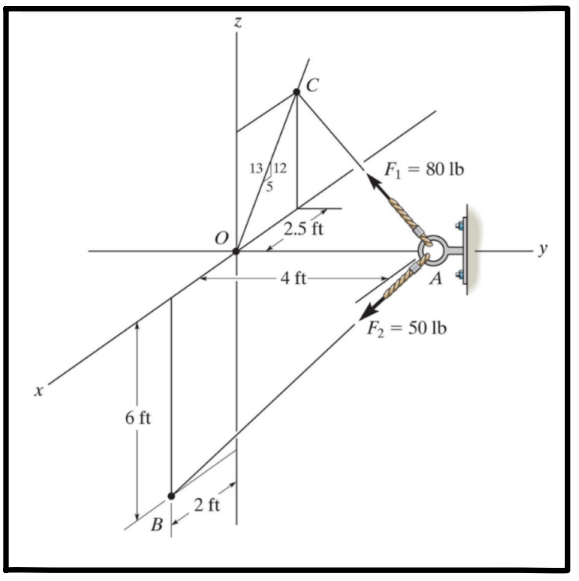
Problem 1. (11 points) 9

a) Express $\mathbf{F}_1$ and $\mathbf{F}_2$ in Cartesian vectors
b) Find the magnitude of the component of $\mathbf{F}_1$ acting along OC
c) Find the resultant force ($\mathbf{F}_R$) at A (express as a Cartesian vector)
d) Find the coordinate angles of $\mathbf{F}_R$
e) Find the resultant moment about O (express as a Cartesian vector)



Solution:

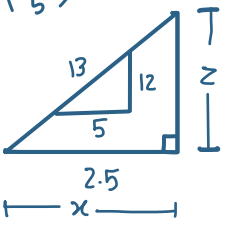
A (0, 4, 0) ft
B (2, 0, −6) ft
C (−2.5, 0, 6) ft

أولًا نجهّز كل ما سنحتاجه

$$\vec{u}_{AC} = \frac{\vec{r}_{AC}}{|r_{AC}|} = \frac{-2.5\,\hat{i} - 4\,\hat{j} + 6\,\hat{k}}{\sqrt{2.5^2 + 4^2 + 6^2}}$$

$$= -0.328\,\hat{i} - 0.524\,\hat{j} + 0.786\,\hat{k}$$

$\tau = (2.5)\left(\frac{12}{5}\right) = 6$



$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{2\,\hat{i} - 4\,\hat{j} - 6\,\hat{k}}{\sqrt{2^2 + 4^2 + 6^2}} = 0.267\,\hat{i} - 0.535\,\hat{j} - 0.802\,\hat{k}$$

$$\vec{u}_{OC} = \frac{\vec{r}_{OC}}{|r_{OC}|} = \frac{-2.5\,\hat{i} + 6\,\hat{k}}{\sqrt{2.5^2 + 6^2}} = -0.385\,\hat{i} + 0.923\,\hat{k}$$

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a) $$\vec{F}_1 = |F_1|\,\vec{u}_{AC} = 80\left[-0.328\,\hat{i} - 0.524\,\hat{j} + 0.786\,\hat{k}\right]$$

$$= -26.24\,\hat{i} - 41.92\,\hat{j} + 62.88\,\hat{k}\ (\text{lb})$$

$$\vec{F}_2 = |F_2|\,\vec{u}_{AB} = 50\left[0.267\,\hat{i} - 0.535\,\hat{j} - 0.802\,\hat{k}\right]$$

$$= 13.35\,\hat{i} - 26.75\,\hat{j} - 40.1\,\hat{k}\ (\text{lb})$$

b) $\vec{F_1} \cdot \vec{u}_{OC}$

$= \left(-26.24\,\hat{i} - 41.92\,\hat{j} + 62.88\,\hat{K}\right) \cdot \left(-0.385\,\hat{i} + 0.923\,\hat{K}\right)$

$= 10.10 + 0 + 58.03$

$= 68.14$ (lb)

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c) $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$= -26.24\,\hat{i} - 41.92\,\hat{j} + 62.88\,\hat{K} + 13.35\,\hat{i} - 26.75\,\hat{j} - 40.1\,\hat{K}$

$= -12.89\,\hat{i} - 68.62\,\hat{j} + 22.78\,\hat{K}$ (lb)

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d) $\dfrac{\vec{F_R}}{|F_R|} = -0.174\,\hat{i} - 0.934\,\hat{j} + 0.31\,\hat{K}$

$\alpha = \cos^{-1}(-0.174) = 100.02^\circ$

$\beta = \cos^{-1}(-0.934) = 159.07^\circ$

$\gamma = \cos^{-1}(0.31) = 71.94^\circ$

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e) $\vec{M_o} = \vec{r_{OA}} \times (\vec{F_1} + \vec{F_2}) = \vec{r_{OA}} \times \vec{F_R}$

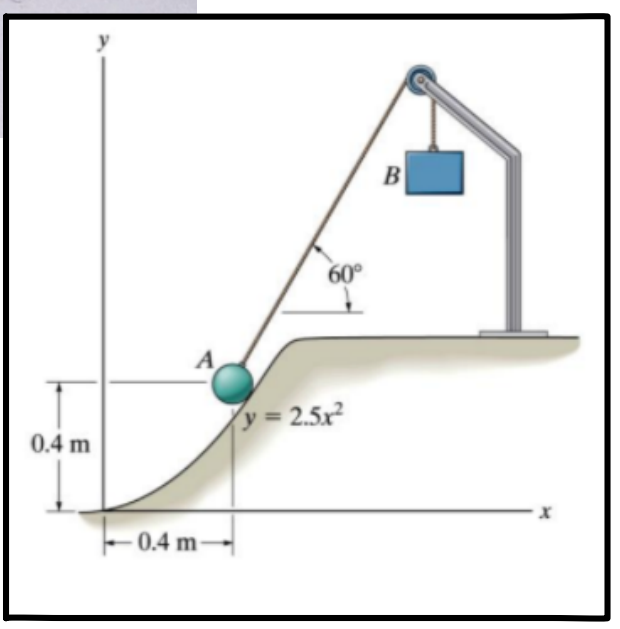
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 0 & 4 & 0 \\ -12.89 & -68.62 & 22.78 \end{vmatrix}$$

$= 91.12\,\hat{i} + 51.56\,\hat{K}$ (lb.ft)

Problem 2. (6 points) 5

A 4-kg sphere rests on the smooth parabolic surface and is held in equilibrium by block B connected to the sphere by a cord, as shown. Determine:

a) The normal force from the surface on the sphere
b) The mass block *B*.



Solution:

أولاً نجد θ عن طريق ميل المماس

$y = \frac{5}{2}x^2$

$y' = 5x$

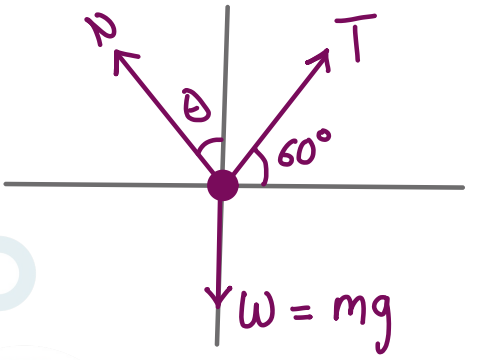
$y' = 5(0.4)$

$= 2$

نقطة التماس هي (0.4, 0.4)

$y' = \tan\theta \rightarrow 2 = \tan\theta$

$\theta = \tan^{-1}(2) = 63.43°$



a) equilibrium $\nearrow \Sigma F_x = 0$, $\searrow \Sigma F_y = 0$

$\Sigma F_x = 0$

$T\cos 60° - N\sin 63.43 = 0$

$T = 1.789\ N$

$\Sigma F_y = 0$

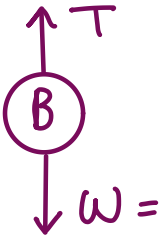
$T\sin 60° + N\cos 63.43 - 39.24 = 0$

بحل المعادلتين

$T = 35.164\ (N)$

$N = 19.66\ (N)$

b)



$\Sigma F_y = 0$

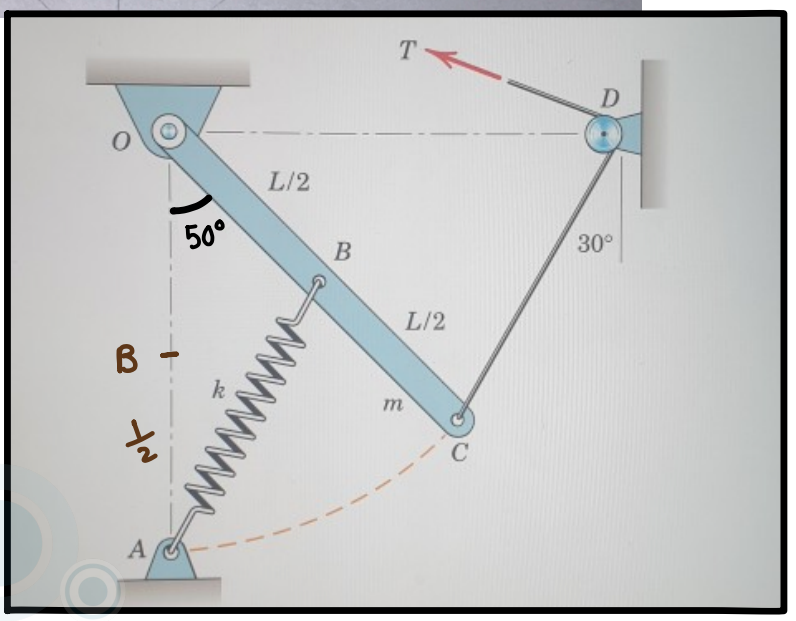
$m_B g = T$

$m_B = \frac{35.164}{9.81}$

$= 3.585\ (kg)$

Problem 3. (8 points) 3

Bar OC has a length L =1 m. The spring has a stiffness k =400 N/m and is unstretched when C is coincident with A. Considering smooth contact at B and neglecting the mass of the bar, determine the reactions at O and B in the position shown for which T=100 N.



Solution:

المطلوب هو ايجاد

$N_B / O_x / O_y$

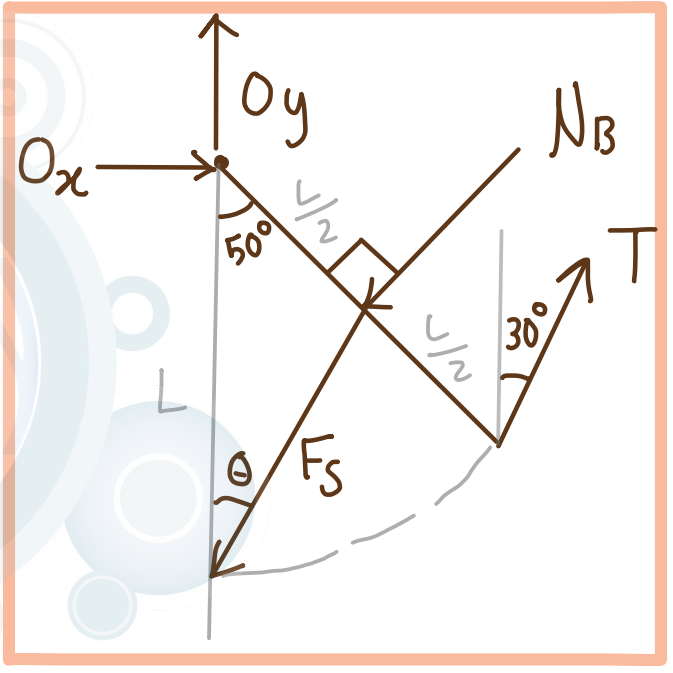
$F_s = KS$

$S = L_{sp}$ when it is streched $- L_{sp}$ when it is unstreched

L_{sp} when it is unstreched $= 0.5\ m$

L_{sp} when it is streched $=$ from the law of cosine $= \sqrt{1^2 + (\frac{1}{2})^2 - (1)(\frac{1}{2})\cos 50^\circ}$

$= 0.779\ (m)$



$S = 0.779 - 0.5 = 0.279$

$F_s = KS = 400(0.279)$

$= 111.6\ N$

Now to find θ we use the law of sine

$$\frac{0.779}{\sin 50^\circ} = \frac{0.5}{\sin\theta} \Longrightarrow \theta = 29.45^\circ$$

$$+\circlearrowleft \Sigma \vec{M}_O = 0$$

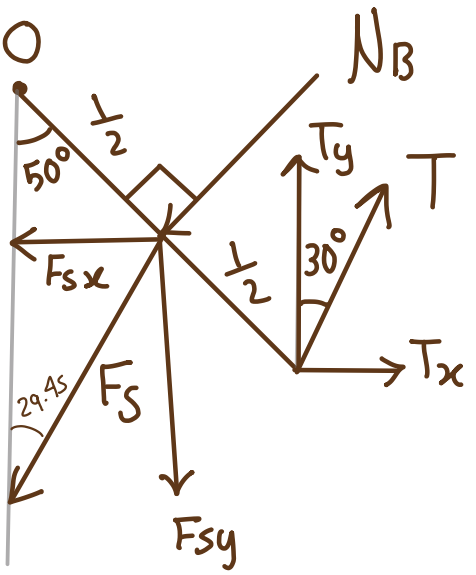
$$-(N_B)(0.5)$$

$$-(111.6 \sin 29.45^\circ)\left(\tfrac{1}{2}\cos 50^\circ\right)$$

$$-(111.6 \cos 29.45^\circ)\left(\tfrac{1}{2}\sin 50^\circ\right)$$

$$+(100 \sin 30^\circ)(1 \cos 50^\circ)$$

$$+(100 \cos 30^\circ)(1 \sin 50^\circ) = 0$$



⇓ بعد حل المعادلة

$$N_B = 87.15 \ (N)$$

$$\uparrow \Sigma F_y = 0$$

$$+ O_y$$

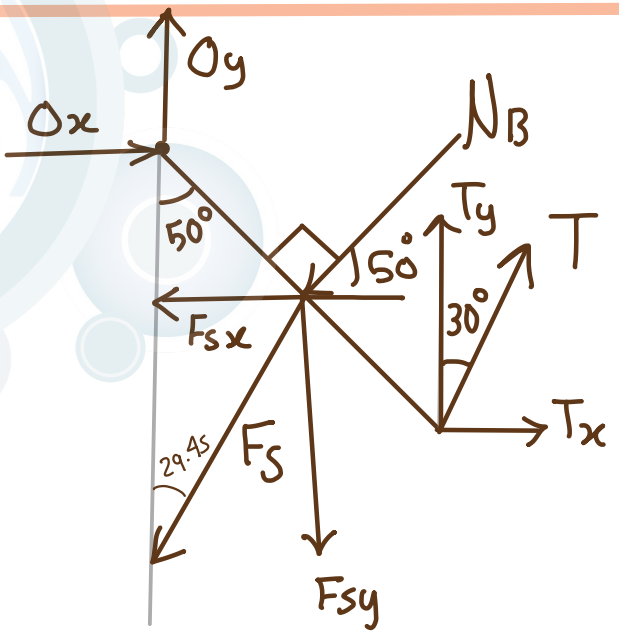
$$-(111.6 \cos 29.45^\circ)$$

$$+(100 \cos 30^\circ)$$

$$-(87.15 \sin 50^\circ) = 0$$

⇓

$$O_y = 77.34 \ (N) \uparrow$$



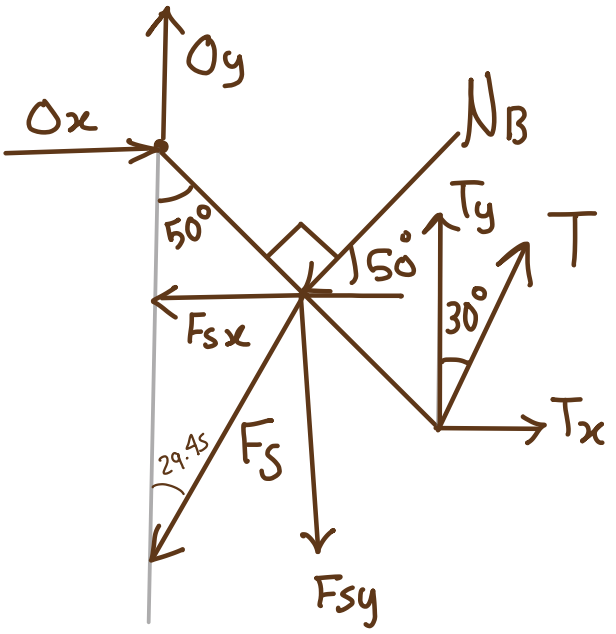
$$\Sigma F_x = 0$$

$$+Ox$$

$$-(111.6 \sin 29.45°)$$

$$+(100 \sin 30°)$$

$$-(87.15 \cos 50°) = 0$$



$$Ox = 60.89 \text{ (N)} \rightarrow$$