Special Topics in Manufacturing Slides With Notes

(1st Semester 2024/2025)
Notes are written by Nada Ababneh

Chapter 1
Introduction

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(مُشَوَّش) (Not clear)

Fuzzy

- The word “fuzzy” means “not sharp, unclear, imprecise, approximate”.
- Fuzziness occurs when the boundary of a piece of information is not clear-cut.
- Fuzzy vs vague (not specific, amorphous).
- Examples:

“see you in a few minutes” vs “see you soon”.

Fuzzy vs vague

(approximetly) مثلاً time limit يوجد

* example:

there is a quiz in 10 minutes

(Nothing Specific)

يعني وقت مش معروف

* example:

there is a quiz

Note:
* Part from the fuzziness comes from the probability

Fuzziness Reasoning

- Real world is too complex!
- Fuzziness introduced to obtain a reasonable model.
- Partial Success with 'quantitative' techniques.
- Expert knowledge' has become too important.
- The fuzziness concept can formulate knowledge in a systematic fashion and give it an engineering dimension. for example: (as a trend)

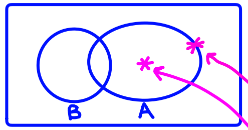
Something Exact — Something Uncertain

Crisp vs Fuzzy Set

Crisp (classical) set

- It is defined by crisp (exact) boundaries (i.e. no uncertainty about the location of the set boundaries).
- Either an element belongs to the set or it does not. Either an element belong or doesn't belong
- Used in digital system.
  as in computer either one or zero

* Example :-
$crisp = \{1, 2, 3, 4\}$

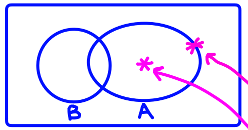


النقطتين زي بعض

Fuzzy set

- It is defined by ambiguous boundaries (i.e. uncertainty about the location of the set boundaries).
- It contains objects that satisfy imprecise properties of membership (degree).
- Used in fuzzy controllers.

* Example :-
$Fuzzy = \{ \frac{0.2}{1}, \frac{0.4}{2}, \frac{0.7}{3} \}$



النقطتين لا ينتمو بنفس الدرجة

Crisp vs Fuzzy Set

Example

- Crisp: Is water colourless? **Yes**

  No

- Fuzzy: Is he honest? **Extremely honest (0.3)**

  Very honest (0.4)

  Sometimes (0.2)

  Dishonest (0.1)

Some examples of fuzzy applications

- Fuzzy Washing Machine:

  ✓The first major consumer product to use ‘fuzzy systems’ (Matsushita Electric Industrial Company in Japan in 1991).

  ✓The fuzzy system included 3 main input variables (the extent of dirt, the dirt type and the load size), which were measured using optical sensors, and one output (choice of the correct cycle).

Some examples of fuzzy applications

- Fuzzy Control of Subway Train (Sendai Subway in Japan):

  ✓Two controllers: Constant speed controller (Starts the train and keeps speed below its safety limit) and Automatic stopping controller (Regulates the speed limit to stop at a target position).

  ✓Sample Fuzzy Rules:

  ➢For Safety

  IF the speed of the train is approaching the limit speed, THEN select the maximum brake notch.

  ➢For Riding Comfort

  IF the speed is in the allowed range, THEN do not change the control notch.

Chapter 2
Crisp (Classical) Sets and Fuzzy Sets

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Crisp Set: Definitions

- Universe of discourse (X): A collection of objects having the same characteristics.
- The elements (x) of a universe are either 1 discrete or 2 continuous.
- Cardinal number (n_x): The total number of elements in a universe.
- Discrete universes have a finite cardinal number, whereas continuous universes have an infinite cardinality.
- Set: Collections of elements within a **UNIVERSE**.
- Subset: Collections of elements within **SETS.**
- The whole set: collection of all elements in the set.

$\in \rightarrow$ belongs to

$\notin \rightarrow$ doesn't belong to

Crisp Set: Notations

- Suppose that A and B consist of collections of some elements in X, then

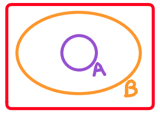
That means that A , B are sets

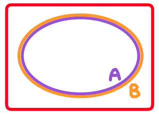
Elements (small letter)

$x \in X$	$\rightarrow$	x belongs to X
$x \in A$	$\rightarrow$	x belongs to A
$x \notin A$	$\rightarrow$	x does not belong to A

Continue…

- Also





$A \subset B$	$\rightarrow$	A is fully contained in B (if $x \in A$, then $x \in B$)
$A \subseteq B$	$\rightarrow$	A is contained in or is equivalent to B
$(A \leftrightarrow B)$	$\rightarrow$	$A \subseteq B$ and $B \subseteq A$ (A is equivalent to B)

يعني الـ elements (X) يلي ينتمو
إلى الـ Set (A)
راح ينتمو كمان لـ Set (B)

- Null set (Ø): The set that contains no elements.
- Power set (P(X)): All possible sets of X.

Continue…

- Example

- X= {1, 2, 3, 4}

Universe of discource

elements

- Cardinal number? (Total # of elements in the universe of discourse (X))

CN = 4

- Power set? (All possible sets of the universe of discource (X))

Power Set = {{1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4}, {2,3,4}, {1,3,4}, {1,2,3,4}, { }}

the set itself

Null set

- Cardinal number of the power set?

CN=16 (Total # of elements in the power set)

اظن ال (CN for power set)

عبارة عن مربع عدد ال elements

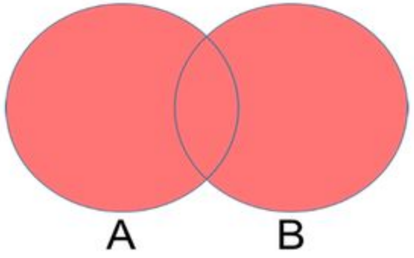
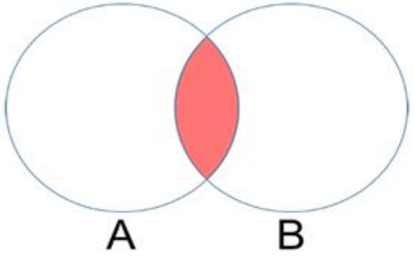
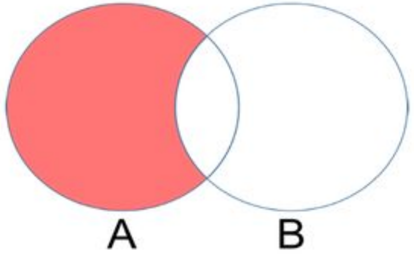
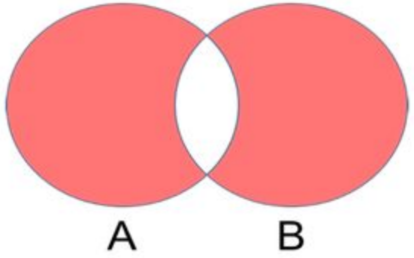
$4^2 = 16$

Crisp Sets: Operations

- Suppose that A and B two sets on the universe X, then

Union	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$
Intersection	$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$
Complement	$\overline{A} = \{x \mid x \notin A, x \in X\}.$
Difference	$A \mid B = \{x \mid x \in A \text{ and } x \notin B\}.$

- Such operations can be easily presented using Venn diagrams.

Set Operation	Venn Diagram	Interpretation
Union		$A \cup B$, is the set of all values that are a member of A, or B, or both.
Intersection		$A \cap B$, is the set of all values that are members of both A and B.
Difference		$A \setminus B$, is the set of all values of A that are not members of B
Symmetric Difference		$A \triangle B$, is the set of all values which are in one of the sets, but not both.

* Example :-

$X = \{ black, white, pink, Blue, Red, Green, \dots\dots\dots \}$

universe of discource

$A = \{ black, white, Red, Green \}$

set

$B = \{ black, Red, blue, pink \}$

Set

$A \cup B = \{ Black, white, Red, blue, green, pink \}$

union

$A \cap B = \{ Black, Red \}$

intersection

$\bar{A} = \{ Pink, Blue, \dots\dots\dots \}$

compliment

$\bar{B} = \{ White, Green, \dots\dots\dots \}$

compliment

الelements
يلي فـ A
وليس فـ B

$A/B = \{ White, Green \}$

difference

$B/A = \{ Blue, pink \}$

difference

Crisp Sets: Properties

التبادل

- **Commutativity:** $A \cup B = B \cup A$

  $A \cap B = B \cap A.$

الترابطية

- **Associativity:** $A \cup (B \cup C) = (A \cup B) \cup C$

  $A \cap (B \cap C) = (A \cap B) \cap C.$

التوزيع

- **Distributivity:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$

∪ اتحاد

∩ تقاطع

Continue…

الفاعلية

- **Idempotency:** $A \cup A = \boxed{A}$

  $A \cap A = \boxed{A}$

- **Identity:** $\boxed{A \cup \emptyset = A}$

  $\boxed{A \cap X = A}$

  $\boxed{A \cap \emptyset = \emptyset.}$

  $\boxed{A \cup X = X.}$

يعني زي كأنه

بال ∪ اتحاد ← بختار الاكبر

بال ∩ تقاطع ← بختار الاصغر

بين ϕ

A

X

Continue…

A is contained in or is equivalent to B

- Transitivity: If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

- Involution: $\overline{\overline{A}} = A$.

- Axiom of excluded middle: $A \cup \overline{A} = X$.

- Axiom of contradiction: $A \cap \overline{A} = \emptyset$.

ينطبقو فقط على ال (crisp)
و لا ينطبقو على ال (Fuzzy)

Continue…

- De Morgan's principles: The complement of a union or an intersection is equal to the intersection or union, respectively.

$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$

Note
for Fuzzy Sets
(not from the slides)

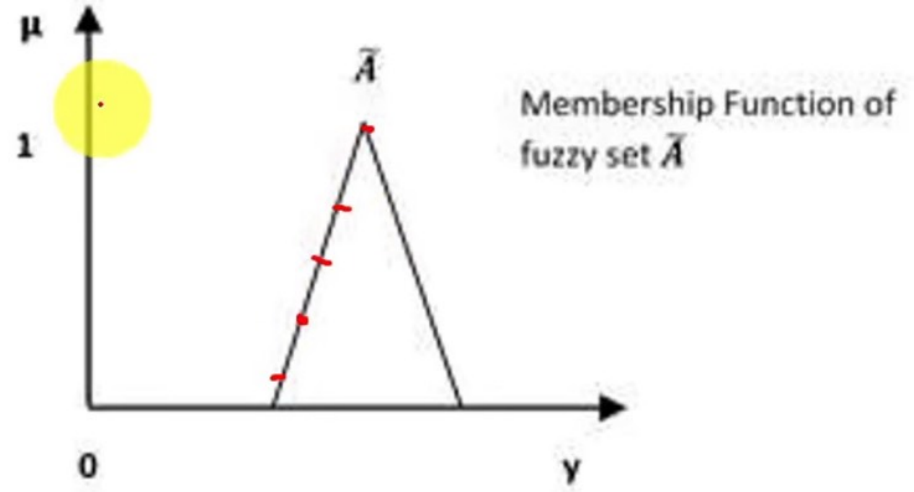
Membership Functions – Introduction

- Fuzzy sets may be viewed as an extension and generalization of the basic concepts of crisp sets.

$$A = \{1, 2, 3, 4\} \quad 5$$

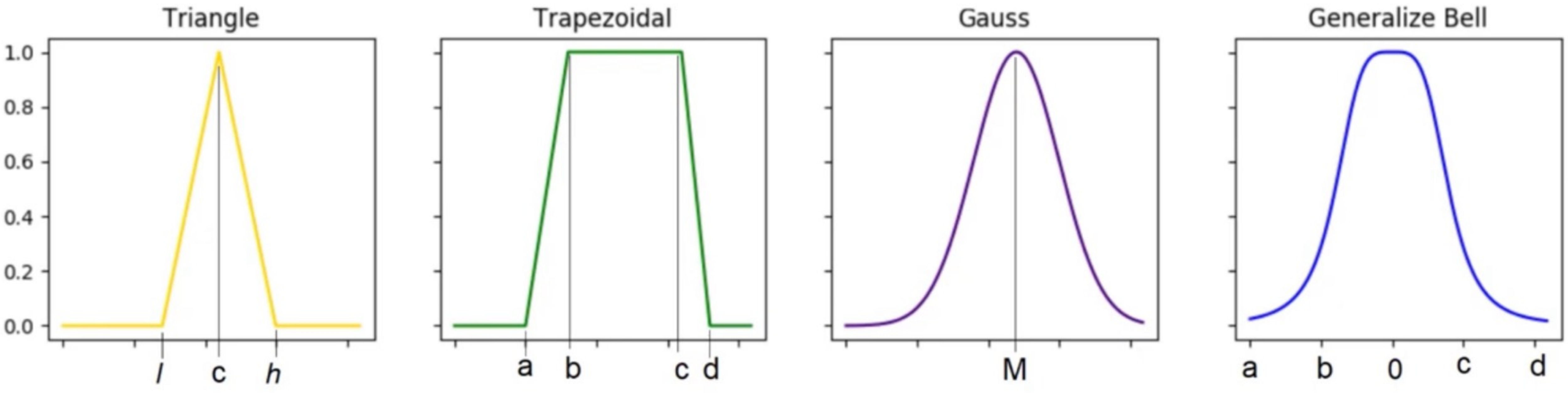
0.6 0.3

- An important property of fuzzy set is that it allows partial membership.
- A fuzzy set is a set having degrees of membership between 0 and 1.
- A **membership function (MF)** is a **curve** that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.



Membership Function

Type of membership functions



- Each of the above membership functions are also known as fuzzy sets
- We normally choose the type of membership function that suites our application

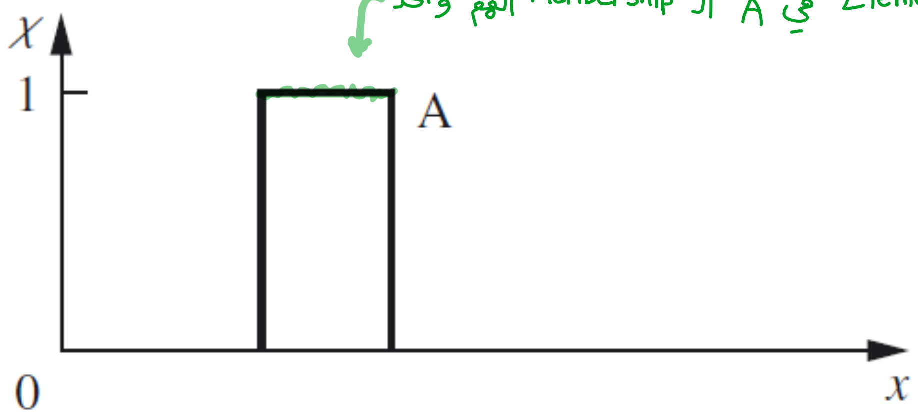
Mapping of Crisp Sets to Functions

- Relating set-theoretic forms to function-theoretic terms.
- Mapping elements (subsets) in an universe of discourse to elements (subsets) in another one.
- Membership function is a mapping for a crisp set:

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Membership Function for set A

* كل ال Elements في A ال Membership الهم واحد



Continue…

- Suppose that A and B are two sets on the universe X, then the function-theoretic terms:

Union $\mathrm{A} \cup \mathrm{B} \longrightarrow \chi_{\mathrm{A}\cup\mathrm{B}}(x) = \chi_{\mathrm{A}}(x) \vee \chi_{\mathrm{B}}(x) = \max(\chi_{\mathrm{A}}(x), \chi_{\mathrm{B}}(x)).$

Intersection $\mathrm{A} \cap \mathrm{B} \longrightarrow \chi_{\mathrm{A}\cap\mathrm{B}}(x) = \chi_{\mathrm{A}}(x) \wedge \chi_{\mathrm{B}}(x) = \min(\chi_{\mathrm{A}}(x), \chi_{\mathrm{B}}(x)).$

Complement $\overline{\mathrm{A}} \longrightarrow \chi_{\overline{\mathrm{A}}}(x) = 1 - \chi_{\mathrm{A}}(x).$

Containment $\mathrm{A} \subseteq \mathrm{B} \longrightarrow \chi_{\mathrm{A}}(x) \leq \chi_{\mathrm{B}}(x).$

* **example ①**

$A = \{orange, green, black\}$

$B = \{green, red\}$

✗ $\boxed{A \cup B}$

$M_A(orange) = 1$

$M_B(orange) = 0$ → ال Union تبعهم يعني Maximum → 1

Membership degree with respect to B

* **Note**:- ussually the compliment is (one - membership degree)

ويكون الجواب واحد أو صفر في ال (crisp)

* **example ②**

$A = \{orange, green, black\}$

$B = \{green\}$

✗ $\boxed{B \subseteq A}$

$M_B(green) \leq M_A(green)$

Note that crisp sets are a special case of fuzzy sets

لأن ال Membership degree في ال (crisp) بالعادة بكون zero / one

اما ال Membership degree في ال (Fuzzy) بالعادة بكون zero / any element in between / one

Fuzzy Sets

- Fuzzy sets contain elements that have varying degrees of membership.
- For discrete universe:

ليست بمعنى الجمع
(الافضل استبدالها بالفاصلة)

$$\underset{\sim}{A} = \left\{ \frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{x_2} + \cdots \right\} = \left\{ \sum_i \frac{\mu_{\underset{\sim}{A}}(x_i)}{x_i} \right\}$$

- For continuous universe:

$$\underset{\sim}{A} = \left\{ \int \frac{\mu_{\underset{\sim}{A}}(x)}{x} \right\}$$

معناها ليس integration

The summation and the integral signs are not algebraic symbols.

(X) universe of discourse is All of the colors

$$A = \left\{ \frac{0.8}{orange}, \frac{0.6}{green}, \frac{0.2}{black} \right\}$$

مثلاً مجموع ال Membership degrees في A

ما في اشي بجبره يكون يساوي واحد

$$B = \left\{ \frac{0.4}{green}, \frac{0.3}{red} \right\}$$

Union $\rightarrow (A \cup B) = \left\{ \frac{0.8}{orange}, \frac{0.6}{green}, \frac{0.3}{red}, \frac{0.2}{black} \right\}$

باخد ال Maximum

intersection $\rightarrow (A \cap B) = \left\{ \frac{0}{orange}, \frac{0.4}{green}, \frac{0}{red}, \frac{0}{black} \right\}$

باخد ال Minimum

عادي لو ما كتبناهم

complement $\rightarrow (\overline{A}) = \left\{ \frac{0.2}{orange}, \frac{0.4}{green}, \frac{0.8}{black}, \frac{1}{red}, \ldots\ldots\ldots \right\}$

لأن هنا ال (universe of discourse) عبارة عن كل الألوان

Fuzzy Sets: Operations

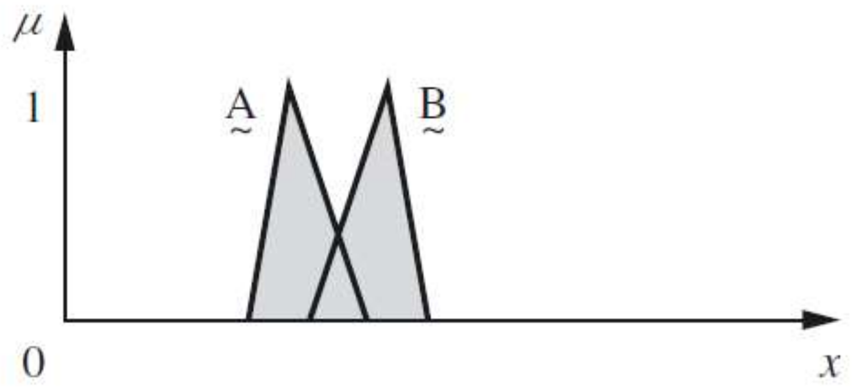
- Suppose that $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are two sets on the universe X, then the function-theoretic terms:

Union $$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x).$$

بيعطيني indication انها (Fuzzy set)

Intersection $$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x).$$

Complement $$\mu_{\overline{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x).$$



Continue…

The excluded middle Axioms

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} = X.$$
$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} = \emptyset.$$

applicable in Crisp only

- All the operations for classical sets are valid for fuzzy sets **EXCEPT for the excluded middle axioms.**
- The excluded middle axioms have been extended for fuzzy sets:

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X.$$
$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \emptyset.$$

in Fuzzy

- De Morgan's principles for crisp sets are valid for fuzzy sets.
- Fuzzy intersections and unions can be represented as t-norms and t- conorms, respectively.

intersection (∩)

Union (∪)

Fuzzy Sets: Question

* $\boxed{\text{Power Set}} \rightarrow$ All possible Sets of A

* $\boxed{\text{Cardinal number}} \rightarrow$ The number of elements in the set

For a collection of $\boxed{\text{fuzzy sets and subsets}}$ on a universe, what is:

- The fuzzy power set? ∞
- The cardinal number of the fuzzy power set? ∞

Fuzzy Sets: $\boxed{\text{Example}}$

اخدناه Quiz بالمحاضرة
(بيجي هيك بالامتحان)

- Suppose that we have two discrete fuzzy sets:

$$\left(\underset{\sim}{A} = \left\{\overset{\frac{0}{1}}{\frac{1}{2}} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\right\}\right) \text{ and } \left(\underset{\sim}{B} = \left\{\overset{\frac{0}{1}}{\frac{0.5}{2}} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\right\}\right)$$

- Note that the membership function of 1 is Zero. → $\frac{0}{1}$
- Calculate: Complement, union, intersection and difference.

الباقي للبسط حتى يوصل ١

ال Max بينهم

ال Min بينهم

Continue…

Complement

$$\bar{\underset{\sim}{A}} = \left\{\frac{1}{1} + \frac{0}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5}\right\}.$$

$$\bar{\underset{\sim}{B}} = \left\{\frac{1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}\right\}.$$

Union

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{\frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5}\right\}.$$

Intersection

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5}\right\}.$$

Difference

$$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \bar{\underset{\sim}{B}} = \left\{\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.3}{4} + \frac{0.2}{5}\right\}.$$

$$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \bar{\underset{\sim}{A}} = \left\{\frac{0}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5}\right\}.$$

Chapter 3
Crisp (Classical) Relations and Fuzzy Relations

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Fuzzy Relations

- $\boxed{\text{Fuzzy relations}}$ map elements of one universe to elements of another universe through the Cartesian product.
- The strength of the relation between ordered pairs of the two universes is measured with a membership function ($\mu_{\underset{\sim}{R}}(x, y)$).
- The cardinality of fuzzy sets is infinity, the cardinality of a fuzzy relation between two or more universes is also infinity.

* CN for Fuzzy Sets $\longrightarrow \infty$

* CN for Fuzzy Relations $\longrightarrow \infty$

$$\underset{\sim}{A} = \left\{ \frac{.8}{green}, \frac{.2}{red} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{.3}{1}, \frac{.2}{2}, \frac{.5}{3} \right\}$$

$$R = \begin{array}{c} \\ gr \\ red \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} .3 & .2 & .5 \\ .2 & .2 & .2 \end{bmatrix} \end{array}$$

(as a Relation)

بقارن ال Membership Function (البسط) بين ال Red و ال 1

و باخذ ال Minimum و هكذا

ممكن يمثل قوة العلاقة بين ال green and 1

membership degree that measure the strength between Red and 2

Fuzzy Relations: Operations

- Let R and S be fuzzy relations on the Cartesian space X×Y, then the following operations apply for the membership values:

Maximum membership degree

Union

$$\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x, y) = \max(\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)).$$

Relation R Relation S

Minimum membership degree

Intersection

$$\mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x, y) = \min(\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)).$$

Complement

$$\mu_{\overline{\underset{\sim}{R}}}(x, y) = 1 - \mu_{\underset{\sim}{R}}(x, y).$$

نادراً نستخدمها
وغالباً بتكون في ال comparison

Containment

$$\underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x, y) \leq \mu_{\underset{\sim}{S}}(x, y).$$

Fuzzy Relations: Properties

- As is the case in crisp relations, the properties of commutativity, associativity, distributivity, involution and idempotency are applicable for fuzzy relations.
- De Morgan's principles are applicable for fuzzy relations.
- Fuzzy relations are not constrained by the excluded middle axioms:

$$\underset{\sim}{R} \cup \overline{\underset{\sim}{R}} \neq \mathbf{E}.$$

اتحاد

$$\underset{\sim}{R} \cap \overline{\underset{\sim}{R}} \neq \mathbf{O}.$$

تقاطع

$$\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

✗ Remember :-

Cartesian Product

A \ B	a	b	c
1	(1,a)	(1,b)	(1,c)
2	(2,a)	(2,b)	(2,c)

A x B

Fuzzy Relations: Cartesian Product

- Suppose A and B are fuzzy sets on universes X and Y, respectively, then the Cartesian product is presented as follows:

$$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R} \subset X \times Y,$$

- The fuzzy relation $\underset{\sim}{R}$ has the following membership function:

$$\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x, y) = \min(\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)).$$

$X_1 \quad X_2 \quad X_3 \longrightarrow$ ممكن يكونو
الوان او اي اشي

Example

- Suppose that

$$\boxed{\underset{\sim}{A} = \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{1}{x_3}} \text{ and } \boxed{\underset{\sim}{B} = \frac{0.3}{y_1} + \frac{0.9}{y_2}.}$$

- $\underset{\sim}{A} \otimes \underset{\sim}{B} = ?$

Cartesian Product

(يعني بدي ال Minimum)

the Solution :-

$$\begin{array}{c|c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.5 \\ 0.3 & 0.9 \end{bmatrix} \end{array}$$

بقارن X_1 و Y_1
وباخذ ال Minimum
و هكذا

وعادي لو حليناها هكذا:

$$\begin{array}{cc} & \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \\ \begin{array}{c} Y_1 \\ Y_2 \end{array} & \begin{bmatrix} & & \\ & & \end{bmatrix} \end{array}$$

Fuzzy Relations: Composition

$X \rightarrow Y \qquad Y \rightarrow Z$

R S

(we use it when we don't have a direct Relation)

- Suppose that $\underset{\sim}{R}$ and $\underset{\sim}{S}$ are fuzzy relations on the Cartesian space (X×Y) and (Y×Z), respectively, and $\underset{\sim}{T}$ is a fuzzy relation on (X×Z), then
- max–min composition can be defined as follows:

composition

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S},$$

$$\mu_{\underset{\sim}{T}}(x, z) = \bigvee_{y \in Y} \left(\mu_{\underset{\sim}{R}}(x, y) \wedge \mu_{\underset{\sim}{S}}(y, z)\right),$$

Max-Min Composition

$X \to Y \quad Y \to Z$

R S

$$\underset{\sim}{R} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & .2 & .2 \\ x_2 & .3 & .5 \\ x_3 & .3 & .9 \end{array}$$

Min = 0.1

Min = 0.2

then Max = 0.2

$$\underset{\sim}{S} = \begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline y_1 & .1 & .2 & .3 & .4 \\ y_2 & .2 & .3 & .4 & .5 \end{array}$$

Two Methods → Maximum, Minimum

$\min(\mu_{x_1,y_1}, \mu_{y_1,z_1}) = .1$

$\min(\mu_{x_1,y_2}, \mu_{y_2,z_1}) = .2$

Max = 0.2

* The Solution :-

$T = R \circ S = \begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline x_1 & .2 & .2 & .2 & .2 \\ x_2 & .2 & .3 & .4 & .5 \\ x_3 & .2 & .3 & .4 & .5 \end{array}$

(x, z)

Continue…

- max–product composition can be defined as follows:

$$\mu_{\underset{\sim}{T}}(x, z) = \bigvee_{y \in Y} (\mu_{\underset{\sim}{R}}(x, y) \bullet \mu_{\underset{\sim}{S}}(y, z)).$$

- Note that fuzzy composition is NOT commutative:

$$\underset{\sim}{R} \circ \underset{\sim}{S} \neq \underset{\sim}{S} \circ \underset{\sim}{R}.$$

Max-product Composition

(بنضرب)

$$\underset{\sim}{R} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & .2 & .2 \\ x_2 & .3 & .5 \\ x_3 & .3 & .9 \end{array}$$

$$\underset{\sim}{S} = \begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline y_1 & .1 & .2 & .3 & .4 \\ y_2 & .2 & .3 & .4 & .5 \end{array}$$

توضيح

$(.02, .04)$

$\Downarrow$ maximum

0.4

The Solution :-

$$T = R \circ S = \begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline x_1 & .04 & .06 & .08 & 0.1 \\ x_2 & 0.1 & 0.15 & 0.2 & 0.25 \\ x_3 & 0.18 & 0.27 & 0.36 & 0.45 \end{array}$$

(x, z)

composition between R and S

Example

- Suppose that $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, and $Z = \{z_1, z_2, z_3\}$
- The fuzzy relations are as follows:

$$\underset{\sim}{R} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{cc} y_1 & y_2 \\ \left[\begin{matrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{matrix}\right] \end{array} \quad \text{and} \quad \underset{\sim}{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{ccc} z_1 & z_2 & z_3 \\ \left[\begin{matrix} 0.9 & 0.6 & 0.2 \\ 0.1 & 0.7 & 0.5 \end{matrix}\right] \end{array}$$

يعني Minimum ثم ال Maximum

- Using the max–min composition and max-product composition find the relation that relates the elements of universe X to the elements of universe Z.

$$\begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{ccc} z_1 & z_2 & z_3 \\ \left[\begin{matrix} .7 & .6 & .5 \\ .8 & .6 & .4 \end{matrix}\right] \end{array}$$

Example

(The Solution)

- Suppose that $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, and $Z = \{z_1, z_2, z_3\}$
- The fuzzy relations are as follows:

$$\underset{\sim}{R} = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \quad \text{and} \quad \underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 0.9 & 0.6 & 0.2 \\ 0.1 & 0.7 & 0.5 \end{bmatrix} \end{array}.$$

- by Max-Min composition

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.7 & 0.6 & 0.5 \\ 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

- by Max product Composition

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.63 & 0.42 & 0.25 \\ 0.72 & 0.48 & 0.20 \end{bmatrix} \end{array}$$

Example(3.7) page(58)

Example 3.7. A certain type of virus attacks cells of the human body. The infected cells can be visualized using a special microscope. The microscope generates digital images that medical doctors can analyze and identify the infected cells. The virus causes the infected cells to have a black spot, within a darker gray region (Figure 3.6).

A digital image process can be applied to the image. This processing generates two variables: the first variable, P, is related to black spot quantity (black pixels) and the second variable, S, is related to the shape of the black spot, that is, if they are circular or elliptic. In these images, it is often difficult to actually count the number of black pixels, or to identify a perfect circular cluster of pixels; hence, both these variables must be estimated in a linguistic way.

Suppose that we have two fuzzy sets: $\underset{\sim}{P}$ that represents the number of black pixels (e.g., none with black pixels, C_1, a few with black pixels, C_2, and a lot of black pixels, C_3) and $\underset{\sim}{S}$ that represents the shape of the black pixel clusters (e.g., S_1 is an ellipse and S_2 is a circle). So, we have

بال Relation بقارن و باخذ ال Min

$$\underset{\sim}{P} = \left\{ \frac{0.1}{C_1} + \frac{0.5}{C_2} + \frac{1.0}{C_3} \right\} \quad \text{and} \quad \underset{\sim}{S} = \left\{ \frac{0.3}{S_1} + \frac{0.8}{S_2} \right\},$$

※of black spot; none; few; a lot; shape of black spot; ellipse; circle

and we want to find the relationship between quantity of black pixels in the virus and the shape of the black pixel clusters. Using a Cartesian product between $\underset{\sim}{P}$ and $\underset{\sim}{S}$ gives

$$\underset{\sim}{R} = \underset{\sim}{P} \times \underset{\sim}{S} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{cc} S_1 & S_2 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.5 \\ 0.3 & 0.8 \end{bmatrix} \end{array}.$$

Relation (cartesian product)

Now, suppose another microscope image is taken and the number of black pixels is slightly different; let the new black pixel quantity be represented by a fuzzy set, $\underset{\sim}{P}'$:

$$\underset{\sim}{P}' = \left\{ \frac{0.4}{C_1} + \frac{0.7}{C_2} + \frac{1.0}{C_3} \right\}.$$

p prime (new ※ of black spots)

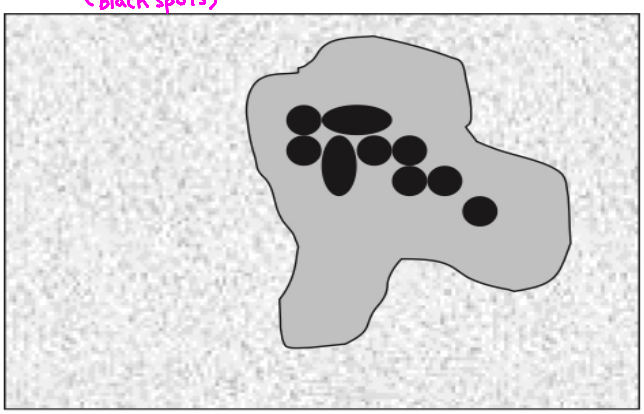

FIGURE 3.6
An infected cell shows black spots with different shapes in a micrograph.

Using max–min composition with the relation $\underset{\sim}{R}$ will yield a new value for the fuzzy set of pixel cluster shapes that are associated with the new black pixel quantity:

الهدف ايجاد S′
S prime

$$\underset{\sim}{S}' = \underset{\sim}{P}' \circ \underset{\sim}{R} = [0.4 \quad 0.7 \quad 1.0] \circ \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.5 \\ 0.3 & 0.8 \end{bmatrix} = [0.3 \quad 0.8]. \Longrightarrow S' = \left\{ \frac{0.3}{S_1}, \frac{0.8}{S_2} \right\}$$

بقارن كل two values و باخذ ال Min بينهم. $\begin{pmatrix} 0.1 \\ 0.3 \\ 0.3 \end{pmatrix}$ ثم نأخذ ال Max بينهم (0.3)

Example(3.8) page(59)

Self Study →

Example 3.8. Suppose we are interested in understanding the speed control of the DC (direct current) shunt motor under no-load condition, as shown diagrammatically in Figure 3.7. Initially, the series resistance R_{se} in Figure 3.7 should be kept in the cut-in position for the following reasons:

1. The back electromagnetic force, given by $E_b = kN\phi$, where k is a constant of proportionality, N is the motor speed, and ϕ is the flux (which is proportional to input voltage, V), is equal to zero because the motor speed is equal to zero initially.
2. We have $V = E_b + I_a(R_a + R_{se})$, therefore $I_a = (V - E_b)/(R_a + R_{se})$, where I_a is the armature current and R_a is the armature resistance. Since E_b is equal to zero initially, the armature current will be $I_a = V/(R_a + R_{se})$, which is going to be quite large initially and may destroy the armature.

On the basis of both cases 1 and 2, keeping the series resistance R_{se} in the cut-in position will restrict the speed to a very low value. Hence, if the rated no-load speed of the motor is 1500 rpm, then the resistance in series with the armature, or the shunt resistance R_{sh}, has to be varied.

Two methods provide this type of control: armature control and field control. For example, in armature control, suppose that ϕ (flux) is maintained at some constant value, then motor speed N is proportional to E_b.

If R_{se} is decreased step by step from its high value, I_a (armature current) increases. Hence, this method increases I_a. On the other hand, as I_a is increased the motor speed N increases. These two possible approaches to control could have been done manually or automatically. Either way, however, results in at least two problems, presuming we do not want to change the design of the armature:

What should be the minimum and maximum level of R_{se}?
What should be the minimum and maximum value of I_a?

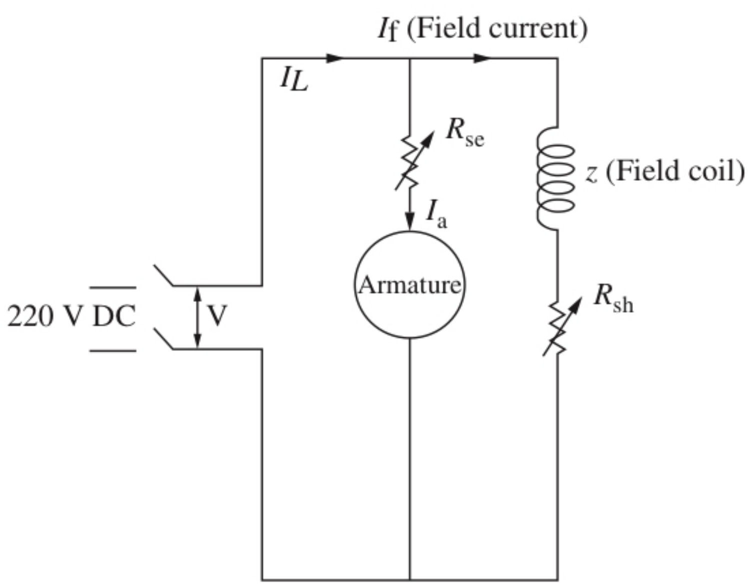

FIGURE 3.7
A DC shunt motor system.

Now let us suppose that load on the motor is taken into consideration. Then the problem of control becomes twofold. First, owing to fluctuations in the load, the armature current may change, resulting in change in the motor speed. Second, as a result of changes in speed, the armature resistance control must be accomplished in order to maintain the motor's rated speed. Such control issues become very important in applications involving electric trains and a large number of consumer appliances making use of small batteries to run their motors.

We wish to use concepts of fuzzy sets to address this problem. Let $\underset{\sim}{\mathrm{R}}_{\mathrm{se}}$ be a fuzzy set representing a number of possible values for series resistance, say s_n values, given as

$$\underset{\sim}{\mathrm{R}}_{\mathrm{se}} = \{R_{s_1}, R_{s_2}, R_{s_3}, \ldots, R_{s_n}\},$$

and let $\underset{\sim}{\mathrm{I}}_{\mathrm{a}}$ be a fuzzy set having a number of possible values of the armature current, say m values, given as

$$\underset{\sim}{\mathrm{I}}_{\mathrm{a}} = \{I_1, I_2, I_3, \ldots, I_m\}.$$

The fuzzy sets $\underset{\sim}{\mathrm{R}}_{\mathrm{se}}$ and $\underset{\sim}{\mathrm{I}}_{\mathrm{a}}$ can be related through a fuzzy relation, say $\underset{\sim}{\mathrm{R}}$, which would allow for the establishment of various degrees of relationship between pairs of resistance and current. In this way, the resistance–current pairings could conform to the modeler's intuition about the trade-offs involved in control of the armature.

Let $\underset{\sim}{\mathrm{N}}$ be another fuzzy set having numerous values for the motor speed, say v values, given as

$$\underset{\sim}{\mathrm{N}} = \{N_1, N_2, N_3, \ldots, N_v\}.$$

Now, we can determine another fuzzy relation, say $\underset{\sim}{\mathrm{S}}$, to relate current to motor speed, that is, $\underset{\sim}{\mathrm{I}}_{\mathrm{a}}$ to $\underset{\sim}{\mathrm{N}}$.

Using the operation of composition, we could then compute a relation, say $\underset{\sim}{\mathrm{T}}$, to be used to relate series resistance to motor speed, that is, $\underset{\sim}{\mathrm{R}}_{\mathrm{se}}$ to $\underset{\sim}{\mathrm{N}}$. The operations needed to develop these relations are as follows – two fuzzy Cartesian products and one composition:

$$\underset{\sim}{\mathrm{R}} = \underset{\sim}{\mathrm{R}}_{\mathrm{se}} \times \underset{\sim}{\mathrm{I}}_{\mathrm{a}},$$
$$\underset{\sim}{\mathrm{S}} = \underset{\sim}{\mathrm{I}}_{\mathrm{a}} \times \underset{\sim}{\mathrm{N}},$$
$$\underset{\sim}{\mathrm{T}} = \underset{\sim}{\mathrm{R}} \circ \underset{\sim}{\mathrm{S}}.$$

Suppose the membership functions for both series resistance $\underset{\sim}{\mathrm{R}}_{\mathrm{se}}$ and armature current $\underset{\sim}{\mathrm{I}}_{\mathrm{a}}$ are given in terms of percentages *of their respective rated values*, that is,

$$\mu_{\mathrm{R_{se}}}(\%\mathrm{se}) = \frac{0.3}{30} + \frac{0.7}{60} + \frac{1.0}{100} + \frac{0.2}{120}$$

and

$$\mu_{\mathrm{I_a}}(\%\mathrm{a}) = \frac{0.2}{20} + \frac{0.4}{40} + \frac{0.6}{60} + \frac{0.8}{80} + \frac{1.0}{100} + \frac{0.1}{120},$$

and the membership value for $\underset{\sim}{\mathrm{N}}$ is given in units of motor speed in rpm,

$$\mu_{\mathrm{N}}(\mathrm{rpm}) = \frac{0.33}{500} + \frac{0.67}{1000} + \frac{1.0}{1500} + \frac{0.15}{1800}.$$

The following relations then result from use of the Cartesian product to determine $\underset{\sim}{R}$ and $\underset{\sim}{S}$:

$$\underset{\sim}{R} = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccccc} 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.3 & 0.3 & 0.3 & 0.1 \\ 0.2 & 0.4 & 0.6 & 0.7 & 0.7 & 0.1 \\ 0.2 & 0.4 & 0.6 & 0.8 & 1 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.1 \end{bmatrix} \end{array}$$

and

$$\underset{\sim}{S} = \begin{array}{c} \\ 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.15 \\ 0.33 & 0.4 & 0.4 & 0.15 \\ 0.33 & 0.6 & 0.6 & 0.15 \\ 0.33 & 0.67 & 0.8 & 0.15 \\ 0.33 & 0.67 & 1 & 0.15 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}.$$

For example, $\mu_{\underset{\sim}{R}}(60, 40) = \min(0.7, 0.4) = 0.4$, $\mu_{\underset{\sim}{R}}(100, 80) = \min(1.0, 0.8) = 0.8$, and $\mu_{\underset{\sim}{S}}(80, 1000) = \min(0.8, 0.67) = 0.67$.

The following relation results from a max–min composition for $\underset{\sim}{T}$:

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.3 & 0.3 & 0.3 & 0.15 \\ 0.33 & 0.67 & 0.7 & 0.15 \\ 0.33 & 0.67 & 1 & 0.15 \\ 0.2 & 0.2 & 0.2 & 0.15 \end{bmatrix} \end{array}.$$

For instance,

$$\begin{aligned} \mu_{\underset{\sim}{T}}(60, 1500) &= \max[\min(0.2, 0.2), \min(0.4, 0.4), \min(0.6, 0.6), \\ &\quad \min(0.7, 0.8), \min(0.7, 1.0), \min(0.1, 0.1)]. \\ &= \max[0.2, 0.4, 0.6, 0.7, 0.7, 0.1] = 0.7. \end{aligned}$$

Crisp Relations: Tolerance and Equivalence Relation

- A relation is considered as an $\boxed{\text{equivalence relation}}$ if it has the following three properties:

① *Reflexivity* $(x_i, x_i) \in \mathrm{R}$ or $\chi_{\mathrm{R}}(x_i, x_i) = 1.$

② *Symmetry* $(x_i, x_j) \in \mathrm{R} \longrightarrow (x_j, x_i) \in \mathrm{R}$

or $\chi_{\mathrm{R}}(x_i, x_j) = \chi_{\mathrm{R}}(x_j, x_i).$ ← membership degree متشابهة

③ *Transitivity* $(x_i, x_j) \in \mathrm{R}$ and $(x_j, x_k) \in \mathrm{R} \longrightarrow (x_i, x_k) \in \mathrm{R}$

3 elements

or $\chi_{\mathrm{R}}(x_i, x_j)$ and $\chi_{\mathrm{R}}(x_j, x_k) = 1 \longrightarrow \chi_{\mathrm{R}}(x_i, x_k) = 1.$

Continue…

- A tolerance (proximity) relation: A relation that exhibits only the properties of ① reflexivity and ② symmetry.

Transitivity ✗
لا تنطبق

- It can be reformed into an equivalence relation by AT MOST (CN−1) compositions with itself, as follows:

cardinal number

$$R_1^{n-1} = R_1 \circ R_1 \circ \cdots \circ R_1 = R$$

Fuzzy Relations: Tolerance and Equivalence Relation

- A fuzzy relation is considered as a fuzzy equivalence relation if it has the following properties:

Reflexivity $\mu_{\underset{\sim}{R}}(x_i, x_i) = 1.$

Symmetry $\mu_{\underset{\sim}{R}}(x_i, x_j) = \mu_{\underset{\sim}{R}}(x_j, x_i).$

in a matrix the upper part = Lower part

Transitivity $\mu_{\underset{\sim}{R}}(x_i, x_j) = \lambda_1$ and $\mu_{\underset{\sim}{R}}(x_j, x_k) = \lambda_2 \longrightarrow \mu_{\underset{\sim}{R}}(x_i, x_k) = \lambda,$

where $\lambda \geq \min[\lambda_1, \lambda_2]$.

عشان اضمن انها اكبر
من اقل وحدة فيهم

Continue…

- A fuzzy tolerance (proximity) relation: A fuzzy relation that exhibits only the properties of reflexivity and symmetry.

Transitivity ✗
لا تنطبق

- It can be reformed into an equivalence fuzzy relation by AT MOST (CN−1) compositions with itself, as follows:

$$\underset{\sim}{R}_1^{n-1} = \underset{\sim}{R}_1 \circ \underset{\sim}{R}_1 \circ \cdots \circ \underset{\sim}{R}_1 = \underset{\sim}{R}$$

Example (3.11)

- A fuzzy relation is as follows:

$$\underset{\sim}{R}_1 = \begin{bmatrix} 1 & 0.8 & 0 & 0.1 & 0.2 \\ 0.8 & 1 & 0.4 & 0 & 0.9 \\ 0 & 0.4 & 1 & 0 & 0 \\ 0.1 & 0 & 0 & 1 & 0.5 \\ 0.2 & 0.9 & 0 & 0.5 & 1 \end{bmatrix}$$

- Is it reflexive, symmetric, transitive?

The Solution :-

where $\lambda \geq \min[\lambda_1, \lambda_2]$.

is reflexive and symmetric. However, it is not transitive, for example,

Main Diagonal is one

in the matrix

upper part = Lower part

$$\mu_{\underset{\sim}{R}}(x_1, x_2) = 0.8, \quad \mu_{\underset{\sim}{R}}(x_2, x_5) = 0.9 \geq 0.8,$$

but

$$\mu_{\underset{\sim}{R}}(x_1, x_5) = 0.2 \leq \min(0.8, 0.9).$$

One composition results in the following relation:

$$\underset{\sim}{R}_1^2 = \underset{\sim}{R}_1 \circ \underset{\sim}{R}_1 = \begin{array}{c} \\ 1 \\ 2 \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} \quad & \;2 & \quad & \quad & \;5 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0.4 & 0.2 & 0.8 \\ 0.8 & 1 & 0.4 & 0.5 & 0.9 \\ 0.4 & 0.4 & 1 & 0 & 0.4 \\ 0.2 & 0.5 & 0 & 1 & 0.5 \\ 0.8 & 0.9 & 0.4 & 0.5 & 1 \end{bmatrix} \end{array},$$

where transitivity still does not result; for example,

لازم كل ال possibilities يتحقق

$$\mu_{\underset{\sim}{R}^2}(x_1, x_2) = 0.8 \geq 0.5 \quad \text{and} \quad \mu_{\underset{\sim}{R}^2}(x_2, x_4) = 0.5,$$

but

$$\mu_{\underset{\sim}{R}^2}(x_1, x_4) = 0.2 \leq \min(0.8, 0.5).$$

Finally, after one or two more compositions, transitivity results:

$$\underset{\sim}{R}_1^3 = \underset{\sim}{R}_1^4 = \underset{\sim}{R} = \begin{bmatrix} 1 & 0.8 & 0.4 & 0.5 & 0.8 \\ 0.8 & 1 & 0.4 & 0.5 & 0.9 \\ 0.4 & 0.4 & 1 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.4 & 1 & 0.5 \\ 0.8 & 0.9 & 0.4 & 0.5 & 1 \end{bmatrix}$$

$$\underset{\sim}{R}_1^3(x_1, x_2) = 0.8 \geq 0.5.$$

$$\underset{\sim}{R}_1^3(x_2, x_4) = 0.5 \geq 0.5.$$

$$\underset{\sim}{R}_1^3(x_1, x_4) = 0.5 \geq 0.5.$$

*Remember :-

Fuzzy :-

$$\begin{bmatrix} .7 & .5 \\ .8 & .4 \end{bmatrix}$$

crisp :-

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Value Assignment Methods

- Cartesian product,
- Closed-form expression,
- Lookup table,
- Linguistic rules of knowledge,
- Classification,
- Automated methods from input/output data,
- Similarity methods in data manipulation.

Cosine Amplitude Method (Similarity Method)

- A similarity metric that uses a collection of data samples.
- It can be presented as follows:

$$r_{ij} = \frac{\left|\sum_{k=1}^{m} x_{ik}x_{jk}\right|}{\sqrt{\left(\sum_{k=1}^{m} x_{ik}^2\right)\left(\sum_{k=1}^{m} x_{jk}^2\right)}},$$

where $i, j = 1, 2, \ldots, n$.

Max-Min Method

- A similarity metric that uses a collection of data samples. It is computationally simpler than the cosine amplitude method.
- It can be presented as follows:

$$r_{ij} = \frac{\sum_{k=1}^{m} \min(x_{ik}, x_{jk})}{\sum_{k=1}^{m} \max(x_{ik}, x_{jk})}, \quad \text{where } i, j = 1, 2, \ldots, n$$

Example (3.12) page (71)

Five separate regions along the San Andreas fault in California have suffered damage from a recent earthquake. For purposes of assessing payouts from insurance companies to building owners, the five regions must be classified as to their damage levels. Expression of the damage in terms of relations will prove helpful. Surveys are conducted of the buildings in each region. All the buildings in each region are described as being in one of three damage states: no damage, medium damage, and serious damage. Each region has each of these three damage states expressed as a percentage (ratio) of the total number of buildings. The following table summarizes the findings of the survey team:

مستحيل الجواب يكون أكبر من واحد

لأنه $\frac{\text{Minimum}}{\text{Maximum}}$

Continue…

Regions	x_1	x_2	x_3	x_4	x_5
x_{i1} – Ratio with no damage	0.3	0.2	0.1	0.7	0.4
x_{i2} – Ratio with medium damage	0.6	0.4	0.6	0.2	0.6
x_{i3} – Ratio with serious damage	0.1	0.4	0.3	0.1	0.0

By using the cosine amplitude and max-min methods, express these data as a fuzzy relation.

Continue…

Cosine amplitude method:

$$\underset{\sim}{R}_1 = \begin{bmatrix} 1 & & & \text{sym} & \\ 0.836 & 1 & & & \\ 0.914 & 0.934 & 1 & & \\ 0.682 & 0.6 & 0.441 & 1 & \\ 0.982 & 0.74 & 0.818 & 0.774 & 1 \end{bmatrix}$$

Max-min method:

$$\underset{\sim}{R}_1 = \begin{bmatrix} 1 & & & \text{sym} & \\ 0.538 & 1 & & & \\ 0.667 & 0.667 & 1 & & \\ 0.429 & 0.333 & 0.250 & 1 & \\ 0.818 & 0.429 & 0.538 & 0.429 & 1 \end{bmatrix}$$

Cosine Amplitude Method

ماني (Strength of Relation) (اكبر من واحد)

Regions	x_1	x_2	x_3	x_4	x_5
x_{i1} – Ratio with no damage	0.3	0.2	0.1	0.7	0.4
x_{i2} – Ratio with medium damage	0.6	0.4	0.6	0.2	0.6
x_{i3} – Ratio with serious damage	0.1	0.4	0.3	0.1	0.0

ايجاد المناطق التي تضررت أكثر وهكذا

$$
R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\
\begin{bmatrix} 1 & & & & \\ 0.836 & 1 & & \text{symmetric} & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \end{array}
$$

$$
r_{ij} = \frac{\left| \sum_{k=1}^{m} x_{ik} x_{jk} \right|}{\sqrt{\left(\sum_{k=1}^{m} x_{ik}^2 \right) \left(\sum_{k=1}^{m} x_{jk}^2 \right)}}
$$

$$
r_{12} = \frac{\left| (.3 \times .2) + (.6 \times .4) + (.1 \times .4) \right|}{\sqrt{(.3^2 + .6^2 + .1^2) \times (.2^2 + .4^2 + .4^2)}} = \frac{0.34}{\sqrt{0.46 \times 0.36}} = 0.836
$$

$$
r_{13} = \frac{\left| (.3 \times .1) + (.6 \times .6) + (.1 \times .3) \right|}{\sqrt{(.3^2 + .6^2 + .1^2) \times (.1^2 + .6^2 + .3)^2}} = 0.914
$$

★ The Final Result :-

$$
\underset{\sim}{R_1} = \begin{bmatrix} 1 & & & & \\ 0.836 & 1 & & \text{sym} & \\ 0.914 & 0.934 & 1 & & \\ 0.682 & 0.6 & 0.441 & 1 & \\ 0.982 & 0.74 & 0.818 & 0.774 & 1 \end{bmatrix}
$$

Max-Min Method

Regions	x_1	x_2	x_3	x_4	x_5
x_{i1} – Ratio with no damage	0.3	0.2	0.1	0.7	0.4
x_{i2} – Ratio with medium damage	0.6	0.4	0.6	0.2	0.6
x_{i3} – Ratio with serious damage	0.1	0.4	0.3	0.1	0.0

$$R = \begin{array}{c|ccccc} & X_1 & X_2 & X_3 & X_4 & X_5 \\ \hline X_1 & 1 & & & & \\ X_2 & 0.5385 & 1 & & & \\ X_3 & 0.667 & 0.667 & 1 & & \\ X_4 & & & & 1 & \\ X_5 & & & & & 1 \end{array}$$

$$r_{ij} = \frac{\sum_{K=1}^{m} \min(X_{iK}, X_{jK})}{\sum_{K=1}^{m} \max(X_{iK}, X_{jK})}$$

$$r_{12} = \frac{0.2+0.4+0.1}{0.3+0.6+0.4} = 0.5385$$

$$r_{13} = \frac{0.1+0.6+0.1}{0.3+0.6+0.3} = 0.667$$

$$r_{23} = \frac{0.1+0.4+0.3}{0.2+0.6+0.4} = 0.667$$

★ The Final Result :-

$$\underset{\sim}{R}_1 = \begin{bmatrix} 1 & & & & \\ 0.538 & 1 & & \text{sym} & \\ 0.667 & 0.667 & 1 & & \\ 0.429 & 0.333 & 0.250 & 1 & \\ 0.818 & 0.429 & 0.538 & 0.429 & 1 \end{bmatrix}$$

Chapter 4
Properties of Membership Functions, Fuzzification and Defuzzification

Chapter 4, 5, 6
talks about Fuzzy Systems

FS → fuzzy System
FIS → Fuzzy inference system

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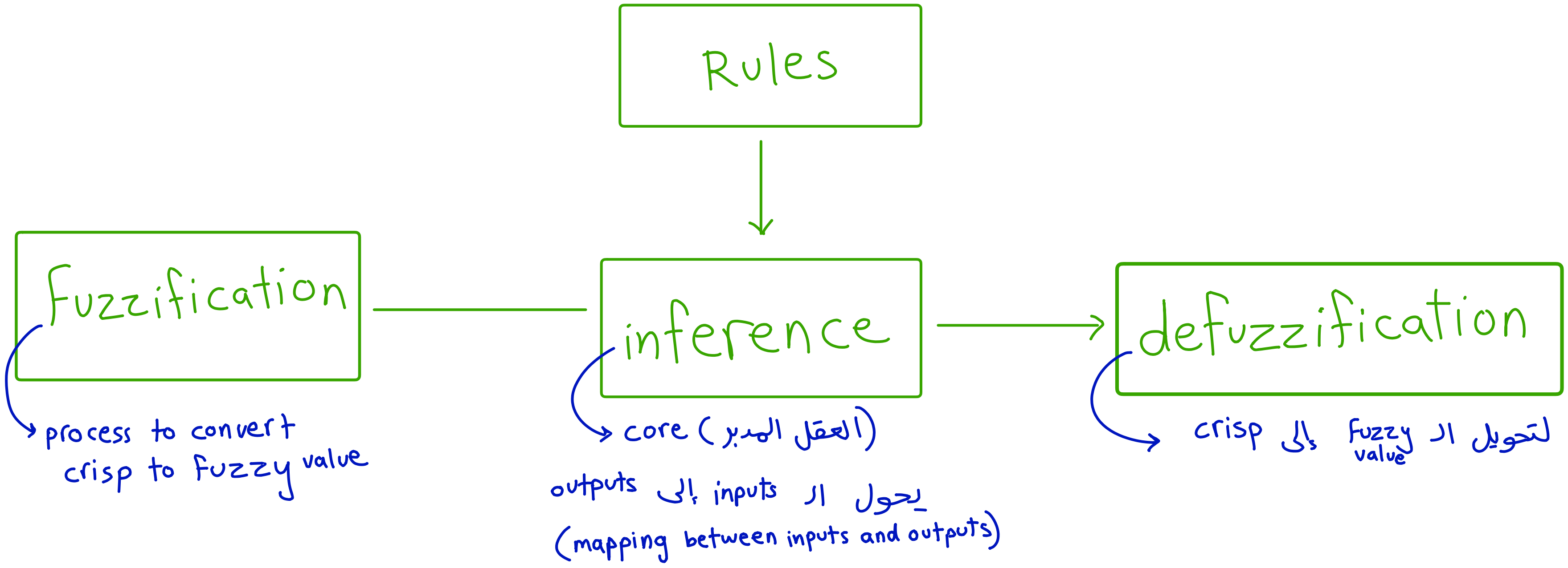
Blocks of fuzzy



Membership Function: Definition

Membership degree → مدى انتماء اشي لإشي

- A membership function: The values assigned to elements of a universal set fall within a specified range.
- It describes the information contained in a fuzzy set.
- Larger values indicate higher degrees of membership.

كل ما كانت ال values أكبر
كل ما اعطتني membership degree أعلى

- The core: The region (element) of a universe that is characterized by full membership in a set.

أكثر من element لهم membership degree واحد

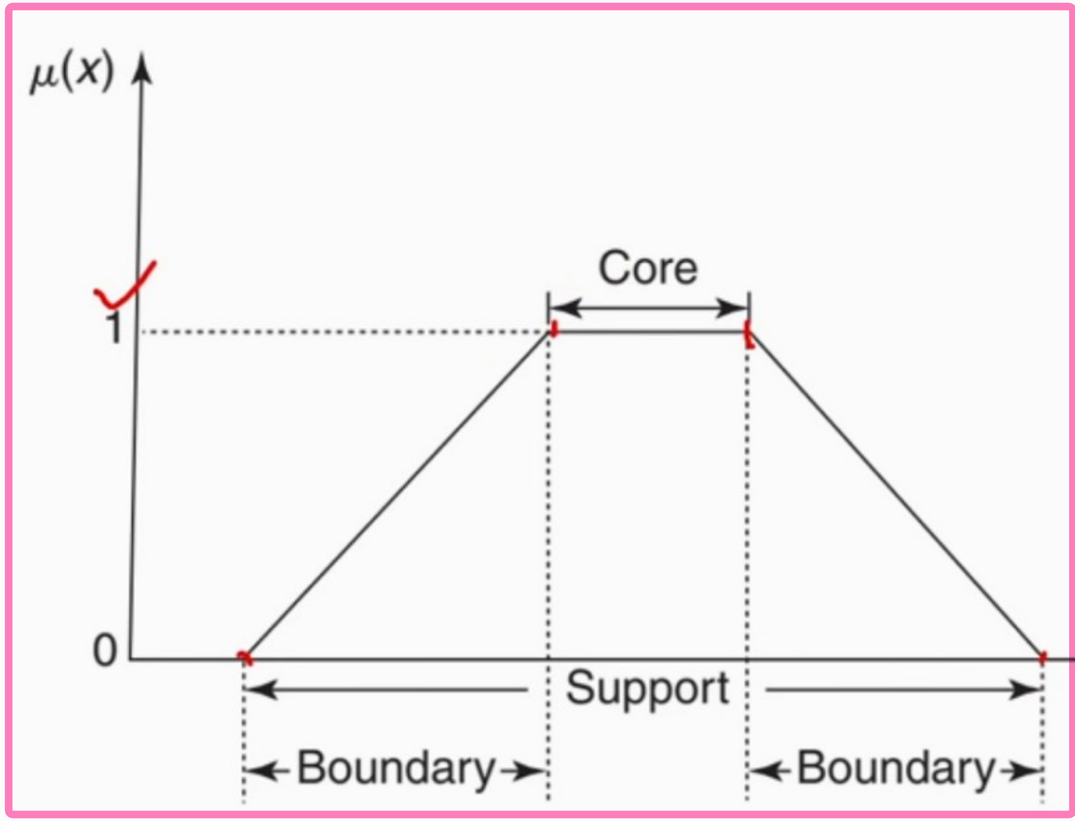
- The support: The region (element) of a universe that is characterized by nonzero membership in a set.

Features of Membership Functions

للفهم :

- **Core:** The core of a membership function for some fuzzy set $\underset{\sim}{A}$ is defined as that region of universe that is characterized by complete membership in the set $\underset{\sim}{A}$.
- The core has elements x of the universe such that

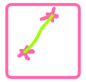
$$\mu_{\underset{\sim}{A}}(x) = 1$$



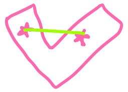
Continue…

- A normal fuzzy set: A set whose membership function has at least one element whose membership value is unity.
- Prototype: It is an element (only one element) that has a membership value that is equal to one.
- A convex fuzzy set: It is a set which is described by a membership function whose values are (1) strictly monotonically increasing, (2) strictly monotonically decreasing, or (3) monotonically increasing then decreasing with increasing the elements values.
- The height of a fuzzy set is the maximum value of the membership function.

Convex Area
(النقطتين بنفس ال Area)



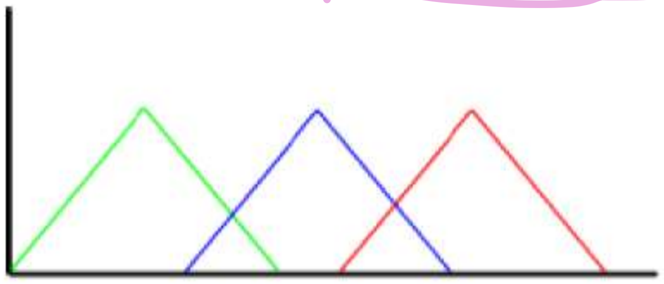
Not convex Area



Membership Functions: Types

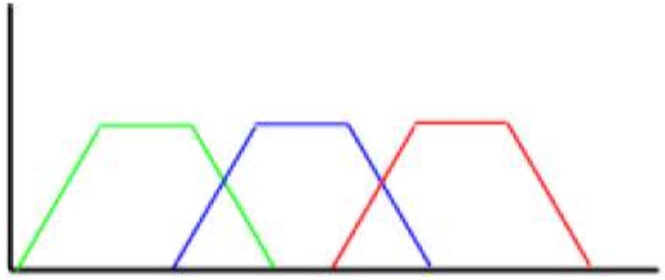
- Membership functions can be symmetrical or asymmetrical.
- They can also be defined as 1D or nD membership functions.

In Matlab, the fuzzy logic toolbox includes 11 built-in membership function types.



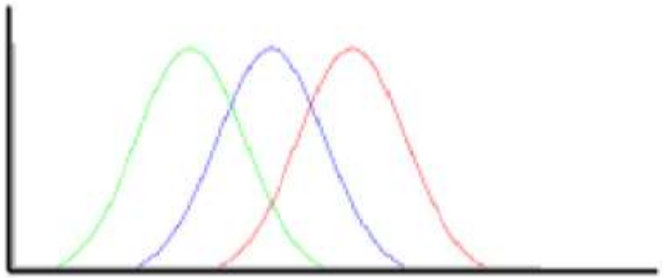
triangular

3 parameter



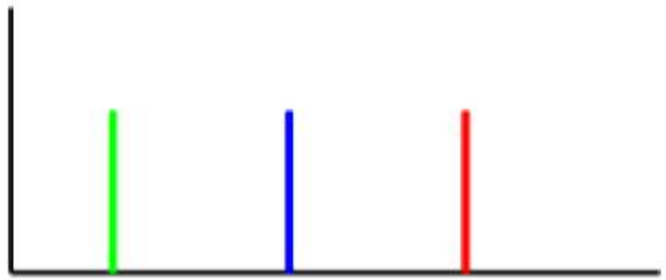
trapezoidal

4 parameters



Gaussian

mean
standard deviation
يعرّف ال



singleton

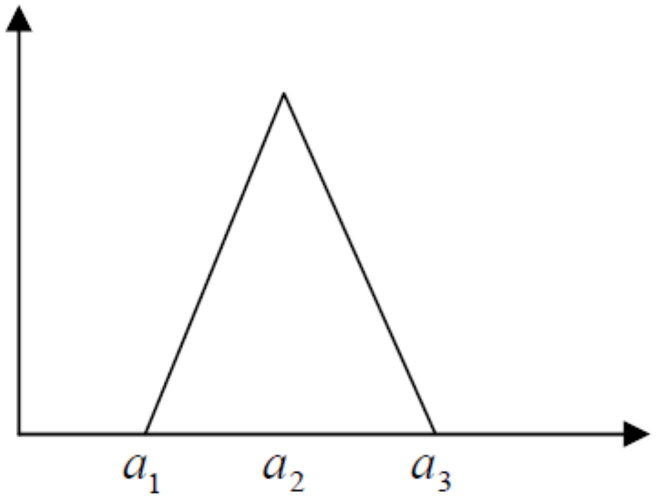
is the crisp type of membership degree function

Fuzzification

- Fuzzification represents the process of mapping crisp values to fuzzy sets.

a) Triangular Membership Function:

3 parameters



النقطة يلي بدي اوجد لها ال membership degree function

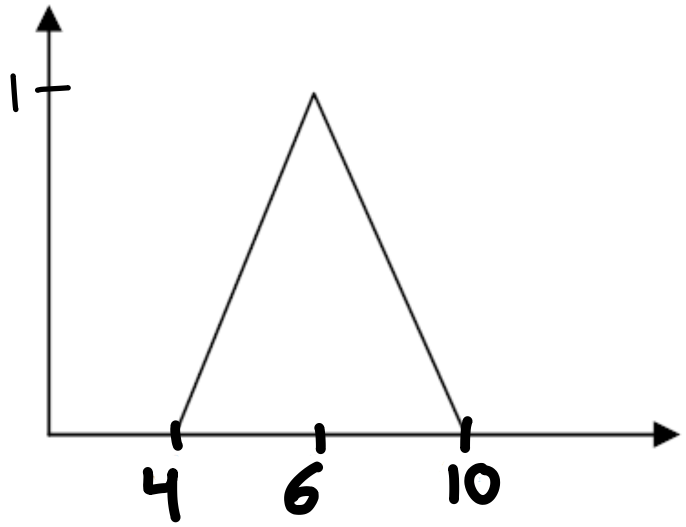
$$\mu_A(u) = \begin{cases} 0 & u < a_1 \\ \dfrac{u - a_1}{a_2 - a_1} & a_1 \le u < a_2 \\ \dfrac{a_3 - u}{a_3 - a_2} & a_2 \le u < a_3 \\ 0 & u \ge a_3 \end{cases}$$

Fuzzification

Example :-

- Fuzzification represents the process of mapping crisp values to fuzzy sets.

a) Triangular Membership Function:



Find

$$\mu(5) = \frac{5-4}{6-4} = 0.5$$

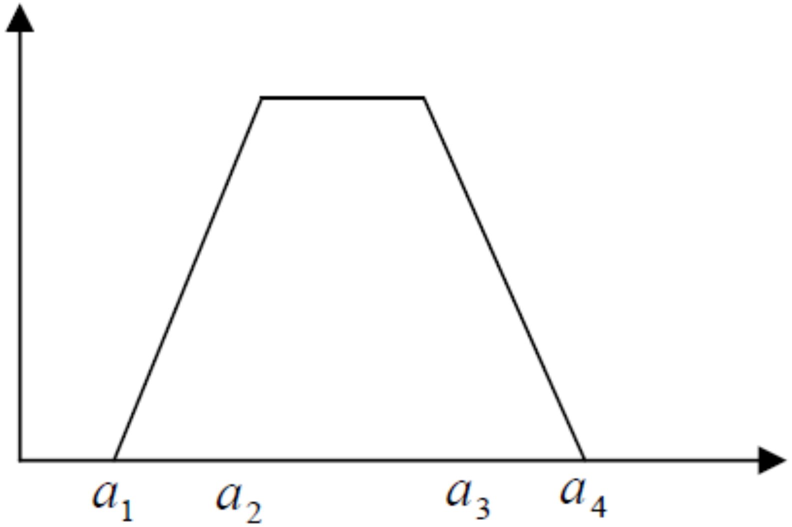
$$\mu(3) = 0 \longrightarrow$$ يعني لا تنتمي لهذه ال set

$$\mu(8) = \frac{10-8}{10-6} = 0.5$$

* Degree of freedom for whom is bigger Triangular or Trapezoidal ?
because it have more parameters to control

Continue…

b) Trapezoidal Membership Function:

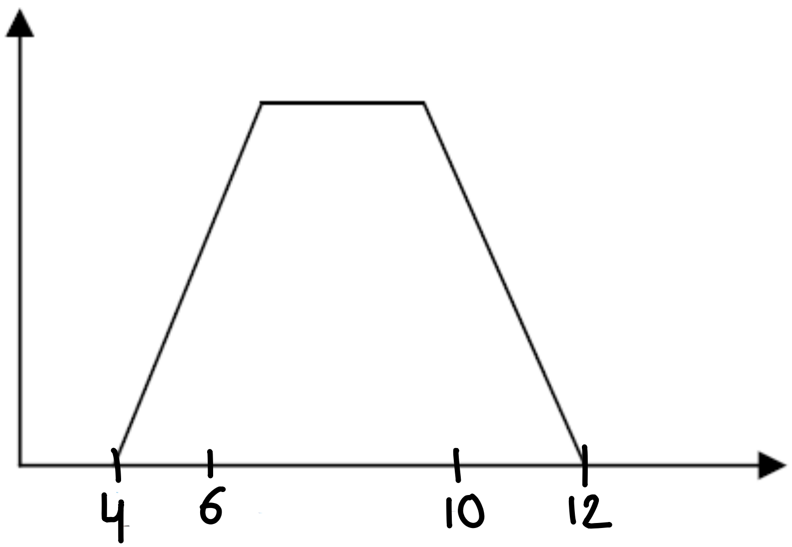


$$\mu_A(u) = \begin{cases} 0 & u < a_1 \\ \dfrac{u - a_1}{a_2 - a_1} & a_1 \le u < a_2 \\ 1 & a_2 \le u < a_3 \\ \dfrac{a_4 - u}{a_4 - a_3} & a_3 \le u < a_4 \\ 0 & u \ge a_4 \end{cases}$$

Continue…

Example :-

b) Trapezoidal Membership Function:



Find

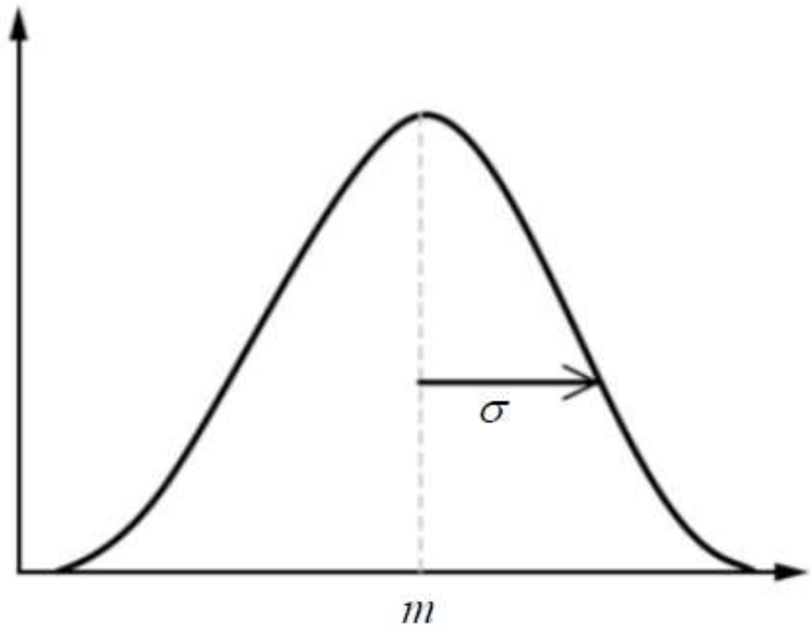
$$\mu(5) = \frac{5-4}{6-4} = 0.5$$

$$\mu(11) = \frac{12-11}{12-10} = 0.5$$

$$\mu(7) = 1$$

Continue…

c) $\boxed{\text{Gaussian Membership Function:}}$



* كأنه المفروض القانون

$\mu = e^{-\frac{1}{2}\left(\frac{u-m}{\sigma}\right)^2}$

$$\mu_A(u) = e^{-\left(\frac{u-m}{\sigma}\right)^2}$$

* if $m = 10$, $\sigma = 2$ find the Gaussian Membership Function

$\mu(7) = e^{-\left(\frac{7-10}{2}\right)^2} = 0.105399$

اذا كان عندي Fuzzy value
كيف الاقي الـ crisp value

Defuzzification

- Defuzzification represents the process of mapping fuzzy values to crisp ones.
- Methods (common):

a) Max membership (the height method): $\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z)$

بالعادة بستخدمها
بس يكون عنا prototype
(نقطة وحدة عالية)

* disadvantage
انه اكثر من اشي اله نفس الـ height

b) Centroid method (centre of area/gravity):

$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z\, dz}{\int \mu_{\underset{\sim}{C}}(z)\, dz}$$

defuzified value

a, d, g → First Maximum
→ Last Maximum
→ Average

Continue…

c) **Weighted average method:**

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\underset{\sim}{C}}(\bar{z})}$$

$$\frac{\sum \left(\text{Membership function of the center} \times z_{\text{prime}} \right)}{\text{Sum of Membership function}}$$

$$z^* = \frac{(2.5 \times 0.3) + (5 \times .5) + (6.5 \times 1)}{0.3 + 0.5 + 1}$$

d) **Mean max membership:** This method is similar to the maximum membership method, except that the locations of the maximum membership can be non-unique.

الفرق بينه وبين Max membership
يفضل استخدامها لما يكون عندي أكثر من قيمة

بنروح لأعلى قيمة Membership و باخذ ال Mean

Continue…

Note: trapezoydal Area $= \frac{1}{2} \times$ (مجموع القاعدتين) $\times$ الارتفاع

e) **Centre of sum:** $$z^* = \frac{\sum_{k=1}^{n} \mu_{C_k}(z) \int_z \bar{z}\, dz}{\sum_{k=1}^{n} \mu_{C_k}(z) \int_z dz}$$

$\sum$ (باخذ ال center في كل رسمة و بضربه بال Area)
تقسيم مجموع ال Areas

$$z^* = \frac{2.5 \times \left(\frac{1}{2} \times (5+3) \times 0.3\right) + 5 \times \left(\frac{1}{2} \times (4+2) \times 0.5\right) + 6.5 \times \left(\frac{1}{2} \times (3+1) \times 1\right)}{\frac{1}{2}(5+3) \times .3 + \frac{1}{2}(2+4) \times 0.5 + \frac{1}{2}(3+1) \times 1}$$

Note: Two drawbacks to this method are that (1) the intersecting areas are added twice, (2) and the method also involves finding the centroids of the individual membership functions.

Continue…

f) Centre of largest area: $z^* = \dfrac{\int \mu_{\underset{\sim}{C}_m}(z) z \, dz}{\int \mu_{\underset{\sim}{C}_m}(z) \, dz}$

it will give me almost the centroid in the convex region

X (we will not use it)

It is a CONVEX sub region

g) First (or last) of maxima: The value of the domain with maximized membership degree.

$$\mathrm{hgt}(\underset{\sim}{C}_k) = \sup_{z \in Z} \mu_{\underset{\sim}{C}_k}(z) \longrightarrow z^* = \inf_{z \in Z}\{z \in Z \mid \mu_{\underset{\sim}{C}_k}(z) = \mathrm{hgt}(\underset{\sim}{C}_k)\}$$

بدنا نعمل طريق و لازم نلاقي عرض الطريق

و عملنا collect data B1 B2 B3

it is asking me to deffuzify each value B1 B2 B3 and choose one

Example

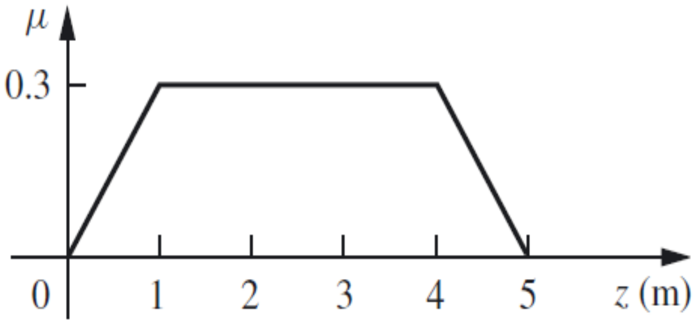
A railroad company intends to lay a new rail line in a particular part of a county. The whole area through which the new line is passing must be purchased for right-of-way considerations. It is surveyed in three stretches, and the data are collected for analysis. The surveyed data for the road are given by the sets: B1, B2 and B3, where the sets are defined on the universe of right-of-way widths, in meters. For the railroad to purchase the land, it must have an assessment of the amount of land to be bought. The three surveys on right-of-way width are ambiguous, however, because some of the land along the proposed railway route is already public domain and will not need to be purchased. Additionally, the original surveys are so old (circa 1860) that some ambiguity exists on boundaries and public right-of-way for old utility lines and old roads. The three fuzzy sets shown in the Figures below, represent the uncertainty in each survey as to the membership of right-of-way width, in meters, in privately owned land.

Continue…

The fuzzy sets:

$\mu = 0.3$

B_1



$\bar{z} = c = 2.5$

center

B_2

slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{0.5-0}{3-2} = 0.5$

* for point (4, 0.5) ← يختار أي نقطة

$y = (m)x + b$

$\mu(z) = 0.5 \times 4 + (\text{int})$

$0.5 = 0.5z + \text{int}$

$\text{int} = -1.5$

معادلة الخط المستقيم

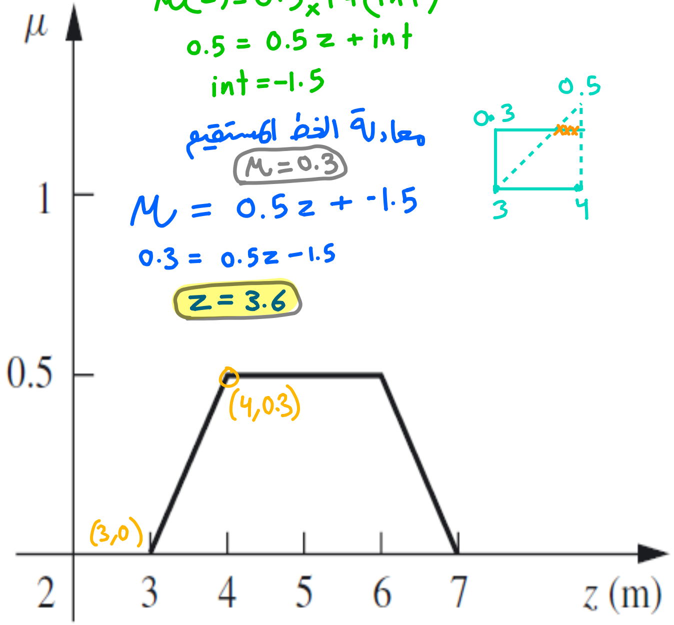
$\mu = 0.3$

$\mu = 0.5z + -1.5$

$0.3 = 0.5z - 1.5$

$\boxed{z = 3.6}$

0.3 0.5 3 4



$\bar{z} = c = 5$

B_3

Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1-0}{5-3} = \dfrac{1}{2} = 0.5$

for point (6, 1)

$1 = 0.5 \times 6 + \text{int}$

$1 = 3 + \text{int}$

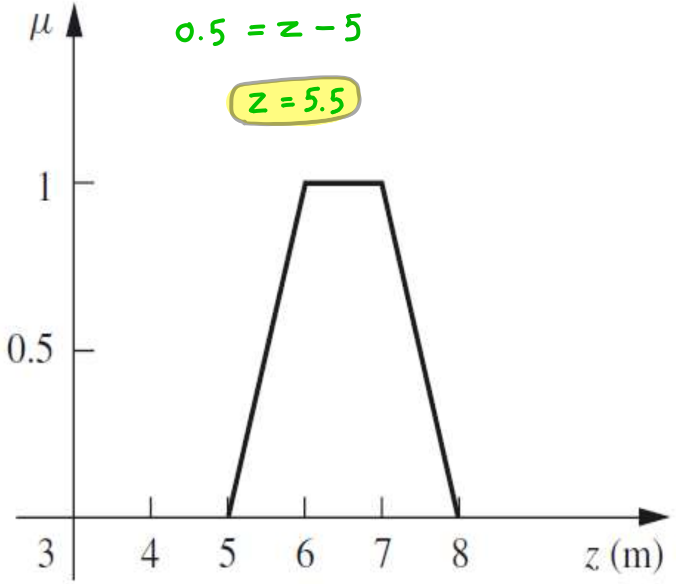
$\text{int} = -2$

at $\mu = 0.5$

$\mu = z + \text{intersection}$

$0.5 = z - 5$

$\boxed{z = 5.5}$



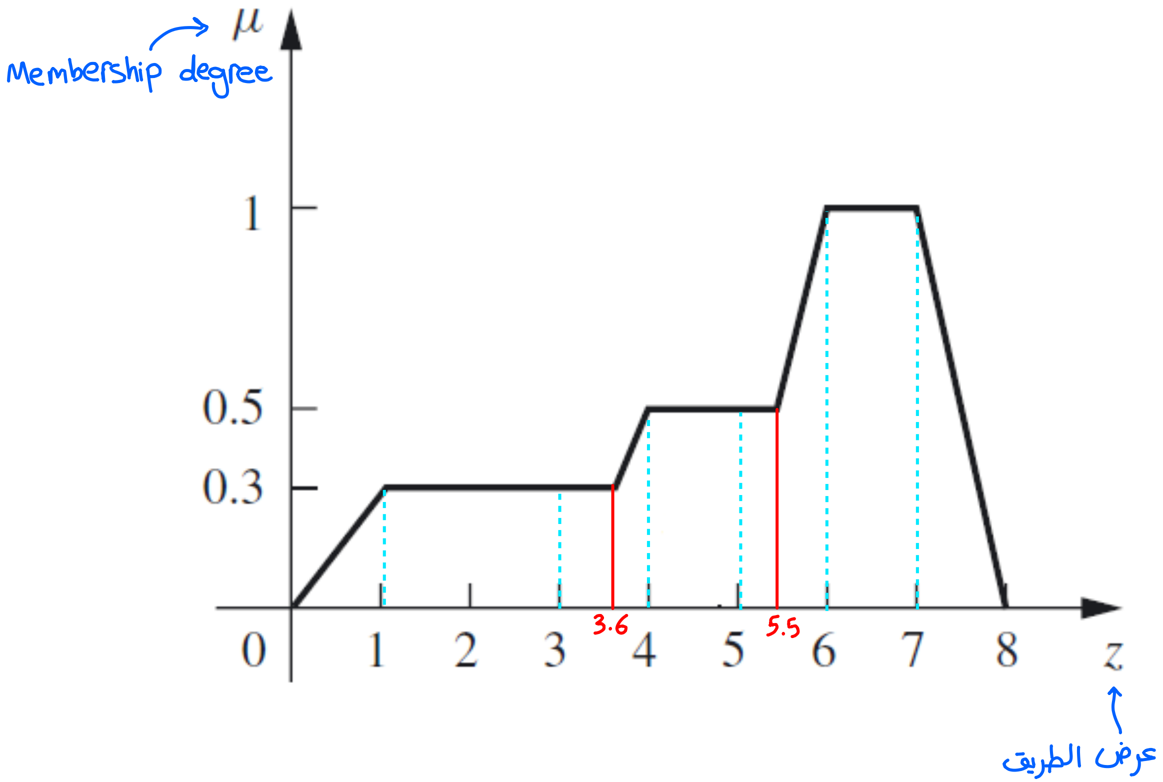
$\bar{z} = c = 6.5$

Continue…

union for the three fuzzy sets

نأخذ ال Membership degree الأعلى

Find the single most nearly representative right-of-way width (z).



Continue…

Method ①

* $\boxed{\text{Max membership}}$ ⟶ في هذه الطريقة أغلب الوقت بتكون مبينة من الرسمات يلي بالسؤال

first Maximum / last Maximum / Average Maximum بشرط ال convex

$\downarrow$ 6 $\quad\downarrow$ 7 $\quad\downarrow$ $\frac{6+7}{2} =$

$\frac{\text{حاصل جمع البسط}}{\text{حاصل جمع المقام}}$

$\boxed{\text{Using the centroid method:}}$

في هذه الطريقة لازم احسب ال union للرسمات

$$z^* = \frac{\int \mu_{\underset{\sim}{B}}(z) \cdot z\, dz}{\int \mu_{\underset{\sim}{B}}(z)\, dz}$$

$$= \left[\int_0^1 (0.3z)z\, dz + \int_1^{3.6} (0.3)z\, dz + \int_{3.6}^4 \left(\frac{z-3.0}{2}\right) z\, dz + \int_4^{5.5} (0.5)z\, dz \right.$$

$$\left. + \int_{5.5}^6 (z-5)z\, dz + \int_6^7 z\, dz + \int_7^8 (8-z)\, z\, dz \right]$$

$$\div \left[\int_0^1 (0.3z)\, dz + \int_1^{3.6} (0.3)\, dz + \int_{3.6}^4 \left(\frac{z-3.6}{2}\right) dz + \int_4^{5.5} (0.5)\, dz \right.$$

$$\left. + \int_{5.5}^6 \left(\frac{z-5.5}{2}\right) dz + \int_6^7 dz + \int_7^8 \left(\frac{7-z}{2}\right) dz \right]$$

$$= 4.9\,\text{m},$$

Continue…

Using the weighted average method:

$$z^* = \frac{(0.3 \times 2.5) + (0.5 \times 5) + (1 \times 6.5)}{0.3 + 0.5 + 1} = 5.41\,\text{m},$$

Using the centre of sum method:

$$z^* = \frac{[2.5 \times 0.5 \times 0.3(3+5) + 5 \times 0.5 \times 0.5(2+4) + 6.5 \times 0.5 \times 1(3+1)]}{[0.5 \times 0.3(3+5) + 0.5 \times 0.5(2+4) + 0.5 \times 1(3+1)]}$$
$$= 5.0\,\text{m},$$

The centre of largest area method provides the same result as the centroid method (z*=4.9).

Continue…

Using the first of maxima method z*= 6.

Using the last of maxima method z∗=7.

بعد دراسة جميع ال Methods ما في طريقة احسن من الثانية
و يلي بحكم معي ال criteria أو الظرف يلي ال sitiuation فيه

$\boxed{4.14}$ H.W

والحل بجميع الطرق

4.14. Often, in chemical processing plants, there will be more than one type of instrumentation measuring the same variable at the same instance during the process. Owing to the nature of measurements, they are almost never exact, and hence can be represented as a fuzzy set. Owing to the differences in instrumentation, the measurements will usually not be the same. Take, for example, two types of temperature sensors, namely, a thermocouple (TC) and a resistance temperature detector (RTD) measuring the same stream temperature. The membership function of the two types of temperature sensors may look as in Figure P4.14.

slope = -1   inf

$\boxed{\mu = -z + 80}$

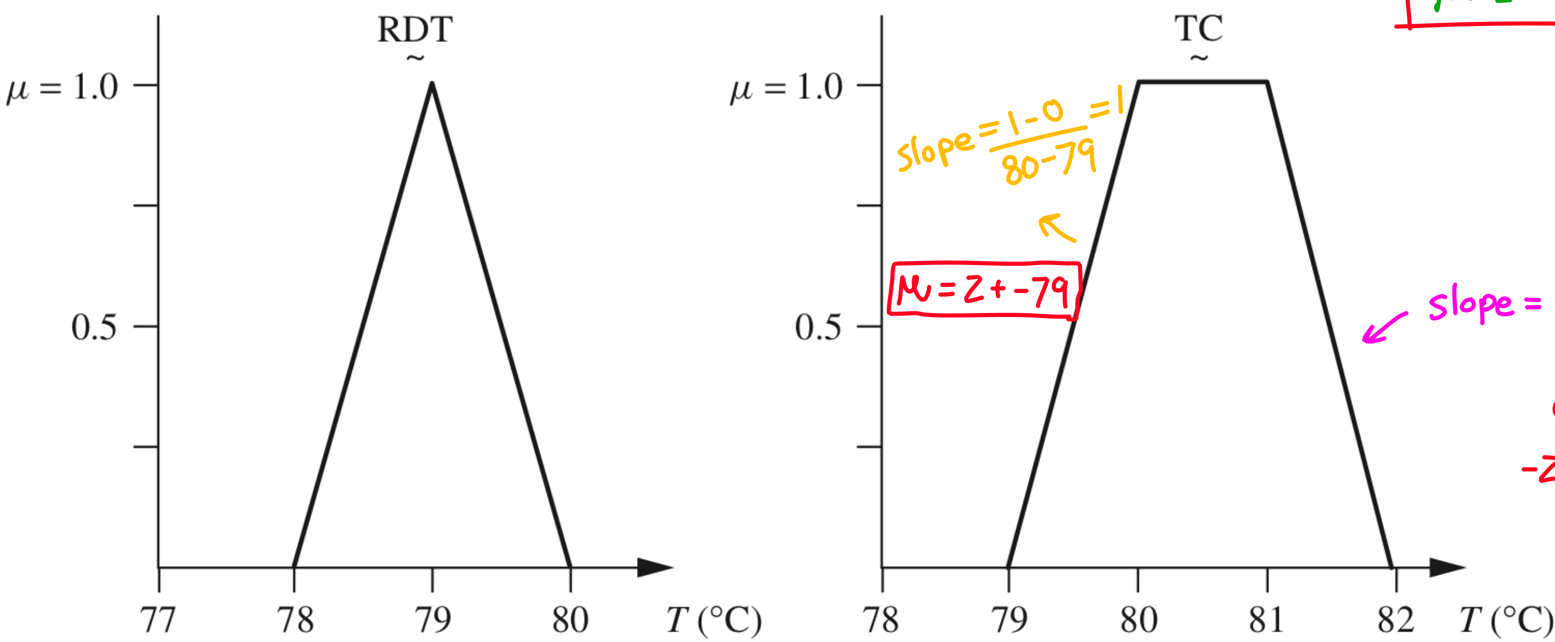


FIGURE P4.14

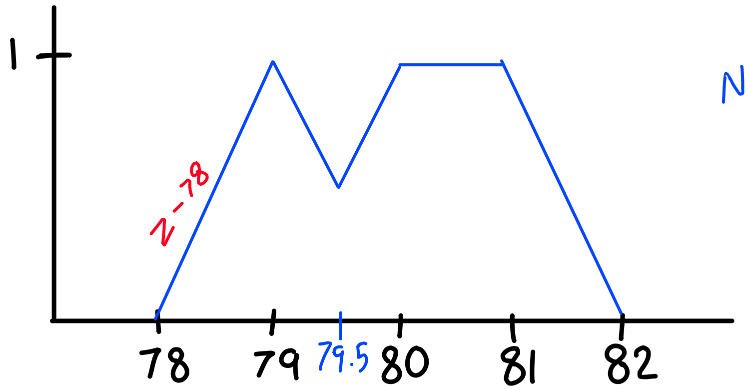
بعد مساوات المعادلتين

$-z + 80 = z + -79$

$159 = 2z$

$\boxed{z = 79.5}$

Max Membership Method → يفضل عدم استخدامها
لأنه not prototype



Not convex Area

centroid Method

المقام نفس البسط لكن بدون ال (z)

$$z^* = \frac{\int_{78}^{79}(z-78)z\,dz + \int_{79}^{79.5}(-z+80)z\,dz + \int_{79.5}^{80}(z-79)z\,dz + \int_{80}^{81}(1)z\,dz + \int_{81}^{82}(-z+82)z\,dz}{\int_{78}^{79}(z-78)\,dz + \int_{79}^{79.5}(-z+80)}$$

weighted Average

$$= \frac{79 \times 1 + 80.5 \times 1}{1+1}$$

Mean Max

$$= \frac{79+80+81}{3} = 80$$

و لازم انتبه عالرسمة اشوف اذا منطقي

applicable
لكن ليست دقيقة

centre of Sum

$$z^* = \frac{79 \times \frac{1}{2} \times 2 \times 1 + 80.5 \times \frac{1}{2} \times (3+1) \times 1}{\frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times (3+1) \times 1}$$

First Maximum → 79

Last Maximum → 81

Chapter 5
Logic and Fuzzy Systems

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Logic: Definition

- Logic is a small part of the human capacity to reason.
- Fuzzy logic is a method to formalize the human capacity of imprecise (approximate) reasoning.
- Reasoning represents the human ability to judge under uncertainty. (Fuzzy Reasoning)
- Interpolative reasoning: the process of interpolating between the binary extremes of true and false is represented by the ability of fuzzy logic to encapsulate partial truths. تضمن

Not Yes or NO it is something in between

- اقْتِراح أو افتراض
- Proposition is associated with the concepts of truth sets, tautologies, and contradictions.

(عبارة دائماً صحيحة)

(عبارة دايماً غلط)

بغض النظر اذا كان (x) ينتمي أو لا

A Statment may be more than one proposition

Crisp Logic

- A proposition (P) is a linguistic (declarative) statement contained within a universe of elements (X) that can be identified as being a collection of elements in X, which are strictly true or strictly false.
- The veracity of an element in the proposition P can be assigned a binary (Boolean) truth value, T (P).
- Assume that U is the universe of all propositions, then one can consider T is a mapping of the elements (u):

$$T : u \in \mathrm{U} \longrightarrow (0, 1)$$

$U = \{black, blue, red, white, green\}$

$P \rightarrow$ red belongs to $U \rightarrow$ so $\underline{\underline{T(P)}} = 1$

Truth value for the proposition

in general

$$T(P) = \begin{cases} 1 & \text{if } x \in U \\ 0 & \text{if } x \notin U \end{cases}$$

* لواجيت عرفت new set

و تحتوى كل ال elements التي تبدأ ب (B)

So Truth Set $= \{black, blue\}$

Falsity set $= \{red, white, green\}$

Truth set for the Universe is the universe (U)

and the falsity set $= \phi$

Continue…

- Truth set, T(P): all elements u in U that are true for proposition P.
- Falsity set: all elements u in U that are false for proposition P.
- So what are:

$$T(\mathrm{U}) = ? \quad = 1$$
$$T(\varnothing) = ? \quad = 0$$

اسموهم هيك عشان بسوو ربط

Crisp Logic: Connectives

- Assume that P and Q are two propositions on the same universe of discourse, such propositions can be combined using the following connectives:

اتحاد (أو) or — disjunction ($\vee$) → Max

تقاطع (و) and — conjunction ($\wedge$) → Min

متممة — negation ($-$) → $1-\square$

تأثير اشي على اشي — implication ($\rightarrow$) → الأول negation واتحاد الثاني

equivalence ($\leftrightarrow$) → اذا زي بعض = 1
اذا غير متشابهين = 0

- Equivalence comes from dual implication.

Continue…

- Let us define sets A and B on a universe X, and propositions P and Q measure the truth of the statement that an element is contained in sets A and B, respectively, or more conventionally:

P : truth that $x \in$ A $\Rightarrow$ if $x \in$ A, T(P) $=$ 1; otherwise, T(P) $=$ 0

Q : truth that $x \in$ B $\Rightarrow$ if $x \in$ B, T(Q) $=$ 1; otherwise, T(Q) $=$ 0

- $\boxed{\text{Using a characteristic function: } \chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}}$

True

False

Continue…

T-Norm ← *Disjunction*

اتحاد
(OR)

$$P \vee Q : x \in A \text{ or } x \in B;$$
$$\text{hence, } T(P \vee Q) = \max(T(P), T(Q)).$$

T-conorm ← *Conjunction*

تقاطع
(AND)

$$P \wedge Q : x \in A \text{ and } x \in B;$$
$$\text{hence, } T(P \wedge Q) = \min(T(P), T(Q)).$$

Negation

If $T(P) = 1$, then $T(\overline{P}) = 0$; if $T(P) = 0$, then $T(\overline{P}) = 1$.

Continue…

Implication → applicable between two propositions $\xrightarrow{\text{مثلاً}}$ P, Q

negation الأولى
و اتحاد الثانية

$$(P \longrightarrow Q) : x \notin A \text{ or } x \in B;$$
$$\text{hence, } T(P \longrightarrow Q) = T(\bar{P} \cup Q).$$

Equivalence

اذا ال Truth set للثنين P Q
متشابهين ← واحد
غير متشابهين ← صفر

$$(P \longleftrightarrow Q) : T(P \longleftrightarrow Q) = \begin{cases} 1, & \text{for } T(P) = T(Q) \\ 0, & \text{for } T(P) \neq T(Q) \end{cases}.$$

- If $T(P) \cap T(Q) = \emptyset$ and the truth of P always implies the falsity of Q and vice versa, then P and Q are mutually exclusive propositions.

They cannot occur at the same time

Continue…

- Truth table for various compound propositions:

$(1 - T(P)) \vee T(Q)$

$\overline{P} \overset{max}{\vee} Q$

اذا متساويين
الجواب ١

$\overline{Q} \vee P$

P	Q	$\overline{P}$	(Max) $P \vee Q$ or	(Min) $P \wedge Q$ and	$P \rightarrow Q$	$P \leftrightarrow Q$	$Q \rightarrow P$
T (1)	T (1)	F (0)	T (1)	T (1)	T (1)	T (1)	1
T (1)	F (0)	F (0)	T (1)	F (0)	F (0)	F (0)	1
F (0)	T (1)	T (1)	T (1)	F (0)	T (1)	F (0)	0
F (0)	F (0)	T (1)	F (0)	F (0)	T (1)	T (1)	1

Truth Table for 3 propositions P Q R

p	q	r	$p \vee q$	$p \vee r$	$q \wedge r$	$(p \vee q) \wedge (p \vee r)$	$p \vee (q \wedge r)$	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$
T	T	T	T	T	T	T	T	T	T	T
T	F	T	T	T	F	T	T	T	F	F
T	T	F	T	T	F	T	T	F	T	F
T	F	F	T	T	F	T	T	F	T	F
F	T	T	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	F	T	F	F
F	T	F	T	F	F	F	F	T	T	T
F	F	F	F	F	F	F	F	T	T	T

max $\vee$

Min $\wedge$

$\to$ الأول negation اتحاد (max) مع الثاني

$\leftrightarrow$ اذا متشابهين واحد
اذا غير متشابهين صفر

لها حالتان فقط في حالة ال Equivelance

$((P \leftrightarrow Q) \leftrightarrow R)$

1 1 1 1 1
0 0 1 0 1

* ما بتفرق بال Truth Table لو كتبت False/True أو 0/1 في ال crisp اما بال Fuzzy بيفرق

★Question :-

if

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 4, 6, 8, 9, 10\}$

$P \rightarrow 3 \in A$

$Q \rightarrow 1 \in B$

Find :-

$\boxed{P \underset{max}{\vee} Q}$

$T(P) \vee T(Q)$

$1 \vee 0$

$= 1$

Crisp Logic: Tautologies

عبارة عن اكثر من Proposition و هي دائماً صحيحة

- Tautologies are compound propositions that are always true irrespective of the truth values of the individual simple propositions.

بغض النظر عن الـProposition الموجودة إذا صحيحة أو لا

- Tautologies are useful for reasoning, proving theorems, and making deductive inferences.

مثلاً Monkey is a human التي غلط و مع ذلك Tautology

"All humans are mammals"

"Prime numbers are not divisible by 6"

- Assignment: Using the truth table, represent a tautology.

Tautology

P	Q	T
1	1	1
1	0	1
0	1	1
0	0	1

Always True

مثال :-
اذا زدنا سرعة القطع و زدنا ال depth of cut ← ال Surface Roughness بتصير سيئة

Continue…

- A valid argument is a list of premises from which the conclusion follows.
  أشياء تؤدي إلى اشي
- Modus ponens: is a very common scheme used in forward-chaining rule-based expert systems. It is an operation to find the truth value of a consequent given the truth value of the antecedent in a rule.

if A then B

Antecedent $\xrightarrow{\text{leads to}}$ Consequent

Form: If A, then B.

A.

Therefore, B.

$(A \wedge (A \longrightarrow B))$

Premisis

conclusion

A	B	A → B	$\boxed{(A \wedge (A \rightarrow B))}$
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

Modus Ponens
it is an (Forward chaining)

Continue…

- Modus Tollens: is a very common scheme used in backward-chaining expert systems.

if B didn't happen so A didn't happen

Form:

If A, then B.

~B.

Therefore, ~A.

$(\overline{B} \wedge (A \longrightarrow B))$

A	B	$A \to B$	$\boxed{(\overline{B} \wedge (A \to B))}$
0	0	1	1
0	1	1	0
1	0	0	0
1	1	1	0

Modus Tollens

it is a (backward chaining)

ال B ما صارت معناته ال A ما صارت

Modus ponens	Modus Tollens
Forward Chaining	Backward Chaining
if (A) then (B)	if (B) didn't happen so (A) didn't happen
$(A \underset{\text{min}}{\wedge} (A \rightarrow B))$	$(\bar{B} \underset{\text{min}}{\wedge} (A \rightarrow B))$
$= (A \underset{\text{min}}{\wedge} (\bar{A} \overset{\text{max}}{\vee} B))$	$= (\bar{B} \underset{\text{min}}{\wedge} (\bar{A} \overset{\text{max}}{\vee} B))$

Crisp Logic: Contradictions and Equivalence

- Contradictions are compound propositions that are always false, regardless of the truth value of the individual propositions constituting the compound proposition.

this statement is always incorrect

بغض النظر عن صحة ال Proposition

Proposition 1 — Proposition 2

"Prime numbers are a multiple of 4" → Always False

- Propositions P and Q are equivalent ($P \leftrightarrow Q$) when both P and Q are true or when both P and Q are false.

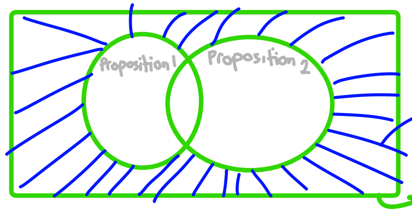
Equivalent

اذا P Q متشابهين ← True

اذا P Q غير متشابهين ← False

P	Q	T	C
1	1	1	0
1	0	1	0
0	1	1	0
0	0	1	0

- Assignment: Using the truth table, represent a contradiction.
- Assignment: plot the Venn diagram for equivalence.



This Shaded Area is the equivalance

universe

Crisp Logic: Example

Equivalent

اذا P ↔ Q متشابهين ← True

اذا P ↔ Q غير متشابهين ← False

and No membership degrees given so it's crisp

1, 8 are included

Example Suppose we consider the universe of positive integers, $X = \{1 \leq n \leq 8\}$. Let P $=$ "n is an even number" and let Q $=$ "$(3 \leq n \leq 7) \wedge (n \neq 6)$." Then $T(P) = \{2, 4, 6, 8\}$ and $T(Q) = \{3, 4, 5, 7\}$. The equivalence P $\leftrightarrow$ Q has the truth set

Truth set for the Proposition (P)

$$T(\mathrm{P} \longleftrightarrow \mathrm{Q}) = (T(P) \cap T(Q)) \cup (\overline{T(P)} \cap \overline{T(Q)}) = \{4\} \cup \{1\} = \{1, 4\}$$

$X = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$P = \{2, 4, 6, 8\}$

$Q = \{3, 4, 5, 7\}$

	P	Q
1	0	0
2		
3		
4	1	1
5		
6		
7		
8		

(1) is False in Q and P and it is included in the Universe

اذا موجود في P and Q

اذا ليس موجود في P and Q

يعني حيكون True للـ element

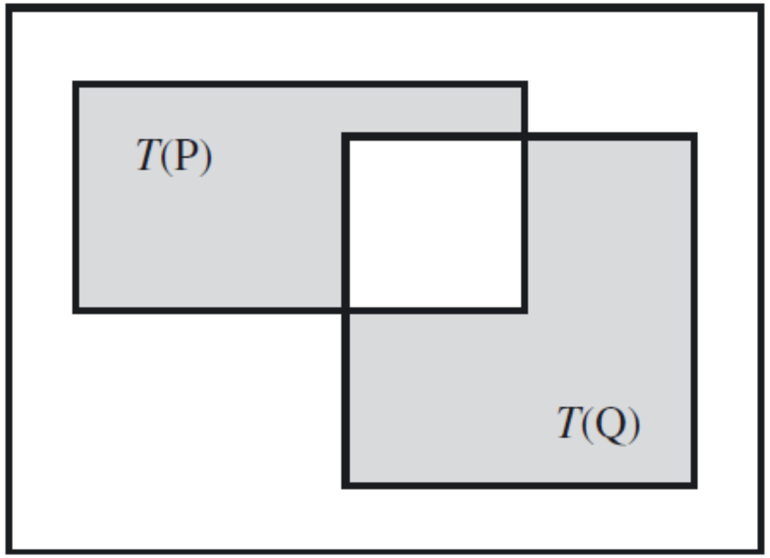
Crisp Logic: Exclusive or (XOR)

وحدة منهم فقط لازم تصير

- Exclusive or (XOR): it arises in many situations involving natural language and human reasoning.
- This situation involves the exclusive or; it does not involve the intersection.
- Assignment: Using the truth table and Venn diagram, represent the "Exclusive or

or → negation for the equivelance

→ $(P \vee Q) \wedge \overline{(P \wedge Q)}$



P	Q	P *XOR* Q
1	1	0
1	0	1
0	1	1
0	0	0

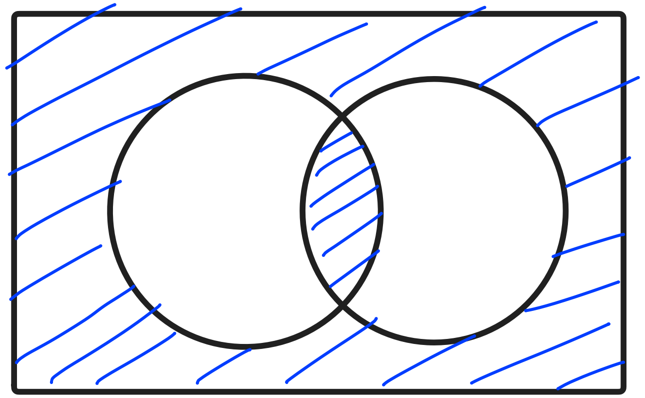
Crisp Logic: Exclusive nor

- Exclusive nor ($\overline{XOR}$) is the complement of the exclusive or.

يا الثنين بالتقاطع
يا الثنين برا لكن جوا الuniverse

- Assignment: Using the truth table and Venn diagram, represent the "Exclusive nor":

by Venn Diagram :-



P	Q	$\overline{P\ XOR\ Q}$
1	1	1
1	0	0
0	1	0
0	0	1

Logical Proofs

- استدلال Inference: the process of making certain conclusions from some given hypotheses.
- How?
  1. The linguistic statement (compound proposition) is made.
  2. The statement is decomposed into its respective single propositions.
  3. The statement is expressed algebraically with logical connectives.
  4. A truth table is used to establish the veracity of the statement.

- Self-reading: Deductive inference. مطلوب دراسته و فهمه

Deductive inferences

The *modus ponens* deduction is used as a tool for making inferences in rule-based systems. A typical if–then rule is used to determine whether an antecedent (cause or action) infers a consequent (effect or reaction). Suppose we have a rule of the form IF A, THEN B, where A is a set defined on universe X and B is a set defined on universe Y. As discussed before, this rule can be translated into a relation between sets A and B; that is, recalling Equation (5.4), $R = (A \times B) \cup (\overline{A} \times Y)$. Now suppose a new antecedent, say A', is known. Can we use *modus ponens* deduction, Equation (5.7), to infer a new consequent, say B', resulting from the new antecedent? That is, can we deduce, in rule form, IF A', THEN B'? The answer, of course, is yes, through the use of the composition operation (defined initially in Chapter 3). Since "A implies B" is defined on the Cartesian space $X \times Y$, B' can be found through the following set-theoretic formulation, again from Equation (5.4):

$$B' = A' \circ R = A' \circ ((A \times B) \cup (\overline{A} \times Y)),$$

where the symbol $\circ$ denotes the composition operation. *Modus ponens* deduction can also be used for the compound rule IF A, THEN B, ELSE C, where this compound rule is equivalent to the relation defined in Equation (5.6) as $R = (A \times B) \cup (\overline{A} \times C)$. For this compound rule, if we define another antecedent A', the following possibilities exist, depending on whether (1) A' is fully contained in the original antecedent A, (2) A' is contained only in the complement of A, or (3) A' and A overlap to some extent as described next:

$$\begin{aligned}
&\text{IF } A' \subset A, \text{ THEN } y = B \\
&\text{IF } A' \subset \overline{A}, \text{ THEN } y = C \\
&\text{IF } A' \cap A \neq \emptyset,\ A' \cap \overline{A} \neq \emptyset, \text{ THEN } y = B \cup C
\end{aligned}.$$

The rule IF A, THEN B (proposition P is defined on set A in universe X, and proposition Q is defined on set B in universe Y), that is, $(P \rightarrow Q) = R = (A \times B) \cup (\overline{A} \times Y)$, is then defined in function-theoretic terms as

$$\chi_R(x, y) = \max[(\chi_A(x) \wedge \chi_B(y)),((1 - \chi_A(x)) \wedge 1)], \tag{5.9}$$

where $\chi()$ is the characteristic function as defined before.

Example 5.7. Suppose we have two universes of discourse for a heat exchanger problem described by the following collection of elements: X = {1, 2, 3, 4} and Y = {1, 2, 3, 4, 5, 6}. Suppose X is a universe of normalized temperatures and Y is a universe of normalized pressures. Define crisp set A on universe X and crisp set B on universe Y as follows: A = {2, 3} and B = {3, 4}. The deductive inference IF A, THEN B (i.e., IF temperature is A, THEN pressure is B) will yield a matrix describing the membership values of the relation R, that is, $\chi_R(x, y)$, through the use of Equation (5.9). That is, the matrix R represents the rule IF A, THEN B as a matrix of characteristic (crisp membership) values.

Crisp sets A and B can be written using Zadeh's notation,

$$\mathrm{A} = \left\{\tfrac{0}{1} + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{0}{4}\right\}.$$
$$\mathrm{B} = \left\{\tfrac{0}{1} + \tfrac{0}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{0}{5} + \tfrac{0}{6}\right\}.$$

If we treat set A as a column vector and set B as a row vector, the following matrix results from the Cartesian product of A × B, using Equation (3.16):

$$\mathrm{A} \times \mathrm{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The Cartesian product $\overline{\mathrm{A}} \times \mathrm{Y}$ can be determined using Equation (3.16) by arranging $\overline{\mathrm{A}}$ as a column vector and the universe Y as a row vector (sets $\overline{\mathrm{A}}$ and Y can be written using Zadeh's notation):

$$\overline{\mathrm{A}} = \left\{\tfrac{1}{1} + \tfrac{0}{2} + \tfrac{0}{3} + \tfrac{1}{4}\right\}.$$
$$\mathrm{Y} = \left\{\tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{6}\right\}.$$
$$\overline{\mathrm{A}} \times \mathrm{Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Then, the full relation R describing the implication IF A, THEN B is the maximum of the two matrices A × B and $\overline{\mathrm{A}} \times \mathrm{Y}$, or, using Equation (5.9),

$$\mathrm{R} = \begin{array}{c c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

The compound rule IF A, THEN B, ELSE C can also be defined in terms of a matrix relation as $\mathrm{R} = (\mathrm{A} \times \mathrm{B}) \cup (\overline{\mathrm{A}} \times \mathrm{C}) \Rightarrow (\mathrm{P} \to \mathrm{Q}) \wedge (\overline{\mathrm{P}} \to \mathrm{S})$, as given by Equations (5.5) and (5.6), where the membership function is determined as

$$\chi_{\mathrm{R}}(x, y) = \max[(\chi_{\mathrm{A}}(x) \wedge \chi_{\mathrm{B}}(y)), ((1 - \chi_{\mathrm{A}}(x)) \wedge \chi_{\mathrm{C}}(y))]. \qquad (5.10)$$

Example 5.8. Continuing with the previous heat exchanger example, suppose we define a crisp set C on the universe of normalized temperatures Y as C = {5, 6}, or, using Zadeh's notation,

$$C = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{1}{5} + \frac{1}{6} \right\},$$

The deductive inference IF A, THEN B, ELSE C (i.e., IF pressure is A, THEN temperature is B, ELSE temperature is C) will yield a relational matrix R, with characteristic values $\chi_R(x, y)$ obtained using Equation (5.10). The first half of the expression in Equation (5.10)

(i.e., A × B) has already been determined in the previous example. The Cartesian product $\overline{A} \times C$ can be determined using Equation (3.16) by arranging the set $\overline{A}$ as a column vector and the set C as a row vector (see set $\overline{A}$ in Example 5.7), or

$$\overline{A} \times C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Then, the full relation R describing the implication IF A, THEN B, ELSE C is the maximum of the two matrices A × B and $\overline{A} \times C$ (Equation (5.10)):

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Logical Proofs: Example

is a compound of propositions

- Hypotheses: Engineers are mathematicians. Logical thinkers do not believe in magic. Mathematicians are logical thinkers.

  $\bar{S}$
- Conclusion: Engineers do not believe in magic.
- Decomposing the hypotheses:

  P : a person is an engineer.

  Q : a person is a mathematician.

  R : a person is a logical thinker.

  S : a person believes in magic.

$$((P \longrightarrow Q) \underset{\text{and}}{\wedge} (R \longrightarrow \bar{S}) \underset{\text{and}}{\wedge} (Q \longrightarrow R)) \longrightarrow (P \longrightarrow \bar{S})$$

Fuzzy Logic

- A fuzzy logic proposition ($\underset{\sim}{\mathrm{P}}$) is a statement involving some concept without clearly defined boundaries.
- The truth value assigned to $\underset{\sim}{\mathrm{P}}$ can be any value on the interval [0, 1].
- Fuzzy propositions are assigned to fuzzy sets. Suppose proposition $\underset{\sim}{\mathrm{P}}$ is assigned to fuzzy set $\underset{\sim}{\mathrm{A}}$, then, the truth value of a proposition is given as follows:

$$T(\underset{\sim}{\mathrm{P}}) = \mu_{\underset{\sim}{\mathrm{A}}}(x), \quad \text{where } 0 \leq \mu_{\underset{\sim}{\mathrm{A}}} \leq 1$$

indicate that its Fuzzy

Fuzzy Logic: $\boxed{\text{Connectives}}$

$T(\underset{\sim}{P})$
Truth Value for proposition $\underset{\sim}{P}$
indicates that it is Fuzzy

- The logical connectives:

Negation

$$\boxed{T(\overline{\underset{\sim}{P}}) = 1 - T(\underset{\sim}{P}).}$$

Disjunction

$$\underset{\sim}{P} \vee \underset{\sim}{Q} : x \text{ is } \underset{\sim}{A} \text{ or } \underset{\sim}{B} \quad \boxed{T(\underset{\sim}{P} \vee \underset{\sim}{Q}) = \max(T(\underset{\sim}{P}), T(\underset{\sim}{Q}))}.$$

Conjunction

$$\underset{\sim}{P} \wedge \underset{\sim}{Q} : x \text{ is } \underset{\sim}{A} \text{ and } \underset{\sim}{B} \quad \boxed{T(\underset{\sim}{P} \wedge \underset{\sim}{Q}) = \min(T(\underset{\sim}{P}), T(\underset{\sim}{Q}))}.$$

Continue...

Implication

$$\underset{\sim}{P} \to \underset{\sim}{Q} : x \text{ is } \underset{\sim}{A}, \text{ then } x \text{ is } \underset{\sim}{B}$$

$$\boxed{T(\underset{\sim}{P} \to \underset{\sim}{Q}) = T(\overline{\underset{\sim}{P}} \vee \underset{\sim}{Q}) = \max(T(\overline{\underset{\sim}{P}}), T(\underset{\sim}{Q}))}$$

- As in the crisp logic, the implication can be modelled in rule-based form:

$$\boxed{\underset{\sim}{P} \to \underset{\sim}{Q} \text{ is IF } x \text{ is } \underset{\sim}{A}, \text{ THEN } y \text{ is } \underset{\sim}{B}}$$

- Note: it is equivalent to $\underset{\sim}{R} = (\underset{\sim}{A} \times \underset{\sim}{B}) \cup (\overline{\underset{\sim}{A}} \times Y)$

Relation Relation

Example

membership degree for one is zero

Example Suppose we are evaluating a new invention to determine its commercial potential. We will use two metrics to make our decisions regarding the innovation of the idea. Our metrics are the "uniqueness" of the invention, denoted by a universe of novelty scales, $X = \{1, 2, 3, 4\}$, and the "market size" of the invention's commercial market, denoted on a universe of scaled market sizes, $Y = \{1, 2, 3, 4, 5, 6\}$. In both universes, the lowest numbers are the "highest uniqueness" and the "largest market," respectively. A new invention in your group, say a compressible liquid of very useful temperature and viscosity conditions, has just received scores of "medium uniqueness," denoted by fuzzy set $\underset{\sim}{A}$, and "medium market size," denoted fuzzy set $\underset{\sim}{B}$. We wish to determine the implication of such a result, that is, IF $\underset{\sim}{A}$, THEN $\underset{\sim}{B}$. We assign the invention the following fuzzy sets to represent its ratings:

$y = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \right\}$

Continue…

$$\underset{\sim}{A} = \text{medium uniqueness} = \left\{ \frac{0}{1} + \frac{0.6}{2} + \frac{1}{3} + \frac{0.2}{4} \right\}.$$

$$\underset{\sim}{B} = \text{medium market size} = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} + \frac{0.3}{5} \right\}.$$

$$\underset{\sim}{C} = \text{diffuse market size} = \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5} + \frac{0.3}{6} \right\}.$$

if $\underset{\sim}{A}$ then $\underset{\sim}{B}$ is represented by :-

Find $\underset{\sim}{R} = (\underset{\sim}{A} \times \underset{\sim}{B}) \cup (\overline{\underset{\sim}{A}} \times Y)$

Continue…

Relation

* important:-
Do Not forget
that the universe
Y is {1,2,3,4,5,6}

$$\underset{\sim}{A} \times \underset{\sim}{B} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0.4 & 0.6 & 0.6 & 0.3 & 0 \\ 3 & 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 4 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0 \end{array}$$

in the relation we
have to write the zeros

$$\overline{\underset{\sim}{A}} \times Y = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{array}$$

باخذ اتحادهم الان
(Maximum)

$$\underset{\sim}{R} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0.4 & 0.4 & 0.6 & 0.6 & 0.4 & 0.4 \\ 3 & 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 4 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{array}$$

if we had a new invention , Assume Market size is $\underset{\sim}{C}$ then what is $\underset{\sim}{A'}$

Continue…

- When the logical conditional implication is of the compound form:

$(\underset{\sim}{A} \times \underset{\sim}{B}) \cup (\bar{\underset{\sim}{A}} \times \cancel{\underset{\sim}{Y}}\ \underset{\sim}{C})$

$$\text{IF } x \text{ is } \underset{\sim}{A}, \text{ THEN } y \text{ is } \underset{\sim}{B}, \boxed{\text{ELSE } y \text{ is } \underset{\sim}{C},}$$

- Then, the fuzzy relation can be presented as

$$\underset{\sim}{R} = (\underset{\sim}{A} \times \underset{\sim}{B}) \underset{\text{Maximum}}{\cup} (\overline{\underset{\sim}{A}} \times \underset{\sim}{C}).$$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix} 0.3 & 0.5 & 0.6 & 0.6 & 0.5 & 0.3 \\ 0.3 & 0.4 & 0.6 & 0.6 & 0.4 & 0.3 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0.3 & 0.5 & 0.6 & 0.6 & 0.5 & 0.3 \end{bmatrix}
\end{array}$$

$$\underset{\sim}{A} = \left\{ \frac{0}{1}, \frac{.6}{2}, \frac{1}{3}, \frac{.2}{4} \right\}$$

$$\bar{\underset{\sim}{A}} = \left\{ \frac{1}{1}, \frac{.4}{2}, \frac{0}{3}, \frac{.8}{4} \right\}$$

بقارن ال (1)
مع من ال 1إلى 6
بال C و باخد
ال Minimum

$$\underset{\sim}{C} = \left\{ \frac{.3}{1}, \frac{.5}{2}, \frac{.6}{3}, \frac{.6}{4}, \frac{.5}{5}, \frac{.3}{6} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0}{1}, \frac{0.4}{2}, \frac{1}{3}, \frac{0.8}{4}, \frac{0.3}{5} \right\}$$

$$y = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \right\}$$

IF x is $\underset{\sim}{A}$, THEN y is $\underset{\sim}{B}$, $\boxed{\text{ELSE } y \text{ is } \underset{\sim}{C}}$,

$$R = \underbrace{(\underset{\sim}{A} \times \underset{\sim}{B})} \overset{max}{\cup} \underbrace{(\bar{\underset{\sim}{A}} \times \underset{\sim}{C})}$$

$$\underset{\sim}{A} \times \underset{\sim}{B} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .4 & .6 & .6 & .3 & 0 \\ 0 & .4 & 1 & .8 & .3 & 0 \\ 0 & .2 & .2 & .2 & .2 & 0 \end{bmatrix} \end{array}$$

max

$$\cup$$

$$\bar{\underset{\sim}{A}} \times \underset{\sim}{C} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0.3 & 0.5 & 0.6 & 0.6 & 0.5 & 0.3 \\ 0.3 & 0.4 & 0.4 & 0.4 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.5 & 0.6 & 0.6 & 0.5 & 0.3 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0.3 & 0.5 & 0.6 & 0.6 & 0.5 & 0.3 \\ 0.3 & 0.4 & 0.6 & 0.6 & 0.4 & 0.3 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0.3 & 0.5 & 0.6 & 0.6 & 0.5 & 0.3 \end{bmatrix} \end{array}$$

Approximate Reasoning

- Approximate reasoning is about imprecise propositions.
- It deals with partial truth.
- Question: suppose we have a rule expressed as follows:

$$\text{IF } x \text{ is } \underset{\sim}{\mathrm{A}},\ \text{THEN } y \text{ is } \underset{\sim}{\mathrm{B}} \longrightarrow (A \times B) \cup (\bar{A} \times U)$$

if x is A
they y is B

If we introduce a new antecedent, is it possible to derive the consequent?

$$\text{IF } x \text{ is } \underset{\sim}{\mathrm{A}}',\ \text{THEN } y \text{ is } \underset{\sim}{\mathrm{B}}'$$

- By using the composition operation ($\underset{\sim}{\mathrm{B}}' = \underset{\sim}{\mathrm{A}}' \circ \underset{\sim}{\mathrm{R}}$), the answer is YES.

ch3
composition

Continue…

- For the previous example, what market size would be associated with a uniqueness score of "almost high uniqueness"

$$\underset{\sim}{A}' = \text{almost high uniqueness} = \left\{ \frac{0.5}{1} + \frac{1}{2} + \frac{0.3}{3} + \frac{0}{4} \right\}$$

how can I find the market size?

- By using the max-min composition:

$$\underset{\sim}{B}' = \underset{\sim}{A}' \circ \underset{\sim}{R} = \left\{ \frac{0.5}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5} + \frac{0.5}{6} \right\}$$

$$A' = \left\{ \frac{0.5}{1}, \frac{1}{2}, \frac{0.3}{3}, \frac{0}{4} \right\}$$

$$R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ .4 & .4 & .6 & .6 & .4 & .4 \\ 0 & .4 & 1 & .8 & .3 & 0 \\ .8 & .8 & .8 & .8 & .8 & .8 \end{bmatrix}$$

Solution

(by Maximum Minimum composition)

$$B' = A' \circ R = \left\{ \frac{0.5}{1}, \frac{0.5}{2}, \frac{0.6}{3}, \frac{0.6}{4}, \frac{0.5}{5}, \frac{0.5}{6} \right\}$$

market size for

almost high uniqueness

Fuzzy Implication Operations

- There are other techniques one can use to obtaining the fuzzy relation ($\underset{\sim}{R}$) based on a fuzzy rule.
- The membership function values of $\underset{\sim}{R}$ can be presented as follows:

$$\mu_{\underset{\sim}{R}}(x, y) = \max[\mu_{\underset{\sim}{B}}(y), 1 - \mu_{\underset{\sim}{A}}(x)]$$

(أقل حسابات)

$$\mu_{\underset{\sim}{R}}(x, y) = \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)]$$

$$\mu_{\underset{\sim}{R}}(x, y) = \min\{1, [1 - \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(y)]\}$$

Continue…

* سلبياتها

اذا كانت ال membership degree لوحدة منهم zero

بيطلع الجواب النهائي zero و هو مو كتير صحيح x

$$\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A}}(x) \overset{\text{multiplication}}{\cdot} \mu_{\underset{\sim}{B}}(y)$$

$$\mu_{\underset{\sim}{R}}(x, y) = \begin{cases} 1, & \text{for } \mu_{\underset{\sim}{A}}(x) \le \mu_{\underset{\sim}{B}}(y); \\ \mu_{\underset{\sim}{B}}(y), & \text{otherwise.} \end{cases}$$

*Example :-

$$\underset{\sim}{A} = \left\{ \frac{0}{1}, \frac{0.6}{2}, \frac{1}{3}, \frac{0.2}{4} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0}{1}, \frac{0.4}{2}, \frac{1}{3}, \frac{0.8}{4}, \frac{0.3}{5}, \frac{0}{6} \right\}$$

$$\mu_{\underset{\sim}{R}}(x, y) = \begin{cases} 1, & \text{for } \mu_{\underset{\sim}{A}}(x) \leq \mu_{\underset{\sim}{B}}(y); \\ \mu_{\underset{\sim}{B}}(y), & \text{otherwise.} \end{cases}$$

$$M_{\underset{\sim}{R}}(x,y) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & .4 & 1 & 1 & .3 & 0 \\ 0 & .4 & 1 & .8 & .3 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

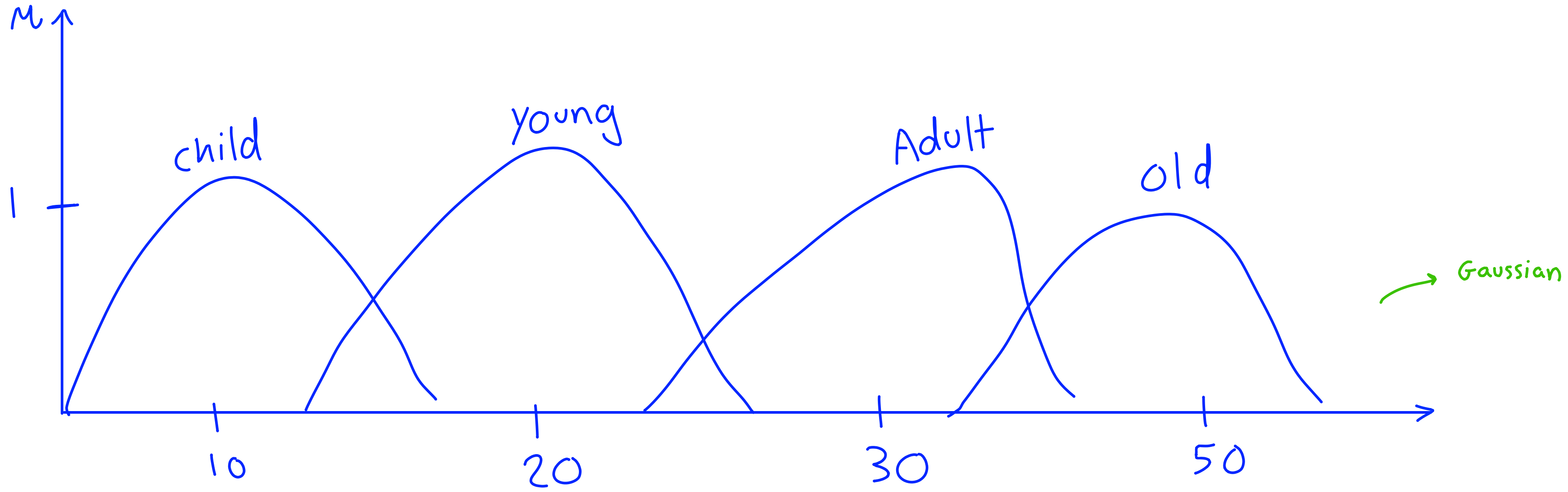
Fuzzy System (FS)

- Natural language: it vague and ambiguous, however, one can understand it.
- Example: “young” is a term that can be linguistically interpreted in terms of age.

$$\mu_{\underset{\sim}{M}}(\text{young}, y) = \begin{cases} \left[1 + \left(\frac{y-25}{5}\right)^2\right]^{-1}, & y > 25 \text{ years;} \\ 1, & y \leq 25 \text{ years.} \end{cases}$$

كلما زاد العمر عن ال 25
كلما قل مدى انتماء الشخص لمرحلة
ال young

* Describe the age of people linguisticly then represent it in a membership Function ?



الرسم و الارقام
تقريبية و بتفرق من شخص لشخص

* A Question in a past Exam

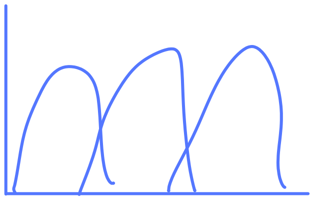
Question 1: Various fuzzy sets for the weather in Jordan can be defined, such as rainy, sunny, windy, etc. Discus and draw membership functions to describe the weather in Jordan, and discuss whether fizzy logic is the best technique to describe it. (Hint: define different universes)

اعرّف ٣ universes

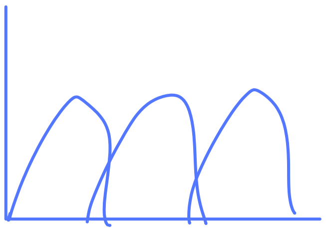
sunny, rainy, windy

Sunny →

windy →



rainy →



الحل ناقص

Continue…

- A composite is a collection (set) of terms combined by various linguistic connectives such as and, or, and not.
- For the two terms α and β, the interpretation of the composite can be defined by using theoretic operations as follows:

$$\cup \quad \alpha \text{ or } \beta: \mu_{\alpha \text{ or } \beta}(y) = \max(\mu_\alpha(y), \mu_\beta(y)),$$

$$\cap \quad \alpha \text{ and } \beta: \mu_{\alpha \text{ and } \beta}(y) = \min(\mu_\alpha(y), \mu_\beta(y)),$$

$$\text{Not } \alpha = \overline{\alpha}: \mu_{\overline{\alpha}}(y) = 1 - \mu_\alpha(y).$$

FS: If-Then Rule-Based System

Fuzzy system

- Knowledge is usually represented using If-Then rule-based form.

الأكثر شيوعاً

IF premise (antecedent), THEN conclusion (consequent)

- The fuzzy rule-based system is useful in modelling complex systems that can be observed by humans, thus linguistic variables can be used to describe the antecedents and consequents.
- The linguistic variables can be naturally represented by fuzzy sets and logical connectives of these sets.

FS: Multiple Conjunctive Antecedents

- Suppose that the rule is as follows:

$$\text{IF } x \text{ is } \underset{\sim}{A}^1 \text{ and } \underset{\sim}{A}^2 \ldots \text{ and } \underset{\sim}{A}^L \text{ THEN } y \text{ is } \underset{\sim}{B}^s$$

- A new fuzzy set can be defined as follows:

$$\underset{\sim}{A}^s = \underset{\sim}{A}^1 \cap \underset{\sim}{A}^2 \cap \cdots \cap \underset{\sim}{A}^L \qquad \mu_{\underset{\sim}{A}^s}(x) = \min[\mu_{\underset{\sim}{A}^1}(x), \mu_{\underset{\sim}{A}^2}(x), \ldots, \mu_{\underset{\sim}{A}^L}(x)]$$

connectives بشبكهم ب

- Then the compound rule can be represented as:

$$\text{IF } \underset{\sim}{A}^s \text{ THEN } \underset{\sim}{B}^s$$

FS: Multiple Disjunctive Antecedents → If I had one Rule (one Fuzzy Rule)

- Suppose that the rule is as follows:

$$\text{IF } x \text{ is } \underset{\sim}{A}^1 \text{ OR } x \text{ is } \underset{\sim}{A}^2 \ldots \text{OR } x \text{ is } \underset{\sim}{A}^L \text{ THEN } y \text{ is } \underset{\sim}{B}^s$$

- A new fuzzy set can be defined as follows:

$$\underset{\sim}{A}^s = \underset{\sim}{A}^1 \cup \underset{\sim}{A}^2 \cup \cdots \cup \underset{\sim}{A}^L \qquad \mu_{\underset{\sim}{A}^s}(x) = \max\left[\mu_{\underset{\sim}{A}^1}(x), \mu_{\underset{\sim}{A}^2}(x), \ldots, \mu_{\underset{\sim}{A}^L}(x)\right]$$

- Then the compound rule can be represented as:

$$\text{IF } \underset{\sim}{A}^s \text{ THEN } \underset{\sim}{B}^s$$

FS: $\boxed{\text{Aggregation of Fuzzy Rules}}$ → For multiple Rules

- Aggregation: The process of obtaining the overall consequent from the individual consequents contributed by each rule.

- Two aggregation strategies:
    1. Conjunctive system of rules → depends on Minimum operation
    2. Disjunctive system of rules → depends on Maximum operation

Continue…

- Conjunctive system of rules

$$y = y^1 \text{ and } y^2 \text{and} \ldots \text{and } y^r \qquad\qquad y = y^1 \cap y^2 \cap \cdots \cap y^r$$

$$\mu_y(y) = \min(\mu_{y^1}(y), \mu_{y^2}(y), \ldots, \mu_{y^r}(y)), \text{ for } y \in Y$$

- Disjunctive system of rules

$$y = y^1 \text{ or } y^2 \text{ or} \ldots \text{or } y^r \qquad\qquad y = y^1 \cup y^2 \cup \cdots \cup y^r$$

$$\mu_y(y) = \max(\mu_{y^1}(y), \mu_{y^2}(y), \ldots, \mu_{y^r}(y)), \text{ for} y \in Y$$

Inference: Graphical Techniques

- Have you read "Deductive inference"?

  Self Reading from the book

- Graphical methods usually make the manual computations of the inference easy and straightforward (with a few rules).
- Three common methods for fuzzy systems based on linguistic rules:
  1. Mamdani systems
  2. Takagi Sugeno systems
  3. Tsukamoto systems

Inference: Mamdani Systems

- Let us consider a simple two-rule system where each rule comprises of two antecedents and one consequent, the Mamdani form is given as:

2 Methods → Max Min → Max product

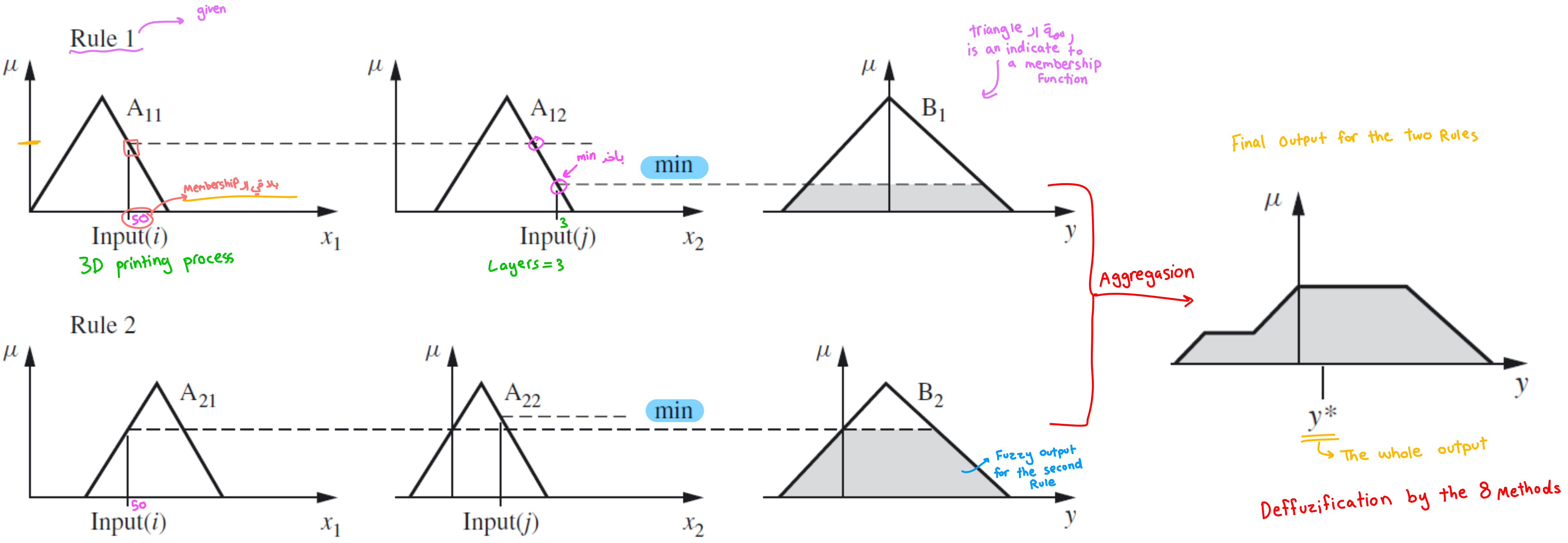
$$\text{IF } x_1 \text{ is } \underset{\sim}{A}_1^k \text{ and } x_2 \text{ is } \underset{\sim}{A}_2^k \text{ THEN } y^k \text{ is } \underset{\sim}{B}^k, \qquad \text{for } k = 1, 2, \ldots, r,$$

- For the Mamdani system, two cases can be considered:
  - A max–min inference method
  - A max–product inference method

The Output is Fuzzy Membership Fuction → So its Mamadani

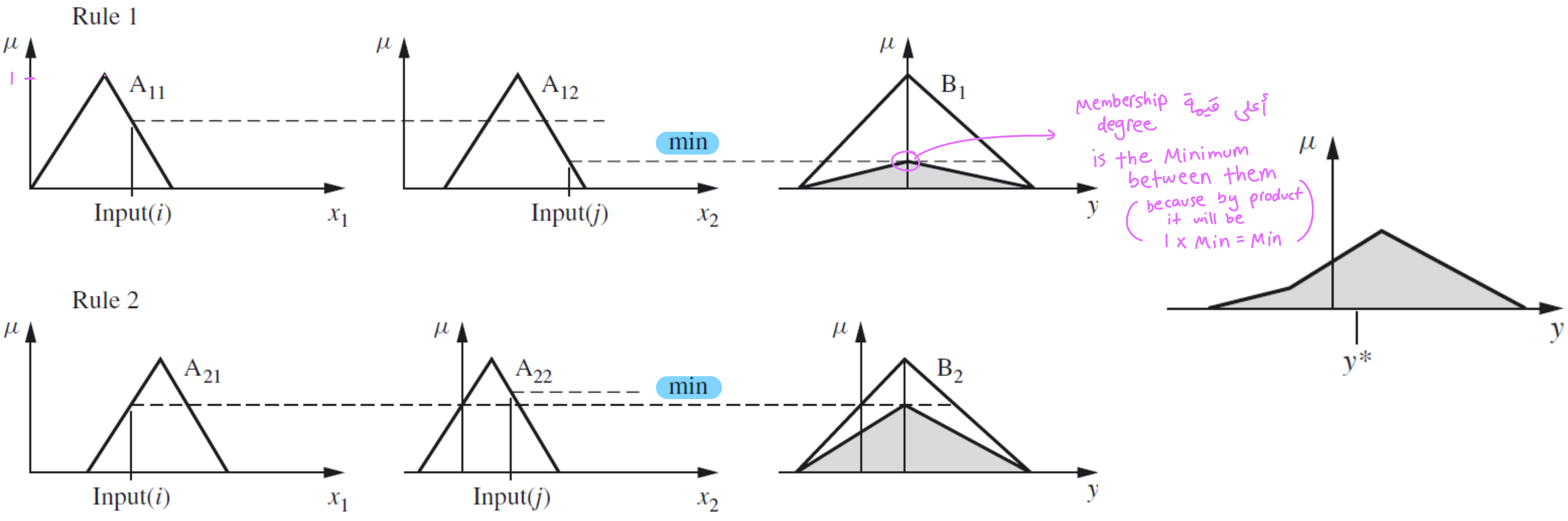
Continue…

A max–min inference method:



Continue…

A max–product inference method



Example

Example In mechanics, the energy of a moving body is called kinetic energy. If an object of mass m (kilograms) is moving with a velocity v (meters per second), then the kinetic energy k (in joules) is given by the equation $\boxed{k = \frac{1}{2}mv^2}$ → The Ideal case. Suppose we model the mass and velocity as inputs to a system (moving body) and the energy as output, then observe the system for a while and deduce the following two disjunctive rules of inference based on our observations:

Minimum

$$\text{Rule 1}: \ \text{IF } x_1 \text{ is } \underset{\sim}{A}_1^1 \text{ (small mass) } \underline{\underline{\textit{and}}} \ x_2 \text{ is } \underset{\sim}{A}_2^1 \text{ (high velocity),}$$

$$\text{THEN } y \text{ is } \underset{\sim}{B}^1 \text{ (medium energy).}$$

Maximum

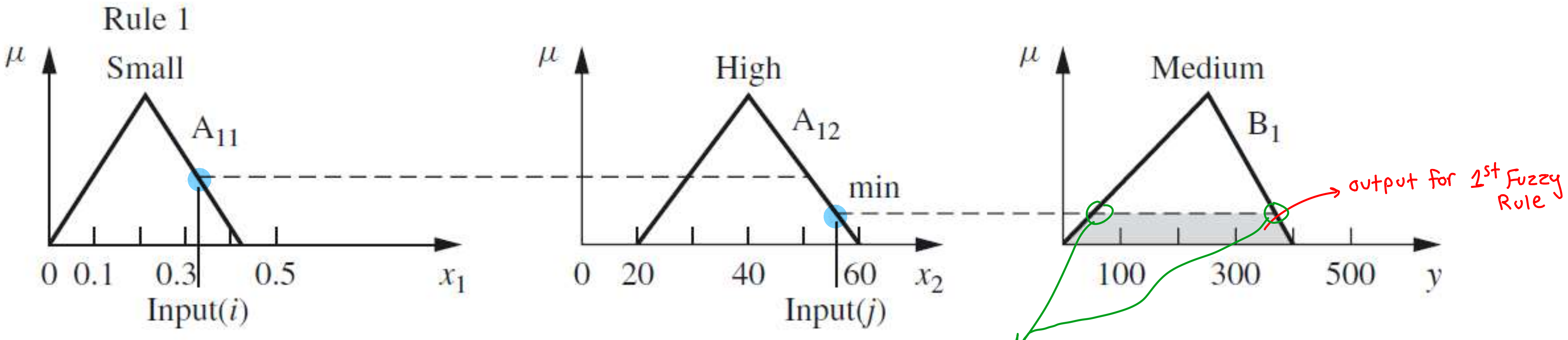
$$\text{Rule 2}: \ \text{IF} x_1 \text{ is } \underset{\sim}{A}_1^2 \text{ (large mass) } \underline{\textit{or}} \ x_2 \text{ is } \underset{\sim}{A}_2^2 \text{ (high velocity),}$$

$$\text{THEN } y \text{ is } \underset{\sim}{B}^2 \text{ (high energy).}$$

Continue…

- Assume that mass=0.35 kg and velocity=55m/s.
- By using the max–min inference:

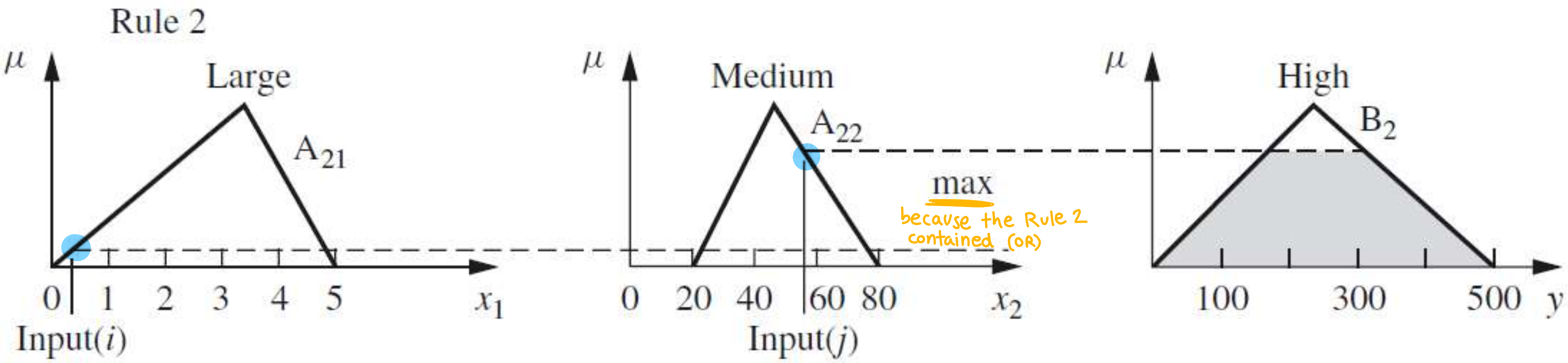
بدي اشوف مدى انتماءهم لـ Membership Degree

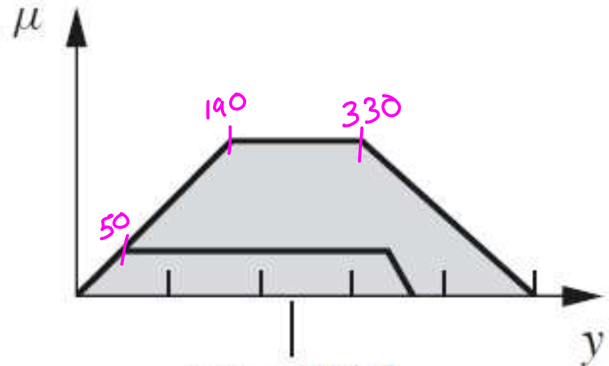


بخبط مرتين فبصير عندي Area

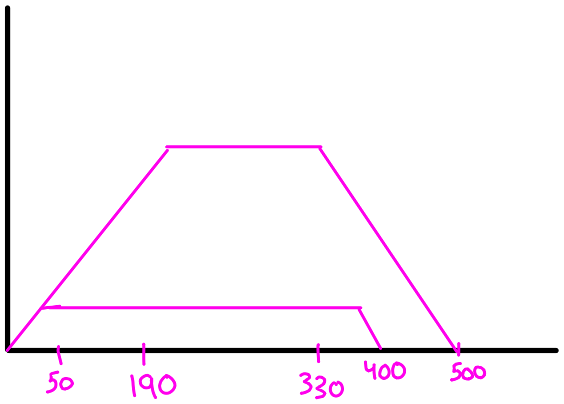
ولازم نعمل deffuzification

Continue…





بشكل أوضح



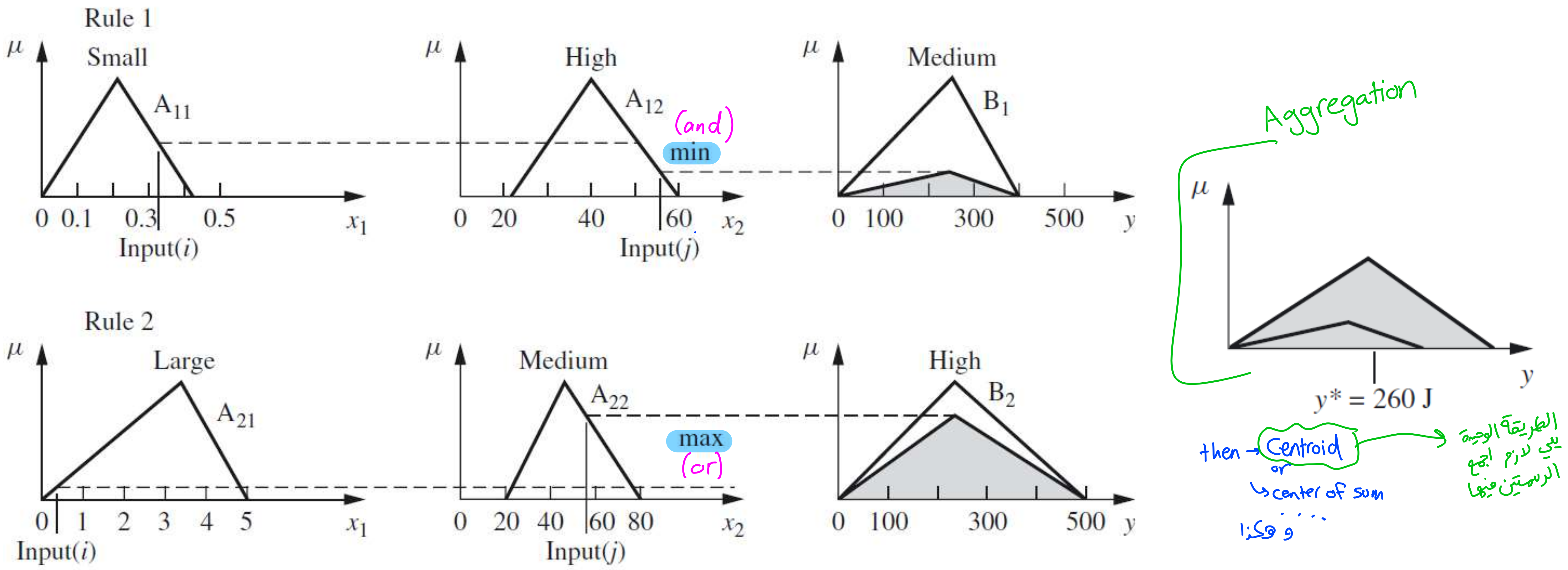
Using the centroid method $y^* = 244$ J

$\frac{1}{2} \times .35 \times 55^2 = 529.375$

The Comment → الطريقة الثانية بتعطيني اقرب لل Ideal number

Continue…

- By using the max–~~min~~ Product inference:



5.32. In finding the Nusselt number (a dimensionless number for determining heat transfer) for an hexagonal cylinder in cross flow, there are two correlations (which are to be used as the consequent terms in a Sugeno inference method):

$$Nu_1 = 0.16Re^{0.638}Pr^{1/3} \quad 5000 < Re < 19\,650,$$
$$Nu_2 = 0.0385Re^{0.728}Pr^{1/3} \quad Re > 19\,650,$$

Re is the Reynolds number and Pr is the Prandtl number. In the equations above, we seek to know whether Nu is low (Nu_1) or is Nu medium (Nu_2)?
The Nusselt number is a function of convective heat transfer (h), diameter of the hexagonal cylinder (D) over which cooling fluid travels, and the conductivity of the material (K):

$$Nu = \frac{hD}{K}.$$

Both Re and Pr can be fuzzy due to uncertainty in the variables in velocity. It would be convenient to find Nu (output) based on Re and Pr (inputs) without having to do all the calculations. More specifically, there is uncertainty in calculating the Reynolds number because velocity is not known exactly:

$$Re = \frac{\rho VD}{\mu},$$

where ρ is the density, V is the velocity, D is the characteristic length (or pipe diameter), and μ is the dynamic viscosity. And there is also uncertainty in the value for the Prandtl number due to its constituents

$$Pr = \frac{v}{\alpha},$$

where v is the kinematic viscosity and α is the specific gravity.
Calculation of Nu is very involved and the incorporation of a rule-base can be used to bypass these calculations; we have the following rules to govern this process:

بدل استخدام
قانون Reynold number

Rules

If Re is high and Pr is low $\longrightarrow$ Then Nu is low.

If Re is low and Pr is low $\longrightarrow$ Then Nu is low.

If Re is high and Pr is high $\longrightarrow$ Then Nu is medium.

If Re is low and Pr is high $\longrightarrow$ Then Nu is medium.

For this problem, conduct a Mamdani and a Sugeno inference, based on the membership functions given in Figure P5.32(a–c), and use the following inputs:

$$\boxed{\begin{aligned} Re &= 1.965 \times 10^4. \\ Pr &= 275. \end{aligned}}$$

Comment on the differences in the results.

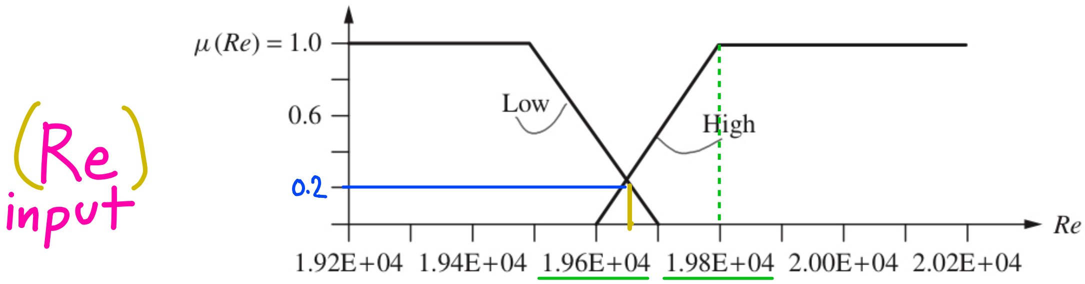


FIGURE P5.32a
Input for Reynolds number

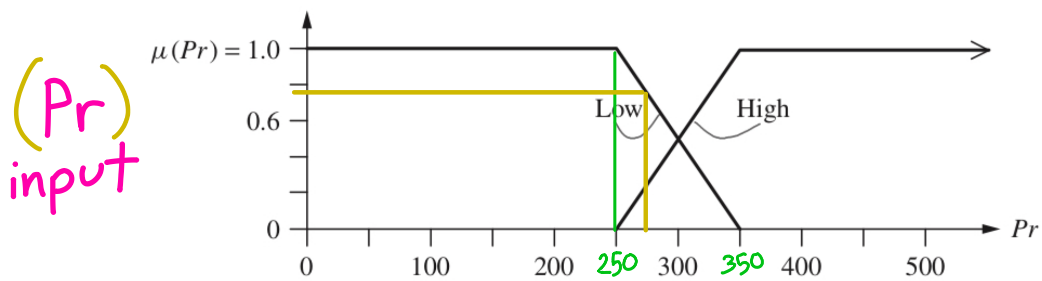


FIGURE P5.32b
Input for Prandtl number

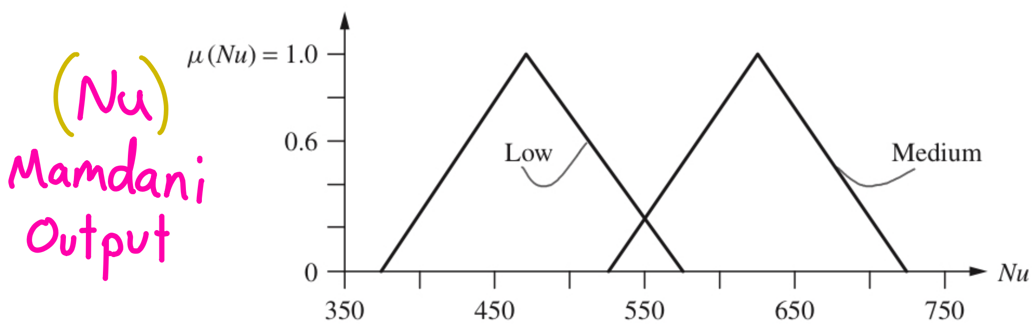


FIGURE P5.32c
Output for Mamdani inference

5.32

1) In finding the Nusselt number (a dimensionless number for determining heat transfer) for a hexagonal cylinder in cross flow, there are two correlations.

$$N_{u1} = 0.16 R_e^{0.638} Pr^{1/3} \qquad 5000 < R_e < 19650$$

$$N_{u2} = 0.0385 R_e^{0.728} Pr^{1/3} \qquad R_e < 19650$$

R_e is the Reynolds number and P_r is the Prandtl number.

The Nusselt number is a function of convective heat transfer (h), diameter of the hexagonal cylinder (D) over which cooling fluid travels, and the conductivity of the material (K).

$$N_u = \frac{hD}{K},$$

R_e's and P_r's both can be fuzzy due to uncertainty in the variables in velocity. It would be convenient to find N_u (Output) based on R_e and P_r (inputs) without having to do all the calculations.

Rules:

If R_e is high and P_r is low $\rightarrow$ N_u is low
If R_e is low and P_r is low $\rightarrow$ N_u is low
If R_e is high and P_r is high $\rightarrow$ N_u is medium
If R_e is low and P_r is high $\rightarrow$ N_u is medium

In the Mandami method:

2) INPUT and P_r $\rightarrow$ propagate minimum to μ(Nu) and use weighted average defuzzification.

INPUT $\quad \text{Re} = 19.65 \times 10^3$
$\mu(\text{Re}) = 0.25$
$\text{Pr} = 275 \qquad \mu(\text{Pr}) = 0.25$

In the Sugeno method:

Use the correlations N_{u1} and N_{u2} to get Z_1 and Z_2

$$Z = \frac{\mu(N_{u1}) z_1 + \mu(N_{u2}) z_2}{\mu(N_{u1}) + \mu(N_{u2})}$$

Mandami

Input $\quad R_e = 19.65\text{E}3$
$\mu(R_{eH}) = \mu(R_{eL}) = 0.25$
$P_r = 275 \qquad \mu(P_{rL}) = 0.75$ and
$\mu(P_{rH}) = 0.25$

Rule1: $N_{uL}\min(0.25,0.75)=0.25$
Rule2: $N_{uL}=\min(0.25,0.75)=0.25$
Rule 1 and 2 : max is 0.25

Defuzzification for $N_{uL}=0.25$ yields to z=487.5

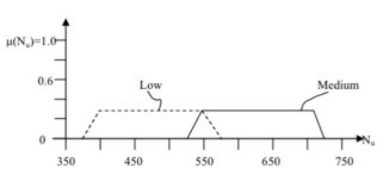
Rule3: $N_{uH}=\min(0.25,0.25)=0.25$
Rule4: $N_{uH}=\min(0.25,0.25)=0.25$
Rule 3 and 4 : max is 0.25

Defuzzification for $N_{uH}=0.25$ yields to z=612.5

Weighted average :

$$z = \frac{0.25 * 487.5 + 0.25 * 612.5}{0.5} = 550$$

3)



For **Sugeno** we have the following input

Input $\quad R_e = 1.965\text{E}4 \quad \mu(R_e)=0.25$
$P_r = 275 \quad \mu(P_{rL})=0.75$ and
$\mu(P_{rH})=0.25$

And from our rule base and the following equations

4)

$$N_{u1} = 0.16 R_e^{0.638} Pr^{1/3} \qquad 5000 < R_e < 19650$$

$$N_{u2} = 0.0385 R_e^{0.728} Pr^{1/3} \qquad R_e < 19650$$

we have the following results:

Nu1 = 560.0993666

Nu2 = 559.5643482

z = 559.8318574

Inference: $\boxed{\text{Takagi Sugeno Systems}}$

↳ most common function is Linear

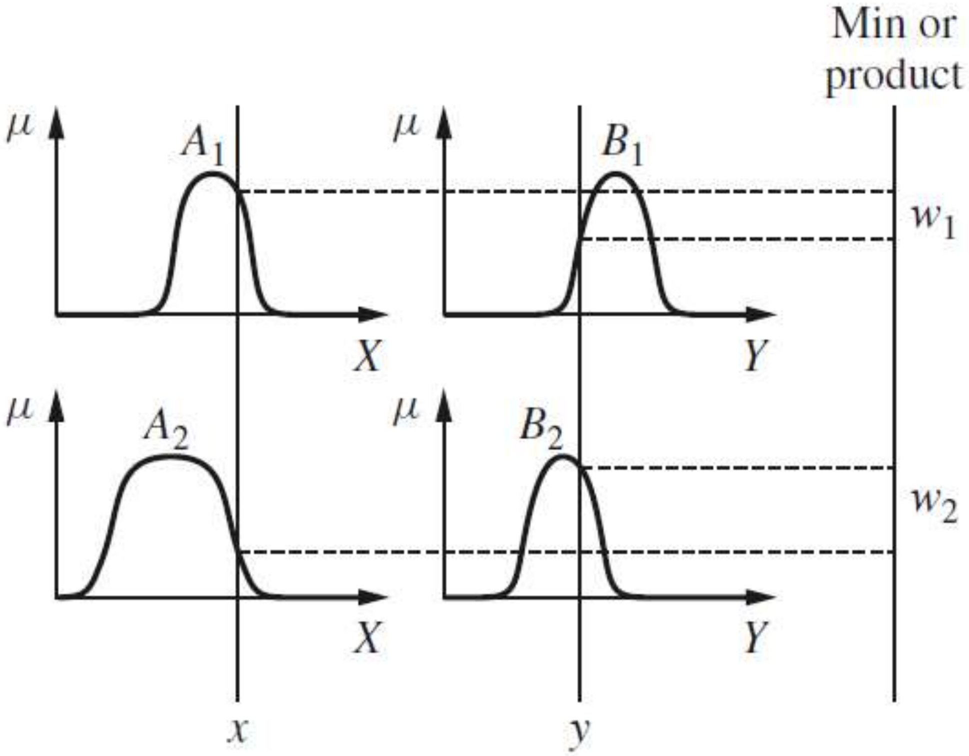
- The Takagi Sugeno rule, which has two inputs x and y and output z, is given as:

$$\text{IF } x \text{ is } \underset{\sim}{A} \textit{ and } y \text{ is } \underset{\sim}{B}, \text{ THEN } z \text{ is } z = f(x, y)$$

- The f (x, y) can be any function that describes the output of the system. A polynomial function is common.
- A zero-order system (special case of Mamdani system): f (x, y) is constant.
- A first-order system: f (x, y) is a linear function.

Continue…

And $\rightarrow$ min
or $\rightarrow$ max



$z_1 = p_1 x + q_1 y + r_1$

$z_2 = p_2 x + q_2 y + r_2$

Deffuzzification

Weighted average

$$\boxed{z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}}$$

Note: Each rule has a crisp output, thus, the overall output is obtained via a weighted average defuzzification

هنا ال output is a function → w, y
and not a membership function

5.31. From thermodynamics it is known that for an ideal gas in an adiabatic reversible process

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

By TaKagi Sugeno

where T_1 and T_2 are temperatures in kelvin (K) and P_1 and P_2 are pressures in bars and, for an ideal gas. For the Sugeno solution, use the following functions for the consequents of the three rules:

$$\text{Rule 1}: T_1 = 320 \text{ K and } \gamma = 1.5$$

$$\text{Rule 2}: T_1 = 300 \text{ K and } \gamma = 1.4$$

$$\text{Rule 3}: T_1 = 300 \text{ K and } \gamma = 1.3$$

For this problem, T_1 will be fixed at 300 K and the fuzzy model will predict P_2 for the given input variables P_1 and T_2. In other words, we are interested in finding the final pressure, P_2, of the system if the temperature of the system is changed to T_2 from an original pressure equal to P_1. A real application could use a similar model built from experimental data to do a prediction on nonideal gases.

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

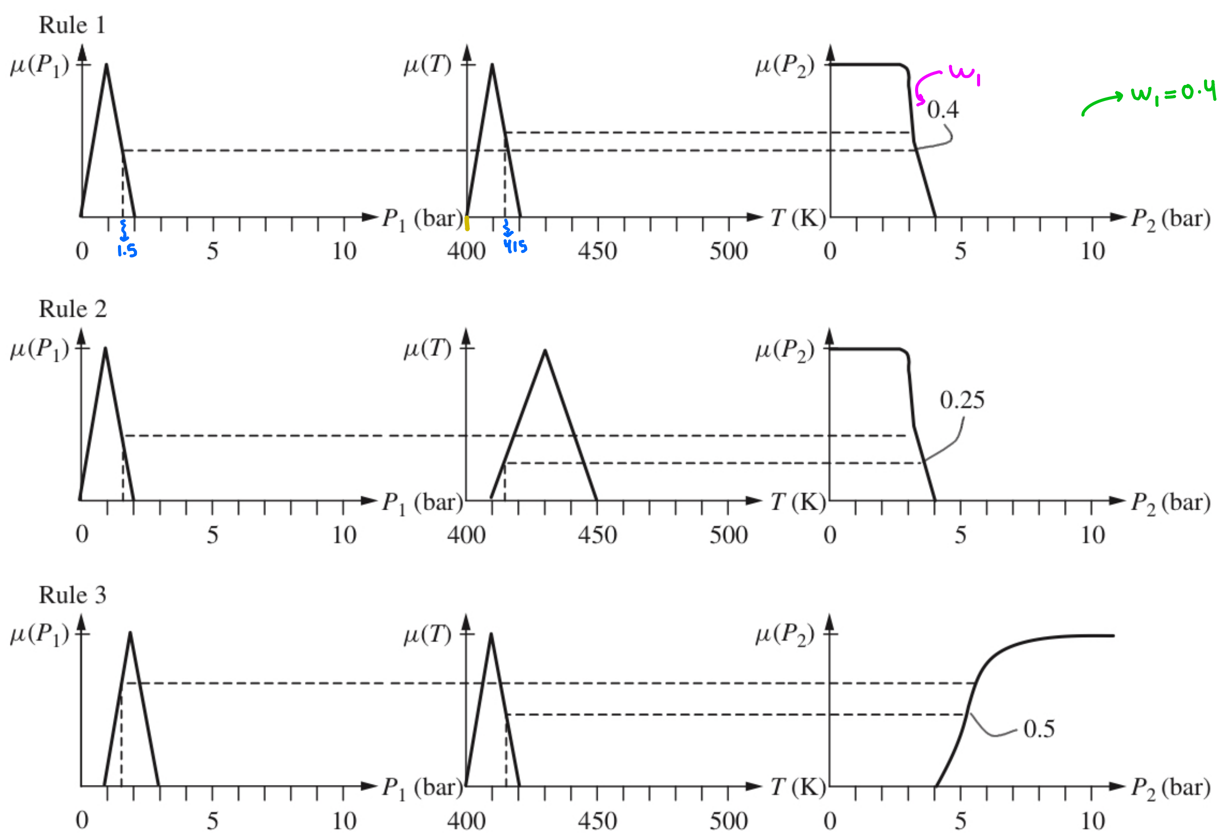


FIGURE P5.31

in Rule $\boxed{1}$ $(w_1 = 0.4)$

$$\frac{415}{320} = \left(\frac{P_2}{1.6}\right)^{\frac{1.5-1}{1.5}}$$

$\rightarrow P_2 = 3.49$

* in Rule $\boxed{2}$ $(w_2 = 0.25)$

$$\frac{415}{300} = \left(\frac{P_2}{1.6}\right)^{\frac{1.4-1}{1.4}}$$

$P_2 = 4.98$

* in Rule $\boxed{3}$ $(w_3 = 0.5)$

$P_3 = 6.53$

The rules used are

Rule 1: IF P_1 = atmP AND T_2 = lowT THEN P_2 = lowP.

Rule 2: IF P_1 = atmP AND T_2 = midT THEN P_2 = lowP.

Rule 3: IF P_1 = lowP AND T_2 = lowT THEN P_2 = very highP.

$P_2 = 0.4 \times 3.49 + 0.25 \times$

Given the rule-base, the membership functions shown in Figure P5.31, and the following pair of input values, $P_1 = 1.6$ bar and $T_2 = 415$ K, conduct a simulation to determine P_2 for the inference methods of Sugeno and Tsukamoto. For the Sugeno consequents use the ideal gas formula, given above. Ussing Taccagi Sogomo

5.31 For Sugeno method rules are as follows:

Rule1: T_1=320K Then γ=1.5

Rule2: T_1=300K Then γ=1.4

Rule3: T_1=300K Then γ=1.3

Solution :-

For problem T_1 is fixed to 300K and the fuzzy model will predict P_2 for the given variable P_1 ad T_2. In other words… what is the final pressure of the system if the temperature is changed to T_2 from a pressure equal to T_1?

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad P_2=P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{and in this problem} \quad T_1=300K$$

Input: P_1=1.6bar and T_2=415K

Rule2 and Rule3 are fired since T_1=300K.

From rule2: γ=1.4 thus P_2=5.0bar

From rule2: γ=1.3 thus P_2=6.5bar

Weighted average:

$$\boxed{P_2^*=\frac{0.25*5.0+0.5*6.5}{0.25+0.5}=6.0bar}$$

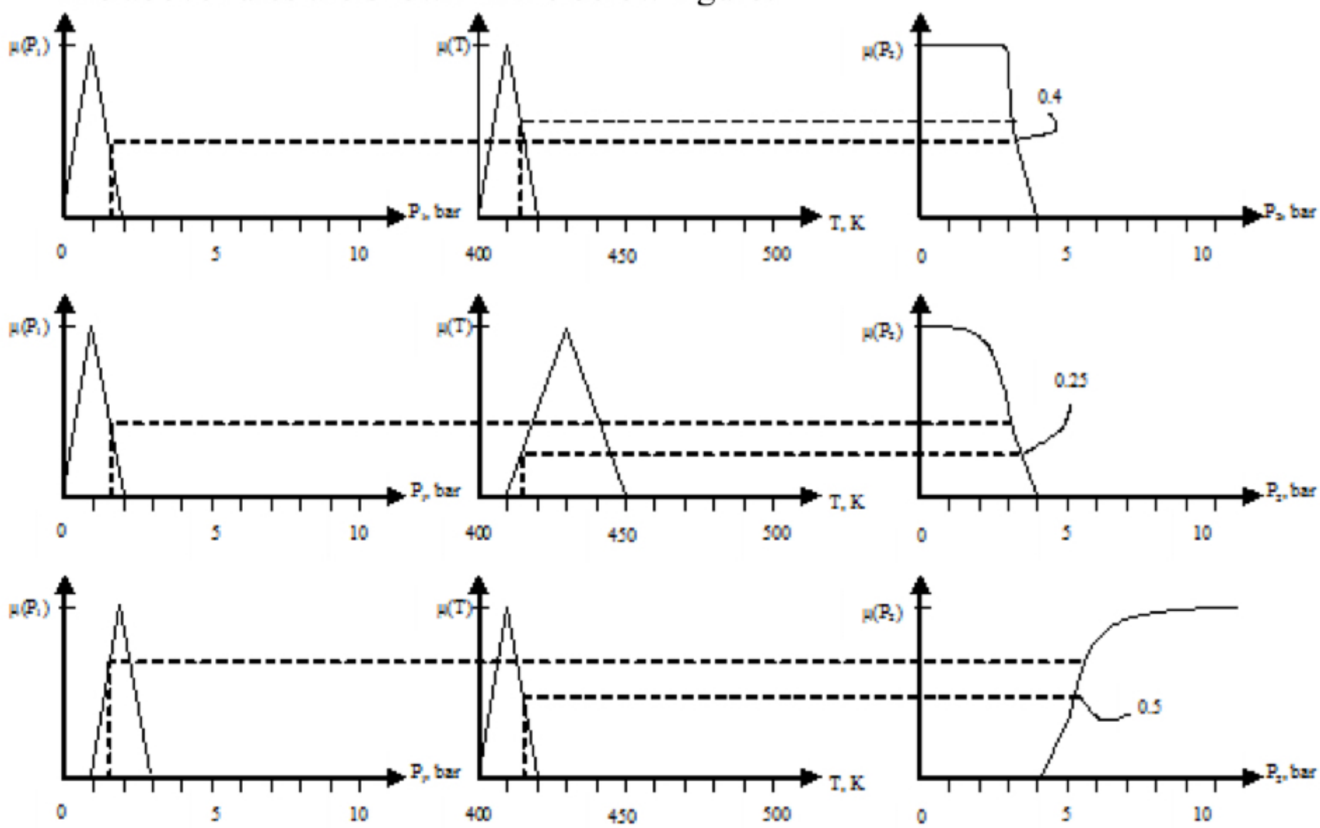
The rules used for Tsukamoto method are:

R1 → IF P_1 =atmP AND T_2 = lowT THEN P_2 = lowP

R2 → IF P_1 =atmP AND T_2 = midT THEN P_2 = midP

R3 → IF P_1 =lowP AND T_2 = lowT THEN P_2 = very highP

The above rules are shown in the below figure:



From the graph of rule2 and rule3 we have (P_2=3.5, μ_2=0.25) and (P_2=5, μ_2=0. 5) respectively.

Thus from the weighted average:

$$\boxed{P_2^*=\frac{0.25*3.5+0.5*5}{0.25+0.5}=4.5bar}$$

Example

in Fuzzy ussually:-

Multi input → single output

- A two-input, single-output Sugeno model with four rules is presented as follows (Jang *et al*.,1997):

IF X is small and Y is small, THEN $z = -x + y + 1$.

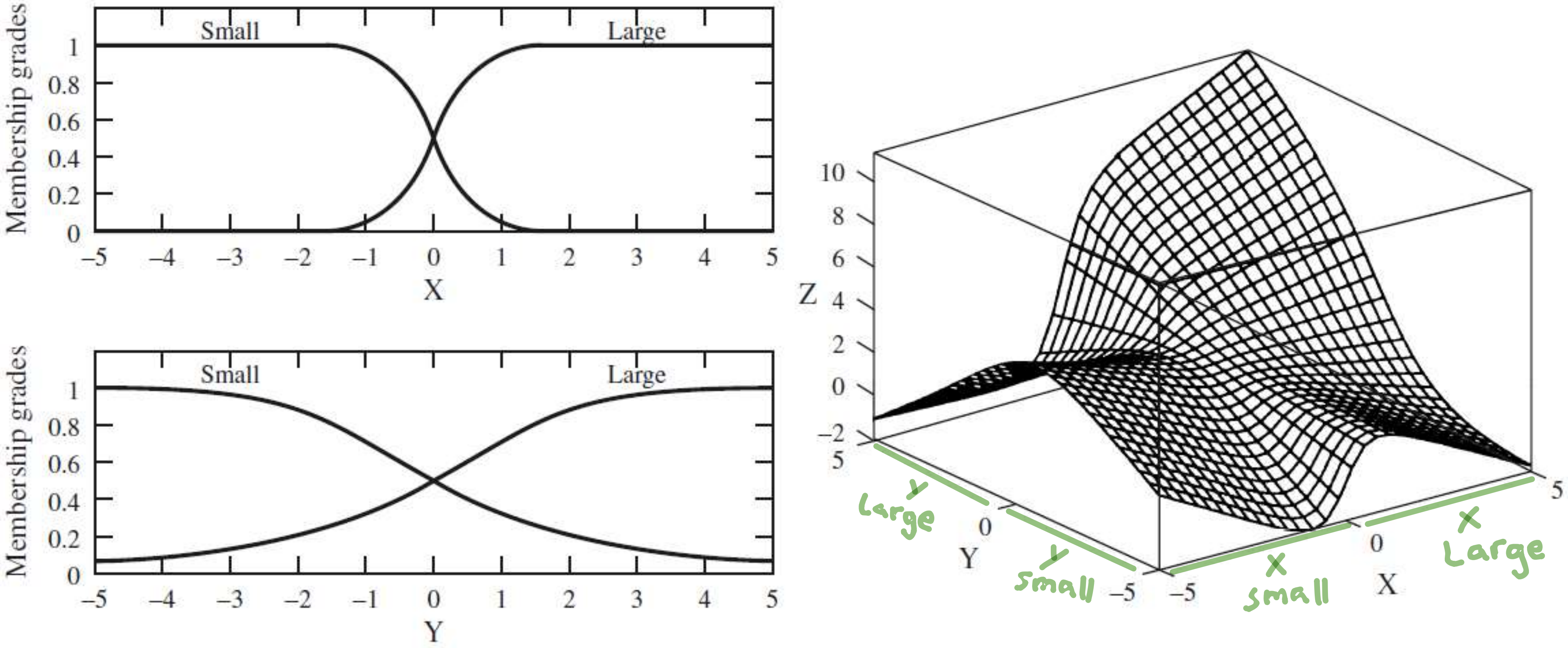
IF X is small and Y is large, THEN $z = -y + 3$.

IF X is large and Y is small, THEN $z = -x + 3$.

IF X is large and Y is large, THEN $z = x + y + 2$.

2 iputs < x y

Continue…



(a)

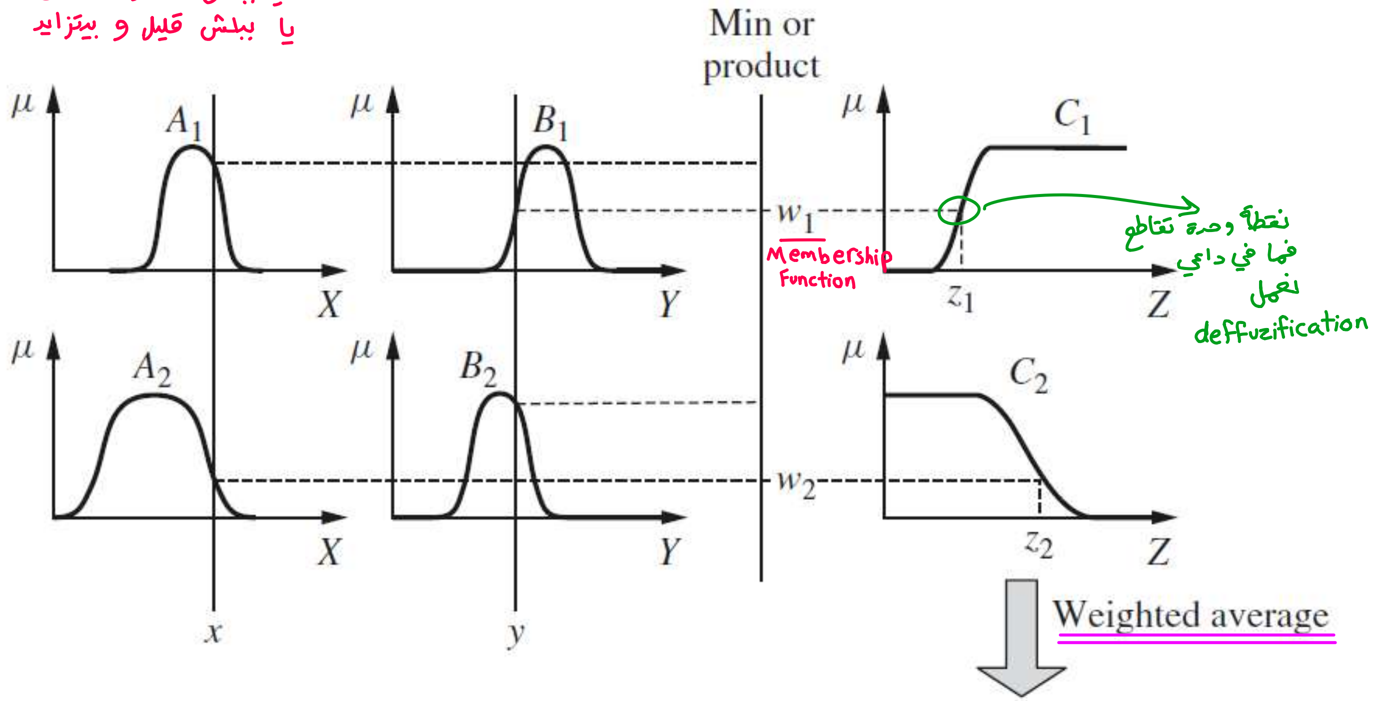
(b)

هنا ما عندي Fuction اعوض فيه

Inference: Tsukamoto Systems

- The consequent of each fuzzy rule is represented by a fuzzy set with a monotonic (shoulder) membership function.

يا ببلش زايد و بتناقص
يا ببلش قليل و بيتزايد



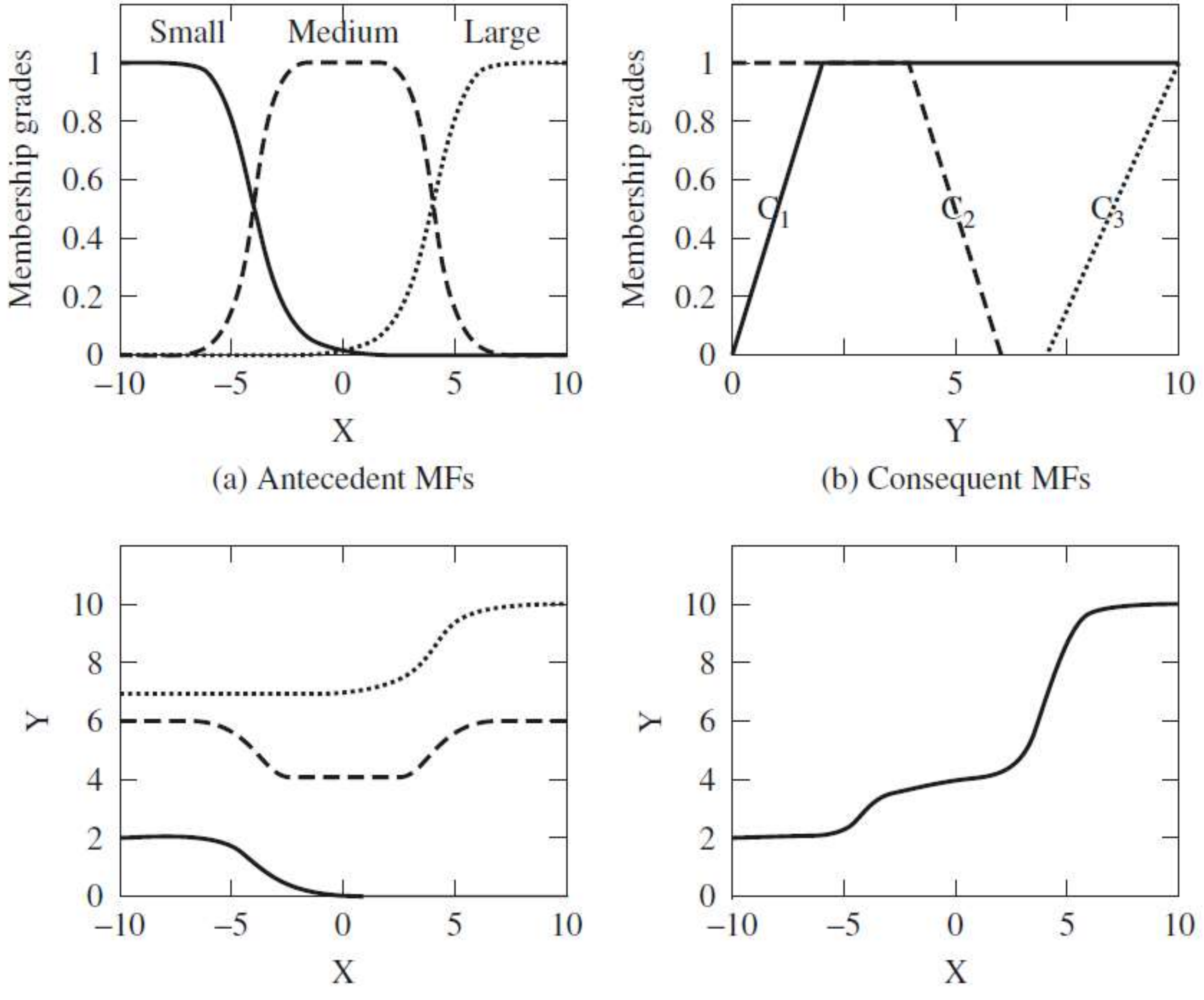
$$z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

Example

- A single-input, single-output Tsukamoto fuzzy model is given as follows:

IF X is small, THEN Y is C_1,
IF X is medium, THEN Y is C_2,
IF X is large, THEN Y is C_3,

Continue…



(a) Antecedent MFs

(b) Consequent MFs

(c) Each rule's output

(d) Overall input-output curve

5.31. From thermodynamics it is known that for an ideal gas in an adiabatic reversible process

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}},$$

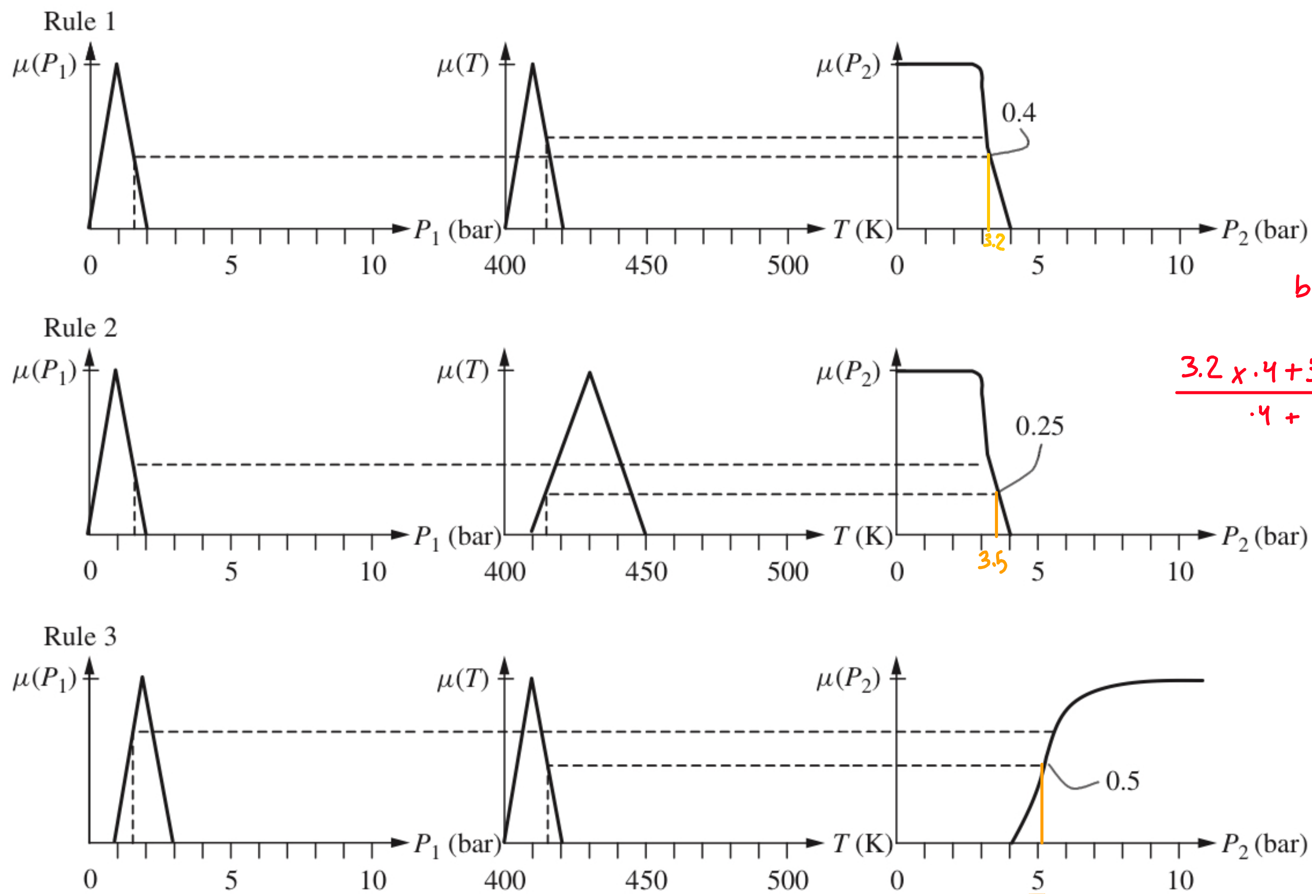
where T_1 and T_2 are temperatures in kelvin (K) and P_1 and P_2 are pressures in bars and, for an ideal gas. For the Sugeno solution, use the following functions for the consequents of the three rules:

$$\text{Rule 1}: T_1 = 320 \text{ K and } \gamma = 1.5$$

$$\text{Rule 2}: T_1 = 300 \text{ K and } \gamma = 1.4$$

$$\text{Rule 3}: T_1 = 300 \text{ K and } \gamma = 1.3$$

For this problem, T_1 will be fixed at 300 K and the fuzzy model will predict P_2 for the given input variables P_1 and T_2. In other words, we are interested in finding the final pressure, P_2, of the system if the temperature of the system is changed to T_2 from an original pressure equal to P_1. A real application could use a similar model built from experimental data to do a prediction on nonideal gases.



by Tsukomato Method

$$\frac{3.2 \times .4 + 3.5 \times .25 + 5 \times .5}{.4 + .25 + .5}$$

$= \boxed{}$

FIGURE P5.31

Chapter 6
Development of Membership Functions

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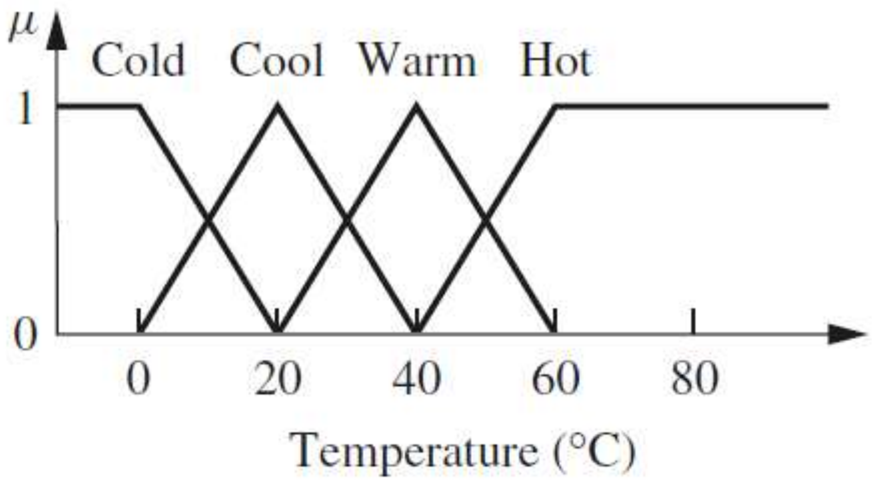
Membership Value Assignments

1 حدسي 2

- The assignment process can be intuitive or based on some algorithmic or logical operations.
- Common methods are:
  1. Intuition
  2. Inference
  3. Rank ordering
  4. Neural networks
  5. Genetic algorithms
  6. Inductive reasoning

Intuition

- The membership values or functions can be derived from the capacity of humans to develop them through their own innate intelligence and understanding.
- Example: Develop fuzzy membership functions for the temperature.
    - Very cold
    - Cold
    - Normal
    - Hot
    - Very hot



its not the only way for defining the membership Function

(مثلاً ممكن بالمانيا غير)

intuition → الرأي الشخصي حسب تجربتنا

inference → مبني على الخبراء (experience Knowlege)

2 Inference

التفكير الاستنتاجي

- Knowledge is utilised to perform deductive reasoning.
- Example: Let U be the universe of triangles, where the inner angles are A, B and C.

متساوي الساقين

$\underset{\sim}{\mathrm{I}}$ Approximate isosceles triangle

$\underset{\sim}{\mathrm{R}}$ Approximate right triangle

$\underset{\sim}{\mathrm{IR}}$ Approximate isosceles *and* right triangle

$\underset{\sim}{\mathrm{E}}$ Approximate equilateral triangle

$\underset{\sim}{\mathrm{T}}$ Other triangles.

$$\mu_{\underset{\sim}{\mathrm{I}}}(A, B, C) = 1 - \frac{1}{60^\circ}\min(A - B, B - C)$$

$$\mu_{\underset{\sim}{\mathrm{R}}}(A, B, C) = 1 - \frac{1}{90^\circ}|A - 90^\circ|$$

كلما كانت الزاوية (90°) يكون اقرب اكثر لل membership degree (واحد)

$$\underset{\sim}{\mathrm{IR}} = \underset{\sim}{\mathrm{I}} \cap \underset{\sim}{\mathrm{R}},$$

$$\mu_{\underset{\sim}{\mathrm{E}}}(A, B, C) = 1 - \frac{1}{180^\circ}(A - C)$$

Question: Assume the Universe is the (GPA)

and we have
- Excellent → (3.65 − 4)
- Very Good → (3 − 3.64)
- Good → (2.5 − 2.99)
- Satisfaction → (2 − 2.49)

by using inference (your own Knowledge)

(we have to write 4 equations of Membership Function)

- $\mu_{Good} = 1 - \frac{1}{2.5} \max(2.99 - c, 2.5)$
-
-
-

3

Rank Ordering

it is easier but the Knowledge is not as accurate as inference

- Preference is determined by pairwise comparisons, and these determine the ordering of the membership.
- Example: Suppose 1000 people respond to a questionare about their pairwise preferences among five colours, X = {red, orange, yellow, green, blue}.

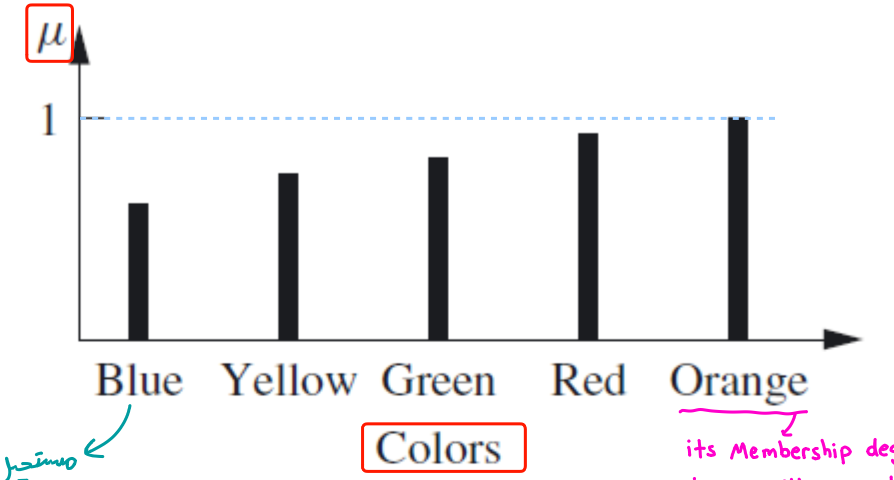
حسب ترتيب الـ Percentages

	Number who preferred							
	Red	Orange	Yellow	Green	Blue	Total	Percentage	Rank order
Red	–	517	525	545	661	2 248	22.5	2
Orange	483	–	841	477	576	2 377	23.8	1
Yellow	475	159	–	534	614	1 782	17.8	4
Green	455	523	466	–	643	2 087	20.9	3
Blue	339	424	386	357	–	1 506	15	5
Total						10 000		

لأنه الشخص
عم ينجمع اكثر من مرة

Continue…

- Membership function for the best colour



zero Membership degree مستحيل يوصل

لأنه برضه عليه طلب

its Membership degree is one with respect to others

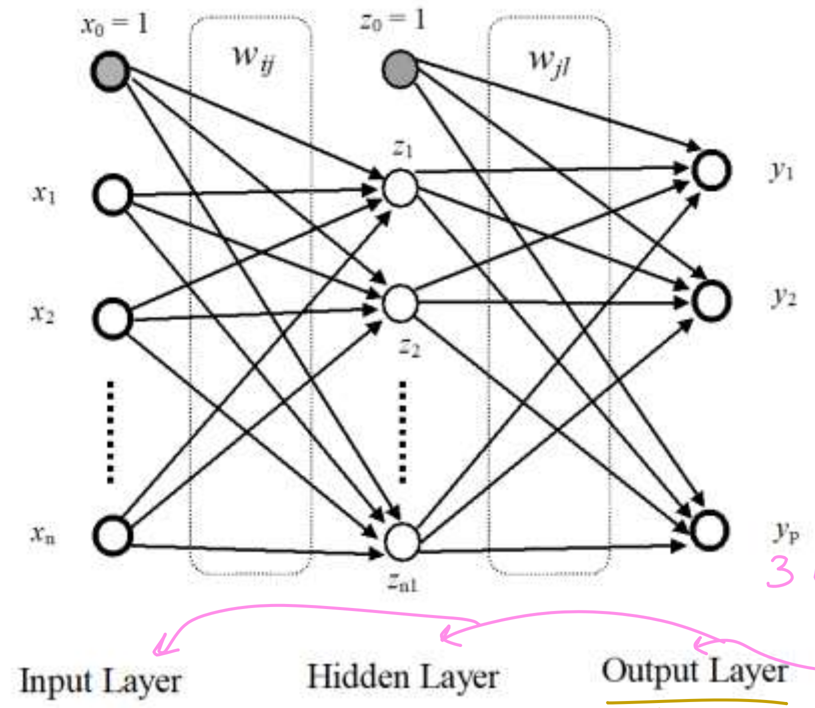
4

Neural Networks

Rarely we use in development of membership Function

There is an input and according to it there will be an output

- A neural network: is a technique that builds an intelligent system by simulating the biological neural network.



3 Layers

يختلفو حسب اختلاف الProcess

$$z_j(k) = f_j\left(\sum_{i=1}^{n} w_{ij} x_i(k) + b_j\right), \quad j = 1,2,\ldots,n_1; \quad k = 1,2,\ldots$$

$$y_l(k) = f_l\left(\sum_{j=1}^{n_1} w_{jl} z_j(k) + b_l\right); \quad l = 1,2,\ldots,p; \quad k = 1,2,\ldots$$

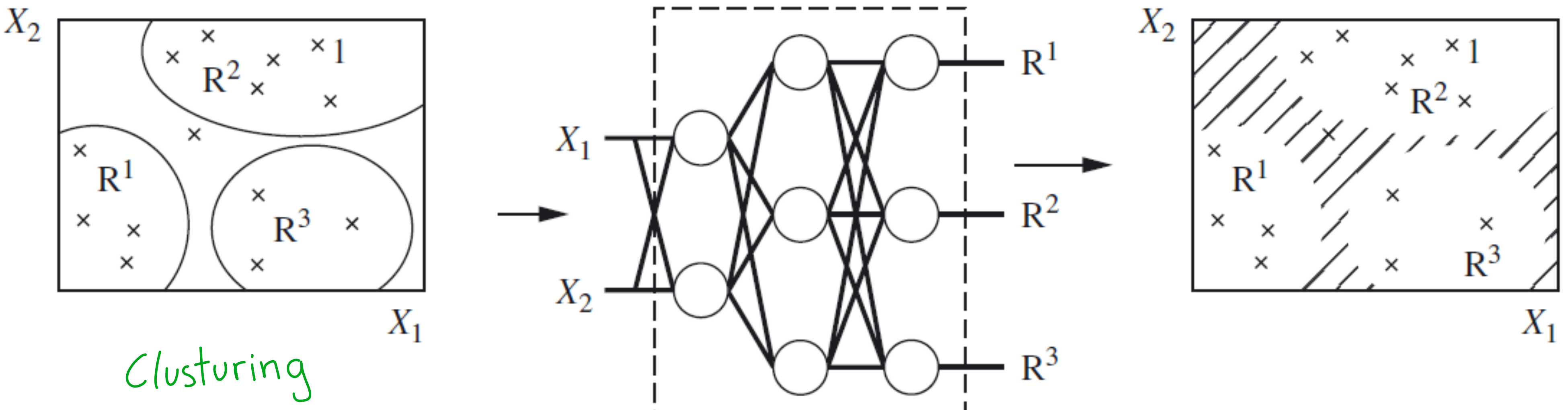
$$E(k) = \frac{1}{2}\sum_{l=1}^{p}\left(y_l(k) - y_l^{t}(k)\right)^2$$

$$w_{jl}(k+1) = w_{jl}(k) - \alpha \nabla_{w_{jl}} E(k)$$

Distributing the error using back-propagation technique

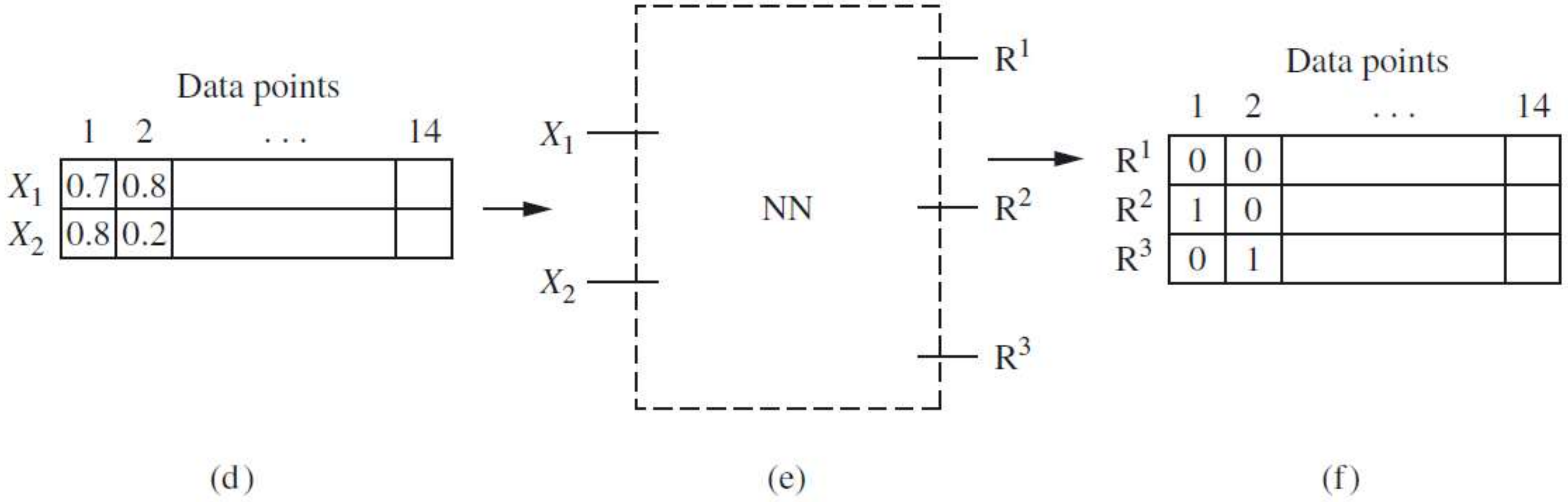
Continue…

- Determining the membership function



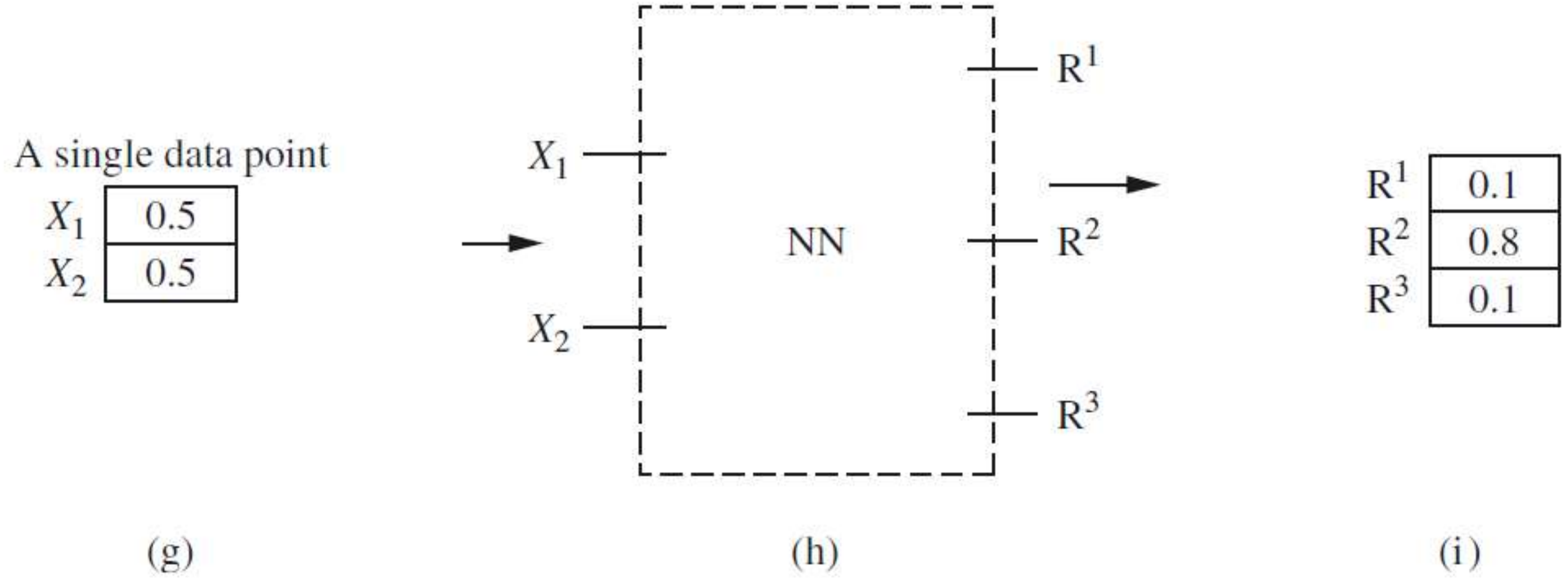
(a) (b) (c)

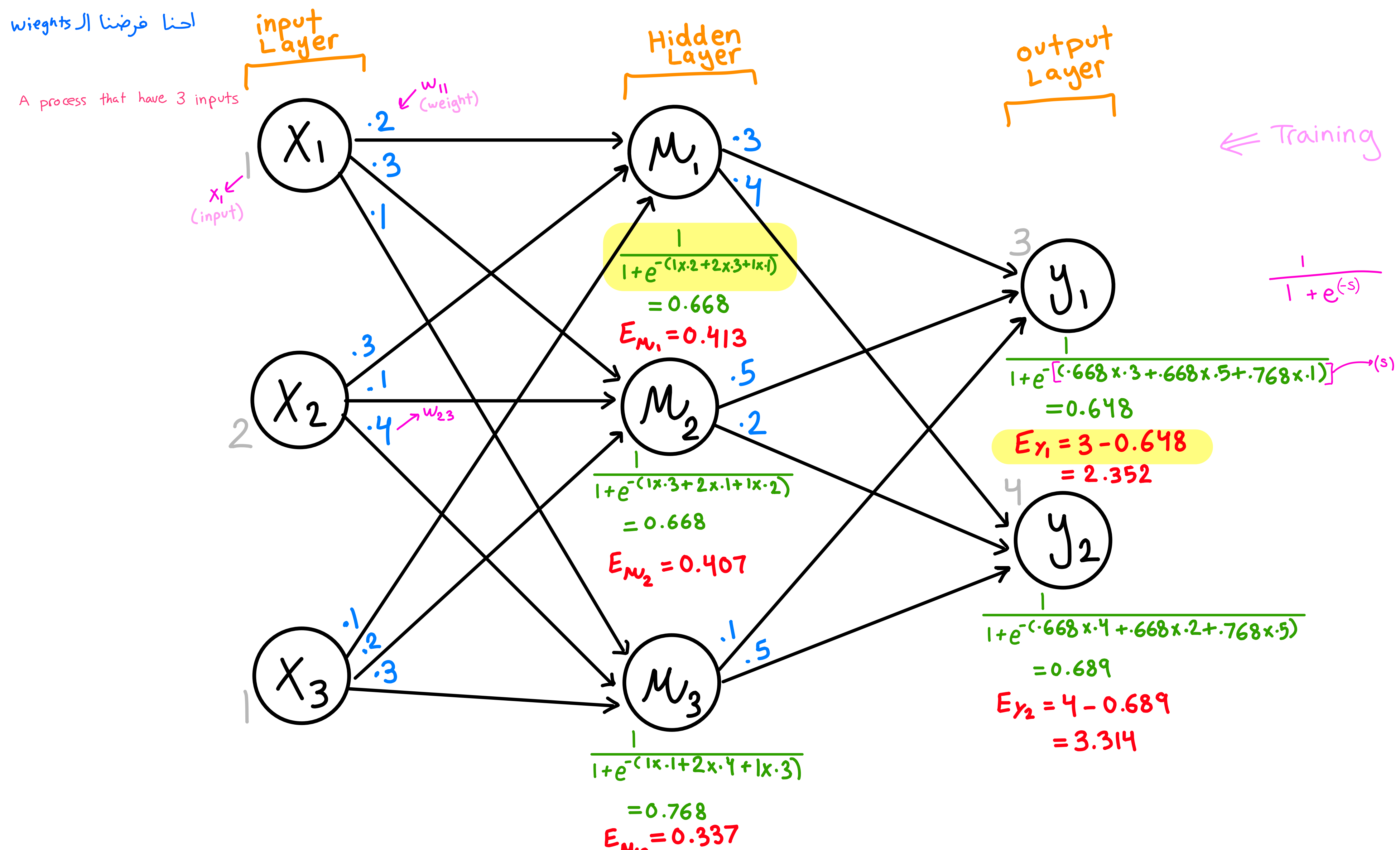
Continue…



Data points
1
2
...
14
X_1
0.7
0.8
X_2
0.8
0.2
(d)
X_1
X_2
NN
R^1
R^2
R^3
(e)
Data points
1
2
...
14
R^1
0
0
R^2
1
0
R^3
0
1
(f)

Continue…


A single data point
X_1
0.5
X_2
0.5
X_1
X_2
NN
R^1
R^2
R^3
R^1
0.1
R^2
0.8
R^3
0.1
(g)
(h)
(i)



Calculations:-

$* \; E_n = O_n(1-O_n) \times \sum W_e$

$E_1 = .668(1-.668)\times[.3\times 2.352+.4\times 3.314] = 0.413 \leftarrow (E_{M_1})$

$E_2 = .668(1-.668)\times[.5\times 2.352+.2\times 3.314] = 0.407 \leftarrow (E_{M_2})$

$E_3 = .768(1-.768)\times[.1\times 2.352+.5\times 3.314] = 0.337 \leftarrow (E_{M_3})$

*Now we will calculate the New Weights given that $\alpha = 0.3$

$$w_{new} = w_{old} + \underbrace{\alpha}_{\text{Learning Factor}} * \underbrace{E}_{\text{Error Neuron for next Layer}} * \underbrace{X}_{\text{input for it}}$$

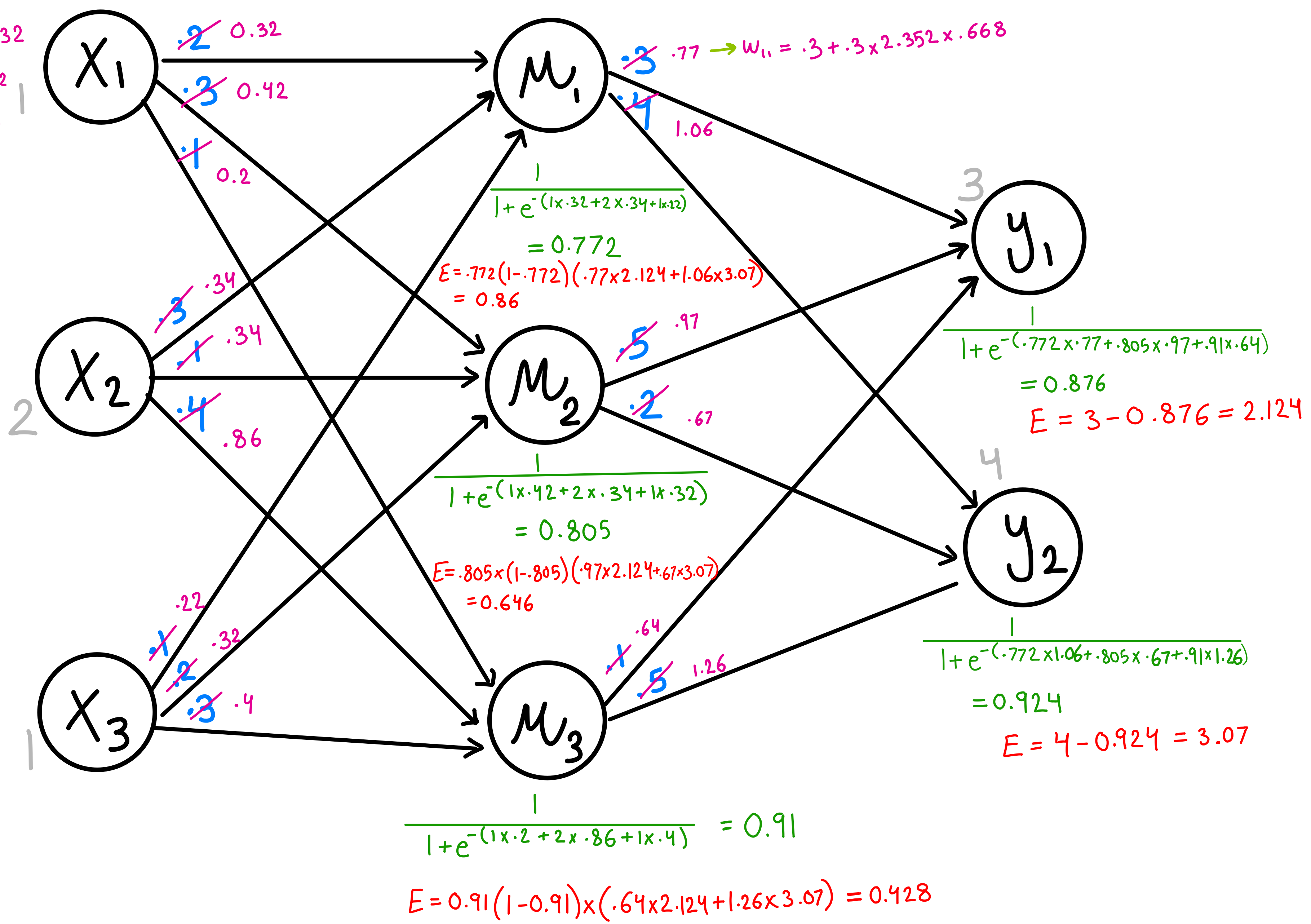
And Do the Forward Calculation for the New weights ~~(H.w)~~

Wnew
لتعديل
الweight

$w_{11\,new} = .2 + .3 \times .413 \times 1 = 0.32$

$w_{12\,new} = .3 + .3 \times .407 \times 1 = 0.42$

$w_{13\,new} = .1 + .3 \times .337 \times 1 = 0.20$



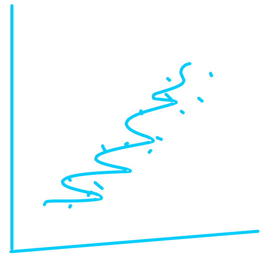
	X_1	X_2	X_3	Y_1	Y_2
1					
⋮					
مثلاً 20					

بقسمها

Training 70%
لتطوير ال Model

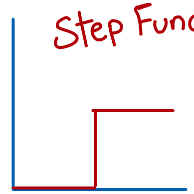
Validation 15%

Testing 15%
بسحب عينات لل Testing
لكن هنا مثلاً
العينات قلال
فلغيناه و
صارت ال Validation
30%



X we will not use it

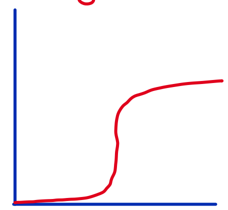
Step Function



Reaction
or
Not

Sigmoid Function ✓

اشتقاقه هو
ال Function نفسه
ضرب (1- Function)



$$\frac{1}{1+e^{(-s)}}$$

it is a must to use the threshold when we have (0,0,0)

Threshold
ال value يلي
بناءاً عليها بصير
Activation
for Reaction

Input and Output Layers

بتحددو بناءاً على الـSimulation يلي بنعمله

بضل اعيد الـ Process لحتى اوصل للـ Error in Output ← أقل من 5%

Normalization Process → نخلي كل الـdata بين الـ صفر والواحد.

و هي بتنعمل قبل الـ Neural Network

مين أفضل ناخد 2 output in neoron network أو 1 output in every neoron network

الجواب 1 output at a time لأنه هيك بيكون أدق

$$\frac{\text{Speed} - \min}{\max - \min}$$

[0,1]

DOC → depth of cut

[0,1]

DOC
→ same [0,1]

DOF
→ .3
.2
.1

(بعاد عن بعض بـ Range)

So in terms of Mathematics we do something similar to Normalization and when we find the output

→ $PV * (\max - \min) + \min$

للتوضيح

DOC	SP
0.1	100
0.2	200
0.3	300

$\frac{.1 - .1}{.3 - .2}$

training error / Validation error / Testing error

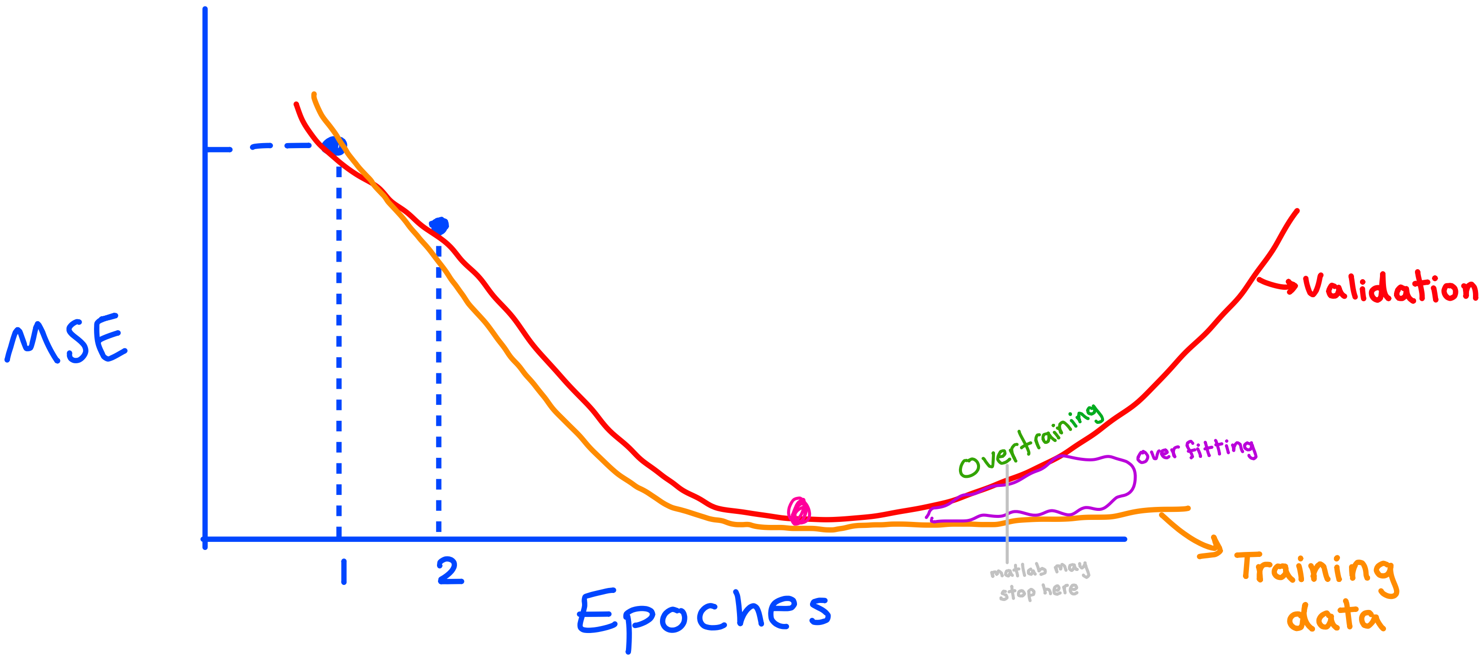
المفروض الـ Error يكون أقل ما يمكن

و يكونو مقتربين من بعض

Error can be represented by many ways →
- MSE
- RMSE
- $\Sigma |E|$

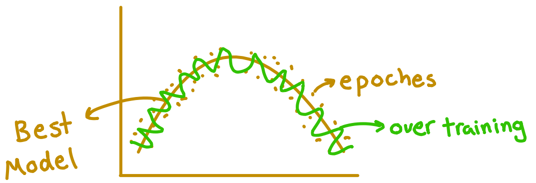
Data →
- traing → لتدريب ال model
- Validation → لتوقيف ال model قبل وصول ال over training
- Testing

Epoche → is an iteration



Validation → بصير يرفع بنهاية كل iteration

Testing → بجربها على ال Best Model



Best epoches → Best Model → بسويله Testing

هاي الأشياء ممكن تحدد متى نوقف

ممكن تغييرهم → Max # of epoches

# of Max epoches → 1000

% of Error → $E \leq 0.05$

* مستحيل ال next stage تكون أسوء من ال Previous stage لأني بضل آخذ الأفضل

Genetic Algorithms

- Genetic algorithms use the concept of Darwin's theory of evolution; "survival of the fittest". → البقاء للأفضل
- New breeds or classes come into existence through the processes of reproduction, crossover, and mutation (طفرة) among existing organisms.

  بعكس كل الجينات يلي عندي (طفرة جينية)
- The algorithms procedure can be summarized as follows:
  1. Different possible solutions are created.
  2. They are then tested for their performance.
  3. Among all of them, a fraction of the good solutions is selected, and the others are eliminated.
  4. The selected solutions undergo the processes of reproduction, crossover, and mutation to create a new generation of possible solutions.

Continue…

مثل فكرة الجينات

- In a genetic algorithm, the parameter set is coded as a $\boxed{\text{finite string}}$, which is presented as a combination of zeros and ones.
- For example, the number 7 requires a 3-bit string, that is, $2^3 - 1 = 7$, and the bit string would look like “111”.
- So the number 10 would look like: “1010”.

0	0	0	1	1	1
2^5	2^4	2^3	2^2	2^1	2^0

→ $2^0 + 2^1 + 2^2 = 7$

0	1	0	1	1	0
2^5	2^4	2^3	2^2	2^1	2^0

String
(it gives me a coded string)

→ $2^1 + 2^2 + 2^4 = 22$

b

mutation for this

1	0	1	0	0	1

0	0	0	1	1	0

6

cross over

0	1	0	1	1	1

23

mutation

1	1	1	0	0	1

→ then I can use it to make
→ crossover
→ Reproduction

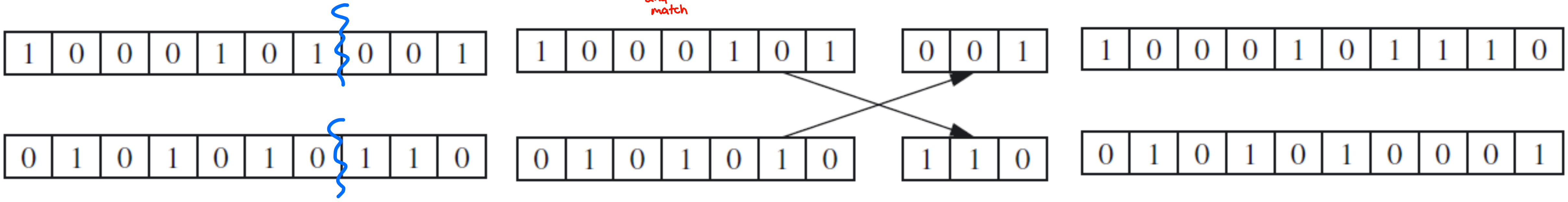
$2^5 \downarrow 32$ $\quad 2^4 \downarrow 16$ $\quad 2^3 \downarrow 8$ $\quad 2^2 \downarrow 4$ $\quad 2^1 \downarrow 2$

Continue…

make a copy for a string

- Reproduction is the process by which strings with better fitness values receive correspondingly better copies in the new generation, to ensure that better solutions persist and contribute to better offspring (new strings).
- Crossover is the process in which the strings are able to mix and match their desirable qualities in a random fashion.

لأني ما بعرف عدد ال bins يلي بدي اسوي بينهم cut and match



(a) (b) (c)

(crossover)

Continue…

- Mutation is the process by which the value at a certain string location is changed; if there is a one originally at a location in the bit string, it is changed to a zero, or vice versa.

  0 → 1
  1 → 0

طفرة
- Mutation takes place very rarely, on the order of once in a thousand bit string locations.

Genetic Algorithms: Example

- Using the data provided in the table below, perform a line fit ($y=C_1x+C_2$).

missing

Actual

Data number	x	y'
1	1.0	1.0
2	2.0	2.0
3	4.0	4.0
4	6.0	6.0

The algorithms procedure can be summarized as follows:

- *Different possible solutions are created.*
- *They are then tested for their performance.*
- *Among all of them, a fraction of the good solutions is selected, and the others are eliminated.*
- *The selected solutions undergo the processes of reproduction, crossover, and mutation to create a new generation of possible solutions.*

Continue…

for String number 1

X	Actual Y	predicted Y
1	1	-1
2	2	-2.22
4	4	-4.66
6	6	-7.11

Maximum ※ I can represent in a String with 6 bit

1	1	1	1	1	1
2^5	2^4	2^3	2^2	2^1	2^0

→ 63

Predicted

to determine which y predicted is better

Error in 1st row → $(1--1)^2+(2--2.22)^2+(4--4.66)^2+(6-7.11)^2=2686$

(1) String number	(2) String	(3) C_1	(4) C_1 (binary)	(5) C_2	(6) C_2 (binary)	(7) y_1	(8) y_2	(9) y_3	(10) y_4		(11) $f(x) = 400 - \Sigma(y_i - y_i')^2$	(12) Expected count $= f/f_{av}$	(13) Actual count
	C_1 C_2	Decimal form b	C_i	Decimal form b	C_i	for 1 $y_1=-1.22x+.22$	for 2	for 4	for 6			→ 307.29	
1	000111 010100	7	−1.22	20 2^2+2^4	0.22	−1.00	−2.22	−4.66	−7.11		~~147.49~~ 268.67	0.48	0 يعني ما راح اخدها
2 Random	010010 001100	18	0.00	12	−0.67	−0.67	−0.67	−0.67	−0.67		~~332.22~~ 131	1.08	1
3	010101 101010	21	0.33	42	2.67	3.00	3.33	~~5.00~~ 3.66	4.67		391.44	1.27	2 أفضل solution
4	100100 001001	36	2.00	9	−1.00	1.00	3.00	3.67	11.00		358.00	1.17	1
										Sum	1229.15		
										Average	$\boxed{307.29}$		
										Maximum	391.44		

L = 6

المفروض ناخذ لغاية 1000

← هنا الارقام غلط

$$\boxed{C_i = C_{\min} + \frac{b}{2^L - 1}(C_{\max_i} - C_{\min_i})}$$

Cmax, Cmin → بكونو معطى في السؤال

The minimum and the maximum values are -2 and 5, respectively. L=6 bits and b is the number in the decimal form.

$C_{min} = -2$
$C_{max} = 5$

※ of bits

※ in the decimal form

To convert the problem into a maximization one with a cut-off value equals to 0.8.

cutoff value (threshold value) أي اشي اقل من 0.8 ما باخذها

Number of copies

$C_i = -2 + \frac{21}{2^6-1}(5--2) = 0.33$

Continue…

كل Table عبارة عن iteration

(1) Selected strings	(2) New strings	(3) C_1	(4) C_1 (binary)	(5) C_2	(6) C_2 (binary)	(7) y_1	(8) y_2	(9) y_3	(10) y_4	(11) $f(x) = 400 - \Sigma(y_i - y_i')^2$	(12) Expected count $= f/f_{av}$	(13) Actual count
0101|01 101010	010110 001100	22	0.44	12	−0.67	−0.22	0.22	1.11	2.00	375.78	1.15	1
0100|10 001100	010001 101010	17	−0.11	42	2.67	2.56	2.44	2.22	2.00	380.78	1.17	2
010101 101|010	010101 101001	21	0.33	41	2.56	2.89	3.22	3.89	4.56	292.06	0.90	1
100100 001|001	100100 001010	36	2.0	10	−0.89	1.11	3.11	7.11	11.11	255.73	0.78	0
									Sum	1304.35		
									Average	326.09		
									Maximum	380.78		

أقل من 0.8
اذا صفر
ما باخدها

Membership Function

Genetic Algorithms: MF

- Membership functions and their shapes are assumed for various variables defined for a problem. They are then coded as bit strings.

Example

- Let us consider that we have a single-input (x), single-output (y) system with input–output values as shown below:

x	1	2	3	4	5
y	1	4	9	16	25

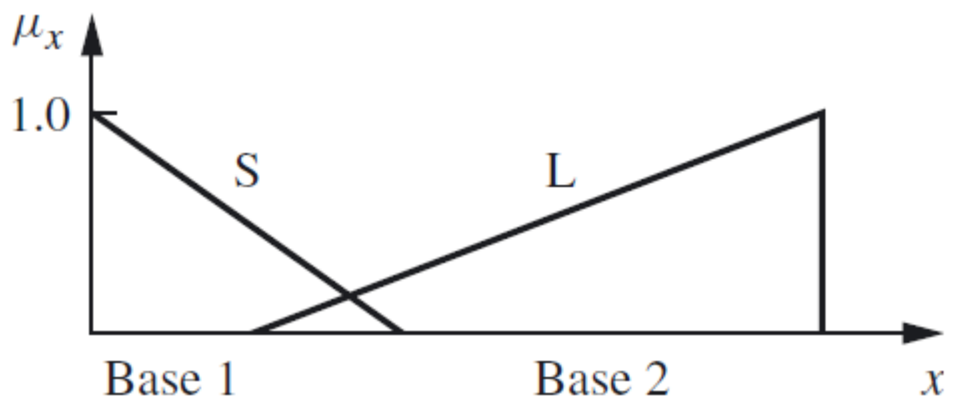
x	S	L
y	S	VL

Continue…

- We assume that the range of the variable x is [0, 5] and that of y is [0, 25].
- Membership function:

* اخترنا Right Triangle لأنه اله أقل عدد parameters من بين كل ال Membership Functions

ال Parameter الوحيد هو القاعدة



Continue…

String number	(1) String	(2) Base 1 (binary)	(3) Base 2 (binary)	(4) Base 3 (binary)	(5) Base 4 (binary)	(6) Base 1	(7) Base 2	(8) Base 3	(9) Base 4	(10) y' $(x = 1)$	(11) y' $(x = 2)$	(12) y' $(x = 3)$	(13) y' $(x = 4)$	(14) y' $(x = 5)$	(15) $1000 - \Sigma(y_i - y_i')^2$	(16) Expected count = f/f_{av}	(17) Actual count
1	000111 010100 010110 110011	7	20	22	51	0.56	1.59	8.73	20.24	0	0	0	12.25	25	887.94	1.24	1
2	010010 001100 101100 100110	18	12	44	38	1.43	0.95	17.46	15.08	12.22	0	0	0	25	521.11	0.73	0
3	010101 101010 001101 101000	21	42	13	40	1.67	3.33	5.16	15.87	3.1	10.72	15.48	20.24	25	890.46	1.25	2
4	100100 001001 101100 100011	36	9	44	35	2.86	0.71	17.46	13.89	6.98	12.22	0	0	25	559.67	0.78	1
														Sum	2859.18		
														Average	714.80		
														Maximum	890.46		

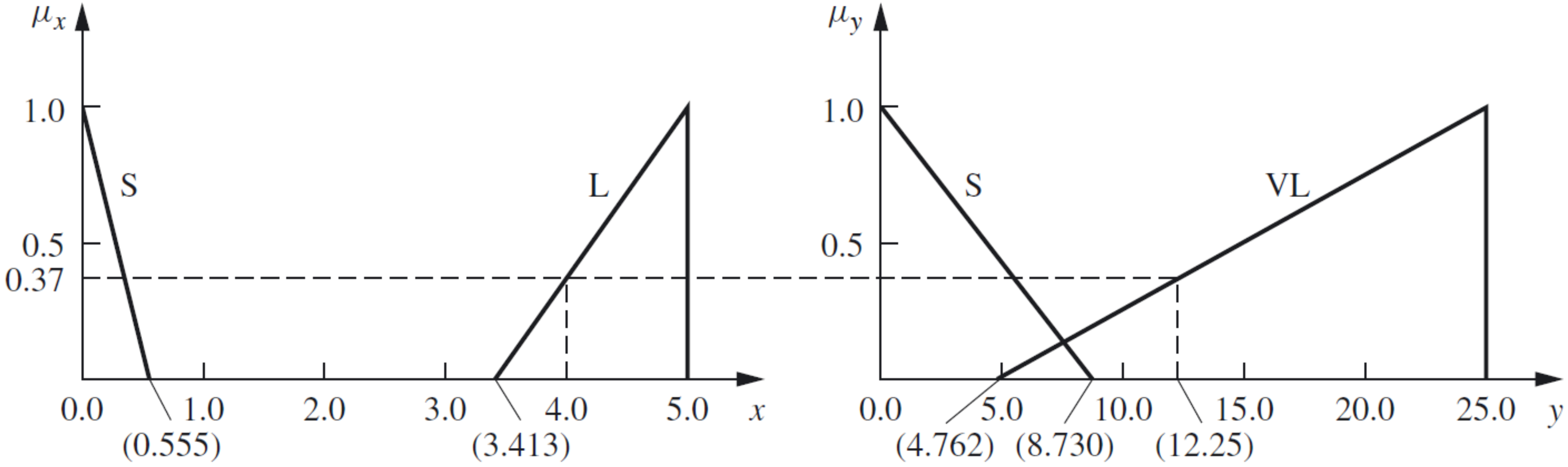
we define at least 300 solutions in the Genetic Algorithm but in this example we defined only 4 just to understand the idea

$$Min + \frac{b}{2^L - 1}(max - Min)$$

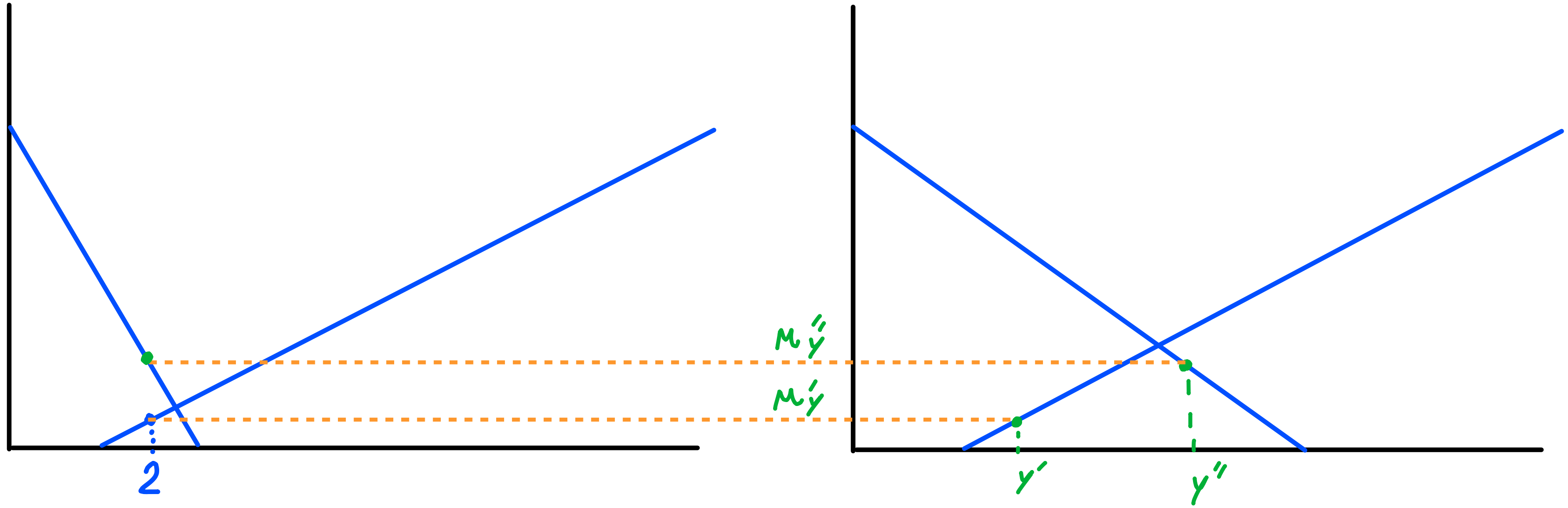
$$0 + \frac{7}{2^6 - 1}(5 - 0) = 0.56$$

※ يلي بصير بالواقع انه بنوصل لل Max of iterations لل Genetic Algorithm
قبل ما نوصل لل Function المناسب

Continue…



μ_x
1.0
0.5
0.37
S
L
0.0
1.0
2.0
3.0
4.0
5.0
x
(0.555)
(3.413)
μ_y
1.0
0.5
S
VL
0.0
5.0
10.0
15.0
20.0
25.0
y
(4.762)
(8.730)
(12.25)



* بدي اعملهم Deffuzification بأي طريقة من ال ٧ يلي درسناهم و عادي لو أخذنا أسهل طريقة يلي هي ال weighted Average

Continue…

<table>
<tr><th>(1)
Selected strings</th><th>(2)
New Strings</th><th>(3)
Base 1 (binary)</th><th>(4)
Base 2 (binary)</th><th>(5)
Base 3 (binary)</th><th>(6)
Base 4 (binary)</th><th>(7)
Base 1</th><th>(8)
Base 2</th><th>(9)
Base 3</th><th>(10)
Base 4</th><th>(11)
y' $(x = 1)$)</th><th>(12)
y' $(x = 2)$</th><th>(13)
y' $(x = 3)$</th><th>(14)
y' $(x = 4)$</th><th>(15)
y' $(x = 5)$</th><th>(16)
1000− $\Sigma(y_i - y_i')^2$</th><th>(17)
Expected count = f/f_{av}</th><th>(18)
Actual count</th></tr>
<tr><td>000111 0101|00 010110 110011</td><td>000111 010110 001101 101000</td><td>7</td><td>22</td><td>13</td><td>40</td><td>0.56</td><td>1.75</td><td>5.16</td><td>15.87</td><td>0</td><td>0</td><td>0</td><td>15.93</td><td>25</td><td>902.00</td><td>1.10</td><td>1</td></tr>
<tr><td>010101 1010|10 001101 101000</td><td>010101 101000 010110 110011</td><td>21</td><td>40</td><td>22</td><td>51</td><td>1.67</td><td>3.17</td><td>8.73</td><td>20.24</td><td>5.24</td><td>5.85</td><td>12.23</td><td>18.62</td><td>25</td><td>961.30</td><td>1.18</td><td>2</td></tr>
<tr><td>010101 101010 001101 101000</td><td>010101 101010 001101 10|0011</td><td>21</td><td>42</td><td>13</td><td>35</td><td>1.67</td><td>3.33</td><td>5.16</td><td>13.89</td><td>3.1</td><td>12.51</td><td>16.68</td><td>20.84</td><td>25</td><td>840.78</td><td>1.03</td><td>1</td></tr>
<tr><td>100100 001001 101100 100011</td><td>100100 001001 101100 10|1000</td><td>36</td><td>9</td><td>44</td><td>40</td><td>2.86</td><td>0.71</td><td>17.46</td><td>15.87</td><td>6.11</td><td>12.22</td><td>0</td><td>0</td><td>25</td><td>569.32</td><td>0.70</td><td>0</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Sum</td><td>3 273.40</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Average</td><td>818.35</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Maximum</td><td>961.30</td><td></td><td></td></tr>
</table>

(Chapter 7)

Type-1 Fuzzy Logic System (T1FLS)

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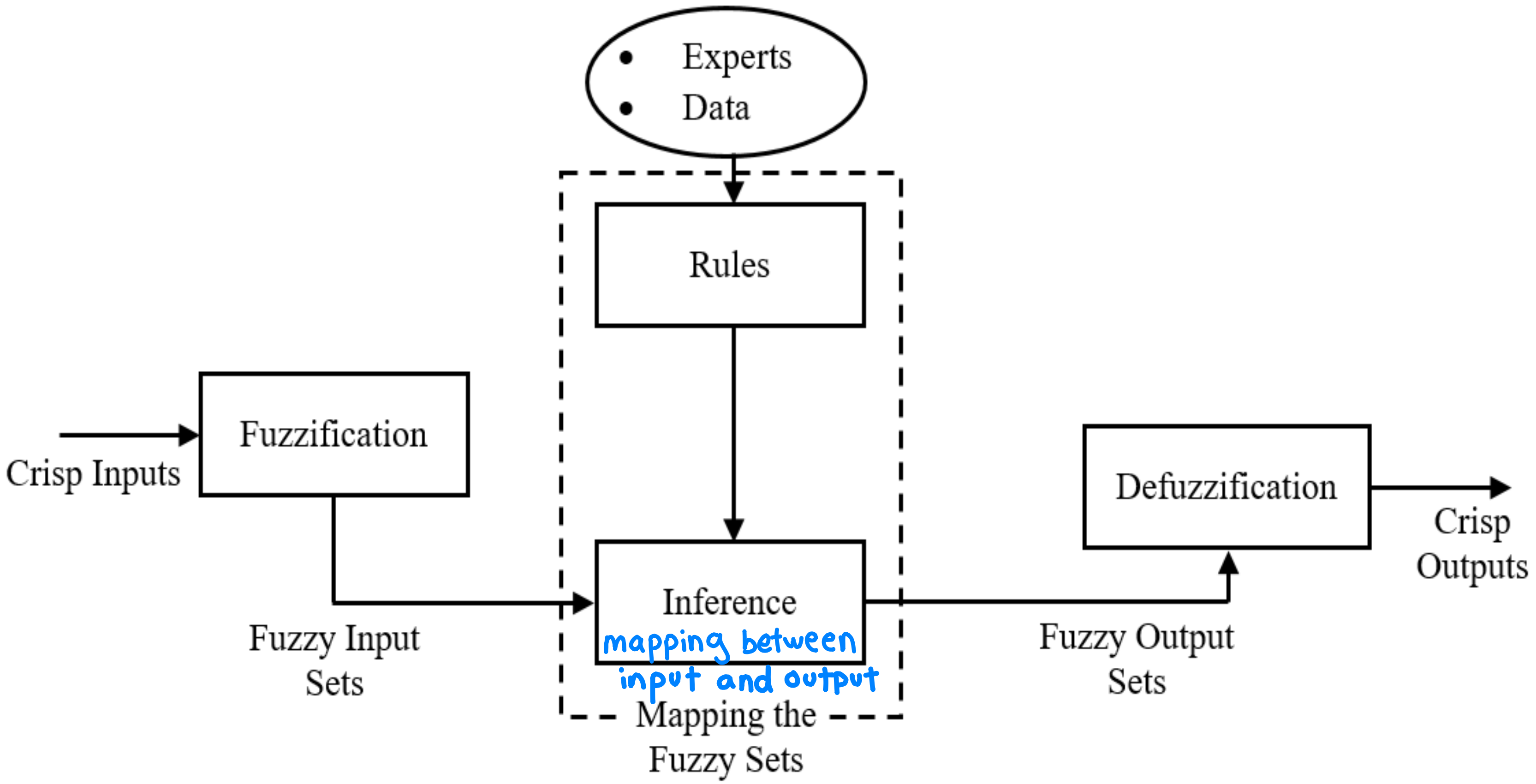
(This Chapter is included in another book)

Type-1 Fuzzy Logic System (T1FLS)



chapter 4

T1FLS: Fuzzification

- Fuzzification step represents the process of mapping the crisp inputs (x_1, x_2 …x_n) to the fuzzy input sets (A_j^i), where A_j^i is the ith fuzzy set for the jth variable.
- The fuzzy sets are usually defined by membership functions. The most commonly used membership function is the Gaussian one:

$$\mu_j^i(x_j) = \exp\left[-\frac{1}{2}\left(\frac{x_j - m^i}{\sigma^i}\right)^2\right]$$

(6) Guassian is better than (3) triangular Depending on the Degree of Freedom

T1FLS: Rules

Data given extraction by Clusturing

- The rules can be provided by experts or can be extracted from a collected data set.
- Both types can be presented as a collection of IF-THEN statements, as follows:

$$\textbf{Rule}^i\textbf{:}\ \textbf{IF}\ x_1 \text{ is } A_1^i \ \ldots \text{ and } x_n \text{ is } A_n^i,\ \textbf{THEN}\ y \text{ is } B^i\,.$$

- Clustering/classification can be utilized to initialize the system parameters.

it is unsupervised
(there is no target to compare with)
(it is the initialilization for parameters in Fuzzy System)

Continue…

- Clustering/classification is a data mining technique used to predict group membership for data instances.
- K-means clustering: clustering N data points into K disjoint subsets.

أسهل method

- How:
  - Specify k, the number of clusters to be generated → يعني لازم يكون عدد ال clustures (K) Known in advance
  - Choose k points at random as cluster centers
  - Assign each instance to its closest cluster center using Euclidean distance
  - Calculate the centroid (mean) for each cluster, use it as a new cluster center
  - Reassign all instances to the closest cluster center
  - Iterate until the cluster centers don't change anymore → بضل اعيد لحتى اوصل لهاي المرحلة

Continue…

- Example:

Subject	A	B
1	1.0	1.0
2	1.5	2.0
3	3.0	4.0
4	5.0	7.0
5	3.5	5.0
6	4.5	5.0
7	3.5	4.5

Simplified Example

$K = 2$

$C_1 = (.2, 150, 10)$

$C_2 = (.25, 100, 7)$

باجي بتطلع على كل نقطة اذا اقرب على ⇐ cluster1 or cluster 2

# of exp	Depth of Cut	Speed	Surface Roughness	D_{c_1}	D_{c_2}	
1	.2	100	5	50.2	2	2
2	.3	200	7	50.08	100	1
3	.2	150	10	0	50.09	1
4	.15	400	8	250	300	1
5	.21	300	7	150.03	200	1
6	.17	200	6	50.01	100	1
7	.25	100	7	50.09	0	2
8	.31	200	10	50	100.05	1
9	.27	300	9	150	200.01	1

$$\text{Distance} = \sqrt{(e_1 - e_{1_c})^2 + (e_2 - e_{2_c})^2 + (\ldots}$$

$$d_{c_1} = \sqrt{(e_1 - .2)^2 + (e_2 - 150)^2 + (e_3 - 10)^2}$$

$$d_{c_2} = \sqrt{(e_1 - .25)^2 + (e_2 - 100)^2 + (e_3 - 7)^2}$$

*for Assigning new Centers

$C_1 =$ as a vector

$$\frac{\left[(.3, 200, 7) + (.2, 150, 10) + (1.15, 400, 8) + (.21, 300, 7) + (.17, 200, 6) + (.31, 200, 10) + (.27, 300, 9)\right]}{7}$$

عددهم لل ١ (prefer Cluster 1)

$\text{Center}_1 = (.23, 250, 8.14)$

$\text{Center}_2 = (.225, 100, 6)$

lets Assume we want 2 Rules so 2 clusters ($K=2$)

اخترنا ٣ و ٧

(Clustering Method)

Continue .... (until the Cluster centers doesn't change)

# of exp	Depth of Cut	Speed	Surface Roughness	D_{c_1}	D_{c_2}	choose cluster	D_{c_1}	D_{c_2}	choose cluster	D_{c_1}	D_{c_2}	choose cluster
1	.2	100	5	50.2	2	2	150.03	1	2	166.6	16.7	2
2	.3	200	7	50.08	100	1	50.01	100	1	66.6	83.4	1
3	.2	150	10	0	50.09	1	100.01	50.15	2	116.6	33.5	2
4	.15	400	8	250	300	1	150	300	1	133.4	283	1
5	.21	300	7	150.03	200	1	50.01	200	1	33.41	183	1
6	.17	200	6	50.01	100	1	50.04	100	1	66.6	83.4	1
7	.25	100	7	50.09	0	2	150	1	2	166.6	16.6	2
8	.31	200	10	50	100.05	1	50.03	100.08	1	66.6	84.4	1
9	.27	300	9	150	200.01	1	50	200	1	33.4	183	1

$C_1 = (.2, 150, 10)$
$C_2 = (.25, 100, 7)$

$C_1 = (.23, 250, 8.14)$
$C_2 = (.225, 100, 6)$

$C_1 = (.235, 266.6, 7.83)$
$C_2 = (.22, 116.6, 7.33)$

$C_1 = (.235, 266.6, 7.83)$
$C_2 = (.22, 116.6, 7.33)$

The centers didn't change so we stop

this is One Pass Method

$$\mu_{gaussian}(x; m, \sigma) = e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

Final Answer

we assumed the Standard deviation
(the numbers are incorrect)
بس للفهم

$C_1 = (.24, 266.6, 7.8) \rightarrow \sigma = (.1, 50, 2)$

$C_2 = (.21, 116.6, 7.3) \rightarrow \sigma = (.12, 30, 1.5)$

for $\text{Exp}_1 = (.2, 100, 5)$ y actual mean for output

Ranges:-

DOC → (.15 - .31)

SP → (100 - 400)

SR → (5 - 10)

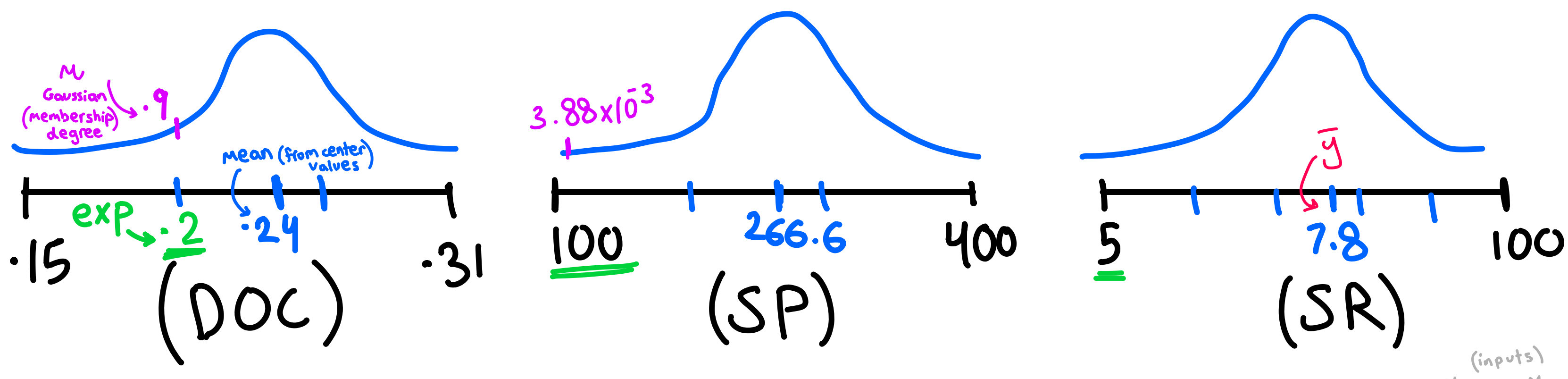
$\mu_{\text{Mean}} = e^{\left(-\frac{1}{2}\frac{(x-m)^2}{(6)^2}\right)}$

Rule ①

$\mu = e^{-.5\left(\frac{.2-.24}{.1}\right)^2} = 0.92$

$\mu = e^{-.5\left(\frac{100-266.6}{50}\right)^2} = 3.88 \times 10^{-3}$

Φ = Firing Level = μ_y (for Output) = (inputs) $\mu_{DOC} \times \mu_{SP}$ $= (.9) \times (3.88 \times 10^{-3})$ $= 3.5 \times 10^{-3}$

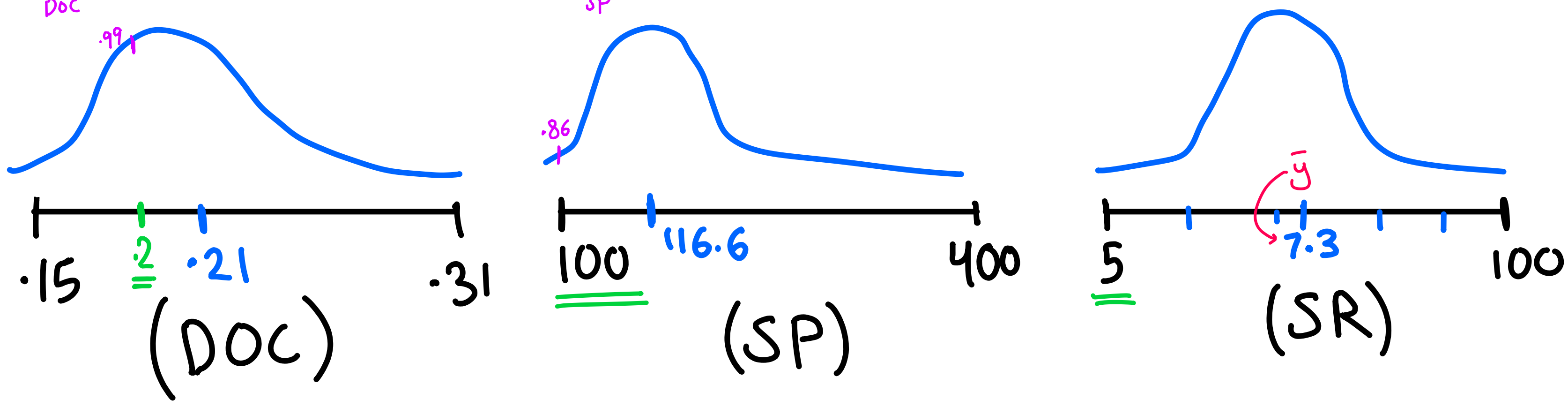


Rule ②

$\mu_{DOC} = e^{-.5\left(\frac{.2-.21}{.12}\right)^2} = .99$

$\mu_{SP} = e^{-.5\left(\frac{100-116.6}{30}\right)^2} = 0.86$

Φ = Firing Level = μ_y (for Output) = (inputs) $\mu_{DOC} \times \mu_{SP}$ $= (.99) \times (.86)$ $= 0.85$



Linguisticly:- when DOC is medium and speed is high then SR is high

Pr DO = Predicted Deffuzified Value for output = $\frac{\sum\left(\mu_y \text{ membership degree for output} \times \text{center value for output}\right)}{\mu_y \text{ membership degree for output (1)} + \mu_y \text{ membership degree for output (2)}}$

$$= \frac{[3.5 \times 10^{-3} \times 7.8] + [0.85 \times 7.3]}{3.5 \times 10^{-3} + 0.85} = 7.3$$ ← The Predicted Output

$C_1 = (.24, 266.6, 7.8) \longrightarrow \sigma = (.1, 50, 2)$

$C_2 = (.21, 116.6, 7.3) \longrightarrow \sigma = (.12, 30, 1.5)$

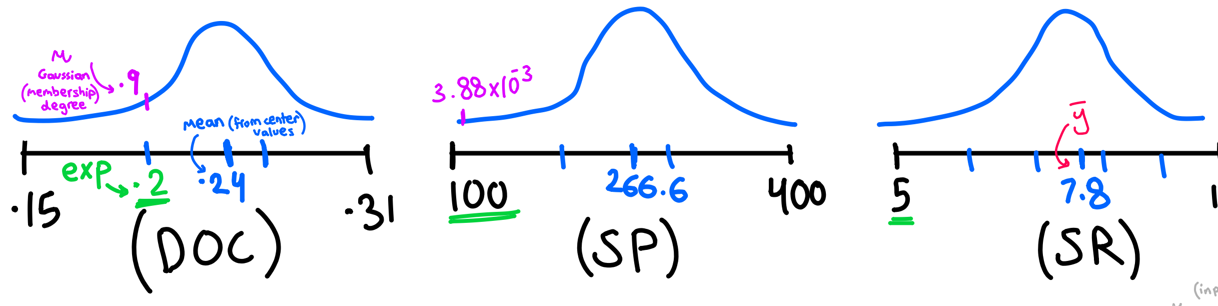
for $Exp_1 = (.2, 100, 5)$ — y actual (mean for output)

Rule ①

$\mu = e^{-.5\left(\frac{.2-.24}{.1}\right)^2} = 0.92$

$\mu = e^{-.5\left(\frac{100-266.6}{50}\right)^2} = 3.88 \times 10^{-3}$

Φ = Firing Level $= \mu_y$ (for output) $= \mu_{DOC} \times \mu_{SP}$ (inputs) $= (.9) \times (3.88 \times 10^{-3}) = 3.5 \times 10^{-3}$

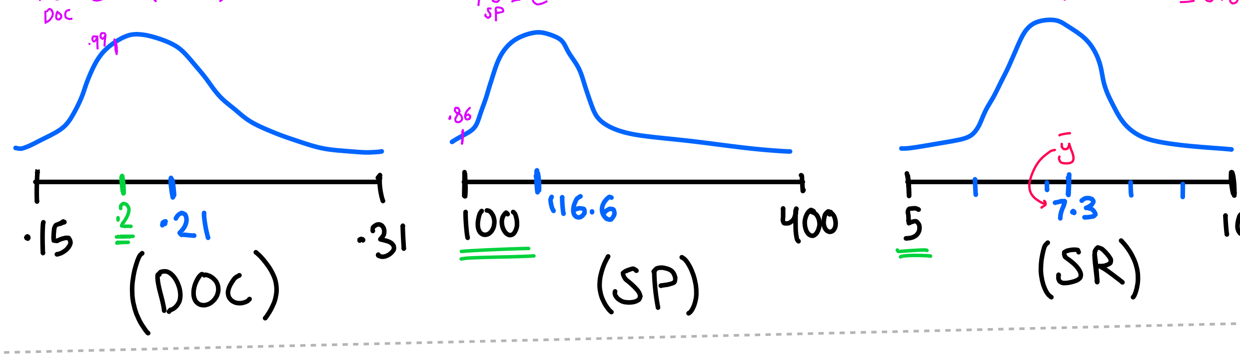


Rule ②

$\mu_{DOC} = e^{-.5\left(\frac{.2-.21}{.12}\right)^2} = .99$

$\mu_{SP} = e^{-.5\left(\frac{100-116.6}{30}\right)^2} = 0.86$

Φ = Firing Level $= \mu_y$ (for output) $= \mu_{DOC} \times \mu_{SP}$ (inputs) $= (.99) \times (.86) = 0.85$



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* Now we will do Fuzzification given ($\alpha = 0.3$)

for input mean only

$$m_{new} = m_{old} - \alpha\,(\text{learning factor}) \left[P_r - y\right] \left[\bar{y} - P_r\right] \times \frac{\left[X_{(input)} - m_{old}\right]}{\sigma^2} * \Phi$$

m أو μ ; P_r: predicted deffuzified value ; y: actual (mean for output) ; Φ: firing level (يفرق حسب ال Rule)

$Exp = (.2, 100, 5)$ — 5: output

P_r Predicted output for first Exp (for Rule 1 and 2) $= 7.3$

كل الحسابات لل Exp1 ال Rule1 Rule2

$\Rightarrow$ DOC R_1: $m_{new} = .24 - .3\left[7.3 - 5\right] \times \left[7.8 - 7.3\right] \times \left[\frac{.2 - .24}{.1^2}\right] \times 3.5 \times 10^{-3}$ (P_r, actual, input for DOC)

$= \boxed{0.244}$

SP R_1: $m_{new} = 266.7 - .3\left[7.3 - 5\right] \times \left[7.8 - 7.3\right] \times \left[\frac{100 - 266.7}{50^2}\right] \times 3.5 \times 10^{-3}$

$= \boxed{266.7}$

SP R_2: $m_{new} = 116.6 - .3\left[7.3 - 5\right] \times \left[7.3 - 7.3\right] \times \left[\frac{100 - 116.6}{30^2}\right] \times 0.85$

$= 116.6 - 0 = \boxed{116.6}$

$$\bar{y}_{new} = \bar{y}_{old} - \alpha * \left[P_r - y_{actual}\right] * \Phi$$

y_{actual}: Mean for output ; Φ: Firing Level يفرق حسب الRule

for R_1: $\bar{y}_{new} = 7.8 - .3 \times \left[7.3 - 5\right] \times 3 \times 10^{-3} = 7.79$

for R_2: $\bar{y}_{new} = 7.3 - .3 \times \left[7.3 - 5\right] \times .85 = 6.71$

باخذ يلي ما عدلتها — باخذ يلي ما عدلتها

$$\sigma_{new} = \sigma_{old} - \alpha\left[P_r - y_{actual}\right] * \left[\bar{y} - P_r\right] * \frac{\left[X - m\right]^2}{\sigma^3} * \Phi$$

Φ: Firing Level

DOC R_1: $\sigma_{new} = .1 - .3\left[7.3 - 5\right] * \left[7.8 - 7.3\right] * \frac{\left[.2 - .24\right]^2}{.1^3} \times 3.5 \times 10^{-3} = .098$

SP R_1: $\sigma_{new} = 50 - .3\left[7.3 - 5\right] * \left[7.8 - 7.3\right] * \frac{\left[100 - 266.7\right]^2}{50^3} \times 3.5 \times 10^{-3} = 49.9$

(.3, 200, 7) Exp Point 2

for practice (بيجي هيك بالفاينل)

$\xrightarrow{Pr}$

$\longleftarrow$

3 Stopping criteria

أكيد 90

1.
2. Error < .05
3. validation less 70% of data

Rules

Fuz. inf. Def.

Steepest Deepest Algorithm $\longrightarrow$ takes error as a function and minimize it

$$Error = \frac{1}{2} [pred - out]^2$$

في ال Fuzzy ما في داعي نعمل Normalization

Inference and Defuzzification

- The inference process combines the defined rules to map the input fuzzy sets to the output fuzzy sets.
- The output fuzzy set is then defuzzified to get a crisp one.
- By using centre average defuzzification method, such a mapping can be represented as follows:

$$f(x|\theta) = \frac{\sum_{i=1}^{R} b_i \prod_{j=1}^{n} \exp\left[-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right]}{\sum_{i=1}^{R} \prod_{j=1}^{n} \exp\left[-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right]},$$

Continue…

- The model parameters need to be optimized by employing an adaptive back-propagation network.
- Assignment: Steepest descent method.

Chapter 9
Decision Making with Fuzzy Information

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Introduction

- The decisions may be binary, however, there should be no restrictions on the usefulness of fuzzy information in the decision making process.
- Being able to make consistent and correct choices is the essence of any decision process imbued with uncertainty.
- The information affecting an issue is likely incomplete or uncertain; hence, the outcomes are uncertain, irrespective of the decision made or the alternative chosen.
- There is a distinct difference between a good decision and a good outcome!

Continue…

- Engineers are primarily concerned with two types of decisions:
  1. Operational decisions: an optimal action is sought to avoid a specific set of hazards;
  2. Strategic decisions: preparation for or anticipation of events in the future.
- Various paradigms for making decisions within a fuzzy environment.

right decision doesn't mean right outcome

Fuzzy Synthetic Evaluation

- Numerical evaluation is often too complex, too unacceptable, and too transient.
- Therefore, the evaluation can be described by natural language (e.g. excellent, good, etc.)
- A fuzzy relation can be found, followed by numerical evaluation.

Example

Suppose we want to measure the value of a microprocessor to a potential client. In conducting this evaluation, the client suggests that certain criteria are important. They can include performance (MIPS), cost ($), availability (AV), and software (SW). Performance is measured by millions of instructions per second (MIPS); a minimum requirement is 10 MIPS. Cost is the cost of the microprocessor, and a cost requirement of "not to exceed" 500 has been set. Availability relates to how much time after the placement of an order the microprocessor vendor can deliver the part; a maximum of eight weeks has been set. Software represents the availability of operating systems, languages, compilers, and tools to be used with this microprocessor. Suppose further that the client is only able to specify a subjective criterion of having "sufficient" software.

for software

A particular microprocessor (CPU) has been introduced into the market. It is measured against these criteria and given ratings categorized as excellent (e), superior (s), adequate (a), and inferior (i).

Continue…

Using the similarity methods presented in Chapter 3, the relation matrix is as follows:

$w = \left[\frac{.4}{P}, \frac{.3}{C}, \frac{.2}{A}, \frac{.1}{S} \right]$

$$
R = \begin{array}{l} \\ \text{MIPS} \\ \$ \\ \text{AV} \\ \text{SW} \end{array}
\begin{array}{c} \begin{array}{cccc} e & s & a & i \end{array} \\
\begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.2 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.6 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.3 & 0.2 \end{bmatrix} \end{array}
$$

If the evaluation team applies a scoring factor of 0.4 for performance, 0.3 for cost, 0.2 for availability, and 0.1 for software, then evaluate such a microprocessor?

by (Max-Min) composition

composition

$$\underset{\sim}{\mathbf{e}} = \underset{\sim}{\mathbf{w}} \circ \underset{\sim}{\mathrm{R}} = \{0.1, 0.3, 0.4, 0.2\}$$

حسب البعد عن ال 10 MIPS

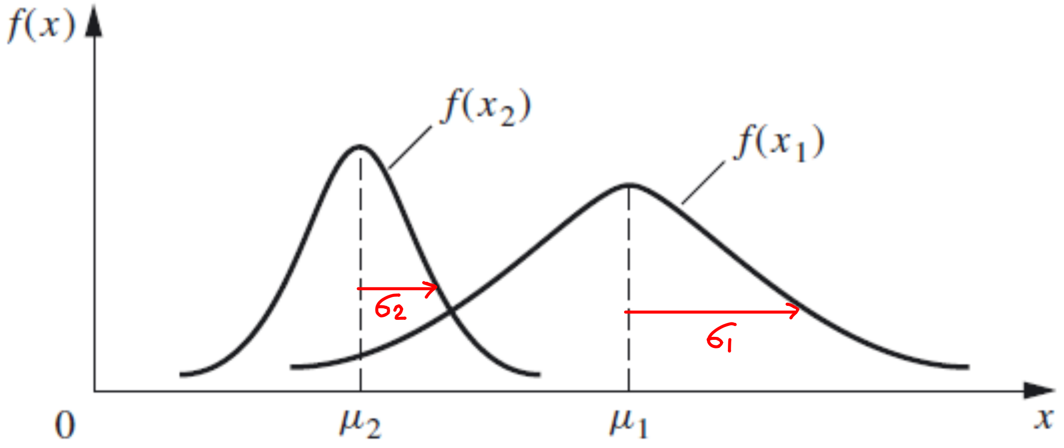
حسب البعد عن ال 500 cost

Fuzzy Ordering

- Decisions are sometimes made on the basis of rank, or ordinal ranking: which issue is best, which is second best, and so forth.
- Issues or actions are associated with randomness of fuzziness uncertainty, therefore, rank ordering may be ambiguous.
- If the uncertainty in rank is random, we can use probability density functions (PDF).

Continue…

- Example: Suppose we have one random variable, x_1, whose uncertainty is characterized by a Gaussian PDF with a mean of μ_1 and a standard deviation of σ_1, and another random variable, x_2, also Gaussian with a mean of μ_2 and standard deviation of σ_2. Suppose that $\sigma_1>\sigma_2$ and $\mu_1>\mu_2$.
- The question of which variable is greater is not clear.



Continue…

- We can assess this by frequency,

$$P(x_1 \geq x_2) = \int_{-\infty}^{\infty} F_{x_2}(x_1)\, \mathrm{d}x_1$$

F is the cumulative function

- If the uncertainty in rank is because of ambiguity, then the ranking is very subjective and not reducible to the elegant form available for some random variables.

Continue…

- If the uncertainty in rank is because of imprecision, then the truth value can be used.
- Suppose we have k fuzzy sets $\underset{\sim}{I}_1, \underset{\sim}{I}_2, \ldots, \underset{\sim}{I}_k$. Then, the truth value of a specified ordinal ranking is given as

$$T(\underset{\sim}{I} \geq \underset{\sim}{I}_1, \underset{\sim}{I}_2, \ldots, \underset{\sim}{I}_k) = T(\underset{\sim}{I} \geq \underset{\sim}{I}_1) \text{ and } T(\underset{\sim}{I} \geq \underset{\sim}{I}_2) \text{ and } \ldots \text{ and } T(\underset{\sim}{I} \geq \underset{\sim}{I}_k)$$

بناخد كل ال possibilities ← I_1 و I_2

I_2 و I_1

I_1 و I_3

I_3 و I_1

I_2 و I_3

I_3 و I_2

و هكذا

Continue…

- Example: Suppose we have three fuzzy sets:

$$\underset{\sim}{I}_1 = \left\{ \frac{1}{3} + \frac{0.8}{7} \right\}, \quad \underset{\sim}{I}_2 = \left\{ \frac{0.7}{4} + \frac{1.0}{6} \right\}, \quad \text{and} \quad \underset{\sim}{I}_3 = \left\{ \frac{0.8}{2} + \frac{1}{4} + \frac{0.5}{8} \right\}$$

لازم بالأول الرقم يكون اكبر من يلي بدي اقارن معهم او يساوي التحت

We can assess the truth value of the inequality, $\underset{\sim}{I}_1 \geq \underset{\sim}{I}_2$, as follows:

I_1 and I_2

بقارن (3) in I_1 مع (4) in I_2 و(6) in I_2 إذا اقل منهم ما باخدها

$$T(\underset{\sim}{I}_1 \geq \underset{\sim}{I}_2) = \max_{x_1 \geq x_2} \{\min(\mu_{\underset{\sim}{I}_1}(x_1), \mu_{\underset{\sim}{I}_2}(x_2))\}$$

بقارن (7) in I_1 مع (4) in I_2 و(6) in I_2 اذا اكبر او يساوي منهم باخد Min لل μ (membership degree) و بعدين ال Max بينهم.

$$= \max\{\min(\mu_{\underset{\sim}{I}_1}(7), \mu_{\underset{\sim}{I}_2}(4)), \min(\mu_{\underset{\sim}{I}_1}(7), \mu_{\underset{\sim}{I}_2}(6))\}$$

$$= \max\{\min(0.8, 0.7), \min(0.8, 1.0)\} \rightarrow \max(.7, .8)$$

$$= 0.8.$$

Truth value that $(I_1 \geq I_2)$

Continue…

Similarly,

$$T(\underset{\sim}{I}_1 \geq \underset{\sim}{I}_3) = 0.8, \quad T(\underset{\sim}{I}_2 \geq \underset{\sim}{I}_1) = \underline{\underline{1.0}}, \quad \text{Truth value that} (I_2 \geqslant I_1)$$

$$T(\underset{\sim}{I}_2 \geq \underset{\sim}{I}_3) = 1.0, \quad T(\underset{\sim}{I}_3 \geq \underset{\sim}{I}_1) = 1.0,$$

$$T(\underset{\sim}{I}_3 \geq \underset{\sim}{I}_2) = 0.7.$$

Then,

Truth value that I_1 is Larger than both I_2, I_3 →

$$T(\underset{\sim}{I}_1 \geq \underset{\sim}{I}_2, \underset{\sim}{I}_3) = 0.8, \rightarrow \min(\underbrace{.8}_{I_1 \geqslant I_2}, \underbrace{.8}_{I_1 \geqslant I_3})$$

$$T(\underset{\sim}{I}_2 \geq \underset{\sim}{I}_1, \underset{\sim}{I}_3) = \underline{1.0}, \rightarrow \min(1, 1)$$

$$T(\underset{\sim}{I}_3 \geq \underset{\sim}{I}_1, \underset{\sim}{I}_2) = 0.7. \rightarrow \min(\underbrace{1}_{I_3 \geqslant I_1}, \underbrace{.7}_{I_3 \geqslant I_2})$$

Then the overall ordering is ……..

I_2, I_1, I_3

Nontransitive Ranking

- When we compare objects that are fuzzy, ambiguous, or vague, we may well encounter a situation where there is a contradiction in the classical notions of ordinal ranking and transitivity in the ranking.
- Example: When comparing red to blue, we prefer red; when comparing blue to yellow, we prefer blue; but when comparing red and yellow we might prefer yellow.
- For nontransitive ranking, the relativity function is introduced.

Continue…

- Let x and y be variables defined on the same universe, and let’s define pairwise functions

$f_y(x)$ as the membership value of x with respect to y

تقريباً كم بنفضل ال x على ال y

$f_x(y)$ as the membership value of y with respect to x

- Then, the relativity function can be written as follows:

Preference

$$f(x \mid y) = \frac{f_y(x)}{\max[f_y(x),\ f_x(y)]}$$

Membership Value
لإخيار x بدل ال y

A measurement of the membership value of choosing x over y.

Continue…

- For more than two variables, the relativity function is given as follows:

$$f(x_i \mid A') = f(x_i \mid \{x_1, x_2, \ldots, \boxed{x_{i-1}, x_{i+1},} \ldots, x_n\})$$
$$= \min\{f(x_i \mid x_1), f(x_i \mid x_2), \ldots, f(x_i \mid x_{i-1}), f(x_i \mid x_{i+1}), \ldots, f(x_i \mid x_n)\}$$

A fuzzy measurement of choosing x_i over all the other elements

- Question: By including x_i in the equation above, what is the output?

Continue…

- Example: In manufacturing, we often try to compare the capabilities of various microprocessors for their appropriateness to certain applications. For instance, suppose we are trying to select from among four microprocessors the one that is best suited for image processing applications. Since many factors, including performance, cost, availability, and software, can affect this decision, coming up with a crisp mathematical model for all these attributes is complicated. Another consideration is that it is much easier to compare these microprocessors subjectively in pairs rather than all four at one time. Suppose the design team is polled to determine which of the four microprocessors, labeled x_1, x_2, x_3, and x_4, is the most preferred when considered as a group rather than when considered as pairs. First, pairwise membership functions are determined. These represent the subjective measurement of the appropriateness of each microprocessor when compared only to one another. The following pairwise functions are determined:

Continue…

تقريباً كم بفضل X_4 على X_1

$$
\begin{array}{llll}
f_{x_1}(x_1) = 1, & f_{x_1}(x_2) = 0.5, & f_{x_1}(x_3) = 0.3, & \underline{f_{x_1}(x_4)} = 0.2. \\
f_{x_2}(x_1) = 0.7, & f_{x_2}(x_2) = 1, & f_{x_2}(x_3) = 0.8, & f_{x_2}(x_4) = 0.9. \\
f_{x_3}(x_1) = 0.5, & f_{x_3}(x_2) = 0.3, & f_{x_3}(x_3) = 1, & f_{x_3}(x_4) = 0.7. \\
f_{x_4}(x_1) = 0.3, & f_{x_4}(x_2) = 0.1, & f_{x_4}(x_3) = 0.3, & f_{x_4}(x_4) = 1.
\end{array}
$$

- Then, the relativity values are:

$f(X_2/X_3) = \dfrac{f_{X_3}(X_2)}{\max(f_{X_3}(X_2), f_{X_2}(X_3))} = \dfrac{.3}{\max(.3,.8)} = \dfrac{.3}{.8} = .375$

$$f(x_i \mid x_j) = f(i\text{th row} \mid j\text{th column})$$

$$
C = \begin{array}{c|cccc}
 & x_1 & x_2 & x_3 & x_4 \\
\hline
x_1 & 1 & 1 & 1 & 1 \\
x_2 & 0.71 & 1 & \underline{0.38} & 0.11 \\
x_3 & 0.6 & 1 & 1 & 0.43 \\
x_4 & 0.67 & 1 & 1 & 1
\end{array}
$$

مش دايماً بيطلعو واحد

لكن ال Diagonal is always one

To determine the overall ranking, we need to find the smallest value in each of the rows of the C matrix.

The order from best to worst is x_1, x_4, x_3, and x_2.

$$f(x/y) = \frac{f_y(x)}{\max(f_y(x), f_x(y))}$$

$$\underbrace{f(x_1/x_1)}_{\text{Prefrence of } x_1 \text{ over } x_2} = \frac{f_{x_2}(x_1)}{\max(f_{x_2}(x_1), f_{x_1}(x_2))} = \frac{.7}{\max(.7, .5)} = 1$$

بتفرج على ال

Minimum Value in every Row

To determine the overall ranking, we need to find the smallest value in each of the rows of the C matrix.

Relativity values →

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.71 & 1 & 0.38 & 0.11 \\ 0.6 & 1 & 1 & 0.43 \\ 0.67 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

[over all Ranking]

ترتيبهم

X_1 best

X_4

X_3

X_2 worse

في اجماع و لا يوجد اشي صح واشي غلط

Preference and Consensus

Self Study (contains things from Linear)
[from book]

- The goal of group decision making typically is to arrive at a consensus concerning a desired action or alternative from among those considered in the decision process.
- The individual preferences of those in the decision group are collected to form a group metric whose properties are used to produce a scalar measure of "degree of consensus."
- First, we need to define a reciprocal relation as a fuzzy relation:

$$
\begin{array}{c|c}
 & \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 0 & .5 & \\ .5 & 0 & \\ & & 0 \end{bmatrix}
\end{array}
$$

انه مستحيل أفضل ال option_1 على ال option_1

$$r_{ii} = 0, \quad \text{for } 1 \le i \le n.$$

$$r_{ij} + r_{ji} = 1, \quad \text{for } i \ne j.$$

ما بقدر اسوي matrix تربيع إلا اذا كانت Square

Continue…

- Two common measures of preference are defined here as average fuzziness and average certainty:

Average Fuzziness

$$F(\underset{\sim}{R}) = \frac{\mathrm{tr}(\underset{\sim}{R}^2)}{n(n-1)/2}.$$

Average Certainty

$$C(\underset{\sim}{R}) = \frac{\mathrm{tr}(\underset{\sim}{R}\,\underset{\sim}{R}^{\mathrm{T}})}{n(n-1)/2}.$$

مجموع ال Diagonal

tr() and $()^{\mathrm{T}}$ denote the trace and transpose, respectively

$$\mathrm{tr}(\underset{\sim}{R}) = \sum_{i=1}^{n} r_{ii}.$$

Remember :-

Find the Trace of a Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Trace =1+5+9
=15

Matrix : $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $\Rightarrow$ Transpose of matrix : $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

Matrix : $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ $\Rightarrow$ Transpose of matrix : $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Continue…

- Three types of consensus:

1. Type I consensus: is a consensus in which there is one clear choice, and the remaining alternatives all have equal secondary preference.

الخيار الأول → A_2

الخيار الثاني → A_1, A_3, A_4

كلهم بنفس ال Level

$$M_1^* = \begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0 & 0 & 0.5 & 0.5 \\ A_2 & 1 & 0 & 1 & 1 \\ A_3 & 0.5 & 0 & 0 & 0.5 \\ A_4 & 0.5 & 0 & 0.5 & 0 \end{array}$$

clear Alternative يوجد هنا

ال option (A_2)

I don't prefer any other Alternative on (A_2)

أنا ما بفضل ال option (1) على ال option (1)

و هكذا .....

Continue…

There is a clear choice and then we don't know the order

2. Type II consensus: there is one clear choice, but the remaining alternatives all have definite secondary preference.

A_2 هو خياري الأول على الأكيد ← my prefered choice

Relation

I prefer A_1 on A_3

I prefer A_3 on A_4

I prefer A_4 on A_1

$$M_2^* = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where alternative 2 has a clear consensus, but where there is no clear ordering after the first choice because alternative 1 is preferred to alternative 3, 3 to 4, but alternative 4 is preferred to alternative 1.

Continue…

قرار بالإجماع

3. **Type fuzzy consensus:** where there is an unanimous decision for the most preferred choice, but the remaining alternatives have infinitely many fuzzy secondary preferences.

A2
A1
A4
A3

but they are Almost the same
(So we have uncertainity)

$$
M_f^* = \begin{array}{c c}
 & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} 0 & 0 & 0.5 & \underline{\underline{0.6}} \\ 1 & 0 & 1 & 1 \\ 0.5 & 0 & 0 & 0.3 \\ 0.4 & 0 & 0.7 & 0 \end{bmatrix}
\end{array}
$$

تفضيل الأول
على الرابع

The matrix shown here has a clear choice for alternative 2, but the other secondary preferences are fuzzy to various degrees.

Continue…

Cardinal Number :The total number of elements in a universe.

- Think about the cardinalities of these matrices.

$$
\begin{aligned}
\left|\mathrm{M}_1^*\right| &= n && \text{(Type I)} \\
\left|\mathrm{M}_2^*\right| &= \left(2^{(n^2-3n+2)/2}\right)(n) && \text{(Type II)} \\
\left|\mathrm{M}_f^*\right| &= \infty && \text{(Type } \textit{fuzzy}\text{)}
\end{aligned}
$$

Continue…

- Distance to consensus metric can be defined as follows:

$$m(\underset{\sim}{R}) = 1 - (2C(\underset{\sim}{R}) - 1)^{1/2}$$

$$m(\underset{\sim}{R}) = 1 - (2/n)^{1/2} \text{ for a Type I } (M_1^*) \text{ consensus relation}$$
$$m(\underset{\sim}{R}) = 0 \text{ for a Type II } (M_2^*) \text{ consensus relation.}$$

- Question: when does $\overset{\text{Type 1}}{m(M_1^*)}$ equal $\overset{\text{Type 2}}{m(M_2^*)}$?

when the Cardinality is 2 $\rightarrow (n=2)$

example

$$\begin{array}{c|cc} & A_1 & A_2 \\ \hline A_1 & 0 & 0 \\ A_2 & 1 & 0 \end{array}$$

I do prefer A_2 on A_1

اذا؟ اكيد صار الترتيب A_2 then A_1

Multi-Objective Decision Making

بدي اعطي كل Alternative حسب ال objective
→ Membership Degree

- Decisions are, more often than not, made in an environment where many objectives need to be considered.
- Two primary issues in multi-objective decision making 1) are to acquire meaningful information regarding the satisfaction of the objectives by the various choices and 2) to rank the relative importance of each of the objectives.
- To evaluate the alternatives, the objectives are usually combined. This process requires subjective information from the decision authority concerning the importance of each objective.

Continue…

- Definitions:
- n alternatives: $A = \{a_1, a_2, \ldots, a_n\}$, r objectives: $O = \{O_1, O_2, \ldots, O_r\}$
- Then the degree of membership of alternative a in O_i , denoted $\mu_{Oi}(a)$, is the degree to which alternative a satisfies the criteria specified for this objective.
- We seek a decision function that simultaneously satisfies all of the decision objectives; hence, the decision function, D, is given by the intersection of all the objective sets:

$$\boxed{D = O_1 \cap O_2 \cap \cdots \cap O_r}$$

Continue…

- The grade of membership that the decision function, D, has for each alternative a is given as:

for a specific Alternative
we take the Minimum value
for this Alternative with respect
to different objectives

$$\boxed{\mu_D(a) = \min[\mu_{O_1}(a), \mu_{O_2}(a), \ldots, \mu_{O_r}(a)]}$$

- The optimum decision will then be the alternative that satisfies:

Now we will have one value for each Alternative
and we will choose the (Max) Membership Degree

$$\boxed{\mu_D(a^*) = \max_{a \in A}(\mu_D(a))}$$

Continue…

- When each objective is associated with a weight expressing its importance, the decision can be given as follows:

$$\boxed{D = M(O_1, b_1) \cap M(O_2, b_2) \cap \cdots \cap M(O_r, b_r)}$$

- Implication is the operation that relates the objective and its importance:

$$M(O_i(a), b_i) = b_i \longrightarrow \boxed{O_i(a) = \overline{b_i} \overset{\text{Max}}{\vee} O_i(a)} \qquad \boxed{D = \bigcap_{i=1}^{r} (\overline{b_i} \cup O_i)}$$

اتحاد

negation for weight

Objective

$$\boxed{C_i = \overline{b_i} \cup O_i,} \qquad \boxed{\mu_{C_i}(a) = \max[\mu_{\overline{b_i}}(a), \mu_{O_i}(a)]}$$

Continue…

Example: A geotechnical engineer on a construction project must prevent a large mass of soil from sliding into a building site during construction and must retain this mass of soil indefinitely after construction to maintain stability of the area around a new facility to be constructed on the site. The engineer therefore must decide which type of retaining wall design to select for the project. Among the many alternative designs available, the engineer reduces the list of candidate retaining wall designs to three: (1) a mechanically stabilized embankment (MSE) wall, (2) a mass concrete spread wall (Conc), and (3) a gabion (Gab) wall. The owner of the facility (the decision maker) has defined four objectives that impact the decision: (1) the cost of the wall (Cost), (2) the maintainability (Main) of the wall, (3) whether the design is a standard one (SD), and (4) the environmental (Env) impact of the wall. Moreover, the owner also decides to rank the preferences for these objectives on the unit interval. Hence, the engineer sets up the problem as follows: (the objectives have different weight)

Continue…

$$A = \{\text{MSE, Conc, Gab}\} = \{a_1, a_2, a_3\}.$$

$$O = \{\text{Cost, Main, SD, Env}\} = \{O_1, O_2, O_3, O_4\}.$$

$$P = \{b_1, b_2, b_3, b_4\} \rightarrow [0, 1].$$

Every objective have a weight

the Range for the weights

- From previous experience with various wall designs, the engineer first rates the retaining walls with respect to the objectives, given here. These ratings are fuzzy sets expressed as follows:

(weights)

Continue…

Membership Degree

Cost objective

$$\underset{\sim}{Q}_1 = \left\{ \frac{0.4}{\text{MSE}} + \frac{1}{\text{Conc}} + \frac{0.1}{\text{Gab}} \right\}.$$

Maintainability objective

$$\underset{\sim}{Q}_2 = \left\{ \frac{0.7}{\text{MSE}} + \frac{0.8}{\text{Conc}} + \frac{0.4}{\text{Gab}} \right\}.$$

Standard objective

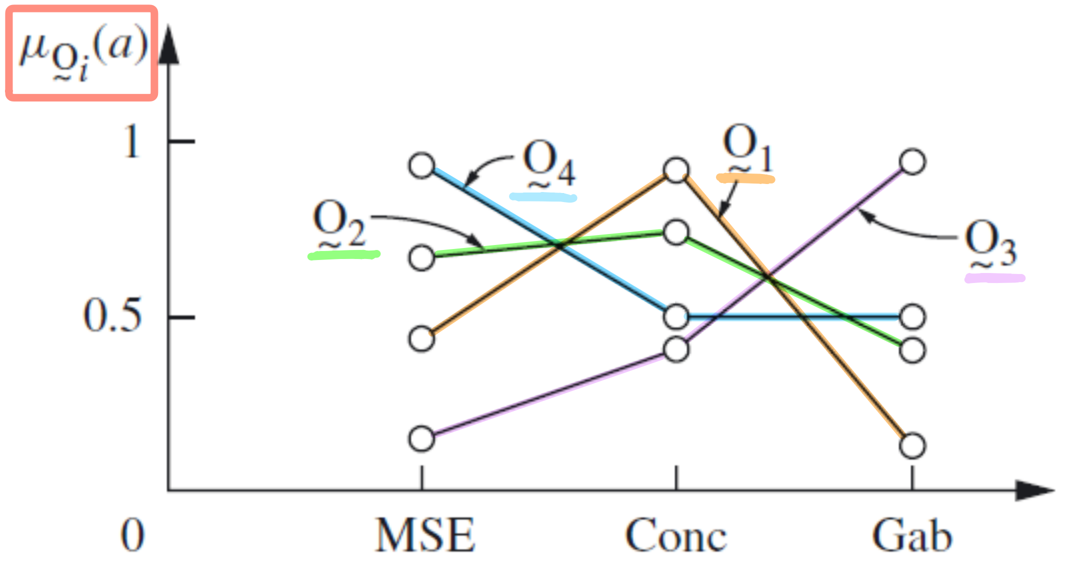
$$\underset{\sim}{Q}_3 = \left\{ \frac{0.2}{\text{MSE}} + \frac{0.4}{\text{Conc}} + \frac{1}{\text{Gab}} \right\}.$$

Enviromental objective

$$\underset{\sim}{Q}_4 = \left\{ \frac{1}{\text{MSE}} + \frac{0.5}{\text{Conc}} + \frac{0.5}{\text{Gab}} \right\}.$$

This Graph represents the Membership degrees

بس السؤال كاتب الFunctions فما اله فائدة

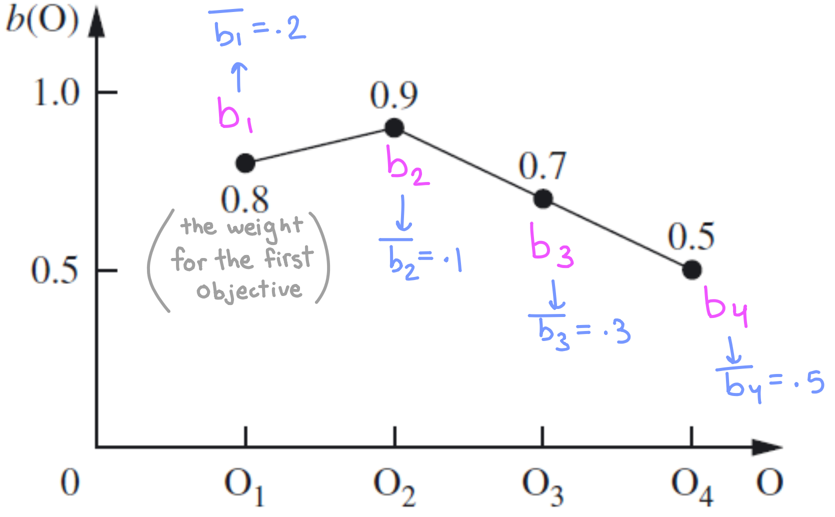


Continue…

Our Goal is to Choose the best Alternative

- The engineer wishes to investigate two decision scenarios. Each scenario propagates a different set of preferences from the owner, who wishes to determine the sensitivity of the optimum solutions to the preference ratings. In the first scenario, the owner lists the preferences for each of the four objectives:

(different weights)



Continue…

- Let’s evaluate the three alternatives:

$$D(a_1) = D(\mathrm{MSE}) = (\overset{\text{Max}}{\bar{b}_1 \cup O_1}) \underset{\text{Min}}{\cap} (\overset{\text{Max}}{\bar{b}_2 \cup O_2}) \underset{\text{Min}}{\cap} (\overset{\text{Max}}{\bar{b}_3 \cup O_3}) \underset{\text{Min}}{\cap} (\overset{\text{Max}}{\bar{b}_4 \cup O_4})$$

Max → $$= (0.2 \vee 0.4) \wedge (0.1 \vee 0.7) \wedge (0.3 \vee 0.2) \wedge (0.5 \vee 1)$$

then Min → $$= 0.4 \wedge 0.7 \wedge 0.3 \wedge 1 = \underline{0.3}$$

Membership Value for the first Alternative (MSE) with respect to All objectives

$$D(a_2) = D(\mathrm{Conc}) = (0.2 \vee 1) \wedge (0.1 \vee 0.8) \wedge (0.3 \vee 0.4) \wedge (0.5 \vee 0.5)$$

$$= 1 \wedge 0.8 \wedge 0.4 \wedge 0.5 = 0.4.$$

We will select the second Alternative because it have the highest Membership Degree

$$D(a_3) = D(\mathrm{Gab}) = (0.2 \vee 0.1) \wedge (0.1 \vee 0.4) \wedge (0.3 \vee 1) \wedge (0.5 \vee 0.5)$$

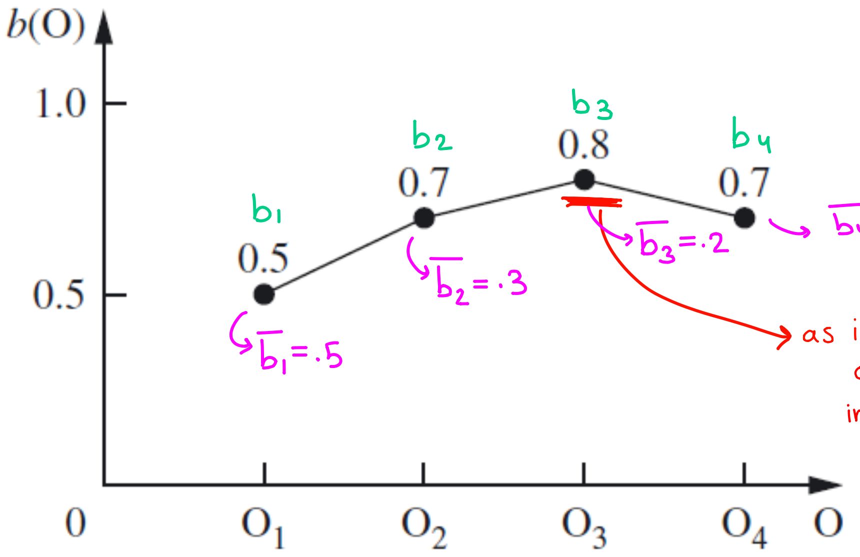
$$= 0.2 \wedge 0.4 \wedge 1 \wedge 0.5 = \underline{0.2}.$$

$$D^* = \max\{D(a_1), D(a_2), D(a_3)\} = \max\{0.3, 0.4, 0.2\} = 0.4.$$

Continue…

- In the second scenario the engineer was given a different set of preferences by the owner as follows:

في هذه الحالة تم تغيير الـ weights لـ objectives



as if we prefer 3rd objective on other objectives in the second scenario

Continue…

$$D(a_1) = D(\text{MSE}) = (\bar{b}_1 \overset{\text{Max}}{\cup} O_1) \underset{\text{Min}}{\cap} (\bar{b}_2 \overset{\text{Max}}{\cup} O_2) \underset{\text{Min}}{\cap} (\bar{b}_3 \overset{\text{Max}}{\cup} O_3) \underset{\text{Min}}{\cap} (\bar{b}_4 \overset{\text{Max}}{\cup} O_4)$$

$$= (0.5 \vee 0.4) \wedge (0.3 \vee 0.7) \wedge (0.2 \vee 0.2) \wedge (0.3 \vee 1)$$

$$= 0.5 \wedge 0.7 \wedge 0.2 \wedge 1 = 0.2.$$

$$D(a_2) = D(\text{Conc}) = (0.5 \vee 1) \wedge (0.3 \vee 0.8) \wedge (0.2 \vee 0.4) \wedge (0.3 \vee 0.5)$$

$$= 1 \wedge 0.8 \wedge 0.4 \wedge 0.5 = 0.4.$$

Same Membership Degrees

$$D(a_3) = D(\text{Gab}) = (0.5 \vee 0.1) \wedge (0.3 \vee 0.4) \wedge (0.2 \vee 1) \wedge (0.3 \vee 0.5)$$

$$= 0.5 \wedge 0.4 \wedge 1 \wedge 0.5 = 0.4.$$

- There is a tie between alternative a_2 and a_3 , what should we do in such a case?

to break this Tie → Remove the Minimum Value from both of them and take the Second Minimum

⟹ So the a_2 will be (.5) x
a_3 will be (.5) x
⟹ So a_2 will become (.8)
a_3 will become $\boxed{(1)}$ → The highest so I will choose a_3 (Gab)

Continue…

between 2^{nd} and 3^{rd} Alternative

- Tie-breaking procedure:

$$\hat{D}(x) = \min_{i \neq k}[C_i(x)] \qquad \text{and} \qquad \hat{D}(y) = \min_{i \neq g}[C_i(y)].$$

- If it persists:

$$\hat{\hat{D}}(x) = \min_{i \neq k,j}[C_i(x)] \qquad \text{and} \qquad \hat{\hat{D}}(y) = \min_{i \neq g,h}[C_i(y)]$$

- The tie-breaking procedure continues until an unambiguous optimum alternative emerges or all of the alternatives have been exhausted. In the latter case, some other tie-breaking procedure can be used.

Continue…

- Step 1:

$$\hat{\mathrm{D}}(a_2) = \hat{\mathrm{D}}(\mathrm{Conc}) = (0.5 \vee 1) \wedge (0.3 \vee 0.8) \wedge (0.3 \vee 0.5)$$
$$= 1 \wedge 0.8 \wedge 0.5 = 0.5.$$
$$\hat{\hat{\mathrm{D}}}(a_3) = \hat{\mathrm{D}}(\mathrm{Gab}) = (0.5 \vee 0.1) \wedge (0.2 \vee 1) \wedge (0.3 \vee 0.5)$$
$$= 0.5 \wedge 1 \wedge 0.5 = 0.5.$$

- Step 2:

$$\hat{\hat{\mathrm{D}}}(a_2) = \hat{\hat{\mathrm{D}}}(\mathrm{Conc}) = (0.5 \vee 1) \wedge (0.3 \vee 0.8) = 0.8.$$
$$\boxed{\hat{\hat{\mathrm{D}}}(a_3)} = \hat{\hat{\mathrm{D}}}(\mathrm{Gab}) = (0.2 \vee 1) = 1.$$

Fuzzy Bayesian Decision Method

it depends on the Probability of something taking place in the future

- Classical statistical decision making involves uncertainties in the future can be characterized probabilistically.

uncertainity مصدر ال
Probability هو ال

- The choice is predicated on information about the future, which is normally discretized into various "states of nature."
- Classical Bayesian decision methods presume that future states of nature can be characterized as probability events.
- Example: Consider the prediction of the annual demand of a product: it could be low, medium or high. These are vague.

For Example the State of Nature
it will Rain Tomorrow
it will Not Rain Tomorrow
و بحسب الـ probability لكل وحدة

Continue…

أشياء راح تصير بالمستقبل

- Let $S=\{s_1, s_2, \ldots, s_n\}$ be a set of possible states of nature, and the probabilities (prior probabilities) of these states:

(مجموع الـ probabilities يساوي واحد)

$$\mathbf{P} = \{p(s_1), p(s_2), \ldots, p(s_n)\}, \qquad \text{where} \qquad \sum_{i=1}^{n} p(s_i) = 1$$

- Assume that there are m alternatives, $A=\{a_1, a_2, \ldots, a_m\}$, and for each alternative a_j, a utility value (u_{ji}) is assigned when the future state of nature turns out to be state s_i.

انه الـ State يلي توقعتها تصير صارت

- The expected utility associated with the jth alternative would be:

$$E(u_j) = \sum_{i=1}^{n} u_{ji}\, p(s_i)$$

Expected Utility Assotiated with each Alternative

حاصل الجمع

Utility Value

Probability

Continue…

- The expected utility associated with the jth alternative would be:

$$E(u_j) = \sum_{i=1}^{n} u_{ji}\, p(s_i)$$

- The best alternative is the one with the maximum expected utility among all the alternatives, as follows:

$$E(u^*) = \max_j E(u_j)$$

to choose the best Alternative

Continue…

$\boxed{\text{Example:}}$ A geological engineer who has been asked by the chief executive officer (CEO) of a large oil firm to help make a decision about whether to drill for natural gas in a particular geographic region of north western New Mexico. There are only two states of nature regarding the existence of natural gas in the region:

States of Nature → $\underline{s_1}$= there is natural gas and $\underline{s_2}$= there is no natural gas

From previous drilling information, the prior probabilities for each of these states is

Probabilities → $p(s_1) = 0.5$ and $p(s_2) = 0.5$.

Continue…

There are two alternatives in this decision:

Alternatives → a_1= drill for gas and a_2= do not drill for gas

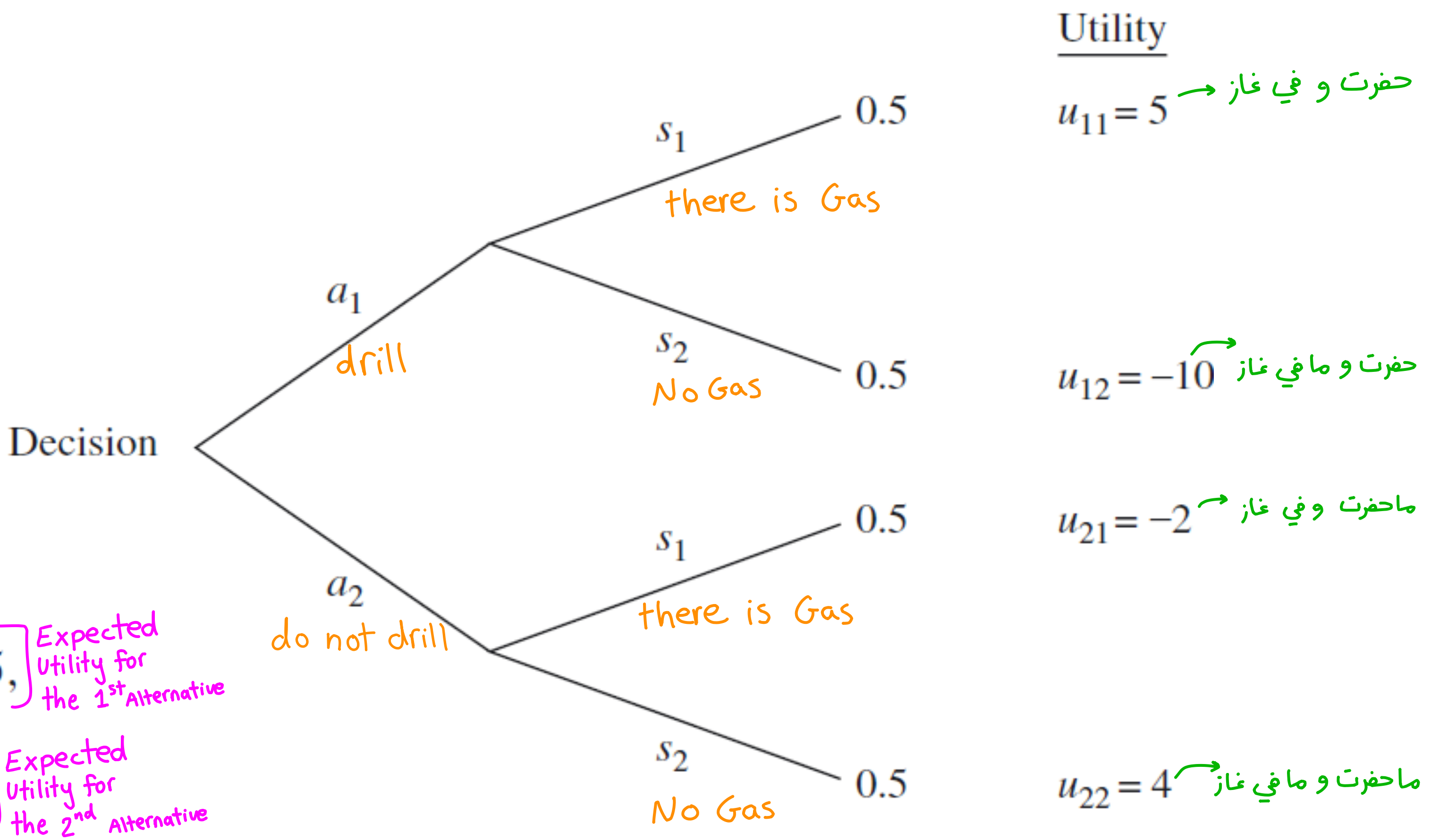
Utility Values → The CEO tells you that the best situation for the firm is to decide to drill for gas, and subsequently find that gas, indeed, was in the geologic formation. The CEO assesses this value (u_{11}) as +5 in nondimensional units; in this case, the CEO would have gambled (drilling costs big money) and won. Moreover, the CEO feels that the worst possible situation would be to drill for gas, and subsequently find that there was no gas in the area. Since this would cost time and money, the CEO determines that the value for this would be $u_{12} = -10$ units; the CEO would have gambled and lost–big. The other two utilities are assessed by the decision maker in nondimensional units as $u_{21}=-2$ and $u_{22}=4$. Hence, the utility matrix for this situation is given as

زي كأنا عم نتوقع الربح و الخسارة للمستقبل (بس مش كأرقام)

يعني زي Utility Values

Continue…

The expected utility associated with the two alternatives?



$$E(u_1) = (0.5)(5) + (0.5)(-10) = -2.5,$$ Expected Utility for the 1st Alternative

✓ $$E(u_2) = (0.5)(-2) + (0.5)(4) = 1.0,$$ Expected Utility for the 2nd Alternative

Our Final Desicion will be the Maximum → (1)
يعني ما يحفر بالمنطقة المحددة

Continue…

- Suppose some new information regarding the true states of nature S is available from r experiments or other observations and is collected in a data vector, $X=\{x_1, x_2, \dots, x_r\}$. This information can be used to update the prior probabilities.
- Thus, given that the piece of new information x_k is true, the probability that the true state of nature is s_i is $p(s_i|x_k)$. The updated probabilities are determined by Bayes's rule:

$$p(s_i \mid x_k) = \frac{p(x_k \mid s_i)}{p(x_k)} p(s_i)$$

Continue…

- $p(x_k)$: is the marginal probability, given as follows:

$$p(x_k) = \sum_{i=1}^{n} p(x_k \mid s_i) \cdot p(s_i)$$

- Now the expected utility for the jth alternative, given the data x_k, is determined from the posterior probabilities:

$$E(u_j \mid x_k) = \sum_{i=1}^{n} u_{ji}\, p(s_i \mid x_k)$$

- The maximum expected utility: $E(u^* \mid x_k) = \max_j E(u_j \mid x_k)$

Continue…

- To determine the unconditional maximum expected utility, we need to weight each of the r conditional expected utilities by the respective marginal probabilities for each datum x_k, that is given as:

$$E(u_x^*) = \sum_{k=1}^{r} E(u^* \mid x_k) \cdot p(x_k)$$

- If there is some uncertainty about the new information, we call the information imperfect information. The value of this imperfect information, V(x), can be given as follows:

$$V(x) = E(u_x^*) - E(u^*)$$

Continue…

- Perfect information is represented by posterior probabilities of 0 or 1:

$$p(s_i \mid x_k) = \begin{cases} 1 \\ 0 \end{cases}$$

- For perfect information, the maximum expected utility is presented as:

$$E(u^*_{x_p}) = \sum_{k=1}^{r} E(u^*_{x_p} \mid x_k) p(x_k)$$

- The value of perfect information is: $V(x_p) = E(u^*_{x_p}) - E(u^*)$

Continue…

- Continuation of the previous example: the CEO provides the utility matrix as follows:

u_{ji}	s_1	s_2
a_1	4	−2
a_2	−1	2

- The CEO has asked you to collect new information by taking eight geological boring samples from the region being considered for drilling. You have a natural gas expert examine the results of these eight tests, and get the expert's opinions about the conditional probabilities in the form of a matrix:

Continue…

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
$p(x_k \mid s_1)$	0	0.05	0.1	0.1	0.2	0.4	0.1	0.05	Σ row $= 1$
$p(x_k \mid s_2)$	0.05	0.1	0.4	0.2	0.1	0.1	0.05	0	Σ row $= 1$

- Moreover, you ask the natural gas expert for an assessment about how the conditional probabilities might change if they were perfect tests capable of providing perfect information. The expert gives you the matrix:

Continue…

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
$p(x_k \mid s_1)$	0	0	0	0	0.2	0.5	0.2	0.1	Σ row = 1
$p(x_k \mid s_2)$	0.1	0.2	0.5	0.2	0	0	0	0	Σ row = 1

- The expected utilities and maximum expected utility based just on prior probabilities are

$$E(a_1) = (4)(0.5) + (-2)(0.5) = 1.0.$$

$$E(a_1) = (-1)(0.5) + (2)(0.5) = 0.5.$$

$$E(u^*) = 1; \qquad \text{hence, you choose alternative } a_1, \text{ drill of natural gas.}$$

Continue…

- The marginal probabilities are calculated as follows:

$$p(x_1) = (0)(0.5) + (0.05)(0.5) = 0.025 \qquad p(x_k) = \sum_{i=1}^{n} p(x_k \mid s_i) \cdot p(s_i)$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
$p(x_k \mid s_1)$	0	0.05	0.1	0.1	0.2	0.4	0.1	0.05	Σ row $= 1$
$p(x_k \mid s_2)$	0.05	0.1	0.4	0.2	0.1	0.1	0.05	0	Σ row $= 1$
$p(x_k)$	0.025	0.075	0.25	0.15	0.15	0.25	0.075	0.025	

Continue…

- The posterior probabilities are calculated as follows:

$$p(s_1 \mid x_2) = \frac{0.05(0.5)}{0.075} = \frac{1}{3}, \qquad p(s_2 \mid x_2) = \frac{0.1(0.5)}{0.075} = \frac{2}{3}, \qquad p(s_i \mid x_k) = \frac{p(x_k \mid s_i)}{p(x_k)} p(s_i)$$

$$p(s_1 \mid x_6) = \frac{0.4(0.5)}{0.25} = \frac{4}{5}, \qquad p(s_2 \mid x_6) = \frac{0.1(0.5)}{0.25} = \frac{1}{5},$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$p(s_1 \mid x_k)$	0	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	1
$p(s_2 \mid x_k)$	1	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	0

Continue…

- The conditional expected utilities are calculated as follows:

$$E(u_1 \mid x_3) = (\tfrac{1}{5})(4) + (\tfrac{4}{5})(-2) = -\tfrac{4}{5} \quad \text{and} \quad E(u_2 \mid x_3) = (\tfrac{1}{5})(-1) + (\tfrac{4}{5})(2) = \tfrac{7}{5}$$

$$E(u_j \mid x_k) = \sum_{i=1}^{n} u_{ji}\, p(s_i \mid x_k)$$

$$E(u^* \mid x_k) = \max_j E(u_j \mid x_k)$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$p(s_1 \mid x_k)$	0	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	1
$p(s_2 \mid x_k)$	1	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	0
$p(x_k)$	0.025	0.075	0.25	0.15	0.15	0.25	0.075	0.025
$E(u^* \mid x_k)$	2	1	$\frac{7}{5}$	1	2	$\frac{14}{5}$	2	4
$a_j \mid x_k$	a_2	a_2	a_2	a_2	a_1	a_1	a_1	a_1

Continue…

- The overall unconditional expected utility for imperfect information is:

$$E(u_x^*) = (0.025)(2) + (0.075)(1) + \cdots + (0.025)(4) = 1.875$$

$$E(u_x^*) = \sum_{k=1}^{r} E(u^* \mid x_k) \cdot p(x_k)$$

- The value of the new imperfect information is:

$$V(x) = E(u_x^*) - E(u^*) = 1.875 - 1 = 0.875.$$

- Finally, which alternative?

Continue…

- For perfect information:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$p(s_1 \mid x_k)$	0	0	0	0	1	1	1	1
$p(s_2 \mid x_k)$	1	1	1	1	0	0	0	0
$p(x_k)$	0.05	0.1	0.25	0.1	0.1	0.25	0.1	0.05
$E(u^* \mid x_k)$	2	2	2	2	4	4	4	4
$a_j \mid x_k$	a_2	a_2	a_2	a_2	a_1	a_1	a_1	a_1

$E(u^*_{x_p}) = (0.05)(2) + (0.1)(2) + \cdots + (0.05)(4).$ $V(x_p) = E(u^*_{x_p}) - E(u^*) = 3 - 1 = 2.0.$

Continue…

- What about fuzzy information!
- Suppose the new information, $X=\{x_1, x_2,..., x_r\}$, is a universe of discourse in the units appropriate for the new information.
- Let's define fuzzy set ($\underset{\sim}{M}$) on the information (e.g. "good", "moderate" information).
- Thus, it has a membership function ($\mu_{\underset{\sim}{M}}(x_k)$).
- The probability of a fuzzy set:

$$P(\underset{\sim}{M}) = \sum_{k=1}^{r} \mu_{\underset{\sim}{M}}(x_k)\, p(x_k)$$

Continue…

- The posterior probability of s_i given fuzzy information $\underset{\sim}{M}$ can be written as:

$$P(s_i \mid \underset{\sim}{M}) = \frac{\sum_{k=1}^{r} p(x_k \mid s_i)\mu_{\underset{\sim}{M}}(x_k)p(s_i)}{P(\underset{\sim}{M})} = \frac{P(\underset{\sim}{M} \mid s_i)p(s_i)}{P(\underset{\sim}{M})},$$

$$p(\underset{\sim}{M} \mid s_i) = \sum_{k=1}^{r} p(x_k \mid s_i)\mu_{\underset{\sim}{M}}(x_k)$$

Continue…

- If the fuzzy events on the new information universe are orthogonal, the Bayesian approach can be extended to consider fuzzy information:

$$E(u_j \mid \underset{\sim}{M}_t) = \sum_{i=1}^{n} u_{ij} \cdot p(s_i \mid \underset{\sim}{M}_t).$$

$$E(u^* \mid \underset{\sim}{M}_t) = \max_j E(u_j \mid \underset{\sim}{M}_t).$$

$$E(u^*_\Phi) = \sum_{t=1}^{g} E(u^* \mid \underset{\sim}{M}_t) \cdot p(\underset{\sim}{M}_t).$$

- The value of the fuzzy information: $V(\Phi) = E(u^*_\Phi) - E(u^*)$

Continue…

- Continuation of the example: Suppose the eight data samples are from overlapping, ill-defined parcels within the drilling property. The orthogonal fuzzy information system:

$$\Phi = \{\underset{\sim}{M}_1, \underset{\sim}{M}_2, \underset{\sim}{M}_3\} = \{\text{fuzzy parcel 1, fuzzy parcel 2, fuzzy parcel 3}\}$$

- The membership functions:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$\mu_{\underset{\sim}{M}_1}(x_k)$	1	1	0.5	0	0	0	0	0
$\mu_{\underset{\sim}{M}_2}(x_k)$	0	0	0.5	1	1	0.5	0	0
$\mu_{\underset{\sim}{M}_3}(x_k)$	0	0	0	0	0	0.5	1	1
$P(x_k)$	0.025	0.075	0.25	0.15	0.15	0.25	0.075	0.025

Continue…

- The marginal probabilities for the fuzzy events:

$$P(\underset{\sim}{M}) = \sum_{k=1}^{r} \mu_{\underset{\sim}{M}}(x_k)\, p(x_k) \qquad p(\underset{\sim}{M}_1) = 0.225, \qquad p(\underset{\sim}{M}_2) = 0.55, \qquad p(\underset{\sim}{M}_3) = 0.225$$

- The fuzzy conditional probabilities:

$$p(\underset{\sim}{M} \mid s_i) = \sum_{k=1}^{r} p(x_k \mid s_i)\, \mu_{\underset{\sim}{M}}(x_k)$$

$$p(\underset{\sim}{M}_1 \mid s_1) = 0.1, \qquad p(\underset{\sim}{M}_2 \mid s_1) = 0.55, \qquad p(\underset{\sim}{M}_3 \mid s_1) = 0.35;$$

$$p(\underset{\sim}{M}_1 \mid s_2) = 0.35, \qquad p(\underset{\sim}{M}_2 \mid s_2) = 0.55, \qquad p(\underset{\sim}{M}_3 \mid s_2) = 0.1;$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
$p(x_k \mid s_1)$	0	0.05	0.1	0.1	0.2	0.4	0.1	0.05	Σ row = 1
$p(x_k \mid s_2)$	0.05	0.1	0.4	0.2	0.1	0.1	0.05	0	Σ row = 1

Continue…

- The fuzzy posterior probabilities:

$$P(s_i \mid \underset{\sim}{M}) = \frac{\sum_{k=1}^{r} p(x_k \mid s_i)\mu_{\underset{\sim}{M}}(x_k)p(s_i)}{P(\underset{\sim}{M})} = \frac{P(\underset{\sim}{M} \mid s_i)p(s_i)}{P(\underset{\sim}{M})},$$

$$p(s_1 \mid \underset{\sim}{M}_1) = 0.222, \qquad p(s_1 \mid \underset{\sim}{M}_2) = 0.5, \qquad p(s_1 \mid \underset{\sim}{M}_3) = 0.778;$$

$$p(s_2 \mid \underset{\sim}{M}_1) = 0.778, \qquad p(s_2 \mid \underset{\sim}{M}_2) = 0.5, \qquad p(s_2 \mid \underset{\sim}{M}_3) = 0.222.$$

Continue…

- The conditional fuzzy expected utilities:

$$
\begin{array}{ll}
\underset{\sim}{M}_1: & E(u_1 \mid \underset{\sim}{M}_1) = (4)(0.222) + (-2)(0.778) = -0.668 \\
 & E(u_2 \mid \underset{\sim}{M}_1) = (-1)(0.222) + (2)(0.778) = 1.334; \\
\underset{\sim}{M}_2: & E(u_1 \mid \underset{\sim}{M}_2) = (4)\,(0.5) + (-2)\,(0.5) = 1.0 \\
 & E(u_2 \mid \underset{\sim}{M}_2) = (-1)\,(0.5) + (2)\,(0.5) = 0.5; \\
\underset{\sim}{M}_3: & E(u_1 \mid \underset{\sim}{M}_3) = (4)(0.778) + (-2)(0.222) = 2.668 \\
 & E(u_2 \mid \underset{\sim}{M}_3) = (-1)(0.778) + (2)(0.222) = -0.334;
\end{array}
$$

- The maximum expected utility and the value of the fuzzy information:

$$E(u^*_\Phi) = (0.225)(1.334) + (0.55)(1) + (0.225)(2.668) = 1.45;$$

$$V(\Phi) = 1.45 - 1 = 0.45.$$

Decision Making Under Fuzzy States and Fuzzy Actions

- The Bayesian method can be further extended to include the possibility that the states of nature are fuzzy and the decision makers' alternatives are also fuzzy.

- Example: Building dike to prevent flooding:

1. build a permanent dike ($\underset{\sim}{A}_1$)
2. build a temporary dike ($\underset{\sim}{A}_2$)
3. do not build a dike ($\underset{\sim}{A}_3$).

Continue…

- The expected utility of fuzzy alternative $\underset{\sim}{A}_j$:

$$E(u_j) = \sum_{s=1}^{n} \mu_{js}\, p(\underset{\sim}{F}_s), \qquad p(\underset{\sim}{F}_s) = \sum_{i=1}^{n} \mu_{\underset{\sim}{F}_s}(s_i)\, p(s_i)$$

- The maximum utility: $E(u^*) = \max_j E(u_j)$.
- The posterior probabilities of fuzzy states $\underset{\sim}{F}_s$ given probabilistic information:

$$p(\underset{\sim}{F}_s \mid x_k) = \frac{\sum_{i=1}^{n} \mu_{\underset{\sim}{F}_s}(s_i)\, p(x_k \mid s_i)\, p(s_i)}{p(x_k)}.$$

Continue…

- The expected utility given probabilistic information:

$$E(u_j \mid x_k) = \sum_{s=1}^{n} u_{js}\, p(\underset{\sim}{F}_s \mid x_k),$$

- The posterior probabilities of fuzzy states $\underset{\sim}{F}_s$ given probabilistic information:

$$p(\underset{\sim}{F}_s \mid \underset{\sim}{M}_t) = \frac{\displaystyle\sum_{i=1}^{n}\sum_{i=1}^{r} \mu_{\underset{\sim}{F}_s}(s_i)\,\mu_{\underset{\sim}{M}_t}(x_k)\,p(x_k \mid s_i)\,p(s_i)}{\displaystyle\sum_{k=1}^{r} \mu_{\underset{\sim}{M}_t}(x_k)\,p(x_k)}.$$

Continue…

- The expected utility given fuzzy information:

$$E(u_j \mid \underset{\sim}{M}_t) = \sum_{s=1}^{n} u_{js}\, p(\underset{\sim}{F}_s \mid \underset{\sim}{M}_t)$$

- The maximum conditional expected utility for probabilistic and fuzzy information:

$$E(u^*_{x_k}) = \max_j E(u_j \mid x_k).$$

$$E(u^*_{\underset{\sim}{M}_t}) = \max_j E(u_j \mid \underset{\sim}{M}_t).$$

Continue…

- The unconditional expected utility for fuzzy states and probabilistic information or fuzzy information:

$$E(u_x^*) = \sum_{k=1}^{r} E(u_{x_k}^*)\,p(x_k).$$

$$E(u_\Phi^*) = \sum_{t=1}^{g} E(u_{\underset{\sim}{M}_t}^*)\,p(\underset{\sim}{M}_t).$$

- The value of the fuzzy information:

$$V(x) = E(u_x^*) - E(u^*)$$

$$V(\Phi) = E(u_\Phi^*) - E(u^*).$$

Example

One of the decisions your project team faces with each new computer product is what type of printed circuit board (PCB) will be required for the unit. Depending on the density of tracks (metal interconnect traces on the PCB that act like wire to connect components together), which is related to the density of the components, we may use a single-layer PCB, a double-layer PCB, a four-layer PCB, or a six-layer PCB. A PCB layer is a two-dimensional plane of interconnecting tracks. The number of layers on a PCB is the number of parallel interconnection layers in the PCB. The greater the density of the interconnections in the design, the greater the number of layers required to fit the design onto a PCB of given size. One measure of board track density is the number of nodes required in the design. A node is created at a location in the circuit where two or more lines (wires, tracks) meet. The decision process will comprise the following steps.

Continue…

1. Define the fuzzy states of nature: The density of the PCB is defined as three fuzzy sets on the singleton states $S=(s_1, s_2, s_3, s_4, s_5)=(s_i)$, $i = 1, 2, \ldots, 5$, where i defines the states in terms of a percentage of our most dense (in terms of components and interconnections) PCB. So, your team defines $s_1=20\%$, $s_2=40\%$, $s_3=60\%$, $s_4=80\%$ and $s_5=100\%$ of the density of the densest PCB; these are singletons on the universe of relative densities. Further, you define the following three fuzzy states that are defined on the universe of relative density states S:

$$\underset{\sim}{F_1} = \text{low-density PCB}$$

$$\underset{\sim}{F_2} = \text{medium-density PCB}$$

$$\underset{\sim}{F_3} = \text{high-density PCB.}$$

Continue…

2. Define fuzzy alternatives: Your decision alternative will represent the type of the PCB we decide to use as follows (these actions are admittedly not very fuzzy, but in general they can be):

$$\underset{\sim}{A}_1 = \text{use a 2-layer PCB for the new design}$$

$$\underset{\sim}{A}_2 = \text{use a 4-layer PCB for the new design}$$

$$\underset{\sim}{A}_3 = \text{use a 6-layer PCB for the new design.}$$

Continue…

3. Define new data samples (information): The universe $X=(x_1, x_2, \ldots, x_5)$ represents the "measured number of nodes in the PCB schematic"; that is, the additional information is the measured number of nodes of the schematic, which can be calculated by a schematic capture system. You propose the following discrete values for number of nodes:

$$
\begin{aligned}
x_1 &= 100 \text{ nodes} \\
x_2 &= 200 \text{ nodes} \\
x_3 &= 300 \text{ nodes} \\
x_4 &= 400 \text{ nodes} \\
x_5 &= 500 \text{ nodes.}
\end{aligned}
$$

Continue…

4. Define orthogonal fuzzy information system: You determine that the ambiguity in defining the density of nodes can be characterized by three linguistic information sets as $(\underset{\sim}{M}_1, \underset{\sim}{M}_2, \underset{\sim}{M}_3)$, where

$$\underset{\sim}{M}_1 = \text{low number of nodes on PCB [generally} < 300 \text{ nodes]}$$
$$\underset{\sim}{M}_2 = \text{average (medium) number of nodes on PCB [about 300 nodes]}$$
$$\underset{\sim}{M}_3 = \text{high number of nodes on PCB [generally} > 300 \text{ nodes].}$$

Continue…

5. Define the prior probabilities: The prior probabilities of the singleton densities (states) are as follows:

$$p(s_1) = 0.2$$
$$p(s_2) = 0.3$$
$$p(s_3) = 0.3$$
$$p(s_4) = 0.1$$
$$p(s_5) = 0.1.$$

The preceding numbers indicate that moderately dense boards are the most probable, followed by low-density boards, and high- to very high–density boards are the least probable.

Continue…

6. Identify the utility values: You propose the nondimensional utility values shown in the table below to represent the fuzzy alternative-fuzzy state relationships.

Utilities for fuzzy states and alternatives.

	$\underset{\sim}{F}_1$	$\underset{\sim}{F}_2$	$\underset{\sim}{F}_3$
$\underset{\sim}{A}_1$	10	3	0
$\underset{\sim}{A}_2$	4	9	6
$\underset{\sim}{A}_3$	1	7	10

Continue…

7. Define membership values for each orthogonal fuzzy state.

Orthogonal fuzzy sets for fuzzy states.

	s_1	s_2	s_3	s_4	s_5
$\underset{\sim}{F}_1$	1	0.5	0	0	0
$\underset{\sim}{F}_2$	0	0.5	1	0.5	0
$\underset{\sim}{F}_3$	0	0	0	0.5	1

Continue…

8. Define membership values for each orthogonal fuzzy set on the fuzzy information system:

Orthogonal fuzzy sets for fuzzy information.

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{M}_1$	1	0.4	0	0	0
$\underset{\sim}{M}_2$	0	0.6	1	0.6	0
$\underset{\sim}{M}_3$	0	0	0	0.4	1

Continue…

9. Define the conditional probabilities (likelihood values) for the uncertain information. The table below shows the conditional probabilities for uncertain (probabilistic) information.

Conditional probabilities $p(x_k \mid s_i)$ for uncertain information.

	x_1	x_2	x_3	x_4	x_5
$p(x_k \mid s_1)$	0.44	0.35	0.17	0.04	0
$p(x_k \mid s_2)$	0.26	0.32	0.26	0.13	0.03
$p(x_k \mid s_3)$	0.12	0.23	0.30	0.23	0.12
$p(x_k \mid s_4)$	0.03	0.13	0.26	0.32	0.26
$p(x_k \mid s_5)$	0	0.04	0.17	0.35	0.44

Continue…

10. Define the conditional probabilities (likelihood values) for the probabilistic perfect information. The table below shows the conditional probabilities for probabilistic perfect information.

Conditional probabilities $p(x_k \mid s_i)$ for fuzzy perfect information.

	x_1	x_2	x_3	x_4	x_5
$p(x_k \mid s_1)$	1	0	0	0	0
$p(x_k \mid s_2)$	0	1	0	0	0
$p(x_k \mid s_3)$	0	0	1	0	0
$p(x_k \mid s_4)$	0	0	0	1	0
$p(x_k \mid s_5)$	0	0	0	0	1

Continue…

Calculation: Crisp states and actions

(i) Utility and optimum decision given no information.

The nondimensional utility values for this nonfuzzy state situation are given as follows:

$E(u_1) = 6.4$

$E(u_2) = 6.3$

$E(u_3) = 4.4$

Utility values for crisp states.

	s_1	s_2	s_3	s_4	s_5
$\underset{\sim}{A}_1$	10	8	6	2	0
$\underset{\sim}{A}_2$	4	6	9	6	4
$\underset{\sim}{A}_3$	1	2	6	8	10

Continue…

(ii) Utility and optimal decision given uncertain and perfect information.

(a) Probabilistic (uncertain) information:

Calculate the unconditional expected utility?

7.37

The value of the uncertain information?

$V(x) = 7.37 - 6.4 = 0.97.$

	x_1	x_2	x_3	x_4	x_5
$p(x_k)$	0.205	0.252	0.245	0.183	0.115
$p(s_1 \mid x_k)$	0.429	0.278	0.139	0.044	0.0
$p(s_2 \mid x_k)$	0.380	0.381	0.318	0.213	0.078
$p(s_3 \mid x_k)$	0.176	0.274	0.367	0.377	0.313
$p(s_4 \mid x_k)$	0.015	0.052	0.106	0.175	0.226
$p(s_5 \mid x_k)$	0.0	0.016	0.069	0.191	0.383
$E(u^* \mid x_k)$	8.42	7.47	6.68	6.66	7.67
$a_j \mid a_k$	1	1	2	2	3

Continue…

(ii) Utility and optimal decision given uncertain and perfect information.

(b) Probabilistic perfect information:

Calculate the unconditional expected utility?

8.9

The value of uncertain Information?

$$V(x_p) = 8.9 - 6.4 = 2.5$$

	x_1	x_2	x_3	x_4	x_5
$p(x_k)$	0.20	0.30	0.30	0.10	0.10
$p(s_1 \mid x_k)$	1.0	0.0	0.0	0.0	0.0
$p(s_2 \mid x_k)$	0.0	1.0	0.0	0.0	0.0
$p(s_3 \mid x_k)$	0.0	0.0	1.0	0.0	0.0
$p(s_4 \mid x_k)$	0.0	0.0	0.0	1.0	0.0
$p(s_5 \mid x_k)$	0.0	0.0	0.0	0.0	1.0
$E(u^* \mid x_k)$	10.0	8.0	9.0	8.0	10.0
$a_j \mid a_k$	1	1	2	3	3

Continue…

Calculation: Fuzzy states and actions

(i) Utility and optimum decision given no information.

$$p(\underset{\sim}{F}_1) = (1)(0.2) + (0.5)(0.3) + (0)(0.3) + (0)(0.1) + (0)(0.1)$$
$$= 0.35.$$

$$p(\underset{\sim}{F}_s) = \sum_{i=1}^{n} \mu_{\underset{\sim}{F}_s}(s_i)\, p(s_i)$$

$p(\underset{\sim}{F}_2) = 0.5$ and $p(\underset{\sim}{F}_3) = 0.15$

Orthogonal fuzzy sets for fuzzy states.

	s_1	s_2	s_3	s_4	s_5
$\underset{\sim}{F}_1$	1	0.5	0	0	0
$\underset{\sim}{F}_2$	0	0.5	1	0.5	0
$\underset{\sim}{F}_3$	0	0	0	0.5	1

Continue…

Calculation: Fuzzy states and actions

(i) Utility and optimum decision given no information.

The expected utility:

$$E(u_j) = \sum_{s=1}^{n} \mu_{js} p(\underset{\sim}{F}_s), \qquad E(u_j) = \begin{bmatrix} 5 \\ 6.8 \\ 5.35 \end{bmatrix}$$

The optimum expected utility of the fuzzy alternatives for the case of no information is 6.8, thus alternative 2 is the optimum.

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(a) Probabilistic (uncertain) information:

$$p(\underset{\sim}{F}_s \mid x_k) = \frac{\sum_{i=1}^{n} \mu_{\underset{\sim}{F}_s}(s_i)\, p(x_k \mid s_i)\, p(s_i)}{p(x_k)}$$

$$p(\underset{\sim}{F}_1 \mid x_1) = \frac{(1)(0.44)(0.2) + (0.5)(0.26)(0.3)}{0.205} = 0.620$$

$$p(s_1) = 0.2$$
$$p(s_2) = 0.3$$
$$p(s_3) = 0.3$$
$$p(s_4) = 0.1$$
$$p(s_5) = 0.1.$$

Orthogonal fuzzy sets for fuzzy states.

	s_1	s_2	s_3	s_4	s_5
$\underset{\sim}{F}_1$	1	0.5	0	0	0
$\underset{\sim}{F}_2$	0	0.5	1	0.5	0
$\underset{\sim}{F}_3$	0	0	0	0.5	1

Conditional probabilities $p(x_k \mid s_i)$ for uncertain information.

	x_1	x_2	x_3	x_4	x_5
$p(x_k \mid s_1)$	0.44	0.35	0.17	0.04	0
$p(x_k \mid s_2)$	0.26	0.32	0.26	0.13	0.03
$p(x_k \mid s_3)$	0.12	0.23	0.30	0.23	0.12
$p(x_k \mid s_4)$	0.03	0.13	0.26	0.32	0.26
$p(x_k \mid s_5)$	0	0.04	0.17	0.35	0.44

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(a) Probabilistic (uncertain) information:

Posterior probabilities for probabilistic information with fuzzy states.

	$\underset{\sim}{F}_1$	$\underset{\sim}{F}_2$	$\underset{\sim}{F}_3$
x_1	0.620	0.373	0.007
x_2	0.468	0.49	0.042
x_3	0.298	0.58	0.122
x_4	0.15	0.571	0.279
x_5	0.039	0.465	0.496

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(a) Probabilistic (uncertain) information: The expected utility values for each of the x_k can be calculated:

$$E(u_j \mid x_k) = \sum_{s=1}^{n} u_{js}\, p(\underset{\sim}{F}_s \mid x_k),$$

Expected utilities for fuzzy alternatives with probabilistic information.

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$
x_1	7.315	5.880	3.305
x_2	6.153	6.534	4.315
x_3	4.718	7.143	5.58
x_4	3.216	7.413	6.934
x_5	1.787	7.317	8.252

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(a) Probabilistic (uncertain) information: The expected utility values for each of the x_k can be calculated:

The optimum expected utilities: $E(u^*_{x_k}) = \max_j E(u_j \mid x_k) = \{7.315, 6.534, 7.143, 7.413, 8.252\}$

The unconditional expected utilities:

$$E(u^*_{\Phi}) = \sum_{k=1}^{r} E(u^*_{x_k})p(x_k)$$
$$= (7.315)(0.205) + (6.534)(0.252) + (7.143)(0.245)$$
$$+ (7.413)(0.183) + (8.252)(0.115) = 7.202.$$

The value of the probabilistic uncertain information

$$V(x) = 7.202 - 6.8 = 0.402$$

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(b) Probabilistic perfect information:

$$p(\underset{\sim}{F}_s \mid x_k) = \frac{\sum_{i=1}^{n} \mu_{\underset{\sim}{F}_s}(s_i)\, p(x_k \mid s_i)\, p(s_i)}{p(x_k)}$$

$$p(\underset{\sim}{F}_1 \mid x_1) = [(1)(1)(0.2) + (0.5)(0)(0.3]/(0.2) = 1.0$$

$p(s_1) = 0.2$
$p(s_2) = 0.3$
$p(s_3) = 0.3$
$p(s_4) = 0.1$
$p(s_5) = 0.1.$

Orthogonal fuzzy sets for fuzzy states.

	s_1	s_2	s_3	s_4	s_5
$\underset{\sim}{F}_1$	1	0.5	0	0	0
$\underset{\sim}{F}_2$	0	0.5	1	0.5	0
$\underset{\sim}{F}_3$	0	0	0	0.5	1

Conditional probabilities $p(x_k \mid s_i)$ for fuzzy perfect information.

	x_1	x_2	x_3	x_4	x_5
$p(x_k \mid s_1)$	1	0	0	0	0
$p(x_k \mid s_2)$	0	1	0	0	0
$p(x_k \mid s_3)$	0	0	1	0	0
$p(x_k \mid s_4)$	0	0	0	1	0
$p(x_k \mid s_5)$	0	0	0	0	1

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(b) Probabilistic perfect information:

Posterior probabilities for probabilistic *perfect* information with fuzzy states.

	$\underset{\sim}{F}_1$	$\underset{\sim}{F}_2$	$\underset{\sim}{F}_3$
x_1	1.0	0.0	0.0
x_2	0.5	0.5	0.0
x_3	0.0	1.0	0.0
x_4	0.0	0.5	0.5
x_5	0.0	0.0	1.0

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(b) Probabilistic perfect information:

The optimum expected utilities:

$$E(u^*_{x_k}) = \max_j E(u_j \mid x_k) = \{10.0, 6.5, 9.0, 8.5, 10.0\}$$

The unconditional expected utilities:

$$E(u^*_{x_p}) = \sum_{k=1}^{r} E(u^*_{x_p} \mid x_k) p(x_k)$$

$$= (10.0)(0.2) + (6.5)(0.3) + (9.0)(0.3) + (8.5)(0.1) + (10.0)(0.1)$$

$$= 8.5.$$

Expected utilities for fuzzy alternatives with probabilistic *perfect* information.

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$
x_1	10.0	4.0	1.0
x_2	6.5	6.5	4.0
x_3	3.0	9.0	7.0
x_4	1.5	7.5	8.5
x_5	0.0	6.0	10.0

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(c) Fuzzy information:

$$p(\underset{\sim}{F}_s \mid \underset{\sim}{M}_t) = \frac{\sum_{i=1}^{n}\sum_{i=1}^{r} \mu_{\underset{\sim}{F}_s}(s_i)\mu_{\underset{\sim}{M}_t}(x_k)p(x_k \mid s_i)p(s_i)}{\sum_{k=1}^{r} \mu_{\underset{\sim}{M}_t}(x_k)p(x_k)}.$$

$$p(\underset{\sim}{F}_1 \mid \underset{\sim}{M}_1) = [(1)(1)(0.44)(0.2) + (1)(0.4)(0.35)(0.2) + (0.5)(1)(0.26)(0.3) + (0.5)(0.4)(0.32)(0.3)] \div [(1)(0.205) + (0.4)(0.252)] = 0.57$$

$$p(s_1) = 0.2$$
$$p(s_2) = 0.3$$
$$p(s_3) = 0.3$$
$$p(s_4) = 0.1$$
$$p(s_5) = 0.1.$$

Orthogonal fuzzy sets for fuzzy states.

	s_1	s_2	s_3	s_4	s_5
$\underset{\sim}{F}_1$	1	0.5	0	0	0
$\underset{\sim}{F}_2$	0	0.5	1	0.5	0
$\underset{\sim}{F}_3$	0	0	0	0.5	1

Orthogonal fuzzy sets for fuzzy information.

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{M}_1$	1	0.4	0	0	0
$\underset{\sim}{M}_2$	0	0.6	1	0.6	0
$\underset{\sim}{M}_3$	0	0	0	0.4	1

Conditional probabilities $p(x_k \mid s_i)$ for uncertain information.

	x_1	x_2	x_3	x_4	x_5
$p(x_k \mid s_1)$	0.44	0.35	0.17	0.04	0
$p(x_k \mid s_2)$	0.26	0.32	0.26	0.13	0.03
$p(x_k \mid s_3)$	0.12	0.23	0.30	0.23	0.12
$p(x_k \mid s_4)$	0.03	0.13	0.26	0.32	0.26
$p(x_k \mid s_5)$	0	0.04	0.17	0.35	0.44

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(c) Fuzzy information:

Posterior probabilities for fuzzy information with fuzzy states.

	$\underset{\sim}{M}_1$	$\underset{\sim}{M}_2$	$\underset{\sim}{M}_3$
$\underset{\sim}{F}_1$	0.570	0.317	0.082
$\underset{\sim}{F}_2$	0.412	0.551	0.506
$\underset{\sim}{F}_3$	0.019	0.132	0.411

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(c) Fuzzy information:

The expected utilities:

$$E(u_j \mid \underset{\sim}{M}_t) = \sum_{s=1}^{n} u_{js}\, p(\underset{\sim}{F}_s \mid \underset{\sim}{M}_t)$$

the optimum expected utility

Posterior probabilities for fuzzy alternatives with fuzzy information.

	$\underset{\sim}{M}_1$	$\underset{\sim}{M}_2$	$\underset{\sim}{M}_3$
$\underset{\sim}{A}_1$	6.932	4.821	2.343
$\underset{\sim}{A}_2$	6.096	7.019	7.354
$\underset{\sim}{A}_3$	3.638	5.496	7.740

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(c) Fuzzy information:

The optimum expected utility: $E(u^*_{\underset{\sim}{M}_t}) = \max_j E(u_j \mid \underset{\sim}{M}_t) = \{6.932, 7.019, 7.740\}$

The marginal probabilities of the fuzzy information sets:

$$P(\underset{\sim}{M}) = \sum_{k=1}^{r} \mu_{\underset{\sim}{M}}(x_k)\, p(x_k)$$

Orthogonal fuzzy sets for fuzzy information.

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{M}_1$	1	0.4	0	0	0
$\underset{\sim}{M}_2$	0	0.6	1	0.6	0
$\underset{\sim}{M}_3$	0	0	0	0.4	1

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(c) Fuzzy information:

The marginal probabilities of the fuzzy information sets:

$$P(\underset{\sim}{M}) = \sum_{k=1}^{r} \mu_{\underset{\sim}{M}}(x_k)\, p(x_k)$$

Orthogonal fuzzy sets for fuzzy information.

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{M}_1$	1	0.4	0	0	0
$\underset{\sim}{M}_2$	0	0.6	1	0.6	0
$\underset{\sim}{M}_3$	0	0	0	0.4	1

	x_1	x_2	x_3	x_4	x_5
$p(x_k)$	0.205	0.252	0.245	0.183	0.115

$$p(\underset{\sim}{M}_t) = \begin{bmatrix} 0.306 \\ 0.506 \\ 0.188 \end{bmatrix}$$

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(c) Fuzzy information:

The unconditional expected utility:

$$p(\underset{\sim}{M}_t) = \begin{bmatrix} 0.306 \\ 0.506 \\ 0.188 \end{bmatrix}$$

$$E(u^*_{\underset{\sim}{M}_t}) = \max_j E(u_j \mid \underset{\sim}{M}_t) = \{6.932, 7.019, 7.740\}$$

$$E(u^*_{\Phi}) = \sum_{t=1}^{g} E(u^*_{\underset{\sim}{M}_t}) p(\underset{\sim}{M}_t) = 7.128$$

$$V(\Phi) = 7.128 - 6.8 = 0.328$$

Continue…

Calculation: Fuzzy states and actions

(ii) Utility and optimum decision given uncertain and perfect information.

(d) Fuzzy perfect information:

$$u(\underset{\sim}{A}_i \mid \underset{\sim}{F}_s) = u(\underset{\sim}{A}_i, \underset{\sim}{F}_s)$$

The optimum fuzzy action:

$$u(A^*_{\underset{\sim}{F}_s} \mid \underset{\sim}{F}_s) = \max_i u(\underset{\sim}{A}_i, \underset{\sim}{F}_s) = \{10.0, 9.0, 10.0\}$$

The unconditional expected utility:

$$E(u^*_{\Phi_p}) = \sum_{j=1}^{3} u(A^*_{\underset{\sim}{F}_s} \mid \underset{\sim}{F}_s)\, p(\underset{\sim}{F}_s) = 10(0.35) + 9(0.5) + 10(0.15) = 9.5$$

The value of fuzzy perfect information is 2.7

Expected utilities for fuzzy alternatives with fuzzy perfect information.

	$\underset{\sim}{F}_1$	$\underset{\sim}{F}_2$	$\underset{\sim}{F}_3$
$\underset{\sim}{A}_1$	10.0	3.0	0.0
$\underset{\sim}{A}_2$	4.0	9.0	6.0
$\underset{\sim}{A}_3$	1.0	7.0	10.0

Continue…

Summary:

Summary of expected utility and value of information for fuzzy states and actions for the example.

Information	Expected utility	Value of information
No information	6.8	–
Probabilistic information, $V(x)$	7.20	0.40
Perfect information, $V(x_p)$	8.5	1.7
Fuzzy probabilistic information, $V(\Phi)$	7.13	0.33
Fuzzy perfect information, $V(\Phi_p)$	9.5	2.7