

* Show that if the following set of vector form basis:

$$v_1 = (6, -3, 5)$$

$$v_2 = (2, 7, 5)$$

$$v_3 = (14, 6, 6)$$

$$\textcircled{1} 6k_1 + 2k_2 + 14k_3 = 0$$

$$-3k_1 + 7k_2 + 6k_3 = 0$$

$$5k_1 + 5k_2 + 6k_3 = 0$$

$$A = \begin{bmatrix} 6 & 2 & 14 \\ -3 & 7 & 6 \\ 5 & 5 & 6 \end{bmatrix} \Rightarrow \det(A) = 6(12) - 2(-48) + 14(-50)$$

$$= -532$$

So its linearly indep

and span $\mathbb{R}^3$, so they are basis

indep $\checkmark$

span $\checkmark$

* Must a basis for P_n contain a polynomial of degree k for each $k=1, 2, \dots, n$?

justify your answer & give an example. $k_0(1) + k_1(x) + k_2(x^2) + \dots + k_n(x^n)$
closed under addition & scalar multiplication

* Show if the following statement is always correct.

"if u is orthogonal to $(w+v)$ then u is orthogonal to v and w "

$$\Rightarrow u \cdot (w+v) = 0$$

$$(u \cdot w) + (u \cdot v) = 0$$

* Assume that V consists of all vectors that are defined in $\mathbb{R}^3$ and have the following form: $\begin{bmatrix} x \\ y \end{bmatrix}$ is V considered as a vector space or not when a) $y = x+1$

$$b) y = x$$

$$a) y = x+1 \Rightarrow \begin{bmatrix} x \\ x+1 \end{bmatrix}$$

$$\text{let } u = \begin{bmatrix} u \\ u+1 \end{bmatrix}, v = \begin{bmatrix} v \\ v+1 \end{bmatrix}$$

$$\text{axiom 1} \Rightarrow u+v = \begin{bmatrix} u+v \\ u+v+2 \end{bmatrix} \text{ not in } V \text{ (not closed under addition)}$$

$$b) \text{ let } u = \begin{bmatrix} u \\ u \end{bmatrix}, v = \begin{bmatrix} v \\ v \end{bmatrix}$$

$$\text{axiom 1: } u+v = \begin{bmatrix} u+v \\ u+v \end{bmatrix} \text{ closed under addition } \checkmark$$

$$\text{axiom 6: } \underset{\text{scalar}}{\downarrow} u = \begin{bmatrix} -u \\ -u \end{bmatrix} \text{ closed under scalar multiplication } \checkmark$$

it is a vector space when $y=x$

* Show that a matrix A is nonsingular iF and only iF A^T is nonsingular.
invertible

inv $\begin{cases} \rightarrow \text{square} \\ \rightarrow \det \neq 0 \end{cases}$

iF A is singular then A^T is singular

iF $\det(A) \neq 0$, then $\det(A^T) \neq 0$ as $\det(A) = \det(A^T)$

* Given that A is a 3×3 upper triangular matrix, show that the corresponding cofactor matrix is a lower one.

$$\text{let } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{bmatrix} \Rightarrow C_{11} = (-1)^2 \begin{vmatrix} a_4 & a_5 \\ 0 & a_6 \end{vmatrix}$$

* Given that A, B and C are 3×3 matrices, show iF the following statement is Always correct or not.

$$\det(A^T B C) = \det(C^T B^T A)$$

$$\Rightarrow \det(A^T B C) = \det(A^T) * \det(B) * \det(C)$$

$$\det(C^T B^T A) = \det(C^T) * \det(B^T) * \det(A)$$

$$\det(A) = \det(A)$$

$$\det(C) = \det(C^T)$$

$$\det(A) \det(B) \det(C) = \det(C) \det(B) \det(A)$$

① inconsistent . ② consistent with ∞ # of solutions

③ consistent with one solution

$$\begin{aligned} * \quad & ax_1 + bx_2 + x_3 = d \\ & x_1 + x_2 + x_3 = c \\ & +e x_3 = 10 \end{aligned}$$

① inconsistent $\Rightarrow a=1, b=1, d=2, c=3, \boxed{e=0}$

2. $\text{let } 0 \neq 10$ solution يكون

② consis. with one solution $\Rightarrow \det \neq 0$ (coef).

$$\left[\begin{array}{ccc|c} a & b & 1 & d \\ 1 & 1 & 1 & c \\ 0 & 0 & e & 10 \end{array} \right]$$

$$\Rightarrow \det = ae - be \neq 0$$

$$\begin{cases} e \neq 0 \\ a \neq b \end{cases}$$

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③ consis. with ∞ # of solutions $\Rightarrow \boxed{\begin{matrix} a=b=1 \\ d=c \end{matrix}}$

* let $u = (3, -2, -4)$, $v = (5, 2, -8)$, $w = (9, 2, 6)$. Find:

① distance betw $-3u$ and $-2v + 4w$

② $u \cdot (v \times w)$

③ a unit vector that is orthogonal to these vectors.

④ Find the vector component of u along w and the vector comp. of u orthogonal to w .

$\Rightarrow$ ① $\text{dist} = \text{norm}(-3u - (-2v + 4w))$

$$-3u = (-9, 6, 12)$$

$$-2v + 4w = (26, 4, 40)$$

$$-3u - (-2v + 4w) = (-35, 2, -28)$$

$$\text{dist} = \sqrt{(-35)^2 + (2)^2 + (-28)^2} = 44.866$$

② $u \cdot (v \times w) \Rightarrow v \times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 2 & -8 \\ 9 & 2 & 6 \end{vmatrix} = 28\hat{i} - 102\hat{j} + -8\hat{k}$

$$u \cdot (v \times w) = 3 \times 28 + -2 \times -102 + -4 \times -8 = 320$$

③ $u \times (v \times w) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -4 \\ 28 & -102 & -8 \end{vmatrix} = -392\hat{i} - 88\hat{j} - 250\hat{k}$

$$\text{norm} = 473.189$$

④ $\text{proj}_w u = \frac{u \cdot w}{\|w\|^2} \cdot w = \frac{3 \times 9 - 2 \times 2 - 4 \times 6}{(\sqrt{9^2 + 2^2 + 6^2})^2} \cdot (9, 2, 6)$
 $= \frac{-1}{121} (9, 2, 6) \Rightarrow \left(\frac{-9}{121}, \frac{-2}{121}, \frac{-6}{121} \right)$

$$u - \text{proj}_w u = (3, -2, -4) - \left(\frac{-9}{121}, \frac{-2}{121}, \frac{-6}{121} \right) = \left(\frac{372}{121}, \frac{-240}{121}, \frac{-478}{121} \right)$$

* must a basis for P_n contain a polynomial of degree k for each $k=1, 2, \dots, n$?
justify your answer & give an example.

$\Rightarrow$ The standard basis for P_n is $\{1, x, x^2, \dots, x^n\}$ so there must be a polynomial for every value of k as if we want the basis for P_2 then it is $\{1, x, x^2\}$ so there is a polynomial of degree k for each ($k=1, 2$) and if there wasn't a polynomial then it will not span P_2 , so it is a must.

* Prove if S is a basis for a vector space V , then for any vectors u, v in V and any scalar k , the following relationships hold:

$$(u+v)S = u(S) + v(S)$$

$$(kv)S = k(v)S$$

$\Rightarrow$ S is a basis so it spans V and linearly indep. u, v are subspaces for S
so both are in addition and scalar multiplication.
(closed under addition and scalar multiplication).

* Show if the following set of vectors form basis:

$$v_1 = (6, -3, 5), v_2 = (2, 7, 5), v_3 = (14, 6, 6)$$

$\Rightarrow$ check for indep. and span

$$6k_1 + 2k_2 + 14k_3 = 0$$

$$-3k_1 + 7k_2 + 6k_3 = 0$$

$$5k_1 + 5k_2 + 6k_3 = 0$$

$$A(\text{coef}) = \begin{bmatrix} 6 & 2 & 14 \\ -3 & 7 & 6 \\ 5 & 5 & 6 \end{bmatrix}$$

$$\det(A) = 6(12) - 2(48) + 14(-50) = -532$$

$\det(A) \neq 0$ so its linearly indep & span
then these vectors form a basis

* If all entries of a square matrix A are integers and $\det(A) = \pm 1$, show that all entries of A^{-1} are integers.

$$\Rightarrow A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \det(A) = \pm 1$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\pm 1} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ so all entries are integers.}$$

* Let A be a square matrix whose $\det$ is integer. Show if the following statement is always correct or not "The $\det(A^n)$ is integer".

$$\Rightarrow \det(A^n) = \underbrace{\det(A)}_{\text{int}} \times \underbrace{\det(A)}_{\text{int}} \dots \text{ so its integer}$$

* Assume that A & B are the same invertible matrices. Show if the following statement is always correct " $A^{-1} + B^{-1}$ is invertible if $A + B$ is invertible ".

$$\begin{aligned} \Rightarrow A^{-1}(A+B) &= (I + A^{-1}B) B^{-1} \\ &= B^{-1} + A^{-1} \quad \text{equivalent.} \end{aligned}$$

* Assume that A & B are 3×3 matrix that can be written as $A = k B^3$. Write the $\det$ of A as a function of the $\det$ of B and k .

$$\Rightarrow A = k B^3$$

$$A = k B * B * B$$

$$\det(A) = \det(kB) * \det(B) * \det(B)$$

$$= k^3 \det(B) * \det(B) * \det(B)$$

$$\det(A) = k^3 (\det(B))^3$$

* $W * \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ Find the value of matrix w that satisfies the following equation:

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$w = \begin{bmatrix} 5 & -11 \\ 3 & -7 \end{bmatrix}$$

* $u = (3, 4, -3)$, $v = (3, 2, -6)$, $w = (3, 0, 4)$ show that if w can be written as LC of u & v .

$$3k_1 + 3k_2 = 3$$

$$4k_1 + 2k_2 = 0$$

$$-3k_1 - 6k_2 = 4$$

$$\Rightarrow \boxed{k_1 = 1 - k_2} \quad \text{subs in (2)}$$

$$\Rightarrow 4(1 - k_2) + 2k_2 = 0 \Rightarrow \boxed{k_2 = 2}, \boxed{k_1 = -1}$$

$$-3(-1) - 6(2) = -9$$

$-9 \neq 4$ so can't be written as LC of u & v

* Use Cramer's rule to solve for $\cos \gamma$ and $\cos \beta$ in terms of a, b and c

$$b \cos \gamma + c \cos \alpha = a$$

$$c \cos \beta + a \cos \gamma = b$$

$$a \cos \alpha + b \cos \beta = c$$

$$\Rightarrow A = \begin{bmatrix} \cos \gamma & \cos \alpha & \cos \beta \\ b & c & 0 \\ a & 0 & c \\ 0 & a & b \end{bmatrix} \quad \text{RHS} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\det(A) = b(-ac) - c(ab) = -2abc$$

$$A_1 = \begin{bmatrix} a & c & 0 \\ b & 0 & c \\ c & a & b \end{bmatrix} \Rightarrow \det(A_1) = a(-ac) - c(b^2 - c^2)$$

$$\cos \gamma = \frac{-a^2c - c(b^2 - c^2)}{-2abc} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$A_3 = \begin{bmatrix} b & c & a \\ a & 0 & b \\ 0 & a & c \end{bmatrix} \Rightarrow \det(A_3) = b(-ab) - c(ac) + a(a^2)$$

$$\cos \beta = \frac{-ab^2 - c^2a + a^3}{-2abc} = \frac{ab^2 + c^2a - a^3}{2abc}$$

* Let A be a 3×3 matrix where determinant value is integer, show if the following statement is always correct or not?

"The $\det(A^n)$ is not prime when n is larger than 2".

$\Rightarrow$ if $n > 2$, it is at least $(\det(A) * \det(A) * \det(A))$ it is not possible to be a prime number so the statement is always correct.

* Determine whether the given set of vectors forms an orthogonal set:

$$u = (2, 5, 8), v = (6, 1, 9), w = (-3, 6, 2)$$

هل متعامدات؟

$$u \cdot v = 89 \quad \text{not orthogonal set.}$$

١) $u \cdot w, u \cdot v, v \cdot w$ كلها

مختلفة عن 0
orthogonal set
ليس متعامدات
set

* Assume V consists of all vector in R^3 $\begin{bmatrix} q+t \\ 2r-t \\ 3s+t \end{bmatrix}$ is V considered as a vector space?

$$\Rightarrow u = \begin{bmatrix} q_1 + t_1 \\ 2r_1 - t_1 \\ 3s_1 + t_1 \end{bmatrix} \quad v = \begin{bmatrix} q_2 + t_2 \\ 2r_2 - t_2 \\ 3s_2 + t_2 \end{bmatrix}$$

① addition $\Rightarrow u+v = \left\{ (q_1+q_2)+2t, (2r_1+2r_2)-2t, (3s_1+3s_2)+2t \right\}$

the result is a vector of the same form as the vectors in V . So, it is closed under addition.

② scalar multiplication $\Rightarrow \alpha u = \left\{ \alpha(q_1+t_1), \alpha(2r_1-t_1), \alpha(3s_1+t_1) \right\}$

closed under scalar multiplication

So V can be considered as a vector space.

* let $W = \{ (x+1), x \in R \}$ is W a vector space or not?

$\Rightarrow$ let $u = u+1, v = v+1$

check axiom 1 $\Rightarrow u+v = u+1 + v+1 = (u+v) + 2$

the result is not a vector that is in the form of vectors in W
so it's not closed under addition.

not a vector space

* Plane 1: $3x + 2y - z = 10$
plane 2: $6x + 4y - 2z = 30$

Find distance betw the two planes?

خطات الحل:

① Point on plane 1: $(0, 0, -10)$ P_0

① إيجاد نقطة على أحد ال planes.

② distance betw P_0 & plane 2

② تطبيق قانون المسافة بين نقطة و plane.

$$D = \frac{|6 \times 0 + 4 \times 0 - 2 \times (-10) - 30|}{\sqrt{6^2 + 4^2 + 2^2}} = 6.681$$

* Assume that A is 3×3 matrix such that column of A sum to zero vector. Assume B is 3×3 matrix $B = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ what is the product of AB ?

$$A = \begin{bmatrix} a & b & c \\ -a & -b & -c \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 0 & a+b+c & 2(a+b+c) \\ 0 & -a-b-c & 2(-a-b-c) \\ 0 & 0 & 0 \end{bmatrix}$$

* Determine whether the given set is linearly independent.

$$u = (2, \sin^2 x, \cos^2 x) \quad v = (1, \sin x, \sin 2x).$$

$$2k_1 + k_2 = 0$$

$$\sin^2 x k_1 + \sin x k_2 = 0$$

$$\cos^2 x k_1 + \sin 2x k_2 = 0$$

$$\begin{bmatrix} 2 & 1 & 0 \\ \sin^2 x & \sin x & 0 \\ \cos^2 x & \sin 2x & 0 \end{bmatrix} \xrightarrow{R_1/2} \begin{bmatrix} 1 & 0.5 & 0 \\ \sin^2 x & \sin x & 0 \\ \cos^2 x & \sin 2x & 0 \end{bmatrix}$$

$$\begin{array}{l} -\sin^2 x R_1 + R_2 \\ -\cos^2 x R_1 + R_3 \end{array} \quad \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & -\frac{1}{2}\sin^2 x + \sin x & 0 \\ 0 & \sin 2x - \frac{1}{2}\cos^2 x & 0 \end{bmatrix} \xrightarrow{R_2/\sin x - \frac{1}{2}\sin^2 x}$$

$$\begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & \sin 2x - \frac{1}{2}\cos^2 x & 0 \end{bmatrix} \xrightarrow{-(\sin 2x - \frac{1}{2}\cos^2 x)R_2 + R_3} \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-0.5R_2 + R_1}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} k_1 = 0 \\ k_2 = 0 \end{array}$$

trivial solution $\Rightarrow$ indep.

$$* 2x_1 + x_2 + \alpha x_3 = \lambda_1$$

$$x_1 + 3x_2 + \beta x_3 = \lambda_2$$

$$\alpha x_1 + \beta x_2 + 2x_3 = \lambda_3$$

$$\textcircled{1} \text{ single sol. } \Rightarrow \begin{bmatrix} 2 & 1 & \alpha \\ 1 & 3 & \beta \\ \alpha & \beta & 2 \end{bmatrix} \det(A) \neq 0$$

$$\alpha=0, \beta=0$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \det(A) = -10 \text{ so its inv.}$$

$\lambda_1, \lambda_2, \lambda_3$ can take any values.

$$\textcircled{2} \text{ No solution } \Leftrightarrow \det(A) = 0$$

inconsistent $\rightarrow$ ∞ # of solution (consistent)

$$\begin{bmatrix} 2 & 1 & \alpha & | & \lambda_1 \\ 1 & 3 & \beta & | & \lambda_2 \\ \alpha & \beta & 2 & | & \lambda_3 \end{bmatrix} \alpha=2, \beta=1 \Rightarrow \begin{bmatrix} 2 & 1 & 2 & | & \lambda_1 \\ 1 & 3 & \beta & | & \lambda_2 \\ \alpha & \beta & 2 & | & \lambda_3 \end{bmatrix}$$

$$\lambda_1 \neq \lambda_3$$

* Find a basis for the nullspace of:

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

طريقة المثل:

REF ①

of parameters ⑤
equal # of vectors.

$$\begin{aligned} R_1/2 \Rightarrow & \begin{bmatrix} 1 & 1 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1.5 & -3 & 1.5 & 0 \\ 0 & 0 & -1.5 & 0 & -1.5 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \\ R_1+R_2 \Rightarrow & \\ -R_1+R_3 \Rightarrow & \end{aligned}$$

$$\begin{aligned} & R_2/1.5 \\ & 1.5R_2+R_3 \\ & -R_2+R_4 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & -0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \end{bmatrix} \begin{aligned} & R_3/-3 \\ & -3R_3+R_4 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & -0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = -t - r$$

$$x_2 = t$$

$$x_3 = -r$$

$$x_4 = 0$$

$$x_5 = r$$

$$v_1 = (-1, 0, -1, 0, 1) \quad v_2 = (-1, 1, 0, 0, 0)$$

$$\textcircled{3} A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix} \xrightarrow{\substack{-3R_1 + R_2 \\ -R_1 + R_2}} \begin{bmatrix} 1 & 3 & 4 & 3 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & -2 & | & 0 \end{bmatrix} \xrightarrow{R_3 / -2} \begin{bmatrix} 1 & 3 & 4 & 3 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \end{bmatrix}$$

Rank = 2, nullity = 2

$$x_1 + 3x_2 + 4x_3 + 3x_4 = 0$$

$$\boxed{x_4 = 0}$$

$$\boxed{x_3 = t}, \boxed{x_2 = r}$$

* $AX = 0$, can we consider the solution of H.S. vector space?

$\Rightarrow$ assume w, u solutions for H.S.

$$AX = 0$$

$$A(w+u) = 0 \Rightarrow Aw + Au = 0 \quad \begin{array}{l} \text{closed under addition} \\ \text{axiom 1} \end{array}$$

$$\begin{array}{c} \text{scalar} \\ \uparrow \\ A(Kw) = 0 \end{array} \Rightarrow A * 0 = 0 \quad \begin{array}{l} \text{closed under scalar multiplication} \\ \text{axiom 6} \end{array}$$

So we can consider the solution of H.S. as vector space.

* $AX = b$, can we consider the solutions of system of linear equ. as a vector space?

$\Rightarrow$ assume w, u solutions for the system.

$$Ax = b$$

$$A(w+u) \stackrel{?}{=} b$$

$$Aw + Au = 2b \Rightarrow 2b \neq b \quad \text{not closed under addition}$$

so can't be considered as a vector space.

* Find the rank and nullity of matrix A:

① $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Solutions: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow[-7R_1+R_3]{-4R_1+R_2} \begin{bmatrix} 1 & 2 & 3 & | & 0 \\ 0 & -3 & -6 & | & 0 \\ 0 & -6 & -12 & | & 0 \end{bmatrix} \xrightarrow[6R_2+R_3]{R_2/-3} \begin{bmatrix} 1 & 2 & 3 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

Rank = # of leaders (indep var) = 2

→ nullity = # of columns(A) - Rank = 1

or $\Rightarrow X_1 + 2X_2 + 3X_3 = 0$

$X_3 = t$

$X_2 + 2X_3 = 0$

$X_2 = -2t$, $X_1 = -2t - 3t \Rightarrow X_1 = -5t$

→ # of parameters = 1

② $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 4 & -1 & 2 \\ -1 & -2 & 5 & 4 \end{bmatrix}$

solution:

$\begin{bmatrix} 1 & 1 & 2 & 3 & | & 0 \\ 3 & 4 & -1 & 2 & | & 0 \\ -1 & -2 & 5 & 4 & | & 0 \end{bmatrix} \xrightarrow[R_1+R_3]{-3R_1+R_2, R_2+R_3} \begin{bmatrix} 1 & 1 & 2 & 3 & | & 0 \\ 0 & 1 & -7 & -7 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$

Rank = 2, nullity $\Rightarrow X_1 + X_2 + 2X_3 + 3X_4 = 0$ $X_1 = -9r - 10t$

$X_2 - 7X_3 - 7X_4 = 0 \Rightarrow X_4 = t, X_3 = r$

$X_2 = 7r + 7t$

of parameters = 2 = nullity