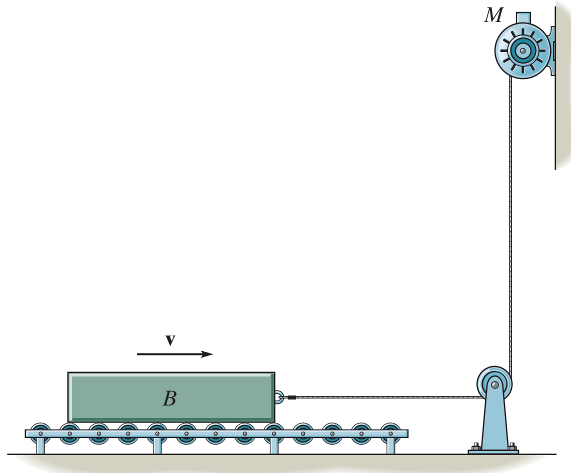
13–34.

The 300-kg bar B, originally at rest, is being towed over a series of small rollers. Determine the force in the cable when $t = 5$ s, if the motor M is drawing in the cable for a short time at a rate of $v = (0.4t^2)$ m/s, where t is in seconds $(0 \leq t \leq 6 \text{ s})$. How far does the bar move in 5 s? Neglect the mass of the cable, pulley, and the rollers.



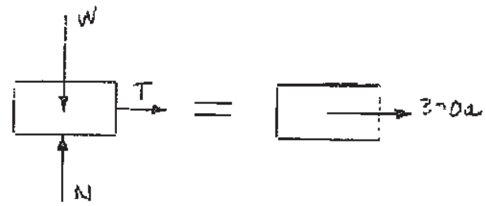
SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T = 300a$

$v = 0.4t^2$

$$a = \frac{dv}{dt} = 0.8t$$

When $t = 5$ s, $a = 4 \text{ m/s}^2$



$T = 300(4) = 1200 \text{ N} = 1.20 \text{ kN}$ **Ans.**

$ds = v\,dt$

$$\int_0^s ds = \int_0^5 0.4t^2\,ds$$

$$s = \left(\frac{0.4}{3}\right)(5)^3 = 16.7 \text{ m}$$ **Ans.**

Ans:
$s = 16.7$ m

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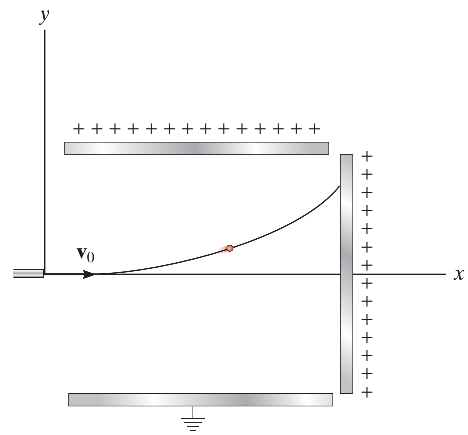
13–35.

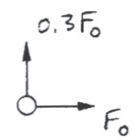
An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 = ma_x$

$+\uparrow \Sigma F_y = ma_y; \qquad 0.3\,F_0 = ma_y$



Thus,

$$\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$$

$$v_x = \frac{F_0}{m}t + v_0$$

$$\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \frac{0.3F_0}{m}t$$

$$v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

$$= \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2} \qquad \textbf{Ans.}$$

$$\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right)dt$$

$$x = \frac{F_0t^2}{2m} + v_0\,t$$

$$\int_0^y dy = \int_0^t \frac{0.3F_0}{m}\,t\,dt$$

$$y = \frac{0.3F_0t^2}{2m}$$

$$t = \left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{F_0}{2m}\left(\frac{2m}{0.3F_0}\right)y + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}} \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{1/2}$$

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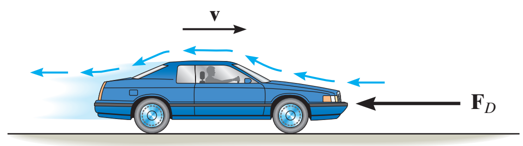
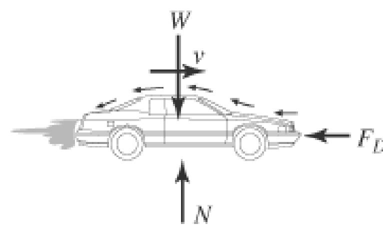
***13–36.** A car of mass m is traveling at a slow velocity v_0. If it is subjected to the drag resistance of the wind, which is proportional to its velocity, i.e., $F_D = kv$, determine the distance and the time the car will travel before its velocity becomes $0.5v_0$. Assume no other frictional forces act on the car.



SOLUTION

$$-F_D = ma$$

$$-kv = ma$$



Find time $\quad a = \frac{d}{dt}v = \frac{-k}{m}v$

$$\frac{-k}{m}\int_0^t 1\,dt = \int_{v_0}^{0.5v_0} \frac{1}{v}\,dv$$

$$t = \frac{m}{k}\ln\left(\frac{v_0}{0.5v_0}\right) \qquad t = \frac{m}{k}\ln(2) \qquad t = 0.693\frac{m}{k} \qquad \textbf{Ans.}$$

Find distance $\quad a = v\frac{d}{dx}v = \frac{-k}{m}v$

$$-\int_0^x k\,dx = \int_{v_0}^{0.5v_0} m\,dv \qquad x = \frac{m}{k}(0.5v_0) \qquad x = 0.5\frac{mv_0}{k} \qquad \textbf{Ans.}$$

Ans:
$t = 0.693\frac{m}{k}$
$x = 0.5\frac{mv_0}{k}$

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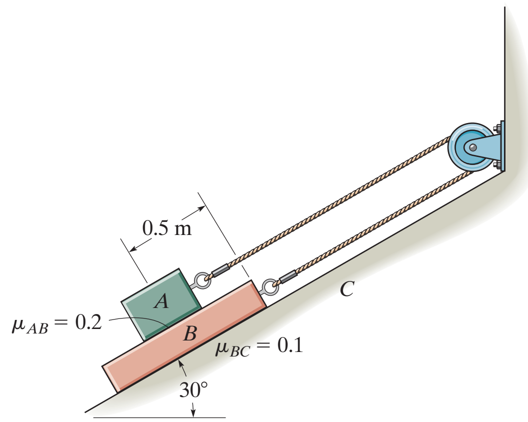
13–37.

The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.



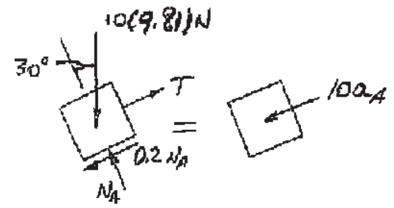
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 10(9.81)\cos 30^\circ = 0 \qquad N_A = 84.96\text{ N}$

$+\swarrow \Sigma F_x = ma_x;\quad -T + 0.2(84.96) + 10(9.81)\sin 30^\circ = 10a_A$

$T - 66.04 = -10a_A$ **(1)**



Block B:

$+\nwarrow \Sigma F_y = ma_y;\quad N_B - 84.96 - 50(9.81)\cos 30^\circ = 0$

$N_B = 509.7\text{ N}$

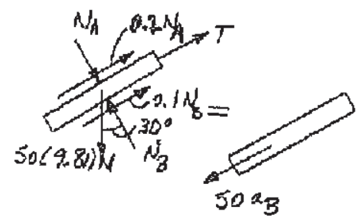
$+\swarrow \Sigma F_x = ma_x;\quad -0.2(84.96) - 0.1(509.7) - T + 50(9.81\sin 30^\circ) = 50a_B$

$177.28 - T = 50a_B$ **(2)**

$s_A + s_B = l$

$\Delta s_A = -\Delta s_B$

$a_A = -a_B$ **(3)**



Solving Eqs. (1) – (3):

$a_B = 1.854\text{ m/s}^2$

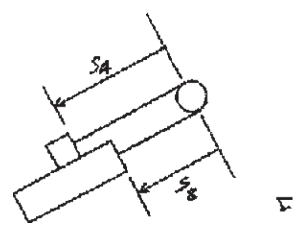
$a_A = -1.854\text{ m/s}^2 \qquad T = 84.58\text{ N}$

In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

$(+\swarrow)\qquad s_B = s_A + s_{B/A}$

$-\Delta s_A = \Delta s_A + 0.5$

$\Delta s_A = -0.25\text{ m}$



$(+\swarrow)\qquad s_A = s_0 + v_0 t + \frac{1}{2}a_A t^2$

$-0.25 = 0 + 0 + \frac{1}{2}(-1.854)t^2$

$t = 0.519\text{ s}$ **Ans.**

Ans:
$t = 0.519\text{ s}$

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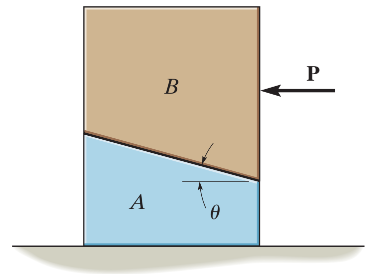
13–38.

Block A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that it will not slide on A. Also, what is the corresponding acceleration? The coefficient of static friction between A and B is μ_s. Neglect any friction between A and the horizontal surface.



SOLUTION

Equations of Motion. Since block B is required to be on the verge to slide on A, $F_f = \mu_s N_B$. Referring to the FBD of block B shown in Fig. a,

$+\uparrow \Sigma F_y = ma_y;\qquad N_B \cos\theta - \mu_s N_B \sin\theta - mg = m(0)$

$$N_B = \frac{mg}{\cos\theta - \mu_s \sin\theta} \qquad \textbf{(1)}$$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\qquad P - N_B \sin\theta - \mu_s N_B \cos\theta = ma$

$$P - N_B(\sin\theta + \mu_s \cos\theta) = ma \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2),

$$P - \left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) mg = ma \qquad \textbf{(3)}$$

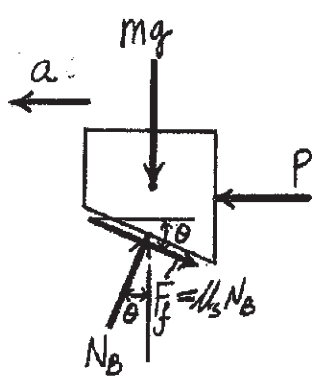
Referring to the FBD of blocks A and B shown in Fig. b

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\qquad P = 2\,ma \qquad \textbf{(4)}$

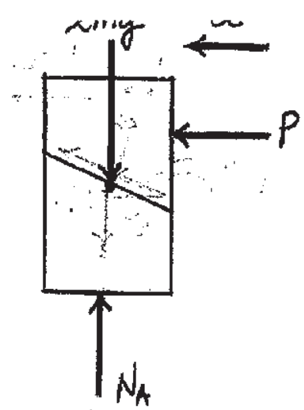
Solving Eqs. (2) into (3),

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) \qquad \textbf{Ans.}$$

$$a = \left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)g \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$

$a = \left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)g$

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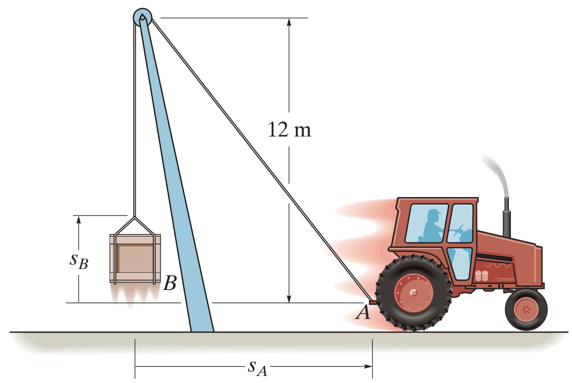
13–39.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



SOLUTION

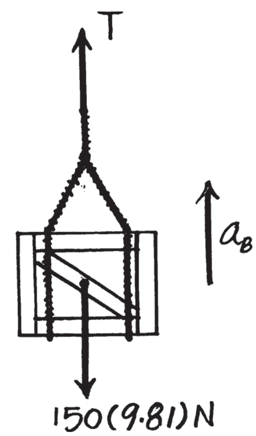
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-s_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$



$+\uparrow \Sigma F_y = m a_y; \qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

Ans:
$T = 1.63$ kN

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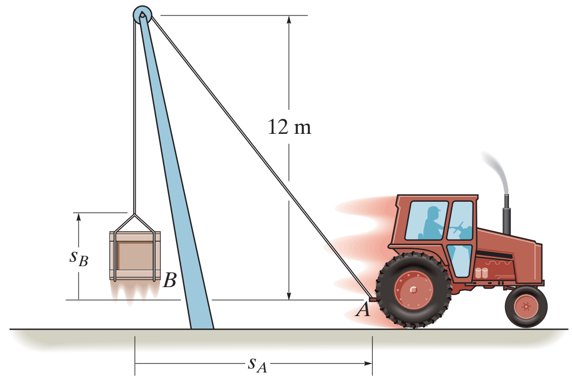
***13–40.**

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



SOLUTION

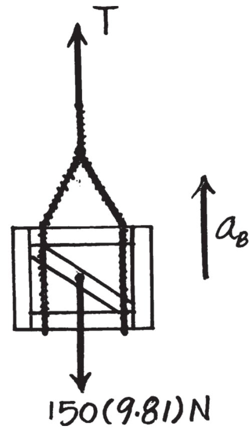
$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A\dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\,\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A\ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$



$+\uparrow \Sigma F_y = ma_y;$ $\quad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans.**

Ans:
$T = 1.80$ kN

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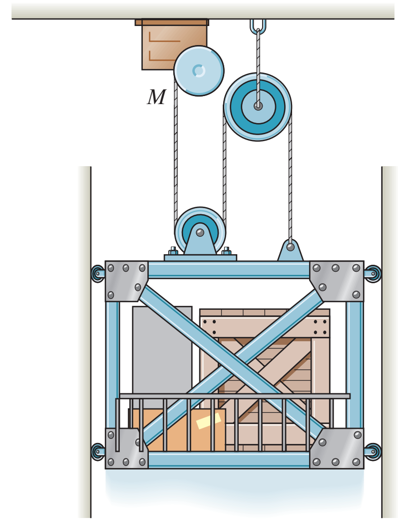
13–41.

A freight elevator, including its load, has a mass of 1 Mg. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 4\text{ kN}$ in its attached cable, determine the velocity of the elevator when it has moved upward 6 m starting from rest. Neglect the mass of the pulleys and cables.



SOLUTION

Equation of Motion. Referring to the FBD of the freight elevator shown in Fig. *a*,

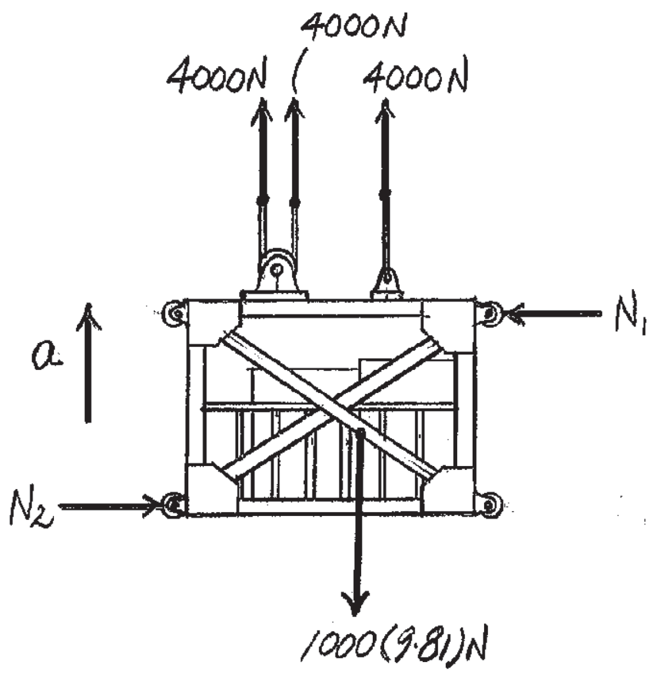
$$+\uparrow \Sigma F_y = ma_y; \qquad 3(4000) - 1000(9.81) = 1000a$$

$$a = 2.19\text{ m/s}^2\uparrow$$

Kinematics. Using the result of **a**,

$$(+\uparrow) \qquad v^2 = v_0^2 + 2as; \qquad v^2 = 0^2 + 2(2.19)(6)$$

$$v = 5.126\text{ m/s} = 5.13\text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 5.13\text{ m/s}$

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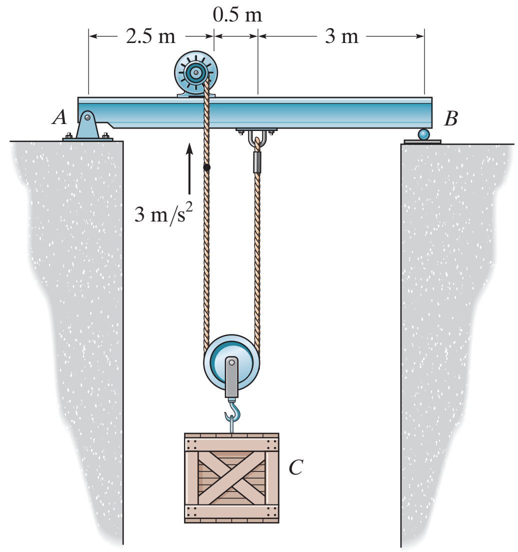
13–42.

If the motor draws in the cable with an acceleration of $3\,\text{m/s}^2$, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



SOLUTION

$S_c + (S_c - S_p)$

$2v_c = v_p$

$2a_c = a_p$

$2a_c = 3\ \text{m/s}^2$

$a_c = 1.5\ \text{m/s}^2$

$+\uparrow \Sigma F_y = ma_y \qquad 2T - 1962 = 200(1.5)$

$T = 1{,}131\ \text{N}$

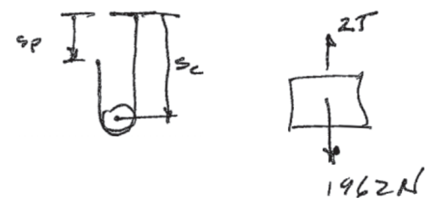
$\zeta + \Sigma M_A = 0; \qquad B_y(6) - (1765.8 + 1{,}131)3 - (1{,}131)(2.5) = 0$

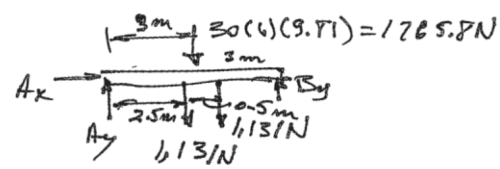
$B_y = 1919.65\ \text{N} = 1.92\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1765.8 - (2)(1{,}131) + 1919.65 = 0$

$A_y = 2108.15\ \text{N} = 2.11\ \text{kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**





Ans:

$B_y = 1.92\ \text{kN}$
$A_y = 2.11\ \text{kN}$
$A_x = 0$

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13–43.

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.

SOLUTION

Free-Body Diagram: The frictional force $\mathbf{F}_f$ is required to act to the left to oppose the motion of the crate which is to the right.

Equations of Motion: Here, $a_y = 0$. Thus,

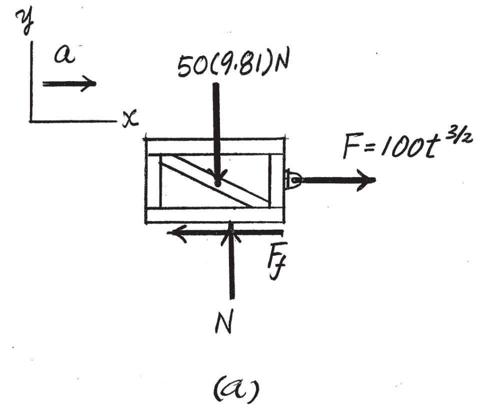
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 50(9.81) = 50(0)$$

$$N = 490.5 \text{ N}$$

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15$ N,

$$+\uparrow \Sigma F_x = ma_x; \qquad 100t^{3/2} - 147.15 = 50a$$

$$a = \left(2t^{3/2} - 2.943\right) \text{ m/s}$$



Equilibrium: For the crate to move, force $\mathbf{F}$ must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100t^{3/2} - 196.2 = 0$$

$$t = 1.567 \text{ s}$$

Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a\,dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$$(\overset{+}{\rightarrow}) \qquad \int dv = \int a dt$$

$$\int_0^v dv = \int_{1.567 \text{ s}}^t \left(2t^{3/2} - 2.943\right)dt$$

$$v = \left(0.8t^{5/2} - 2.943t\right)\Big|_{1.567 \text{ s}}^t$$

$$v = \left(0.8t^{5/2} - 2.943t + 2.152\right) \text{ m/s}$$

When $t = 5$ s,

$$v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16 \text{ m/s} = 32.2 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 32.2$ m/s

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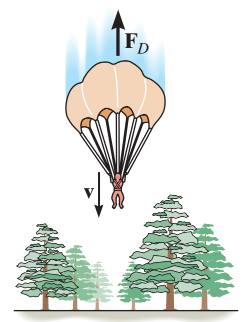
***13–44.**

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



SOLUTION

$+\downarrow \Sigma F_z = ma_z; \qquad mg - kv^2 = m\dfrac{dv}{dt}$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$\sqrt{\frac{mg}{k}}e^{2t\sqrt{\frac{mg}{k}}} - v\,e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

When $t \rightarrow \infty \qquad v_t = \sqrt{\dfrac{mg}{k}}$ **Ans.**

Ans:

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{mg/k}} - 1}{e^{2t\sqrt{mg/k}} + 1}\right]$$

$$v_t = \sqrt{\frac{mg}{k}}$$

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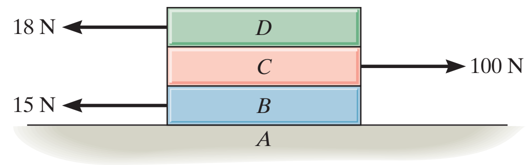
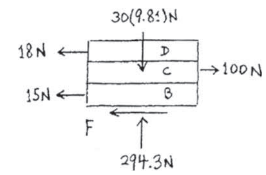
13–45.

Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



SOLUTION

Plates *B, C* and *D*

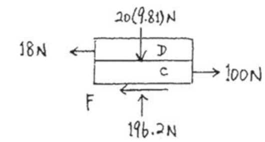


$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 100 - 15 - 18 - F = 0$

$F = 67 \text{ N}$

$F_{max} = 0.3(294.3) = 88.3 \text{ N} > 67 \text{ N}$

Plate *B* will not slip.



$a_B = 0$ **Ans.**

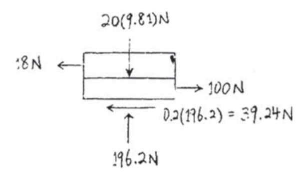
Plates *D* and *C*

$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 100 - 18 - F = 0$

$F = 82 \text{ N}$

$F_{max} = 0.3(196.2) = 58.86 \text{ N} < 82\text{N}$

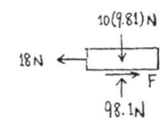
Slipping between *B* and *C*.



Assume no slipping between *D* and *C*,

$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 100 - 39.24 - 18 = 20\, a_x$

$a_x = 2.138 \text{ m/s}^2 \rightarrow$

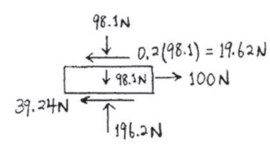


Check slipping between *D* and *C*.

$\stackrel{+}{\rightarrow} \Sigma F_x = m\, a_x; \qquad F - 18 = 10(2.138)$

$F = 39.38 \text{ N}$

$F_{max} = 0.3(98.1) = 29.43 \text{ N} < 39.38 \text{ N}$



Slipping between *D* and *C*.

Plate *C*:

$\stackrel{+}{\rightarrow} \Sigma F_x = m\, a_x; \qquad 100 - 39.24 - 19.62 = 10\, a_c$

$a_C = 4.11 \text{ m/s}^2 \rightarrow$ **Ans.**

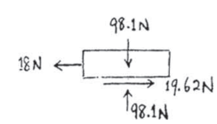


Plate *D*:

$\stackrel{+}{\rightarrow} \Sigma F_x = m\, a_x; \qquad 19.62 - 18 = 10\, a_D$

$a_D = 0.162 \text{m/s}^2 \rightarrow$ **Ans.**

Ans:
$a_B = 0$
$a_C = 4.11 \text{ m/s}^2 \rightarrow$
$a_D = 0.162 \text{ m/s}^2 \rightarrow$

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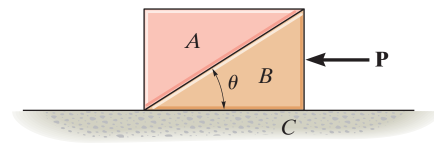
13–46.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not move relative to B. All surfaces are smooth.

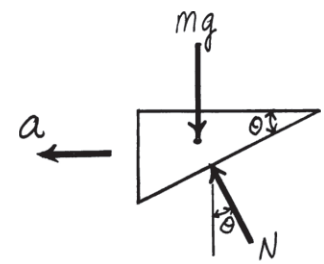


SOLUTION

Require

$a_A = a_B = a$

Block A:

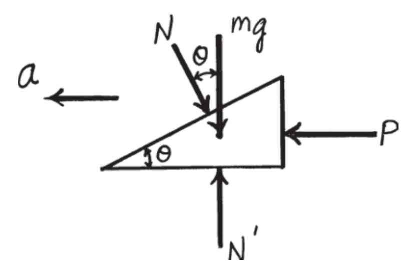


$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - mg = 0$

$\xleftarrow{+} \Sigma F_x = ma_x; \qquad N\sin\theta = ma$

$a = g\tan\theta$

Block B:



$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P - N\sin\theta = ma$

$P - mg\tan\theta = mg\tan\theta$

$P = 2mg\tan\theta$ **Ans.**

Ans:

$P = 2mg\tan\theta$

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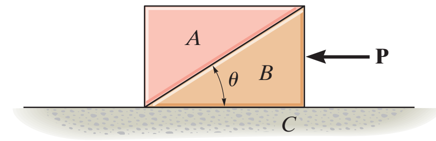
13–47.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

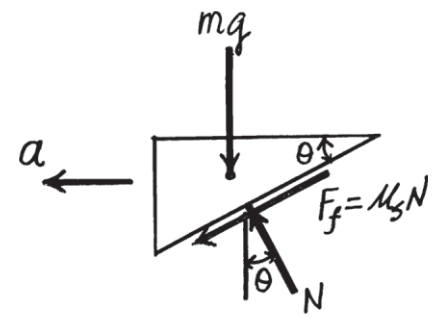


SOLUTION

Require

$a_A = a_B = a$

Block A:



$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N \sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

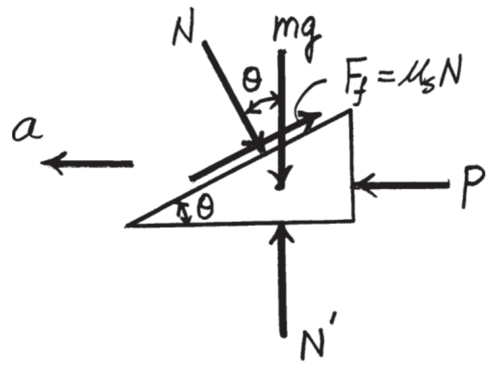
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$ **Ans.**



Ans:

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

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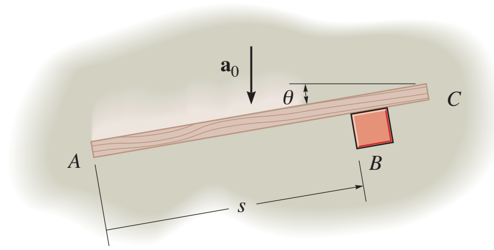
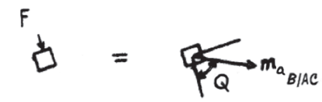
***13–48.**

The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



SOLUTION



$\nearrow+\Sigma F_x = m\,a_x;\qquad 0 = m\,a_B \sin\phi$

$$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$$

$$\mathbf{a}_B = \mathbf{a_0} + \mathbf{a}_{B/AC}$$

$\nearrow+\qquad a_B\ \sin\phi = -a_0\ \sin\theta + a_{B/AC}$

Thus,

$$0 = m(-a_0 \sin\theta + a_{B/AC})$$

$$a_{B/AC} = a_0\ \sin\theta$$

$$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0\ \sin\theta\, dt$$

$$v_{B/AC} = a_0 \sin\theta\, t \qquad \textbf{Ans.}$$

$$s_{B/AC} = s = \int_0^t a_0\ \sin\theta\, t\, dt$$

$$s = \frac{1}{2}a_0 \sin\theta\, t^2 \qquad \textbf{Ans.}$$

Ans:

$v_{B/AC} = a_0 \sin\theta\, t$

$s = \frac{1}{2}a_0 \sin\theta\, t^2$

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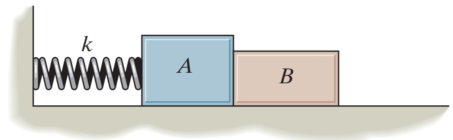
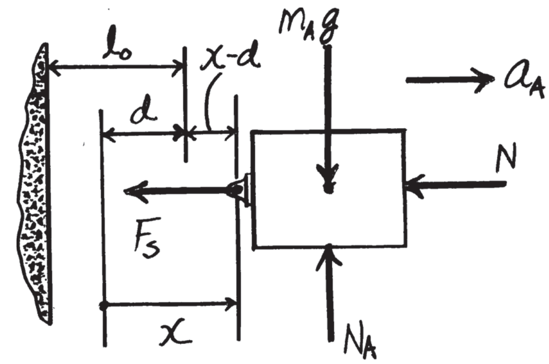
13–49.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



SOLUTION



Block A:

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -k(x - d) - N = m_A a_A$

Block B:

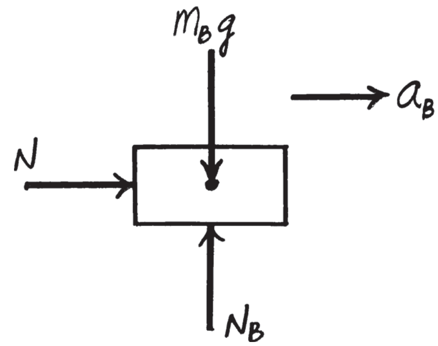
$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad N = m_B a_B$

Since $a_A = a_B = a$,

$-k(x - d) - m_B a = m_A a$

$$a = \frac{k(d - x)}{(m_A + m_B)} \qquad N = \frac{km_B(d - x)}{(m_A + m_B)}$$

$N = 0$ when $\quad d - x = 0, \quad$ or $\qquad x = d$ **Ans.**



$v\,dv = a\,dx$

$$\int_0^v v\,dv = \int_0^d \frac{k(d - x)}{(m_A + m_B)}\,dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A + m_B)}$$

$$v = \sqrt{\frac{kd^2}{(m_A + m_B)}}$$ **Ans.**

Ans:
$x = d$
$v = \sqrt{\dfrac{kd^2}{m_A + m_B}}$

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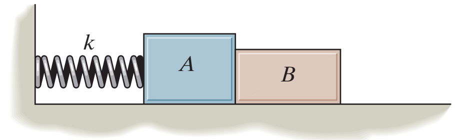
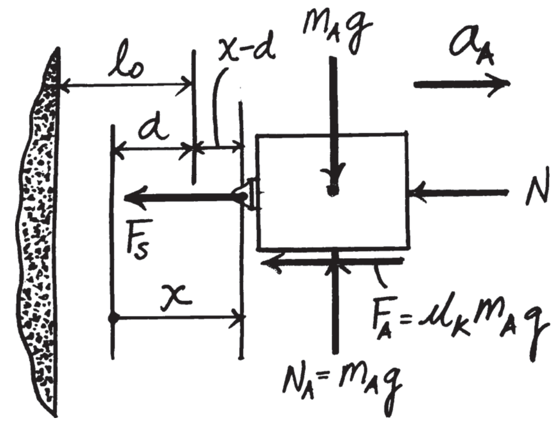
13–50.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



SOLUTION



Block A:

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad -k(x-d) - N - \mu_k m_A g = m_A a_A$

Block B:

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad N - \mu_k m_B g = m_B a_B$

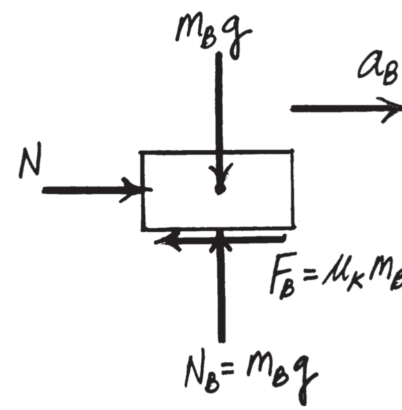
Since $a_A = a_B = a$,

$$a = \frac{k(d-x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

$$N = \frac{km_B(d-x)}{(m_A + m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans.**

At the moment of separation:



$v\,dv = a\,dx$

$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d-x)}{(m_A + m_B)} - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$

Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0$$

Thus,

$$kd > 2\mu_k g(m_A + m_B)$$

$$d > \frac{2\mu_k g}{k}(m_A + m_B)$$ **Q.E.D.**

Ans:
$x = d$ for separation.

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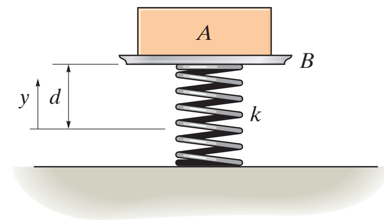
13–51.

The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

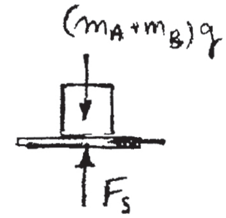


SOLUTION

For Equilibrium

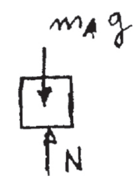
$$+\uparrow \Sigma F_y = ma_y; \qquad F_s = (m_A + m_B)g$$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$



Block:

$$+\uparrow \Sigma F_y = ma_y; \qquad -m_A g + N = m_A a$$



Block and pan

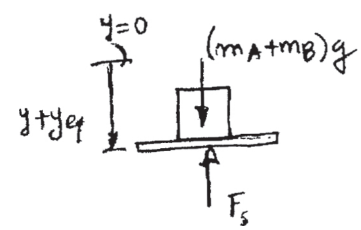
$$+\uparrow \Sigma F_y = ma_y; \qquad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d, N = 0$

$$kd = -(m_A + m_B)g$$



Since d is downward,

$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans.}$$

Ans:
$d = \dfrac{(m_A + m_B)g}{k}$

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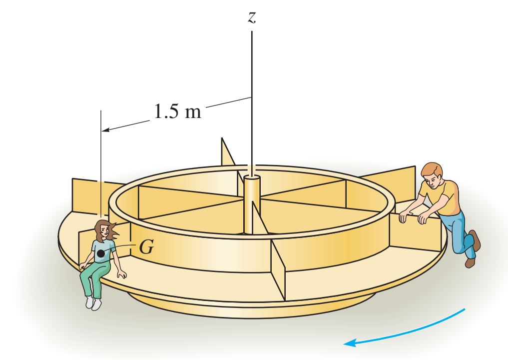
***13–52.**

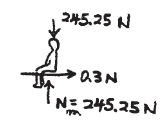
A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.



SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$$

$$v = 2.10 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 2.10$ m/s

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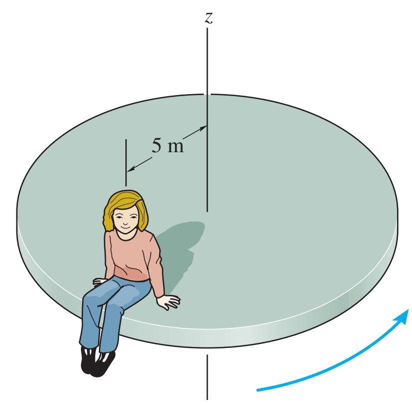
13–53.

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



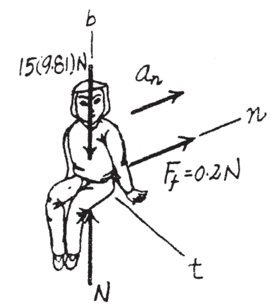
SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–7, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15 \text{ N}$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 3.13$ m/s

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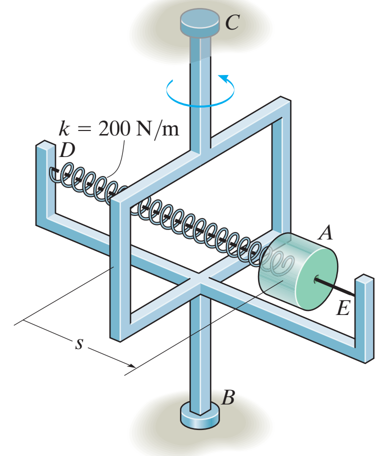
13–54.

The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of $k = 200$ N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $s = 0$, determine the constant speed of the collar in order that $s = 100$ mm. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



SOLUTION

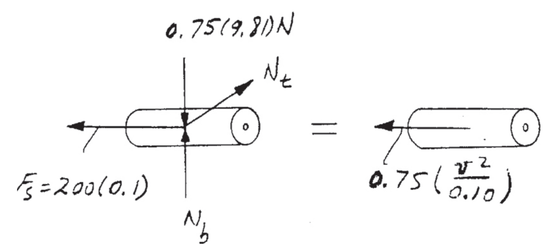
$\Sigma F_b = 0; \qquad N_b - 0.75(9.81) = 0 \qquad N_b = 7.36$

$\Sigma F_n = ma_n; \qquad 200(0.1) = 0.75\left(\dfrac{v^2}{0.10}\right)$

$\Sigma F_t = ma_t; \qquad N_t = 0$

$v = 1.63 \text{ m/s}$ **Ans.**

$N = \sqrt{(7.36)^2 + (0)} = 7.36 \text{ N}$ **Ans.**



Ans:
$v = 1.63$ m/s
$N = 7.36$ N

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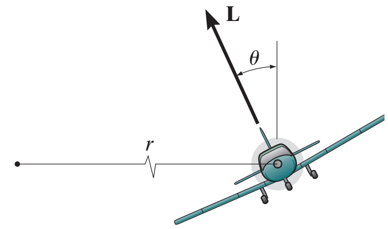
13–55.

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000$ m. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.



SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

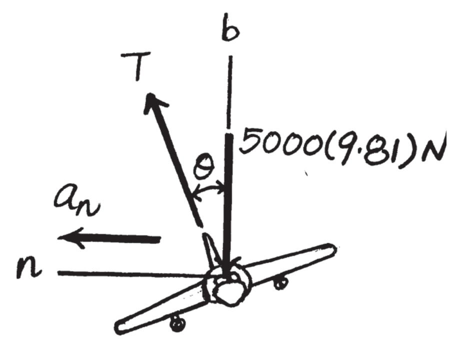
Equations of Motion: The speed of the airplane is $v = \left(350 \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{3000} = 3.151 \text{ m/s}^2$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\qquad T \cos\theta - 5000(9.81) = 0$ **(1)**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\qquad T \sin\theta = 5000(3.151)$ **(2)**

Solving Eqs. (1) and (2) yields

$\theta = 17.8^\circ \qquad T = 51517.75 = 51.5 \text{ kN}$ **Ans.**



(a)

Ans:

$T = 51.5$ kN

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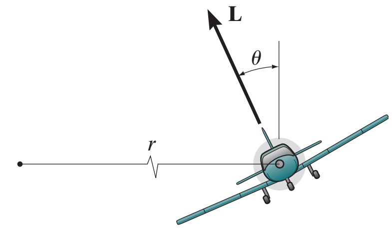
***13–56.**

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15°$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

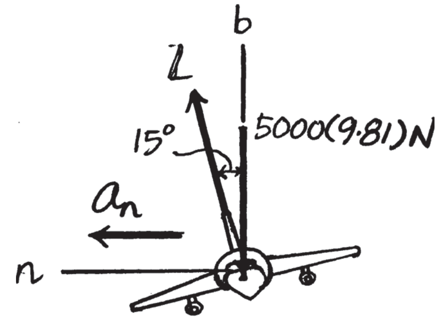
Equations of Motion: The speed of the airplane is $v = \left(350\,\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000\text{ m}}{1\text{ km}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{97.22^2}{r}$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\qquad L\cos 15° - 5000(9.81) = 0$

$L = 50780.30\text{ N} = 50.8\text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\qquad 50780.30\sin 15° = 5000\left(\dfrac{97.22^2}{r}\right)$

$r = 3595.92\text{ m} = 3.60\text{ km}$ **Ans.**



(a)

Ans:
$L = 50.8$ kN
$r = 3.60$ km

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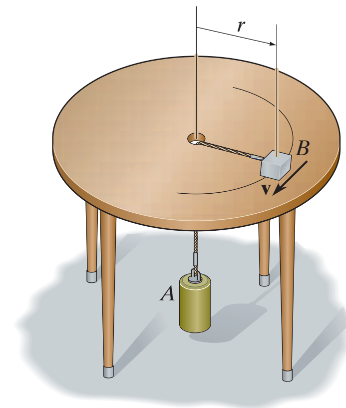
13–57.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.



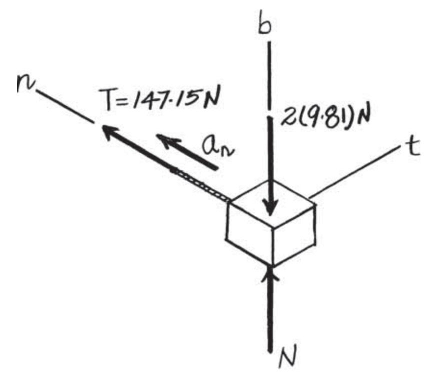
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

$$r = 1.36\text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$r = 1.36\text{ m}$

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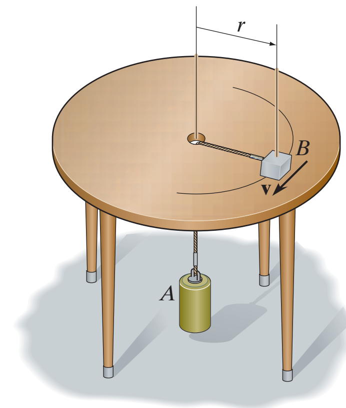
13–58.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.



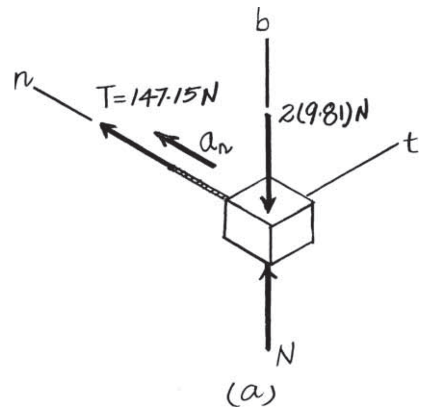
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\text{ m/s}$$ **Ans.**



Ans:
$v = 10.5\text{ m/s}$

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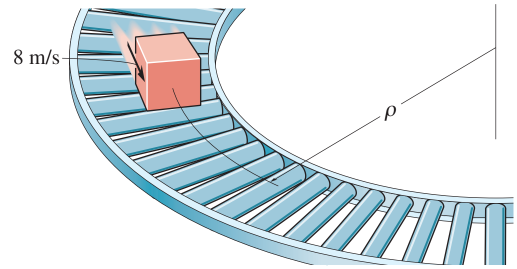
13–59.

Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



SOLUTION

$+\uparrow \Sigma F_b = m\,a_b; \qquad N - W = 0$

$N = W$

$F_x = 0.7W$

$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad 0.7W = \frac{W}{9.81}\left(\frac{8^2}{\rho}\right)$

$\rho = 9.32 \text{ m}$ **Ans.**

Ans:
$\rho = 9.32$ m

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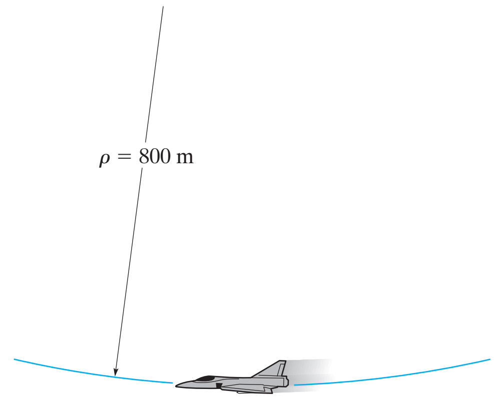
***13–60.**

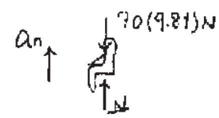
Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 800$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5\ \text{m/s}^2$. If he has a mass of 70 kg, determine the normal force he exerts on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.



SOLUTION

$$a_n = \frac{v^2}{\rho}; \qquad 78.5 = \frac{v^2}{800}$$

$v = 251$ m/s **Ans.**



$$+\uparrow \Sigma F_n = ma_n; \qquad N - 70(9.81) = 70(78.5)$$

$N = 6.18$ kN **Ans.**

Ans:
$N = 6.18$ kN

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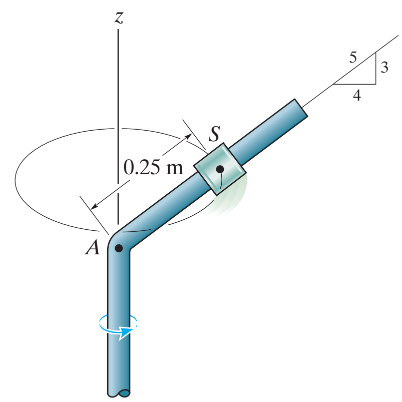
13–61.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.

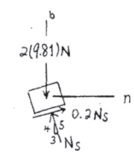


SOLUTION

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2\text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $$N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

$+\uparrow \Sigma F_b = m\,a_b;$ $$N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$



$$N_s = 21.3\text{ N}$$

$$v = 0.969\text{ m/s}$$ **Ans.**

Ans:
$v = 0.969$ m/s

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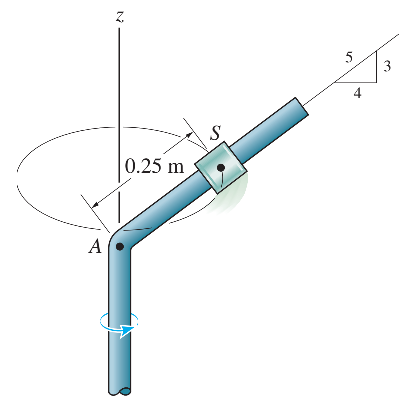
13–62.

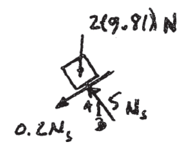
The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



SOLUTION

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n; \qquad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$



$$+\uparrow \Sigma F_b = m\,a_b; \qquad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 1.48 \text{ m/s}$

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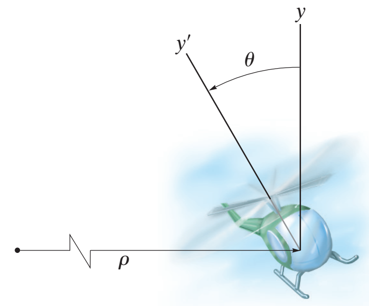
13–63. The 1.40-Mg helicopter is traveling at a constant speed of 40 m/s along the horizontal curved path while banking at $\theta = 30^\circ$. Determine the force acting normal to the blade, i.e., in the y' direction, and the radius of curvature of the path.



SOLUTION

Guesses $\quad F_N = 1\ \text{kN} \qquad \rho = 1\ \text{m}$

Given $\quad F_N\cos(\theta) - Mg = 0$

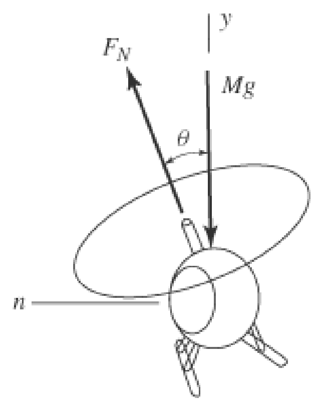
$$F_N\sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

$v = 40\ \text{m/s}$

$M = 1.4 \times 10^3\ \text{kg}$

$\theta = 30^\circ$

$g = 9.81\ \text{m/s}^2$



$$\begin{pmatrix} F_N \\ \rho \end{pmatrix} = \text{Find}(F_N, \rho) \qquad F_N = 15.86\ \text{kN} \quad \textbf{Ans.}$$

$$\rho = 282\ \text{m} \quad \textbf{Ans.}$$

Ans:
$F_N = 15.86$ kN
$\rho = 282$ m

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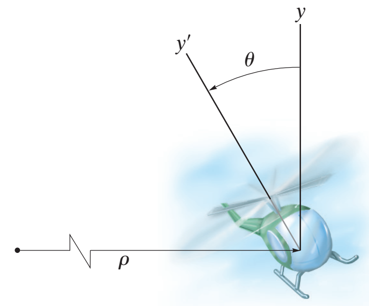
***13–64.** The 1.40-Mg helicopter is traveling at a constant speed of 33 m/s along the horizontal curved path having a radius of curvature of $\rho = 300$ m. Determine the force the blade exerts on the frame and the bank angle θ.



SOLUTION

Guesses $\quad F_N = 1\ \text{kN} \qquad \theta = 1^\circ$

Given $\quad F_N\cos(\theta) - Mg = 0$

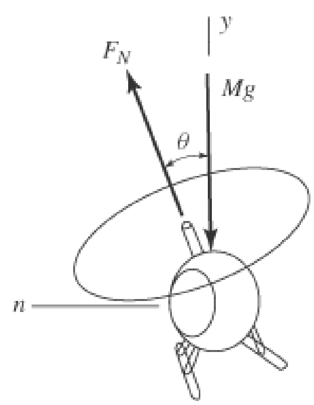
$$F_N\sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

$v = 33\ \text{m/s}$

$M = 1.4 \times 10^3\ \text{kg}$

$\rho = 300\ \text{m}$

$g = 9.81\ \text{m/s}^2$



$$\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta) \qquad F_N = 14.64\ \text{kN} \quad \textbf{Ans.}$$

$$\theta = 20^\circ \quad \textbf{Ans.}$$

Ans:
$F_N = 14.64$ kN
$\theta = 20^\circ$

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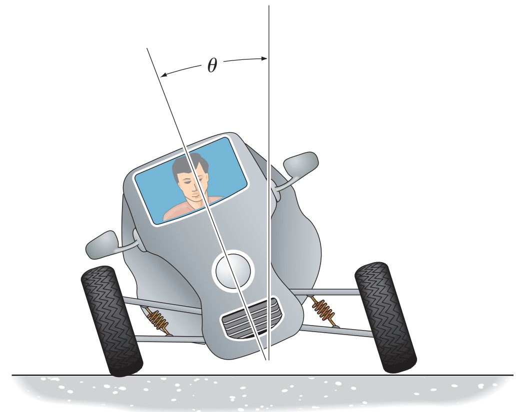
13–65.

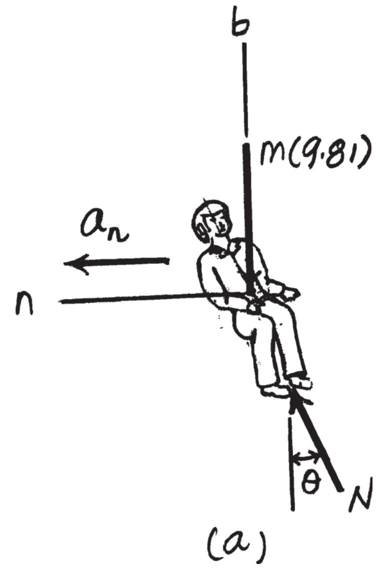
The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



SOLUTION

Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),



$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

$$\theta = 26.7^\circ$$ **Ans.**

Ans:
$\theta = 26.7^\circ$

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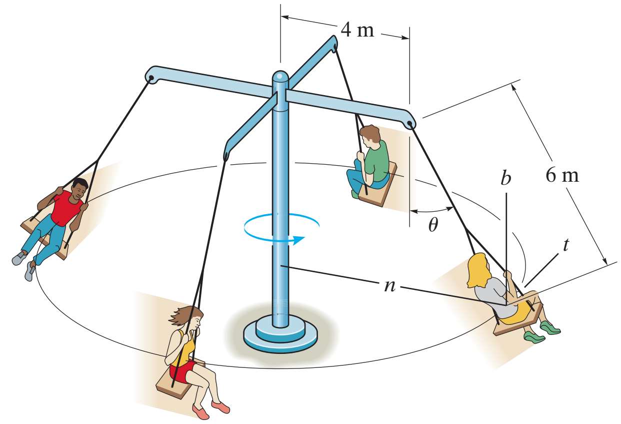
13–66.

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the *n*, *t*, and *b* directions which the chair exerts on a 50-kg passenger during the motion?



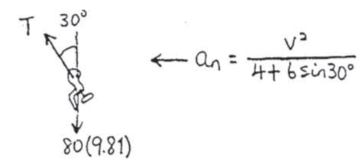
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\quad T \sin 30° = 80\left(\dfrac{v^2}{4 + 6 \sin 30°}\right)$

$+\uparrow \Sigma F_b = 0;$ $\quad T \cos 30° - 80(9.81) = 0$

$T = 906.2 \text{ N}$

$v = 6.30 \text{ m/s}$ **Ans.**

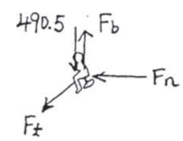


$\Sigma F_n = m\,a_n;$ $\quad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283 \text{ N}$ **Ans.**

$\Sigma F_t = m\,a_t;$ $\quad F_t = 0$ **Ans.**

$\Sigma F_b = m\,a_b;$ $\quad F_b - 490.5 = 0$

$F_b = 490 \text{ N}$ **Ans.**



Ans:
$v = 6.30 \text{ m/s}$
$F_n = 283 \text{ N}$
$F_t = 0$
$F_b = 490 \text{ N}$

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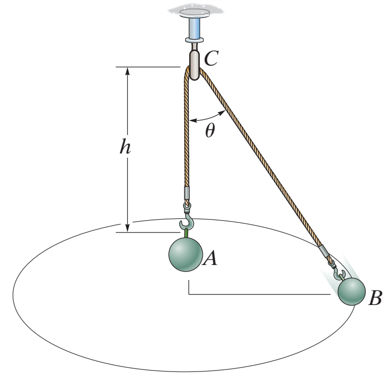
13–67.

Bobs A and B of mass m_A and m_B $(m_A > m_B)$ are connected to an inextensible light string of length l that passes through the smooth ring at C. If bob B moves as a conical pendulum such that A is suspended a distance of h from C, determine the angle θ and the speed of bob B. Neglect the size of both bobs.



SOLUTION

Free-Body Diagram: The free-body diagram of bob B is shown in Fig. a. The tension developed in the string is equal to the weight of bob A, i.e., $T = m_A g$. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = (l - h)\sin\theta$. Thus, $a_n = \dfrac{v^2}{\rho} = \dfrac{v_B{}^2}{(l - h)\sin\theta}$. By referring to Fig. a,

$+\uparrow \Sigma F_b = 0; \qquad m_A g\cos\theta - m_B g = 0$

$$\theta = \cos^{-1}\left(\frac{m_B}{m_A}\right) \qquad \textbf{Ans.}$$

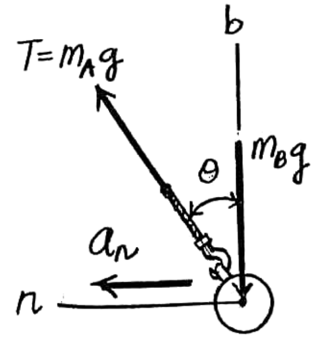
$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad m_A g\sin\theta = m_B\left[\dfrac{v_B{}^2}{(l - h)\sin\theta}\right]$

$$v_B = \sqrt{\frac{m_A g(l - h)}{m_B}}\sin\theta \qquad \textbf{(1)}$$

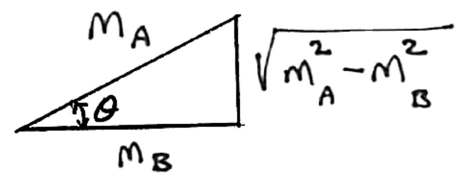
From Fig. b, $\sin\theta = \dfrac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}$. Substituting this value into Eq. (1),

$$v_B = \sqrt{\frac{m_A g(l - h)}{m_B}\left(\frac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}\right)}$$

$$= \sqrt{\frac{g(l - h)(m_A{}^2 - m_B{}^2)}{m_A m_B}} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$\theta = \cos^{-1}\left(\dfrac{m_B}{m_A}\right)$

$v_B = \sqrt{\dfrac{g(l - h)(m_A^2 - m_B^2)}{m_A m_B}}$

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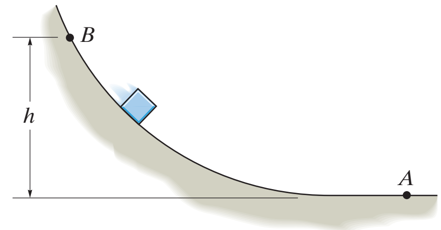
***13–68.**

Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



SOLUTION

$+\searrow \Sigma F_t = ma_t; \qquad mg \sin\theta = ma_t \qquad a_t = g\sin\theta$

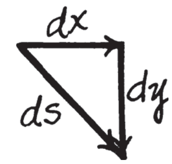
$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad$ However $\quad dy = ds\sin\theta$

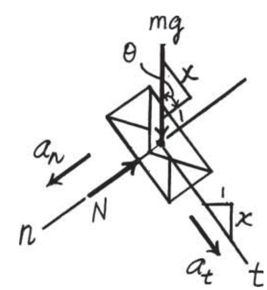
$$\int_0^v v\,dv = \int_0^h g\,dy$$

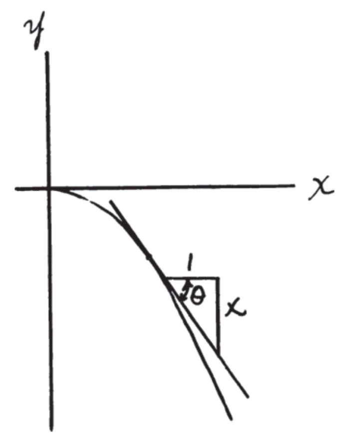
$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh}$$

Q.E.D.







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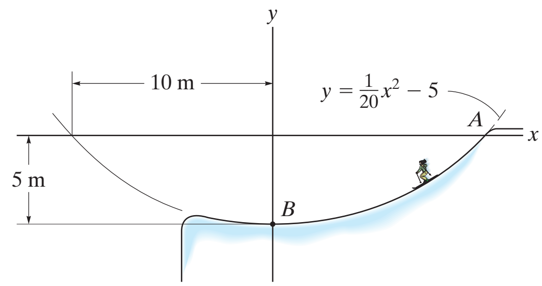
13–69.

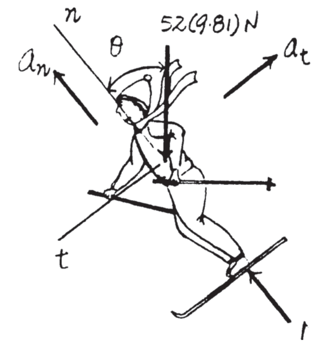
The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier.



SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{10}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \qquad \theta = 0^\circ$$



and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{ m}} = 10.0\text{ m}$$

Equations of Motion:

$+\swarrow\Sigma F_t = ma_t; \qquad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$

$+\nwarrow\Sigma F_n = ma_n; \qquad N - 52(9.81)\cos\theta = m\left(\dfrac{v^2}{\rho}\right) \qquad$ **(1)**

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}dx = \sqrt{1 + \frac{1}{100}x^2}dx$

Here, $\tan\theta = \dfrac{1}{10}x$. Then, $\sin\theta = \dfrac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+\ ) \qquad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^0 \left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \tfrac{1}{100}x^2}dx\right)$$

$$v^2 = 9.81\text{ m}^2/\text{s}^2$$

Substituting $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0^\circ$, and $\rho = 10.0$ m into Eq.(1) yields

$$N - 52(9.81)\cos 0^\circ = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN}$$ **Ans.**

Ans:
$N = 1.02$ kN

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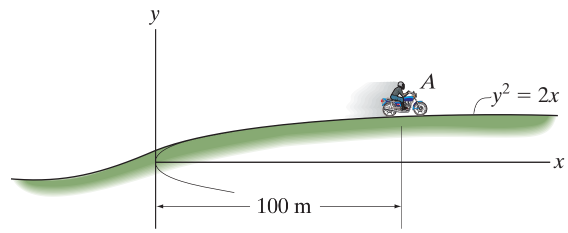
13–70.

The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.



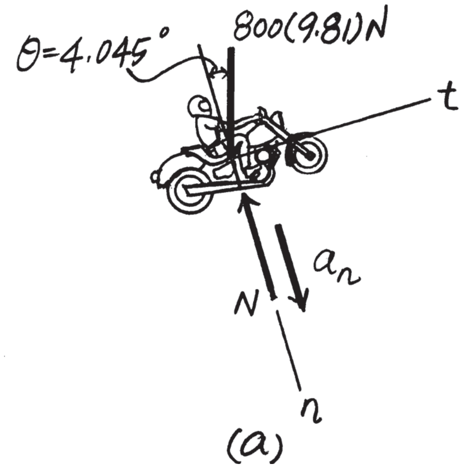
SOLUTION

Geometry: Here, $y = \sqrt{2}x^{1/2}$. Thus, $\dfrac{dy}{dx} = \dfrac{\sqrt{2}}{2x^{1/2}}$ and $\dfrac{d^2y}{dx^2} = -\dfrac{\sqrt{2}}{4x^{3/2}}$. The angle that the hill slope at A makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=100\text{ m}} = \tan^{-1}\left(\frac{\sqrt{2}}{2x^{1/2}}\right)\bigg|_{x=100\text{ m}} = 4.045^\circ$$

The radius of curvature of the hill at A is given by

$$\rho_A = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ m}} = \frac{\left[1 + \left(\dfrac{\sqrt{2}}{2(100^{1/2})}\right)^2\right]^{3/2}}{\left|-\dfrac{\sqrt{2}}{4(100^{3/2})}\right|} = 2849.67\text{ m}$$



Free-Body Diagram: The free-body diagram of the motorcycle is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the motorcycle is

$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$$

Thus, $a_n = \dfrac{v^2}{\rho_A} = \dfrac{22.22^2}{2849.67} = 0.1733\text{ m/s}^2$. By referring to Fig. (a),

$$\searrow + \Sigma F_n = ma_n; \qquad 800(9.81)\cos 4.045^\circ - N = 800(0.1733)$$

$$N = 7689.82\text{ N} = 7.69\text{ kN}$$ **Ans.**

Ans:
$N = 7.69\text{ kN}$

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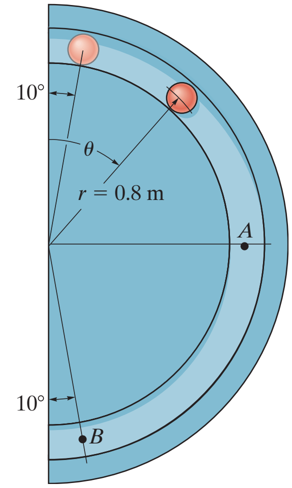
13–71. A ball having a mass 2 kg and negligible size moves within a smooth vertical circular slot. If it is released from rest when $\theta = 10°$, determine the force of the slot on the ball when the ball arrives at points A and B.

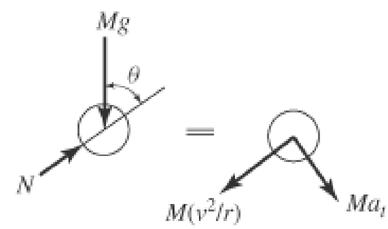


SOLUTION

Given:

$M = 2$ kg $\quad \theta = 90° \quad \theta_1 = 10°$

$g = 9.81\ \text{m/s}^2 \quad r = 0.8$ m



$$Mg\sin(\theta) = Ma_t \qquad a_t = g\sin(\theta)$$

At A $\quad \theta_A = 90°$

$$v_A = \sqrt{2g\left(\int_{\theta_1}^{\theta_A} \sin(\theta)\, r\, d\theta\right)}$$

$$N_A - Mg\cos(\theta_A) = -M\left(\frac{v_A^{\,2}}{r}\right)$$

$$N_A = Mg\cos(\theta_A) - M\left(\frac{v_A^{\,2}}{r}\right) \qquad N_A = -38.6\ \text{N} \qquad \textbf{Ans.}$$

At B $\quad \theta_B = 180° - \theta_1$

$$v_B = \sqrt{2g\left(\int_{\theta_1}^{\theta_B} \sin(\theta)\, r\, d\theta\right)}$$

$$N_B - Mg\cos(\theta_B) = -M\left(\frac{v_B^{\,2}}{r}\right)$$

$$N_B = Mg\cos(\theta_B) - M\left(\frac{v_B^{\,2}}{r}\right) \qquad N_B = -96.6\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$N_A = -38.6$ N
$N_B = -96.6$ N

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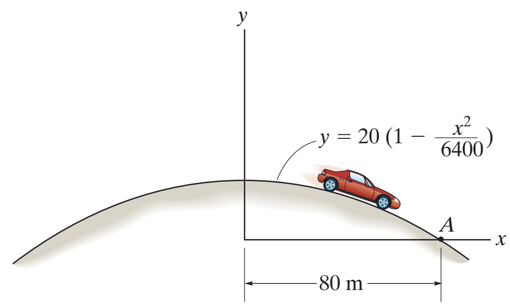
***13–72.**

The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



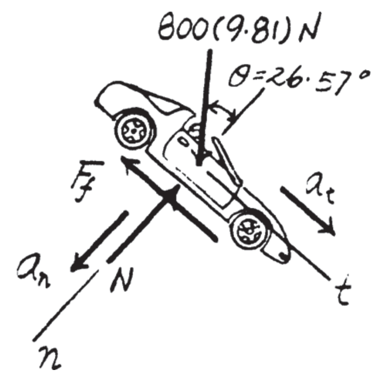
SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–7 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57° - F_f = 800(0)$$

$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_f = 3.51$ kN
$N = 6.73$ kN

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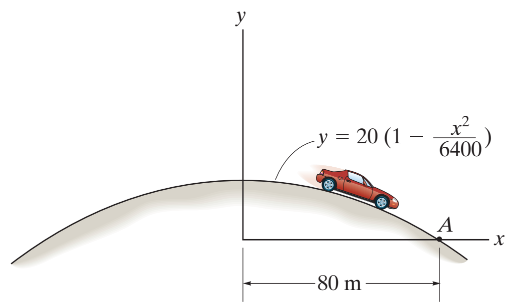
13–73.

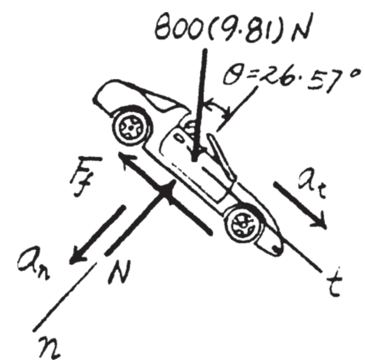
The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$



and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equation of Motion: Applying Eq. 13–7 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$

$F_f = 1109.73\text{ N} = 1.11\text{ kN}$ **Ans.**

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

$N = 6729.67\text{ N} = 6.73\text{ kN}$ **Ans.**

Ans:

$F_f = 1.11$ kN
$N = 6.73$ kN

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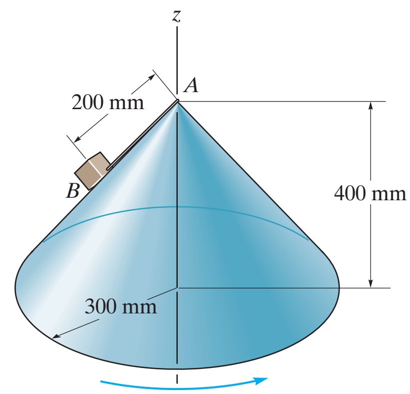
13–74. The block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the z axis such that the block attains a speed of 0.5 m/s. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block and the effect of friction.



SOLUTION

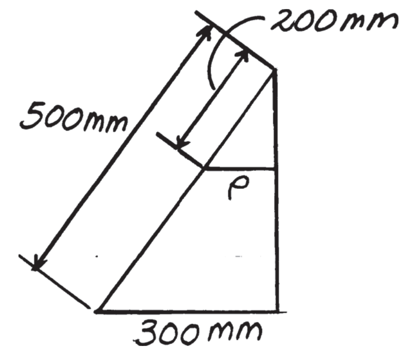
$$\frac{\rho}{200} = \frac{300}{500}; \qquad \rho = 120 \text{ mm} = 0.120 \text{ m}$$

$$+\nearrow \Sigma F_y = ma_y; \qquad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$$

$$T = 1.82 \text{ N} \qquad \textbf{Ans.}$$

$$+\nwarrow \Sigma F_x = ma_x; \qquad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$$

$$N_B = 0.844 \text{ N} \qquad \textbf{Ans.}$$



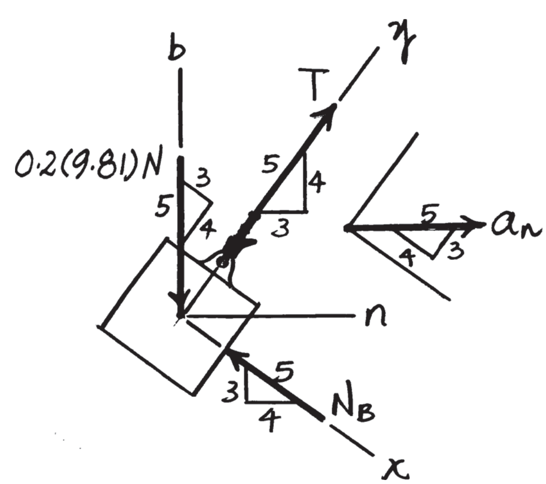
Also,

$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$$

$$+\uparrow \Sigma F_b = 0; \qquad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$$

$$T = 1.82 \text{ N} \qquad \textbf{Ans.}$$

$$N_B = 0.844 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$T = 1.82$ N
$N_B = 0.844$ N

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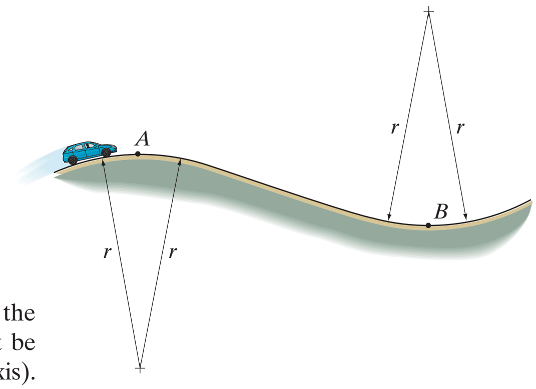
13–75.

Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



SOLUTION

Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

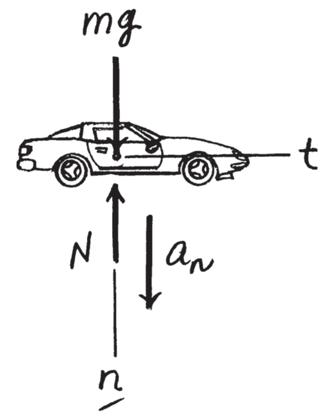
Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad\qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

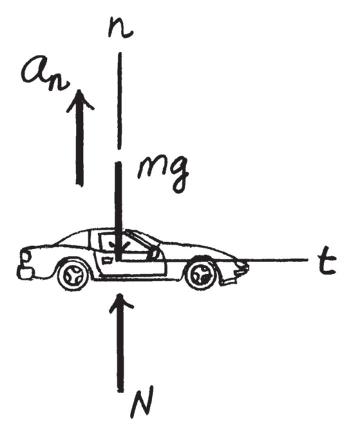
Using the result of v, the normal component of car acceleration is $a_n = \dfrac{v^2}{\rho} = \dfrac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$v = \sqrt{gr}$
$N = 2mg$

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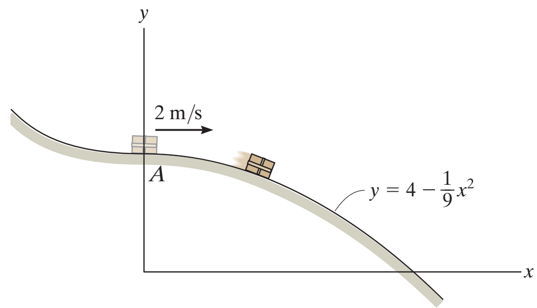
***13–76.** The 35-kg box has a speed 2 m/s when it is at A on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant $x = 3$ m. Also, what is the rate of increase in its speed at this instant?



SOLUTION

Given:

$M = 35\ \text{kg} \qquad a = 4\ \text{m}$

$v_0 = 2\ \text{m/s} \qquad b = \dfrac{1}{9}\ \text{m}^{-1}$

$x_1 = 3\ \text{m}$

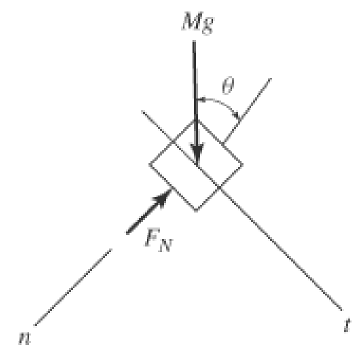
$y(x) = a - bx^2$

$y'(x) = -2bx$

$y''(x) = -2b$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$\theta(x) = \text{atan}(y'(x))$



Find the velocity

$$v_1 = \sqrt{v_0^2 + 2g\left(y(0\ \text{m}) - y(x_1)\right)} \qquad v_1 = 4.859\ \text{m/s}$$

Guesses $\quad F_N = 1\ \text{N} \qquad v' = 1\ \text{m/s}^2$

Given $\quad F_N - Mg\cos\left(\theta(x_1)\right) = M\left(\dfrac{v_1^2}{\rho(x_1)}\right) \qquad -Mg\sin\left(\theta(x_1)\right) = Mv'$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} = \text{Find}(F_N, v') \qquad F_N = 179.8\ \text{N} \qquad v' = 5.44\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$F_N = 179.8$ N
$v' = 5.44$ m/s^2

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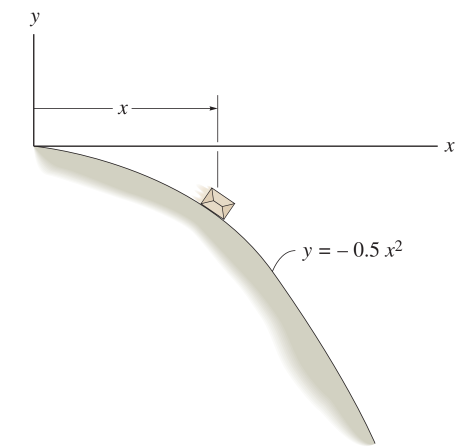
13–77.

The box has a mass m and slides down the smooth chute having the shape of a parabola. If it has an initial velocity of v_0 at the origin, determine its velocity as a function of x. Also, what is the normal force on the box, and the tangential acceleration as a function of x?



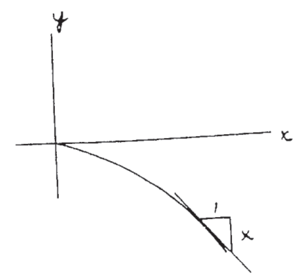
SOLUTION

$$y = -\frac{1}{2}x^2$$

$$\frac{dy}{dx} = -x$$

$$\frac{d^2y}{dx^2} = -1$$

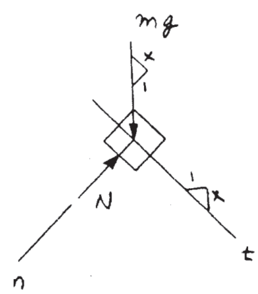
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + x^2\right]^{\frac{3}{2}}}{|-1|} = \left(1 + x^2\right)^{\frac{3}{2}}$$



$+\swarrow \Sigma F_n = ma_n;\qquad mg\left(\frac{1}{\sqrt{1 + x^2}}\right) - N = m\left(\frac{v^2}{(1 + x^2)^{\frac{3}{2}}}\right)$ **(1)**

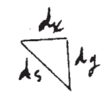
$+\searrow \Sigma F_t = ma_t;\qquad mg\left(\frac{x}{\sqrt{1 + x^2}}\right) = ma_t$

$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$ **Ans.**



$$v\,dv = a_t\,ds = g\left(\frac{x}{\sqrt{1 + x^2}}\right)ds$$

$$ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx = \left(1 + x^2\right)^{\frac{1}{2}} dx$$



$$\int_{v_0}^{v} v\,dv = \int_0^x gx\,dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = g\left(\frac{x^2}{2}\right)$$

$v = \sqrt{v_0^2 + gx^2}$ **Ans.**

From Eq. (1):

$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{(v_0^2 + gx^2)}{(1 + x^2)}\right]$ **Ans.**

Ans:

$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$

$v = \sqrt{v_0^2 + gx^2}$

$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{v_0^2 + gx^2}{1 + x^2}\right]$

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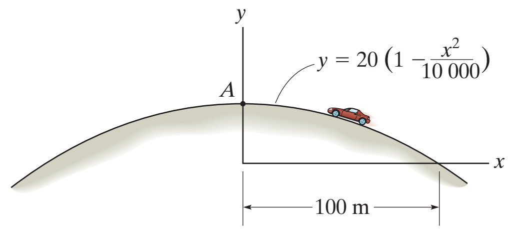
13–78.

Determine the maximum constant speed at which the 2-Mg car can travel over the crest of the hill at A without leaving the surface of the road. Neglect the size of the car in the calculation.



SOLUTION

Geometry. The radius of curvature of the road at A must be determined first. Here

$$\frac{dy}{dx} = 20\left(-\frac{2x}{10000}\right) = -0.004x$$

$$\frac{d^2y}{dx^2} = -0.004$$

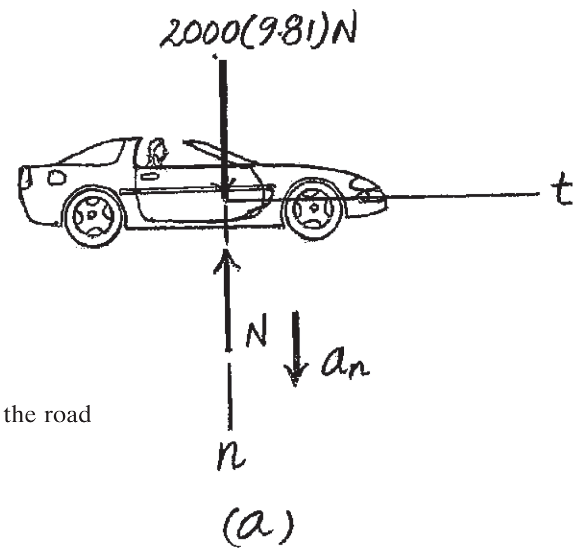
At point A, $x = 0$. Thus, $\left.\frac{dy}{dx}\right|_{x=0} = 0$. Then

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + 0^2)^{3/2}}{0.004} = 250 \text{ m}$$

Equation of Motion. Since the car is required to be on the verge to leave the road surface, $N = 0$.

$$\Sigma F_n = ma_n; \qquad 2000(9.81) = 2000\left(\frac{v^2}{250}\right)$$

$$v = 49.52 \text{ m/s} = 49.5 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 49.5$ m/s

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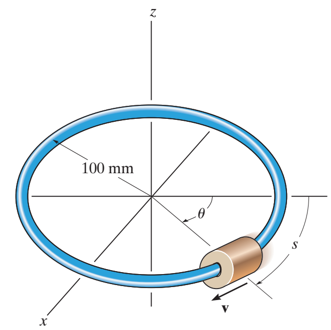
13–79. A collar having a mass 0.75 kg and negligible size slides over the surface of a horizontal circular rod for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the collar is given a speed of 4 m/s and then released at $\theta = 0°$, determine how far, s, it slides on the rod before coming to rest.



SOLUTION

$$N_{Cz} - Mg = 0$$

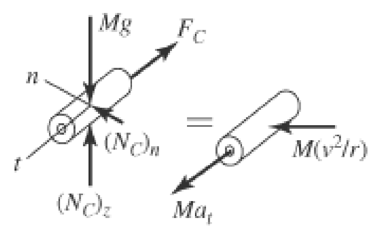
$$N_{Cn} = M\left(\frac{v^2}{r}\right)$$

$$N_C = \sqrt{N_{Cz}^{\ 2} + N_{Cn}^{\ 2}}$$

$$F_C = \mu_k N_C = -Ma_t$$

$$a_t(v) = -\mu_k\sqrt{g^2 + \frac{v^4}{r^2}}$$

Given:



$M = 0.75$ kg $\quad r = 100$ mm

$\mu_k = 0.3 \quad g = 9.81$ m/s^2

$v_1 = 4$ m/s

$$d = \int_{v_1}^{0} \frac{v}{a_t(v)}\,dv \qquad d = 0.581\text{ m} \qquad \textbf{Ans.}$$

Ans:
$d = 0.581$ m

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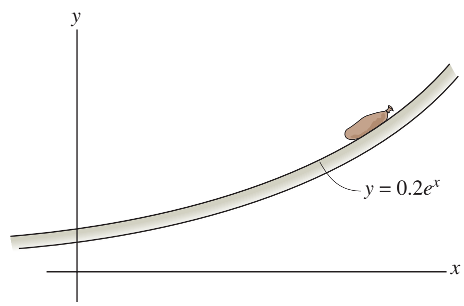
***13–80.**

The 8-kg sack slides down the smooth ramp. If it has a speed of 1.5 m/s when $y = 0.2$ m, determine the normal reaction the ramp exerts on the sack and the rate of increase in the speed of the sack at this instant.



SOLUTION

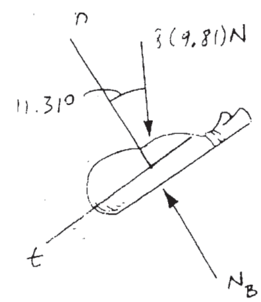
$y = 0.2 \qquad x = 0$

$y = 0.2e^x$

$$\frac{dy}{dx} = 0.2e^x \bigg|_{x=0} = 0.2$$

$$\frac{d^2y}{dx^2} = 0.2e^x \bigg|_{x=0} = 0.2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (0.2)^2\right]^{\frac{3}{2}}}{|0.2|} = 5.303$$



$\theta = \tan^{-1}(0.2) = 11.31°$

$+\nwarrow \Sigma F_n = ma_n; \qquad N_B - 8(9.81)\cos 11.31° = 8\left(\frac{(1.5)^2}{5.303}\right)$

$N_B = 80.4$ N **Ans.**

$+\swarrow \Sigma F_t = ma_t; \qquad 8(9.81)\sin 11.31° = 8a_t$

$a_t = 1.92$ m/s^2 **Ans.**

Ans:
$N_B = 80.4$ N
$a_t = 1.92$ m/s^2

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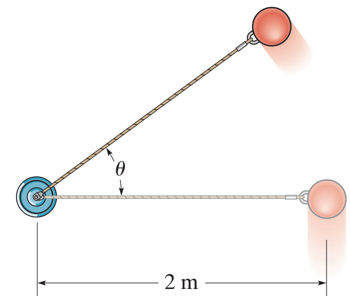
13–81.

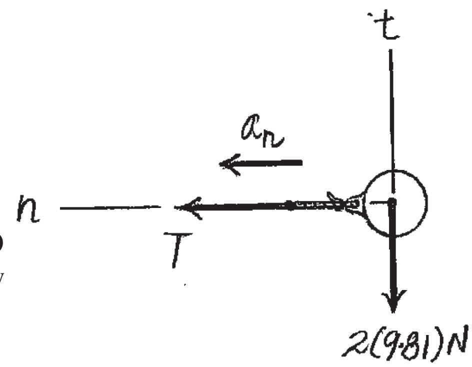
The 2-kg pendulum bob moves in the vertical plane with a velocity of 8 m/s when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches $\theta = 30°$. Neglect the size of the bob.



SOLUTION

Equations of Motion. Referring to the FBD of the bob at position $\theta = 0°$, Fig. *a*,

$$\Sigma F_n = ma_n; \qquad T = 2\left(\frac{8^2}{2}\right) = 64.0 \text{ N} \qquad \textbf{Ans.}$$



(a)

For the bob at an arbitrary position θ, the FBD is shown in Fig. *b*.

$$\Sigma F_t = ma_t; \qquad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T + 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$

$$T = v^2 - 19.62\sin\theta \qquad \textbf{(1)}$$

Kinematics. The velocity of the bob at the position $\theta = 30°$ can be determined by integrating $vdv = a_t\,ds$. However, $ds = rd\theta = 2d\theta$.

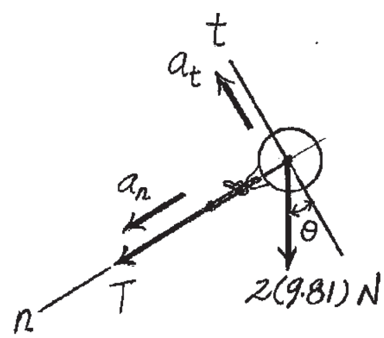
Then,

$$\int_{8\text{ m/s}}^{v} vdv = \int_{0°}^{30°} -9.81\cos\theta(zd\theta)$$

$$\left.\frac{v^2}{2}\right|_{8\text{ m/s}}^{v} = -19.62\sin\theta\Big|_{0°}^{30°}$$

$$\frac{v^2}{2} - \frac{8^2}{2} = -19.62(\sin 30° - 0)$$

$$v^2 = 44.38 \text{ m}^2/\text{s}^2$$



(b)

Substitute this result and $\theta = 30°$ into Eq. (1),

$$T = 44.38 - 19.62\sin 30°$$

$$= 34.57 \text{ N} = 34.6 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$T = 64.0$ N
$T = 34.6$ N

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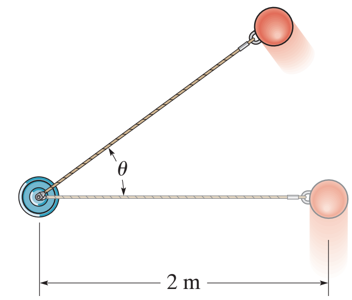
13–82.

The 2-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0°$. Determine the angle θ where the tension in the cord becomes zero.

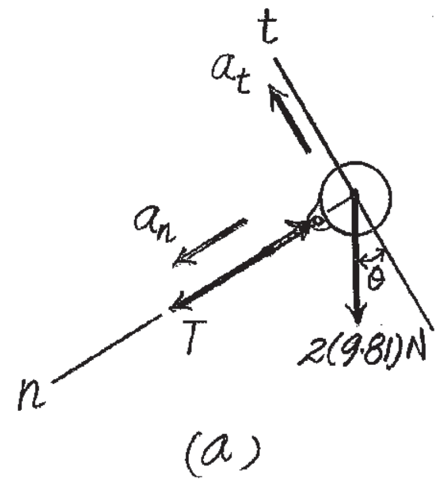


SOLUTION

Equation of Motion. The FBD of the bob at an arbitrary position θ is shown in Fig. *a*. Here, it is required that $T = 0$.

$$\Sigma F_t = ma_t; \qquad -2(9.81)\cos\theta = 2a_t$$
$$a_t = -9.81\cos\theta$$
$$\Sigma F_n = ma_n; \qquad 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$
$$v^2 = 19.62\sin\theta \qquad \textbf{(1)}$$

Kinematics. The velocity of the bob at an arbitrary position θ can be determined by integrating $v\,dv = a_t\,ds$. However, $ds = r\,d\theta = 2d\theta$.



Then

$$\int_{6\text{ m/s}}^{v} v\,dv = \int_{0°}^{\theta} -9.81\cos\theta(2d\theta)$$
$$\left.\frac{v^2}{2}\right|_{6\text{ m/s}}^{v} = -19.62\sin\theta\Big|_{0°}^{\theta}$$
$$v^2 = 36 - 39.24\sin\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

$$19.62\sin\theta = 36 - 39.24\sin\theta$$
$$58.86\sin\theta = 36$$
$$\theta = 37.71° = 37.7° \qquad \textbf{Ans.}$$

Ans:
$\theta = 37.7°$

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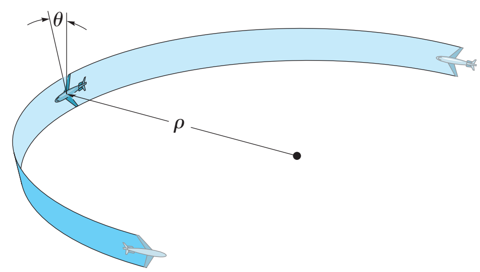
13–83.

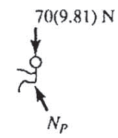
The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.



SOLUTION

$+\uparrow \sum F_b = ma_b; \qquad N_P \sin 15° - 70(9.81) = 0$

$N_P = 2.65 \text{ kN}$ **Ans.**



$\overset{+}{\leftarrow} \sum F_n = ma_n; \qquad N_P \cos 15° = 70\left(\dfrac{50^2}{\rho}\right)$

$\rho = 68.3 \text{ m}$ **Ans.**

Ans:
$N_P = 2.65 \text{ kN}$
$\rho = 68.3 \text{ m}$

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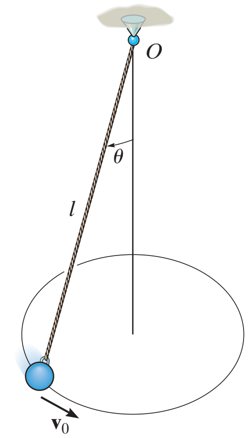
***13–84.**

The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



SOLUTION

$\overset{+}{\rightarrow}\Sigma F_n = ma_n;\qquad T\sin\theta = m\left(\dfrac{v_0^2}{r}\right)$

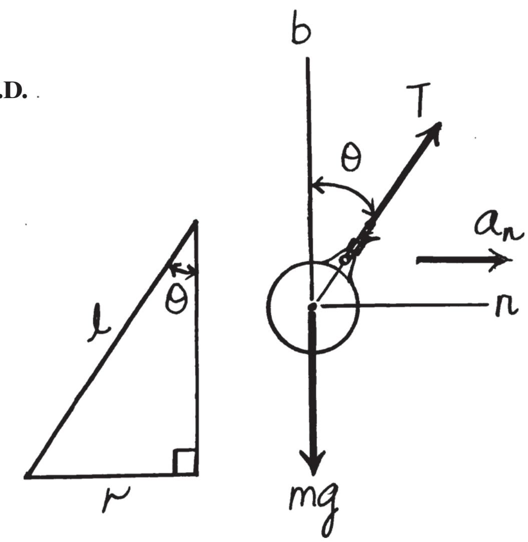
$+\uparrow\Sigma F_b = 0;\qquad T\cos\theta - mg = 0$

Since $r = l\sin\theta \qquad T = \dfrac{mv_0^2}{l\sin^2\theta}$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

Q.E.D.



Ans:

$\tan\theta\sin\theta = \dfrac{v_0^2}{gl}$

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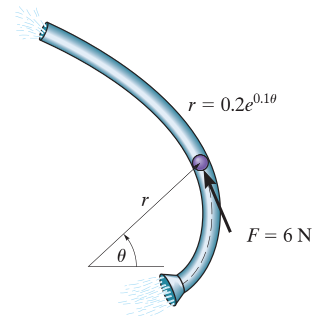
13–85. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. What direction does it act in?



SOLUTION

Given:

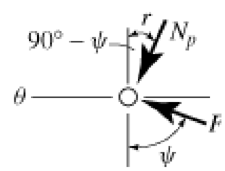
$M = 0.5\ \text{kg}$

$a = 0.2\ \text{m}$

$b = 0.1$

$F = 6\ \text{N}$

$\theta_1 = \dfrac{\pi}{2}$



$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b} \qquad \psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.29° \qquad \textbf{Ans.}$$

$$F = M v' \qquad v' = \frac{F}{M} \qquad v' = 12.00\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\psi = 84.29°$
$v' = 12.00\ \text{m/s}^2$

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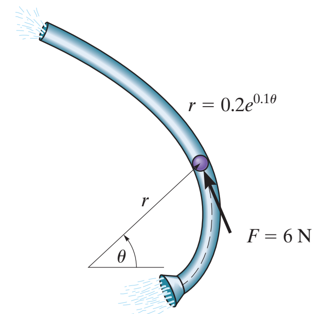
13–86. Solve Prob. 13–85 if the tube lies in a *vertical plane*.



SOLUTION

Given:

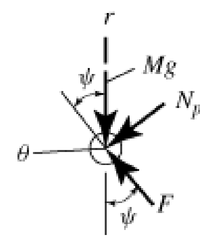
$M = 0.5\ \text{kg}$

$a = 0.2\ \text{m}$

$b = 0.1$

$F = 6\ \text{N}$

$\theta_1 = \dfrac{\pi}{2}$



$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b} \qquad \psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.29° \qquad \textbf{Ans.}$$

$$F - Mg\cos(\psi) = Mv' \qquad v' = \frac{F}{M} - g\cos(\psi) \qquad v' = 11.02\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$\psi = 84.29°$
$v' = 11.02\ \text{m/s}^2$

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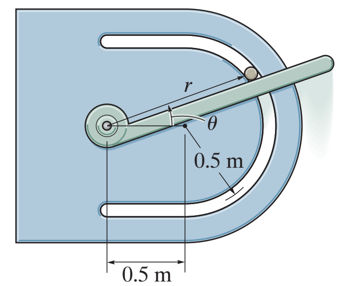
13–87. The 0.75-kg smooth can is guided along the circular path using the arm guide. If the arm has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 0.4$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the can. Motion occurs in the *horizontal plane.*

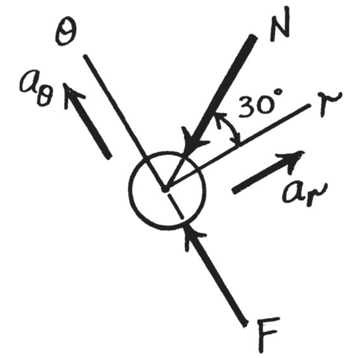


SOLUTION

$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ m}$$

$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ m/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ m/s}^2$$



Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660\left(2^2\right) = -7.128 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ m/s}^2$$

Equations of Motion: By referring to Fig. (a),

$\Sigma F_r = ma_r;$ $\quad -N\cos 30° = 0.75(-7.128)$ $\quad N = 5.346$ N

$\Sigma F_\theta = ma_\theta;$ $\quad F - 5.346\sin 30° = 0.75(-0.5359)$ $\quad F = 2.271$ N $\quad$ **Ans.**

Ans:
$F = 2.271$ N

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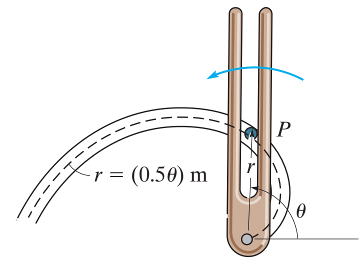
***13–88.**

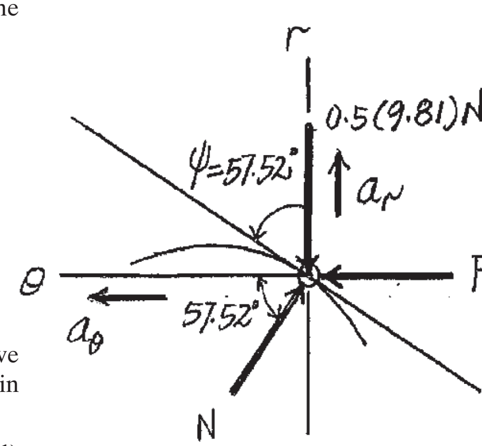
Using a forked rod, a 0.5-kg smooth peg P is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (\frac{\pi}{8}t^2)$ rad, where t is in seconds, determine the force of the rod on the peg and the normal force of the slot on the peg at the instant $t = 2$ s. The peg is in contact with only *one edge* of the rod and slot at any instant.



SOLUTION

Equation of Motion. Here, $r = 0.5\theta$. Then $\frac{dr}{d\theta} = 0.5$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5\theta}{0.5} = \theta$$



At the instant $t = 25$, $\theta = \frac{\pi}{8}(2^2) = \frac{\pi}{2}$ rad

$$\tan\psi = \frac{\pi}{2} \qquad \psi = 57.52^\circ$$

The positive sign indicates that ψ is measured from extended radial line in positive sense of θ (counter clockwise) to the tangent. Then the FBD of the peg shown in Fig. a can be drawn.

$$\Sigma F_r = ma_r; \quad N\sin 57.52^\circ - 0.5(9.81) = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \quad F - N\cos 57.52^\circ = 0.5a_\theta \qquad \textbf{(2)}$$

Kinematics. Using the chain rule, the first and second derivatives of r and θ with respect to t are

$$r = 0.5\theta = 0.5\left(\frac{\pi}{8}t^2\right) = \frac{\pi}{16}t^2 \qquad \theta = \frac{\pi}{8}t^2$$

$$\dot{r} = \frac{\pi}{8}t \qquad \dot{\theta} = \frac{\pi}{4}t$$

$$\ddot{r} = \frac{\pi}{8} \qquad \ddot{\theta} = \frac{\pi}{4}$$

When $t = 2$ s,

$$r = \frac{\pi}{16}(2^2) = \frac{\pi}{4}\text{ m} \qquad \theta = \frac{\pi}{8}(2^2) = \frac{\pi}{2}\text{ rad}$$

$$\dot{r} = \frac{\pi}{8}(2) = \frac{\pi}{4}\text{ m/s} \qquad \dot{\theta} = \frac{\pi}{4}(2) = \frac{\pi}{2}\text{ rad/s}$$

$$\ddot{r} = \frac{\pi}{8}\text{ m/s}^2 \qquad \ddot{\theta} = \frac{\pi}{4}\text{ rad/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{\pi}{8} - \frac{\pi}{4}\left(\frac{\pi}{2}\right)^2 = -1.5452\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{\pi}{4}\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = 3.0843\text{ m/s}^2$$

Substitute these results in Eqs. (1) and (2)

$$N = 4.8987\text{ N} = 4.90\text{ N} \qquad \textbf{Ans.}$$

$$F = 4.173\text{ N} = 4.17\text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 4.90$ N
$F = 4.17$ N

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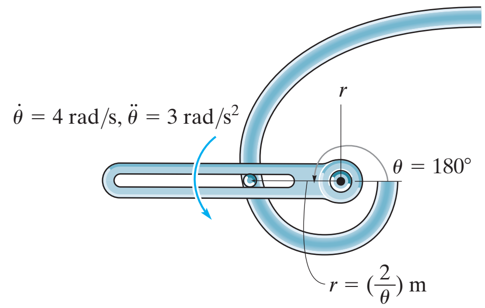
13–89.

The arm is rotating at a rate of $\dot{\theta} = 4$ rad/s when $\ddot{\theta} = 3$ rad/s^2 and $\theta = 180°$. Determine the force it must exert on the 0.5-kg smooth cylinder if it is confined to move along the slotted path. Motion occurs in the horizontal plane.



SOLUTION

Equation of Motion. Here, $r = \dfrac{2}{\theta}$. Then $\dfrac{dr}{d\theta} = -\dfrac{2}{\theta^2}$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2/\theta}{-2/\theta^2} = -\theta$$

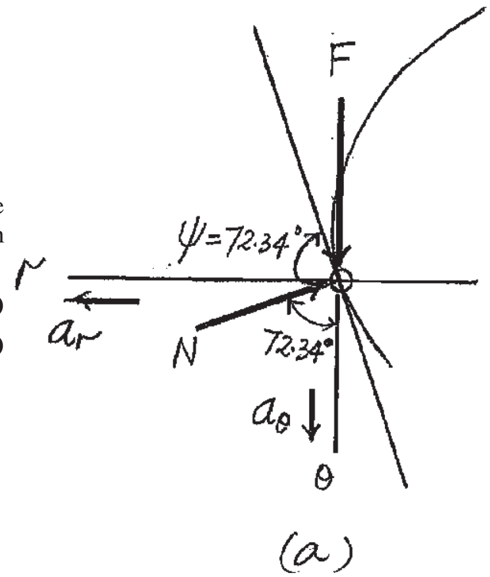
At $\theta = 180° = \pi$ rad,

$$\tan\psi = -\pi \qquad \psi = -72.34°$$

The negative sign indicates that ψ is measured from extended radial line in the negative sense of θ (clockwise) to the tangent. Then, the FBD of the peg shown in Fig. *a* can be drawn.

$$\Sigma F_r = ma_r; \qquad -N\sin 72.34° = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F - N\cos 72.34° = 0.5a_\theta \qquad \textbf{(2)}$$



Kinematics. Using the chain rule, the first and second time derivatives of r are

$$r = 2\theta^{-1}$$

$$\dot{r} = -2\theta^{-2}\dot{\theta} = -\left(\frac{2}{\theta^2}\right)\dot{\theta}$$

$$\ddot{r} = -2\left(-2\theta^{-3}\dot{\theta}^2 + \theta^{-2}\ddot{\theta}\right) = \frac{2}{\theta^3}\left(2\dot{\theta}^2 - \theta\ddot{\theta}\right)$$

When $\theta = 180° = \pi$ rad, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 3$ rad/s^2. Thus

$$r = \frac{2}{\pi}\text{ m} = 0.6366\text{ m}$$

$$\dot{r} = -\left(\frac{2}{\pi^2}\right)(4) = -0.8106\text{ m/s}$$

$$\ddot{r} = \frac{2}{\pi^3}\left[2(4^2) - \pi(3)\right] = 1.4562\text{ m/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 1.4562 - 0.6366(4^2) = -8.7297\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6366(3) + 2(-0.8106)(4) = -4.5747\text{ m/s}^2$$

Substitute these result into Eqs. (1) and (2),

$$N = 4.5807\text{ N}$$

$$F = -0.8980\text{ N} = -0.898\text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that **F** acts in the sense opposite to that shown in the FBD.

Ans:
$F = -0.898$ N

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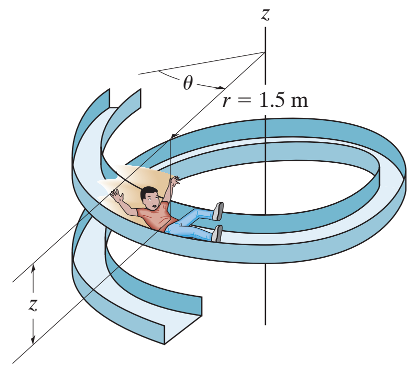
13–90.

The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



SOLUTION

$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

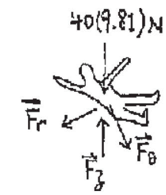
$\dot{r} = \ddot{r} = 0 \qquad \dot{\theta} = 0.7 \qquad \dot{z} = -0.5$

$\ddot{\theta} = 0 \qquad \ddot{z} = 0$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$



$\Sigma F_r = ma_r; \qquad F_r = 40(-0.735) = -29.4 \text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 0$ **Ans.**

$\Sigma F_z = ma_z; \qquad F_z - 40(9.81) = 0$

$F_z = 392 \text{ N}$ **Ans.**

Ans:
$F_r = -29.4$ N
$F_\theta = 0$
$F_z = 392$ N

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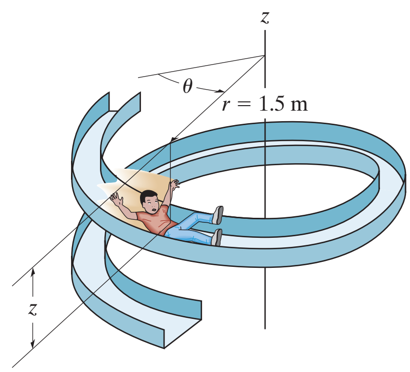
13–91.

The 40-kg boy is sliding down the smooth spiral slide such that $z = -2$ m/s and his speed is 2 m/s. Determine the r, θ, z components of force the slide exerts on him at this instant. Neglect the size of the boy.



SOLUTION

$r = 1.5$ m

$\dot{r} = 0$

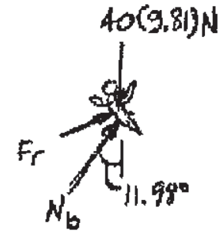
$\ddot{r} = 0$

$v_\theta = 2 \cos 11.98° = 1.9564$ m/s

$v_z = -2 \sin 11.98° = -0.41517$ m/s

$v_\theta = r\dot{\theta}; \qquad 1.9564 = 1.5\,\dot{\theta}$

$\dot{\theta} = 1.3043$ rad/s



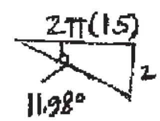
$\Sigma F_r = ma_r; \qquad -F_r = 40(0 - 1.5(1.3043)^2)$

$F_r = 102$ N **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad N_b \sin 11.98° = 40(a_\theta)$

$\Sigma F_z = ma_z; \qquad -N_b \cos 11.98° + 40(9.81) = 40a_z$

Require $\tan 11.98° = \dfrac{a_z}{a_\theta}, \qquad a_\theta = 4.7123a_z$



Thus,

$a_z = 0.423$ m/s^2

$a_\theta = 1.99$ m/s^2

$N_b = 383.85$ N

$N_z = 383.85 \cos 11.98° = 375$ N **Ans.**

$N_\theta = 383.85 \sin 11.98° = 79.7$ N **Ans.**

Ans:
$F_r = 102$ N
$F_z = 375$ N
$F_\theta = 79.7$ N

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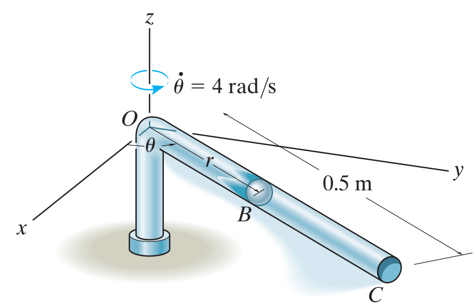
***13–92.** The tube rotates in the horizontal plane at a constant rate of $\dot{\theta} = 4$ rad/s. If a 0.2-kg ball B starts at the origin O with an initial radial velocity $\dot{r} = 1.5$ m/s and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C, $r = 0.5$ m. *Hint:* Show that the equation of motion in the r direction is $\ddot{r} - 16r = 0$. The solution is of the form $r = Ae^{-4t} + Be^{4t}$. Evaluate the integration constants A and B, and determine the time t when $r = 0.5$ m. Proceed to obtain v_r and v_θ.



SOLUTION

$$0 = M\left(r'' - r\theta'^2\right)$$

$$r(t) = A\,e^{\theta' t} + B\,e^{-\theta' t}$$

$$r'(t) = \theta'\left(A\,e^{\theta' t} - B\,e^{-\theta' t}\right)$$

Guess $A = 1\ \text{m}$ $B = 1\ \text{m}$

$t = 1\ \text{s}$

Given $0 = A + B$ $r'_0 = \theta'(A - B)$ $r_1 = A\,e^{\theta' t} + B\,e^{-\theta' t}$

Given:

$\theta' = 4\ \text{rad/s}$ $M = 0.2\ \text{kg}$ $r'_0 = 1.5\ \text{m/s}$ $r_1 = 0.5\ \text{m}$

$$\begin{pmatrix} A \\ B \\ t_1 \end{pmatrix} = \text{Find}(A, B, t) \qquad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.188 \\ -0.188 \end{pmatrix}\text{m} \qquad t_1 = 0.275\ \text{s}$$

$$r(t) = A\,e^{\theta' t} + B\,e^{-\theta' t} \qquad r'(t) = \theta'\left(A\,e^{\theta' t} - B\,e^{-\theta' t}\right)$$

$$v_r = r'(t_1) \qquad v_\theta = r(t_1)\,\theta'$$

$$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2 \end{pmatrix}\text{m/s}$$ **Ans.**

Ans:
$v_r = 2.5\ \text{m/s}$
$v_\theta = 2\ \text{m/s}$

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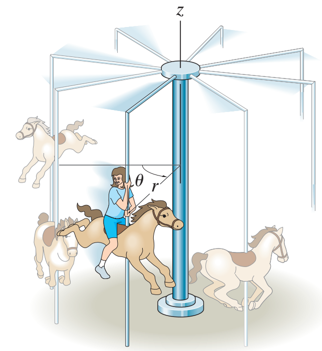
13–93. The girl has a mass of 50 kg. She is seated on the horse of the merry-go-round which undergoes constant rotational motion $\dot{\theta} = 1.5$ rad/s. If the path of the horse is defined by $r = 4$ m, $z = (0.5 \sin \theta)$ m, determine the maximum and minimum force F_z the horse exerts on her during the motion.



SOLUTION

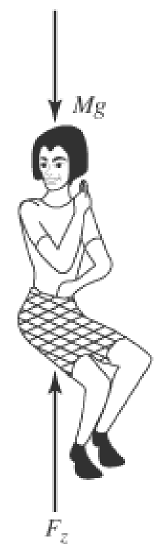
Given:

$M = 50$ kg

$\theta' = 1.5$ rad/s

$r_0 = 4$ m

$b = 0.5$ m



$z = b\sin(\theta)$

$z' = b\cos(\theta)\theta'$

$z'' = -b\sin(\theta)\,\theta'^2$

$F_z - Mg = Mz''$

$F_z = M\left(g - b\sin(\theta)\,\theta'^2\right)$

$F_{zmax} = M\left(g + b\theta'^2\right)$ $\qquad F_{zmax} = 547\,\text{N}$ **Ans.**

$F_{zmin} = M\left(g - b\theta'^2\right)$ $\qquad F_{zmin} = 434\,\text{N}$ **Ans.**

Ans:

$F_{zmax} = 547$ N

$F_{zmin} = 434$ N

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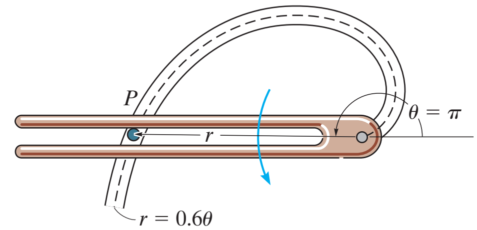
13–94.

Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.



SOLUTION

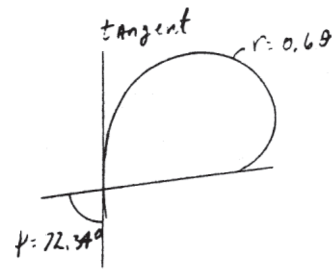
$r = 0.6\theta \qquad \dot{r} = 0.6\dot{\theta} \qquad \ddot{r} = 0.6\ddot{\theta}$

$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$

$v^2 = \dot{r}^2 + \left(r\theta\right)^2$

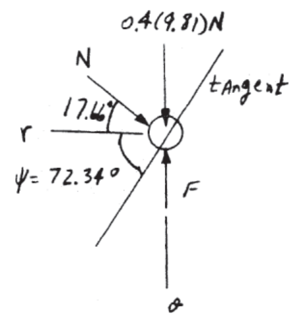
$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\theta^2}}$

$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\dfrac{\theta\dot{\theta}^2}{1+\theta^2}$



At $\theta = \pi$ rad, $\qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\pi^2}} = 1.011$ rad/s

$\ddot{\theta} = -\dfrac{(\pi)(1.011)^2}{1+\pi^2} = -0.2954 \text{ rad/s}^2$



$r = 0.6(\pi) = 0.6\,\pi$ m $\qquad \dot{r} = 0.6(1.011) = 0.6066$ m/s

$\ddot{r} = 0.6(-0.2954) = -0.1772 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698 \text{ m/s}^2$

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34^\circ$

$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad -N\cos 17.66^\circ = 0.4(-2.104) \qquad N = 0.883$ N **Ans.**

$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66^\circ = 0.4(0.6698)$

$F = 3.92$ N **Ans.**

Ans:
$N = 0.883$ N
$F = 3.92$ N

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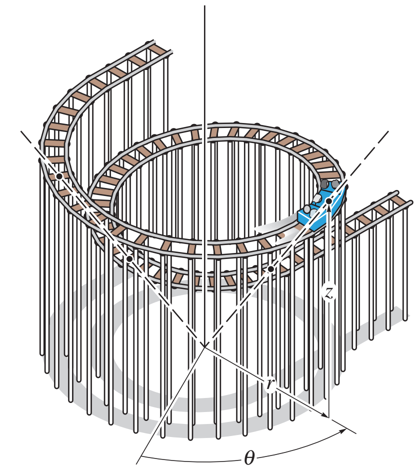
13–95.

A car of a roller coaster travels along a track which for a short distance is defined by a conical spiral, $r = \frac{3}{4}z$, $\theta = -1.5z$, where r and z are in meters and θ in radians. If the angular motion $\dot{\theta} = 1$ rad/s is always maintained, determine the r, θ, z components of reaction exerted on the car by the track at the instant $z = 6$ m. The car and passengers have a total mass of 200 kg.



SOLUTION

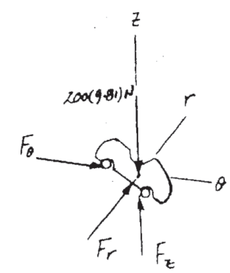
$$r = 0.75z \qquad \dot{r} = 0.75\dot{z} \qquad \ddot{r} = 0.75\ddot{z}$$

$$\theta = -1.5z \qquad \dot{\theta} = -1.5\dot{z} \qquad \ddot{\theta} = -1.5\ddot{z}$$

$$\dot{\theta} = 1 = -1.5\dot{z} \qquad \dot{z} = -0.6667 \text{ m/s} \qquad \ddot{z} = 0$$

At $z = 6$ m,

$$r = 0.75(6) = 4.5 \text{ m} \qquad \dot{r} = 0.75(-0.6667) = -0.5 \text{ m/s} \qquad \ddot{r} = 0.75(0) = 0 \qquad \ddot{\theta} = 0$$



$$a_t = \ddot{r} - r\dot{\theta}^2 = 0 - 4.5(1)^2 = -4.5 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.5(0) + 2(-0.5)(1) = -1 \text{ m/s}^2$$

$$a_z = \ddot{z} = 0$$

$\Sigma F_t = ma_r;$ $\quad F_r = 200(-4.5)$ $\quad F_r = -900$ N **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 200(-1)$ $\quad F_\theta = -200$N **Ans.**

$\Sigma F_z = ma_z;$ $\quad F_z - 200(9.81) = 0$ $\quad F_z = 1962 \text{ N} = 1.96$ kN **Ans.**

Ans:
$F_r = -900$ N
$F_\theta = -200$ N
$F_z = 1.96$ kN

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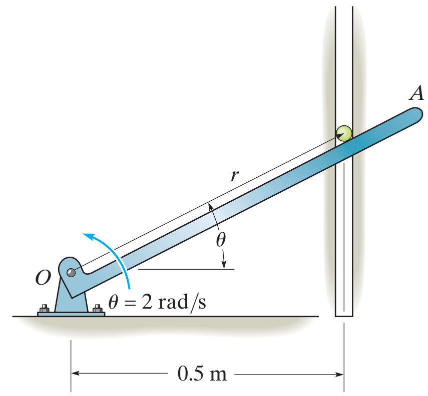
***13–96.**

The particle has a mass of 0.5 kg and is confined to move along the smooth vertical slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



SOLUTION

$$r = \frac{0.5}{\cos\theta} = 0.5 \sec\theta, \qquad \dot{r} = 0.5 \sec\theta \tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5 \sec\theta \tan\theta\ddot{\theta} + 0.5 \sec^3\theta\dot{\theta}^2 + 0.5 \sec\theta \tan^2\theta\dot{\theta}^2$$

At $\theta = 30°$.

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

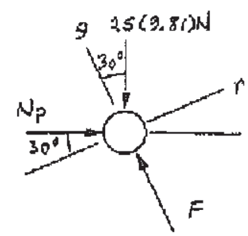
$r = 0.5774$ m

$\dot{r} = 0.6667$ m/s

$\ddot{r} = 3.8490$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.8490 - 0.5774(2)^2 = 1.5396$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.6667)(2) = 2.667$ m/s^2



$+\nearrow\Sigma F_r = ma_r;\qquad N_P \cos 30° - 0.5(9.81)\sin 30° = 0.5(1.5396)$

$N_P = 3.7208 = 3.72$ N **Ans.**

$+\nwarrow\Sigma F_\theta = ma_\theta;\qquad F - 3.7208 \sin 30° - 0.5(9.81) \cos 30° = 0.5(2.667)$

$F = 7.44$ N **Ans.**

Ans:
$N_P = 3.72$ N
$F = 7.44$ N

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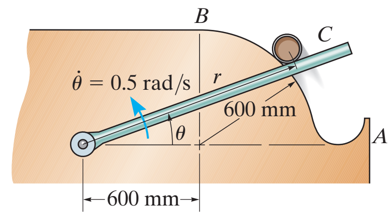
13–97.

A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the can at the instant $\theta = 30°$. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2 \cos \theta)$ m. The ramp from A to B is circular, having a radius of 600 mm.



SOLUTION

$r = 2(0.6 \cos \theta) = 1.2 \cos \theta$

$\dot{r} = -1.2 \sin \theta \dot{\theta}$

$\ddot{r} = -1.2 \cos \theta \dot{\theta}^2 - 1.2 \sin \theta \ddot{\theta}$

At $\theta = 30°, \dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

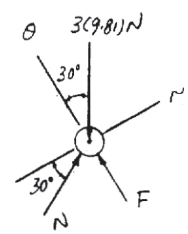
$r = 1.2 \cos 30° = 1.0392$ m

$\dot{r} = -1.2 \sin 30°(0.5) = -0.3$ m/s

$\ddot{r} = -1.2 \cos 30°(0.5)^2 - 1.2 \sin 30°(0) = -0.2598$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2598 - 1.0392(0.5)^2 = -0.5196$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.0392(0) + 2(-0.3)(0.5) = -0.3$ m/s^2



$+\nearrow \Sigma F_r = ma_r;$ $\quad N \cos 30° - 3(9.81) \sin 30° = 3(-0.5196) \quad N = 15.19$ N

$+\nwarrow \Sigma F_\theta = ma_\theta;$ $\quad F + 15.19 \sin 30° - 3(9.81) \cos 30° = 3(-0.3)$

$F = 17.0$ N **Ans.**

Ans:
$F = 17.0$ N

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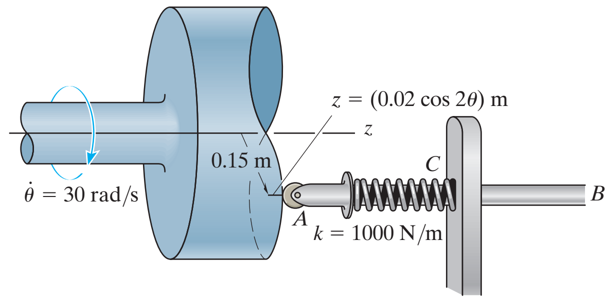
13–98.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the force component F_z at the end A of the follower when $\theta = 30°$. The spring is uncompressed when $\theta = 90°$. Neglect friction at the bearing C.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

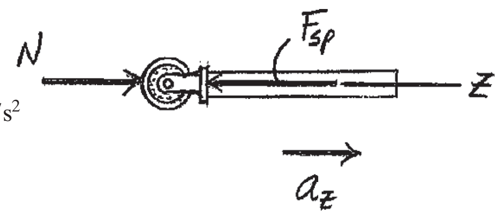
$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = [-0.04(\sin 2\theta)\dot{\theta}] \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + (\sin 2\theta)\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here, $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$



(a)

Equation of Motion. When $\theta = 30°$, the spring compresses $x = 0.02 + 0.02 \cos 2(30°) = 0.03$ m. Thus, $F_{sp} = kx = 1000(0.03) = 30$ N. Also, at this position $a_z = \ddot{z} = -72 \cos 2(30°) = -36.0 \text{ m/s}^2$. Referring to the FBD of the follower, Fig. *a*,

$$\Sigma F_z = ma_z; \qquad N - 30 = 0.5(-36.0)$$

$$N = 12.0 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 12.0$ N

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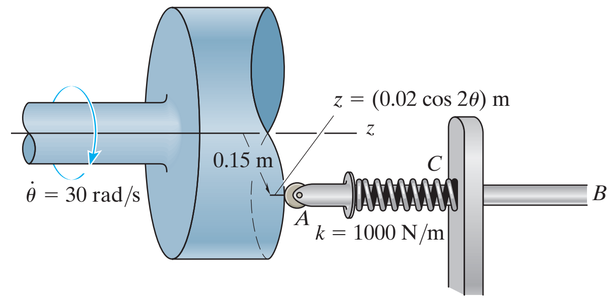
13–99.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the maximum and minimum force components F_z the follower exerts on the cam if the spring is uncompressed when $\theta = 90°$.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

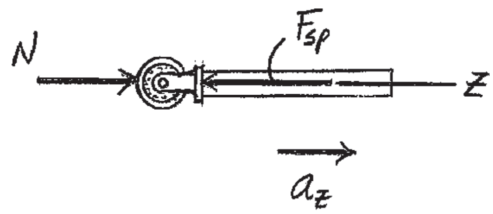
$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = (-0.04 \sin 2\theta\dot{\theta}) \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + \sin 2\theta\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then,

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$



(a)

Equation of Motion. At any arbitrary θ, the spring compresses $x = 0.02(1 + \cos 2\theta)$. Thus, $F_{sp} = kx = 1000[0.02(1 + \cos 2\theta)] = 20\,(1 + \cos 2\theta)$. Referring to the FBD of the follower, Fig. *a*,

$$\Sigma F_z = ma_z; \qquad N - 20(1 + \cos 2\theta) = 0.5(-72 \cos 2\theta)$$

$$N = (20 - 16 \cos 2\theta) \text{ N}$$

N is maximum when $\cos 2\theta = -1$. Then

$(N)_{max} = 36.0$ N **Ans.**

N is minimum when $\cos 2\theta = 1$. Then

$(N)_{min} = 4.00$ N **Ans.**

Ans:
$(N)_{max} = 36.0$ N
$(N)_{min} = 4.00$ N

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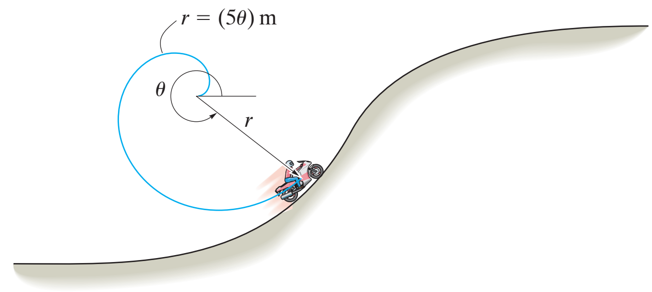
***13–100.**

Determine the normal and frictional driving forces that the partial spiral track exerts on the 200-kg motorcycle at the instant $\theta = \frac{5}{3}\pi$ rad, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2. Neglect the size of the motorcycle.



SOLUTION

$$\theta = \left(\frac{5}{3}\pi\right) = 300^\circ \qquad \dot{\theta} = 0.4 \qquad \ddot{\theta} = 0.8$$

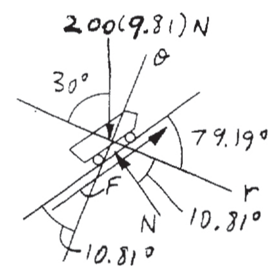
$$r = 5\theta = 5\left(\frac{5}{3}\pi\right) = 26.18$$

$$\dot{r} = 5\dot{\theta} = 5(0.4) = 2$$

$$\ddot{r} = 5\ddot{\theta} = 5(0.8) = 4$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 26.18(0.4)^2 = -0.1888$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 26.18(0.8) + 2(2)(0.4) = 22.54$$



$$\tan\psi = \frac{r}{dr/d\theta} = \frac{5\left(\frac{5}{3}\pi\right)}{5} = 5.236 \qquad \psi = 79.19^\circ$$

$+\searrow\Sigma F_r = ma_r;\qquad F\sin 10.81^\circ - N\cos 10.81^\circ + 200(9.81)\cos 30^\circ = 200(-0.1888)$

$+\nearrow\Sigma F_\theta = ma_\theta;\qquad F\cos 10.81^\circ - 200(9.81)\sin 30^\circ + N\sin 10.81^\circ = 200(22.54)$

$F = 5.07$ kN **Ans.**

$N = 2.74$ kN **Ans.**

Ans:
$F = 5.07$ kN
$N = 2.74$ kN

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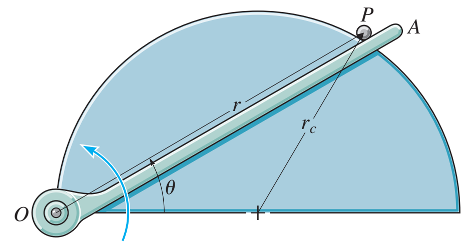
13–101. The 0.5-kg ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ m.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivative of r are

$$r = 2(0.4)\cos\theta = 0.8\cos\theta \text{ m}$$

$$\dot{r} = -0.8\sin\theta\dot{\theta}$$

$$\ddot{r} = -0.8(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})$$

At $\theta = 30°, \dot{\theta} = 0.4$ rad/s and $\ddot{\theta} = 0.8$ rad/s^2.

$$r = 0.8\cos 30° = 0.6928 \text{ m}$$

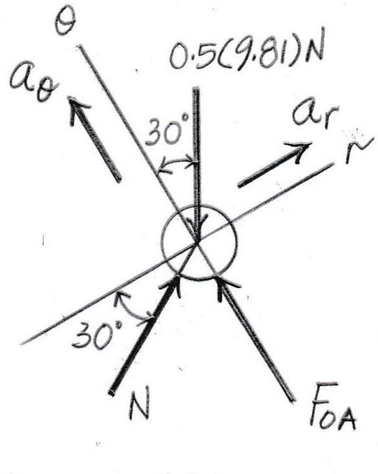
$$\dot{r} = -0.8\sin 30°(0.4) = -0.16 \text{ m/s}$$

$$\ddot{r} = -0.8\,[(\cos 30°)(0.4^2) + \sin 30°(0.8)] = -0.4309 \text{ m/s}^2$$

Then

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4^2) = -0.5417 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.6928)(0.8) + 2(-0.16)(0.4) = 0.4263 \text{ m/s}^2$$



(a)

Equation of motion. Referring to the FBD of the ball, Fig. *a*,

$\Sigma F_r = ma_r;\quad N\cos 30° - 0.5(9.81)\sin 30° = 0.5(-0.5417)\quad N = 2.5192$ N

$\Sigma F_\theta = ma_\theta;\quad F_{OA} + 2.5192\sin 30° - 0.5(9.81)\cos 30° = 0.5(0.4263)$

$$F_{OA} = 3.2014 \text{ N} = 3.20 \text{ N}$$ **Ans.**

Ans:
$F_{OA} = 3.20$ N

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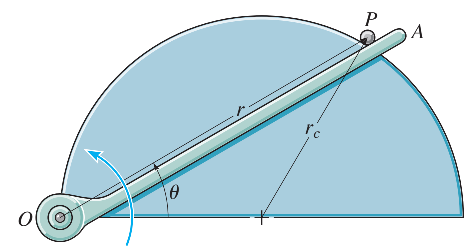
13–102.

The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45^\circ$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



SOLUTION

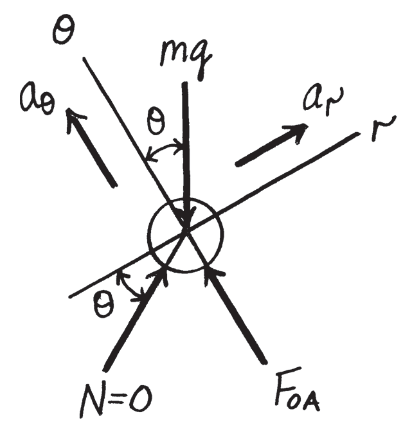
$r = 2r_c \cos\theta$

$\dot{r} = -2r_c \sin\theta\dot{\theta}$

$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$



$+\nearrow\Sigma F_r = ma_r; \qquad -mg \sin\theta = m(-4r_c \cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\,\dot{\theta}_0^2}{g} \qquad \theta = \tan^{-1}\left(\frac{4r_c\,\dot{\theta}_0^2}{g}\right) \qquad \textbf{Ans.}$$

Ans:

$\theta = \tan^{-1}\left(\frac{4r_c\dot{\theta}_0^2}{g}\right)$

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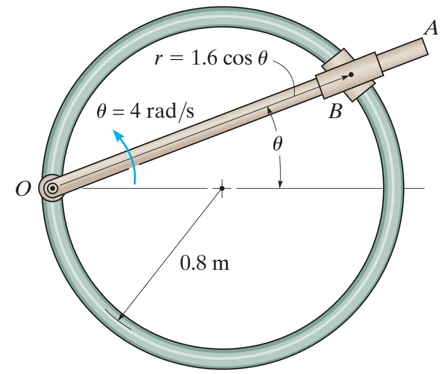
13–103.

Rod OA rotates counterclockwise at a constant angular rate $\dot{\theta} = 4\text{ rad/s}$. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other collar slides over the circular rod described by the equation $r = (1.6\cos\theta)$ m. If *both* collars have a mass of 0.5 kg, determine the force which the circular rod exerts on one of the collars and the force that OA exerts on the other collar at the instant $\theta = 45°$. Motion is in the horizontal plane.



SOLUTION

$r = 1.6\cos\theta$

$\dot{r} = -1.6\sin\theta\dot{\theta}$

$\ddot{r} = -1.6\cos\theta\dot{\theta}^2 - 1.6\sin\theta\ddot{\theta}$

At $\theta = 45°, \dot{\theta} = 4\text{ rad/s}$ and $\ddot{\theta} = 0$

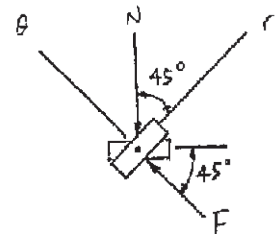
$r = 1.6\cos 45° = 1.1314\text{ m}$

$\dot{r} = -1.6\sin 45°(4) = -4.5255\text{ m/s}$

$\ddot{r} = -1.6\cos 45°(4)^2 - 1.6\sin 45°(0) = -18.1019\text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20\text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20\text{ m/s}^2$



$\nearrow + \Sigma F_r = ma_r;\qquad -N_C\cos 45° = 0.5(-36.20)\qquad N_C = 25.6\text{ N}$ **Ans.**

$+\nwarrow \Sigma F_\theta = ma_\theta;\qquad F_{OA} - 25.6\sin 45° = 0.5(-36.20)\qquad F_{OA} = 0$ **Ans.**

Ans:
$N_C = 25.6\text{ N}$
$F_{OA} = 0$

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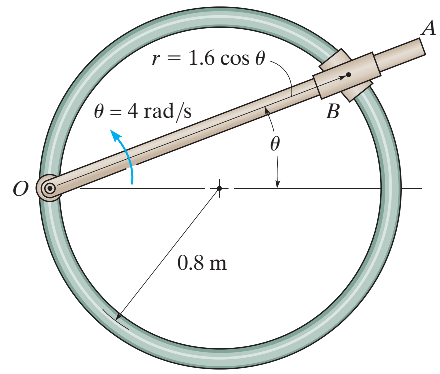
***13–104.**

Solve Prob. 13–103 if motion is in the vertical plane.



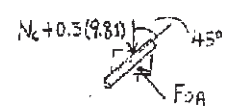
SOLUTION

$r = 1.6 \cos \theta$

$\dot{r} = -1.6 \sin \theta \dot{\theta}$

$\ddot{r} = -1.6 \cos \theta \dot{\theta}^2 - 1.6 \sin \theta \ddot{\theta}$

At $\theta = 45°$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 0$



$r = 1.6 \cos 45° = 1.1314$ m

$\dot{r} = -1.6 \sin 45°(4) = -4.5255$ m/s

$\ddot{r} = -1.6 \cos 45°(4)^2 - 1.6 \sin 45°(0) = -18.1019 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20 \text{ m/s}^2$

$+\swarrow \Sigma F_r = ma_r; \quad -N_C \cos 45° - 4.905 \cos 45° = 0.5(-36.204)$

$+\nwarrow \Sigma F_\theta = ma_\theta; \quad F_{OA} - N_C \sin 45° - 4.905 \sin 45° = 0.5(-36.204)$

$N_C = 20.7$ N **Ans.**

$F_{OA} = 0$ **Ans.**

Ans:
$N_C = 20.7$ N
$F_{OA} = 0$

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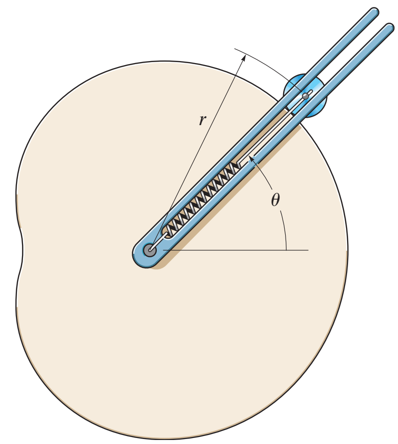
13–105. The smooth surface of the vertical cam is defined in part by the curve $r = (0.2\cos\theta + 0.3)$ m. The forked rod is rotating with an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$, and when $\theta = 45°$ the angular velocity is $\dot{\theta} = 6\ \text{rad/s}$. Determine the force the cam and the rod exert on the 2-kg roller at this instant. The attached spring has a stiffness $k = 100$ N/m and an unstretched length of 0.1 m.



SOLUTION

Given:

$$a = 0.2\ \text{m} \qquad k = 100\ \frac{\text{N}}{\text{m}} \qquad \theta = 45°$$

$$b = 0.3\ \text{m} \qquad l = 0.1\ \text{m} \qquad \theta' = 6\ \text{rad/s}$$

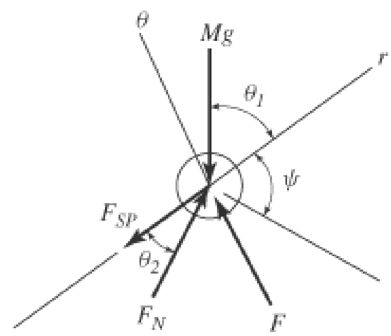
$$g = 9.81\ \text{m/s}^2 \quad M = 2\ \text{kg} \qquad \theta'' = 2\ \text{rad/s}^2$$

$$r = a\cos(\theta) + b$$

$$r' = -(a)\sin(\theta)\theta'$$

$$r'' = -(a)\cos(\theta)\theta'^2 - (a)\sin(\theta)\theta''$$

$$\psi = \operatorname{atan}\left(\frac{r\theta'}{r'}\right) + \pi$$



Guesses $\quad F_N = 1\ \text{N} \qquad F = 1\ \text{N}$

Given

$$F_N\sin(\psi) - Mg\sin(\theta) - k(r - l) = M\left(r'' - r\theta'^2\right)$$

$$F - F_N\cos(\psi) - Mg\cos(\theta) = M(r\theta'' + 2r'\theta')$$

$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \operatorname{Find}(F, F_N) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} -6.483 \\ 5.76 \end{pmatrix}\text{N} \quad \textbf{Ans.}$$

Ans:

$F = -6.483$ N
$F_N = 5.76$ N

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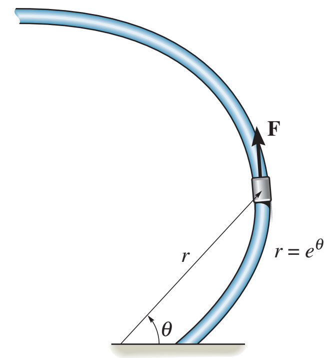
13–106.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\,\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

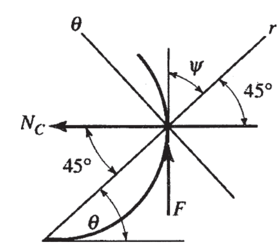
$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$$\tan\psi = \frac{r}{\left(\dfrac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$\psi = \theta = 45°$



$\nearrow + \sum F_r = ma_r; \qquad -N_C \cos 45° + F\cos 45° = 2(0)$

$+\nwarrow \sum F_{\theta} = ma_{\theta}; \qquad F\sin 45° + N_C \sin 45° = 2(17.5462)$

$N = 24.8 \text{ N}$ **Ans.**

$F = 24.8 \text{ N}$ **Ans.**

Ans:
$N = 24.8$ N
$F = 24.8$ N

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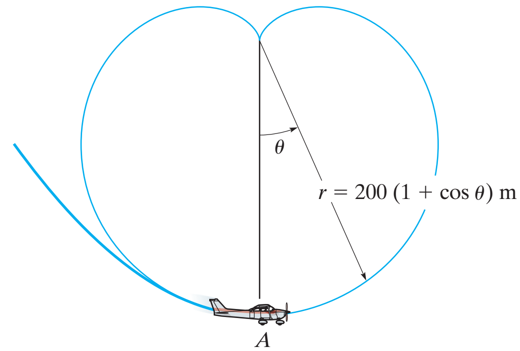
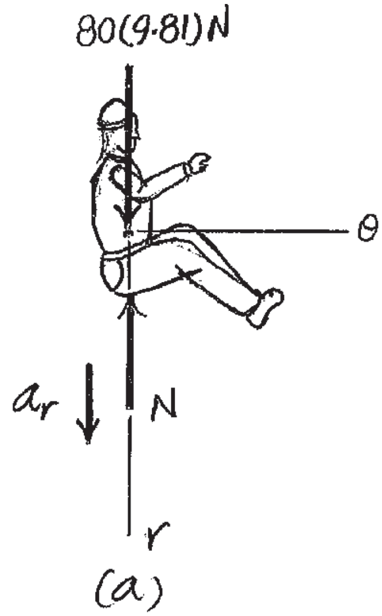
13–107.

The pilot of the airplane executes a vertical loop which in part follows the path of a cardioid, $r = 200(1 + \cos\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 85$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He has a mass of 80 kg. *Hint:* To determine the time derivatives necessary to calculate the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 200(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$.



SOLUTION



Kinematic. Using the chain rule, the first and second time derivatives of r are

$$r = 200(1 + \cos\theta)$$

$$\dot{r} = 200(-\sin\theta)(\dot{\theta}) = -200(\sin\theta)\dot{\theta}$$

$$\ddot{r} = -200[(\cos\theta)(\dot{\theta})^2 + (\sin\theta)(\ddot{\theta})]$$

When $\theta = 0°$,

$$r = 200(1 + \cos 0°) = 400 \text{ m}$$

$$\dot{r} = -200(\sin 0°)\,\dot{\theta} = 0$$

$$\ddot{r} = -200\left[(\cos 0°)(\dot{\theta})^2 + (\sin 0°)(\ddot{\theta})\right] = -200\dot{\theta}^2$$

Using Eq. 12–26

$$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2$$

$$85^2 = 0^2 + (400\dot{\theta})^2$$

$$\dot{\theta} = 0.2125 \text{ rad/s}$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -200\left(0.2125^2\right) - 400\left(0.2125^2\right) = -27.09 \text{ m/s}^2$$

Equation of Motion. Referring to the FBD of the pilot, Fig. a,

$$\downarrow + \Sigma F_r = ma_r; \qquad 80(9.81) - N = 80(-27.09)$$

$$N = 2952.3 \text{ N} = 2.95 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N = 2.95$ N

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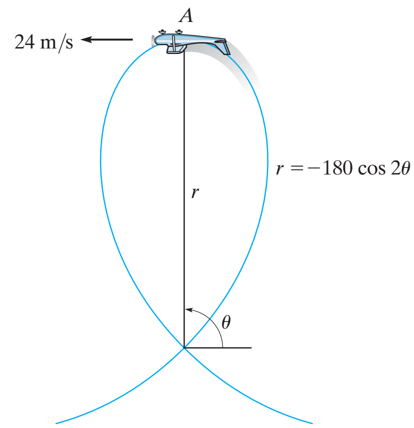
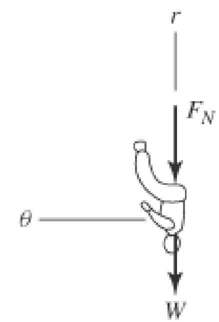
***13–108.** The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = (-180 \cos 2\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 24$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He weights 650 N. *Hint:* To determine the time derivatives necessary to compute the acceleration components a_r and a_θ, take the first and second time derivatives of $r = -180(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$ to determine $\ddot{\theta}$.



SOLUTION

$$\theta = 90°$$

$$r = (a)\cos(2\theta)$$



Guesses

$$r' = 1 \text{ m/s} \qquad r'' = 1 \text{ m/s}^2 \qquad \theta' = 1 \text{ rad/s} \qquad \theta'' = 1 \text{ rad/s}^2$$

Given Note that v_p is constant so $dv_p/dt = 0$

$$r' = -(a)\sin(2\theta)2\theta' \qquad r'' = -(a)\sin(2\theta)2\theta'' - (a)\cos(2\theta)4\theta'^2$$

$$v_p = \sqrt{r'^2 + (r\theta')^2} \qquad 0 = \frac{r'r'' + r\theta'(r\theta'' + r'\theta')}{\sqrt{r'^2 + (r\theta')^2}}$$

Given:

$$a = -180 \text{ m} \qquad W = 650 \text{ N}$$

$$v_p = 24 \text{ m/s} \qquad g = 9.81 \text{ m/s}^2$$

$$\begin{pmatrix} r' \\ r'' \\ \theta' \\ \theta'' \end{pmatrix} = \text{Find}(r', r'', \theta', \theta'') \qquad r' = 0 \text{ m/s} \qquad r'' = -12.8 \text{ m/s}^2$$

$$\theta' = 0.133 \text{ rad/s} \qquad \theta'' = 0 \text{ rad/s}^2$$

$$-F_N - W = M(r'' - r\theta'^2) \qquad F_N = -W - \left(\frac{W}{g}\right)(r'' - r\theta'^2) \qquad F_N = 410.1 \text{ N} \quad \textbf{Ans.}$$

Ans:
$F_N = 410.1$ N

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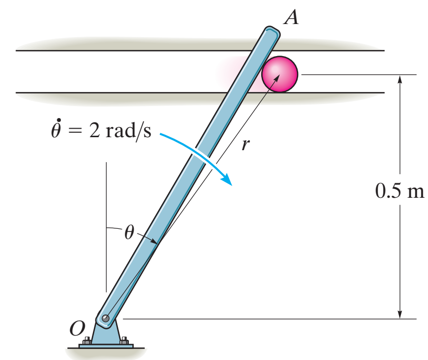
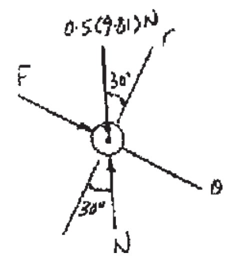
13–109.

The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5 \sec\theta$$

$$\dot{r} = 0.5 \sec\theta \tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}]$$

When $\theta = 30°, \dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$

$$r = 0.5 \sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5 \sec 30° \tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30° \tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30° \tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r$; $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N **Ans.**

$+\searrow \Sigma F_\theta = ma_\theta$; $\quad F + 0.5(9.81)\sin 30° - 5.79 \sin 30° = 0.5(2.667)$

$F = 1.78$ N **Ans.**

Ans:
$F_r = 1.78$ N
$N_s = 5.79$ N

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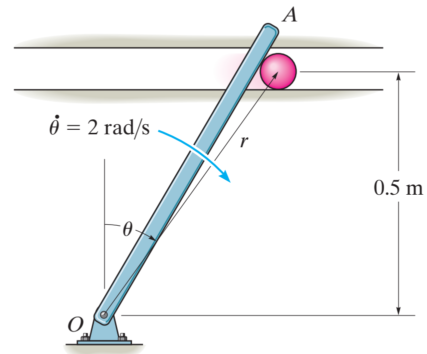
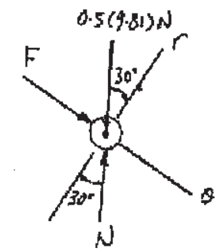
13–110.

Solve Prob. 13–109 if the arm has an angular acceleration of $\ddot{\theta} = 3\text{ rad/s}^2$ when $\dot{\theta} = 2\text{ rad/s}$ at $\theta = 30°$.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2\text{ rad/s}$ and $\ddot{\theta} = 3\text{ rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\text{ m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$N = 6.3712 = 6.37\text{ N}$ **Ans.**

$+\searrow \Sigma F_\theta = ma_\theta;\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$F = 2.93\text{ N}$ **Ans.**

Ans:
$F_r = 2.93\text{ N}$
$N_s = 6.37\text{ N}$

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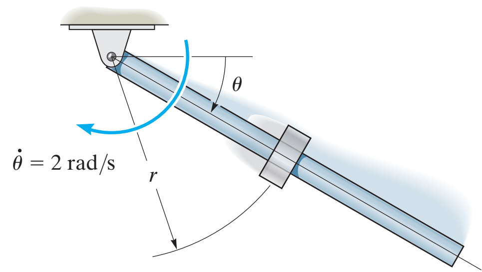
13–111.

A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 to determine r at the instant $\theta = \pi/4$ rad.

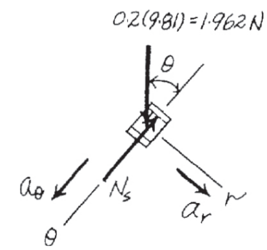


SOLUTION

Kinematic: Here, $\dot{\theta}. = 2$ rad/s and $\dot{\theta} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$



Equation of Motion: Applying Eq. 13–8, we have

$\Sigma F_r = ma_r;$ $\qquad 1.962\sin\theta = 0.2(\ddot{r} - 4r)$

$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(1)}$$

$\Sigma F_\theta = ma_\theta;$ $\qquad 1.962\cos\theta - N_s = 0.2(4\dot{r})$

$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(2)}$$

Since $\theta. = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^1 2dt, \theta = 2t$. The solution of the differential equation (Eq.(1)) is given by

$$r = C_1\, e^{-2t} + C_2\, e^{2t} - \frac{9.81}{8}\sin 2t \qquad \textbf{(3)}$$

Thus,

$$\dot{r} = -2\,C_1 e^{-2t} + 2C_2\, e^{2t} - \frac{9.81}{4}\cos 2t \qquad \textbf{(4)}$$

At $t = 0, r = 0$. From Eq.(3) $0 = C_1(1) + C_2(1) - 0$ **(5)**

At $t = 0, \dot{r} = 0$. From Eq.(4) $0 = -2\,C_1(1) + 2C_2(1) - \frac{9.81}{4}$ **(6)**

Solving Eqs. (5) and (6) yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$

$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$

$$= \frac{9.81}{8}(\sin\text{h}\, 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\qquad r = \frac{9.81}{8}\left(\sin\text{h}\frac{\pi}{4} - \sin\frac{\pi}{4}3\right) = 0.198$ m **Ans.**

Ans:
$r = 0.198$ m

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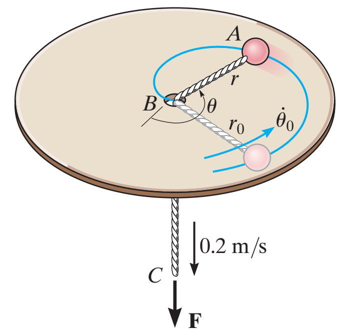
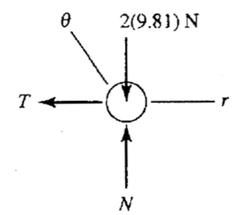
***13–112.**

The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



SOLUTION



$$\sum F_\theta = ma_\theta; \qquad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$$

Thus,

$$d(r^2\dot{\theta}) = 0$$

$$r^2\dot{\theta} = C$$

$$(0.5)^2(1) = C = (0.25)^2\dot{\theta}$$

$$\dot{\theta} = 4.00 \text{ rad/s} \qquad \textbf{Ans.}$$

Since $\dot{r} = -0.2$ m/s, $\quad \ddot{r} = 0$

$$a_r = \ddot{r} - \dot{r}(\theta)^2 = 0 - 0.25(4.00)^2 = -4 \text{ m/s}^2$$

$$\sum F_r = ma_r; \quad -T = 2(-4)$$

$$T = 8 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 4.00$ rad/s
$T = 8$ N

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13–113.

The earth has an orbit with eccentricity $e = 0.0167$ around the sun. Knowing that the earth's minimum distance from the sun is $146(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

SOLUTION

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\,v_0^2}\right) \text{ and } h = r_0\,v_0$$

$$e = \frac{1}{GM_S\,r_0}\left(1 - \frac{GM_S}{r_0\,v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0\,v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S\,(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0167 + 1)}{146(10^9)}} = 30409 \text{ m/s} = 30.4 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\,v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{146(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30409)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[146(10^9)\right]^2(30409)^2}$$

$$\frac{1}{r} = 0.348(10^{-12})\cos\theta + 6.74(10^{-12}) \qquad \textbf{Ans.}$$

Ans:
$v_o = 30.4$ km/s
$\frac{1}{r} = 0.348\left(10^{-12}\right)\cos\theta + 6.74\left(10^{-12}\right)$

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13–114.

A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

SOLUTION

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right.}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$$h = 35.87(10^6)\text{ m} = 35.9\text{ Mm} \qquad v_S = 3072.32\text{ m/s} = 3.07\text{ km/s} \quad \textbf{Ans.}$$

Ans:
$h = 35.9$ mm
$v_s = 3.07$ km/s

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13–115.

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–24. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

SOLUTION

For a 800-km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_0 = 7.45 \text{ km/s}$

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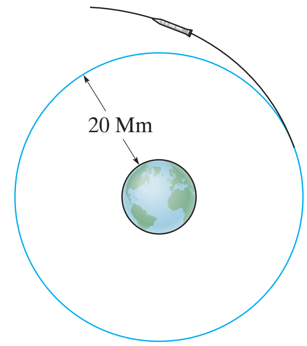
***13–116.**

The rocket is in circular orbit about the earth at an altitude of 20 Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

The speed of the rocket in circular orbit is

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 3888.17 \text{ m/s}$$

To escape the earth's gravitational field, the rocket must enter the parabolic trajectory, which require its speed to be

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 5498.70 \text{ m/s}$$

The required increment in speed is

$$\Delta v = v_e - v_c = 5498.70 - 3888.17$$
$$= 1610.53 \text{ m/s}$$
$$= 1.61(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\Delta v = 1.61(10^3)$ m/s

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13–117.

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–18, 13–27, 13–28, and 13–30.

SOLUTION

From Eq. 13–18,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–27 and 13–28,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–30,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$ **Q.E.D.**

Ans:

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$

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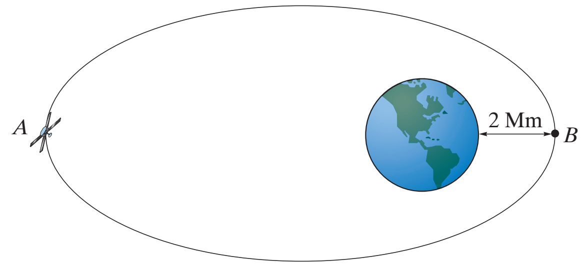
13–118.

The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



SOLUTION

$$e = \frac{Ch^2}{GM_e}$$

where $C = \dfrac{1}{r_0}\left(1 - \dfrac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0 v_0$.

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

where $r_0 = r_p = 2\left(10^6\right) + 6378\left(10^3\right) = 8.378\left(10^6\right)$ m.

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25 + 1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

$$r_a = \frac{r_0}{\dfrac{2GM_e}{r_0 v_0} - 1} = \frac{8.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96\left(10^6\right) \text{ m}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.71$ km/s
$v_A = 4.63$ km/s

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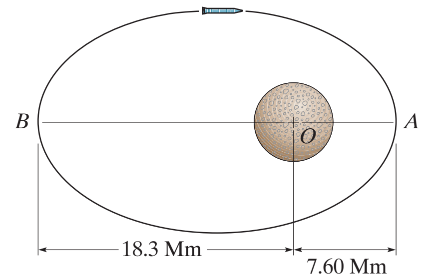
13–119.

The rocket is traveling in free flight along the elliptical orbit. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's speed when it is at A and at B.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM\, r_a}{r_p(r_p + r_a)}}$$

The elliptical orbit has $r_p = 7.60(10^6)$ m, $r_a = 18.3(10^6)$ m and $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[0.6(5.976)\left(10^{24}\right)\right]\left[18.3\left(10^6\right)\right]}{7.60\left(10^6\right)\left[7.60\left(10^6\right) + 18.3\left(10^6\right)\right]}}$$

$$= 6669.99 \text{ m/s} = 6.67\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

In this case,

$$h = r_p v_A = r_a v_B$$

$$7.60\left(10^6\right)(6669.99) = 18.3\left(10^6\right) v_B$$

$$v_B = 2770.05 \text{ m/s} = 2.77\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 6.67\left(10^3\right)$ m/s
$v_B = 2.77\left(10^3\right)$ m/s

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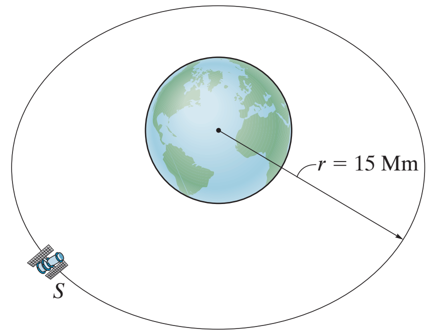
***13–120.**

Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



SOLUTION

$F = G\dfrac{m_s m_e}{r^2}$ Also $F = m_s\left(\dfrac{v_s^2}{r}\right)$ Hence

$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v = 5.16$ km/s

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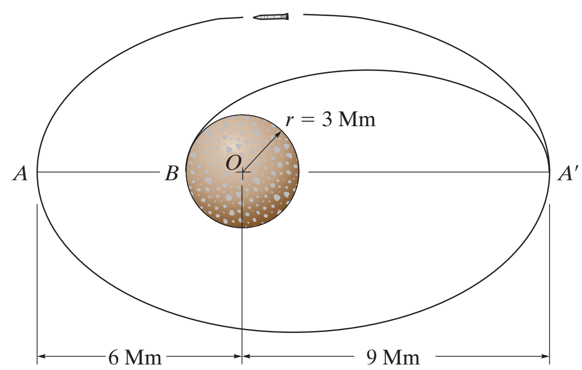
13–121.

The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



SOLUTION

Central-Force Motion: Use $r_a = \dfrac{r_0}{\left(2\,GM/r_0\,v_0^2\right)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_A = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 7.47$ km/s

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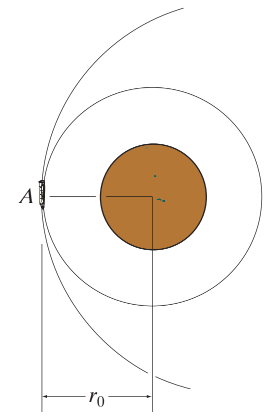
13–122.

The Viking Explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.



SOLUTION

When the Viking explorer approaches point A on a parabolic trajectory, its velocity at point A is given by

$$v_A = \sqrt{\frac{2GM_M}{r_0}}$$

$$\left[10(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \sqrt{\frac{2(66.73)(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{r_0}}$$

$$r_0 = 11.101(10^6)\text{ m} = 11.1\text{ Mm} \qquad \textbf{Ans.}$$

When the explorer travels along a circular orbit of $r_0 = 11.101(10^6)$ m, its velocity is

$$v_{A'} = \sqrt{\frac{GM_r}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{11.101(10^6)}}$$

$$= 1964.19\text{ m/s}$$

Thus, the required sudden decrease in the explorer's velocity is

$$\Delta v_A = v_A - v_{A'}$$

$$= 10(10^6)\left(\frac{1}{3600}\right) - 1964.19$$

$$= 814\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$r_0 = 11.1$ Mm
$\Delta v_A = 814$ m/s

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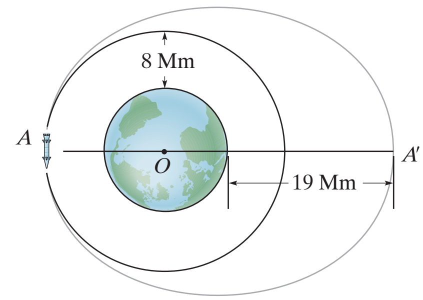
13–123.

The rocket is initially in free-flight circular orbit around the earth. Determine the speed of the rocket at A. What change in the speed at A is required so that it can move in an elliptical orbit to reach point A'?



SOLUTION

The required speed to remain in circular orbit containing point A of which $r_0 = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m can be determined from

$$(v_A)_C = \sqrt{\frac{GM_e}{r_0}}$$

$$= \sqrt{\frac{[66.73(10^{-12})][5.976(10^{24})]}{14.378(10^6)}}$$

$$= 5266.43 \text{ m/s} = 5.27(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

To more from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–26.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = (v_A)_e$. Then

$$(v_A)_e = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][25.378(10^6)]}{14.378(10^6)[14.378(10^6) + 25.378(10^6)]}}$$

$$= 5950.58 \text{ m/s}$$

Thus, the required change in speed is

$$\Delta v = (v_A)_e - (v_A)_c = 5950.58 - 5266.43 = 684.14 \text{ m/s} = 684 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(v_A)_C = 5.27(10^3)$ m/s
$\Delta v = 684$ m/s

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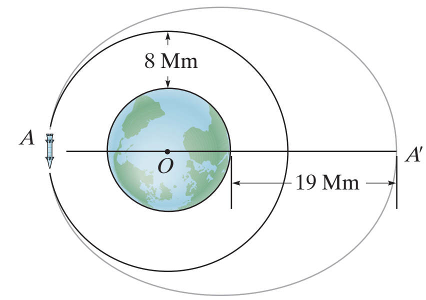
***13–124.**

The rocket is in free-flight circular orbit around the earth. Determine the time needed for the rocket to travel from the inner orbit at A to the outer orbit at A'.



SOLUTION

To move from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–26.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[25.378(10^6)\right]}{14.378(10^6)\left[14.378(10^6) + 25.378(10^6)\right]}} = 5950.58 \text{ m/s}$$

Then

$$h = v_A r_p = 5950.58\left[14.378(10^6)\right] = 85.5573(10^9) \text{ m}^2\text{/s}$$

The period of this elliptical orbit can be determined using Eq. 13–30.

$$\begin{aligned} T &= \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a} \\ &= \frac{\pi}{85.5573(10^9)}\left[14.378(10^6) + 25.378(10^6)\right]\sqrt{\left[14.378(10^6)\right]\left[25.378(10^6)\right]} \\ &= 27.885(10^3) \text{ s} \end{aligned}$$

Thus, the time required to travel from A to A' is

$$t = \frac{T}{2} = \frac{27.885(10^3)}{2} = 13.94(10^3) \text{ s} = 3.87 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$t = 3.87$ h

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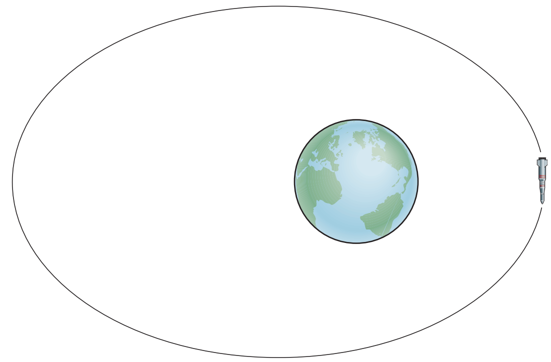
13–125.

A rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.



SOLUTION

To escape the earth's gravitational field, the rocket has to make a parabolic trajectory.

ParabolicTrajectory:

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical Orbit:

$$e = \frac{Ch^2}{GM_e} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e (e+1)}{r_0}}$$

$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e (e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans.}$$

Ans:

$$\Delta v = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right)$$

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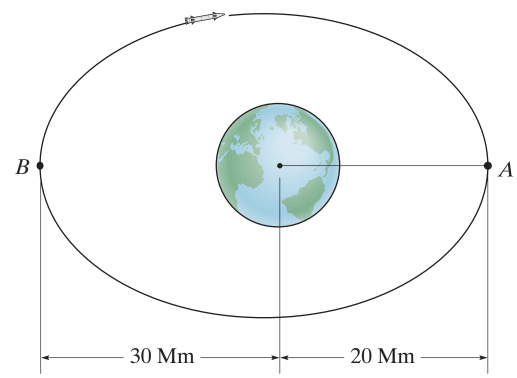
13–126.

The rocket is traveling around the earth in free flight along the elliptical orbit. If the rocket has the orbit shown, determine the speed of the rocket when it is at A and at B.



SOLUTION

Here $r_p = 20(10^6)$ m and $r_a = 30(10^6)$ m. Applying Eq. 13–26,

$$r_a = \frac{r_p}{\left(2GM_e/r_p v_p^2\right) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_P = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[30(10^6)\right]}{20(10^6)\left[20(10^6) + 30(10^6)\right]}}$$

$$= 4891.49 \text{ m/s} = 4.89(10^3) \text{ m/s}$$ **Ans.**

For the same orbit h is constant. Thus,

$$h = r_p v_p = r_a v_a$$

$$\left[20(10^6)\right](4891.49) = \left[30(10^6)\right]v_B$$

$$v_B = 3261.00 \text{ m/s} = 3.26(10^3) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 4.89(10^3)$ m/s
$v_B = 3.26(10^3)$ m/s

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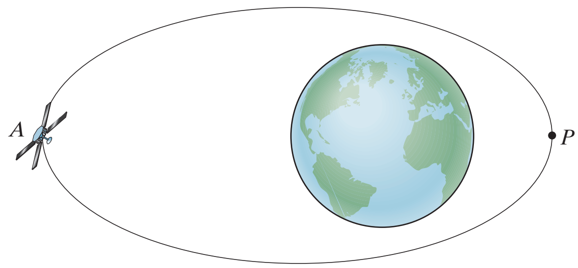
13–127.

An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



SOLUTION

$$e = 0.130$$

$$v_p = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$$

$$= 25.96 \text{ Mm}$$

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$$= 33.71(10^6) \text{ m} = 33.7 \text{ Mm}$$

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$$= 11.5 \text{ Mm/h} \qquad \textbf{Ans.}$$

$$d = 33.71(10^6) - 6.378(10^6)$$

$$= 27.3 \text{ Mm} \qquad \textbf{Ans.}$$

Ans:
$v_A = 11.5$ Mm/h
$d = 27.3$ Mm

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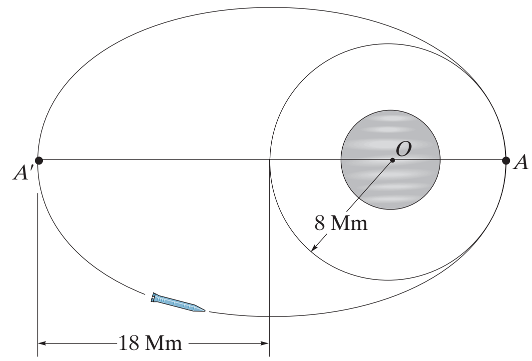
***13–128.**

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are 8 Mm and 26 Mm, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes an 8-Mm free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is 0.816 times the mass of the earth.



SOLUTION

a) $$M_\nu = 0.816(5.976(10^{24})) = 4.876(10^{24})$$

$$OA' = \frac{OA}{\left(\dfrac{2GM\nu}{OA\,\nu_A^2} - 1\right)}$$

$$26(10)^6 = \frac{8(10^6)}{\left(\dfrac{2(66.73)(10^{-12})4.876(10^{24})}{8(10^6)\nu_A^2} - 1\right)}$$

$$\frac{81.35(10^6)}{\nu_A^2} = 1.307$$

$$\nu_A = 7887.3 \text{ m/s} = 7.89 \text{ km/s}$$

$$\nu_A = \frac{OA\,\nu_A}{OA'} = \frac{8(10^6)(7887.3)}{26(10^6)} = 2426.9 \text{ m/s} = 2.43 \text{ m/s} \qquad \textbf{Ans.}$$

b) $$\nu_{A''} = \sqrt{\frac{GM_\nu}{OA'}} = \sqrt{\frac{66.73(10^{-12})4.876(10^{24})}{8(10^6)}}$$

$$\nu_{A''} = 6377.7 \text{ m/s} = 6.38 \text{ km/s} \qquad \textbf{Ans.}$$

c) Circular orbit:

$$T_c = \frac{2\pi OA}{\nu_{A''}} = \frac{2\pi 8(10^6)}{6377.7} = 7881.41 \text{ s} = 2.19 \text{ h} \qquad \textbf{Ans.}$$

Elliptic orbit:

$$T_e = \frac{\pi}{OA\nu_A}(OA + OA')\sqrt{(OA)(OA')} = \frac{\pi}{8(10^6)(7886.8)}(8 + 26)\left(10^6\right)\left(\sqrt{(8)(26)}\right)\left(10^6\right)$$

$$T_e = 24414.2 \text{ s} = 6.78 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$\nu_A = 2.43$ m/s
$\nu_{A''} = 6.38$ km/s
$T_c = 2.19$ h
$T_e = 6.78$ h

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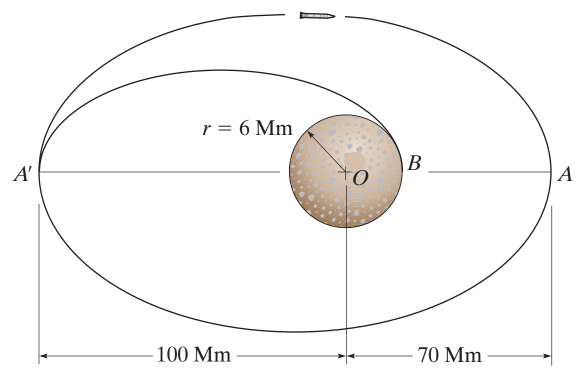
13–129.

The rocket is traveling in a free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}}$$

The rocket is traveling around the elliptical orbit with $r_p = 70\left(10^6\right)$ m, $r_a = 100\left(10^6\right)$ m and $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[0.6(5.976)\left(10^{24}\right)\right]\left[100\left(10^6\right)\right]}{70\left(10^6\right)\left[70\left(10^6\right) + 100\left(10^6\right)\right]}}$$

$$= 2005.32 \text{ m/s} = 2.01\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 2.01\left(10^3\right)$ m/s

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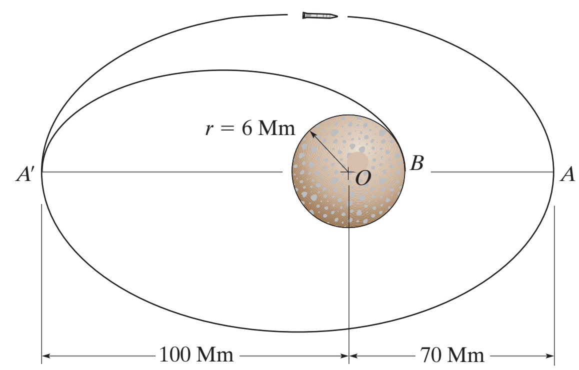
13–130.

If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p\left(r_p + r_a\right)}}$$

To land on B, the rocket has to follow the elliptical orbit $A'B$ with $r_p = 6\left(10^6\right)$, $r_a = 100\left(10^6\right)$ m and $v_p = v_B$.

$$v_B = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[0.6\left(5.976\right)\left(10^{24}\right)\right]\left[100\left(10^6\right)\right]}{6\left(10^6\right)\left[6\left(10^6\right) + 100\left(10^6\right)\right]}} = 8674.17 \text{ m/s}$$

In this case

$$h = r_p v_B = r_a v_{A'}$$

$$6\left(10^6\right)\left(8674.17\right) = 100\left(10^6\right)v_{A'}$$

$$v_{A'} = 520.45 \text{ m/s} = 521 \text{ m/s} \qquad \textbf{Ans.}$$

The period of the elliptical orbit can be determined using Eq. 13–30.

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{6\left(10^6\right)\left(8674.17\right)}\left[6\left(10^6\right) + 100\left(10^6\right)\right]\sqrt{\left[6\left(10^6\right)\right]\left[100\left(10^6\right)\right]}$$

$$= 156.73\left(10^3\right) \text{ s}$$

Thus, the time required to travel from A' to B is

$$t = \frac{T}{2} = 78.365\left(10^3\right) \text{ s} = 21.8 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$v_{A'} = 521$ m/s
$t = 21.8$ h

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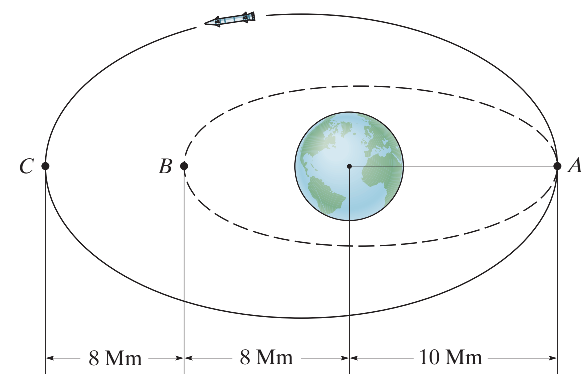
13–131.

The rocket is traveling around the earth in free flight along an elliptical orbit AC. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.



SOLUTION

For orbit AC, $r_p = 10(10^6)$ m and $r_a = 16(10^6)$ m. Applying Eq. 13–26

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][16(10^6)]}{10(10^6)[10(10^6) + 16(10^6)]}}$$

$$= 7005.74 \text{ m/s} = 7.01(10^3) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 7.01(10^3)$ m/s

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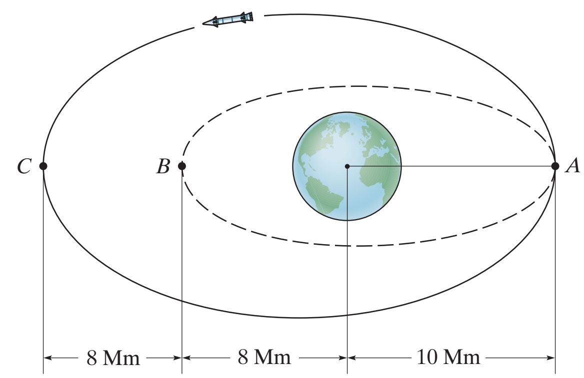
***13–132.**

The rocket is traveling around the earth in free flight along the elliptical orbit AC. Determine its change in speed when it reaches A so that it travels along the elliptical orbit AB.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_a v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

For orbit AC, $r_p = 10(10^6)$ m, $r_a = 16(10^6)$ m and $v_p = (v_A)_{AC}$. Then

$$(v_A)_{AC} = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][16(10^6)]}{10(10^6)[10(10^6) + 16(10^6)]}} = 7005.74 \text{ m/s}$$

For orbit AB, $r_p = 8(10^6)$ m, $r_a = 10(10^6)$ m and $v_p = v_B$. Then

$$v_B = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][10(10^6)]}{8(10^6)[8(10^6) + 10(10^6)]}} = 7442.17 \text{ m/s}$$

Since h is constant at any position of the orbit,

$$h = r_p v_p = r_a v_a$$

$$8(10^6)(7442.17) = 10(10^6)(v_A)_{AB}$$

$$(v_A)_{AB} = 5953.74 \text{ m/s}$$

Thus, the required change in speed is

$$\Delta v = (v_A)_{AB} - (v_A)_{AC} = 5953.74 - 7005.74$$

$$= -1052.01 \text{ m/s} = -1.05 \text{ km/s} \qquad \textbf{Ans.}$$

The negative sign indicates that the speed must be decreased.

Ans:
$\Delta v = -1.05$ km/s

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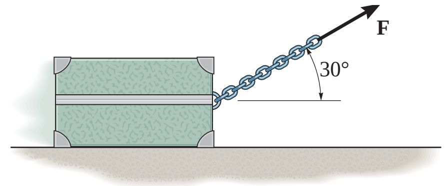
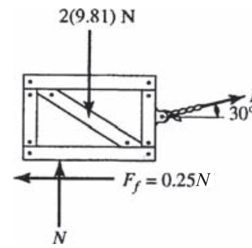
14–1.

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30° - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(20)(8^2) + \int_{15\text{ m}}^{25\text{ m}} 100 \cos 30°\, ds$$

$$- \int_{15\text{ m}}^{25\text{ m}} 36.55\, ds = \frac{1}{2}(20)v^2$$

$v = 10.7$ m/s **Ans.**

Ans:
$v = 10.7$ m/s

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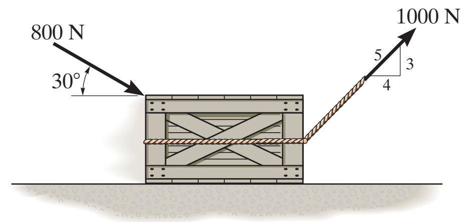
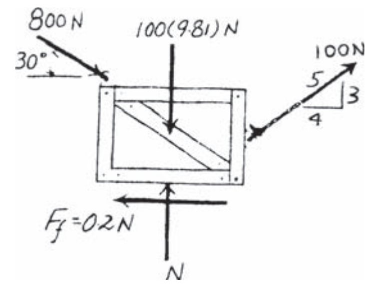
14–2.

The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION



Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30° - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$

Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800 \cos 30°(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m} \qquad \textbf{Ans.}$$

Ans:
$s = 1.35$ m

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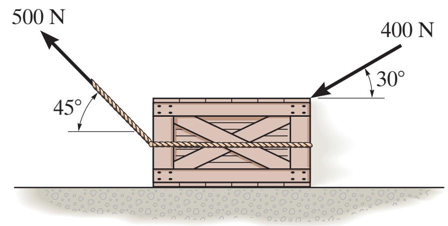
14–3.

The 100-kg crate is subjected to the forces shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 8\text{ m/s}$. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION

Work. Consider the force equilibrium along the y axis by referring to the FBD of the crate, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N + 500 \sin 45° - 100(9.81) - 400 \sin 30° = 0$$

$$N = 827.45 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.2(827.45) = 165.49$ N. Here, F_1 and F_2 do positive work whereas F_f does negative work. W and N do no work

$$U_{F_1} = 400 \cos 30° \text{ s} = 346.41 \text{ s}$$

$$U_{F_2} = 500 \cos 45° \text{ s} = 353.55 \text{ s}$$

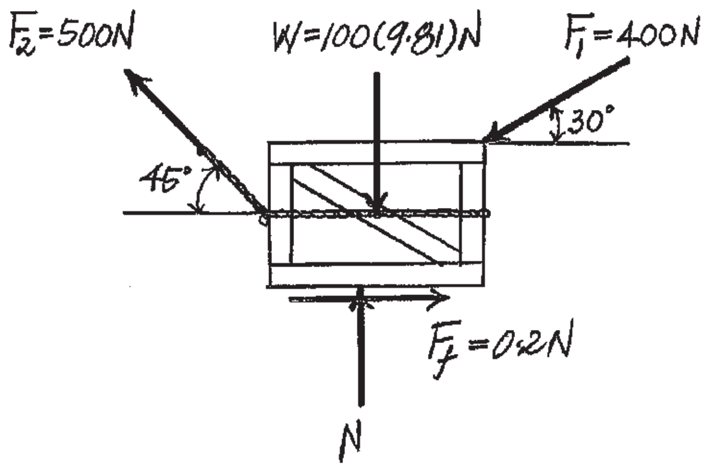
$$U_{F_f} = -165.49 \text{ s}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 346.41 \text{ s} + 353.55 \text{ s} + (-165.49 \text{ s}) = \frac{1}{2}(100)(8^2)$$

$$s = 5.987 \text{ m} = 5.99 \text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$s = 5.99$ m

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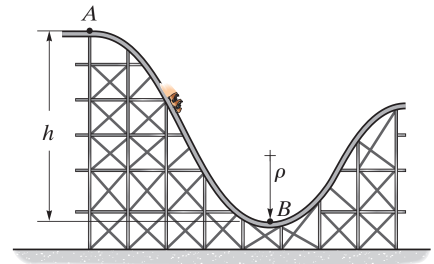
***14–4.**

Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill A it will reach a speed of 100 km/h when it comes to the bottom B. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than $4mg = (39.24m)$ N? Neglect the size of the car and passenger.



SOLUTION

$$100 \text{ km/h} = \frac{100(10^3)}{3600} = 27.778 \text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m(9.81)h = \frac{1}{2}m(27.778)^2$$

$$h = 39.3 \text{ m}$$ **Ans.**

$$+\uparrow \Sigma F_n = ma_n; \qquad 39.24\,m - mg = m\left(\frac{(27.778)^2}{\rho}\right)$$

$$\rho = 26.2 \text{ m}$$ **Ans.**

Ans:
$h = 39.3$ m
$\rho = 26.2$ m

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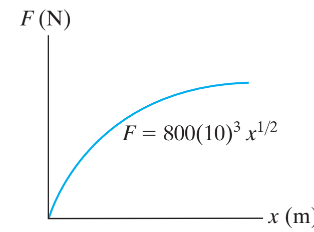
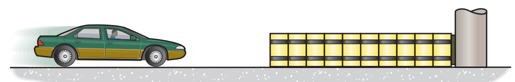
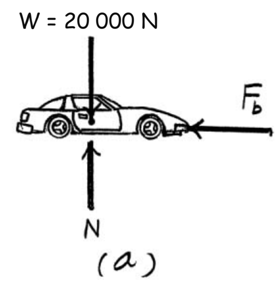
14–5. For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = [800(10^3)x^{1/2}]$ N, where x is in m, determine the car's maximum penetration in the barrier. The car has a mass of 2 Mg and it is traveling with a speed of 20 m/s just before it hits the barrier.





SOLUTION

Principle of Work and Energy: The speed of the car just before it crashes into the barrier is $v_1 = 20$ m/s. The maximum penetration occurs when the car is brought to a stop, i.e., $v_2 = 0$. Referring to the free-body diagram of the car, Fig. *a*, **W** and **N** do no work; however, $\mathbf{F}_b$ does negative work.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{20\,000}{9.81}\right)(20^2) + \left[-\int_0^{x_{max}} 800(10^3)x^{1/2}dx\right] = 0$$

$$x^{max} = 0.825\,\text{m}$$ **Ans.**

Ans:
$x^{max} = 0.825$ m

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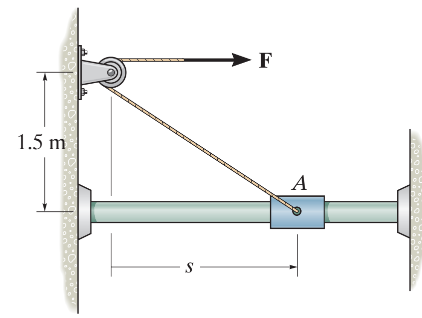
14–6.

The force of $F = 50$ N is applied to the cord when $s = 2$ m. If the 6-kg collar is orginally at rest, determine its velocity at $s = 0$. Neglect friction.



SOLUTION

Work. Referring to the FBD of the collar, Fig. *a*, we notice that force F does positive work but W and N do no work. Here, the displacement of F is $s = \sqrt{2^2 + 1.5^2} - 1.5 = 1.00$ m

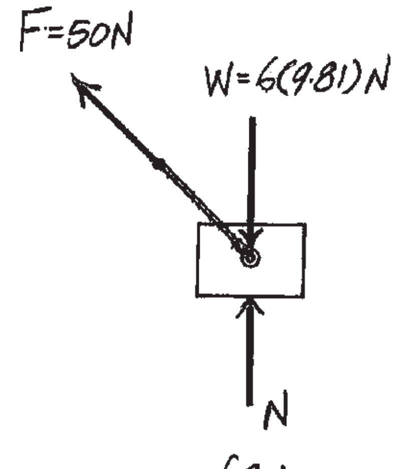
$$U_F = 50(1.00) = 50.0 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 50 = \frac{1}{2}(6)v^2$$

$$v = 4.082 \text{ m/s} = 4.08 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 4.08$ m/s

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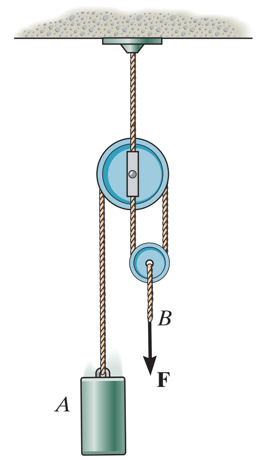
14–7.

A force of $F = 250$ N is applied to the end at B. Determine the speed of the 10-kg block when it has moved 1.5 m, starting from rest.



SOLUTION

Work. with reference to the datum set in Fig. a,

$$S_W + 2s_F = l$$

$$\delta S_W + 2\delta s_F = 0 \qquad \textbf{(1)}$$

Assuming that the block moves upward 1.5 m, then $\delta S_W = -1.5$ m since it is directed in the negative sense of S_W. Substituted this value into Eq. (1),

$$-1.5 + 2\delta s_F = 0 \qquad \delta s_F = 0.75 \text{ m}$$

Thus,

$$U_F = F\delta S_F = 250(0.75) = 187.5 \text{ J}$$

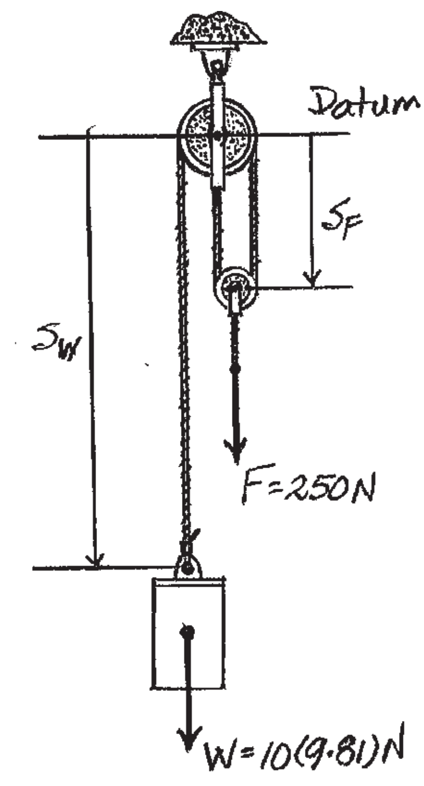
$$U_W = -W\delta S_W = -10(9.81)(1.5) = -147.15 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + U_{1-2} = T_2$$

$$0 + 187.5 + (-147.15) = \frac{1}{2}(10)v^2$$

$$v = 2.841 \text{ m/s} = 2.84 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 2.84$ m/s

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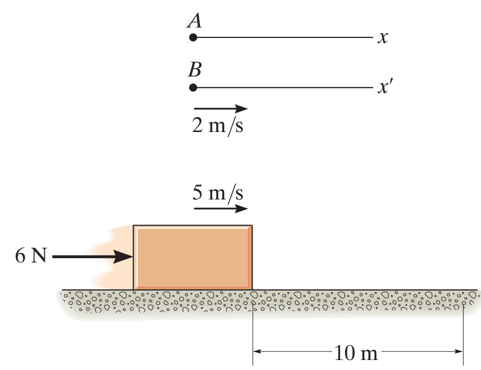
***14–8.**

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



SOLUTION

Observer A:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

$$v_2 = 6.08 \text{ m/s}$$ **Ans.**

Observer B:

$$F = ma$$

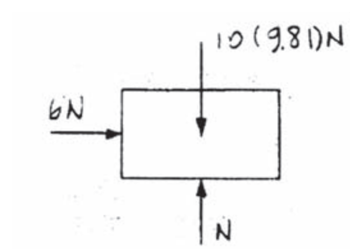
$$6 = 10a \qquad a = 0.6 \text{ m/s}^2$$

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805 \text{ s}$$



At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

$$v_2 = 4.08 \text{ m/s}$$ **Ans.**

Note that this result is 2 m/s less than that observed by A.

Ans:
Observer A: $v_2 = 6.08$ m/s
Observer B: $v_2 = 4.08$ m/s

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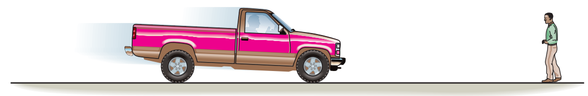
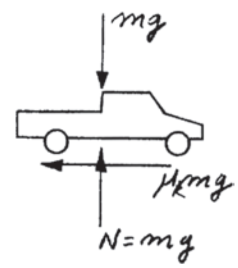
14–9.

When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



SOLUTION



$40\ \text{km/h} = \dfrac{40(10^3)}{3600} = 11.11\ \text{m/s} \qquad 80\ \text{km/h} = 22.22\ \text{m/s}$

$T_1 + \Sigma U_{1-2} = T_2$

$\dfrac{1}{2}m(11.11)^2 - \mu_k\, mg(3) = 0$

$\mu_k g = 20.576$

$T_1 + \Sigma U_{1-2} = T_2$

$\dfrac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$

$d = 12\ \text{m}$ **Ans.**

Ans:
$d = 12\ \text{m}$

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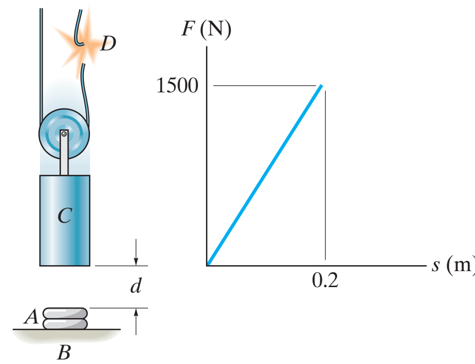
14–10.

The "air spring" A is used to protect the support B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the air spring as a function of its deflection is shown by the graph. If the block has a mass of 20 kg and is suspended a height $d = 0.4$ m above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



SOLUTION

Work. Referring to the FBD of the tensioning weight, Fig. a, W does positive work whereas force F does negative work. Here the weight displaces downward $S_W = 0.4 + x_{max}$ where x_{max} is the maximum compression of the air spring. Thus

$$U_W = 20(9.81)\left(0.4 + x_{max}\right) = 196.2\left(0.4 + x_{max}\right)$$

The work of F is equal to the area under the F-S graph shown shaded in Fig. b, Here $\dfrac{F}{x_{max}} = \dfrac{1500}{0.2}$; $F = 7500x_{max}$. Thus

$$U_F = -\frac{1}{2}(7500\,x_{max})(x_{max}) = -3750x_{max}^2$$

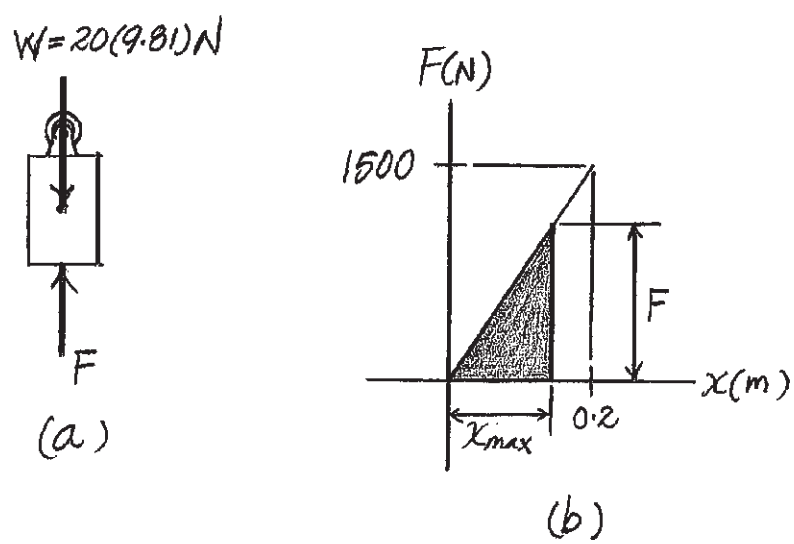
Principle of Work And Energy. Since the block is at rest initially and is required to stop momentarily when the spring is compressed to the maximum, $T_1 = T_2 = 0$. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 196.2(0.4 + x_{max}) + \left(-3750x_{max}^2\right) = 0$$

$$3750x_{max}^2 - 196.2x_{max} - 78.48 = 0$$

$$x_{max} = 0.1732 \text{ m} = 0.173 \text{ m} < 0.2 \text{ m} \quad \textbf{(O.K!)} \qquad \textbf{Ans.}$$



Ans:
$x_{max} = 0.173$ m

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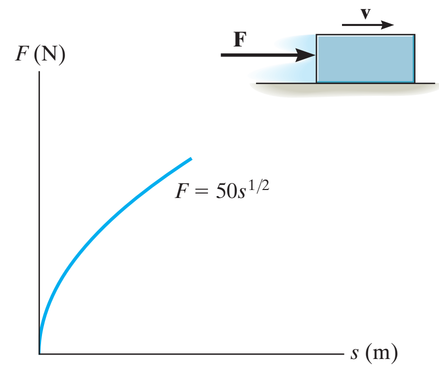
14–11.

The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block must slide before its velocity becomes 15 m/s. When $s = 0$ the block is moving to the right at $v = 6\text{ m/s}$. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



SOLUTION

Work. Consider the force equilibrium along y axis, by referring to the FBD of the block, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 20(9.81) = 0 \qquad N = 196.2\text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.3(196.2) = 58.86\text{ N}$. Here, force F does positive work whereas friction F_f does negative work. The weight W and normal reaction N do no work.

$$U_F = \int F ds = \int_0^s 50s^{\frac{1}{2}}\, ds = \frac{100}{3}s^{\frac{3}{2}}$$

$$U_{F_f} = -58.86\, s$$

Principle of Work And Energy. Applying Eq. 14–7,

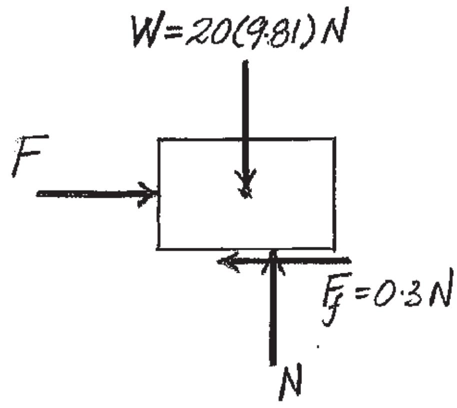
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(6^2) + \frac{100}{3}s^{\frac{3}{2}} + (-58.86s) = \frac{1}{2}(20)(15^2)$$

$$\frac{100}{3}s^{\frac{3}{2}} - 58.86s - 1890 = 0$$

Solving numerically,

$$s = 20.52\text{ m} = 20.5\text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$s = 20.5\text{ m}$

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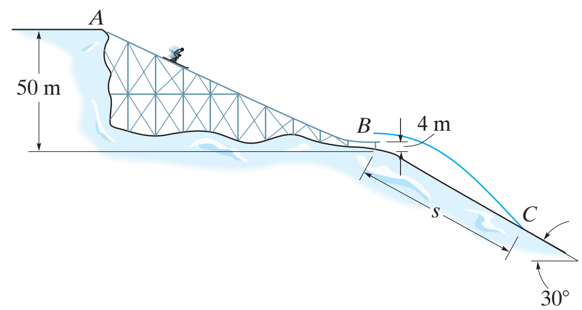
***14–12.**

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.

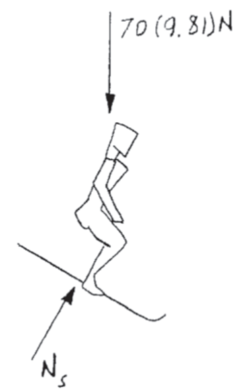


SOLUTION

$T_A + \Sigma U_{A-B} = T_B$

$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$

$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$ **Ans.**



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$s \cos 30° = 0 + 30.04t$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s \sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$

Eliminating t,

$s^2 - 122.67s - 981.33 = 0$

Solving for the positive root

$s = 130 \text{ m}$ **Ans.**

Ans:
$v_B = 30.0 \text{ m/s}$
$s = 130 \text{ m}$

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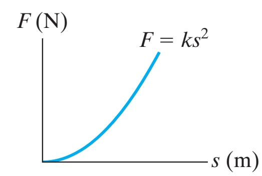
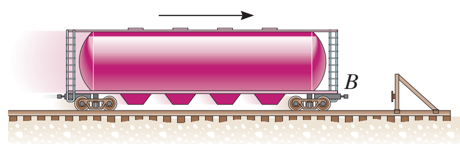
14–13.

Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



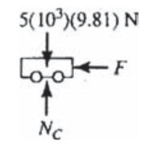


SOLUTION

$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\,ds = 0$$

$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

$$k = 15.0\ \text{MN/m}^2 \qquad \textbf{Ans.}$$



Ans:
$k = 15.0\ \text{MN/m}^2$

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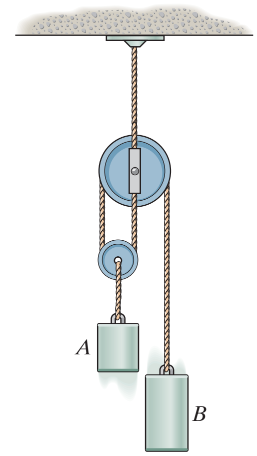
14–14.

The 8-kg cylinder A and 3-kg cylinder B are released from rest. Determine the speed of A after it has moved 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

Kinematics: Express the length of cord in terms of position coordinates s_A and s_B by referring to Fig. *a*

$$2s_A + s_B = l \qquad \textbf{(1)}$$

Thus

$$2\Delta s_A + \Delta s_B = 0 \qquad \textbf{(2)}$$

If we assume that cylinder A is moving downward through a distance of $\Delta s_A = 2$ m, Eq. (2) gives

$$(+\downarrow) \qquad 2(2) + \Delta s_B = 0 \qquad \Delta s_B = -4 \text{ m} = 4 \text{ m} \uparrow$$

Taking the time derivative of Eq. (1),

$$(+\downarrow) \qquad 2v_A + v_B = 0 \qquad \textbf{(3)}$$

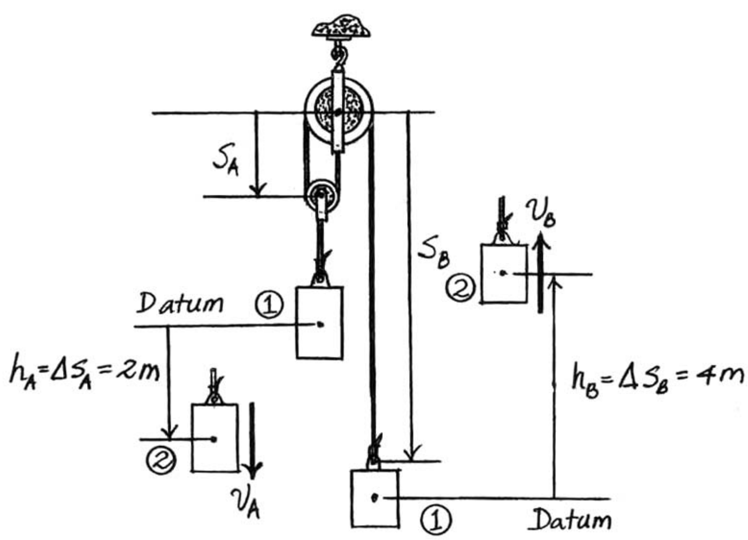
$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$0 + 8(2)9.81 - 3(4)9.81 = \frac{1}{2}(8)v_A^2 + \frac{1}{2}(3)v_B^2$$

Positive net work on left means assumption of A moving down is correct. Since $v_B = -2v_A$,

$$v_A = 1.98 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

$$v_B = -3.96 \text{ m/s} = 3.96 \text{ m/s} \uparrow$$



(a)

Ans:
$v_A = 1.98 \text{ m/s} \downarrow$
$v_B = 3.96 \text{ m/s} \uparrow$

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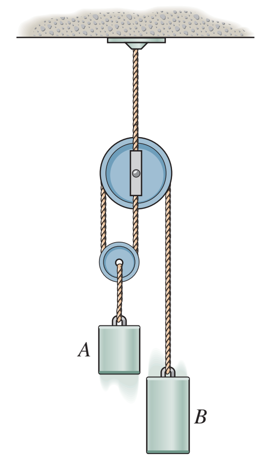
14–15. Cylinder A has a mass of 3 kg and cylinder B has a mass of 8 kg. Determine the speed of A after it has moved 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

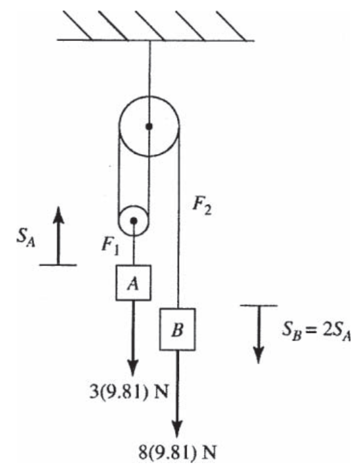
$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$0 + 2[F_1 - 3(9.81)] + 4[8(9.81) - F_2] = \frac{1}{2}(3)v_A^2 + \frac{1}{2}(8)v_B^2$$

Also, $v_B = 2v_A$, and because the pulleys are massless, $F_1 = 2F_2$. The F_1 and F_2 terms drop out and the work-energy equation reduces to

$$255.06 = 17.5v_A^2$$

$v_A = 3.82$ m/s **Ans.**



Ans:
$v_A = 3.82$ m/s

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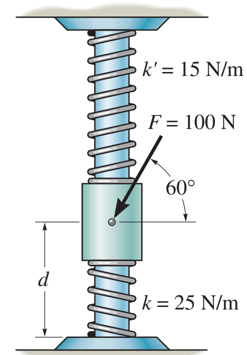
***14–16.**

The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

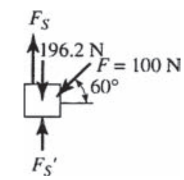


SOLUTION

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2$$

$$- \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$



$v_C = 2.36$ m/s **Ans.**

Ans:
$v_C = 2.36$ m/s

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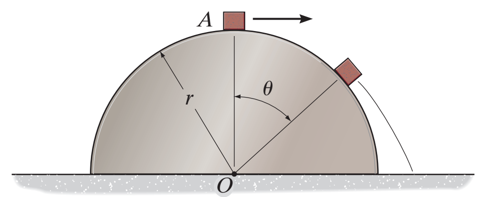
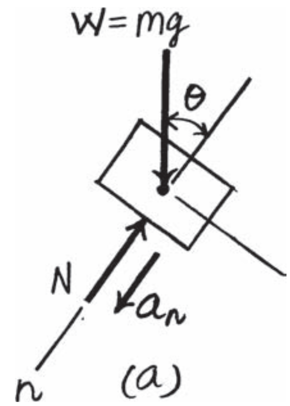
14–17.

A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



SOLUTION



Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. a, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of $h = r - r\cos\theta$.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mg(r - r\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = gr\left(\frac{9}{4} - 2\cos\theta\right) \tag{1}$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\frac{9}{4} - 2\cos\theta\right)$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad mg\cos\theta - N = m\left[g\left(\frac{9}{4} - 2\cos\theta\right)\right]$$

$$N = mg\left(3\cos\theta - \frac{9}{4}\right)$$

It is required that the block leave the track. Thus, $N = 0$.

$$0 = mg\left(3\cos\theta - \frac{9}{4}\right)$$

Since $mg \neq 0$,

$$3\cos\theta - \frac{9}{4} = 0$$

$$\theta = 41.41° = 41.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 41.4°$

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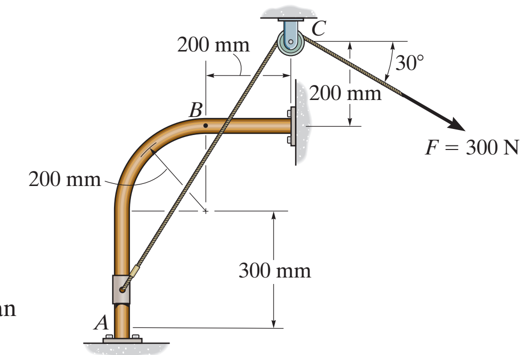
14–18.

If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

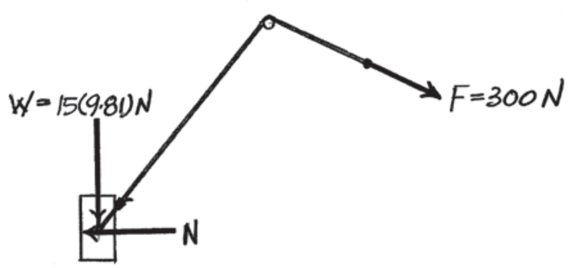
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)$ m $= 0.5$ m, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234$ m. Here, the work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

$$v_B = 3.335 \text{ m/s} = 3.34 \text{ m/s}$$ **Ans.**



(a)

Ans:
$v_B = 3.34$ m/s

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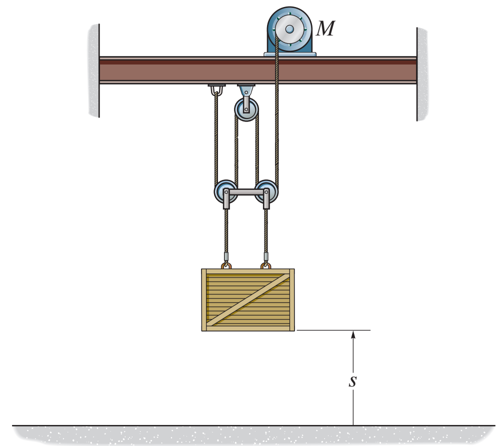
14–19.

If the force exerted by the motor M on the cable is 250 N, determine the speed of the 100-kg crate when it is hoisted to $s = 3$ m. The crate is at rest when $s = 0$.

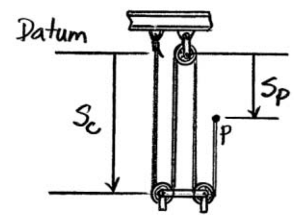


SOLUTION

Kinematics: Expressing the length of the cable in terms of position coordinates s_C and s_P referring to Fig. a,

$$3s_C + (s_C - s_P) = l$$

$$4s_C - s_P = l \qquad \textbf{(1)}$$



(a)

Using Eq. (1), the change in position of the crate and point P on the cable can be written as

$$(+\downarrow) \qquad 4\Delta s_C - \Delta s_P = 0$$

Here, $\Delta s_C = -3$ m. Thus,

$$(+\downarrow) \qquad 4(-3) - \Delta s_P = 0 \qquad\qquad \Delta s_p = -12 \text{ m} = 12 \text{ m} \uparrow$$

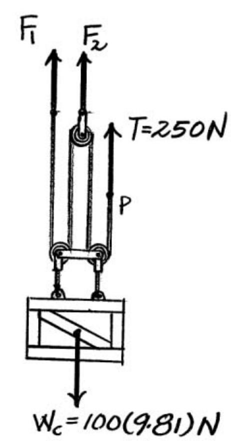
Principle of Work and Energy: Referring to the free-body diagram of the pulley system, Fig. b, $\mathbf{F}_1$ and $\mathbf{F}_2$ do no work since it acts at the support; however, $\mathbf{T}$ does positive work and $\mathbf{W}_C$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + T\Delta s_P + [-W_C \Delta s_C] = \frac{1}{2} m_C v^2$$

$$0 + 250(12) + [-100(9.81)(3)] = \frac{1}{2}(100)v^2$$

$$v = 1.07 \text{ m/s} \qquad \textbf{Ans.}$$



(b)

Ans:
$v = 1.07$ m/s

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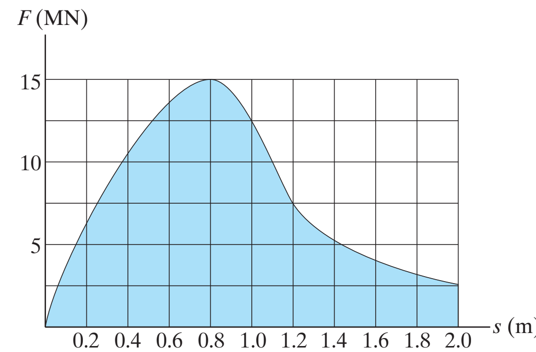
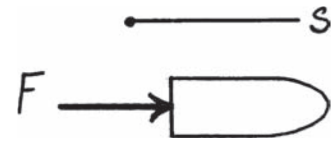
***14–20.**

When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



SOLUTION

The work done is measured as the area under the force–displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5(10^6)(0.2)$,



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[(31.5)(2.5)\left(10^6\right)(0.2)\right] = \frac{1}{2}(7)(v_2)^2$$

$v_2 = 2121 \text{ m/s} = 2.12 \text{ km/s}$ (approx.) **Ans.**

Ans:
$v_2 = 2.12$ km/s

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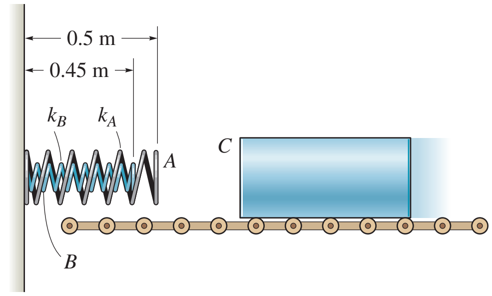
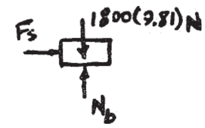
14–21.

The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. If the stiffness of the outer spring is $k_A = 5$ kN/m, determine the required stiffness k_B of the inner spring so that the motion of the ingot is stopped at the moment the front, C, of the ingot is 0.3 m from the wall.



SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$



$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)(0.5 - 0.3)^2 - \frac{1}{2}(k_B)(0.45 - 0.3)^2 = 0$$

$k_B = 11.1$ kN/m **Ans.**

Ans:
$k_B = 11.1$ kN/m

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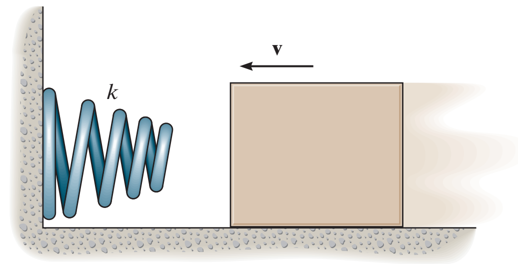
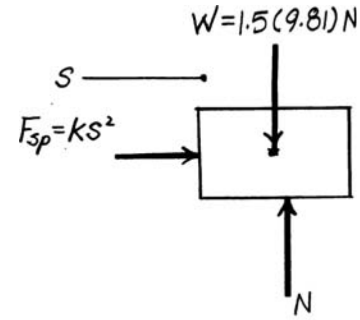
14–22. The 1.5-kg block slides along a smooth plane and strikes a *nonlinear spring* with a speed of $v = 4$ m/s. The spring is termed "nonlinear" because it has a resistance of $F_s = ks^2$, where $k = 900$ N/m^2. Determine the speed of the block after it has compressed the spring $s = 0.2$ m.



SOLUTION

Principle of Work and Energy: The spring force F_{sp} which acts in the opposite direction to that of displacement does *negative* work. The normal reaction N and the weight of the block do not displace hence do no work. Applying Eq. 14–7, we have



$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(1.5)\left(4^2\right) + \left[-\int_0^{0.2\text{ m}} 900s^2\,ds\right] = \frac{1}{2}(1.5)\,v^2$$

$$v = 3.58 \text{ m/s}$$ **Ans.**

Ans:
$v = 3.58$ m/s

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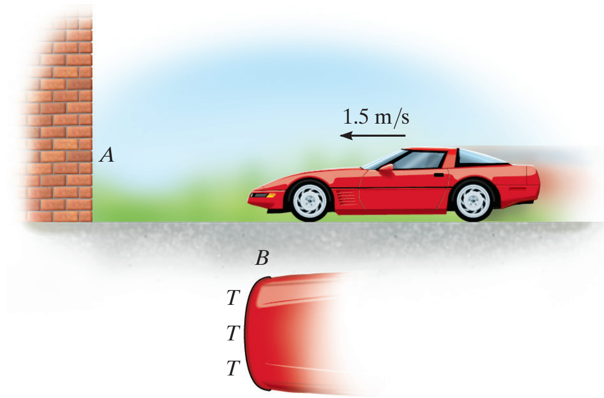
14–23. A car is equipped with a bumper B designed to absorb collisions. The bumper is mounted to the car using pieces of flexible tubing T. Upon collision with a rigid barrier at A, a constant horizontal force $\mathbf{F}$ is developed which causes a car deceleration of $3g = 29.43\ \text{m/s}^2$ (the highest safe deceleration for a passenger without a seatbelt). If the car and passenger have a total mass of 1.5 Mg and the car is initially coasting with a speed of 1.5 m/s, determine the magnitude of $\mathbf{F}$ needed to stop the car and the deformation x of the bumper tubing.



Units Used:

$$\text{Mm} = 10^3\ \text{kg}$$

$$\text{kN} = 10^3\ \text{N}$$

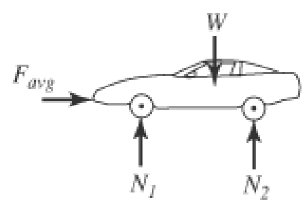
SOLUTION

Given:

$$M = 1.5\,10^3\ \text{kg}$$

$$v = 1.5\ \frac{\text{m}}{\text{s}}$$

$$k = 3$$



The average force needed to decelerate the car is

$$F_{avg} = Mkg \qquad F_{avg} = 44.1\ \text{kN} \qquad \textbf{Ans.}$$

The deformation is

$$T_1 + U_{12} = T_2$$

$$\frac{1}{2}Mv^2 - F_{avg}x = 0$$

$$x = \frac{1}{2}M\left(\frac{v^2}{F_{avg}}\right) \qquad x = 38.2\ \text{mm} \qquad \textbf{Ans.}$$

Ans:

$F_{avg} = 44.1$ kN
$x = 38.2$ mm

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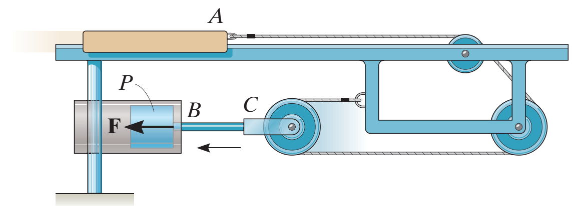
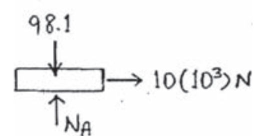
***14–24.**

The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.





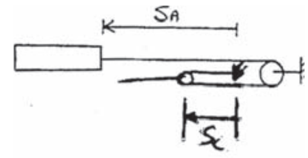
SOLUTION

$2\,s_C + s_A = l$

$2\,\Delta\,s_C + \Delta\,s_A = 0$

$2(0.2) = -\,\Delta\,s_A$

$-0.4 = \Delta\,s_A$



$T_1 + \Sigma U_{1-2} = T_2$

$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$v_A = 28.3$ m/s **Ans.**

Ans:
$v_A = 28.3$ m/s

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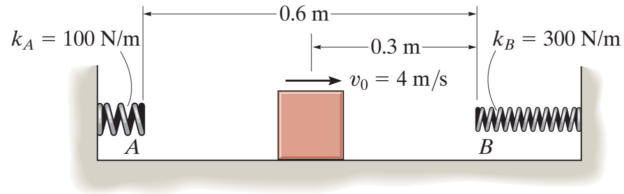
14–25. The 12-kg block has an initial speed of $v_0 = 4$ m/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



SOLUTION

Principle of Work and Energy: Here, the friction is $f_f = M_k N = 0.4[12(9.81)] = 47.088$ N. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14–7, we have

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(12)(4^2) - 47.088(0.3 + s_1) - \frac{1}{2}(300)\, s_1^2 = 0$$

$$s_1 = 0.5983 \text{ m}$$

Assume the block bounces back and stops without striking spring A. The spring force does positive work since it acts in the direction of displacement. Applying Eq. 14–7, we have

$$T_2 + \Sigma U_{2-3} = T_3$$

$$0 + \frac{1}{2}(300)(0.5983^2) - 47.088(0.5983 + s_2) = 0$$

$$s_2 = 0.5421 \text{ m}$$

Since $s_2 = 0.5421 \text{ m} < 0.6$ m, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 0.3 = 2(0.5983) + 0.5421 + 0.3 = 2.039 \text{ m} = 2.04 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_{\text{Tot}} = 2.04$ m

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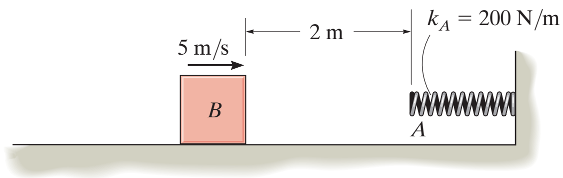
14–26.

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



SOLUTION

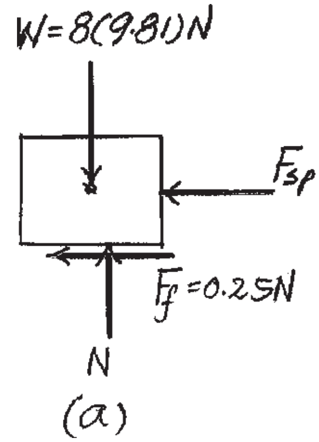
Work. Consider the force equilibrium along y axis by referring to the FBD of the block, Fig. a

$$+\uparrow \Sigma F_y = 0; \qquad N - 8(9.81) = 0 \qquad N = 78.48 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.25(78.48) = 19.62$ N and $F_{sp} = kx = 200\,x$. Here, the spring force F_{sp} and F_f both do negative work. The weight W and normal reaction N do no work.

$$U_{F_{sp}} = -\int_0^x 200\,x\,dx = -100\,x^2$$

$$U_{F_f} = -19.62(x + 2)$$



Principle of Work And Energy. It is required that the block stopped momentarily, $T_2 = 0$. Applying Eq. 14–7

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(8)(5^2) + (-100x^2) + [-19.62(x + 2)] = 0$$

$$100x^2 + 19.62x - 60.76 = 0$$

Solved for positive root,

$$x = 0.6875 \text{ m} = 0.688 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$x = 0.688$ m

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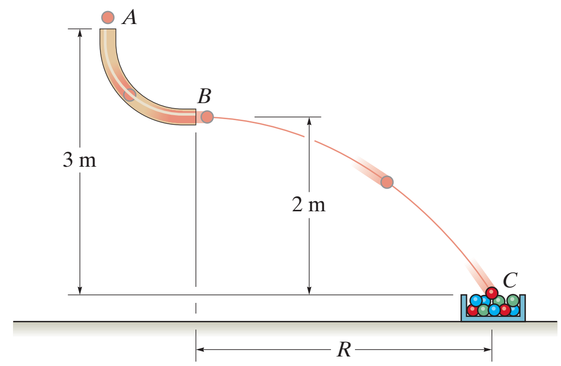
14–27.

Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.

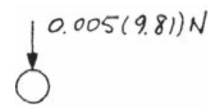


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$



$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$$

$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_1$$

$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

Ans:
$R = 2.83$ m
$v_C = 7.67$ m/s

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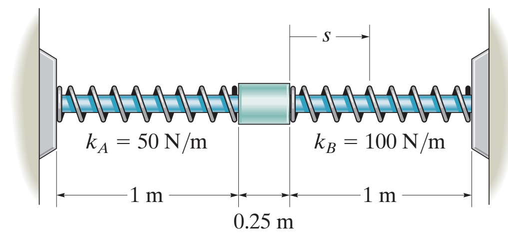
***14–28.**

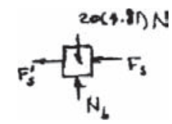
The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$



$s = 0.730$ m **Ans.**

Ans:
$s = 0.730$ m

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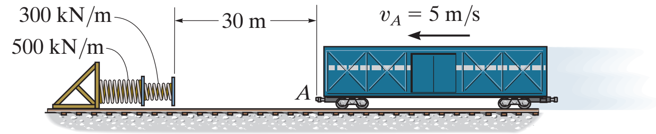
14–29.

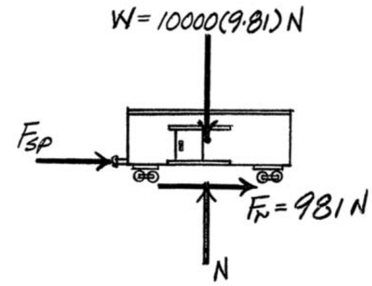
The train car has a mass of 10 Mg and is traveling at 5 m/s when it reaches A. If the rolling resistance is 1/100 of the weight of the car, determine the compression of each spring when the car is momentarily brought to rest.



SOLUTION

Free-Body Diagram: The free-body diagram of the train in contact with the spring is shown in Fig. a. Here, the rolling resistance is $F_r = \frac{1}{100}[10\,000(9.81)] = 981$ N. The compression of springs 1 and 2 at the instant the train is momentarily at rest will be denoted as s_1 and s_2. Thus, the force developed in springs 1 and 2 are $\left(F_{sp}\right)_1 = k_1 s_1 = 300(10^3)s_1$ and $\left(F_{sp}\right)_2 = 500(10^3)s_2$. Since action is equal to reaction,

$$\left(F_{sp}\right)_1 = \left(F_{sp}\right)_2$$
$$300(10^3)s_1 = 500(10^3)s_2$$
$$s_1 = 1.6667s_2$$



(a)

Principle of Work and Energy: Referring to Fig. a, **W** and **N** do no work, and $\mathbf{F}_{sp}$ and $\mathbf{F}_r$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10\,000)(5^2) + [-981(30 + s_1 + s_2)] + \left\{-\frac{1}{2}\left[300(10^3)\right]s_1^2\right\} + \left\{-\frac{1}{2}\left[500(10^3)\right]s_2^2\right\} = 0$$

$$150(10^3)s_1^2 + 250(10^3)s_2^2 + 981(s_1 + s_2) - 95570 = 0$$

Substituting Eq. (1) into Eq. (2),

$$666.67(10^3)s_2^2 + 2616s_2 - 95570 = 0$$

Solving for the positive root of the above equation,

$$s_2 = 0.3767 \text{ m} = 0.377 \text{ m}$$

Substituting the result of s_2 into Eq. (1),

$$s_1 = 0.6278 \text{ m} = 0.628 \text{ m}$$ **Ans.**

Ans:
$s_1 = 0.628$ m

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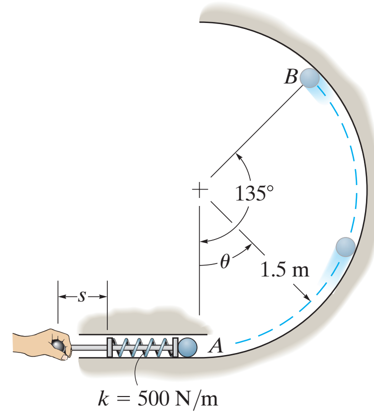
14–30.

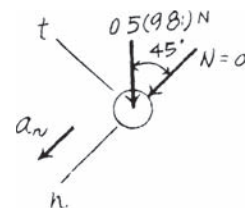
The 0.5-kg ball is fired up the smooth vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.



SOLUTION

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad 0.5(9.81)\cos 45° = 0.5\left(\frac{v_B^2}{1.5}\right) \qquad v_B^2 = 10.41\ \text{m}^2/\text{s}^2$$



Principle of Work and Energy: Here, the weight of the ball is being displaced vertically by $s = 1.5 + 1.5\sin 45° = 2.561$ m and so it does *negative* work. The spring force, given by $F_{sp} = 500(s + 0.08)$, does positive work. Since the ball is at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-B} = T_B$$

$$0 + \int_0^s 500(s + 0.08)\,ds - 0.5(9.81)(2.561) = \frac{1}{2}(0.5)(10.41)$$

$$s = 0.1789\ \text{m} = 179\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$s = 179$ mm

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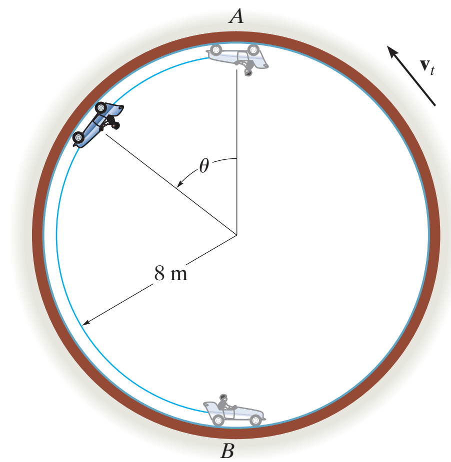
14–31.

The "flying car" is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car's brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, $v_t = 3$ m/s. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass of 250 kg and the center of mass of the car and rider moves along a circular path having a radius of 8 m.

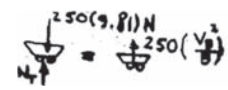


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(250)(3)^2 + 250(9.81)(16) = \frac{1}{2}(250)(v_B)^2$$

$$v_B = 17.97 = 18.0 \text{ m/s}$$ **Ans.**



$$+\uparrow \Sigma F_n = ma_n \qquad N_B - 250(9.81) = 250\left(\frac{(17.97)^2}{8}\right)$$

$$N_B = 12.5 \text{ kN}$$ **Ans.**

Ans:
$v_B = 18.0$ m/s
$N_B = 12.5$ kN

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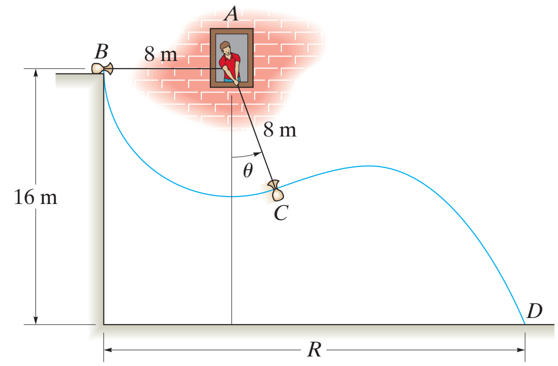
***14–32.**

The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



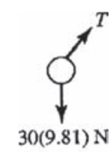
SOLUTION

$$T_B + \Sigma U_{B-C} = T_C$$

$$0 + 30(9.81)8\cos 30° = \frac{1}{2}(30)v_C^2$$

$$v_C = 11.659 \text{ m/s}$$

$$T_B + \Sigma U_{B-D} = T_D$$



$$0 + 30(9.81)(16) = \frac{1}{2}(30)\,v_D^2$$

$$v_D = 17.7 \text{ m/s}$$ **Ans.**

During free flight:

$$(+\downarrow)\; s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8\cos 30° - 11.659\sin 30° t + \frac{1}{2}(9.81)t^2$$

$$t^2 - 1.18848\,t - 1.8495 = 0$$

Solving for the positive root:

$$t = 2.0784 \text{ s}$$

$$(\overset{+}{\rightarrow})\; s = s_0 + v_0\,t$$

$$s = 8\sin 30° + 11.659\cos 30°(2.0784)$$

$$s = 24.985 \text{ m}$$

Thus,

$$R = 8 + 24.985 = 33.0 \text{ m}$$ **Ans.**

Also,

$$(v_D)_x = 11.659\cos 30° = 10.097 \text{ m/s}$$

$$(+\downarrow)\;(v_D)_x = -11.659\sin 30° + 9.81(2.0784) = 14.559 \text{ m/s}$$

$$v_D = \sqrt{(10.097)^2 + (14.559)^2} = 17.7 \text{ m/s}$$ **Ans.**

Ans:
$v_D = 17.7$ m/s
$R = 33.0$ m
$v_D = 17.7$ m/s

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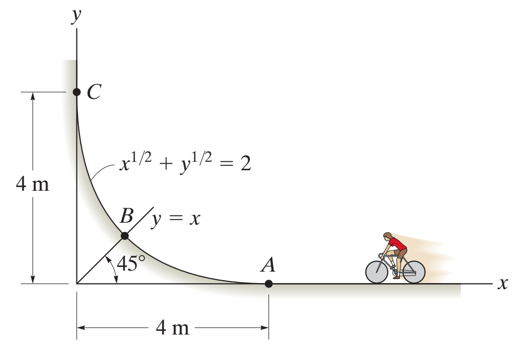
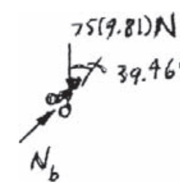
14–33.

The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4$ m/s. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



SOLUTION



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

$$y = 0.81549 \text{ m} = 0.815 \text{ m} \qquad \textbf{Ans.}$$

$$x^{1/2} + (0.81549)^{1/2} = 2$$

$$x = 1.2033 \text{ m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{-(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

$$\theta = -39.46^\circ$$

$$\nearrow + \Sigma F_n = m\,a_n; \qquad N_b - 9.81(75)\cos 39.46^\circ = 0$$

$$N_b = 568 \text{ N} \qquad \textbf{Ans.}$$

$$+\searrow \Sigma F_t = m\,a_t; \qquad 75(9.81)\sin 39.46^\circ = 75\,a_t$$

$$a = a_t = 6.23 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$y = 0.815$ m
$N_b = 568$ N
$a = a_t = 6.23$ m/s^2

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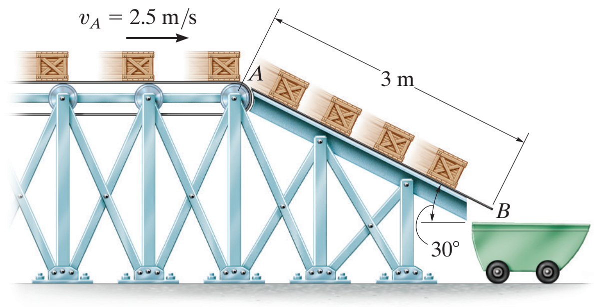
14–34. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's velocity is $v_A = 2.5\ \text{m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs.



SOLUTION

Given:

$M = 12\ \text{kg}$

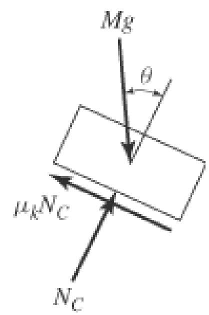
$v_A = 2.5\ \text{m/s}$

$\mu_k = 0.3$

$g = 9.81\ \text{m/s}^2$

$\theta = 30°$

$a = 3\ \text{m}$



$$N_c = Mg\cos(\theta)$$

$$\frac{1}{2}Mv_A^{\ 2} + (Mga)\sin(\theta) - \mu_k N_c a = \frac{1}{2}Mv_B^{\ 2}$$

$$v_B = \sqrt{v_A^{\ 2} + (2ga)\sin(\theta) - (2\mu_k g)\cos(\theta)a}$$

$v_B = 4.52\ \text{m/s}$ **Ans.**

Ans:

$v_B = 4.52\ \text{m/s}$

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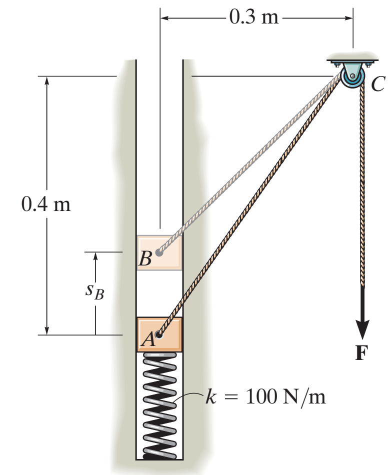
14–35.

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of $\mathbf{F}$ can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



SOLUTION

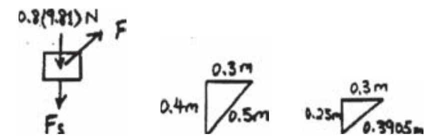
$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5$ m

$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905$ m

$T_A + \Sigma U_{A-B} = T_B$

$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

$F = 43.9$ N **Ans.**



Ans:
$F = 43.9$ N

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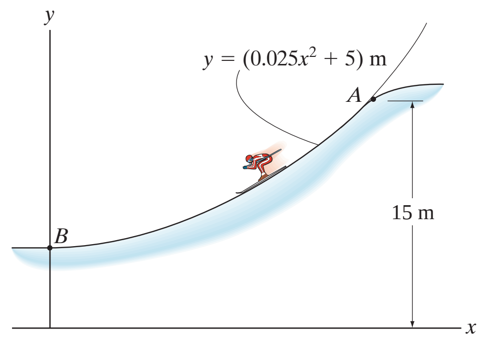
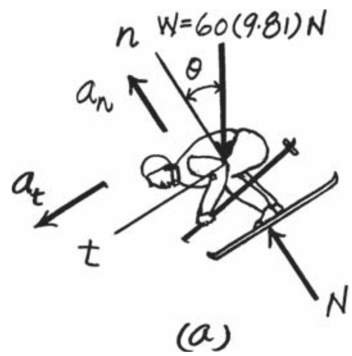
***14–36.**

If the 60-kg skier passes point A with a speed of $5\,\text{m/s}$, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the skier at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 15 - \left[0.025\left(0^2\right) + 5\right] = 10$ m and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(60)(5^2) + \left[60(9.81)(10)\right] = \frac{1}{2}(60)v_B{}^2$$

$$v_B = 14.87 \text{ m/s} = 14.9 \text{ m/s} \qquad \textbf{Ans.}$$

$dy/dx = 0.05x$

$d^2y/dx^2 = 0.05$

$$\rho = \frac{[1 + 0]^{3/2}}{0.5} = 20 \text{ m}$$

$$+\uparrow \Sigma F_n = ma_n; \qquad N - 60(9.81) = 60\left(\frac{(14.87)^2}{20}\right)$$

$$N = 1.25 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$v_B = 14.9\,\text{m/s}$
$N = 1.25$ kN

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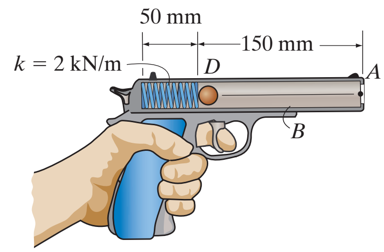
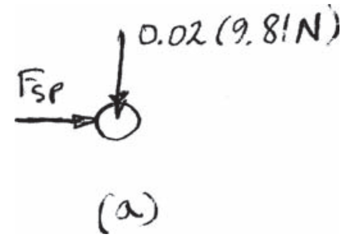
14–37.

The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



SOLUTION

Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. *a*, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[\frac{1}{2}ks_1{}^2 - \frac{1}{2}ks_2{}^2\right] = \frac{1}{2}mv_A{}^2$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A{}^2$$

$v_A = 10.5$ m/s **Ans.**

Ans:
$v_A = 10.5$ m/s

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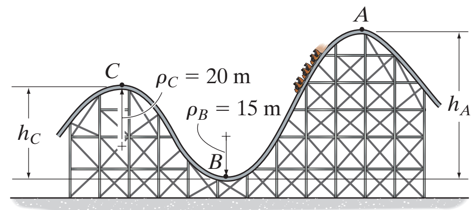
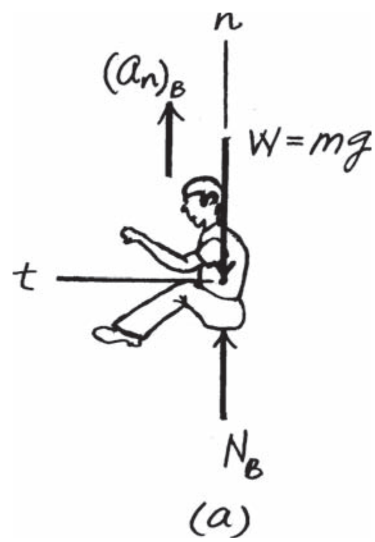
14–38.

If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.



Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B{}^2}{15}\right)$$

$$v_B{}^2 = 45g$$

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C{}^2}{20}\right)$$

$$v_C{}^2 = 20g$$

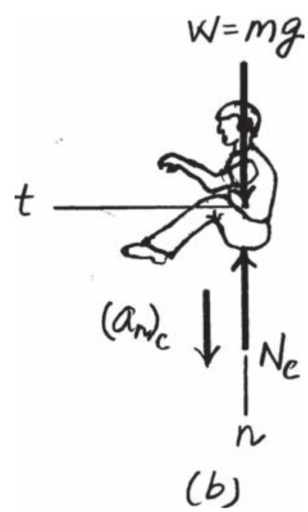
Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A\text{–}B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$



When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

$$T_A + \Sigma U_{A\text{–}B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h_A = 22.5$ m
$h_C = 12.5$ m

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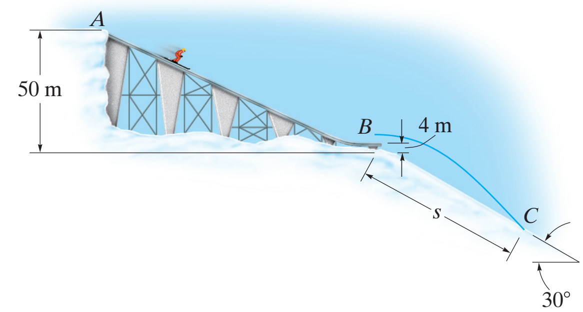
14–39. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass 75 kg.



SOLUTION

Guesses $\quad v_B = 1\ \frac{\text{m}}{\text{s}} \qquad t = 1\ \text{s} \qquad d = 1\ \text{m}$

Given $\quad M g(h_1 - h_2) = \frac{1}{2} M v_B^2 \qquad v_B t = d\cos(\theta) \qquad -h_2 - d\sin(\theta) = \frac{-1}{2} g t^2$

Given:

$M = 75\ \text{kg}$

$h_1 = 50\ \text{m}$

$h_2 = 4\ \text{m}$

$\theta = 30°$

$$\begin{pmatrix} v_B \\ t \\ d \end{pmatrix} = \text{Find}(v_B, t, d) \qquad t = 3.754\ \text{s} \qquad v_B = 30.0\ \text{m/s} \qquad d = 130.2\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v_B = 30.0\ \text{m/s}$
$d = 130.2\ \text{m}$

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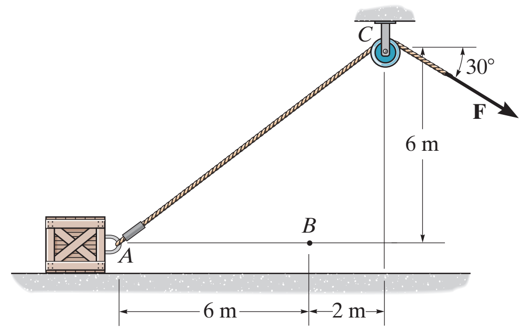
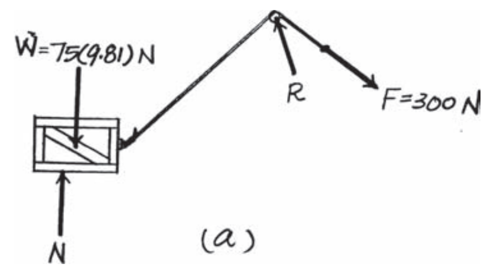
***14–40.**

If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a.*



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 300(3.675) = \frac{1}{2}(75)v_B{}^2$$

$$v_B = 5.42 \text{ m/s}$$ **Ans.**

Ans:
$v_B = 5.42$ m/s

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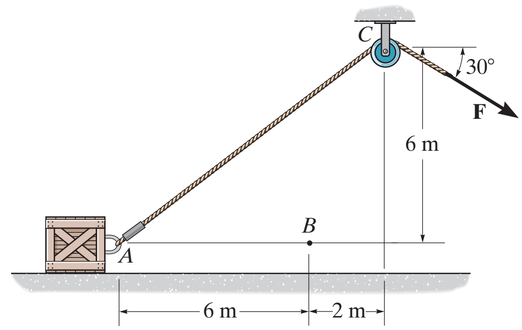
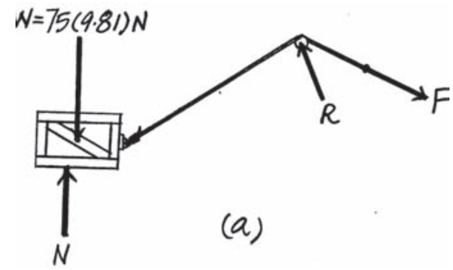
14–41.

If the 75-kg crate starts from rest at A, and its speed is $6\,\mathrm{m/s}$ when it passes point B, determine the constant force **F** exerted on the cable. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675\ \mathrm{m}$. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + F(3.675) = \frac{1}{2}(75)(6^2)$$

$$F = 367\ \mathrm{N} \qquad \textbf{Ans.}$$

Ans:
$F = 367\ \mathrm{N}$

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14–42. A spring having a stiffness of 5 kN/m is compressed 400 mm. The stored energy in the spring is used to drive a machine which requires 90 W of power. Determine how long the spring can supply energy at the required rate.

SOLUTION

Given: $k = 5 \text{ kN/m}$ $\quad \delta = 400 \text{ mm}$ $\quad P = 90 \text{ W}$

$$U_{12} = \frac{1}{2}k\delta^2 = P\,t \qquad t = \frac{1}{2}k\left(\frac{\delta^2}{P}\right) \qquad t = 4.44 \text{ s} \quad \textbf{Ans.}$$

Ans:
$t = 4.44$ s

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14–43.

To dramatize the loss of energy in an automobile, consider a car having a weight of 25 000 N that is traveling at 56 km/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy.

SOLUTION

Energy: Here, the speed of the car is $v = \left(\frac{56\text{ km}}{\text{h}}\right) \times \left(\frac{1000\text{ m}}{1\text{ km}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 15.56 m/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{25\,000}{9.81}\right)\left(15.56^2\right) = 308\,504\text{ J}$$

The power of the bulb is $P_{bulb} = 100$ W.

Thus,

$$t = \frac{U}{P_{bulb}} = \frac{308\,504}{100} = 3085.04\text{ s} = 51.4\text{ min} \qquad \textbf{Ans.}$$

Ans:
$t = 51.4$ min

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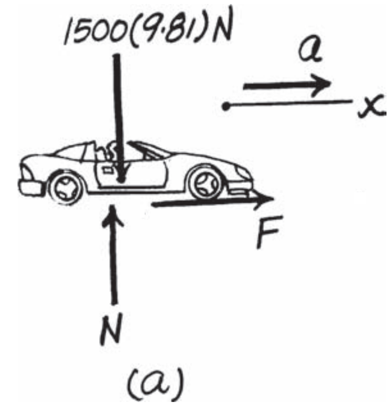
***14–44.** If the engine of a 1.5-Mg car generates a constant power of 15 kW, determine the speed of the car after it has traveled a distance of 200 m on a level road starting from rest. Neglect friction.



SOLUTION

Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the car shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad F = 1500\left(v\frac{dv}{ds}\right)$$

Power:

$$P_{\text{out}} = \mathbf{F}\cdot\mathbf{v}$$

$$15(10^3) = 1500\left(v\frac{dv}{ds}\right)v$$

$$\int_0^{200\text{ m}} 10ds = \int_0^{v} v^2\,dv$$

$$10s\Big|_0^{200\text{ m}} = \frac{v^3}{3}\Big|_0^{v}$$

$$v = 18.7\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 18.7$ m/s

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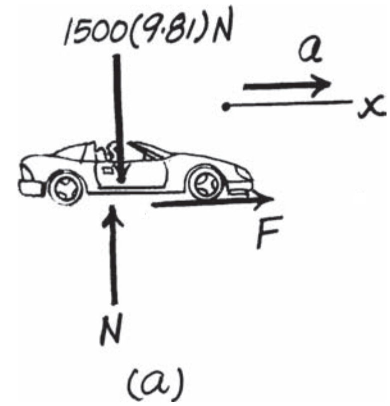
14–45. If the engine of a 1.5-Mg car generates a constant power of 15 kW, determine the speed of the car after it has traveled a distance of 200 m on a level road starting from rest. Neglect friction.



SOLUTION

Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the car shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 1500\left(v\frac{dv}{ds}\right)$$

Power:

$$P_{\text{out}} = \mathbf{F} \cdot \mathbf{v}$$

$$15(10^3) = 1500\left(v\frac{dv}{ds}\right)v$$

$$\int_0^{200\text{ m}} 10ds = \int_0^v v^2\, dv$$

$$10s\Big|_0^{200\text{ m}} = \frac{v^3}{3}\Big|_0^v$$

$$v = 18.7\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 18.7\text{ m/s}$

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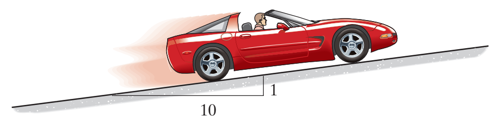
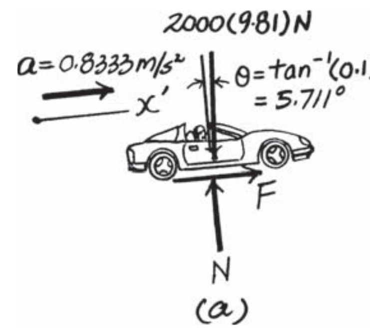
14–46.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\varepsilon = 0.8$. Also, find the average power supplied by the engine.



SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

$$(P_{\text{out}})_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

Ans:
$P_{\max} = 113$ kW
$P_{\text{avg}} = 56.5$ kW

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14–47. A car has a mass m and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

SOLUTION

Power: Since the power output is constant, then the traction force F varies with v. Applying Eq. 14-10, we have

$$P = Fv \qquad F = \frac{P}{v}$$

Equation of Motion: $\dfrac{P}{v} = ma \qquad a = \dfrac{P}{Mv}$

Kinematics: Applying equation $ds = \dfrac{v\,dv}{a}$, we have

$$\int_0^s 1\,ds = \int_0^v \frac{mv^2}{P}\,dv \qquad s = \frac{mv^3}{3P} \qquad \textbf{Ans.}$$

Ans:

$$s = \frac{mv^3}{3P}$$

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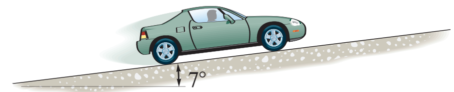
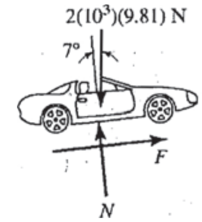
***14–48.**

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100\ \text{km/h}$. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\varepsilon = 0.65$.



SOLUTION

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08\ \text{N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\ \text{m}}{\text{h}}\right] \times \left(\dfrac{1\ \text{h}}{3600\ \text{s}}\right) = 27.78\ \text{m/s}$. The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 2391.08(27.78) = 66.418(10^3)\ \text{W} = 66.418\ \text{kW}$$

Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102\ \text{kW} \qquad \textbf{Ans.}$$

Ans:
power input $= 102\ \text{kW}$

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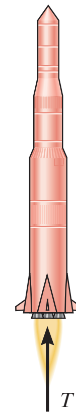
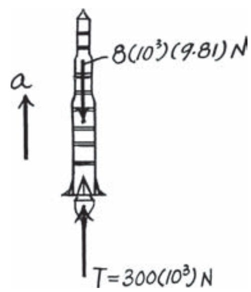
14–49. A rocket having a total mass of 8 Mg is fired vertically from rest. If the engines provide a constant thrust of $T = 300$ kN, determine the power output of the engines as a function of time. Neglect the effect of drag resistance and the loss of fuel mass and weight.



SOLUTION

$+\uparrow \Sigma F_y = ma_y;\quad 300(10^3) - 8(10^3)(9.81) = 8(10^3)a \qquad a = 27.69 \text{ m/s}^2$



$(+\uparrow)\qquad v = v_0 + a_c t$

$= 0 + 27.69t = 27.69t$

$P = \mathbf{T} \cdot \mathbf{v} = 300\,(10^3)\,(27.69t) = 8.31t \text{ MW}$ **Ans.**

Ans:

$P = 8.31t$ MW

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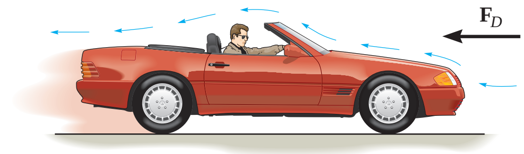
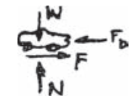
14–50.

The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\varepsilon = 0.68$.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad F - 0.3v^2 = 2.3(10^3)(5)$

$$F = 0.3v^2 + 11.5(10^3)$$

At $v = 28$ m/s

$$F = 11\,735.2 \text{ N}$$

$P_O = (11\,735.2)(28) = 328.59$ kW

$P_i = \dfrac{P_O}{\varepsilon} = \dfrac{328.59}{0.68} = 438$ kW **Ans.**

Ans:
$P_i = 483$ kW

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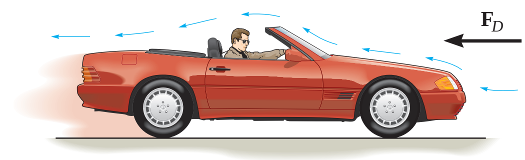
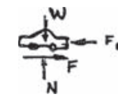
14–51.

The sports car has a mass of 2.3 Mg and accelerates at $6\ \text{m/s}^2$, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\varepsilon = 0.68$.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x; \qquad F - 10v = 2.3(10^3)(6)$

$$F = 13.8(10^3) + 10\,v$$

$(\overset{+}{\rightarrow})\ v = v_0 + a_c\,t$

$$v = 0 + 6(5) = 30\ \text{m/s}$$

$P_O = \mathbf{F}\cdot\mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0\ \text{kW}$

$P_i = \dfrac{P_O}{\varepsilon} = \dfrac{423.0}{0.68} = 622\ \text{kW}$ **Ans.**

Ans:
$P_i = 622\ \text{kW}$

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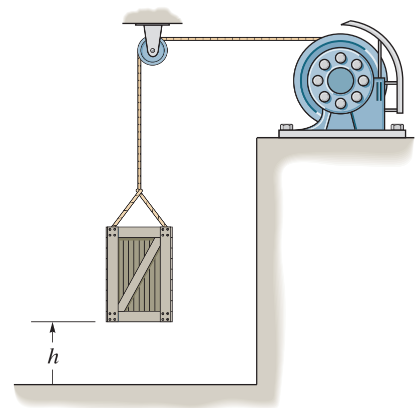
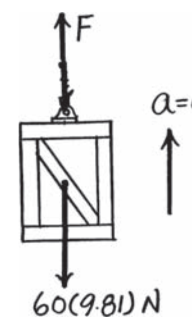
***14–52.** A motor hoists a 60-kg crate at a constant velocity to a height of $h = 5$ m in 2 s. If the indicated power of the motor is 3.2 kW, determine the motor's efficiency.



SOLUTION

Equations of Motion:

$$+\uparrow \Sigma F_y = ma_y; \qquad F - 60(9.81) = 60(0) \qquad F = 588.6 \text{ N}$$



Power: The crate travels at a constant speed of $v = \frac{5}{2} = 2.50$ m/s. The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 588.6\,(2.50) = 1471.5 \text{ W}$$

Thus, from Eq. 14–11, the efficiency of the motor is given by

$$\varepsilon = \frac{\text{power output}}{\text{power input}} = \frac{1471.5}{3200} = 0.460 \qquad \textbf{Ans.}$$

Ans:
$\varepsilon = 0.460$

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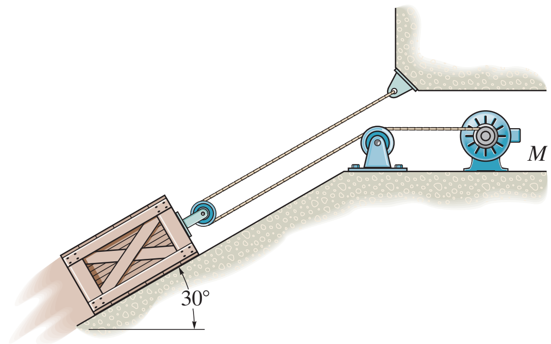
14–53. The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at the instant. Neglect friction along the plane. The motor has an efficiency of $\varepsilon = 0.74$.

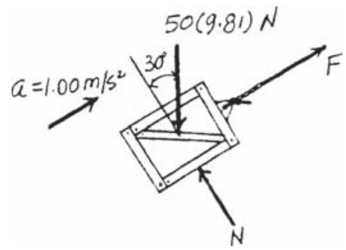


SOLUTION

Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00\ \text{m/s}^2$$

Equations of Motion:



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 50(9.81)\sin 30° = 50(1.00) \qquad F = 295.25\ \text{N}$$

Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 295.25\,(4) = 1181\ \text{W} = 1.181\ \text{kW}$$

Using Eq. 14–11, the required power input to the motor in order to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.181}{0.74} = 1.60\ \text{kW}$$ **Ans.**

Ans:
$P_{in} = 1.60$ kW

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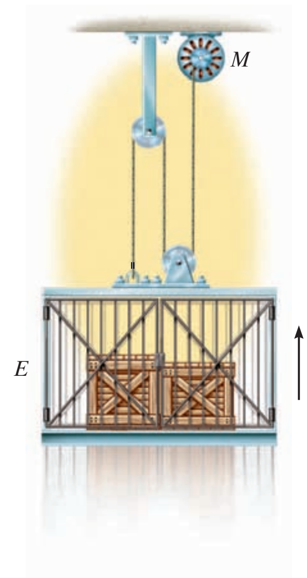
14–54.

The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\ \text{m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



SOLUTION

$+\uparrow \Sigma F_y = m\,a_y;\qquad 3T - 500(9.81) = 500(2)$

$T = 1968.33\ \text{N}$

$3s_E - s_P = l$

$3\,v_E = v_P$

When $t = 3$ s,

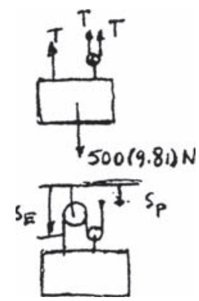
$(+\uparrow)\ v_0 + a_c\,t$

$v_E = 0 + 2(3) = 6\ \text{m/s}$

$v_P = 3(6) = 18\ \text{m/s}$

$P_O = 1968.33(18)$

$P_O = 35.4\ \text{kW}$ **Ans.**



Ans:
$P_O = 35.4\ \text{kW}$

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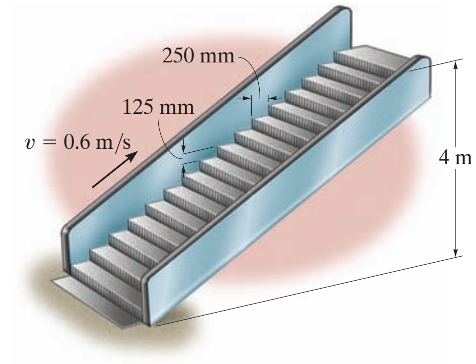
14–55.

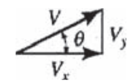
The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$



Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

Ans:
$P = 12.6$ kW

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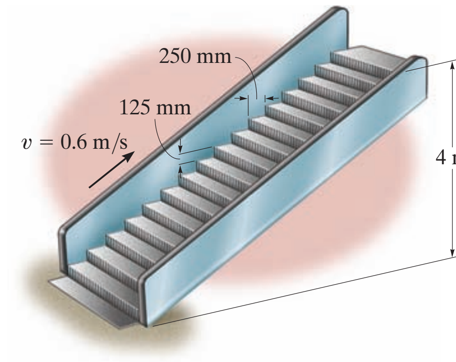
***14–56.**

If the escalator in Prob. 14–55 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



SOLUTION

$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4 \text{ s}$$

$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285 \text{ m/s}$$ **Ans.**

Ans:
$v = 0.285$ m/s

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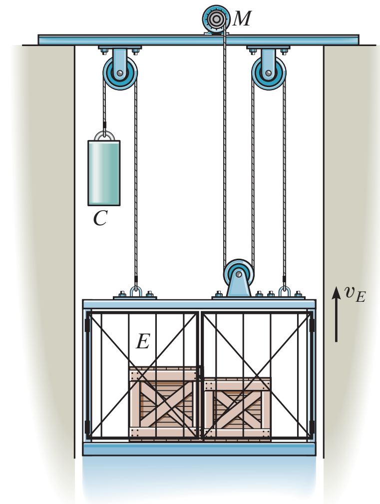
14–57.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\varepsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$$

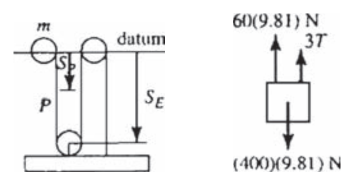
$$T = 1111.8 \text{ N}$$

$2s_E + (s_E - s_P) = l$

$3v_E = v_P$

Since $v_E = -4$ m/s, $\quad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_P}{\varepsilon} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW} \qquad \textbf{Ans.}$$



Ans:
$P_i = 22.2$ kW

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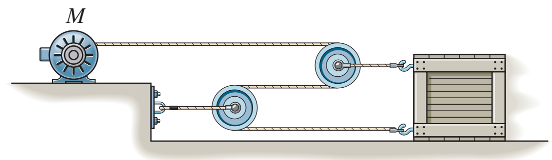
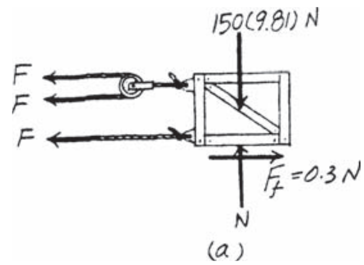
14–58.

The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



SOLUTION

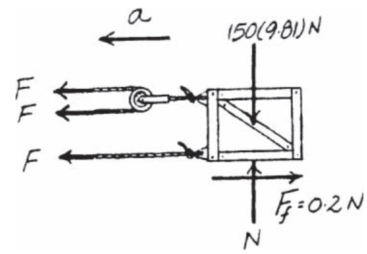


Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867\text{ s}$$

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150\,(0) \qquad N = 1471.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2\,(1471.5) - 3\left(8t^2 + 20\right) = 150\,(-a)$$

$$a = \left(0.160t^2 - 1.562\right)\text{ m/s}^2$$

Kinematics: Applying $dv = a\,dt$, we have

$$\int_0^v dv = \int_{3.9867\text{ s}}^{5} \left(0.160t^2 - 1.562\right) dt$$

$$v = 1.7045\text{ m/s}$$

Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 3\,(220)\,(1.7045) = 1124.97\text{ W} = 1.12\text{ kW}$$ **Ans.**

Ans:
$P = 1.12$ kW

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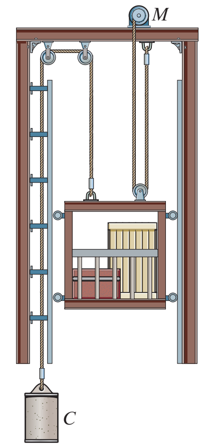
14–59. The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. At a given instant, the hoist has an upward velocity of 2 m/s and an acceleration of 1.5 m/s^2. Determine the power generated by the motor M at this instant if it operates with an efficiency of $\varepsilon = 0.8$.



SOLUTION

Equations of Motion: Here, $a = 1.5$ m/s^2. By referring to the free-body diagram of the hoist and counterweight shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(1.5) \qquad \textbf{(1)}$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(1.5)$$

Solving,

$$T' = 1246.5 \text{ N}$$

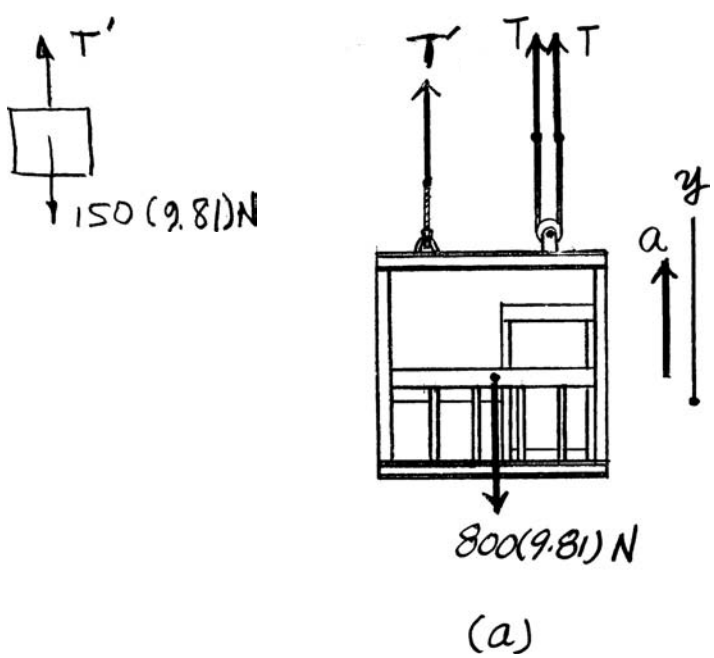
$$T = 3900.75 \text{ N}$$

Power:

$$P_{\text{out}} = 2\mathbf{T} \cdot \mathbf{v} = 2(3900.75)(2) = 15\,603 \text{ W}$$

Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{15603}{0.8} = 19.5(10^3) \text{ W} = 19.5 \text{ kW} \qquad \textbf{Ans.}$$



(a)

Ans:
$P_{\text{in}} = 19.5$ kW

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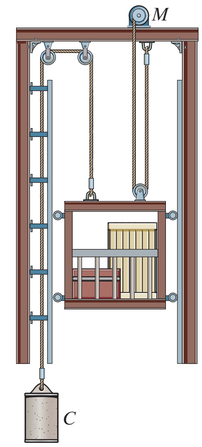
***14–60.** The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. If the upward speed of the hoist increases uniformly from 0.5 m/s to 1.5 m/s in 1.5 s, determine the average power generated by the motor M during this time. The motor operates with an efficiency of $\varepsilon = 0.8$.



SOLUTION

Kinematics: The acceleration of the hoist can be determined from

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0.5 + a(1.5)$$

$$a = 0.6667 \text{ m/s}^2$$

Equations of Motion: Using the result of **a** and referring to the free-body diagram of the hoist and block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(0.6667)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(0.6667)$$

Solving,

$$T' = 1371.5 \text{ N}$$

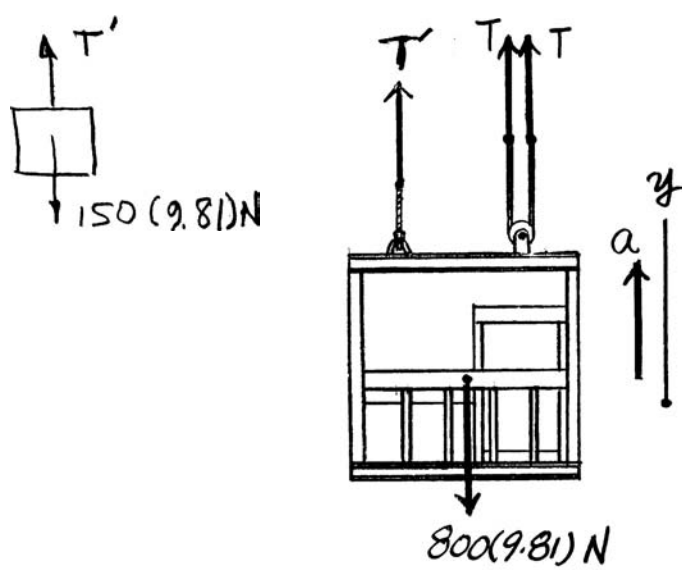
$$T = 3504.92 \text{ N}$$

Power:

$$(P_{\text{out}})_{\text{avg}} = 2\mathbf{T} \cdot \mathbf{v}_{\text{avg}} = 2(3504.92)\left(\frac{1.5 + 0.5}{2}\right) = 7009.8 \text{ W}$$

Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{7009.8}{0.8} = 8762.3 \text{ W} = 8.76 \text{ kW} \qquad \textbf{Ans.}$$



(a)

Ans:
$P_{\text{in}} = 8.76$ kW

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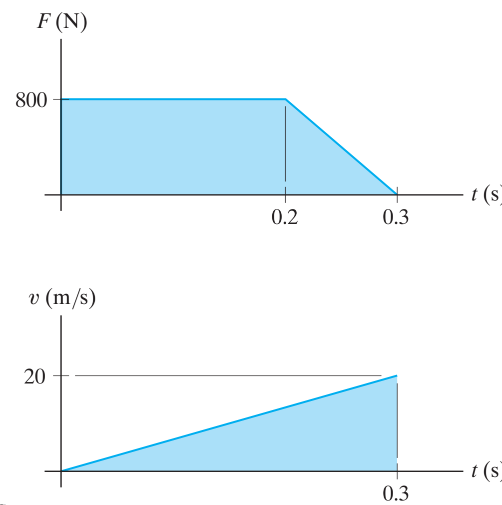
14–61.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.



SOLUTION

For $0 \leq t \leq 0.2$

$F = 800$ N

$$v = \frac{20}{0.3}t = 66.67t$$

$\boldsymbol{P} = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans.**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000t$

$v = 66.67t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2)$ kW **Ans.**

$$U = \int_0^{0.3} P\, dt$$

$$U = \int_0^{0.2} 53.3t\, dt + \int_{0.2}^{0.3} (160t - 533t^2)\, dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans.**

Ans:

$$P = \left\{160\,t - 533t^2\right\} \text{kW}$$

$U = 1.69$ kJ

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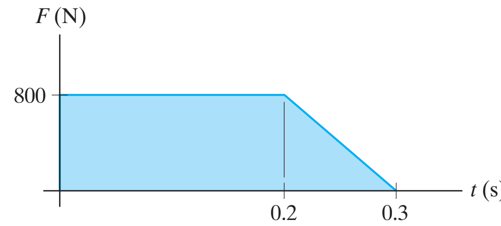
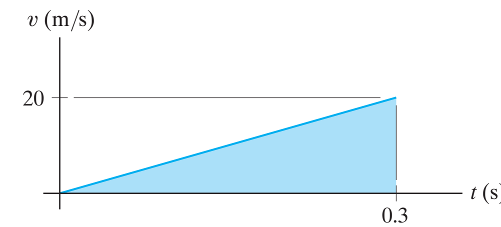
14–62.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.





SOLUTION

See solution to Prob. 14–62.

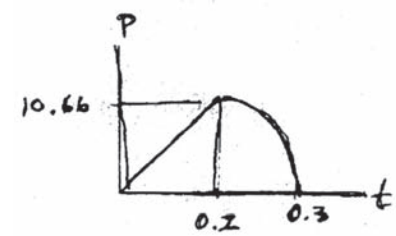
$$P = 160\,t - 533\,t^2$$

$$\frac{dP}{dt} = 160 - 1066.6\,t = 0$$

$t = 0.15\text{ s} < 0.2\text{ s}$

Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7$ kW **Ans.**



Ans:
$P_{\max} = 10.7$ kW

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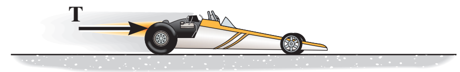
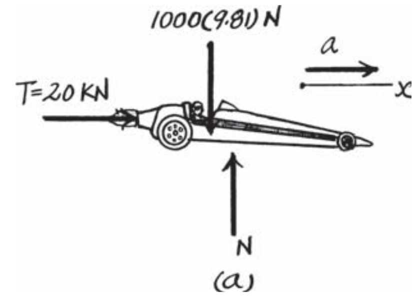
14–63.

If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.



SOLUTION



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20 \text{ m/s}^2$$

Kinematics: The velocity of the dragster can be determined from

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t) \text{ m/s}$$

Power:

$$P = \mathbf{F} \cdot \mathbf{v} = 20(10^3)(20t)$$

$$= \left[400(10^3)t\right] \text{ W} \qquad \textbf{Ans.}$$

Ans:

$$P = \left\{400(10^3)t\right\} \text{ W}$$

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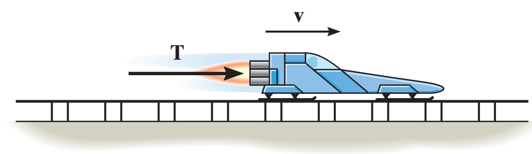
***14–64.**

The rocket sled has a mass of 4 Mg and travels from rest along the horizontal track for which the coefficient of kinetic friction is $\mu_k = 0.20$. If the engine provides a constant thrust $T = 150$ kN, determine the power output of the engine as a function of time. Neglect the loss of fuel mass and air resistance.



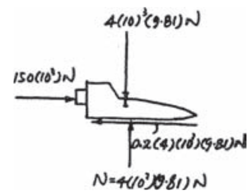
SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 150(10)^3 - 0.2(4)(10)^3(9.81) = 4(10)^3\, a$

$a = 35.54 \text{ m/s}^2$

$\left(\xrightarrow{+}\right)\ v = v_0 + a_c t$

$= 0 + 35.54t = 35.54t$



$P = \mathbf{T} \cdot \mathbf{v} = 150(10)^3\,(35.54t) = 5.33t$ MW **Ans.**

Ans:
$P = 5.33t$ MW

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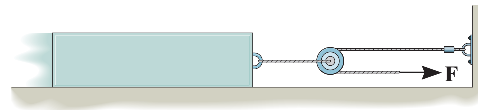
14–65.

The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.

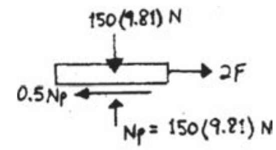


SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F - 0.5(150)(9.81) = 0$

$$F = 367.875 = 60t^2$$

$$t = 2.476 \text{ s}$$

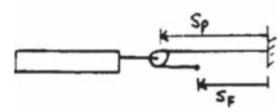


$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$$a_p = 0.8t^2 - 3.924$$

$dv = a\,dt$

$$\int_0^v dv = \int_{2.476}^5 \left(0.8t^2 - 3.924\right) dt$$



$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t\Big|_{2.476}^{5} = 19.38 \text{ m/s}$$

$s_P + (s_P - s_F) = l$

$2v_P = v_F$

$v_F = 2(19.38) = 38.76$ m/s

$F = 60(5)^2 = 1500$ N

$P = \mathbf{F} \cdot \mathbf{v} = 1500(38.76) = 58.1$ kW **Ans.**

Ans:
$P = 58.1$ kW

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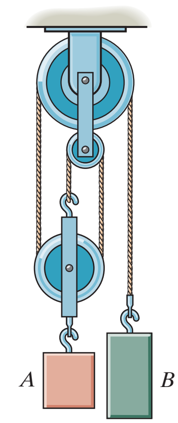
14–66.

The assembly consists of two blocks A and B, which have a mass of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



SOLUTION

$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

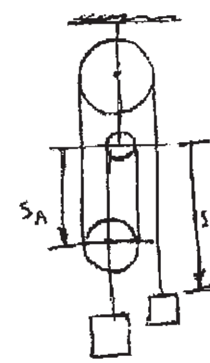
$3v_A = -v_B$

$v_B = -9\text{ m/s}$

$T_1 + V_1 = T_2 + V_2$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(3)^2 + \frac{1}{2}(30)(-9)^2 + 20(9.81)\left(\frac{s_B}{3}\right) - 30(9.81)(s_B)$$

$s_B = 5.70\text{ m}$ **Ans.**



Ans:
$s_B = 5.70\text{ m}$

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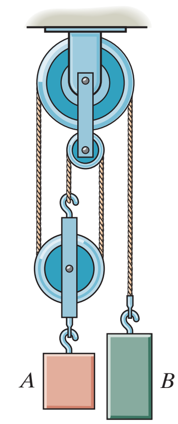
14–67.

The assembly consists of two blocks A and B which have a mass of 20 kg and 30 kg, respectively. Determine the speed of each block when B descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



SOLUTION

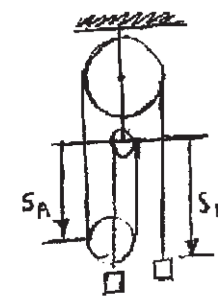
$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

$3v_A = -v_B$

$T_1 + V_1 = T_2 + V_2$

$$(0+0)+(0+0) = \frac{1}{2}(20)(v_A)^2 + \frac{1}{2}(30)(-3v_A)^2 + 20(9.81)\left(\frac{1.5}{3}\right) - 30(9.81)(1.5)$$



$v_A = 1.54$ m/s **Ans.**

$v_B = 4.62$ m/s **Ans.**

Ans:
$v_A = 1.54$ m/s
$v_B = 4.62$ m/s

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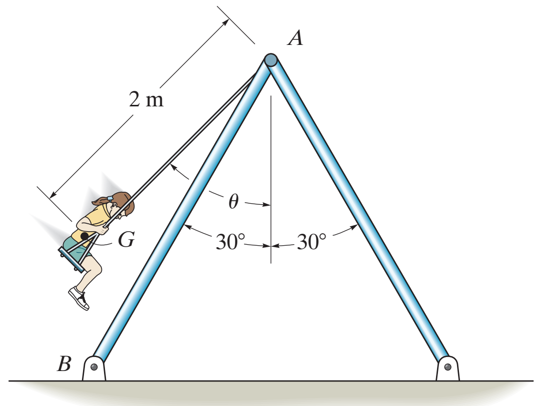
***14–68.**

The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



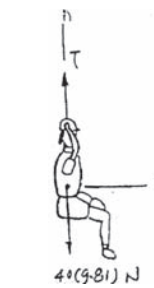
SOLUTION

The maximum tension in the cable occurs when $\theta = 0°$.

$$T_1 + V_1 = T_2 + V_2$$

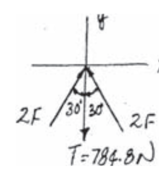
$$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$$

$$v = 4.429 \text{ m/s}$$



$$+\uparrow \Sigma F_n = ma_n; \qquad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right) \qquad T = 784.8 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(2F)\cos 30° - 784.8 = 0 \qquad F = 227 \text{ N}$$ **Ans.**



Ans:
$F = 227$ N

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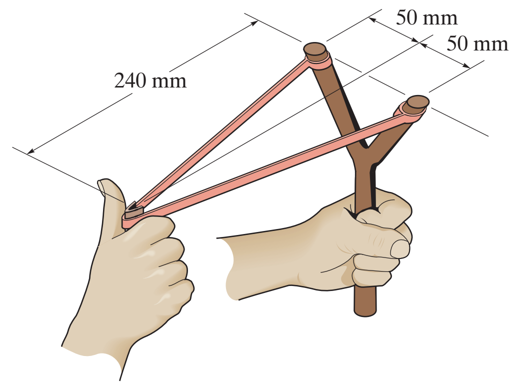
14–69. Each of the two elastic rubber bands of the slingshot has an unstretched length of 180 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 30-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 80$ N/m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left(\frac{1}{2}\right)(80)[\sqrt{(0.05)^2 + (0.240)^2} - 0.18]^2 = 0 + 0.030(9.81)h$$

$$h = 1.154 \text{ m} = 1154 \text{ mm}$$ **Ans.**

Ans:
$h = 1154$ mm

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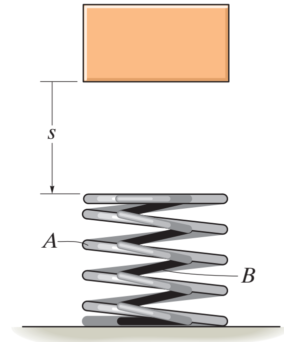
14–70.

Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2(9.81)(0.5 + 0.2) + \frac{1}{2}(400)(0.2)^2 + \frac{1}{2}(k_B)(0.2)^2$$

$$k_B = 287 \text{ N/m}$$ **Ans.**

Ans:
$k_B = 287$ N/m

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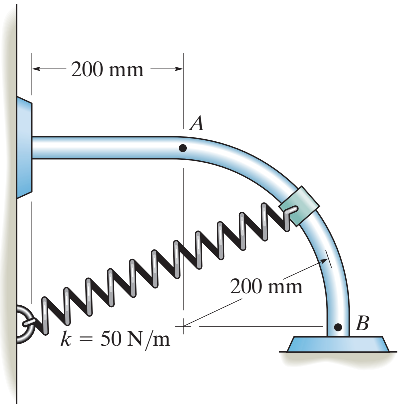
14–71.

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels down along the smooth guide. Determine the speed of the collar when it reaches point B, which is located just before the end of the curved portion of the rod. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81 \text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828 \text{ m}$ and $x_B = 0.4 - 0.1 = 0.3 \text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25 \text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

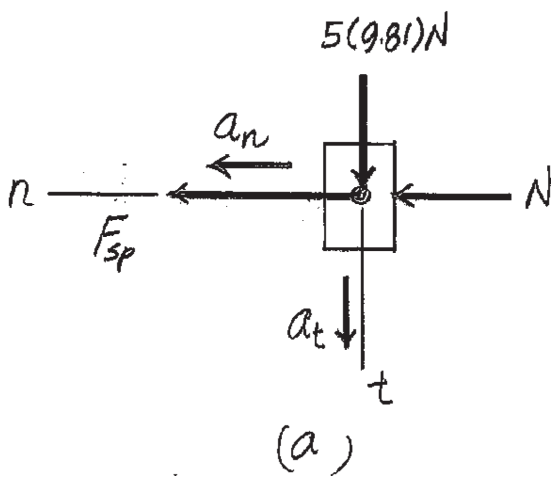
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325 \text{ m/s} = 5.33 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15 \text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95 \text{ N} = 694 \text{ N} \qquad \textbf{Ans.}$$



(a)

Ans:
$v_B = 5.33$ m/s
$N = 694$ N

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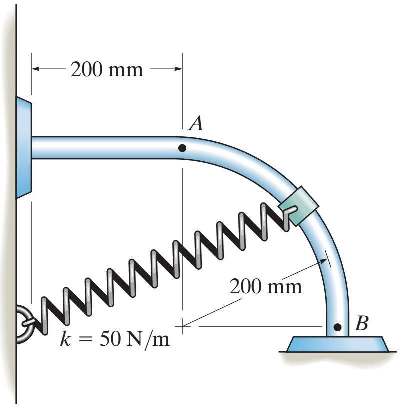
***14–72.**

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels along the smooth guide. Determine its speed when its center reaches point B and the normal force it exerts on the rod at this point. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81 \text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828 \text{ m}$ and $x_B = 0.4 - 0.1 = 0.3 \text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25 \text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

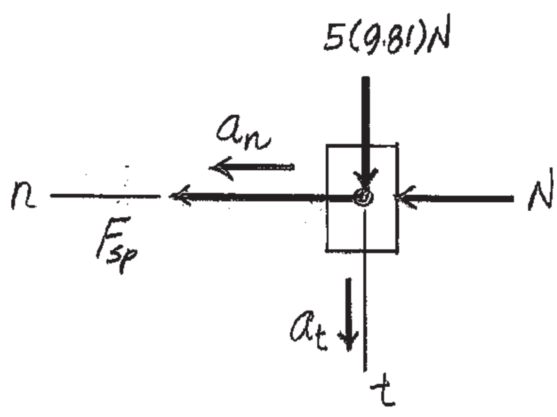
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325 \text{ m/s} = 5.33 \text{ m/s}$$ **Ans.**

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15 \text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95 \text{ N} = 694 \text{ N}$$ **Ans.**



(a)

Ans:
$v_B = 5.33 \text{ m/s}$
$N = 694 \text{ N}$

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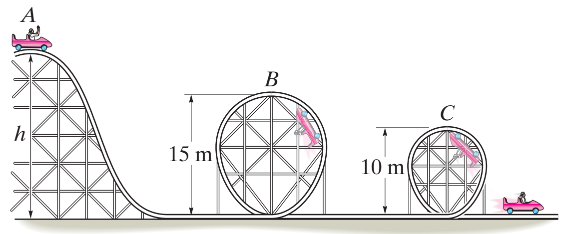
14–73.

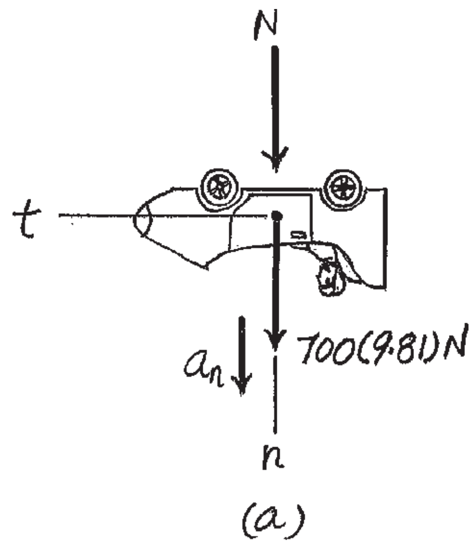
The roller coaster car has a mass of 700 kg, including its passenger. If it starts from the top of the hill A with a speed $v_A = 3\text{ m/s}$, determine the minimum height h of the hill crest so that the car travels around the inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5\text{ m}$ and $\rho_C = 5\text{ m}$.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. *a*,

$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$



When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5\ m$ and $\rho_C = 5\ m$. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575\text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05\text{ m}^2/\text{s}^2$$

Judging from the above results, the coster car will not leave the loop at C if it safely passes through B. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. With $v_B^2 = 73.575\text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(700)(3^2) + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.29\text{ m} = 18.3\text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_c^2 + 700(9.81)(10)$$

$$v_c^2 = 171.675\text{ m}^2/\text{s}^2 > 49.05\text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5\text{ m}$,

$$N_c + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17(10^3)\text{ N} = 17.2\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 0$
$h = 18.3\text{ m}$
$N_c = 17.2\text{ kN}$

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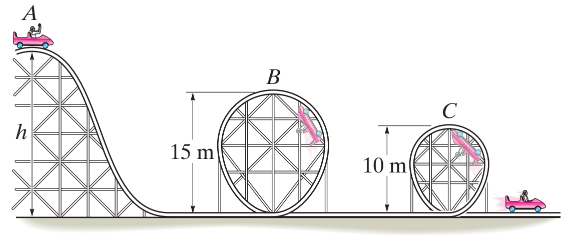
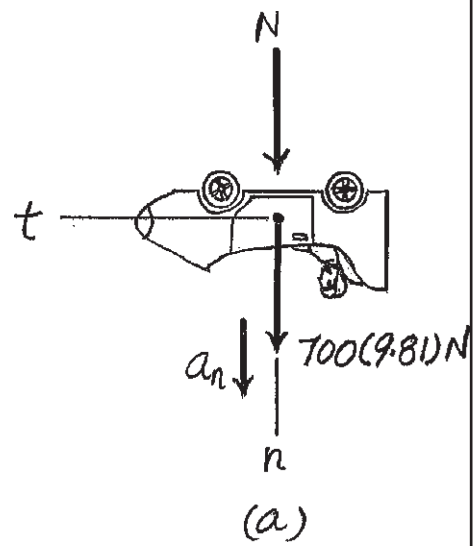
14–74.

The roller coaster car has a mass of 700 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside the loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.



SOLUTION



Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5$ m and $\rho_C = 5$ m. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05 \text{ m}^2/\text{s}^2$$

Judging from the above result the coaster car will not leave the loop at C provided it passes through B safely. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. Applying Eq. 14– from A to B with $v_B^2 = 73.575 \text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$0 + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.75 \text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_C^2 + 700(9.81)(10)$$

$$v_C^2 = 171.675 \text{ m}^2/\text{s}^2 > 49.05 \text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5$ m,

$$N_C + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17\left(10^3\right)N = 17.2 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 0$
$h = 18.75$ m
$N_C = 17.2$ kN

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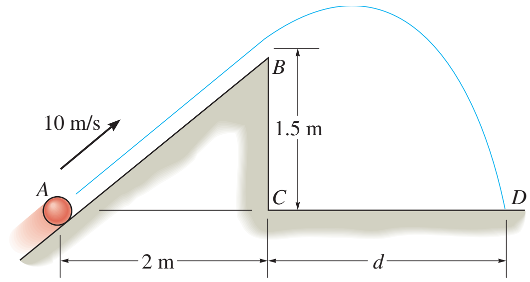
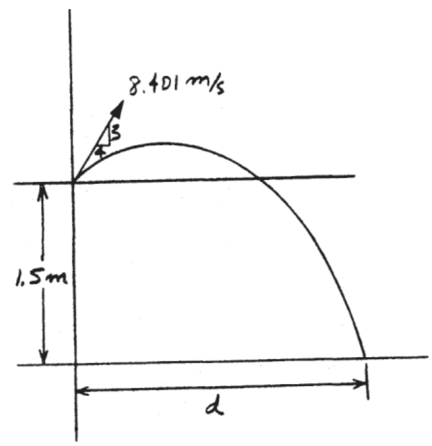
14–75.

The 2-kg ball of negligible size is fired from point A with an initial velocity of 10 m/s up the smooth inclined plane. Determine the distance from point C to where it hits the horizontal surface at D. Also, what is its velocity when it strikes the surface?



SOLUTION



Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_B)^2 + 2(9.81)(1.5)$$

$$v_B = 8.401 \text{ m/s}$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$d = 0 + 8.401\left(\frac{4}{5}\right)t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-1.5 = 0 + 8.401\left(\frac{3}{5}\right)t + \frac{1}{2}(-9.81)t^2$$

$$-4.905t^2 + 5.040t + 1.5 = 0$$

Solving for the positive root,

$$t = 1.269 \text{ s}$$

$$d = 8.401\left(\frac{4}{5}\right)(1.269) = 8.53 \text{ m} \qquad \textbf{Ans.}$$

Datum at A:

$$T_A + V_A = T_D + V_D$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_D)^2 + 0$$

$$v_D = 10 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$d = 8.53$ m
$v_D = 10$ m/s

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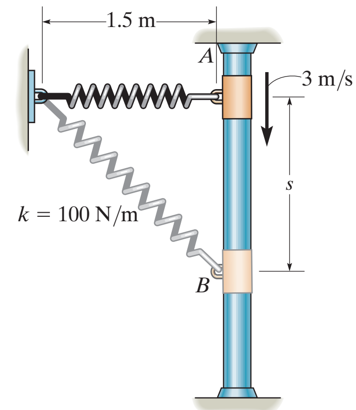
***14–76.**

The 4-kg smooth collar has a speed of 3 m/s when it is at $s = 0$. Determine the maximum distance s it travels before it stops momentarily. The spring has an unstretched length of 1 m.



SOLUTION

Potential Energy. With reference to the datum set through A the gravitational potential energies of the collar at A and B are

$$(V_g)_A = 0 \qquad (V_g)_B = -mgh_B = -4(9.81)\,S_{max} = -39.24\,S_{max}$$

At A and B, the spring stretches $x_A = 1.5 - 1 = 0.5$ m and $x_B = \sqrt{S_{max}^2 + 1.5^2} - 1$. Thus, the elastic potential Energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(100)\left(0.5^2\right) = 12.5\text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(100)\left(\sqrt{S_{max}^2 + 1.5^2} - 1\right)^2 = 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

Conservation of Energy. Since the collar is required to stop momentarily at B, $T_B = 0$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(4)\left(3^2\right) + 0 + 12.5 = 0 + (-39.24\,S_{max}) + 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

$$50\,S_{max}^2 - 100\sqrt{S_{max}^2 + 1.5^2} - 39.24\,S_{max} + 132 = 0$$

Solving numerically,

$$S_{max} = 1.9554\text{ m} = 1.96\text{ m} \qquad \textbf{Ans.}$$

Ans:
$S_{max} = 1.96$ m

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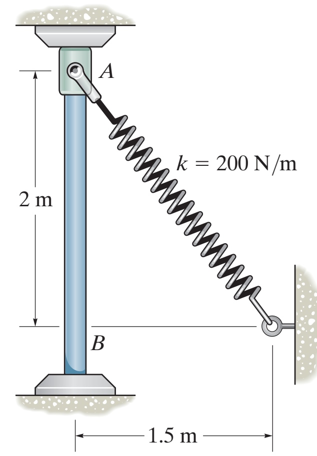
14–77.

The spring has a stiffness $k = 200\ \text{N/m}$ and an unstretched length of 0.5 m. If it is attached to the 3-kg smooth collar and the collar is released from rest at A, determine the speed of the collar when it reaches B. Neglect the size of the collar.



SOLUTION

Potential Energy. With reference to the datum set through B, the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 3(9.81)(2) = 58.86\ \text{J}$$
$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{1.5^2 + 2^2} - 0.5 = 2.00\ \text{m}$ and $x_B = 1.5 - 0.5 = 1.00\ \text{m}$. Thus, the elastic potential energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(200)\left(2.00^2\right) = 400\ \text{J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(200)\left(1.00^2\right) = 100\ \text{J}$$

Conservation of Energy. Since the collar is released from rest at A, $T_A = 0$.

$$T_A + V_A = T_B + V_B$$

$$0 + 58.86 + 400 = \frac{1}{2}(3)v_B^2 + 0 + 100$$

$$v_B = 15.47\ \text{m/s} = 15.5\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 15.5\ \text{m/s}$

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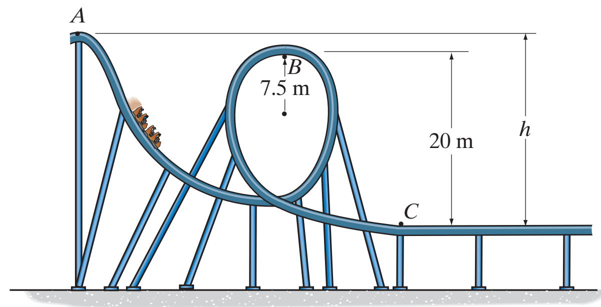
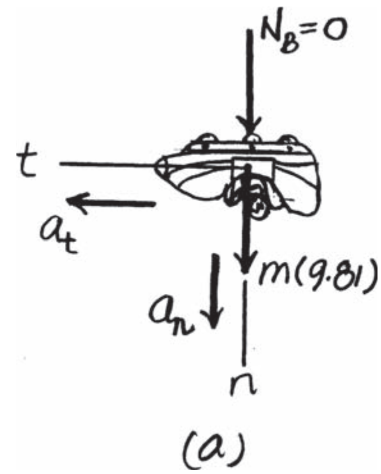
14–78.

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



SOLUTION

Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \dfrac{v_B^2}{\rho_B} = \dfrac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $\left(V_g\right)_A = mgh_A = m(9.81)h = 9.81mh$, $\left(V_g\right)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $\left(V_g\right)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + \left(V_g\right)_A = \frac{1}{2}mv_B^2 + \left(V_g\right)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m} \qquad \textbf{Ans.}$$

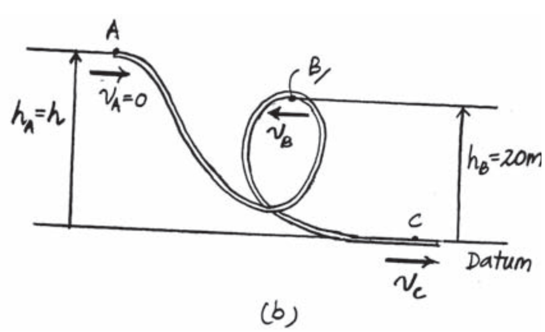
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + \left(V_g\right)_B = \frac{1}{2}mv_C^2 + \left(V_g\right)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

$$v_C = 21.6 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$h = 23.75$ m
$v_C = 21.6$ m/s

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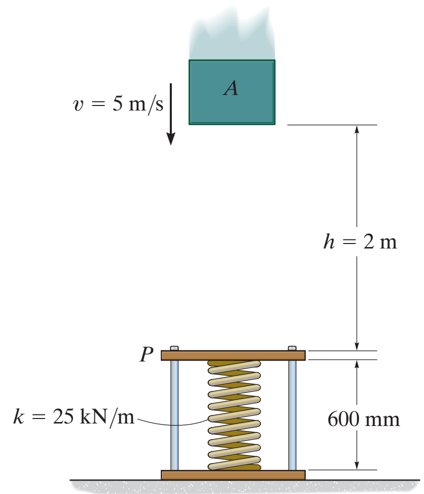
14–79.

A 750-mm-long spring is compressed and confined by the plate P, which can slide freely along the vertical 600-mm-long rods. The 40-kg block is given a speed of $v = 5$ m/s when it is $h = 2$ m above the plate. Determine how far the plate moves downwards when the block momentarily stops after striking it. Neglect the mass of the plate.

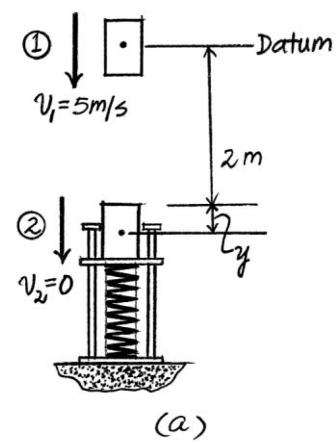


SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the block at positions (1) and (2) are $(V_g)_1 = mgh_1 = 40(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 40(9.81)[-(2+y)] = [-392.4(2+y)]$, respectively. The compression of the spring when the block is at positions (1) and (2) are $s_1 = (0.75 - 0.6) = 0.15$ m and $s_2 = s_1 + y = (0.15 + y)$ m. Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(25)(10^3)(0.15^2) = 281.25 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(25)(10^3)(0.15 + y)^2$$



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + \left[(V_g)_1 + (V_e)_1\right] = \frac{1}{2}mv_2^2 + \left[(V_g)_2 + (V_e)_2\right]$$

$$\frac{1}{2}(40)(5^2) + (0 + 281.25) = 0 + [-392.4(2 + y)] + \frac{1}{2}(25)(10^3)(0.15 + y)^2$$

$$12500y^2 + 3357.6y - 1284.8 = 0$$

Solving for the positive root of the above equation,

$$y = 0.2133 \text{ m} = 213 \text{ mm}$$ **Ans.**

Ans:
$y = 213$ mm

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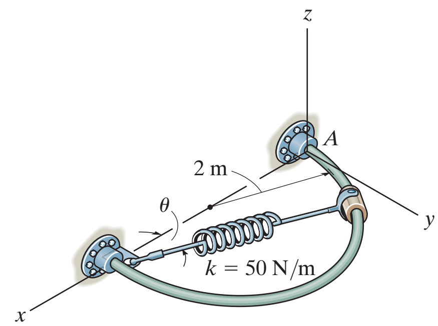
***14–80.**

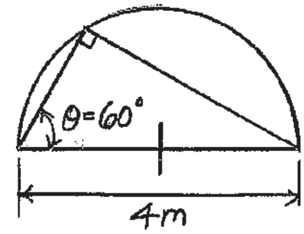
The spring has a stiffness $k = 50$ N/m and an unstretched length of 0.3 m. If it is attached to the 2-kg smooth collar and the collar is released from rest at A ($\theta = 0°$), determine the speed of the collar when $\theta = 60°$. The motion occurs in the horizontal plane. Neglect the size of the collar.



SOLUTION

Potential Energy. Since the motion occurs in the horizontal plane, there will be no change in gravitational potential energy when $\theta = 0°$, the spring stretches $x_1 = 4 - 0.3 = 3.7$ m. Referring to the geometry shown in Fig. *a*, the spring stretches $x_2 = 4\cos 60° - 0.3 = 1.7$ m. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(50)(3.7^2) = 342.25 \text{ J}$$



(a)

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(50)(1.7^2) = 72.25 \text{ J}$$

Conservation of Energy. Since the collar is released from rest when $\theta = 0°$, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 342.25 = \frac{1}{2}(2)v^2 + 72.25$$

$$v = 16.43 \text{ m/s} = 16.4 \text{ m/s}$$ **Ans.**

Ans:
$v = 16.4$ m/s

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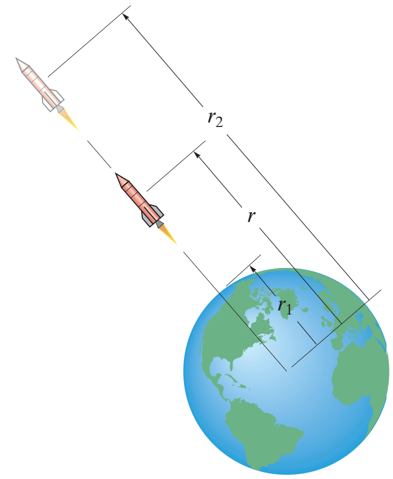
14–81.

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_em/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_em/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.



SOLUTION

The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m\int_{r1}^{r2}\frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \to \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

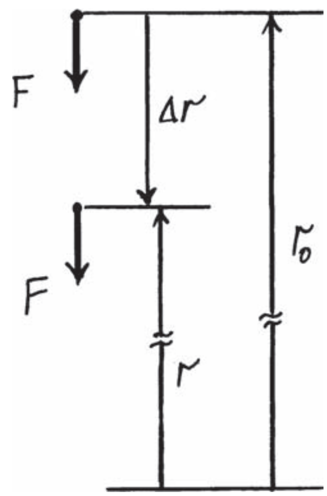
$$V_g \to \frac{-G\,M_e\,m}{r}$$

To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

$$= \frac{-G\,M_e\,m}{r^2}$$

Q.E.D.



Ans:

$$F = \frac{-G\,M_e\,m}{r^2}$$

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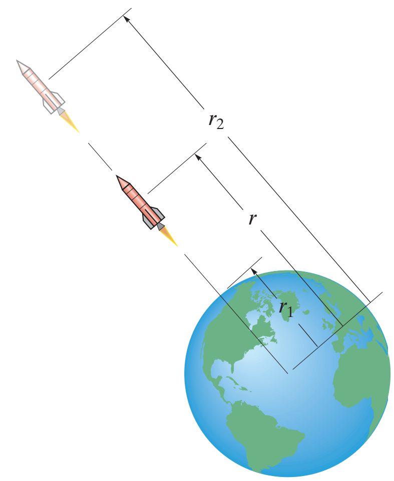
14–82.

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



SOLUTION

$$F = G\frac{M_e m}{r^2}$$

$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$ **Ans.**

Ans:
$F = GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

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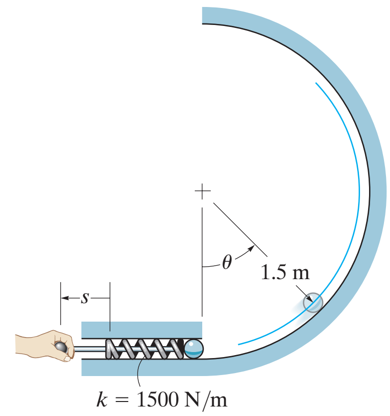
14–83.

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg ball and the normal reaction of the circular track on the ball when $\theta = 60°$. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Potential Energy. With reference to the datum set through the center of the circular track, the gravitational potential energies of the ball when $\theta = 0°$ and $\theta = 60°$ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = -mgh_2 = -0.3(9.81)(1.5 \cos 60°) = -2.20725 \text{ J}$$

When $\theta = 0°$, the spring compress $x_1 = 0.1$ m and is unstretched when $\theta = 60°$. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)\left(0.1^2\right) = 7.50 \text{ J}$$

$$(V_e)^2 = 0$$

Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

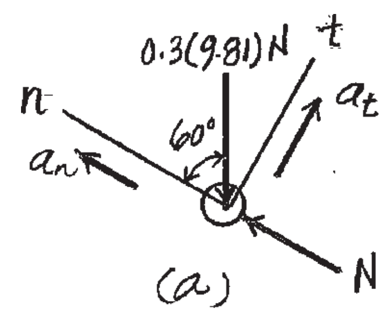
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + (-2.20725) + 0$$

$$v^2 = 35.285 \text{ m}^2/\text{s}^2$$

$$v = 5.94 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion. Referring to the FBD of the ball, Fig. *a*,



$$\Sigma F_n = ma_n; \qquad N - 0.3(9.81)\cos 60° = 0.3\left(\frac{35.285}{1.5}\right)$$

$$N = 8.5285 \text{ N} = 8.53 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$v = 5.94$ m/s
$N = 8.53$ N

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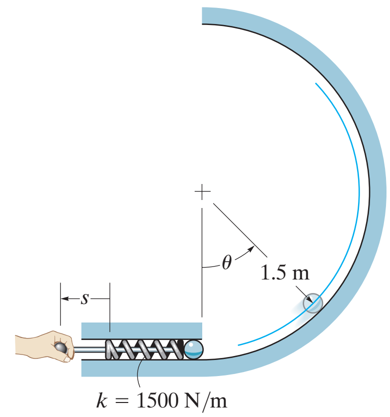
***14–84.**

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the maximum angle θ the ball will travel without leaving the circular track. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Equation of Motion. It is required that the ball leaves the track, and this will occur provided $\theta > 90°$. When this happens, $N = 0$. Referring to the FBD of the ball, Fig. *a*

$$\Sigma F_n = ma_n; \qquad 0.3(9.81)\sin(\theta - 90°) = 0.3\left(\frac{v^2}{1.5}\right)$$

$$v^2 = 14.715 \sin(\theta - 90°) \qquad \textbf{(1)}$$

Potential Energy. With reference to the datum set through the center of the circular track Fig. *b*, the gravitational potential Energies of the ball when $\theta = 0°$ and θ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = mgh_2 = 0.3(9.81)[1.5 \sin(\theta - 90°)]$$

$$= 4.4145 \sin(\theta - 90°)$$

When $\theta = 0°$, the spring compresses $x_1 = 0.1$ m and is unstretched when the ball is at θ for max height. Thus, the elastic potential energies in the spring when $\theta = 0°$ and θ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)(0.1^2) = 7.50 \text{ J}$$

$$(V_e)_2 = 0$$

Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + 4.4145 \sin(\theta - 90°) + 0$$

$$v^2 = 20.57 - 29.43 \sin(\theta - 90°) \qquad \textbf{(2)}$$

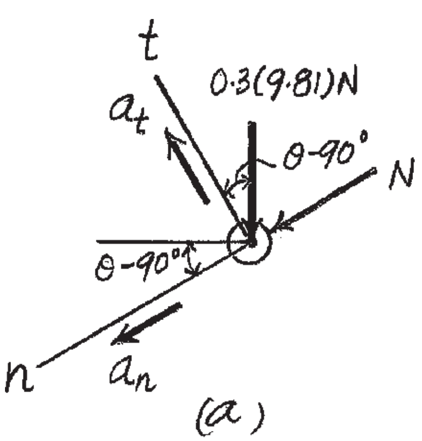
Equating Eqs. (1) and (2),

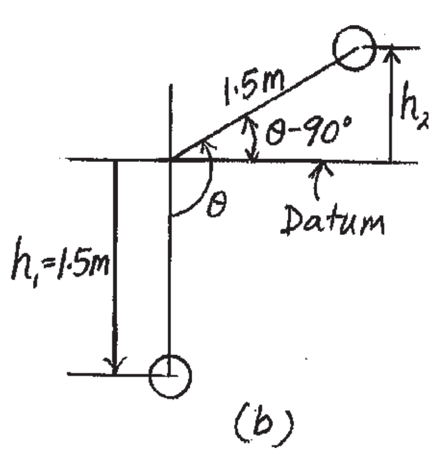
$$14.715 \sin(\theta - 90°) = 20.57 - 29.43 \sin(\theta - 90°)$$

$$\sin(\theta - 90°) = 0.4660$$

$$\theta - 90° = 27.77°$$

$$\theta = 117.77° = 118° \qquad \textbf{Ans.}$$





Ans:
$\theta = 118°$

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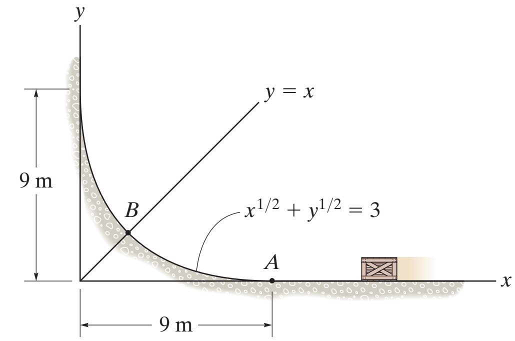
14–85.

When the 5-kg box reaches point A it has a speed $v_A = 10\ \text{m/s}$. Determine the normal force the box exerts on the surface when it reaches point B. Neglect friction and the size of the box.



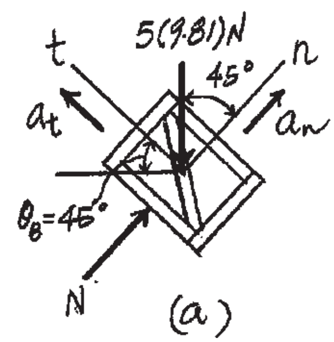
SOLUTION

Conservation of Energy. At point B, $y = x$

$$x^{\frac{1}{2}} + x^{\frac{1}{2}} = 3$$

$$x = \frac{9}{4}\ \text{m}$$

Then $y = \dfrac{9}{4}$ m. With reference to the datum set to coincide with the x axis, the gravitational potential energies of the box at points A and B are



$$(V_g)_A = 0 \qquad (V_g)_B = mgh_B = 5(9.81)\left(\frac{9}{4}\right) = 110.3625\ \text{J}$$

Applying the energy equation,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(5)\left(10^2\right) + 0 = \frac{1}{2}(5)v_B^2 + 110.3625$$

$$v_B^2 = 55.855\ \text{m}^2/\text{s}^2$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{-\frac{1}{2}}\right) = \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{-\frac{3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$. At point B, $x = \dfrac{9}{4}$ m. Thus,

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_{x=\frac{9}{4}\ \text{m}} = 1 - \frac{3}{\left(\frac{9}{4}\right)^{\frac{1}{2}}} = -1 \qquad \theta_B = -45° = 45°$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{9}{4}\ \text{m}} = \frac{3}{2\left(\frac{9}{4}\right)^{\frac{3}{2}}} = 0.4444$$

The radius of curvature at B is

$$P_B = \frac{\left[1 + (dy/dx)^2\right]^{\frac{3}{2}}}{\left|d^2y/dx^2\right|} = \frac{\left[1 + (-1)^2\right]^{\frac{3}{2}}}{0.4444} = 6.3640\ \text{m}$$

Referring to the FBD of the box, Fig. *a*

$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 45° = 5\left(\frac{55.855}{6.3640}\right)$$

$$N = 78.57\ \text{N} = 78.6\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$N = 78.6\ \text{N}$

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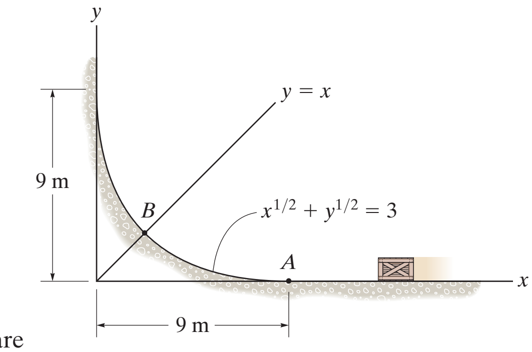
14–86.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine how high the box reaches up the surface before it comes to a stop. Also, what is the resultant normal force on the surface at this point and the acceleration? Neglect friction and the size of the box.



SOLUTION

Conservation of Energy. With reference to the datum set coincide with x axis, the gravitational potential energy of the box at A and C (at maximum height) are

$$(V_g)_A = 0 \qquad (V_g)_C = mgh_c = 5(9.81)(y) = 49.05y$$

It is required that the box stop at C. Thus, $T_c = 0$

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}(5)\left(10^2\right) + 0 = 0 + 49.05y$$

$$y = 5.0968\text{ m} = 5.10\text{ m} \qquad \textbf{Ans.}$$

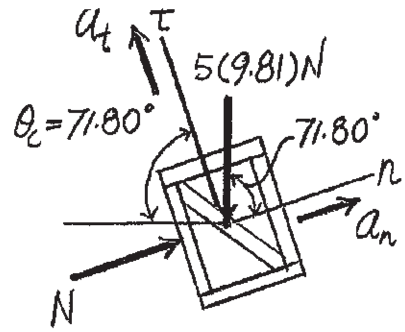
Then,

$$x^{\frac{1}{2}} + 5.0968^{\frac{1}{2}} = 3 \qquad x = 0.5511\text{ m}$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{\frac{1}{2}}\right)$ $= \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{\frac{-3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$. At point C, $x = 0.5511$ m.

Thus

$$\tan\theta_c = \left.\frac{dy}{dx}\right|_{x=0.5511\text{ m}} = 1 - \frac{3}{0.5511^{\frac{1}{2}}} = -3.0410 \qquad \theta_C = -71.80° = 71.80°$$



(a)

Referring to the FBD of the box, Fig. a,

$$\Sigma F_n = ma_n\,; \qquad N - 5(9.81)\cos 71.80° = 5\left(\frac{0^2}{\rho_C}\right)$$

$$N = 15.32\text{ N} = 15.3\text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_t = ma_t\,; \qquad -5(9.81)\sin 71.80° = 5a_t$$

$$a_t = -9.3191\text{ m/s}^2 = 9.32\text{ m/s}^2 \searrow$$

Since $a_n = 0$, Then

$$a = a_t = 9.32\text{ m/s}^2 \searrow \qquad \textbf{Ans.}$$

Ans:
$y = 5.10$ m
$N = 15.3$ N
$a = 9.32\text{ m/s}^2 \searrow$

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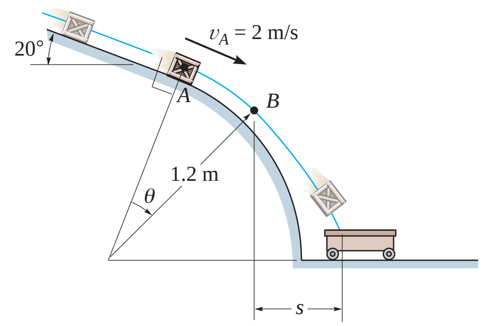
14–87.

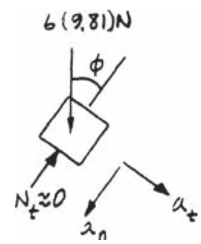
When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



SOLUTION

At point B:

$$+\swarrow \Sigma F_n = ma_n; \qquad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \qquad \textbf{(1)}$$



Datum at bottom of curve:

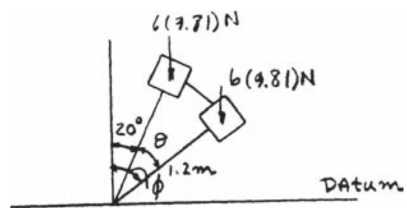
$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20^\circ) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

$$13.062 = 0.5v_B^2 + 11.772\cos\phi \qquad \textbf{(2)}$$

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$$v_B = 2.951 \text{ m/s}$$



Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29^\circ$

$$\theta = \phi - 20^\circ = 22.3^\circ \qquad \textbf{Ans.}$$

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-1.2\cos 42.29^\circ = 0 - 2.951(\sin 42.29^\circ)t + \frac{1}{2}(-9.81)t^2$$

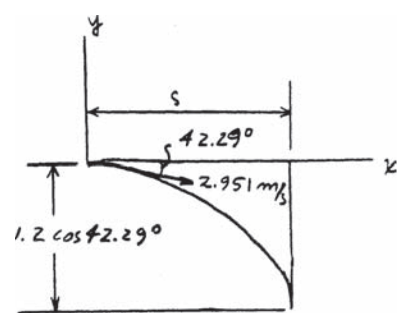
$$4.905t^2 + 1.9857t - 0.8877 = 0$$

Solving for the positive root: $t = 0.2687$ s

$(\xrightarrow{+}) \qquad s = s_0 + v_0 t$

$$s = 0 + (2.951\cos 42.29^\circ)(0.2687)$$

$$s = 0.587 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$\theta = 22.3^\circ$
$s = 0.587$ m

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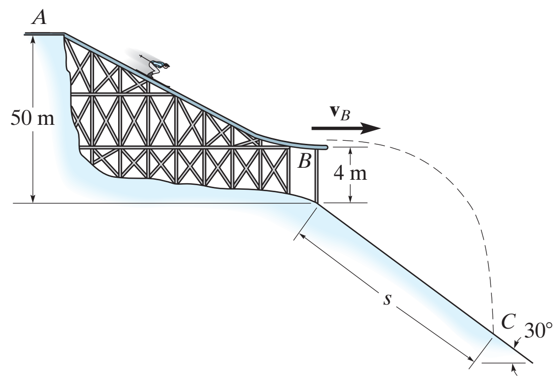
***14–88.**

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



SOLUTION

$$T_A + V_A = T_B + V_B$$

$$0 + 70(9.81)\,(46) = \frac{1}{2}(70)v^2 + 0$$

$v = 30.04\text{ m/s} = 30.0\text{ m/s}$ **Ans.**

$$(+\downarrow)\; s_y = (s_y)_0 + (v_0)_y t + \frac{1}{2}a_c t^2$$

$$4 + s\sin 30^\circ = 0 + 0 + \frac{1}{2}(9.81)t^2 \qquad \textbf{(1)}$$

$$(\overset{+}{\leftarrow})\, s_x = v_x\, t$$

$$s\cos 30^\circ = 30.04t \qquad \textbf{(2)}$$

$s = 130\text{ m}$ **Ans.**

$t = 3.75\text{ s}$

Ans:
$v = 30.0\text{ m/s}$
$s = 130\text{ m}$

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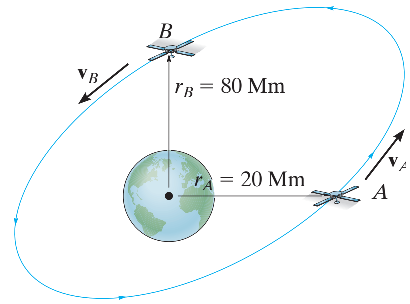
14–89.

A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–81, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.



SOLUTION

$$v_A = 40\ \text{Mm/h} = 11\,111.1\ \text{m/s}$$

$$\text{Since } V = -\frac{GM_e m}{r}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$$v_B = 9672\ \text{m/s} = 34.8\ \text{Mm/h}$$ **Ans.**

Ans:
$v_B = 34.8$ Mm/h

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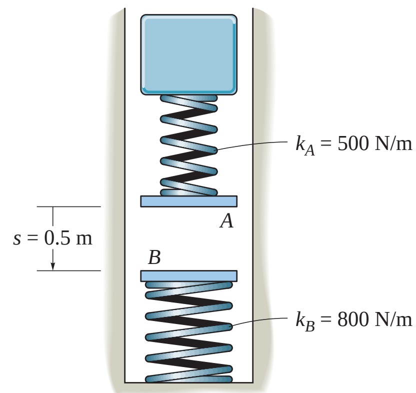
14–90.

The block has a mass of 20 kg and is released from rest when $s = 0.5$ m. If the mass of the bumpers A and B can be neglected, determine the maximum deformation of each spring due to the collision.



SOLUTION

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \frac{1}{2}(500)s_A^2 + \frac{1}{2}(800)s_B^2 + 20(9.81)\left[-(s_A + s_B) - 0.5\right] \qquad \textbf{(1)}$$

Also, $F_s = 500s_A = 800s_B \qquad s_A = 1.6s_B$ **(2)**

Solving Eqs. (1) and (2) yields:

$s_B = 0.638$ m **Ans.**

$s_A = 1.02$ m **Ans.**

Ans:
$s_B = 0.638$ m
$s_A = 1.02$ m

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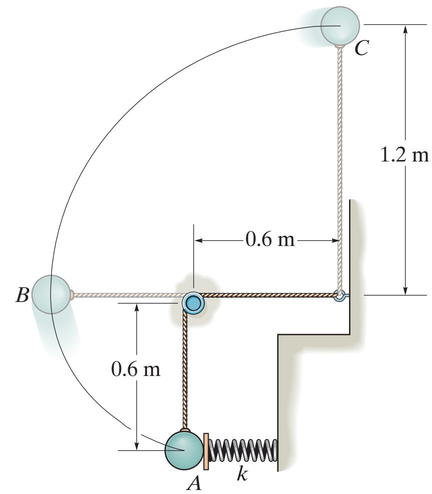
14–91. The 0.75-kg bob of a pendulum is fired from rest at position A by a spring which has a stiffness $k = 6$ kN/m and is compressed 125 mm. Determine the speed of the bob and the tension in the cord when the bob is at positions B and C. Point B is located on the path where the radius of curvature is still 0.6 m, i.e., just before the cord becomes horizontal.



SOLUTION

Given:

$M = 0.75$ kg

$k = 6$ kN/m

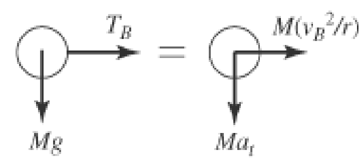
$\delta = 125$ mm

$r = 0.6$ m

At B:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}Mv_B^2 + Mgr$$

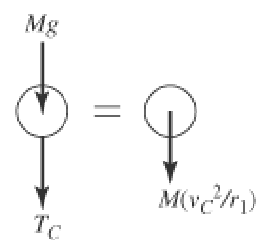
$$v_B = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 2gr} \qquad v_B = 10.6 \text{ m/s} \quad \textbf{Ans.}$$



$$T_B = M\left(\frac{v_B^2}{r}\right) \qquad T_B = 142 \text{ N} \quad \textbf{Ans.}$$

At C:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}Mv_C^2 + Mg3r$$



$$v_C = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 6gr} \qquad v_C = 9.47 \text{ m/s} \quad \textbf{Ans.}$$

$$T_C + Mg = M\left(\frac{v_C^2}{2r}\right)$$

$$T_C = M\left(\frac{v_C^2}{2r} - g\right) \qquad T_C = 48.7 \text{ N} \quad \textbf{Ans.}$$

Ans:
$v_B = 10.6$ m/s, $T_B = 142$ N
$v_C = 9.47$ m/s, $T_C = 48.7$ N

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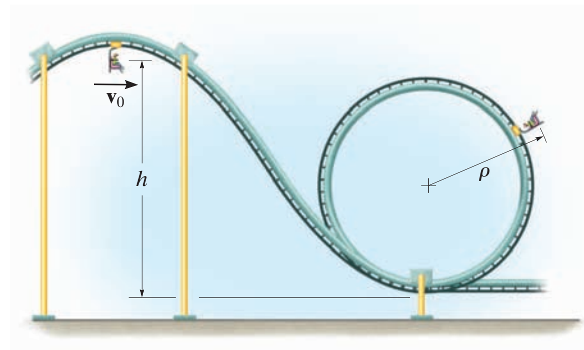
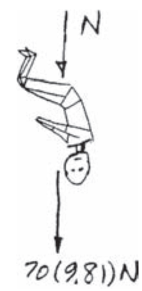
***14–92.**

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at $v_0 = 4$ m/s when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the 70-kg passenger on his seat at this instant. The car has a mass of 50 kg. Take $h = 12$ m, $\rho = 5$ m. Neglect friction and the size of the car and passenger.



SOLUTION



Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(120)(4)^2 + 120(9.81)(12) = \frac{1}{2}(120)(v_1)^2 + 120(9.81)(10)$$

$v_1 = 7.432$ m/s **Ans.**

$$+\downarrow \Sigma F_n = ma_n; \qquad 70(9.81) + N = 70\left(\frac{(7.432)^2}{5}\right)$$

$N = 86.7$ N **Ans.**

Ans:
$v_1 = 7.432$ m/s
$N = 86.7$ N

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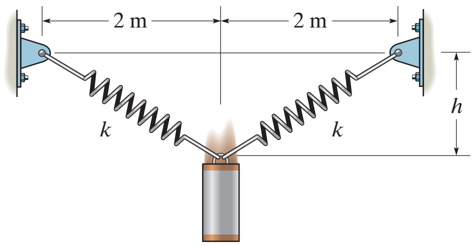
14–93.

If the 20-kg cylinder is released from rest at $h = 0$, determine the required stiffness k of each spring so that its motion is arrested or stops when $h = 0.5$ m. Each spring has an unstretched length of 1 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k(2-1)^2\right] = 0 - 20(9.81)(0.5) + 2\left[\frac{1}{2}k\left(\sqrt{(2)^2 + (0.5)^2} - 1\right)^2\right]$$

$$k = -98.1 + 1.12689\,k$$

$$k = 773 \text{ N/m}$$ **Ans.**

Ans:
$k = 773$ N/m

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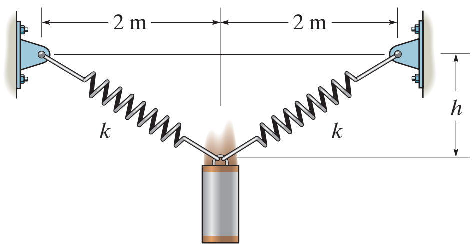
14–94.

The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. Each spring has a stiffness $k = 40$ N/m and an unstretched length of 2 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + 2\left[\frac{1}{2}(40)\left(\sqrt{3^2 + 2^2} - 2^2\right)\right] - 20(9.81)(3) + \frac{1}{2}(20)v^2$$

$$v = 6.97 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 6.97$ m/s

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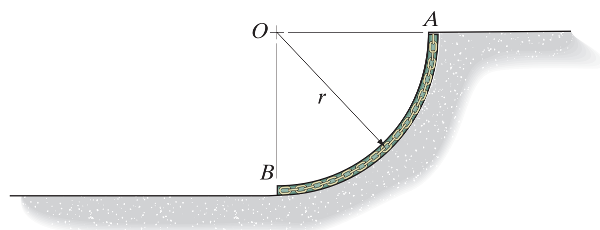
14–95.

A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



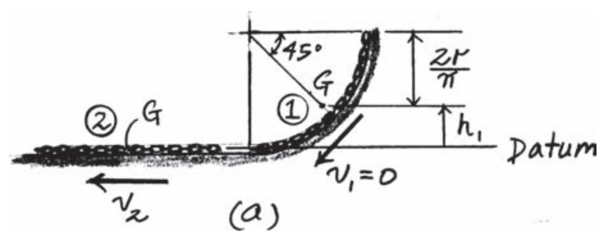
SOLUTION

Potential Energy: The location of the center of gravity G of the chain at positions (1) and (2) are shown in Fig. *a*. The mass of the chain is $m = m_0\left(\frac{\pi}{2}r\right) = \frac{\pi}{2}m_0 r$. Thus, the center of mass is at $h_1 = r - \frac{2r}{\pi} = \left(\frac{\pi - 2}{\pi}\right)r$. With reference to the datum set in Fig. *a* the gravitational potential energy of the chain at positions (1) and (2) are

$$\left(V_g\right)_1 = mgh_1 = \left(\frac{\pi}{2}m_0 rg\right)\left(\frac{\pi - 2}{\pi}\right)r = \left(\frac{\pi - 2}{2}\right)m_0 r^2 g$$

and

$$\left(V_g\right)_2 = mgh_2 = 0$$



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^{\,2} + \left(V_g\right)_1 = \frac{1}{2}mv_2^{\,2} + \left(V_g\right)_2$$

$$0 + \left(\frac{\pi - 2}{2}\right)m_0 r^2 g = \frac{1}{2}\left(\frac{\pi}{2}m_0 r\right)v_2^{\,2} + 0$$

$$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr} \qquad \textbf{Ans.}$$

Ans:

$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr}$

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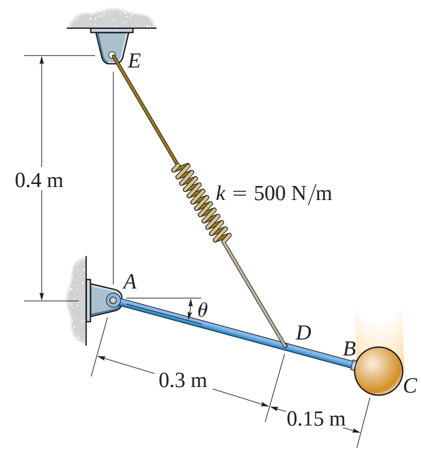
***14–96.**

The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



SOLUTION

Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the sphere at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0.45) = 44.145$ J and $\left(V_g\right)_2 = mgh_2 = 10(9.81)(0) = 0$. When the sphere is at position (1), the spring stretches $s_1 = \dfrac{100}{500} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = \sqrt{0.3^2 + 0.4^2} - 0.2 = 0.3$ m, and the elastic potential energy of the spring is $\left(V_e\right)_1 = \frac{1}{2}ks_1^{\,2} = \frac{1}{2}(500)(0.2^2) = 10$ J. When the sphere is at position (2), the spring stretches $s_2 = 0.7 - 0.3 = 0.4$ m, and the elastic potential energy of the spring is $\left(V_e\right)_2 = \frac{1}{2}ks_2^{\,2} = \frac{1}{2}(500)(0.4^2) = 40$ J.

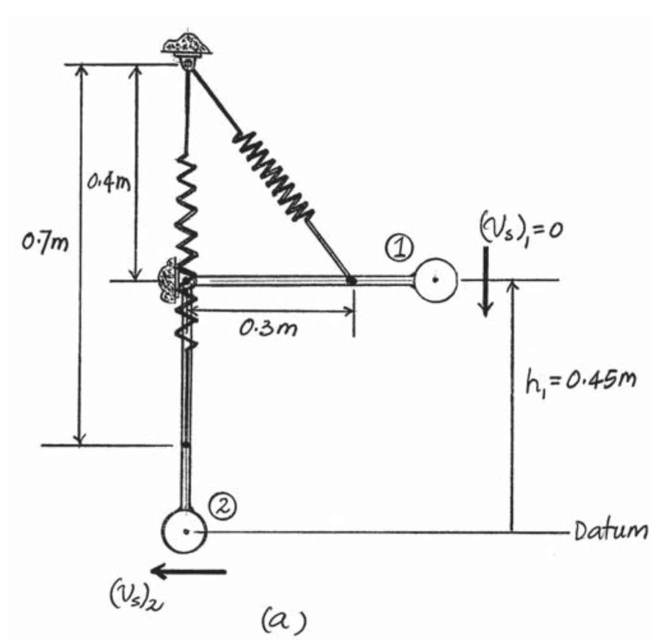
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_s\left(v_s\right)_1^{\,2} + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_s\left(v_s\right)_2^{\,2} + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (44.145 + 10) = \frac{1}{2}(10)(v_s)_2^{\,2} + (0 + 40)$$

$$(v_s)_2 = 1.68 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 1.68$ m/s

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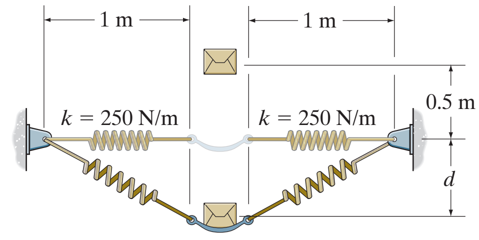
14–97.

A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the box at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)[-(0.5 + d)] = -98.1(0.5 + d)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $(V_e)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$(V_e)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2 + 1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

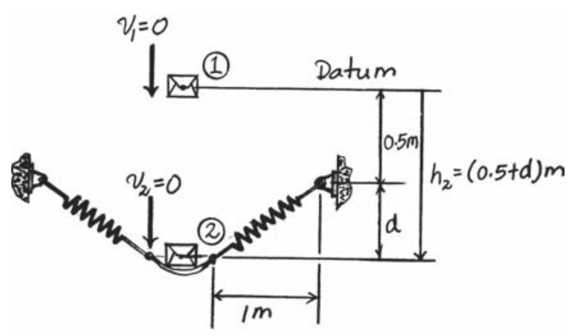
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (0 + 10) = 0 + \left[-98.1(0.5 + d) + 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2 + 1} + 350.95 = 0$$

Solving the above equation by trial and error,

$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$d = 1.34$ m

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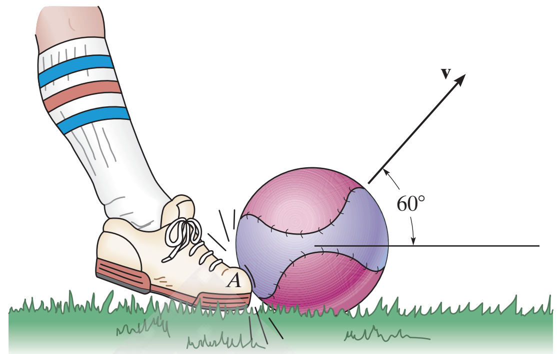
15–1.

A man kicks the 150-g ball such that it leaves the ground at an angle of 60° and strikes the ground at the same elevation a distance of 12 m away. Determine the impulse of his foot on the ball at A. Neglect the impulse caused by the ball's weight while it's being kicked.



SOLUTION

Kinematics. Consider the vertical motion of the ball where

$(s_0)_y = s_y = 0$, $(v_0)_y = v \sin 60° \uparrow$ and $a_y = 9.81\ \text{m/s}^2 \downarrow$,

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2; \quad 0 = 0 + v \sin 60° t + \frac{1}{2}(-9.81)t^2$$

$$t(v \sin 60° - 4.905t) = 0$$

Since $t \neq 0$, then

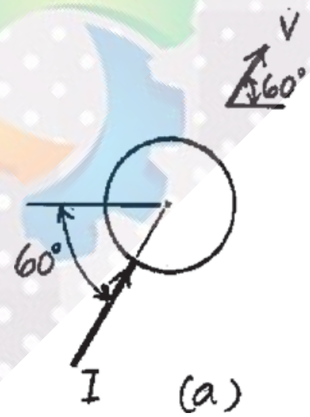
$$v \sin 60° - 4.905t = 0$$

$$t = 0.1766\, v \qquad \textbf{(1)}$$

Then, consider the horizontal motion where $(v_0)_x = v \cos 60°$, and $(s_0)_x = 0$,

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t; \quad 12 = 0 + v \cos 60° t$$

$$t = \frac{24}{v} \qquad \textbf{(2)}$$



Equating Eqs. (1) and (2)

$$0.1766\, v = \frac{24}{v}$$

$$v = 11.66\ \text{m/s}$$

Principle of Impulse and Momentum.

$$(+\nearrow) \quad mv_1 + \Sigma\int_{t_1}^{t_2} F\,dt = mv_2$$

$$0 + I = 0.15\,(11.66)$$

$$I = 1.749\ \text{N}\cdot\text{s} = 1.75\ \text{N}\cdot\text{s} \qquad \textbf{Ans.}$$

Ans:
$I = 1.75\ \text{N}\cdot\text{s}$

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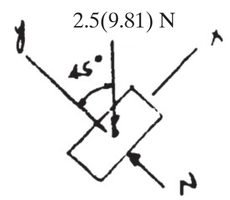
15–2. A 2.5-kg block is given an initial velocity of 3 m/s up a 45° smooth slope. Determine the time for it to travel up the slope before it stops.



SOLUTION

$$(\nearrow +) \qquad m(v_{x'})_1 + \Sigma \int_{t_1}^{-t_2} F_x\, dt = m(v_{x'})_2$$

$$2.5(3) + (-2.5(9.81)\sin 45°)t = 0$$

$$t = 0.432 \text{ s}$$ **Ans.**

Ans:
$t = 0.432$ s

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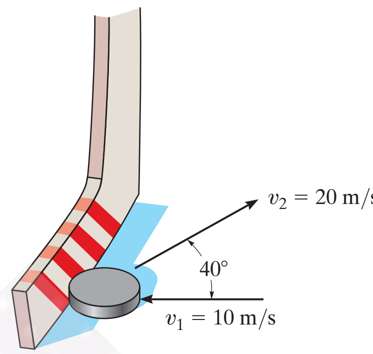
15–3.

A hockey puck is traveling to the left with a velocity of $v_1 = 10\ \text{m/s}$ when it is struck by a hockey stick and given a velocity of $v_2 = 20\ \text{m/s}$ as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.



SOLUTION

$$v_1 = \{-10\mathbf{i}\}\ \text{m/s}$$

$$v_2 = \{20\cos 40°\mathbf{i} + 20\sin 40°\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{I} = m\Delta v = (0.2)\,\{[20\cos 40° - (-10)]\mathbf{i} + 20\sin 40°\mathbf{j}\}$$

$$= \{5.0642\mathbf{i} + 2.5712\mathbf{j}\}\ \text{kg}\cdot\text{m/s}$$

$$\mathbf{I} = \sqrt{(5.0642)^2 + (2.5712)^2}$$

$$= 5.6795 = 5.68\ \text{kg}\cdot\text{m/s} \qquad \textbf{Ans.}$$

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Ans:
$I = 5.68\ \text{N}\cdot\text{s}$

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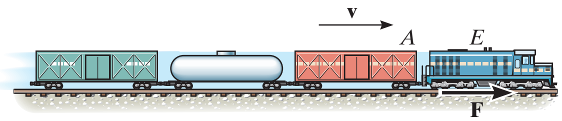
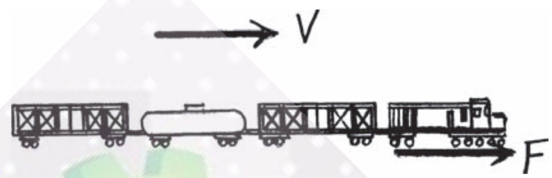
***15–4.**

A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.





SOLUTION

$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN} \qquad \textbf{Ans.}$$

Three cars:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

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Ans:
$F = 19.4$ kN
$T = 12.5$ kN

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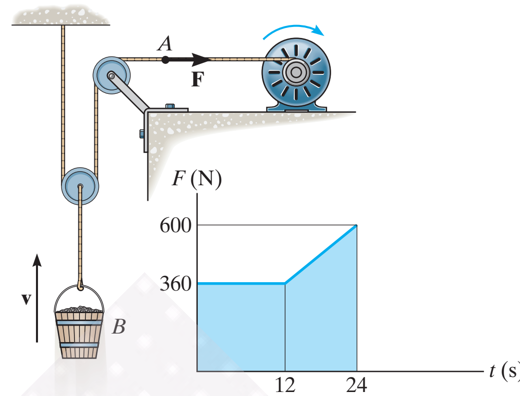
15–5.

The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.



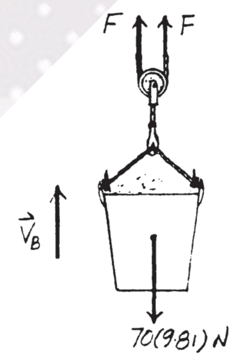
SOLUTION

Principle of Linear Impulse and Momentum: For the time period $12\text{ s} \leq t < 18\text{ s}$, $\dfrac{F - 360}{t - 12} = \dfrac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15–4 to bucket B, we have

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$(+\uparrow) \qquad 70(3) + 2\left[360(12) + \int_{12\text{ s}}^{18\text{ s}} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

$$v_2 = 21.8 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v_2 = 21.8$ m/s

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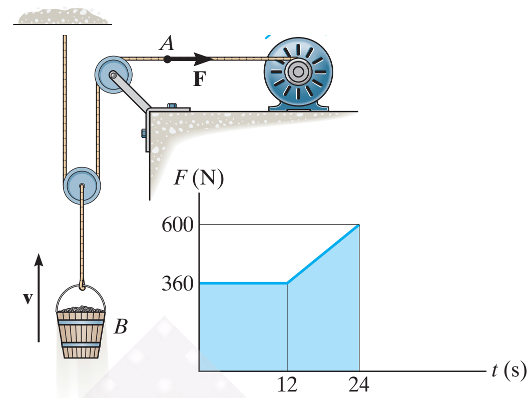
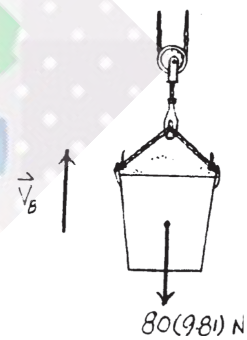
15–6.

The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket when $t = 24$ s. Originally the bucket is released from rest.



SOLUTION

Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma \int_{t_1}^{t_2} F_y\, dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right] = 20160\ \text{N}\cdot\text{s}$. Applying Eq. 15–4 to the bucket B, we have



$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$(+\uparrow) \qquad 80(0) + 20160 - 80(9.81)(24) = 80v_2$$

$$v_2 = 16.6 \text{ m/s} \qquad \textbf{Ans.}$$

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Ans:
$v_2 = 16.6$ m/s

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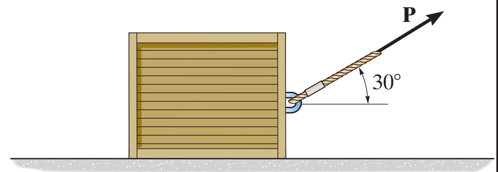
15–7.

The 50-kg crate is pulled by the constant force **P**. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of **P**. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Impulse and Momentum Diagram: The frictional force acting on the crate is $F_f = \mu_k N = 0.2N$.

Principle of Impulse and Momentum:

$(+\uparrow)$ $$m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$0 + N(5) + P(5)\sin 30° - 50(9.81)(5) = 0$$

$$N = 490.5 - 0.5P \qquad \textbf{(1)}$$

$(\overset{+}{\rightarrow})$ $$m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

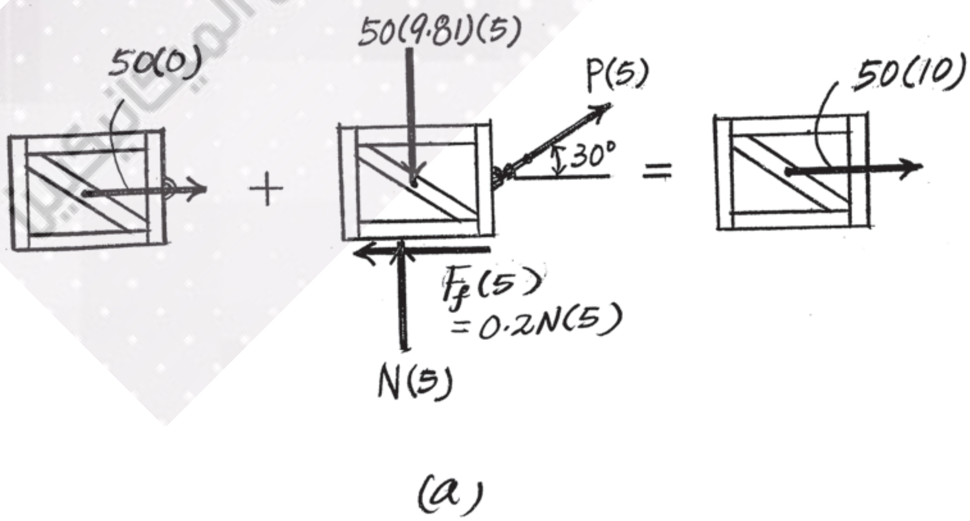
$$50(0) + P(5)\cos 30° - 0.2N(5) = 50(10)$$

$$4.3301P - N = 500 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$N = 387.97 \text{ N}$$

$$P = 205 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$P = 205$ N

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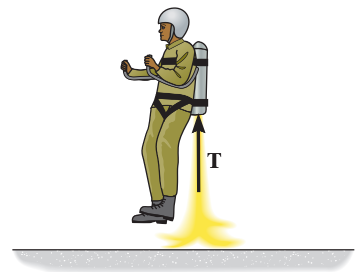
***15–8.**

If the jets exert a vertical thrust of $T = (500t^{3/2})$N, where t is in seconds, determine the man's speed when $t = 3$ s. The total mass of the man and the jet suit is 100 kg. Neglect the loss of mass due to the fuel consumed during the lift which begins from rest on the ground.



SOLUTION

Free-Body Diagram: The thrust **T** must overcome the weight of the man and jet before they move. Considering the equilibrium of the free-body diagram of the man and jet shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad 500t^{3/2} - 100(9.81) = 0 \qquad t = 1.567 \text{ s}$

Principle of Impulse and Momentum: Only the impulse generated by thrust **T** after $t = 1.567$ s contributes to the motion. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$100(0) + \int_{1.567 \text{ s}}^{3 \text{ s}} 500t^{3/2} dt - 100(9.81)(3 - 1.567) = 100v$$

$$\left(200t^{5/2}\right)\Big|_{1.567 \text{ s}}^{3 \text{ s}} - 1405.55 = 100v$$

$v = 11.0$ m/s **Ans.**

Ans:
$v = 11.0$ m/s

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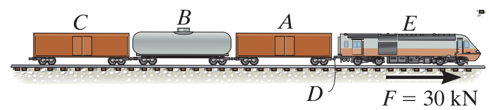
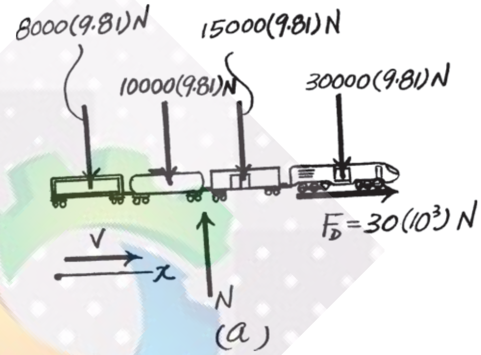
15–9.

The train consists of a 30-Mg engine E, and cars A, B, and C, which have a mass of 15 Mg, 10 Mg, and 8 Mg, respectively. If the tracks provide a traction force of $F = 30$ kN on the engine wheels, determine the speed of the train when $t = 30$ s, starting from rest. Also, find the horizontal coupling force at D between the engine E and car A. Neglect rolling resistance.



SOLUTION

Principle of Impulse and Momentum: By referring to the free-body diagram of the entire train shown in Fig. *a*, we can write

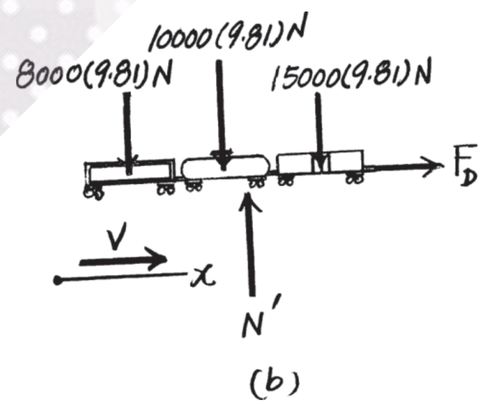


$(\overset{+}{\rightarrow})$ $$m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$63\,000(0) + 30(10^3)(30) = 63\,000v$$

$$v = 14.29 \text{ m/s}$$ **Ans.**

Using this result and referring to the free-body diagram of the train's car shown in Fig. *b*,



$(\overset{+}{\rightarrow})$ $$m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_2)_x$$

$$33000(0) + F_D(30) = 33\,000(14.29)$$

$$F_D = 15\,714.29 \text{ N} = 15.7 \text{ kN}$$ **Ans.**

Ans:
$v = 14.29$ m/s
$F_D = 15.7$ kN

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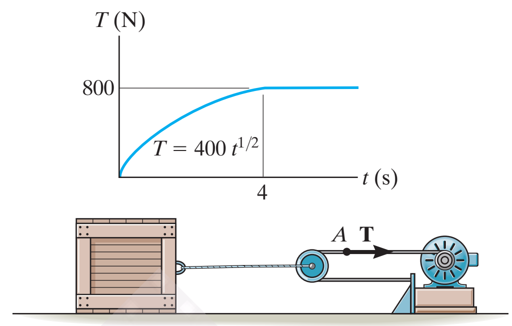
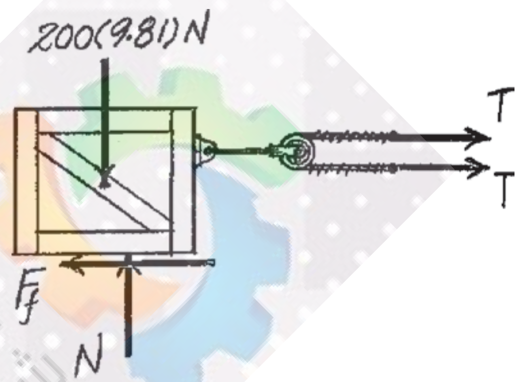
15–10.

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $\boldsymbol{T}$ to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the crate.



SOLUTION

Equilibrium. The time required to move the crate can be determined by considering the equilibrium of the crate. Since the crate is required to be on the verge of sliding, $F_f = \mu_s N = 0.5$ N. Referring to the FBD of the crate, Fig. *a*,



(a)

$$+\uparrow \Sigma F_y = 0; \quad N - 200(9.81) = 0 \quad N = 1962 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 2\left(400t^{\frac{1}{2}}\right) - 0.5(1962) = 0 \quad t = 1.5037 \text{ s}$$

Principle of Impulse and Momentum. Since the crate is sliding, $F_f = \mu_k N = 0.4(1962) = 784.8$ N. Referring to the FBD of the crate, Fig. *a*

$$(\xrightarrow{+}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$0 + 2\int_{1.5037 \text{ s}}^{4 \text{ s}} 400t^{\frac{1}{2}} \, dt - 784.8(4 - 1.5037) = 200v$$

$$v = 6.621 \text{ m/s} = 6.62 \text{ m/s} \qquad \textbf{Ans.}$$

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Ans:
$v = 6.62$ m/s

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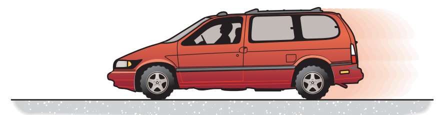
15–11.

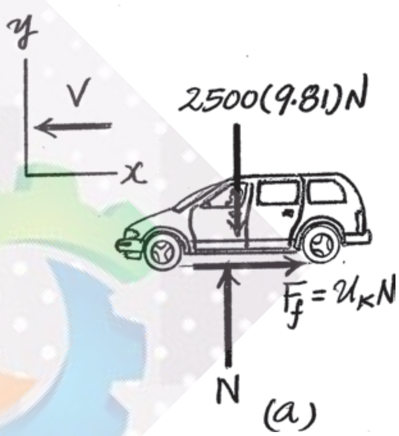
The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.



SOLUTION

Free-Body Diagram: The free-body diagram of the van is shown in Fig. *a*. The frictional force is $F_f = \mu_k N$ since all the wheels of the van are locked and will cause the van to slide.

Principle of Impulse and Momentum: The initial and final speeds of the van are $v_1 = \left[100(10^3)\,\dfrac{\text{m}}{\text{h}}\right]\left[\dfrac{1\text{ h}}{3600\text{ s}}\right] = 27.78\text{ m/s}$ and $v_2 = \left[40(10^3)\,\dfrac{\text{m}}{\text{h}}\right]\left[\dfrac{1\text{ h}}{3600\text{ s}}\right] = 11.11\text{ m/s}$. Referring to Fig. *a*,



$(+\uparrow)$ $$m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

$$2500(0) + N(5) - 2500(9.81)(5) = 2500(0)$$

$$N = 24\,525\text{ N}$$

$(\overset{+}{\leftarrow})$ $$m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

$$2500(27.78) + [-\mu_k(24525)(5)] = 2500(11.1)$$

$$\mu_k = 0.340$$ **Ans.**

Ans:
$\mu_k = 0.340$

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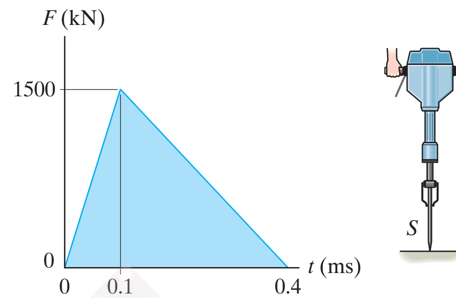
***15–12.**

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.



SOLUTION

Principle of Impulse and Momentum. The impulse of the force $\boldsymbol{F}$ is equal to the area under the F–t graph. Referring to the FBD of the spike, Fig. a

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$2(-90) + \frac{1}{2}\left[0.4\left(10^{-3}\right)\right]\left[1500\left(10^{3}\right)\right] = 2v$$

$$v = 60.0 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

(a)

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Ans:
$v = 60.0 \text{ m/s}$

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15–13.

For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.

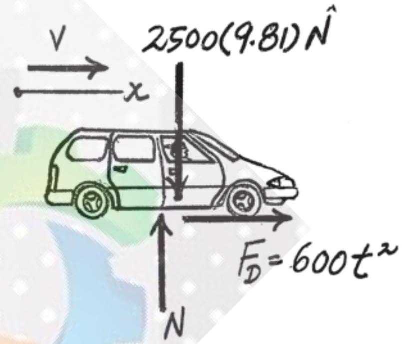
SOLUTION

Principle of Impulse and Momentum: The initial speed of the van is $v_1 = \left[20(10^3)\dfrac{\text{m}}{\text{h}}\right]\left[\dfrac{1\text{ h}}{3600\text{ s}}\right] = 5.556$ m/s. Referring to the free-body diagram of the van shown in Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$2500(5.556) + \int_0^{5\text{ s}} 600t^2 dt = 2500\, v_2$$

$$v_2 = 15.6\text{ m/s}$$ **Ans.**



(a)

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Ans:
$v_2 = 15.6$ m/s

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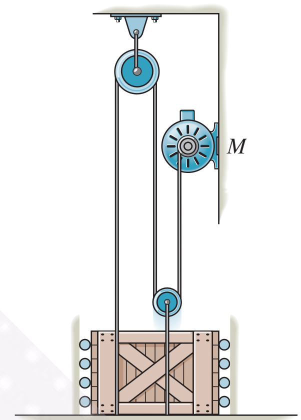
15–14.

The motor, M, pulls on the cable with a force $F = (10t^2 + 300)$ N, where t is in seconds. If the 100 kg crate is originally at rest at $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



SOLUTION

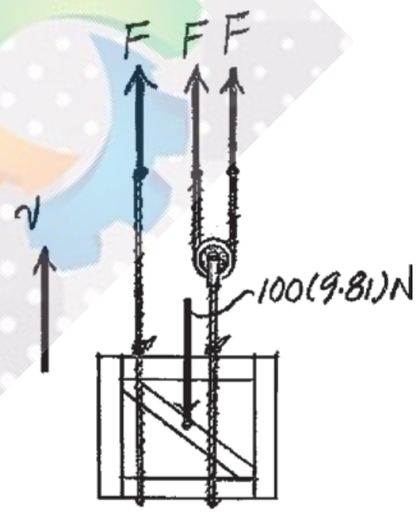
Principle of Impulse and Momentum. The crate will only move when $3\left(10t^2 + 300\right) = 100(9.81)$. Thus, this instant is $t = 1.6432$ s. Referring to the FBD of the crate, Fig. a,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$0 + \int_{1.6432\text{ s}}^{4\text{ s}} 3\left(10t^2 + 300\right) dt - 100(9.81)(4 - 1.6432) = 100v$$

$$3\left(\frac{10t^3}{3} + 300t\right)\bigg|_{1.6432\text{ s}}^{4\text{ s}} - 2312.05 = 100v$$

$$v = 4.047 \text{ m/s} = 4.05 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

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Ans:
$v = 4.05$ m/s

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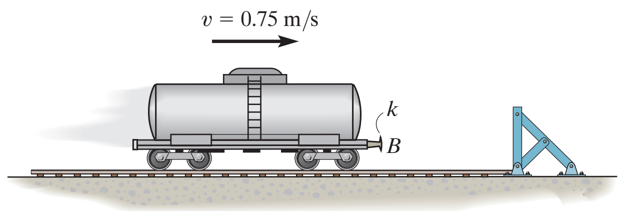
15–15.

A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper B has a stiffness (a) $k \rightarrow \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.



SOLUTION

a) b) $\quad (\overset{+}{\rightarrow}) \quad mv_1 + \Sigma \int F\,dt = mv_2$

$$20(10^3)(0.75) - \int F\,dt = 0$$

$$\int F\,dt = 15 \text{ kN}\cdot\text{s}$$ **Ans.**

The impulse is the same for both cases. For the spring having a stiffness $k = 15$ kN/m, the impulse is applied over a longer period of time than for $k \rightarrow \infty$.

Ans:
$I = 15$ kN·s in both cases.

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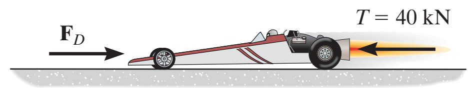
***15–16.**

Under a constant thrust of $T = 40$ kN, the 1.5-Mg dragster reaches its maximum speed of 125 m/s in 8 s starting from rest. Determine the average drag resistance $\mathbf{F}_D$ during this period of time.



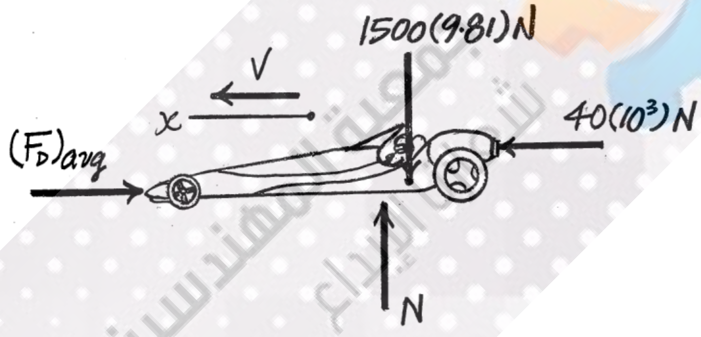
SOLUTION

Principle of Impulse and Momentum: The final speed of the dragster is $v_2 = 125$ m/s. Referring to the free-body diagram of the dragster shown in Fig. *a*,

$$(\overset{+}{\leftarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$1500(0) + 40(10^3)(8) - (F_D)_{\text{avg}}(8) = 1500(125)$$

$$(F_D)_{\text{avg}} = 16\,562.5 \text{ N} = 16.6 \text{ kN} \qquad \textbf{Ans.}$$



(a)

Ans:

$(F_D)_{\text{avg}} = 16.6$ kN

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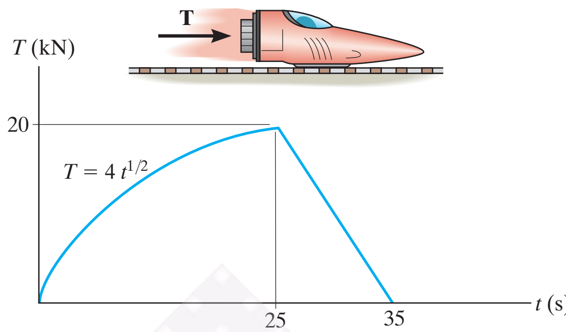
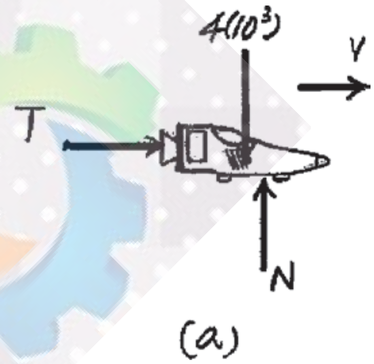
15–17.

The thrust on the 4-Mg rocket sled is shown in the graph. Determine the sleds maximum velocity and the distance the sled travels when $t = 35$ s. Neglect friction.



SOLUTION

Principle of Impulse And Momentum. The FBD of the rocket sled is shown in Fig. *a*. For $0 \le t < 25$ s,



$$(\xrightarrow{+}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + \int_0^t 4(10^3)t^{\frac{1}{2}}dt = 4(10^3)\,v$$

$$4(10^3)\left(\frac{2}{3}t^{\frac{3}{2}}\right)\Big|_0^t = 4(10^3)\,v$$

$$v = \left\{\frac{2}{3}t^{\frac{3}{2}}\right\} \text{ m/s}$$

At $t = 25$ s,

$$v = \frac{2}{3}(25)^{\frac{3}{2}} = 83.33 \text{ m/s}$$

For $25 \text{ s} < t < 35$ s, $\dfrac{T - 0}{t - 35} = \dfrac{20(10^3) - 0}{25 - 35}$ or $T = 2(10^3)(35 - t)$.

Here, $(v_x)_1 = 83.33$ m/s and $t_1 = 25$ s.

$$(\xrightarrow{+}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$4(10^3)(83.33) + \int_{25\text{ s}}^t 2(10^3)(35 - t)dt = 4(10^3)\,v$$

$$v = \{-0.25t^2 + 17.5t - 197.9167\} \text{ m/s}$$

The maximum velocity occurs at $t = 35$ s, Thus,

$$v_{\max} = -0.25(35^2) + 17.5(35) - 197.9167$$
$$= 108.33 \text{ m/s} = 108 \text{ m/s}$$ **Ans.**

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15–17. Continued

Kinematics. The displacement of the sled can be determined by integrating $ds = vdt$. For $0 \leq t < 25$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{2}{3} t^{\frac{3}{2}}\, dt$$

$$s\Big|_0^s = \frac{2}{3}\left(\frac{2}{5}\right)t^{\frac{5}{2}}\Big|_0^t$$

$$s = \left\{\frac{4}{15}t^{\frac{5}{2}}\right\} \text{ m}$$

At $t = 25$ s,

$$s = \frac{4}{15}(25)^{\frac{5}{2}} = 833.33 \text{ m}$$

For $25 < t \leq 35$ s, the initial condition is $s = 833.33$ at $t = 25$ s.

$$\int_{833.33\text{ m}}^{S} ds = \int_{25\text{ s}}^{t} (-0.25t^2 + 17.5t - 197.9167)\, dt$$

$$s\Big|_{833.33\text{ m}}^{S} = \left(-0.08333t^3 + 8.75t^2 - 197.9167t\right)\Big|_{25\text{ s}}^{t}$$

$$s = \{-0.08333t^3 + 8.75t^2 - 197.9167t + 1614.58\} \text{ m}$$

At $t = 35$ s,

$$s = -0.08333(35^3) + 8.75(35^2) - 197.9167(35) + 1614.58$$

$$= 1833.33 \text{ m} = 1833 \text{ m}$$ **Ans.**

Ans:
$v_{max} = 108$ m/s
$s = 1.83$ km

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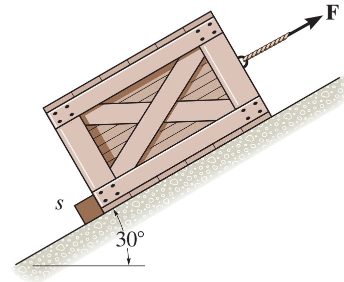
15–18. A 50-kg crate rests against a stop block s, which prevents the crate from moving down the plane. If the coefficients of static and kinetic friction between the plane and the crate are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the time needed for the force **F** to give the crate a speed of 2 m/s up the plane. The force always acts parallel to the plane and has a magnitude of $F = (300t)$ N, where t is in seconds. *Hint*: First determine the time needed to overcome static friction and start the crate moving.



SOLUTION

Guesses $t_1 = 1 \text{ s}$ $N_C = 1 \text{ N}$ $t_2 = 1 \text{ s}$

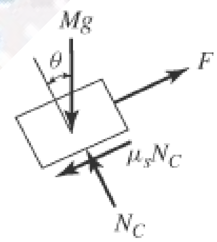
Given:

$M = 50 \text{ kg}$ $\theta = 30°$ $g = 9.81 \text{ m/s}^2$

$v = 2 \text{ m/s}$ $\mu_s = 0.3$

$\mu_k = 0.2$

$a = 300 \text{ N/s}$



Given $N_C - Mg\cos(\theta) = 0$

$$a t_1 - \mu_s N_C - Mg\sin(\theta) = 0$$

$$\int_{t_1}^{t_2} \left(a t - Mg\sin(\theta) - \mu_k N_C\right) \mathrm{d}t = M v$$

$$\begin{pmatrix} t_1 \\ t_2 \\ N_C \end{pmatrix} = \text{Find}\left(t_1, t_2, N_C\right) \qquad t_1 = 1.242 \text{ s} \qquad t_2 = 1.929 \text{ s} \qquad \textbf{Ans.}$$

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Ans:
$t_1 = 1.242$ s
$t_2 = 1.929$ s

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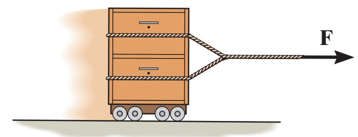
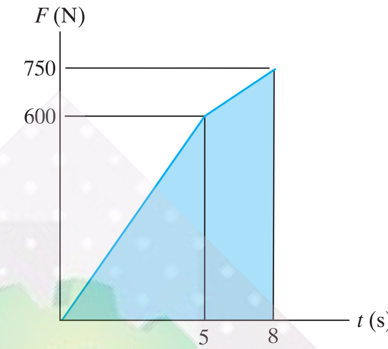
15–19.

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?





SOLUTION

Principle of Impulse and Momentum. The FBD of the safe is shown in Fig. *a*. For $0 \le t < 5$ s, $F = \dfrac{600}{5}t = 120t$.

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + \int_0^t 120t\,dt = 400v$$

$$v = \{0.15t^2\}\ \text{m/s}$$

At $t = 5$ s,

$$v = 0.15(5^2) = 3.75\ \text{m/s}$$

For $5\text{ s} < t \le 8$ s, $\dfrac{F - 600}{t - 5} = \dfrac{750 - 600}{8 - 5}$, $F = 50t + 350$. Here,

$(v_x)_1 = 3.75$ m/s and $t_1 = 5$ s.

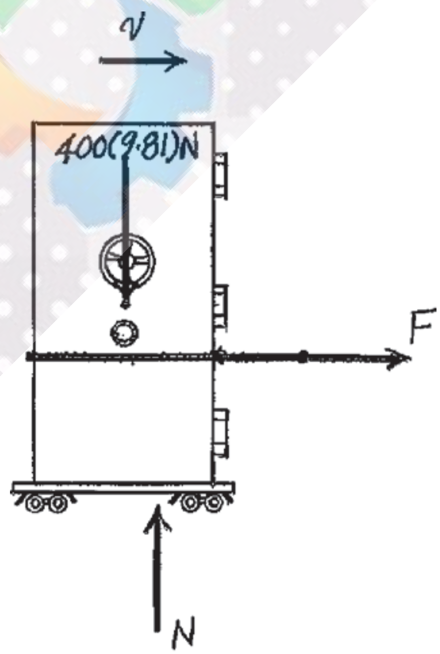
$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$400(3.75) + \int_{5\text{ s}}^{t} (50t + 350)\,dt = 400v$$

$$v = \{0.0625t^2 + 0.875t - 2.1875\}\ \text{m/s}$$

At $t = 8$ s,

$$v = 0.0625(8^2) + 0.875(8) - 2.1875 = 8.8125\ \text{m/s} = 8.81\ \text{m/s} \quad \textbf{Ans.}$$



(a)

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15–19. Continued

Kinematics. The displacement of the safe can be determined by integrating $ds = v\,dt$. For $0 \le t < 5$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = \{0.05t^3\}\text{ m}$$

At $t = 5$ s,

$$s = 0.05(5^3) = 6.25\text{ m}$$

For $5\text{ s} < t \le 8$ s, the initial condition is $s = 6.25$ m at $t = 5$ s.

$$\int_{6.25\text{ m}}^s ds = \int_{5\text{ s}}^t \left(0.0625t^2 + 0.875\,t - 2.1875\right) dt$$

$$s - 6.25 = \left(0.02083t^3 + 0.4375t^2 - 2.1875t\right)\Big|_{5\text{ s}}^t$$

$$s = \{0.02083t^3 + 0.4375t^2 - 2.1875t + 3.6458\}\text{ m}$$

At $t = 8$ s,

$$s = 0.02083(8^3) + 0.4375(8^2) - 2.1875(8) + 3.6458$$

$$= 24.8125\text{ m} = 24.8\text{ m}$$ **Ans.**

Ans:
$v = 8.81$ m/s
$s = 24.8$ m

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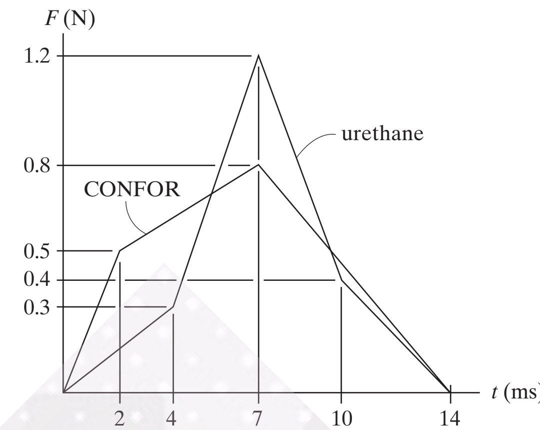
***15–20.**

The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.



SOLUTION

CONFOR foam:

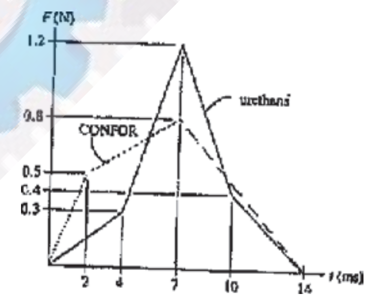
$$I_c = \int F\,dt = \left[\frac{1}{2}(2)(0.5) + \frac{1}{2}(0.5 + 0.8)(7 - 2) + \frac{1}{2}(0.8)(14 - 7)\right](10^{-3})$$

$$= 6.55\ \text{N}\cdot\text{ms}$$ **Ans.**

Urethane foam:

$$I_v = \int F\,dt = \left[\frac{1}{2}(4)(0.3) + \frac{1}{2}(1.2 + 0.3)(7 - 4) + \frac{1}{2}(1.2 + 0.4)(10 - 7) + \frac{1}{2}(14 - 10)(0.4)\right](10^{-3})$$

$$= 6.05\ \text{N}\cdot\text{ms}$$ **Ans.**



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Ans:
$I_c = 6.55\ \text{N}\cdot\text{ms}$
$I_v = 6.05\ \text{N}\cdot\text{ms}$

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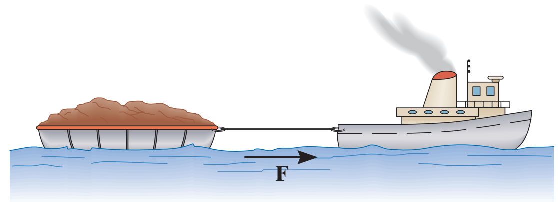
15–21.

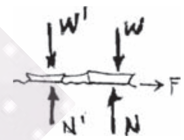
If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.



SOLUTION

$$25\left(\frac{1000}{3600}\right) = 6.944 \text{ m/s}$$

System:

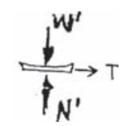


$(\xrightarrow{+})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$[0+0] + F(35) = (50 + 75)(10^3)(6.944)$$

$F = 24.8$ kN **Ans.**

Barge:



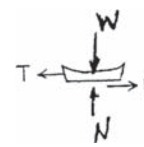
$(\xrightarrow{+})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$0 + T(35) = (75)(10^3)(6.944)$$

$T = 14.881 = 14.9$ kN **Ans.**

Also, using this result for T,

Tugboat:



$(\xrightarrow{+})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$$

$F = 24.8$ kN **Ans.**

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Ans:
$T = 14.9$ kN
$F = 24.8$ kN

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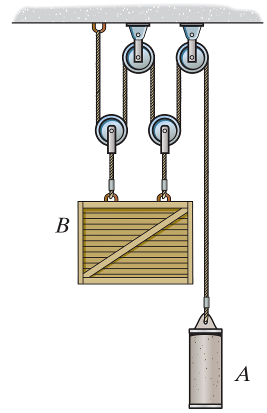
15–22.

The crate B and cylinder A have a mass of 200 kg and 75 kg, respectively. If the system is released from rest, determine the speed of the crate and cylinder when $t = 3$ s. Neglect the mass of the pulleys.



SOLUTION

Free-Body Diagram: The free-body diagrams of cylinder A and crate B are shown in Figs. b and c. $\mathbf{v}_A$ and $\mathbf{v}_B$ must be assumed to be directed downward so that they are consistent with the positive sense of s_A and s_B shown in Fig. a.

Principle of Impulse and Momentum: Referring to Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$75(0) + 75(9.81)(3) - T(3) = 75v_A$$

$$v_A = 29.43 - 0.04T \qquad \textbf{(1)}$$

From Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$200(0) + 2500(9.81)(3) - 4T(3) = 200v_B$$

$$v_B = 29.43 - 0.06T \qquad \textbf{(2)}$$

Kinematics: Expressing the length of the cable in terms of s_A and s_B and referring to Fig. a,

$$s_A + 4s_B = l \qquad \textbf{(3)}$$

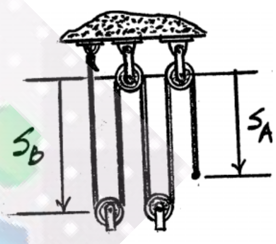
Taking the time derivative,

$$v_A + 4v_B = 0 \qquad \textbf{(4)}$$

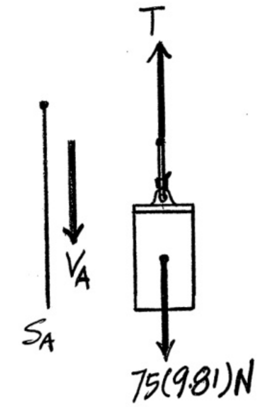
Solving Eqs. (1), (2), and (4) yields

$v_B = -2.102 \text{ m/s} = 2.10 \text{ m/s} \uparrow \qquad v_A = 8.409 \text{ m/s} = 8.41 \text{ m/s} \downarrow$ **Ans.**

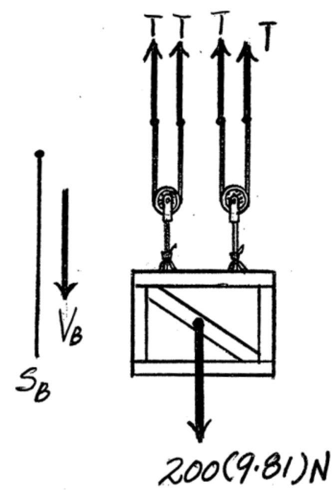
$T = 525.54$ N



(a)



(b)



(C)

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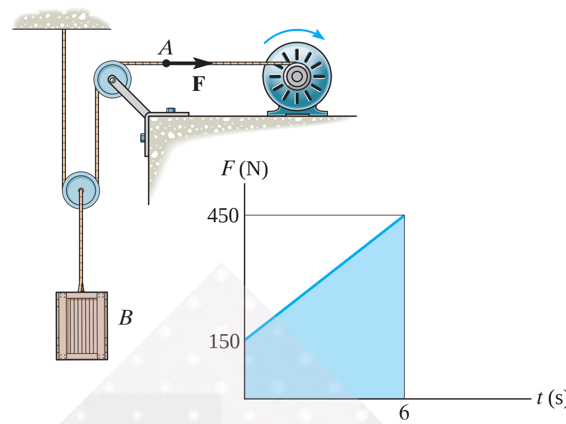
15–23.

The motor exerts a force $\boldsymbol{F}$ on the 40-kg crate as shown in the graph. Determine the speed of the crate when $t = 3$ s and when $t = 6$ s. When $t = 0$, the crate is moving downward at 10 m/s.



SOLUTION

Principle of Impulse and Momentum. The impulse of force $\boldsymbol{F}$ is equal to the area under the F–t graph. At $t = 3$ s, $\dfrac{F - 150}{3 - 0} = \dfrac{450 - 150}{6 - 0}$ $F = 300$ N. Referring to the FBD of the crate, Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$40(-10) + 2\left[\frac{1}{2}(150 + 300)(3)\right] - 40(9.81)(3) = 40v$$

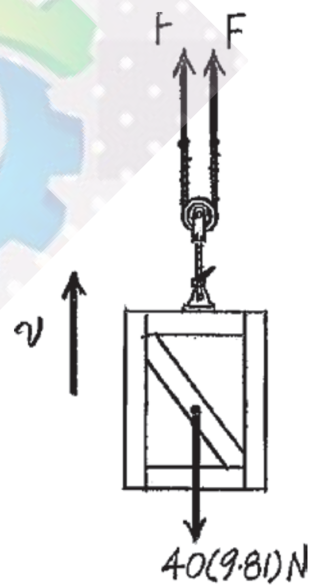
$$v = -5.68 \text{ m/s} = 5.68 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

At $t = 6$ s,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$40(-10) + 2\left[\frac{1}{2}(450 + 150)(6)\right] - 40(9.81)(6) = 40v$$

$$v = 21.14 \text{ m/s} = 21.1 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

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Ans:
$v|_{t=3\text{ s}} = 5.68 \text{ m/s} \downarrow$
$v|_{t=6\text{ s}} = 21.1 \text{ m/s} \uparrow$

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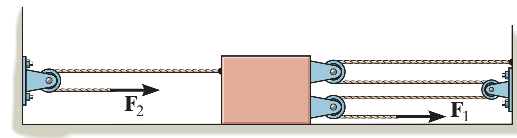
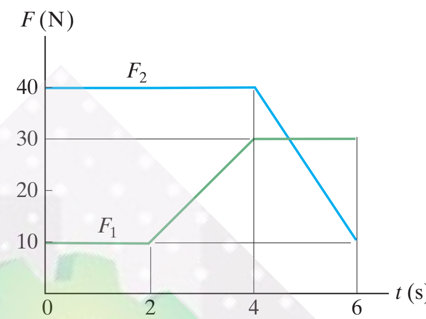
***15–24.**

The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





SOLUTION

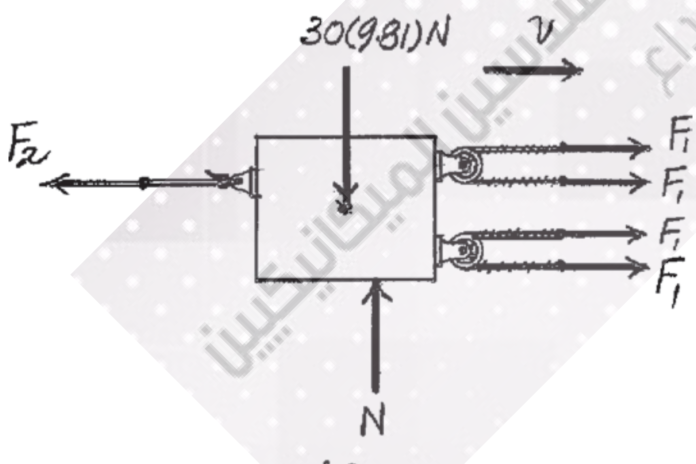
Principle of Impulse and Momentum. The impulses produced by $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal to the area under the respective F–t graph. Referring to the FBD of the block Fig. a,

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dx = m(v_x)_2$$

$$-30(5) + 4\left[10(2) + \frac{1}{2}(10 + 30)(4 - 2) + 30(6 - 4)\right]$$

$$+ \left[-40(4) - \frac{1}{2}(10 + 40)(6 - 4)\right] = 30v$$

$$v = 4.00 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 4.00$ m/s

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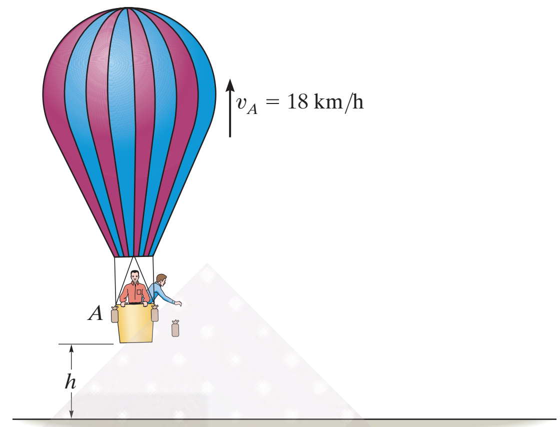
15–25.

The balloon has a total mass of 400 kg including the passengers and ballast. The balloon is rising at a constant velocity of 18 km/h when $h = 10$ m. If the man drops the 40-kg sand bag, determine the velocity of the balloon when the bag strikes the ground. Neglect air resistance.



SOLUTION

Kinematic. When the sand bag is dropped, it will have an upward velocity of $v_0 = \left(18\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5\text{ m/s}\uparrow$. When the sand bag strikes the ground $s = 10\text{ m}\downarrow$. The time taken for the sand bag to strike the ground can be determined from

$$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2;$$

$$-10 = 0 + 5t + \frac{1}{2}(-9.81t^2)$$

$$4.905t^2 - 5t - 10 = 0$$

Solve for the positive root,

$$t = 2.0258\text{ s}$$

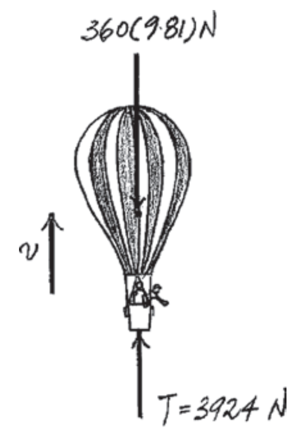
Principle of Impulse and Momentum. The FBD of the ballon when the ballon is rising with the constant velocity of 5 m/s is shown in Fig. *a*

$$(+\uparrow)\quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$400(5) + T(t) - 400(9.81)t = 400(5)$$

$$T = 3924\text{ N}$$

When the sand bag is dropped, the thrust $T = 3924$ N is still maintained as shown in the FBD, Fig. *b*.



$$(+\uparrow)\quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$360(5) + 3924(2.0258) - 360(9.81)(2.0258) = 360v$$

$$v = 7.208\text{ m/s} = 7.21\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

Ans:
$v = 7.21\text{ m/s}\uparrow$

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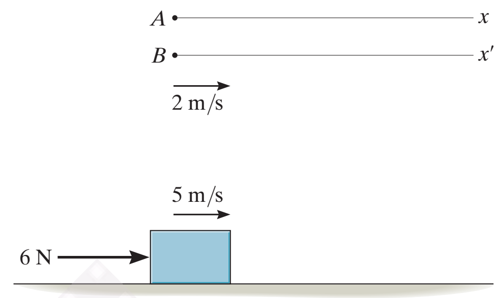
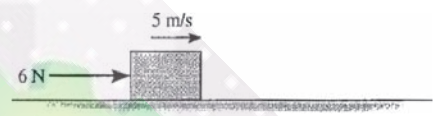
15–26.

As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which slides along the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



SOLUTION



Observer A:

$$(\overset{+}{\rightarrow}) \qquad m\,v_1 + \sum \int F\,dt = m\,v_2$$

$$10(5) + 6(4) = 10v$$

$$v = 7.40 \text{ m/s}$$ **Ans.**

Observer B:

$$(\overset{+}{\rightarrow}) \qquad m\,v_1 + \sum \int F\,dt = m\,v_2$$

$$10(3) + 6(4) = 10v$$

$$v = 5.40 \text{ m/s}$$ **Ans.**

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Ans:
Observer A: $v = 7.40$ m/s
Observer B: $v = 5.40$ m/s

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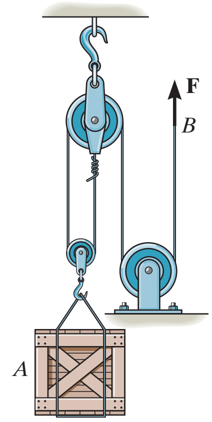
15–27.

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine the speed of the crate when $t = 3$ s, starting from rest.



SOLUTION

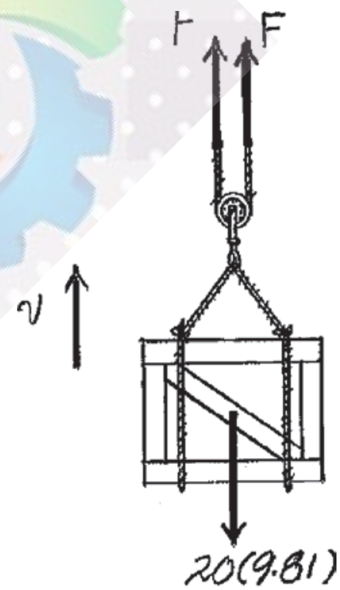
Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force **F** is applied. Referring to the FBD of the crate, Fig. *a*,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^{3\text{ s}} (100 + 5t^2)dt - 20(9.81)(3) = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Bigg|_0^{3\text{ s}} - 588.6 = 20v$$

$$v = 5.07\text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$v = 5.07$ m/s

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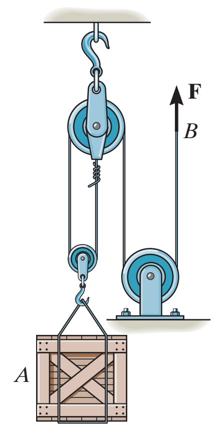
***15–28.**

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.



SOLUTION

Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200 \text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force F is applied. Referring to the FBD of the crate, Fig. *a*

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^t (100 + 5t^2)dt - 20(9.81)t = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^t - 196.2t = 20v$$

$$v = \{0.1667t^3 + 0.19t\} \text{ m/s}$$

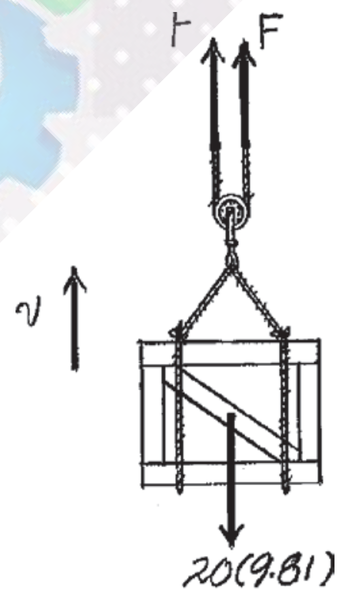
Kinematics. The displacement of the crate can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (0.1667t^3 + 0.19t)\,dt$$

$$s = \{0.04167t^4 + 0.095t^2\} \text{ m}$$

At $t = 3$ s,

$$s = 0.04167(3^4) + 0.095(3^2) = 4.23 \text{ m} \qquad \textbf{Ans.}$$



(a)

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Ans:
$s = 4.23$ m

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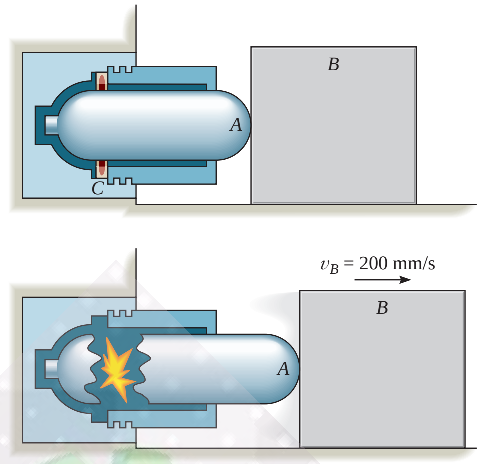
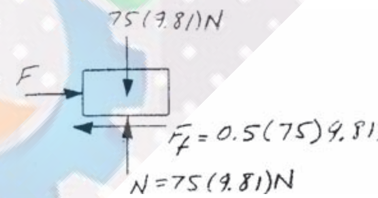
15–29.

In case of emergency, the gas actuator is used to move a 75-kg block B by exploding a charge C near a pressurized cylinder of negligible mass. As a result of the explosion, the cylinder fractures and the released gas forces the front part of the cylinder, A, to move B forward, giving it a speed of 200 mm/s in 0.4 s. If the coefficient of kinetic friction between B and the floor is $\mu_k = 0.5$, determine the impulse that the actuator imparts to B.



SOLUTION

Principle of Linear Impulse and Momentum: In order for the package to rest on top of the belt, it has to travel at the same speed as the belt. Applying Eq. 15–4, we have



$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$(+\uparrow)$ $$6(0) + Nt - 6(9.81)\,t = 6(0)$$

$$N = 58.86\text{ N}$$

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$(\overset{+}{\rightarrow})$ $$6(3) + [-0.2(58.86)t] = 6(1)$$

$$t = 1.02\text{ s}$$ **Ans.**

$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

$$0 + \int F\,dt - (0.5)(9.81)(75)(0.4) = 75(0.2)$$

$$\int F\,dt = 162\text{ N}\cdot\text{s}$$ **Ans.**

Ans:
$t = 1.02$ s
$I = 162$ N · s

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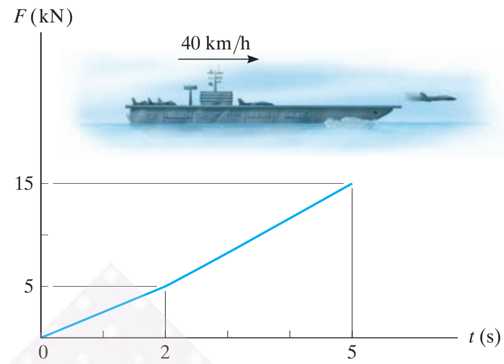
15–30.

A jet plane having a mass of 7 Mg takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed of 40 km/h, determine the plane's airspeed after 5 s.

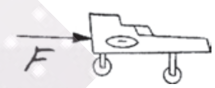


SOLUTION

The impulse exerted on the plane is equal to the area under the graph.

$v_1 = 40 \text{ km/h} = 11.11 \text{ m/s}$

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$



$$(7)\left(10^3\right)(11.11) - \frac{1}{2}(2)(5)(10^3) + \frac{1}{2}(15 + 5)(5 - 2)(10^3) = 7(10^3)_{v_2}$$

$v_2 = 16.1 \text{ m/s}$ **Ans.**

Ans:
$v = 16.1 \text{ m/s}$

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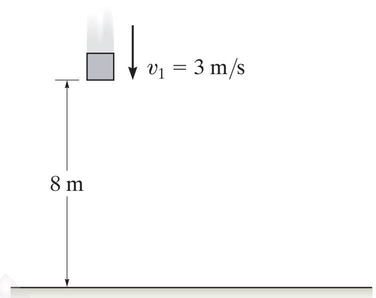
15–31. The 6-kg block is moving downward at $v_1 = 3$ m/s when it is 8 m from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



SOLUTION

Given:

$M = 6$ kg

$v_1 = 3$ m/s

$h = 8$ m

$g = 9.81$ m/s^2

Just before impact $\quad v_2 = \sqrt{v_1^2 + 2gh} \quad v_2 = 12.88$ m/s

Collision $\quad Mv_2 - I = 0 \quad I = Mv_2 \quad I = 77.3$ N·s **Ans.**

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Ans:
$I = 77.3$ N·s

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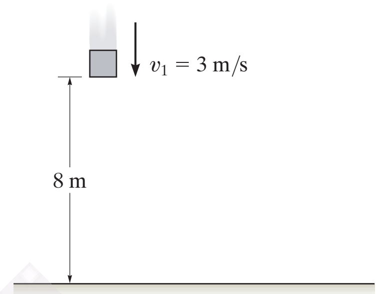
***15–32.** The 6-kg block is falling downward at $v_1 = 3$ m/s when it is 8 m from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in time 1.2 s once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



SOLUTION

Given:

$M = 6$ kg

$v_1 = 3$ m/s

$\Delta t = 0.9$ s

$h = 8$ m

$g = 9.81$ m/s^2

Just before impact $\quad v_2 = \sqrt{v_1^2 + 2gh} \quad v_2 = 12.88$ m/s

Collision $\quad Mv_2 - F\Delta t = 0 \quad F = \dfrac{Mv_2}{\Delta t} \quad F = 64.4$ N **Ans.**

Ans:
$F = 64.4$ N

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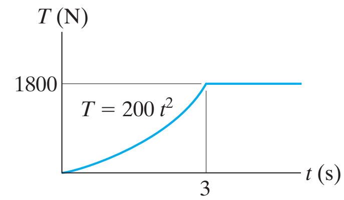
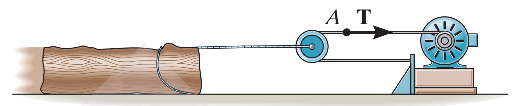
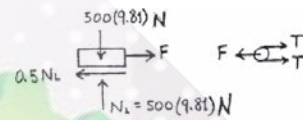
15–33.

The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.



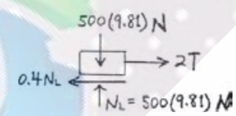


SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad F - 0.5(500)(9.81) = 0$

$$F = 2452.5 \text{ N}$$



Thus,

$$2T = F$$

$$2(200t^2) = 2452.5$$

$$t = 2.476 \text{ s to start log moving}$$

$(\xrightarrow{+}) \qquad m v_1 + \Sigma \int F\, dt = m v_2$

$$0 + 2\int_{2.476}^{3} 200t^2\, dt + 2(1800)(5-3) - 0.4(500)(9.81)(5 - 2.476) = 500v_2$$

$$400\left(\frac{t^3}{3}\right)\Big|_{2.476}^{3} + 2247.91 = 500v_2$$

$$v_2 = 7.65 \text{ m/s} \qquad \textbf{Ans.}$$

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Ans:
$v = 7.65$ m/s

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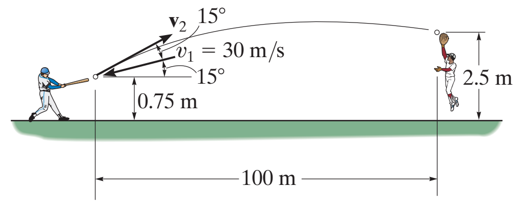
15–34.

The 0.15-kg baseball has a speed of $v = 30$ m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.



SOLUTION

Kinematics. First, we will consider the horizontal motion. Here, the horizontal component of the initial velocity is $(v_0)_x = v_2 \cos 30° = \frac{\sqrt{3}}{2} v_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 100$ m

$$(\overset{+}{\rightarrow})\ S_x = (S_0)_x + (v_0)_x t$$

$$100 = 0 + \left(\frac{\sqrt{3}}{2} v_2\right) t$$

$$v_2 = \frac{200}{\sqrt{3}t} \qquad (1)$$

For the vertical motion, the vertical component of the initial velocity is $(v_0)_y = v_2 \sin 30° = \frac{1}{2} v_2$ and the initial and find positions are $(s_0)_y = 0.75$ m and $s_y = 2.5$ m.

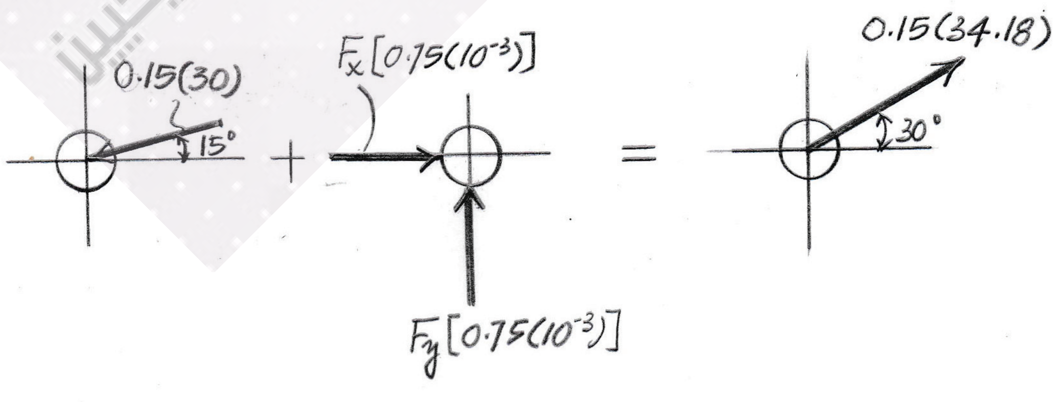
$$(+\uparrow) S_y = (S_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y t^2$$

$$2.5 = 0.75 + \left(\frac{1}{2} v_2\right) t + \frac{1}{2}(-9.81)\, t^2$$

$$4.905t^2 - 0.5v_2 t + 1.75 = 0 \qquad (2)$$

Solving Eqs. 1 and 2,

$v_2 = 34.18$ m/s $\quad t = 3.378$ s



(a)

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15–34. Continued

Principle of Impulse and Momentum. Referring to the impulse and momentum diagram shown in Fig. *a*,

$$M(v_1)_x + \Sigma\int_{t_1}^{t_z} F_x dt = m(v_2)_x$$

$$(\overset{+}{\rightarrow})\ -[0.15(30)]\cos 15^\circ + F_x[0.75(10^{-3})] = [0.15(34.18)]\cos 30^\circ$$

$$F_x = 11.715(10^3)\text{ N} = 11.715\text{ kN}$$

$$M(v_1)_y + \Sigma\int_{t_1}^{t_z} F_y dt = m(v_2)_y$$

$$(+\uparrow)\ -[0.15(30)]\sin 15^\circ + F_y[0.75(10^3)] = [0.15(34.18)]\sin 30^\circ$$

$$F_y = 4.971(10^3)\text{ N} = 4.971\text{ kN}$$

Thus, the magnitude of the average impulsive force on the ball is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{11.715^2 + 4.971^2} = 12.73\text{ kN} = 12.7\text{ kN}$$ **Ans.**

Ans:
$F = 12.7$ kN

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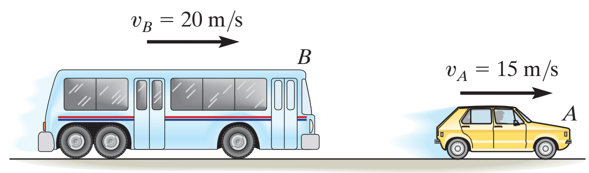
15–35.

The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

Conservation of Linear Momentum.

$(\xrightarrow{+})$ $$m_A v_A + m_B v_B = (m_A + m_B)v$$

$$\left[5(10^3)\right](20) + \left[2(10^3)\right](15) = \left[5(10^3) + 2(10^3)\right]v$$

$$v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow$$ **Ans.**

Ans:
$v = 18.6$ m/s $\rightarrow$

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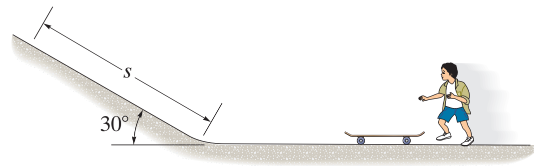
***15–36.**

The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.



SOLUTION

Free-Body Diagram: The free-body diagram of the boy and skateboard system is shown in Fig. *a*. Here, $\mathbf{W}_b, \mathbf{W}_{sb}$, and $\mathbf{N}$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ resulting from the impact during landing cancel each other out since they are internal to the system.

Conservation of Linear Momentum: Since the resultant of the impulsive force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m_b(v_b)_1 + m_{sb}(v_{sb})_1 = (m_b + m_{sb})v$$

$$50(5) + 5(0) = (50 + 5)v$$

$$v = 4.545 \text{ m/s}$$

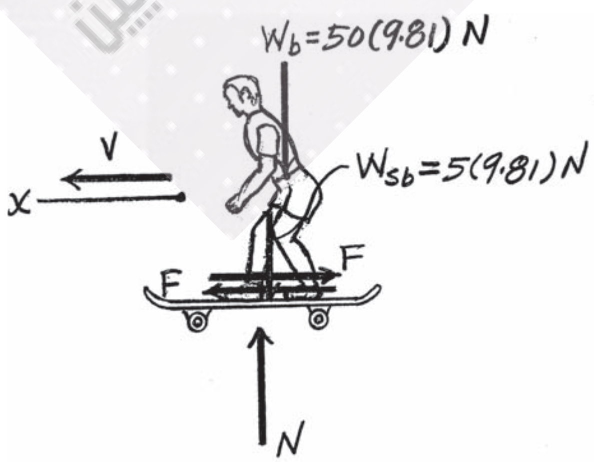
Conservation of Energy: With reference to the datum set in Fig. *b*, the gravitational potential energy of the boy and skateboard at positions A and B are $\left(V_g\right)_A = (m_b + m_{sb})gh_A = 0$ and $\left(V_g\right)_B = (m_b + m_{sb})gh_B = (50 + 5)(9.81)(s \sin 30°) = 269.775s$.

$$T_A + V_A = T_B + V_B$$

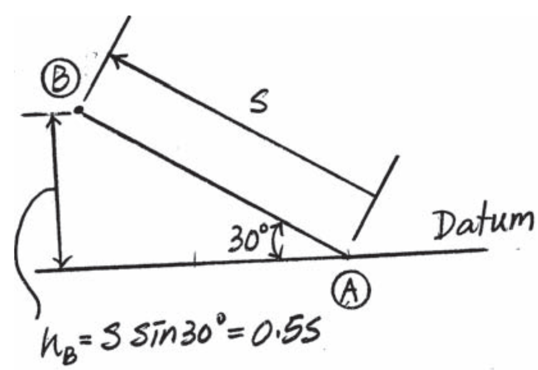
$$\frac{1}{2}(m_b + m_{sb})v_A{}^2 + \left(V_g\right)_A = \frac{1}{2}(m_b + m_{sb})v_B{}^2 + \left(V_g\right)_B$$

$$\frac{1}{2}(50 + 5)\left(4.545^2\right) + 0 = 0 + 269.775s$$

$$s = 2.11 \text{ m}$$ **Ans.**



(a)



(b)

Ans:
$s = 2.11$ m

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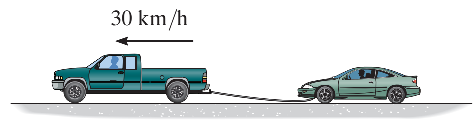
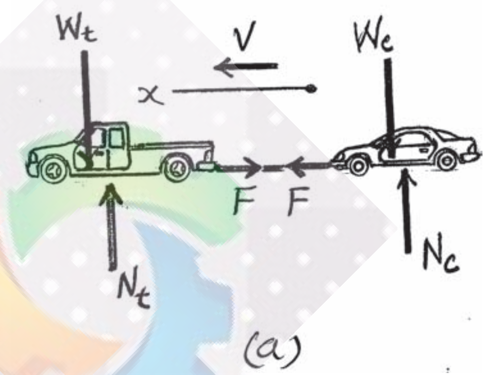
15–37.

The 2.5-Mg pickup truck is towing the 1.5-Mg car using a cable as shown. If the car is initially at rest and the truck is coasting with a velocity of 30 km/h when the cable is slack, determine the common velocity of the truck and the car just after the cable becomes taut. Also, find the loss of energy.



SOLUTION

Free-Body Diagram: The free-body diagram of the truck and car system is shown in Fig. *a*. Here, $\mathbf{W}_t$, $\mathbf{W}_C$, $\mathbf{N}_t$, and $\mathbf{N}_C$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ generated at the instant the cable becomes taut are internal to the system and thus cancel each other out.



Conservation of Linear Momentum: Since the resultant of the impulsive force is zero, the linear momentum of the system is conserved along the x axis. The initial speed of the truck is $(v_t)_1 = \left[30(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 8.333$ m/s.

$$(\overset{+}{\leftarrow}) \qquad m_t(v_t)_1 + m_C(v_C)_1 = (m_t + m_C)v_2$$

$$2500(8.333) + 0 = (2500 + 1500)v_2$$

$$v_2 = 5.208\text{ m/s} = 5.21\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Kinetic Energy: The initial and final kinetic energy of the system is

$$T_1 = \frac{1}{2}m_t(v_t)_1^2 + \frac{1}{2}m_C(v_C)_1^2$$

$$= \frac{1}{2}(2500)(8.333^2) + 0$$

$$= 86\,805.56\text{ J}$$

and

$$T_2 = (m_t + m_C)v_2^2$$

$$= \frac{1}{2}(2500 + 1500)(5.208^2)$$

$$= 54\,253.47$$

Thus, the loss of energy during the impact is

$$\Delta T = T_1 - T_2 = 86\,805.56 - 54\,253.47 = 32.55(10^3)\text{ J} = 32.6\text{ kJ} \qquad \textbf{Ans.}$$

Ans:
$v = 5.21$ m/s $\leftarrow$
$\Delta T = -32.6$ kJ

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15–38.

A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

SOLUTION

$(\xrightarrow{+})\quad \Sigma m v_1 = \Sigma m v_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$$v_2 = 0.5 \text{ m/s} \qquad \textbf{Ans.}$$

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

$$\Delta T = T_2 - T_1$$

$$= 3.375 - 20.25 = -16.9 \text{ kJ} \qquad \textbf{Ans.}$$

This energy is dissipated as noise, shock, and heat during the coupling.

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Ans:
$v = 0.5$ m/s
$\Delta T = -16.9$ kJ

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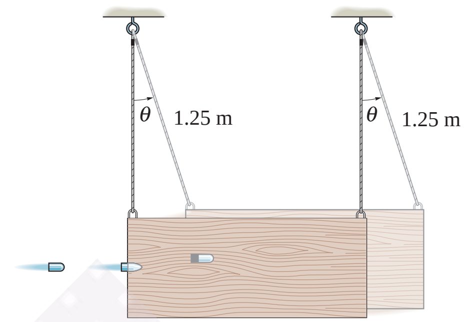
15–39.

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the speed of the bullet.



SOLUTION

Just after impact:

Datum at lowest point.

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(4 + 0.002)(v_B)_2^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6°)$$

$$(v_B)_2 = 0.3665 \text{ m/s}$$

For the system of bullet and block:

$$(\overset{+}{\rightarrow}) \qquad \Sigma mv_1 = \Sigma mv_2$$

$$0.002(v_B)_1 = (4 + 0.002)(0.3665)$$

$$(v_B)_1 = 733 \text{ m/s}$$ **Ans.**

Ans:
$v = 733$ m/s

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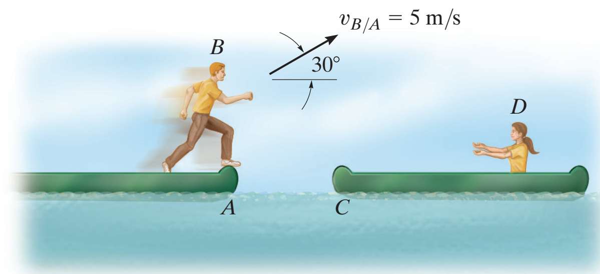
***15–40.** The boy B jumps off the canoe at A with a velocity 5 m/s relative to the canoe as shown. If he lands in the second canoe C, determine the final speed of both canoes after the motion. Each canoe has a mass of 40 kg. The boy's mass is 30 kg, and the girl D has a mass of 25 kg. Both canoes are originally at rest.



SOLUTION

Guesses $\quad v_A = 1\ \frac{\text{m}}{\text{s}} \qquad v_C = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad 0 = M_c v_A + M_B\left(v_A + v_{BA}\cos(\theta)\right)$

$$M_B\left(v_A + v_{BA}\cos(\theta)\right) = \left(M_c + M_B + M_D\right)v_C$$

Given:

$M_c = 40\ \text{kg}$

$M_B = 30\ \text{kg}$

$M_D = 25\ \text{kg}$

$v_{BA} = 5\ \text{m/s}$

$\theta = 30°$

$$\begin{pmatrix} v_A \\ v_C \end{pmatrix} = \text{Find}(v_A, v_C) \qquad \begin{pmatrix} v_A \\ v_C \end{pmatrix} = \begin{pmatrix} -1.856 \\ 0.781 \end{pmatrix}\ \text{m/s}$$ **Ans.**

Ans:
$v_A = -1.856\ \text{m/s}$
$v_C = 0.781\ \text{m/s}$

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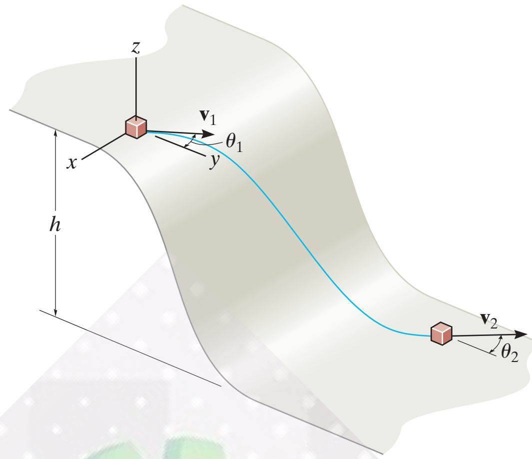
15–41.

The block of mass m is traveling at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



SOLUTION

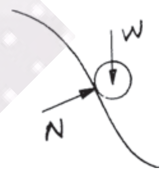
There are no impulses in the v direction:

$$mv_1 \sin\theta_1 = mv_2 \sin\theta_2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh}$$ **Ans.**



$$\sin\theta_2 = \frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}$$

$$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}\right)$$ **Ans.**

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Ans:
$v_2 = \sqrt{v_1^2 + 2gh}$
$\theta_2 = \sin^{-1}\left(\frac{v_1\sin\theta}{\sqrt{v_1^2 + 2gh}}\right)$

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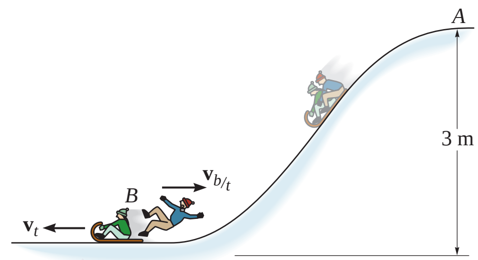
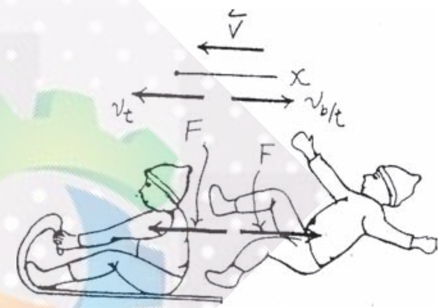
15–42.

A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of $v_{b/t} = 2$ m/s, measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.



SOLUTION



Conservation of Energy: The datum is set at the lowest point B. When the toboggan and its rider is at A, their position is 3 m *above* the datum and their gravitational potential energy is $(10 + 40 + 45)(9.81)(3) = 2795.85$ N · m. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2795.85 = \frac{1}{2}(10 + 40 + 45)\, v_B^2 + 0$$

$$v_B = 7.672 \text{ m/s}$$

Relative Velocity: The relative velocity of the falling boy with respect to the toboggan is $v_{b/t} = 2$ m/s. Thus, the velocity of the boy falling off the toboggan is

$$v_b = v_t + v_{b/t}$$

$$(\overset{+}{\leftarrow}) \qquad v_b = v_t - 2 \qquad \textbf{[1]}$$

Conservation of Linear Momentum: If we consider the tobbogan and the riders as a system, then the impulsive force caused by the push is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$m_T v_B = m_b v_b + \left(m_t + m_g\right) v_t$$

$$(\overset{+}{\leftarrow}) \qquad (10 + 40 + 45)(7.672) = 45 v_b + (10 + 40)\, v_t \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_t = 8.62 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_b = 6.619 \text{ m/s}$$

Ans:
$v_t = 8.62$ m/s

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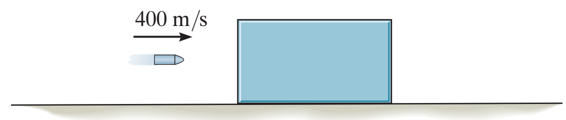
15–43.

The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



SOLUTION

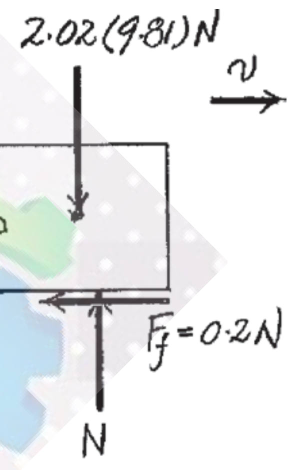
Conservation of Momentum.

$$(\xrightarrow{+}) \qquad m_b v_b + m_B v_B = (m_b + m_B)v$$

$$0.02(400) + 0 = (0.02 + 2)v$$

$$v = 3.9604 \text{ m/s}$$

Principle of Impulse and Momentum. Here, friction $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the blocks, Fig. *a*,



(a)

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + N(t) - 2.02(9.81)(t) = 0$$

$$N = 19.8162 \text{ N}$$

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$2.02(3.9604) + [-0.2(19.8162)t] = 2.02v$$

$$v = \{3.9604 - 1.962t\} \text{ m/s}$$

Thus, the stopping time can be determined from

$$0 = 3.9604 - 1.962t$$

$$t = 2.0186 \text{ s}$$

Kinematics. The displacement of the block can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (3.9604 - 1.962t)\,dt$$

$$s = \{3.9604t - 0.981t^2\} \text{ m}$$

The block stopped at $t = 2.0186$ s. Thus

$$s = 3.9604(2.0186) - 0.981(2.0186^2)$$

$$= 3.9971 \text{ m} = 4.00 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s = 4.00$ m

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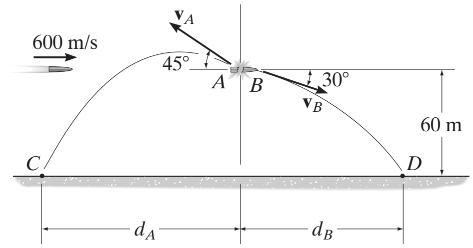
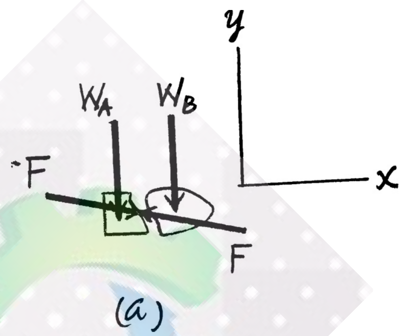
***15–44** A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_A where segment A strikes the ground at C.



SOLUTION

Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. *a*, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



(a)

$(\overset{+}{\rightarrow})$ $$mv_x = m_A(v_A)_x + m_B(v_B)_x$$

$$4(600) = -1.5v_A\cos 45^\circ + 2.5v_B\cos 30^\circ$$

$$2.165v_B - 1.061v_A = 2400 \qquad \textbf{(1)}$$

$(+\uparrow)$ $$mv_y = m_A(v_A)_y + m_B(v_B)_y$$

$$0 = 1.5v_A\sin 45^\circ - 2.5v_B\sin 30^\circ$$

$$v_B = 0.8485v_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96\text{ m/s} = 3.09(10^3)\text{ m/s} \qquad \textbf{Ans.}$$

$$v_B = 2622.77\text{ m/s} = 2.62(10^3)\text{ m/s} \qquad \textbf{Ans.}$$

By considering the x and y motion of segment A,

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$-60 = 0 + 3090.96\sin 45^\circ\, t_{AC} + \frac{1}{2}(-9.81)t_{AC}{}^2$$

$$4.905t_{AC}{}^2 - 2185.64t_{AC} - 60 = 0$$

Solving for the positive root of this equation,

$$t_{AC} = 445.62\text{ s}$$

and

$(\overset{+}{\leftarrow})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d_A = 0 + 3090.96\cos 45^\circ(445.62)$$

$$= 973.96\left(10^3\right)\text{ m} = 974\text{ km} \qquad \textbf{Ans.}$$

Ans:
$v_A = 3.09(10^3)$ m/s
$v_B = 2.62(10^3)$ m/s
$d_A = 974$ km

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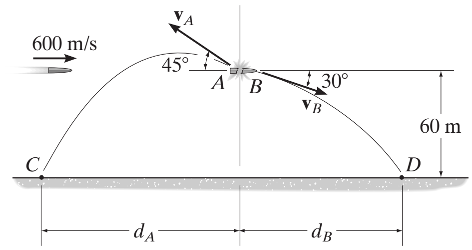
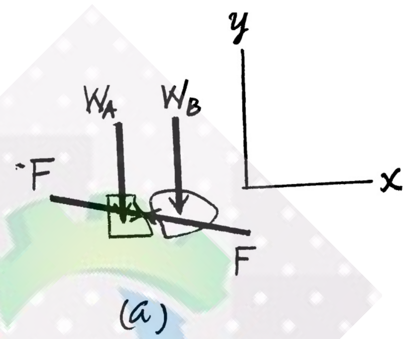
15–45. A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_B where segment B strikes the ground at D.



SOLUTION

Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. a, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$(\xrightarrow{+})$ $$mv_x = m_A(v_A)_x + m_B(v_B)_x$$

$$4(600) = -1.5v_A \cos 45° + 2.5v_B \cos 30°$$

$$2.165v_B - 1.061v_A = 2400 \qquad (1)$$

$(+\uparrow)$ $$mv_y = m_A(v_A)_y + m_B(v_B)_y$$

$$0 = 1.5v_A \sin 45° - 2.5v_B \sin 30°$$

$$v_B = 0.8485v_A \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

By considering the x and y motion of segment B,

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$$

$$-60 = 0 - 2622.77 \sin 30°\, t_{BD} + \frac{1}{2}(-9.81)t_{BD}{}^2$$

$$4.905t_{BD}{}^2 + 1311.38t_{BD} - 60 = 0$$

Solving for the positive root of the above equation,

$$t_{BD} = 0.04574 \text{ s}$$

and

$(\xrightarrow{+})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d_B = 0 + 2622.77 \cos 30°(0.04574)$$

$$= 103.91 \text{ m} = 104 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$v_A = 3.09(10^3)$ m/s
$v_B = 2.62(10^3)$ m/s
$d_B = 104$ m

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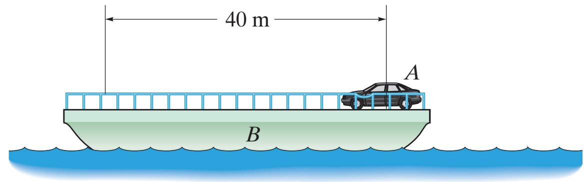
15–46.

The 10-Mg barge B supports a 2-Mg automobile A. If someone drives the automobile to the other side of the barge, determine how far the barge moves. Neglect the resistance of the water.



SOLUTION

Conservation of Momentum. Assuming that $\mathbf{V}_B$ is to the left,

$$(\overset{+}{\leftarrow}) \quad m_A v_A + m_B v_B = 0$$

$$2(10^3)v_A + 10(10^3)v_B = 0$$

$$2\,v_A + 10\,v_B = 0$$

Integrate this equation,

$$2\,s_A + 10\,s_B = 0 \qquad \textbf{(1)}$$

Kinematics. Here, $s_{A/B} = 40\text{ m} \leftarrow$, using the relative displacement equation by assuming that $\mathbf{s}_B$ is to the left,

$$(\overset{+}{\leftarrow}) \quad \mathbf{s}_A = \mathbf{s}_B + \mathbf{s}_{A/B}$$

$$\mathbf{s}_A = \mathbf{s}_B + 40 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2),

$$s_B = -6.6667\text{ m} = 6.67\text{ m} \rightarrow \qquad \textbf{Ans.}$$

$$s_A = 33.33\text{ m} \leftarrow$$

The negative sign indicates that $\mathbf{s}_B$ is directed to the right which is opposite to that of the assumed.

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Ans:
$s_B = 6.67\text{ m} \rightarrow$

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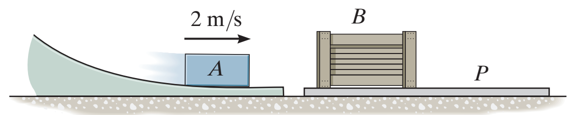
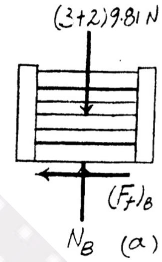
15–47. Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.4$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.5$.



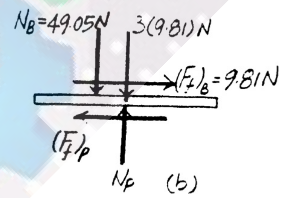
SOLUTION



Equations of Equilibrium: From FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad N_B - (3+2)(9.81) = 0 \qquad N_B = 49.05\text{ N}$$

When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).



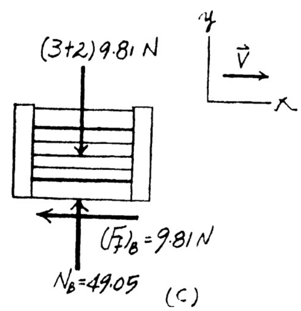
$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81\text{ N}$$

Since $(F_f)_P < \left[(F_f)_P\right]_{\max} = \mu_s' N_P = 0.5(78.48) = 39.24$ N, plate P does not move. Thus

$$s_P = 0 \qquad \textbf{Ans.}$$

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along the x axis.



$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R)\, v_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(2) + 0 = (2+3)\, v_2$$

$$v_2 = 0.800\text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(0.8) + \left[-9.81(t)\right] = 5(0)$$

$$t = 0.408\text{ s} \qquad \textbf{Ans.}$$

Ans:
$s_P = 0$
$t = 0.408$ s

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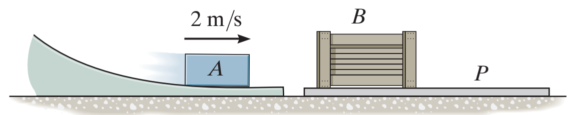
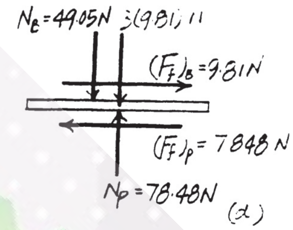
***15–48.** Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.1$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.12$.



SOLUTION



Equations of Equilibrium: From FBD(a),

$$+\uparrow \Sigma F_x = 0; \qquad N_B - (3 + 2)(9.81) = 0 \qquad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P. $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).

$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P > \left[(F_f)_P\right]_{\max} = \mu_s{}' N_P = 0.12(78.48) = 9.418\text{N}$, plate P slides. Thus, $(F_f)_P = \mu_k{}' N_P = 0.1(78.48) = 7.848$ N.

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along x axis.

$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R)\, v_2$$

$$(\xrightarrow{+}) \qquad 2(2) + 0 = (2 + 3)\, v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: In order for box B to stop sliding on plate P, both box B and plate P must have same speed v_3. Applying Eq. 15–4 to box B (FBD(c)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\xrightarrow{+}) \qquad 5(0.8) + \left[-9.81(t_1)\right] = 5v_3 \qquad \textbf{[1]}$$

Applying Eq. 15–4 to plate P[FBD(d)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\xrightarrow{+}) \qquad 3(0) + 9.81(t_1) - 7.848(t_1) = 3v_3 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$t_1 = 0.3058 \text{ s} \qquad v_3 = 0.200 \text{ m/s}$$

Equation of Motion: From FBD(d), the acceleration of plate P when box B still slides on top of it is given by

$$\xrightarrow{+} \Sigma Fx = ma_x; \qquad 9.81 - 7.848 = 3(a_P)_1 \qquad (a_P)_1 = 0.654 \text{ m/s}^2$$

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***15–48. Continued**

When box B stop slid ling on top of box B, $(F_f)_B = 0$. From this instant onward plate P and box B act as a unit and slide together. From FBD(d), the acceleration of plate P and box B is given by

$$\overset{+}{\rightarrow} \Sigma Fx = ma_x; \qquad -7.848 = 8(a_P)_2 \qquad (a_P)_2 = -0.981 \text{ m/s}^2$$

Kinematics: Plate P travels a distance s_1 before box B stop sliding.

$$(\overset{+}{\rightarrow}) \qquad s_1 = (v_0)_P\, t_1 + \frac{1}{2}(a_P)_1\, t_1^2$$

$$= 0 + \frac{1}{2}(0.654)\left(0.3058^2\right) = 0.03058 \text{ m}$$

The time t_2 for plate P to stop after box B stop slidding is given by

$$(\overset{+}{\rightarrow}) \qquad v_4 = v_3 + (a_P)_2\, t_2$$

$$0 = 0.200 + (-0.981)t_2 \qquad t_2 = 0.2039 \text{ s}$$

The distance s_2 traveled by plate P after box B stop sliding is given by

$$(\overset{+}{\rightarrow}) \qquad v_4^2 = v_3^2 + 2(a_P)_2\, s_2$$

$$0 = 0.200^2 + 2(-0.981)s_2 \qquad s_2 = 0.02039 \text{ m}$$

The total distance travel by plate P is

$$s_P = s_1 + s_2 = 0.03058 + 0.02039 = 0.05097 \text{ m} = 51.0 \text{ mm} \qquad \textbf{Ans.}$$

The total time taken to cease all the motion is

$$t_{\text{Tot}} = t_1 + t_2 = 0.3058 + 0.2039 = 0.510 \text{ s} \qquad \textbf{Ans.}$$

Ans:

$s_P = 51.0$ mm
$t_{\text{Tot}} = 0.510$ s

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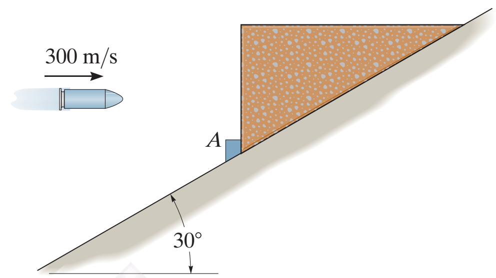
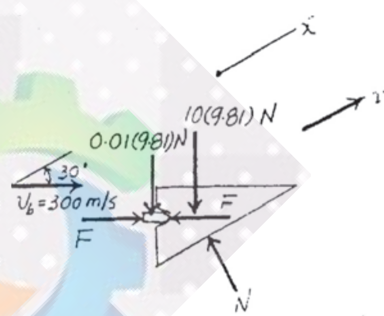
15–49.

The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



SOLUTION

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)\,v_{x'}$$

$$0.01(300 \cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy: The datum is set at the blocks initial position. When the block and the embedded bullet is at their highest point they are h *above* the datum. Their gravitational potential energy is $(10 + 0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10 + 0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm}$$ **Ans.**

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Ans:
$d = 6.87$ mm

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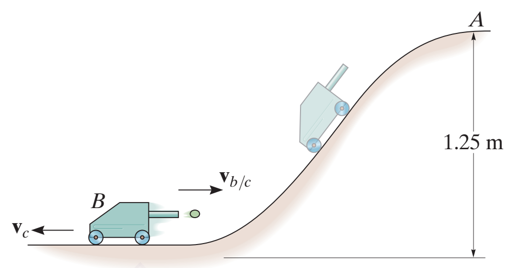
15–50.

The cart has a mass of 3 kg and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a 0.5-kg ball out the back with a horizontal velocity of $v_{b/c} = 0.6$ m/s, measured relative to the cart. Determine the final velocity of the cart.



SOLUTION

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + (3 + 0.5)(9.81)(1.25) = \frac{1}{2}(3 + 0.5)(\nu_B)_2^2 + 0$$

$$\nu_B = 4.952 \text{ m/s}$$

$(\overset{+}{\leftarrow})$ $\quad \Sigma m\nu_1 = \Sigma m\nu_2$

$$(3 + 0.5)(4.952) = (3)\nu_c - (0.5)\nu_b \qquad \textbf{(1)}$$

$(\overset{+}{\leftarrow})$ $\quad \nu_b = \nu_c + \nu_{b/c}$

$$-\nu_b = \nu_c - 0.6 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$\nu_c = 5.04 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$\nu_b = -4.44 \text{ m/s} = 4.44 \text{ m/s} \leftarrow$$

Ans:
$v_c = 5.04$ m/s $\leftarrow$

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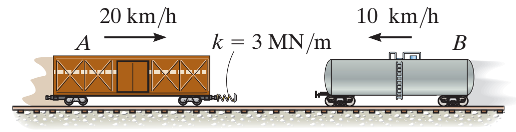
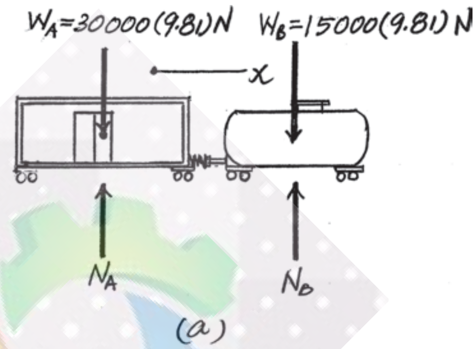
15–51.

The 30-Mg freight car A and 15-Mg freight car B are moving towards each other with the velocities shown. Determine the maximum compression of the spring mounted on car A. Neglect rolling resistance.



SOLUTION



Conservation of Linear Momentum: Referring to the free-body diagram of the freight cars A and B shown in Fig. *a*, notice that the linear momentum of the system is conserved along the x axis. The initial speed of freight cars A and B are $(v_A)_1 = \left[20(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5.556\text{ m/s}$ and $(v_B)_1 = \left[10(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 2.778\text{ m/s}$. At this instant, the spring is compressed to its maximum, and no relative motion occurs between freight cars A and B and they move with a common speed.

$$(\overset{+}{\rightarrow})\qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$30(10^3)(5.556) + \left[-15(10^3)(2.778)\right] = \left[30(10^3) + 15(10^3)\right]v_2$$

$$v_2 = 2.778\text{ m/s}\rightarrow$$

Conservation of Energy: The initial and final elastic potential energy of the spring is $(V_e)_1 = \frac{1}{2}ks_1^2 = 0$ and $(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(3)(10^6)s_{max}^2 = 1.5(10^6)s_{max}^2$.

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$\left[\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2\right] + (V_e)_1 = \frac{1}{2}(m_A + m_B)v_2^2 + (V_e)_2$$

$$\frac{1}{2}(30)\left(10^3\right)(5.556^2) + \frac{1}{2}(15)\left(10^3\right)\left(2.778^2\right) + 0$$

$$= \frac{1}{2}\left[30\left(10^3\right) + 15\left(10^3\right)\right]\left(2.778^2\right) + 1.5\left(10^6\right)s_{max}^2$$

$s_{max} = 0.4811\text{ m} = 481\text{ mm}$ **Ans.**

Ans:
$s_{max} = 481$ mm

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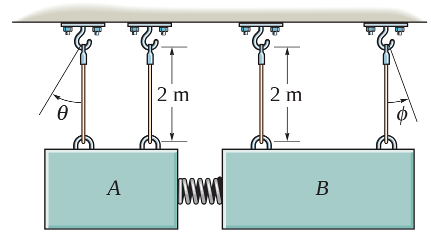
***15–52.**

The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



SOLUTION

$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348 \text{ m/s}$$

For A or B:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$\theta = \phi = 9.52°$ **Ans.**

Ans:
$\theta = \phi = 9.52°$

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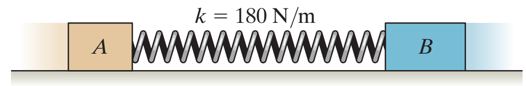
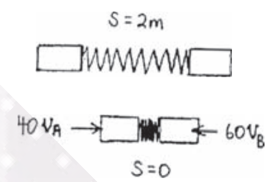
15–53.

Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



SOLUTION



$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 40\,v_{\text{A}} - 60\,v_{\text{B}}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$$

$v_A = 3.29$ m/s **Ans.**

$v_B = 2.19$ m/s **Ans.**

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Ans:
$v_A = 3.29$ m/s
$v_B = 2.19$ m/s

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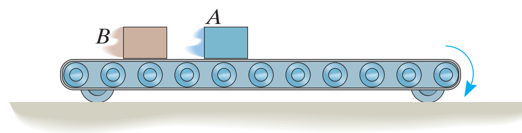
15–54.

Two boxes A and B, each having a mass of 80 kg, sit on the 250-kg conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 1 m/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



SOLUTION

a) Let v_b be the velocity of A and B.

$(\overset{+}{\rightarrow})$ $$\Sigma m v_1 = \Sigma m v_2$$

$$0 = (160)(v_b) - (250)(v_c)$$

$(\overset{+}{\rightarrow})$ $$v_b = v_c + v_{b/c}$$

$$v_b = -v_c + 1$$

Thus, $v_b = 0.610 \text{ m/s} \rightarrow \qquad v_c = 0.390 \text{ m/s} \leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

a) $v_c = 0.390 \text{ m/s} \leftarrow$ **Ans.**

b) $v_c = 0.390 \text{ m/s} \leftarrow$ **Ans.**

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Ans:
$v_C = 0.390 \text{ m/s} \leftarrow$
$v_C = 0.390 \text{ m/s} \leftarrow$

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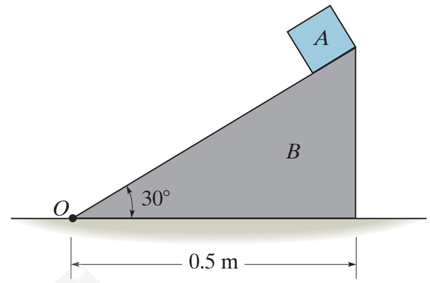
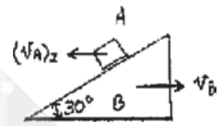
15–55.

Block A has a mass of 5 kg and is placed on the smooth triangular block B having a mass of 30 kg. If the system is released from rest, determine the distance B moves from point O when A reaches the bottom. Neglect the size of block A.



SOLUTION



$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$0 = 30 v_B - 5(v_A)_x$$

$$(v_A)_x = 6 v_B$$

$$v_B = v_A + v_{B/A}$$

$(\overset{+}{\rightarrow})$ $v_B = -(v_A)_x + (v_{B/A})_x$

$$v_B = -6 v_B + (v_{B/A})_x$$

$$(v_{B/A})_x = 7 v_B$$

Integrate

$$(s_{B/A})_x = 7 s_B$$

$$(s_{B/A})_x = 0.5 \text{ m}$$

Thus,

$$s_B = \frac{0.5}{7} = 0.0714 \text{ m} = 71.4 \text{ mm} \rightarrow$$ **Ans.**

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Ans:
$s_B = 71.4 \text{ mm} \rightarrow$

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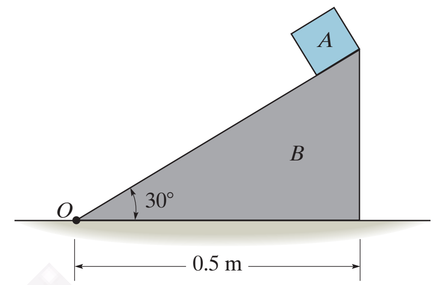
***15–56.**

Solve Prob. 15–55 if the coefficient of kinetic friction between A and B is $\mu_k = 0.3$. Neglect friction between block B and the horizontal plane.



SOLUTION

$+\nwarrow\Sigma F_y = 0; \qquad N_A - 5(9.81)\cos 30° = 0$

$N_A = 42.4785\text{ N}$

$\nearrow+\Sigma F_x = 0; \qquad F_A - 5(9.81)\sin 30° = 0$

$F_A = 24.525\text{ N}$

$F_{max} = \mu N_A = 0.3(42.4785) = 12.74\text{ N} < 24.525\text{ N}$

Block indeed slides.

Solution is the same as in Prob. 15–55. Since F_A is internal to the system.

$s_B = 71.4\text{ mm} \rightarrow$ **Ans.**

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Ans:
$s_B = 71.4\text{ mm} \rightarrow$

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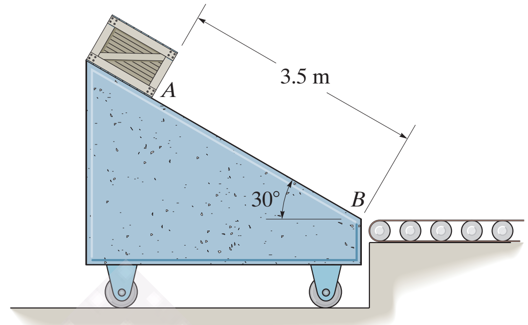
15–57.

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



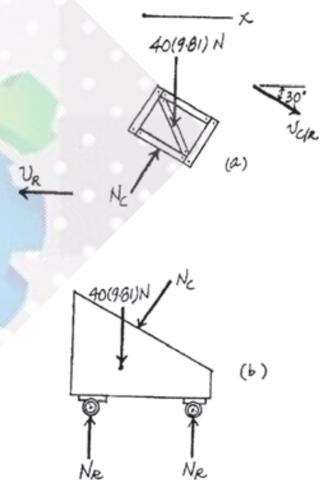
SOLUTION

Conservation of Energy: The datum is set at lowest point B. When the crate is at point A, it is $3.5 \sin 30^\circ = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675\ \text{N}\cdot\text{m}$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)v_C^2 + \frac{1}{2}(40)v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad \textbf{(1)}$$



Relative Velocity: The velocity of the crate is given by

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_R + \mathbf{v}_{C/R}\\ &= -v_R\mathbf{i} + (v_{C/R}\cos 30^\circ\mathbf{i} - v_{C/R}\sin 30^\circ\mathbf{j})\\ &= (0.8660\,v_{C/R} - v_R)\mathbf{i} - 0.5\,v_{C/R}\mathbf{j} \qquad \textbf{(2)}\end{aligned}$$

The magnitude of v_C is

$$\begin{aligned}v_C &= \sqrt{(0.8660\,v_{C/R} - v_R)^2 + (-0.5\,v_{C/R})^2}\\ &= \sqrt{v_{C/R}^2 + v_R^2 - 1.732\,v_R v_{C/R}} \qquad \textbf{(3)}\end{aligned}$$

Conservation of Linear Momentum: If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660\,v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660\,v_{C/R} - 50\,v_R \qquad \textbf{(4)}$$

Solving Eqs. (1), (3), and (4) yields

$$v_R = 1.101\text{ m/s} = 1.10\text{ m/s} \quad v_C = 5.43\text{ m/s} \qquad \textbf{Ans.}$$

$$v_{C/R} = 6.356\text{ m/s}$$

From Eq. (2)

$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\}\text{ m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8^\circ \quad ⦪\phi \qquad \textbf{Ans.}$$

Ans:
$v_{C/R} = 6.356$ m/s
$\phi = 35.8^\circ$ ⦪

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15–58.

Disk A has a mass of 250 g and is sliding on a *smooth* horizontal surface with an initial velocity $(v_A)_1 = 2\text{ m/s}$. It makes a direct collision with disk B, which has a mass of 175 g and is originally at rest. If both disks are of the same size and the collision is perfectly elastic ($e = 1$), determine the velocity of each disk just after collision. Show that the kinetic energy of the disks before and after collision is the same.

SOLUTION

$(\xrightarrow{+})\qquad (0.250)(2) + 0 = (0.250)(v_A)_2 + (0.175)(v_B)_2$

$(\xrightarrow{+})\qquad e = 1 = \dfrac{(v_B)_2 - (v_A)_2}{2 - 0}$

Solving

$(v_A)_2 = 0.353\text{ m/s}$ **Ans.**

$(v_B)_2 = 2.35\text{ m/s}$ **Ans.**

$$T_1 = \frac{1}{2}(0.25)(2)^2 = 0.5\text{ J}$$

$$T_2 = \frac{1}{2}(0.25)(0.353)^2 + \frac{1}{2}(0.175)(2.35)^2 = 0.5\text{ J}$$

$T_1 = T_2$ **QED**

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Ans:
$(v_A)_2 = 0.353\text{ m/s}$
$(v_B)_2 = 2.35\text{ m/s}$

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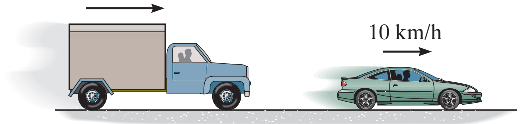
15–59.

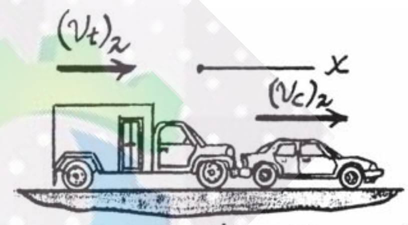
The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.



SOLUTION

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 2.778\text{ m/s}$.



Just after impact
(a)

By referring to Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m_t(v_t)_1 + m_c(v_c)_1 = m_t(v_t)_2 + m_c(v_c)_2$$

$$5000(8.333) + 2000(2.778) = 5000(v_t)_2 + 2000(v_c)_2$$

$$5(v_t)_2 + 2(v_c)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$. Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$(\overset{+}{\rightarrow}) \qquad (v_c)_2 = (v_t)_2 + 4.167$$

$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

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15–59. Continued

Substituting Eq. (2) into Eq. (3),

$$e = \frac{4.167}{8.333 - 2.778} = 0.75$$ **Ans.**

Solving Eqs. (1) and (2) yields

$$(v_t)_2 = 5.556 \text{ m/s}$$

$$(v_c)_2 = 9.722 \text{ m/s}$$

Kinetic Energy: The kinetic energy of the system just before and just after the collision are

$$T_1 = \frac{1}{2} m_t (v_t)_1{}^2 + \frac{1}{2} m_c (v_c)_1{}^2$$

$$= \frac{1}{2}(5000)(8.333^2) + \frac{1}{2}(2000)(2.778^2)$$

$$= 181.33\left(10^3\right) \text{ J}$$

$$T_2 = \frac{1}{2} m_t (v_t)_2{}^2 + \frac{1}{2} m_c (v_c)_2{}^2$$

$$= \frac{1}{2}(5000)(5.556^2) + \frac{1}{2}(2000)(9.722^2)$$

$$= 171.68\left(10^3\right) \text{J}$$

Thus,

$$\Delta T = T_1 - T_2 = 181.33\left(10^3\right) - 171.68\left(10^3\right)$$

$$= 9.645\left(10^3\right) \text{ J}$$

$$= 9.65 \text{ kJ}$$ **Ans.**

Ans:
$e = 0.75$
$\Delta T = -9.65 \text{ kJ}$

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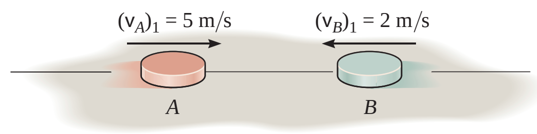
***15–60.**

Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.

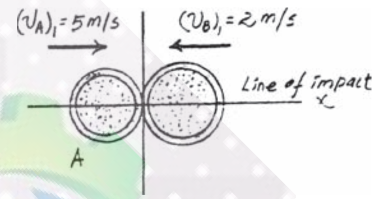


SOLUTION

Conservation of Momentum :

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad \textbf{(1)}$$



Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = -1.53 \text{ m/s} = 1.53 \text{ m/s} \leftarrow \qquad (v_B)_2 = 1.27 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

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Ans:
$(v_A)_2 = 1.53$ m/s $\leftarrow$
$(v_B)_2 = 1.27$ m/s $\rightarrow$

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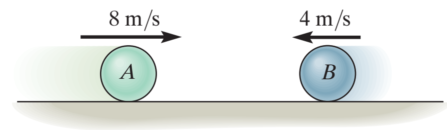
15–61.

Ball A has a mass of 3 kg and is moving with a velocity of 8 m/s when it makes a direct collision with ball B, which has a mass of 2 kg and is moving with a velocity of 4 m/s. If $e = 0.7$, determine the velocity of each ball just after the collision. Neglect the size of the balls.



SOLUTION

Conservation of Momentum. The velocity of balls A and B before and after impact are shown in Fig. a

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$3(8) + 2(-4) = 3v_A + 2v_B$$

$$3v_A + 2v_B = 16 \qquad \textbf{(1)}$$

Coefficient of Restitution.

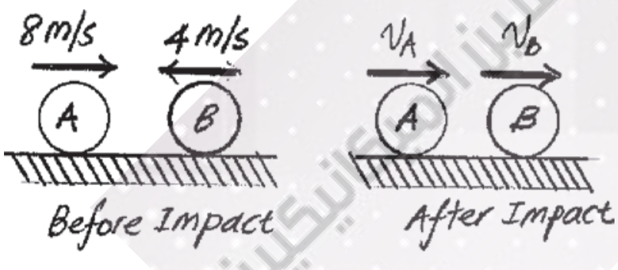
$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \quad 0.7 = \frac{v_B - v_A}{8 - (-4)}$$

$$v_B - v_A = 8.4 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$v_B = 8.24 \text{ m/s} \rightarrow$ **Ans.**

$v_A = -0.16 \text{ m/s} = 0.160 \text{ m/s} \leftarrow$ **Ans.**



(a)

Ans:
$v_B = 8.24 \text{ m/s} \rightarrow$
$v_A = 0.160 \text{ m/s} \leftarrow$

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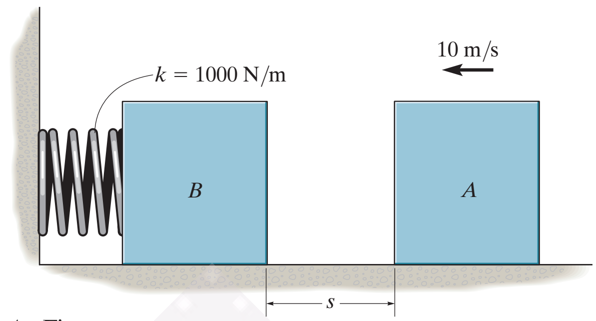
15–62.

The 15-kg block A slides on the surface for which $m_k = 0.3$. The block has a velocity $v = 10\text{ m/s}$ when it is $s = 4\text{ m}$ from the 10-kg block B. If the unstretched spring has a stiffness $k = 1000\text{ N/m}$, determine the maximum compression of the spring due to the collision. Take $e = 0.6$.



SOLUTION

Principle of Work and Energy. Referring to the FBD of block A, Fig. a, motion along the y axis gives $N_A = 15(9.81) = 147.15\text{ N}$. Thus the friction is $F_f = \mu_k N_A = 0.3(147.15) = 44.145\text{ N}$.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(15)\left(10^2\right) + (-44.145)(4) = \frac{1}{2}(15)(v_A)_1^2$$

$$(v_A)_1 = 8.7439\text{ m/s} \leftarrow$$

Conservation of Momentum.

$$(\overset{+}{\leftarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$15(8.7439) + 0 = 15(v_A)_2 + 10(v_B)_2$$

$$3(v_A)_2 + 2(v_B)_2 = 26.2317 \qquad \textbf{(1)}$$

Coefficient of Restitution.

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.6 = \frac{(v_B)_2 - (v_A)_2}{8.7439 - 0}$$

$$(v_B)_2 - (v_A)_2 = 5.2463 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

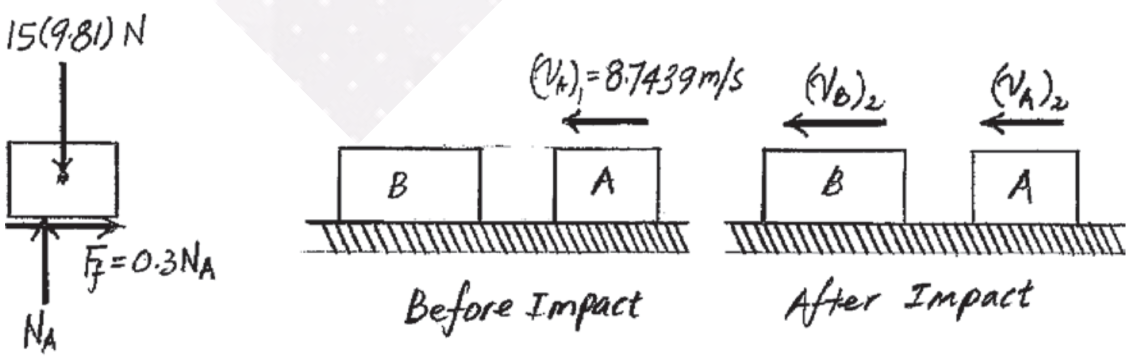
$$(v_B)_2 = 8.3942\text{ m/s} \leftarrow \quad (v_A)_2 = 3.1478\text{ m/s} \leftarrow$$

Conservation of Energy. When block B stops momentarily, the compression of the spring is maximum. Thus, $T_2 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(10)\left(8.3942^2\right) + 0 = 0 + \frac{1}{2}(1000)x_{\max}^2$$

$$x_{\max} = 0.8394\text{ m} = 0.839\text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$x_{\max} = 0.839\text{ m}$

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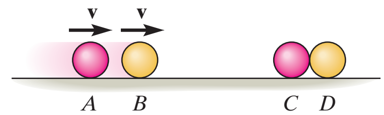
15–63.

The four smooth balls each have the same mass m. If A and B are rolling forward with velocity $\mathbf{v}$ and strike C, explain why after collision C and D each move off with velocity $\mathbf{v}$. Why doesn't D move off with velocity $2\mathbf{v}$? The collision is elastic, $e = 1$. Neglect the size of each ball.



SOLUTION

Collision will occur in the following sequence;

B strikes C

$(\overset{+}{\rightarrow}) \qquad mv = -mv_B + mv_C$

$$v = -v_B + v_C$$

$(\overset{+}{\rightarrow}) \qquad e = 1 = \dfrac{v_C + v_B}{v}$

$$v_C = v, \qquad v_B = 0$$

C strikes D

$(\overset{+}{\rightarrow}) \qquad mv = -mv_C + mv_D$

$(\overset{+}{\rightarrow}) \qquad e = 1 = \dfrac{v_D + v_C}{v}$

$v_C = 0, \qquad v_D = v$ **Ans.**

A strikes B

$(\overset{+}{\rightarrow}) \qquad mv = -mv_A + mv_B$

$(\overset{+}{\rightarrow}) \qquad e = 1 = \dfrac{v_B + v_A}{v}$

$v_B = v, \qquad v_A = 0$ **Ans.**

Finally, B strikes C

$(\overset{+}{\rightarrow}) \qquad mv = -mv_B + mv_C$

$(\overset{+}{\rightarrow}) \qquad e = 1 = \dfrac{v_C + v_B}{v}$

$v_C = v, \qquad v_B = 0$ **Ans.**

Note: If D rolled off with twice the velocity, its kinetic energy would be twice the energy available from the original two A and B: $\left(\frac{1}{2}mv^2 + \frac{1}{2}mv^2 \neq \frac{1}{2}(2v)^2\right)$

Ans:
$v_C = 0, v_D = v$
$v_B = v, v_A = 0$
$v_C = v, v_B = 0$

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***15–64.**

The four balls each have the same mass m. If A and B are rolling forward with velocity $\mathbf{v}$ and strike C, determine the velocity of each ball after the first three collisions. Take $e = 0.5$ between each ball.

SOLUTION

Collision will occur in the following sequence;

B strikes C

$(\overset{+}{\rightarrow})$ $$mv = mv_B + mv_c$$

$$v = v_B + v_C$$

$(\overset{+}{\rightarrow})$ $$e = 0.5 = \frac{v_C - v_B}{v}$$

$$v_C = 0.75v \rightarrow, \qquad v_B = 0.25v \rightarrow$$

C strikes D

$(\overset{+}{\rightarrow})$ $$m(0.75v) = mv_C + mv_D$$

$(\overset{+}{\rightarrow})$ $$e = 0.5 = \frac{v_D - v_C}{0.75v}$$

$$v_C = 0.1875v \rightarrow$$ **Ans.**

$$v_D = 0.5625v \rightarrow$$ **Ans.**

A strikes B

$(\overset{+}{\rightarrow})$ $$mv + m(0.25v) = mv_A + mv_B$$

$(\overset{+}{\rightarrow})$ $$e = 0.5 = \frac{v_B - v_A}{(v - 0.25v)}$$

$$v_B = 0.8125v \rightarrow, \qquad v_A = 0.4375v \rightarrow$$ **Ans.**

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Ans:
$v_C = 0.1875v \rightarrow$
$v_D = 0.5625v \rightarrow$
$v_B = 0.8125v \rightarrow$
$v_A = 0.4375v \rightarrow$

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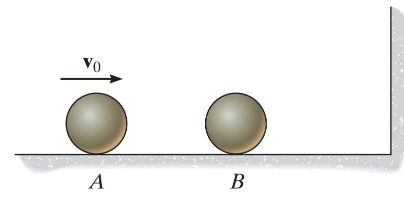
15–65.

Two smooth spheres A and B each have a mass m. If A is given a velocity of v_0, while sphere B is at rest, determine the velocity of B just after it strikes the wall. The coefficient of restitution for any collision is e.



SOLUTION

Impact: The first impact occurs when sphere A strikes sphere B. When this occurs, the linear momentum of the system is conserved along the x axis (line of impact). Referring to Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m_A v_A + m_B v_B = m_A (v_A)_1 + m_B (v_B)_1$$

$$m v_0 + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = v_0 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_1 - (v_A)_1}{v_A - v_B}$$

$$e = \frac{(v_B)_1 - (v_A)_1}{v_0 - 0}$$

$$(v_B)_1 - (v_A)_1 = e v_0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

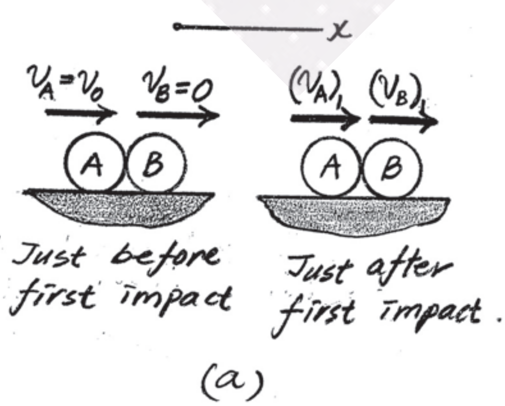
$$(v_B)_1 = \left(\frac{1+e}{2}\right) v_0 \rightarrow \qquad (v_A)_1 = \left(\frac{1-e}{2}\right) v_0 \rightarrow$$

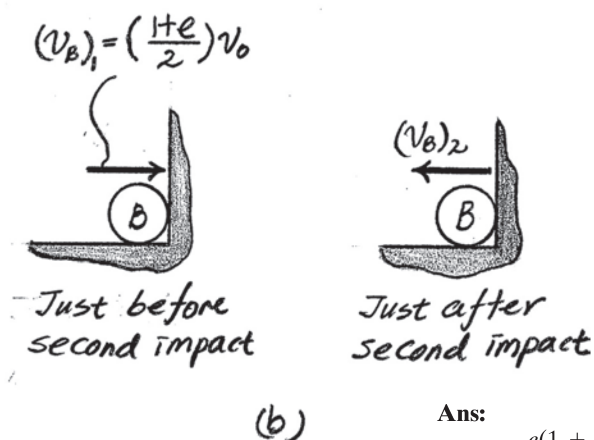
The second impact occurs when sphere B strikes the wall, Fig. b. Since the wall does not move during the impact, the coefficient of restitution can be written as

$$(\overset{+}{\rightarrow}) \qquad e = \frac{0 - \left[-(v_B)_2\right]}{(v_B)_1 - 0}$$

$$e = \frac{0 + (v_B)_2}{\left[\dfrac{1+e}{2}\right] v_0 - 0}$$

$$(v_B)_2 = \frac{e(1+e)}{2} v_0 \qquad \textbf{Ans.}$$





Ans:

$$(v_B)_2 = \frac{e(1+e)}{2} v_0$$

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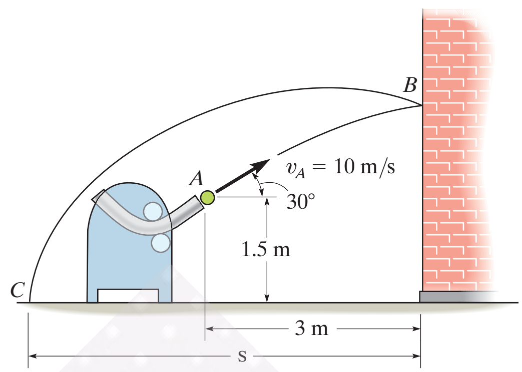
15–66.

A pitching machine throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10\text{ m/s}$ as shown. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

(a)

$$(v_B)_{x1} = 10\cos 30^\circ = 8.660\text{ m/s}\rightarrow$$

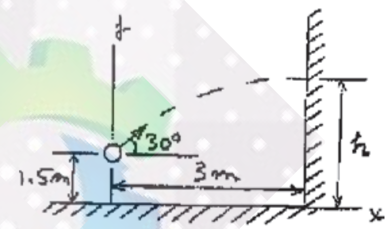
$$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$$

$$3 = 0 + 10\cos 30^\circ t$$

$$t = 0.3464\text{ s}$$

$$(+\uparrow)\qquad v = v_0 + a_c t$$

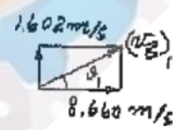
$$(v_B)_{y\,t} = 10\ \sin 30^\circ - 9.81(0.3464) = 1.602\text{ m/s}\uparrow$$



$$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

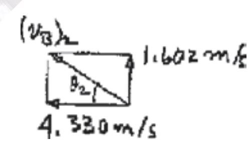
$$h = 1.5 + 10\sin 30^\circ(0.3464) - \frac{1}{2}(9.81)(0.3464)^2$$

$$= 2.643\text{ m}$$



$$(v_B)_1 = \sqrt{(1.602)^2 + (8.660)^2} = 8.81\text{ m/s}$$ **Ans.**

$$\theta_1 = \tan^{-1}\left(\frac{1.602}{8.660}\right) = 10.5^\circ\ \nearrow$$ **Ans.**



(b)

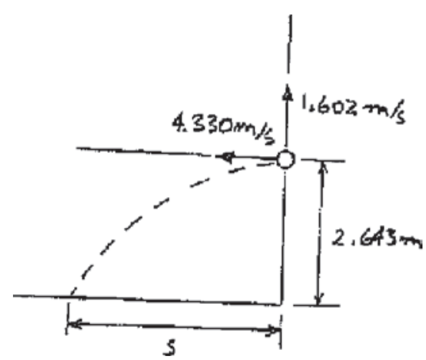
$$(\overset{+}{\rightarrow})\qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1};\qquad 0.5 = \frac{(v_{Bx})_2 - 0}{0 - (8.660)}$$

$$(v_{Bx})_2 = 4.330\text{ m/s}\leftarrow$$

$$(v_{By})_2 = (v_{By})_1 = 1.602\text{ m/s}\uparrow$$

$$(v_B)_2 = \sqrt{(4.330)^2 + (1.602)^2} = 4.62\text{ m/s}$$ **Ans.**

$$\theta_2 = \tan^{-1}\left(\frac{1.602}{4.330}\right) = 20.3^\circ\ \nwarrow$$ **Ans.**



(c)

$$(+\uparrow)\qquad s = s_0 + v_{Bt} + \frac{1}{2}a_c t^2$$

$$-2.643 = 0 + 1.602(t) - \frac{1}{2}(9.81)(t)^2$$

$$t = 0.9153\text{ s}$$

$$(\overset{+}{\leftarrow})\qquad s = s_0 + v_0 t$$

$$s = 0 + 4.330(0.9153) = 3.96\text{ m}$$ **Ans.**

Ans:
$(v_B)_1 = 8.81\text{ m/s}$
$\theta_1 = 10.5^\circ\ \nearrow$
$(v_B)_2 = 4.62\text{ m/s}$
$\theta_2 = 20.3^\circ\ \nwarrow$
$s = 3.96\text{ m}$

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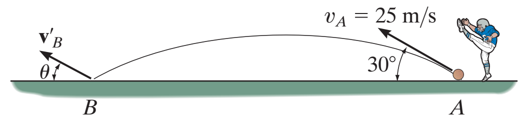
15–67.

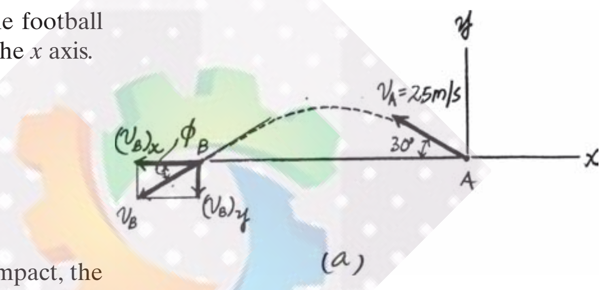
A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



SOLUTION

Kinematics: The parabolic trajectory of the football is shown in Fig. *a*. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25$ m/s and $\phi = 30°$.

Conservation of Linear Momentum: Since no impulsive force acts on the football along the x axis, the linear momentum of the football is conserved along the x axis.



$$\left(\overset{+}{\leftarrow}\right) \qquad m\left(v_B\right)_x = m\left(v'_B\right)_x$$

$$0.3\left(25\cos 30°\right) = 0.3\left(v'_B\right)_x$$

$$\left(v'_B\right)_x = 21.65 \text{ m/s} \leftarrow$$

Coefficient of Restitution: Since the ground does not move during the impact, the coefficient of restitution can be written as

$$(+\uparrow) \qquad e = \frac{0 - \left(v'_B\right)_y}{\left(v_B\right)_y - 0}$$

$$0.4 = \frac{-\left(v'_B\right)_y}{-25\sin 30°}$$

$$\left(v'_B\right)_y = 5 \text{ m/s} \uparrow$$

Thus, the magnitude of $\mathbf{v}'_B$ is

$$v'_B = \sqrt{\left(v'_B\right)_x + \left(v'_B\right)_y} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \qquad \textbf{Ans.}$$

and the angle of $\mathbf{v}'_B$ is

$$\theta = \tan^{-1}\left[\frac{\left(v'_B\right)_y}{\left(v'_B\right)_x}\right] = \tan^{-1}\left(\frac{5}{21.65}\right) = 13.0° \qquad \textbf{Ans.}$$

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Ans:
$v'_B = 22.2$ m/s
$\theta = 13.0°$

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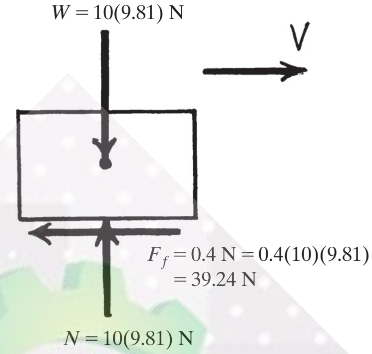
***15–68.** The 1-kg ball A is traveling horizontally at 20 m/s when it strikes a 10-kg block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the distance block B slides on the plane before it stops sliding.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$(1)(20) + 0 = (1)(v_A)_2 + (10)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ m/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ m/s} = 9.091 \text{ m/s} \leftarrow$

Block B:

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(10)(2.909)^2 - 39.24d = 0$

$d = 1.078 \text{ m}$ **Ans.**

Ans:
$d = 1.078$ m

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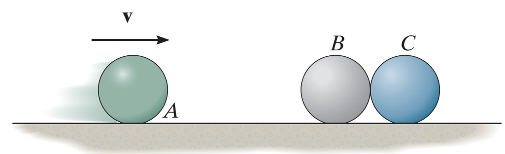
15–69.

The three balls each have a mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each pair of balls e. Neglect the size of each ball.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = \frac{v(1 - e)}{2} \qquad (v_B)_2 = \frac{v(1 + e)}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B\,(v_B)_2 + m_C\,(v_C)_1 = m_B\,(v_B)_3 + m_C\,(v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1 + e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{(3)}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1 + e)}{2} - 0} \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_C)_2 = \frac{v(1 + e)^2}{4} \qquad \textbf{Ans.}$$

$$(v_B)_3 = \frac{v(1 - e^2)}{4}$$

Ans:

$$(v_C)_2 = \frac{v(1 + e)^2}{4}$$

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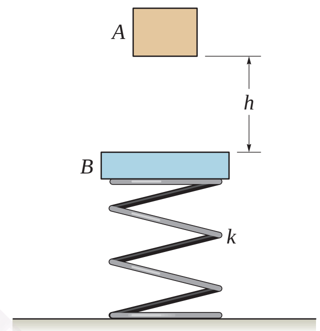
15–70.

Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



SOLUTION

Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$(+\downarrow)$ $$e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad \textbf{(1)}$$

$(+\downarrow)$ $$\Sigma mv_1 = \Sigma mv_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_B)_2$ yields;

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}(1 + e)$$ **Ans.**

Ans:

$(v_B)_2 = \frac{1}{3}\sqrt{2gh}\,(1 + e)$

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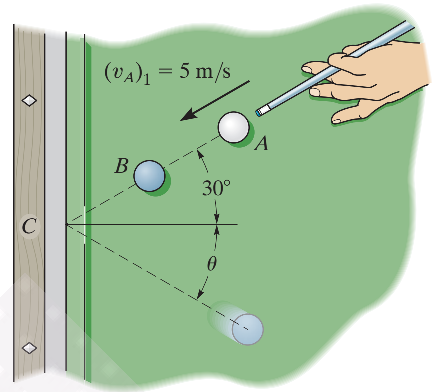
15–71.

The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect their size.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$(v_A)_2 = 0.500$ m/s $\quad (v_B)_2 = 4.50$ m/s

Conservation of "y" Momentum: When ball B strikes the cushion at C, we have

$$m_B(v_{B_y})_2 = m_B(v_{B_y})_3$$

$$(+\downarrow) \qquad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \qquad \textbf{(3)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_C)_2 - (v_{B_x})_3}{(v_{B_x})_2 - (v_C)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.6 = \frac{0 - [-(v_B)_3 \, \cos\theta]}{4.50 \cos 30° - 0} \qquad \textbf{(4)}$$

Solving Eqs. (1) and (2) yields

$(v_B)_3 = 3.24$ m/s $\quad \theta = 43.9°$ **Ans.**

Ans:
$(v_B)_3 = 3.24$ m/s
$\theta = 43.9°$

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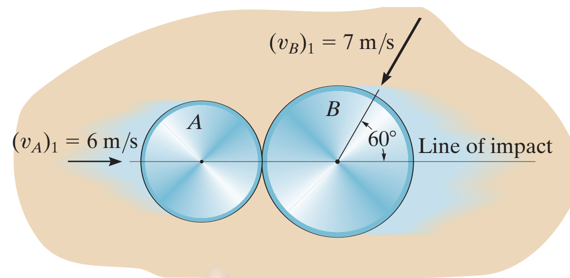
***15–72.**

The two disks A and B have a mass of 3 kg and 5 kg, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is $e = 0.65$.



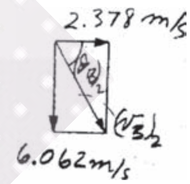
SOLUTION

$(v_{Ax}) = 6$ m/s $\qquad (v_{Ay})_1 = 0$

$(v_{Bx})_1 = -7\cos 60° = -3.5$ m/s $\qquad (v_{By})_1 = -7\cos 60° = -6.062$ m/s

$$(\overset{+}{\rightarrow}) \qquad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$3(6) - 5(3.5) = 3(v_A)_{x2} + 5(v_B)_{x2}$$



$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \qquad 0.65 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{6-(-3.5)}$$

$(v_{Bx})_2 - (v_{Ax})_2 = 6.175$

Solving,

$(v_{Ax})_2 = -3.80$ m/s $\qquad (v_{Bx})_2 = 2.378$ m/s

$$(+\uparrow) \qquad m_A(v_{Ay})_1 + m_A(v_{Ay})_2$$

$$(v_{Ay})_2 = 0$$

$$(+\uparrow) \qquad m_B(v_{By})_1 + m_B(v_{Ay})_2$$

$$(v_{By})_2 = -6.062 \text{ m/s}$$

$(v_A)_2 = \sqrt{(3.80)^2 + (0)^2 = } = 3.80 \text{ m/s} \leftarrow$ **Ans.**

$(v_B)_2 = \sqrt{(2.378)^2 + (-6.062)^2} = 6.51$ m/s **Ans.**

$(\theta_B)_2 = \tan^{-1}\left(\dfrac{6.062}{2.378}\right) = 68.6°$ **Ans.**

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Ans:
$(v_A)_2 = 3.80 \text{ m/s} \leftarrow$
$(v_B)_2 = 6.51$ m/s
$(\theta_B)_2 = 68.6°$

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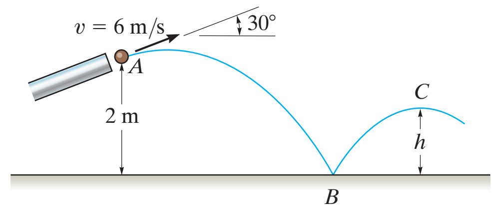
15–73.

The 0.5-kg ball is fired from the tube at A with a velocity of $v = 6$ m/s. If the coefficient of restitution between the ball and the surface is $e = 0.8$, determine the height h after it bounces off the surface.



SOLUTION

Kinematics. Consider the vertical motion from A to B.

$$(+\uparrow)\qquad (v_B)_y^2 = (v_A)_y^2 + 2a_y[(s_B)_y - (s_A)_y]\,;$$

$$(v_B)_y^2 = (6\sin 30°)^2 + 2(-9.81)(-2-0)$$

$$(v_B)_y = 6.9455 \text{ m/s}\downarrow$$

Coefficient of Restitution. The y-component of the rebounding velocity at B is $(v'_B)_y$ and the ground does not move. Then

$$(+\uparrow)\qquad e = \frac{(v_g)_2 - (v'_B)_y}{(v_B)_y - (v_g)_1}\,; \qquad 0.8 = \frac{0 - (v'_B)_y}{-6.9455 - 0}$$

$$(v'_B)_y = 5.5564 \text{ m/s}\uparrow$$

Kinematics. When the ball reach the maximum height h at C, $(v_c)_y = 0$.

$$(+\uparrow)\qquad (v_c)_y^2 = (v'_B)_y^2 + 2a_c[(s_c)_y - (s_B)_y]\,;$$

$$0^2 = 5.5564^2 + 2(-9.81)(h-0)$$

$$h = 1.574 \text{ m} = 1.57 \text{ m} \qquad \textbf{Ans.}$$

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Ans:
$h = 1.57$ m

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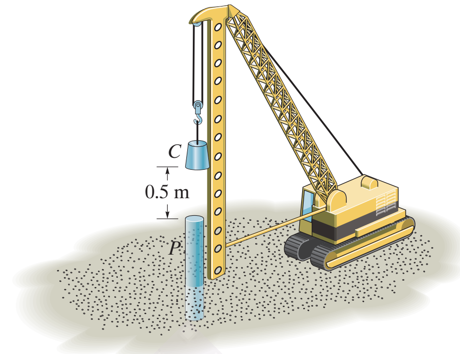
15–74. The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the initial speed of the pile just after it is struck by the hammer. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



SOLUTION

The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,

Counter weight: Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

$(v_P)_2 = 0.940$ m/s **Ans.**

$(v_C)_2 = 0.626$ m/s

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Ans:
$(v_P)_2 = 0.940$ m/s

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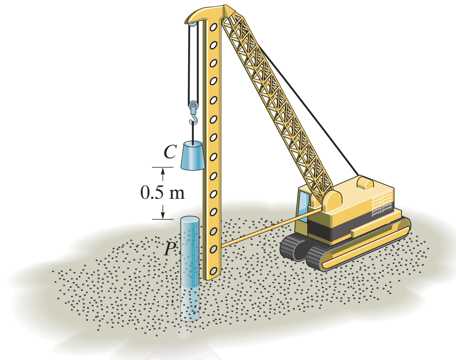
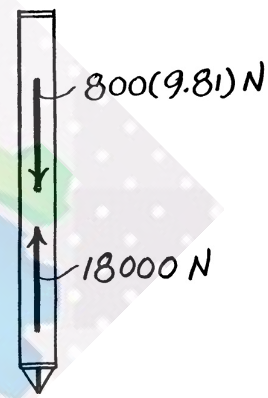
15–75. The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the distance the pile is driven into the sand after one blow if the sand offers a frictional resistance against the pile of 18 kN. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



SOLUTION

The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,



Counter weight: Datum at lowest point,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m\, v_1 = \Sigma m\, v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

$$(v_P)_2 = 0.9396 \text{ m/s}$$

$$(v_C)_2 = 0.6264 \text{ m/s}$$

Pile:

$$T_2 + \Sigma U_{2-3} = T_3$$

$$\frac{1}{2}(800)(0.9396)^2 + 800(9.81)d - 18\,000d = 0$$

$$d = 0.0348 \text{ m} = 34.8 \text{ mm} \qquad \textbf{Ans.}$$

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Ans:
$d = 34.8$ mm

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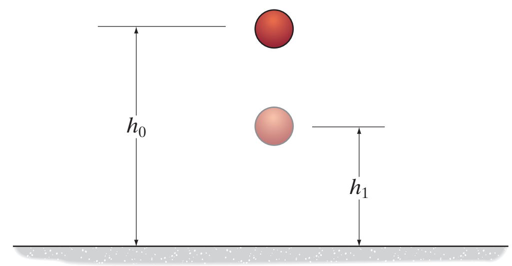
***15–76.**

A ball of mass m is dropped vertically from a height h_0 above the ground. If it rebounds to a height of h_1, determine the coefficient of restitution between the ball and the ground.



SOLUTION

Conservation of Energy: First, consider the ball's fall from position A to position B. Referring to Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + (V_g)_A = \frac{1}{2}mv_B{}^2 + (V_g)_B$$

$$0 + mg(h_0) = \frac{1}{2}m(v_B)_1{}^2 + 0$$

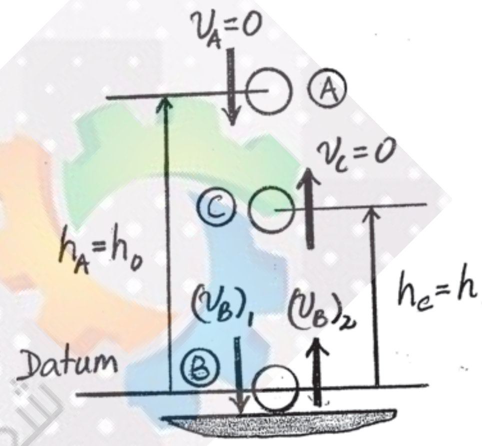
Subsequently, the ball's return from position B to position C will be considered.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B{}^2 + (V_g)_B = \frac{1}{2}mv_C{}^2 + (V_g)_C$$

$$\frac{1}{2}m(v_B)_2{}^2 + 0 = 0 + mgh_1$$

$$(v_B)_2 = \sqrt{2gh_1}\uparrow$$



(a)

Coefficient of Restitution: Since the ground does not move,

$$(+\uparrow) \qquad e = -\frac{(v_B)_2}{(v_B)_1}$$

$$e = -\frac{\sqrt{2gh_1}}{-\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}} \qquad \textbf{Ans.}$$

Ans:

$$e = \sqrt{\frac{h_1}{h_0}}$$

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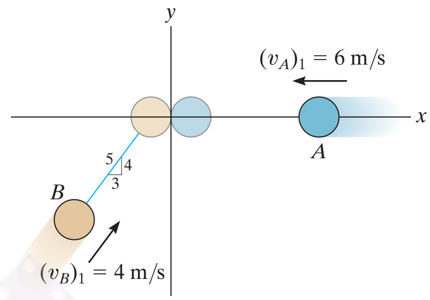
15–77.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4\left(\frac{3}{5}\right) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow$$

$$(v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

$(+\uparrow)$ $\quad m v_1 = m v_2$

$$0.5\left(\frac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$v_A = 1.35 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

$$v_B = \sqrt{(4.59)^2 + (3.20)^2} = 5.89 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{3.20}{4.95} = 32.9 \quad \measuredangle \qquad \textbf{Ans.}$$

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Ans:
$v_A = 1.35$ m/s $\rightarrow$
$v_B = 5.89$ m/s
$\theta = 32.9° \measuredangle$

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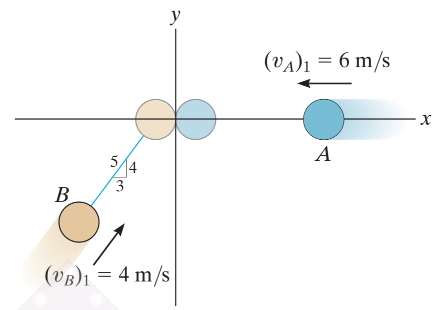
15–78.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



SOLUTION

$\Sigma m v_1 = \Sigma m v_2$

$$(\overset{+}{\rightarrow}) \qquad 0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

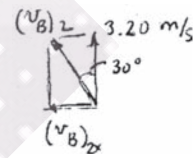
$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$$(+\uparrow) \qquad 0.5(4)\left(\frac{4}{5}\right) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$(v_B)_{2x} = 3.20 \tan 30° = 1.8475 \text{ m/s} \leftarrow$$

$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$



$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752-(-1.8475)}{4(\frac{3}{5})-(-6)} = 0.0113$$ **Ans.**

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Ans:
$e = 0.0113$

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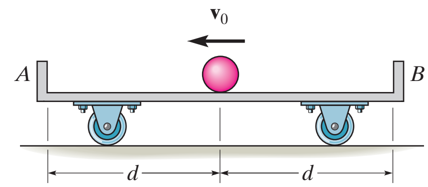
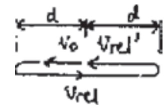
15–79.

A ball of negligible size and mass m is given a velocity of $\mathbf{v}_0$ on the center of the cart which has a mass M and is originally at rest. If the coefficient of restitution between the ball and walls A and B is e, determine the velocity of the ball and the cart just after the ball strikes A. Also, determine the total time needed for the ball to strike A, rebound, then strike B, and rebound and then return to the center of the cart. Neglect friction.



SOLUTION



After the first collision;

$(\overset{+}{\leftarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + m v_0 = m v_b + M v_c$$

$(\overset{+}{\leftarrow})\qquad e = \dfrac{v_c - v_b}{v_0}$

$$m v_0 = m v_b + \frac{M}{m} v_c$$

$$e v_0 = v_c - v_b$$

$$v_0(1 + e) = \left(1 + \frac{M}{m}\right) v_c$$

$$v_c = \frac{v_0(1 + e)m}{(m + M)} \qquad \textbf{Ans.}$$

$$v_b = \frac{v_0(1 + e)m}{(m + M)} - e v_0$$

$$= v_0\left[\frac{m + me - em - eM}{m + M}\right]$$

$$= v_0\left(\frac{m - eM}{m + M}\right) \qquad \textbf{Ans.}$$

The relative velocity on the cart after the first collision is

$$e = \frac{v_{\text{ref}}}{v_0}$$

$$v_{\text{ref}} = e v_0$$

Similarly, the relative velocity after the second collision is

$$e = \frac{v_{\text{ref}}}{e v_0}$$

$$v_{ref} = e^2 v_0$$

Total time is

$$t = \frac{d}{v_0} + \frac{2d}{e v_0} + \frac{d}{e^2 v_0}$$

$$= \frac{d}{v_0}\left(1 + \frac{1}{e}\right)^2 \qquad \textbf{Ans.}$$

Ans:

$$v_c = \frac{v_0(1 + e)m}{(m + M)}$$

$$v_b = v_0\left(\frac{m - eM}{m + M}\right)$$

$$t = \frac{d}{v_0}\left(1 + \frac{1}{e}\right)^2$$

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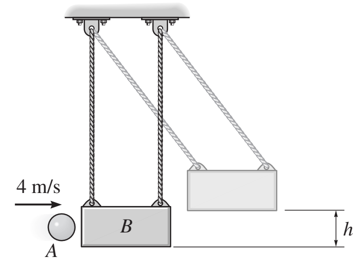
***15–80.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height h to which the block will swing before it momentarily stops.



SOLUTION

System:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block:

Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$

$h = 0.0218 \text{ m} = 21.8 \text{ mm}$ **Ans.**

Ans:
$h = 21.8$ mm

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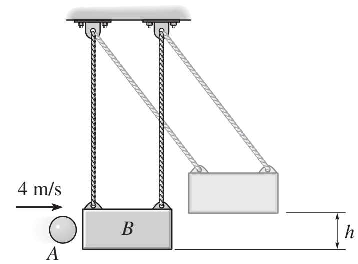
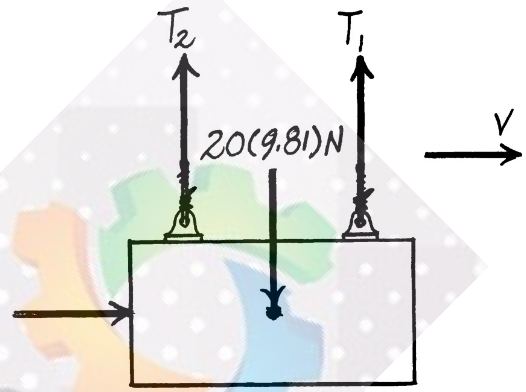
15–81. The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.



SOLUTION



System:

$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 + (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block:

$(\overset{+}{\rightarrow})$ $\quad m v_1 + \Sigma \int F\,dt = m v_2$

$$0 + F(0.005) = 20(0.6545)$$

$F = 2618 \text{ N} = 2.62 \text{ kN}$ **Ans.**

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Ans:
$F = 2.62$ kN

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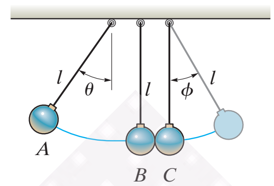
15-82.

The three balls each have the same mass m. If A is released from rest at θ, determine the angle ϕ to which C rises after collision. The coefficient of restitution between each ball is e.



SOLUTION

Energy

$$0 + l\left(1 - \cos\left(\theta\right)\right) m g = \frac{1}{2} m v_A{}^2$$

$$v_A = \sqrt{2\left(1 - \cos\left(\theta\right)\right) g\, l}$$

Collision of ball A with B:

$$m v_A + 0 = m v'_A + m v'_B \qquad e v_A = v'_B - v'_A \qquad v'_B = \frac{1}{2}(1 + e)\, v'_B$$

Collision of ball B with C:

$$m v'_B + 0 = m v''_B + m v''_C \qquad e v'_B = v''_C - v''_B \qquad v''_C = \frac{1}{4}(1 + e)^2 v_A$$

Energy

$$\frac{1}{2} m v''_c{}^2 + 0 = 0 + l\left(1 - \cos\left(\phi\right)\right) m g$$

$$\frac{1}{2}\left(\frac{1}{16}\right)(1 + e)^4 (2)\left(1 - \cos\left(\theta\right)\right) = \left(1 - \cos\left(\phi\right)\right)$$

$$\left(\frac{1 + e}{2}\right)^4 \left(1 - \cos\left(\theta\right)\right) = 1 - \cos\left(\phi\right)$$

$$\phi = \operatorname{acos}\left[1 - \left(\frac{1 + e}{2}\right)^4 \left(1 - \cos\left(\theta\right)\right)\right] \qquad \textbf{Ans.}$$

Ans:

$$\phi = \operatorname{acos}\left[1 - \left(\frac{1 + e}{2}\right)^4 (1 - \cos(\theta))\right]$$

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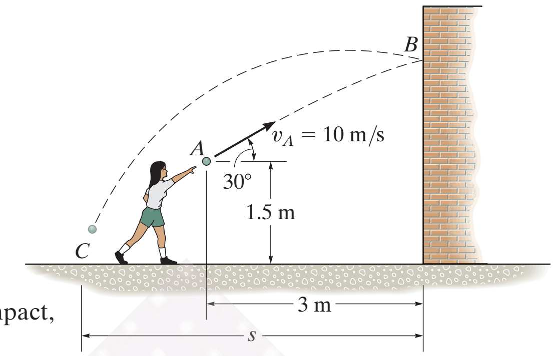
15–83.

The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

Kinematics: By considering the horizontal motion of the ball before the impact, we have

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$3 = 0 + 10 \cos 30° t \qquad t = 0.3464 \text{ s}$$

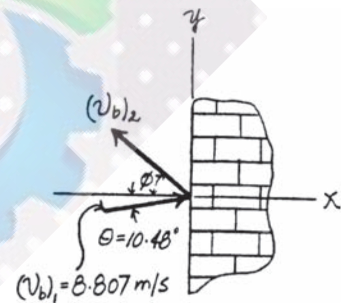
By considering the vertical motion of the ball before the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + (a_c)_y t$$

$$= 10 \sin 30° + (-9.81)(0.3464)$$

$$= 1.602 \text{ m/s}$$

The vertical position of point B above the ground is given by



$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$(s_B)_y = 1.5 + 10 \sin 30°(0.3464) + \frac{1}{2}(-9.81)\left(0.3464^2\right) = 2.643 \text{ m}$$

Thus, the magnitude of the velocity and its directional angle are

$$(v_b)_1 = \sqrt{(10 \cos 30°)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{1.602}{10 \cos 30°} = 10.48° = 10.5° \qquad \textbf{Ans.}$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of $(v_b)_1 = 8.807$ m/s, it rebounds with a speed of $(v_b)_2$.

$$m_b\left(v_{b_y}\right)_1 = m_b\left(v_{b_y}\right)_2$$

$$(\overset{+}{\leftarrow}) \qquad m_b(1.602) = m_b\left[(v_b)_2 \sin \phi\right]$$

$$(v_b)_2 \sin \phi = 1.602 \qquad \textbf{(1)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_w)_2 - \left(v_{b_x}\right)_2}{\left(v_{b_x}\right)_1 - (v_w)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.5 = \frac{0 - \left[-(v_b)_2 \cos \phi\right]}{10 \cos 30° - 0} \qquad \textbf{(2)}$$

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15–83. Continued

Solving Eqs. (1) and (2) yields

$$\phi = 20.30° = 20.3° \qquad (v_b)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the ball after the impact, we have

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2.643 = 0 + 4.617 \sin 20.30° t_1 + \frac{1}{2}(-9.81)t_1^2$$

$$t_1 = 0.9153 \text{ s}$$

By considering the horizontal motion of the ball after the impact, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30°(0.9153) = 3.96 \text{ m} \qquad \textbf{Ans.}$$

Ans:
(a) $(v_B)_1 = 8.81 \text{ m/s}, \theta = 10.5°$
(b) $(v_B)_2 = 4.62 \text{ m/s}, \phi = 20.3°$
(c) $s = 3.96 \text{ m}$

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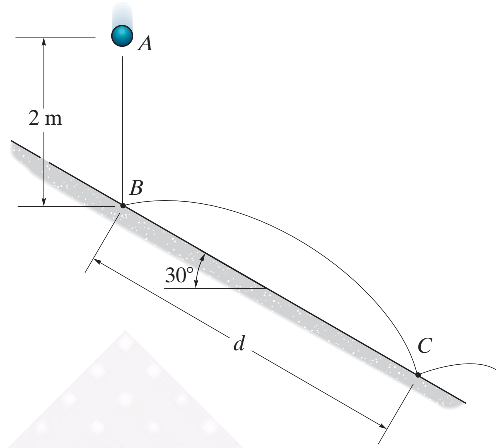
***15–84.**

The 1-kg ball is dropped from rest at point A, 2 m above the smooth plane. If the coefficient of restitution between the ball and the plane is $e = 0.6$, determine the distance d where the ball again strikes the plane.



SOLUTION

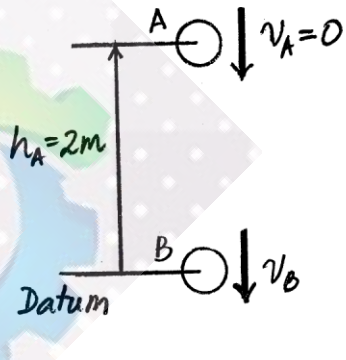
Conservation of Energy: By considering the ball's fall from position (1) to position (2) as shown in Fig. *a*,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_A v_A^2 + (V_g)_A = \frac{1}{2}m_B v_B^2 + (V_g)_B$$

$$0 + 1(9.81)(2) = \frac{1}{2}(1)v_B^2 + 0$$

$$v_B = 6.264 \text{ m/s} \downarrow$$



(a)

Conservation of Linear Momentum: Since no impulsive force acts on the ball along the inclined plane (x' axis) during the impact, linear momentum of the ball is conserved along the x' axis. Referring to Fig. *b*,

$$m_B(v_B)_{x'} = m_B(v'_B)_{x'}$$

$$1(6.264)\sin 30° = 1(v'_B)\cos\theta$$

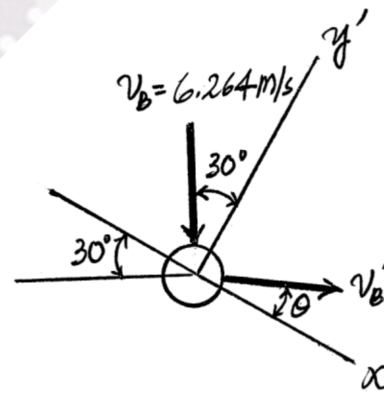
$$v'_B \cos\theta = 3.1321 \qquad \textbf{(1)}$$

Coefficient of Restitution: Since the inclined plane does not move during the impact,

$$e = \frac{0 - (v'_B)_{y'}}{(v'_B)_{y'} - 0}$$

$$0.6 = \frac{0 - v'_B \sin\theta}{-6.264\cos 30° - 0}$$

$$v'_B \sin\theta = 3.2550 \qquad \textbf{(2)}$$



(b)

Solving Eqs. (1) and (2) yields

$$\theta = 46.10° \qquad v'_B = 4.517 \text{ m/s}$$

Kinematics: By considering the x and y motion of the ball after the impact, Fig. *c*,

$(\overset{+}{\rightarrow})$ $\quad s_x = (s_0)_x + (v'_B)_x t$

$$d\cos 30° = 0 + 4.517\cos 16.10° t$$

$$t = 0.1995d \qquad \textbf{(3)}$$

$(+\uparrow)$ $\quad s_y = (s_0)_y + (v'_B)_y t + \frac{1}{2}a_y t^2$

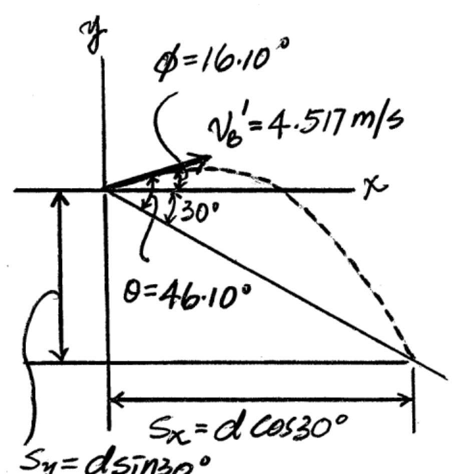
$$-d\sin 30° = 0 + 4.517\sin 16.10° t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 1.2528t - 0.5d = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$d = 3.84 \text{ m} \qquad \textbf{Ans.}$$

$$t = 0.7663 \text{ s}$$



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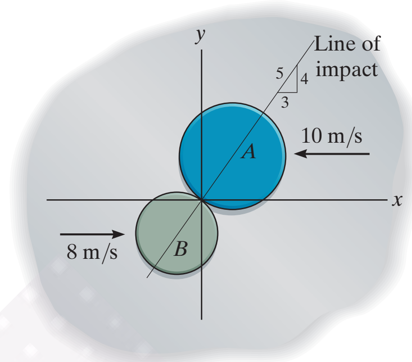
15–85.

Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



SOLUTION

Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow \; m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v_A'\right)_n + m_B\left(v_B'\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v_A' \cos\phi_A + 10v_B' \cos\phi_B$$

$$15v_A' \cos\phi_A + 10v_B' \cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow \; m_A\left(v_A\right)_t = m_A\left(v_A'\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v_A' \sin\phi_A$$

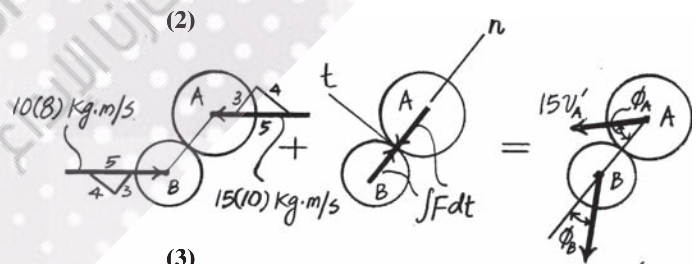
$$v_A' \sin\phi_A = 8 \qquad \textbf{(2)}$$

and

$$+\nwarrow \; m_B\left(v_B\right)_t = m_B\left(v_B'\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\,v_B' \sin\phi_B$$

$$v_B' \sin\phi_B = 6.4 \qquad \textbf{(3)}$$



(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

$$0.8 = \frac{v_B' \cos\phi_B - v_A' \cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v_B' \cos\phi_B - v_A' \cos\phi_A = 8.64 \qquad \textbf{(4)}$$

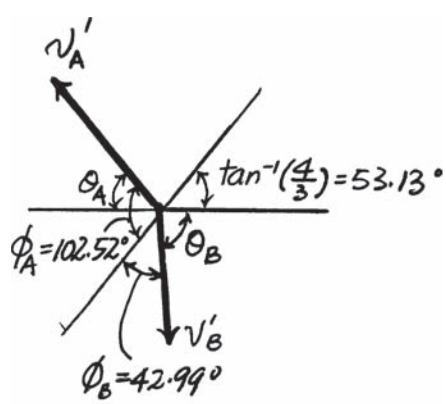
Solving Eqs. (1), (2), (3), and (4), yeilds

$v_A' = 8.19$ m/s **Ans.**

$\phi_A = 102.52°$

$v_B' = 9.38$ m/s **Ans.**

$\phi_B = 42.99°$



(b)

Ans:
$(v_A)_2 = 8.19$ m/s
$(v_B)_2 = 9.38$ m/s

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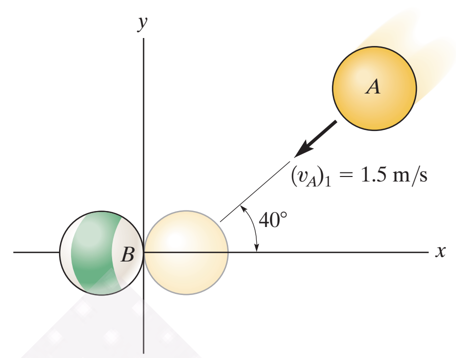
15–86.

Two smooth billiard balls A and B each have a mass of 200 g. If A strikes B with a velocity $(v_A)_1 = 1.5\text{ m/s}$ as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.85$. Neglect the size of each ball.



SOLUTION

$$(v_{A_x})_1 = -1.5\cos 40° = -1.1491\text{ m/s}$$

$$(v_{A_y})_1 = -1.5\sin 40° = -0.9642\text{ m/s}$$

$$(\xrightarrow{+})\qquad m_A(v_{A_x})_1 + m_B(v_{B_x})_1 = m_A(v_{A_x})_2 + m_B(v_{B_x})_2$$

$$-0.2(1.1491) + 0 = 0.2(v_{A_x})_2 + 0.2(v_{B_x})_2$$

$$(\xrightarrow{+})\qquad e = \frac{(v_{A_x})_2 - (v_{B_x})_2}{(v_{B_x})_1 - (v_{A_x})_1};\qquad 0.85 = \frac{(v_{A_x})_2 - (v_{B_x})_2}{1.1491}$$

Solving,

$$(v_{A_x})_2 = -0.08618\text{ m/s}$$

$$(v_{B_x})_2 = -1.0629\text{ m/s}$$

For A:

$$(+\downarrow)\qquad m_A(v_{A_y})_1 = m_A(v_{A_y})_2$$

$$(v_{A_y})_2 = 0.9642\text{ m/s}$$

For B:

$$(+\uparrow)\qquad m_B(v_{B_y})_1 = m_B(v_{B_y})_2$$

$$(v_{B_y})_2 = 0$$

Hence.

$$(v_B)_2 = (v_{B_x})_2 = 1.06\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$(v_A)_2 = \sqrt{(-0.08618)^2 + (0.9642)^2} = 0.968\text{ m/s} \qquad \textbf{Ans.}$$

$$(\theta_A)_2 = \tan^{-1}\left(\frac{0.08618}{0.9642}\right) = 5.11° \measuredangle \qquad \textbf{Ans.}$$

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Ans:
$(v_B)_2 = 1.06\text{ m/s} \leftarrow$
$(v_A)_2 = 0.968\text{ m/s}$
$(\theta_A)_2 = 5.11° \measuredangle$

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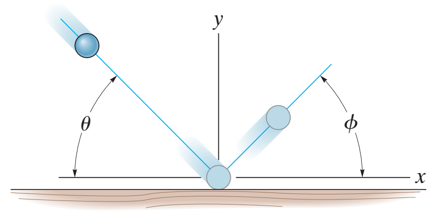
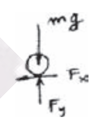
15–87.

A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



SOLUTION

$$(+\downarrow) \qquad e = \frac{0 - [-v_2 \sin \phi]}{v_1 \sin \theta - 0} \qquad e = \frac{v_2 \sin \phi}{v_1 \sin \theta} \qquad \textbf{(1)}$$



$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \int_{t_1}^{t_2} F_x \, dx = m(v_x)_2$$

$$mv_1 \cos \theta - F_x \Delta t = mv_2 \cos \phi$$

$$F_x = \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} \qquad \textbf{(2)}$$

$$(+\downarrow) \qquad m(v_y)_1 + \int_{t_1}^{t_2} F_y \, dx = m(v_y)_2$$

$$mv_1 \sin \theta - F_y \Delta t = -mv_2 \sin \phi$$

$$F_y = \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} = \frac{\mu(mv_1 \sin \theta + mv_2 \sin \phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$e = \frac{\sin \phi}{\sin \theta}\left(\frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi}\right) \qquad \textbf{Ans.}$$

Ans:

$$e = \frac{\sin \phi}{\sin \theta}\left(\frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi}\right)$$

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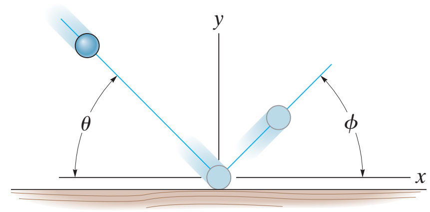
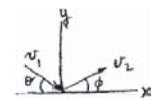
***15–88.**

A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



SOLUTION

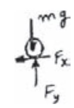


$(+\downarrow)$ $$e = \frac{0 - [-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad \textbf{(1)}$$

$(\xrightarrow{+})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad \textbf{(2)}$$



$(+\uparrow)$ $$m\left(v_y\right)_1 + \int_{t_1}^{t_2} F_y dx = m\left(v_y\right)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu\sin\theta}{\mu\sin\phi + \cos\phi}\right)$$

$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1-\mu}{1+\mu} \qquad \mu = 0.25$$ **Ans.**

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Ans:
$\mu_k = 0.25$

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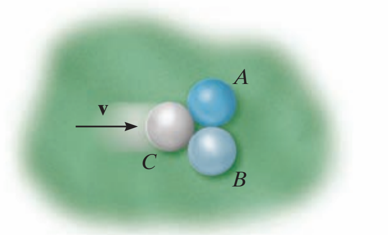
15–89.

The two billiard balls A and B are originally in contact with one another when a third ball C strikes each of them at the same time as shown. If ball C remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.

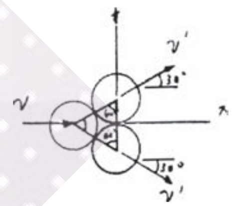


SOLUTION

Conservation of "x" momentum:

$$(\xrightarrow{+})\qquad mv = 2mv' \cos 30^\circ$$

$$v = 2v' \cos 30^\circ \qquad (1)$$

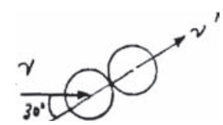


Coefficient of restitution:

$$(+\nearrow)\qquad e = \frac{v'}{v \cos 30^\circ} \qquad (2)$$

Substituting Eq. (1) into Eq. (2) yields:

$$e = \frac{v'}{2v' \cos^2 30^\circ} = \frac{2}{3} \qquad \textbf{Ans.}$$



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Ans:
$e = \frac{2}{3}$

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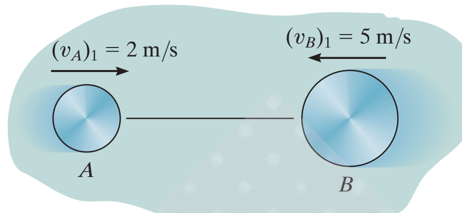
15–90. Disks A and B have masses of 2 kg and 4 kg, respectively. If they have the velocities shown, and $e = 0.4$, determine their velocities just after direct central impact.



SOLUTION

Guesses $\quad v_{A2} = 1\ \text{m/s} \qquad v_{B2} = 1\ \text{m/s}$

Given:

$M_A = 2\ \text{kg}$

$M_B = 4\ \text{kg}$

$e = 0.4$

$v_{A1} = 2\ \text{m/s}$

$v_{B1} = 5\ \text{m/s}$

Given $\quad M_A v_{A1} - M_B v_{B1} = M_A v_{A2} + M_B v_{B2}$

$$e(v_{A1} + v_{B1}) = v_{B2} - v_{A2}$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -4.533 \\ -1.733 \end{pmatrix} \text{m/s}$$ **Ans.**

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Ans:
$v_{A2} = -4.533\ \text{m/s}$
$v_{B2} = -1.733\ \text{m/s}$

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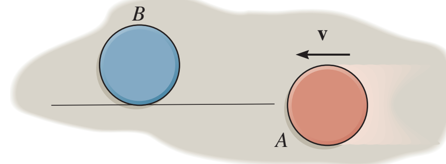
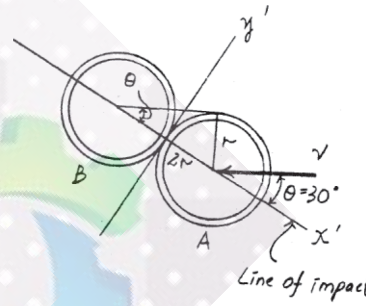
15–91.

If disk A is sliding along the tangent to disk B and strikes B with a velocity $\mathbf{v}$, determine the velocity of B after the collision and compute the loss of kinetic energy during the collision. Neglect friction. Disk B is originally at rest. The coefficient of restitution is e, and each disk has the same size and mass m.



SOLUTION

Impact: This problem involves *oblique impact* where the *line of impact* lies along x' axis (line jointing the mass center of the two impact bodies). From the geometry $\theta = \sin^{-1}\left(\frac{r}{2r}\right) = 30°$. The x' and y' components of velocity for disk A just before impact are



$$\left(v_{A_{x'}}\right)_1 = -v\cos 30° = -0.8660v \qquad \left(v_{A_{y'}}\right)_1 = -v\sin 30° = -0.5\,v$$

Conservation of "x′" Momentum:

$$m_A\left(v_{A_{x'}}\right)_1 + m_B\left(v_{B_{x'}}\right)_1 = m_A\left(v_{A_{x'}}\right)_2 + m_B\left(v_{B_{x'}}\right)_2$$

$$(\searrow +) \qquad m(-0.8660v) + 0 = m\left(v_{A_{x'}}\right)_2 + m\left(v_{B_{x'}}\right)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution (x′):

$$e = \frac{\left(v_{B_{x'}}\right)_2 - \left(v_{A_{x'}}\right)_2}{\left(v_{A_{x'}}\right)_1 - \left(v_{B_{x'}}\right)_1}$$

$$e = \frac{\left(v_{B_{x'}}\right)_2 - \left(v_{A_{x'}}\right)_2}{-0.8660v - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\left(v_{B_{x'}}\right)_2 = -\frac{\sqrt{3}}{4}(1+e)\,v \qquad \left(v_{A_{x'}}\right)_2 = \frac{\sqrt{3}}{4}(e-1)\,v$$

Conservation of "y′" Momentum: The momentum is conserved along y' axis for both disks A and B.

$$(+\nearrow) \qquad m_B\left(v_{B_{y'}}\right)_1 = m_B\left(v_{B_{y'}}\right)_2; \qquad \left(v_{B_{y'}}\right)_2 = 0$$

$$(+\nearrow) \qquad m_A\left(v_{A_{y'}}\right)_1 = m_A\left(v_{A_{y'}}\right)_2; \qquad \left(v_{A_{y'}}\right)_2 = -0.5\,v$$

Thus, the magnitude the velocity for disk B just after the impact is

$$(v_B)_2 = \sqrt{(v_{B_{x'}})_2^2 + (v_{B_{y'}})_2^2}$$

$$= \sqrt{\left(-\frac{\sqrt{3}}{4}(1+e)\,v\right)^2 + 0} = \frac{\sqrt{3}}{4}(1+e)\,v \qquad \textbf{Ans.}$$

and directed toward **negative x' axis**. **Ans.**

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15–91. continued

The magnitude of the velocity for disk A just after the impact is

$$(v_A)_2 = \sqrt{(v_{A_{x'}})_2^2 + (v_{A_{y'}})_2^2}$$

$$= \sqrt{\left[\frac{\sqrt{3}}{4}(e-1)\,v\right]^2 + (-0.5\,v)^2}$$

$$= \sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}$$

Loss of Kinetic Energy: Kinetic energy of the system before the impact is

$$U_k = \frac{1}{2}mv^2$$

Kinetic energy of the system after the impact is

$$U_k{}' = \frac{1}{2}m\left[\sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}\right]^2 + \frac{1}{2}m\left[\frac{\sqrt{3}}{4}(1+e)\,v\right]^2$$

$$= \frac{mv^2}{32}\left(6e^2 + 10\right)$$

Thus, the kinetic energy loss is

$$\Delta U_k = U_k - U_k{}' = \frac{1}{2}mv^2 - \frac{mv^2}{32}(6e^2 + 10)$$

$$= \frac{3mv^2}{16}(1 - e^2) \qquad \textbf{Ans.}$$

Ans:

$(v_B)_2 = \dfrac{\sqrt{3}}{4}(1+e)v$,
Negative x' axis.
$\Delta U_k = \dfrac{3mv^2}{16}(1 - e^2)$

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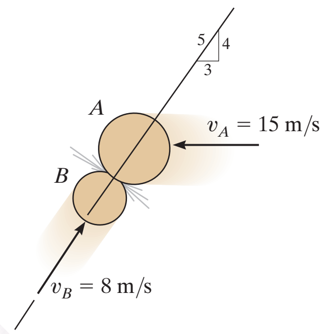
***15–92.**

Two smooth disks A and B have the initial velocities shown just before they collide. If they have masses $m_A = 4$ kg and $m_B = 2$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.8$.



SOLUTION

Impact. The line of impact is along the line joining the centers of disks A and B represented by y axis in Fig. a. Thus

$$[(v_A)_1]_y = 15\left(\frac{3}{5}\right) = 9 \text{ m/s} \swarrow \qquad [(v_A)_1]_x = 15\left(\frac{4}{5}\right) = 12 \text{ m/s} \nwarrow$$

$$[(v_B)_1]_y = 8 \text{ m/s} \nearrow \qquad [(v_B)_1]_x = 0$$

Coefficient of Restitution. Along the line of impact (y axis),

$$(+\nearrow) \quad e = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{[(v_A)_1]_y - [(v_B)_1]_y}; \qquad 0.8 = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{-9 - 8}$$

$$[(v_A)_2]_y - [(v_B)_2]_y = 13.6 \qquad \textbf{(1)}$$

Conservation of 'y' Momentum.

$$(+\nearrow) \quad m_A[(v_A)_1]_y + m_B[(v_B)_1]_y = m_A[(v_A)_2]_y + m_B[(v_B)_2]_y$$

$$4(-9) + 2(8) = 4[(v_A)_2]_y + 2[(v_B)_2]_y$$

$$2[(v_A)_2]_y + [(v_B)_2]_y = -10 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$[(v_A)_2]_y = 1.20 \text{ m/s} \nearrow \qquad [(v_B)_2]_y = -12.4 \text{ m/s} = 12.4 \text{ m/s} \swarrow$$

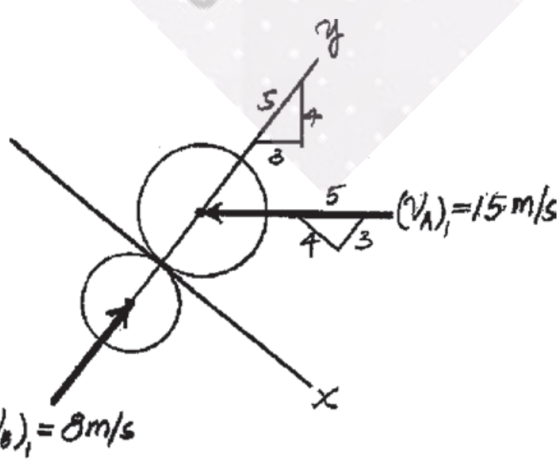
Conservation of 'x' Momentum. Since no impact occurs along the x axis, the component of velocity of each disk remain constant before and after the impact. Thus

$$[(v_A)_2]_x = [(v_A)_1]_x = 12 \text{ m/s} \nwarrow \qquad [(v_B)_2]_x = [(v_B)_1]_x = 0$$

Thus, the magnitude of the velocity of disks A and B just after the impact is

$$(v_A)_2 = \sqrt{[(v_A)_2]_x^2 + [(v_A)_2]_y^2} = \sqrt{12^2 + 1.20^2} = 12.06 \text{ m/s} = 12.1 \text{ m/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = \sqrt{[(v_B)_2]_x^2 + [(v_B)_2]_y^2} = \sqrt{0^2 + 12.4^2} = 12.4 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$(v_A)_2 = 12.1$ m/s
$(v_B)_2 = 12.4$ m/s

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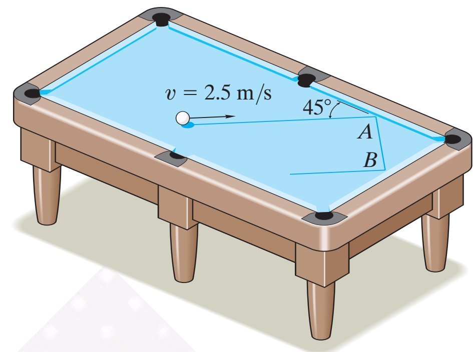
15–93.

The 200-g billiard ball is moving with a speed of 2.5 m/s when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is $e = 0.6$, determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.



SOLUTION

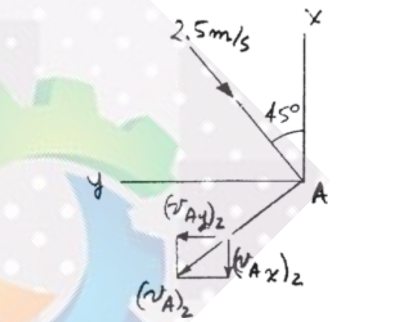
At A:

$$(v_A)_{y1} = 2.5(\sin 45°) = 1.7678 \text{ m/s} \rightarrow$$

$$e = \frac{(v_{A_y})_2}{(v_{A_y})_1}; \qquad 0.6 = \frac{(v_{A_y})_2}{1.7678}$$

$$(v_{A_y})_2 = 1.061 \text{ m/s} \leftarrow$$

$$(v_{A_x})_2 = (v_{A_x})_1 = 2.5 \cos 45° = 1.7678 \text{ m/s} \downarrow$$

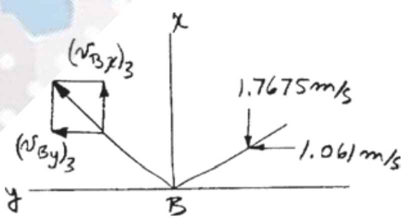


At B:

$$e = \frac{(v_{B_x})_3}{(v_{B_x})_2}; \qquad 0.6 = \frac{(v_{B_x})_3}{1.7678}$$

$$(v_{B_x})_3 = 1.061 \text{ m/s}$$

$$\left(v_{B_y}\right)_3 = \left(v_{A_y}\right)_2 = 1.061 \text{ m/s}$$



Hence,

$$(v_B)_3 = \sqrt{(1.061)^2 + (1.061)^2} = 1.50 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(v_B)_3 = 1.50$ m/s

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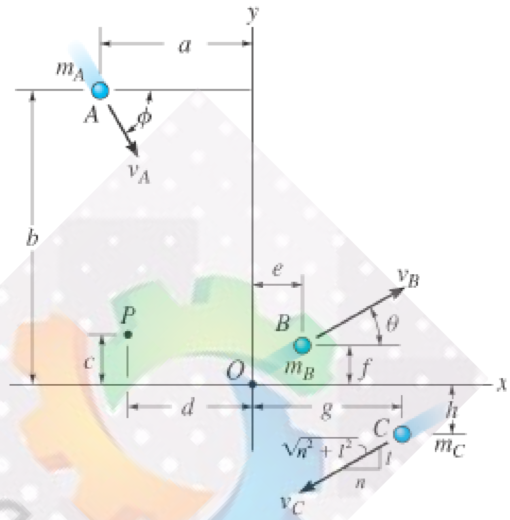
15-94.

Determine the angular momentum $\mathbf{H_O}$ of each of the particles about point O.



SOLUTION

Given: $\theta = 30°$ $\quad \phi = 60°$

$m_A = 6\text{ kg}$ $\quad c = 2\text{ m}$

$m_B = 4\text{ kg}$ $\quad d = 5\text{ m}$

$m_C = 2\text{ kg}$ $\quad e = 2\text{ m}$

$v_A = 4\text{ m/s}$ $\quad f = 1.5\text{ m}$

$v_B = 6\text{ m/s}$ $\quad g = 6\text{ m}$

$v_C = 2.6\text{ m/s}$ $\quad h = 2\text{ m}$

$a = 8\text{ m}$ $\quad l = 5$

$b = 12\text{ m}$ $\quad n = 12$

$$\mathbf{H_{AO}} = a m_A v_A \sin(\phi) - b m_A v_A \cos(\phi)$$

$\mathbf{H_{AO}} = 22.3\text{ kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$$\mathbf{H_{BO}} = -f m_B v_B \cos(\theta) + e m_B v_B \sin(\theta)$$

$\mathbf{H_{BO}} = -7.18\text{ kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$$\mathbf{H_{CO}} = -h m_C\left(\frac{n}{\sqrt{l^2+n^2}}\right)v_C - g m_C\left(\frac{l}{\sqrt{l^2+n^2}}\right)v_C$$

$\mathbf{H_{CO}} = -21.60\text{ kg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:

$\mathbf{H_{AO}} = 22.3\text{ kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{BO}} = -7.18\text{ kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{CO}} = -21.60\text{ kg}\cdot\text{m}^2/\text{s}$

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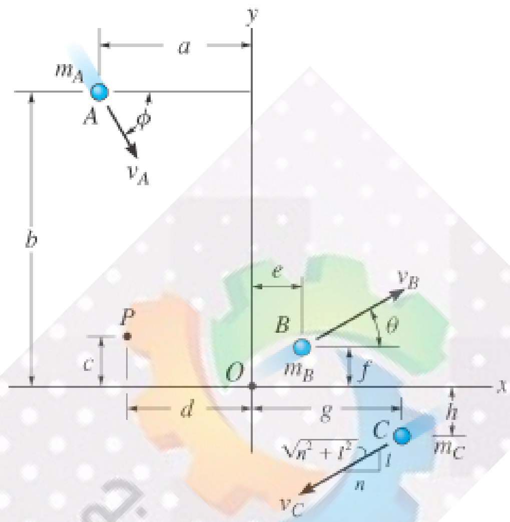
15-95.

Determine the angular momentum $\mathbf{H_P}$ of each of the particles about point P.



SOLUTION

Given: $\theta = 30°$ $\quad \phi = 60°$

$m_A = 6\ \text{kg}$ $\quad c = 2\ \text{m}$

$m_B = 4\ \text{kg}$ $\quad d = 5\ \text{m}$

$m_C = 2\ \text{kg}$ $\quad e = 2\ \text{m}$

$v_A = 4\ \text{m/s}$ $\quad f = 1.5\ \text{m}$

$v_B = 6\ \text{m/s}$ $\quad g = 6\ \text{m}$

$v_C = 2.6\ \text{m/s}$ $\quad h = 2\ \text{m}$

$a = 8\ \text{m}$ $\quad l = 5$

$b = 12\ \text{m}$ $\quad n = 12$

$$\mathbf{H_{AP}} = m_A v_A \sin(\phi)(a-d) - m_A v_A \cos(\phi)(b-c)$$

$\mathbf{H_{AP}} = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$$\mathbf{H_{BP}} = m_B v_B \cos(\theta)(c-f) + m_B v_B \sin(\theta)(d+e)$$

$\mathbf{H_{BP}} = 94.4\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$$\mathbf{H_{CP}} = -m_C\left(\frac{n}{\sqrt{l^2+n^2}}\right)v_C(c+h) - m_C\left(\frac{l}{\sqrt{l^2+n^2}}\right)v_C(d+g)$$

$\mathbf{H_{CP}} = -41.2\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:
$\mathbf{H_{AP}} = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{BP}} = 94.4\ \text{kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{CP}} = -41.2\ \text{kg}\cdot\text{m}^2/\text{s}$

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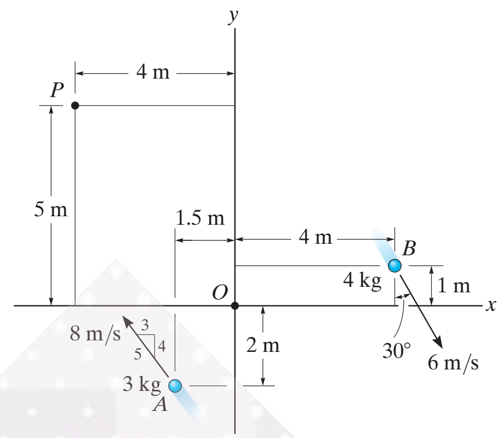
***15–96.**

Determine the angular momentum $\mathbf{H}_o$ of each of the two particles about point O.



SOLUTION

$$\zeta + (H_A)_O = (-1.5)\left[3(8)\left(\frac{4}{5}\right)\right] - (2)\left[3(8)\left(\frac{3}{5}\right)\right] = -57.6 \text{ kg} \cdot \text{m}^2/\text{s}$$

$$\zeta + (H_B)_O = (-1)[4(6 \sin 30°)] - (4)[4\,(6 \cos 30°)] = -95.14 \text{ kg} \cdot \text{m}^2/\text{s}$$

Thus

$$(H_A)_O = \{-57.6\mathbf{k}\} \text{ kg} \cdot \text{m}^2/\text{s} \qquad \textbf{Ans.}$$

$$(H_B)_O = \{-95.1\mathbf{k}\} \text{ kg} \cdot \text{m}^2/\text{s} \qquad \textbf{Ans.}$$

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Ans:

$(H_A)_O = \{-57.6\mathbf{k}\} \text{ kg} \cdot \text{m}^2/\text{s}$

$(H_B)_O = \{-95.1\mathbf{k}\} \text{ kg} \cdot \text{m}^2/\text{s}$

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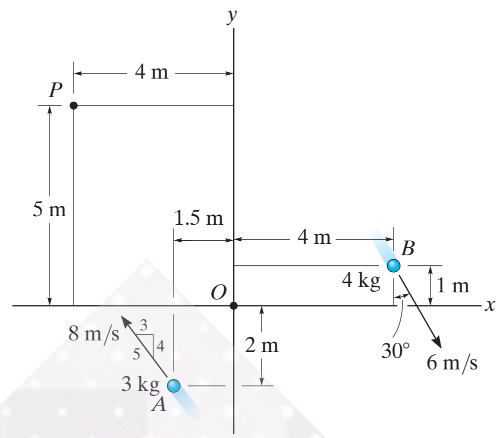
15–97.

Determine the angular momentum $\mathbf{H}_p$ of each of the two particles about point P.



SOLUTION

$$\zeta+ (\boldsymbol{H}_A)_p = (2.5)\left[3(8)\left(\frac{4}{5}\right)\right] - (7)\left[3(8)\left(\frac{3}{5}\right)\right] = -52.8 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$\zeta+ (\boldsymbol{H}_B)_p = (4)[4(6 \sin 30°)] - 8[4\,(6 \cos 30°)] = -118.28 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$(\boldsymbol{H}_A)_p = \{-52.8\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$(\boldsymbol{H}_B)_p = \{-118\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:
$(H_A)_P = \{-52.8\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$
$(H_B)_P = \{-118\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$

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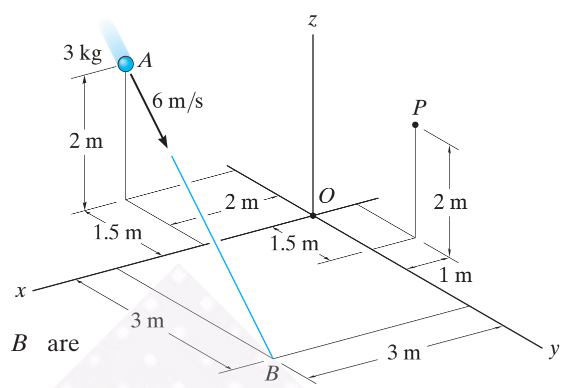
15–98.

Determine the angular momentum $\mathbf{H}_O$ of the 3-kg particle about point O.



SOLUTION

Position and Velocity Vectors. The coordinates of points A and B are $A(2, -1.5, 2)$ m and $B(3, 3, 0)$.

$$\mathbf{r}_{OB} = \{3\mathbf{i} + 3\mathbf{j}\}\ \text{m} \qquad \mathbf{r}_{OA} = \{2\mathbf{i} - 1.5\mathbf{j} + 2\mathbf{k}\}\ \text{m}$$

$$V_A = v_A\left(\frac{r_{AB}}{r_{AB}}\right) = (6)\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0.2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\ \text{m/s}$$

Angular Momentum about Point O. Applying Eq. 15

$$\mathbf{H}_O = \mathbf{r}_{OB} \times mV_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 0 \\ 3\left(\dfrac{6}{\sqrt{25.25}}\right) & 3\left(\dfrac{27}{\sqrt{25.25}}\right) & 3\left(-\dfrac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Also,

$$\mathbf{H}_O = \mathbf{r}_{OA} \times mV_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1.5 & 2 \\ 3\left(\dfrac{6}{\sqrt{25.25}}\right) & 3\left(\dfrac{27}{\sqrt{25.25}}\right) & 3\left(-\dfrac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$\{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\ \text{kg}\cdot\text{m}^2/\text{s}$

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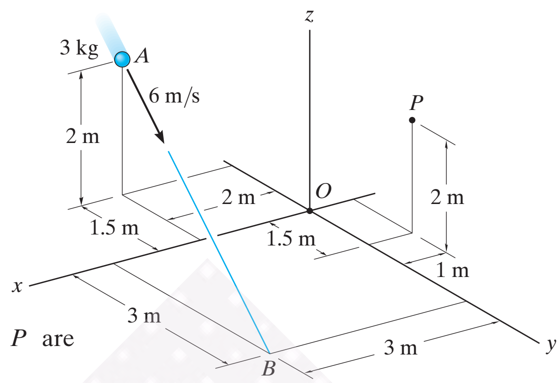
15–99.

Determine the angular momentum $\mathbf{H}_P$ of the 3-kg particle about point P.



SOLUTION

Position and Velocity Vectors. The coordinates of points A, B and P are $A(2, -1.5, 2)$ m, $B(3, 3, 0)$ m and $P(-1, 1.5, 2)$ m.

$$\boldsymbol{r}_{PA} = [2 - (-1)]\mathbf{i} + (-1.5 - 1.5)\mathbf{j} + (2 - 2)\mathbf{k} = \{3\mathbf{i} - 3\mathbf{j}\}\text{ m}$$

$$\boldsymbol{r}_{PB} = [3 - (-1)]\mathbf{i} + (3 - 1.5)\mathbf{j} + (0 - 2)\mathbf{k} = \{4\mathbf{i} + 1.5\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

$$\boldsymbol{V}_A = v_A\left(\frac{\boldsymbol{r}_{AB}}{\boldsymbol{r}_{AB}}\right) = 6\left[\frac{(3-2)\mathbf{i} + [3 - (-1.5)]\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\text{ m/s}$$

Angular Momentum about Point P. Applying Eq. 15

$$\boldsymbol{H}_p = \boldsymbol{r}_{pA} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 0 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Also,

$$\boldsymbol{H}_p = \boldsymbol{r}_{PB} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 1.5 & -2 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$\{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}$ kg·m²/s

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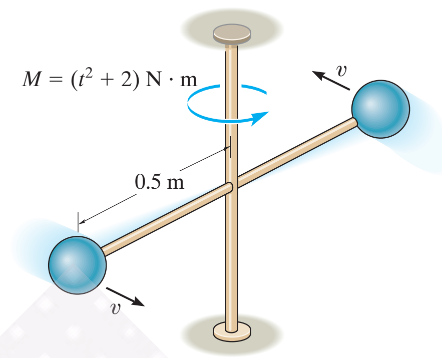
***15–100.**

Each ball has a negligible size and a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (t^2 + 2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the speed of each ball when $t = 3$ s. Each ball has a speed $v = 2$ m/s when $t = 0$.



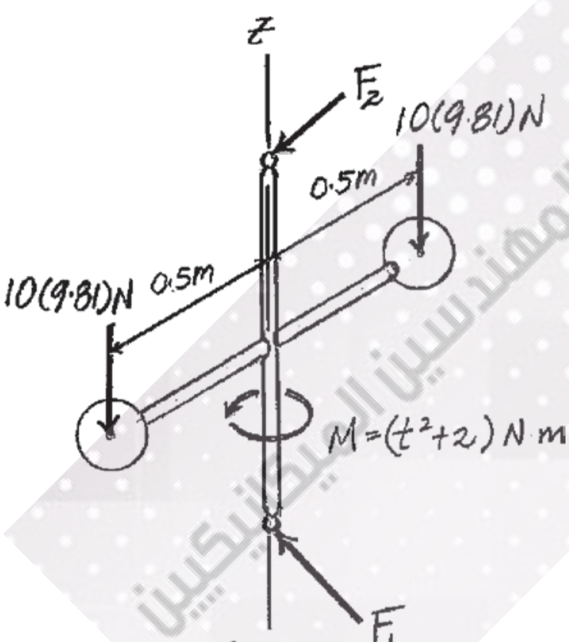
SOLUTION

Principle of Angular Impulse and Momentum. Referring to the FBD of the assembly, Fig. *a*

$$(H_Z)_1 + \Sigma \int_{t_1}^{t_2} M_Z\, dt = (H_Z)_2$$

$$2[0.5(10)(2)] + \int_0^{3\text{ s}} (t^2 + 2)dt = 2[0.5(10v)]$$

$$v = 3.50 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

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Ans:
$v = 3.50$ m/s

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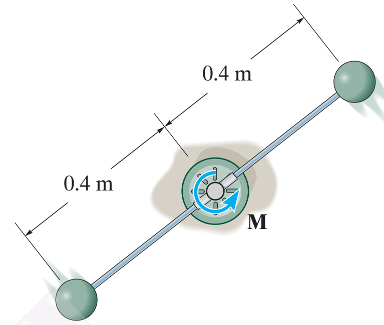
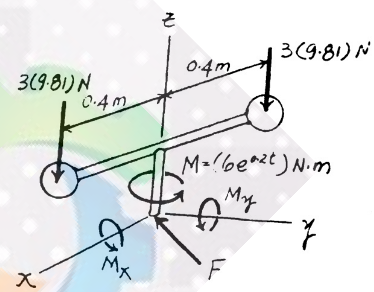
15–101. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. If a torque $M = (6e^{0.2t})\ \text{N} \cdot \text{m}$, where t is in seconds, is applied to the rod as shown, determine the speed of each of the spheres in 2 s, starting from rest.



SOLUTION

Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have



$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$2[0.4\ (3)\ (0)] + \int_0^{2\text{ s}} \left(6e^{0.2t}\right)\, dt = 2\ [0.4\ (3)\ v]$$

$$v = 6.15\text{ m/s} \qquad \textbf{Ans.}$$

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Ans:
$v = 6.15$ m/s

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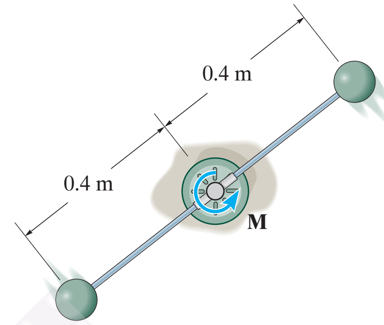
15–102. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. Determine the time the torque $M = (8t)$ N · m, where t is in seconds, must be applied to the rod so that each sphere attains a speed of 3 m/s starting from rest.



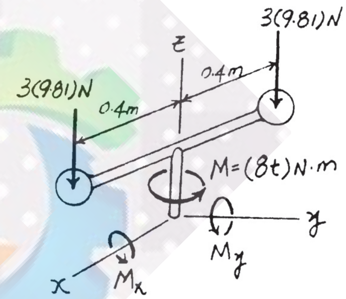
SOLUTION

Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z \, dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^t (8t)\, dt = 2[0.4\,(3)\,(3)]$$

$$t = 1.34 \text{ s}$$ **Ans.**



Ans:
$t = 1.34$ s

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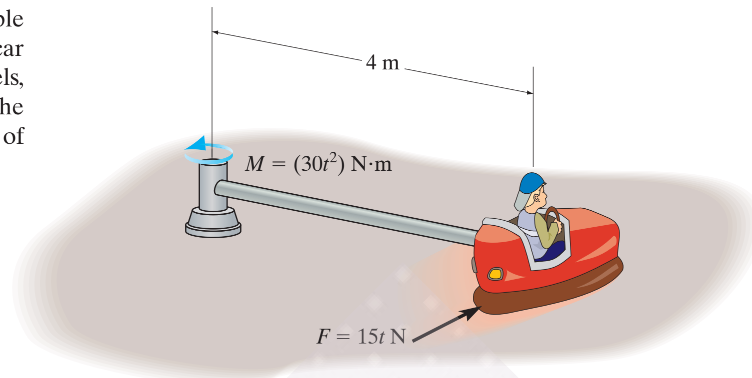
15–103.

If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)\ \text{N}\cdot\text{m}$ and the engine of the car supplies a traction force of $F = (15t)\ \text{N}$ to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



SOLUTION

Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of $\mathbf{W}$ and $\mathbf{N}$ are parallel to the z axis, they produce no angular impulse about the z axis.

Principle of Angular Impulse and Momentum:

$$(H_1)_z + \Sigma \int_{t_2}^{t_1} M_z\, dt = (H_2)_z$$

$$0 + \int_0^{5\text{ s}} 30t^2\, dt + \int_0^{5\text{ s}} 15t(4)dt = 150v(4)$$

$v = 3.33$ m/s **Ans.**

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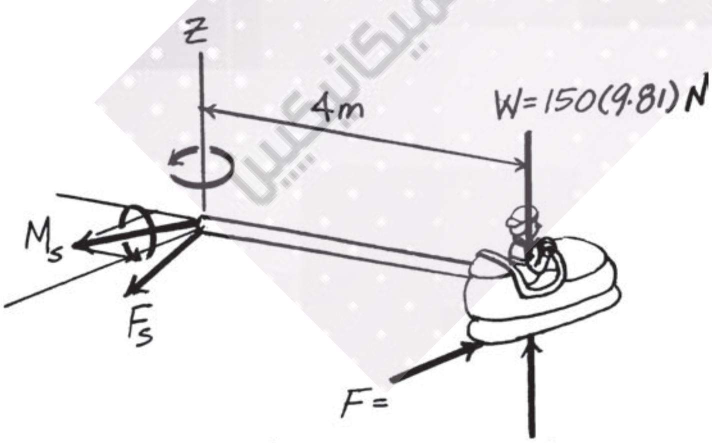

(a)

Ans:
$v = 3.33$ m/s

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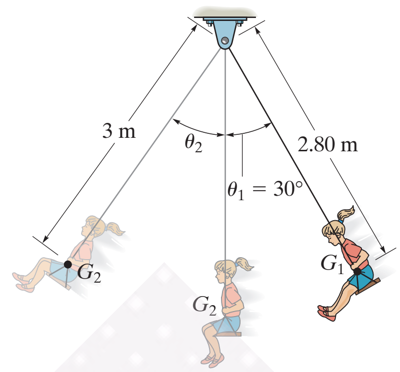
***15–104.**

A child having a mass of 50 kg holds her legs up as shown as she swings downward from rest at $\theta_1 = 30°$. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.



SOLUTION

First before $\theta = 30°$;

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.80(1 - \cos 30°)(50)(9.81) = \frac{1}{2}(50)(v_1)^2 + 0$$

$$v_1 = 2.713 \text{ m/s}$$

$$H_1 = H_2$$

$$50(2.713)(2.80) = 50(v_2)(3)$$

$$v_2 = 2.532 = 2.53 \text{ m/s} \qquad \textbf{Ans.}$$

Just after $\theta = 0°$;

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(50)(2.532)^2 + 0 = 0 + 50(9.81)(3)(1 - \cos \theta_2)$$

$$0.1089 = 1 - \cos \theta_2$$

$$\theta_2 = 27.0° \qquad \textbf{Ans.}$$

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Ans:
$v_2 = 2.53$ m/s
$\theta_2 = 27.0°$

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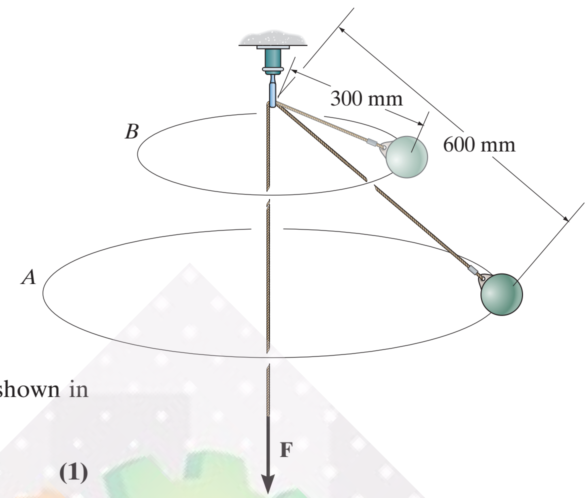
15–105.

When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to rotate around the horizontal circular path A. If the force $\mathbf{F}$ on the cord is increased, the bob rises and then rotates around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force $\mathbf{F}$.

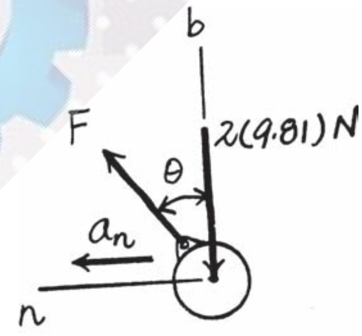


SOLUTION

Equations of Motion: By referring to the free-body diagram of the bob shown in Fig. a,

$$+\uparrow \Sigma F_b = 0; \qquad F\cos\theta - 2(9.81) = 0 \qquad \textbf{(1)}$$

$$\leftarrow \Sigma F_n = ma_n; \qquad F\sin\theta = 2\left(\frac{v^2}{l\sin\theta}\right) \qquad \textbf{(2)}$$



(a)

Eliminating F from Eqs. (1) and (2) yields

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{9.81l}$$

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{9.81l} \qquad \textbf{(3)}$$

When $l = 0.6$ m, $v = v_1 = 5$ m/s. Using Eq. (3), we obtain

$$\frac{1-\cos^2\theta_1}{\cos\theta_1} = \frac{1.5^2}{9.81(0.6)}$$

$$\cos^2\theta_1 + 0.3823\cos\theta_1 - 1 = 0$$

Solving for the root < 1, we obtain

$$\theta_1 = 34.21°$$

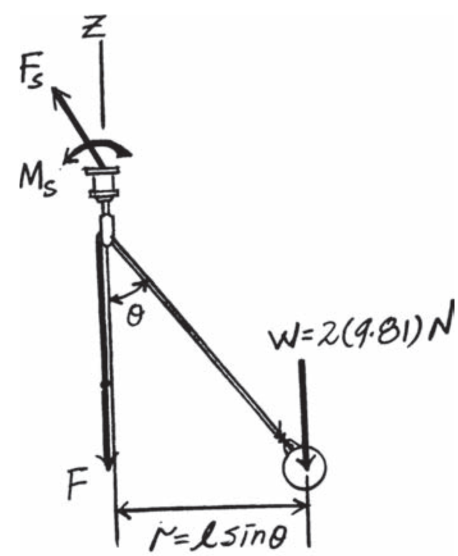
Conservation of Angular Momentum: By observing the free-body diagram of the system shown in Fig. b, notice that $\mathbf{W}$ and $\mathbf{F}$ are parallel to the z axis, $\mathbf{M}_S$ has no z component, and $\mathbf{F}_S$ acts through the z axis. Thus, they produce no angular impulse about the z axis. As a result, the angular momentum of the system is conserved about the z axis. When $\theta = \theta_1 = 34.21°$ and $\theta = \theta_2$, $r = r_1 = 0.6\sin 34.21° = 0.3373$ m and $r = r_2 = 0.3\sin\theta_2$. Thus,

$$(H_z)_1 = (H_z)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$0.3373(2)(1.5) = 0.3\sin\theta_2\,(2)v_2$$

$$v_2\sin\theta_2 = 1.6867 \qquad \textbf{(4)}$$



(b)

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15–105. Continued

Substituting $l = 0.3$ and $\theta = \theta_2$ $v = v_2$ into Eq. (3) yields

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{v^2}{9.81(0.3)}$$

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{{v_2}^2}{2.943} \qquad \textbf{(5)}$$

Eliminating v_2 from Eqs. (4) and (5),

$$\sin^3\theta_2 \tan\theta_2 - 0.9667 = 0$$

Solving the above equation by trial and error, we obtain

$$\theta_2 = 57.866°$$

Substituting the result of θ_2 into Eq. (4), we obtain

$$v_2 = 1.992 \text{ m/s} = 1.99 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy: When θ changes from θ_1 to θ_2, **W** displaces vertically upward $h = 0.6 \cos 34.21° - 0.3 \cos 57.866° = 0.3366$ m. Thus, **W** does negatives work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} m{v_1}^2 + U_F + \left(-Wh\right) = \frac{1}{2} m{v_2}^2$$

$$\frac{1}{2}(2)(1.5^2) + U_F - 2(9.81)(0.3366) = \frac{1}{2}(2)(1.992)^2$$

$$U_F = 8.32 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Ans:
$v_2 = 1.99$ m/s
$U_F = 8.32$ N · m

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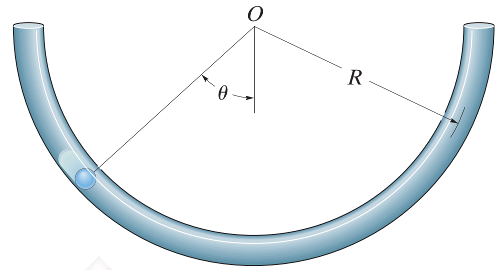
15–106.

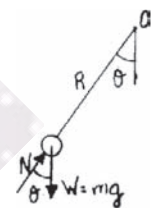
A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O ($\Sigma M_O = \dot{H}_O$), and show that the motion of the particle is governed by the differential equation $\ddot{\theta} + (g/R)\sin\theta = 0$.



SOLUTION

$$\circlearrowleft +\Sigma M_O = \frac{dH_O}{dt}; \qquad -Rmg\sin\theta = \frac{d}{dt}(mvR)$$

$$g\sin\theta = -\frac{dv}{dt} = -\frac{d^2s}{dt^2}$$



But, $s = R\theta$

Thus, $g\sin\theta = -R\ddot{\theta}$

or, $\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$ **Q.E.D.**

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Ans:
$\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$

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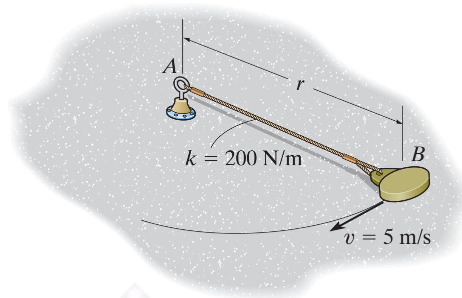
15–107. At the instant $r = 1.5$ m, the 5-kg disk is given a speed of $v = 5$ m/s, perpendicular to the elastic cord. Determine the speed of the disk and the rate of shortening of the elastic cord at the instant $r = 1.2$ m. The disk slides on the smooth horizontal plane. Neglect its size. The cord has an unstretched length of 0.5 m.



SOLUTION

Conservation of Energy: The intial and final stretch of the elastic cord is $s_1 = 1.5 - 0.5 = 1$ m and $s_2 = 1.2 - 0.5 = 0.7$ m. Thus,

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1{}^2 + \frac{1}{2}ks_1{}^2 = \frac{1}{2}mv_2{}^2 = \frac{1}{2}ks_2{}^2$$

$$\frac{1}{2}5(5^2) + \frac{1}{2}(200)(1^2) = \frac{1}{2}(5)v_2{}^2 + \frac{1}{2}(200)(0.7^2)$$

$$v_2 = 6.738 \text{ m/s} \qquad \textbf{Ans.}$$

Conservation of Angular Momentum: Since no angular impulse acts on the disk about an axis perpendicular to the page passing through point O, its angular momentum of the system is conserved about this z axis. Thus,

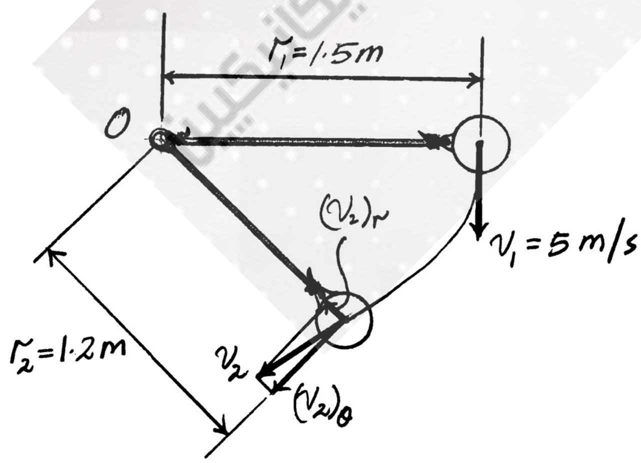
$$(H_O)_1 = (H_O)_2$$

$$r_1mv_1 = r_2m(v_2)_\theta$$

$$(v_2)_\theta = \frac{r_1v_1}{r_2} = \frac{1.5(5)}{1.2} = 6.25 \text{ m/s}$$

Since $v_2{}^2 = (v_2)_\theta{}^2 + (v_2)_r{}^2$, then

$$(v_2)_r = \sqrt{v_2{}^2 - (v_2)_\theta{}^2} = \sqrt{6.738^2 - 6.25^2} = 2.52 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v_2 = 6.738$ m/s
$(v_2)_r = 2.52$ m/s

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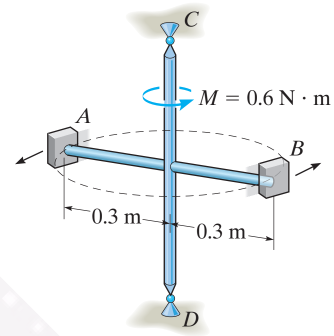
***15–108.**

The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)\ \text{N} \cdot \text{m}$ is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.



SOLUTION

$$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o dt = (H_o)_2$$

$$2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]$$

$$v = 9.50 \text{ m/s}$$ **Ans.**

Ans:
$v = 9.50$ m/s

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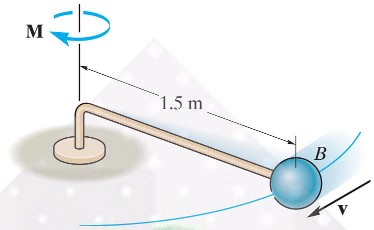
15–109. The ball B has mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (3t^2 + 5t + 2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.



SOLUTION

Principle of angular impulse momentum

Given:

$M = 10\ \text{kg}$

$a = 3\ \text{N}\cdot\text{m/s}^2$

$b = 5\ \text{N}\cdot\text{m/s}$

$c = 2\ \text{N}\cdot\text{m}$

$t_1 = 2\ \text{s}$

$v_0 = 2\ \text{m/s}$

$L = 1.5\ \text{m}$

$$M v_0 L + \int_0^{t_1} a t^2 + b t + c \, dt = M v_1 L$$

$$v_1 = v_0 + \frac{1}{ML}\int_0^{t_1} a t^2 + b t + c \, dt \qquad v_1 = 3.47\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_1 = 3.47$ m/s

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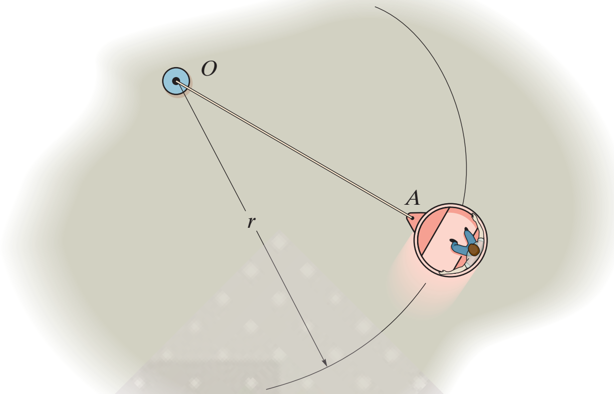
15–110.

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.



SOLUTION

Conservation of Angular Momentum. At $t = 4$ s, $r_2 = 8 - 0.5(4) = 6$ m.

$$(H_0)_1 = (H_0)_2$$

$$r_1 m v_1 = r_2 m (v_2)_t$$

$$8[200(3)] = 6[200(v_2)_t]$$

$$(v_2)_t = 4.00 \text{ m/s}$$

Here, $(v_2)_t = 0.5$ m/s. Thus

$$v_2 = \sqrt{(v_2)_t^2 + (v_2)_r^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(200)(3^2) + \Sigma U_{1-2} = \frac{1}{2}(200)(4.031)^2$$

$$\Sigma U_{1-2} = 725 \text{ J} \qquad \textbf{Ans.}$$

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Ans:
$v_2 = 4.03$ m/s
$\Sigma U_{1-2} = 725$ J

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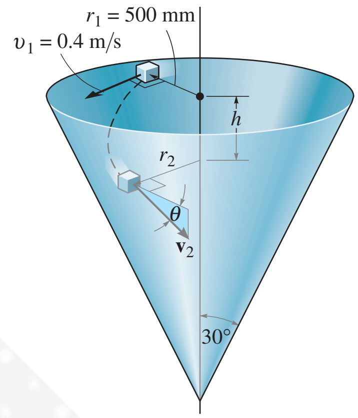
15–111.

A small block having a mass of 0.1 kg is given a horizontal velocity of $v_1 = 0.4$ m/s when $r_1 = 500$ mm. It slides along the smooth conical surface. Determine the distance h it must descend for it to reach a speed of $v_2 = 2$ m/s. Also, what is the angle of descent θ, that is, the angle measured from the horizontal to the tangent of the path?



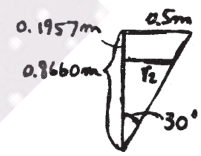
SOLUTION

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(0.1)(0.4)^2 + 0.1(9.81)(h) = \frac{1}{2}(0.1)(2)^2$

$h = 0.1957 \text{ m} = 196 \text{ mm}$ **Ans.**

From similar triangles



$$r_2 = \frac{(0.8660 - 0.1957)}{0.8660}(0.5) = 0.3870 \text{ m}$$

$(H_0)_1 = (H_0)_2$

$0.5(0.1)(0.4) = 0.3870(0.1)(v_2{}')$

$v_2{}' = 0.5168$ m/s

$v_2 = \cos\theta = v_2{}'$

$2\cos\theta = 0.5168$

$\theta = 75.0°$ **Ans.**

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Ans:
$h = 196$ mm
$\theta = 75.0°$

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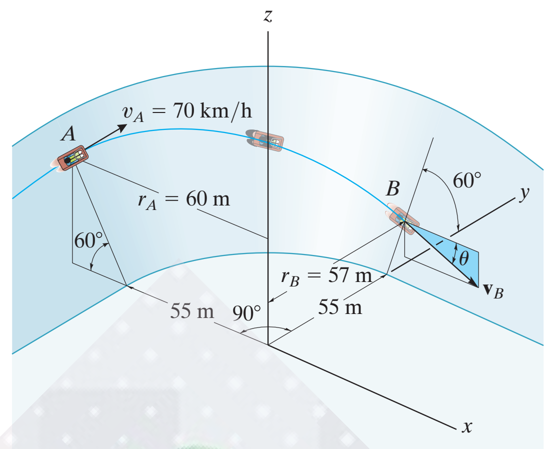
***15–112.**

A toboggan and rider, having a total mass of 150 kg, enter horizontally tangent to a 90° circular curve with a velocity of $v_A = 70$ km/h. If the track is flat and banked at an angle of 60°, determine the speed v_B and the angle θ of "descent," measured from the horizontal in a vertical x–z plane, at which the toboggan exists at B. Neglect friction in the calculation.



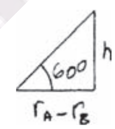
SOLUTION

$$v_A = 70 \text{ km/h} = 19.44 \text{ m/s}$$

$$(H_A)_z = (H_B)_z$$

$$150(19.44)(60) = 150(v_B)\cos\theta(57) \qquad \textbf{(1)}$$

Datum at B:



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(150)(19.44)^2 + 150(9.81)h = \frac{1}{2}(150)(v_B)^2 + 0 \qquad \textbf{(2)}$$

Since $h = (r_A - r_B)\tan 60° = (60 - 57)\tan 60° = 5.196$

Solving Eq. (1) and Eq (2):

$$v_B = 21.9 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = 20.9 \qquad \textbf{Ans.}$$

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Ans:
$v_B = 21.9$ m/s
$\theta = 20.9$

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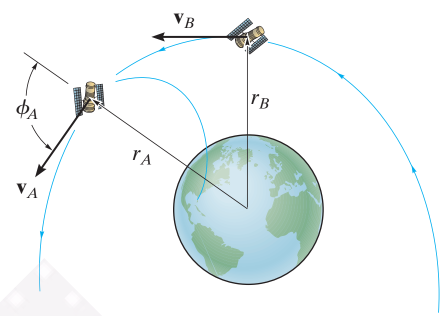
15–113.

An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



SOLUTION

$(H_O)_1 = (H_O)_2$

$m_s\,(v_A \sin\phi_A) r_A = m_s\,(v_B) r_B$

$$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \qquad \textbf{(1)}$$

$T_A + V_A = T_B + V_B$

$$\frac{1}{2} m_s\,(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2} m_s\,(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

Solving,

$v_B = 10.2$ km/s **Ans.**

$r_B = 13.8$ Mm **Ans.**

Ans:
$v_B = 10.2$ km/s
$r_B = 13.8$ Mm

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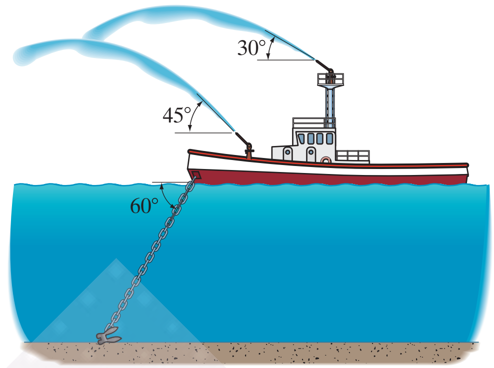
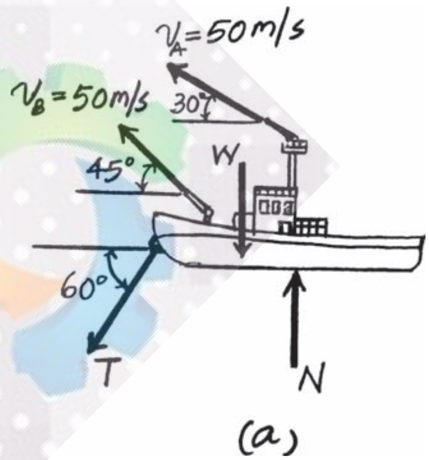
15–114.

The fire boat discharges two streams of seawater, each at a flow of $0.25\ \text{m}^3/\text{s}$ and with a nozzle velocity of 50 m/s. Determine the tension developed in the anchor chain needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ \text{kg/m}^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate of the sea water at nozzles A and B are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho sw\, Q = 1020(0.25) = 225$ kg/s. Since the sea water is collected from the larger reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume (the boat),



$$\overset{+}{\leftarrow}\ \Sigma F_x = \frac{dm_A}{dt}\left(v_A\right)_x + \frac{dm_B}{dt}\left(v_B\right)_x;$$

$$T\cos 60° = 225(50\cos 30°) + 225(50\cos 45°)$$

$$T = 40\,114.87\ \text{N} = 40.1\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$T = 40.1$ kN

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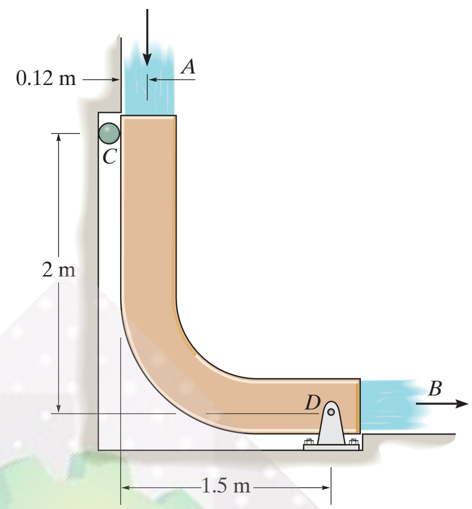
15–115.

The chute is used to divert the flow of water, $Q = 0.6\ \text{m}^3/\text{s}$. If the water has a cross-sectional area of $0.05\ \text{m}^2$, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

Equations of Steady Flow: Here, the flow rate $Q = 0.6\ \text{m}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.6}{0.05} = 12.0\ \text{m/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = 1000\,(0.6) = 600\ \text{kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{DB}\, v_B - d_{DA}\, v_A);$$

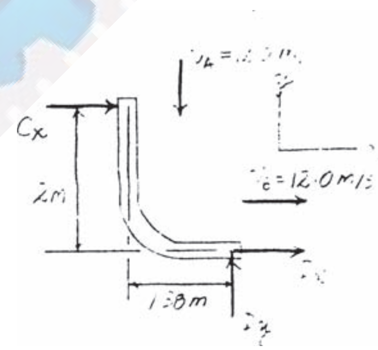
$$-C_x\,(2) = 600\,[0 - 1.38(12.0)] \qquad C_x = 4968\ \text{N} = 4.97\ \text{kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right);$$

$$D_x + 4968 = 600\,(12.0 - 0) \qquad D_x = 2232\text{N} = 2.23\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \Sigma\frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);$$

$$D_y = 600[0 - (-12.0)] \qquad D_y = 7200\ \text{N} = 7.20\ \text{kN} \qquad \textbf{Ans.}$$



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Ans:
$C_x = 4.97\ \text{kN}$
$D_x = 2.23\ \text{kN}$
$D_y = 7.20\ \text{kN}$

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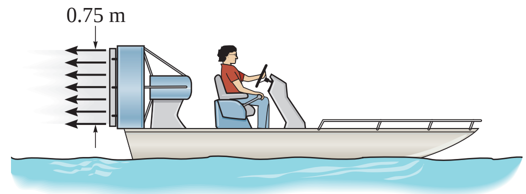
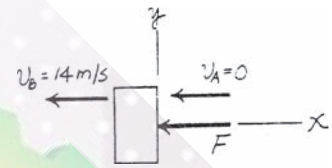
***15–116.**

The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$ and that the entering air is essentially at rest. Neglect the drag resistance of the water.



SOLUTION



Equations of Steady Flow: Initially, the boat is at rest hence $v_B = v_{a/b} = 14$ m/s. Then, $Q = v_B A = 14\left[\frac{\pi}{4}\left(0.75^2\right)\right] = 6.185\ \text{m}^3/\text{s}$ and $\frac{dm}{dt} = \rho_a Q = 1.22(6.185) = 7.546$ kg/s. Applying Eq. 15–25, we have

$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}); \qquad -F = 7.546(-14 - 0) \qquad F = 105.64\ \text{N}$$

Equation of Motion :

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 105.64 = 200a \qquad a = 0.528\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.528\ \text{m/s}^2$

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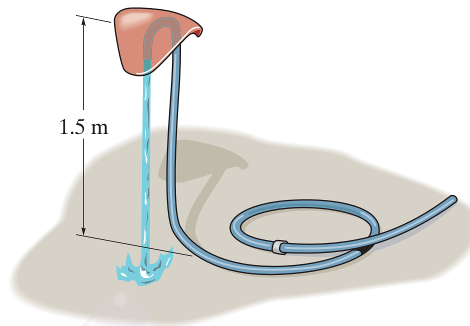
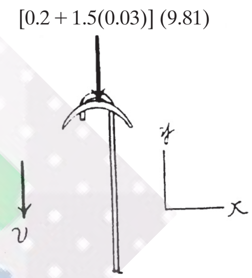
15–117. The toy sprinkler for children consists of a 0.2-kg cap and a hose that has a mass per length of 30 g/m. Determine the required rate of flow of water through the 5-mm-diameter tube so that the sprinkler will lift 1.5 m from the ground and hover from this position. Neglect the weight of the water in the tube. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION



Equations of Steady Flow: Here, $v = \dfrac{Q}{\frac{\pi}{4}(0.005^2)} = \dfrac{Q}{6.25(10^{-6})\pi}$ and $\dfrac{dm}{dt} = \rho_w Q = 1000Q$. Applying Eq. 15–25, we have

$$\Sigma F_y = \frac{dm}{dt}\left(v_{B_y} - v_{A_y}\right); \quad -[0.2 + 1.5\,(0.03)]\,(9.81) = 1000Q\left(-\frac{Q}{6.25\,(10^{-6})\,\pi} - 0\right)$$

$$Q = 0.217\left(10^{-3}\right)\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$Q = 0.217\left(10^{-3}\right)\ \text{m}^3/\text{s}$

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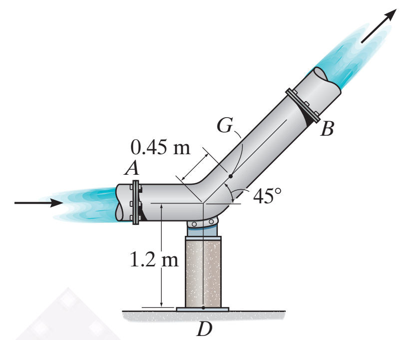
15-118. The bend is connected to the pipe at flanges A and B as shown. If the diameter of the pipe is 0.3 m and it carries a discharge of 1.35 m^3/s, determine the horizontal and vertical components of force reaction and the moment reaction exerted at the fixed base D of the support. The total weight of the bend and the water within it is 2500 N, with a mass center at point G. The gauge pressure of the water at the flanges at A and B are 120 kN/m^2 and 96 kN/m^2, respectively. Assume that no force is transferred to the flanges at A and B. The specific weight of water is $\gamma_w = 10\ kN/m^3$.



SOLUTION

Free-Body Diagram: The free-body of the control volume is shown in Fig. *a*. The force exerted on sections A and B due to the water pressure is $F_A = P_A A_A = 120\left[\frac{\pi}{4}(0.3^2)\right] = 8.4823$ kN and $F_B = P_B A_B = 96\left[\frac{\pi}{4}(0.3^2)\right] = 6.7858$ kN. The speed of the water at, sections A and B are $v_A = v_B = \frac{Q}{A} = \frac{1.35}{\frac{\pi}{4}(0.3^2)} = 19.10$ m/s. Also, the mass flow rate at these two sections are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_W Q = \frac{10(10^3)}{9.81}(1.35) = 1376.1$ kg/s.

Steady Flow Equation: The moment steady flow equation will be written about point D to eliminate D_x and D_y.

$$\zeta + \quad \Sigma M_D = \frac{dm_B}{dt} dv_B - \frac{dm_A}{dt} dv_A;$$

$$M_D + 6.7858(10^3)\cos 45°(1.2) - 2500(0.45\cos 45°) - 8.4823(10^3)(1.2)$$

$$= -1376.1(1.2)(19.10\cos 45°) - [-1376.1(1.2)(19.10)]$$

$$M_D = 14\,454.23\ \text{N}\cdot\text{m} = 14.45\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Writing the force steady flow equation along the x and y axes,

$$(+\uparrow)\ \Sigma F_y = \frac{dm}{dt}\left[(v_B)_y - (v_A)_y\right];$$

$$D_y - 2500 - 6.7858(10^3)\sin 45° = 1376.1(19.10\sin 45° - 0)$$

$$D_y = 25\,883.53\ \text{N} = 25.88\ \text{kN} \qquad \textbf{Ans.}$$

$$(\overset{+}{\rightarrow})\ \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right];$$

$$8.4823(10^3) - 6.7858(10^3)\cos 45° - D_x = 1376.1[19.1\cos 45° - 19.1]$$

$$D_x = 11382.28\ \text{N} = 11.38\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$M_D = 14.45$ kN · m
$D_y = 25.88$ kN
$D_x = 11.38$ kN

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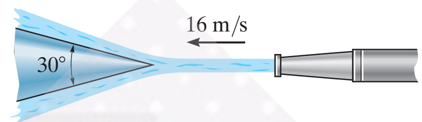
15-119.

Water is discharged at speed v against the fixed cone diffuser. If the opening diameter of the nozzle is d, determine the horizontal force exerted by the water on the diffuser.



SOLUTION

Given:

$$v = 16 \text{ m/s}$$

$$d = 40 \text{ mm}$$

$$\theta = 30°$$

$$\rho_w = 1 \text{ Mg/m}^3$$

$$Q = \frac{\pi}{4} d^2 v$$

$$m' = \rho_w Q$$

$$F_x = m'\left(-v\cos\left(\frac{\theta}{2}\right) + v\right)$$

$F_x = 11.0\text{ N}$ **Ans.**

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Ans:
$F_x = 11.0 \text{ N}$

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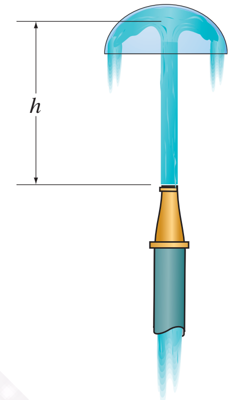
***15–120.** The hemispherical bowl of mass m is held in equilibrium by the vertical jet of water discharged through a nozzle of diameter d. If the discharge of the water through the nozzle is Q, determine the height h at which the bowl is suspended. The water density is ρ_w. Neglect the weight of the water jet.



SOLUTION

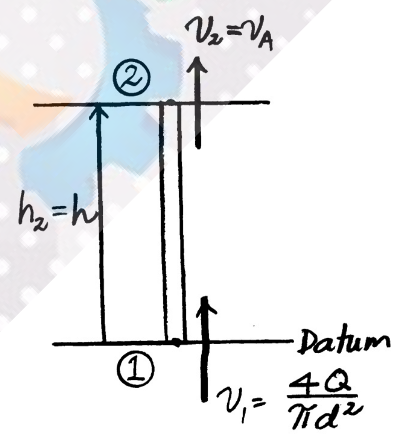
Conservation of Energy: The speed at which the water particle leaves the nozzle is $v_1 = \dfrac{Q}{A} = \dfrac{Q}{\frac{\pi}{4}d^2} = \dfrac{4Q}{\pi d^2}$. The speed of particle v_A when it comes in contact with the bowl can be determined using conservation of energy. With reference to the datum set in Fig. a,

$$T_1 + V_1 = T_2 + V_2 0$$

$$\frac{1}{2}mv_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2{}^2 + \left(V_g\right)_2$$

$$\frac{1}{2}m\left(\frac{4Q}{\pi d^2}\right)^2 + 0 = \frac{1}{2}mv_A{}^2 + mgh$$

$$v_A = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$$



(a)

Steady Flow Equation: The mass flow rate of the water jet that enters the control volume at A is $\dfrac{dm_A}{dt} = \rho_w Q$, and exits from the control volume at B is $\dfrac{dm_B}{dt} = \dfrac{dm_A}{dt} = \rho_w Q$. Thus, $v_B = v_A = \sqrt{\dfrac{16Q^2}{\pi^2 d^4} - 2gh}$. Here, the vertical force acting on the control volume is equal to the weight of the bowl. By referring to the free - body diagram of the control volume, Fig. b,

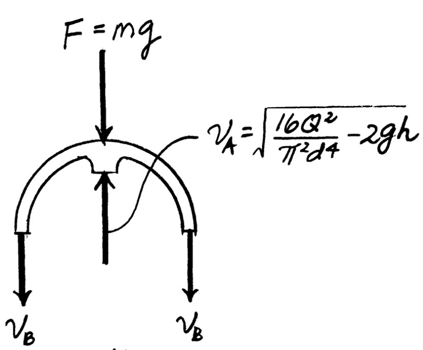
$$+\uparrow \Sigma F_y = 2\frac{dm_B}{dt}v_B - \frac{dm_A}{dt}v_A;$$

$$-mg = -(\rho_w Q)\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right) - \rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$mg = 2\rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$m^2 g^2 = 4\rho_w{}^2 Q^2\left(\frac{16Q^2}{\pi^2 d^4} - 2gh\right)$$

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2} \qquad \textbf{Ans.}$$



(b)

Ans:

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2}$$

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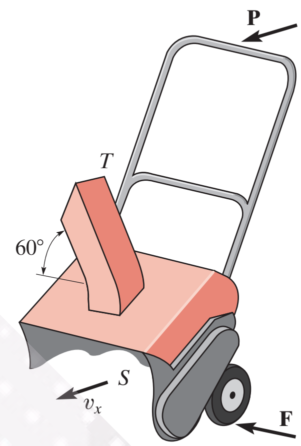
15–121.

A snowblower having a scoop S with a cross-sectional area of $A_s = 0.12\ \text{m}^3$ is pushed into snow with a speed of $v_s = 0.5\ \text{m/s}$. The machine discharges the snow through a tube T that has a cross-sectional area of $A_T = 0.03\ \text{m}^2$ and is directed 60° from the horizontal. If the density of snow is $\rho_s = 104\ \text{kg/m}^3$, determine the horizontal force P required to push the blower forward, and the resultant frictional force F of the wheels on the ground, necessary to prevent the blower from moving sideways. The wheels roll freely.



SOLUTION

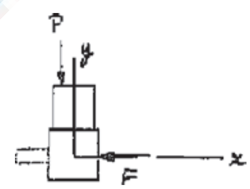
$$\frac{dm}{dt} = \rho v_s A_s = (104)(0.5)(0.12) = 6.24\ \text{kg/s}$$

$$v_s = \frac{dm}{dt}\left(\frac{1}{\rho A_r}\right) = \left(\frac{6.24}{104(0.03)}\right) = 2.0\ \text{m/s}$$

$$\Sigma F_x = \frac{dm}{dt}(v_{T_2} - v_{S_2})$$

$$-F = 6.24(-2\cos 60° - 0)$$

$$F = 6.24\ \text{N}$$ **Ans.**



$$\Sigma F_y = \frac{dm}{dt}(v_{T_2} - v_{S_2})$$

$$-P = 6.24(0 - 0.5)$$

$$P = 3.12\ \text{N}$$ **Ans.**

Ans:
$F = 6.24\ \text{N}$
$P = 3.12\ \text{N}$

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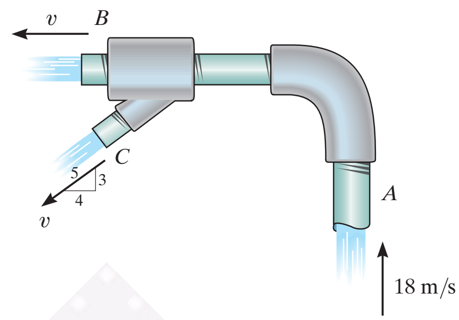
15–122.

The gauge pressure of water at A is 150.5 kPa. Water flows through the pipe at A with a velocity of 18 m/s, and out the pipe at B and C with the same velocity v. Determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 50 mm at A, and at B and C the diameter is 30 mm. $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Continuity. The flow rate at B and C are the same since the pipe have the same diameter there. The flow rate at A is

$$Q_A = v_A A_A = (18)[\pi(0.025^2)] = 0.01125\pi\ \text{m}^3/\text{s}$$

Continuity negatives that

$$Q_A = Q_B + Q_C; \qquad 0.01125\pi = 2Q$$

$$Q = 0.005625\pi\ \text{m}^3/\text{s}$$

Thus,

$$v_c = v_B = \frac{Q}{A} = \frac{0.005625\pi}{\pi(0.015^2)} = 25\ \text{m/s}$$

Equation of Steady Flow. The force due to the pressure at A is $P = \rho_A A_A = (150.5)(10^3)[\pi(0.025^2)] = 94.0625\pi$ N. Here, $\dfrac{dm_A}{dt} = \rho_w Q_A$ $= 1000(0.01125\pi) = 11.25\pi$ kg/s and $\dfrac{dm_A}{dt} = \dfrac{dM_c}{dt} = \rho_w Q = 1000(0.005625\pi)$ $= 5.625\pi$ kg/s.

$$\overset{+}{\leftarrow}\Sigma F_x = \frac{dm_B}{dt}(v_B)_x + \frac{dm_c}{dt}(v_c)_x - \frac{dm_A}{dt}(v_A)_x;$$

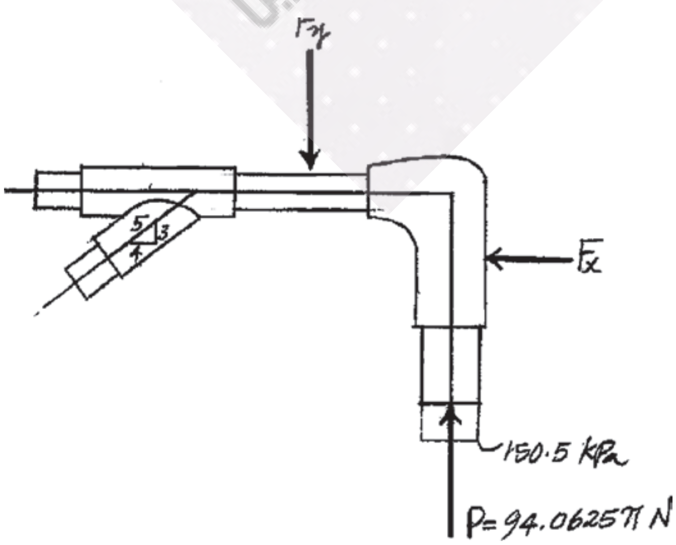
$$F_x = (5.625\pi)(25) + (5.625\pi)\left[25\left(\frac{4}{5}\right)\right] - (11.25\pi)(0)$$

$$= 795.22\ \text{N} = 795\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = \frac{dm_B}{dt}(v_B)_y + \frac{dm_C}{dt}(v_C)_y - \frac{dm_A}{dt}(v_A)_y;$$

$$94.0625\pi - F_y = (5.625\pi)(0) + (5.625\pi)\left[-25\left(\frac{3}{5}\right)\right] - (11.25\pi)(18)$$

$$F_y = 1196.75\ \text{N} = 1.20\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$F_x = 795$ N
$F_y = 1.20$ kN

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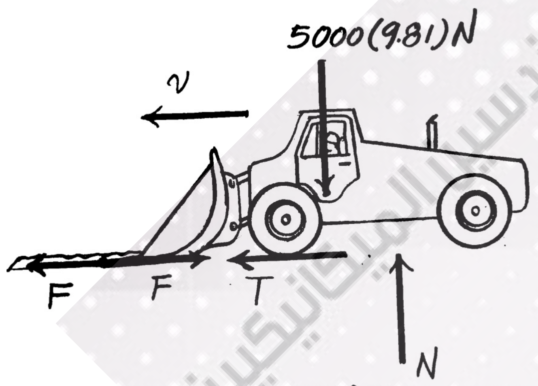
15–123. A scoop in front of the tractor collects snow at a rate of 200 kg/s. Determine the resultant traction force **T** that must be developed on all the wheels as it moves forward on level ground at a constant speed of 5 km/h. The tractor has a mass of 5 Mg.

SOLUTION

Here, the tractor moves with the constant speed of $v = \left[5\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right]$ $= 1.389$ m/s. Thus, $v_{D/s} = v = 1.389$ m/s since the snow on the ground is at rest. The rate at which the tractor gains mass is $\frac{dm_s}{dt} = 200$ kg/s. Since the tractor is moving with a constant speeds $\frac{dv}{dt} = 0$. Referring to Fig. *a*,

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad T = 0 + 1.389(200)$$

$$T = 278 \text{ N} \qquad \textbf{Ans.}$$



(a)

Ans:
$T = 278$ N

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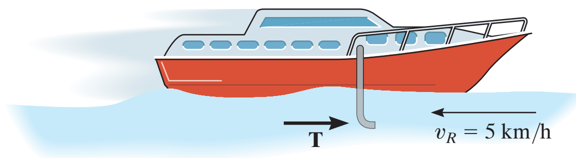
***15–124.**

The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \frac{40}{80} = 0.5\ \text{kg/s}$$

$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444\ \text{m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$T = 0 + 19.444(0.5) = 9.72\ \text{N}$$ **Ans.**

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Ans:
$T = 9.72$ N

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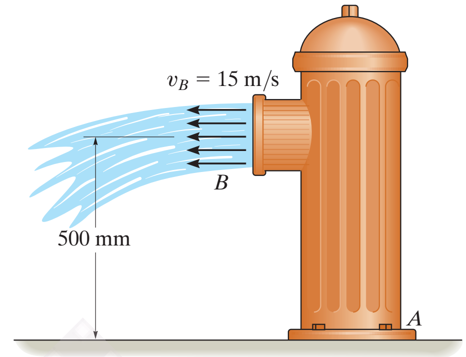
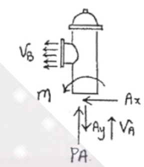
15–125.

Water is flowing from the 150-mm-diameter fire hydrant with a velocity $v_B = 15$ m/s. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is 50 kPa. The diameter of the fire hydrant at A is 200 mm. $\rho_w = 1$ Mg/m^3.



SOLUTION



$$\frac{dm}{dt} = \rho v_A A_B = 1000(15)(\pi)(0.075)^2$$

$$\frac{dm}{dt} = 265.07 \text{ kg/s}$$

$$v_A = \left(\frac{dm}{dt}\right)\frac{1}{\rho A_A} = \frac{265.07}{1000(\pi)(0.1)^2}$$

$$v_A = 8.4375 \text{ m/s}$$

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$$A_x = 265.07(15-0) = 3.98 \text{ kN}$$ **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

$$-A_y + 50(10^3)(\pi)(0.1)^2 = 265.07(0-8.4375)$$

$$A_y = 3.81 \text{ kN}$$ **Ans.**

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{AB}\, v_B - d_{AA}\, v_A)$$

$$M = 265.07(0.5(15) - 0)$$

$$M = 1.99 \text{ kN} \cdot \text{m}$$ **Ans.**

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Ans:
$A_x = 3.98$ kN
$A_y = 3.81$ kN
$M = 1.99$ kN·m

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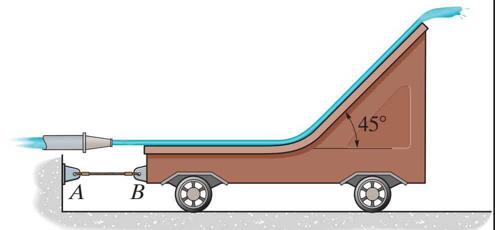
15–126.

Water is discharged from a nozzle with a velocity of 12 m/s and strikes the blade mounted on the 20-kg cart. Determine the tension developed in the cord, needed to hold the cart stationary, and the normal reaction of the wheels on the cart. The nozzle has a diameter of 50 mm and the density of water is $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate at sections A and B of the control volume is $\dfrac{dm}{dt} = \rho_W Q = \rho_W A v = 1000\left[\dfrac{\pi}{4}(0.05^2)\right](12) = 7.5\pi\ \text{kg/s}$

Referring to the free-body diagram of the control volume shown in Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -F_x = 7.5\pi(12\cos 45^\circ - 12)$$

$$F_x = 82.81\ \text{N}$$

$$+\uparrow\Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad F_y = 7.5\pi(12\sin 45^\circ - 0)$$

$$F_y = 199.93\ \text{N}$$

Equilibrium: Using the results of $\mathbf{F}_x$ and $\mathbf{F}_y$ and referring to the free-body diagram of the cart shown in Fig. b,

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 82.81 - T = 0 \qquad T = 82.8\ \text{N}$ **Ans.**

$+\uparrow\Sigma F_y = 0; \qquad N - 20(9.81) - 199.93 = 0 \qquad N = 396\ \text{N}$ **Ans.**

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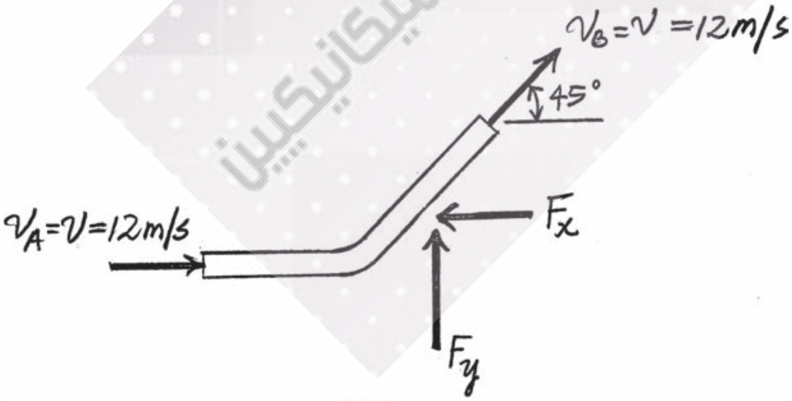
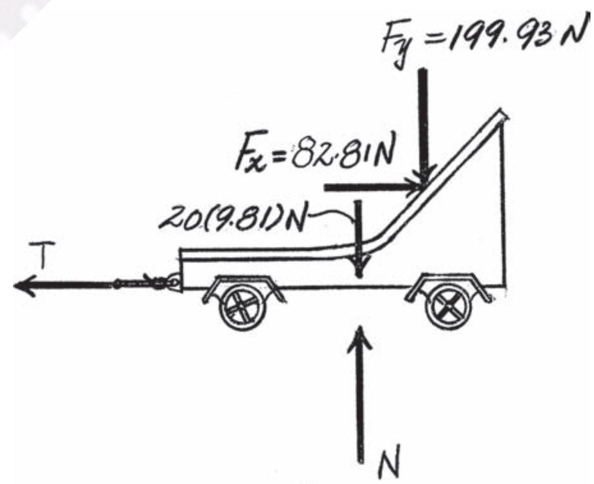


(a)



(b)

Ans:
$T = 82.8\ \text{N}$
$N = 396\ \text{N}$

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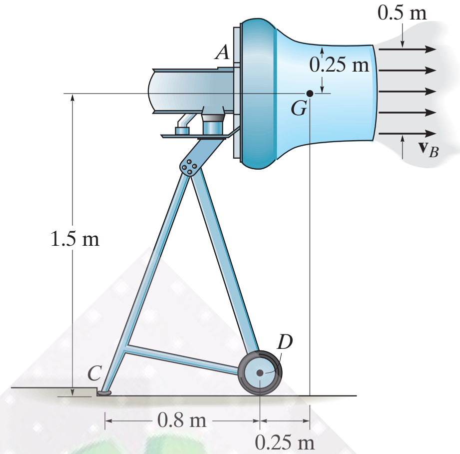
15–127. When operating, the air-jet fan discharges air with a speed of $v_B = 20$ m/s into a slipstream having a diameter of 0.5 m. If air has a density of 1.22 kg/m^3, determine the horizontal and vertical components of reaction at C and the vertical reaction at each of the two wheels, D, when the fan is in operation. The fan and motor have a mass of 20 kg and a center of mass at G. Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



SOLUTION

$$\frac{dm}{dt} = \rho v A = 1.22(20)(\pi)(0.25)^2 = 4.791 \text{ kg/s}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x})$$

$$C_x = 4.791(20 - 0)$$

$$C_x = 95.8 \text{ N}$$ **Ans.**

$$+\uparrow \Sigma F_y = 0; \qquad C_y + 2D_y - 20(9.81) = 0$$

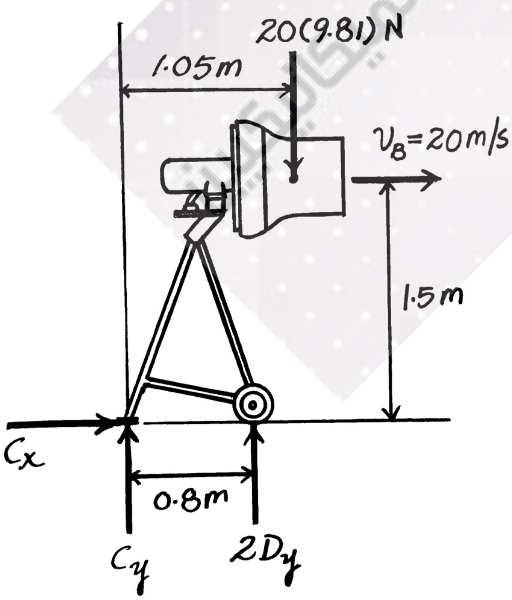
$$\zeta + \Sigma M_C = \frac{dm}{dt}(d_{CG}\, v_B - d_{CG}\, v_A)$$

$$2D_y(0.8) - 20(9.81)(1.05) = 4.791(-1.5(20) - 0)$$

Solving:

$$D_y = 38.9 \text{ N}$$ **Ans.**

$$C_y = 118 \text{ N}$$ **Ans.**



Ans:
$C_x = 95.8$ N
$D_y = 38.9$ N
$C_y = 118$ N

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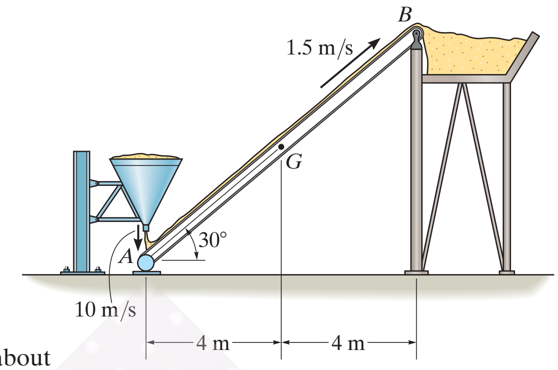
***15–128.**

Sand is discharged from the silo at A at a rate of 50 kg/s with a vertical velocity of 10 m/s onto the conveyor belt, which is moving with a constant velocity of 1.5 m/s. If the conveyor system and the sand on it have a total mass of 750 kg and center of mass at point G, determine the horizontal and vertical components of reaction at the pin support B roller support A. Neglect the thickness of the conveyor.



SOLUTION

Steady Flow Equation: The moment steady flow equation will be written about point B to eliminate $\mathbf{B}_x$ and $\mathbf{B}_y$. Referring to the free-body diagram of the control volume shown in Fig. a,

$$+\Sigma M_B = \frac{dm}{dt}(dv_B - dv_A); \qquad 750(9.81)(4) - A_y(8) = 50[0 - 8(5)]$$

$$A_y = 4178.5 \text{ N} = 4.18 \text{ kN} \qquad \textbf{Ans.}$$

Writing the force steady flow equation along the x and y axes,

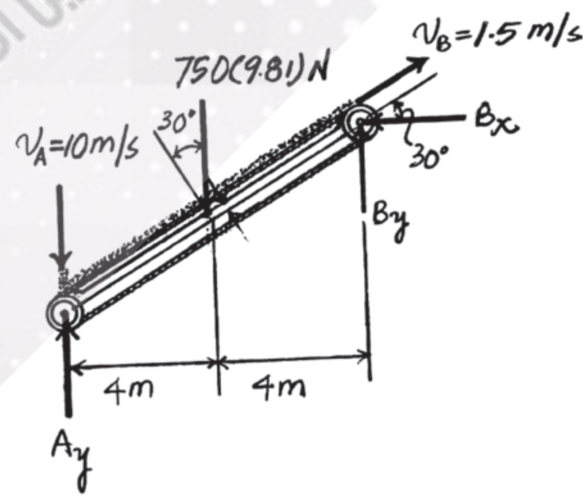
$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -B_x = 50(1.5 \cos 30° - 0)$$

$$B_x = |-64.95 \text{ N}| = 65.0 \text{ N} \rightarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad B_y + 4178.5 - 750(9.81)$$

$$= 50[1.5 \sin 30° - (-10)]$$

$$B_y = 3716.25 \text{ N} = 3.72 \text{ kN} \uparrow \qquad \textbf{Ans.}$$



(a)

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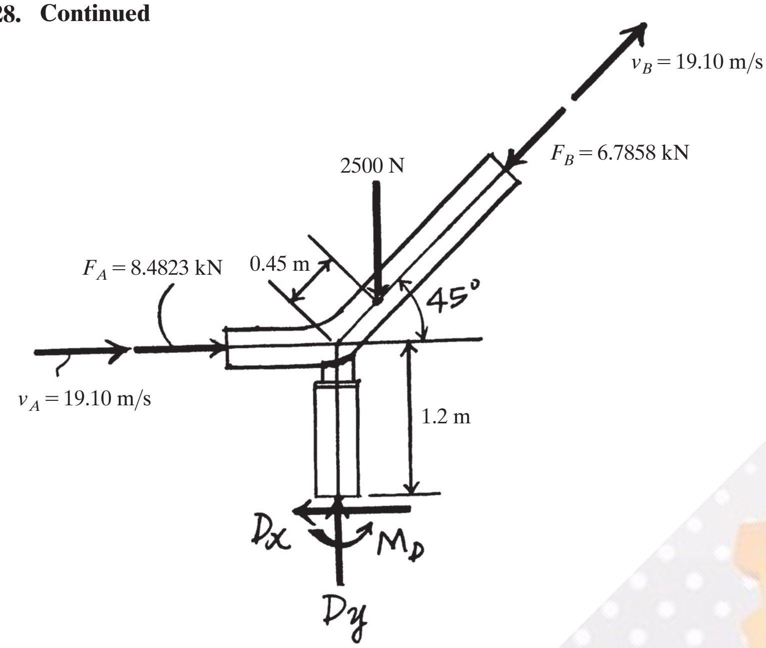
***15–128. Continued**



(a)

Ans:
$A_y = 4.18$ kN
$B_x = 65.0$ N $\rightarrow$
$B_y = 3.72$ kN $\uparrow$

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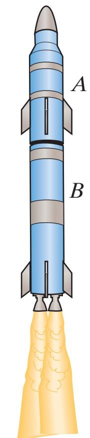
15–129.

Each of the two stages A and B of the rocket has a mass of 2 Mg when their fuel tanks are empty. They each carry 500 kg of fuel and are capable of consuming it at a rate of 50 kg/sand eject it with a constant velocity of 2500 m/s, measured with respect to the rocket. The rocket is launched vertically from rest by first igniting stage B. Then stage A is ignited immediately after all the fuel in B is consumed and A has separated from B. Determine the maximum velocity of stage A. Neglect drag resistance and the variation of the rocket's weight with altitude.



SOLUTION

The mass of the rocket at any instant t is $m = (M + m_0) - qt$. Thus, its weight at the same instant is $W = mg = [(M + m_0) - qt]g$.

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \quad -[(M + m_0) - qt]g = [(M + m_0) - qt]\frac{dv}{dt} - v_{D/e}q$$

$$\frac{dv}{dt} = \frac{v_{D/e}q}{(M + m_0) - qt} - g$$

During the first stage, $M = 4000$ kg, $m_0 = 1000$ kg, $q = 50$ kg/s, and $v_{D/e} = 2500$ m/s. Thus,

$$\frac{dv}{dt} = \frac{2500(50)}{(4000 + 1000) - 50t} - 9.81$$

$$\frac{dv}{dt} = \left(\frac{2500}{100 - t} - 9.81\right) \text{m/s}^2$$

The time that it take to complete the first stage is equal to the time for all the fuel in the rocket to be consumed, i.e., $t = \frac{500}{50} = 10$ s. Integrating,

$$\int_0^{v_1} dv = \int_0^{10\text{ s}} \left(\frac{2500}{100 - t} - 9.81\right)dt$$

$$v_1 = [-2500\ln(100 - t) - 9.81t]\Big|_0^{10\text{ s}}$$

$$= 165.30 \text{ m/s}$$

During the second stage of launching, $M = 2000$ kg, $m_0 = 500$ kg, $q = 50$ kg/s, and $v_{D/e} = 2500$ m/s. Thus, Eq. (1) becomes

$$\frac{dv}{dt} = \frac{2500(50)}{(2000 + 500) - 50t} - 9.81$$

$$\frac{dv}{dt} = \left(\frac{2500}{50 - t} - 9.81\right) \text{m/s}^2$$

The maximum velocity of rocket A occurs when it has consumed all the fuel. Thus, the time taken is given by $t = \frac{500}{50} = 10$ s. Integrating with the initial condition $v = v_1 = 165.30$ m/s when $t = 0$ s,

$$\int_{165.30\text{ m/s}}^{v_{max}} dv = \int_0^{10\text{ s}} \left(\frac{2500}{50 - t} - 9.81\right)dt$$

$$v_{max} - 165.30 = [-2500\ln(50 - t) - 9.81t]\Big|_0^{10\text{ s}}$$

$$v_{max} = 625 \text{ m/s}$$ **Ans.**

Ans:
$v_{max} = 625$ m/s

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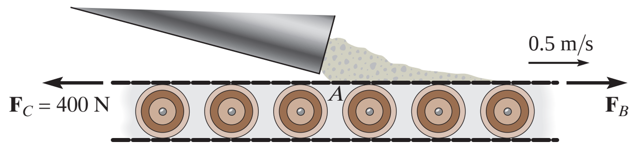
15–130.

Sand is deposited from a chute onto a conveyor belt which is moving at 0.5 m/s. If the sand is assumed to fall vertically onto the belt at A at the rate of 4 kg/s, determine the belt tension F_B to the right of A. The belt is free to move over the conveyor rollers and its tension to the left of A is $F_C = 400$ N.



SOLUTION

$$(\overset{+}{\rightarrow})\ \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$$F_B - 400 = 4(0.5 - 0)$$

$$F_B = 2 + 400 = 402 \text{ N} \qquad \textbf{Ans.}$$

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Ans:
$F_B = 402$ N

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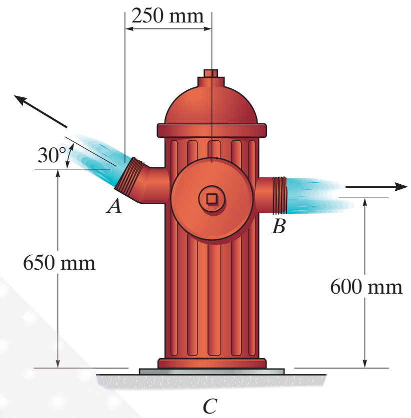
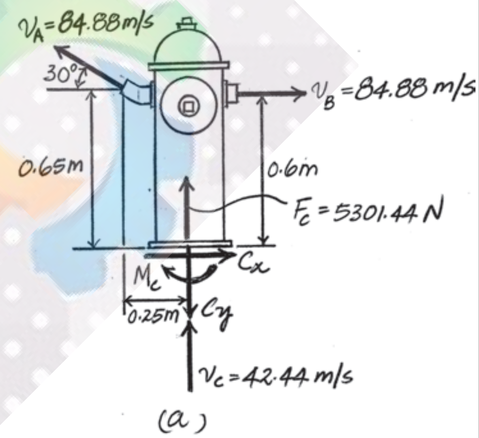
15–131.

The water flow enters below the hydrant at C at the rate of $0.75\ \text{m}^3/\text{s}$. It is then divided equally between the two outlets at A and B. If the gauge pressure at C is 300 kPa, determine the horizontal and vertical force reactions and the moment reaction on the fixed support at C. The diameter of the two outlets at A and B is 75 mm, and the diameter of the inlet pipe at C is 150 mm. The density of water is $\rho_w = 1000\ \text{kg/m}^3$. Neglect the mass of the contained water and the hydrant.



SOLUTION

Free-Body Diagram: The free-body diagram of the control volume is shown in Fig. *a*. The force exerted on section A due to the water pressure is $F_C = p_C A_C = 300(10^3)\left[\frac{\pi}{4}(0.15^2)\right] = 5301.44$ N. The mass flow rate at sections A, B, and C, are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_W\left(\frac{Q}{2}\right) = 1000\left(\frac{0.75}{2}\right) = 375$ kg/s and $\frac{dm_C}{dt} = \rho_W Q = 1000(0.75) = 750$ kg/s.



The speed of the water at sections A, B, and C are

$$v_A = v_B = \frac{Q/2}{A_A} = \frac{0.75/2}{\frac{\pi}{4}(0.075^2)} = 84.88\ \text{m/s} \qquad v_C = \frac{Q}{A_C} = \frac{0.75}{\frac{\pi}{4}(0.15^2)} = 42.44\ \text{m/s}.$$

Steady Flow Equation: Writing the force steady flow equations along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm_A}{dt}(v_A)_x + \frac{dm_B}{dt}(v_B)_x - \frac{dm_C}{dt}(v_C)_x;$$

$$C_x = -375(84.88\cos 30°) + 375(84.88) - 0$$

$$C_x = 4264.54\ \text{N} = 4.26\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = \frac{dm_A}{dt}(v_A)_y + \frac{dm_B}{dt}(v_B)_y - \frac{dm_C}{dt}(v_C)_y;$$

$$-C_y + 5301.44 = 375(84.88\sin 30°) + 0 - 750(42.44)$$

$$Cy = 21\ 216.93\ \text{N} = 2.12\ \text{kN} \qquad \textbf{Ans.}$$

Writing the steady flow equation about point C,

$$+\Sigma M_C = \frac{dm_A}{dt}dv_A + \frac{dm_B}{dt}dv_B - \frac{dm_C}{dt}dv_C;$$

$$-M_C = 375(0.65)(84.88\cos 30°) - 375(0.25)(84.88\sin 30°) + [-375(0.6)(84.88)] - 0$$

$$M_C = 5159.28\ \text{N}\cdot\text{m} = 5.16\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$C_x = 4.26$ kN
$C_y = 2.12$ kN
$M_C = 5.16\ \text{kN}\cdot\text{m}$

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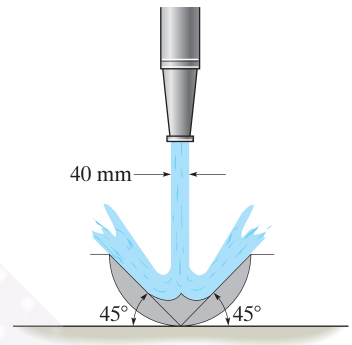
***15–132.**

The nozzle has a diameter of 40 mm. If it discharges water uniformly with a downward velocity of 20 m/s against the fixed blade, determine the vertical force exerted by the water on the blade. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \rho v A = (1000)(20)(\pi)(0.02)^2 = 25.13\ \text{kg/s}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

$$F = (25.13)(20 \sin 45° - (-20))$$

$$F = 858\ \text{N}$$ **Ans.**

Ans:
$F = 858\ \text{N}$

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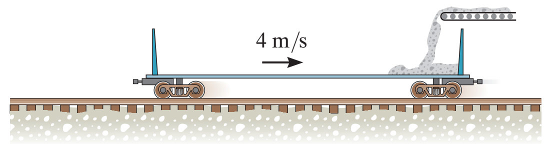
15–133.

Sand drops onto the 2-Mg empty rail car at 50 kg/s from a conveyor belt. If the car is initially coasting at 4 m/s, determine the speed of the car as a function of time.



SOLUTION

Gains Mass System. Here the sand drops vertically onto the rail car. Thus $(v_i)_x = 0$. Then

$$\mathbf{V}_D = \mathbf{V}_i + \mathbf{V}_{D/i}$$

$$(\xrightarrow{+})\quad v = (v_i)_x + (v_{D/i})_x$$

$$v = 0 + (v_{D/i})_x$$

$$(v_{D/i})_x = v$$

Also, $\dfrac{dm_i}{dt} = 50\text{ kg/s}$ and $m = 2000 + 50t$

$$\Sigma F_x = m\frac{dv}{dt} + (v_{D/i})_x\frac{dm_i}{dt};$$

$$0 = (2000 + 50t)\frac{dv}{dt} + v(50)$$

$$\frac{dv}{v} = -\frac{50\,dt}{2000 + 50t}$$

Integrate this equation with initial condition $v = 4\text{ m/s}$ at $t = 0$.

$$\int_{4\text{ m/s}}^{v}\frac{dv}{v} = -50\int_0^t\frac{dt}{2000 + 50t}$$

$$\ln v\Big|_{4\text{ m/s}}^{v} = -\ln(2000 + 50t)\Big|_0^t$$

$$\ln\frac{v}{4} = \ln\left(\frac{2000}{2000 + 50t}\right)$$

$$\frac{v}{4} = \frac{2000}{2000 + 50t}$$

$$v = \left\{\frac{8000}{2000 + 50t}\right\}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:

$$v = \left\{\frac{8000}{2000 + 50t}\right\}\text{ m/s}$$

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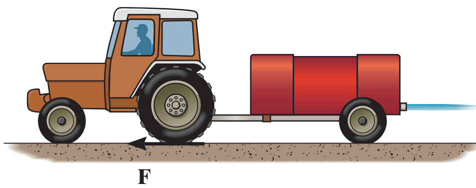
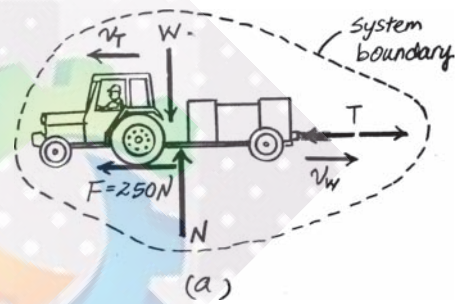
15–134.

The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.



SOLUTION

The free-body diagram of the tractor and water jet is shown in Fig. *a*. The pair of thrust **T** cancel each other since they are internal to the system. The mass of the tractor and the tank at any instant *t* is given by $m = (4000 + 2000) - 50t = (6000 - 50t)\text{kg}$.



$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 250 = (6000 - 50t)\frac{dv}{dt} - 5(50)$$

$$a = \frac{dv}{dt} = \frac{10}{120 - t} \qquad \textbf{(1)}$$

The time taken to empty the tank is $t = \dfrac{2000}{50} = 40$ s. Substituting the result of t into Eq. (1),

$$a = \frac{10}{120 - 40} = 0.125\text{m/s}^2 \qquad \textbf{Ans.}$$

Integrating Eq. (1),

$$\int_0^v dv = \int_0^{40\text{ s}} \frac{10}{120 - t}dt$$

$$v = -10\ln(120 - t)\Big|_0^{40\text{ s}}$$

$$= 4.05\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$a = 0.125\text{ m/s}^2$
$v = 4.05\text{ m/s}$

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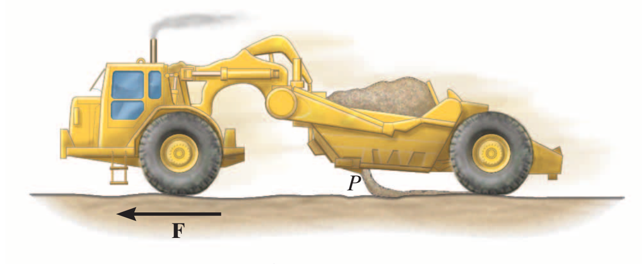
15 135.

The earthmover initially carries $10\ \text{m}^3$ of sand having a density of $1520\ \text{kg/m}^3$. The sand is unloaded horizontally through a $2.5\ \text{m}^2$ dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is $0.1\ \text{m/s}^2$ when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



SOLUTION

When half the sand remains,

$$m = 30\ 000 + \frac{1}{2}(10)(1520) = 37\ 600\ \text{kg}$$

$$\frac{dm}{dt} = 900\ \text{kg/s} = \rho\, v_{D/e}\, A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237\ \text{m/s}$$

$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\ 600(0.1) - 900(0.237)$$

$$F = 3.55\ \text{kN}$$ **Ans.**

Ans:
$F = 3.55$ kN

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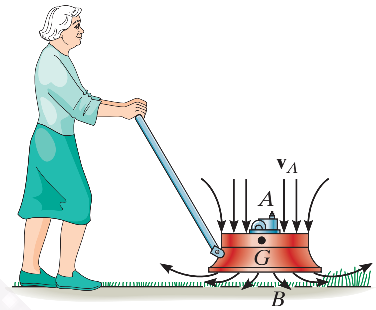
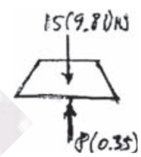
***15–136.**

A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION



$$\frac{dm}{dt} = \rho\, A_A\, v_A = 1.22(0.25)(6) = 1.83\ \text{kg/s}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

$$\text{pressure} = (0.35) - 15(9.81) = 1.83(0 - (-6))$$

$$\text{pressure} = 452\ \text{Pa}$$ **Ans.**

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Ans:
452 Pa

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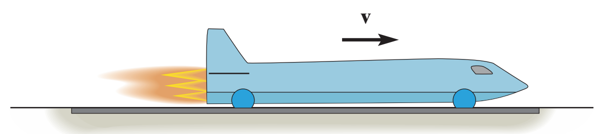
15–137.

The rocket car has a mass of 2 Mg (empty) and carries 120 kg of fuel. If the fuel is consumed at a constant rate of 6 kg/s and ejected from the car with a relative velocity of 800 m/s, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = (6.8v^2)$ N, where v is the speed in m/s.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - v_{\text{[placeholder]}}\text{[placeholder]}$$

At time t_1 the mass of the car is $m_0 - ct_1$ where $c = \dfrac{dm_c}{dt} = 6$ kg/s

Set $F = kv^2$, then

$$-kv^2 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_0^t \frac{dv}{(cv_{D/e} - kv^2)} = \int_0^t \frac{dt}{(m_0 - ct)}$$

$$\left(\frac{1}{2\sqrt{cv_{D/e}k}}\right)\ln\left[\frac{\sqrt{\frac{cv_{D/e}}{k}} + v}{\sqrt{\frac{cv_{D/e}}{k}} - v}\right]_0^v = -\frac{1}{c}\ln(m_0 - ct)\Big|_0^t$$

$$\left(\frac{1}{2\sqrt{cv_{D/e}k}}\right)\ln\left(\frac{\sqrt{\frac{cv_{D/e}}{k}} + v}{\sqrt{\frac{cv_{D/e}}{k}} - v}\right) = -\frac{1}{c}\ln\left(\frac{m_0 - ct}{m_0}\right)$$

Maximum speed occurs at the instant the fuel runs out

$$t = \frac{120}{6} = 20 \text{ s}$$

Thus,

$$\left(\frac{1}{2\sqrt{(6)(800)(6.8)}}\right)\ln\left(\frac{\sqrt{\frac{(6)(800)}{6.8}} + v}{\sqrt{\frac{(6)(800)}{6.8}} - v}\right) = -\frac{1}{6}\ln\left(\frac{2120 - 6(20)}{2120}\right)$$

Solving,

$v = 25.0$ m/s **Ans.**

Ans:
$v = 25.0$ m/s

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15-138.

The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed v_{er}, determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.

SOLUTION

$$a_0 = \frac{d}{dt}v$$

$$+\uparrow \quad \Sigma F_s = m\frac{d}{dt}v - v_{er}\frac{d}{dt}m_e$$

$$-mg = ma_o - v_{er}\frac{d}{dt}m$$

$$v_{er}\frac{dm}{m} = (a_0 + g)dt$$

Since v_{er} is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{er}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)t \qquad \frac{m}{m_0} = e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

The time rate fuel consumption is determined from Eq.[1]

$$\frac{d}{dt}m = m\frac{a_0 + g}{v_{er}} \qquad \frac{d}{dt}m = m_0\left(\frac{a_0 + g}{v_{er}}\right)e^{\left(\frac{a_0+g}{v_{er}}\right)t} \qquad \textbf{Ans.}$$

Note : v_{er} must be considered a negative quantity.

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Ans:

$$\frac{d}{dt}m = m_0\left(\frac{a_0 + g}{v_{er}}\right)e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

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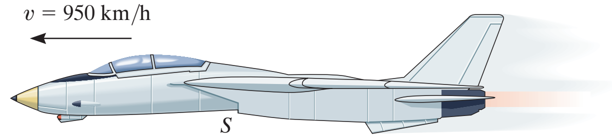
15-139.

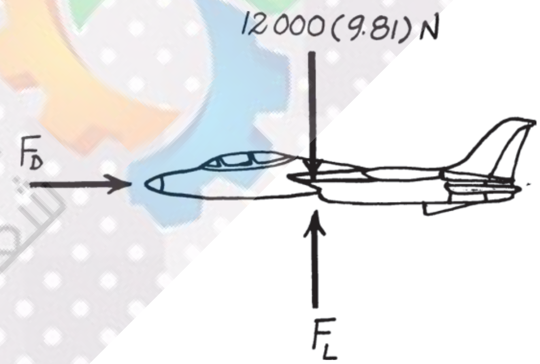
The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}.$$



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad \textbf{(1)}$$



$$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$$

$$v_{D/E} = 0.45 \text{ km/s}$$

$$v_{D/t} = 0.2639 \text{ km/s}$$

$$\frac{dm_t}{dt} = 50(1.22) = 61.0 \text{ kg/s}$$

$$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$$

Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

$$(\overset{+}{\leftarrow}) \quad -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_D = 11.5$ kN

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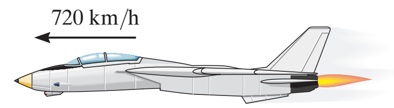
***15–140.**

The jet is traveling at a speed of 720 km/h. If the fuel is being spent at 0.8 kg/s, and the engine takes in air at 200 kg/s, whereas the exhaust gas (air and fuel) has a relative speed of 12 000 m/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (55\ v^2)$, where the speed is measured in m/s. The jet has a mass of 7 Mg.



SOLUTION

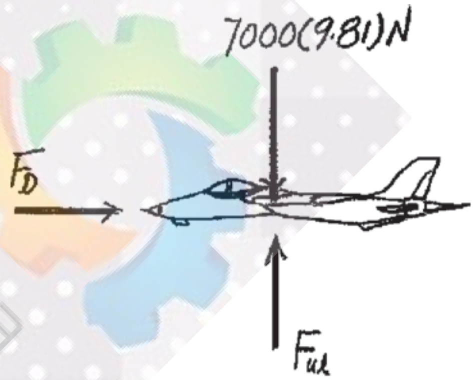
Since the mass enters and exits the plane at the same time, we can combine Eqs. 15-29 and 15-30 which resulted in

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

Here $m = 7000$ kg, $\dfrac{dv}{dt} = a$, $v_{D/e} = 12000$ m/s, $\dfrac{dm_e}{dt} = 0.8 + 200 = 200.8$ kg/s

$$v = \left(720\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 200\text{ m/s},\ v_{D/i} = v = 200\text{ m/s},$$

$$\frac{dm_i}{dt} = 200\text{ kg/s}$$



(a)

and $F_D = 55(200^2) = 2.2(10^6)$ N. Referring to the FBD of the jet, Fig. *a*

$$(\overset{+}{\leftarrow})\ \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt};$$

$$-2.2(10^6) = 7000a - 12000(200.8) + 200(200)$$

$$a = 24.23\text{ m/s}^2 = 24.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 24.2\text{ m/s}^2$

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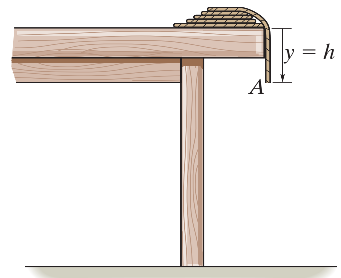
15–141.

The rope has a mass m' per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end A for any position y, as the rope uncoils and begins to fall.



SOLUTION

$$+\downarrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m'y$ and $\dfrac{dm_i}{dt} = \dfrac{m'dy}{dt} = m'v$. Here, $v_{D/i} = v$, $\dfrac{dv}{dt} = g$.

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$gy = y\frac{dv}{dt} + v^2 \qquad \text{since } v = \frac{dy}{dt}, \text{ then } dt = \frac{dy}{v}$$

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply both sides by $2ydy$

$$2gy^2\,dy = 2vy^2\,dv + 2yv^2\,dy$$

$$\int 2gy^2\,dy = \int d\left(v^2y^2\right)$$

$$\tfrac{2}{3}gy^3 + C = v^2y^2$$

$$v = 0 \text{ at } y = h \qquad \tfrac{2}{3}gh^3 + C = 0 \qquad C = -\tfrac{2}{3}gh^3$$

$$\tfrac{2}{3}gy^3 - \tfrac{2}{3}gh^3 = v^2y^2$$

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)} \qquad \textbf{Ans.}$$

Ans:

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$

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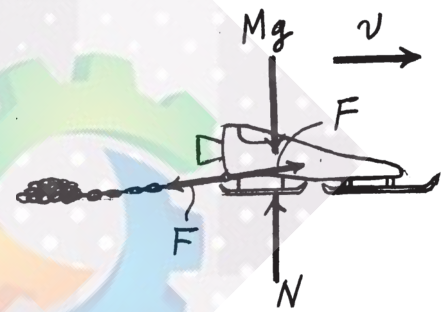
15–142.

A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/2)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

SOLUTION

Observing the free-body diagram of the system shown in Fig. a, notice that the pair of forces **F,** which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\frac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. a,



(a)

$$(\overset{+}{\rightarrow})\quad \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \qquad \textbf{(1)}$$

Since $\frac{dx}{dt} = v$ or $dt = \frac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \qquad \textbf{(2)}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \frac{1}{2}v_0$ at $x = s$,

$$\int_{v_0}^{\frac{1}{2}v_0} \frac{dv}{v} = -\int_0^s \left(\frac{m'}{m'x + M}\right)dx$$

$$\ln v\Big|_{v_0}^{\frac{1}{2}v_0} = -\ln\left(m'x + M\right)\Big|_0^s$$

$$\frac{1}{2} = \frac{M}{m's + M}$$

$$m' = \frac{M}{s} \qquad \textbf{Ans.}$$

Ans:

$$m' = \frac{M}{s}$$

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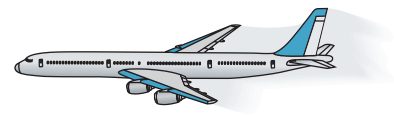
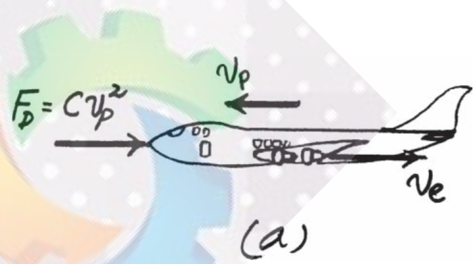
15–143.

A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.



SOLUTION

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is



$$\left(\overset{+}{\rightarrow}\right) \qquad v_e + v_p + v_{e/p}$$

When the four engines are in operation, the airplane has a constant speed of $v_p = \left[800(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 222.22$ m/s. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad v_e = -222.22 + 775 = 552.78 \text{ m/s} \rightarrow$$

Referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(222.22^2) = 4\frac{dm}{dt}(552.78 - 0)$$

$$C = 0.044775\,\frac{dm}{dt}$$

When only two engines are in operation, the exit speed of the air is

$$\left(\overset{+}{\rightarrow}\right) \qquad v_e = -v_p + 775$$

Using the result for C,

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \quad \left(0.044775\,\frac{dm}{dt}\right)(v_p{}^2) = 2\frac{dm}{dt}\left[-v_p + 775) - 0\right]$$

$$0.044775v_p{}^2 + 2v_p - 1550 = 0$$

Solving for the positive root,

$v_p = 165.06 \text{ m/s} = 594 \text{ km/h}$ **Ans.**

Ans:
$v_P = 594$ km/h

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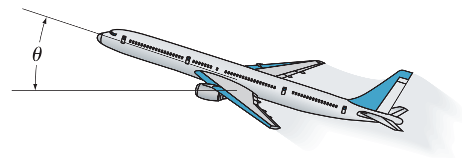
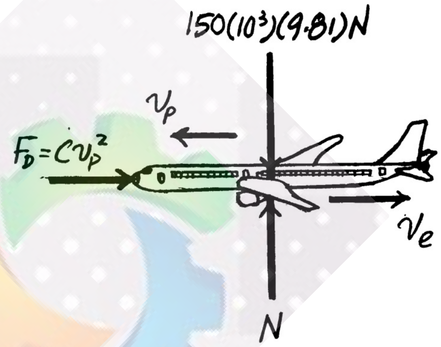
***15–144.** A commercial jet aircraft has a mass of 150 Mg and is cruising at a constant speed of 850 km/h in level flight ($\theta = 0°$). If each of the two engines draws in air at a rate of 1000 kg/s and ejects it with a velocity of 900 m/s, relative to the aircraft, determine the maximum angle of inclination θ at which the aircraft can fly with a constant speed of 750 km/h. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. The engines are operating with the same power in both cases. Neglect the amount of fuel consumed.



SOLUTION



(a)

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is given by

$$v_e = v_p + v_{e/p}$$

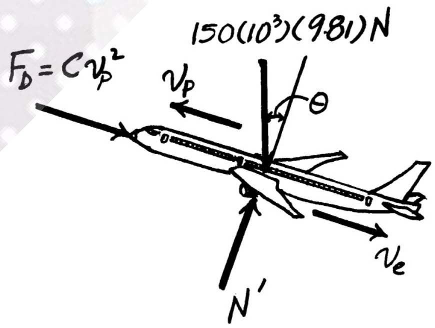
When the airplane is in level flight, it has a constant speed of $v_p = \left[850(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 236.11\text{m/s}$. Thus,

$(\overset{+}{\rightarrow})$ $\qquad v_e = -236.11 + 900 = 663.89\text{ m/s} \rightarrow$

By referring to the free-body diagram of the airplane shown in Fig. *a*,

$$(\overset{+}{\rightarrow})\ \Sigma F_x = \frac{dm}{dt}\left[\left(v_B\right)_x - \left(v_A\right)_x\right]; \qquad C(236.11^2) = 2(1000)(663.89 - 0)$$

$$C = 23.817\text{ kg}\cdot\text{s/m}$$



(b)

When the airplane is in the inclined position, it has a constant speed of $v_p = \left[750\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 208.33\text{ m/s}$. Thus,

$$v_e = -208.33 + 900 = 691.67\text{ m/s}$$

By referring to the free-body diagram of the airplane shown in Fig. *b* and using the result of C, we can write

$$\nwarrow + \Sigma F_{x'} = \frac{dm}{dt}\left[\left(v_B\right)_{x'} - \left(v_A\right)_{x'}\right]; \qquad 23.817\left(208.33^2\right) + 150\left(10^3\right)(9.81)\sin\theta$$
$$= 2(1000)(691.67 - 0)$$

$$\theta = 13.7° \qquad \textbf{Ans.}$$

Ans:
$\theta = 13.7°$

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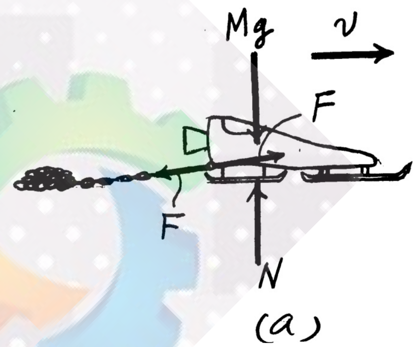
15–145. A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/4)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

SOLUTION



Observing the free-body diagram of the system shown in Fig. *a*, notice that the pair of forces **F,** which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\frac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. *a*,

$$(\overset{+}{\rightarrow}) \quad \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \qquad \textbf{(1)}$$

Since $\frac{dx}{dt} = v$ or $dt = \frac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \qquad \textbf{(2)}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \frac{1}{4}v_0$ at $x = s$,

$$\int_{v_0}^{\frac{1}{4}v_0} \frac{dv}{v} = -\int_0^s \left(\frac{m'}{m'x + M}\right)dx$$

$$1\text{n}\, v\Big|_{v_0}^{\frac{1}{4}v_0} = -1\text{n}\left(m'x + M\right)\Big|_0^s$$

$$\frac{1}{4} = \frac{M}{m's + M}$$

$$m' = \frac{3M}{s} \qquad \textbf{Ans.}$$

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Ans:
$m' = \frac{3M}{s}$

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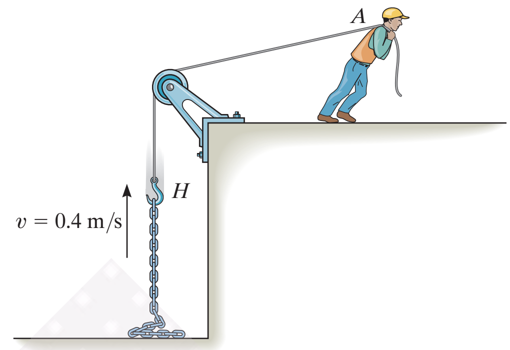
15–146.

Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.



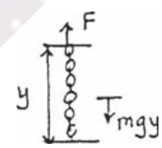
SOLUTION

$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$



$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\text{ N} \qquad \textbf{Ans.}$$

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Ans:
$F = \{7.85t + 0.320\}$ N

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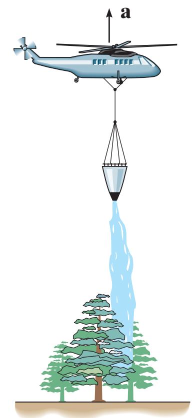
15–147.

The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to thehelicopter, determine the initial upward accelerationthe helicopter experiences as the water is being released.



SOLUTION

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

$$0 = 10.5(10^3)\,a - (10)(50)$$

$$a = 0.0476 \text{ m/s}^2$$ **Ans.**

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Ans:
$a = 0.0476 \text{ m/s}^2$

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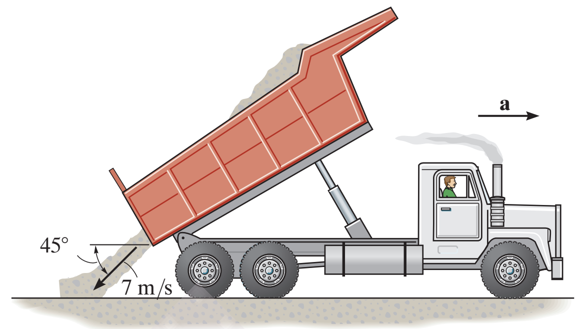
***15–148.**

The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520\ \text{kg/m}^3$.



SOLUTION

A System That Loses Mass: Initially, the total mass of the truck is $m = 50(10^3) + 5(1520) = 57.6(10^3)$ kg and $\dfrac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15–28, we have

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 0 = 57.6(10^3)a - (0.8\cos 45°)(1216)$$

$$a = 0.104\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.104\ \text{m/s}^2$

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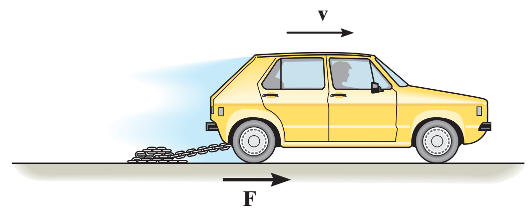
15–149.

The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m_0 + ct$, where $c = \dfrac{dm_i}{dt} = \dfrac{m'dx}{dt} = m'v$.

Here, $v_{D/i} = v, \quad \dfrac{dv}{dt} = 0.$

$$F = (m_0 - m'v)(0) + v(m'v) = m'v^2 \qquad \textbf{Ans.}$$

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Ans:
$F = m'v^2$

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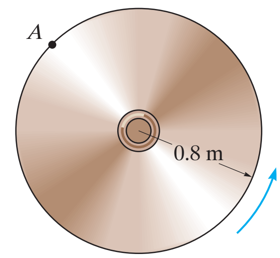
16–1.

The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.



SOLUTION

$$\omega = (5\,t^2 + 2)\text{ rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 10\,t$$

$$t = 0.5\text{ s}$$

$$\omega = 3.25\text{ rad/s}$$

$$\alpha = 5\text{ rad/s}^2$$

$$v_A = \omega r = 3.25(0.8) = 2.60\text{ m/s}$$ **Ans.**

$$a_z = \alpha r = 5(0.8) = 4\text{ m/s}^2$$

$$a_n = \omega^2 r = (3.25)^2(0.8) = 8.45\text{ m/s}^2$$

$$a_A = \sqrt{(4)^2 + (8.45)^2} = 9.35\text{ m/s}^2$$ **Ans.**

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Ans:
$v_A = 2.60$ m/s
$a_A = 9.35$ m/s^2

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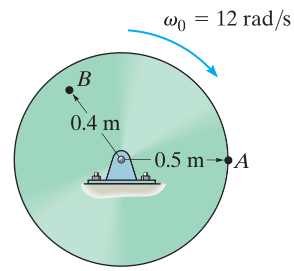
16–2. The angular acceleration of the disk is defined by $\alpha = (3t^2 + 12)$ rad/s^2, where t is in seconds. If the disk is originally rotating at $\omega_0 = 12$ rad/s, determine the magnitude of the velocity and the n and t components of acceleration of point A on the disk when $t = 2$ s.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 12$ rad/s at $t = 0$.

$$\int_{12\text{ rad/s}}^{\omega} d\omega = \int_0^{2\text{ s}} (3t^2 + 12)dt$$

$$\omega - 12 = (t^3 + 12t)\Big|_0^{2\text{ s}}$$

$$\omega = 44.0 \text{ rad/s}$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 44.0(0.5) = 22.0 \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 2$ s, $\alpha = 3(2^2) + 12 = 24$ rad/s^2. Thus, the tangential and normal components of the acceleration are

$$(a_A)_t = \alpha r_A = 24(0.5) = 12.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(a_A)_n = \omega^2 r_A = (44.0^2)(0.5) = 968 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_A = 22.0$ m/s
$(a_A)_t = 12.0$ m/s^2
$(a_A)_n = 968$ m/s^2

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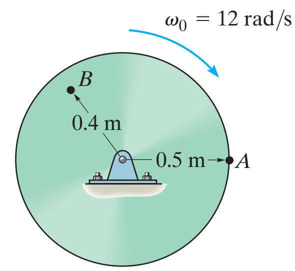
16–3.

The disk is originally rotating at $\omega_0 = 12$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 20\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 2$ s.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega = \omega_0 + \alpha_c t; \qquad \omega = 12 + 20(2) = 52\ \text{rad/s}$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 52(0.5) = 26.0\ \text{m/s}$$ **Ans.**

The tangential and normal component of acceleration are

$$(a_A)_t = \alpha r = 20(0.5) = 10.0\ \text{m/s}^2$$ **Ans.**

$$(a_A)_n = \omega^2 r = (52^2)(0.5) = 1352\ \text{m/s}^2$$ **Ans.**

Ans:
$v_A = 26.0$ m/s
$(a_A)_t = 10.0\ \text{m/s}^2$
$(a_A)_n = 1352\ \text{m/s}^2$

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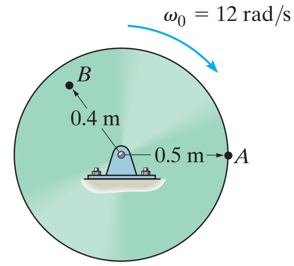
***16–4.**

The disk is originally rotating at $\omega_0 = 12\ \text{rad/s}$. If it is subjected to a constant angular acceleration of $\alpha = 20\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point B when the disk undergoes 2 revolutions.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0); \qquad \omega^2 = 12^2 + 2(20)[2(2\pi) - 0]$$

$$\omega = 25.43\ \text{rad/s}$$

Motion of Point B. The magnitude of the velocity is

$$v_B = \omega r_B = 25.43(0.4) = 10.17\ \text{m/s} = 10.2\ \text{m/s} \qquad \textbf{Ans.}$$

The tangential and normal components of acceleration are

$$(a_B)_t = \alpha r_B = 20(0.4) = 8.00\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_n = \omega^2 r_B = (25.43^2)(0.4) = 258.66\ \text{m/s}^2 = 259\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 10.2\ \text{m/s}$
$(a_B)_t = 8.00\ \text{m/s}^2$
$(a_B)_n = 259\ \text{m/s}^2$

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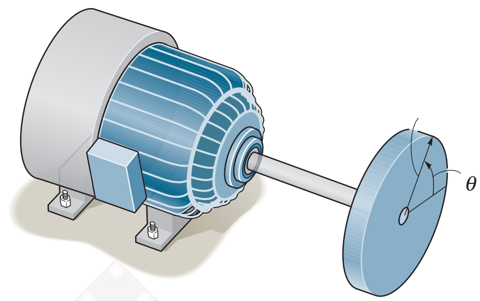
16–5.

The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



SOLUTION

Angular Displacement: At $t = 90$ s,

$$\theta = 20(90) + 4\left(90^2\right) = (34200\text{ rad}) \times \left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 5443\text{ rev} \qquad \textbf{Ans.}$$

Angular Velocity: Applying Eq. 16–1, we have

$$\omega = \frac{d\theta}{dt} = 20 + 8t\Big|_{t=90\,s} = 740\text{ rad/s} \qquad \textbf{Ans.}$$

Angular Acceleration: Applying Eq. 16–2, we have

$$\alpha = \frac{d\omega}{dt} = 8\text{ rad/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$\theta = 5443$ rev
$\omega = 740$ rad/s
$\alpha = 8$ rad/s^2

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16–6.

A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s^2. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

SOLUTION

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$(15)^2 = (10)^2 + 2(3)(\theta - 0)$$

$$\theta = 20.83 \text{ rad} = 20.83\left(\frac{1}{2\pi}\right) = 3.32 \text{ rev}$$ **Ans.**

$$\omega = \omega_0 + \alpha_c t$$

$$15 = 10 + 3t$$

$$t = 1.67 \text{ s}$$ **Ans.**

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Ans:
$\theta = 3.32$ rev
$t = 1.67$ s

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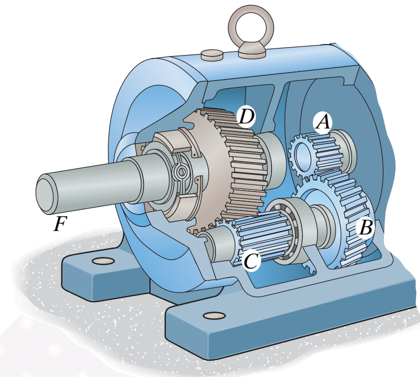
16–7.

If gear A rotates with a constant angular acceleration of $\alpha_A = 90\text{ rad/s}^2$, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Gear B is in mesh with gear A. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(90) = 27\text{ rad/s}^2$$

Since gears C and B share the same shaft, $\alpha_C = \alpha_B = 27\text{ rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(27) = 9\text{ rad/s}^2$$

The final angular velocity of gear D is $\omega_D = \left(\dfrac{600\text{ rev}}{\text{min}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 20\pi\text{ rad/s}$. Applying the constant acceleration equation,

$$\omega_D = (\omega_D)_0 + \alpha_D t$$

$$20\pi = 0 + 9t$$

$$t = 6.98\text{ s}$$ **Ans.**

and

$$\omega_D{}^2 = (\omega_D)_0{}^2 + 2\alpha_D[\theta_D - (\theta_D)_0]$$

$$(20\pi)^2 = 0^2 + 2(9)(\theta_D - 0)$$

$$\theta_D = (219.32\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)$$

$$= 34.9\text{ rev}$$ **Ans.**

Ans:
$t = 6.98\text{ s}$
$\theta_D = 34.9\text{ rev}$

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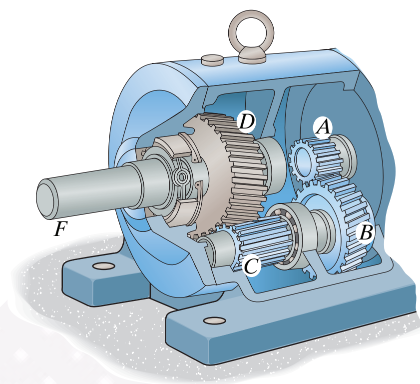
***16–8.**

If gear A rotates with an angular velocity of $\omega_A = (\theta_A + 1)$ rad/s, where θ_A is the angular displacement of gear A, measured in radians, determine the angular acceleration of gear D when $\theta_A = 3$ rad, starting from rest. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Motion of Gear A:

$$\alpha_A d\theta_A = \omega_A d\omega_A$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d(\theta_A + 1)$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d\theta_A$$

$$\alpha_A = (\theta_A + 1)$$

At $\theta_A = 3$ rad,

$$\alpha_A = 3 + 1 = 4 \text{ rad/s}^2$$

Motion of Gear D: Gear A is in mesh with gear B. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(4) = 1.20 \text{ rad/s}^2$$

Since gears C and B share the same shaft $\alpha_C = \alpha_B = 1.20$ rad/s^2. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(1.20) = 0.4 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_D = 0.4 \text{ rad/s}^2$

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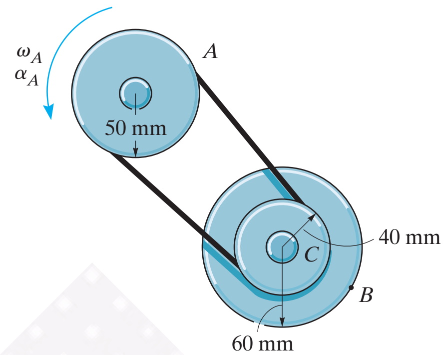
16–9.

At the instant $\omega_A = 5\ \text{rad/s}$, pulley A is given an angular acceleration $\alpha = (0.8\theta)\ \text{rad/s}^2$, where θ is in radians. Determine the magnitude of acceleration of point B on pulley C when A rotates 3 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. The angular velocity of pulley A can be determined by integrating $\omega\, d\omega = \alpha\, d\theta$ with the initial condition $\omega_A = 5\ \text{rad/s}$ at $\theta_A = 0$.

$$\int_{5\ \text{rad/s}}^{\omega_A} \omega\, d\omega = \int_0^{\theta_A} 0.8\theta d\theta$$

$$\left.\frac{\omega^2}{2}\right|_{5\ \text{rad/s}}^{\omega_A} = \left.\left(0.4\theta^2\right)\right|_0^{\theta_A}$$

$$\frac{\omega_A^2}{2} - \frac{5^2}{2} = 0.4\theta_A^2$$

$$\omega_A = \left\{\sqrt{0.8\theta_A^2 + 25}\right\}\ \text{rad/s}$$

At $\theta_A = 3(2\pi) = 6\pi$ rad,

$$\omega_A = \sqrt{0.8(6\pi)^2 + 25} = 17.585\ \text{rad/s}$$

$$\alpha_A = 0.8(6\pi) = 4.8\pi\ \text{rad/s}^2$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 17.585(50)$$

$$\omega_C = 21.982\ \text{rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = (4.8\pi)(50)$$

$$\alpha_C = 6\pi\ \text{rad/s}^2$$

Motion of Point B. The tangential and normal components of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 6\pi(0.06) = 1.1310\ \text{m/s}^2$$

$$(a_B)_n = \omega_C^2 r_B = \left(21.982^2\right)(0.06) = 28.9917\ \text{m/s}^2$$

Thus, the magnitude of a_B is

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{(1.1310)^2 + (28.9917)^2}$$

$$= 29.01\ \text{m/s}^2 = 29.0\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_B = 29.0\ \text{m/s}^2$

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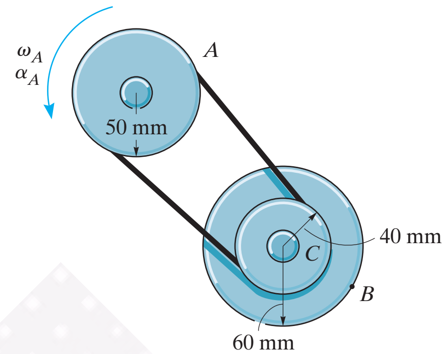
16–10.

At the instant $\omega_A = 5$ rad/s, pulley A is given a constant angular acceleration $\alpha_A = 6$ rad/s^2. Determine the magnitude of acceleration of point B on pulley C when A rotates 2 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. Since the angular acceleration of pulley A is constant, we can apply

$$\omega_A^2 = (\omega_A)_0^2 + 2\alpha_A[\theta_A - (\theta_A)_0]$$

$$\omega_A^2 = 5^2 + 2(6)[2(2\pi) - 0]$$

$$\omega_A = 13.2588 \text{ rad/s}$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 13.2588(50)$$

$$\omega_C = 16.5735 \text{ rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = 6(50)$$

$$\alpha_C = 7.50 \text{ rad/s}^2$$

Motion of Point B. The tangential and normal component of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 7.50(0.06) = 0.450 \text{ m/s}^2$$

$$(a_B)_n = \omega_C^2 r_B = (16.5735^2)(0.06) = 16.4809 \text{ m/s}^2$$

Thus, the magnitude of a_B is

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{(0.450)^2 + (16.4809)^2}$$

$$= 16.4871 \text{ m/s}^2 = 16.5 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_B = 16.5 \text{ m/s}^2$

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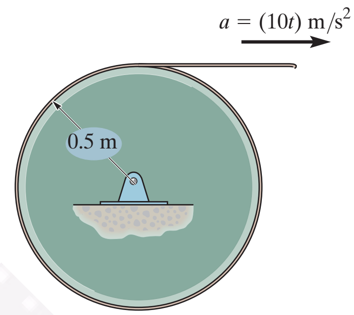
16–11.

The cord, which is wrapped around the disk, is given an acceleration of $a = (10t)\ \text{m/s}^2$, where t is in seconds. Starting from rest, determine the angular displacement, angular velocity, and angular acceleration of the disk when $t = 3$ s.



SOLUTION

Motion of Point P. The tangential component of acceleration of a point on the rim is equal to the acceleration of the cord. Thus

$$(a_t) = \alpha r; \qquad 10t = \alpha(0.5)$$

$$\alpha = \{20t\}\ \text{rad/s}^2$$

When $t = 3$ s,

$$\alpha = 20(3) = 60\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 0$ at $t = 0$.

$$\int_0^{\omega} d\omega = \int_0^{t} 20t\, dt$$

$$\omega = \{10t^2\}\ \text{rad/s}$$

When $t = 3$ s,

$$\omega = 10(3^2) = 90.0\ \text{rad/s} \qquad \textbf{Ans.}$$

The angular displacement of the disk can be determined by integrating $d\theta = \omega\, dt$ with the initial condition $\theta = 0$ at $t = 0$.

$$\int_0^{\theta} d\theta = \int_0^{t} 10t^2\, dt$$

$$\theta = \left\{\frac{10}{3}t^3\right\}\text{rad}$$

When $t = 3$ s,

$$\theta = \frac{10}{3}(3^3) = 90.0\ \text{rad} \qquad \textbf{Ans.}$$

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Ans:
$\alpha = 60\ \text{rad/s}^2$
$\omega = 90.0\ \text{rad/s}$
$\theta = 90.0\ \text{rad}$

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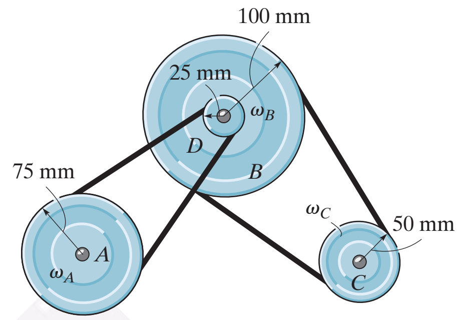
***16–12.**

The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = (20t + 40)$ rad/s, where t is in seconds, determine the angular velocities of the generator pulley B and the air-conditioning pulley C when $t = 3$ s.



SOLUTION

When $t = 3$ s

$$\omega_A = 20(3) + 40 = 100 \text{ rad/s}$$

The speed of a point P on the belt wrapped around A is

$$v_P = \omega_A r_A = 100(0.075) = 7.5 \text{ m/s}$$

$$\omega_B = \frac{v_P}{r_D} = \frac{7.5}{0.025} = 300 \text{ rad/s} \qquad \textbf{Ans.}$$

The speed of a point P' on the belt wrapped around the outer periphery of B is

$$v'_p = \omega_B r_B = 300(0.1) = 30 \text{ m/s}$$

Hence, $\omega_C = \dfrac{v'_P}{r_C} = \dfrac{30}{0.05} = 600 \text{ rad/s}$ **Ans.**

Ans:
$\omega_B = 300$ rad/s
$\omega_C = 600$ rad/s

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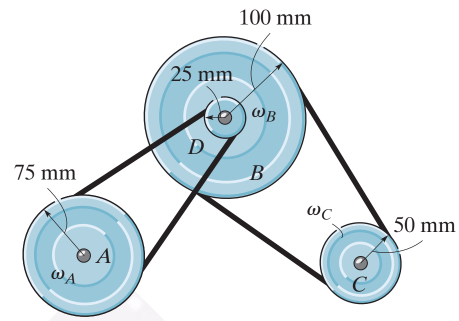
16–13.

The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = 60$ rad/s, determine the angular velocities of the generator pulley B and the air-conditioning pulley C. The hub at D is rigidly *connected* to B and turns with it.



SOLUTION

The speed of a point P on the belt wrapped around A is

$$v_P = \omega_A r_A = 60(0.075) = 4.5 \text{ m/s}$$

$$\omega_B = \frac{v_P}{r_D} = \frac{4.5}{0.025} = 180 \text{ rad/s} \qquad \textbf{Ans.}$$

The speed of a point P' on the belt wrapped around the outer periphery of B is

$$v'_P = \omega_B r_B = 180(0.1) = 18 \text{ m/s}$$

Hence, $\omega_C = \dfrac{v'_P}{r_C} = \dfrac{18}{0.05} = 360 \text{ rad/s}$ **Ans.**

Ans:
$\omega_B = 180$ rad/s
$\omega_C = 360$ rad/s

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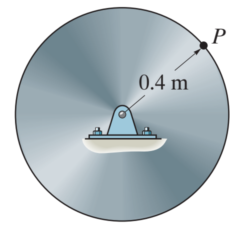
16–14.

The disk starts from rest and is given an angular acceleration $\alpha = (2t^2)\ \text{rad/s}^2$, where t is in seconds. Determine the angular velocity of the disk and its angular displacement when $t = 4$ s.



SOLUTION

$$\alpha = \frac{d\omega}{dt} = 2t^2$$

$$\int_0^{\omega} d\omega = \int_0^{t} 2t^2 dt$$

$$\omega = \frac{2}{3}t^3\Big|_0^t$$

$$\omega = \frac{2}{3}t^3$$

When $t = 4$ s,

$$\omega = \frac{2}{3}(4)^3 = 42.7\ \text{rad/s}$$ **Ans.**

$$\int_0^{\theta} d\theta = \int_0^{t} \frac{2}{3}t^3 dt$$

$$\theta = \frac{1}{6}t^4$$

When $t = 4$ s,

$$\theta = \frac{1}{6}(4)^4 = 42.7\ \text{rad}$$ **Ans.**

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Ans:
$\omega = 42.7$ rad/s
$\theta = 42.7$ rad

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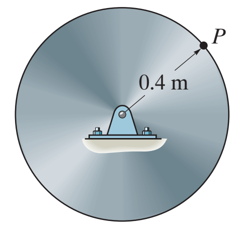
16–15.

The disk starts from rest and is given an angular acceleration $\alpha = (5t^{1/2})\ \text{rad/s}^2$, where t is in seconds. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 2$ s.



SOLUTION

Motion of the Disk: Here, when $t = 0, \omega = 0$.

$$d\omega = a dt$$

$$\int_0^{\omega} d\omega = \int_0^t 5t^{\frac{1}{2}} dt$$

$$\omega\Big|_0^{\omega} = \frac{10}{3} t^{\frac{3}{2}}\Big|_0^t$$

$$\omega = \left\{\frac{10}{3} t^{\frac{3}{2}}\right\} \text{rad/s}$$

When $t = 2$ s,

$$\omega = \frac{10}{3}\left(2^{\frac{3}{2}}\right) = 9.428 \text{ rad/s}$$

When $t = 2$ s,

$$\alpha = 5\left(2^{\frac{1}{2}}\right) = 7.071 \text{ rad/s}^2$$

Motion of point P: The tangential and normal components of the acceleration of point P when $t = 2$ s are

$$a_t = \alpha r = 7.071(0.4) = 2.83 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = \omega^2 r = 9.428^2(0.4) = 35.6 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_t = 2.83 \text{ m/s}^2$
$a_n = 35.6 \text{ m/s}^2$

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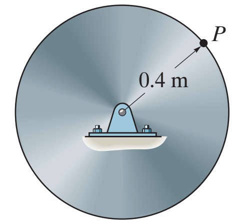
***16–16.**

The disk starts at $\omega_0 = 1$ rad/s when $\theta = 0$, and is given an angular acceleration $\alpha = (0.3\theta)$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $\theta = 1$ rev.



SOLUTION

$\alpha = 0.3\theta$

$$\int_1^{\omega} \omega d\omega = \int_0^{\theta} 0.3\theta d\theta$$

$$\frac{1}{2}\omega^2\Big|_1^{\omega} = 0.15\theta^2\Big|_0^{\theta}$$

$$\frac{\omega^2}{2} - 0.5 = 0.15\theta^2$$

$$\omega = \sqrt{0.3\theta^2 + 1}$$

At $\theta = 1 \text{ rev} = 2\pi$ rad

$$\omega = \sqrt{0.3(2\pi)^2 + 1}$$

$$\omega = 3.584 \text{ rad/s}$$

$$a_t = \alpha r = 0.3(2\pi) \text{ rad/s}^2(0.4 \text{ m}) = 0.7540 \text{ m/s}^2$$ **Ans.**

$$a_n = \omega^2 r = (3.584 \text{ rad/s})^2(0.4 \text{ m}) = 5.137 \text{ m/s}^2$$ **Ans.**

$$a_p = \sqrt{(0.7540)^2 + (5.137)^2} = 5.19 \text{ m/s}^2$$

Ans:
$a_t = 0.7540 \text{ m/s}^2$
$a_n = 5.137 \text{ m/s}^2$

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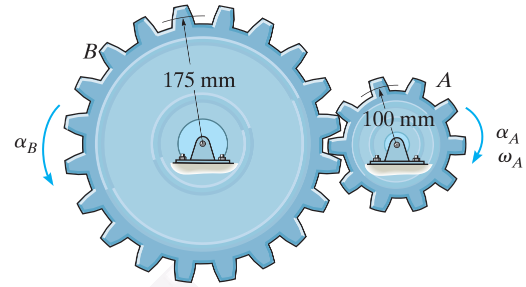
16–17.

A motor gives gear A an angular acceleration of $\alpha_A = (2 + 0.006\,\theta^2)$ rad/s^2, where θ is in radians. If this gear is initially turning at $\omega_A = 15$ rad/s, determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.



SOLUTION

Angular Motion. The angular velocity of the gear A can be determined by integrating $\omega\, d\omega = \alpha\, d\theta$ with initial condition $\omega_A = 15$ rad/s at $\theta_A = 0$.

$$\int_{15\text{ rad/s}}^{\omega_A} \omega\, d\omega = \int_0^{\theta_A} \left(2 + 0.006\,\theta^2\right) d\theta$$

$$\left.\frac{\omega^2}{2}\right|_{15\text{ rad/s}}^{\omega_A} = \left.\left(2\theta + 0.002\,\theta^3\right)\right|_0^{\theta_A}$$

$$\frac{\omega_A^2}{2} - \frac{15^2}{2} = 2\theta_A + 0.002\,\theta_A^3$$

$$\omega_A = \sqrt{0.004\,\theta_A^3 + 4\,\theta + 225}\ \text{rad/s}$$

At $\theta_A = 10(2\pi) = 20\pi$ rad,

$$\omega_A = \sqrt{0.004(20\pi)^3 + 4(20\pi) + 225}$$

$$= 38.3214\ \text{rad/s}$$

Since gear B is meshed with gear A,

$$\omega_B r_B = \omega_A r_A; \qquad \omega_B(175) = 38.3214(100)$$

$$\omega_B = 21.8979\ \text{rad/s}$$

$$= 21.9\ \text{rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

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Ans:
$\omega_B = 21.9$ rad/s $\circlearrowleft$

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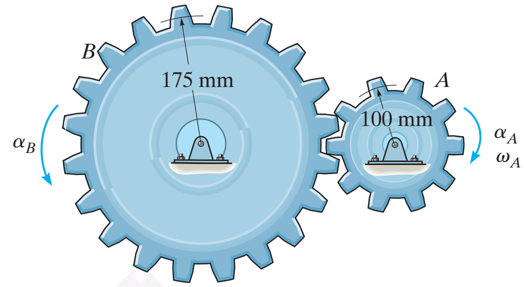
16–18.

A motor gives gear A an angular acceleration of $\alpha_A = (2t^3)\ \text{rad/s}^2$, where t is in seconds. If this gear is initially turning at $\omega_A = 15\ \text{rad/s}$, determine the angular velocity of gear B when $t = 3$ s.



SOLUTION

Angular Motion. The angular velocity of gear A can be determined by integrating $d\omega = \alpha\, dt$ with initial condition $\omega_A = 15$ rad/s at $t = 0$ s.

$$\int_{15\ \text{rad/s}}^{\omega_A} d\omega = \int_0^t 2t^3\, dt$$

$$\omega_A - 15 = \frac{1}{2}t^4\Big|_0^t$$

$$\omega_A = \left\{\frac{1}{2}t^4 + 15\right\}\ \text{rad/s}$$

At $t = 3$ s,

$$\omega_A = \frac{1}{2}(3^4) + 15 = 55.5\ \text{rad/s}$$

Since gear B meshed with gear A,

$$\omega_B r_B = \omega_A r_A; \qquad \omega_B(175) = 55.5(100)$$

$$\omega_B = 31.7143\ \text{rad/s}$$

$$= 31.7\ \text{rad/s}\ ↻ \qquad \textbf{Ans.}$$

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Ans:
$\omega_B = 31.7\ \text{rad/s}$ ↻

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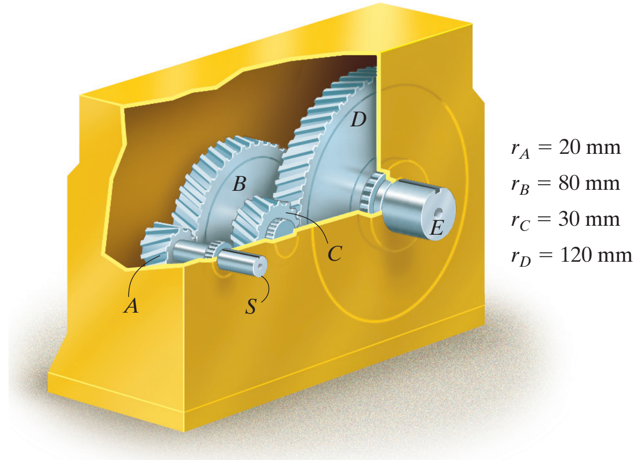
16–19. Morse Industrial manufactures the speed reducer shown. If a motor drives the gear shaft S with an angular acceleration $\alpha = (4\omega^{-3})\ \text{rad/s}^2$, where ω is in rad/s, determine the angular velocity of shaft E at time $t = 2$ s after starting from an angular velocity 1 rad/s when $t = 0$. The radius of each gear is listed in the figure. Note that gears B and C are fixed connected to the same shaft.



SOLUTION

Given:

$r_A = 20$ mm

$r_B = 80$ mm

$r_C = 30$ mm

$r_D = 120$ mm

$\omega_0 = 1$ rad/s

$k = 4$ rad/s^5

$t_1 = 2$ s

Guess $\omega_1 = 1$ rad/s Given $\displaystyle\int_0^{t_1} k\,dt = \int_{\omega_0}^{\omega_1} \omega^3\,d\omega$ $\omega_1 = \text{Find}(\omega_1)$

$\omega_1 = 2.397$ rad/s $\omega_E = \left(\dfrac{r_A}{r_B}\right)\left(\dfrac{r_C}{r_D}\right)\omega_1$ $\omega_E = 0.150$ rad/s **Ans.**

Ans:
$\omega_E = 0.150$ rad/s

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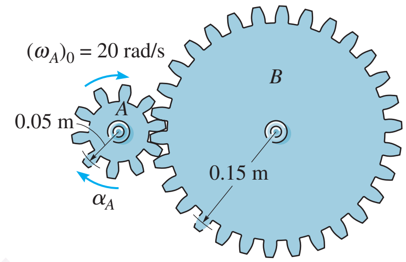
***16–20.**

A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)\ \text{rad/s}^2$, where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20\ \text{rad/s}$, determine the angular velocity of gear B when $t = 2$ s.



SOLUTION

$\alpha_A = 4\,t^3$

$d\omega = \alpha\,dt$

$$\int_{20}^{\omega_A} d\omega_A = \int_0^t \alpha_A\,dt = \int_0^t 4\,t^3\,dt$$

$\omega_A = t^4 + 20$

When $t = 2$ s,

$\omega_A = 36\ \text{rad/s}$

$\omega_A\,r_A = \omega_B\,r_B$

$36(0.05) = \omega_B(0.15)$

$\omega_B = 12\ \text{rad/s}$ **Ans.**

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Ans:
$\omega_B = 12\ \text{rad/s}$

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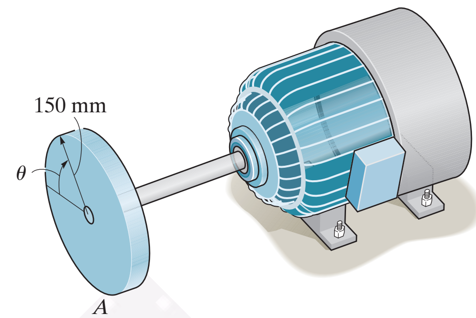
16–21.

The motor turns the disk with an angular velocity of $\omega = (5t^2 + 3t)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and the n and t components of acceleration of the point A on the disk when $t = 3$ s.



SOLUTION

Angular Motion. At $t = 3$ s,

$$\omega = 5(3^2) + 3(3) = 54 \text{ rad/s}$$

The angular acceleration of the disk can be determined using

$$\alpha = \frac{d\omega}{dt}; \qquad \alpha = \{10t + 3\} \text{ rad/s}^2$$

At $t = 3$ s,

$$\alpha = 10(3) + 3 = 33 \text{ rad/s}^2$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 54(0.15) = 8.10 \text{ m/s} \qquad \textbf{Ans.}$$

The tangential and normal component of acceleration are

$$(a_A)_t = \alpha r_A = 33(0.15) = 4.95 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(a_A)_n = \omega^2 r_A = (54^2)(0.15) = 437.4 \text{ m/s}^2 = 437 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_A = 8.10$ m/s
$(a_A)_t = 4.95$ m/s^2
$(a_A)_n = 437$ m/s^2

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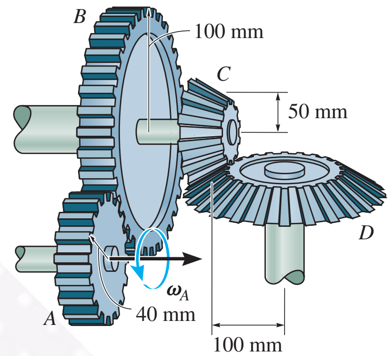
16–22.

If the motor turns gear A with an angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$ when the angular velocity is $\omega_A = 20\ \text{rad/s}$, determine the angular acceleration and angular velocity of gear D.



SOLUTION

Angular Motion: The angular velocity and acceleration of gear B must be determined first. Here, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{40}{100}\right)(20) = 8.00\ \text{rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{40}{100}\right)(2) = 0.800\ \text{rad/s}^2$$

Since gear C is attached to gear B, then $\omega_C = \omega_B = 8\ \text{rad/s}$ and $\alpha_C = \alpha_B = 0.8\ \text{rad/s}^2$. Realizing that $\omega_C r_C = \omega_D r_D$ and $\alpha_C r_C = \alpha_D r_D$, then

$$\omega_D = \frac{r_C}{r_D}\omega_C = \left(\frac{50}{100}\right)(8.00) = 4.00\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\alpha_D = \frac{r_C}{r_D}\alpha_C = \left(\frac{50}{100}\right)(0.800) = 0.400\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_D = 4.00\ \text{rad/s}$
$\alpha_D = 0.400\ \text{rad/s}^2$

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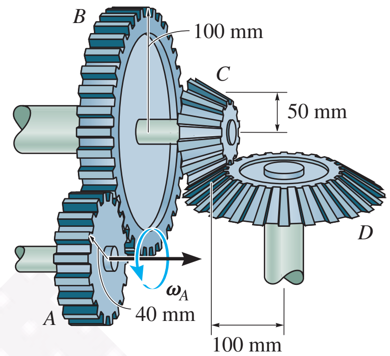
16–23.

If the motor turns gear A with an angular acceleration of $\alpha_A = 3\ \text{rad/s}^2$ when the angular velocity is $\omega_A = 60\ \text{rad/s}$, determine the angular acceleration and angular velocity of gear D.



SOLUTION

Angular Motion: The angular velocity and acceleration of gear B must be determined first. Here, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{40}{100}\right)(60) = 24.0\ \text{rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{40}{100}\right)(3) = 1.20\ \text{rad/s}^2$$

Since gear C is attached to gear B, then $\omega_C = \omega_B = 24.0\ \text{rad/s}$ and $\alpha_C = \alpha_B = 1.20\ \text{rad/s}^2$. Realizing that $\omega_C r_C = \omega_D r_D$ and $\alpha_C r_C = \alpha_D r_D$, then

$$\omega_D = \frac{r_C}{r_D}\omega_C = \left(\frac{50}{100}\right)(24.0) = 12.0\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\alpha_D = \frac{r_C}{r_D}\alpha_C = \left(\frac{50}{100}\right)(1.20) = 0.600\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$\omega_D = 12.0\ \text{rad/s}$
$\alpha_D = 0.600\ \text{rad/s}^2$

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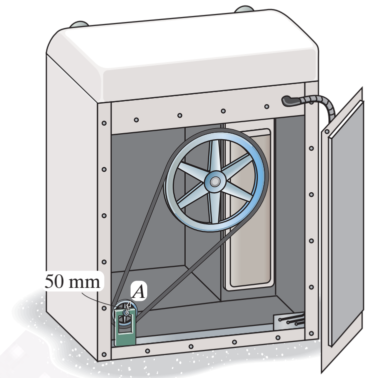
***16–24.** The 50-mm-radius pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (27\theta_A^{1/2})$ rad/s^2, where θ_A is in radians. Determine its angular acceleration when $t = 1$ s, starting from rest.



SOLUTION

Motion of Pulley A: The angular velocity of pulley A can be determined from

$$\int \omega_A \, d\omega_A = \int \alpha_A \, d\theta_A$$

$$\int_0^{\omega_A} \omega_A d\omega_A = \int_0^{\theta_A} 27\theta_A{}^{1/2} d\theta_A$$

$$\left.\frac{\omega_A{}^2}{2}\right|_0^{\omega_A} = 18\theta_A{}^{3/2}\Big|_0^{\theta_A}$$

$$\omega_A = \left(6\theta_A{}^{3/4}\right) \text{ rad/s}$$

Using this result, the angular displacement of A as a function of t can be determined from

$$\int dt = \int \frac{d\theta_A}{\omega_A}$$

$$\int_0^t dt = \int_0^{\theta_A} \frac{d\theta_A}{6\theta_A{}^{3/4}}$$

$$t\Big|_0^t = \left.\frac{2}{3}\theta_A{}^{1/4}\right|_0^{\theta_A}$$

$$t = \left(\frac{2}{3}\theta_A{}^{1/4}\right) \text{s}$$

$$\theta_A = \left(\frac{3}{2}t\right)^4 \text{ rad}$$

When $t = 1$ s

$$\theta_A = \left[\frac{3}{2}(1)\right]^4 = 5.0625 \text{ rad}$$

Thus, when $t = 1$ s, α_A is

$$\alpha_A = 27\left(5.0625^{1/2}\right) = 60.8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$\alpha_A = 60.8$ rad/s^2

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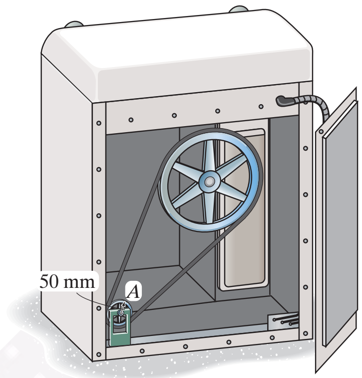
16–25. If the 50-mm-radius motor pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (10 + 50t)$ rad/s^2, where t is in seconds, determine its angular velocity when $t = 3$ s, starting from rest.



SOLUTION

Motion of Pulley *A*: The angular velocity of pulley A can be determined from

$$\int d\omega_A = \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (10 + 50t)dt$$

$$\omega_A\Big|_0^{\omega_A} = \left(10t + 25t^2\right)\Big|_0^t$$

$$\omega_A = \left(10t + 25t^2\right)\text{rad/s}$$

When $t = 3$ s

$$\omega_A = 10(3) + 25\left(3^2\right) = 225 \text{ rad/s} \qquad \textbf{Ans.}$$

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Ans:
$\omega_A = 225$ rad/s

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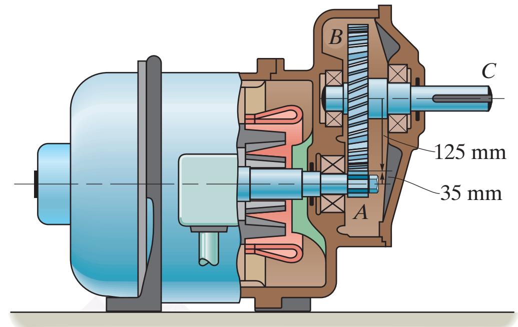
16–26.

The pinion gear A on the motor shaft is given a constant angular acceleration $\alpha = 3\ \text{rad/s}^2$. If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when $t = 2$ s starting from rest. The shaft is fixed to B and turns with it.



SOLUTION

$$\omega = \omega_0 + \alpha_c t$$

$$\omega_A = 0 + 3(2) = 6\ \text{rad/s}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta_A = 0 + 0 + \frac{1}{2}(3)(2)^2$$

$$\theta_A = 6\ \text{rad}$$

$$\omega_A r_A = \omega_B r_B$$

$$6(35) = \omega_B(125)$$

$$\omega_C = \omega_B = 1.68\ \text{rad/s}$$ **Ans.**

$$\theta_A r_A = \theta_B r_B$$

$$6(35) = \theta_B(125)$$

$$\theta_C = \theta_B = 1.68\ \text{rad}$$ **Ans.**

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Ans:
$\omega_C = 1.68\ \text{rad/s}$
$\theta_C = 1.68\ \text{rad}$

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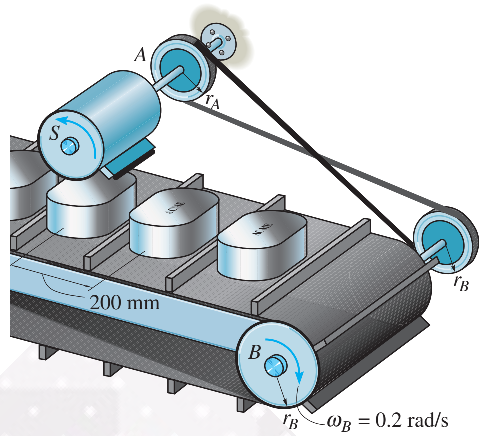
16–27.

A stamp S, located on the revolving drum, is used to label canisters. If the canisters are centered 200 mm apart on the conveyor, determine the radius r_A of the driving wheel A and the radius r_B of the conveyor belt drum so that for each revolution of the stamp it marks the top of a canister. How many canisters are marked per minute if the drum at B is rotating at $\omega_B = 0.2$ rad/s? Note that the driving belt is twisted as it passes between the wheels.



SOLUTION

$l = 2\pi(r_A)$

$$r_A = \frac{200}{2\pi} = 31.8 \text{ mm} \qquad \textbf{Ans.}$$

For the drum at B:

$l = 2\pi(r_B)$

$$r_B = \frac{200}{2\pi} = 31.8 \text{ mm} \qquad \textbf{Ans.}$$

In $t = 60$ s

$\theta = \theta_0 + \omega_0 t$

$\theta = 0 + 0.2(60) = 12$ rad

$l = \theta r_B = 12(31.8) = 382.0$ mm

Hence,

$$n = \frac{382.0}{200} = 1.91 \text{ canisters marked per minute} \qquad \textbf{Ans.}$$

Ans:
$r_A = 31.8$ mm
$r_B = 31.8$ mm
1.91 canisters per minute

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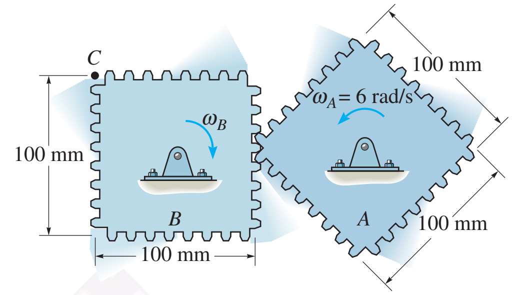
***16–28.**

At the instant shown, gear A is rotating with a constant angular velocity of $\omega_A = 6\ \text{rad/s}$. Determine the largest angular velocity of gear B and the maximum speed of point C.



SOLUTION

$$(r_B)_{max} = (r_A)_{max} = 50\sqrt{2}\ \text{mm}$$

$$(r_B)_{min} = (r_A)_{min} = 50\ \text{mm}$$

When r_A is max., r_B is min.

$$\omega_B(r_B) = \omega_A r_A$$

$$(\omega_B)_{max} = 6\left(\frac{r_A}{r_B}\right) = 6\left(\frac{50\sqrt{2}}{50}\right)$$

$$(\omega_B)_{max} = 8.49\ \text{rad/s} \qquad \textbf{Ans.}$$

$$v_C = (\omega_B)_{max}\, r_C = 8.49\left(0.05\sqrt{2}\right)$$

$$v_C = 0.6\ \text{m/s} \qquad \textbf{Ans.}$$

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Ans:
$(\omega_B)_{\text{max}} = 8.49\ \text{rad/s}$
$(v_C)_{\text{max}} = 0.6\ \text{m/s}$

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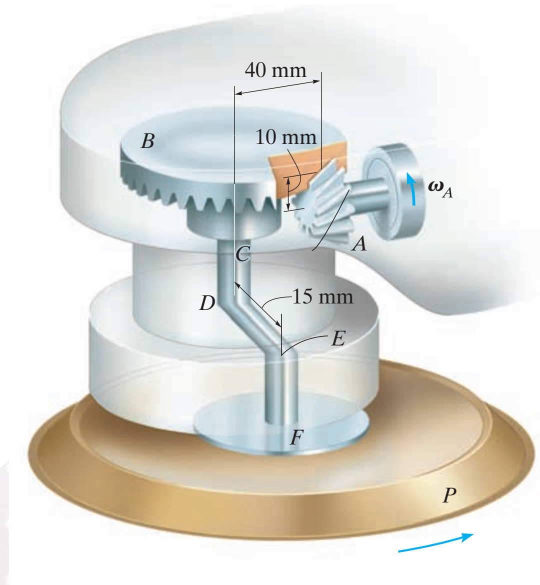
16–29.

For a short time a motor of the random-orbit sander drives the gear A with an angular velocity of $\omega_A = 40(t^3 + 6t)$ rad/s, where t is in seconds. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius of 15 mm. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle EF when $t = 2$ s after starting from rest.



SOLUTION

$$\omega_A r_A = \omega_B r_B$$

$$\omega_A(10) = \omega_B(40)$$

$$\omega_B = \frac{1}{4}\omega_A$$

$$v_E = \omega_B r_E = \frac{1}{4}\omega_A(0.015) = \frac{1}{4}(40)\left(t^3 + 6t\right)(0.015)\Big|_{t=2}$$

$$v_E = 3 \text{ m/s} \qquad \textbf{Ans.}$$

$$\alpha_A = \frac{d\omega_A}{dt} = \frac{d}{dt}\left[40\left(t^3 + 6t\right)\right] = 120t^2 + 240$$

$$\alpha_A r_A = \alpha_B r_B$$

$$\alpha_A(10) = \alpha_B(40)$$

$$\alpha_B = \frac{1}{4}\alpha_A$$

$$(a_E)_t = \alpha_B r_E = \frac{1}{4}\left(120t^2 + 240\right)(0.015)\Big|_{t=2}$$

$$(a_E)_t = 2.70 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(a_E)_n = \omega_B^2 r_E = \left[\frac{1}{4}(40)\left(t^3 + 6t\right)\right]^2(0.015)\Big|_{t=2}$$

$$(a_E)_n = 600 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$v_E = 3 \text{ m/s}$
$(a_E)_t = 2.70 \text{ m/s}^2$
$(a_E)_n = 600 \text{ m/s}^2$

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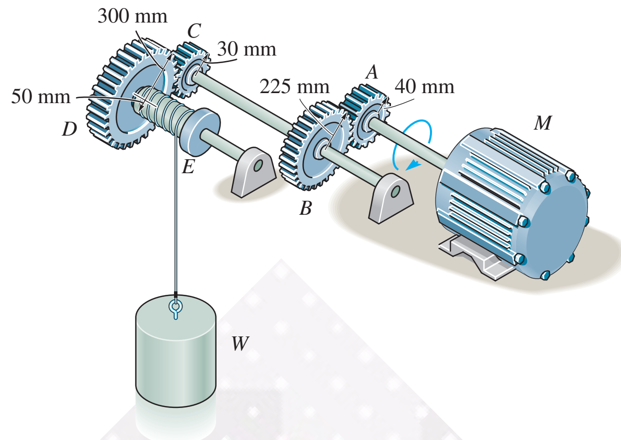
16–30.

Determine the distance the load W is lifted in $t = 5$ s using the hoist. The shaft of the motor M turns with an angular velocity $\omega = 100(4 + t)$ rad/s, where t is in seconds.



SOLUTION

Angular Motion: The angular displacement of gear A at $t = 5$ s must be determined first. Applying Eq. 16–1, we have

$$d\theta = \omega dt$$

$$\int_0^{\theta_A} d\theta = \int_0^{5\text{ s}} 100(4 + t)\, dt$$

$$\theta_A = 3250 \text{ rad}$$

Here, $r_A\,\theta_A = r_B\,\theta_B$. Then, the angular displacement of gear B is given by

$$\theta_B = \frac{r_A}{r_B}\theta_A = \left(\frac{40}{225}\right)(3250) = 577.78 \text{ rad}$$

Since gear C is attached to the same shaft as gear B, then $\theta_C = \theta_B = 577.78$ rad. Also, $r_D\,\theta_D = r_C\,\theta_C$, then, the angular displacement of gear D is given by

$$\theta_D = \frac{r_C}{r_D}\theta_C = \left(\frac{30}{300}\right)(577.78) = 57.78 \text{ rad}$$

Since shaft E is attached to gear D, $\theta_E = \theta_D = 57.78$ rad. The distance at which the load W is lifted is

$$s_W = r_E\theta_E = (0.05)(57.78) = 2.89 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_W = 2.89$ m

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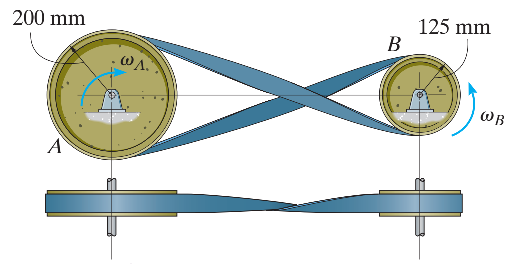
16–31.

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If the angular displacement of A is $\theta_A = (5t^3 + 10t^2)$ rad, where t is in seconds, determine the angular velocity and angular acceleration of B when $t = 3$ s.



SOLUTION

Motion of Wheel A: The angular velocity and angular acceleration of wheel A can be determined from

$$\omega_A = \frac{d\theta_A}{dt} = \left(15t^2 + 20t\right) \text{ rad/s}$$

and

$$\alpha_A = \frac{d\omega_A}{dt} = \left(30t + 20\right) \text{ rad/s}^2$$

When $t = 3$ s,

$$\omega_A = 15\left(3^2\right) + 20(3) = 195 \text{ rad/s}$$

$$\alpha_A = 30(3) + 20 = 110 \text{ rad/s}^2$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(195) = 312 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(110) = 176 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 312$ rad/s
$\alpha_B = 176$ rad/s^2

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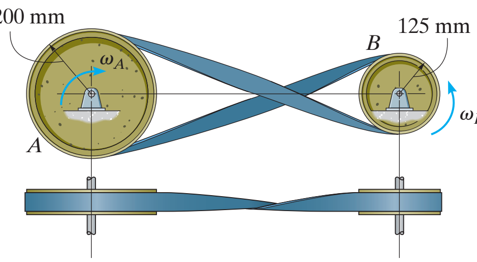
***16–32.**

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If A has a constant angular acceleration of $\alpha_A = 30\ \text{rad/s}^2$, determine the tangential and normal components of acceleration of a point located at the rim of B when $t = 3$ s, starting from rest.



SOLUTION

Motion of Wheel A: Since the angular acceleration of wheel A is constant, its angular velocity can be determined from

$$\omega_A = (\omega_A)_0 + \alpha_C t$$

$$= 0 + 30(3) = 90\ \text{rad/s}$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(90) = 144\ \text{rad/s}$$

and

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(30) = 48\ \text{rad/s}^2$$

Thus, the tangential and normal components of the acceleration of point P located at the rim of wheel B are

$$(a_p)_t = \alpha_B r_B = 48(0.125) = 6\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$(a_p)_n = \omega_B{}^2 r_B = (144^2)(0.125) = 2592\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$(a_p)_t = 6\ \text{m/s}^2$
$(a_p)_n = 2592\ \text{m/s}^2$

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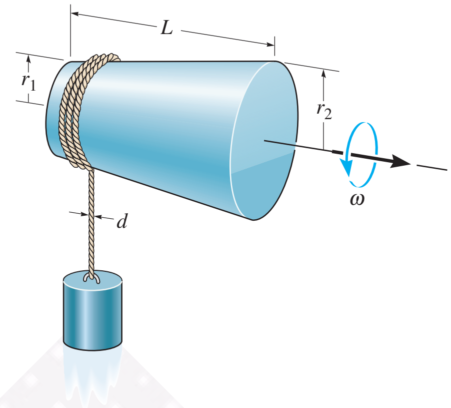
16–33.

The rope of diameter d is wrapped around the tapered drum which has the dimensions shown. If the drum is rotating at a constant rate of ω, determine the upward acceleration of the the block. Neglect the small horizontal displacement of the block.



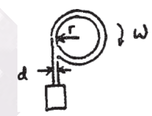
SOLUTION

$$v = \omega r$$

$$a = \frac{d(\omega r)}{dt}$$

$$= \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$$

$$= \omega \left(\frac{dr}{dt}\right)$$



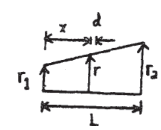
$$r = r_1 + \left(\frac{r_2 - r_1}{L}\right)x$$

$$dr = \left(\frac{r_2 - r_1}{L}\right)dx$$

But $dx = \dfrac{d\theta}{2\pi} \cdot d$

Thus $\dfrac{dr}{dt} = \dfrac{1}{2\pi}\left(\dfrac{r_2 - r_1}{L}\right) d\left(\dfrac{d\theta}{dt}\right)$

$$= \frac{1}{2\pi}\left(\frac{r_2 - r_1}{L}\right) d\omega$$



Thus, $a = \dfrac{\omega^2}{2\pi}\left(\dfrac{r_2 - r_1}{L}\right)d$ **Ans.**

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Ans:
$$a = \frac{\omega^2}{2\pi}\left(\frac{r_2 - r_1}{L}\right)d$$

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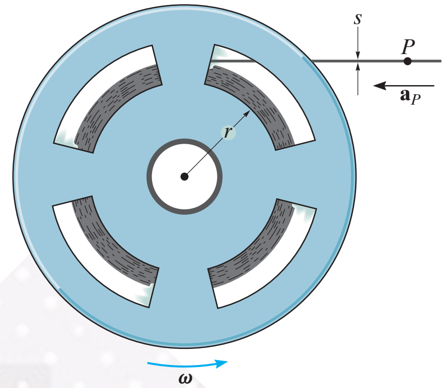
16–34.

A tape having a thickness s wraps around the wheel which is turning at a constant rate ω. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point P of the unwrapped tape when the radius of the wrapped tape is r. *Hint*: Since $v_P = \omega r$, take the time derivative and note that $dr/dt = \omega(s/2\pi)$.



SOLUTION

$$v_P = \omega r$$

$$a = \frac{dv_P}{dt} = \frac{d\omega}{dt}r + \omega\frac{dr}{dt}$$

Since $\dfrac{d\omega}{dt} = 0$,

$$a = \omega\left(\frac{dr}{dt}\right)$$

In one revolution r is increased by s, so that

$$\frac{2\pi}{\theta} = \frac{s}{\Delta r}$$

Hence,

$$\Delta r = \frac{s}{2\pi}\theta$$

$$\frac{dr}{dt} = \frac{s}{2\pi}\omega$$

$$a = \frac{s}{2\pi}\omega^2 \qquad \textbf{Ans.}$$

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Ans:
$a = \dfrac{s}{2\pi}\omega^2$

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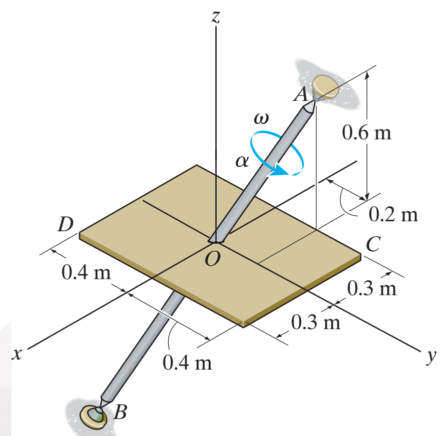
16–35.

If the shaft and plate rotates with a constant angular velocity of $\omega = 14$ rad/s, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

Since ω is constant

$$\alpha = 0$$

For convenience, $\mathbf{r}_C = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point C can be determined from

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$

$$= [-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \alpha \times \mathbf{r}_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_c)$$

$$= 0 + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$\mathbf{v}_C = \{-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}\}$ m/s^2

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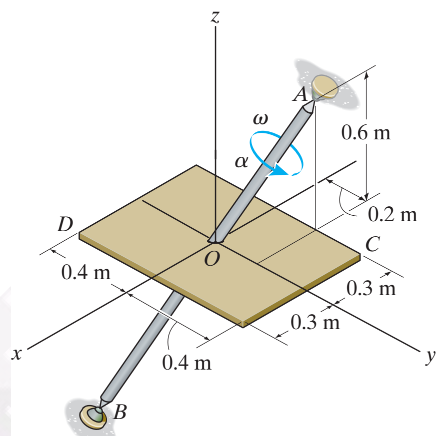
***16–36.**

At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14$ rad/s and angular acceleration of $\alpha = 7$ rad/s^2. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω and α is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}]\text{ rad/s}$$

$$\alpha = \alpha\mathbf{u}_{OA} = 7\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}]\text{ rad/s}$$

For convenience, $\mathbf{r}_D = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point D can be determined from

$$\mathbf{v}_D = \omega \times r_D$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j})$$

$$= [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]\text{ m/s}$$ **Ans.**

and

$$\mathbf{a}_D = \alpha \times \mathbf{r}_D - \omega^2\,\mathbf{r}_D$$

$$= (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}]\text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_D = [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]$ m/s
$\mathbf{a}_D = [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}]$ m/s^2

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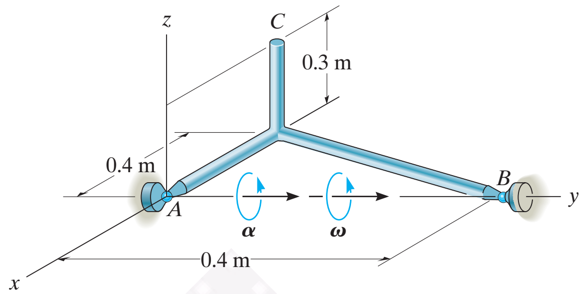
16–37.

The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with an angular velocity $\omega = 5$ rad/s and has an angular acceleration $\alpha = 8$ rad/s^2. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16–9 and 16–13.



SOLUTION

$$v_C = \boldsymbol{\omega} \times \mathbf{r}$$

$$v_C = 5\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) = \{1.5\mathbf{i} + 2\mathbf{k}\} \text{ m/s}$$

$$v_C = \sqrt{1.5^2 + 2^2} = 2.50 \text{ m/s}$$ **Ans.**

$$a_C = \mathbf{a} \times \mathbf{r} - \omega^2\mathbf{r}$$

$$= 8\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) - 5^2(-0.4\mathbf{i} + 0.3\mathbf{k})$$

$$= \{12.4\mathbf{i} - 4.3\mathbf{k}\} \text{ m/s}^2$$

$$a_C = \sqrt{(12.4)^2 + (-4.3)^2} = 13.1 \text{ m/s}^2$$ **Ans.**

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Ans:
$v_C = 2.50$ m/s
$a_C = 13.1$ m/s^2

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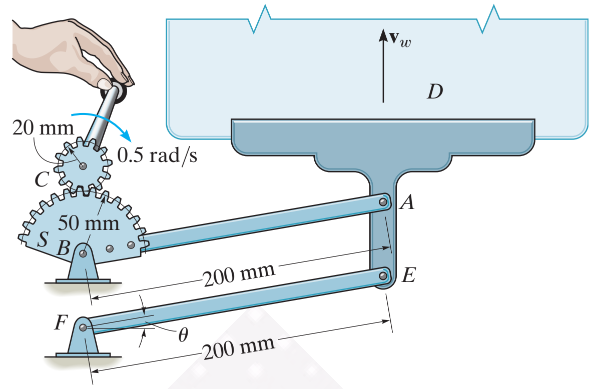
16–38. The mechanism for a car window winder is shown in the figure. Here the handle turns the small cog C, which rotates the spur gear S, thereby rotating the fixed-connected lever AB which raises track D in which the window rests. The window is free to slide on the track. If the handle is wound at 0.5 rad/s, determine the speed of points A and E and the speed v_w of the window at the instant $\theta = 30°$.



SOLUTION

Given:

$\omega_c = 0.5 \text{ rad/s} \quad r_C = 20 \text{ mm}$

$\theta = 30° \quad r_s = 50 \text{ mm}$

$r_A = 200 \text{ mm}$

$v_C = \omega_c \, r_C$

$v_C = 0.01 \text{ m/s}$

$$\omega_s = \frac{v_C}{r_s}$$

$\omega_s = 0.2 \text{ rad/s}$

$v_A = v_E = \omega_s \, r_A$

$v_A = \omega_s \, r_A \qquad v_A = 40 \text{ mm/s} \qquad$ **Ans.**

Points A and E move along circular paths. The vertical component closes the window.

$v_w = v_A \cos(\theta) \qquad v_w = 34.6 \text{ mm/s} \qquad$ **Ans.**

Ans:
$v_A = 40 \text{ mm/s}$
$v_w = 34.6 \text{ mm/s}$

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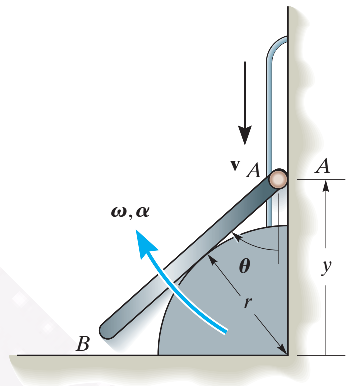
16–39.

The end A of the bar is moving downward along the slotted guide with a constant velocity $\mathbf{v}_A$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar as a function of its position y.



SOLUTION

Position coordinate equation:

$$\sin\theta = \frac{r}{y}$$

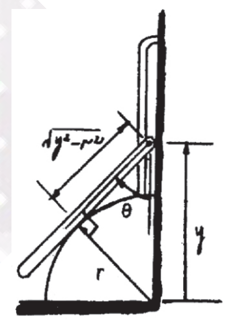
Time derivatives:

$$\cos\theta\dot{\theta} = -\frac{r}{y^2}\dot{y} \text{ however, } \cos\theta = \frac{\sqrt{y^2 - r^2}}{y} \text{ and } \dot{y} = -v_A, \dot{\theta} = \omega$$

$$\left(\frac{\sqrt{y^2 - r^2}}{y}\right)\omega = \frac{r}{y^2}v_A \qquad \omega = \frac{rv_A}{y\sqrt{y^2 - r^2}} \qquad \textbf{Ans.}$$

$$\alpha = \dot{\omega} = rv_A\left[-y^{-2}\dot{y}\left(y^2 - r^2\right)^{-\frac{1}{2}} + \left(y^{-1}\right)\left(-\tfrac{1}{2}\right)\left(y^2 - r^2\right)^{-\frac{3}{2}}(2y\dot{y})\right]$$

$$\alpha = \frac{rv_A^2\,(2y^2 - r^2)}{y^2(y^2 - r^2)^{\frac{3}{2}}} \qquad \textbf{Ans.}$$



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Ans:

$$\omega = \frac{rv_A}{y\sqrt{y^2 - r^2}}$$

$$\alpha = \frac{rv_A^2\left(2y^2 - r^2\right)}{y^2\left(y^2 - r^2\right)^{3/2}}$$

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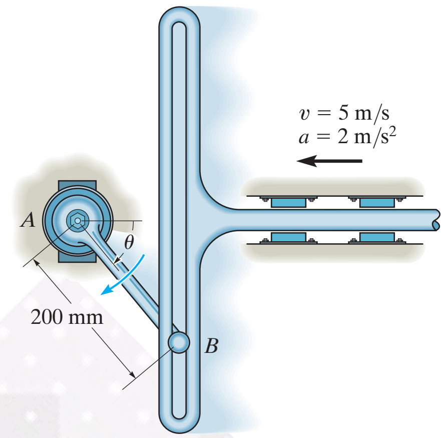
***16–40.**

At the instant $\theta = 60°$, the slotted guide rod is moving to the left with an acceleration of $2\ \text{m/s}^2$ and a velocity of $5\ \text{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the guide rod can be related to the angular motion of the crank by relating x and θ using the geometry shown in Fig. a, which is

$$x = 0.2 \cos \theta \text{ m}$$

Time Derivatives. Using the chain rule,

$$\dot{x} = -0.2(\sin \theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{x} = -0.2[(\cos \theta)\dot{\theta}^2 + (\sin \theta)\ddot{\theta}] \qquad \textbf{(2)}$$

Here $\dot{x} = v$, $\ddot{x} = a$, $\dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$ when $\theta = 60°$. Realizing that the velocity and acceleration of the guide rod are directed toward the negative sense of x, $v = -5\ \text{m/s}$ and $a = -2\ \text{m/s}^2$. Then Eq (1) gives

$$-s = (-0.2(\sin 60°)\omega$$

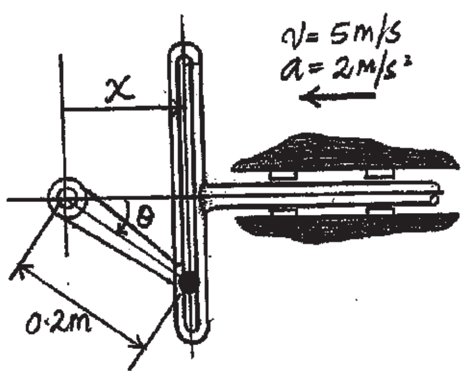
$$\omega = 28.87\ \text{rad/s} = 28.9\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Subsequently, Eq. (2) gives

$$-2 = -0.2[\cos 60°(28.87^2) + (\sin 60°)\alpha]$$

$$\alpha = -469.57\ \text{rad/s}^2 = 470\ \text{rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

The negative sign indicates that $\boldsymbol{\alpha}$ is directed in the negative sense of θ.



(a)

Ans:
$\omega = 28.9\ \text{rad/s} \circlearrowright$
$\alpha = 470\ \text{rad/s}^2 \circlearrowleft$

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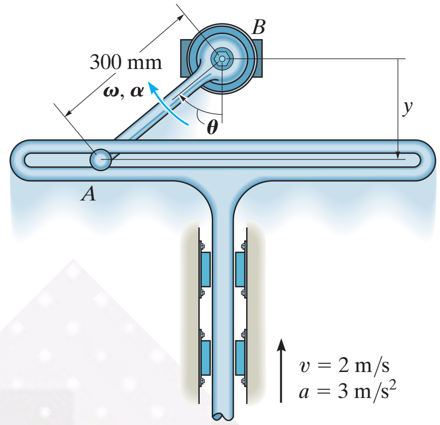
16–41.

At the instant $\theta = 50°$, the slotted guide is moving upward with an acceleration of $3\ \text{m/s}^2$ and a velocity of $2\ \text{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant. *Note:* The upward motion of the guide is in the negative y direction.

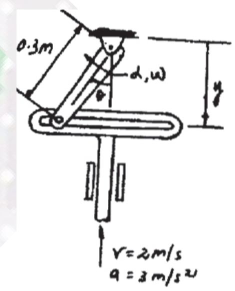


SOLUTION

$$y = 0.3 \cos \theta$$

$$\dot{y} = v_y = -0.3 \sin \theta \dot{\theta}$$

$$\ddot{y} = a_y = -0.3\left(\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2\right)$$



Here $v_y = -2\ \text{m/s}$, $a_y = -3\ \text{m/s}^2$, and $\dot{\theta} = \omega, \dot{\theta} = \omega, \ddot{\theta} = \alpha, \theta = 50°$.

$-2 = -0.3 \sin 50°(\omega)$ $\qquad \omega = 8.70\ \text{rad/s}$ **Ans.**

$-3 = -0.3[\sin 50°(\alpha) + \cos 50°(8.70)^2]$ $\qquad \alpha = -50.5\ \text{rad/s}^2$ **Ans.**

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Ans:
$\omega = 8.70\ \text{rad/s}$
$\alpha = -50.5\ \text{rad/s}^2$

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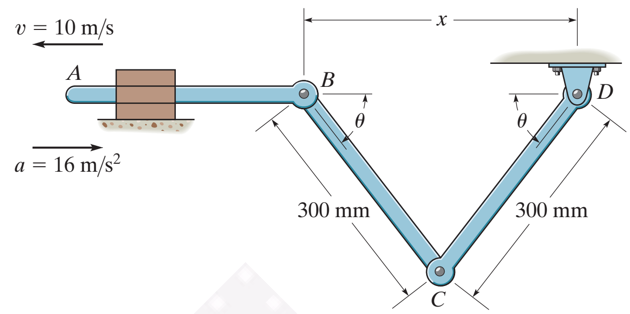
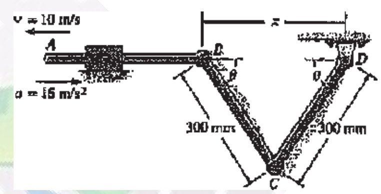
16–42.

At the instant shown, $\theta = 60°$, and rod AB is subjected to a deceleration of $16\ \text{m/s}^2$ when the velocity is $10\ \text{m/s}$. Determine the angular velocity and angular acceleration of link CD at this instant.



SOLUTION

$x = 2(0.3)\cos\theta$



$x = -0.6\sin\theta(\dot{\theta})$ **(1)**

$\ddot{x} = -0.6\cos\theta(\dot{\theta})^2 - 0.6\sin\theta(\ddot{\theta})$ **(2)**

Using Eqs. (1) and (2) at $\theta = 60°$, $\dot{x} = 10\ \text{m/s}$, $\ddot{x} = -16\ \text{m/s}^2$.

$10 = -0.6\sin 60°(\omega)$

$\omega = -19.245 = -19.2\ \text{rad/s}$ **Ans.**

$-16 = -0.6\cos 60°(-19.245)^2 - 0.6\sin 60°(\alpha)$

$\alpha = -183\ \text{rad/s}^2$ **Ans.**

Ans:
$\omega = -19.2\ \text{rad/s}$
$\alpha = -183\ \text{rad/s}^2$

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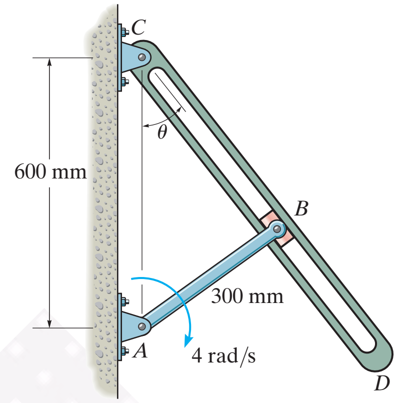
16–43.

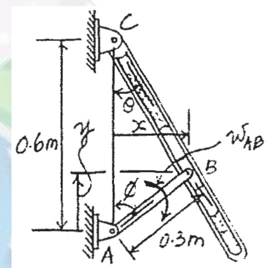
The crank AB is rotating with a constant angular velocity of 4 rad/s. Determine the angular velocity of the connecting rod CD at the instant $\theta = 30°$.



SOLUTION

Position Coordinate Equation: From the geometry,

$$0.3 \sin \phi = (0.6 - 0.3 \cos \phi) \tan \theta \qquad \textbf{[1]}$$



Time Derivatives: Taking the time derivative of Eq. [1], we have

$$0.3 \cos \phi \frac{d\phi}{dt} = 0.6\sec^2\theta \frac{d\theta}{dt} - 0.3\left(\cos \theta \sec^2 \theta \frac{d\theta}{dt} - \tan \theta \sin \theta \frac{d\phi}{dt}\right)$$

$$\frac{d\theta}{dt} = \left[\frac{0.3(\cos \phi - \tan \theta \sin \phi)}{0.3\sec^2\theta(2 - \cos \phi)}\right]\frac{d\phi}{dt} \qquad \textbf{[2]}$$

However, $\frac{d\theta}{dt} = \omega_{BC}$, $\frac{d\phi}{dt} = \omega_{AB} = 4$ rad/s. At the instant $\theta = 30°$, from Eq. [3], $\phi = 60.0°$. Substitute these values into Eq. [2] yields

$$\omega_{BC} = \left[\frac{0.3(\cos 60.0° - \tan 30° \sin 60.0°)}{0.3 \sec^2 30° \, (2 - \cos 60.0°)}\right](4) = 0 \qquad \textbf{Ans.}$$

Ans:
$\omega_{AB} = 0$

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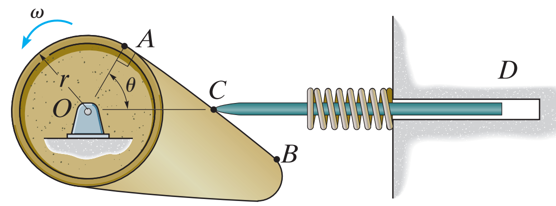
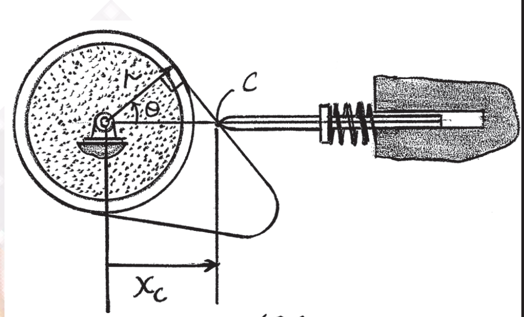
***16–44.**

Determine the velocity and acceleration of the follower rod *CD* as a function of θ when the contact between the cam and follower is along the straight region *AB* on the face of the cam. The cam rotates with a constant counterclockwise angular velocity $\boldsymbol{\omega}$.



SOLUTION



(a)

Position Coordinate: From the geometry shown in Fig. *a*,

$$x_C = \frac{r}{\cos\theta} = r\sec\theta$$

Time Derivative: Taking the time derivative,

$$v_{CD} = \dot{x}_C = r\sec\theta\tan\theta\dot{\theta} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = +\omega$ since ω acts in the positive rotational sense of θ. Thus, Eq. (1) gives

$$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow \qquad \textbf{Ans.}$$

The time derivative of Eq. (1) gives

$$a_{CD} = \ddot{x}_C = r\{\sec\theta\tan\theta\ddot{\theta} + \dot{\theta}[\sec\theta(\sec^2\theta\dot{\theta}) + \tan\theta(\sec\theta\tan\theta\dot{\theta})]\}$$

$$a_{CD} = r[\sec\theta\tan\theta\ddot{\theta} + (\sec^3\theta + \sec\theta\tan^2\theta)\dot{\theta}^2]$$

Since $\dot{\theta} = \omega$ is constant, $\ddot{\theta} = \alpha = 0$. Then,

$$a_{CD} = r[\sec\theta\tan\theta(0) + (\sec^3\theta + \sec\theta\tan^2\theta)\omega^2]$$

$$= r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow \qquad \textbf{Ans.}$$

Ans:
$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow$
$a_{CD} = r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow$

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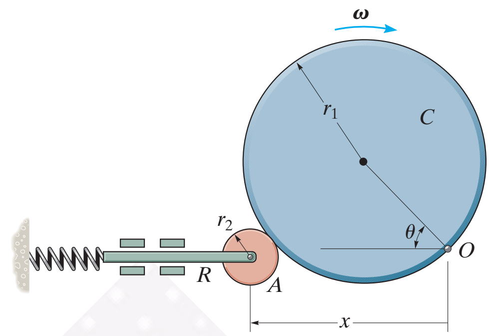
16–45.

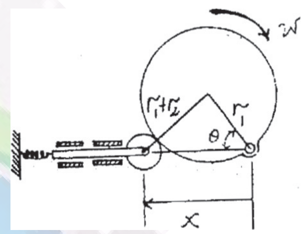
Determine the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of A on C.



SOLUTION

Position Coordinate Equation: Using law of cosine.

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x \cos\theta \qquad \textbf{(1)}$$



Time Derivatives: Taking the time derivative of Eq. (1). we have

$$0 = 2x\frac{dx}{dt} - 2r_1\left(-x\sin\theta\frac{d\theta}{dt} + \cos\theta\frac{dx}{dt}\right) \qquad \textbf{(2)}$$

However $v = \dfrac{dx}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.(2),

$$0 = xv - r_1(v\cos\theta - x\omega\sin\theta)$$

$$v = \frac{r_1 x\omega\sin\theta}{r_1\cos\theta - x} \qquad \textbf{(3)}$$

However, the positive root of Eq.(1) is

$$x = r_1\cos\theta + \sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}$$

Substitute into Eq.(3), we have

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}} + r_1\omega\sin\theta\right) \qquad \textbf{Ans.}$$

Note: Negative sign indicates that v is directed in the opposite direction to that of positive x.

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Ans:

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}} + r_1\omega\sin\theta\right)$$

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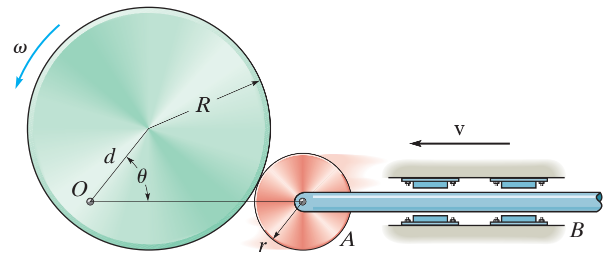
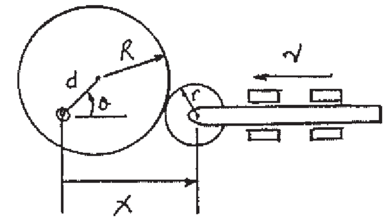
16–46.

The circular cam rotates about the fixed point O with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity v of the follower rod AB as a function of θ.



SOLUTION

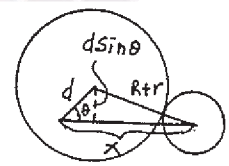
$$x = d\cos\theta + \sqrt{(R+r)^2 - (d\sin\theta)^2}$$



$$\dot{x} = v_{AB} = -d\sin\theta\dot{\theta} - \frac{d^2\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\dot{\theta}$$ Where $\dot{\theta} = \omega$ and $v_{AB} = -v$

$$-v = -d\sin\theta(\omega) - \frac{d^2\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\omega$$

$$v = \omega d\left(\sin\theta + \frac{d\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\right)$$ **Ans.**



Ans:

$$v = \omega d\left(\sin\theta + \frac{d\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\right)$$

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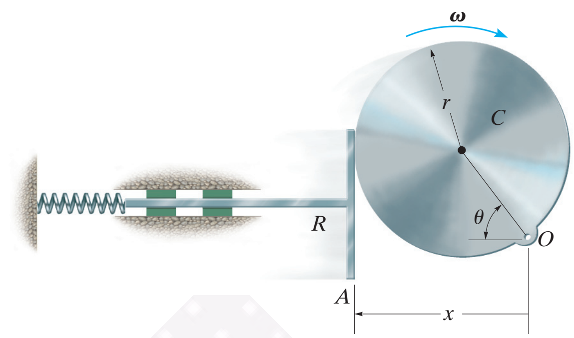
16–47.

Determine the velocity of the rod R for any angle θ of cam C as the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of plate A on C.



SOLUTION

$$x = r + r\cos\theta$$

$$x = -r\sin\theta\theta$$

$$v = -r\omega\sin\theta$$ **Ans.**

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Ans:
$v = -r\omega\sin\theta$

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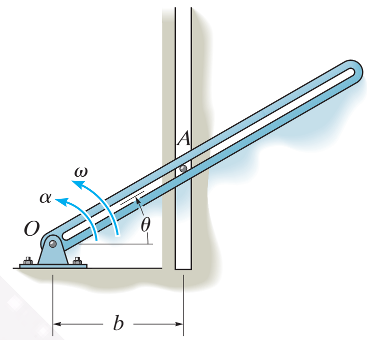
***16–48.**

Determine the velocity and acceleration of the peg A which is confined between the vertical guide and the rotating slotted rod.



SOLUTION

Position Coordinate Equation. The rectilinear motion of peg A can be related to the angular motion of the slotted rod by relating y and θ using the geometry shown in Fig. a, which is

$$y = b \tan \theta$$

Time Derivatives. Using the chain rule,

$$\dot{y} = b(\sec^2 \theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{y} = b[2 \sec \theta(\sec \theta \tan \theta\dot{\theta})\dot{\theta} + \sec^2 \theta\ddot{\theta}]$$

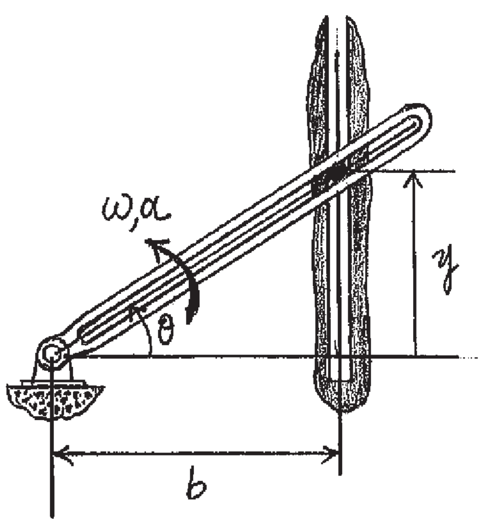
$$\ddot{y} = b(2 \sec^2 \theta \tan \theta\dot{\theta}^2 + \sec^2 \theta\ddot{\theta})$$

$$\ddot{y} = b \sec^2 \theta(2 \tan \theta\dot{\theta}^2 + \ddot{\theta}) \qquad \textbf{(2)}$$

Here, $\dot{y} = v, \ddot{y} = a, \dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$. Then Eqs. (1) and (2) become

$$v = \omega b \sec^2 \theta \qquad \textbf{Ans.}$$

$$a = b \sec^2 \theta(2\omega^2 \tan \theta + \alpha) \qquad \textbf{Ans.}$$



(a)

Ans:
$v = \omega b \sec^2 \theta$
$a = b \sec^2 \theta(2\omega^2 \tan \theta + \alpha)$

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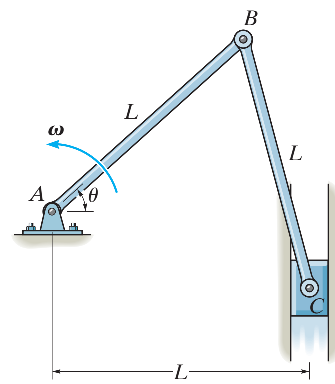
16–49.

Bar AB rotates uniformly about the fixed pin A with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of block C, at the instant $\theta = 60°$.



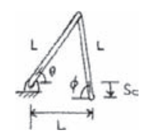
SOLUTION

$$L\cos\theta + L\cos\phi = L$$

$$\cos\theta + \cos\phi = 1$$

$$\sin\theta\,\dot{\theta} + \sin\phi\,\dot{\phi} = 0 \qquad \textbf{(1)}$$

$$\cos\theta(\dot{\theta})^2 + \sin\theta\ddot{\theta} + \sin\phi\,\ddot{\phi} + \cos\phi\,(\dot{\phi})^2 = 0 \qquad \textbf{(2)}$$



When $\theta = 60°, \phi = 60°$,

thus, $\dot{\theta} = -\dot{\phi} = \omega$ (from Eq. (1))

$\ddot{\theta} = 0$

$\ddot{\phi} = -1.155\omega^2$ (from Eq.(2))

Also, $s_C = L\sin\phi - L\sin\theta$

$$v_C = L\cos\phi\,\dot{\phi} - L\cos\theta\,\dot{\theta}$$

$$a_C = -L\sin\phi\,(\dot{\phi})^2 + L\cos\phi\,(\ddot{\phi}) - L\cos\theta(\ddot{\theta}) + L\sin\theta(\dot{\theta})^2$$

At $\theta = 60°, \phi = 60°$

$$s_C = 0$$

$$v_C = L(\cos 60°)(-\omega) - L\cos 60°(\omega) = -L\omega = L\omega\uparrow \qquad \textbf{Ans.}$$

$$a_C = -L\sin 60°(-\omega)^2 + L\cos 60°(-1.155\omega^2) + 0 + L\sin 60°(\omega)^2$$

$$a_C = -0.577\,L\omega^2 = 0.577\,L\omega^2\uparrow \qquad \textbf{Ans.}$$

Ans:
$v_C = L\omega\uparrow$
$a_C = 0.577\,L\omega^2\uparrow$

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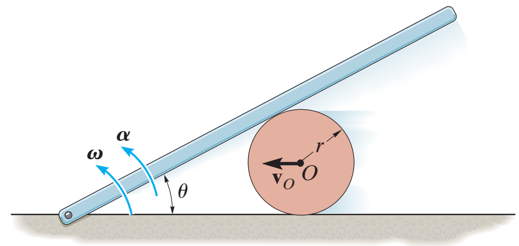
16–50.

The center of the cylinder is moving to the left with a constant velocity $\mathbf{v}_0$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar. Neglect the thickness of the bar.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the cylinder can be related to the angular motion of the rod by relating x and θ using the geometry shown in Fig. a, which is

$$x = \frac{r}{\tan\theta/2} = r\cot\theta/2$$

Time Derivatives. Using the chain rule,

$$\dot{x} = r\left[\left(-\csc^2\theta/2\right)\left(\frac{1}{2}\dot{\theta}\right)\right]$$

$$\dot{x} = -\frac{r}{2}\left(\csc^2\theta/2\right)\dot{\theta} \tag{1}$$

$$\ddot{x} = -\frac{r}{2}\left[2\csc\theta/2\left(-\csc\theta/2\cot\theta/2\right)\left(\frac{1}{2}\dot{\theta}\right)\dot{\theta} + \left(\csc^2\theta/2\right)\ddot{\theta}\right]$$

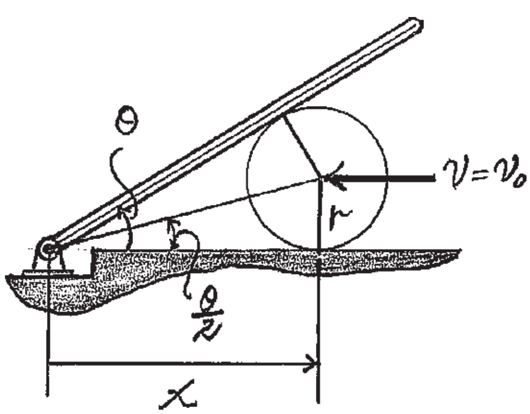
$$\ddot{x} = \frac{r}{2}\left[\left(\csc^2\theta/2\cot\theta/2\right)\dot{\theta}^2 - \left(\csc^2\theta/2\right)\ddot{\theta}\right]$$

$$\ddot{x} = \frac{r\csc^2\theta/2}{2}\left[\left(\cot\theta/2\right)\dot{\theta}^2 - \ddot{\theta}\right] \tag{2}$$

Here $\dot{x} = -v_0$ since $\mathbf{v}_0$ is directed toward the negative sense of x and $\dot{\theta} = \omega$. Then Eq. (1) gives,

$$-v_0 = -\frac{r}{2}\left(\csc^2\theta/2\right)\omega$$

$$\omega = \frac{2v_0}{r}\sin^2\theta/2 \qquad \textbf{Ans.}$$



(a)

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15–50. Continued

Also, $\ddot{x} = 0$ since v is constant and $\ddot{\theta} = \alpha$. Substitute the results of ω into Eq. (2):

$$0 = \frac{r \csc^2 \theta/2}{2}\left[(\cot \theta/2)\left(\frac{2v_0}{r}\sin^2 \theta/2\right)^2 - \alpha\right]$$

$$\alpha = (\cot \theta/2)\left(\frac{2r_0}{r}\sin^2 \theta/2\right)^2$$

$$\alpha = \left(\frac{\cos \theta/2}{\sin \theta/2}\right)\left(\frac{4v_0^2}{r^2}\sin^4 \theta/2\right)$$

$$\alpha = \frac{4v_0^2}{r^2}(\sin^3 \theta/2)(\cos \theta/2)$$

$$\alpha = \frac{2v_0^2}{r^2}(2 \sin \theta/2 \cos \theta/2)(\sin^2 \theta/2)$$

Since $\sin \theta = 2 \sin \theta/2 \cos \theta/2$, then

$$\alpha = \frac{2v_0^2}{r^2}(\sin \theta)(\sin^2 \theta/2)$$ **Ans.**

Ans:
$\omega = \dfrac{2v_0}{r}\sin^2 \theta/2$
$\alpha = \dfrac{2v_0^2}{r^2}(\sin \theta)(\sin^2 \theta/2)$

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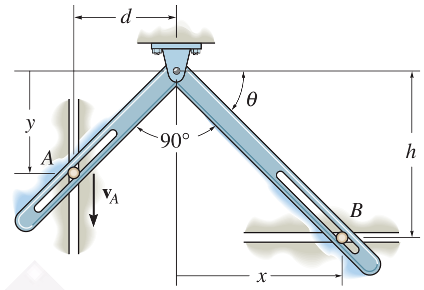
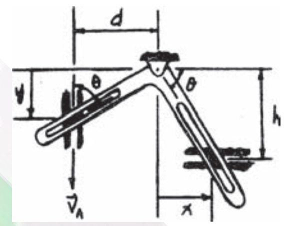
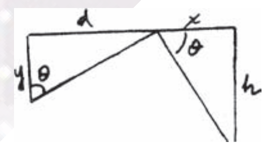
16–51.

The pins at A and B are confined to move in the vertical and horizontal tracks. If the slotted arm is causing A to move downward at $\mathbf{v}_A$, determine the velocity of B at the instant shown.



SOLUTION





Position coordinate equation:

$$\tan\theta = \frac{h}{x} = \frac{d}{y}$$

$$x = \left(\frac{h}{d}\right)y$$

Time derivatives:

$$\dot{x} = \left(\frac{h}{d}\right)\dot{y}$$

$$v_B = \left(\frac{h}{d}\right)v_A \qquad \textbf{Ans.}$$

Ans:
$v_B = \left(\frac{h}{d}\right)v_A$

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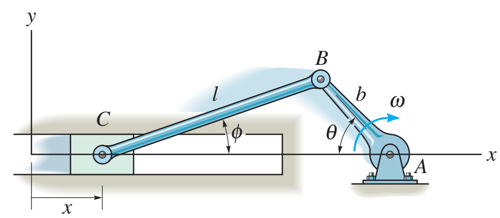
***16–52.**

The crank AB has a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of the slider at C as a function of θ. *Suggestion:* Use the x coordinate to express the motion of C and the ϕ coordinate for CB. $x = 0$ when $\phi = 0°$.



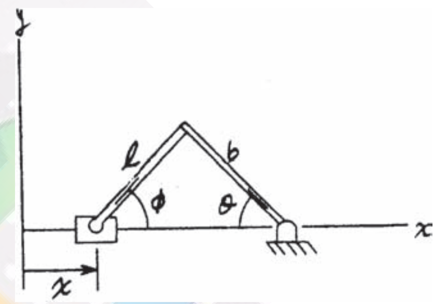
SOLUTION

$$x = l + b - (L\cos\phi + b\cos\theta)$$

$$l\sin\phi = b\sin\theta \text{ or } \sin\phi = \frac{b}{l}\sin\theta$$

$$v_C = \dot{x} = l\sin\phi\dot{\phi} + b\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

$$\cos\phi\dot{\phi} = \frac{b}{l}\cos\theta\dot{\theta} \qquad \textbf{(2)}$$



Since $\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \left(\frac{b}{l}\right)^2 \sin^2\theta}$

then,

$$\dot{\phi} = \frac{\left(\frac{b}{l}\right)\cos\theta\omega}{\sqrt{1 - \left(\frac{b}{l}\right)^2 \sin^2\theta}} \qquad \textbf{(3)}$$

$$v_C = b\omega\left[\frac{\left(\frac{b}{l}\right)\sin\theta\cos\theta}{\sqrt{1 - \left(\frac{b}{l}\right)^2 \sin^2\theta}}\right] + b\omega\sin\theta \qquad \textbf{Ans.}$$

From Eq. (1) and (2):

$$a_C = \dot{v}_C = l\ddot{\phi}\sin\phi + l\dot{\phi}\cos\phi\dot{\phi} + b\cos\theta\left(\dot{\theta}\right)^2 \qquad \textbf{(4)}$$

$$-\sin\phi\dot{\phi}^2 + \cos\phi\ddot{\phi} = -\left(\frac{b}{l}\right)\sin\theta\dot{\theta}^2$$

$$\ddot{\phi} = \frac{\dot{\phi}^2\sin\phi - \frac{b}{l}\omega^2\sin\theta}{\cos\phi} \qquad \textbf{(5)}$$

Substituting Eqs. (1), (2), (3) and (5) into Eq. (4) and simplifying yields

$$a_C = b\omega^2\left[\frac{\left(\frac{b}{l}\right)\left(\cos 2\theta + \left(\frac{b}{l}\right)^2 \sin^4\theta\right)}{\left(1 - \left(\frac{b}{l}\right)^2 \sin^2\theta\right)^{\frac{3}{2}}} + \cos\theta\right] \qquad \textbf{Ans.}$$

Ans:

$$v_C = b\omega\left[\frac{\left(\frac{b}{l}\right)\sin\theta\cos\theta}{\sqrt{1 - \left(\frac{b}{l}\right)^2 \sin^2\theta}}\right] + b\omega\sin\theta$$

$$a_C = b\omega^2\left[\frac{\left(\frac{b}{l}\right)\left(\cos 2\theta + \left(\frac{b}{l}\right)^2 \sin^4\theta\right)}{\left(1 - \left(\frac{b}{l}\right)^2 \sin^2\theta\right)^{\frac{3}{2}}} + \cos\theta\right]$$

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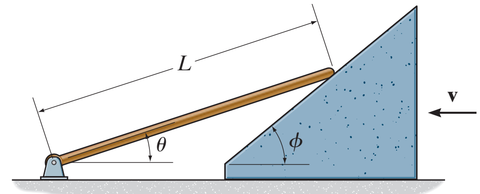
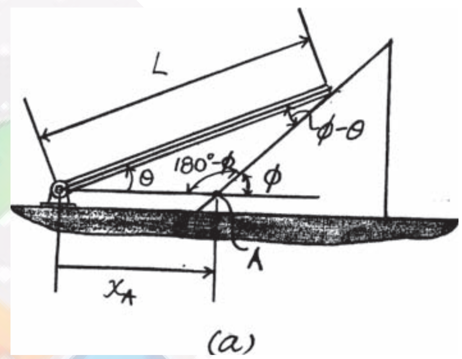
16–53.

If the wedge moves to the left with a constant velocity **v**, determine the angular velocity of the rod as a function of θ.



SOLUTION

Position Coordinates: Applying the law of sines to the geometry shown in Fig. a,



$$\frac{x_A}{\sin(\phi - \theta)} = \frac{L}{\sin\left(180° - \phi\right)}$$

$$x_A = \frac{L \sin(\phi - \theta)}{\sin\left(180° - \phi\right)}$$

However, $\sin\left(180° - \phi\right) = \sin\phi$. Therefore,

$$x_A = \frac{L \sin\,(\phi - \theta)}{\sin\,\phi}$$

Time Derivative: Taking the time derivative,

$$\dot{x}_A = \frac{L \cos\,(\phi - \theta)(-\dot{\theta})}{\sin\,\phi}$$

$$v_A = \dot{x}_A = -\frac{L \cos\,(\phi - \theta)\dot{\theta}}{\sin\,\phi} \qquad \textbf{(1)}$$

Since point A is on the wedge, its velocity is $v_A = -v$. The negative sign indicates that $\mathbf{v}_A$ is directed towards the negative sense of x_A. Thus, Eq. (1) gives

$$\dot{\theta} = \frac{v \sin\,\phi}{L \cos\,(\phi - \theta)} \qquad \textbf{Ans.}$$

Ans:
$$\dot{\theta} = \frac{v \sin\,\phi}{L \cos\,(\phi - \theta)}$$

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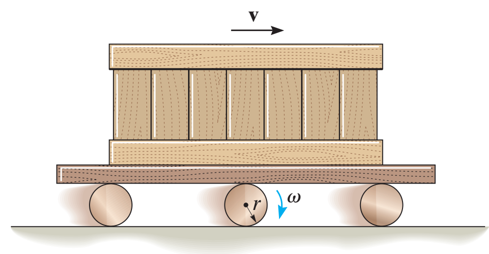
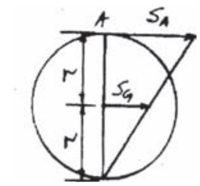
16–54.

The crate is transported on a platform which rests on rollers, each having a radius r. If the rollers do not slip, determine their angular velocity if the platform moves forward with a velocity **v**.



SOLUTION

Position coordinate equation: From Example 16.4, $s_G = r\theta$. Using similar triangles



$$s_A = 2s_G = 2r\theta$$

Time derivatives:

$$s_A = v = 2r\dot{\theta} \quad \text{Where } \dot{\theta} = \omega$$

$$\omega = \frac{v}{2r}$$ **Ans.**

Ans:
$\omega = \dfrac{v}{2r}$

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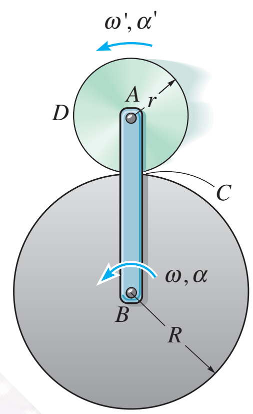
16–55.

Arm AB has an angular velocity of $\boldsymbol{\omega}$ and an angular acceleration of $\boldsymbol{\alpha}$. If no slipping occurs between the disk D and the fixed curved surface, determine the angular velocity and angular acceleration of the disk.



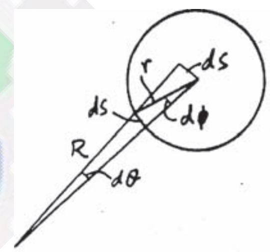
SOLUTION

$$ds = (R + r)\,d\theta = r\,d\phi$$

$$(R + r)\left(\frac{d\theta}{dt}\right) = r\left(\frac{d\phi}{dt}\right)$$

$$\omega' = \frac{(R + r)\omega}{r}$$ **Ans.**

$$\alpha' = \frac{(R + r)\alpha}{r}$$ **Ans.**



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Ans:

$$\omega' = \frac{(R + r)\omega}{r}$$

$$\alpha' = \frac{(R + r)\alpha}{r}$$

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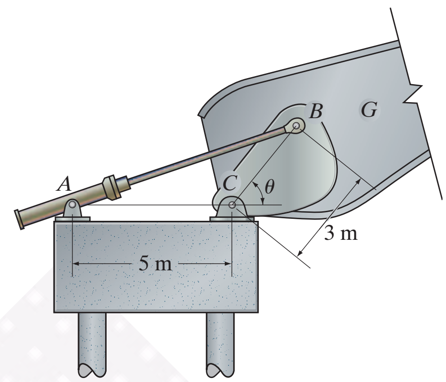
***16–56.**

The bridge girder G of a bascule bridge is raised and lowered using the drive mechanism shown. If the hydraulic cylinder AB shortens at a constant rate of 0.15 m/s, determine the angular velocity of the bridge girder at the instant $\theta = 60°$.



SOLUTION

Position Coordinates: Applying the law of cosines to the geometry shown in Fig. *a*,

$$s^2 = 3^2 + 5^2 - 2(3)(5)\cos\left(180°-\theta\right)$$

$$s^2 = 34 - 30\cos\left(180°-\theta\right)$$

However, $\cos\left(180°-\theta\right) = -\cos\theta$. Thus,

$$s^2 = 34 + 30\cos\theta$$

Time Derivatives: Taking the time derivative,

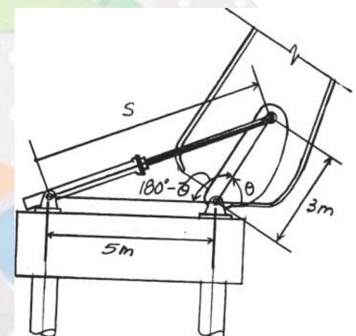
$$2s\dot{s} = 0 + 30\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = -15\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

When $\theta = 60°$, $s = \sqrt{34 + 30\cos 60°} = 7$ m. Also, $\dot{s} = -0.15$ m/s since $\dot{s}$ is directed towards the negative sense of s. Thus, Eq. (1) gives

$$7\left(-0.15\right) = -15\sin 60°\dot{\theta}$$

$$\omega = \dot{\theta} = 0.0808 \text{ rad/s} \qquad \textbf{Ans.}$$



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Ans:
$\omega = 0.0808$ rad/s

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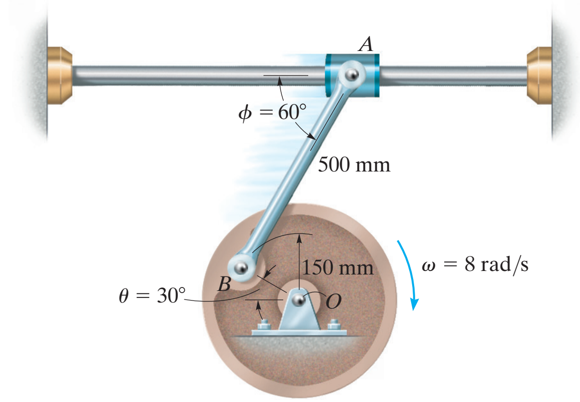
16–57. The wheel is rotating with an angular velocity $\omega = 8$ rad/s. Determine the velocity of the collar A at the instant $\theta = 30°$ and $\phi = 60°$. Also, sketch the location of bar AB when $\theta = 0°$, $30°$, and $60°$ to show its general plane motion.



SOLUTION

Given:

$\theta = 30°$

$\phi = 60°$

$\omega = 8$ rad/s

$r_A = 500$ mm

$r_B = 150$ mm

Guesses $\qquad \omega_{AB} = 1$ rad/s $\qquad v_A = 1$ m/s

Given
$$\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r_B\cos(\theta) \\ r_B\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_A\cos(\phi) \\ r_A\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{AB} \\ v_A \end{pmatrix} = \text{Find}(\omega_{AB}, v_A) \qquad \omega_{AB} = -4.16 \text{ rad/s} \qquad v_A = 2.4 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 2.4$ m/s

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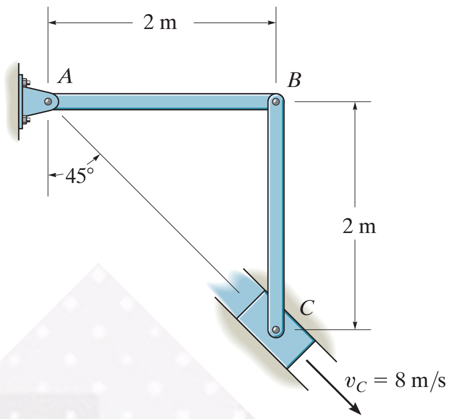
16–58.

The slider block C moves at 8 m/s down the inclined groove. Determine the angular velocities of links AB and BC, at the instant shown.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$\mathbf{v}_B = (-\omega_{AB}\mathbf{k}) \times (2\mathbf{i}) = -2\omega_{AB}\mathbf{j}$$

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

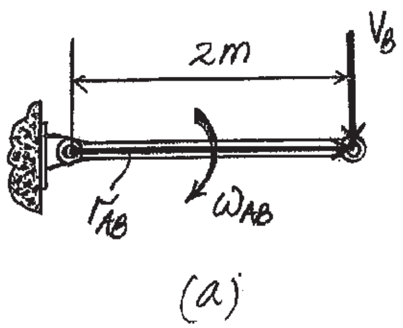
$$-2\omega_{AB}\mathbf{j} = (8 \sin 45°\mathbf{i} - 8 \cos 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (2\mathbf{j})$$

$$-2\omega_{AB}\mathbf{j} = (8 \sin 45° - 2\omega_{BC})\mathbf{i} - 8 \cos 45°\mathbf{j}$$

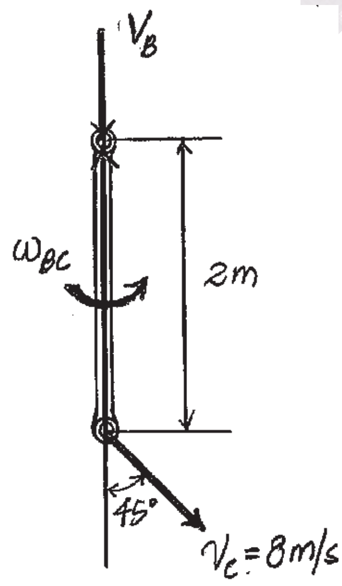
Equating **i** and **j** components,

$$0 = 8 \sin 45° - 2\omega_{BC} \qquad \omega_{BC} = 2.828 \text{ rad/s} = 2.83 \text{ rad/s} \circlearrowleft$$ **Ans.**

$$-2\omega_{AB} = -8 \cos 45° \qquad \omega_{AB} = 2.828 \text{ rad/s} = 2.83 \text{ rad/s} \circlearrowright$$ **Ans.**



(a)



(b)

Ans:
$\omega_{BC} = 2.83 \text{ rad/s} \circlearrowleft$
$\omega_{AB} = 2.83 \text{ rad/s} \circlearrowright$

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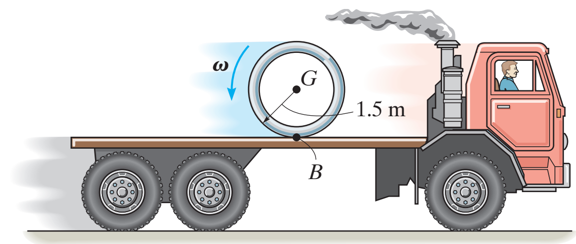
16–59. At the instant shown, the truck is traveling to the right at 3 m/s, while the pipe is rolling counterclockwise at angular $\omega = 8$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



SOLUTION

Given:

$v = 3$ m/s

$\omega = 8$ rad/s

$r = 1.5$ m

$v_G = v + v_{GB}$ $\quad$ $v_G = v - \omega r$ $\quad$ $v_G = -9.00$ m/s **Ans.**

Ans:
$v_G = -9.00$ m/s

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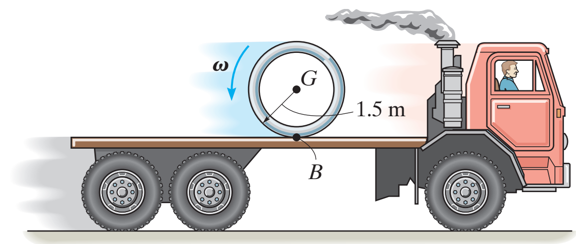
***16–60.** At the instant shown, the truck is traveling to the right at 8 m/s. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary.



SOLUTION

Given:

$v = 8 \text{ m/s}$

$r = 1.5 \text{ m}$

$v_G = v + v_{GB} \qquad 0 = v - \omega r \qquad \omega = \dfrac{v}{r} \qquad \omega = 5.33 \text{ rad/s}$ **Ans.**

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Ans:
$\omega = 5.33 \text{ rad/s}$

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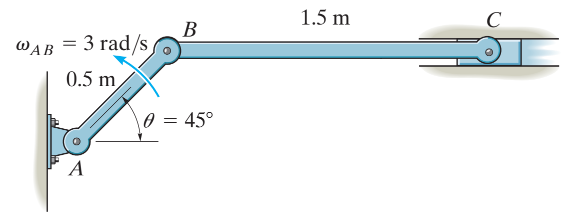
16–61.

The link AB has an angular velocity of 3 rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 45°$. Also, sketch the position of link BC when $\theta = 60°$, 45°, and 30° to show its general plane motion.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. *a*.

$$\begin{aligned}\mathbf{v}_B &= \omega_{AB} \times \mathbf{r}_{AB}\\ &= (3\mathbf{k}) \times (0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j})\\ &= \{-1.0607\mathbf{i} + 1.0607\mathbf{j}\} \text{ m/s}\end{aligned}$$

General Plane Motion. For link BC, refer to Fig. *b*. Applying the relative velocity equation,

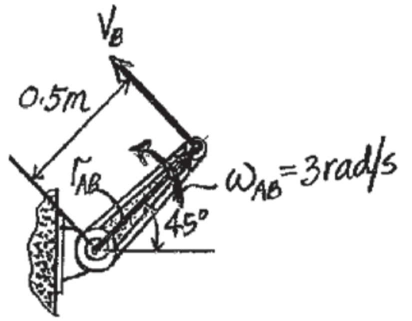
$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}\\ -v_C\mathbf{i} &= (-1.0607\mathbf{i} + 1.0607\mathbf{j}) + (-\omega_{BC}\mathbf{k}) \times (1.5\mathbf{i})\\ -v_C\mathbf{i} &= -1.0607\mathbf{i} + (1.0607 - 1.5\omega_{BC})\mathbf{j}\end{aligned}$$

Equating **i** and **j** components;

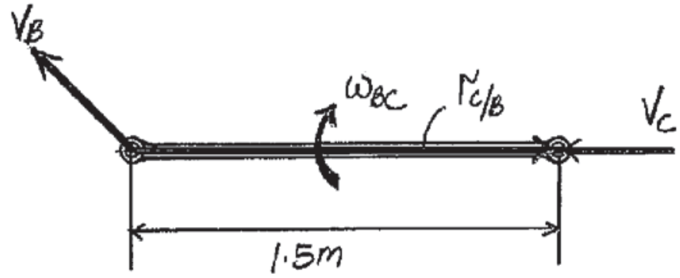
$$-v_C = -1.0607 \qquad v_C = 1.0607 \text{ m/s} = 1.06 \text{ m/s} \qquad \textbf{Ans.}$$

$$0 = 1.0607 - 1.5\omega_{BC} \qquad \omega_{BC} = 0.7071 \text{ rad/s} = 0.707 \text{ rad/s} \qquad \textbf{Ans.}$$

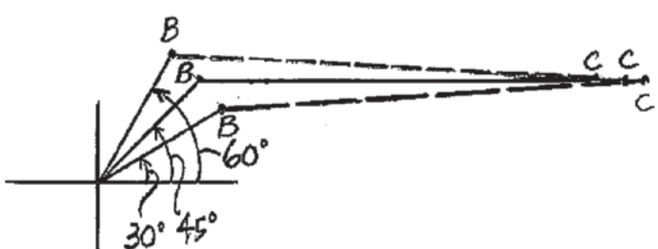
The general plane motion of link BC is described by its orientation when $\theta = 30°$, 45° and 60° shown in Fig. *c*.



(a)



(b)



(c)

Ans:
$v_C = 1.06$ m/s ←
$\omega_{BC} = 0.707$ rad/s ↺

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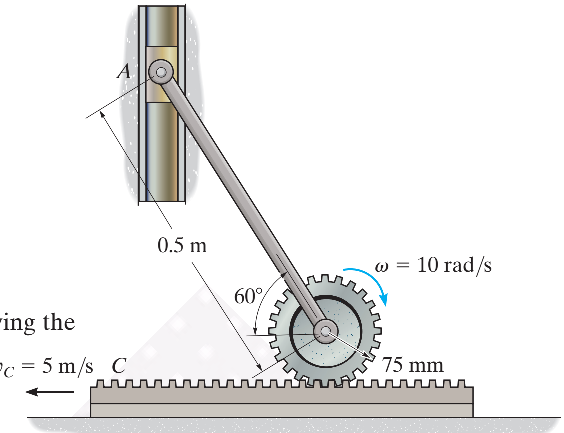
16–62.

If the gear rotates with an angular velocity of $\omega = 10$ rad/s and the gear rack moves at $v_C = 5$ m/s, determine the velocity of the slider block A at the instant shown.



SOLUTION

General Plane Motion: Referring to the diagram shown in Fig. *a* and applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{B/C}$$

$$= -5\mathbf{i} + (-10\mathbf{k}) \times (0.075\mathbf{j})$$

$$= [-4.25\mathbf{i}] \text{ m/s}$$

Then, applying the relative velocity equation to link AB shown in Fig. b,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{j} = -4.25\mathbf{i} + (-\omega_{AB}\mathbf{k}) \times (-0.5\cos 60°\,\mathbf{i} + 0.5\sin 60°\,\mathbf{j})$$

$$v_A\mathbf{j} = (0.4330\omega_{AB} - 4.25)\mathbf{i} + 0.25\omega_{AB}\mathbf{j}$$

Equating the **i** and **j** components, yields

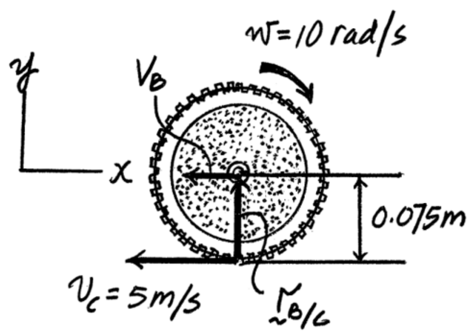
$$0 = 0.4330\omega_{AB} - 4.25 \qquad \textbf{(1)}$$

$$v_A = 0.25\omega_{AB} \qquad \textbf{(2)}$$

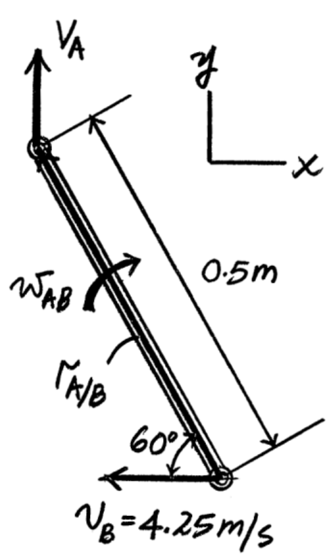
Solving Eqs. (1) and (2) yields

$$\omega_{AB} = 9.815 \text{ rad/s}$$

$$v_A = 2.45 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$v_A = 2.45$ m/s $\uparrow$

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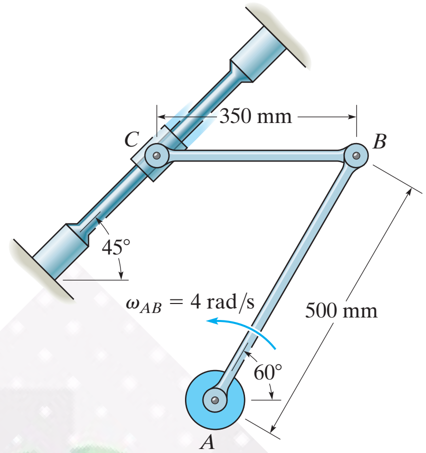
16–63.

Knowing that angular velocity of link AB is $\omega_{AB} = 4$ rad/s, determine the velocity of the collar at C and the angular velocity of link CB at the instant shown. Link CB is horizontal at this instant.



SOLUTION

$$v_B = \omega_{AB} r_{AB}$$
$$= 4(0.5) = 2 \text{ m/s}$$

$\mathbf{v}_B = \{-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\}$ m/s $\qquad \mathbf{v}_C = -v_C\cos 45°\mathbf{i} - v_C\sin 45°\mathbf{j}$

$\omega = \omega_{BC}\mathbf{k} \qquad \mathbf{r}_{C/B} = \{-0.35\mathbf{i}\}$ m

$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$-v_C\cos 45°\mathbf{i} - v_C\sin 45°\mathbf{j} = (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.35\mathbf{i})$$

$$-v_C\cos 45°\mathbf{i} - v_C\sin 45°\mathbf{j} = -2\cos 30°\mathbf{i} + (2\sin 30° - 0.35\omega_{BC})\mathbf{j}$$

Equating the **i** and **j** components yields:

$-v_C\cos 45° = -2\cos 30° \qquad v_C = 2.45$ m/s **Ans.**

$-2.45\sin 45° = 2\sin 30° - 0.35\omega_{BC} \qquad \omega_{BC} = 7.81$ rad/s **Ans.**

Ans:
$v_C = 2.45$ m/s
$\omega_{BC} = 7.81$ rad/s

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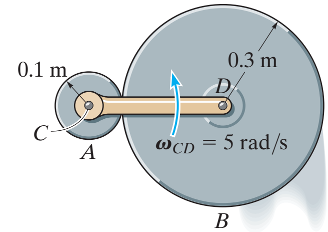
***16–64.** The cylinder B rolls on the *fixed cylinder* A without slipping. If the connected bar CD is rotating with an angular velocity of $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder B.



SOLUTION

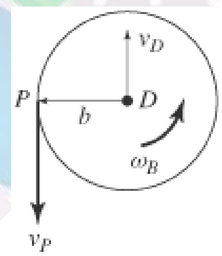
Given:

$$\omega_{CD} = 5 \text{ rad/s}$$

$$a = 0.1 \text{ m}$$

$$b = 0.3 \text{ m}$$

$$v_D = \omega_{CD}(a+b)$$



$$\omega_B = \frac{v_D}{b} \qquad \omega_B = 6.67 \text{ rad/s} \quad \textbf{Ans.}$$

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Ans:
$\omega_B = 6.67$ rad/s

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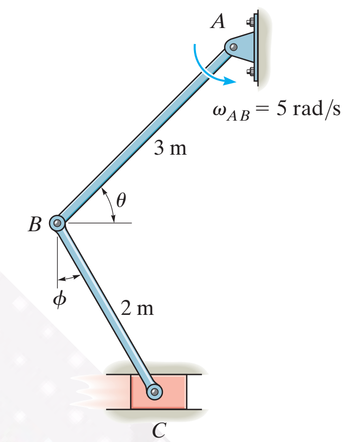
16–65.

The angular velocity of link AB is $\omega_{AB} = 5$ rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 45°$ and $\phi = 30°$. Also, sketch the position of link CB when $\theta = 45°, 60°$, and $75°$ to show its general plane motion.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. *a*.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$= (5\mathbf{k}) \times (-3\cos 45°\mathbf{i} - 3\sin 45°\mathbf{j})$$

$$= \left\{\frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}\right\} \text{ m/s}$$

General Plane Motion. For link BC, refer to Fig. *b*. Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$v_C\,\mathbf{i} = \left(\frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}\right) + (\omega_{BC}\mathbf{k}) \times (2\sin 30°\,\mathbf{i} - 2\cos 30°\mathbf{j})$$

$$v_C\,\mathbf{i} = \left(\frac{15\sqrt{2}}{2} + \sqrt{3}\,\omega_{BC}\right)\mathbf{i} + \left(\omega_{BC} - \frac{15\sqrt{2}}{2}\right)\mathbf{j}$$

Equating $\mathbf{j}$ components,

$$0 = \omega_{BC} - \frac{15\sqrt{2}}{2};\ \omega_{BC} = \frac{15\sqrt{2}}{2} \text{ rad/s} = 10.6 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Then, equating $\mathbf{i}$ components,

$$v_C = \frac{15\sqrt{2}}{2} + \sqrt{3}\left(\frac{15\sqrt{2}}{2}\right) = 28.98 \text{ m/s} = 29.0 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

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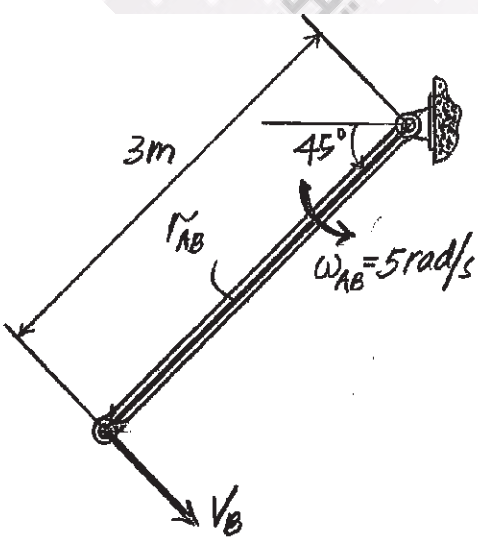
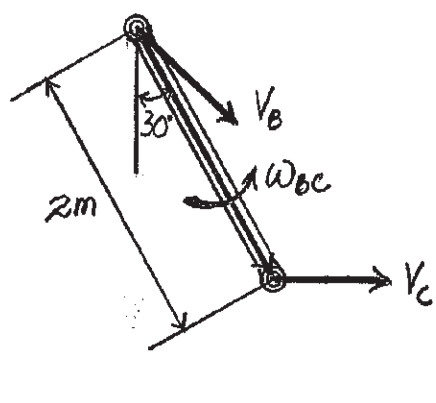


(a)



(b)

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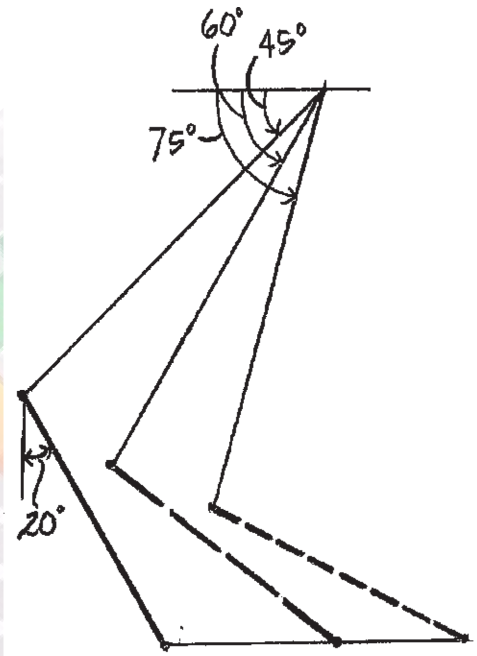
16–65. Continued

The general plane motion of link BC is described by its orientation when $\theta = 45°$, $60°$ and $75°$ shown in Fig. c



(c)

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Ans:
$\omega_{BC} = 10.6$ rad/s ↺
$v_C = 29.0$ m/s →

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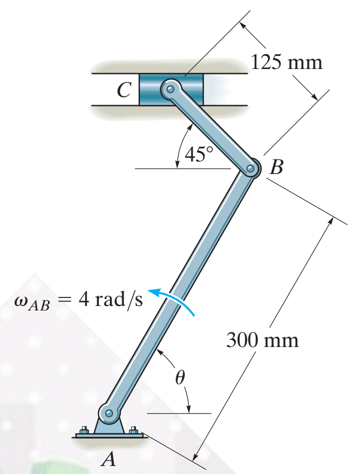
16–66. The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant $\theta = 45°$, if link AB is rotating at 4 rad/s.



SOLUTION

Given:

$\theta = 60°$

$\phi = 45°$

$\omega_{AB} = 4 \text{ rad/s}$

$a = 300 \text{ mm}$

$b = 125 \text{ mm}$

Guesses $\quad \omega_{BC} = 1 \text{ rad/s} \quad v_C = 1 \text{ m/s}$

Given $$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 6.79 \text{ rad/s} \qquad v_C = -1.64 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_C = -1.64$ m/s

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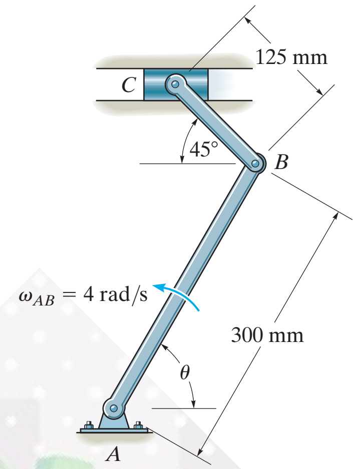
16–67. Determine the velocity of the slider block at C at the instant $\theta = 45°$, if link AB is rotating at 4 rad/s.



SOLUTION

Given:

$\theta = 45°$

$\phi = 45°$

$\omega_{AB} = 4$ rad/s

$a = 300$ mm

$b = 125$ mm

Guesses $\quad \omega_{BC} = 1$ rad/s $\quad v_C = 1$ m/s

Given $\quad \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 1.00$ rad/s $\qquad v_C = -1.70$ m/s $\qquad$ **Ans.**

Ans:
$v_C = -1.70$ m/s

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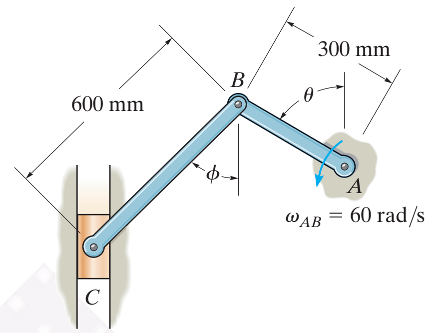
***16–68.**

Rod AB is rotating with an angular velocity of $\omega_{AB} = 60$ rad/s. Determine the velocity of the slider C at the instant $\theta = 60°$ and $\phi = 45°$. Also, sketch the position of bar BC when $\theta = 30°$, $60°$ and $90°$ to show its general plane motion.



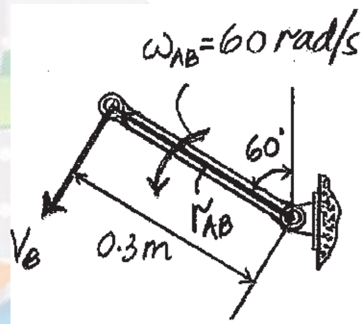
SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{V}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$= (60\mathbf{k}) \times (-0.3 \sin 60°\mathbf{i} + 0.3 \cos 60°\mathbf{j})$$

$$= \{-9\mathbf{i} - 9\sqrt{3}\mathbf{j}\} \text{ m/s}$$



(a)

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{V}_C = \mathbf{V}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$-v_C\mathbf{j} = (-9\mathbf{i} - 9\sqrt{3}\,\mathbf{j}\,) + (\omega_{BC}\mathbf{k}) \times (-0.6 \sin 45°\mathbf{i} - 0.6 \cos 45°\mathbf{j})$$

$$-v_C\mathbf{j} = \left(0.3\sqrt{2}\omega_{BC} - 9\right)\mathbf{i} + \left(-0.3\sqrt{2}\omega_{BC} - 9\sqrt{3}\right)\mathbf{j}$$

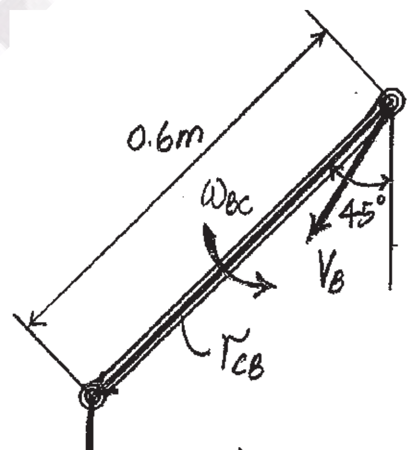
Equating **i** components,

$$0 = 0.3\sqrt{2}\omega_{BC} - 9; \qquad \omega_{BC} = 15\sqrt{2} \text{ rad/s} = 21.2 \text{ rad/s} \circlearrowleft$$

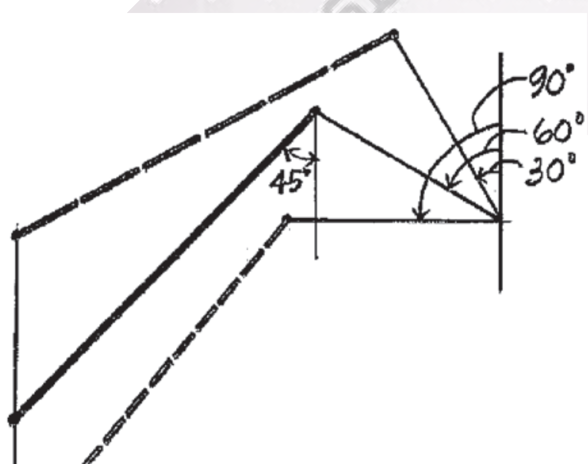
Then, equating **j** components,

$$-v_C = \left(-0.3\sqrt{2}\right)\left(15\sqrt{2}\right) - 9\sqrt{3}; \quad v_C = 24.59 \text{ m/s} = 24.6 \text{ m/s}\downarrow \quad \textbf{Ans.}$$

The general plane motion of link BC is described by its orientation when $\theta = 30°$, $60°$ and $90°$ shown in Fig. c.



(b)



(c)

Ans:
$v_C = 24.6$ m/s$\downarrow$

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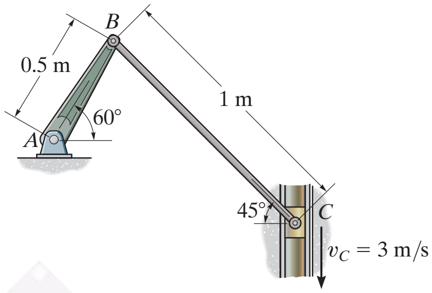
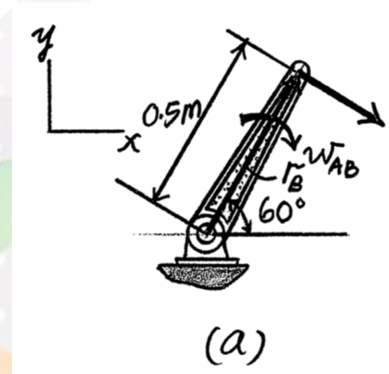
16–69.

If the slider block C is moving at $v_C = 3$ m/s, determine the angular velocity of BC and the crank AB at the instant shown.



SOLUTION

Rotation About a Fixed Axis: Referring to Fig. a,



$$v_B = \omega_{AB} \times \mathbf{r}_B$$

$$= (-\omega_{AB}\,\mathbf{k}) \times (0.5\cos 60^\circ\,\mathbf{i} + 0.5\sin 60^\circ\,\mathbf{j})$$

$$= 0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j}$$

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of link BC shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j} = -3\mathbf{j} + (-\omega_{BC}\mathbf{k}) \times (-1\cos 45^\circ\,\mathbf{i} + 1\sin 45^\circ\,\mathbf{j})$$

$$0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j} = 0.7071\omega_{BC}\mathbf{i} + (0.7071\omega_{BC} - 3)\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields,

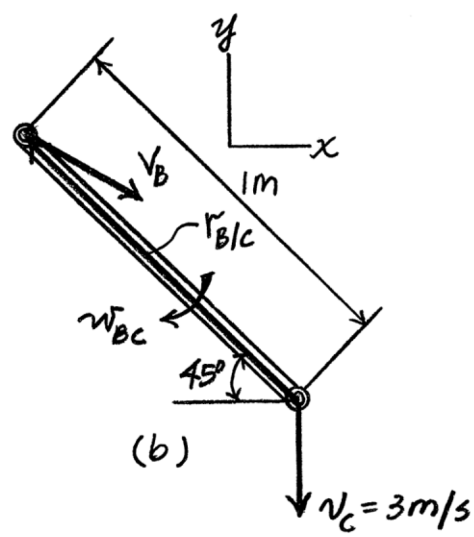
$$0.4330\omega_{AB} = 0.7071\omega_{BC}$$

$$-0.25\omega_{AB} = 0.7071\omega_{BC} - 3$$

Solving,

$$\omega_{BC} = 2.69 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\omega_{AB} = 4.39 \text{ rad/s} \qquad \textbf{Ans.}$$



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Ans:
$\omega_{BC} = 2.69$ rad/s
$\omega_{AB} = 4.39$ rad/s

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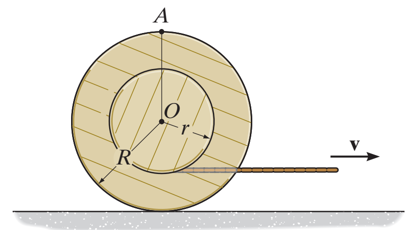
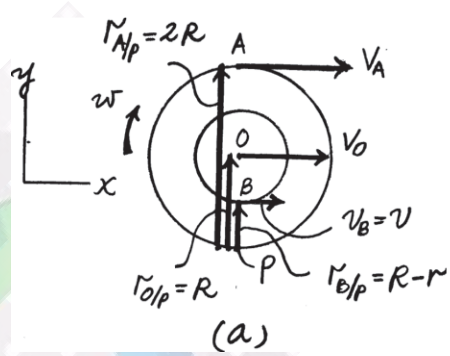
16–70.

Determine the velocity of the center O of the spool when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



SOLUTION

Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. a.



General Plane Motion: Applying the relative velocity equation and referring to Fig. a,

$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

$$\mathbf{v}_O = \mathbf{v}_P + \omega \times \mathbf{r}_{O/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times R\mathbf{j}$$

$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow \qquad \textbf{Ans.}$$

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Ans:

$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow$$

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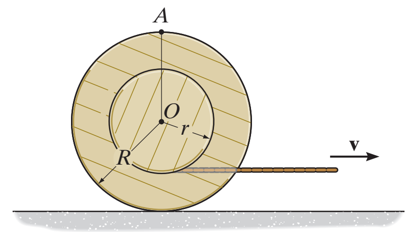
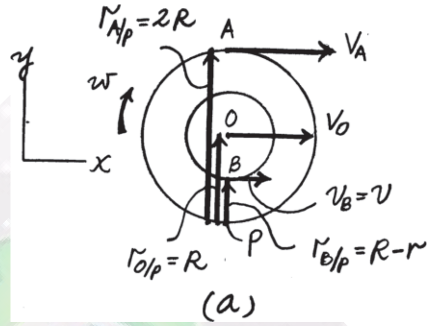
16–71.

Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



SOLUTION

Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.



General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

$$\mathbf{v}_B = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

$$\mathbf{v}_A = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{A/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times 2R\mathbf{j}$$

$$= \left[\left(\frac{2R}{R - r}\right)v\right]\mathbf{i}$$

Thus,

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow$$ **Ans.**

Ans:

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow$$

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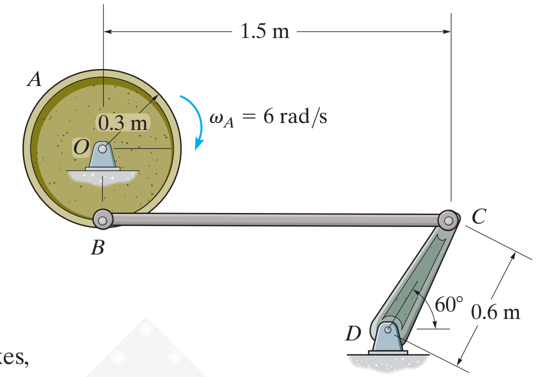
***16–72.**

If the flywheel is rotating with an angular velocity of $\omega_A = 6$ rad/s, determine the angular velocity of rod BC at the instant shown.



SOLUTION

Rotation About a Fixed Axis: Flywheel A and rod CD rotate about fixed axes, Figs. a and b. Thus, the velocity of points B and C can be determined from

$$v_B = \omega_A \times \mathbf{r}_B = (-6\mathbf{k}) \times (-0.3\mathbf{j}) = [-1.8\mathbf{i}] \text{ m/s}$$

$$v_C = \omega_{CD} \times \mathbf{r}_C = (\omega_{CD}\mathbf{k}) \times (0.6\cos 60°\,\mathbf{i} + 0.6\sin 60°\,\mathbf{j})$$

$$= -0.5196\omega_{CD}\mathbf{i} + 0.3\omega_{CD}\mathbf{j}$$

General Plane Motion: By referring to the kinematic diagram of link BC shown in Fig. c and applying the relative velocity equation, we have

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-1.8\mathbf{i} = -0.5196\omega_{CD}\mathbf{i} + 0.3\omega_{CD}\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (-1.5\mathbf{i})$$

$$-1.8\mathbf{i} = -0.5196\omega_{CD}\mathbf{i} + (0.3\omega_{CD} - 1.5\omega_{BC})\mathbf{j}$$

Equating the **i** and **j** components

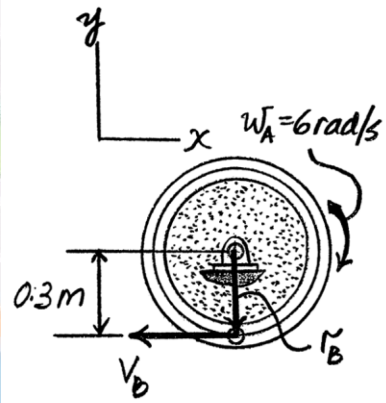
$$-1.8 = -0.5196\omega_{CD}$$

$$0 = 0.3\omega_{CD} - 1.5\omega_{BC}$$

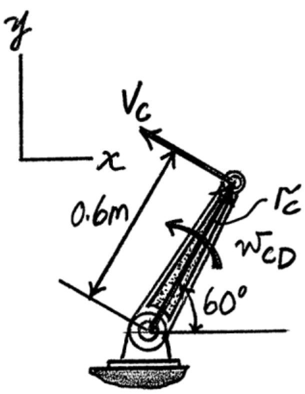
Solving,

$$\omega_{CD} = 3.46 \text{ rad/s}$$

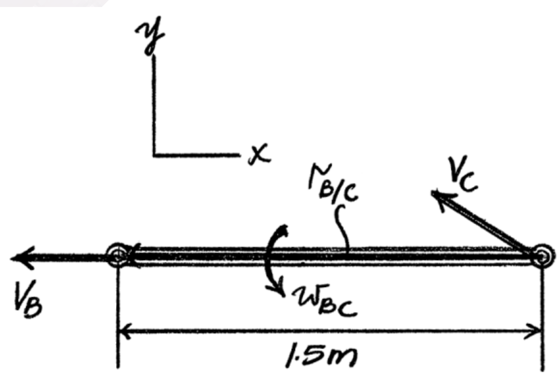
$$\omega_{BC} = 0.693 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$\omega_{BC} = 0.693$ rad/s

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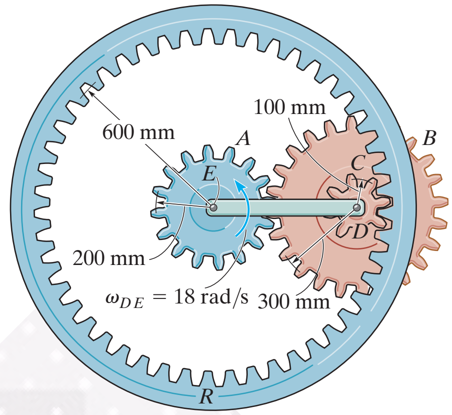
16–73.

The epicyclic gear train consists of the sun gear A which is in mesh with the planet gear B. This gear has an inner hub C which is fixed to B and in mesh with the fixed ring gear R. If the connecting link DE pinned to B and C is rotating at $\omega_{DE} = 18$ rad/s about the pin at E, determine the angular velocities of the planet and sun gears.



SOLUTION

$v_D = r_{DE}\,\omega_{DE} = (0.5)(18) = 9 \text{ m/s} \uparrow$

The velocity of the contact point P with the ring is zero.

$\mathbf{v}_D = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{D/P}$

$9\mathbf{j} = 0 + (-\omega_B\,\mathbf{k}) \times (-0.1\mathbf{i})$

$\omega_B = 90 \text{ rad/s} \;\circlearrowright$ **Ans.**

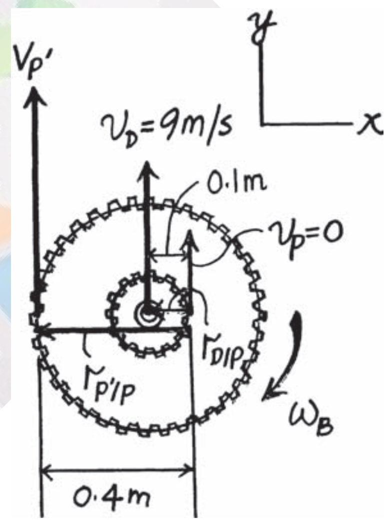
Let P' be the contact point between A and B.

$\mathbf{v}_{P'} = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{P'/P}$

$v_{P'}\,\mathbf{j} = \mathbf{0} + (-90\mathbf{k}) \times (-0.4\mathbf{i})$

$v_{P'} = 36 \text{ m/s} \uparrow$

$\omega_A = \dfrac{v_{P'}}{r_A} = \dfrac{36}{0.2} = 180 \text{ rad/s} \;\circlearrowleft$ **Ans.**



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Ans:
$\omega_B = 90 \text{ rad/s} \;\circlearrowright$
$\omega_A = 180 \text{ rad/s} \;\circlearrowleft$

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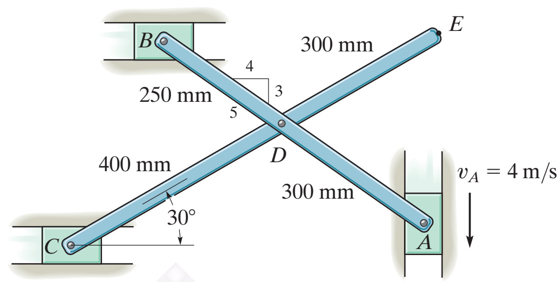
16–74.

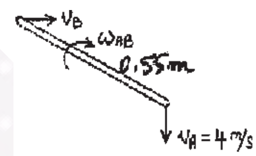
If the slider block A is moving downward at $v_A = 4\text{ m/s}$, determine the velocities of blocks B and C at the instant shown.



SOLUTION

$$v_B = v_A + v_{B/A}$$

$$\underset{\rightarrow}{v_B} = 4\downarrow + \omega_{AB}(0.55)$$



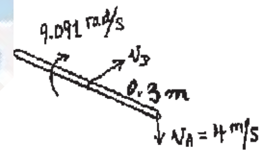
$$(\overset{+}{\rightarrow}) \qquad v_B = 0 + \omega_{AB}(0.55)\left(\frac{3}{5}\right)$$

$$(+\uparrow) \qquad 0 = -4 + \omega_{AB}(0.55)\left(\frac{4}{5}\right)$$

Solving,

$$\omega_{AB} = 9.091\text{ rad/s}$$

$$v_B = 3.00\text{ m/s} \qquad \textbf{Ans.}$$

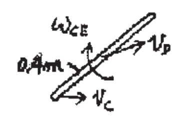


$$v_D = v_A + v_{D/A}$$

$$\mathbf{v}_D = \underset{\downarrow}{4} + \underset{{}_4\nearrow^3_5}{[(0.3)(9.091) = 2.727]}$$

$$v_C = v_D + v_{C/D}$$

$$\underset{\rightarrow}{v_C} = \underset{\downarrow}{4} + \underset{{}^5\nearrow^3_4}{2.727} + \underset{\nwarrow 30^\circ}{\omega_{CE}(0.4)}$$



$$(\overset{+}{\rightarrow}) \qquad v_C = 0 + 2.727\left(\frac{3}{5}\right) - \omega_{CE}(0.4)(\sin 30^\circ)$$

$$(+\uparrow) \qquad 0 = -4 + 2.727\left(\frac{4}{5}\right) + \omega_{CE}(0.4)(\cos 30^\circ)$$

$$\omega_{CE} = 5.249\text{ rad/s}$$

$$v_C = 0.587\text{ m/s} \qquad \textbf{Ans.}$$

Also:

$$v_B = v_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -4\mathbf{j} + (-\omega_{AB}\mathbf{k}) \times \left\{\frac{-4}{5}(0.55)\mathbf{i} + \frac{3}{5}(0.55)\mathbf{j}\right\}$$

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16–74. Continued

$$v_B = \omega_{AB}(0.33)$$

$$0 = -4 + 0.44\omega_{AB}$$

$$\omega_{AB} = 9.091 \text{ rad/s}$$

$$v_B = 3.00 \text{ m/s}$$ **Ans.**

$$\mathbf{v}_D = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$v_D = -4\mathbf{j} + (-9.091\mathbf{k}) \times \left\{\frac{-4}{5}(0.3)\mathbf{i} + \frac{3}{5}(0.3)\mathbf{j}\right\}$$

$$v_D = \{1.636\mathbf{i} - 1.818\mathbf{j}\} \text{ m/s}$$

$$v_C = v_D + \omega_{CE} \times \mathbf{r}_{C/D}$$

$$v_C\mathbf{i} = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-\omega_{CE}\mathbf{k}) \times (-0.4\cos 30°\mathbf{i} - 0.4\sin 30°\mathbf{j})$$

$$v_C = 1.636 - 0.2\omega_{CE}$$

$$0 = -1.818 - 0.346\omega_{CE}$$

$$\omega_{CE} = 5.25 \text{ rad/s}$$

$$v_C = 0.587 \text{ m/s}$$ **Ans.**

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Ans:
$v_B = 3.00$ m/s
$v_C = 0.587$ m/s
$v_B = 3.00$ m/s
$v_C = 0.587$ m/s

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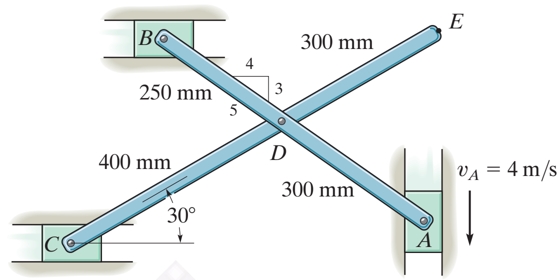
16–75.

If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocity of point E at the instant shown.

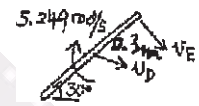


SOLUTION

See solution to Prob. 16–89.

$$v_E = v_D + v_{E/D}$$

$$\overrightarrow{v_E} = 4\downarrow + \underset{4\,\nearrow^{3}_{5}}{2.727} + \underset{\searrow 30^\circ}{(5.249)(0.3)}$$



$$(\overset{+}{\rightarrow}) \qquad (v_E)_x = 0 + 2.727\left(\frac{3}{5}\right) + 5.249(0.3)(\sin 30^\circ)$$

$$(+\downarrow) \qquad (v_E)_y = 4 - 2.727\left(\frac{4}{5}\right) + 5.249(0.3)(\cos 30^\circ)$$

$$(v_E)_x = 2.424 \text{ m/s} \rightarrow$$

$$(v_E)_y = 3.182 \text{ m/s} \downarrow$$

$$\mathbf{v}_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{3.182}{2.424}\right) = 52.7^\circ \qquad \textbf{Ans.}$$

Also:

See solution to Prob. 16–89.

$$\mathbf{v}_E = \mathbf{v}_D + \omega_{CE} \times \mathbf{r}_{E/D}$$

$$\mathbf{v}_E = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-5.25\mathbf{k}) \times \{\cos 30^\circ(0.3)\mathbf{i} - 0.4 \sin 30^\circ(0.3)\mathbf{j}\}$$

$$\mathbf{v}_E = \{2.424\mathbf{i} - 3.182\mathbf{j}\} \text{ m/s}$$

$$v_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{3.182}{2.424}\right) = 52.7^\circ \qquad \textbf{Ans.}$$

Ans:
$v_E = 4.00$ m/s
$\theta = 52.7^\circ \measuredangle$

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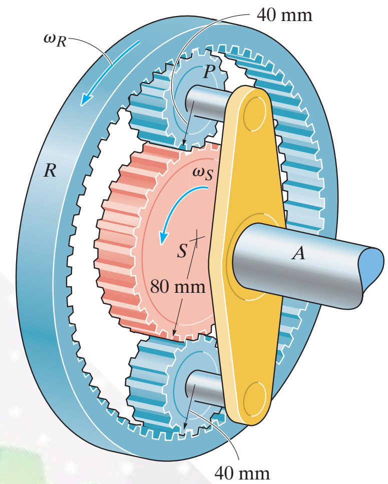
***16–76.**

The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



SOLUTION

$v_A = 5(80) = 400 \text{ mm/s} \leftarrow$

$v_B = 0$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$0 = -400\mathbf{i} + (\omega_p \mathbf{k}) \times (80\mathbf{j})$

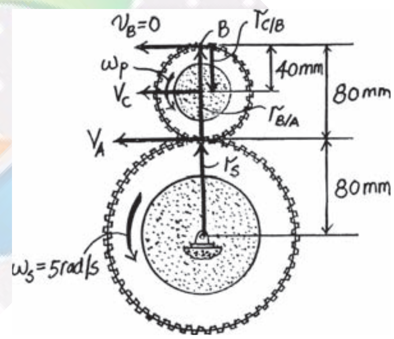
$0 = -400\mathbf{i} - 80\omega_p \mathbf{i}$

$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$ **Ans.**

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$

$\omega_A = \dfrac{200}{120} = 1.67 \text{ rad/s}$ **Ans.**



Ans:
$\omega_P = 5$ rad/s
$\omega_A = 1.67$ rad/s

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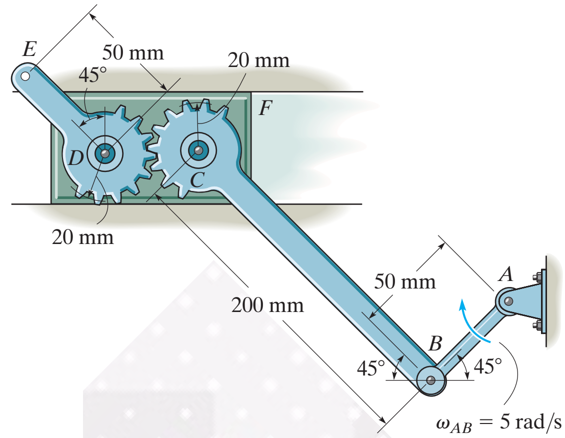
16–77.

The mechanism is used on a machine for the manufacturing of a wire product. Because of the rotational motion of link AB and the sliding of block F, the segmental gear lever DE undergoes general plane motion. If AB is rotating at $\omega_{AB} = 5$ rad/s, determine the velocity of point E at the instant shown.



SOLUTION

$v_B = \omega_{AB} r_{AB} = 5(50) = 250$ mm/s 45° ⦣

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$v_C = 250 + \omega_{BC}(200)$

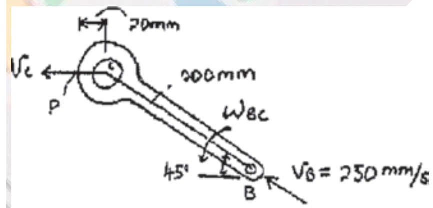
$\leftarrow \quad 45^\circ \quad 45^\circ$

$(+\uparrow) \quad 0 = 250 \sin 45^\circ - \omega_{BC}(200) \sin 45^\circ$

$(\overset{+}{\leftarrow}) \quad v_C = 250 \cos 45^\circ + \omega_{BC}(200) \cos 45^\circ$

Solving,

$v_C = 353.6$ mm/s; $\qquad \omega_{BC} = 1.25$ rad/s



$\mathbf{v}_p = \mathbf{v}_C + \mathbf{v}_{p/C}$

$\underset{\rightarrow}{v_p} = \underset{\leftarrow}{353.6} + [(1.25)\underset{\downarrow}{(20)} = 25]$

$\mathbf{v}_D = \mathbf{v}_p + \mathbf{v}_{D/p}$

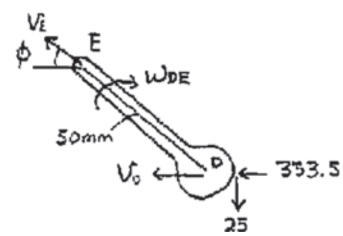
$\underset{\leftarrow}{v_D} = (\underset{\leftarrow}{353.6} + \underset{\downarrow}{25}) + \underset{\uparrow}{20\omega_{DE}}$

$(\overset{+}{\leftarrow}) \quad v_D = 353.6 + 0 + 0$

$(+\downarrow) \quad 0 = 0 + (1.25)(20) - \omega_{DE}(20)$

Solving,

$v_D = 353.6$ mm/s; $\qquad \omega_{DE} = 1.25$ rad/s



$\mathbf{v}_E = \mathbf{v}_D + \mathbf{v}_{E/D}$

$v_E = 353.6 + 1.25(50)$

$\phi \quad \leftarrow \quad \angle 45^\circ$

$(\overset{+}{\leftarrow}) \quad v_E \cos \phi = 353.6 - 1.25(50) \cos 45^\circ$

$(+\uparrow) \quad v_E \sin \phi = 0 + 1.25(50) \sin 45^\circ$

Solving,

$v_E = 312$ mm/s **Ans.**

$\phi = 8.13^\circ$ **Ans.**

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16–77. Continued

Also;

$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = (-5\mathbf{k}) \times (-0.05 \cos 45°\mathbf{i} - 0.05 \sin 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.2 \cos 45°\mathbf{i} + 0.2 \sin 45°\mathbf{j})$

$-v_C = -0.1768 - 0.1414\omega_{BC}$

$0 = 0.1768 - 0.1414\omega_{BC}$

$\omega_{BC} = 1.25 \text{ rad/s}, \qquad v_C = 0.254 \text{ m/s}$

$\mathbf{v}_p = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{p/C}$

$\mathbf{v}_D = \mathbf{v}_p + \omega_{DE} \times \mathbf{r}_{D/p}$

$\mathbf{v}_D = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{p/C} + \omega_{DE} \times \mathbf{r}_{D/p}$

$v_D\mathbf{i} = -0.354\mathbf{i} + (1.25\mathbf{k}) \times (-0.02\mathbf{i}) + (\omega_{DE}\mathbf{k}) \times (-0.02\mathbf{i})$

$v_D = -0.354$

$0 = -0.025 - \omega_{DE}(0.02)$

$v_D = 0.354 \text{ m/s}, \qquad \omega_{DE} = 1.25 \text{ rad/s}$

$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$

$(v_E)_x\mathbf{i} + (v_E)_y\mathbf{j} = -0.354\mathbf{i} + (-1.25\mathbf{k}) \times (-0.05 \cos 45°\mathbf{i} + 0.05 \sin 45°\mathbf{j})$

$(v_E)_x = -0.354 + 0.0442 = -0.3098$

$(v_E)_y = 0.0442$

$v_E = \sqrt{(-0.3098)^2 + (0.0442)^2} = 312 \text{ mm/s}$ **Ans.**

$\phi = \tan^{-1}\left(\dfrac{0.0442}{0.3098}\right) = 8.13°$ **Ans.**

Ans:
$v_E = 312 \text{ mm/s}$
$\phi = 8.13°$
$v_E = 312 \text{ mm/s}$
$\phi = 8.13°$

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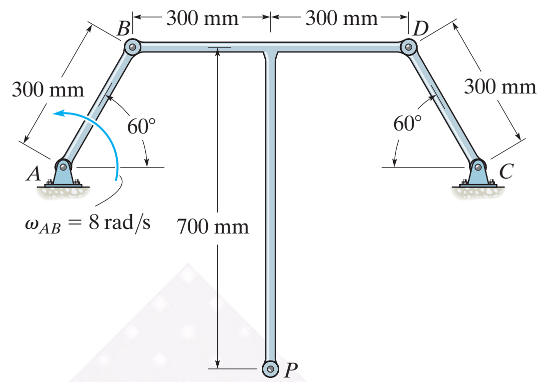
16–78.

The similar links AB and CD rotate about the fixed pins at A and C. If AB has an angular velocity $\omega_{AB} = 8$ rad/s, determine the angular velocity of BDP and the velocity of point P.



SOLUTION

$$\mathbf{v}_D = \mathbf{v}_B + \omega \times \mathbf{r}_{D/B}$$

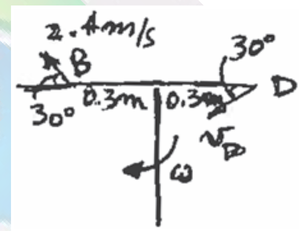
$$-v_D \cos 30°\mathbf{i} - v_D \sin 30°\mathbf{j} = -2.4 \cos 30°\mathbf{i} + 2.4 \sin 30°\mathbf{j} + (\omega\mathbf{k}) \times (0.6\mathbf{i})$$

$$-v_D \cos 30° = -2.4 \cos 30°$$

$$-v_D \sin 30° = 2.4 \sin 30° + 0.6\omega$$

$$v_D = 2.4 \text{ m/s}$$

$$\omega = -4 \text{ rad/s}$$ **Ans.**



$$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$$

$$\mathbf{v}_P = -2.4 \cos 30°\mathbf{i} + 2.4 \sin 30°\mathbf{j} + (-4\mathbf{k}) \times (0.3\mathbf{i} - 0.7\mathbf{j})$$

$$(v_P)_x = -4.88 \text{ m/s}$$

$$(v_P)_y = 0$$

$$v_P = 4.88 \text{ m/s} \leftarrow$$ **Ans.**

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Ans:
$v_P = 4.88$ m/s $\leftarrow$

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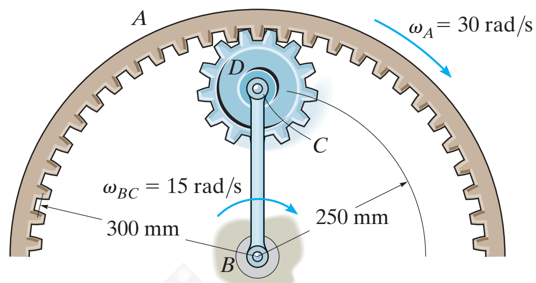
16–79.

If the ring gear A rotates clockwise with an angular velocity of $\omega_A = 30$ rad/s, while link BC rotates clockwise with an angular velocity of $\omega_{BC} = 15$ rad/s, determine the angular velocity of gear D.



SOLUTION

Rotation About A Fixed Axis. The magnitudes of the velocity of Point E on the rim and center C of gear D are

$$v_E = \omega_A r_A = 30(0.3) = 9 \text{ m/s}$$

$$v_C = \omega_{BC} r_{BC} = 15(0.25) = 3.75 \text{ m/s}$$

General Plane Motion. Applying the relative velocity equation by referring to Fig. *a*,

$$\mathbf{v}_E = \mathbf{v}_C + \boldsymbol{\omega}_D \times \mathbf{r}_{E/C}$$

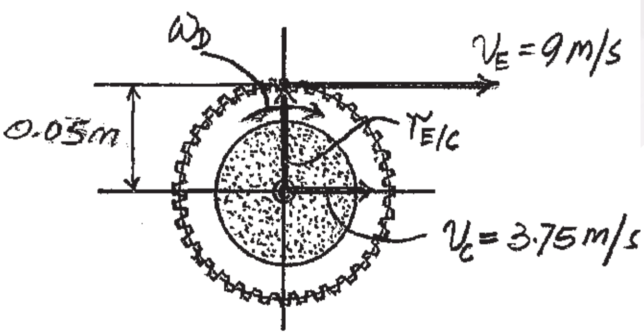
$$9\mathbf{i} = 3.75\mathbf{i} + (-\omega_D \mathbf{k}) \times (0.05\mathbf{j})$$

$$9\mathbf{i} = (3.75 + 0.05\omega_D)\mathbf{i}$$

Equating **i** component,

$$9 = 3.75 + 0.05\omega_D$$

$$\omega_D = 105 \text{ rad/s} \curvearrowright$$ **Ans.**



(a)

Ans:
$\omega_D = 105$ rad/s $\curvearrowright$

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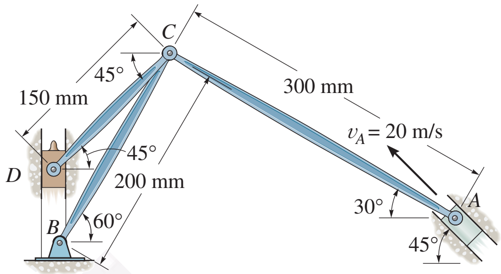
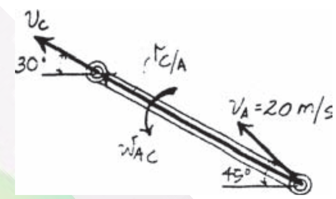
***16–80.**

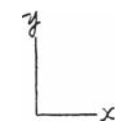
The mechanism shown is used in a riveting machine. It consists of a driving piston A, three links, and a riveter which is attached to the slider block D. Determine the velocity of D at the instant shown, when the piston at A is traveling at $v_A = 20$ m/s.



SOLUTION

Kinematic Diagram: Since link BC is rotating about fixed point B, then $\mathbf{v}_C$ is always directed perpendicular to link BC. At the instant shown. $\mathbf{v}_C = -v_C \cos 30°\mathbf{i} + v_C \sin 30°\mathbf{j} = -0.8660\,v_C\mathbf{i} + 0.500\,v_C\,\mathbf{j}$. Also, block D is moving towards the *negative y* axis due to the constraint of the guide. Then. $\mathbf{v}_D = -v_D\mathbf{j}$.





Velocity Equation: Here, $\mathbf{v}_A = \{-20 \cos 45°\mathbf{i} + 20 \sin 45°\mathbf{j}\}$m/s $= \{-14.14\mathbf{i} + 14.14\mathbf{j}\}$ m/s and $\mathbf{r}_{C/A} = \{-0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j}\}$m $= \{-0.2598\mathbf{i} + 0.150\mathbf{j}\}$ m. Applying Eq. 16–16 to link AC, we have

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A}$$

$$-0.8660\,v_C\mathbf{i} + 0.500\,v_C\mathbf{j} = -14.14\mathbf{i} + 14.14\mathbf{j} + (\omega_{AC}\mathbf{k}) \times (-0.2598\mathbf{i} + 0.150\mathbf{j})$$

$$-0.8660\,v_C\mathbf{i} + 0.500\,v_C\mathbf{j} = -(14.14 + 0.150\,\omega_{AC})\mathbf{i} + (14.14 - 0.2598\omega_{AC})\mathbf{j}$$

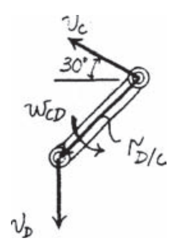
Equating **i** and **j** components gives

$$-0.8660\,v_C = -(14.14 + 0.150\,\omega_{AC}) \qquad \textbf{[1]}$$

$$0.500\,v_C = 14.14 - 0.2598\,\omega_{AC} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\omega_{AC} = 17.25 \text{ rad/s} \qquad v_C = 19.32 \text{ m/s}$$



Thus, $\mathbf{v}_C = \{-19.32 \cos 30°\mathbf{i} + 19.32 \sin 30°\mathbf{j}\}$ m/s $= \{-16.73\mathbf{i} + 9.659\mathbf{j}\}$ m/s and $\mathbf{r}_{D/C} = \{-0.15 \cos 45°\mathbf{i} - 0.15 \sin 45°\mathbf{j}\}$m $= \{-0.1061\mathbf{i} - 0.1061\mathbf{j}\}$ m. Applying Eq. 16–16 to link CD, we have

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$-v_D\mathbf{j} = -16.73\mathbf{i} + 9.659\mathbf{j} + (\omega_{CD}\mathbf{k}) \times (-0.1061\mathbf{i} - 0.1061\mathbf{j})$$

$$-v_D\mathbf{j} = (0.1061\omega_{CD} - 16.73)\mathbf{i} + (9.659 - 0.1061\omega_{CD})\mathbf{j}$$

Equating **i** and **j** components gives

$$0 = 0.1061\omega_{CD} - 16.73 \qquad \textbf{[3]}$$

$$-v_D = 9.659 - 0.1061\,\omega_{CD} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

$$\omega_{CD} = 157.74 \text{ rad/s}$$

$$v_D = 7.07 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_D = 7.07$ m/s

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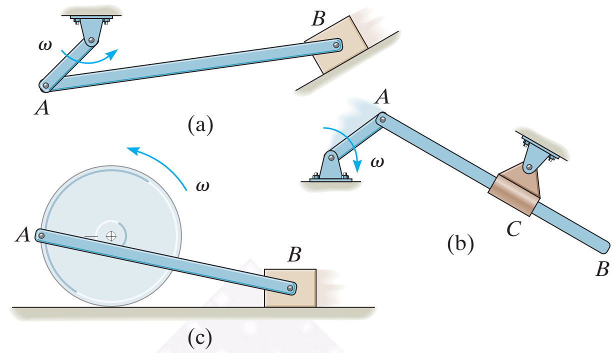
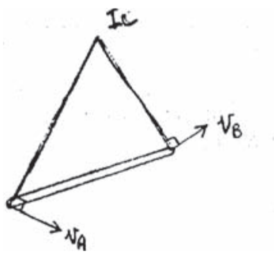
16–81.

In each case show graphically how to locate the instantaneous center of zero velocity of link AB. Assume the geometry is known.

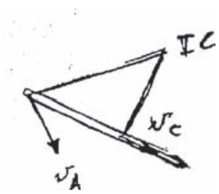


SOLUTION

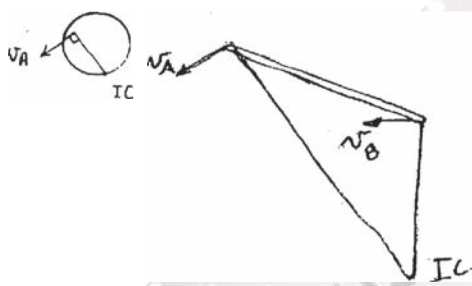
a)



b)



c)



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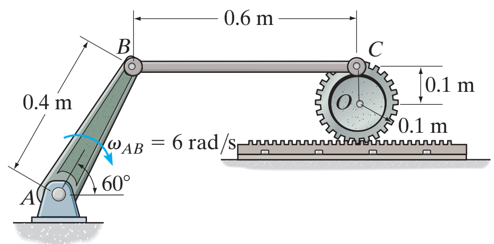
16–82.

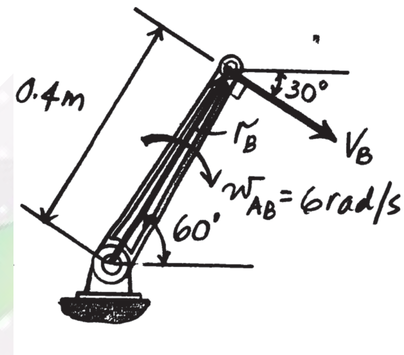
If crank AB is rotating with an angular velocity of $\omega_{AB} = 6$ rad/s, determine the velocity of the center O of the gear at the instant shown.



SOLUTION

Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega_{AB}\, r_B = 6(0.4) = 2.4 \text{ m/s}$$



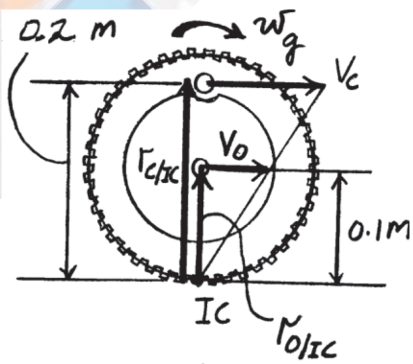
(a)

General Plane Motion: Since the gear rack is stationary, the IC of the gear is located at the contact point between the gear and the rack, Fig. *b*. Thus, $\mathbf{v}_O$ and $\mathbf{v}_C$ can be related using the similar triangles shown in Fig. *b*,

$$\omega_g = \frac{v_C}{r_{C/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_C}{0.2} = \frac{v_O}{0.1}$$

$$v_C = 2v_O$$



(b)

The location of the IC for rod BC is indicated in Fig. *c*. From the geometry shown,

$$r_{B/IC} = \frac{0.6}{\cos 60^\circ} = 1.2 \text{ m}$$

$$r_{C/IC} = 0.6 \tan 60^\circ = 1.039 \text{ m}$$

Thus, the angular velocity of rod BC can be determined from

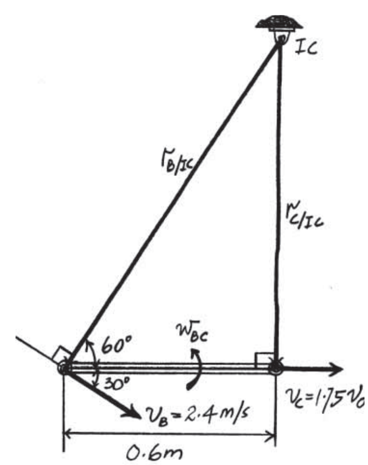
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.4}{1.2} = 2 \text{ rad/s}$$

Then,

$$v_C = \omega_{BC}\, r_{C/IC}$$

$$2v_O = 2(1.039)$$

$$v_O = 1.04 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



(c)

Ans:
$v_O = 1.04$ m/s $\rightarrow$

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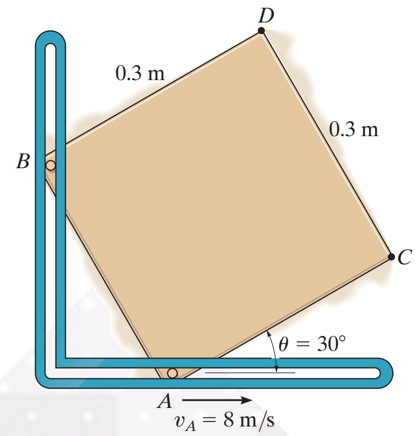
16–83. The square plate is confined within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point C at this instant.



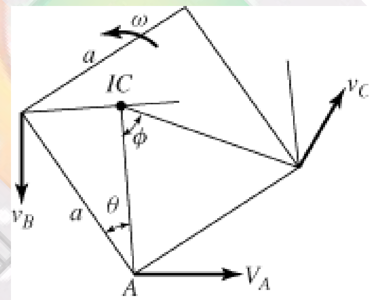
SOLUTION

Given:

$\theta = 30°$

$v_A = 8 \text{ m/s}$

$a = 0.3 \text{ m}$



$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79 \text{ rad/s}$$

$$v_C = \omega\sqrt{(a\cos(\theta))^2 + (a\cos(\theta) - a\sin(\theta))^2} \qquad v_C = 8.69 \text{ m/s} \quad \textbf{Ans.}$$

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Ans:
$v_C = 8.69$ m/s

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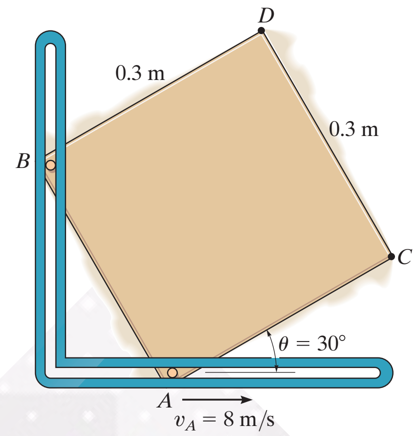
***16–84.** The square plate is confined within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point D at this instant.



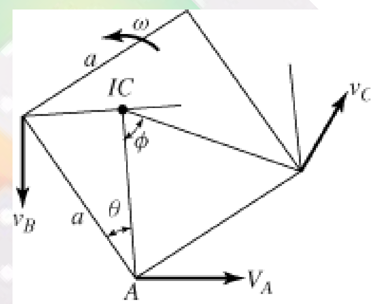
SOLUTION

Given:

$\theta = 30°$

$v_A = 8 \text{ m/s}$

$a = 0.3 \text{ m}$



$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79 \text{ rad/s}$$

$$v_D = \omega\sqrt{(-a\sin(\theta) + a\cos(\theta))^2 + (a\sin(\theta))^2} \qquad v_D = 5.72 \text{ m/s} \quad \textbf{Ans.}$$

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Ans:
$v_D = 5.72 \text{ m/s}$

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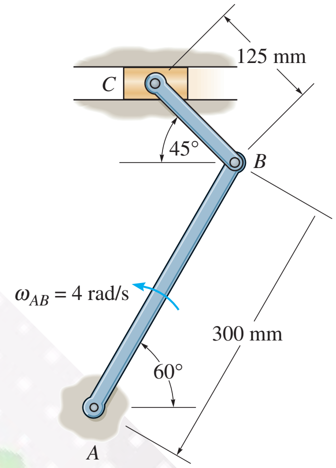
16–85.

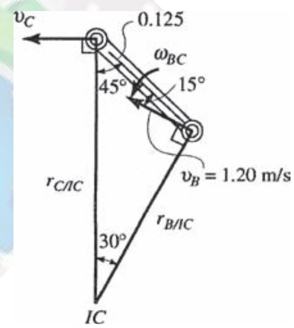
The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the angular velocity of the link CB at the instant shown, if the link AB is rotating at 4 rad/s.



SOLUTION

Kinematic Diagram: Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 4(0.3) = 1.20$ m/s. At the instant shown, $\mathbf{v}_B$ is directed at an angle 30° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.

Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have



$$\frac{r_{B/IC}}{\sin 45°} = \frac{0.125}{\sin 30°} \qquad r_{B/IC} = 0.1768 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105°} = \frac{0.125}{\sin 30°} \qquad r_{C/IC} = 0.2415 \text{ m}$$

The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.1768} = 6.79 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_{BC} = 6.79$ rad/s

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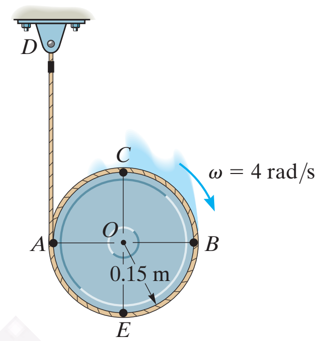
16–86.

At the instant shown, the disk is rotating at $\omega = 4$ rad/s. Determine the velocities of points A, B, and C.



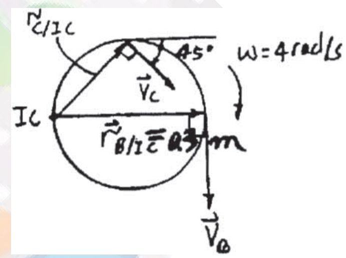
SOLUTION

The instantaneous center is located at point A. Hence, $v_A = 0$ **Ans.**

$$r_{C/IC} = \sqrt{0.15^2 + 0.15^2} = 0.2121 \text{ m} \qquad r_{B/IC} = 0.3 \text{ m}$$

$$v_B = \omega\, r_{B/IC} = 4(0.3) = 1.2 \text{ m/s}$$ **Ans.**

$$v_C = \omega\, r_{C/IC} = 4(0.2121) = 0.849 \text{ m/s} \;\; \searrow 45^\circ$$ **Ans.**



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Ans:
$v_A = 0$,
$v_B = 1.2$ m/s
$v_C = 0.849$ m/s $\searrow 45^\circ$

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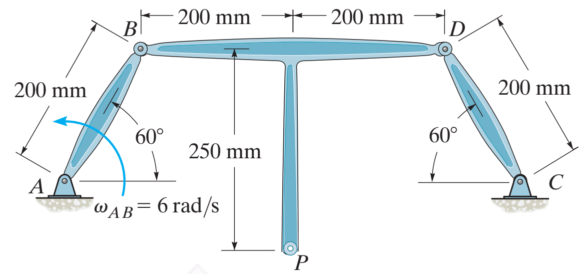
16–87

Member AB is rotating at $\omega_{AB} = 6\text{ rad/s}$. Determine the velocity of point D and the angular velocity of members BPD and CD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20\text{ m/s} \qquad v_D = \omega_{CD}(0.2)$$

And their directions are indicated in Figs. *a* and *b*.

General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as show in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = r_{D/IC} = 0.4\text{ m}$$

Then, the kinematics gives

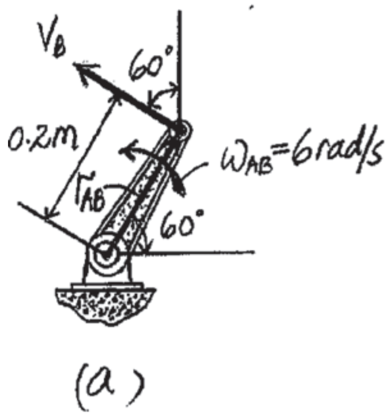
$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00\text{ rad/s } \circlearrowright \qquad \textbf{Ans.}$$

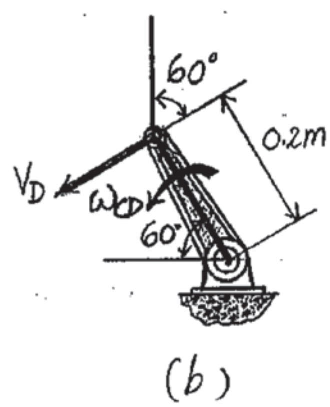
$$v_D = \omega_{BPD} r_{D/IC} = (3.00)(0.4) = 1.20\text{ m/s} \swarrow \qquad \textbf{Ans.}$$

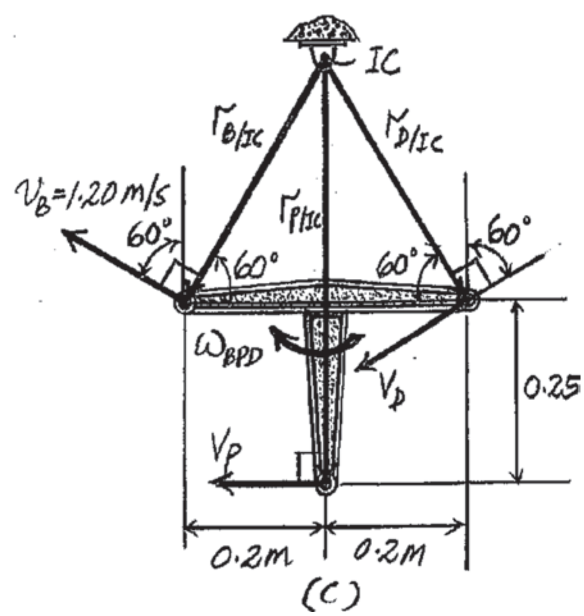
Thus,

$$v_D = \omega_{CD}(0.2); \qquad 1.2 = \omega_{CD}(0.2)$$

$$\omega_{CD} = 6.00\text{ rad/s } \circlearrowleft \qquad \textbf{Ans.}$$







Ans:
$\omega_{BPD} = 3.00\text{ rad/s } \circlearrowright$
$v_D = 1.20\text{ m/s} \swarrow$
$\omega_{CD} = 6.00\text{ rad/s } \circlearrowleft$

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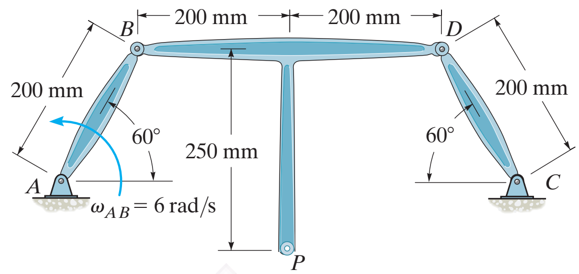
***16–88.**

Member AB is rotating at $\omega_{AB} = 6\ \text{rad/s}$. Determine the velocity of point P, and the angular velocity of member BPD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20\ \text{m/s} \qquad v_D = \omega_{CD}(0.2)$$

And their direction are indicated in Fig. *a* and *b*

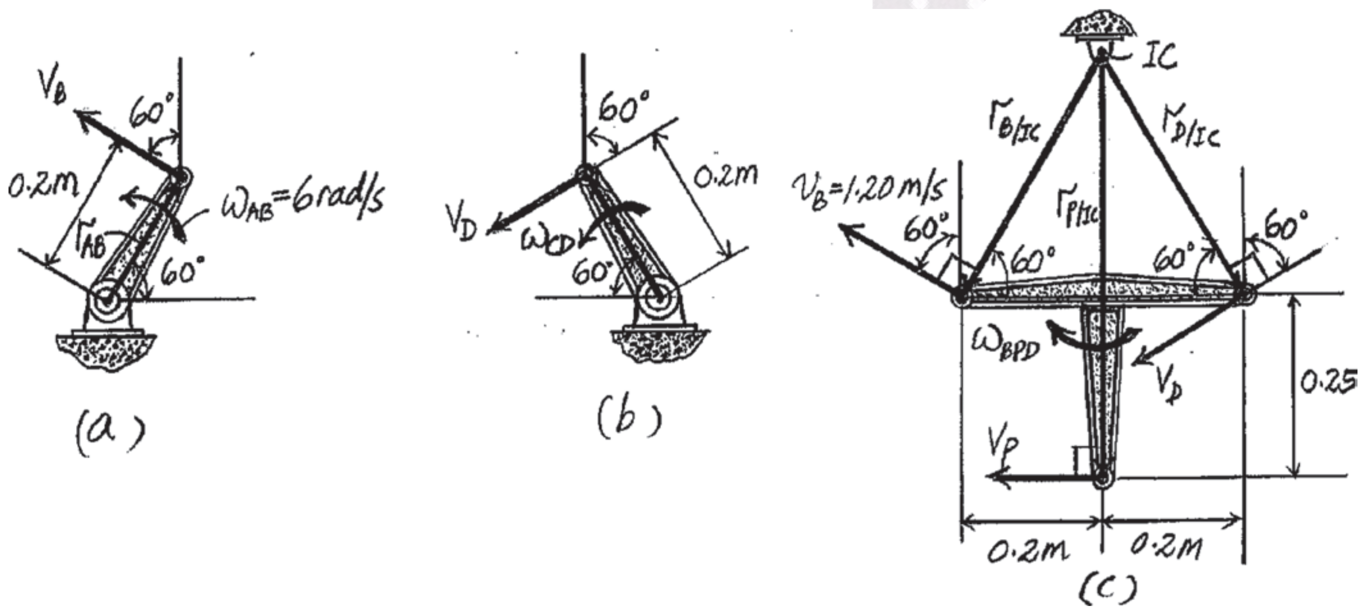
General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as shown in Fig. *c*. From the geometry of this figure

$$r_{B/IC} = 0.4\ \text{m} \qquad r_{P/IC} = 0.25 + 0.2 \tan 60° = 0.5964\ \text{m}$$

Then the kinematics give

$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$$v_P = \omega_{BPD} r_{P/IC} = (3.00)(0.5964) = 1.7892\ \text{m/s} = 1.79\ \text{m/s} \leftarrow \qquad \textbf{Ans.}$$



Ans:
$\omega_{BPD} = 3.00\ \text{rad/s} \circlearrowright$
$v_P = 1.79\ \text{m/s} \leftarrow$

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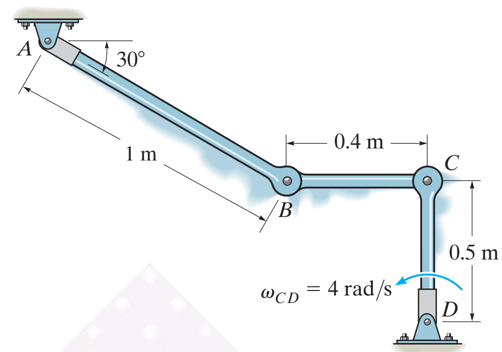
16–89.

If rod CD is rotating with an angular velocity $\omega_{CD} = 4$ rad/s, determine the angular velocities of rods AB and CB at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of C and D are

$$v_C = \omega_{CD} r_{CD} = 4(0.5) = 2.00 \text{ m/s}$$

$$v_B = \omega_{AB} r_{AB} = \omega_{AB}(1)$$

And their direction are indicated in Fig. *a* and *b*.

General Plane Motion. With the results of $\mathbf{v}_C$ and $\mathbf{v}_B$, the IC for link BC can be located as shown in Fig. *c*. From the geometry of this figure,

$$r_{C/IC} = 0.4 \tan 30° = 0.2309 \text{ m} \qquad r_{B/IC} = \frac{0.4}{\cos 30°} = 0.4619 \text{ m}$$

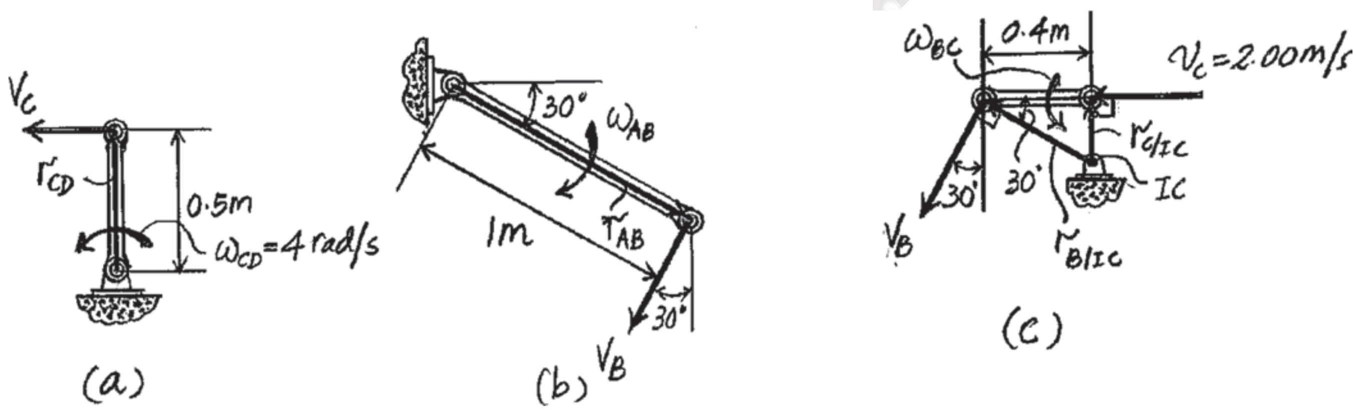
Then, the kinematics gives

$v_C = \omega_{BC} r_{C/IC};\quad 2.00 = \omega_{BC}(0.2309)$

$$\omega_{BC} = 8.6603 \text{ rad/s} = 8.66 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$v_B = \omega_{BC} r_{B/IC};\quad \omega_{AB}(1) = 8.6603(0.4619)$

$$\omega_{AB} = 4.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



Ans:
$\omega_{BC} = 8.66$ rad/s $\circlearrowleft$
$\omega_{AB} = 4.00$ rad/s $\circlearrowright$

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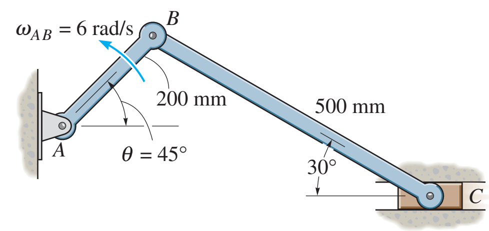
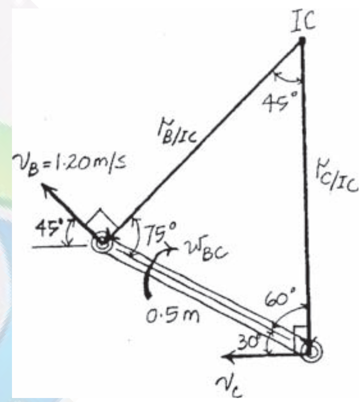
16–90.

If bar AB has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block C at the instant shown.



SOLUTION

Kinematic Diagram: Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20$ m/s. At the instant shown. $\mathbf{v}_B$ is directed with an angle 45° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.



Instantaneous Center: The instantaneous center of zero velocity of bar BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sine, we have

$$\frac{r_{B/IC}}{\sin 60°} = \frac{0.5}{\sin 45°} \qquad r_{B/IC} = 0.6124 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 75°} = \frac{0.5}{\sin 45°} \qquad r_{C/IC} = 0.6830 \text{ m}$$

The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.6124} = 1.960 \text{ rad/s}$$

Thus, the velocity of block C is

$$v_C = \omega_{BC}\, r_{C/IC} = 1.960(0.6830) = 1.34 \text{ m/s} \leftarrow$$ **Ans.**

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Ans:
$v_C = 1.34$ m/s $\leftarrow$

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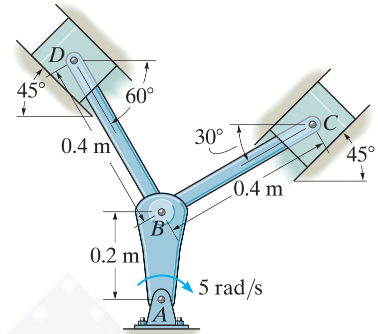
16–91. The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity $\omega_{AB} = 5$ rad/s.



SOLUTION

Given:

$$\omega_{AB} = 5 \text{ rad/s}$$

$$a = 0.2 \text{ m}$$

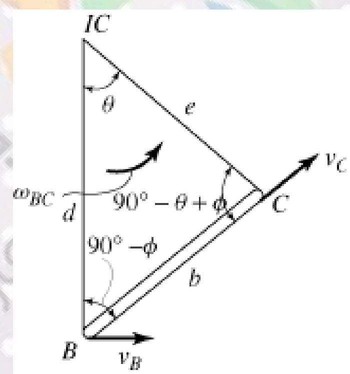
$$b = 0.4 \text{ m}$$

$$c = 0.4 \text{ m}$$

$$\theta = 45°$$

$$\phi = 30°$$

$$\beta = 45°$$



$$d = b\left(\frac{\sin(90° - \phi)}{\sin(\theta)}\right) \qquad d = 0.49 \text{ m}$$

$$e = b\left(\frac{\sin(90° + \phi - \theta)}{\sin(\theta)}\right) \qquad e = 0.55 \text{ m}$$

$$v_B = \omega_{AB}\, a \qquad v_B = 1.00 \text{ m/s}$$

$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83 \text{ rad/s}$$

$$v_C = \omega_{BC} d \qquad v_C = 0.897 \text{ m/s}$$ **Ans.**

Ans:
$v_C = 0.897$ m/s

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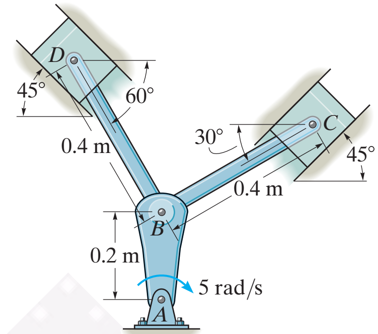
***16–92.** The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at D the instant the crank is in the position shown and has an angular velocity $\omega_{AB} = 5$ rad/s.



SOLUTION

Given:

$\omega_{AB} = 5\ \text{rad/s}$

$a = 0.2\ \text{m}$

$b = 0.4\ \text{m}$

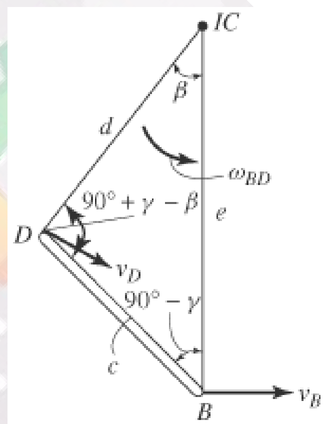
$c = 0.4\ \text{m}$

$\theta = 45°$

$\phi = 30°$

$\gamma = 60°$

$\beta = 45°$



$d = c\left(\dfrac{\sin(90° - \gamma)}{\sin(\beta)}\right)$ $\quad d = 0.28\ \text{m}$

$e = c\left(\dfrac{\sin(90° + \gamma - \beta)}{\sin(\beta)}\right)$ $\quad e = 0.55\ \text{m}$

$v_B = \omega_{AB}\, a$ $\quad v_B = 1.00\ \text{m/s}$

$\omega_{BC} = \dfrac{v_B}{e}$ $\quad \omega_{BC} = 1.83\ \text{rad/s}$

$v_D = \omega_{BC} d$ $\quad v_D = 0.518\ \text{m/s}$ **Ans.**

Ans:
$v_D = 0.518\ \text{m/s}$

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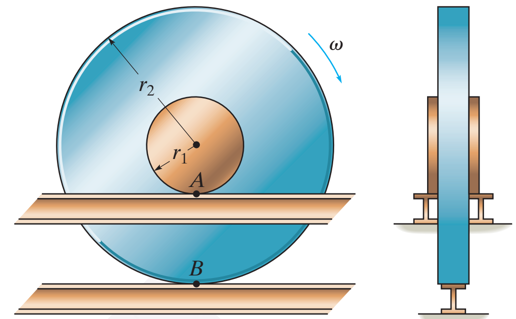
16–93.

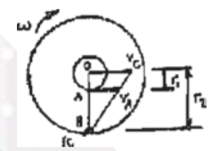
Show that if the rim of the wheel and its hub maintain contact with the three tracks as the wheel rolls, it is necessary that slipping occurs at the hub A if no slipping occurs at B. Under these conditions, what is the speed at A if the wheel has angular velocity $\boldsymbol{\omega}$?



SOLUTION

IC is at B.

$v_A = \omega(r_2 - r_1) \rightarrow$ **Ans.**



Ans:
$v_A = \omega(r_2 - r_1)$

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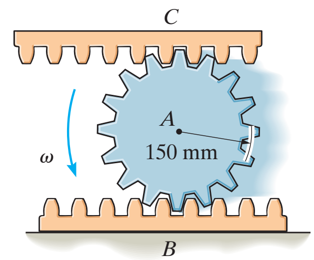
16–94.

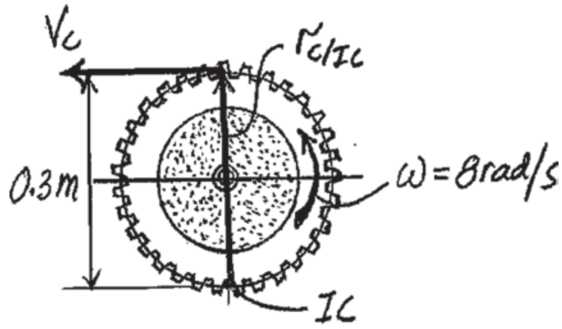
The pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 8$ rad/s. Determine the velocity of the gear rack C.



SOLUTION

General Plane Motion. The location of IC for the gear is at the bottom of the gear where it meshes with gear rack B as shown in Fig. a. Thus,

$$v_C = \omega r_{C/IC} = 8(0.3) = 2.40 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$



(a)

Ans:
$v_C = 2.40$ m/s $\leftarrow$

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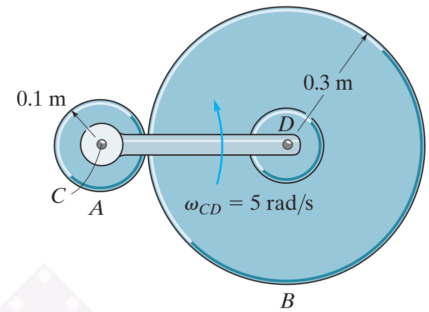
16–95.

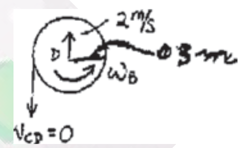
The cylinder B rolls on the fixed cylinder A without slipping. If connected bar CD is rotating with an angular velocity $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder B. Point C is a fixed point.



SOLUTION

$v_D = 5(0.4) = 2$ m/s

$$\omega_B = \frac{2}{0.3} = 6.67 \text{ rad/s}$$ **Ans.**



Ans:
$\omega_B = 6.67$ rad/s

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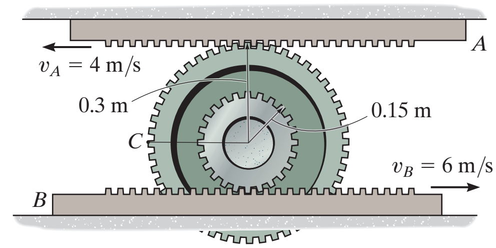
***16–96.**

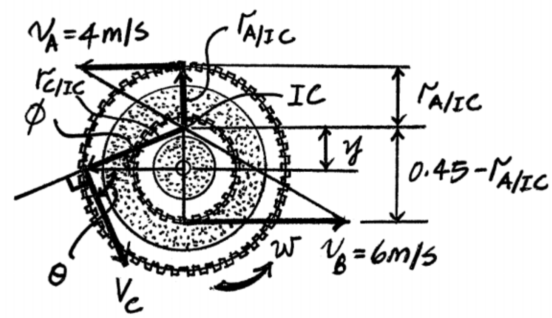
Determine the angular velocity of the double-tooth gear and the velocity of point C on the gear.



SOLUTION

General Plane Motion: The location of the IC can be found using the similar triangles shown in Fig. a.

$$\frac{r_{A/IC}}{4} = \frac{0.45 - r_{A/IC}}{6} \qquad r_{A/IC} = 0.18 \text{ m}$$



(a)

Then,

$$y = 0.3 - r_{A/IC} = 0.3 - 0.18 = 0.12 \text{ m}$$

and

$$r_{C/IC} = \sqrt{0.3^2 + 0.12^2} = 0.3231 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{0.12}{0.3}\right) = 21.80°$$

Thus, the angular velocity of the gear can be determined from

$$\omega = \frac{v_A}{r_{A/IC}} = \frac{4}{0.18} = 22.22 \text{ rad/s} = 22.2 \text{ rad/s} \qquad \textbf{Ans.}$$

Then

$$v_C = \omega r_{C/IC} = 22.2(0.3231) = 7.18 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is

$$\phi = 90° - \phi = 90° - 21.80° = 68.2° \qquad \textbf{Ans.}$$

Ans:
$\omega = 22.2$ rad/s
$v_C = 7.18$ m/s
$\phi = 68.2°$

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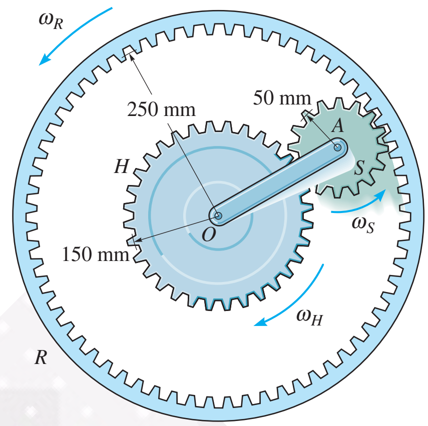
16–97.

If the hub gear H and ring gear R have angular velocities $\omega_H = 5$ rad/s and $\omega_R = 20$ rad/s, respectively, determine the angular velocity ω_S of the spur gear S and the angular velocity of its attached arm OA.

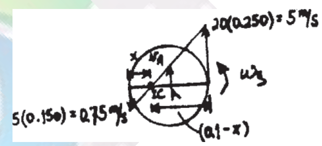


SOLUTION

$$\frac{5}{0.1 - x} = \frac{0.75}{x}$$

$$x = 0.01304 \text{ m}$$

$$\omega_S = \frac{0.75}{0.01304} = 57.5 \text{ rad/s} \circlearrowright$$ **Ans.**



$$v_A = 57.5(0.05 - 0.01304) = 2.125 \text{ m/s}$$

$$\omega_{OA} = \frac{2.125}{0.2} = 10.6 \text{ rad/s} \circlearrowright$$ **Ans.**

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Ans:
$\omega_S = 57.5$ rad/s $\circlearrowright$
$\omega_{OA} = 10.6$ rad/s $\circlearrowright$

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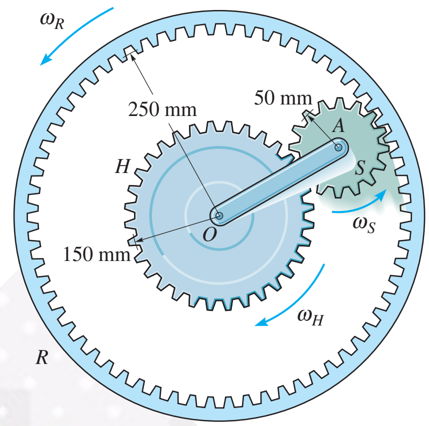
16–98.

If the hub gear H has an angular velocity $\omega_H = 5$ rad/s, determine the angular velocity of the ring gear R so that the arm OA attached to the spur gear S remains stationary ($\omega_{OA} = 0$). What is the angular velocity of the spur gear?

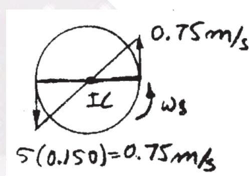


SOLUTION

The IC is at A.

$$\omega_S = \frac{0.75}{0.05} = 15.0 \text{ rad/s}$$ **Ans.**

$$\omega_R = \frac{0.75}{0.250} = 3.00 \text{ rad/s}$$ **Ans.**



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Ans:
$\omega_S = 15.0$ rad/s
$\omega_R = 3.00$ rad/s

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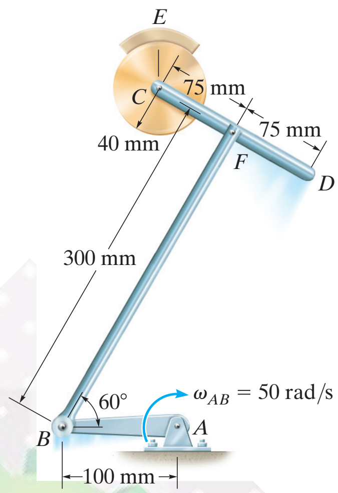
16–99.

The crankshaft AB rotates at $\omega_{AB} = 50$ rad/s about the fixed axis through point A, and the disk at C is held fixed in its support at E. Determine the angular velocity of rod CD at the instant shown.



SOLUTION

$$r_{B/IC} = \frac{0.3}{\sin 30^\circ} = 0.6 \text{ m}$$

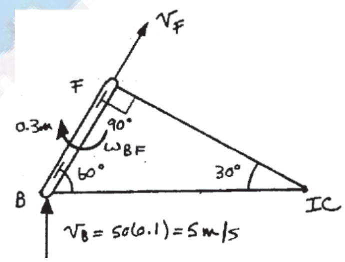
$$r_{F/IC} = \frac{0.3}{\tan 30^\circ} = 0.5196 \text{ m}$$

$$\omega_{BF} = \frac{5}{0.6} = 8.333 \text{ rad/s}$$

$$v_F = 8.333(0.5196) = 4.330 \text{ m/s}$$

Thus,

$$\omega_{CD} = \frac{4.330}{0.075} = 57.7 \text{ rad/s} \circlearrowleft$$ **Ans.**



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Ans:
$\omega_{CD} = 57.7 \text{ rad/s} \circlearrowleft$

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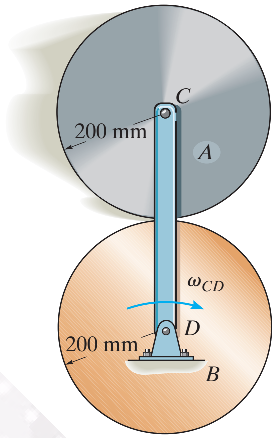
***16–100.**

Cylinder A rolls on the *fixed cylinder B* without slipping. If bar CD is rotating with an angular velocity of $\omega_{CD} = 3$ rad/s, determine the angular velocity of A.



SOLUTION

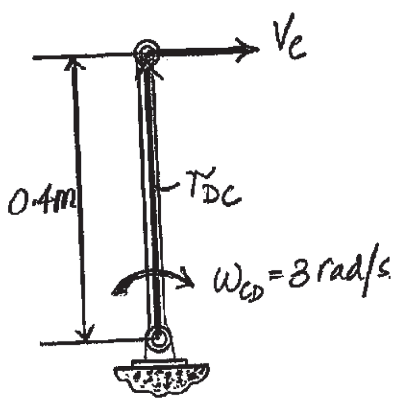
Rotation About A Fixed Axis. The magnitude of the velocity of C is

$$v_C = \omega_{CD} r_{DC} = 3(0.4) = 1.20 \text{ m/s} \rightarrow$$

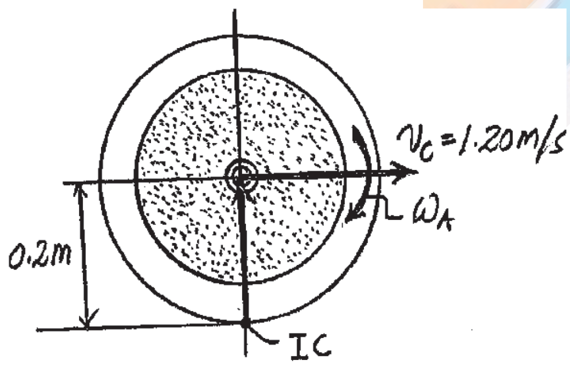
General Plane Motion. The IC for cylinder A is located at the bottom of the cylinder where it contacts with cylinder B, since no slipping occurs here, Fig. b.

$$v_C = \omega_A\, r_{C/IC}; \qquad 1.20 = \omega_A(0.2)$$

$$\omega_A = 6.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\omega_A = 6.00$ rad/s $\circlearrowright$

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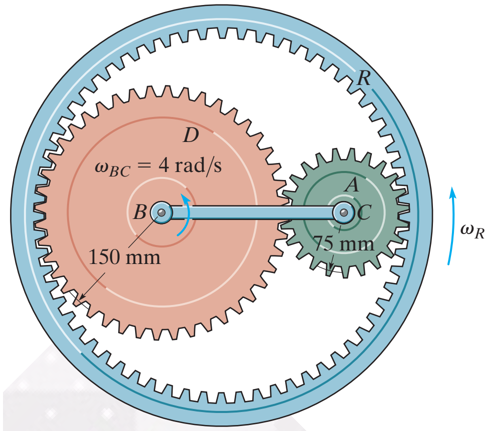
16–101.

The planet gear A is pin connected to the end of the link BC. If the link rotates about the fixed point B at 4 rad/s, determine the angular velocity of the ring gear R. The sun gear D is fixed from rotating.



SOLUTION

Gear A:

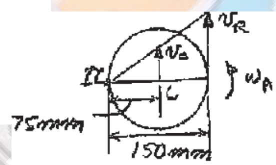
$$v_C = 4(225) = 900 \text{ mm/s}$$

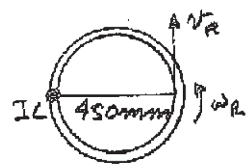
$$\omega_A = \frac{900}{75} = \frac{v_R}{150}$$

$$v_R = 1800 \text{ mm/s}$$

Ring gear:

$$\omega_R = \frac{1800}{450} = 4 \text{ rad/s}$$ **Ans.**





Ans:
$\omega_R = 4$ rad/s

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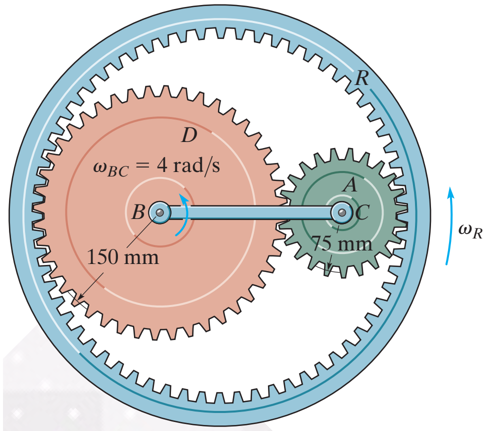
16–102.

Solve Prob. 16–101 if the sun gear D is rotating clockwise at $\omega_D = 5$ rad/s while link BC rotates counterclockwise at $\omega_{BC} = 4$ rad/s.



SOLUTION

Gear A:

$$v_P = 5(150) = 750 \text{ mm/s}$$

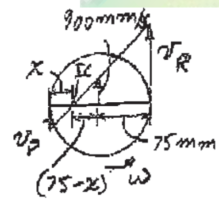
$$v_C = 4(225) = 900 \text{ mm/s}$$

$$\frac{x}{750} = \frac{75 - x}{900}$$

$$x = 34.09 \text{ mm}$$

$$\omega = \frac{750}{34.09} = 22.0 \text{ rad/s}$$

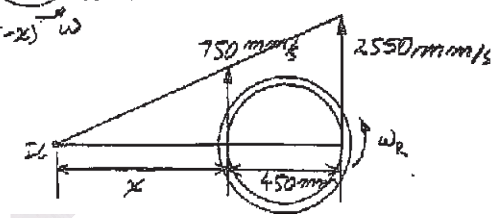
$$v_R = [75 + (75 - 34.09)](22) = 2550 \text{ mm/s}$$



Ring gear:

$$\frac{750}{x} = \frac{2550}{x + 450}$$

$$x = 187.5 \text{ mm}$$



$$\omega_R = \frac{750}{187.5} = 4 \text{ rad/s } \circlearrowleft$$ **Ans.**

Ans:
$\omega_R = 4$ rad/s

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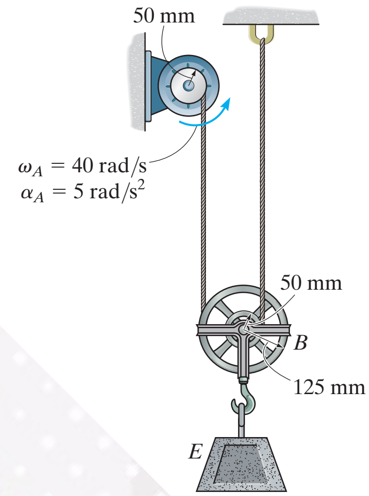
16–103.

Pulley A rotates with the angular velocity and angular acceleration shown. Determine the angular acceleration of pulley B at the instant shown.



SOLUTION

Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

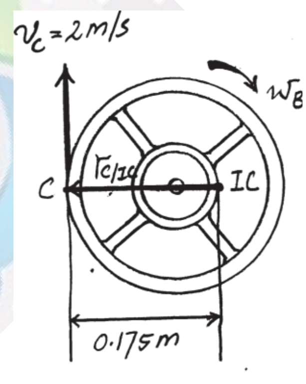
$$(a_D)_n \mathbf{i} = (a_C)_n \mathbf{i} + 0.25\mathbf{j} + (-\alpha_B \mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n \mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

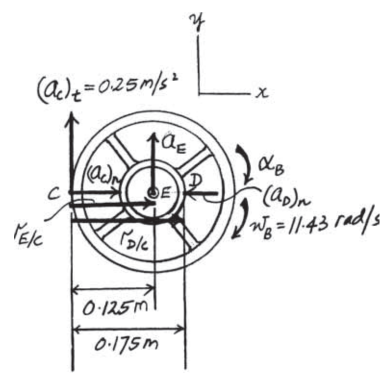
Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.43 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\alpha_B = 1.43 \text{ rad/s}^2$

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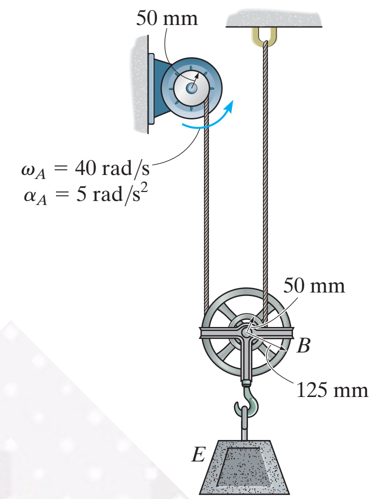
***16–104.**

Pulley A rotates with the angular velocity and angular acceleration shown. Determine the acceleration of block E at the instant shown.



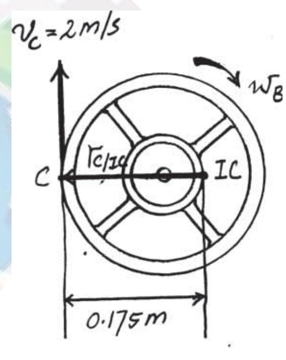
SOLUTION

Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A\, r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

$$(a_D)_n\, \mathbf{i} = (a_C)_n\, \mathbf{i} + 0.25\mathbf{j} + (-\alpha_B \mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\, \mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

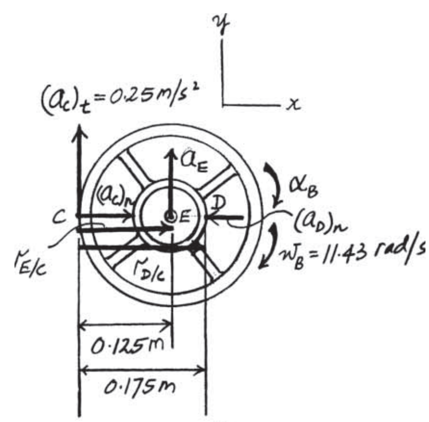
$$\alpha_B = 1.429 \text{ rad/s} = 1.43 \text{ rad/s}^2$$

Using this result, the relative acceleration equation applied to points C and E, Fig. b, gives

$$\mathbf{a}_E = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{E/C} - \omega_B{}^2 r_{E/C}$$

$$a_E\, \mathbf{j} = [(a_C)_n \mathbf{i} + 0.25\mathbf{j}] + (-1.429\mathbf{k}) \times (0.125\mathbf{i}) - 11.43^2(0.125\mathbf{i})$$

$$a_E\, \mathbf{j} = [(a_C)_n - 16.33]\mathbf{i} + 0.0714\mathbf{j}$$



(b)

Equating the $\mathbf{j}$ components,

$$a_E = 0.0714 \text{ m/s}^2 \uparrow \qquad \textbf{Ans.}$$

Ans:
$a_E = 0.0714$ m/s$^2\uparrow$

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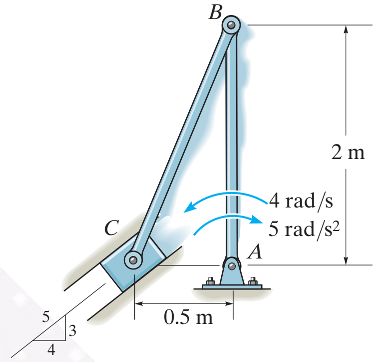
16–105.

Member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



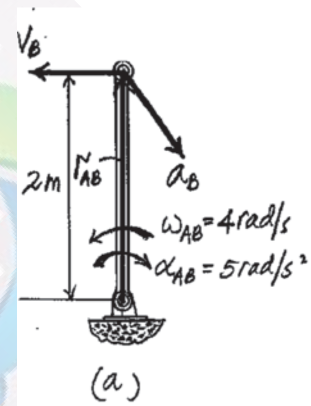
SOLUTION

Rotation About A Fixed Axis. For member AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 4(2) = 8 \text{ m/s} \leftarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-5\mathbf{k}) \times (2\mathbf{j}) - 4^2(2\mathbf{j}) = \{10\mathbf{i} - 32\mathbf{j}\} \text{ m/s}^2$$



General Plane Motion. The IC for member BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *b*. From the geometry of this figure

$$\phi = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ \qquad \theta = 90^\circ - \phi = 53.13^\circ$$

Then

$$\frac{r_{B/IC} - 2}{0.5} = \tan 53.13; \qquad r_{B/IC} = 2.6667 \text{ m}$$

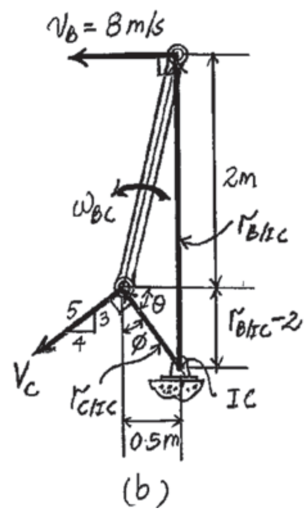
$$\frac{0.5}{r_{C/IC}} = \cos 53.13; \qquad r_{C/IC} = 0.8333 \text{ m}$$

The kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 8 = \omega_{BC}(2.6667)$$

$$\omega_{BC} = 3.00 \text{ rad/s} \circlearrowleft$$

$$v_C = \omega_{BC} r_{C/IC} = 3.00(0.8333) = 2.50 \text{ m/s} \swarrow \qquad \textbf{Ans.}$$



Applying the relative acceleration equation by referring to Fig. *c*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = (10\mathbf{i} - 32\mathbf{j}) + \alpha_{BC}\mathbf{k} \times (-0.5\mathbf{i} - 2\mathbf{j}) - \left(3.00^2\right)(-0.5\mathbf{i} - 2\mathbf{j})$$

$$-\frac{4}{5}a_C\mathbf{i} - \frac{3}{5}a_C\mathbf{j} = (2\alpha_{BC} + 14.5)\mathbf{i} + (-0.5\alpha_{BC} - 14)\mathbf{j}$$

Equating **i** and **j** components

$$-\frac{4}{5}a_C = 2\alpha_{BC} + 14.5 \qquad \textbf{(1)}$$

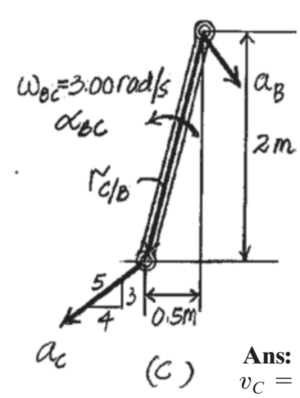
$$-\frac{3}{5}a_C = -0.5\alpha_{BC} - 14 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$a_C = 12.969 \text{ m/s}^2 = 13.0 \text{ m/s}^2 \swarrow \qquad \textbf{Ans.}$$

$$\alpha_{BC} = -12.4375 \text{ rad/s}^2 = 12.4 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

The negative sign indicates that α_{BC} is directed in the opposite sense from what is shown in Fig. (*c*).



Ans:
$v_C = 2.50 \text{ m/s} \swarrow$
$a_C = 13.0 \text{ m/s}^2 \swarrow$
$\alpha_{BC} = 12.4 \text{ rad/s}^2 \circlearrowright$

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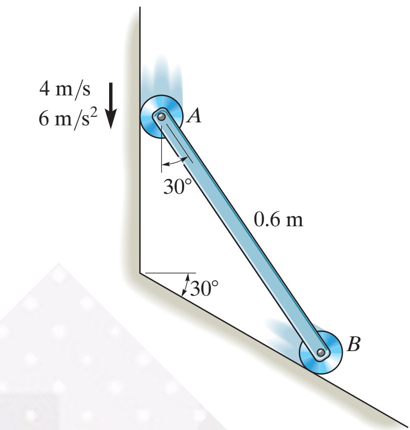
16–106.

At a given instant the roller A on the bar has the velocity and acceleration shown. Determine the velocity and acceleration of the roller B, and the bar's angular velocity and angular acceleration at this instant.



SOLUTION

General Plane Motion. The IC of the bar can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. *a*. From the geometry of this figure,

$$r_{A/IC} = r_{B/IC} = 0.6\text{ m}$$

Thus, the kinematics give

$$v_A = \omega r_{A/IC}; \qquad 4 = \omega(0.6)$$

$$\omega = 6.667\text{ rad/s} = 6.67\text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$$v_B = \omega r_{B/IC} = 6.667(0.6) = 4.00\text{ m/s} \searrow \qquad \textbf{Ans.}$$

Applying the relative acceleration equation, by referring to Fig. *b*,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B \cos 30°\mathbf{i} - a_B \sin 30°\mathbf{j} = -6\mathbf{j} + (\alpha\mathbf{k}) \times (0.6 \sin 30°\mathbf{i} - 0.6 \cos 30°\mathbf{j}) - \left(6.667^2\right)(0.6 \sin 30°\mathbf{i} - 0.6 \cos 30°\mathbf{j})$$

$$\frac{\sqrt{3}}{2} a_B\mathbf{i} - \frac{1}{2}a_B\mathbf{j} = \left(0.3\sqrt{3}\alpha - 13.33\right)\mathbf{i} + (0.3\alpha + 17.09)\mathbf{j}$$

Equating **i** and **j** components,

$$\frac{\sqrt{3}}{2} a_B = 0.3\sqrt{3}\alpha - 13.33 \qquad \textbf{(1)}$$

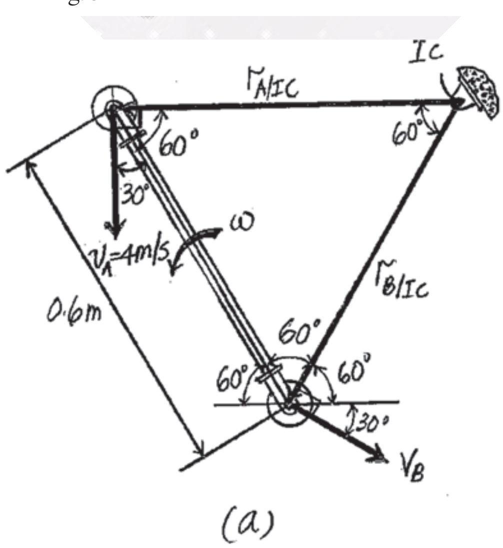
$$-\frac{1}{2}a_B = 0.3\alpha + 17.09 \qquad \textbf{(2)}$$

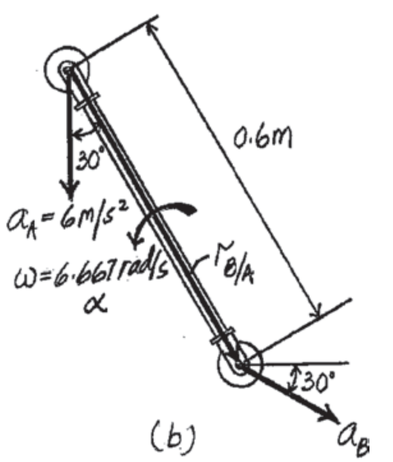
Solving Eqs. (1) and (2)

$$\alpha = -15.66\text{ rad/s}^2 = 15.7\text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$a_B = -24.79\text{ m/s}^2 = 24.8\text{ m/s}^2 \nwarrow \qquad \textbf{Ans.}$$

The negative signs indicate that $\boldsymbol{\alpha}$ and $\mathbf{a}_B$ are directed in the senses that opposite to those shown in Fig. *b*





Ans:
$\omega = 6.67\text{ rad/s} \circlearrowright$
$v_B = 4.00\text{ m/s} \searrow$
$\alpha = 15.7\text{ rad/s}^2 \circlearrowright$
$a_B = 24.8\text{ m/s}^2 \nwarrow$

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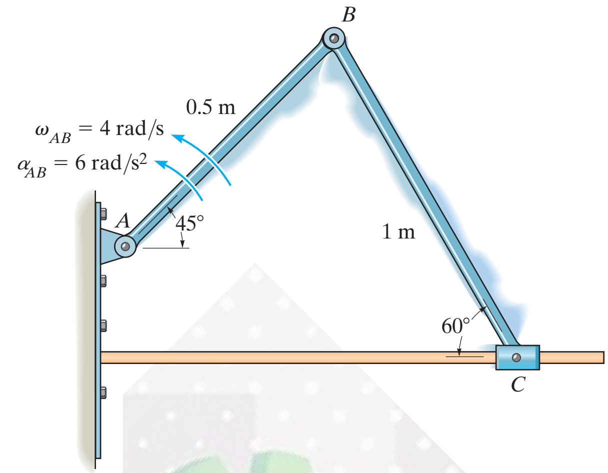
16–107.

Bar AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



SOLUTION

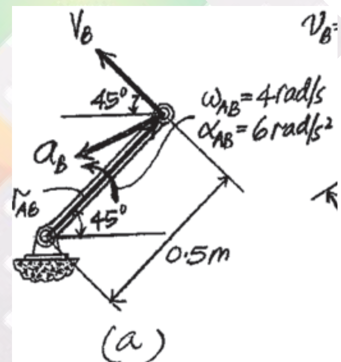
Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 4(0.5) = 2.00 \text{ m/s } \measuredangle 45°$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= 6\mathbf{k} \times (0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j}) - 4^2(0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j})$$

$$= \{-5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}\} \text{ m/s}^2$$



General Plane Motion. The IC of link BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. b. From the geometry of this figure,

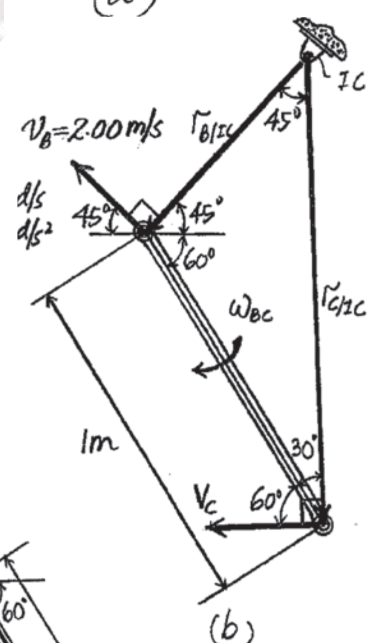
$$\frac{r_{B/IC}}{\sin 30°} = \frac{1}{\sin 45°}; \qquad r_{B/IC} = \frac{\sqrt{2}}{2} \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105°} = \frac{1}{\sin 45°}; \qquad r_{C/IC} = 1.3660 \text{ m}$$

Then the kinematics gives,

$$v_B = \omega_{BC}\, r_{B/IC}; \qquad 2 = \omega_{BC}\left(\frac{\sqrt{2}}{2}\right) \qquad \omega_{BC} = 2\sqrt{2} \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{B/IC}; \qquad v_C = \left(2\sqrt{2}\right)(1.3660) = 3.864 \text{ m/s} = 3.86 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$



Applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\, \mathbf{r}_{C/B}$$

$$-a_C\mathbf{i} = \left(-5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}\right) + (-\alpha_{BC}\mathbf{k}) \times (1 \cos 60°\mathbf{i} - 1 \sin 60°\mathbf{j})$$

$$- \left(2\sqrt{2}\right)^2(1 \cos 60°\mathbf{i} - 1 \sin 60°\mathbf{j})$$

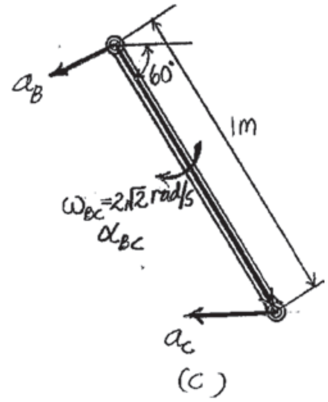
$$-a_C\mathbf{i} = \left(-\frac{\sqrt{3}}{2}\alpha_{BC} - 11.7782\right)\mathbf{i} + (3.3927 - 0.5\alpha_{BC})\mathbf{j}$$

Equating $\mathbf{j}$ components,

$$0 = 3.3927 - 0.5\alpha_{BC}; \qquad \alpha_{BC} = 6.7853 \text{ rad/s}^2 \circlearrowright$$

Then, $\mathbf{i}$ component gives

$$-a_C = -\frac{\sqrt{3}}{2}(6.7853) - 11.7782; \qquad a_C = 17.65 \text{ m/s}^2 = 17.7 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$



Ans:
$v_C = 3.86$ m/s $\leftarrow$
$a_C = 17.7$ m/s² $\leftarrow$

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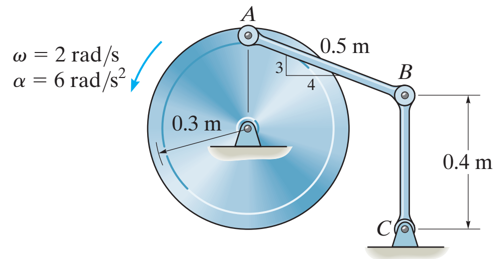
***16–108.** The flywheel rotates with angular velocity $\omega = 2$ rad/s and angular acceleration $\alpha = 6$ rad/s^2. Determine the angular acceleration of links AB and BC at this instant.



SOLUTION

Given:

$\omega = 2$ rad/s $\quad a = 0.4$ m

$\alpha = 6$ rad/s^2 $\quad b = 0.5$ m

$r = 0.3$ m $\quad e = 3$

$d = 4$

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \frac{b}{\sqrt{e^2 + d^2}}\begin{pmatrix} d \\ -e \\ 0 \end{pmatrix} \qquad \mathbf{r_3} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1$ rad/s $\quad \omega_{BC} = 1$ rad/s $\quad \alpha_{AB} = 1$ rad/s^2 $\quad \alpha_{BC} = 1$ rad/s^2

Given

$$\omega \mathbf{k} \times \mathbf{r_1} + \omega_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{BC}\mathbf{k} \times \mathbf{r_3} = 0$$

$$\alpha \mathbf{k} \times \mathbf{r_1} + \omega \mathbf{k} \times (\omega \mathbf{k} \times \mathbf{r_1}) + \alpha_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{AB}\mathbf{k} \times (\omega_{AB}\mathbf{k} \times \mathbf{r_2}) \ \ldots = 0$$
$$+ \alpha_{BC}\mathbf{k} \times \mathbf{r_3} + \omega_{BC}\mathbf{k} \times (\omega_{BC}\mathbf{k} \times \mathbf{r_3})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 1.50 \end{pmatrix} \text{rad/s}$$

$$\begin{pmatrix} \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 3.94 \end{pmatrix} \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_{AB} = 0.75$ rad/s^2
$\alpha_{BC} = 3.94$ rad/s^2

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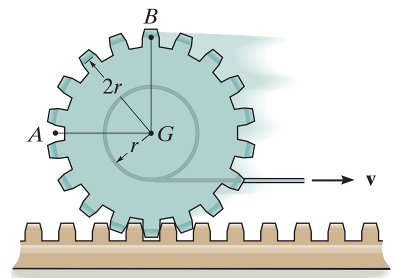
16–109.

A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity **v**, determine the velocities and accelerations of points A and B. The gear rolls on the fixed gear rack.



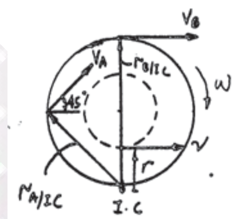
SOLUTION

Velocity analysis:

$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow$$ **Ans.**

$$v_A = \omega\, r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \qquad \measuredangle 45^\circ$$ **Ans.**

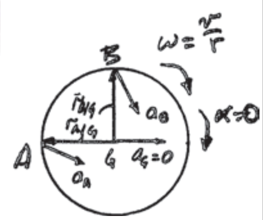


Acceleration equation: From Example 16–3, Since $a_G = 0, \alpha = 0$

$$\mathbf{r}_{B/G} = 2r\,\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\,\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2\mathbf{r}_{B/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2(2r\mathbf{j}) = -\frac{2v^2}{r}\mathbf{j}$$



$$a_B = \frac{2v^2}{r}\downarrow$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2\mathbf{r}_{A/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2(-2r\mathbf{i}) = \frac{2v^2}{r}\mathbf{i}$$

$$a_A = \frac{2v^2}{r}\rightarrow$$ **Ans.**

Ans:
$v_B = 4v \rightarrow$
$v_A = 2\sqrt{2}v$
$\theta = 45^\circ \measuredangle$
$a_B = \dfrac{2v^2}{r}\downarrow$
$a_A = \dfrac{2v^2}{r}\rightarrow$

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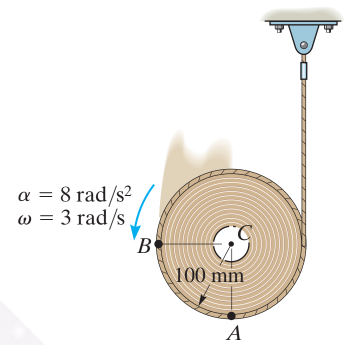
16–110.

The reel of rope has the angular motion shown. Determine the velocity and acceleration of point A at the instant shown.



SOLUTION

General Plane Motion. The IC of the reel is located as shown in Fig. *a*. Here,

$$r_{A/IC} = \sqrt{0.1^2 + 0.1^2} = 0.1414\ \text{m}$$

Then, the Kinematics give

$$v_A = \omega r_{A/IC} = 3(0.1414) = 0.4243\ \text{m/s} = 0.424\ \text{m/s} \ \searrow 45^\circ \qquad \textbf{Ans.}$$

Here $a_C = \alpha r = 8(0.1) = 0.8\ \text{m/s}^2 \downarrow$. Applying the relative acceleration equation by referring to Fig. *b*,

$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

$$\mathbf{a}_A = -0.8\mathbf{j} + (8\mathbf{k}) \times (-0.1\mathbf{j}) - 3^2(-0.1\mathbf{j})$$

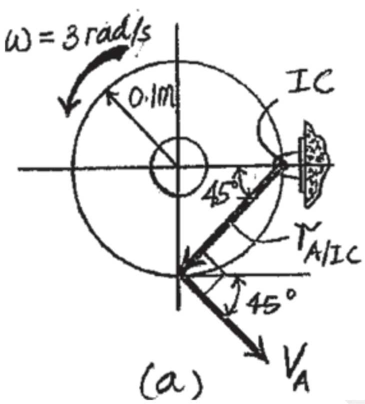
$$= \{0.8\mathbf{i} + 0.1\mathbf{j}\}\ \text{m/s}^2$$

The magnitude of $\mathbf{a}_A$ is

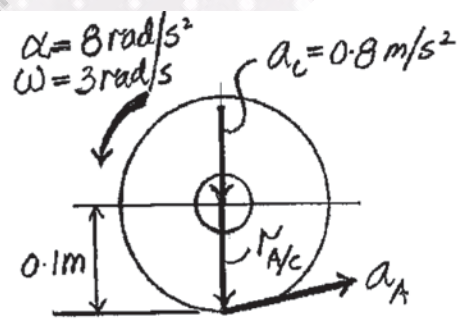
$$a_A = \sqrt{0.8^2 + 0.1^2} = 0.8062\ \text{m/s}^2 = 0.806\ \text{m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{0.1}{0.8}\right) = 7.125^\circ = 7.13^\circ \ \measuredangle \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$v_A = 0.424\ \text{m/s}$
$\theta_v = 45^\circ \searrow$
$a_A = 0.806\ \text{m/s}^2$
$\theta_a = 7.13^\circ \ \measuredangle$

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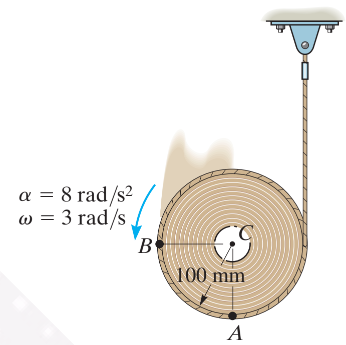
16–111.

The reel of rope has the angular motion shown. Determine the velocity and acceleration of point B at the instant shown.



SOLUTION

General Plane Motion. The *IC* of the reel is located as shown in Fig. *a*. Here, $r_{B/FC} = 0.2$ m. Then the kinematics gives

$$v_B = \omega r_{B/IC} = (3)(0.2) = 0.6 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Here, $a_C = \alpha r = 8(0.1) = 0.8 \text{ m/s}^2 \downarrow$. Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$\mathbf{a}_B = -0.8\mathbf{j} + (8\mathbf{k}) \times (-0.1\mathbf{i}) - 3^2(-0.1\mathbf{i})$$

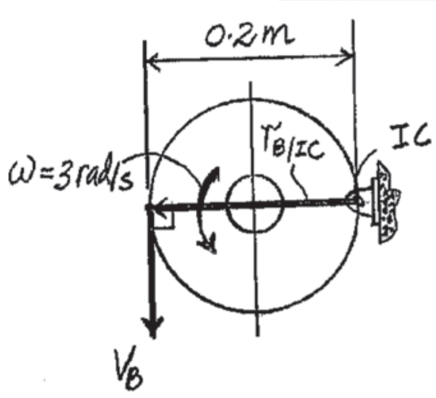
$$= \{0.9\mathbf{i} - 1.6\mathbf{j}\} \text{ m/s}^2$$

The magnitude of $\mathbf{a}_B$ is

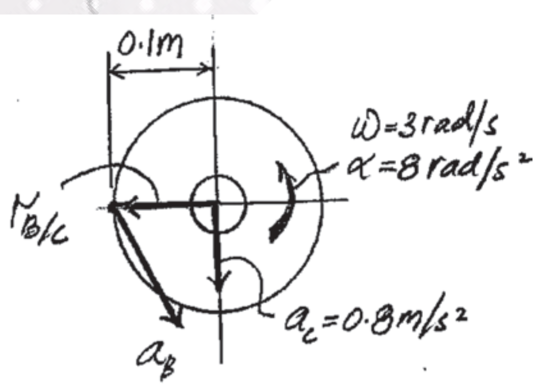
$$a_B = \sqrt{0.9^2 + (-1.6)^2} = 1.8358 \text{ m/s}^2 = 1.84 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{1.6}{0.9}\right) = 60.64° = 60.6° \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$v_B = 0.6 \text{ m/s} \downarrow$
$a_B = 1.84 \text{ m/s}^2$
$\theta = 60.6°$

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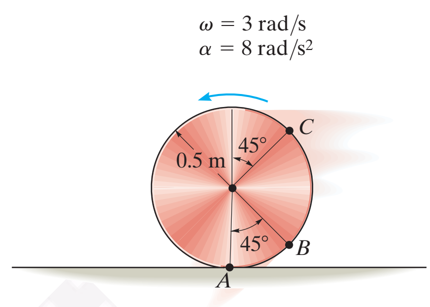
***16–112.**

The disk has an angular acceleration $\alpha = 8 \text{ rad/s}^2$ and angular velocity $\omega = 3 \text{ rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



SOLUTION

General Plane Motion. Since the disk rolls without slipping, $a_O = \alpha r = 8(0.5) = 4 \text{ m/s}^2 \leftarrow$. Applying the relative acceleration equation by referring to Fig. *a*,

$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$\mathbf{a}_B = (-4\mathbf{i}) + (8\mathbf{k}) \times (0.5 \sin 45°\mathbf{i} - 0.5 \cos 45°\mathbf{j}) - 3^2(0.5 \sin 45°\mathbf{i} - 0.5 \cos 45°\mathbf{j})$$

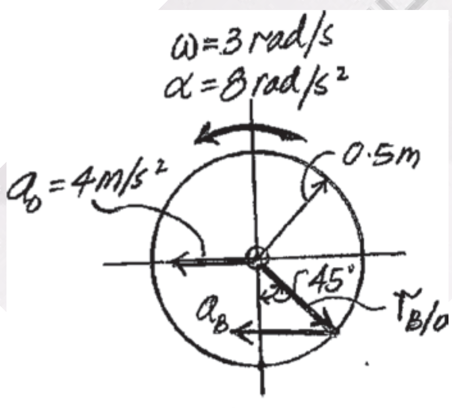
$$a_B = \{-4.354\mathbf{i} + 6.010\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

$$a_B = \sqrt{(-4.354)^2 + 6.010^2} = 7.4215 \text{ m/s}^2 = 7.42 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is given by

$$\theta = \tan^{-1}\left(\frac{6.010}{4.354}\right) = 54.08° = 54.1° \qquad \textbf{Ans.}$$



(a)

Ans:
$a_B = 7.42 \text{ m/s}^2$
$\theta = 54.1°$

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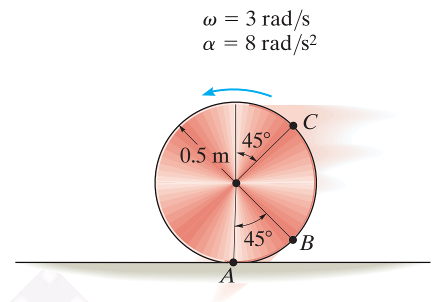
16–113.

The disk has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point C.



SOLUTION

General Plane Motion. Since the disk rolls without slipping, $a_O = \alpha r = 8(0.5) = 4\ \text{m/s}^2 \leftarrow$. Applying the relative acceleration equation by referring to Fig. *a*,

$$\mathbf{a}_C = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{C/O} - \omega^2\,\mathbf{r}_{C/O}$$

$$\mathbf{a}_C = (-4\mathbf{i}) + (8\mathbf{k}) \times (0.5 \sin 45°\mathbf{i} + 0.5 \cos 45°\mathbf{j}) - 3^2(0.5 \sin 45°\mathbf{i} + 0.5 \cos 45°\mathbf{j})$$

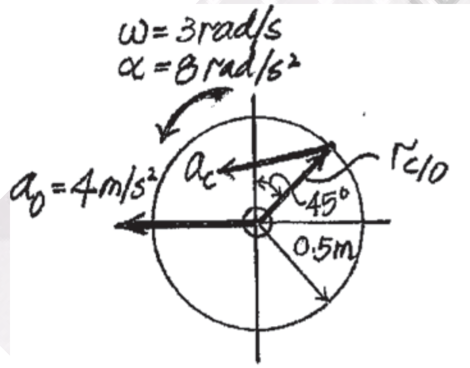
$$a_C = \{-10.0104\mathbf{i} - 0.3536\mathbf{j}\}\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{(-10.0104)^2 + (-0.3536)^2} = 10.017\ \text{m/s}^2 = 10.0\ \text{m/s}^2 \quad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left\{\frac{0.3536}{10.0104}\right\} = 2.023° = 2.02° \measuredangle \quad \textbf{Ans.}$$



(a)

Ans:
$a_C = 10.0\ \text{m/s}^2$
$\theta = 2.02° \measuredangle$

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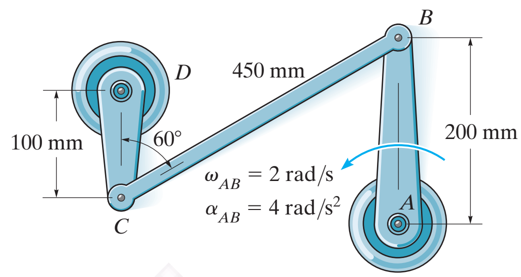
16–114.

Member AB has the angular motions shown. Determine the angular velocity and angular acceleration of members CB and DC.



SOLUTION

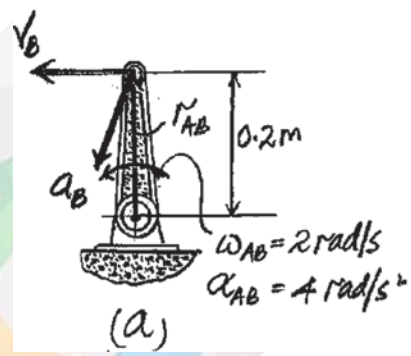
Rotation About A Fixed Axis. For crank AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 2(0.2) = 0.4 \text{ m/s} \leftarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (4\mathbf{k}) \times (0.2\mathbf{j}) - 2^2(0.2\mathbf{j})$$

$$= \{-0.8\mathbf{i} - 0.8\mathbf{j}\} \text{ m/s}^2$$



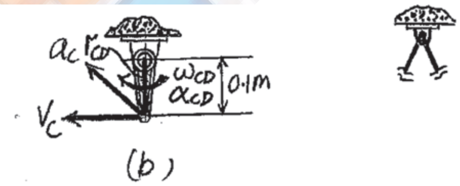
For link CD, refer to Fig. *b*.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.1)$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$= (-\alpha_{CD}\mathbf{k}) \times (-0.1\mathbf{j}) - \omega_{CD}^2(-0.1\mathbf{j})$$

$$= -0.1\alpha_{CD}\mathbf{i} + 0.1\omega_{CD}^2\mathbf{j}$$

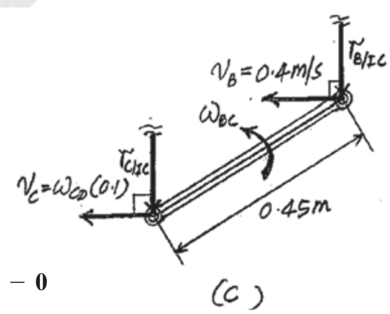


General Plane Motion. The IC of link CD can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ of which in this case is at infinity as indicated in Fig. *c*. Thus, $r_{B/IC} = r_{C/IC} = \infty$. Thus, kinematics gives

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{0.4}{\infty} = 0 \qquad \textbf{Ans.}$$

Then

$$v_C = v_B; \qquad \omega_{CD}(0.1) = 0.4 \qquad \omega_{CD} = 4.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.1\alpha_{CD}\mathbf{i} + 0.1(4.00^2)\mathbf{j} = (-0.8\mathbf{i} - 0.8\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (-0.45 \sin 60°\mathbf{i} - 0.45 \cos 60°\mathbf{j}) - \mathbf{0}$$

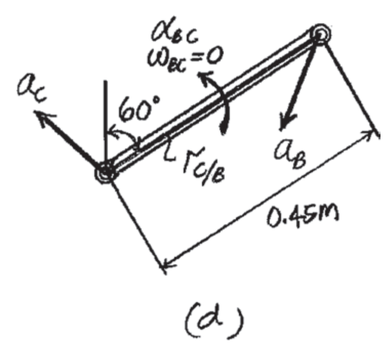
$$-0.1\alpha_{CD}\mathbf{i} + 1.6\mathbf{j} = (0.225\alpha_{BC} - 0.8)\mathbf{i} + (-0.8 - 0.3897\alpha_{BC})\mathbf{j}$$

Equating $\mathbf{j}$ components,

$$1.6 = -0.8 - 0.3897\alpha_{BC}; \qquad \alpha_{BC} = -6.1584 \text{ rad/s}^2 = 6.16 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Then $\mathbf{i}$ components give

$$-0.1\alpha_{CD} = 0.225(-6.1584) - 0.8; \qquad \alpha_{CD} = 21.86 \text{ rad/s}^2 = 21.9 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$



Ans:
$\omega_{BC} = 0$
$\omega_{CD} = 4.00 \text{ rad/s} \circlearrowright$
$\alpha_{BC} = 6.16 \text{ rad/s}^2 \circlearrowright$
$\alpha_{CD} = 21.9 \text{ rad/s}^2 \circlearrowright$

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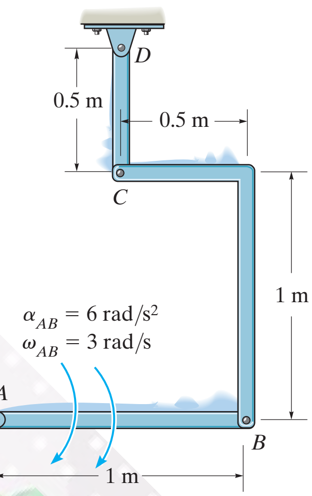
16–115.

Determine the angular acceleration of link CD if link AB has the angular velocity and angular acceleration shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(1) = 3.00\ \text{m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}{}^2\,\mathbf{r}_{AB}$$

$$= (-6\mathbf{k}) \times (1\mathbf{i}) - 3^2(1\mathbf{i})$$

$$= \{-9\mathbf{i} - 6\mathbf{j}\}\ \text{m/s}$$

For link CD, refer to Fig. b

$$v_C = \omega_{CD}\, r_{DC} = \omega_{CD}(0.5) \rightarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{DC} - \omega_{CD}{}^2\,\mathbf{r}_{DC}$$

$$= (\alpha_{CD}\mathbf{k}) \times (-0.5\mathbf{j}) - \omega_{CD}{}^2(-0.5\mathbf{j})$$

$$= 0.5\alpha_{CD}\mathbf{i} + 0.5\omega_{CD}^2\mathbf{j}$$

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. c. Thus

$$r_{B/IC} = 0.5\ \text{m} \qquad r_{C/IC} = 1\ \text{m}$$

Then, the kinematics gives

$$v_B = \omega_{BC}\, r_{B/IC}; \quad 3 = \omega_{BC}(0.5) \quad \omega_{BC} = 6.00\ \text{rad/s} \circlearrowright$$

$$v_C = \omega_{BC}\, r_{C/IC}; \quad \omega_{CD}(0.5) = 6.00(1) \quad \omega_{CD} = 12.0\ \text{rad/s} \circlearrowleft$$

Applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times r_{C/B} - \omega_{BC}{}^2\,\mathbf{r}_{C/B}$$

$$0.5\alpha_{CD}\mathbf{i} + 0.5\big(12.0^2\big)\mathbf{j} = (-9\mathbf{i} - 6\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5\mathbf{i} + \mathbf{j})$$
$$-6.00^2(-0.5\mathbf{i} + \mathbf{j})$$

$$0.5\alpha_{CD}\mathbf{i} + 72\mathbf{j} = (\alpha_{BC} + 9)\mathbf{i} + (0.5\alpha_{BC} - 42)\mathbf{j}$$

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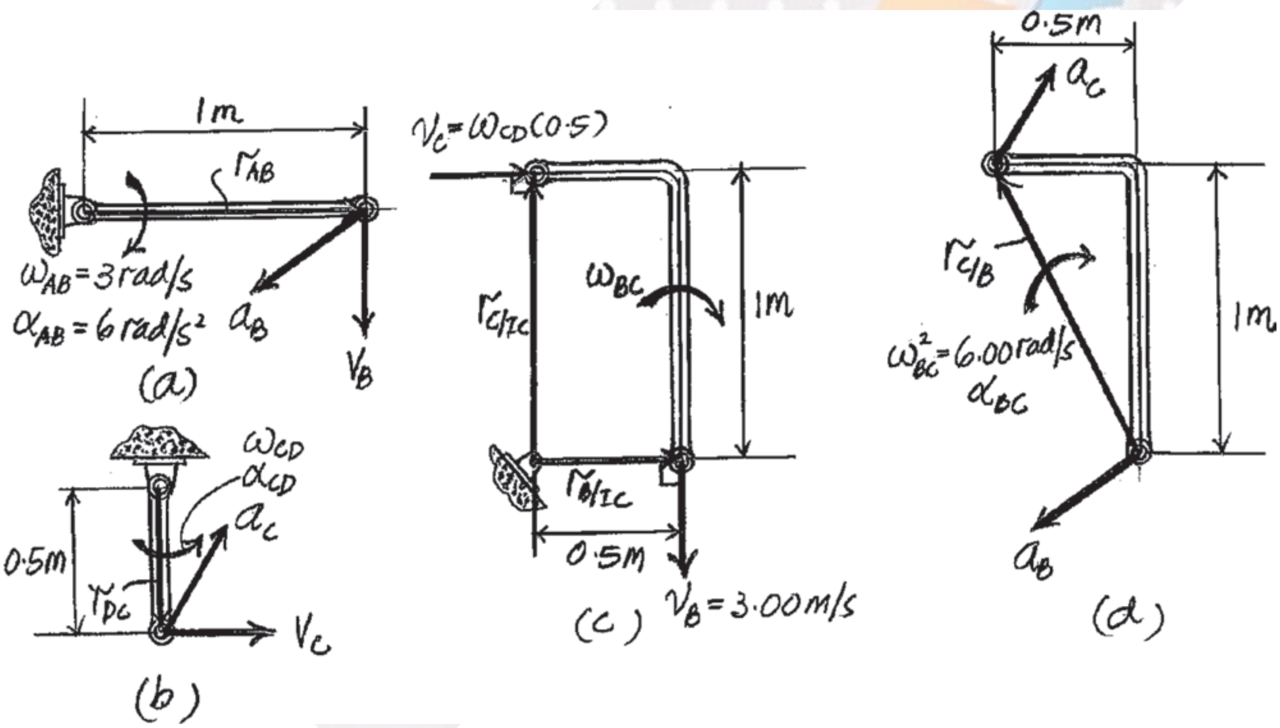
16–115. Continued

Equating **j** components,

$72 = (0.5\alpha_{BC} - 42);\quad \alpha_{BC} = 228 \text{ rad/s}^2$ ↻

Then **i** component gives

$0.5\alpha_{CD} = 228 + 9;\quad \alpha_{CD} = 474 \text{ rad/s}^2$ ↺ **Ans.**



Ans:
$\alpha_{CD} = 474 \text{ rad/s}^2$ ↺

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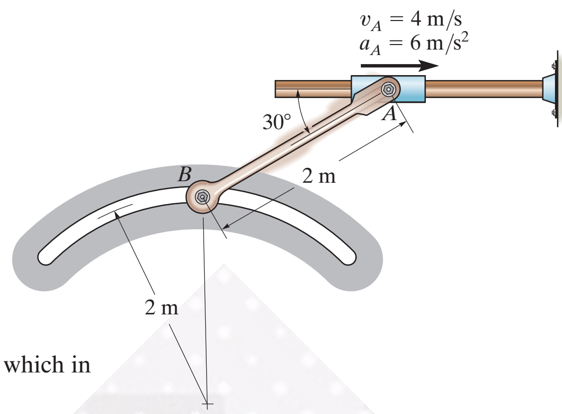
***16–116.**

At a given instant the slider block A is moving to the right with the motion shown. Determine the angular acceleration of link AB and the acceleration of point B at this instant.



SOLUTION

General Plane Motion. The IC of the link can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity as shown in Fig. *a*. Thus

$$r_{A/IC} = r_{B/IC} = \infty$$

Then the kinematics gives

$$v_A = \omega\, r_{A/IC}; \quad 4 = \omega\,(\infty) \quad \omega = 0$$

$$v_B = v_A = 4 \text{ m/s}$$

Since B moves along a circular path, its acceleration will have tangential and normal components. Hence $(a_B)_n = \dfrac{v_B^2}{r_B} = \dfrac{4^2}{2} = 8 \text{ m/s}^2$

Applying the relative acceleration equation by referring to Fig. *b*,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = 6\mathbf{i} + (\alpha\mathbf{k}) \times (-2\cos 30°\mathbf{i} - 2\sin 30°\mathbf{j}) - 0$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = (\alpha + 6)\mathbf{i} - \sqrt{3}\alpha\mathbf{j}$$

Equating **i** and **j** componenets,

$$-8 = -\sqrt{3}\alpha; \quad \alpha = \frac{8\sqrt{3}}{3} \text{ rad/s}^2 = 4.62 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

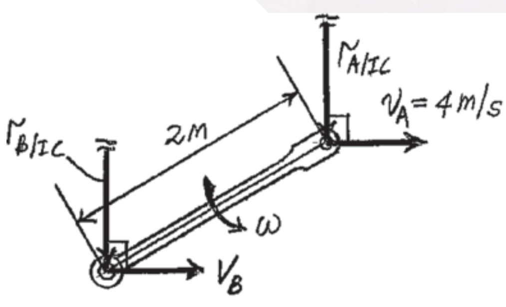
$$(a_B)_t = \alpha + 6; \quad (a_B)_t = \frac{8\sqrt{3}}{3} + 6 = 10.62 \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

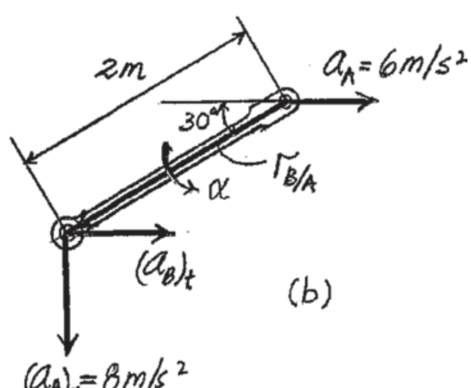
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{10.62^2 + 8^2} = 13.30 \text{ m/s}^2 = 13.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{8}{10.62}\right) = 36.99° = 37.0° \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\alpha_{AB} = 4.62 \text{ rad/s}^2 \circlearrowleft$
$a_B = 13.3 \text{ m/s}^2$
$\theta = 37.0°$

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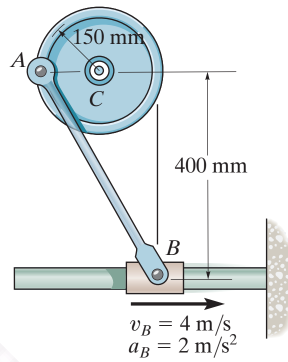
16–117.

The slider block has the motion shown. Determine the angular velocity and angular acceleration of the wheel at this instant.



SOLUTION

Rotation About A Fixed Axis. For wheel C, refer to Fig. a.

$$v_A = \omega_C r_C = \omega_C(0.15) \downarrow$$

$$\mathbf{a}_A = \boldsymbol{\alpha}_C \times \mathbf{r}_C - \omega_C^2 \mathbf{r}_C$$

$$\mathbf{a}_A = (\alpha_C \mathbf{k}) \times (-0.15\mathbf{i}) - \omega_C^2(-0.15\mathbf{i})$$

$$= 0.15\,\omega_C^2 \mathbf{i} - 0.15\alpha_C \mathbf{j}$$

General Plane Motion. The IC for crank AB can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. b. Here

$$r_{A/IC} = 0.3 \text{ m} \quad r_{B/IC} = 0.4 \text{ m}$$

Then the kinematics gives

$$v_B = \omega_{AB}\, r_{B/IC}; \quad 4 = \omega_{AB}(0.4) \quad \omega_{AB} = 10.0 \text{ rad/s} \circlearrowright$$

$$v_A = \omega_{AB}\, r_{A/IC}; \quad \omega_C(0.15) = 10.0(0.3) \quad \omega_C = 20.0 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$2\mathbf{i} = 0.15\left(20.0^2\right)\mathbf{i} - 0.15\alpha_C \mathbf{j} + (\alpha_{AB}\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j})$$
$$-10.0^2(0.3\mathbf{i} - 0.4\mathbf{j})$$

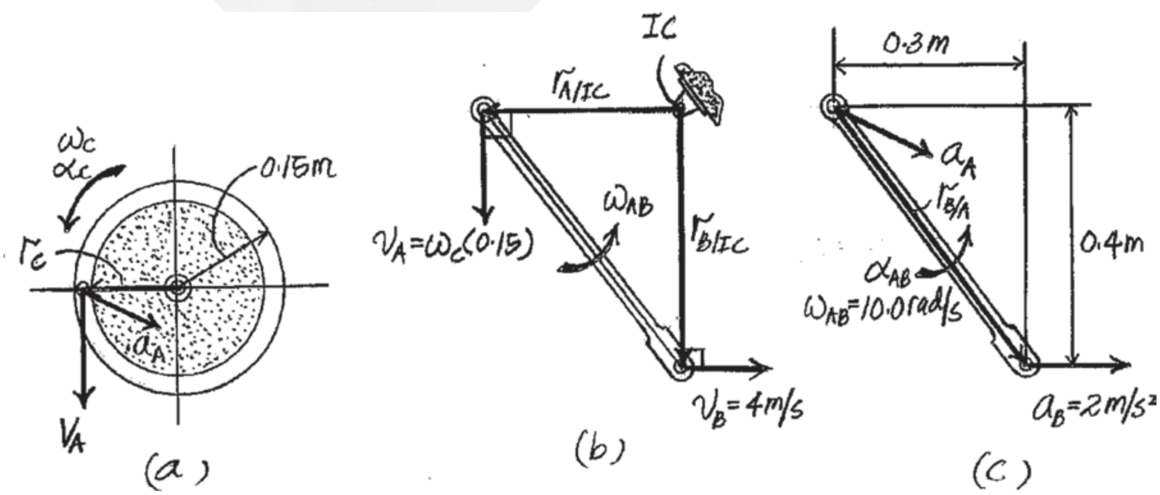
$$2\mathbf{i} = (0.4\alpha_{AB} + 30)\mathbf{i} + (0.3\alpha_{AB} - 0.15\alpha_C + 40)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components,

$$2 = 0.4\alpha_{AB} + 30; \quad \alpha_{AB} = -70.0 \text{ rad/s}^2 = 70.0 \text{ rad/s}^2 \circlearrowright$$

$$0 = 0.3(-70.0) + 0.15\alpha_C + 40; \quad \alpha_C = -126.67 \text{ rad/s}^2 = 127 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

The negative signs indicate that $\boldsymbol{\alpha}_C$ and $\boldsymbol{\alpha}_{AB}$ are directed in the sense that those shown in Fig. a and c.



Ans:
$\omega_C = 20.0 \text{ rad/s} \circlearrowright$
$\alpha_C = 127 \text{ rad/s}^2 \circlearrowright$

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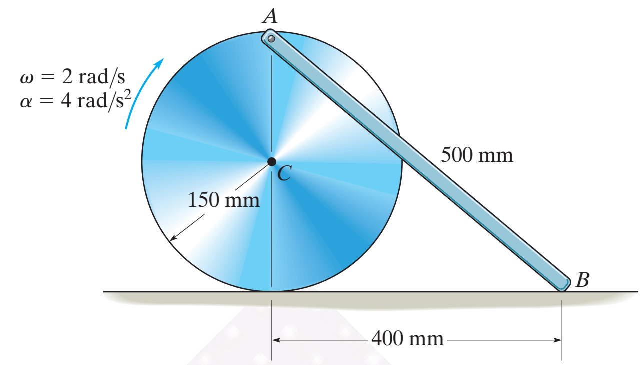
16–118.

The disk rolls without slipping such that it has an angular acceleration of $\alpha = 4\ \text{rad/s}^2$ and angular velocity of $\omega = 2\ \text{rad/s}$ at the instant shown. Determine the acceleration of points A and B on the link and the link's angular acceleration at this instant. Assume point A lies on the periphery of the disk, 150 mm from C.



SOLUTION

The IC is at ∞, so $\omega = 0$.

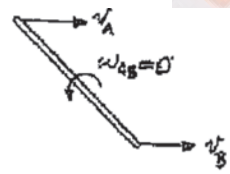
$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

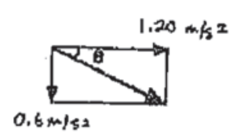
$$\mathbf{a}_A = 0.6\mathbf{i} + (-4\mathbf{k}) \times (0.15\mathbf{j}) - (2)^2(0.15\mathbf{j})$$

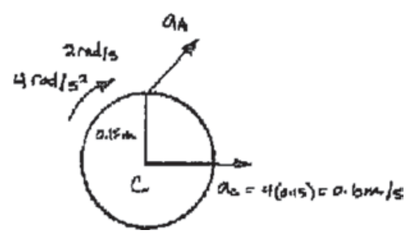
$$\mathbf{a}_A = (1.20\mathbf{i} - 0.6\mathbf{j})\ \text{m/s}^2$$

$$a_A = \sqrt{(1.20)^2 + (-0.6)^2} = 1.34\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{0.6}{1.20}\right) = 26.6^\circ \searrow \qquad \textbf{Ans.}$$







Ans:
$a_A = 1.34\ \text{m/s}^2$
$\theta = 26.6^\circ \searrow$

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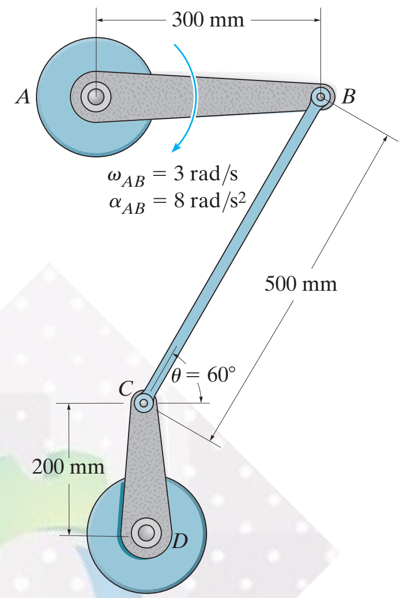
16–119.

If member AB has the angular motion shown, determine the angular velocity and angular acceleration of member CD at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

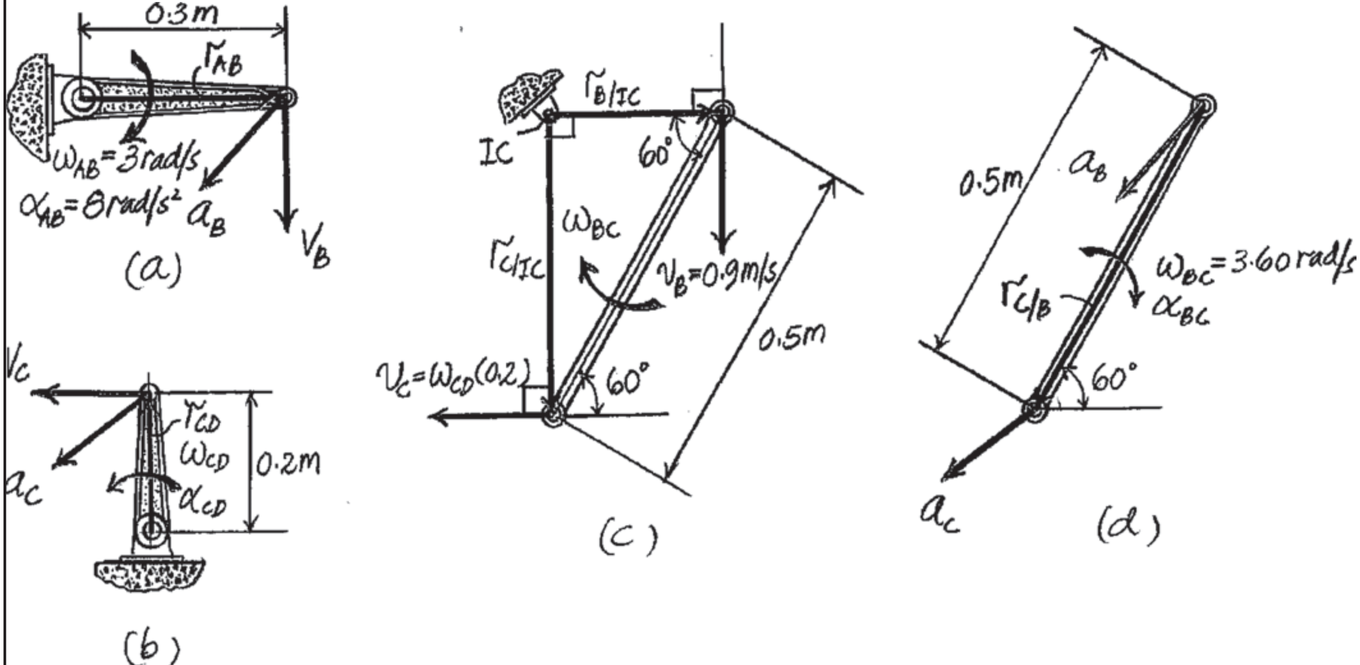
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



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16–119. Continued

General Plane Motion. The *IC* of link *BC* can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60° = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60° = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 0.9 = \omega_{BC}(0.25) \qquad \omega_{BC} = 3.60 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{C/IC}; \qquad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the **j** components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2 \circlearrowleft$$

Then the **i** component gives

$$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 7.79 \text{ rad/s} \circlearrowleft$
$\alpha_{CD} = 136 \text{ rad/s}^2 \circlearrowright$

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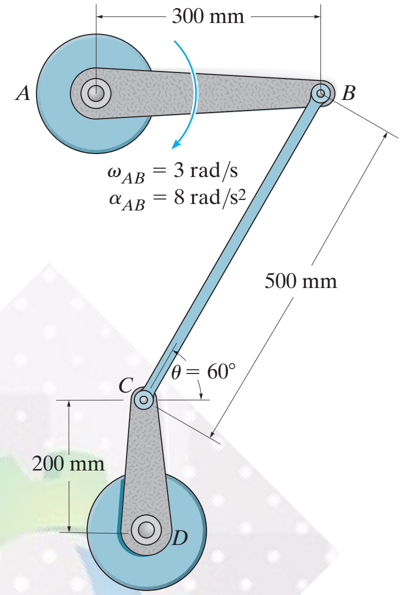
***16–120.**

If member AB has the angular motion shown, determine the velocity and acceleration of point C at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

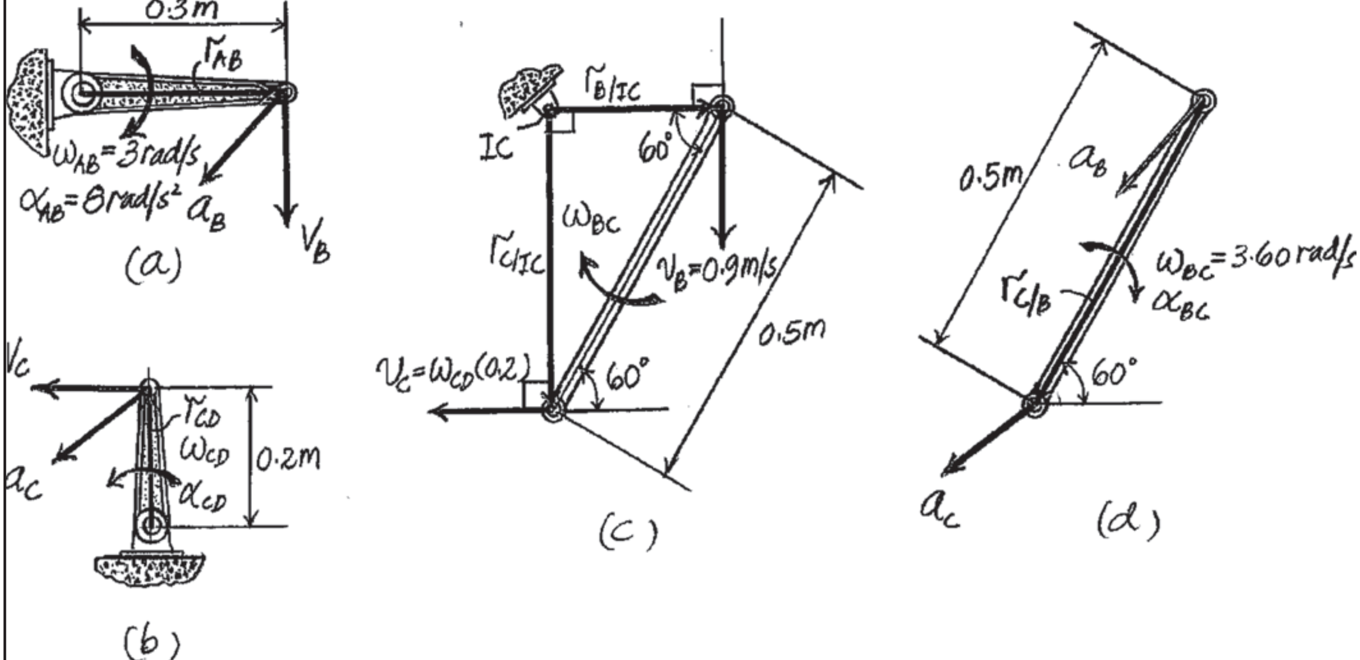
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



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***16–120. Continued**

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. c. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60° = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60° = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 0.9 = \omega_{BC}(0.25) \qquad \omega_{BC} = 3.60 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{C/IC}; \qquad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the **j** components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2 \circlearrowleft$$

Then the **i** component gives

$$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \quad \textbf{Ans.}$$

From the angular motion of CD,

$$v_C = w_{CD}(0.2) = (7.7942)(0.2) = 1.559 \text{ m/s} = 1.56 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = -0.2(-135.74)\mathbf{i} - 12.15\mathbf{j}$$
$$= \{27.15\mathbf{i} - 12.15\mathbf{j}\} \text{ m/s}$$

The magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{27.15^2 + (-12.15)^2} = 29.74 \text{ m/s}^2 = 29.7 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{12.15}{27.15}\right) = 24.11° = 24.1° \qquad \textbf{Ans.}$$

Ans:
$v_C = 1.56 \text{ m/s} \leftarrow$
$a_C = 29.7 \text{ m/s}^2$
$\theta = 24.1°$

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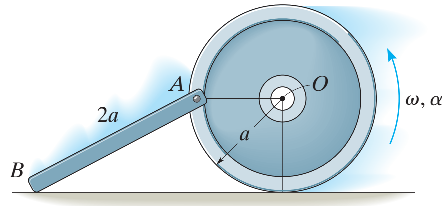
16–121.

The wheel rolls without slipping such that at the instant shown it has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of point B on the rod at this instant.



SOLUTION

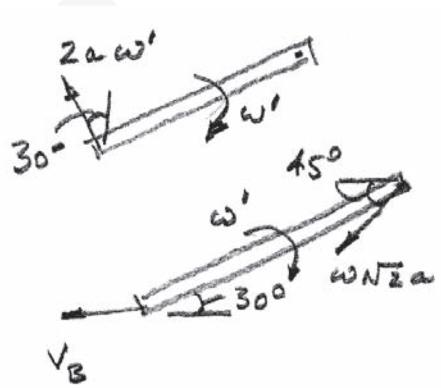
$$\overline{v}_B = \overline{v}_A + \overline{v}_{B/A\,(Pin)}$$

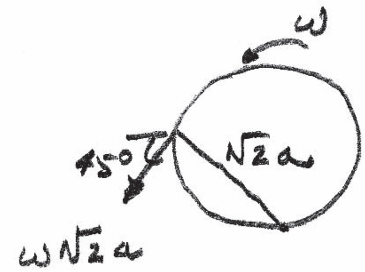
$$\stackrel{+}{\leftarrow}\; v_B = \frac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\frac{1}{2}\right)$$

$$+\uparrow\; O = -\frac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\frac{\sqrt{3}}{2}\right)$$

$$\omega' = \frac{\omega}{\sqrt{3}}$$

$$v_B = 1.58\,\omega a \qquad \textbf{Ans.}$$





$$\overline{a}_A = \overline{a}_O + \overline{a}_{A/O\,(Pin)}$$

$$\underset{\leftarrow}{(a_A)_x} + \underset{\downarrow}{(a_A)_y} = \underset{\leftarrow}{\alpha a} + \underset{\downarrow}{\alpha(a)} + \underset{\rightarrow}{\omega^2 a}$$

$$(a_A)_x = \alpha a - \omega^2 a$$

$$(a_A)_y = \alpha a$$

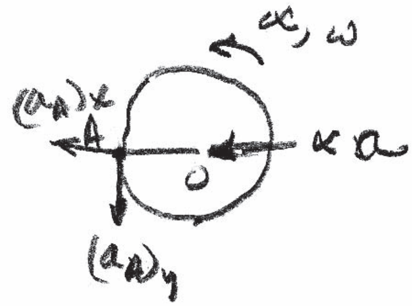
$$\overline{a}_B = \overline{a}_A + \overline{a}_{B/A\,(Pin)}$$

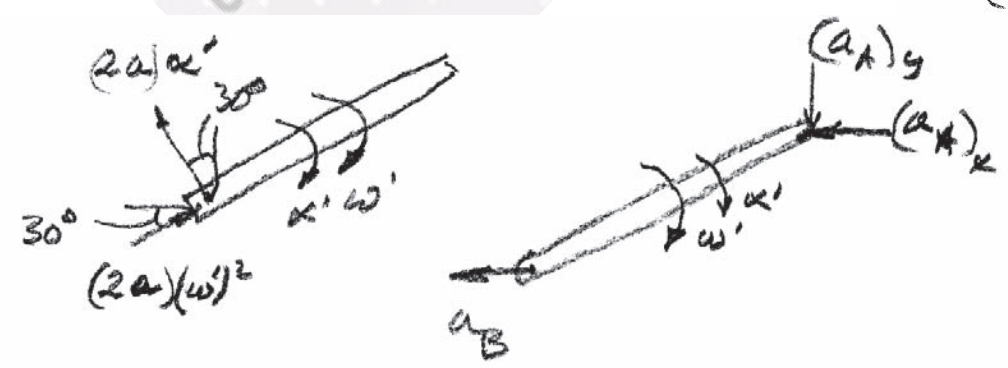
$$a_B = \alpha a - \omega^2 a + 2a(\alpha')\left(\frac{1}{2}\right) - 2a\left(\frac{\omega}{\sqrt{3}}\right)^2\frac{\sqrt{3}}{2}$$

$$O = -\alpha a + 2a\alpha'\left(\frac{2}{\sqrt{3}}\right) + 2a\left(\frac{\omega}{\sqrt{3}}\right)^2\left(\frac{1}{2}\right)$$

$$\alpha' = 0.577\alpha - 0.1925\omega^2$$

$$a_B = 1.58\alpha a - 1.77\omega^2 a \qquad \textbf{Ans.}$$





Ans:
$v_B = 1.58\omega a$
$a_B = 1.58\alpha a - 1.77\omega^2 a$

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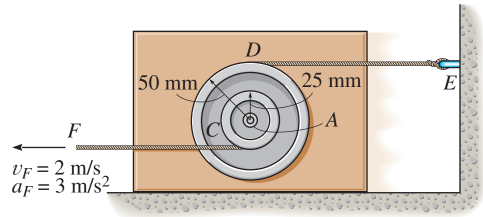
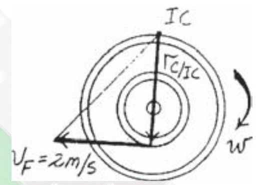
16–122.

A single pulley having both an inner and outer rim is pin-connected to the block at A. As cord CF unwinds from the inner rim of the pulley with the motion shown, cord DE unwinds from the outer rim. Determine the angular acceleration of the pulley and the acceleration of the block at the instant shown.



SOLUTION

Velocity Analysis: The angular velocity of the pulley can be obtained by using the method of instantaneous center of zero velocity. Since the pulley rotates without slipping about point D, i.e: $v_D = 0$, then point D is the location of the instantaneous center.

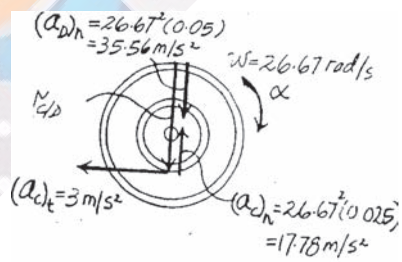


$$v_F = \omega r_{C/IC}$$

$$2 = \omega(0.075)$$

$$\omega = 26.67 \text{ rad/s}$$

Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion points C and D. Applying Eq. 16–18 with $\mathbf{r}_{C/D} = \{-0.075\mathbf{j}\}$ m, we have



$$\mathbf{a}_C = \mathbf{a}_D + \alpha \times \mathbf{r}_{C/D} - \omega^2 \mathbf{r}_{C/D}$$

$$-3\mathbf{i} + 17.78\mathbf{j} = -35.56\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.075\mathbf{j}) - 26.67^2(-0.075\mathbf{j})$$

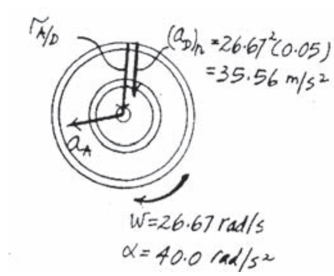
$$-3\mathbf{i} + 17.78\mathbf{j} = -0.075\,\alpha\mathbf{i} + 17.78\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$-3 = -0.075\alpha \qquad \alpha = 40.0 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$17.78 = 17.78 \; (\textbf{\textit{Check!}})$$

The acceleration of point A can be obtained by analyzing the angular motion points A and D. Applying Eq. 16–18 with $\mathbf{r}_{A/D} = \{-0.05\mathbf{j}\}$ m. we have



$$\mathbf{a}_A = \mathbf{a}_D + \alpha \times \mathbf{r}_{A/D} - \omega^2 \mathbf{r}_{A/D}$$

$$= -35.56\mathbf{j} + (-40.0\mathbf{k}) \times (-0.05\mathbf{j}) - 26.67^2(-0.05\mathbf{j})$$

$$= \{-2.00\mathbf{i}\} \text{ m/s}^2$$

Thus,

$$a_A = 2.00 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

Ans:
$\alpha = 40.0$ rad/s²
$a_A = 2.00$ m/s² $\leftarrow$

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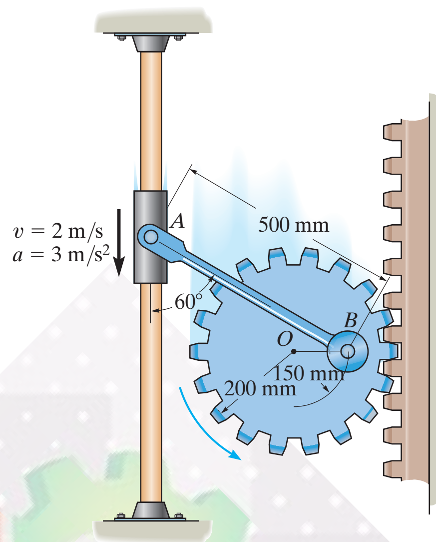
16–123.

The collar is moving downward with the motion shown. Determine the angular velocity and angular acceleration of the gear at the instant shown as it rolls along the fixed gear rack.



SOLUTION

General Plane Motion. For gear C, the location of its IC is indicate in Fig. a. Thus

$$v_B = \omega_C\, r_{B/(IC)_1} = \omega_C(0.05)\downarrow \qquad \textbf{(1)}$$

The IC of link AB can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity. Thus

$$\omega_{AB} = \frac{v_A}{r_{A/(IC)_2}} = \frac{2}{\infty} = 0$$

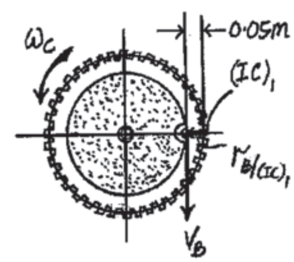
Then

$$v_B = v_A = 2\text{ m/s}\downarrow$$

Substitute the result of v_B into Eq. (1)

$$2 = \omega_C(0.05)$$

$$\omega_C = 40.0\text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



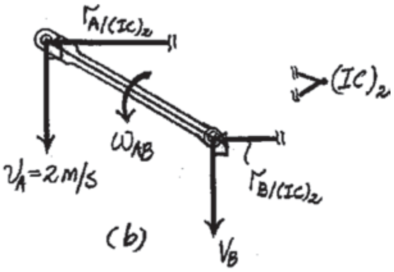
(a)

Applying the relative acceleration equation to gear C, Fig. c, with $a_O = \alpha_C r_C = \alpha_C(0.2)\downarrow$,

$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha}_C \times \mathbf{r}_{B/O} - \omega_C^2\mathbf{r}_{B/O}$$

$$\mathbf{a}_B = -\alpha_C(0.2)\mathbf{j} + (\alpha_C\mathbf{k}) \times (0.15\mathbf{i}) - 40.0^2(0.15\mathbf{i})$$

$$= -240\mathbf{i} - 0.05\alpha_C\mathbf{j}$$



(b)

For link AB, Fig. d,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

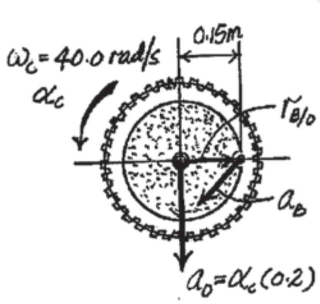
$$-240\mathbf{i} - 0.05\alpha_C\mathbf{j} = (-3\mathbf{j}) + (\alpha_{AB}\mathbf{k}) \times (0.5\sin 60°\mathbf{i} - 0.5\cos 60°\mathbf{j}) - \mathbf{0}$$

$$-240\mathbf{i} - 0.05\alpha_C = 0.25\alpha_{AB}\mathbf{i} + (0.25\sqrt{3}\alpha_{AB} - 3)\mathbf{j}$$

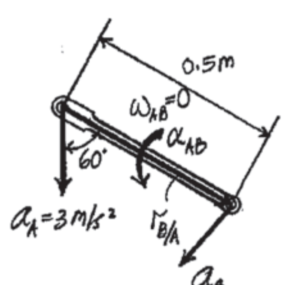
Equating $\mathbf{i}$ and $\mathbf{j}$ components

$$-240 = 0.25\alpha_{AB};\quad \alpha_{AB} = -960\text{ rad/s}^2 = 960\text{ rad/s}^2 \circlearrowright$$

$$-0.05\alpha_C = \left(0.25\sqrt{3}\right)(-960) - 3;\quad \alpha_C = 8373.84\text{ rad/s}^2 = 8374\text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$



(c)



(d)

Ans:
$\omega_C = 40.0$ rad/s $\circlearrowright$
$\alpha_C = 8374$ rad/s² $\circlearrowright$

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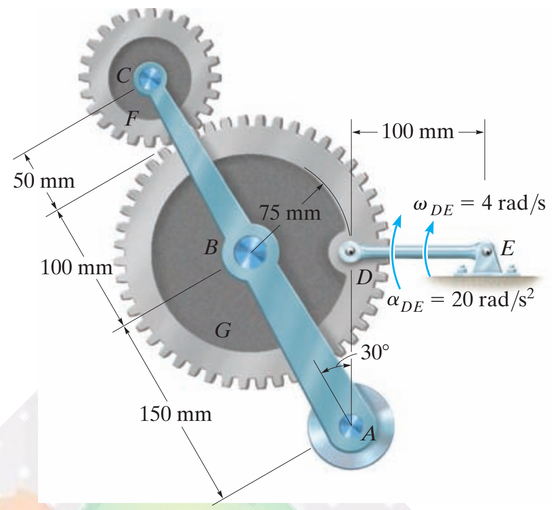
***16–124.**

The tied crank and gear mechanism gives rocking motion to crank AC, necessary for the operation of a printing press. If link DE has the angular motion shown, determine the respective angular velocities of gear F and crank AC at this instant, and the angular acceleration of crank AC.



SOLUTION

Velocity analysis:

$v_D = \omega_{DE} r_{D/E} = 4(0.1) = 0.4 \text{ m/s} \uparrow$

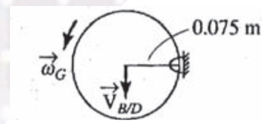
$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$

$$\underset{\measuredangle 30^\circ}{v_B} = \underset{\uparrow}{0.4} + \underset{\downarrow}{(\omega_G)(0.075)}$$

$(\xrightarrow{+})\qquad v_B \cos 30^\circ = 0, \qquad v_B = 0$

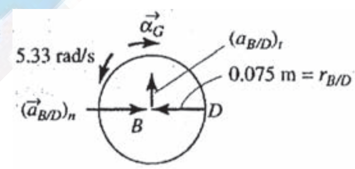
$(+\uparrow)\quad \omega_G = 5.33 \text{ rad/s}$

Since $v_B = 0, \qquad v_C = 0, \qquad \omega_{AC} = 0$ **Ans.**



$\omega_F r_F = \omega_G r_G$

$\omega_F = 5.33\left(\dfrac{100}{50}\right) = 10.7 \text{ rad/s}$ **Ans.**



Acceleration analysis:

$(a_D)_n = (4)^2(0.1) = 1.6 \text{ m/s}^2 \rightarrow$

$(a_D)_t = (20)(0.1) = 2 \text{ m/s}^2 \uparrow$

$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = (\mathbf{a}_D)_n + (\mathbf{a}_D)_t + (\mathbf{a}_{B/D})_n + (\mathbf{a}_{B/D})_t$

$$0 + \underset{\measuredangle 30^\circ}{(a_B)_t} = \underset{\rightarrow}{1.6} + \underset{\uparrow}{2} + \underset{\rightarrow}{(5.33)^2(0.075)} + \underset{\uparrow}{\alpha_G(0.075)}$$

$(+\uparrow)\quad (a_B)_t \sin 30^\circ = 0 + 2 + 0 + \alpha_G(0.075)$

$(\xrightarrow{+})\quad (a_B)_t \cos 30^\circ = 1.6 + 0 + (5.33)^2(0.075) + 0$

Solving,

$(a_B)_t = 4.31 \text{ m/s}^2, \qquad \alpha_G = 2.052 \text{ rad/s}^2 \circlearrowright$

Hence,

$\alpha_{AC} = \dfrac{(a_B)_t}{r_{B/A}} = \dfrac{4.31}{0.15} = 28.7 \text{ rad/s}^2 \circlearrowright$ **Ans.**

Ans:
$\omega_{AC} = 0$
$\omega_F = 10.7 \text{ rad/s} \circlearrowright$
$\alpha_{AC} = 28.7 \text{ rad/s}^2 \circlearrowright$

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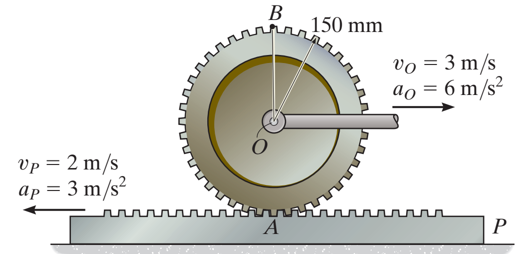
16–125.

The center O of the gear and the gear rack P move with the velocities and accelerations shown. Determine the angular acceleration of the gear and the acceleration of point B located at the rim of the gear at the instant shown.



SOLUTION

Angular Velocity: The location of the IC is indicated in Fig. a. Using similar triangles,

$$\frac{3}{r_{O/IC}} = \frac{2}{0.15 - r_{O/IC}} \qquad r_{O/IC} = 0.09 \text{ m}$$

Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.09} = 33.33 \text{ rad/s}$$

Acceleration and Angular Acceleration: Applying the relative acceleration equation to points O and A and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2 r_{A/O}$$

$$-3\mathbf{i} + (a_A)_n\mathbf{j} = 6\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.15\mathbf{j}) - 33.33^2(-0.15\mathbf{j})$$

$$-3\mathbf{i} + (a_A)_n\mathbf{j} = (6 - 0.15\alpha)\mathbf{i} + 166.67\mathbf{j}$$

Equating the **i** components,

$$-3 = 6 - 0.15\alpha$$

$$\alpha = 60 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Using this result, the relative acceleration equation is applied to points O and B, Fig. b, which gives

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 r_{B/O}$$

$$(a_B)_t\mathbf{i} - (a_B)_n\mathbf{j} = 6\mathbf{i} + (-60\mathbf{k}) \times (0.15\mathbf{j}) - 33.33^2(0.15\mathbf{j})$$

$$(a_B)_t\mathbf{i} - (a_B)_n\mathbf{j} = 15\mathbf{i} - 166.67\mathbf{j}$$

Equating the **i** and **j** components,

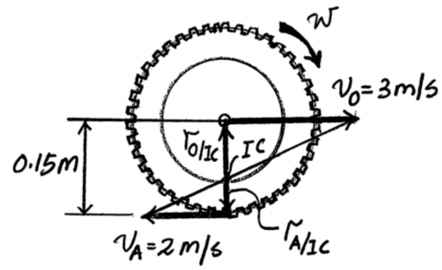
$$(a_B)_t = 15 \text{ m/s}^2 \qquad (a_B)_n = 166.67 \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

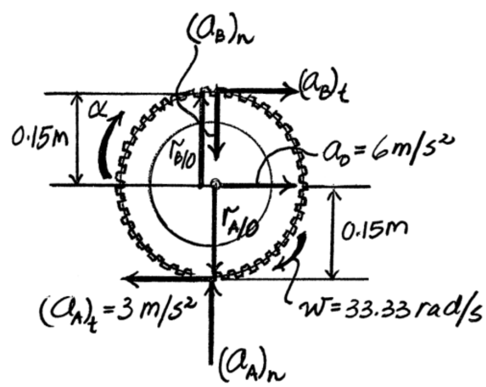
$$a_B = \sqrt{(a_B)_t{}^2 + (a_B)_n{}^2} = \sqrt{15^2 + 166.67^2} = 167 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{166.67}{15}\right) = 84.9^\circ \measuredangle$$



(a)



(b)

Ans:
$\alpha = 60$ rad/s^2
$a_B = 167$ m/s^2

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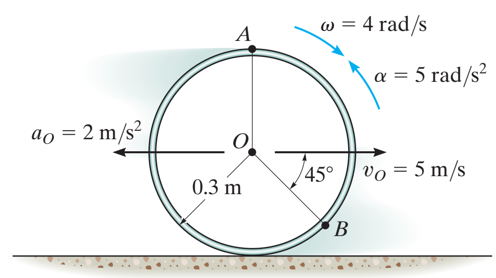
16–126. The hoop is cast on the rough surface such that it has angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity $v_0 = 5$ m/s and a deceleration $a_0 = 2$ m/s^2. Determine the acceleration of point A at this instant.



SOLUTION

Given:

$\omega = 4 \text{ rad/s} \quad a_0 = 2 \text{ m/s}^2$

$\alpha = 5 \text{ rad/s}^2 \quad r = 0.3 \text{ m}$

$v_0 = 5 \text{ m/s} \quad \phi = 45°$

$$\mathbf{a_A} = \begin{pmatrix} -a_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_A} = \begin{pmatrix} -3.50 \\ -4.80 \\ 0.00 \end{pmatrix} \text{m/s}^2 \qquad |\mathbf{a_A}| = 5.94 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$\mathbf{a_A} = \{-3.50\mathbf{i} - 4.80\mathbf{j}\}$ m/s^2
$|\mathbf{a_A}| = 5.94$ m/s^2

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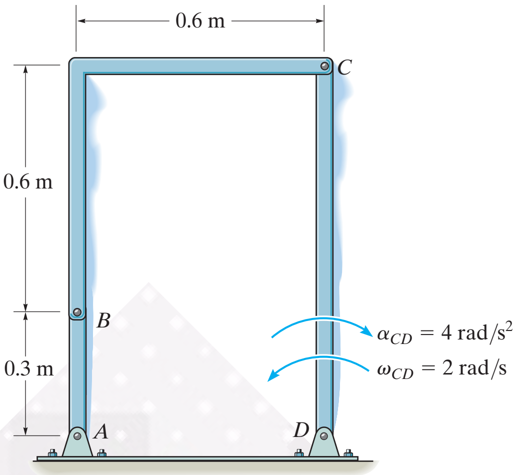
16–127. Determine the angular acceleration of link AB if link CD has the angular velocity and angular deceleration shown.



SOLUTION

Given:

$$\alpha_{CD} = 4\ \text{rad/s}^2 \qquad a = 0.3\ \text{m}$$

$$b = 0.6\ \text{m}$$

$$\omega_{CD} = 2\ \text{rad/s} \qquad c = 0.6\ \text{m}$$

$$\omega_{BC} = 0 \qquad \omega_{AB} = \omega_{CD}\frac{a+b}{a}$$

Guesses

$$\alpha_{AB} = 1\ \text{rad/s}^2 \qquad \alpha_{BC} = 1\ \text{rad/s}^2$$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\alpha_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ a+b \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ a+b \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -b \\ 0 \end{pmatrix} \ldots = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{m/s}^2$$

$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\alpha_{AB}, \alpha_{BC}) \qquad \alpha_{BC} = 12\ \text{rad/s}^2 \qquad \alpha_{AB} = -36\ \text{rad/s}^2$$ **Ans.**

Ans:
$\alpha_{AB} = -36\ \text{rad/s}^2$

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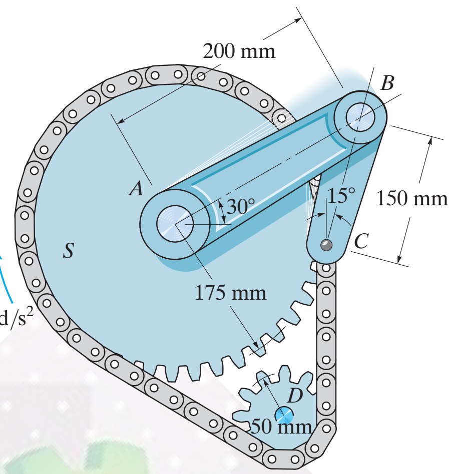
***16–128.**

The mechanism produces intermittent motion of link AB. If the sprocket S is turning with an angular acceleration $\alpha_S = 2\ \text{rad/s}^2$ and has an angular velocity $\omega_S = 6\ \text{rad/s}$ at the instant shown, determine the angular velocity and angular acceleration of link AB at this instant. The sprocket S is mounted on a shaft which is *separate* from a collinear shaft attached to AB at A. The pin at C is attached to one of the chain links such that it moves vertically downward.

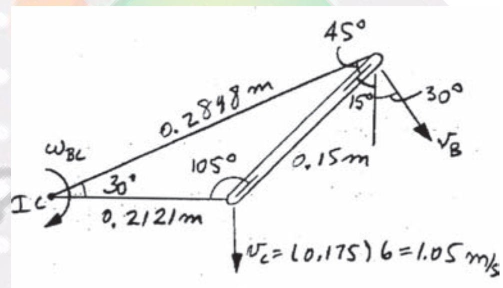


SOLUTION

$$\omega_{BC} = \frac{1.05}{0.2121} = 4.950\ \text{rad/s}$$

$$v_B = (4.95)(0.2898) = 1.434\ \text{m/s}$$

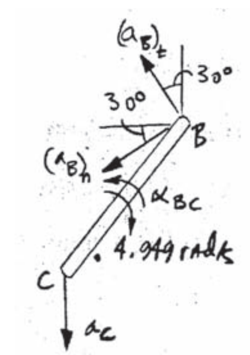
$$\omega_{AB} = \frac{1.435}{0.2} = 7.1722\ \text{rad/s} = 7.17\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$



$$a_C = \alpha_S r_S = 2(0.175) = 0.350\ \text{m/s}^2$$

$$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = \mathbf{a}_C + (\mathbf{a}_{B/C})_n + (\mathbf{a}_{B/C})_t$$

$$\begin{bmatrix} (7.172)^2(0.2) \\ 30^\circ \nearrow \end{bmatrix} + \begin{bmatrix} (a_B)_t \\ \nwarrow 30^\circ \end{bmatrix} = \begin{bmatrix} 0.350 \\ \downarrow \end{bmatrix} + \begin{bmatrix} (4.949)^2\ (0.15) \\ 15^\circ \end{bmatrix} + \begin{bmatrix} \alpha_{BC}\ (0.15) \\ 15^\circ \end{bmatrix}$$



$$(\overset{+}{\rightarrow}) \qquad -(10.29)\cos 30^\circ - (a_B)_t \sin 30^\circ = 0 - (4.949)^2(0.15)\sin 15^\circ - \alpha_{BC}(0.15)\cos 15^\circ$$

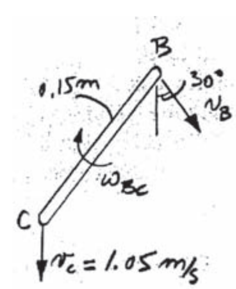
$$(+\uparrow) \qquad -(10.29)\sin 30^\circ + (a_B)_t \cos 30^\circ = -0.350 - (4.949)^2(0.15)\cos 15^\circ + \alpha_{BC}(0.15)\sin 15^\circ$$

$$\alpha_{BC} = 70.8\ \text{rad/s}^2, \qquad (a_B)_t = 4.61\ \text{m/s}^2$$

Hence,

$$\alpha_{AB} = \frac{(a_B)_t}{r_{B/A}} = \frac{4.61}{0.2} = 23.1\ \text{rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

Also,



$$v_C = \omega_S r_S = 6(0.175) = 1.05\ \text{m/s} \downarrow$$

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$v_B \sin 30^\circ \mathbf{i} - v_B \cos 30^\circ \mathbf{j} = -1.05\mathbf{j} + (-\omega_{BC}\mathbf{k}) \times (0.15 \sin 15^\circ \mathbf{i} + 0.15 \cos 15^\circ \mathbf{j})$$

$$(\overset{+}{\rightarrow}) \qquad v_B \sin 30^\circ = 0 + \omega_{BC}(0.15)\cos 15^\circ$$

$$(+\uparrow) \qquad -v_B \cos 30^\circ = -1.05 - \omega_{BC}(0.15)\sin 15^\circ$$

$$v_B = 1.434\ \text{m/s}, \qquad \omega_{BC} = 4.950\ \text{rad/s}$$

$$\omega_{AB} = \frac{v_B}{r_{B/A}} = \frac{1.434}{0.2} = 7.172 = 7.17\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

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***16–128. Continued**

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$(\alpha_{AB}\mathbf{k}) \times (0.2 \cos 30°\mathbf{i} + 0.2 \sin 30°\mathbf{j}) - (7.172)^2(0.2 \cos 30°\mathbf{i} + 0.2 \sin 30°\mathbf{j})$$

$$= -(2)(0.175)\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (0.15 \sin 15°\mathbf{i} + 0.15 \cos 15°\mathbf{j}) - (4.950)^2(0.15 \sin 15°\mathbf{i} + 0.15 \cos 15°\mathbf{j})$$

$$\left(\xrightarrow{+}\right) \qquad -\alpha_{AB}(0.1) - 8.9108 = -0.1449\alpha_{BC} - 0.9512$$

$$(+\uparrow) \qquad \alpha_{AB}(0.1732) - 5.143 = -0.350 + 0.0388\alpha_{BC} - 3.550$$

$\alpha_{AB} = 23.1 \text{ rad/s}^2$ ↺ **Ans.**

$\alpha_{BC} = 70.8 \text{ rad/s}^2$

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Ans:
$\omega_{AB} = 7.17 \text{ rad/s}$ ↻
$\alpha_{AB} = 23.1 \text{ rad/s}^2$ ↺

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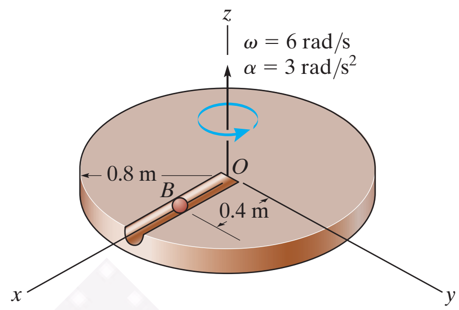
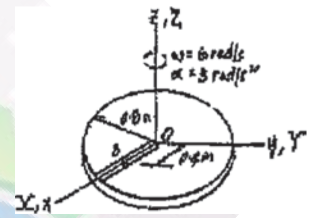
16–129.

At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 150 mm/s^2, both measured relative to the disk and directed away from O. If at the same instant the disk has the angular velocity and angular acceleration shown, determine the velocity and acceleration of the ball at this instant.



SOLUTION



Kinematic Equations:

$$v_B = v_O + \Omega \times \mathbf{r}_{B/O} + (v_{B/O})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (v_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \quad \textbf{(2)}$$

$$v_O = 0$$

$$\mathbf{a}_O = 0$$

$$\Omega = \{6\mathbf{k}\} \text{ rad/s}$$

$$\dot{\Omega} = \{3\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_{B/O} = \{0.4\mathbf{i}\} \text{ m}$$

$$(\boldsymbol{v}_{B/O})_{xyz} = \{0.6\mathbf{i}\} \text{ m/s}$$

$$(\mathbf{a}_{B/O})_{xyz} = \{0.15\mathbf{i}\} \text{ m/s}^2$$

Substitute the date into Eqs. (1) and (2) yields:

$$\boldsymbol{v}_B = 0 + (6\mathbf{k}) \times (0.4\mathbf{i}) + (0.6\mathbf{i}) = \{0.6\mathbf{i} + 2.4\mathbf{j}\} \text{ m/s} \quad \textbf{Ans.}$$

$$\mathbf{a}_B = 0 + (3\mathbf{k}) \times (0.4\mathbf{i}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.4\mathbf{i})] + 2(6\mathbf{k}) \times (0.6\mathbf{i}) + (0.15\mathbf{i})$$

$$= \{-14.2\mathbf{i} + 8.40\mathbf{j}\} \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:
$\mathbf{v}_B = \{0.6\mathbf{i} + 2.4\mathbf{j}\}$ m/s
$\mathbf{a}_B = \{-14.2\mathbf{i} + 8.40\mathbf{j}\}$ m/s^2

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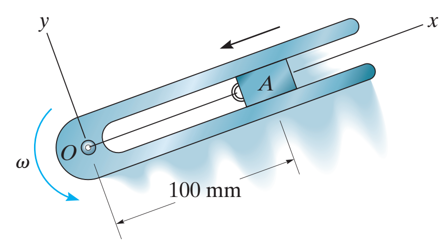
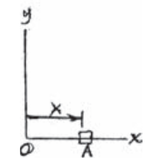
16–130.

Block A, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at O with an acceleration of 4 m/s^2 and its velocity is 2 m/s. Determine the acceleration of the block at this instant. The rod rotates about O with a constant angular velocity $\omega = 4$ rad/s.



SOLUTION

Motion of moving reference.



$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = 4\mathbf{k}$

$\dot{\Omega} = \mathbf{0}$

Motion of A with respect to moving reference.

$\mathbf{r}_{A/O} = 0.1\mathbf{i}$

$\mathbf{v}_{A/O} = -2\mathbf{i}$

$\mathbf{a}_{A/O} = -4\mathbf{i}$

Thus,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (4\mathbf{k}) \times (4\mathbf{k} \times 0.1\mathbf{i}) + 2(4\mathbf{k} \times (-2\mathbf{i})) - 4\mathbf{i}$$

$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\text{ m/s}^2$ **Ans.**

Ans:
$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\text{ m/s}^2$

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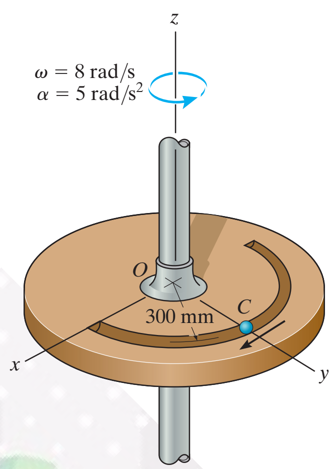
16–131. Ball C moves with a speed of $3\ \text{m/s}$, which is increasing at a constant rate of $1.5\ \text{m/s}^2$, both measured relative to the circular plate and directed as shown. At the same instant the plate rotates with the angular velocity and angular acceleration shown. Determine the velocity and acceleration of the ball at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [8\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \alpha = [5\mathbf{k}]\ \text{rad/s}^2$$

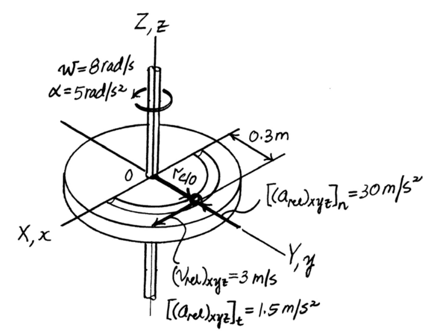
For the motion of ball C with respect to the xyz frame, we have

$$\mathbf{r}_{C/O} = [0.3\mathbf{j}]\ \text{m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [3\mathbf{i}]\ \text{m/s}$$

The normal component of $\left(\mathbf{a}_{\text{rel}}\right)_{xyz}$ is $\left[\left(a_{\text{rel}}\right)_{xyz}\right]_n = \dfrac{\left(v_{\text{rel}}\right)_{xyz}{}^2}{\rho} = \dfrac{3^2}{0.3} = 30\ \text{m/s}^2$. Thus,

$$\left(\mathbf{a}_{\text{rel}}\right)_{xyz} = [1.5\mathbf{i} - 30\mathbf{j}]\ \text{m/s}$$



(a)

Velocity: Applying the relative velocity equation,

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz}\\ &= \mathbf{0} + (8\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{i})\\ &= [0.6\mathbf{i}]\ \text{m/s} \qquad \textbf{Ans.}\end{aligned}$$

Acceleration: Applying the relative acceleration equation.

$$\begin{aligned}\mathbf{a}_C &= \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}\\ &= \mathbf{0} + (5\mathbf{k}) \times (0.3\mathbf{j}) + (8\mathbf{k}) \times [(8\mathbf{k}) \times (0.3\mathbf{j})] + \mathbf{2}(8\mathbf{k}) \times (3\mathbf{i}) + (1.5\mathbf{i} - 30\mathbf{j})\\ &= [-1.2\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}\end{aligned}$$

Ans:
$\mathbf{v}_C = [0.6\mathbf{i}]\ \text{m/s}$
$\mathbf{a}_C = [-1.2\mathbf{j}]\ \text{m/s}^2$

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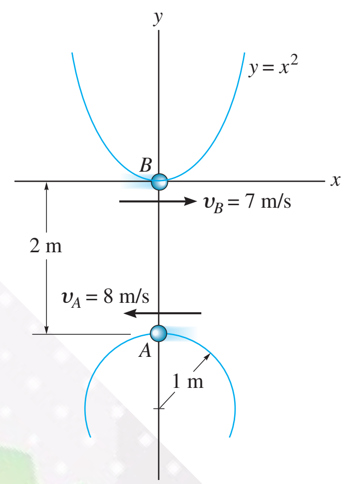
***16–132.**

Particles B and A move along the parabolic and circular paths, respectively. If B has a velocity of 7 m/s in the direction shown and its speed is increasing at 4 m/s^2, while A has a velocity of 8 m/s in the direction shown and its speed is decreasing at 6 m/s^2, determine the relative velocity and relative acceleration of B with respect to A.



SOLUTION

$$\Omega = \frac{8}{1} = 8 \text{ rad/s}^2, \qquad \Omega = \{-8\mathbf{k}\} \text{ rad/s}$$

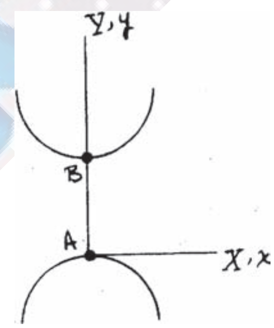
$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$7\mathbf{i} = -8\mathbf{i} + (8\mathbf{k}) \times (2\mathbf{j}) + (\mathbf{v}_{B/A})_{xyz}$$

$$7\mathbf{i} = -8\mathbf{i} - 16\mathbf{i} + (\mathbf{v}_{B/A})_{xyz}$$

$$(\mathbf{v}_{B/A})_{xyz} = \{31.0\mathbf{i}\} \text{ m/s}$$ **Ans.**

$$\dot{\Omega} = \frac{6}{1} = 6 \text{ rad/s}^2, \qquad \dot{\Omega} = \{-6\mathbf{k}\} \text{ rad/s}^2$$



$$(a_A)_n = \frac{(v_A)^2}{1} = \frac{(8)^2}{1} = 64 \text{ m/s}^2 \downarrow$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x\bigg|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = 2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{2} = \frac{1}{2}$$

$$(a_B)_n = \frac{(7)^2}{\frac{1}{2}} = 98 \text{ m/s}^2 \uparrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$4\mathbf{i} + 98\mathbf{j} = 6\mathbf{i} - 64\mathbf{j} + (-6\mathbf{k}) \times (2\mathbf{j}) + (8\mathbf{k}) \times (8\mathbf{k} \times 2\mathbf{j}) + 2(8\mathbf{k}) \times (31\mathbf{i}) + (\mathbf{a}_{B/A})_{xyz}$$

$$4\mathbf{i} + 98\mathbf{j} = 6\mathbf{i} - 64\mathbf{j} + 12\mathbf{i} - 128\mathbf{j} + 496\mathbf{j} + (\mathbf{a}_{B/A})_{xyz}$$

$$(\mathbf{a}_{B/A})_{xyz} = \{-14.0\mathbf{i} - 206\mathbf{j}\} \text{ m/s}^2$$ **Ans.**

Ans:
$(\mathbf{v}_{B/A})_{xyz} = \{31.0\mathbf{i}\}$ m/s
$(\mathbf{a}_{B/A})_{xyz} = \{-14.0\mathbf{i} - 206\mathbf{j}\}$ m/s^2

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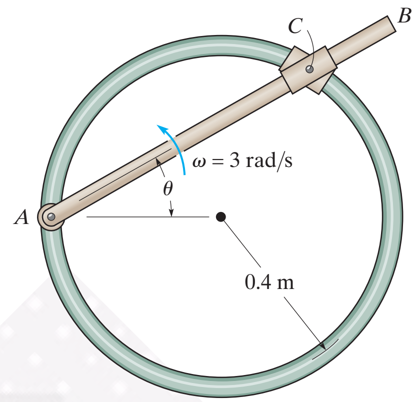
16–133.

Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity of point C located on the double collar when $\theta = 30°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.



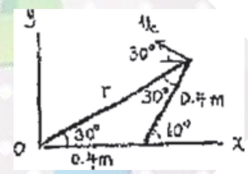
SOLUTION

$r = 2(0.4 \cos 30°) = 0.6928$ m

$\mathbf{r}_{C/A} = 0.6928 \cos 30°\mathbf{i} + 0.6928 \sin 30°\mathbf{j}$

$= \{0.600\mathbf{i} + 0.3464\mathbf{j}\}$ m

$\mathbf{v}_C = -0.866 v_C \mathbf{i} + 0.5 v_C \mathbf{j}$



$v_C = v_A + \Omega \times \mathbf{r}_{C/A} + (v_{C/A})_{xyz}$

$-0.866 v_C \mathbf{i} + 0.5 v_C \mathbf{j} = 0 + (3\mathbf{k}) \times (0.600\mathbf{i} + 0.3464\mathbf{j}) + (v_{C/A} \cos 30°\mathbf{i} + v_{C/A} \sin 30°\mathbf{j})$

$-0.866 v_C \mathbf{i} + 0.5 v_C \mathbf{j} = 0 - 1.039\mathbf{i} + 1.80\mathbf{j} + 0.866 v_{C/A}\mathbf{i} + 0.5 v_{C/A}\mathbf{j}$

$-0.866 v_C = -1.039 + 0.866 v_{C/A}$

$0.5 v_C = 1.80 + 0.5 v_{C/A}$

$v_C = 2.40$ m/s **Ans.**

$v_{C/A} = -1.20$ m/s

جمعية المهندسين الميكانيكيين
شارك الإبداع

Ans:
$v_C = 2.40$ m/s
$\theta = 60°$

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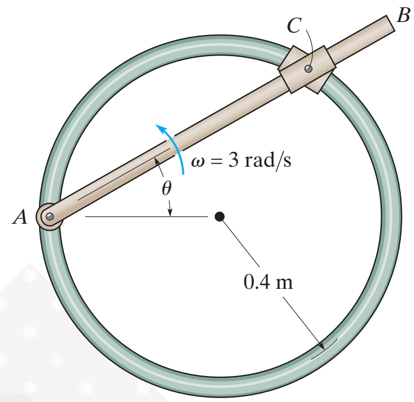
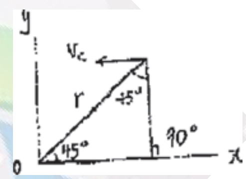
16–134.

Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity and acceleration of point C located on the double collar when $\theta = 45°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.



SOLUTION

$\mathbf{r}_{C/A} = \{0.400\mathbf{i} + 0.400\mathbf{j}\}$



$\mathbf{v}_C = -v_C\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-v_C\mathbf{i} = 0 + (3\mathbf{k}) \times (0.400\mathbf{i} + 0.400\mathbf{j}) + (v_{C/A}\cos 45°\mathbf{i} + v_{C/A}\sin 45°\mathbf{j})$

$-v_C\mathbf{i} = 0 - 1.20\mathbf{i} + 1.20\mathbf{j} + 0.707v_{C/A}\mathbf{i} + 0.707v_{C/A}\mathbf{j}$

$-v_C = -1.20 + 0.707v_{C/A}$

$0 = 1.20 + 0.707v_{C/A}$

$v_C = 2.40$ m/s **Ans.**

$v_{C/A} = -1.697$ m/s

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$$-(a_C)_t\mathbf{i} - \frac{(2.40)^2}{0.4}\mathbf{j} = 0 + 0 + 3\mathbf{k} \times [3\mathbf{k} \times (0.4\mathbf{i} + 0.4\mathbf{j})] + 2(3\mathbf{k}) \times [0.707(-1.697)\mathbf{i} + 0.707(-1.697)\mathbf{j}] + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$$

$-(a_C)_t\mathbf{i} - 14.40\mathbf{j} = 0 + 0 - 3.60\mathbf{i} - 3.60\mathbf{j} + 7.20\mathbf{i} - 7.20\mathbf{j} + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$

$-(a_C)_t = -3.60 + 7.20 + 0.707a_{C/A}$

$-14.40 = -3.60 - 7.20 + 0.707a_{C/A}$

$a_{C/A} = -5.09$ m/s^2

$(a_C)_t = 0$

Thus,

$$a_C = (a_C)_n = \frac{(2.40)^2}{0.4} = 14.4 \text{ m/s}^2$$

$a_C = \{-14.4\mathbf{j}\}$ m/s^2 **Ans.**

Ans:
$v_C = 2.40$ m/s
$a_C = \{-14.4\mathbf{j}\}$ m/s^2

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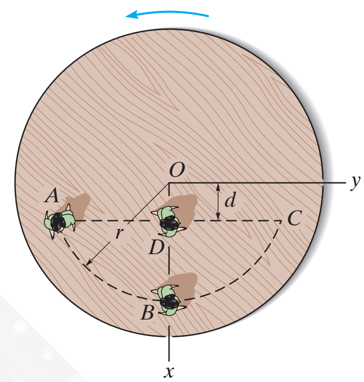
16–135.

A girl stands at A on a platform which is rotating with an angular acceleration $\alpha = 0.2\ \text{rad/s}^2$ and at the instant shown has an angular velocity $\omega = 0.5\ \text{rad/s}$. If she walks at a constant speed $v = 0.75\ \text{m/s}$ measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, $d = 1$ m; and (b) when she reaches point B if she follows the path ABC, $r = 3$ m.



SOLUTION

(a)

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{D/O} + \Omega \times (\Omega \times \mathbf{r}_{D/O}) + 2\Omega \times (\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \qquad \textbf{(1)}$$

Motion of moving reference	*Motion of D with respect to moving reference*
$\mathbf{a}_O = 0$	$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$

Substitute the data into Eq.(1):

$$\mathbf{a}_B = \mathbf{0} + (0.2\mathbf{k}) \times (1\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (1\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + \mathbf{0}$$

$$= \{-1\mathbf{i} + 0.2\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference	*Motion of B with respect to moving reference*
$\mathbf{a}_O = \mathbf{0}$	$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\,\mathbf{i} + (a_{B/O})_t\,\mathbf{j}$
	$= -\left(\frac{0.75^2}{3}\right)\mathbf{i}$
	$= \{-0.1875\mathbf{i}\}$ m/s

Substitute the data into Eq.(2):

$$\mathbf{a}_B = 0 + (0.2\mathbf{k}) \times (3\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (3\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + (-0.1875\mathbf{i})$$

$$= \{-1.69\mathbf{i} + 0.6\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{a}_B = \{-1\mathbf{i} + 0.2\mathbf{j}\}\ \text{m/s}^2$
$\mathbf{a}_B = \{-1.69\mathbf{i} + 0.6\mathbf{j}\}\ \text{m/s}^2$

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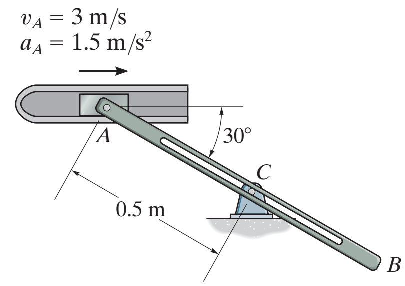
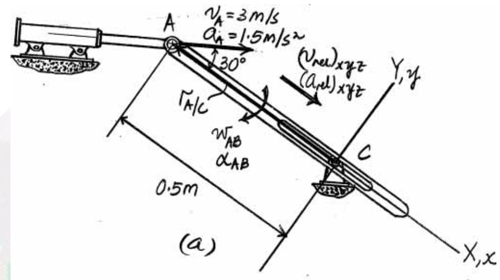
***16–136.**

If the piston is moving with a velocity of $v_A = 3$ m/s and acceleration of $a_A = 1.5\ \text{m/s}^2$, determine the angular velocity and angular acceleration of the slotted link at the instant shown. Link AB slides freely along its slot on the fixed peg C.



SOLUTION

Reference Frame: The xyz reference frame centered at C rotates with link AB and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is



$$\mathbf{v}_C = \mathbf{a}_C = \mathbf{0} \qquad \boldsymbol{\omega}_{AB} = -\omega_{AB}\mathbf{k} \qquad \boldsymbol{\alpha}_{AB} = -\alpha_{AB}\mathbf{k}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/C} = [-0.5\mathbf{i}]\ \text{m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\mathbf{i}$$

The motion of point A with respect to the XYZ frame is

$$\mathbf{v}_A = 3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j} = [2.598\mathbf{i} + 1.5\mathbf{j}]\ \text{m/s}$$

$$a_A = 1.5\cos 30°\mathbf{i} + 1.5\sin 30°\mathbf{j} = [1.299\mathbf{i} + 0.75\mathbf{j}]\ \text{m/s}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$2.598\mathbf{i} + 1.5\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (-0.5\mathbf{i}) + (v_{\text{rel}})_{xyz}\mathbf{i}$$

$$2.598\mathbf{i} + 1.5\mathbf{j} = (v_{\text{rel}})_{xyz}\mathbf{i} + 0.5\omega_{AB}\mathbf{j}$$

Equating the **i** and **j** components,

$$(v_{\text{rel}})_{xyz} = 2.598\ \text{m/s}$$

$$0.5\omega_{AB} = 1.5 \qquad \omega_{AB} = 3\ \text{rad/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\boldsymbol{\omega}}_{AB} \times \mathbf{r}_{A/C} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times r_{A/C}) + 2\boldsymbol{\omega}_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$1.299\mathbf{i} + 0.75\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (-0.5\mathbf{i}) + (-3\mathbf{k}) \times [(-3\mathbf{k}) \times (-0.5\mathbf{i})] + 2(-3\mathbf{k}) \times (2.598\mathbf{i}) + (a_{\text{rel}})_{xyz}\mathbf{i}$$

$$1.299\mathbf{i} + 0.75\mathbf{j} = \left[4.5 + (a_{\text{rel}})_{xyz}\right]\mathbf{i} + (0.5\alpha_{AB} - 15.59)\mathbf{j}$$

Equating the **j** components,

$$0.75 = 0.5\alpha_{AB} - 15.59$$

$$\alpha_{AB} = 32.68\ \text{rad/s}^2 = 32.7\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_{AB} = 3$ rad/s
$\alpha_{AB} = 32.7$ rad/s^2

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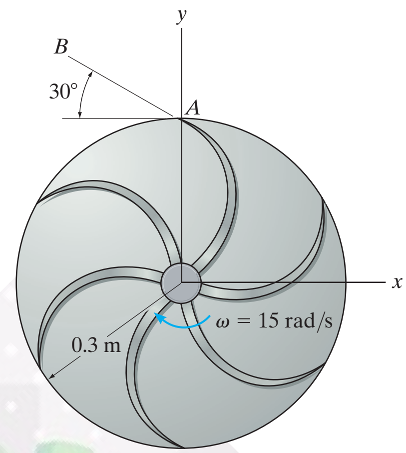
16–137.

Water leaves the impeller of the centrifugal pump with a velocity of 25 m/s and acceleration of 30 m/s^2, both measured relative to the impeller along the blade line AB. Determine the velocity and acceleration of a water particle at A as it leaves the impeller at the instant shown. The impeller rotates with a constant angular velocity of $\omega = 15$ rad/s.

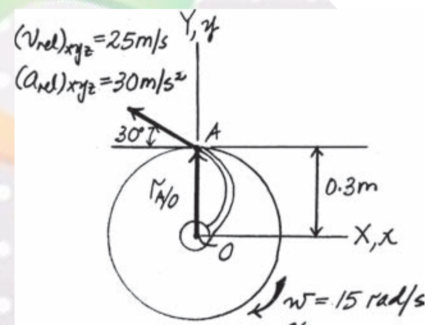


SOLUTION

Reference Frame: The xyz rotating reference frame is attached to the impeller and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [-15\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \mathbf{0}$$

The motion of point A with respect to the xyz frame is



(a)

$$\mathbf{r}_{A/O} = [0.3\mathbf{j}]\ \text{m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-25\cos 30°\,\mathbf{i} + 25\sin 30°\,\mathbf{j}) = [-21.65\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-30\cos 30°\,\mathbf{i} + 30\sin 30°\,\mathbf{j}) = [-25.98\mathbf{i} + 15\mathbf{j}]\ \text{m/s}^2$$

Velocity: Applying the relative velocity equation.

$$\mathbf{v}_A = \mathbf{v}_O + \omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-15\mathbf{k}) \times (0.3\mathbf{j}) + (-21.65\mathbf{i} + 12.5\mathbf{j})$$

$$= [-17.2\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{A/O} + \omega \times (\omega \times \mathbf{r}_{A/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-15\mathbf{k}) \times [(-15\mathbf{k}) \times (0.3\mathbf{j})] + 2(-15\mathbf{k}) \times (-21.65\mathbf{i} + 12.5\mathbf{j}) + (-25.98\mathbf{i} + 15\mathbf{j})$$

$$= [349\mathbf{i} + 597\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-17.2\mathbf{i} + 12.5\mathbf{j}\}$ m/s
$\mathbf{a}_A = \{349\mathbf{i} + 597\mathbf{j}\}$ m/s^2

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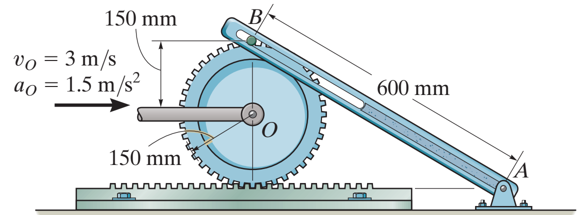
16–138.

Peg B on the gear slides freely along the slot in link AB. If the gear's center O moves with the velocity and acceleration shown, determine the angular velocity and angular acceleration of the link at this instant.



SOLUTION

Gear Motion: The IC of the gear is located at the point where the gear and the gear rack mesh, Fig. a. Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.15} = 20 \text{ rad/s}$$

Then,

$$v_B = \omega r_{B/IC} = 20(0.3) = 6 \text{ m/s} \rightarrow$$

Since the gear rolls on the gear rack, $\alpha = \dfrac{a_O}{r} = \dfrac{1.5}{0.15} = 10$ rad/s. By referring to Fig. b,

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$(a_B)_t\,\mathbf{i} - (a_B)_n\,\mathbf{j} = 1.5\mathbf{i} + (-10\mathbf{k}) \times 0.15\mathbf{j} - 20^2(0.15\mathbf{j})$$

$$(a_B)_t\,\mathbf{i} - (a_B)_n\,\mathbf{j} = 3\mathbf{i} - 60\mathbf{j}$$

Thus,

$$(a_B)_t = 3 \text{ m/s}^2 \qquad (a_B)_n = 60 \text{ m/s}^2$$

Reference Frame: The $x'y'z'$ rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame, Figs. c and d. Thus, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ frame is

$$\mathbf{v}_B = [6 \sin 30°\mathbf{i} - 6 \cos 30°\,\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_B = (3 \sin 30° - 60 \cos 30°)\mathbf{i} + (-3 \cos 30° - 60 \sin 30°)\mathbf{j}$$

$$= [-50.46\mathbf{i} - 32.60\mathbf{j}] \text{ m/s}^2$$

For motion of the $x'y'z'$ frame with reference to the XYZ reference frame,

$$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0} \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

For the motion of point B with respect to the $x'y'z'$ frame is

$$\mathbf{r}_{B/A} = [0.6\mathbf{j}]\text{m} \qquad (\mathbf{v}_{\text{rel}})_{x'y'z'} = (v_{\text{rel}})_{x'y'z'}\,\mathbf{j} \qquad (\mathbf{a}_{\text{rel}})_{x'y'z'} = (a_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{x'y'z'}$$

$$3\mathbf{i} - 5.196\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (v_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

$$3\mathbf{i} - 5.196\mathbf{j} = 0.6\omega_{AB}\,\mathbf{i} + (v_{\text{rel}})_{x'y'z'}\mathbf{j}$$

Equating the **i** and **j** components yields

$$3 = 0.6\omega_{AB} \qquad \omega_{AB} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(v_{\text{rel}})_{x'y'z'} = -5.196 \text{ m/s}$$

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{x'y'z'} + (\mathbf{a}_{\text{rel}})_{x'y'z'}$$

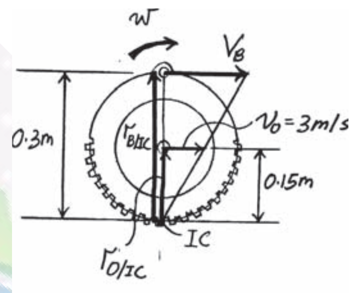
$$-50.46\mathbf{i} - 32.60\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (0.6\mathbf{j})] + 2(-5\mathbf{k}) \times (-5.196\mathbf{j}) + (a_{\text{rel}})_{x'y'z'}\mathbf{j}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = (0.6\alpha_{AB} - 51.96)\mathbf{i} + \left[(a_{\text{rel}})_{x'y'z'} - 15\right]\mathbf{j}$$

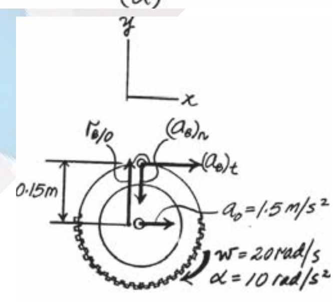
Equating the **i** components,

$$-50.46 = 0.6\alpha_{AB} - 51.96$$

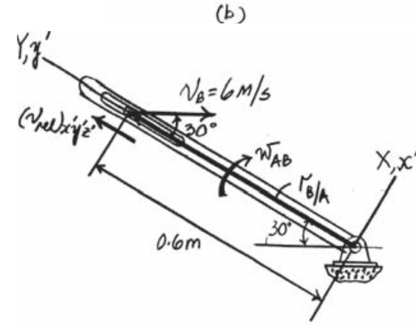
$$\alpha_{AB} = 2.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



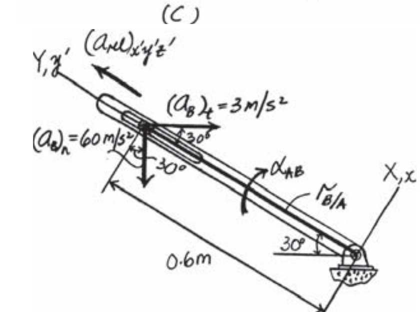
(a)



(b)



(c)



(d)

Ans:
$\omega_{AB} = 5$ rad/s
$\alpha_{AB} = 2.5$ rad/s^2

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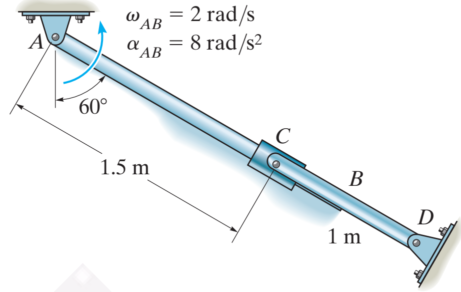
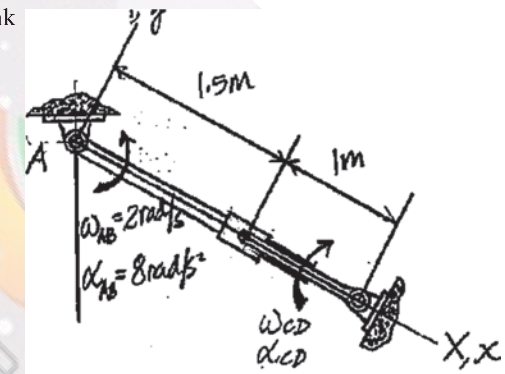
16–139.

The collar C is pinned to rod CD while it slides on rod AB. If rod AB has an angular velocity of 2 rad/s and an angular acceleration of 8 rad/s^2, both acting counterclockwise, determine the angular velocity and the angular acceleration of rod CD at the instant shown.



SOLUTION

The fixed and rotating $X - Y$ and $x - y$ coordinate systems are set to coincide with origin at A as shown in Fig. a. Here, the $x - y$ coordinate system is attached to link AC. Thus,



(a)

Motion of moving Reference	**Motion of collar C with respect to moving Reference**
$\mathbf{v}_A = \mathbf{0}$	$\mathbf{r}_{C/A} = \{1.5\mathbf{i}\}$ m
$\mathbf{a}_A = \mathbf{0}$	
$\mathbf{\Omega} = \boldsymbol{\omega}_{AB} = \{2\mathbf{k}\}$ rad/s	$(\mathbf{v}_{C/A})_{xyz} = (\mathbf{v}_{C/A})_{xyz}\mathbf{i}$
$\dot{\mathbf{\Omega}} = \boldsymbol{\alpha}_{AB} = \{8\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{C/A})_{xyz} = (\mathbf{a}_{C/A})_{xyz}\mathbf{i}$

The motions of collar C in the fixed system are

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = (-\omega_{CD}\mathbf{k}) \times (-\mathbf{i}) = \omega_{CD}\mathbf{j}$$

$$a_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}{}^2\,\mathbf{r}_{C/D} = (-\alpha_{C/D}\mathbf{k}) \times (-\mathbf{i}) - \omega_{CD}^2(-\mathbf{i}) = \omega_{CD}^2\mathbf{i} + \alpha_{CD}\mathbf{j}$$

Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$\omega_{CD}\mathbf{j} = \mathbf{0} + (2\mathbf{k}) \times (1.5\mathbf{i}) = (v_{C/A})_{xyz}\mathbf{i}$$

$$\omega_{CD}\mathbf{j} = (v_{C/A})_{xyz}\mathbf{i} + 3\mathbf{j}$$

Equating **i** and **j** components

$$(v_{C/A})_{xyz} = 0$$

$$\omega_{CD} = 3.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Applying the relative acceleration equation,

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$3.00^2\mathbf{i} + \alpha_{CD}\mathbf{j} = \mathbf{0} + (8\mathbf{k}) \times (1.5\mathbf{i}) + (2\mathbf{k}) \times (2\mathbf{k} \times 1.5\mathbf{i}) + 2(2\mathbf{k}) \times \mathbf{0} + (a_{C/A})_{xyz}\mathbf{i}$$

$$9\mathbf{i} + \alpha_{CD}\mathbf{j} = \left[(a_{C/A})_{xyz} - 6\right]\mathbf{i} + 12\mathbf{j}$$

Equating **i** and **j** components,

$$9 = (a_{C/A})_{xyz} - 6; \quad (a_{C/A})_{xyz} = 15 \text{ m/s}^2$$

$$\alpha_{CD} = 12.0 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 3.00$ rad/s $\circlearrowright$
$\alpha_{CD} = 12.0$ rad/s^2 $\circlearrowright$

جمعية المهندسين الميكانيكيين
شعاع الإبداع

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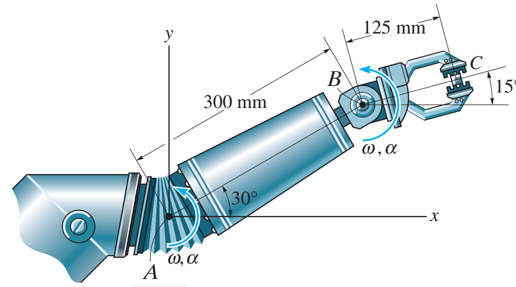
***16–140.**

At the instant shown, the robotic arm AB is rotating counterclockwise at $\omega = 5$ rad/s and has an angular acceleration $\alpha = 2$ rad/s^2. Simultaneously, the grip BC is rotating counterclockwise at $\omega' = 6$ rad/s and $\alpha' = 2$ rad/s^2, both measured relative to a *fixed* reference. Determine the velocity and acceleration of the object held at the grip C.

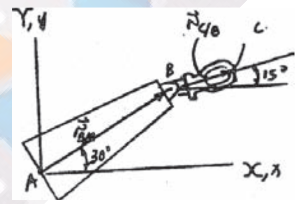


SOLUTION

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference	***Motion of C with respect to moving reference***
	$\mathbf{r}_{C/B} = \{0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}\}$ m
$\Omega = \{6\mathbf{k}\}$ rad/s	$(\mathbf{v}_{C/B})_{xyz} = 0$
$\dot{\Omega} = \{2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{C/B})_{xyz} = 0$



Motion of B:

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A}$$
$$= (5\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-0.75\mathbf{i} + 1.2990\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$
$$= (2\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j}) - (5)^2(0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-6.7952\mathbf{i} - 3.2304\mathbf{j}\} \text{ m/s}^2$$

Substitute the data into Eqs. (1) and (2) yields:

$$\mathbf{v}_C = (-0.75\mathbf{i} + 1.2990\mathbf{j}) + (6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}) + 0$$
$$= \{-0.944\mathbf{i} + 2.02\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = (-6.79527\mathbf{i} - 3.2304\mathbf{j}) + (2\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})$$
$$+ (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})] + 0 + 0$$
$$= \{-11.2\mathbf{i} - 4.15\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-0.944\mathbf{i} + 2.02\mathbf{j}\}$ m/s
$\mathbf{a}_C = \{-11.2\mathbf{i} - 4.15\mathbf{j}\}$ m/s^2

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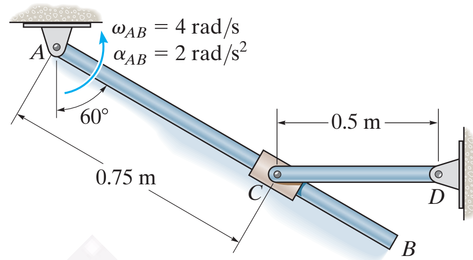
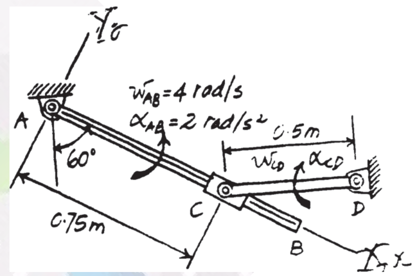
16–141.

At the instant shown rod AB has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin connected to CD and slides freely along AB.



SOLUTION

Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point A. The x, y, z moving frame is attached to and rotate with rod AB since collar C slides along rod AB.



Kinematic Equation: Applying Eqs. 16–26 and 16–29, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference

$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = 4\mathbf{k}$ rad/s

$\dot{\Omega} = 2\mathbf{k}$ rad/s^2

Motion of C with respect to moving reference

$r_{C/A} = \{0.75\mathbf{i}\}$m

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

The velocity and acceleration of collar C can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}\text{m} = \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$
$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$
$$= -\alpha_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$
$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.(1) yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$
$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$
$$-0.250\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\,\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$
$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 6.93$ rad/s

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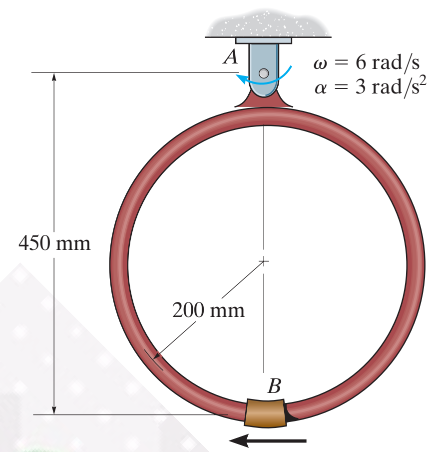
16–142.

Collar B moves to the left with a speed of 5 m/s, which is increasing at a constant rate of 1.5 m/s^2, relative to the hoop, while the hoop rotates with the angular velocity and angular acceleration shown. Determine the magnitudes of the velocity and acceleration of the collar at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the hoop and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$v_A = a_A = \mathbf{0} \qquad \omega = [-6\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \alpha = [-3\mathbf{k}] \text{ rad/s}^2$$

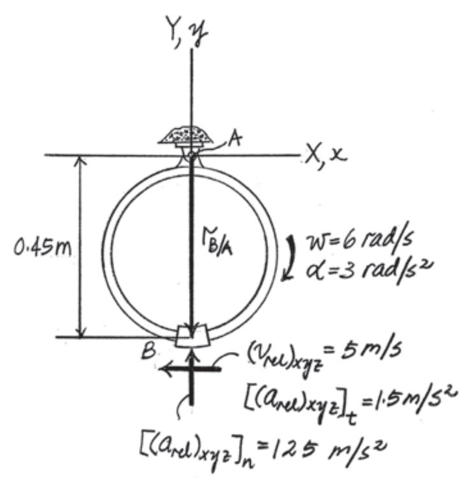
For the motion of collar B with respect to the xyz frame,

$$\mathbf{r}_{B/A} = [-0.45\mathbf{j}] \text{ m}$$

$$(v_{\text{rel}})_{xyz} = [-5\mathbf{i}] \text{ m/s}$$

The normal components of $(\mathbf{a}_{\text{rel}})_{xyz}$ is $[(a_{\text{rel}})_{xyz}]_n = \dfrac{(v_{\text{rel}})_{xyz}{}^2}{\rho} = \dfrac{5^2}{0.2} = 125 \text{ m/s}^2$. Thus,

$$(\mathbf{a}_{\text{rel}})_{xyz} = [-1.5\mathbf{i} + 125\mathbf{j}] \text{ m/s}$$



(a)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$
$$= \mathbf{0} + (-6\mathbf{k}) \times (-0.45\mathbf{j}) + (-5\mathbf{i})$$
$$= [-7.7\mathbf{i}] \text{ m/s}$$

Thus,

$$v_B = 7.7 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$
$$= \mathbf{0} + (-3\mathbf{k}) \times (-0.45\mathbf{j}) + (-6\mathbf{k}) \times [(-6\mathbf{k}) \times (-0.45\mathbf{j})] + 2(-6\mathbf{k}) \times (-5\mathbf{i}) + (-1.5\mathbf{i} + 125\mathbf{j})$$
$$= [-2.85\mathbf{i} + 201.2\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is therefore

$$a_B = \sqrt{2.85^2 + 201.2^2} = 201 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.7$ m/s
$a_B = 201$ m/s^2

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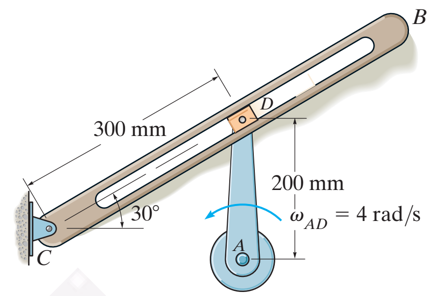
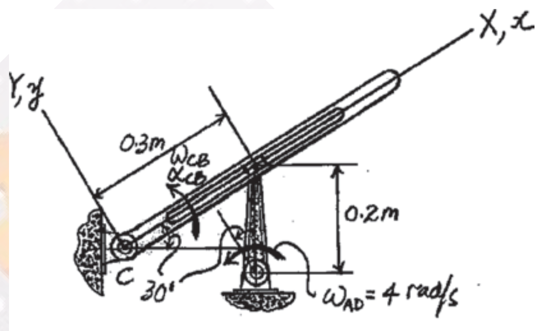
16–143.

Block D of the mechanism is confined to move within the slot of member CB. If link AD is rotating at a constant rate of $\omega_{AD} = 4$ rad/s, determine the angular velocity and angular acceleration of member CB at the instant shown.



SOLUTION

The fixed and rotating $X - Y$ and $x - y$ coordinate system are set to coincide with origin at C as shown in Fig. a. Here the $x - y$ coordinate system is attached to member CB. Thus



(a)

Motion of moving Reference	**Motion of Block D with respect to moving Reference**
$\mathbf{v}_C = \mathbf{0}$	$\mathbf{r}_{D/C} = \{0.3\mathbf{i}\}$ m
$\mathbf{a}_C = \mathbf{0}$	
$\mathbf{\Omega} = \boldsymbol{\omega}_{CB} = \omega_{CB}\mathbf{k}$	$(\mathbf{v}_{D/C})_{xyz} = (\mathbf{v}_{D/C})_{xyz}\mathbf{i}$
$\dot{\mathbf{\Omega}} = \boldsymbol{\alpha}_{CB} = \alpha_{CB}\mathbf{k}$	$(\mathbf{a}_{D/C})_{xyz} = (\mathbf{a}_{D/C})_{xyz}\mathbf{i}$

The Motions of Block D in the fixed frame are,

$$\mathbf{v}_D = \boldsymbol{\omega}_{A/D} \times \mathbf{r}_{D/A} = (4\mathbf{k}) \times (0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j}) = \{-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_D = \boldsymbol{\alpha}_{AD} \times \mathbf{r}_{D/A} - \omega_{AD}^2(\mathbf{r}_{D/A}) = 0 - 4^2(0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j})$$
$$= \{-1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j}\} \text{ m/s}^2$$

Applying the relative velocity equation,

$$\mathbf{v}_D = \mathbf{v}_C + \mathbf{\Omega} \times \mathbf{r}_{D/C} + (\mathbf{v}_{D/C})_{xyz}$$

$$-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j} = \mathbf{0} + (\omega_{CB}\mathbf{k}) \times (0.3\mathbf{i}) + (v_{D/C})_{xyz}\mathbf{i}$$

$$-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j} = (v_{D/C})_{xyz}\mathbf{i} + 0.3\,\omega_{CB}\mathbf{j}$$

Equating **i** and **j** components,

$$(v_{D/C})_{xyz} = -0.4\sqrt{3} \text{ m/s}$$

$0.4 = 0.3\,\omega_{CB}$; $\omega_{CB} = 1.3333$ rad/s $= 1.33$ rad/s ↺ **Ans.**

Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_C + \dot{\mathbf{\Omega}} \times \mathbf{r}_{D/C} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{D/C}) + 2\mathbf{\Omega} \times (\mathbf{v}_{D/C})_{xyz} + (\mathbf{a}_{D/C})_{xyz}$$

$$-1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j} = 0 + (\alpha_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + (1.3333\mathbf{k}) \times (1.3333\mathbf{k} \times 0.3\mathbf{i})$$
$$+ 2(1.3333\mathbf{k}) \times (-0.4\sqrt{3}\mathbf{i}) + (a_{D/C})_{xyz}\mathbf{i}$$

$$1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j} = [(a_{D/C})_{xyz} - 0.5333]\mathbf{i} + (0.3\alpha_{CD} - 1.8475)\mathbf{j}$$

Equating **i** and **j** components

$1.6 = [(a_{D/C})_{xyz} - 0.5333]$; $(a_{D/C})_{xyz} = 2.1333 \text{ m/s}^2$

$-1.6\sqrt{3} = 0.3\,\alpha_{CD} - 1.8475$; $\alpha_{CD} = -3.0792 \text{ rad/s}^2 = 3.08 \text{ rad/s}^2$ ↻ **Ans.**

Ans:
$\omega_{CB} = 1.33$ rad/s ↺
$\alpha_{CD} = 3.08 \text{ rad/s}^2$ ↻

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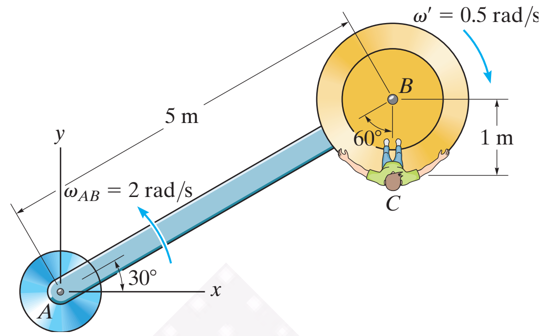
***16–144.** A ride in an amusement park consists of a rotating arm AB having a constant angular velocity $\omega_{AB} = 2$ rad/s about point A and a car mounted at the end of the arm which has a constant angular velocity $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at C.



SOLUTION

Given:

$$\omega_{AB} = 2 \text{ rad/s}$$

$$a = 5 \text{ m}$$

$$\omega' = 0.5 \text{ rad/s}$$

$$r = 1 \text{ m}$$

$$\theta = 30°$$

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -3.5 \\ 8.66 \\ 0 \end{pmatrix} \text{m/s} \quad \textbf{Ans.}$$

$$\mathbf{a_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_C} = \begin{pmatrix} -17.32 \\ -7.75 \\ 0.00 \end{pmatrix} \text{m/s}^2 \quad \textbf{Ans.}$$

Ans:
$\mathbf{v_C} = \{-3.5\mathbf{i} + 8.66\mathbf{j}\}$ m/s^2
$\mathbf{a_C} = \{-17.32\mathbf{i} - 7.75\mathbf{j}\}$ m/s^2

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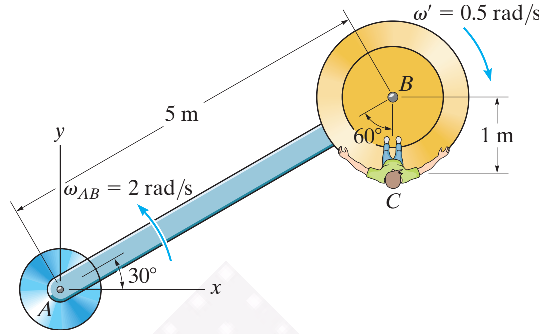
16–145. A ride in an amusement park consists of a rotating arm AB that has an angular acceleration of $\alpha_{AB} = 1\ \text{rad/s}^2$ when $\omega_{AB} = 2\ \text{rad/s}$ at the instant shown. Also at this instant the car mounted at the end of the arm has a relative angular acceleration of $\boldsymbol{\alpha}' = \{-0.6\mathbf{k}\}\ \text{rad/s}^2$ when $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}\ \text{rad/s}$. Determine the velocity and acceleration of the passenger C at this instant.



SOLUTION

Given:

$\omega_{AB} = 2\ \text{rad/s} \qquad \alpha_{AB} = 1\ \text{rad/s}^2$

$\omega' = 0.5\ \text{rad/s} \qquad \alpha' = 0.6\ \text{rad/s}^2$

$a = 5\ \text{m} \qquad r = 1\ \text{m}$

$\theta = 30°$

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -3.5 \\ 8.66 \\ 0 \end{pmatrix}\ \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a_C} = \left[\begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}\right]\right] \ldots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} - \alpha' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_C} = \begin{pmatrix} -19.42 \\ -3.42 \\ 0.00 \end{pmatrix}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v_C} = \{-3.5\mathbf{i} + 8.66\mathbf{j}\}\ \text{m/s}^2$
$\mathbf{a_C} = \{-19.42\mathbf{i} - 3.42\mathbf{j}\}\ \text{m/s}^2$

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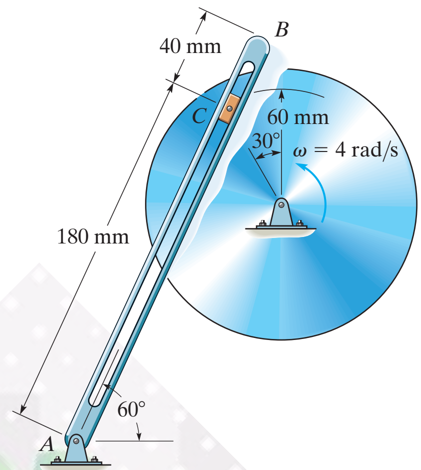
16–146.

If the slider block C is fixed to the disk that has a constant counterclockwise angular velocity of 4 rad/s, determine the angular velocity and angular acceleration of the slotted arm AB at the instant shown.

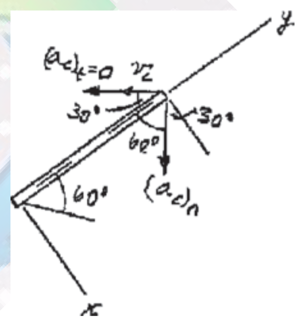


SOLUTION

$\mathbf{v}_C = -(4)(60)\sin 30°\mathbf{i} - 4(60)\cos 30°\mathbf{j} = -120\mathbf{i} - 207.85\mathbf{j}$

$\mathbf{a}_C = (4)^2(60)\sin 60°\mathbf{i} - (4)^2(60)\cos 60°\mathbf{j} = 831.38\mathbf{i} - 480\mathbf{j}$

Thus,



$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-120\mathbf{i} - 207.85\mathbf{j} = \mathbf{0} + \left(\omega_{AB}\mathbf{k}\right) \times (180\mathbf{j}) - v_{C/A}\mathbf{j}$

$-120 = -180\omega_{AB}$

$\omega_{AB} = 0.667 \text{ rad/s} \circlearrowright$ **Ans.**

$-207.85 = -v_{C/A}$

$v_{C/A} = 207.85 \text{ mm/s}$

$\mathbf{a}_C = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/A} + \mathbf{\Omega} \times \left(\mathbf{\Omega} \times \mathbf{r}_{C/A}\right) + 2\mathbf{\Omega} \times \left(v_{C/A}\right)_{xyz} + \left(\mathbf{a}_{C/A}\right)_{xyz}$

$831.38\mathbf{i} - 480\mathbf{j} = 0 + (\alpha_{AB}\mathbf{k}) \times (180\mathbf{j}) + (0.667\mathbf{k}) \times [(0.667\mathbf{k}) \times (180\mathbf{j})]$

$\quad + 2(0.667\mathbf{k}) \times (-207.85\mathbf{j}) - a_{C/A}\mathbf{j}$

$831.38\mathbf{i} - 480\mathbf{j} = -180\,\alpha_{AB}\mathbf{i} - 80\mathbf{j} + 277.13\mathbf{i} - a_{C/A}\mathbf{j}$

$831.38 = -180\alpha_{AB} + 277.13$

$\alpha_{AB} = -3.08$

Thus,

$\alpha_{AB} = 3.08 \text{ rad/s}^2 \circlearrowright$ **Ans.**

$-480 = -80 - a_{C/A}$

$a_{C/A} = 400 \text{ mm/s}^2$

Ans:
$\omega_{AB} = 0.667 \text{ rad/s} \circlearrowright$
$\alpha_{AB} = 3.08 \text{ rad/s}^2 \circlearrowright$

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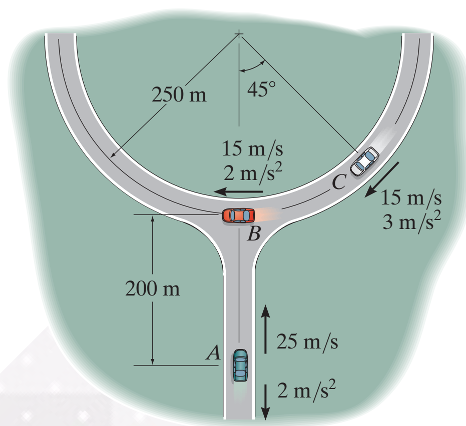
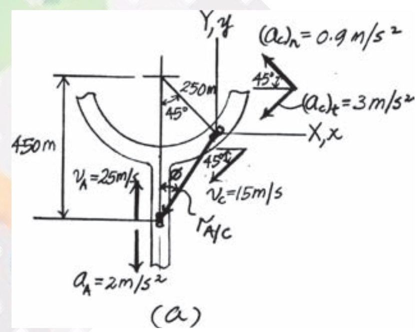
16–147.

At the instant shown, car A travels with a speed of 25 m/s, which is decreasing at a constant rate of 2 m/s^2, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of 3 m/s^2. Determine the velocity and acceleration of car A with respect to car C.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to car C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since car C moves along the circular road, its normal component of acceleration is $(a_C)_n = \dfrac{v_C^{\,2}}{\rho} = \dfrac{15^2}{250} = 0.9\text{ m/s}^2$. Thus, the motion of car C with respect to the XYZ frame is



$$\mathbf{v}_C = -15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j} = [-10.607\mathbf{i} - 10.607\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\mathbf{j}]\text{ m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame is

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\text{ rad/s} \qquad \boldsymbol{\omega} = [-0.06\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\text{ rad/s}^2 \qquad \dot{\boldsymbol{\omega}} = [-0.012\mathbf{k}]\text{ rad/s}^2$$

The velocity and accdeleration of car A with respect to the XYZ frame is

$$\mathbf{v}_A = [25\mathbf{j}]\text{ m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}]\text{ m/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{A/C} = -250\sin 45°\mathbf{i} - (450 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 273.22\mathbf{j}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\boldsymbol{\omega}} \times r_{A/C} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/C}) + 2\boldsymbol{\omega} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})$$
$$+ (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = -2.4\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]$ m/s
$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]$ m/s^2

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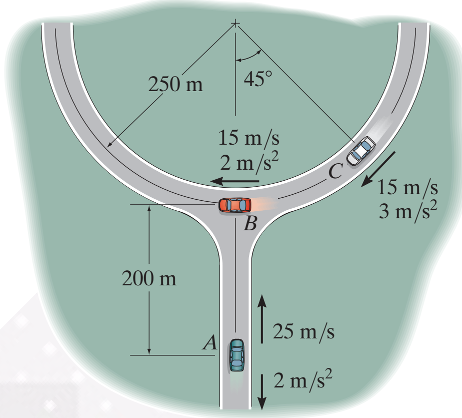
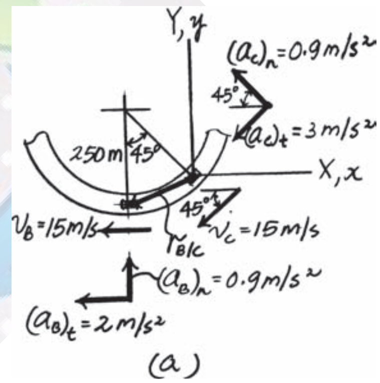
***16–148.**

At the instant shown, car B travels with a speed of 15 m/s, which is increasing at a constant rate of $2\,\text{m/s}^2$, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of $3\ \text{m/s}^2$. Determine the velocity and acceleration of car B with respect to car C.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since B and C move along the circular road, their normal components of acceleration are $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$ and $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$. Thus, the motion of cars B and C with respect to the XYZ frame are



$$\mathbf{v}_B = [-15\mathbf{i}]\ \text{m/s}$$

$$\mathbf{v}_C = [-15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j}] = [-10.607\mathbf{i} - 10.607\mathbf{j}]\ \text{m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}]\ \text{m/s}^2$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\,\mathbf{j}]\ \text{m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\ \text{rad/s} \qquad \omega = [-0.06\mathbf{k}]\ \text{rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\ \text{rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\ \text{rad/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{B/C} = -250\sin 45°\mathbf{i} - (250 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 73.22\,\mathbf{j}]\ \text{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times r_{B/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = -15\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = \mathbf{0} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \dot{\omega} \times \mathbf{r}_{B/C} + \omega \times (\omega \times \mathbf{r}_{B/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})$$
$$+(-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times \mathbf{0} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = -3\mathbf{i} + 0.9\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(a_{\text{rel}})_{xyz} = [1\mathbf{i}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{\text{rel}})_{xyz} = \mathbf{0}$
$(\mathbf{a}_{\text{rel}})_{xyz} = \{1\mathbf{i}\}\ \text{m/s}^2$

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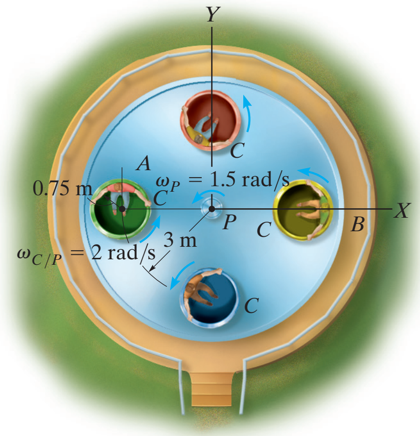
16–149. A ride in an amusement park consists of a rotating platform P, having constant angular velocity $\omega_P = 1.5$ rad/s, and four cars, C, mounted on the platform, which have constant angular velocities $\omega_{C/P} = 2$ rad/s measured relative to the platform. Determine the velocity and acceleration of the passenger at B at the instant shown.



SOLUTION

Given: $\omega_P = 1.5$ rad/s $\quad r = 0.75$ m

$\omega_{CP} = 2$ rad/s $\quad R = 3$ m

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v_B} = \begin{pmatrix} 0.00 \\ 7.13 \\ 0.00 \end{pmatrix} \text{m/s} \qquad |\mathbf{v_B}| = 7.13 \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_B} = \begin{pmatrix} -15.94 \\ 0.00 \\ 0.00 \end{pmatrix} \text{m/s}^2 \qquad |\mathbf{a_B}| = 15.94 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$|\mathbf{v_B}| = 7.13 \text{ m/s}^2$

$|\mathbf{a_B}| = 15.94 \text{ m/s}^2$

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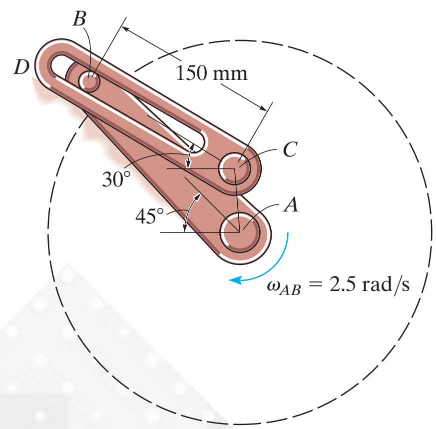
16–150.

The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s, determine the angular velocity of CD (output) at this instant.



SOLUTION

$$\frac{r_{BA}}{\sin 120^\circ} = \frac{0.15\text{ m}}{\sin 45^\circ}$$

$$r_{BA} = 0.1837\text{ m}$$

$$\mathbf{v}_C = \mathbf{0}$$

$$\mathbf{a}_C = \mathbf{0}$$

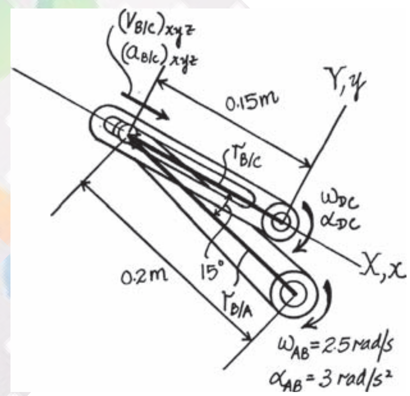
$$\Omega = -\omega_{DC}\mathbf{k}$$

$$\dot{\Omega} = -\alpha_{DC}\mathbf{k}$$

$$\mathbf{r}_{B/C} = \{-0.15\,\mathbf{i}\}\text{ m}$$

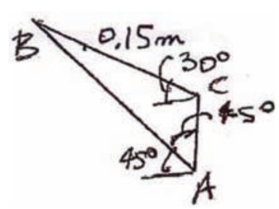
$$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\mathbf{i}$$

$$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\mathbf{i}$$



$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.1837\cos 15^\circ\mathbf{i} + 0.1837\sin 15^\circ\mathbf{j})$$

$$= \{0.1189\mathbf{i} + 0.4436\mathbf{j}\}\text{ m/s}$$



$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$$

$$0.1189\mathbf{i} + 0.4436\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\mathbf{i}$$

$$0.1189\mathbf{i} + 0.4436\mathbf{j} = (v_{B/C})_{xyz}\mathbf{i} + 0.15\omega_{DC}\mathbf{j}$$

Solving:

$$(v_{B/C})_{xyz} = 0.1189\text{ m/s}$$

$$\omega_{DC} = 2.96\text{ rad/s} \circlearrowright$$ **Ans.**

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Ans:
$\omega_{DC} = 2.96$ rad/s $\circlearrowright$

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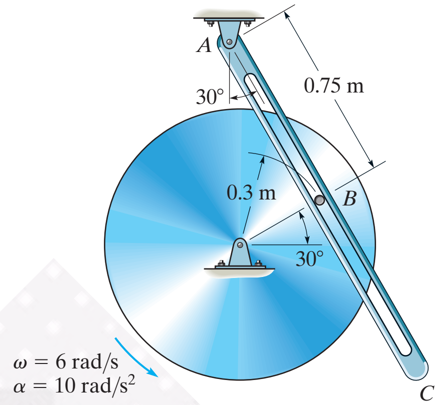
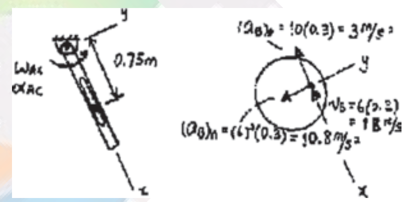
16–151.

The disk rotates with the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link AC at this instant. The peg at B is fixed to the disk.



SOLUTION



$\mathbf{v}_B = -6(0.3)\mathbf{i} = -1.8\mathbf{i}$

$\mathbf{a}_B = -10(0.3)\mathbf{i} - (6)^2(0.3)\mathbf{j} = -3\mathbf{i} - 10.8\mathbf{j}$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (v_{B/A})_{xyz}$

$-1.8\mathbf{i} = 0 + (\omega_{AC}\mathbf{k}) \times (0.75\mathbf{i}) - (v_{B/A})_{xyz}\mathbf{i}$

$-1.8\mathbf{i} = -(v_{B/A})_{xyz}$

$(v_{B/A})_{xyz} = 1.8 \text{ m/s}$

$0 = \omega_{AC}(0.75)$

$\omega_{AC} = 0$ **Ans.**

$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (v_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

$-3\mathbf{i} - 10.8\mathbf{j} = \mathbf{0} + \alpha_{AC}\mathbf{k} \times (0.75\mathbf{i}) + \mathbf{0} + \mathbf{0} - a_{A/B}\mathbf{i}$

$-3 = -a_{A/B}$

$a_{A/B} = 3 \text{ m/s}^2$

$-10.8 = \alpha_{A/C}(0.75)$

$\alpha_{A/C} = 14.4 \text{ rad/s}^2 \circlearrowright$ **Ans.**

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Ans:
$\omega_{AC} = 0$
$\alpha_{AC} = 14.4 \text{ rad/s}^2 \circlearrowright$

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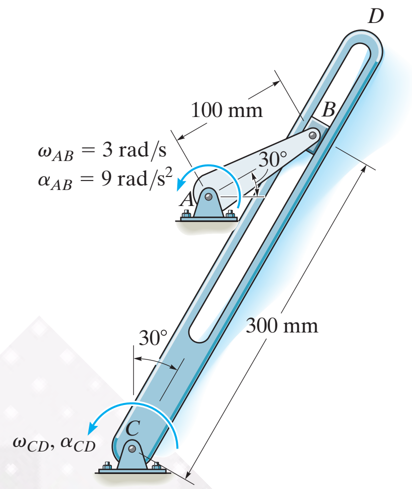
***16–152.**

The "quick-return" mechanism consists of a crank AB, slider block B, and slotted link CD. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



SOLUTION

$v_B = 3(0.1) = 0.3$ m/s

$(a_B)_t = 9(0.1) = 0.9$ m/s^2

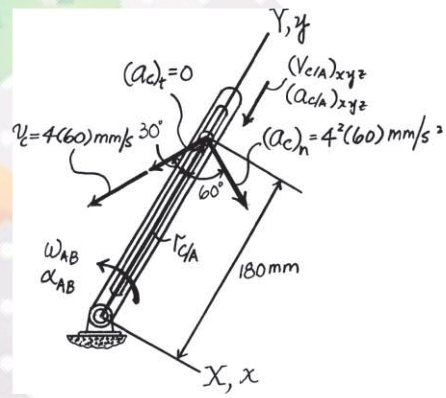
$(a_B)_n = (3)^2\,(0.1) = 0.9$ m/s^2

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.3 \cos 60°\mathbf{i} + 0.3 \sin 60°\mathbf{j} = \mathbf{0} + (\omega_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + v_{B/C}\,\mathbf{i}$

$v_{B/C} = 0.15$ m/s

$\omega_{CD} = 0.866$ rad/s ↺ **Ans.**



$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

$0.9 \cos 30°\mathbf{i} - 0.9 \cos 30°\mathbf{i} + 0.9 \sin 60°\mathbf{j} + 0.9 \sin 30°\mathbf{j} = \mathbf{0} + (\alpha_{CD}\,\mathbf{k}) \times (0.3\mathbf{i})$

$+(0.866\mathbf{k}) \times (0.866\mathbf{k} \times 0.3\mathbf{i}) + 2(0.866\mathbf{k} \times 0.15\mathbf{i}) + a_{B/C}\,\mathbf{i}$

$-0.3294\mathbf{i} + 1.2294\mathbf{j} = 0.3\alpha_{CD}\mathbf{j} - 0.225\mathbf{i} + 0.2598\mathbf{j} + a_{B/C}\,\mathbf{i}$

$a_{B/C} = -0.104$ m/s^2

$\alpha_{CD} = 3.23$ rad/s^2 ↺ **Ans.**

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Ans:
$\omega_{CD} = 0.866$ rad/s ↺
$\alpha_{CD} = 3.23$ rad/s^2 ↺

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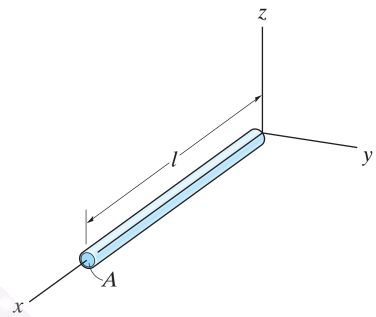
17–1.

Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



SOLUTION

$$I_y = \int_M x^2\,dm$$

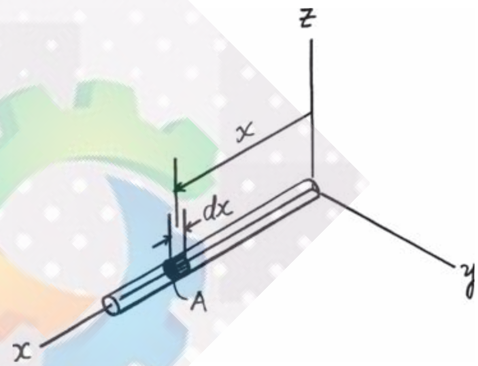
$$= \int_0^l x^2\,(\rho\,A\,dx)$$

$$= \frac{1}{3}\rho\,A\,l^3$$

$$m = \rho\,A\,l$$

Thus,

$$I_y = \frac{1}{3}m\,l^2 \qquad \textbf{Ans.}$$



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Ans:
$I_y = \frac{1}{3}m\,l^2$

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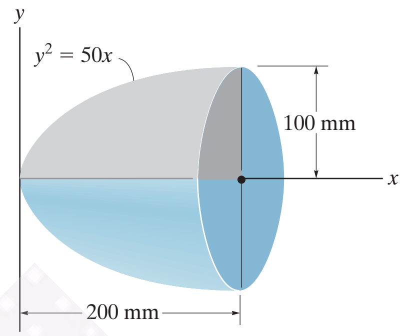
17–2.

The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \mathrm{Mg/m^3}$.



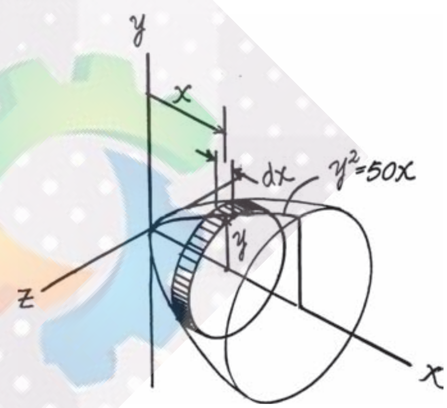
SOLUTION

$dm = \rho\,\pi\,y^2\,dx = \rho\,\pi\,(50x)\,dx$

$$I_x = \int \frac{1}{2} y^2\,dm = \frac{1}{2}\int_0^{200} 50\,x\,\{\pi\,\rho\,(50x)\}\,dx$$

$$= \rho\,\pi\left(\frac{50^2}{2}\right)\left[\frac{1}{3}x^3\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50^2}{6}\right)(200)^3$$



$$m = \int dm = \int_0^{200} \pi\,\rho\,(50x)\,dx$$

$$= \rho\,\pi\,(50)\left[\frac{1}{2}x^2\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50}{2}\right)(200)^2$$

$k_x = \sqrt{\dfrac{I_x}{m}} = \sqrt{\dfrac{50}{3}(200)} = 57.7\ \mathrm{mm}$ **Ans.**

Ans:
$k_x = 57.7\ \mathrm{mm}$

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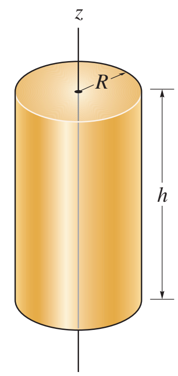
17–3.

The solid cylinder has an outer radius R, height h, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where k and a are constants. Determine the mass of the cylinder and its moment of inertia about the z axis.



SOLUTION

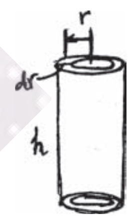
Consider a shell element of radius r and mass

$$dm = \rho\, dV = \rho(2\pi\, r\, dr)h$$

$$m = \int_0^R (k + ar^2)(2\pi\, r\, dr)h$$

$$m = 2\pi\, h\left(\frac{kR^2}{2} + \frac{aR^4}{4}\right)$$

$$m = \pi\, h\, R^2\left(k + \frac{aR^2}{2}\right) \qquad \textbf{Ans.}$$



$$dI = r^2\, dm = r^2(\rho)(2\pi\, r\, dr)h$$

$$I_z = \int_0^R r^2(k + ar^2)(2\pi\, r\, dr)\, h$$

$$I_z = 2\pi h \int_0^R (k\, r^3 + a\, r^5)\, dr$$

$$I_z = 2\pi h\left[\frac{k\, R^4}{4} + \frac{aR^6}{6}\right]$$

$$I_z = \frac{\pi\, h\, R^4}{2}\left[k + \frac{2\, aR^2}{3}\right] \qquad \textbf{Ans.}$$

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Ans:

$$m = \pi\, h R^2\left(k + \frac{aR^2}{2}\right)$$

$$I_z = \frac{\pi\, h R^4}{2}\left[k + \frac{2\, aR^2}{3}\right]$$

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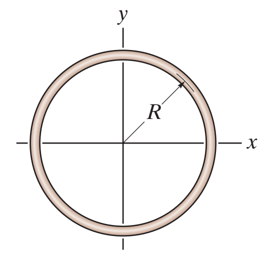
***17–4.**

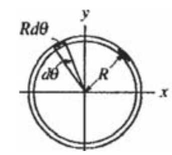
Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



SOLUTION

$$I_z = \int_0^{2\pi} \rho A(R\,d\theta)R^2 = 2\pi\,\rho\,A\,R^3$$

$$m = \int_0^{2\pi} \rho\,A\,R\,d\theta = 2\pi\,\rho\,A\,R$$



Thus,

$$I_z = m R^2 \qquad \textbf{Ans.}$$

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Ans:
$I_z = mR^2$

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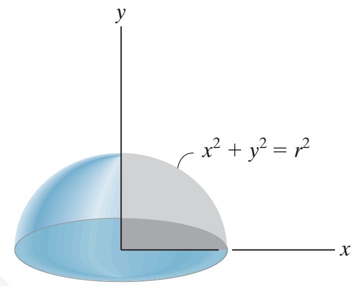
17–5.

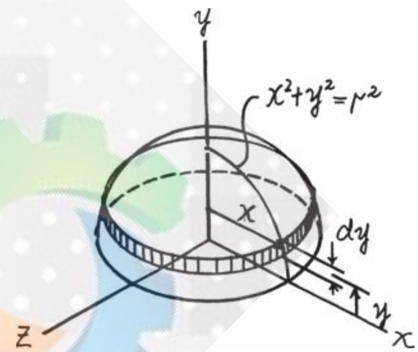
The hemisphere is formed by rotating the shaded area around the y axis. Determine the moment of inertia I_y and express the result in terms of the total mass m of the hemisphere. The material has a constant density ρ.



SOLUTION

$$m = \int_V \rho\, dV = \rho \int_0^r \pi\, x^2\, dy = \rho\pi \int_0^r (r^2 - y^2) dy$$

$$= \rho\pi \left[r^2\, y - \frac{1}{3}\, y^3 \right]_0^r = \frac{2}{3} \rho\pi\, r^3$$



$$I_y = \int_m \frac{1}{2}(dm)\, x^2 = \frac{\rho}{2} \int_0^r \pi x^4\, dy = \frac{\rho\pi}{2} \int_0^r (r^2 - y^2)^2\, dy$$

$$= \frac{\rho\pi}{2} \left[r^4 y - \frac{2}{3} r^2\, y^3 + \frac{y^5}{5} \right]_0^r = \frac{4\rho\pi}{15}\, r^5$$

Thus,

$$I_y = \frac{2}{5} m\, r^2$$ **Ans.**

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Ans:
$I_y = \frac{2}{5} m r^2$

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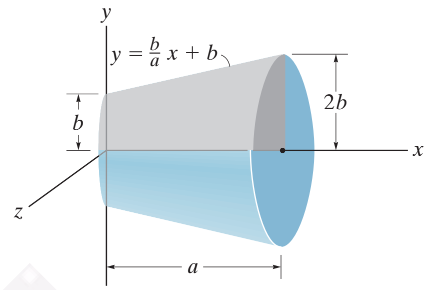
17–6.

The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density ρ.

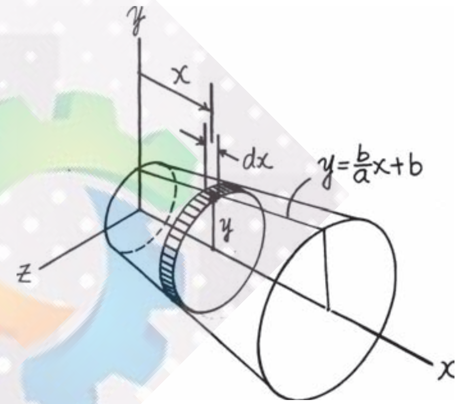


SOLUTION

$$dm = \rho\, dV = \rho\pi y^2\, dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\rho\pi y^4\, dx$$

$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4\,b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$



$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4\,b^4}{a}x + b^4\right)dx$$

$$= \frac{31}{10}\rho\pi ab^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi ab^2$$

$$I_x = \frac{93}{70}mb^2$$ **Ans.**

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Ans:
$I_x = \frac{93}{70}mb^2$

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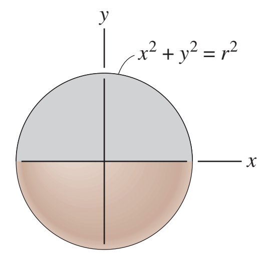
17–7.

The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the sphere. The material has a constant density ρ.



SOLUTION

$$dI_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\,\pi(r^2 - x^2)\,dx$$

$$dI_x = \frac{1}{2}\rho\,\pi(r^2 - x^2)^2\ dx$$

$$I_x = \int_{-r}^{r} \frac{1}{2}\rho\,\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho\,r^5$$

$$m = \int_{-r}^{r} \rho\,\pi(r^2 - x^2)\,dx$$

$$= \frac{4}{3}\rho\,\pi\,r^3$$

Thus,

$$I_x = \frac{2}{5}m\,r^2 \qquad \textbf{Ans.}$$

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Ans:
$I_x = \frac{2}{5}m\,r^2$

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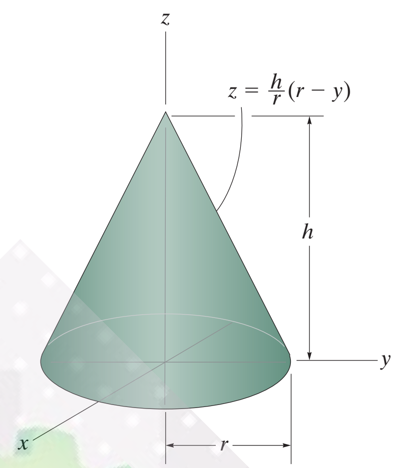
***17–8.**

Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



SOLUTION

Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho\, dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \frac{r_o}{h}z$. Thus, $dm = \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is

$$dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

Mass: The mass of the cone can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Big|_0^h = \frac{1}{3}\rho\pi r_o^2 h$$

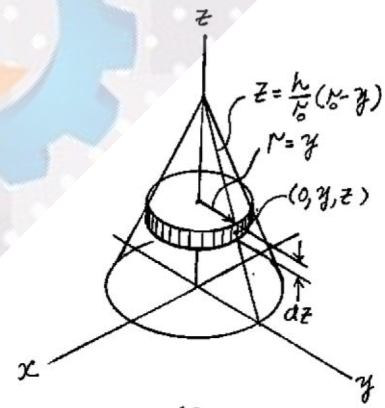
Mass Moment of Inertia: Integrating dI_z, we obtain

$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Big|_0^h = \frac{1}{10}\rho\pi r_o^4 h$$

From the result of the mass, we obtain $\rho\pi r_o^2 h = 3m$. Thus, I_z can be written as

$$I_z = \frac{1}{10}\left(\rho\pi r_o^2 h\right)r_o^2 = \frac{1}{10}(3m)r_o^2 = \frac{3}{10}mr_o^2 \qquad \textbf{Ans.}$$



(a)

Ans:

$I_z = \frac{3}{10}mr_o^2$

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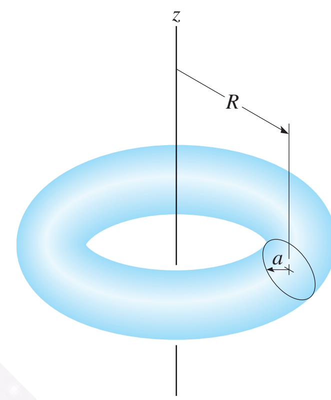
17–9.

Determine the moment of inertia I_z of the torus. The mass of the torus is m and the density ρ is constant. *Suggestion:* Use a shell element.



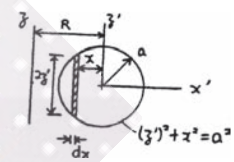
SOLUTION

$$dm = 2\pi(R - x)(2z'\rho\, dx)$$

$$dI_z = (R - x)^2 dm$$

$$= 4\pi\rho[(R^3 - 3R^2x + 3Rx^2 - x^3)\sqrt{a^2 - x^2}\, dx]$$

$$I_z = 4\pi\rho[R^3\int_{-a}^{a}\sqrt{a^2 - x^2}\, dx - 3R^2\int_{-a}^{a}x^3\sqrt{a^2 - x^2}\, dx + 3R\int_{-a}^{a}\sqrt{a^2 - x^2}\, dx - \int_{-a}^{a}x^3\sqrt{a^2 - x^2}$$



$$= 2\pi^2\rho Ra^2(R^2 + \frac{3}{4}a^2)$$

Since $m = \rho V = 2\pi R\rho\pi a^2$

$$I_z = m(R^2 + \frac{3}{4}a^2)$$ **Ans.**

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Ans:

$I_z = m(R^2 + \frac{3}{4}a^2)$

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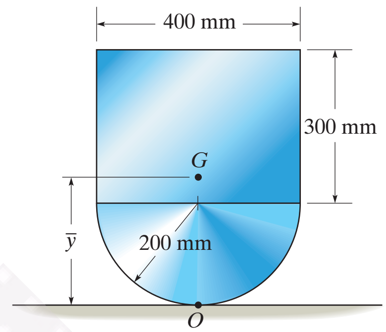
17–10.

Determine the location $\bar{y}$ of the center of mass G of the assembly and then calculate the moment of inertia about an axis perpendicular to the page and passing through G. The block has a mass of 3 kg and the semicylinder has a mass of 5 kg.



SOLUTION

Moment inertia of the semicylinder about its center of mass:

$$(I_G)_{cyc} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199mR^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{\left[0.2 - \frac{4(0.2)}{3\pi}\right](5) + 0.35(3)}{5 + 3} = 0.2032 \text{ m} = 0.203 \text{ m}$$ **Ans.**

$$I_G = 0.3199(5)(0.2)^2 + 5\left[0.2032 - \left(0.2 - \frac{4(0.2)}{3\pi}\right)\right]^2 + \frac{1}{12}(3)\left(0.3^2 + 0.4^2\right) + 3(0.35 - 0.2032)^2$$

$$= 0.230 \text{ kg}\cdot\text{m}^2$$ **Ans.**

IME

Ans:
$I_G = 0.230 \text{ kg}\cdot\text{m}^2$

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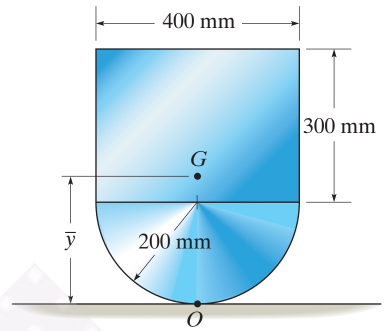
17–11.

Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point O. The block has a mass of 3 kg, and the semicylinder has a mass of 5 kg.



SOLUTION

$$(I_G)_{cyl} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199\,mR^2$$

$$I_O = 0.3199(5)(0.2)^2 + 5\left(0.2 - \frac{4(0.2)}{3\pi}\right)^2 + \frac{1}{12}(3)\left((0.3)^2 + (0.4)^2\right) + 3(0.350)^2$$

$$= 0.560 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Also from the solution to Prob. 17–22,

$$I_O = I_G + md^2$$

$$= 0.230 + 8(0.2032)^2$$

$$= 0.560 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_O = 0.560 \text{ kg} \cdot \text{m}^2$

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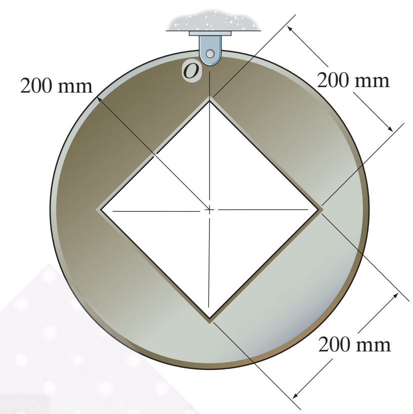
***17–12.**

Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \mathrm{kg/m^2}$.



SOLUTION

Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point O for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\ \mathrm{kg \cdot m^2}$$ **Ans.**

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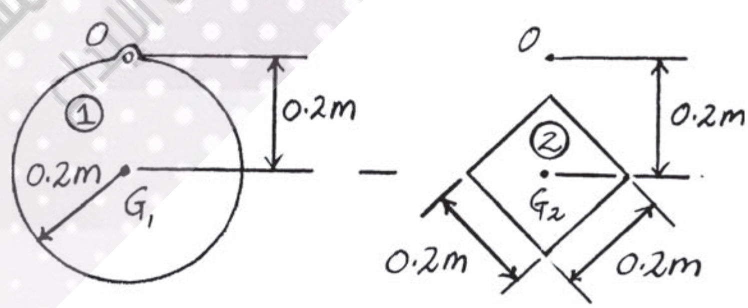

(a)

Ans:
$I_O = 0.113\ \mathrm{kg \cdot m^2}$

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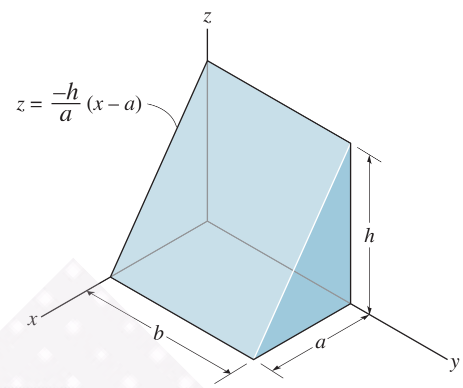
17–13.

Determine the moment of inertia of the homogeneous triangular prism with respect to the y axis. Express the result in terms of the mass m of the prism. *Hint:* For integration, use thin plate elements parallel to the x–y plane and having a thickness dz.



SOLUTION

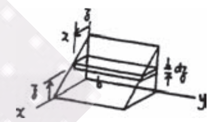
$$dV = bx\,dz = b(a)\left(1 - \frac{z}{h}\right) dz$$

$$dI_y = dI_y + (dm)\left[\left(\frac{x}{2}\right)^2 + z^2\right]$$

$$= \frac{1}{12}\,dm(x^2) + dm\left(\frac{x^2}{4}\right) + dm z^2$$

$$= dm\left(\frac{x^2}{3} + z^2\right)$$

$$= \left[b(a)\left(1 - \frac{z}{h}\right)dz\right](\rho)\left[\frac{a^2}{3}\left(1 - \frac{z}{h}\right)^2 + z^2\right]$$



$$I_y = ab\rho \int_0^k \left[\frac{a^3}{3}\left(\frac{h - z}{h}\right)^3 + z^2\left(1 - \frac{z}{h}\right)\right]dz$$

$$= ab\rho\left[\frac{a^2}{3h^3}\left(h^4 - \frac{3}{2}h^4 + h^4 - \frac{1}{4}h^4\right) + \frac{1}{h}\left(\frac{1}{3}h^4 - \frac{1}{4}h^4\right)\right]$$

$$= \frac{1}{12}\,abh\rho(a^2 + h^2)$$

$$m = \rho V = \frac{1}{2}\,abh\rho$$

Thus,

$$I_y = \frac{m}{6}(a^2 + h^2)$$ **Ans.**

Ans:

$$I_y = \frac{m}{6}\left(a^2 + h^2\right)$$

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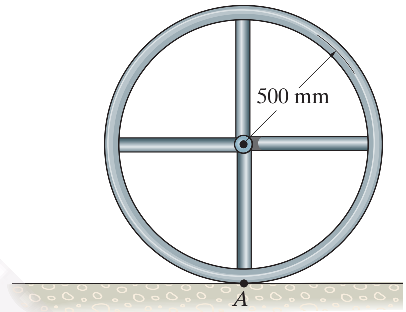
17–14.

The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods, each having a mass of 2 kg. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.



SOLUTION

$$I_A = I_o + md^3$$

$$= \left[2\left[\frac{1}{12}(4)(1)^2\right] + 10(0.5)^2\right] + 18(0.5)^2$$

$$= 7.67 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_A = 7.67 \text{ kg} \cdot \text{m}^2$

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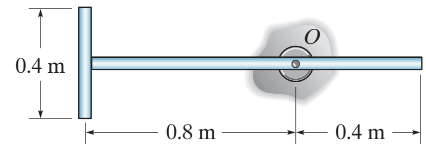
17–15.

The assembly is made of the slender rods that have a mass per unit length of 3 kg/m. Determine the mass moment of inertia of the assembly about an axis perpendicular to the page and passing through point O.



SOLUTION

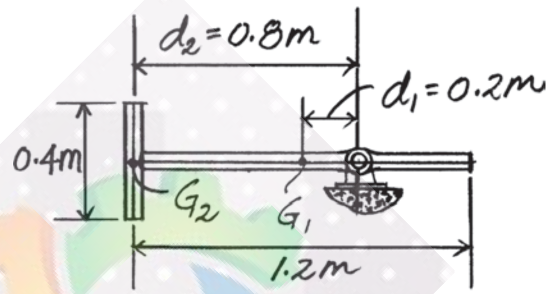
Using the parallel axis theorem by referring to Fig. a,

$$I_O = \Sigma\left(I_G + md^2\right)$$

$$= \left\{\frac{1}{12}[3(1.2)]\left(1.2^2\right) + [3(1.2)]\left(0.2^2\right)\right\}$$

$$+ \left\{\frac{1}{12}[3(0.4)]\left(0.4^2\right) + [3(0.4)]\left(0.8^2\right)\right\}$$

$$= 1.36 \text{ kg} \cdot \text{m}^2$$ **Ans.**



(a)

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Ans:
$I_O = 1.36 \text{ kg} \cdot \text{m}^2$

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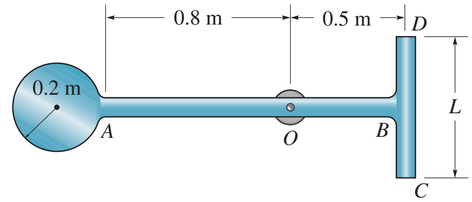
***17–16.** The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass per unit length of 2 kg/m. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through O?



SOLUTION

Measured from the right side,

$$y = \frac{6(1.5) + 2(1.3)(0.65)}{6 + 1.3(2) + L(2)} = 0.5$$

$L = 6.39\text{ m}$ **Ans.**

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(6.39)(6.39)^2 + 2(6.39)(0.5)^2$$

$I_O = 53.2\text{ kg}\cdot\text{m}^2$ **Ans.**

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Ans:

$L = 6.39\text{ m}$

$I_O = 53.2\text{ kg}\cdot\text{m}^2$

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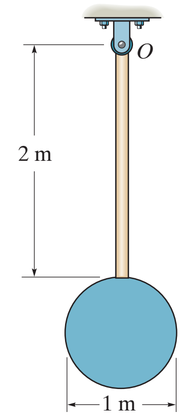
17–17.

The pendulum consists of a 4-kg circular plate and a 2-kg slender rod. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



SOLUTION

Using the parallel axis theorem by referring to Fig. *a*,

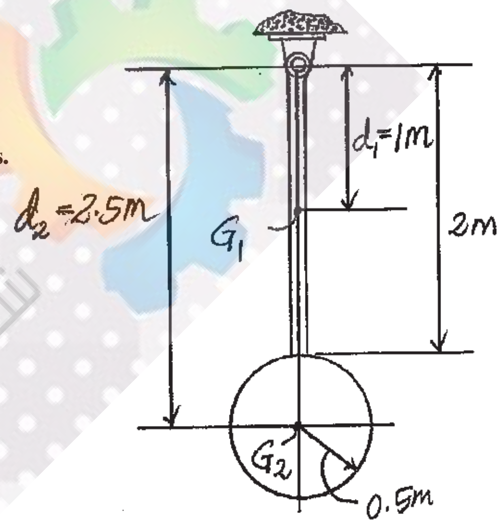
$$I_O = \Sigma\left(I_G + md^2\right)$$

$$= \left[\frac{1}{12}(2)\left(2^2\right) + 2\left(1^2\right)\right] + \left[\frac{1}{2}(4)\left(0.5^2\right) + 4\left(2.5^2\right)\right]$$

$$= 28.17\ \text{kg}\cdot\text{m}^2$$

Thus, the radius of gyration is

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{28.17}{4 + 2}} = 2.167\ \text{m} = 2.17\ \text{m}$$ **Ans.**



(a)

Ans:
$k_O = 2.17$ m

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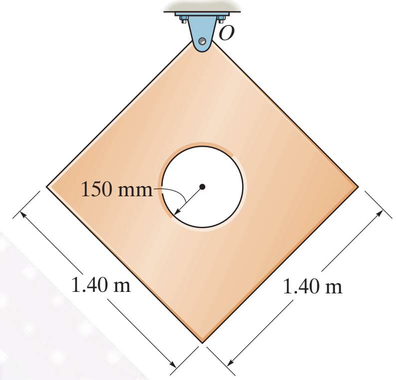
17–18.

Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density $\rho = 50\ \text{kg/m}^3$.



SOLUTION

$$I_G = \frac{1}{12}\left[50(1.4)(1.4)(0.05)\right]\left[(1.4)^2 + (1.4)^2\right] - \frac{1}{2}\left[50(\pi)(0.15)^2(0.05)\right](0.15)^2$$

$$= 1.5987\ \text{kg}\cdot\text{m}^2$$

$$I_O = I_G + md^2$$

$$m = 50(1.4)(1.4)(0.05) - 50(\pi)(0.15)^2(0.05) = 4.7233\ \text{kg}$$

$$I_O = 1.5987 + 4.7233(1.4\sin 45°)^2 = 6.23\ \text{kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_O = 6.23\ \text{kg}\cdot\text{m}^2$

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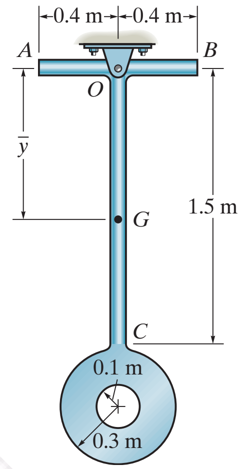
17–19. The pendulum consists of two slender rods AB and OC which have a mass per unit length of 3 kg/m. The thin circular plate has a mass per unit area of 12 kg/m^2. Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{1.5(3)(0.75) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12) + 0.8(3)}$$

$$= 0.8878 \text{ m} = 0.888 \text{ m} \qquad \textbf{Ans.}$$

$$I_G = \frac{1}{12}(0.8)(3)(0.8)^2 + 0.8(3)(0.8878)^2$$

$$+\frac{1}{12}(1.5)(3)(1.5)^2 + 1.5(3)(0.75 - 0.8878)^2$$

$$+\frac{1}{2}[\pi(0.3)^2(12)(0.3)^2 + [\pi(0.3)^2(12)](1.8 - 0.8878)^2$$

$$-\frac{1}{2}[\pi(0.1)^2(12)(0.1)^2 - [\pi(0.1)^2(12)](1.8 - 0.8878)^2$$

$$I_G = 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = 0.888$ m
$I_G = 5.61$ kg · m^2

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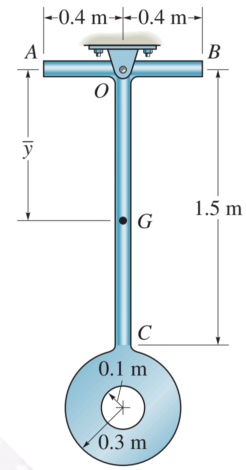
*__17–20.__ The pendulum consists of two slender rods AB and OC which have a mass per unit length of 3 kg/m. The thin circular plate has a mass per unit area of 12 kg/m^2. Determine the moment of inertia of the pendulum about an axis perpendicular to the page and passing through the pin at O.



SOLUTION

$$I_o = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 = 13.4 \text{ kg}\cdot\text{m}^2$$ **Ans.**

Also, from the solution to Prob. 17–16,

$$m = 3(0.8 + 1.5) + 12[\pi(0.3)^2 - \pi(0.1)^2] = 9.916 \text{ kg}$$

$$I_o = I_G + m\,d^2$$

$$= 5.61 + 9.916(0.8878)^2$$

$$= 13.4 \text{ kg}\cdot\text{m}^2$$ **Ans.**

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Ans:
$I_o = 13.4 \text{ kg}\cdot\text{m}^2$

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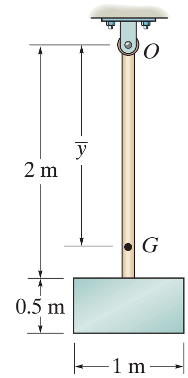
17–21.

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{\Sigma\,\bar{y}m}{\Sigma\,m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m}$$ **Ans.**

$$I_G = \Sigma \bar{I}_G + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg}\cdot\text{m}^2$$ **Ans.**

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Ans:
$\bar{y} = 1.78$ m
$I_G = 4.45$ kg $\cdot$ m^2

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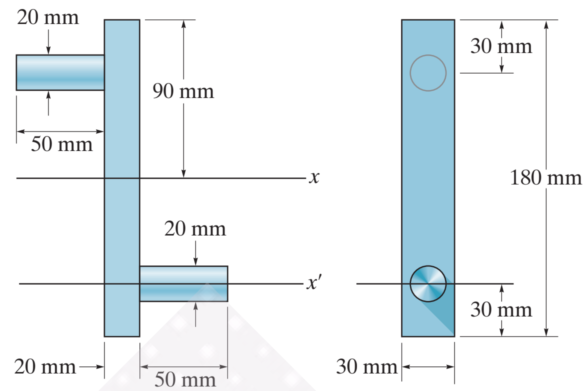
17–22.

Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a destiny of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

$$m_c = 7.85(10^3)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_\rho = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

$$+ \left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big)\right]$$

$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_x = 3.25\ \text{g}\cdot\text{m}^2$

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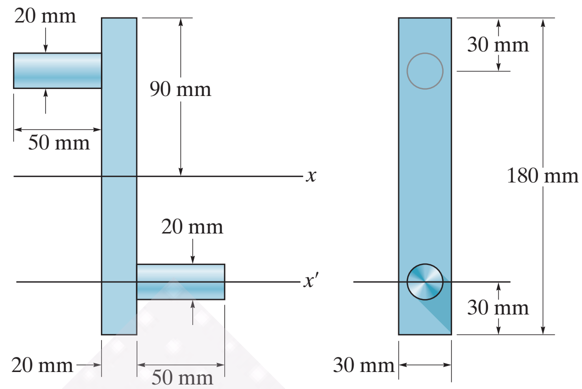
17–23.

Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a destiny of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

$$m_c = 7.85\left(10^3\right)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

$$m_p = 7.85\left(10^3\right)\left((0.03)(0.180)(0.02)\right) = 0.8478\ \text{kg}$$

$$I_{x'} = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_{x'} = 7.19\ \text{g}\cdot\text{m}^2$

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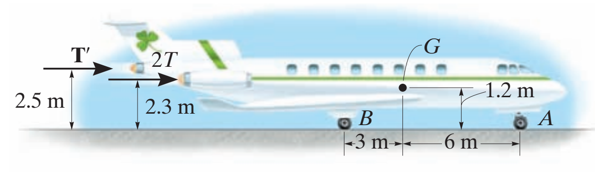
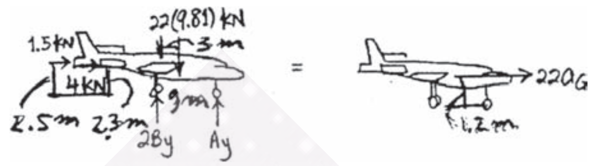
***17–24.**

The jet aircraft has a total mass of 22 Mg and a center of mass at G. Initially at take-off the engines provide a thrust $2T = 4$ kN and $T' = 1.5$ kN. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.



SOLUTION



$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 1.5 + 4 = 22a_G$

$+\uparrow \Sigma F_y = 0; \qquad 2B_y + A_y - 22(9.81) = 0$

$\circlearrowleft + \Sigma M_B = \Sigma(M_K)_B; \qquad 4(2.3) - 1.5(2.5) - 22(9.81)(3) + A_y(9) = -22a_G(1.2)$

$A_y = 72.6\text{ kN}$ **Ans.**

$B_y = 71.6\text{ kN}$ **Ans.**

$a_G = 0.250\text{ m/s}^2$ **Ans.**

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Ans:
$A_y = 72.6\text{ kN}$
$B_y = 71.6\text{ kN}$
$a_G = 0.250\text{ m/s}^2$

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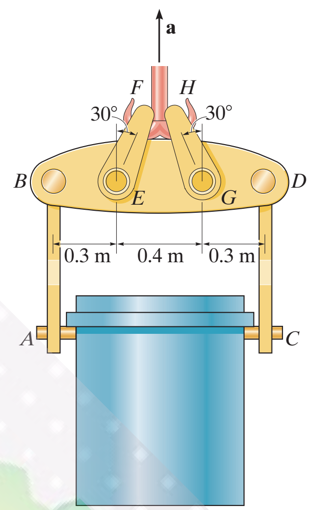
17–25. The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the force in each of the links AB, CD, EF, and GH when the system is lifted with an acceleration of $a = 2\ \text{m/s}^2$ for a short period of time.



SOLUTION

Canister:

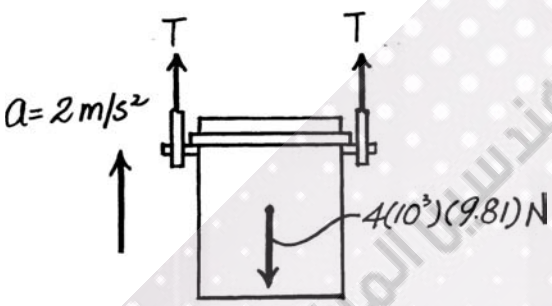
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T - 4(10^3)(9.81) = 4(10^3)(2)$$

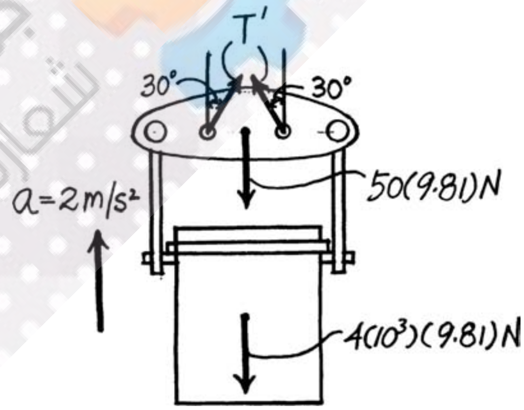
$$T_{AB} = T_{CD} = T = 23.6 \text{ kN} \qquad \textbf{Ans.}$$

System:

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T' \cos 30° - 4050(9.81) = 4050(2)$$

$$T_{EF} = T_{GH} = T' = 27.6 \text{ kN} \qquad \textbf{Ans.}$$





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Ans:

$T_{AB} = T_{CD} = T = 23.6$ kN
$T_{EF} = T_{GH} = T' = 27.6$ kN

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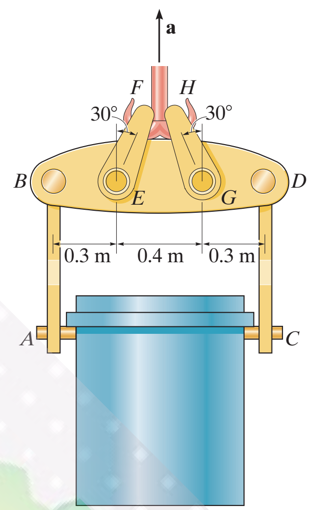
17–26. The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the largest vertical acceleration **a** of the system so that each of the links AB and CD are not subjected to a force greater than 30 kN and links EF and GH are not subjected to a force greater than 34 kN.



SOLUTION

Canister:

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2(30)(10^3) - 4(10^3)(9.81) = 4(10^3)a$

$a = 5.19\ \text{m/s}^2$

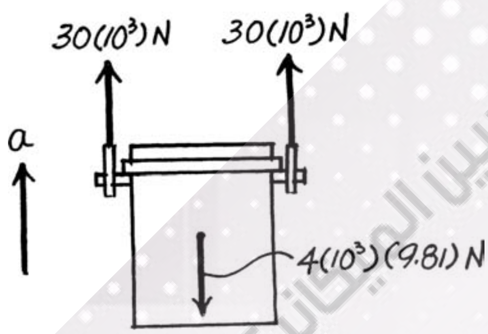
System:

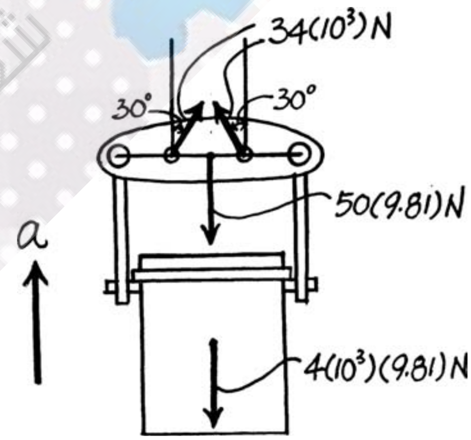
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2\left[34(10^3)\cos 30^\circ\right] - 4050(9.81) = 4050a$

$a = 4.73\ \text{m/s}^2$

Thus,

$$a_{max} = 4.73\ \text{m/s}^2 \qquad \textbf{Ans.}$$





Ans:
$a_{max} = 4.73\ \text{m/s}^2$

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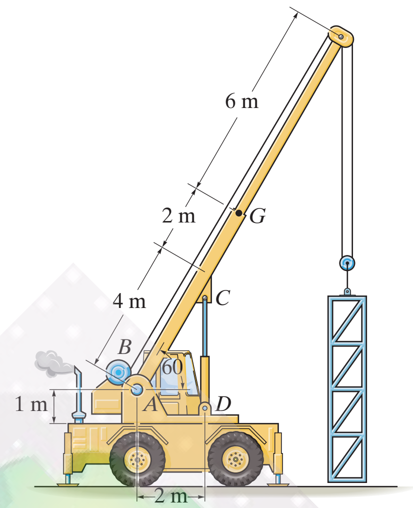
17–27.

The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at B draws in the cable with an acceleration of 2 m/s^2, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at G.



SOLUTION

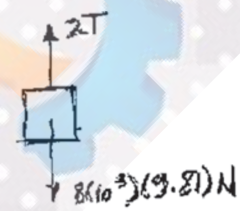
$s_B + 2s_L = l$

$a_B = -2a_L$

$2 = -2a_L$

$a_L = -1\ \text{m/s}^2$

Assembly:



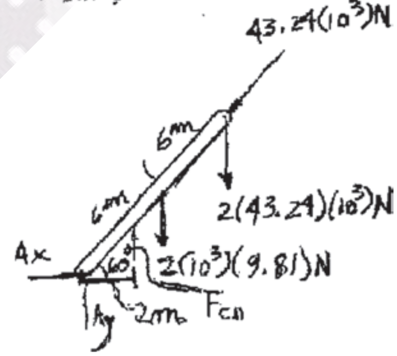
$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 8(10^3)(9.81) = 8(10^3)(1)$

$T = 43.24\ \text{kN}$

Boom:

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(2) - 2(10^3)(9.81)(6 \cos 60°) - 2(43.24)(10^3)(12 \cos 60°) = 0$

$F_{CD} = 289\ \text{kN}$ **Ans.**



Ans:
$F_{CD} = 289\ \text{kN}$

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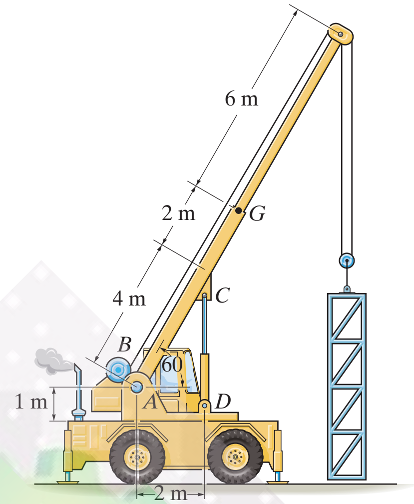
***17–28.**

The assembly has a mass of 4 Mg and is hoisted using the winch at B. Determine the greatest acceleration of the assembly so that the compressive force in the hydraulic cylinder supporting the boom does not exceed 180 kN. What is the tension in the supporting cable? The boom has a mass of 2 Mg and mass center at G.



SOLUTION

Boom:

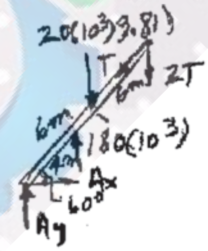
$\zeta + \Sigma M_A = 0;$ $\quad 180(10^3)(2) - 2(10^3)(9.81)(6\cos 60°) - 2T(12\cos 60°) = 0$

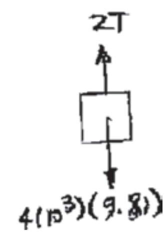
$T = 25\,095\text{ N} = 25.1\text{ kN}$ **Ans.**

Assembly:

$+\uparrow \Sigma F_y = ma_y;$ $\quad 2(25\,095) - 4(10^3)(9.81) = 4(10^3)\,a$

$a = 2.74\text{ m/s}^2$ **Ans.**





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Ans:
$a = 2.74\text{ m/s}^2$
$T = 25.1\text{ kN}$

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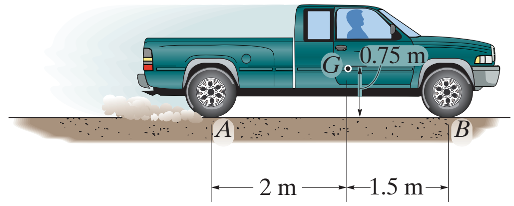
17–29.

Determine the shortest time possible for the rear-wheel drive, 2-Mg truck to achieve a speed of 16 m/s with a constant acceleration starting from rest. The coefficient of static friction between the wheels and the road surface is $\mu_s = 0.8$. The front wheels are free to roll. Neglect the mass of the wheels.



SOLUTION

Equations of Motion: The maximum acceleration of the truck occurs when its rear wheels are on the verge of slipping. Thus, $F_A = \mu_s N_A = 0.8N_A$. Referring to the free-body diagram of the truck shown in Fig. *a*, we can write

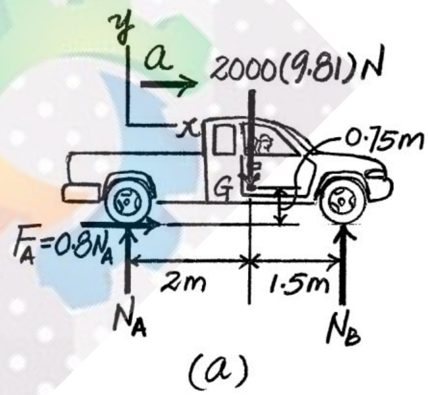
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.8N_A = 2000a \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 2000(9.81) = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_G = 0; \quad N_B(1.5) + 0.8N_A(0.75) - N_A(2) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$N_A = 10\,148.28$ N $\qquad N_B = 9471.72$ N $\qquad a = 4.059$ m/s^2



Kinematics: Since the acceleration of the truck is constant, we can apply

$$(\xrightarrow{+}) \qquad v = v_0 + at$$

$$16 = 0 + 4.059t$$

$$t = 3.94 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 3.94$ s

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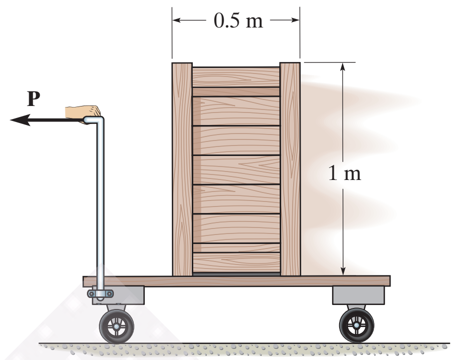
17–30.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to slip or tip on the cart. The coefficient of static friction between the crate and cart is $\mu_s = 0.2$.



SOLUTION

Equation of Motion. Assuming that the crate slips before it tips, then $F_f = \mu_s N = 0.2\text{ N}$. Referring to the FBD and kinetic diagram of the crate, Fig. *a*

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150\,(9.81) = 150\,(0) \qquad N = 1471.5\text{ N}$$

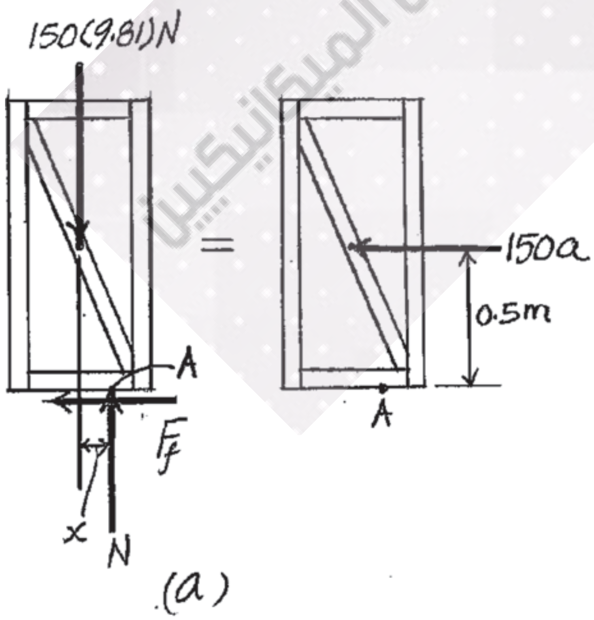
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.2(1471.5) = 150\,a \qquad a = 1.962\text{ m/s}^2$$

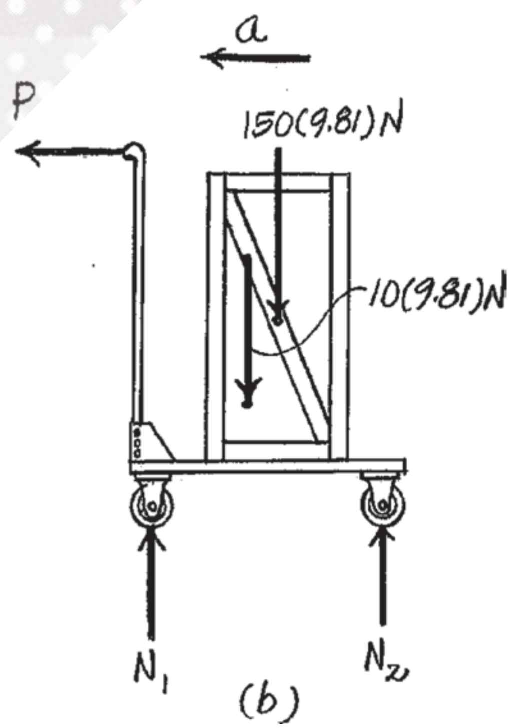
$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 150(9.81)(x) = 150(1.962)(0.5)$$

$$x = 0.1\text{ m}$$

Since $x = 0.1\text{ m} < 0.25\text{ m}$, the crate indeed slips before it tips. Using the result of a and refer to the FBD of the crate and cart, Fig. *b*,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = (150 + 10)(1.962) = 313.92\text{ N} = 314\text{ N} \qquad \textbf{Ans.}$$





Ans:
$P = 314\text{ N}$

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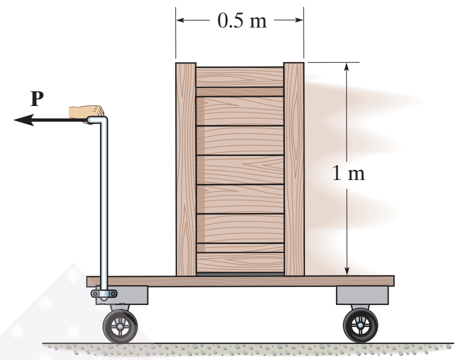
17–31.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to tip on the cart. Slipping does not occur.



SOLUTION

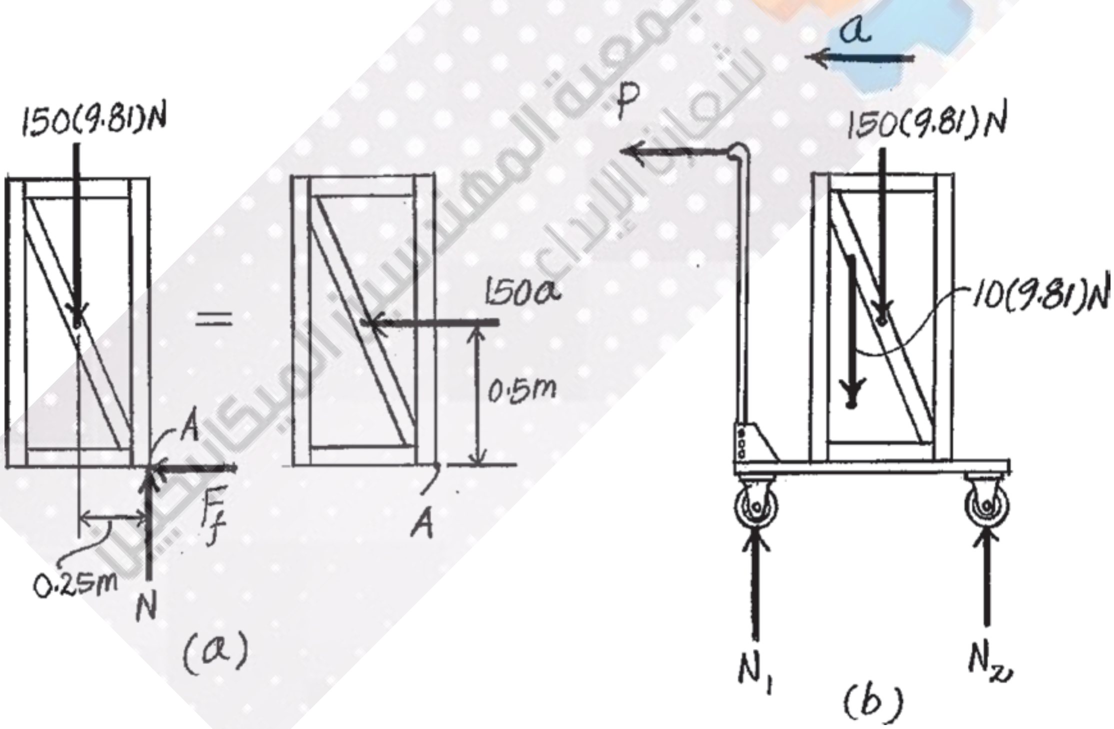
Equation of Motion. Tipping will occur about edge A. Referring to the FBD and kinetic diagram of the crate, Fig. *a*,

$$\zeta + \Sigma M_A = \Sigma (M_K)_A; \qquad 150(9.81)(0.25) = (150a)(0.5)$$

$$a = 4.905 \text{ m/s}^2$$

Using the result of a and refer to the FBD of the crate and cart, Fig. *b*,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x \qquad P = (150 + 10)(4.905) = 784.8 \text{ N} = 785 \text{ N}$$ **Ans.**



Ans:
$P = 785$ N

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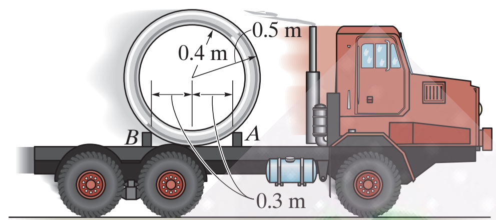
***17–32.**

The pipe has a mass M and is held in place on the truck bed using the two boards A and B. Determine the acceleration of the truck so that the pipe begins to lose contact at A and the bed of the truck and starts to pivot about B. Assume board B will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board B exert on the pipe during the acceleration?



SOLUTION

Units Used: $\quad kN = 10^3\ N$

Given:

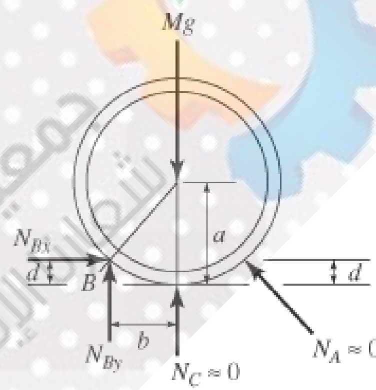
$$M = 460\ \text{kg}$$

$$a = 0.5\ \text{m}$$

$$b = 0.3\ \text{m}$$

$$c = 0.4\ \text{m}$$

$$g = 9.81\ \text{m/s}^2$$



$$\theta = \text{asin}\left(\frac{b}{a}\right)$$

Guesses

$$N_{Bx} = 1\ \text{N} \qquad N_{By} = 1\ \text{N} \qquad a_t = 1\ \text{m/s}^2$$

Given

$$N_{Bx} = M a_t \qquad N_{By} - M g = 0 \qquad N_{Bx}(a)\cos(\theta) - N_{By}\, b = 0$$

$$\begin{pmatrix} N_{Bx} \\ N_{By} \\ a_t \end{pmatrix} = \text{Find}(N_{Bx}, N_{By}, a_t) \qquad \begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix} = \begin{pmatrix} 3.384 \\ 4.513 \end{pmatrix} \text{kN} \qquad a_t = 7.36\ \text{m/s}^2 \quad \textbf{Ans.}$$

$$\left|\begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix}\right| = 5.64\ \text{kN} \quad \textbf{Ans.}$$

Ans:

$a_t = 7.36\ \text{m/s}^2$

$N_{Bx} = 5.64\ \text{kN}$

$N_{By} = 5.64\ \text{kN}$

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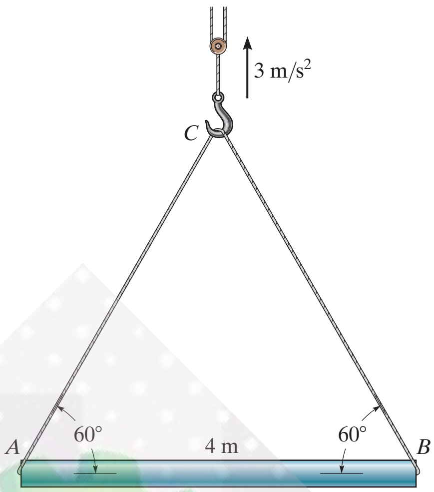
17–33.

The uniform girder AB has a mass of 8 Mg. Determine the internal axial, shear, and bending-moment loadings at the center of the girder if a crane gives it an upward acceleration of 3 m/s^2.



SOLUTION

Girder:

$+\uparrow\Sigma F_y = ma_y; \qquad 2T \sin 60° - 8000(9.81) = 8000(3)$

$T = 59\,166.86$ N

Segment:

$\xrightarrow{+}\Sigma F_x = ma_x; \qquad 59\,166.86 \cos 60° - N = 0$

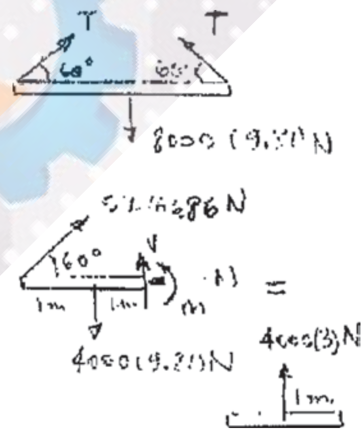
$N = 29.6$ kN **Ans.**

$+\uparrow\Sigma F_y = ma_y; \qquad 59\,166.86 \sin 60° - 4000(9.81) + V = 4000(3)$

$V = 0$ **Ans.**

$\circlearrowleft +\Sigma M_C = \Sigma(M_k)_C; \qquad M + 4000(9.81)(1) - 59\,166.86 \sin 60°(2) = -4000(3)(1)$

$M = 51.2$ kN·m **Ans.**



Ans:
$N = 29.6$ kN
$V = 0$
$M = 51.2$ kN·m

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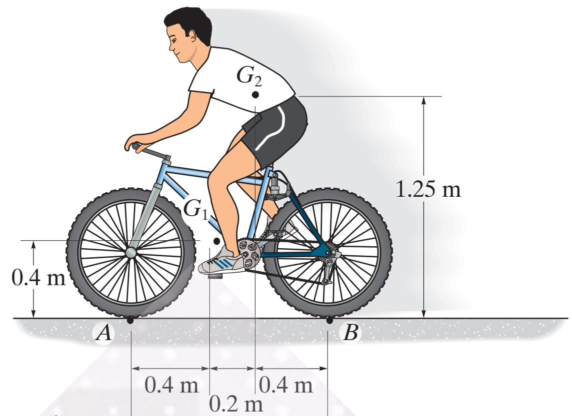
17–34.

The mountain bike has a mass of 40 kg with center of mass at point G_1, while the rider has a mass of 60 kg with center of mass at point G_2. Determine the maximum deceleration when the brake is applied to the front wheel, without causing the rear wheel B to leave the road. Assume that the front wheel does not slip. Neglect the mass of all the wheels.

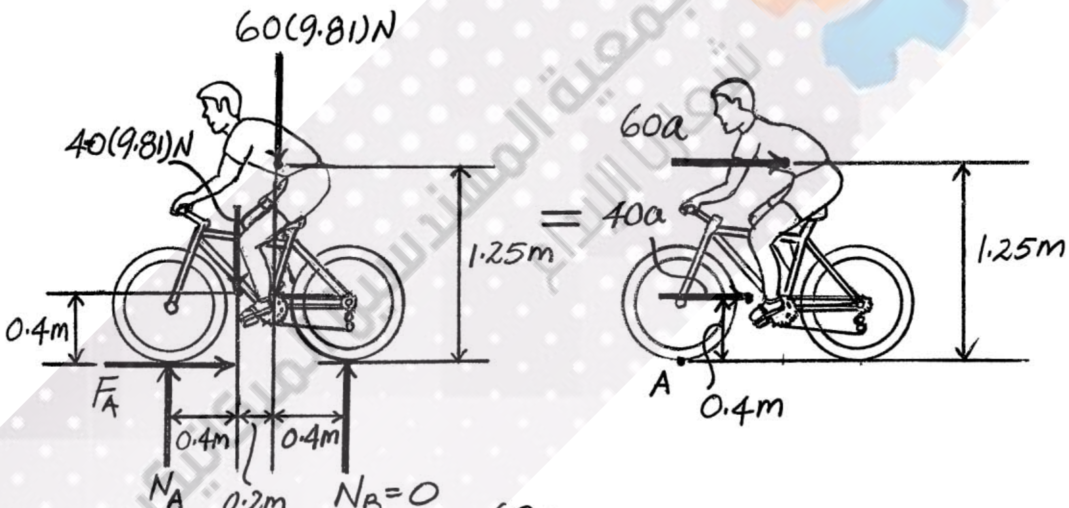


SOLUTION

Equations of Motion: Since the rear wheel B is required to just leave the road, $N_B = 0$. Thus, the acceleration **a** of the bike can be obtained directly by writing the moment equation of motion about point A.

$$+\Sigma M_A = (M_k)_A; \quad -40(9.81)(0.4) - 60(9.81)(0.6) = -40a(0.4) - 60a(1.25)$$

$$a = 5.606 \text{ m/s}^2 = 5.61 \text{ m/s}^2 \qquad \textbf{Ans.}$$



(a)

Ans:
$a = 5.61 \text{ m/s}^2$

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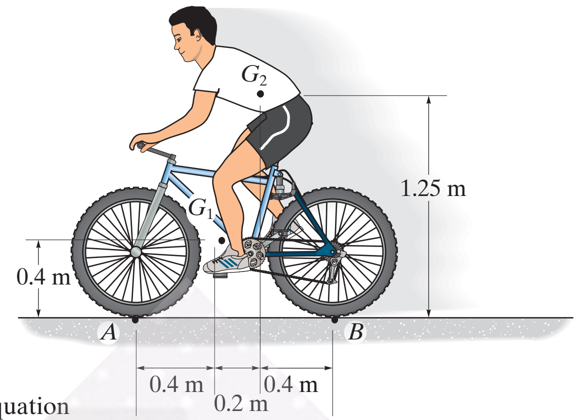
17–35.

The mountain bike has a mass of 40 kg with center of mass at point G_1, while the rider has a mass of 60 kg with center of mass at point G_2. When the brake is applied to the front wheel, it causes the bike to decelerate at a constant rate of 3 m/s^2. Determine the normal reaction the road exerts on the front and rear wheels. Assume that the rear wheel is free to roll. Neglect the mass of all the wheels.



SOLUTION

Equations of Motion: $\mathbf{N}_B$ can be obtained directly by writing the moment equation of motion about point A.

$+\Sigma M_A = (M_k)_A;$

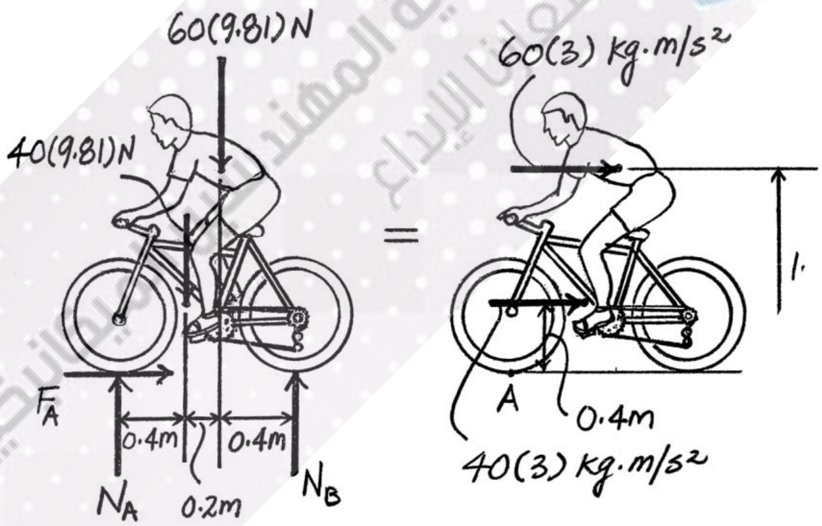
$$N_B(1) - 40(9.81)(0.4) - 60(9.81)(0.6) = -60(3)(1.25) - 40(3)(0.4)$$

$$N_B = 237.12 \text{ N} = 237 \text{ N} \qquad \textbf{Ans.}$$

Using this result and writing the force equations of motion along the y axis,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + 237.12 - 40(9.81) - 60(9.81) = 0$$

$$N_A = 743.88 \text{ N} = 744 \text{ N} \qquad \textbf{Ans.}$$



Ans:

$N_B = 237$ N
$N_A = 744$ N

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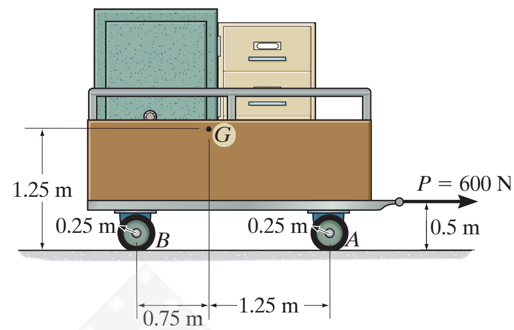
***17–36.**

The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.



SOLUTION

Equations of Motion: Writing the force equation of motion along the x axis,

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 600 = 150a \qquad\qquad a = 4 \text{ m/s}^2 \rightarrow$ **Ans.**

Using this result to write the moment equation about point A,

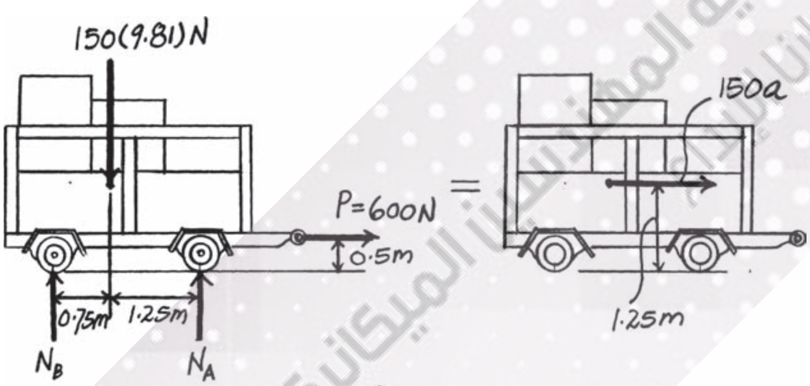
$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)$

$$N_B = 1144.69 \text{ N} = 1.14 \text{ kN}$$ **Ans.**

Using this result to write the force equation of motion along the y axis,

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1144.69 - 150(9.81) = 150(0)$

$$N_A = 326.81 \text{ N} = 327 \text{ N}$$ **Ans.**



(a)

Ans:
$a = 4 \text{ m/s}^2 \rightarrow$
$N_B = 1.14$ kN
$N_A = 327$ N

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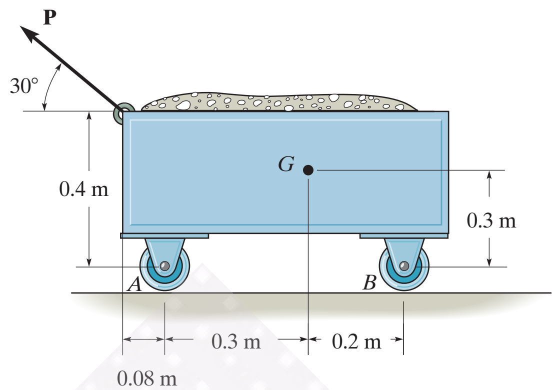
17–37.

A force of $P = 300$ N is applied to the 60-kg cart. Determine the reactions at both the wheels at A and both the wheels at B. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Referring to the FBD of the cart, Fig. *a*,

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 300 \cos 30° = 60a$

$a = 4.3301 \text{ m/s}^2 = 4.33 \text{ m/s}^2 \leftarrow$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + N_B + 300 \sin 30° - 60(9.81) = 60(0)$ **(1)**

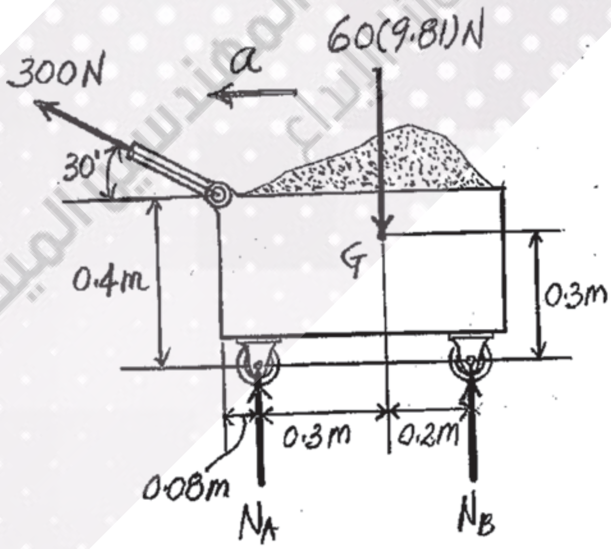
$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(0.2) - N_A(0.3) + 300 \cos 30°(0.1)$

$- 300 \sin 30°(0.38) = 0$ **(2)**

Solving Eqs. (1) and (2),

$N_A = 113.40 \text{ N} = 113 \text{ N}$ **Ans.**

$N_B = 325.20 \text{ N} = 325 \text{ N}$ **Ans.**



(a)

Ans:
$a = 4.33 \text{ m/s}^2 \leftarrow$
$N_A = 113$ N
$N_B = 325$ N

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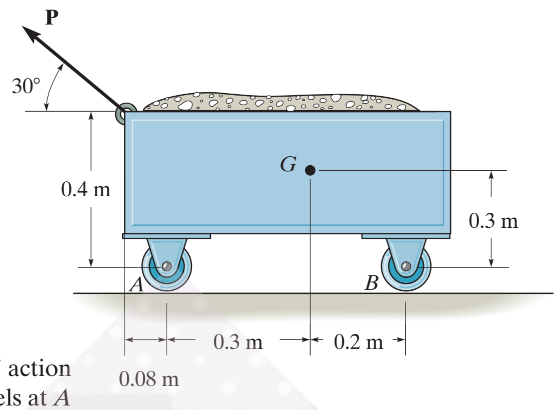
17–38.

Determine the largest force **P** that can be applied to the 60-kg cart, without causing one of the wheel reactions, either at A or at B, to be zero. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Since $(0.38 \text{ m}) \tan 30° = 0.22 \text{ m} > 0.1 \text{ m}$, the line of action of **P** passes *below* G. Therefore, **P** tends to rotate the cart clockwise. The wheels at A will leave the ground before those at B. Then, it is required that $N_A = 0$. Referring, to the FBD of the cart, Fig. *a*

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B + P \sin 30° - 60(9.81) = 60(0) \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad P \cos 30°(0.1) - P \sin 30°(0.38) + N_B(0.2) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$P = 578.77 \text{ N} = 579 \text{ N} \qquad \textbf{Ans.}$$

$$N_B = 299.22 \text{ N}$$

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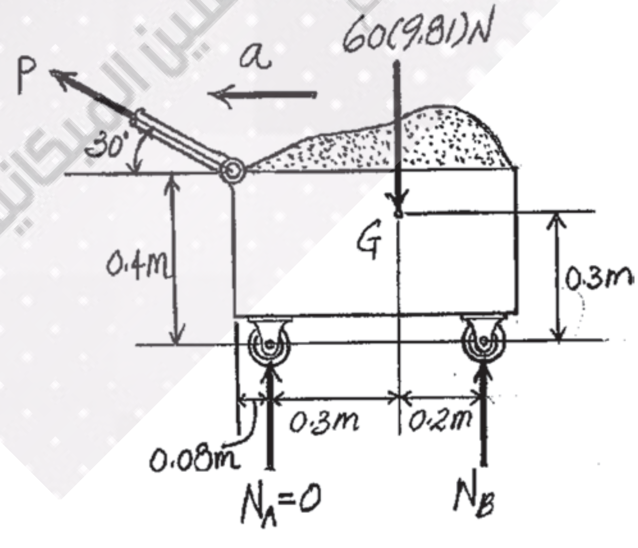

(a)

Ans:
$P = 579$ N

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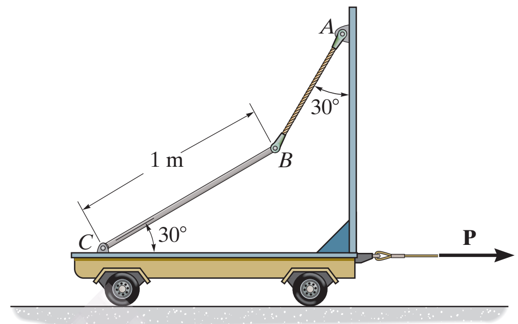
17–39.

If the cart's mass is 30 kg and it is subjected to a horizontal force of $P = 90$ N, determine the tension in cord AB and the horizontal and vertical components of reaction on end C of the uniform 15-kg rod BC.



SOLUTION

Equations of Motion: The acceleration **a** of the cart and the rod can be determined by considering the free-body diagram of the cart and rod system shown in Fig. *a*.

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 90 = (15 + 30)a \qquad a = 2 \text{ m/s}^2$

The force in the cord can be obtained directly by writing the moment equation of motion about point C by referring to Fig. *b*.

$+\Sigma M_C = (M_k)_C; \qquad F_{AB} \sin 30°(1) - 15(9.81) \cos 30°(0.5) = -15(2) \sin 30°(0.5)$

$F_{AB} = 112.44 \text{ N} = 112 \text{ N}$ **Ans.**

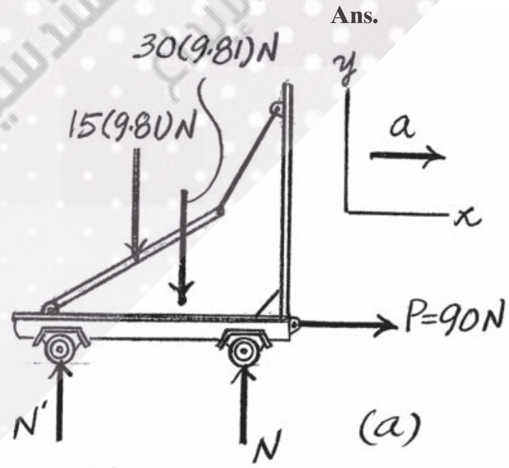
Using this result and applying the force equations of motion along the x and y axes,

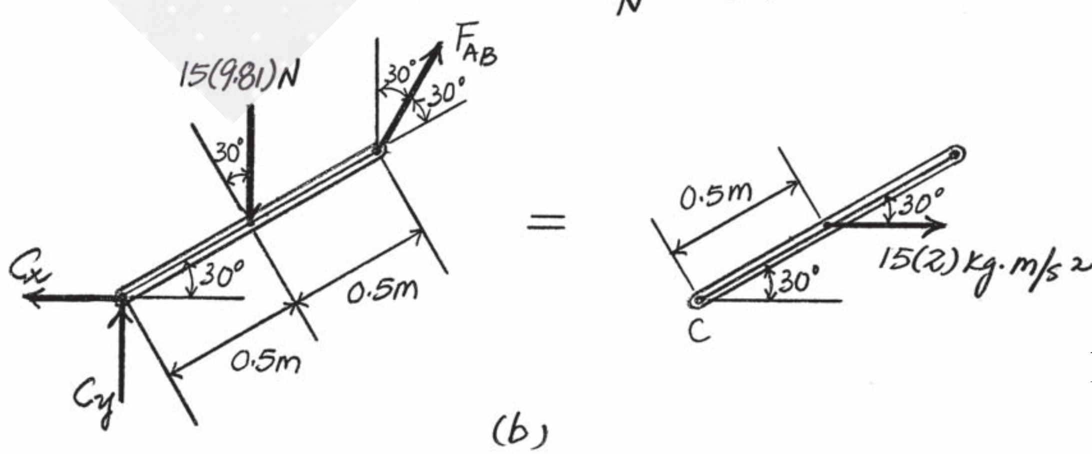
$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad -C_x + 112.44 \sin 30° = 15(2)$

$C_x = 26.22 \text{ N} = 26.2 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y + 112.44 \cos 30° - 15(9.81) = 0$

$C_y = 49.78 \text{ N} = 49.8 \text{ N}$ **Ans.**





Ans:
$F_{AB} = 112$ N
$C_x = 26.2$ N
$C_y = 49.8$ N

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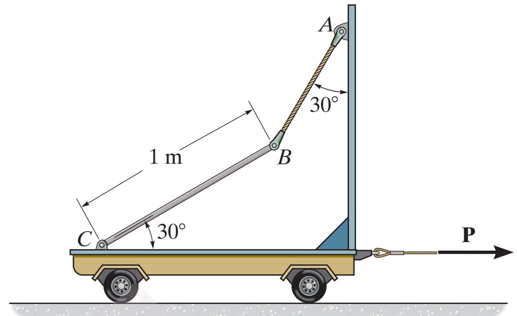
***17–40.**

If the cart's mass is 30 kg, determine the horizontal force P that should be applied to the cart so that the cord AB just becomes slack. The uniform rod BC has a mass of 15 kg.



SOLUTION

Equations of Motion: Since cord AB is required to be on the verge of becoming slack, $F_{AB} = 0$. The corresponding acceleration **a** of the rod can be obtained directly by writing the moment equation of motion about point C. By referring to Fig. a.

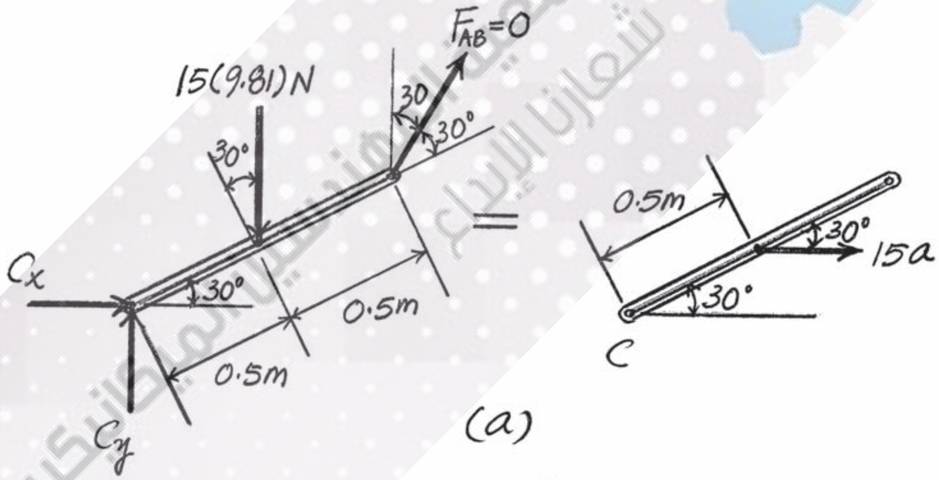
$$+\Sigma M_C = \Sigma(M_C)_A; \qquad -15(9.81)\cos 30°(0.5) = -15a\sin 30°(0.5)$$

$$a = 16.99 \text{ m/s}^2$$

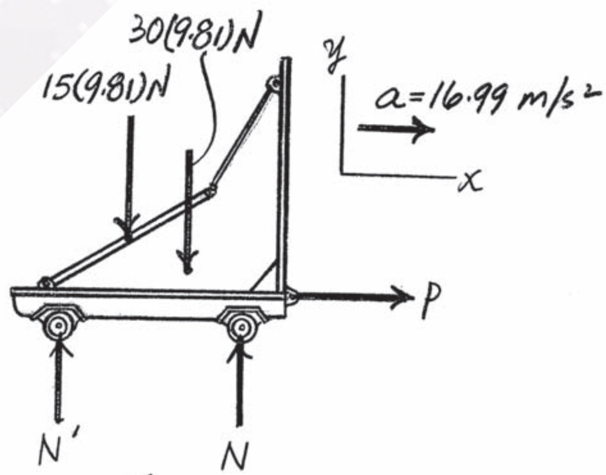
Using this result and writing the force equation of motion along the x axis and referring to the free-body diagram of the cart and rod system shown in Fig. b,

$$(\xrightarrow{+})\Sigma F_x = m(a_G)_x; \qquad P = (30 + 15)(16.99)$$

$$= 764.61\text{ N} = 765\text{ N} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$P = 765$ N

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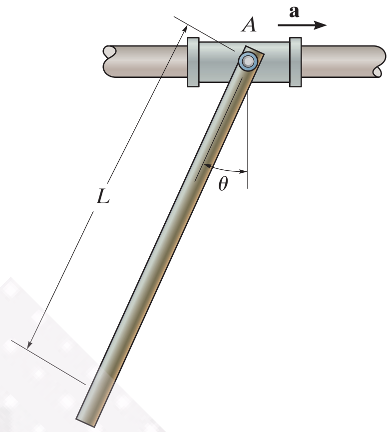
17–41.

The uniform bar of mass m is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of **a**, determine the bar's inclination angle θ. Neglect the collar's mass.

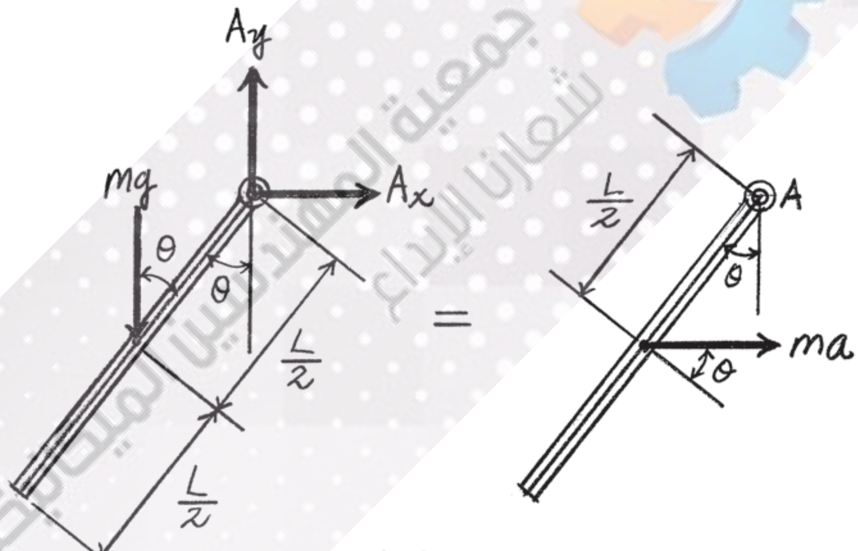


SOLUTION

Equations of Motion: Writing the moment equation of motion about point A,

$$+\Sigma M_A = (M_k)_A; \quad mg \sin\theta\left(\frac{L}{2}\right) = ma\cos\theta\left(\frac{L}{2}\right)$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \qquad \textbf{Ans.}$$



(a)

Ans:
$\theta = \tan^{-1}\left(\frac{a}{g}\right)$

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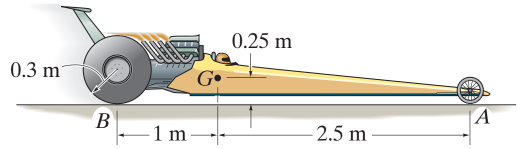
17–42. The dragster has a mass of 1500 kg and a center of mass at G. If the coefficient of kinetic friction between the rear wheels and the pavement is $\mu_k = 0.6$, determine if it is possible for the driver to lift the front wheels, A, off the ground while the rear drive wheels are slipping. Neglect the mass of the wheels and assume that the front wheels are free to roll.



SOLUTION

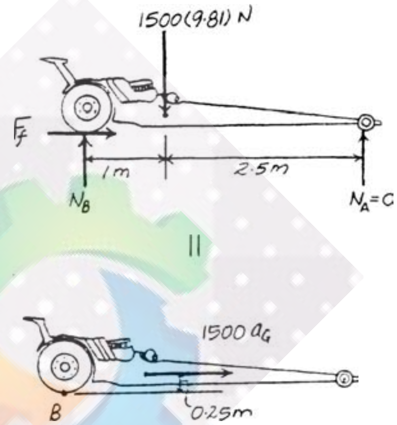
If the front wheels A lift off the ground, then $N_A = 0$.

$\zeta + \Sigma M_B = \Sigma(M_k)_B;\qquad -1500(9.81)(1) = -1500a_G(0.25)$

$$a_G = 39.24\ \text{m/s}^2$$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad F_f = 1500(39.24) = 58860\ \text{N}$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_B - 1500(9.81) = 0 \qquad N_B = 14715\ \text{N}$



Since the required friction $F_f > (F_f)_{\max} = \mu_k N_B = 0.6(14715) = 8829$ N, **it is not possible to lift the front wheels off the ground**. **Ans.**

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Ans:

Since the required friction $F_f > (F_f)_{\max} = \mu_k N_B = 0.6(14715) = 8829$ N, **it is not possible to lift the front wheels off the ground**.

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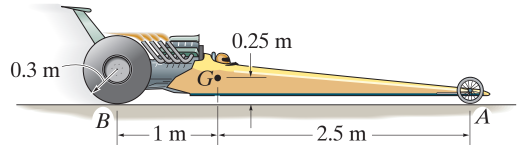
17–43. The dragster has a mass of 1500 kg and a center of mass at G. If no slipping occurs, determine the frictional force $\mathbf{F}_B$ which must be developed at each of the rear drive wheels B in order to create an acceleration of $a = 6\ \mathrm{m/s^2}$. What are the normal reactions of each wheel on the ground? Neglect the mass of the wheels and assume that the front wheels are free to roll.



SOLUTION

$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B;\qquad 2N_A(3.5) - 1500(9.81)(1) = -1500(6)(0.25)$

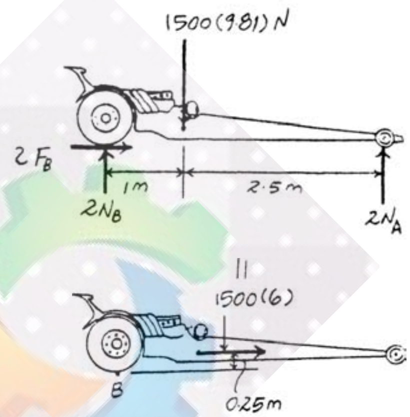
$N_A = 1780.71\ \mathrm{N} = 1.78\ \mathrm{kN}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2N_B + 2(1780.71) - 1500(9.81) = 0$

$N_B = 5576.79\ \mathrm{N} = 5.58\ \mathrm{kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 2F_B = 1500(6)$

$F_B = 4500\ \mathrm{N} = 4.50\ \mathrm{kN}$ **Ans.**



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Ans:
$N_A = 1.78\ \mathrm{kN}$
$N_B = 5.58\ \mathrm{kN}$
$F_B = 4.50\ \mathrm{kN}$

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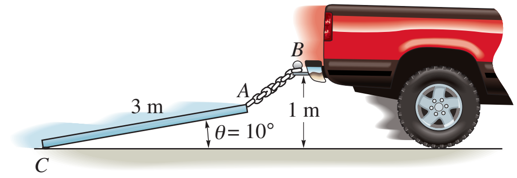
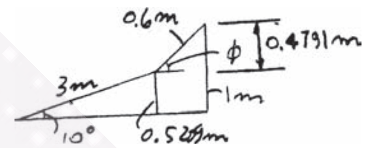
***17–44.**

The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6-m-long chain AB. If the coefficient of kinetic friction at C is $\mu_k = 0.4$, determine the acceleration of the truck if the angle $\theta = 10°$ with the road as shown.



SOLUTION



$$\phi = \sin^{-1}\left(\frac{0.4791}{0.6}\right) = 52.98°$$

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad T\cos 52.98° - 0.4N_C = 500a_G$$

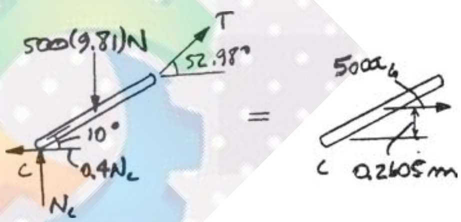
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_C - 500(9.81) + T\sin 52.98° = 0$$

$$\circlearrowleft +\Sigma M_C = \Sigma(M_k)_C; \qquad -500(9.81)(1.5\cos 10°) + T\sin(52.98° - 10°)(3) = -500a_G(0.2605)$$

$$T = 3.39\text{ kN}$$

$$N_C = 2.19\text{ kN}$$

$$a_G = 2.33\text{ m/s}^2 \qquad \textbf{Ans.}$$



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Ans:
$a_G = 2.33\text{ m/s}^2$

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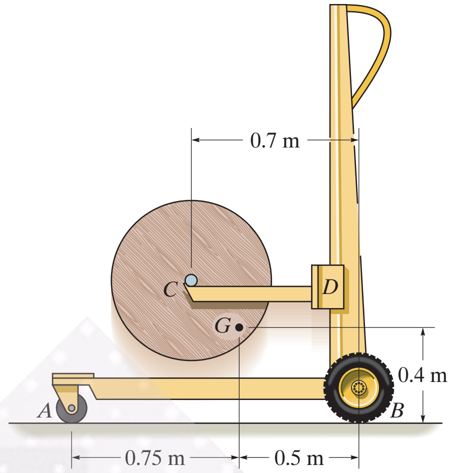
17–45.

The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of $3\ \text{m/s}^2$, determine the reactions on each of the four wheels. The loading is symmetric. Neglect the mass of the movable arm CD.



SOLUTION

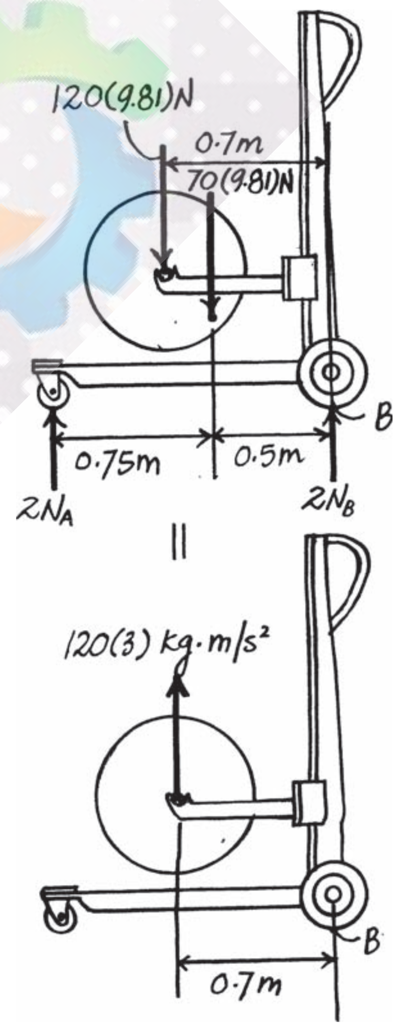
$\zeta + \Sigma M_B = \Sigma (M_k)_B;\qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25)$

$= -120(3)(0.7)$

$N_A = 567.76\ \text{N} = 568\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$

$N_B = 544\ \text{N}$ **Ans.**



Ans:

$N_A = 568\ \text{N}$
$N_B = 544\ \text{N}$

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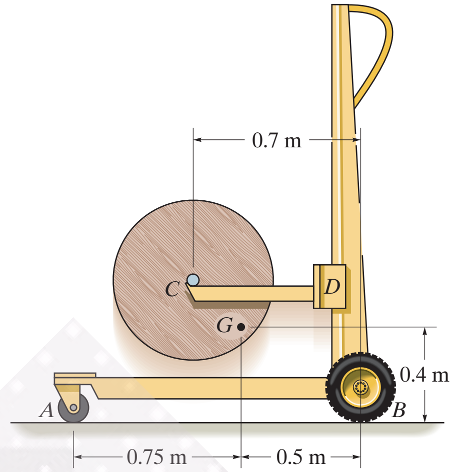
17–46.

The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction on the wheels exceeds 600 N.



SOLUTION

Assume $N_A = 600$ N.

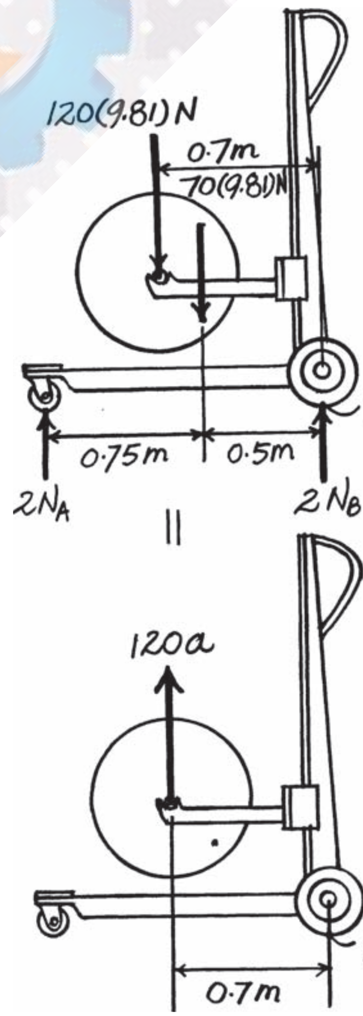
$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B;$ $\quad 70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$

$$a = 3.960 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$

$N_B = 570 \text{ N} < 600 \text{ N}$ **OK**

Thus $a = 3.96 \text{ m/s}^2$ **Ans.**



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Ans:
$a = 3.96 \text{ m/s}^2$

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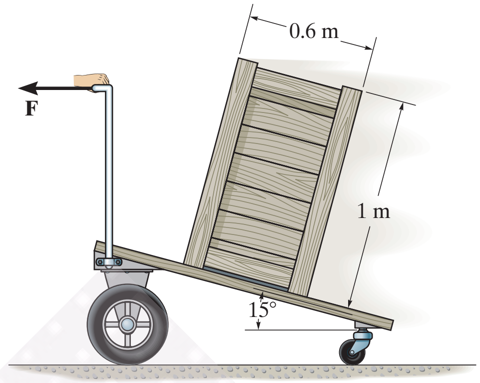
17–47.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



SOLUTION

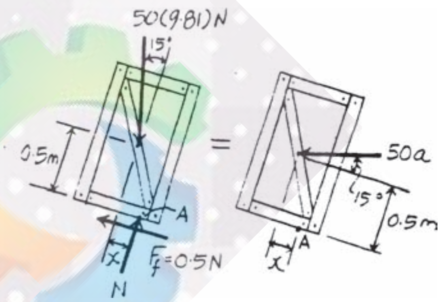
Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5)$

$$= 50a\cos 15°(0.5) + 50a\sin 15°(x) \qquad \textbf{(1)}$$

$+\nearrow\Sigma F_{y'} = m(a_G)_{y'};\quad N - 50(9.81)\cos 15° = -50a\sin 15° \qquad$ **(2)**

$\searrow + \Sigma F_{x'} = m(a_G)_{x'};\quad 50(9.81)\sin 15° - 0.5N = -50a\cos 15° \qquad$ **(3)**



Solving Eqs. (1), (2), and (3) yields

$$N = 447.81\text{ N} \qquad x = 0.250\text{ m}$$

$$a = 2.01\text{ m/s}^2 \qquad \textbf{Ans.}$$

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

Ans:
$a = 2.01\text{ m/s}^2$
The crate slips.

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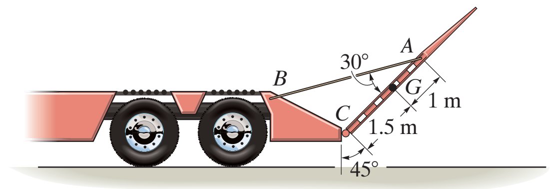
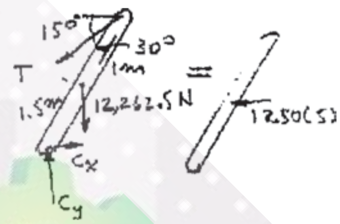
***17–48.**

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at 5 m/s^2. Also, what are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\zeta + \Sigma M_C = \Sigma(M_k)_C;\quad T \sin 30°(2.5) - 12\,262.5(1.5 \cos 45°) = 1250(5)(1.5 \sin 45°)$

$T = 15\,708.4 \text{ N} = 15.7 \text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad -C_x + 15\,708.4 \cos 15° = 1250(5)$

$C_x = 8.92 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad C_y - 12\,262.5 - 15\,708.4 \sin 15° = 0$

$C_y = 16.3 \text{ kN}$ **Ans.**

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Ans:
$T = 15.7$ kN
$C_x = 8.92$ kN
$C_y = 16.3$ kN

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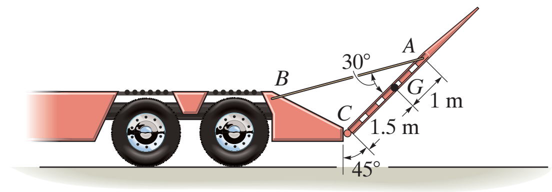
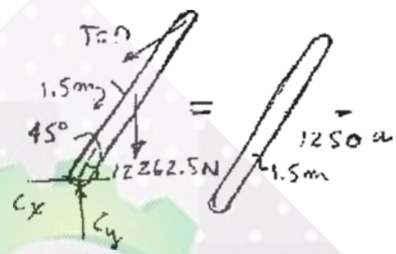
17–49.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the maximum deceleration of the truck so that the gate does not begin to rotate forward. What are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\circlearrowleft + \Sigma M_C = \Sigma(M_k)_C;\qquad -12\,262.5(1.5\cos 45°) = -1250(a)(1.5\sin 45°)$

$a = 9.81\ \text{m/s}^2$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad C_x = 1250(9.81)$

$C_x = 12.3\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad C_y - 12\,262.5 = 0$

$C_y = 12.3\ \text{kN}$ **Ans.**

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Ans:
$a = 9.81\ \text{m/s}^2$
$C_x = 12.3\ \text{kN}$
$C_y = 12.3\ \text{kN}$

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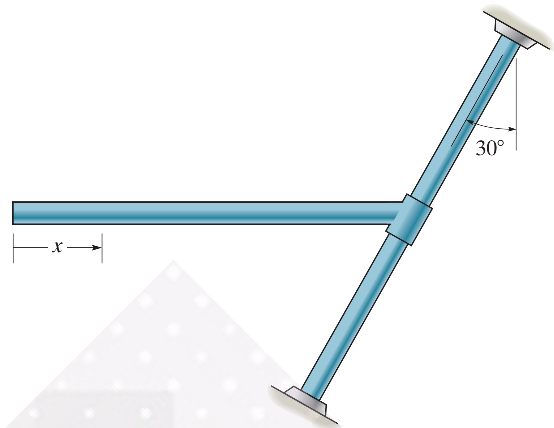
17–50.

The bar has a weight per length w and is supported by the smooth collar. If it is released from rest, determine the internal normal force, shear force, and bending moment in the bar as a function of x.



SOLUTION

Entire bar:

$$\Sigma F_{x'} = m(a_G)_{x'}; \qquad wl \cos 30° = \frac{wl}{g}(a_G)$$

$$a_G = g \cos 30°$$

Segment:

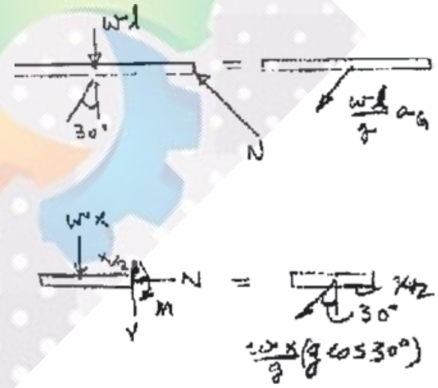
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad N = (wx \cos 30°) \sin 30° = 0.433wx \qquad \textbf{Ans.}$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad wx - V = wx \cos 30°(\cos 30°)$$

$$V = 0.25wx \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_S = \Sigma(M_k)_S; \qquad wx\left(\frac{x}{2}\right) - M = wx \cos 30°(\cos 30°)\left(\frac{x}{2}\right)$$

$$M = 0.125wx^2 \qquad \textbf{Ans.}$$



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Ans:
$N = 0.433wx$
$V = 0.25wx$
$M = 0.125wx^2$

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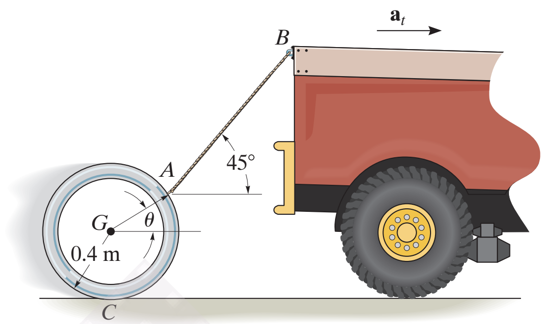
17–51.

The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5\ \text{m/s}^2$, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.

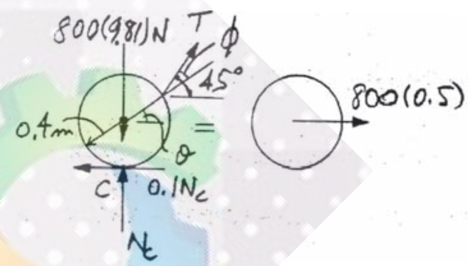


SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -0.1N_C + T\cos 45° = 800(0.5)$

$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 800(9.81) + T\sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad -0.1N_C(0.4) + T\sin\phi(0.4) = 0$



$N_C = 6770.9\ \text{N}$

$T = 1523.24\ \text{N} = 1.52\ \text{kN}$ **Ans.**

$\sin\phi = \dfrac{0.1(6770.9)}{1523.24} \qquad \phi = 26.39°$

$\theta = 45° - \phi = 18.6°$ **Ans.**

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Ans:
$T = 1.52\ \text{kN}$
$\theta = 18.6°$

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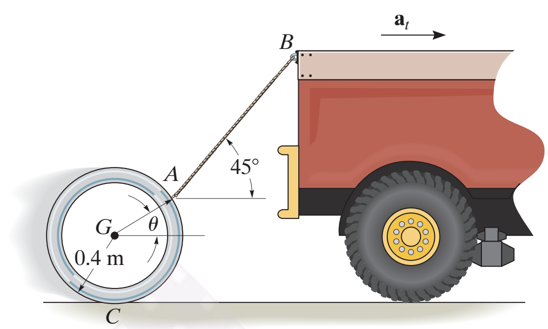
***17–52.**

The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T\cos 45° - 0.1N_C = 800a$

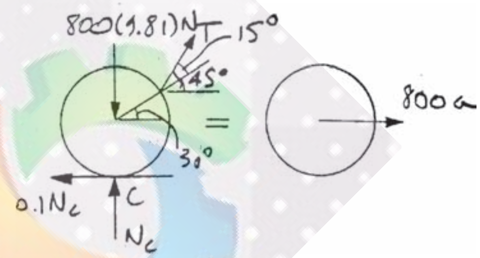
$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 800(9.81) + T\sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad T\sin 15°(0.4) - 0.1N_C(0.4) = 0$

$N_C = 6161 \text{ N}$

$T = 2382 \text{ N} = 2.38 \text{ kN}$ **Ans.**

$a = 1.33 \text{ m/s}^2$ **Ans.**



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Ans:
$T = 2.38 \text{ kN}$
$a = 1.33 \text{ m/s}^2$

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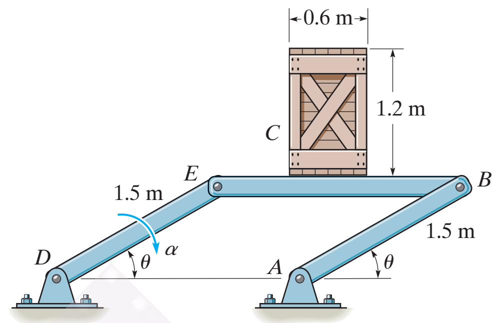
17–53.

The 100-kg uniform crate C rests on the elevator floor where the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest at $\theta = 90°$, without causing the crate to slip. No tipping occurs.



SOLUTION

Equations of Motion. The crate undergoes curvilinear translation. At $\theta = 90°$, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. However; $(a_G)_t = \alpha r = \alpha(1.5)$. Assuming that the crate slides before it tips, then, $F_f = \mu_s N = 0.4\ N$.

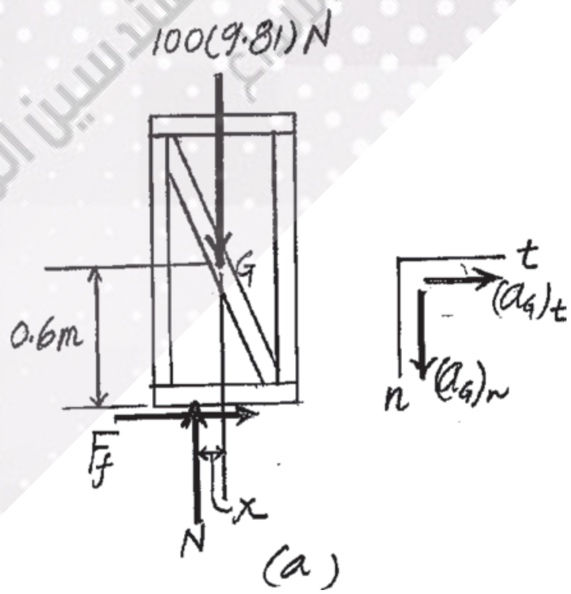
$\Sigma F_n = m(a_G)_n;\qquad 100(9.81) - N = 100(0)\qquad N = 981\text{ N}$

$\Sigma F_t = m(a_G)_t;\qquad 0.4(981) = 100[\alpha(1.5)]\qquad \alpha = 2.616\text{ rad/s}^2 = 2.62\text{ rad/s}^2$ **Ans.**

$\circlearrowleft + \Sigma M_G = 0;\qquad 0.4(981)(0.6) - 981(x) = 0$

$$x = 0.24\text{ m}$$

Since $x < 0.3$ m, the crate indeed slides before it tips, as assumed.



Ans:
$\alpha = 2.62\text{ rad/s}^2$

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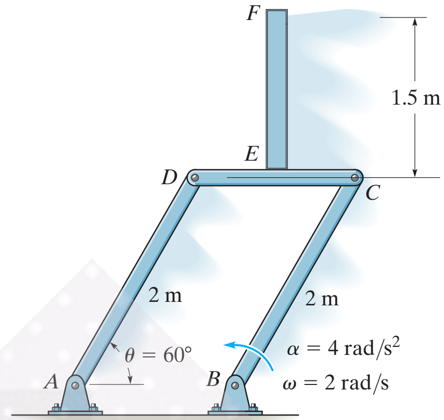
17–54.

The two uniform 4-kg bars DC and EF are fixed (welded) together at E. Determine the normal force N_E, shear force V_E, and moment M_E, which DC exerts on EF at E if at the instant $\theta = 60°$ BC has an angular velocity $\omega = 2$ rad/s and an angular acceleration $\alpha = 4$ rad/s^2 as shown.



SOLUTION

Equations of Motion. The rod assembly undergoes curvilinear motion. Thus, $(a_G)_t = \alpha r = 4(2) = 8$ m/s^2 and $(a_G)_n = \omega^2 r = (2^2)(2) = 8$ m/s^2. Referring to the FBD and kinetic diagram of rod EF, Fig. *a*

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad V_E = 4(8)\cos 30° + 4(8)\cos 60°$$

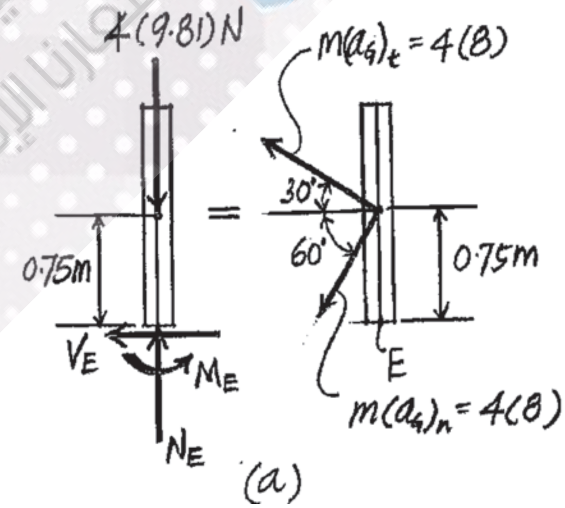
$$= 43.71 \text{ N} = 43.7 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_E - 4(9.81) = 4(8)\sin 30° - 4(8)\sin 60°$$

$$N_E = 27.53 \text{ N} = 27.5 \text{ N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_E = \Sigma(M_k)_E; \qquad M_E = 4(8)\cos 30°(0.75) + 4(8)\cos 60°(0.75)$$

$$= 32.78 \text{ N}\cdot\text{m} = 32.8 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



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Ans:
$V_E = 43.7$ N
$N_E = 27.5$ N
$M_E = 32.8$ N·m

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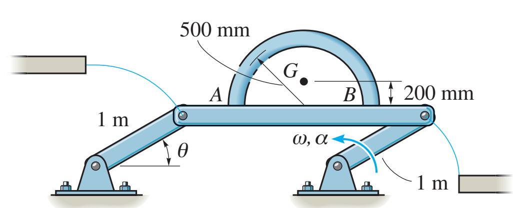
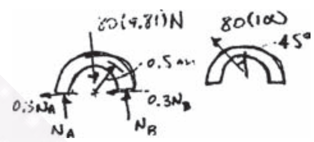
17–55.

The arched pipe has a mass of 80 kg and rests on the surface of the platform for which the coefficient of static friction is $\mu_s = 0.3$. Determine the greatest angular acceleration α of the platform, starting from rest when $\theta = 45°$, without causing the pipe to slip on the platform.



SOLUTION



$a_G = (a_G)_t = (1)(\alpha)$

$\zeta\, +\Sigma M_A = \Sigma(M_k)_A;\qquad N_B(1) - 80(9.81)(0.5) = 80(1\alpha)(\sin 45°)(0.2) + 80(1\alpha)(\cos 45°)(0.5)$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 0.3N_A + 0.3N_B = 80(1\alpha)\sin 45°$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + N_B - 80(9.81) = 80(1\alpha)\cos 45°$

Solving,

$\alpha = 5.95 \text{ rad/s}^2$ **Ans.**

$N_A = 494 \text{ N}$

$N_B = 628 \text{ N}$

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Ans:
$\alpha = 5.95 \text{ rad/s}^2$

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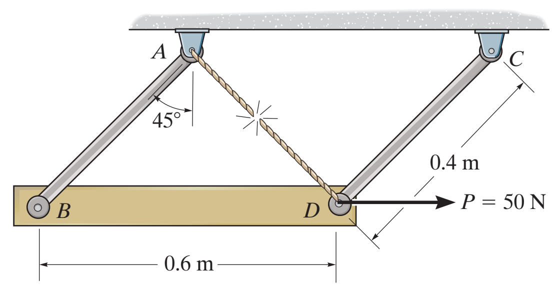
***17–56.**

Determine the force developed in the links and the acceleration of the bar's mass center immediately after the cord fails. Neglect the mass of links AB and CD. The uniform bar has a mass of 20 kg.



SOLUTION

Equations of Motion: Since the bar is still at rest at the instant the cord fails, $v_G = 0$. Thus, $(a_G)_n = \dfrac{v_G^2}{r} = 0$. Referring to the free-body diagram of the bar, Fig. *a*,

$\Sigma F_n = m(a_G)_n;$ $\quad T_{AB} + T_{CD} - 20(9.81)\cos 45° + 50\cos 45° = 0$

$\Sigma F_t = m(a_G)_t;$ $\quad 20(9.81)\sin 45° + 50\sin 45° = 20(a_G)_t$

$+\Sigma M_G = 0;$ $\quad T_{CD}\cos 45°(0.3) - T_{AB}\cos 45°(0.3) = 0$

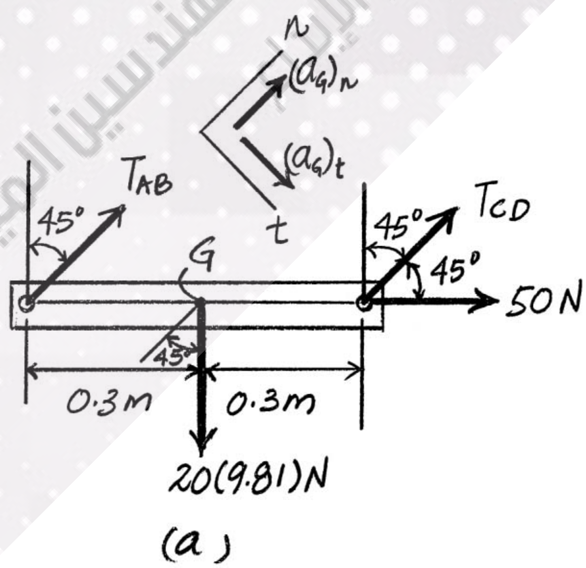
Solving,

$$T_{AB} = T_{CD} = 51.68\text{ N} = 51.7\text{ N}$$ **Ans.**

$$(a_G)_t = 8.704\text{ m/s}^2$$

Since $(a_G)_n = 0$, then

$$a_G = (a_G)_t = 8.70\text{ m/s}^2 \searrow$$ **Ans.**



Ans:
$T_{AB} = T_{CD} = 51.7\text{ N}$
$a_G = (a_G)_t = 8.70\text{ m/s}^2 \searrow$

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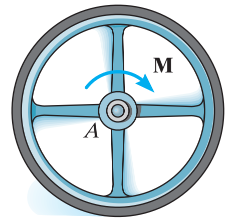
17–57.

The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N·m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

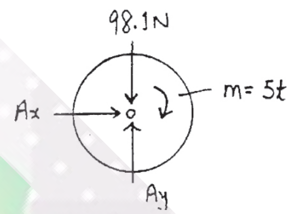


SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad A_x = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 10(9.81) = 0$

$\circlearrowleft + \Sigma M_A = I_a\alpha; \qquad 5t = 10(0.2)^2\alpha$



$$\alpha = \frac{d\omega}{dt} = 12.5t$$

$$\omega = \int_0^3 12.5t\,dt = \frac{12.5}{2}(3)^2$$

$\omega = 56.2$ rad/s **Ans.**

$A_x = 0$ **Ans.**

$A_y = 98.1$ N **Ans.**

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Ans:
$\omega = 56.2$ rad/s
$A_x = 0$
$A_y = 98.1$ N

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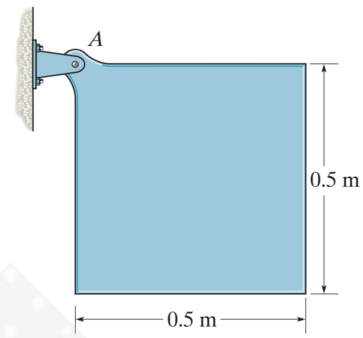
17–58.

The uniform 24-kg plate is released from rest at the position shown. Determine its initial angular acceleration and the horizontal and vertical reactions at the pin A.



SOLUTION

Equations of Motion. The mass moment of inertia of the plate about its center of gravity G is $I_G = \frac{1}{12}(24)\left(0.5^2 + 0.5^2\right) = 1.00\ \text{kg}\cdot\text{m}^2$. Since the plate is at rest initially $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Here $r_G = \sqrt{0.25^2 + 0.25^2} = 0.25\sqrt{2}\ \text{m}$. Thus, $(a_G)_t = \alpha r_G = \alpha\left(0.25\sqrt{2}\right)$. Referring to the FBD and kinetic diagram of the plate,

$$\zeta + \Sigma M_A = (M_k)_A; \qquad -24(9.81)(0.25) = -24\left[\alpha\left(0.25\sqrt{2}\right)\right]\left(0.25\sqrt{2}\right) - 1.00\,\alpha$$

$$\alpha = 14.715\ \text{rad/s}^2 = 14.7\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Also, the same result can be obtained by applying $\Sigma M_A = I_A\alpha$ where
$I_A = \frac{1}{12}(24)\left(0.5^2 + 0.5^2\right) + 24\left(0.25\sqrt{2}\right)^2 = 4.00\ \text{kg}\cdot\text{m}^2$:

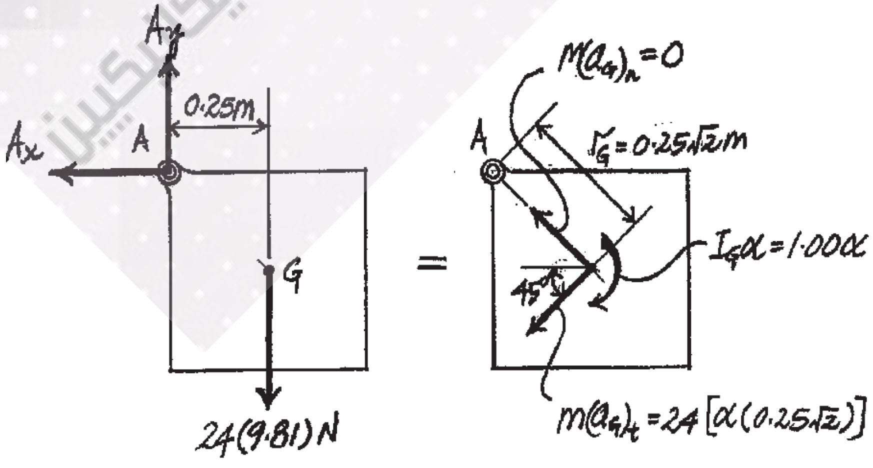
$$\zeta + \Sigma M_A = I_A\,\alpha; \qquad -24(9.81)(0.25) = -4.00\,\alpha$$

$$\alpha = 14.715\ \text{rad/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad A_x = 24\left[14.715\left(0.25\sqrt{2}\right)\right]\cos 45^\circ = 88.29\ \text{N} = 88.3\ \text{N} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 24(9.81) = -24\left[14.715\left(0.25\sqrt{2}\right)\right]\sin 45^\circ$$

$$A_y = 147.15\ \text{N} = 147\ \text{N} \qquad \textbf{Ans.}$$



(a)

Ans:
$\alpha = 14.7\ \text{rad/s}^2$
$A_x = 88.3\ \text{N}$
$A_y = 147\ \text{N}$

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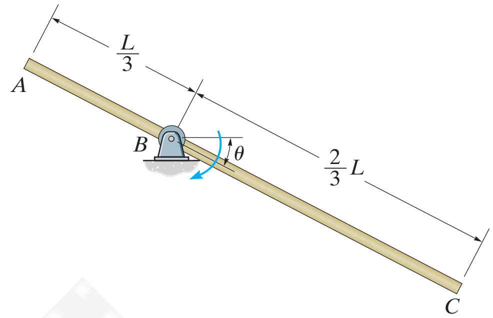
17–59.

The uniform slender rod has a mass m. If it is released from rest when $\theta = 0°$, determine the magnitude of the reactive force exerted on it by pin B when $\theta = 90°$.



SOLUTION

Equations of Motion: Since the rod rotates about a fixed axis passing through point B, $(a_G)_t = \alpha r_G = \alpha\left(\frac{L}{6}\right)$ and $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{6}\right)$. The mass moment of inertia of the rod about its G is $I_G = \frac{1}{12}mL^2$. Writing the moment equation of motion about point B,

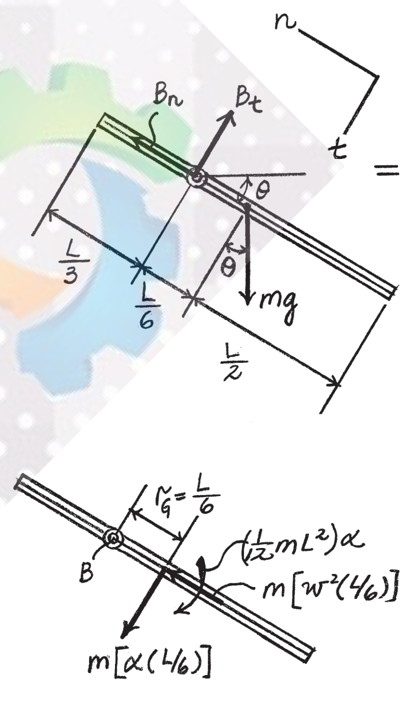
$$+\Sigma M_B = \Sigma(M_k)_B; \qquad -mg\cos\theta\left(\frac{L}{6}\right) = -m\left[\alpha\left(\frac{L}{6}\right)\right]\left(\frac{L}{6}\right) - \left(\frac{1}{12}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

This equation can also be obtained by applying $\Sigma M_B = I_B\alpha$, where $I_B = \frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{9}mL^2$. Thus,

$$+\Sigma M_B = I_B\alpha; \qquad -mg\cos\theta\left(\frac{L}{6}\right) = -\left(\frac{1}{9}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$



(a)

Using this result and writing the force equation of motion along the n and t axes,

$$\Sigma F_t = m(a_G)_t; \qquad mg\cos\theta - B_t = m\left[\left(\frac{3g}{2L}\cos\theta\right)\left(\frac{L}{6}\right)\right]$$

$$B_t = \frac{3}{4}mg\cos\theta \qquad \textbf{(1)}$$

$$\Sigma F_n = m(a_G)_n; \qquad B_n - mg\sin\theta = m\left[\omega^2\left(\frac{L}{6}\right)\right]$$

$$B_n = \frac{1}{6}m\omega^2 L + mg\sin\theta \qquad \textbf{(2)}$$

Kinematics: The angular velocity of the rod can be determined by integrating

$$\int \omega d\omega = \int \alpha d\theta$$

$$\int_0^\omega \omega d\omega = \int_0^\theta \frac{3g}{2L}\cos\theta\, d\theta$$

$$\omega = \sqrt{\frac{3g}{L}\sin\theta}$$

When $\theta = 90°$, $\omega = \sqrt{\frac{3g}{L}}$. Substituting this result and $\theta = 90°$ into Eqs. (1) and (2),

$$B_t = \frac{3}{4}mg\cos 90° = 0$$

$$B_n = \frac{1}{6}m\left(\frac{3g}{L}\right)(L) + mg\sin 90° = \frac{3}{2}mg$$

$$F_A = \sqrt{A_t^2 + A_n^2} = \sqrt{0^2 + \left(\frac{3}{2}mg\right)^2} = \frac{3}{2}mg \qquad \textbf{Ans.}$$

Ans:

$F_A = \frac{3}{2}mg$

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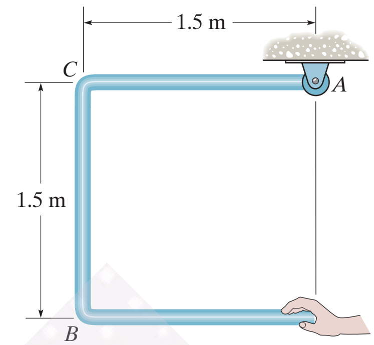
***17–60.**

The bent rod has a mass of 2 kg/m. If it is released from rest in the position shown, determine its initial angular acceleration and the horizontal and vertical components of reaction at A.



SOLUTION

Equations of Motion. Referring to Fig. a, the location of center of gravity G of the bent rod is at

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{2[0.75(1.5)(2)] + 1.5(2)(1.5)}{3(1.5)(2)} = 1.00 \text{ m}$$

$$\bar{y} = \frac{1.5}{2} = 0.75 \text{ m}$$

The mass moment of inertia of the bent rod about its center of gravity is $I_G = 2\left[\frac{1}{12}(3)(1.5^2) + 3(0.25^2 + 0.75^2)\right] + \left[\frac{1}{12}(3)(1.5^2) + 3(0.5^2)\right] = 6.1875 \text{ kg} \cdot \text{m}^2$. Here, $r_G = \sqrt{1.00^2 + 0.75^2} = 1.25$ m. Since the bent rod is at rest initially, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.25)$. Referring to the FBD and kinetic diagram of the plate,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 9(9.81)(1) = 9[\alpha(1.25)](1.25) + 6.1875\,\alpha$$

$$\alpha = 4.36 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Also, the same result can be obtained by applying $\Sigma M_A = I_A \alpha$ where

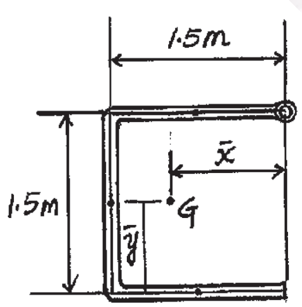
$$I_A = \frac{1}{12}(3)(1.5^2) + 3(0.75^2) + \frac{1}{12}(3)(1.5^2) + 3(1.5^2 + 0.75^2)$$
$$+ \frac{1}{12}(3)(1.5^2) + 3(1.5^2 + 0.75^2) = 20.25 \text{ kg} \cdot \text{m}^2:$$

$$\circlearrowleft + \Sigma M_A = I_A \alpha, \qquad 9(9.81)(1) = 20.25\,\alpha \qquad \alpha = 4.36 \text{ rad/s}^2$$

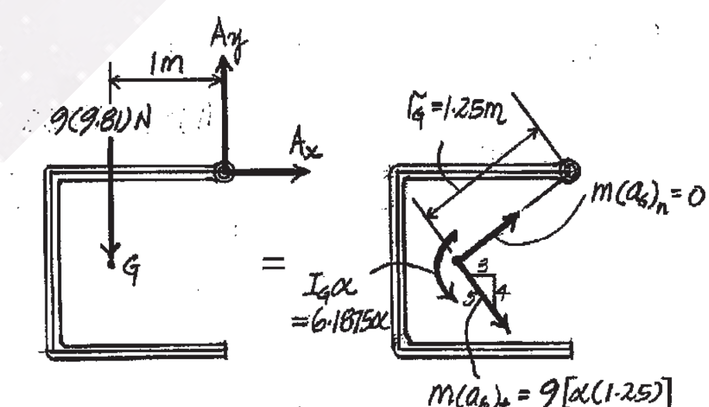
$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad A_x = 9[4.36(1.25)]\left(\frac{3}{5}\right) = 29.43 \text{ N} = 29.4 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 9(9.81) = -9[4.36(1.25)]\left(\frac{4}{5}\right)$$

$$A_y = 49.05 \text{ N} = 49.1 \text{ N} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\alpha = 4.36 \text{ rad/s}^2 \circlearrowright$
$A_x = 29.4$ N
$A_y = 49.1$ N

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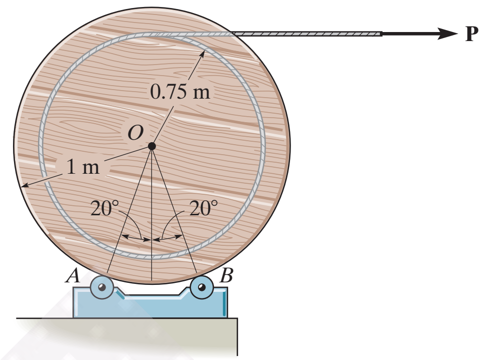
17–61.

If a horizontal force of $P = 100$ N is applied to the 300-kg reel of cable, determine its initial angular acceleration. The reel rests on rollers at A and B and has a radius of gyration of $k_O = 0.6$ m.



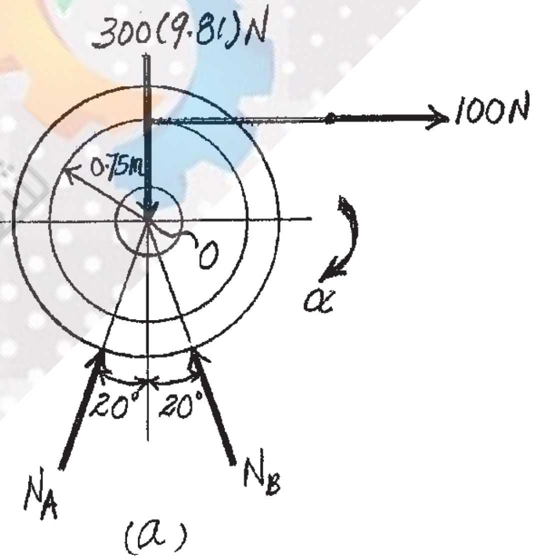
SOLUTION

Equations of Motions. The mass moment of inertia of the reel about O is $I_O = Mk_O^2 = 300(0.6^2) = 108\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. a,

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad -100(0.75) = 108(-\alpha)$$

$$\alpha = 0.6944\ \text{rad/s}^2$$

$$= 0.694\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



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Ans:
$\alpha = 0.694\ \text{rad/s}^2$

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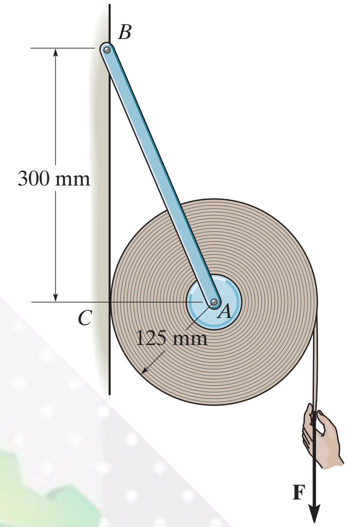
17–62.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$ and a vertical force $F = 30$ N is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad N_C - T_{AB}\cos 67.38° = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_{AB}\sin 67.38° - 0.2N_C - 20(9.81) - 30 = 0$

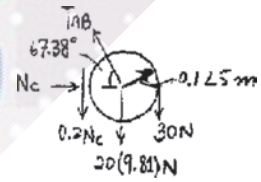
$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad -0.2N_C(0.125) + 30(0.125) = 20(0.09)^2\alpha$

Solving:

$N_C = 103$ N

$T_{AB} = 267$ N

$\alpha = 7.28 \text{ rad/s}^2$ **Ans.**



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Ans:
$\alpha = 7.28 \text{ rad/s}^2$

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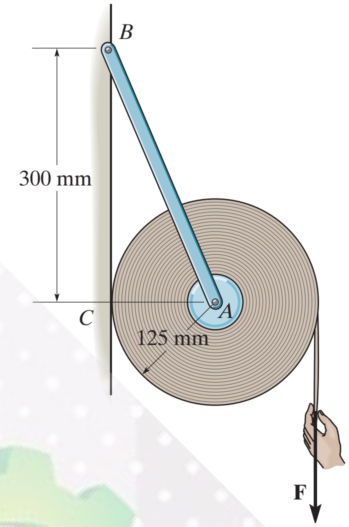
17–63.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$, determine the constant vertical force F that must be applied to the roll to pull off 1 m of paper in $t = 3$ s starting from rest. Neglect the mass of paper that is removed.



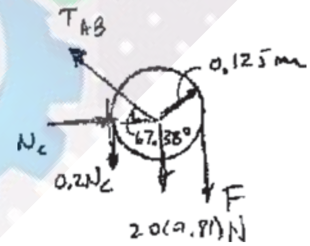
SOLUTION

$$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_C t^2$$

$$1 = 0 + 0 + \frac{1}{2}a_C(3)^2$$

$$a_C = 0.222\ \text{m/s}^2$$

$$\alpha = \frac{a_C}{0.125} = 1.778\ \text{rad/s}^2$$



$\xrightarrow{+}\ \Sigma F_x = m(a_{G\,x});\qquad N_C - T_{AB}\cos 67.38^\circ = 0$

$+\uparrow\ \Sigma F_y = m(a_G)_y;\qquad T_{AB}\sin 67.38^\circ - 0.2N_C - 20(9.81) - F = 0$

$\circlearrowleft + \ \Sigma M_A = I_A\alpha;\qquad -0.2N_C(0.125) + F(0.125) = 20(0.09)^2(1.778)$

Solving:

$$N_C = 99.3\ \text{N}$$

$$T_{AB} = 258\ \text{N}$$

$$F = 22.1\ \text{N} \qquad \textbf{Ans.}$$

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Ans:
$F = 22.1$ N

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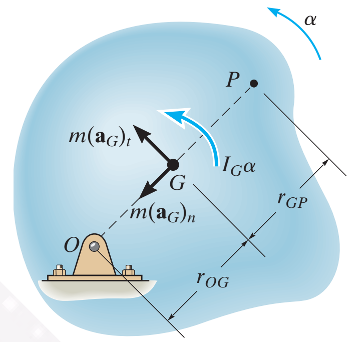
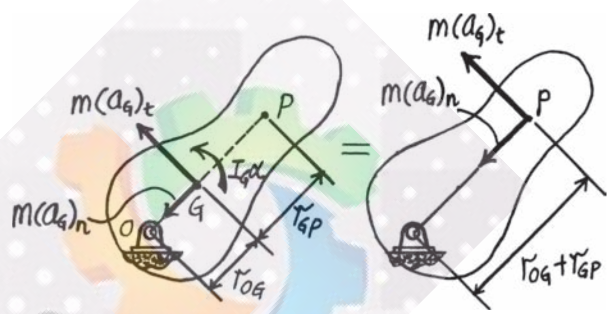
***17–64.**

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through O is shown in the figure. Show that $I_G\boldsymbol{\alpha}$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about an axis passing through G. The point P is called the *center of percussion* of the body.



SOLUTION

$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + \left(mk_G^2\right)\alpha$$



However,

$$k_G^2 = r_{OG}\, r_{GP} \text{ and } \alpha = \frac{(a_G)_t}{r_{OG}}$$

$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + (mr_{OG}\, r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

$$= m(a_G)_t(r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$

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Ans:
$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t(r_{OG} + r_{GP})$

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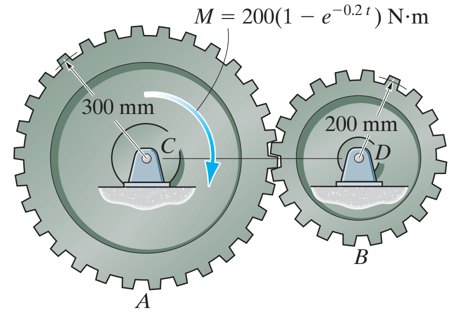
17–65.



Gears A and B have a mass of 50 kg and 15 kg, respectively. Their radii of gyration about their respective centers of mass are $k_C = 250$ mm and $k_D = 150$ mm. If a torque of $M = 200(1 - e^{-0.2t})$ N · m, where t is in seconds, is applied to gear A, determine the angular velocity of both gears when $t = 3$ s, starting from rest.

SOLUTION

Equations of Motion: Since gear B is in mesh with gear A, $\alpha_B = \left(\dfrac{r_A}{r_B}\right)\alpha_A = \left(\dfrac{0.3}{0.2}\right)\alpha_A = 1.5\alpha_A$. The mass moment of inertia of gears A and B about their respective centers are $I_C = m_A k_C{}^2 = 50(0.25^2) = 3.125 \text{ kg}\cdot\text{m}^2$ and $I_D = m_B k_D{}^2 = 15(0.15^2) = 0.3375 \text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about the gears' center using the free-body diagrams of gears A and B, Figs. *a* and *b*,

$$\circlearrowleft+\Sigma M_C = I_C\alpha_A; \quad F(0.3) - 200(1 - e^{-0.2t}) = -3.125\alpha_A \qquad \textbf{(1)}$$

and

$$\circlearrowleft+\Sigma M_D = I_D\alpha_B; \quad F(0.2) = 0.3375(1.5\alpha_A) \qquad \textbf{(2)}$$

Eliminating F from Eqs. (1) and (2) yields

$$\alpha_A = 51.49(1 - e^{-0.2t}) \text{ rad/s}^2$$

Kinematics: The angular velocity of gear A can be determined by integration.

$$\int d\omega_A = \int \alpha_A\, dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t 51.49(1 - e^{-0.2t})dt$$

$$\omega_A = 51.49(t + 5e^{-0.2t} - 5) \text{ rad/s}$$

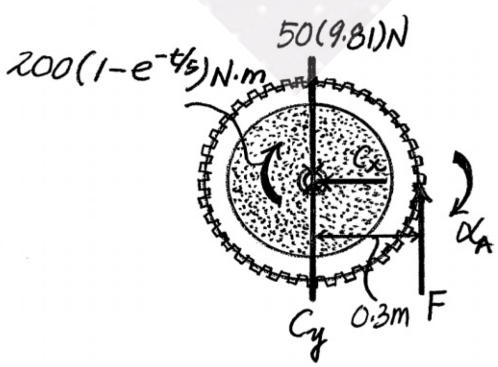
When $t = 3$ s,

$$\omega_A = 51.49(3 + 5e^{-0.2(3)} - 5) = 38.31 \text{ rad/s} = 38.3 \text{ rad/s} \qquad \textbf{Ans.}$$

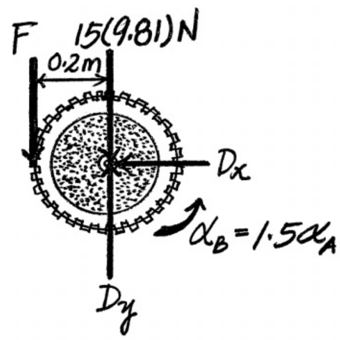
Then

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{0.3}{0.2}\right)(38.31)$$

$$= 57.47 \text{ rad/s} = 57.5 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\omega_A = 38.3$ rad/s
$\omega_B = 57.5$ rad/s

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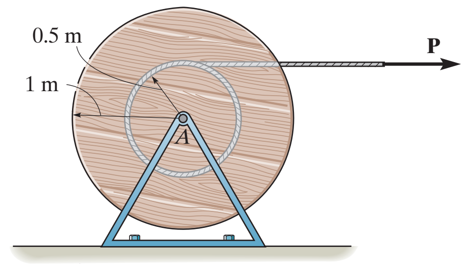
17–66.

The reel of cable has a mass of 400 kg and a radius of gyration of $k_A = 0.75$ m. Determine its angular velocity when $t = 2$ s, starting from rest, if the force $\mathbf{P} = (20t^2 + 80)$ N, when t is in seconds. Neglect the mass of the unwound cable, and assume it is always at a radius of 0.5 m.



SOLUTION

Equations of Motion. The mass moment of inertia of the reel about A is $I_A = Mk_A^2 = 400(0.75^2) = 225\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. *a*

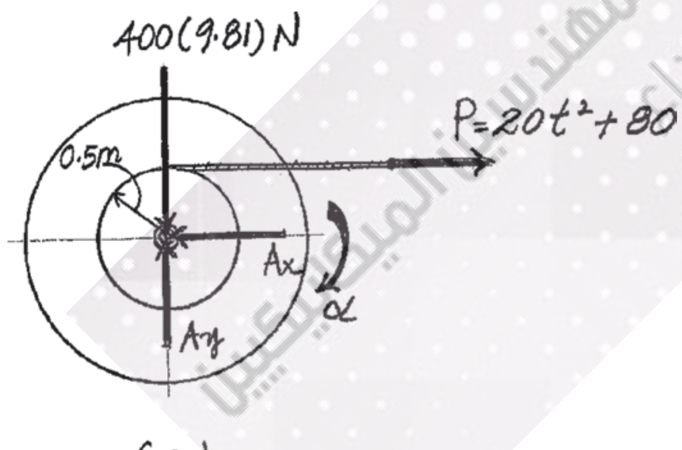
$\circlearrowleft + \Sigma M_A = I_A\alpha; \quad -(20t^2 + 80)(0.5) = 225(-\alpha)$

$$\alpha = \frac{2}{45}(t^2 + 4)\ \text{rad/s}^2$$

Kinematics. Using the result of α, integrate $d\omega = \alpha dt$, with the initial condition $\omega = 0$ at $t = 0$,

$$\int_0^{\omega} d\omega = \int_0^{2s} \frac{2}{45}(t^2 + 4)\, dt$$

$$\omega = 0.4741\ \text{rad/s} = 0.474\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$\omega = 0.474$ rad/s

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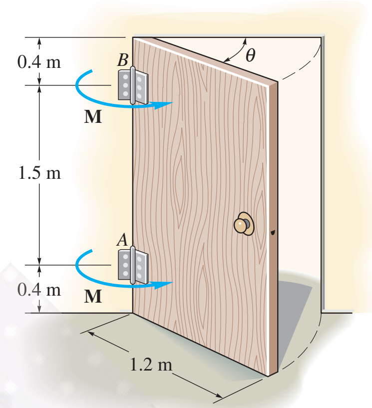
17–67.

The door will close automatically using torsional springs mounted on the hinges. Each spring has a stiffness $k = 50\ \text{N}\cdot\text{m/rad}$ so that the torque on each hinge is $M = (50\theta)\ \text{N}\cdot\text{m}$, where θ is measured in radians. If the door is released from rest when it is open at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



SOLUTION

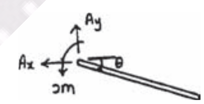
$$I_{AB} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(70)(1.2)^2 + 70(0.6)^2 = 33.6\ \text{kg}\cdot\text{m}^2$$

$$\Sigma M_{AB} = I_{AB}\,\alpha; \qquad 2(50\theta) = -33.6(\alpha) \qquad \alpha = -2.9762\theta$$

$$\omega d\omega = \alpha d\theta$$

$$\int_0^{\omega} \omega d\omega = -\int_{\frac{\pi}{2}}^{0} 2.9762\theta d\theta$$

$$\omega = 2.71\ \text{rad/s}$$ **Ans.**



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Ans:

$\omega = 2.71\ \text{rad/s}$

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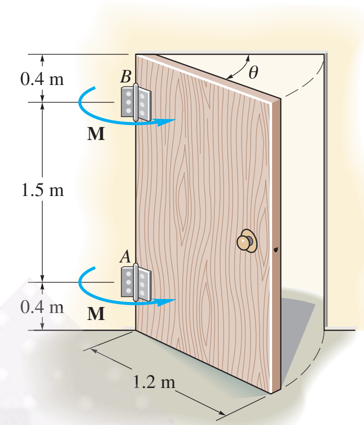
***17–68.**

The door will close automatically using torsional springs mounted on the hinges. If the torque on each hinge is $M = k\theta$, where θ is measured in radians, determine the required torsional stiffness k so that the door will close ($\theta = 0°$) with an angular velocity $\omega = 2$ rad/s when it is released from rest at $\theta = 90°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



SOLUTION

$$\Sigma M_A = I_A\alpha; \qquad 2M = -\left[\frac{1}{12}(70)(1.2)^2 + 70(0.6)^2\right](\alpha)$$

$$M = -16.8\alpha$$

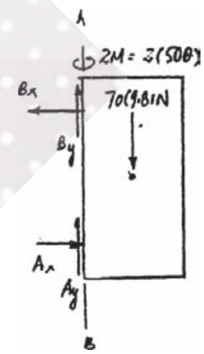
$$k\theta = -16.8\alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$-k\int_{\frac{\pi}{2}}^{0} \theta d\theta = 16.8\int_0^2 \omega\, d\omega$$

$$\frac{k}{2}\left(\frac{\pi}{2}\right)^2 = \frac{16.8}{2}(2)^2$$

$$k = 27.2 \text{ N}\cdot\text{m/rad} \qquad \textbf{Ans.}$$



Ans:
$k = 27.2$ N·m/rad

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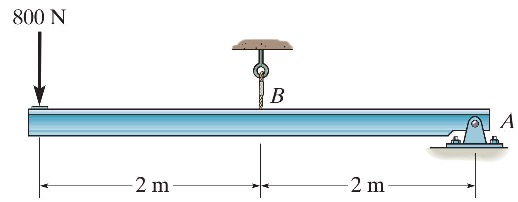
17–69.

If the cord at B suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin A, and the angular acceleration of the 120-kg beam. Treat the beam as a uniform slender rod.



SOLUTION

Equations of Motion. The mass moment of inertia of the beam about A is $I_A = \frac{1}{12}(120)(4^2) + 120(2^2) = 640\ \text{kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(2) = 2\alpha$. Referring to the FBD of the beam, Fig. *a*

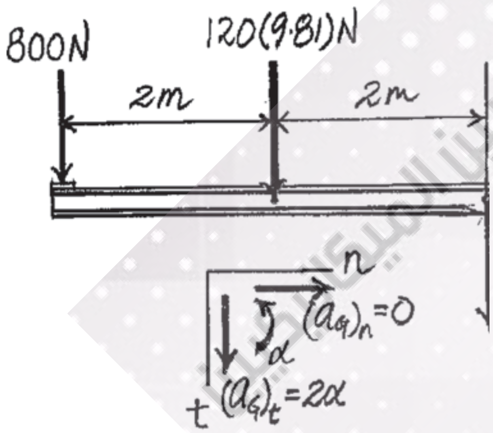
$\zeta + \Sigma M_A = I_A\alpha;$ $\quad 800(4) + 120(9.81)(2) = 640\,\alpha$

$\alpha = 8.67875\ \text{rad/s}^2 = 8.68\ \text{rad/s}^2$ **Ans.**

$\Sigma F_n = m(a_G)_n;$ $\quad A_n = 0$ **Ans.**

$\Sigma F_t = m(a_G)_t;$ $\quad 800 + 120(9.81) + A_t = 120[2(8.67875)]$

$A_t = 105.7\ \text{N} = 106\ \text{N}$ **Ans.**



Ans:
$\alpha = 8.68\ \text{rad/s}^2$
$A_n = 0$
$A_t = 106\ \text{N}$

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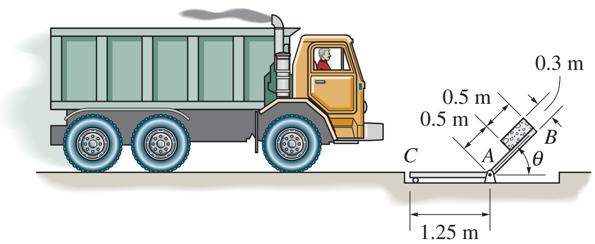
17–70.

The device acts as a pop-up barrier to prevent the passage of a vehicle. It consists of a 100-kg steel plate AC and a 200-kg counterweight solid concrete block located as shown. Determine the moment of inertia of the plate and block about the hinged axis through A. Neglect the mass of the supporting arms AB. Also, determine the initial angular acceleration of the assembly when it is released from rest at $\theta = 45°$.

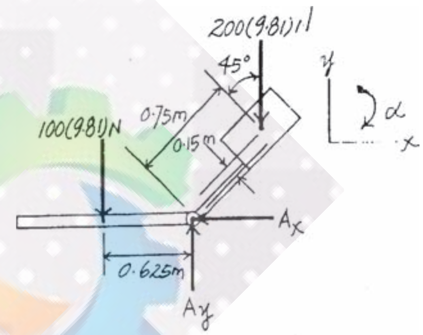


SOLUTION

Mass Moment of Inertia:

$$I_A = \frac{1}{12}(100)\left(1.25^2\right) + 100\left(0.625^2\right) + \frac{1}{12}(200)\left(0.5^2 + 0.3^2\right) + 200\left(\sqrt{0.75^2 + 0.15^2}\right)^2$$

$$= 174.75 \text{ kg}\cdot\text{m}^2 = 175 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



Equation of Motion: Applying Eq. 17–16, we have

$$\circlearrowleft +\Sigma M_A = I_A\,\alpha; \qquad 100(9.81)(0.625) + 200(9.81)\sin 45°(0.15) - 200(9.81)\cos 45°(0.75) = -174.75\alpha$$

$$\alpha = 1.25 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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Ans:
$I_A = 175 \text{ kg}\cdot\text{m}^2$
$\alpha = 1.25 \text{ rad/s}^2$

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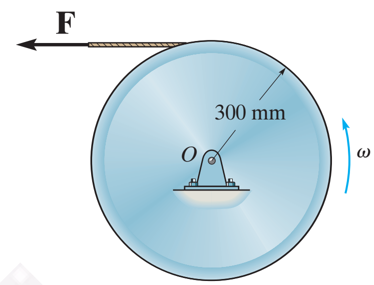
17–71.

A cord is wrapped around the outer surface of the 8-kg disk. If a force of $F = \left(\frac{1}{4}\theta^2\right)$ N, where θ is in radians, is applied to the cord, determine the disk's angular acceleration when it has turned 5 revolutions. The disk has an initial angular velocity of $\omega_0 = 1$ rad/s.



SOLUTION

Equations of Motion. The mass moment inertia of the disk about O is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(8)\left(0.3^2\right) = 0.36\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the disk, Fig. a,

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad \left(\frac{1}{4}\theta^2\right)(0.3) = 0.36\,\alpha$$

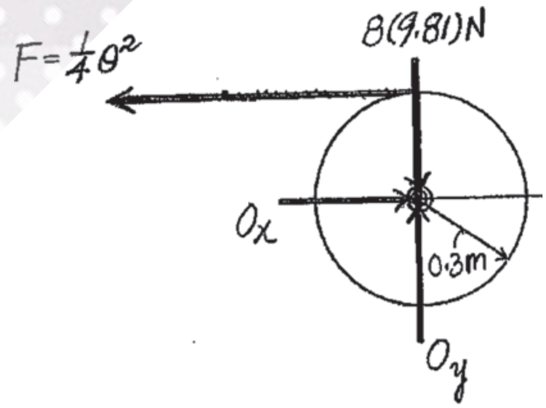
$$\alpha = \left(0.2083\,\theta^2\right)\ \text{rad/s}^2$$

Kinematics. Using the result of α, integrate $\omega d\omega = \alpha d\theta$ with the initial condition $\omega = 0$ when $\theta = 0$,

$$\int_1^{\omega} \omega d\omega = \int_0^{5(2\pi)} 0.2083\,\theta^2\,d\theta$$

$$\left(\frac{1}{2}\right)\left(\omega_2 - 1\right) = 0.06944\,\theta^3\Big|_0^{5(2\pi)}$$

$$\omega = 65.63\ \text{rad/s} = 65.6\ \text{rad/s} \qquad \textbf{Ans.}$$



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Ans:
$\omega = 65.6$ rad/s

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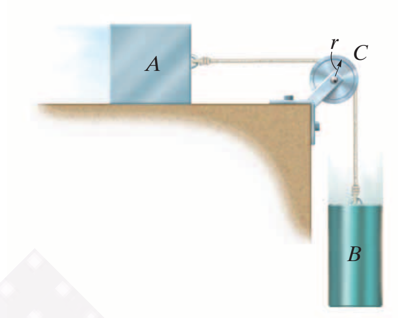
***17–72.**

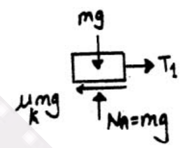
Block A has a mass m and rests on a surface having a coefficient of kinetic friction μ_k. The cord attached to A passes over a pulley at C and is attached to a block B having a mass $2m$. If B is released, determine the acceleration of A. Assume that the cord does not slip over the pulley. The pulley can be approximated as a thin disk of radius r and mass $\frac{1}{4}m$. Neglect the mass of the cord.



SOLUTION

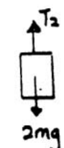
Block A:

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T_1 - \mu_k mg = ma$ **(1)**



Block B:

$+\downarrow \Sigma F_y = ma_y;\qquad 2mg - T_2 = 2ma$ **(2)**

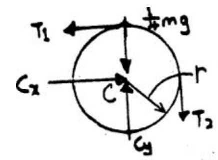


Pulley C:

$\circlearrowleft + \Sigma M_C = I_G\alpha;\qquad T_2 r - T_1 r = \left[\frac{1}{2}\left(\frac{1}{4}m\right)r^2\right]\left(\frac{a}{r}\right)$

$$T_2 - T_1 = \frac{1}{8}ma \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (2) into (3),



$$2mg - 2ma - (ma + \mu_k mg) = \frac{1}{8}ma \qquad (2 - \mu_k)g = \frac{25}{8}a$$

$$2mg - \mu_k mg = \frac{1}{8}ma + 3ma \qquad a = \frac{8}{25}(2 - \mu_k)g \qquad \textbf{Ans.}$$

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Ans:

$a = \frac{8}{25}(2 - \mu_k)g$

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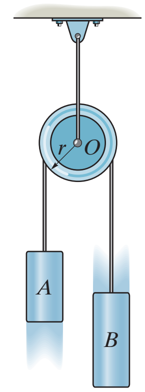
17–73.

The two blocks A and B have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



SOLUTION

Kinematics: Since the pulley rotates about a fixed axis passes through point O, its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

The mass moment of inertia of the pulley about point O is

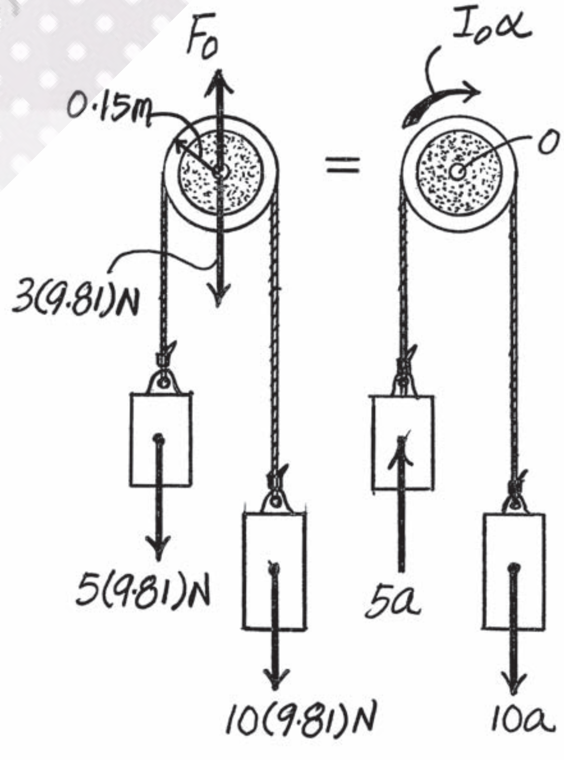
$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375 \text{ kg} \cdot \text{m}^2$$

Equation of Motion: Write the moment equation of motion about point O by referring to the free-body and kinetic diagram of the system shown in Fig. a,

$$\circlearrowleft + \Sigma M_o = \Sigma (M_k)_o; \qquad 5(9.81)(0.15) - 10(9.81)(0.15)$$

$$= -0.03375(6.6667a) - 5a(0.15) - 10a(0.15)$$

$$a = 2.973 \text{ m/s}^2 = 2.97 \text{ m/s}^2 \qquad \textbf{Ans.}$$



(a)

Ans:
$a = 2.97 \text{ m/s}^2$

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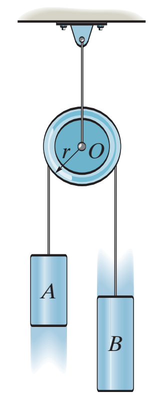
17–74.

The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



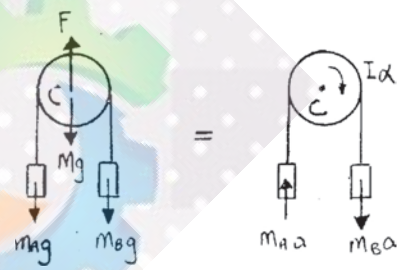
SOLUTION

$$a = \alpha r$$

$$\circlearrowleft +\Sigma M_C = \Sigma (M_k)_C; \qquad m_B\, g(r) - m_A g(r) = \left(\frac{1}{2}Mr^2\right)\alpha + m_B r^2\alpha + m_A r^2 \alpha$$

$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2}M + m_B + m_A\right)}$$

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)} \qquad \textbf{Ans.}$$



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Ans:

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)}$$

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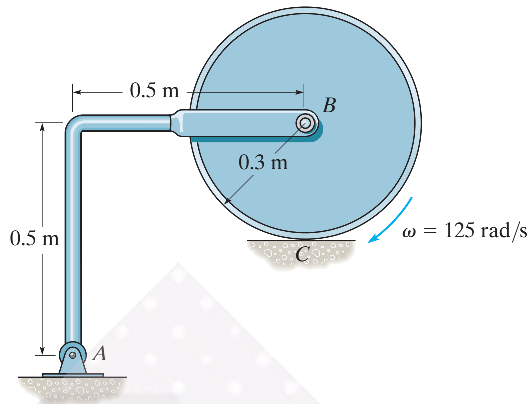
17–75.

The 30-kg disk is originally spinning at $\omega = 125$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of force which the member AB exerts on the pin at A during this time? Neglect the mass of AB.



SOLUTION

Equations of Motion. The mass moment of inertia of the disk about B is $I_B = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.3^2) = 1.35\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, $F_f = \mu_C N_C = 0.5\ N_C$. Referring to the FBD of the disk, Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 0.5N_C - F_{AB}\cos 45° = 30(0) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_C - F_{AB}\sin 45° - 30(9.81) = 30(0) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$N_C = 588.6\ \text{N} \qquad F_{AB} = 416.20\ \text{N}$$

Subsequently,

$$\circlearrowleft + \Sigma M_B = I_B\alpha; \qquad 0.5(588.6)(0.3) = 1.35\alpha$$

$$\alpha = 65.4\ \text{rad/s}^2 \circlearrowright$$

Referring to the FBD of pin A, Fig. b,

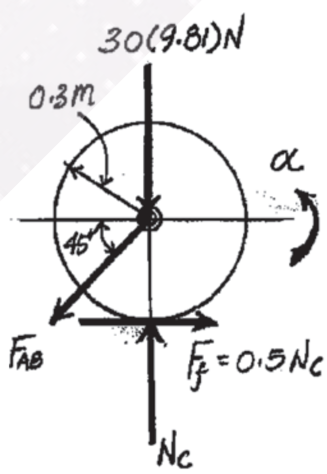
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 416.20\cos 45° - A_x = 0 \qquad A_x = 294.3\ \text{N} = 294\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 416.20\sin 45° - A_y = 0 \qquad A_y = 294.3\ \text{N} = 294\ \text{N} \qquad \textbf{Ans.}$$

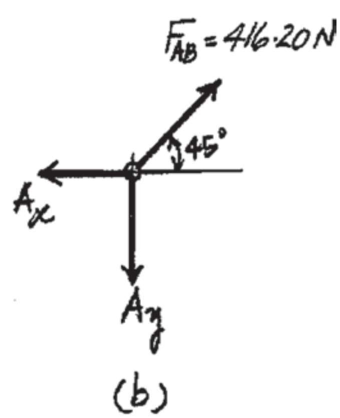
Kinematic. Using the result of α,

$$+\circlearrowright\ \omega = \omega_0 + \alpha t; \qquad 0 = 125 + (-65.4)t$$

$$t = 1.911\ \text{s} = 1.91\ \text{s} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$A_x = 294$ N
$A_y = 294$ N
$t = 1.91$ s

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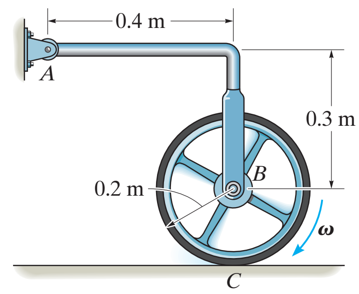
***17–76.**

The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.



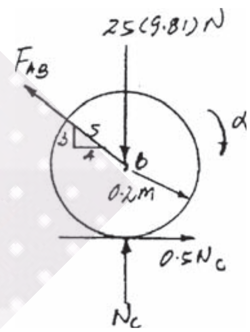
SOLUTION

$$I_B = mk_B^2 = 25(0.15)^2 = 0.5625\ \text{kg}\cdot\text{m}^2$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad \left(\tfrac{3}{5}\right) F_{AB} + N_C - 25(9.81) = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 0.5N_C - \left(\tfrac{4}{5}\right)F_{AB} = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_B = I_B\alpha; \qquad 0.5N_C(0.2) = 0.5625(-\alpha) \qquad \textbf{(3)}$$



Solvings Eqs. (1), (2) and (3) yields:

$$F_{AB} = 111.48\ \text{N} \qquad N_C = 178.4\ \text{N}$$

$$\alpha = -31.71\ \text{rad/s}^2$$

$$A_x = \tfrac{4}{5}F_{AB} = 0.8(111.48) = 89.2\ \text{N} \qquad \textbf{Ans.}$$

$$A_y = \tfrac{3}{5}F_{AB} = 0.6(111.48) = 66.9\ \text{N} \qquad \textbf{Ans.}$$

$$\omega = \omega_0 + \alpha_c t$$

$$0 = 40 + (-31.71)\,t$$

$$t = 1.26\ \text{s} \qquad \textbf{Ans.}$$

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Ans:
$A_x = 89.2$ N
$A_y = 66.9$ N
$t = 1.25$ s

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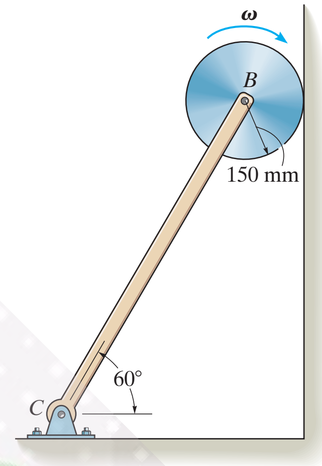
17–77.

The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, where the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad F_{CB}\sin 30^\circ - N_A = 0$

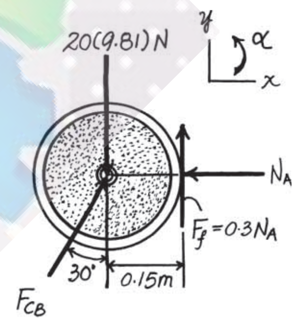
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad F_{CB}\cos 30^\circ - 20(9.81) + 0.3N_A = 0$

$\circlearrowleft + \Sigma M_B = I_B\,\alpha;\qquad 0.3N_A(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$$N_A = 96.6 \text{ N}$$

$$F_{CB} = 193 \text{ N} \qquad \textbf{Ans.}$$

$$\alpha = 19.3 \text{ rad/s}^2$$



$\circlearrowright +\qquad \omega = \omega_0 + \alpha_c t$

$$0 = 60 + (-19.3)\,t$$

$$t = 3.11 \text{ s} \qquad \textbf{Ans.}$$

Ans:

$F_{CB} = 193$ N
$t = 3.11$ s

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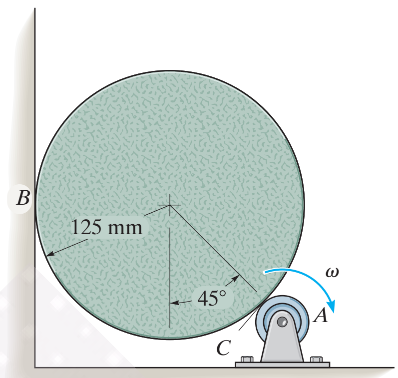
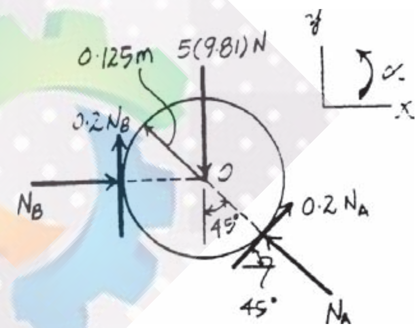
17–78.

The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



SOLUTION

Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad N_B + 0.2N_A \cos 45° - N_A \sin 45° = 0$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 0.2N_B + 0.2N_A \sin 45° + N_A \cos 45° - 5(9.81) = 0$ **(2)**

$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 0.2N_A\,(0.125) - 0.2N_B\,(0.125) = 0.0390625\alpha$ **(3)**

Solving Eqs. (1), (2), and (3) yields;

$$N_A = 51.01\ \text{N} \qquad N_B = 28.85\ \text{N}$$

$$\alpha = 14.2\ \text{rad/s}^2$$ **Ans.**

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Ans:
$\alpha = 14.2\ \text{rad/s}^2$

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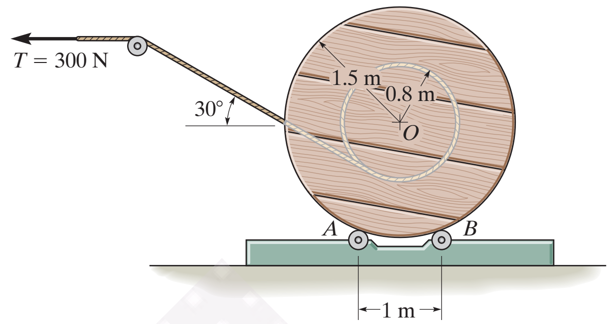
17–79.

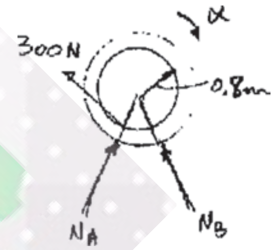
Cable is unwound from a spool supported on small rollers at A and B by exerting a force $T = 300$ N on the cable. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.



SOLUTION

$I_O = mk_O^2 = 600(1.2)^2 = 864\ \text{kg}\cdot\text{m}^2$

$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 300(0.8) = 864(\alpha) \qquad \alpha = 0.2778\ \text{rad/s}^2$



The angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad.

$$\theta = \theta_0 + \omega_0\, r + \frac{1}{2}\alpha_c t^2$$

$$6.25 = 0 + 0 + \frac{1}{2}(0.27778)t^2$$

$$t = 6.71\ \text{s}$$ **Ans.**

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Ans:
$t = 6.71$ s

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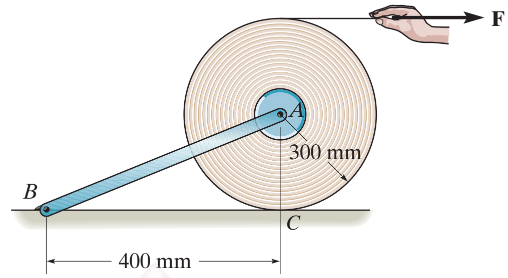
***17–80.**

The 20-kg roll of paper has a radius of gyration $k_A = 120$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. The roll rests on the floor, for which the coefficient of kinetic friction is $\mu_k = 0.2$. If a horizontal force $F = 60$ N is applied to the end of the paper, determine the initial angular acceleration of the roll as the paper unrolls.



SOLUTION

Equations of Motion. The mass moment of inertia of the paper roll about A is $I_A = mk_A^2 = 20(0.12^2) = 0.288\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, the friction is $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the paper roll, Fig. a

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 0.2\text{ N} - F_{AB}\left(\frac{4}{5}\right) + 60 = 20(0) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad \text{N} - F_{AB}\left(\frac{3}{5}\right) - 20(9.81) = 20(0) \qquad \textbf{(2)}$$

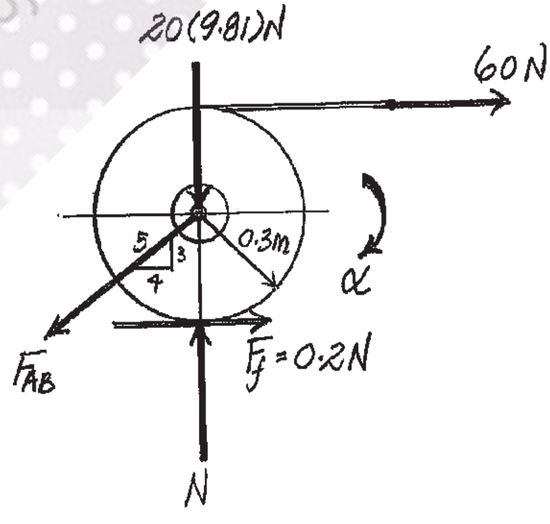
Solving Eqs. (1) and (2)

$$F_{AB} = 145.94\text{ N} \qquad \text{N} = 283.76\text{ N}$$

Subsequently

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad 0.2(283.76)(0.3) - 60(0.3) = 0.288(-\alpha)$$

$$\alpha = 3.3824\text{ rad/s}^2 = 3.38\text{ rad/s}^2 \qquad \textbf{Ans.}$$



(a)

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Ans:
$\alpha = 3.38\text{ rad/s}^2$

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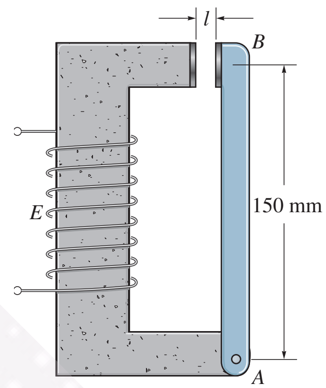
17–81.

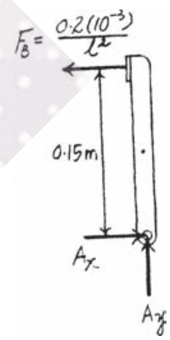
The armature (slender rod) AB has a mass of 0.2 kg and can pivot about the pin at A. Movement is controlled by the electromagnet E, which exerts a horizontal attractive force on the armature at B of $F_B = (0.2(10^{-3})l^{-2})$ N, where l in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at B the instant $l = 0.01$ m. Originally $l = 0.02$ m.



SOLUTION

Equation of Motion: The mass moment of inertia of the armature about point A is given by $I_A = I_G + mr_G^2 = \frac{1}{12}(0.2)\left(0.15^2\right) + 0.2\left(0.075^2\right) = 1.50\left(10^{-3}\right)\text{kg}\cdot\text{m}^2$

Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad \frac{0.2(10^{-3})}{l^2}(0.15) = 1.50\left(10^{-3}\right)\alpha$$

$$\alpha = \frac{0.02}{l^2}$$

Kinematic: From the geometry, $l = 0.02 - 0.15\theta$. Then $dl = -0.15\,d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

$$\frac{v}{0.15}\left(\frac{dv}{0.15}\right) = \alpha\left(-\frac{dl}{0.15}\right)$$

$$v\,dv = -0.15\,\alpha\,dl$$

$$\int_0^v v\,dv = \int_{0.02\text{ m}}^{0.01\text{ m}} -0.15\left(\frac{0.02}{l^2}\right)dl$$

$$v = 0.548\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 0.548$ m/s

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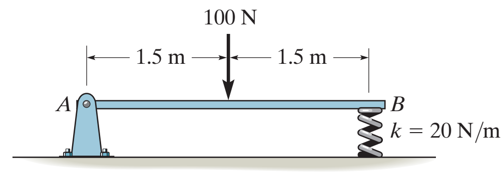
17–82.

The 4-kg slender rod is initially supported horizontally by a spring at B and pin at A. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the 100-N force is applied.



SOLUTION

Equation of Motion. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(4)(3^2) + 4(1.5^2) = 12.0\ \text{kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.5)$. The force developed in the spring before the application of the 100 N force is $F_{sp} = \frac{4(9.81)\ \text{N}}{2} = 19.62$ N. Referring to the FBD of the rod, Fig. *a*,

$$\circlearrowleft + M_A = I_A\alpha; \qquad 19.62(3) - 100(1.5) - 4(9.81)(1.5) = 12.0(-\alpha)$$

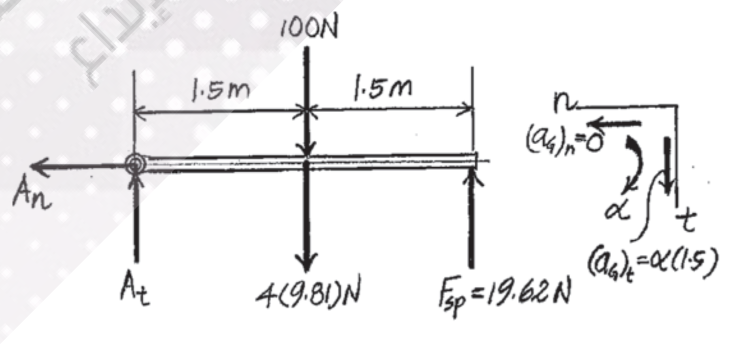
$$\alpha = 12.5\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Then

$$(a_G)_t = 12.5(1.5) = 18.75\ \text{m/s}^2 \downarrow$$

Since $(a_G)_n = 0$. Then

$$a_G = (a_G)_t = 18.75\ \text{m/s}^2 \downarrow \qquad \textbf{Ans.}$$



(a)

Ans:
$\alpha = 12.5\ \text{rad/s}^2 \circlearrowright$
$a_G = 18.75\ \text{m/s}^2 \downarrow$

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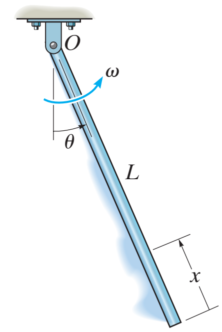
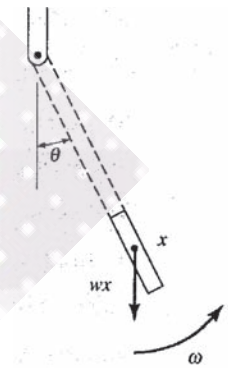
17–83.

The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.



SOLUTION

$$a = \omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow}$$



Forces:

$$\frac{wx}{g}\omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow} = N\overset{\theta}{\nwarrow} + S\measuredangle\theta + wx\downarrow \qquad \textbf{(1)}$$

Moments:

$$I\alpha = M - S\left(\frac{x}{2}\right)$$

$$O = M - \frac{1}{2}Sx \qquad \textbf{(2)}$$

Solving (1) and (2),

$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right] \qquad \textbf{Ans.}$$

$$V = wx\sin\theta \qquad \textbf{Ans.}$$

$$M = \frac{1}{2}wx^2\sin\theta \qquad \textbf{Ans.}$$

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Ans:

$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right]$$

$$V = wx\sin\theta$$

$$M = \frac{1}{2}wx^2\sin\theta$$

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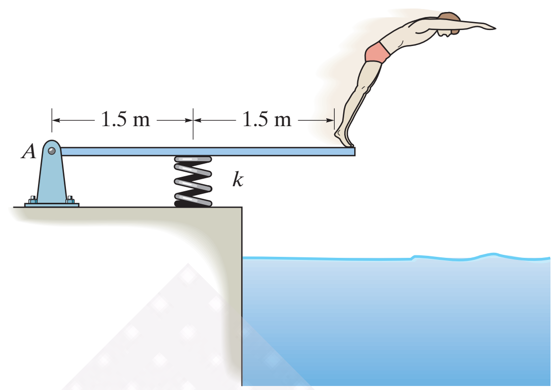
***17–84.**

Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



SOLUTION

$$\circlearrowleft + \sum M_A = I_A\alpha; \qquad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t\,; \qquad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

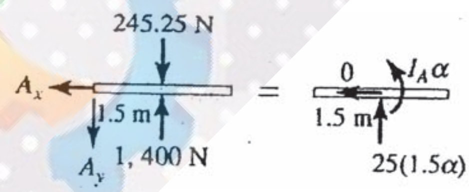
$$\xleftarrow{+} \sum F_n = m(a_G)_n\,; \qquad A_x = 0$$

Solving,

$A_x = 0$ **Ans.**

$A_y = 289$ N **Ans.**

$\alpha = 23.1$ rad/s^2 **Ans.**



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Ans:
$A_x = 0$
$A_y = 289$ N
$\alpha = 23.1$ rad/s^2

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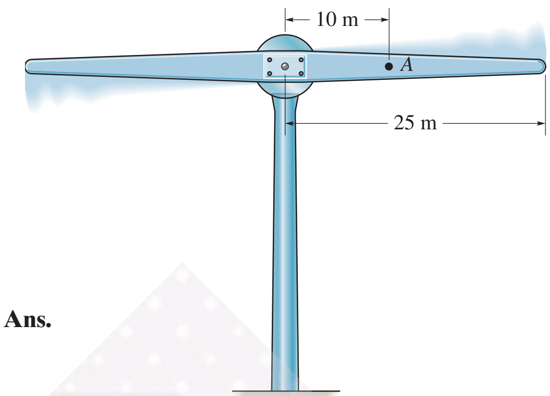
17–85.

The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and a clockwise angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.



SOLUTION

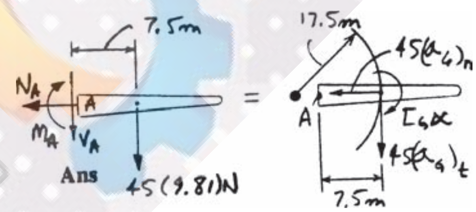
$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad N_A = 45(15)^2(17.5) = 177\text{kN}$ **Ans.**

$+\downarrow \Sigma F_t = m(a_G)_t; \qquad V_A + 45(9.81) = 45(8)(17.5)$

$V_A = 5.86 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad M_A + 45(9.81)(7.5) = \left[\frac{1}{12}(45)(15)^2\right](8) + [45(8)(17.5)](7.5)$

$M_A = 50.7 \text{ kN}\cdot\text{m}$ **Ans.**



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Ans:
$N_A = 177$ kN
$V_A = 5.86$ kN
$M_A = 50.7$ kN·m

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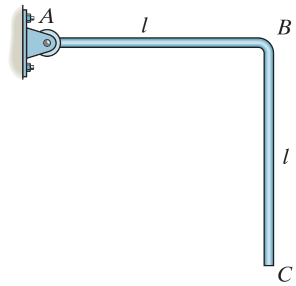
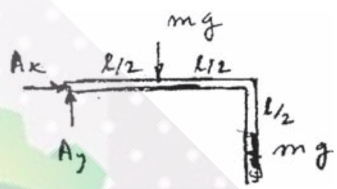
17–86.

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass m and length l.



SOLUTION

Assembly:



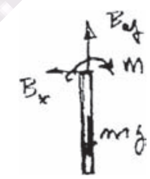
$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}(m)(l)^2 + m(l^2 + (\frac{l}{2})^2)$$

$$= 1.667\ ml^2$$

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad mg(\frac{l}{2}) + mg(l) = (1.667ml^2)\alpha$$

$$\alpha = \frac{0.9\ g}{l}$$

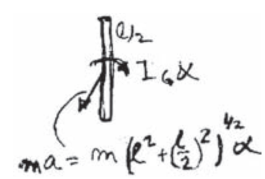
Segment BC:



$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \qquad M = \left[\frac{1}{12}ml^2\right]\alpha + m(l^2 + (\frac{l}{2})^2)^{1/2}\,\alpha(\frac{l/2}{l^2 + (\frac{l}{2})^2})(\frac{l}{2})$$

$$M = \frac{1}{3}ml^2\,\alpha = \frac{1}{3}ml^2(\frac{0.9g}{l})$$

$$M = 0.3gml \qquad \textbf{Ans.}$$



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Ans:
$M = 0.3gml$

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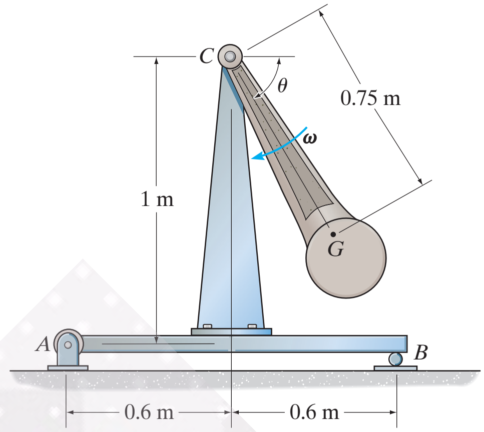
17–87.

The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 90°$ when the pendulum is rotating at $\omega = 8$ rad/s. Neglect the weight of the beam and the support.



SOLUTION

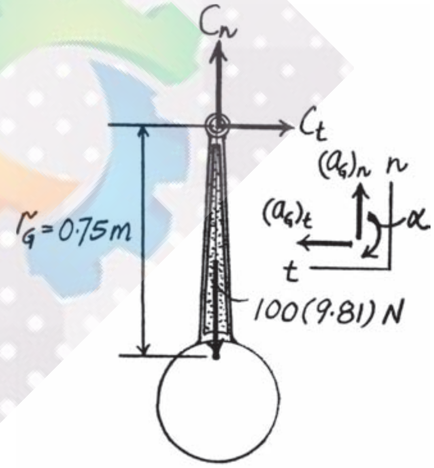
Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 8^2(0.75) = 48\ \text{m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25)^2 + 100(0.75^2) = 62.5\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram of the pendulum, Fig. a, we have

$$\circlearrowleft + \Sigma M_C = I_C\alpha; \qquad 0 = 62.5\alpha \qquad \alpha = 0$$

Using this result to write the force equations of motion along the n and t axes,

$$\xleftarrow{+} \Sigma F_t = m(a_G)_t; \qquad -C_t = 100[0(0.75)] \qquad C_t = 0$$

$$+\uparrow \Sigma F_n = m(a_G)_n; \qquad C_n - 100(9.81) = 100(48) \qquad C_n = 5781\ \text{N}$$



(a)

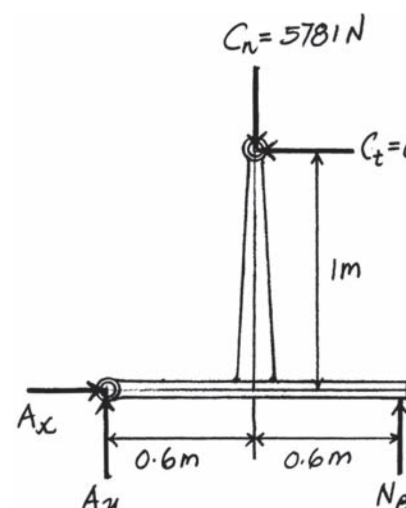
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b, we have

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 5781(0.6) = 0 \qquad N_B = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 2890.5 - 5781 = 0 \qquad A_y = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$



(b)

Ans:
$N_B = 2.89$ kN
$A_x = 0$
$A_y = 2.89$ kN

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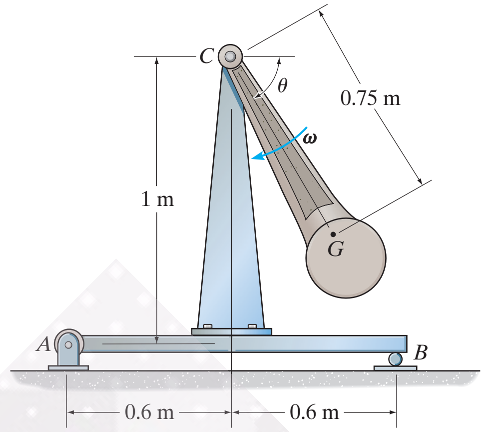
***17–88.**

The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 0°$ when the pendulum is rotating at $\omega = 4$ rad/s. Neglect the weight of the beam and the support.



SOLUTION

Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 4^2(0.75) = 12\text{ m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25^2) + 100(0.75)^2 = 62.5\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram shown in Fig. *a*,

$$\circlearrowleft +\Sigma M_C = I_C\alpha; \qquad -100(9.81)(0.75) = -62.5\alpha \qquad \alpha = 11.772\text{ rad/s}^2$$

Using this result to write the force equations of motion along the n and t axes, we have

$$+\uparrow \Sigma F_t = m(a_G)_t; \qquad C_t - 100(9.81) = -100[11.772(0.75)] \qquad C_t = 98.1\text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad C_n = 100(12) \qquad C_n = 1200\text{ N}$$

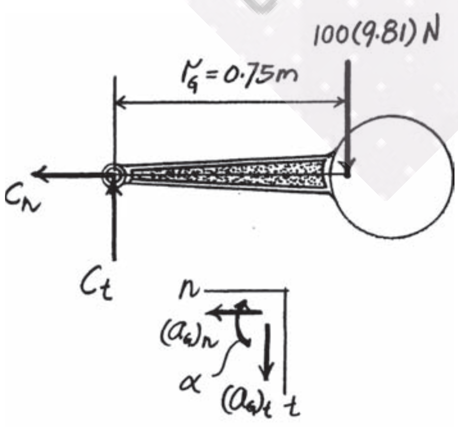
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. *b*,

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 98.1(0.6) - 1200(1) = 0 \qquad N_B = 1049.05\text{ N} = 1.05\text{ kN} \qquad \textbf{Ans.}$$

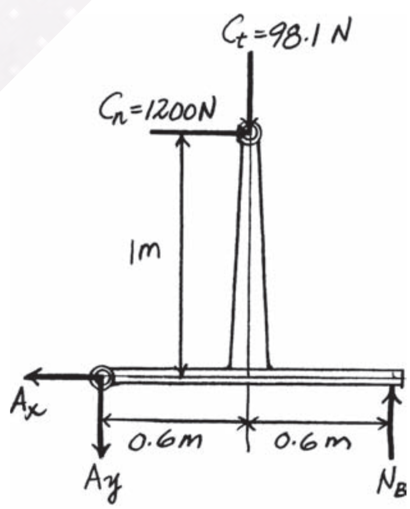
Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1200 - A_x = 0 \qquad A_x = 1200\text{ N} = 1.20\text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1049.05 - 98.1 - A_y = 0 \qquad A_y = 950.95\text{ N} = 951\text{ N} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$N_B = 1.05$ kN
$A_x = 1.20$ kN
$A_y = 951$ N

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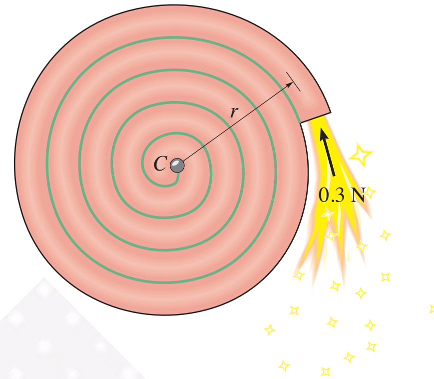
17–89.

The "Catherine wheel" is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such as that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of r = 75 mm. For the calculation, consider the wheel to always be a thin disk.

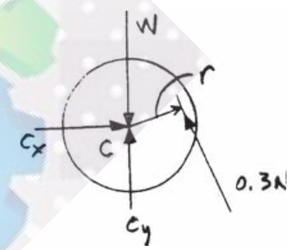


SOLUTION

Mass of wheel when 75% of the powder is burned = 0.025 kg

$$\text{Time to burn off 75 \%} = \frac{0.075\text{ kg}}{0.02\text{ kg/s}} = 3.75\text{ s}$$

$$m(t) = 0.1 - 0.02t$$



Mass of disk per unit area is

$$\rho_0 = \frac{m}{A} = \frac{0.1\text{ kg}}{\pi(0.075\text{ m})^2} = 5.6588\text{ kg/m}^2$$

At any time t,

$$5.6588 = \frac{0.1 - 0.02t}{\pi r^2}$$

$$r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}$$

$$+\Sigma M_C = I_C\alpha; \qquad 0.3r = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{0.6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\dfrac{0.1 - 0.02t}{\pi(5.6588)}}}$$

$$\alpha = 0.6\left(\sqrt{\pi(5.6588)}\right)[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$\alpha = 2.530[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$d\omega = \alpha\, dt$$

$$\int_0^{\omega} d\omega = 2.530\int_0^{t}[0.1 - 0.02t]^{-\frac{3}{2}}\, dt$$

$$\omega = 253\left[(0.1 - 0.02t)^{-\frac{1}{2}} - 3.162\right]$$

For $t = 3.75$ s,

$$\omega = 800\text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 800$ rad/s

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17–90.

If the disk in Fig. 17–19 *rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

SOLUTION

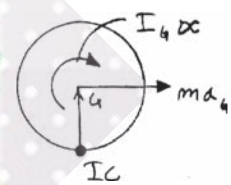
$\curvearrowleft + \Sigma M_{IC} = \Sigma(M_K)_{IC}; \qquad \Sigma M_{IC} = I_G\alpha + (ma_G)r$

Since there is no slipping, $a_G = \alpha r$

Thus, $\Sigma M_{IC} = \left(I_G + mr^2\right)\alpha$

By the parallel–axis thoerem, the term in parenthesis represents I_{IC}. Thus,

$\Sigma M_{IC} = I_{IC}\alpha$ **Q.E.D**.



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Ans:
$\Sigma M_{IC} = I_{IC}\alpha$

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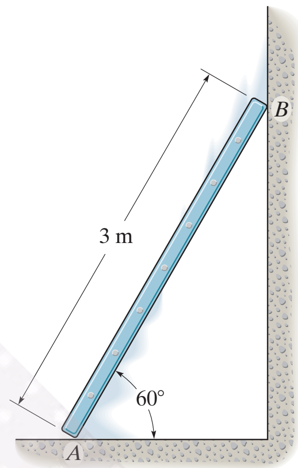
17–91.

The slender 12-kg bar has a clockwise angular velocity of $\omega = 2$ rad/s when it is in the position shown. Determine its angular acceleration and the normal reactions of the smooth surface A and B at this instant.



SOLUTION

Equations of Motion. The mass moment of inertia of the rod about its center of gravity G is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(3^2) = 9.00\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the rod, Fig. a

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad N_B = 12(a_G)_x \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 12(9.81) = -12(a_G)_y \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = (M_k)_O; \quad -12(9.81)(1.5\cos 60°) = -12(a_G)_x(1.5\sin 60°)$$

$$-12(a_G)_y(1.5\cos 60°) - 9.00\alpha$$

$$\sqrt{3}(a_G)_x + (a_G)_y + \alpha = 9.81 \qquad \textbf{(3)}$$

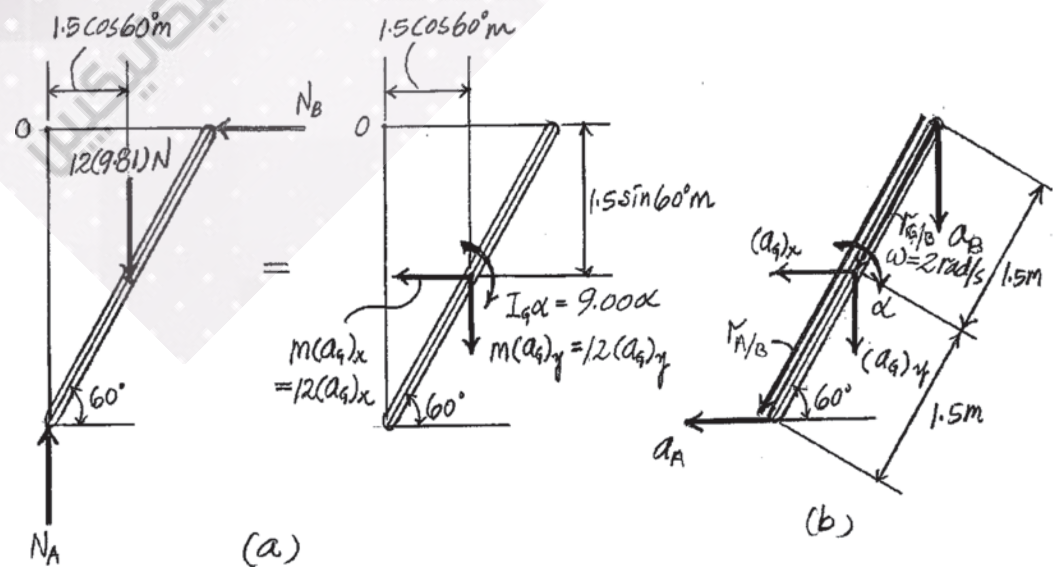
Kinematics. Applying the relative acceleration equation relating $\mathbf{a}_G$ and $\mathbf{a}_B$ by referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{G/B} - \omega^2 \mathbf{r}_{G/B}$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = -a_B\mathbf{j} + (-\alpha\mathbf{k}) \times (-1.5\cos 60°\mathbf{i} - 1.5\sin 60°\mathbf{j})$$

$$-2^2(-1.5\cos 60°\mathbf{i} - 1.5\sin 60°\mathbf{j})$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (3 - 0.75\sqrt{3}\alpha)\mathbf{i} + (0.75\alpha - a_B + 3\sqrt{3})\mathbf{j}$$



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17–91. Continued

Equating **i** and **j** components,

$$-(a_G)_x = 3 - 0.75\sqrt{3}\alpha \qquad \textbf{(4)}$$

$$-(a_G)_y = 0.75\alpha - a_B + 3\sqrt{3} \qquad \textbf{(5)}$$

Also, relate $\mathbf{a}_B$ and $\mathbf{a}_A$,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = -a_B\mathbf{j} + (-\alpha\mathbf{k}) \times (-3\cos 60°\mathbf{i} - 3\sin 60°\mathbf{j})$$

$$-2^2(-3\cos 60°\mathbf{i} - 3\sin 60°\mathbf{j})$$

$$-a_A\mathbf{i} = (6 - 1.5\sqrt{3}\alpha)\mathbf{i} + (1.5\alpha - a_B + 6\sqrt{3})\mathbf{j}$$

Equating **j** components,

$$0 = 1.5\alpha - a_B + 6\sqrt{3}; \qquad a_B = 1.5\alpha + 6\sqrt{3} \qquad \textbf{(6)}$$

Substituting Eq. (6) into (5)

$$(a_G)_y = 0.75\alpha + 3\sqrt{3} \qquad \textbf{(7)}$$

Substituting Eq. (4) and (7) into (3)

$$\sqrt{3}\left(0.75\sqrt{3}\alpha - 3\right) + 0.75\alpha + 3\sqrt{3} + \alpha = 9.81$$

$$\alpha = 2.4525 \text{ rad/s}^2 = 2.45 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (4) and (7)

$$-(a_G)_x = 3 - (0.75\sqrt{3})(2.4525); \quad (a_G)_x = 0.1859 \text{ m/s}^2$$

$$(a_G)_y = 0.75(2.4525) + 3\sqrt{3}; \quad (a_G)_y = 7.0355 \text{ m/s}^2$$

Substituting these results into Eqs. (1) and (2)

$$N_B = 12(0.1859); \quad N_B = 2.2307 \text{ N} = 2.23 \text{ N} \qquad \textbf{Ans.}$$

$$N_A - 12(9.81) = -12(7.0355); \quad N_A = 33.2937 \text{ N} = 33.3 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$\alpha = 2.45 \text{ rad/s}^2$
$N_B = 2.23$ N
$N_A = 33.3$ N

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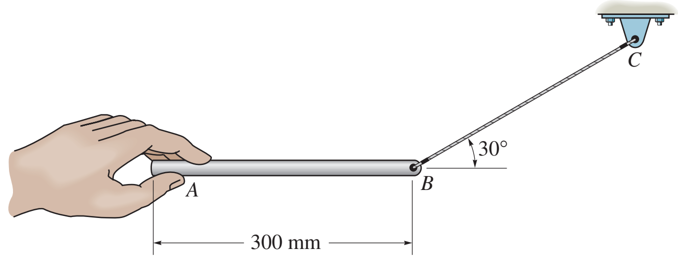
***17–92.**

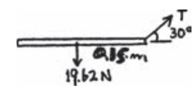
The 2-kg slender bar is supported by cord BC and then released from rest at A. Determine the initial angular acceleration of the bar and the tension in the cord.



SOLUTION

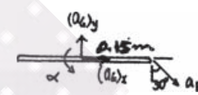
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad T\cos 30° = 2(a_G)_x$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T\sin 30° - 19.62 = 2(a_G)_y$$



$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad T\sin 30°(0.15) = [\frac{1}{12}(2)(0.3)^2]\alpha$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$



$$a_B \sin 30°\mathbf{i} - a_B\cos 30°\mathbf{j} = (a_G)_x\mathbf{i} + (a_G)_y\,\mathbf{j} + \alpha\,(0.15)\mathbf{j}$$

$$(\xrightarrow{+}) \qquad (a_B)\sin 30° = (a_G)_x$$

$$(+\uparrow) \qquad (a_B)\cos 30° = -(a_G)_y - \alpha\,(0.15)$$

Thus,

$$1.7321(a_G)_x = -(a_G)_y - 0.15\alpha$$

$T = 5.61$ N **Ans.**

$$(a_G)_x = 2.43 \text{ m/s}^2$$

$$(a_G)_y = -8.41 \text{ m/s}^2$$

$\alpha = 28.0$ rad/s^2 **Ans.**

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Ans:

$T = 5.61$ N
$\alpha = 28.0$ rad/s^2

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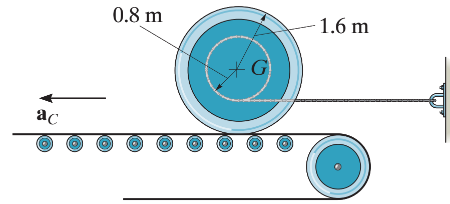
17–93.

The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.4$. If the conveyor accelerates at $a_C = 1\text{ m/s}^2$, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.

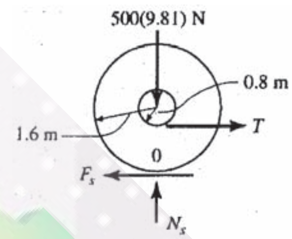


SOLUTION

$\xrightarrow{+} \sum F_x = m(a_G)_x; \qquad -F_s + T = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \qquad N_s - 500(9.81) = 0$

$\circlearrowright + \sum M_G = I_G\alpha; \qquad F_s(1.6) - T(0.8) = 500(1.30)^2\alpha$



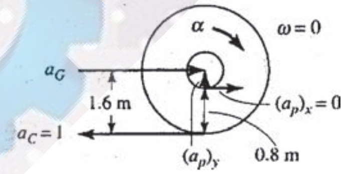
$$\mathbf{a}_p = \mathbf{a}_G + \mathbf{a}_{p/G}$$

$$(a_p)_y\mathbf{j} = a_G\,\mathbf{i} - 0.8\alpha\mathbf{i}$$

$$\alpha_G = 0.8\alpha$$

$$N_s = 4905\text{ N}$$

Assume no slipping



$$\alpha = \frac{ac}{0.8} = \frac{1}{0.8} = 1.25\text{ rad/s} \qquad \textbf{Ans.}$$

$$a_G = 0.8(1.25) = 1\text{ m/s}^2$$

$$T = 2.32\text{ kN} \qquad \textbf{Ans.}$$

$$F_s = 1.82\text{ kN}$$

Since

$$(F_s)_{\max} = 0.5(4.905) = 2.45 > 1.82$$

(No slipping occurs)

Ans:
$\alpha = 1.25\text{ rad/s}$
$T = 2.32\text{ N}$

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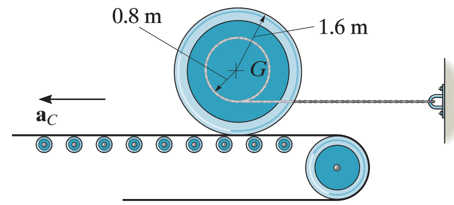
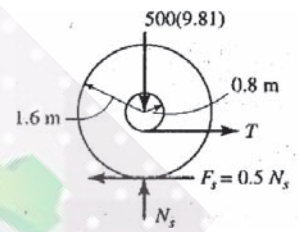
17–94.

The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$. Determine the greatest acceleration a_C of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



SOLUTION



$\xrightarrow{+} \sum F_x = m(a_G)_x; \qquad T - 0.5N_S = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \qquad N_s - 500(9.81) = 0$

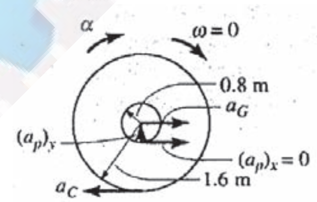
$\circlearrowleft + \sum M_G = I_G\alpha, \qquad 0.5N_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$a_p = a_C + a_{p/G}$

$(a_p)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$

Solving;



$N_s = 4905$ N

$T = 3.13$ kN **Ans.**

$\alpha = 1.684$ rad/s **Ans.**

$a_G = 1.347\text{ m/s}^2$

Since no slipping

$\mathbf{a}_C = \mathbf{a}_G + \mathbf{a}_{C/G}$

$ac = 1.347\mathbf{i} - (1.684)(1.6)\mathbf{i}$

$a_C = 1.35\text{ m/s}^2$ **Ans.**

Also,

$\circlearrowleft + \sum M_{IC} = I_{IC}\alpha; \qquad 0.5N_s(0.8) = [500(1.30)^2 + 500(0.8)^2]\alpha$

Since $N_S = 4905$ N

$\alpha = 1.684$ rad/s

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Ans:
$T = 3.13$ kN,
$\alpha = 1.684$ rad/s
$a_C = 1.35\text{ m/s}^2$

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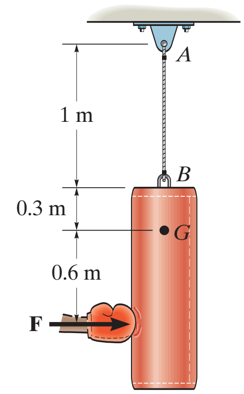
17–95.

The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



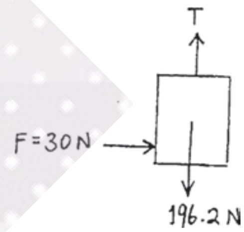
SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 30 = 20(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 30(0.6) = 20(0.4)^2\alpha$

$$\alpha = 5.62 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



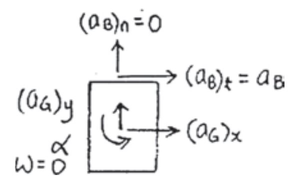
$$(a_G)_x = 1.5 \text{ m/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

$$a_B\mathbf{i} = (a_G)_y\mathbf{j} + (a_G)_x\mathbf{i} - \alpha(0.3)\mathbf{i}$$

$$(+\uparrow) \qquad (a_G)_y = 0$$

Thus,



$$T = 196 \text{ N} \qquad \textbf{Ans.}$$

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Ans:
$\alpha = 5.62 \text{ rad/s}^2$
$T = 196$ N

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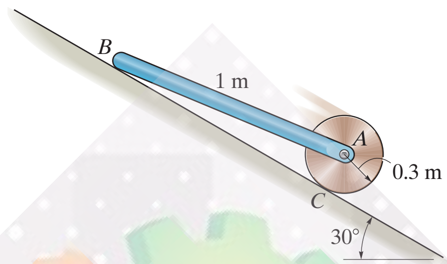
***17–96.** The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.



SOLUTION

$m_D = 8 \text{ kg}$ $\quad L = 1 \text{ m}$

$m_b = 10 \text{ kg}$ $\quad r = 0.3 \text{ m}$

$\mu_s = 0.6$ $\quad \theta = 30°$

$\mu_k = 0.4$ $\quad g = 9.81 \text{ m/s}^2$

$$\phi = \operatorname{asin}\left(\frac{r}{L}\right)$$

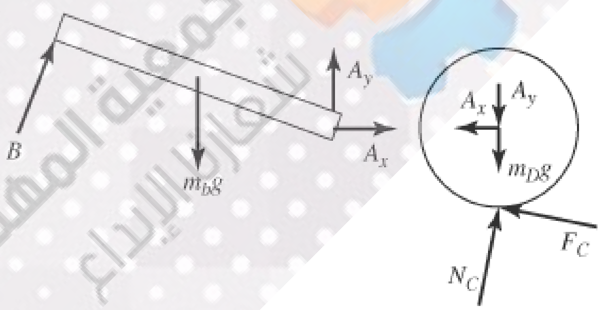
Assume no slip

Guesses

$N_C = 1 \text{ N}$ $\quad F_C = 1 \text{ N}$

$\alpha = 1 \text{ rad/s}^2$ $\quad a_A = 1 \text{ m/s}^2$

$F_{max} = 1 \text{ N}$



Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \operatorname{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 109.04 \\ 16.05 \\ 65.43 \end{pmatrix}\text{N} \qquad \alpha = 13.38 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Since $F_C = 16.05 \text{ N} < F_{max} = 65.43 \text{ N}$ then our no-slip assumption is correct.

Ans:

$\alpha = 13.38 \text{ rad/s}^2$

Since $F_C = 16.05 \text{ N} < F_{max} = 65.43 \text{ N}$ then our no-slip assumption is correct.

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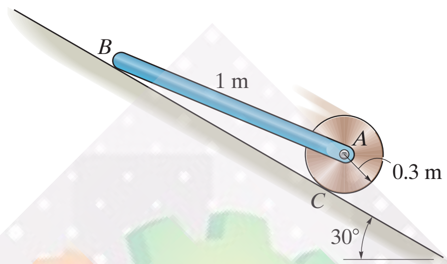
17–97. Solve Prob. 17–96 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.



SOLUTION

$m_D = 8 \text{ kg} \qquad L = 1 \text{ m}$

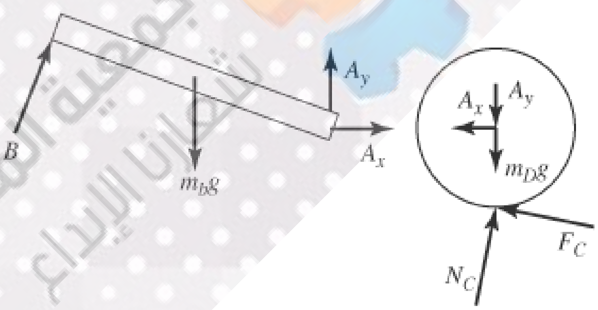
$m_b = 0 \text{ kg} \qquad r = 0.3 \text{ m}$

$\mu_s = 0.15 \qquad \theta = 30°$

$\mu_k = 0.1 \qquad g = 9.81 \text{ m/s}^2$

$\phi = \operatorname{asin}\left(\dfrac{r}{L}\right)$

Assume no slip



Guesses

$N_C = 1 \text{ N} \qquad F_C = 1 \text{ N}$

$\alpha = 1 \text{ rad/s}^2 \qquad a_A = 1 \text{ m/s}^2$

$F_{max} = 1 \text{ N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A \frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.97 \\ 13.08 \\ 10.19 \end{pmatrix} \text{N} \qquad \alpha = 10.90 \text{ rad/s}^2$$

Since $F_C = 13.08 \text{ N} \quad > F_{max} = 10.19 \text{ N}$ then our no-slip assumption is wrong and we know that slipping does occur.

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17–97. Continued

Guesses

$N_C = 1\text{ N} \quad F_C = 1\text{ N} \quad \alpha = 1\text{ rad/s}^2 \quad a_A = 1\text{ m/s}^2 \quad F_{max} = 1\text{ N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta-\phi) - m_b g\frac{L}{2}\cos(\theta-\phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad F_{max} = \mu_s N_C \qquad F_C = \mu_k N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.97 \\ 6.80 \\ 10.19 \end{pmatrix}\text{N} \qquad \alpha = 5.66\text{ rad/s}^2 \quad \textbf{Ans.}$$

Ans:
$\alpha = 5.66\text{ rad/s}^2$
Since $F_C = 13.08\text{ N} > F_{max} = 10.19\text{ N}$
then our no-slip assumption is wrong and we know that slipping does occur.

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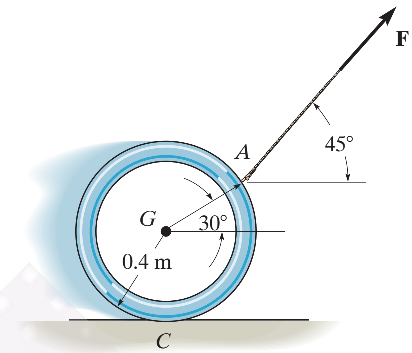
17–98.

A force of $F = 10$ N is applied to the 10-kg ring as shown. If slipping does not occur, determine the ring's initial angular acceleration, and the acceleration of its mass center, G. Neglect the thickness of the ring.



SOLUTION

Equations of Motion. The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10(0.4^2) = 1.60 \text{ kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the ring, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = (\mu_k)_C; \quad (10 \sin 45°)(0.4 \cos 30°) - (10 \cos 45°)[0.4(1 + \sin 30°)]$$

$$= -(10a_G)(0.4) - 1.60\alpha$$

$$4a_G + 1.60\alpha = 1.7932 \qquad \textbf{(1)}$$

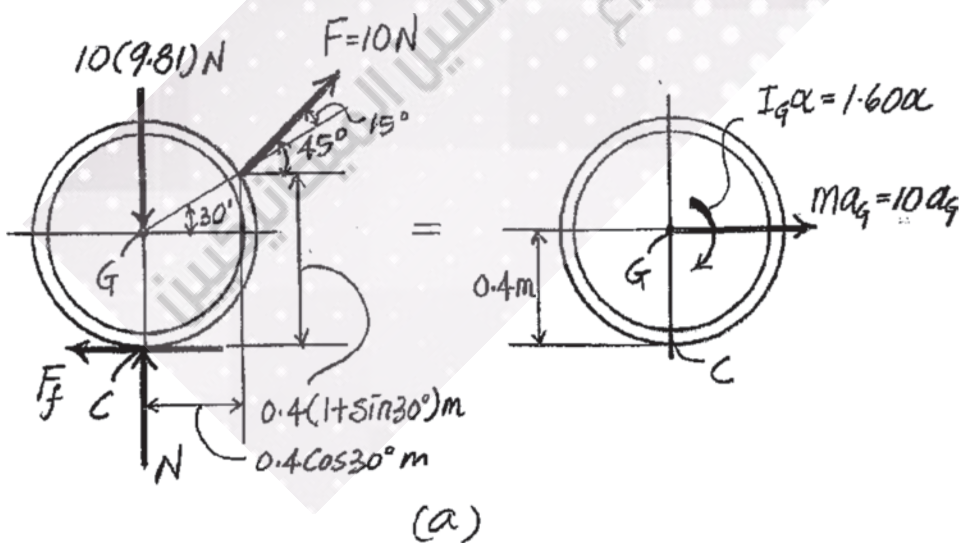
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\alpha = 0.5604 \text{ rad/s}^2 = 0.560 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$a_G = 0.2241 \text{ m/s}^2 = 0.224 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans.}$$



(a)

Ans:
$\alpha = 0.560 \text{ rad/s}^2 \circlearrowright$
$a_G = 0.224 \text{ m/s}^2 \rightarrow$

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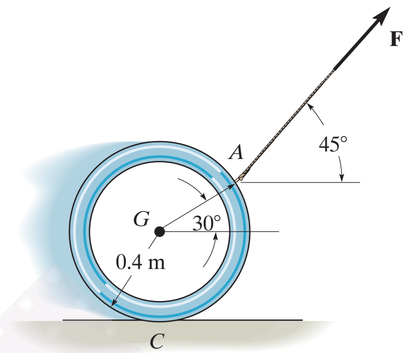
17–99.

If the coefficient of static friction at C is $\mu_s = 0.3$, determine the largest force **F** that can be applied to the 5-kg ring, without causing it to slip. Neglect the thickness of the ring.



SOLUTION

Equations of Motion: The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10(0.4^2) = 1.60\ \text{kg}\cdot\text{m}^2$. Here, it is required that the ring is on the verge of slipping at C, $F_f = \mu_s N = 0.3\ \text{N}$. Referring to the FBD and kinetic diagram of the ring, Fig. *a*

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F \sin 45° + N - 10(9.81) = 10(0) \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F \cos 45° - 0.3\ \text{N} = 10a_G \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad F \sin 15°(0.4) - 0.3\ \text{N}(0.4) = -1.60\alpha \qquad \textbf{(3)}$$

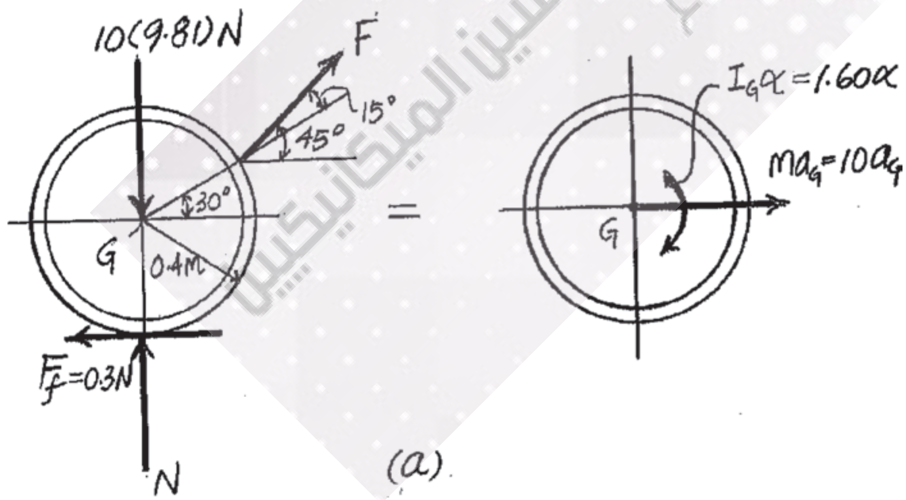
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4),

$$F = 42.34\ \text{N} = 42.3\ \text{N} \qquad \textbf{Ans.}$$

$N = 68.16\ \text{N} \qquad \alpha = 2.373\ \text{rad/s}^2 \circlearrowright \qquad a_G = 0.9490\ \text{m/s}^2 \rightarrow$



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Ans:
$F = 42.3\ \text{N}$

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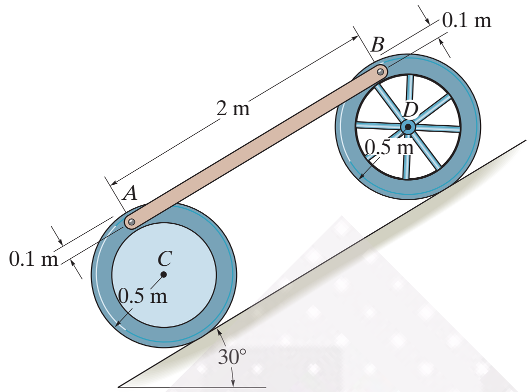
***17–100.**

Wheel C has a mass M_1 and a radius of gyration k_C, whereas wheel D has a mass M_2 and a radius of gyration k_D. Determine the angular acceleration of each wheel at the instant shown. Neglect the mass of the link and assume that the assembly does not slip on the plane.



Given:

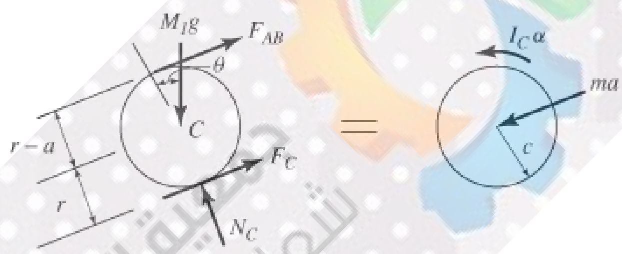
$M_1 = 60\ \text{kg}$ $\quad r = 0.5\ \text{m}$

$M_2 = 40\ \text{kg}$ $\quad a = 0.1\ \text{m}$

$k_C = 0.4\ \text{m}$ $\quad b = 2\ \text{m}$

$k_D = 0.35\ \text{m}$ $\quad \theta = 30°$

Solution:

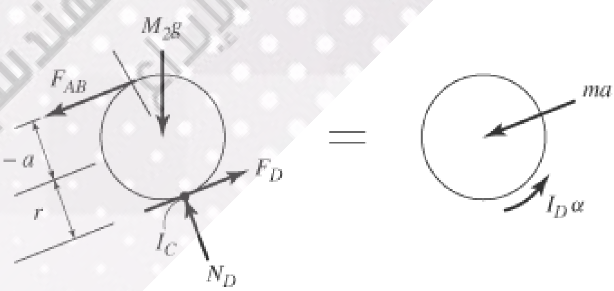


Both wheels have the same angular acceleration.

Guesses

$F_{AB} = 1\ \text{N}$

$\alpha = 1\ \text{rad}/\text{s}^2$



Given

$$-F_{AB}(2r - a) + M_1 g \sin(\theta) r = M_1 k_C^2 \alpha + M_1 (r\alpha) r$$

$$F_{AB}(2r - a) + M_2 g \sin(\theta) r = M_2 k_D^2 \alpha + M_2 (r\alpha) r$$

$$\begin{pmatrix} F_{AB} \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, \alpha) \qquad F_{AB} = -6.21\ \text{N} \qquad \alpha = 6.21\ \text{rad}/\text{s}^2 \quad \textbf{Ans.}$$

Ans:
$\alpha = 6.21\ \text{rad}/\text{s}^2$

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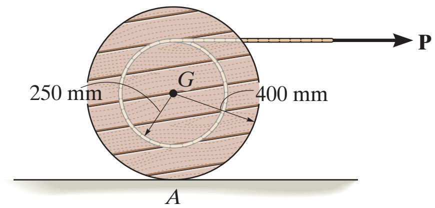
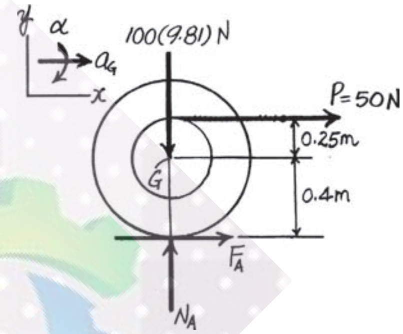
17–101.

The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowright + \Sigma M_G = I_G\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 1.30 \text{ rad/s}^2$ **Ans.**

$a_G = 0.520 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 2.00 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**

Ans:
$\alpha = 1.30 \text{ rad/s}^2$

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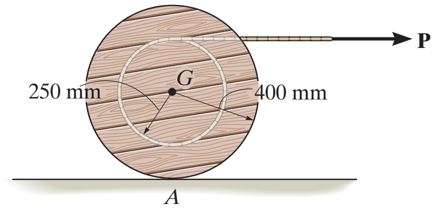
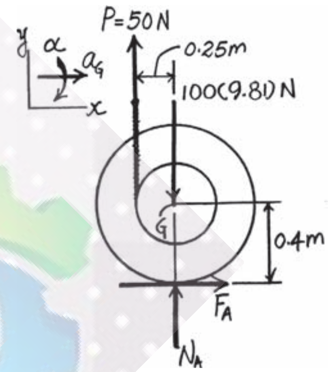
17–102.

Solve Prob. 17–101 if the cord and force $P = 50$ N are directed vertically upwards.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)x; \quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 50 - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$

$\alpha = 0.500 \text{ rad/s}^2$ **Ans.**

$a_G = 0.2 \text{ m/s}^2 \qquad N_A = 931 \text{ N} \qquad F_A = 20 \text{ N}$

Since $(F_A)_{max} = 0.2(931) = 186.2 \text{ N} > 20 \text{ N}$ **OK**

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Ans:
$\alpha = 0.500 \text{ rad/s}^2$

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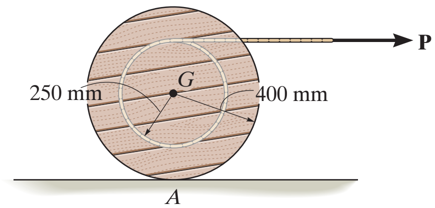
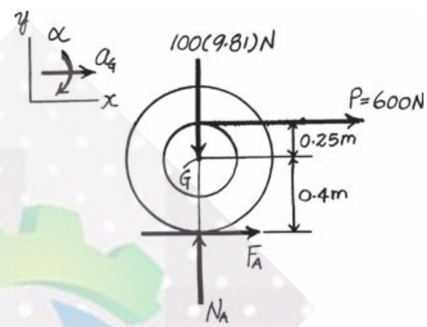
17–103.

The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 600 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 100(9.81) = 0$

$\circlearrowright + \Sigma M_G = I_G\,\alpha; \qquad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 15.6 \text{ rad/s}^2$ **Ans.**

$a_G = 6.24 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 24.0 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}$ **OK**

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Ans:
$\alpha = 15.6 \text{ rad/s}^2$

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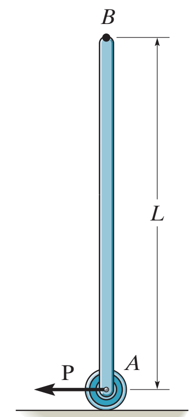
***17–104.**

The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



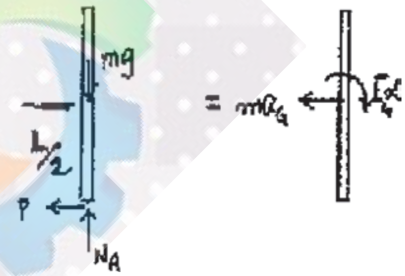
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = ma_G$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad P\left(\frac{L}{2}\right) = \left(\frac{1}{12}mL^2\right)\alpha$

$P = \frac{1}{6}mL\alpha$

$\alpha = \frac{6P}{mL}$ **Ans.**



$a_G = \frac{P}{m}$

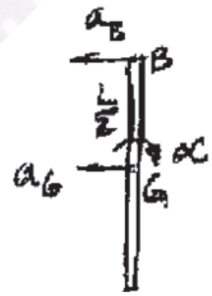
$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = \frac{-P}{m}\mathbf{i} + \frac{L}{2}\alpha\,\mathbf{i}$

$(\overset{+}{\leftarrow}) \qquad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

$= \frac{P}{m} - \frac{L}{2}\left(\frac{6P}{mL}\right)$

$a_B = -\frac{2P}{m} = \frac{2P}{m}$ **Ans.**



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Ans:

$\alpha = \frac{6P}{mL}$

$a_B = \frac{2P}{m}$

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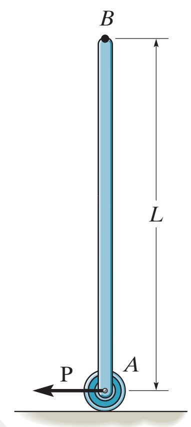
17–105.

Solve Prob. 17–104 if the roller is removed and the coefficient of kinetic friction at the ground is μ_k.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P - \mu_k N_A = m a_G$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad (P - \mu_k N_A)\dfrac{L}{2} = \left(\dfrac{1}{12} m L^2\right)\alpha$

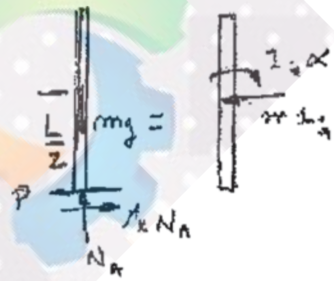
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - mg = 0$

Solving,

$$N_A = mg$$

$$a_G = \frac{L}{6}\alpha$$

$$\alpha = \frac{6(P - \mu_k mg)}{mL} \qquad \textbf{Ans.}$$



$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$(\overset{+}{\rightarrow}) \quad a_B = -\dfrac{L}{6}\alpha + \dfrac{L}{2}\alpha$

$a_B = \dfrac{L\alpha}{3}$

$$a_B = \frac{2(P - \mu_k mg)}{m} \qquad \textbf{Ans.}$$

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Ans:

$\alpha = \dfrac{6(P - \mu_k mg)}{mL}$

$a_B = \dfrac{2(P - \mu_k mg)}{m}$

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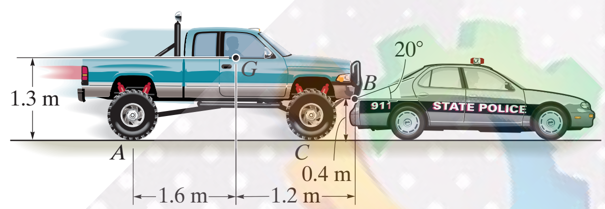
17–106. A "lifted" truck can become a road hazard since the bumper is high enough to ride up a standard car in the event the car is rear-ended. As a model of this case consider the truck to have a mass of 2.70 Mg, a mass center G, and a radius of gyration about G of $k_G = 1.45$ m. Determine the horizontal and vertical components of acceleration of the mass center G, and the angular acceleration of the truck, at the moment its front wheels at C have just left the ground and its smooth front bumper begins to ride up the back of the stopped car so that point B has a velocity of $v_B = 8$ m/s at 20º from the horizontal. Assume the wheels are free to roll, and neglect the size of the wheels and the deformation of the material.



SOLUTION

$\text{Mg} = 10^3 \text{ kg}$

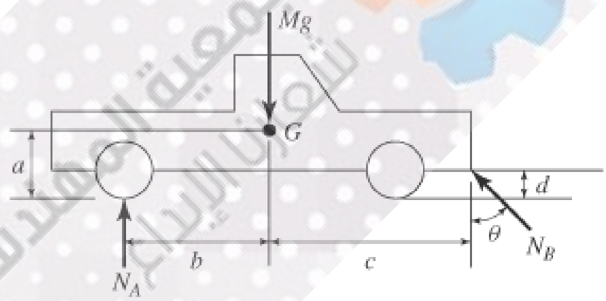
$\text{kN} = 10^3 \text{ N}$

$M = 2.70 \text{ Mg} \quad a = 1.3 \text{ m}$

$\theta = 20 \text{ deg} \quad b = 1.6 \text{ m}$

$k_G = 1.45 \text{ m} \quad c = 1.2 \text{ m}$

$v_B = 8 \text{ m/s} \quad d = 0.4 \text{ m}$



Guesses

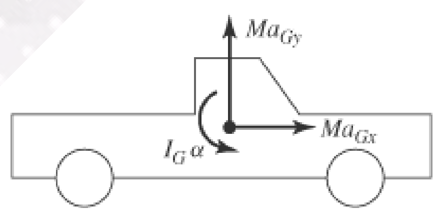
$v_A = 1 \text{ m/s} \quad \omega = 1 \text{ rad/s}$

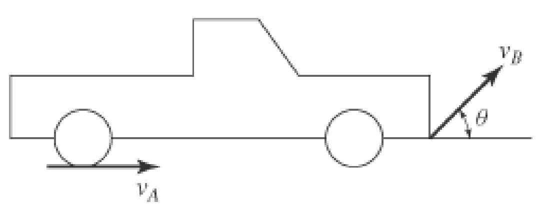
$a_B = 1 \text{ m/s}^2 \quad \alpha = 1 \text{ rad/s}^2$

$a_{Gx} = 1 \text{ m/s}^2 \quad a_{Gy} = 1 \text{ m/s}^2$

$N_A = 1 \text{ N} \quad N_B = 1 \text{ N}$

$a_A = 1 \text{ m/s}^2$





Given

$$N_A + N_B\cos(\theta) - Mg = Ma_{Gy} \qquad -N_B\sin(\theta) = Ma_{Gx}$$

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17–106. Continued

$$N_B\cos(\theta)c - N_B\sin(\theta)(a-d) - N_A b = M k_G^{\,2}\alpha$$

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} v_B\cos(\theta) \\ v_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ \omega \\ a_A \\ a_B \\ a_{Gx} \\ a_{Gy} \\ \alpha \\ N_A \\ N_B \end{pmatrix} = \text{Find}(v_A, \omega, a_A, a_B, a_{Gx}, a_{Gy}, \alpha, N_A, N_B)$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \end{pmatrix} = \begin{pmatrix} -1.82 \\ -1.69 \end{pmatrix} \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\alpha = -0.283 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 8.38 \\ 14.39 \end{pmatrix} \text{kN}$$

$$\omega = 0.977 \text{ rad/s}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -0.663 \\ -3.431 \end{pmatrix} \text{m/s}^2$$

Ans:
$a_{Gx} = -1.82\ \text{m/s}^2$
$a_{Gy} = -1.69\ \text{m/s}^2$
$\alpha = -0.283\ \text{rad/s}^2$

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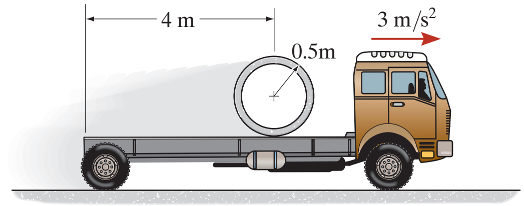
17–107.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of $3\,\text{m/s}^2$, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



SOLUTION

Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A using Fig. *a*,

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3\ \text{m/s}^2$. Applying the relative acceleration equation and referring to Fig. *b*,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} = 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

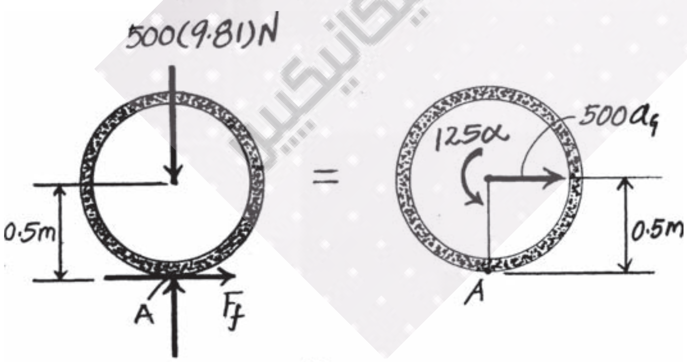
Equating the **i** components,

$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

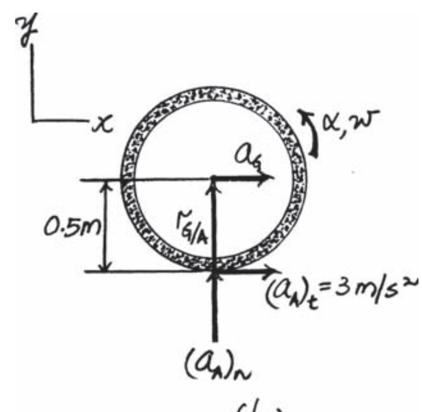
Solving Eqs. (1) and (2) yields

$$a_G = 1.5\ \text{m/s}^2 \rightarrow$$

$$\alpha = 3\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\alpha = 3\ \text{rad/s}^2$

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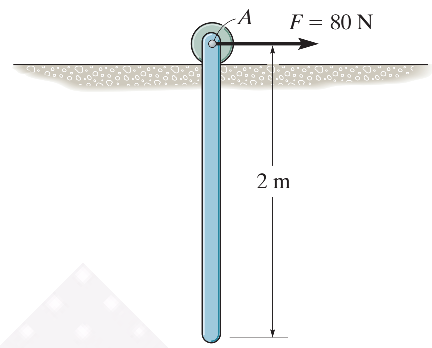
***17–108.**

The 12-kg uniform bar is supported by a roller at A. If a horizontal force of $F = 80$ N is applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. Neglect the weight and the size of the roller.



SOLUTION

Equations of Motion. The mass moment of inertia of the bar about its center of gravity G is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(2^2) = 4.00\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the bar, Fig. a

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 80 = 12(a_G)_x \qquad (a_G)_x = 6.6667\ \text{m/s}^2 \rightarrow$$

$$\circlearrowleft + \Sigma M_A = (\mu_k)_A; \qquad 0 = 12(6.6667)(1) - 4.00\,\alpha \qquad \alpha = 20.0\ \text{rad/s}^2 \circlearrowright$$

Kinematic. Since the bar is initially at rest, $\omega = 0$. Applying the relative acceleration equation by referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

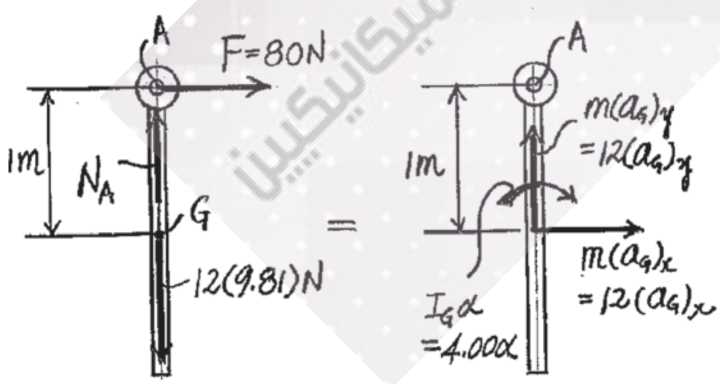
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = a_A\mathbf{i} + (-20.0\mathbf{k}) \times (-\mathbf{j}) - \mathbf{0}$$

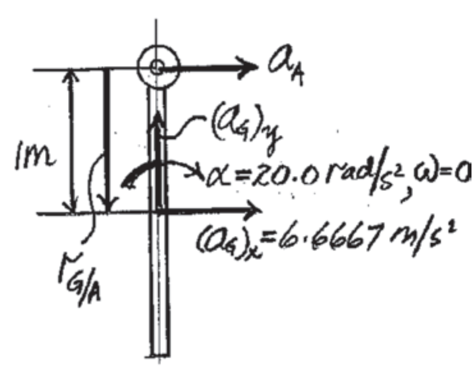
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = (a_A - 20)\mathbf{i}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components,

$6.6667 = a_A - 20; \qquad a_A = 26.67\ \text{m/s}^2 = 26.7\ \text{m/s}^2 \rightarrow$ **Ans.**

$(a_G)_y = 0$





(b)

Ans:
$a_A = 26.7\ \text{m/s}^2 \rightarrow$

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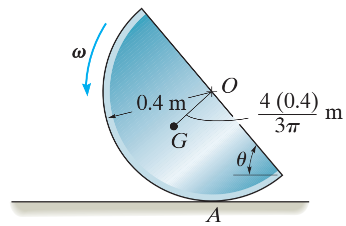
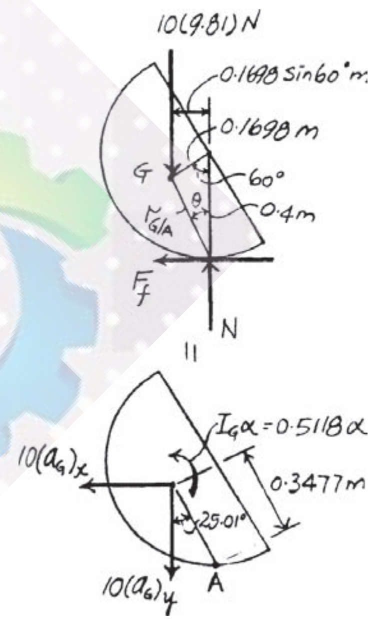
17–109.

The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



SOLUTION

Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\left(0.1698^2\right) = 0.5118 \text{ kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60°} = 0.3477$ m Also, using law of sines, $\dfrac{\sin\theta}{0.1698} = \dfrac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have

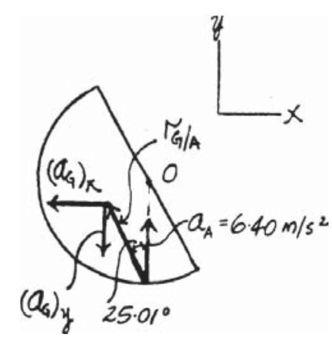


$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha$$
$$+ 10(a_G)_x \cos 25.01°(0.3477)$$
$$+ 10(a_G)_y \sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have



$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\,\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

$$\alpha = 13.85 \text{ rad/s}^2 \qquad (a_G)_x = 2.012 \text{ m/s}^2 \qquad (a_G)_y = 0.6779 \text{ m/s}^2$$

$$F_f = 20.12 \text{ N} \qquad N = 91.32 \text{ N}$$

Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**. **Ans.**

Ans:
Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**.

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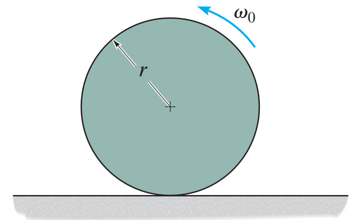
17–110.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the initial angular acceleration of the disk and the acceleration of its mass center. The coefficient of kinetic friction between the disk and the floor is μ_k.



SOLUTION

Equations of Motion. Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$. We have

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - mg = 0 \qquad N = mg$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad \mu_k(mg) = ma_G \qquad a_G = \mu_k g \leftarrow$ **Ans.**

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad -\mu_k(mg)r = \left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r} \circlearrowright$ **Ans.**

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Ans:
$a_G = \mu_k g \leftarrow$
$\alpha = \frac{2\mu_k g}{r} \circlearrowright$

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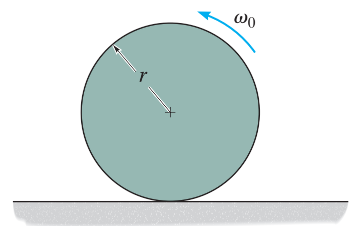
17–111.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the time before it starts to roll without slipping. What is the angular velocity of the disk at this instant? The coefficient of kinetic friction between the disk and the floor is μ_k.



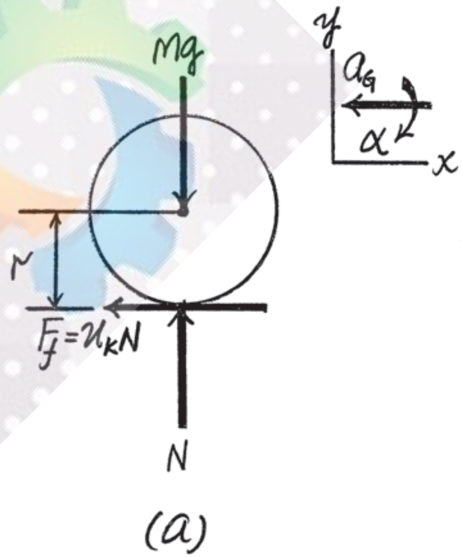
SOLUTION

Equations of Motion: Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$.

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - mg = 0 \qquad N = mg$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad \mu_k(mg) = ma_G \qquad a_G = \mu_k g$$

$$+\Sigma M_G = I_G\alpha; \qquad -\mu_k(mg)r = -\left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r}$$



Kinematics: At the instant when the disk rolls without slipping, $v_G = \omega r$. Thus,

$$(\overset{+}{\leftarrow}) \qquad v_G = (v_G)_0 + a_G t$$

$$\omega r = 0 + \mu_k g t$$

$$t = \frac{\omega r}{\mu_k g} \qquad \textbf{(1)}$$

and

$$\omega = \omega_0 + \alpha t$$

$$(\circlearrowleft +) \qquad \omega = \omega_0 + \left(-\frac{2\mu_k g}{r}\right)t \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\omega = \frac{1}{3}\omega_0 \qquad t = \frac{\omega_0 r}{3\mu_k g} \qquad \textbf{Ans.}$$

Ans:

$\omega = \frac{1}{3}\omega_0$

$t = \frac{\omega_0 r}{3\mu_k g}$

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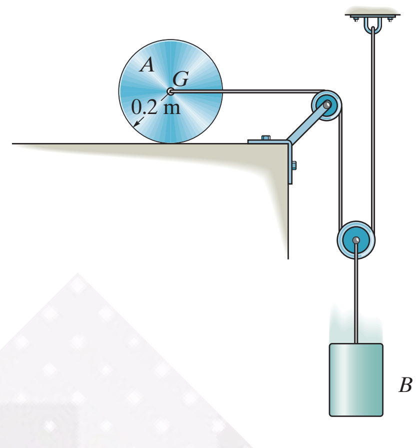
***17–112.** The 20-kg disk A is attached to the 10-kg block B using the cable and pulley system shown. If the disk rolls without slipping, determine its angular acceleration and the acceleration of the block when they are released. Also, what is the tension in the cable? Neglect the mass of the pulleys.



SOLUTION

Equation of Motions:

Disk:

$$\circlearrowleft +\Sigma M_{IC} = \Sigma(M_k)_{IC}; \qquad T(0.2) = -\left[\frac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right]\alpha \qquad \textbf{(1)}$$

Block:

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 10(9.81) - 2T = 10a_B \qquad \textbf{(2)}$$

Kinematics:

$$2s_B + s_A = l$$

$$2a_B = -a_A$$

Also,

$$a_A = 0.2\alpha$$

Thus,

$$a_B = -0.1\alpha \qquad \textbf{(3)}$$

Note the direction for α and a_B are the same for all equations.

Solving Eqs. (1) through (3):

$$a_B = 0.755 \text{ m/s}^2 = 0.755 \text{ m/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$\alpha = -7.55 \text{ rad/s}^2 = 7.55 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$T = 45.3 \text{ N} \qquad \textbf{Ans.}$$

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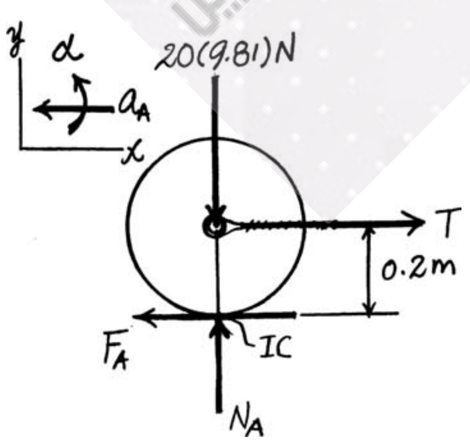

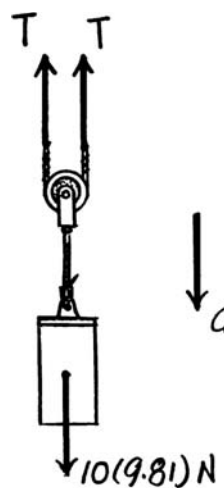

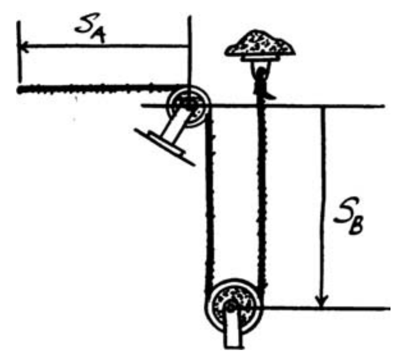

Ans:
$a_B = 0.755 \text{ m/s}^2 \downarrow$
$\alpha = 7.55 \text{ rad/s}^2 \circlearrowright$
$T = 45.3$ N

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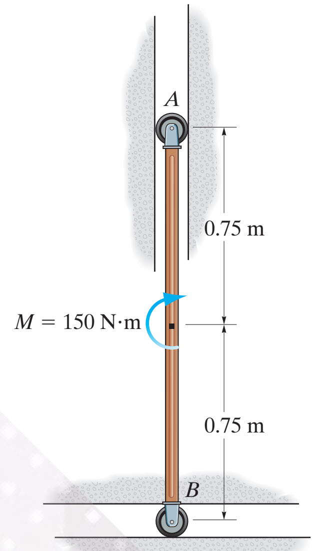
17–113.

The 30-kg uniform slender rod AB rests in the position shown when the couple moment of $M = 150\ \text{N}\cdot\text{m}$ is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.



SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625\ \text{kg}\cdot\text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of $\mathbf{N}_A$ and $\mathbf{N}_B$ and using, Fig. *a*,

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -150 = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 150 \qquad \textbf{(1)}$$

Kinematics: Applying the relative acceleration equation to points A and G, Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -a_A\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.75\mathbf{j}) - \mathbf{0}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -0.75\alpha\mathbf{i} - a_A\mathbf{j}$$

Equating the $\mathbf{i}$ components,

$$(a_G)_x = -0.75\alpha \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1),

$$\alpha = 6.667\ \text{rad/s}^2 = 6.67\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

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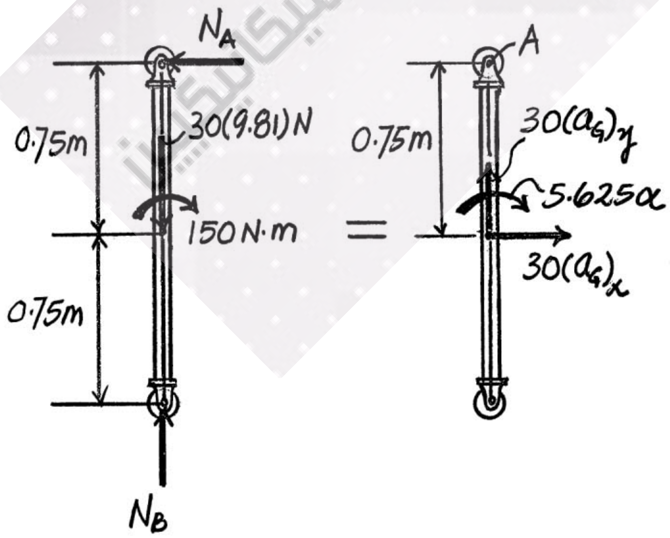
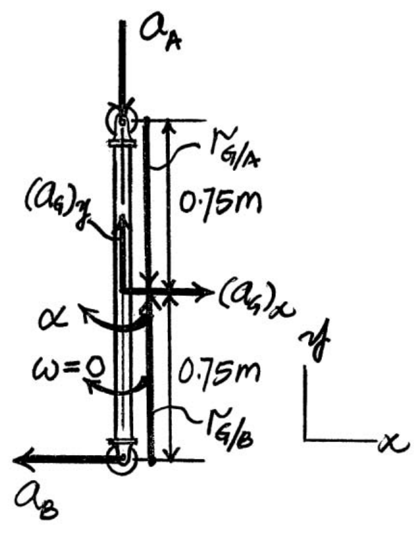


(a)



(b)

Ans:
$\alpha = 6.67\ \text{rad/s}^2$

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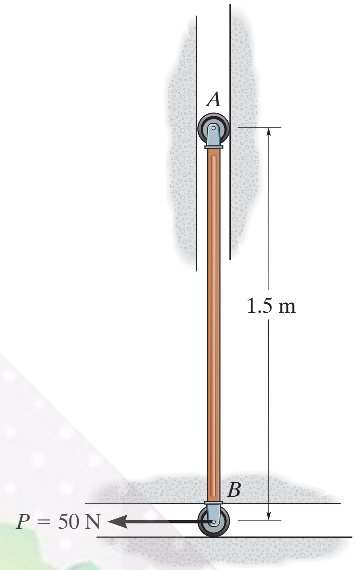
17–114.

The 30-kg slender rod AB rests in the position shown when the horizontal force $P = 50\text{ N}$ is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.



SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625\text{ kg}\cdot\text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of $\mathbf{N}_A$ and $\mathbf{N}_B$ and using, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -50(0.15) = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 75 \qquad \textbf{(1)}$$

Kinematics: Applying the relative acceleration equation to points A and G, Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -a_A\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.75\mathbf{j}) - \mathbf{0}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -0.75\alpha\mathbf{i} - a_A\mathbf{j}$$

Equating the **i** components,

$$(a_G)_x = -0.75\alpha \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1),

$$\alpha = 3.333\text{ rad/s}^2 = 3.33\text{ rad/s}^2 \qquad \textbf{Ans.}$$

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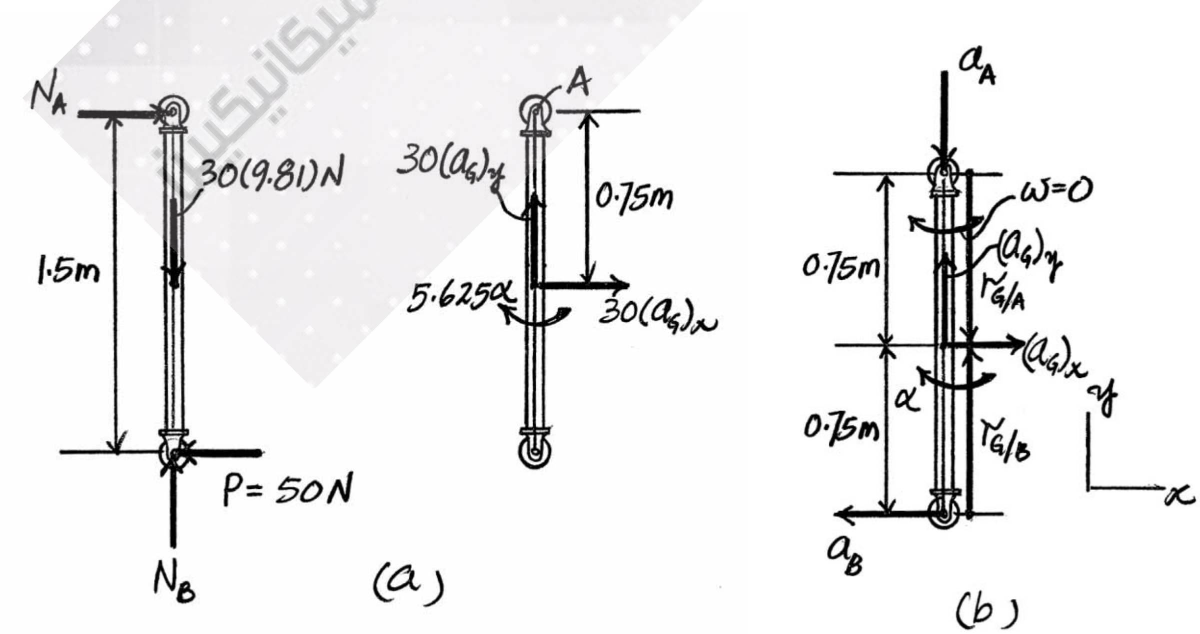

Ans:
$\alpha = 3.33\text{ rad/s}^2$

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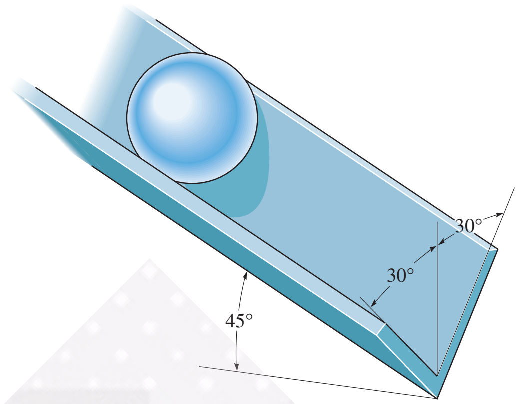
17–115.

The solid ball of radius r and mass m rolls without slipping down the 60° trough. Determine its angular acceleration.

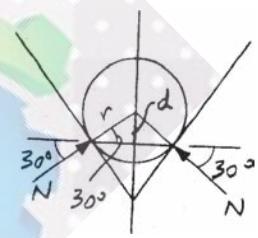


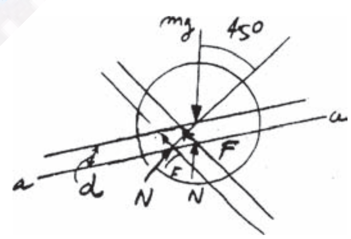
SOLUTION

$$d = r \sin 30^\circ = \frac{r}{2}$$

$$\Sigma M_{a-a} = \Sigma(M_k)_{a-a}; \qquad mg \sin 45^\circ\left(\frac{r}{2}\right) = \left[\frac{2}{5}mr^2 + m\left(\frac{r}{2}\right)^2\right]\alpha$$

$$\alpha = \frac{10g}{13\sqrt{2}\,r} \qquad \textbf{Ans.}$$





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Ans:
$$\alpha = \frac{10g}{13\sqrt{2}\,r}$$

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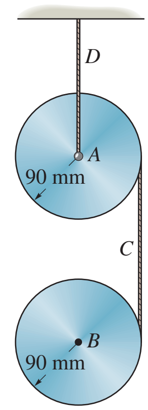
***17–116.**

A cord is wrapped around each of the two 10-kg disks. If they are released from rest, determine the angular acceleration of each disk and the tension in the cord C. Neglect the mass of the cord.



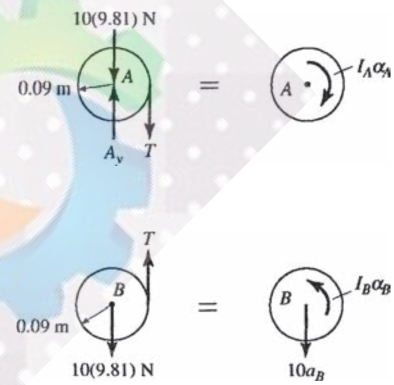
SOLUTION

For A:

$$\circlearrowleft +\Sigma M_A = I_A\alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \qquad \textbf{(1)}$$

For B:

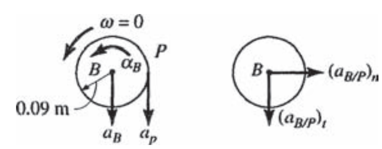
$$\circlearrowright +\Sigma M_B = I_B\alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \qquad \textbf{(2)}$$



$$+\downarrow \Sigma F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \qquad \textbf{(3)}$$

$$\mathbf{a}_B = \mathbf{a}_P + (\mathbf{a}_{B/P})_t + (\mathbf{a}_{B/P})_n$$

$$(+\downarrow)\ a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \qquad \textbf{(4)}$$



Solving,

$a_B = 7.85\ \text{m/s}^2$

$\alpha_A = 43.6\ \text{rad/s}^2$ **Ans.**

$\alpha_B = 43.6\ \text{rad/s}^2$ **Ans.**

$T = 19.6\ \text{N}$ **Ans.**

$A_y = 10(9.81) + 19.62$

$\quad = 118\ \text{N}$

Ans:
$\alpha_A = 43.6\ \text{rad/s}^2 \circlearrowright$
$\alpha_B = 43.6\ \text{rad/s}^2 \circlearrowleft$
$T = 19.6\ \text{N}$

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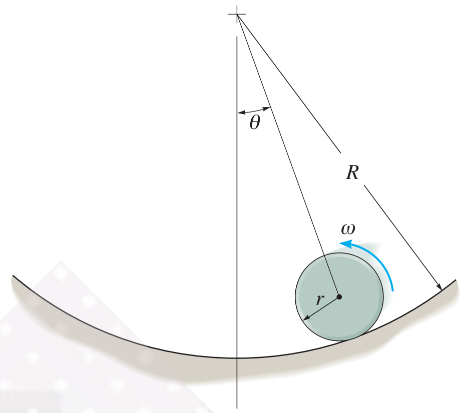
17–117.

The disk of mass m and radius r rolls without slipping on the circular path. Determine the normal force which the path exerts on the disk and the disk's angular acceleration if at the instant shown the disk has an angular velocity of $\boldsymbol{\omega}$.

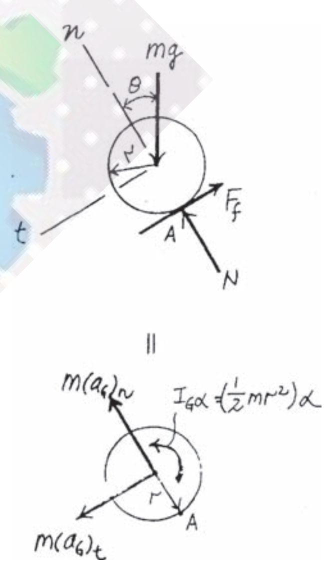


SOLUTION

Equation of Motion: The mass moment of inertia of the disk about its center of mass is given by $I_G = \frac{1}{2}mr^2$. Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(a_G)_t\,(r) \qquad \textbf{[1]}$$

$$\Sigma F_n = m(a_G)_n; \qquad N - mg\cos\theta = m(a_G)_n \qquad \textbf{[2]}$$



Kinematics: Since the semicircular disk does not slip at A, then $v_G = \omega r$ and $(a_G)_t = \alpha r$. Substitute $(a_G)_t = \alpha r$ into Eq. [1] yields

$$mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(\alpha r)(r)$$

$$\alpha = \frac{2g}{3r}\sin\theta \qquad \textbf{Ans.}$$

Also, the center of the mass for the disk moves around a circular path having a radius of $\rho = R - r$. Thus, $(a_G)_n = \frac{v_G^2}{\rho} = \frac{\omega^2 r^2}{R - r}$. Substitute into Eq. [2] yields

$$N - mg\cos\theta = m\left(\frac{\omega^2 r^2}{R - r}\right)$$

$$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right) \qquad \textbf{Ans.}$$

Ans:
$\alpha = \frac{2g}{3r}\sin\theta$
$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right)$

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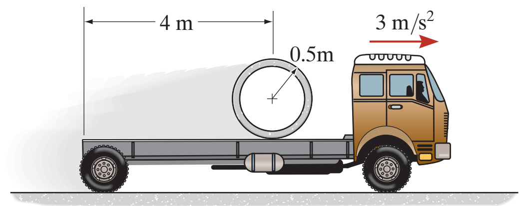
17–118.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of $3\,\text{m/s}^2$, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



SOLUTION

Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500(0.5^2) = 125\,\text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A using Fig. *a*,

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3\,\text{m/s}^2$. Applying the relative acceleration equation and referring to Fig. *b*,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} - 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

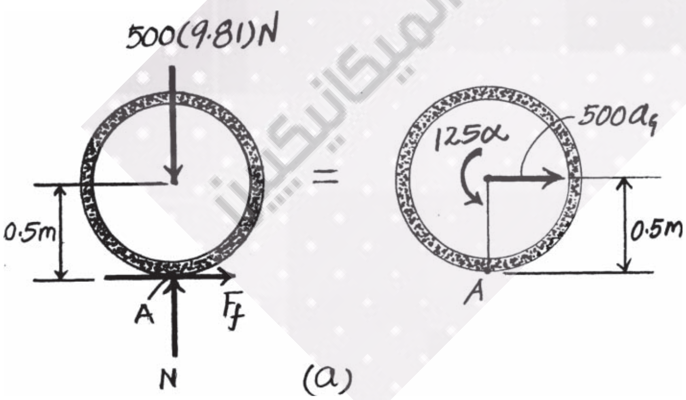
Equating the **i** components,

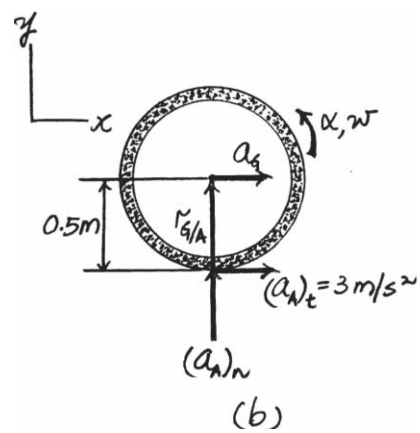
$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 1.5\,\text{m/s}^2 \rightarrow$$

$$\alpha = 3\,\text{rad/s}^2 \qquad \textbf{Ans.}$$





Ans:
$\alpha = 3\,\text{rad/s}^2$

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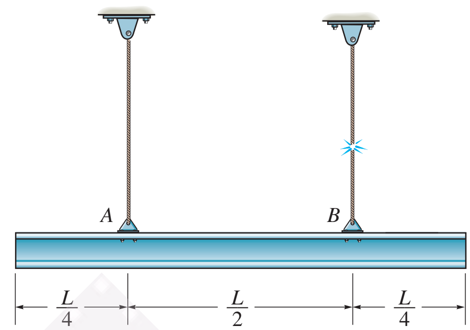
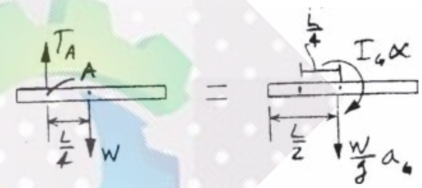
17–119.

The uniform beam has a weight W. If it is originally at rest while being supported at A and B by cables, determine the tension in cable A if cable B suddenly fails. Assume the beam is a slender rod.



SOLUTION



$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad W\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha + \frac{W}{g}\left(\frac{L}{4}\right)\alpha\left(\frac{L}{4}\right)$$

$$1 = \frac{1}{g}\left(\frac{L}{4} + \frac{L}{3}\right)\alpha$$

Since $a_G = \alpha\left(\frac{L}{4}\right)$.

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = W - \frac{W}{g}(\alpha)\left(\frac{L}{4}\right) = W - \frac{W}{g}\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)\left(\frac{L}{4}\right)$$

$$T_A = \frac{4}{7}W \qquad \textbf{Ans.}$$

Also,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_G = I_G\,\alpha; \qquad T_A\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha$$

Since $a_G = \frac{L}{4}\alpha$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\alpha$$

$$\frac{1}{3}\left(\frac{W}{g}\right)L\alpha - W = -\frac{W}{g}\left(\frac{L}{4}\right)\alpha$$

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)$$

$$T_A = \frac{4}{7}W \qquad \textbf{Ans.}$$

Ans:
$T_A = \frac{4}{7}W$

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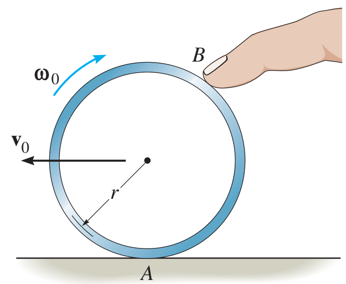
***17–120.**

By pressing down with the finger at B, a thin ring having a mass m is given an initial velocity $\mathbf{v_0}$ and a backspin $\boldsymbol{\omega}_0$ when the finger is released. If the coefficient of kinetic friction between the table and the ring is μ_κ, determine the distance the ring travels forward before backspinning stops.



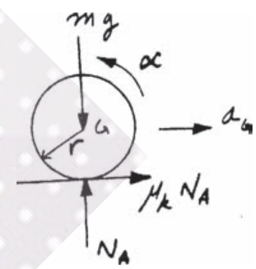
SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad N_A - mg = 0$$

$$N_A = mg$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad \mu_k\,(mg) = m(a_G)$$

$$a_G = \mu_k\, g$$



$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad \mu_k\,(mg)r = mr^2\alpha$$

$$\alpha = \frac{\mu_k\, g}{r}$$

$$(\circlearrowleft +) \qquad \omega = \omega_0 + \alpha_c\, t$$

$$0 = \omega_0 - \left(\frac{\mu_k\, g}{r}\right) t$$

$$t = \frac{\omega_0\, r}{\mu_k\, g}$$

$$(\xleftarrow{+}) \qquad s = s_0 + v_0 t + \tfrac{1}{2} a_c\, t^2$$

$$s = 0 + v_0\left(\frac{\omega_0\, r}{\mu_k\, g}\right) - \left(\frac{1}{2}\right)(\mu_k\, g)\left(\frac{\omega_0^2\, r^2}{\mu_k^2\, g^2}\right)$$

$$s = \left(\frac{\omega_0\, r}{\mu_k\, g}\right)\left(v_0 - \frac{1}{2}\,\omega_0\, r\right)$$ **Ans.**

Ans:

$$s = \left(\frac{\omega_0\, r}{\mu_k\, g}\right)\left(v_0 - \frac{1}{2}\,\omega_0\, r\right)$$