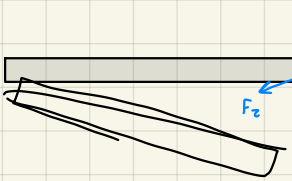
: mechanics ل من مطلوب

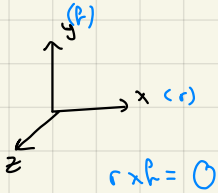
- stress & strain
- loads → normal load
  - shear load
  - Bending moment
  - Torque (Torsion)



bending moment $r \times f$

$\vec{M}_z = \vec{r} \times \vec{F}_y$

$\vec{M}_y = \vec{r} \times \vec{F}_z$



$r \times f = 0$



$\epsilon = \frac{\Delta L}{L_0}$ engineering strain

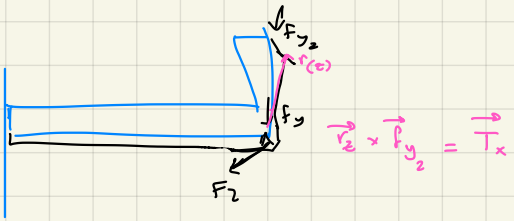
$\epsilon_x = \ln(1+\epsilon)$ elastic strain

critical section : has on it the maximum load. (stress)

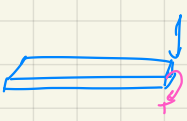
principal stresses :-

max. generated stresses

$\sigma_{max} = \sigma_1$, τ_{max} $\sigma_{min} = \sigma_2$



$\vec{r_2} \times \vec{f}_{y_2} = \vec{T}_x$



$\vec{T}_x = \vec{r_2} \times f_{y_2}$

$M_z = \vec{r_x} \times f_{y_2}$

* structural design : for the structure (building, جسر) to not be a failure.
هاد للجزء بس الجزء عشان ما يفشل .

↓ material selection + percentage

* mechanical engineering design : to design mechanical systems, and to not be a failed system
↳ the one we are intersted in.

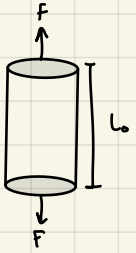
- وهذا شغل الـ mechanical eng. design :

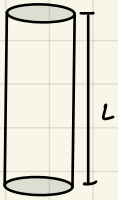
① Power transmission. نقل power من مكان لآخر
in a car the motor rotates → torque → mechanical system
to trans the power to the عجلات

---

deformation in material.

tensile load :





زيادة في الطول
cross-section نقصان بال

Compressive load:-

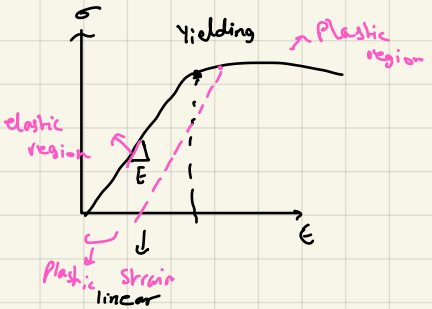
→ volume has to be constat





Stress $\sigma = \frac{F}{A}$

Strain $\epsilon = \frac{\Delta L}{L_o}$



$\sigma \propto \epsilon$

Hooke's law $\Rightarrow$ $\sigma = E \times \epsilon$

E : young's modules $\quad E = \frac{\Delta \sigma}{\Delta \epsilon}$

Plastic deformation

* if severe change of the shape of the material occurs $\Rightarrow$ it losses function
  ↳ we want to stop this

---

* strength: magnitude of stres at which it occurs

- Yield strength: a level of stress at which yielding occurs. (plastic deformation begins)

---

* types of loads (that produces stres)

① tensile
② compressive
③ shear

two styles:

- Static loads: constant with time (static failure) → Yielding
- time varying (fluctuating load) → load → fatigue failure (التعب المعدني)

- we are intrested in the Yield strength

force > Yield strength ⇒ Plastic deformation

↓

the ans in table

0-20 are mean

Yeilding = failure.

$\sigma = \frac{F}{A}$

$= \frac{N}{m^2} = Pa$

we don't use m^2 ↓ mm^2

$\frac{N}{mm^2} = MPa$

$\sigma = \frac{F}{A}$

$= \frac{Pound}{in^2}$

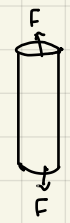
$= psi$

↳ $kpsi$ ← $kilo = 10^3$

$Mega = 10^6$

$\epsilon = \frac{\Delta L}{L_o}$ ← change in length / original length

$= \frac{mm}{mm} = \underline{unitless}$



tension



compression

$\epsilon = \frac{\Delta L}{L_o} = \frac{L_f - L_o}{L_o}$

tension → $L_f > L_o$ → ΔL +ve → ϵ +ve ⇒ σ → +ve

compression → $L_o > L_f$ → ΔL -ve → ϵ -ve ⇒ σ → -ve.

in tension stress is positive

in compression stres is negative.

---

* Factor of Safety (FOS) :-

ratio between Yield strength of material & max applied stress

two types :- ① deterministic , ② probabilistic

1 Deterministic (n) :-

$n = \frac{\text{Yield strength}}{\text{max stress}} = \frac{S_y}{\text{max } \sigma}$, Yield factor of safety

n →
- $n > 1$ (no failure) $S_y > \sigma$
- $n < 1$ (failure) $\sigma > S_y$
- $n = 1$ $S_y = \sigma$

→ $S_y > \sigma$ → no failure (yielding)

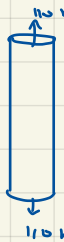
→ $S_y < \sigma$ → failure.

n range : 1.2 → 4

this is stronger

ex $n = 1.1$ → no failure $n = 2.5$ → no failure

example:-



$D = 20\,mm$ (for cross section)

$S_y = 480\ MPa$

find factor of safety (n)

$$\max \sigma = \frac{F}{A} = \frac{110 \times 10^3\ N}{\frac{\pi}{4}(20)^2} = 350.14\ MPa$$

$$n = \frac{S_y}{\sigma} = \frac{480}{350.14} = 1.37 > 1$$

no failure (survival)

---

2 probabilistic (stochastic) factor of safety :-

$S_y \rightarrow$ continuous random var.

$\sigma \rightarrow$ continuous random var.

- both are normaly distributed.

$P(S_y > \sigma)$ survival (Reliability (1-failure))

Just two probability

$P(\sigma > S_y)$ failure

$P(\text{failure}) + P(\text{survival}) = 100\%$.

$M_s = \ldots \quad \hat{\sigma}_s = \ldots$

$M_\sigma = \ldots \quad \hat{\sigma}_\sigma = \ldots$

normal distribution ⇐ reliability نسبة إلى $P(S_y > \sigma)$ $P(\text{survival})$ إلى

$P(S_y > \sigma) \Rightarrow P(S_y - \sigma > 0)$

merge in a

$m = S_y - \sigma$

$P(S_y > \sigma) = P(m > 0)$ survival

$P(m < 0)$ failure.

mean

$\hat{M}_m = M_s - M_\sigma$

$V_m = V_s + V_\sigma$

variance

$\hat{\sigma} = \sqrt{V} \rightarrow V = \hat{\sigma}^2$

$\hat{\sigma}_m = \sqrt{\hat{\sigma}^2_{(S_y)} + \hat{\sigma}^2_\sigma}$

$P(m > 0) \rightarrow$ reliability of survival

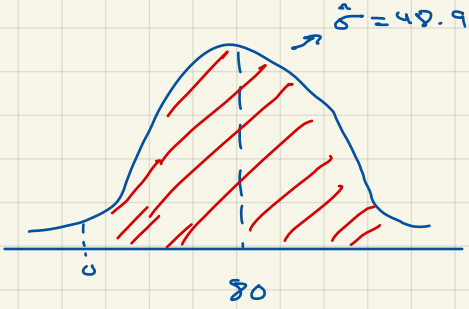
$\Downarrow$

Z

example :

$M_s = 553$ Mpa , $\hat{\sigma}_s = 42.7$ Mpa

$M_\sigma = 473$ Mpa , $\hat{\sigma}_\sigma = 23.8$ Mpa



① $n = \frac{M_s}{M_\sigma} = \frac{553}{473} = 1.17$

② a) $m = S_y - \sigma$

$M_m = 553 - 473 = 80$ Mpa

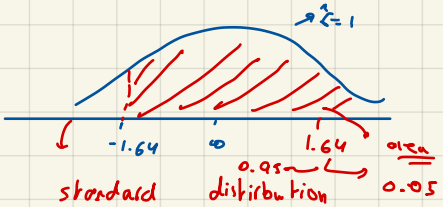
$\hat{\sigma}_m = \sqrt{42.7^2 + 23.8^2} = 48.9$ Mpa

لحساب الاحتمال نحول z

b) reliability $P(m>0)$

c) calculations (standardization to Z)

$$z = \frac{x - M_m}{\hat{\sigma}_m} \rightarrow P\left(\frac{m - M_m}{\hat{\sigma}_m} > \frac{0 - M_m}{\hat{\sigma}_m}\right)$$



$P(z > -1.64)$ ⇐ هاد اللي بدنا نجيبه

$z \rightarrow M=0 \rightarrow \sigma=1$

$P(z < \text{--}) \rightarrow$ area to the right

table $\rightarrow P(z > -1.64)$

$P(z > 0) = 0.95$

$R = 0.95$

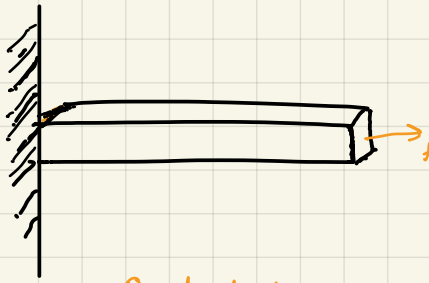
$P(z < -1.64) = P(z > 1.64)$

$= 1 - P(z > 1.64)$

Chapter 3 :-

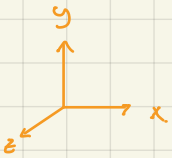
① normal force (tension +ve, compression -ve)

axial force. (normal on the cross section)



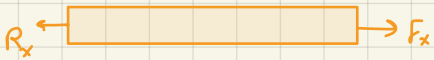
rectangle cross section

① decide the coordinate axis (draw it)



- the second force to balance it is the reaction from the wall.

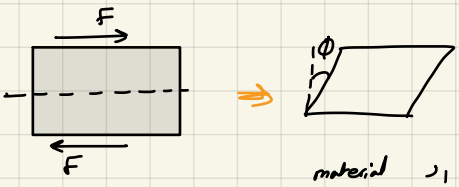
② FBD



---

② shear force

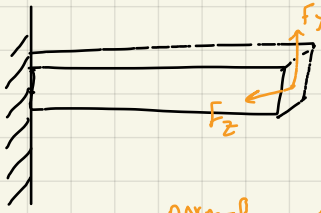
two equal forces in opposite directions

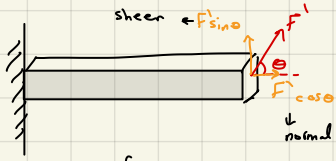


ideally speaking

بنقول ع المثالي

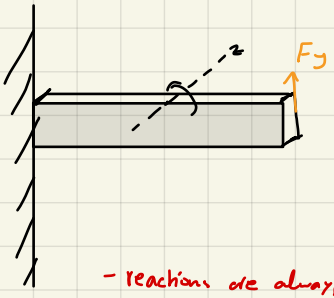
material بنقول ع

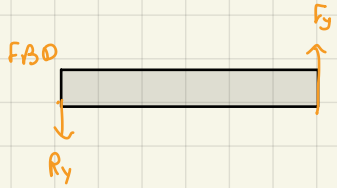




force effect with a degree on c.s → تكليل

- normal on C.S → normal
- parallel to C.S → shear





$R_y = F_y$ ⇒ shear force ⇓ one couple.

- reactions are always force و بس

---

* shear force ⇒ transaition motion

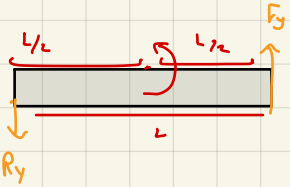
* Moment bending ⇒ rotational motion

$M = Fd$. [N.m]

F للقوة اللي بدي احسب عزمها — مسافة عامودية من

rotational — Scalar

C.W → -ve. — C.C.W → +ve



the moment is rotating around an axis so لحتى اعرف اتجاه الـ moment لازم عين على الـ axis .

① sense of rotation

② moment of vector ⇐ force و شو بيأثر

vector operations



Right hand rule

$$\vec{M} = \vec{r} \times \vec{F}$$

بالترتيب

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

r is measured from the point we want to calculate M in it To the force.

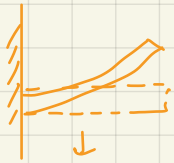


- the force has moment everwhere unless on the line of action

$F_z \rightarrow M_{z\,ME} = 0$

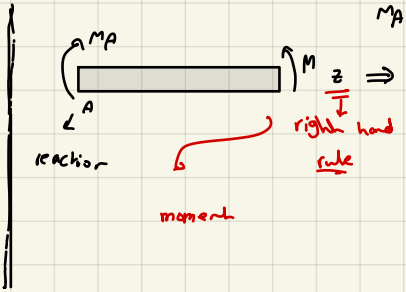
$\Sigma M = 0$

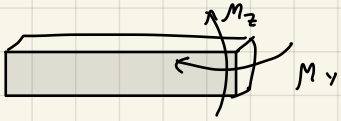
$\Sigma M_{c.w} = \Sigma M_{c.c.w}$



we.

bending

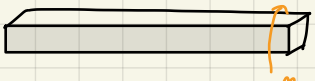




when the moment is around an axis wich is parallel to c.s

⇓

Bending moment



- moment around an axis wich is normal on c.s

⇓ Torisional moment (Tourqe)

(twist) it not bending

* Types of forces :-

① normal force (P)
↓ force normal on C.S
(longitudinal / major axis) ← along His

② shear force (V)
↓ parallel to C.S

③ bending moment (M)
moment around axis parallel to C.S

④ Torque (T)
moment around an axis normal to C.S
around the longitudinal axis

---

* Types of stresses :-

① normal stress
↓ from normal force
$\sigma = \frac{F_{normal}}{A}$

② shear stress
↓ from shear force
$\tau = \frac{F_{shear}}{A}$

Hooks' Law :

$\sigma = E \epsilon$
σ : normal stress
ϵ : normal strain
E : modulus of elasticity

$\tau = G \gamma$ → gama → strain
↓ stress shear
modulus of rigidity

---

* normal force (P) ⇒ normal stress $\sigma = \frac{P}{A}$
* shear force (V) ⇒ shear force $\tau = \frac{V}{A}$
* bending moment (M) ⇒ normal stress
* Tourge (T) ⇒ shear stress (displacement)

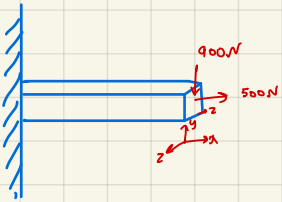
* Chapter 3-9 :- uniformly distributed stress

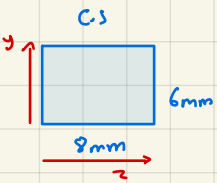
$\sigma = \frac{F}{A}$ → normal $\sigma = \frac{P}{A}$

→ shear $\tau = \frac{V}{A}$

---

example 8-





∥ parallel

⊥ normal

1) determine the normal stress caused by $F_x = 500N$

2) determine the avarge shear stress caused by $F_y = 900N$

① $\sigma = \frac{500}{8\times10^{-3} \times 6\times10^{-3}} = 10.4\ MPa$

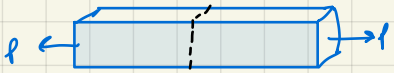
② * he asked for avarge shear , why AVARGE *

↓ we will know under this

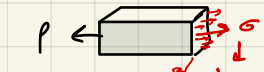
$\tau = \frac{900}{8\times10^{-3}\times6\times10^{-3}} = 13.75\ MPa$

---

in normal stress:



→ in equilibriem



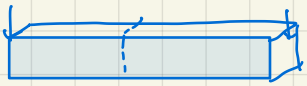
→ it has to be in equilibriem so when we cut it we have internal force.

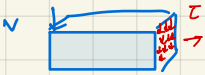
any point on the C.s has the same stress (σ)

internal ↓ it's uniformly distributed.

$\sigma = \frac{P}{A}$

in shear stress:





shear stresses don't be uniformally distributed, not all points have the same (τ), there is a point with max τ 

$$\text{avg } \tau = \frac{V}{A}$$

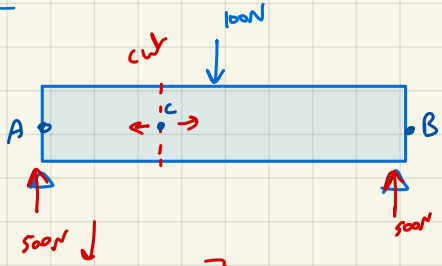
but in one specific point there's another rule.

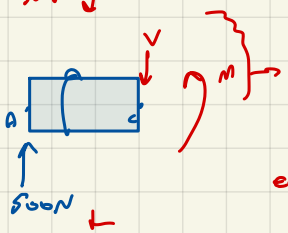
3.2 : shear force & bending moment:-

- internal loading:- if a beam has external load then the material from inside will have a loading to resist the external load.

---

ex :-





internal loading

V internal shear force
M internal bending moment

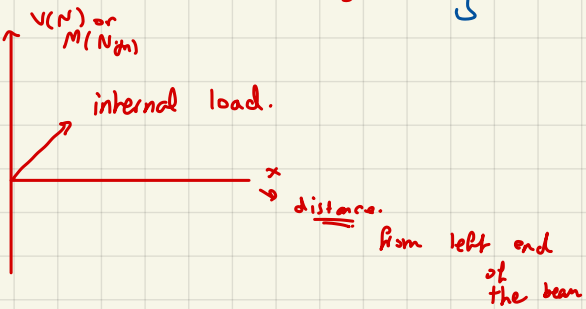
vary from point to point

equilibrium

equilibrium → $\Sigma F = 0$, $\Sigma M = 0$

when we cut we always take the left segment
stress is from the internal load. not the external.

* Shear & moment diagram } always for internal loading NOT external.



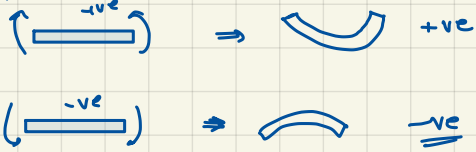
---

* 3.2 8- how to make shear force & bending moment diagrams.

① $V = \frac{dM}{dx}$ ② $M_2 - M_1 = \int_{x_1}^{x_2} V\,dx$

- Sign convention :

M :



V: ↑ +ve ↓ -ve

---

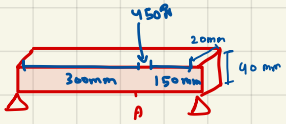
example :

do the shear force & bending moment diagram.

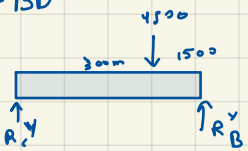
$V = \frac{dM}{dx}$ $\Delta M = \int_{x_1}^{x_2} V\,dx$

من خلال العلاقتين نستنتج, نستنتج

بناء, diagram من diagram



① FBD



shear force diagram

$\Sigma F = 0$, $\Sigma M = 0$

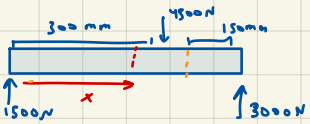
$R_B^y + R_C^y = 4500$

$R_C^y = 1500$

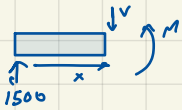
$\Sigma M_C = 0$

$-4500(300) + R_B^y(450) = 0$

$\rightarrow R_B^y = 3000\,N$



$0 < x < 300$



$\Sigma F = 0$

$V = 1500$

$\Sigma M = 0$

$M = -1500\,x$

$\min \rightarrow x = 0 \Rightarrow M = 0$

$\max \rightarrow M = -1500 \times 300$

bending moment يا

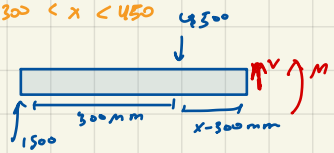
جسيم صوب او بالعكس ٥

rotation

clockwise ← positive

shear ↓

$300 < x < 450$



c.c.w ← +ve

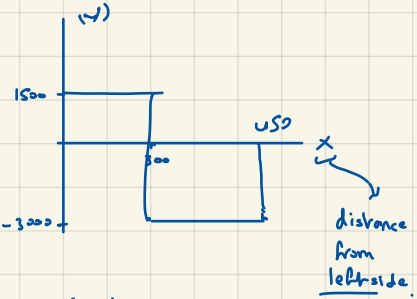
shear ↓ -ve

$V = 4500 - 1500 = 3000$

$M = -4500(300) - 3000(x-300)$

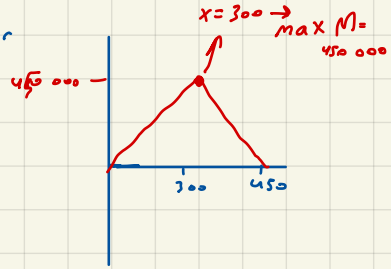
$\min \rightarrow x = 300 = -4500(300) - 0$

$\max \rightarrow x = 450 \Rightarrow -4500(300) - 3000(150)$



$V = \frac{dM}{dx}$ ← slope

$M_2 - M_1 = \int_{x_1}^{x_2} V\,dx$ ← area under



$\Delta M = 300(1500) = 450\,000$

من 0 الى 300

$M_{450} - M_{300} = \int_{300}^{450} V\,dx$

$\Delta M = 150(3000) = 450\,000$

negative area

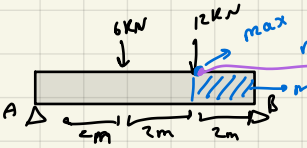
negative بعني النزل

① FBD

② shear diag ③ moment diagram

من ٥

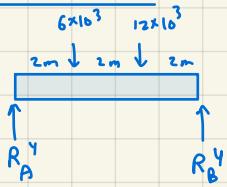
ex: draw shear & moment diagrams :-



max bending moment → max stress (stress analysis)

max shear force (القيمة المطلقة) ← بس ايه

① FBD



$R_A^y + R_B^y = 18\times10^3$

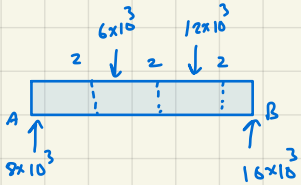
$\sum M_{R_A^y} = 0$

$-6\times10^3(2) - 12\times10^3 \times 4 + R_B^y(6) = 0$

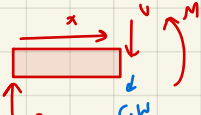
$-12\times10^3 - 48\times10^3 + R_B^y(6) = 0$

$\frac{60\times10^3}{6} = R_B^y \Rightarrow R_B^y = 10\times10^3$

$\Rightarrow R_A^y = 8\times10^3$



① $0 < x < 2$



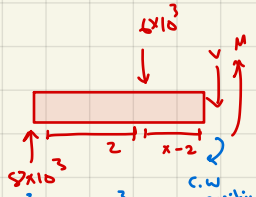
$V = 8\times10^3$

$M = -8\times10^3(x)$

$x=0$ min $= 0$

$x=2$ max $= -16\times10^3$

② $2 < x < 4$



$V = 8\times10^3 - 6\times10^3 = 2\times10^3$

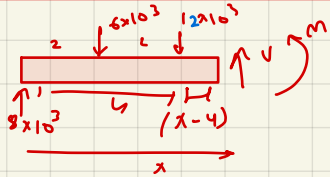
$M = -6\times10^3(2) - 2\times10^3(x-2)$

$(x=2)$ min $= -12\times10^3$

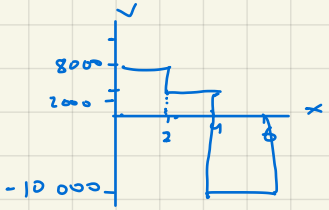
$(x=4)$ max $= -12\times10^3 - 2\times10^3(2)$

$= -16\times10^3$

③ $4 < x < 6$



$V = 18\times10^3 - 8\times10^3 = 10\times10^3 \uparrow$



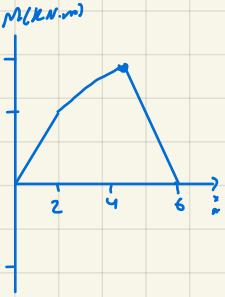
$M_2 - M_0 = 8000(2) = 16000$

$M_4 - M_2 = 2(2000) = 4000$

$M_6 - M_4 = 2(10\,000) = 20\,000$ in negative

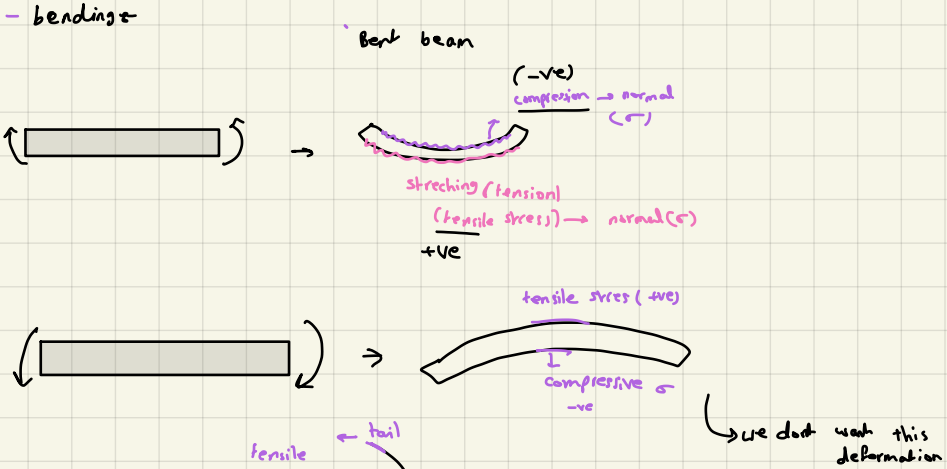
← عادة القيمة بتبقى الزيادة في الاتجاه السالب

(-ve)



3.10 normal stress for beams in bending

- bending:



*

stress هون بس نحسب ال
انا بدي اعرف الاشارة
=ve ⇐ head ال
+ve ⇐ tail ال

- انا بحاول اوصل لمكان بدي اشتغل فيه

↑ load → ↑ stress , so

I need to find the section where ⟵ the previous lesson

ib max F & max σ

$$\sigma = \frac{MC}{I}$$

flexure formula

bending M ← stress ال
distance from N.A
I

*C : distance from neutral axis

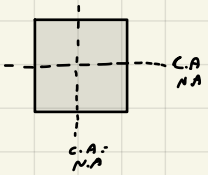
perpindicular

stress = 0 ⇐ N.A ال

ثابتة بكل شكل

*I : moment of inertia

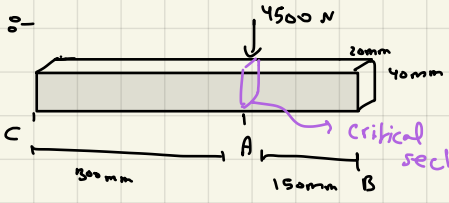
- How to know where the N.A is ??
  - for straight beams :→ N.A = C.A

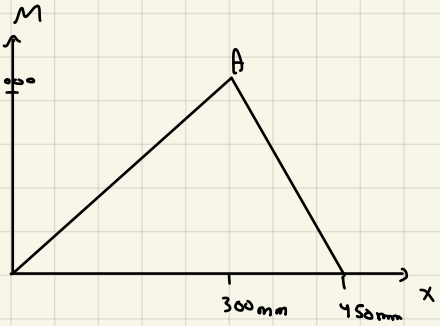


- we want the max value for bending moment → max σ

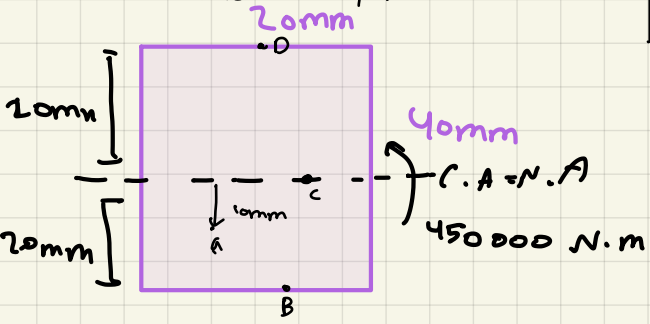
---

example :-





we solved this question :-



max M → A point

max σ → A point

for rectangular :-

$$I = \frac{1}{12}(b)(h)^3$$

b : عرض المقطع العمودي على N.A

h : البعد بين أعلى وأسفل نقطة

- at point a :

$$\sigma = \frac{Mc}{I} = \frac{450\,000 \times 10}{(\frac{1}{12})(20)(40)^3} = 42.19 \text{ MPa}$$

- at point b :

$$\sigma = \frac{Mc}{I} = \frac{450000 \times 20}{(\frac{1}{12})(20)(40)^3} = 84.38 \text{ MPa}$$

Positive because tension (with the tail of moment)

- at point c :

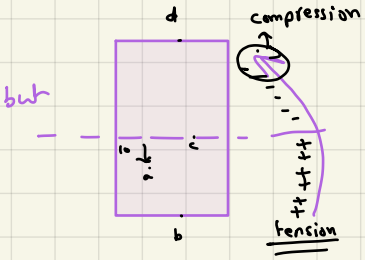
$$\sigma = \frac{M\,c}{I} = \frac{450000 \times (0)}{\frac{1}{12}(20)(40)^3} = 0 \text{ MPa}$$

b & d ↓ critical points

- at point d :

$$\sigma = \frac{M\,c}{I} = \frac{450\,000\,(20)}{\frac{1}{12}(20)(40)^3} = 84.38$$

but $= -84.38$

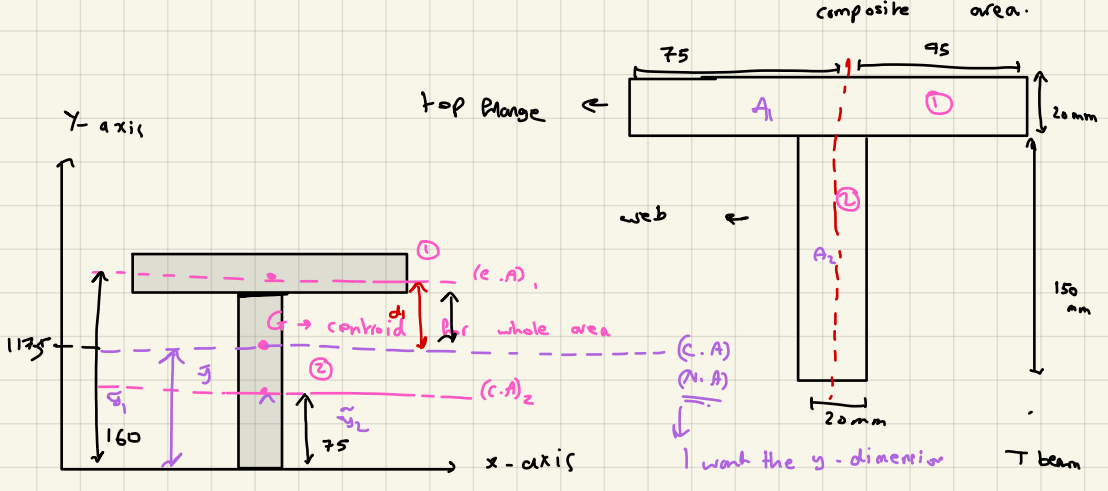


- the bending moment stress → at C.A & N.A = zero

but on the barriers of the critical section is the max & min ↑ critical points ↓ stress

- we have to not forget the sign of compression & tension

① find critical cross. section (diagrams)
② find the critical points (on the barriers)
③ C.A & N.A → ④ I → ⑤ Flexure Formula



①

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2} = \frac{160(20\times150) + 75(20\times150)}{2\times(20\times150)}$$

$$\bar{y} = 117.5\,mm$$

---

② $$I = \frac{1}{12}(b)(h)^3$$

$$I_1 = \frac{1}{12}(150)(20)^3 = 100\,000\ mm^4$$

$$I_2 = \frac{1}{12}(20)(150)^3 = 5625\,000\ mm^4$$

*parallel axis theorem

$$\bar{I} = I + Ad^2$$

$\bar{I}$: moment of inertia about C.A; I: about (C.A)$_1$; A: area$_1$; d: perpindicular distance between two axis (C.A) & (C.A)$_1$

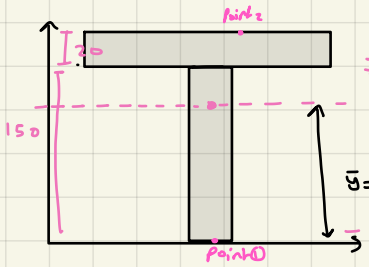
$$\bar{I}_1 = 100\,000 + (20\times150)(160 - 117.5)^2 \quad (42.5)$$

$$= 5518\,750\ mm^4$$

$$\bar{I}_2 = 5625\,000 + 20(150)(117.5 - 75)^2 \quad (42.5)$$

$$= 11\,043\,750\ mm^4$$

$$I = \bar{I}_1 + \bar{I}_2 = 16\,562\,500\ mm^4$$



1600 N.m = 1600×10^3 N.mm

N.A

$I = 16562\ 500\ mm^4$

→ find the max tensile stress + max compressive stress due to M=1600

max tensile stress

max .C ← تحت

point① ← distance from N.A

point② ⇐ max distance أبعد عن N.A

but compressive فوق

max tensile stress:

$$\sigma_1 = \frac{(1600 \times 10^3)(117.5)}{16562\ 500}$$

tensile σ_{max} = 11.35 Mpa

max comp stress

$$\sigma_2 = \frac{(1600 \times 10^3)(170 - 117.5)}{16562\ 500}$$ (52.5)

max comp σ = 5.07 (−)

اقيم بـ الـ

I for rectangle:

$\frac{1}{12} b h^3$

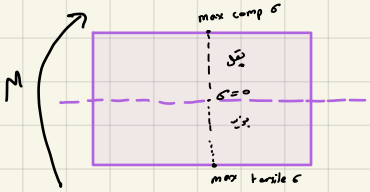
for circular:

$\frac{\pi D^4}{64}$

in appendix table A-18

max stress (tensile or compressive) on the border

stress on N.A(C.G) = zero

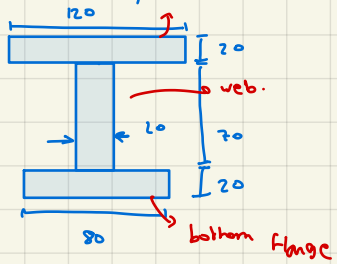


Inertia + Centroid

- صعب أعرف احسب

بسرعة

example :- top flange



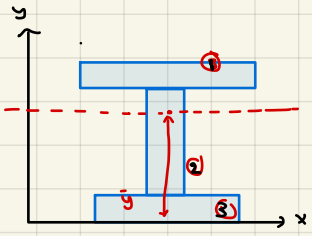


table ← I_1 قيمة

معطيني = هي بحسبه

table ← I نروح ع هي

<ع أساس I_1 هي

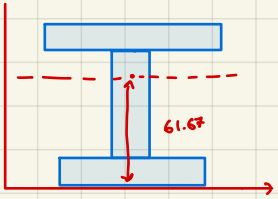
$$\bar{y} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2 + \tilde{y}_3 A_3}{A_1 + A_2 + A_3}$$

ارتفاع segment

segment	$\tilde{y}$	A	$\tilde{y}A$	I	d	Ad^2	$\bar{I}$
1	100	120(20) = 2400	240×10^3	458×10^3	38.33	3526053	3606053
2	55	20(90) = 1400	77×10^3	571.67×10^3	6.67	62284	633984
3	10	20(80) = 1600	16×10^3	53.33×10^3	51.67	4271662	4324992
		$\Sigma A = 5400$	$\Sigma \tilde{y}A = 333 \times 10^3$				$\Sigma \bar{I} = 8565029$ mm

$$\bar{I} = I + Ad^2$$

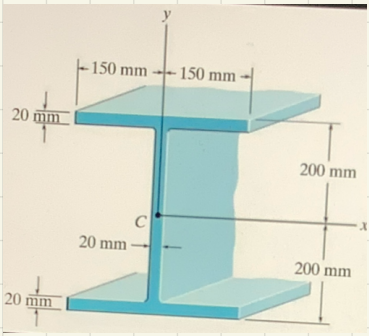
$$\bar{y} = \frac{333 \times 10^3}{4500} = 61.67$$



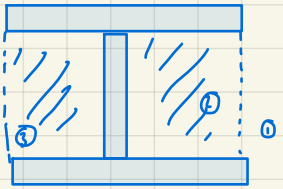
↳ upper flange & bottom flange are not equal

if upper is7 → centroid is closer to upper flange.

example.



→ exactly in middle because upper flenge = bottom flenge.



$I = I_1 - I_2 - I_3$

or the rule in the appendix for the symmetric I_{beam} But just symmetric flanges the not symmetric we have to do it the previous way

second method:

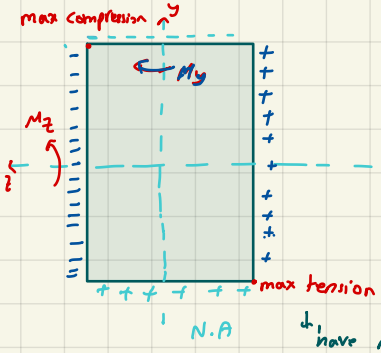
$$I = \frac{1}{12}(300)(400)^2 - 2\left[\frac{1}{12}(140)(360)^2\right]$$

$$I = 511\,360\,000\ mm^4$$

$\sigma = \frac{Mc}{I}$

bending ↓ (sag)

* two plane bending :



$\sigma = \frac{MC}{I}$

, principle of superposition

two stress بيطلعلي ⇐ beam على ال point ده عايز اعرف ال

one from each moment

وقال ان ال principle بيقول

ان ال stress اللي بيطلعلي من كل moment لوحده

positive → tension

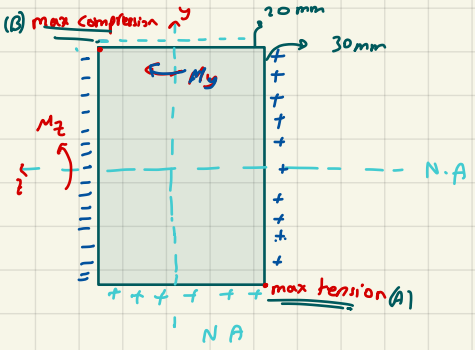
negative → compression

- In rectangular C.S the critical points are always in the corners

beam على شكل مستطيل او مربع ال critical points بتكون دايما في ال corners

---

(B) max compression



$M_y = 180\ N.m$

$M_z = 250\ N.m$

we want max tension & max comp stress

① max tension σ :- (A)

$$\sigma_{My} = \frac{(My)(c)}{I_y} = \frac{180 \times 10^3 (10)}{\frac{1}{12}(30)(20)^2}$$

$$= 90\ MPa$$

$$\sigma_{Mz} = \frac{M_z(c)}{I_z} = \frac{250 \times 10^3 \times 15}{\frac{1}{12}(20)(30)^3}$$

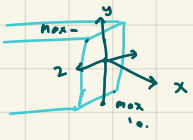
$\sigma_x = 83.33\ MPa$

max tension $\sigma_x = 90 + 83.33 = 173.33$

max comp $\sigma_x = -173.33$

- c is distance from the axis that the moment is rotating around.

- when the I_y → base(b) high (h) axis على حسب ال



Example 8

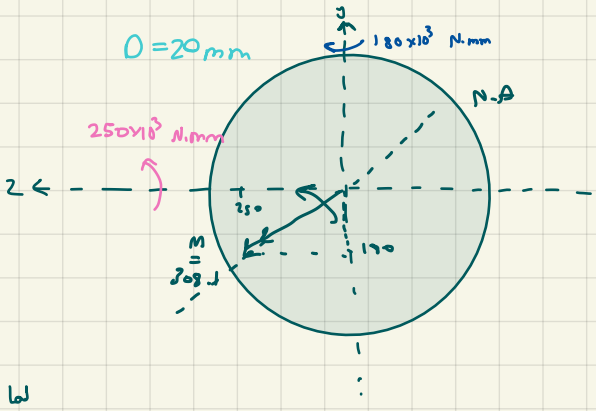
$I = \frac{\pi D^4}{64}$

moment لف الجسم مع الـ

$\vec{M}_y = (-180\hat{j})$ N.m

$\vec{M}_z = (+250\hat{i})$ N.m

.... لأن i و j لما تجمع مع بعض بتجيب حاجة تانية



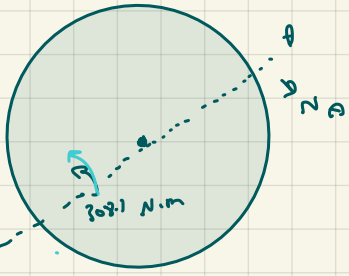
$\vec{M}_{res} = (-180\hat{j} + 250\hat{k})$ N.m

$|\vec{M}_{res}| = \sqrt{180^2 + 250^2}$ -- 308.1 N.m

- the circle : has infinite number of C.A
- the only rule is that it has to go through the center

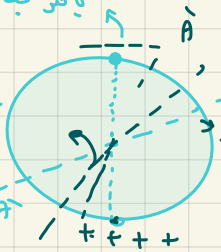
$\sigma_x = \frac{M_{resultant}\ c}{I}$ ← max distance

$\frac{308.1\ (10)}{\frac{\pi}{64}(20)^4} = 392.23$ MPa

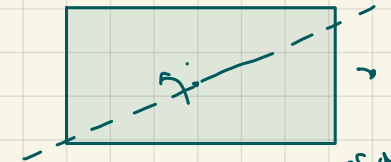


بالين بك خوصيا

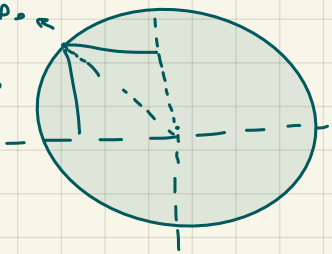
max perpindicular point and the value is The radius



the nutral axis that the resultant moment rotates around.



طريقة الدائرة مع المستطيل كتير صعب

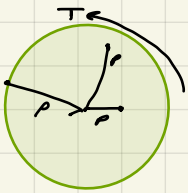


صعب طريقة الـ superposition مع بعض

3.12 - Torsion

torison formula

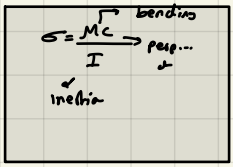
$$\tau = \frac{Tp}{J}$$



T : internal tourge

p : radial distance

J : polar moment of inertia



AT table A-18 ▭ I_y

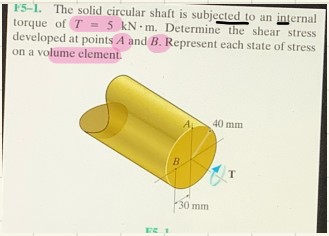
○ I_y

* moment of Inertia for circular



$$J = \frac{\pi D^4}{32} \qquad I = \frac{\pi D^4}{64}$$

F5–1. The solid circular shaft is subjected to an internal torque of $T = 5$ kN·m. Determine the shear stress developed at points A and B. Represent each state of stress on a volume element.



* $\tau = \frac{Tp}{J}$

* $J = \frac{\pi\, 80}{32}$

$$\tau_A = \frac{5\times10^6 \times 40}{J} = 37.3\,MPa$$

$$I_B = \frac{5\times10^6 \times 30}{J} = 49.74\,MPa$$

Polar moment of Inertia is different from solid to hollow

5×10^3 N.m = 5×10^6 N.mm

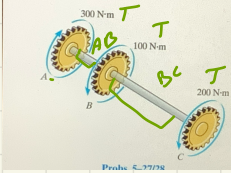
-τ at A is bigger the difference distance and it's maximum is at the border

-T for the same c.s is the same (equality)

$$\tau_{max} = \frac{Tp_{max}}{J} = \frac{Tr}{J}$$

5–27. The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the maximum shear stress developed in the segments *AB* and *BC*. The shaft has a diameter of 40 mm.

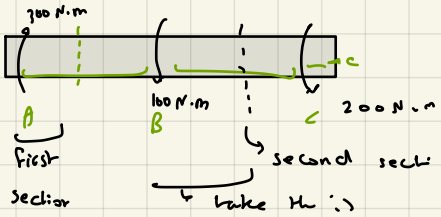
*5–28. The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the required diameter of the shaft to the nearest mm if $\tau_{allow} = 60$ MPa.

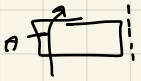


Probs. 5–27/28

D = 40 mm

τ_{max} in segments AB & BC ?







internal T

$T' = 300$ N·m

AB

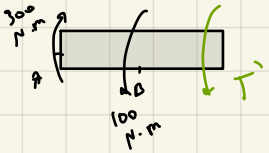
$T' = 300$ N·m

$$\tau_{max} = \frac{300 \times 10^3 (20)}{\frac{\pi}{32}(40)^4}$$

$$\tau_{max_{AB}} = 23.87$$

When we cut we always take the left

BC



$T' = 200$ N·m

$$\tau_{max_{BC}} = \frac{200 \times 10^3 (20)}{\frac{\pi}{32}(40)^4} = 15.92 \text{ MPa}$$

- we will take the internal torsion because that's what is in the rule

- When designing I have to take the max τ in all the body → τ_{BC}

Q: 5-28 from the photo:-

$$\tau_{max} = \frac{T\, r}{J}$$

هو صعلني انه $\tau_{max} = 60$ MPa

so from A → C

صفوف عم أكبر

- أول خطوة: تحديد أين يظهر فيها T' أكبر مطلقا على سؤي

so AB

$T' = 300 \times 10^3$ N.mm

$$60 = \frac{300 \times 10^3 \times \frac{D}{2}}{\frac{\pi}{32}(D)^4} \Rightarrow$$

$$D = 29.42 \text{ mm}$$

$\uparrow \Delta\tau = \frac{T\, r}{\frac{\pi}{32} D^4} \uparrow \downarrow$

لـ اذا صار أكبر من هيك عادي بصير عندي τ أصغر

- اذا صار أقل لا τ أكبر و انا عندي τ_{max} بتكون 60 MPa

so this D is the minimum it can be

- مكتوب بالسؤال : to the nearest mm

الحل : $D = 29.42$

- we have standard sizes →

so if we don't have the standard

so

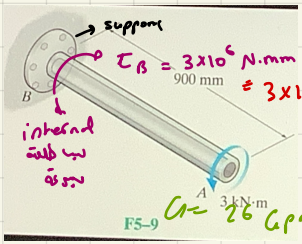
Preferred sizes ⇐ TABLE A-18

$D = 30$

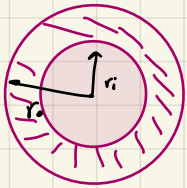
29 → 30 nearest mm

because 29.42 is the min

Continue on Torsion



* determine the angle of twist of A with respect to B



$r_i = 30$ mm
$r_o = 40$ mm
$D_i = 60$ mm
$D_o = 80$ mm

$$\theta = \frac{TL}{GJ}$$

$$= \frac{3\times10^3\left(\frac{900}{1000}\right)}{26\times10^9\left(\frac{\pi(80^4-60^4)}{32}\right)}$$

Internal load is the same here. because no external diff

$\theta_{A/B} = 0.0379$ rad

deg. J تحويل

radians → degrees
$\times \frac{180}{\pi}$

* G :- is given with Question
$G = 26\times10^9$ pa

* Hollow circular cross section :-

$$J = \frac{\pi(D_o^4 - D_i^4)}{32}$$

$$= \frac{\pi D_o^4}{32} - \frac{\pi D_i^4}{32}$$

degree → rad
$\times \frac{\pi}{180}$

if this is N.mm, mm

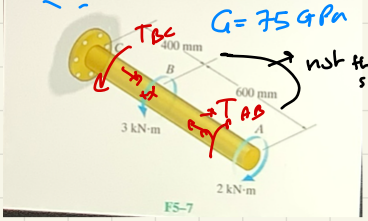
$\frac{TL}{GJ}$ mm⁴

Mpa

for A θ طلب هو
with respect to B

length يعني باخذ
من B ← A

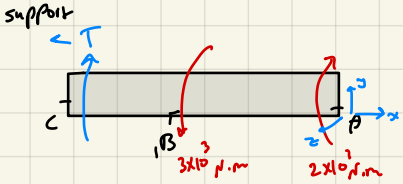
F5–7. The 60-mm-diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of end A with respect to C.



F5–7

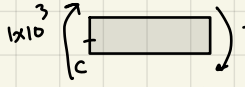
$G = 75\ GPa$

must the same



$T = 1 \times 10^3\ N.m$

here the internal load is diffrent in each section so we will part the θ for the two sech



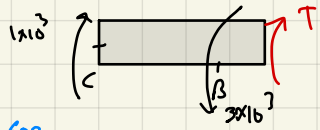
between B, C

$T' = 1 \times 10^3\ N.m$

around + x-axis so the sign is +ve

$$\theta_{A/C} = \theta_{A/B} + \theta_{B/C}$$

$$= \frac{T_{AB} l_{AB}}{GJ} + \frac{T_{BC} l_{BC}}{GJ}$$



$T' = -2 \times 10^3$

around − x-axis

so negative

$$= \frac{1 \times 10^3 \times \frac{400}{1000}}{75 \times 10^9 \left(\frac{\pi}{32} \times \frac{60^4}{1000}\right)} + \frac{-2 \times 10^3 \times \frac{600}{1000}}{75 \times 10^9 \left(\frac{\pi}{32} \times \left(\frac{60}{1000}\right)^4\right)}$$

$$= \frac{1}{75 \times 10^9 \left(\frac{\pi}{32}(0.06)^4\right)} \times \left[-2 \times 10^3 (0.6) + 1 \times 10^3 (0.4)\right]$$

* note the angle of twist sign is the same as $T' \rightarrow$ (direction)

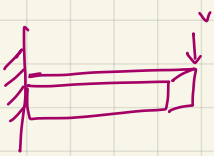
$$\theta_{A/C} = -0.00838\ rad \times \frac{180}{\pi} = -0.48^\circ$$

So the direction of the angle is anticlock from +

angle of twist should be REALLY small like less than 5°

in this example its negative because it's C.W

3.11 % Shear stresses for beam in bending:-



↓ causes bending & stress

Transverse shear formula

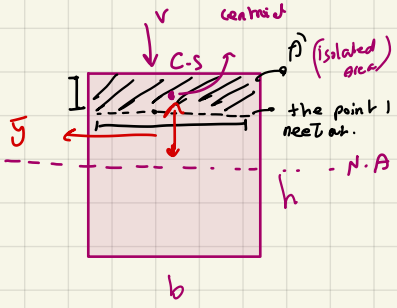
$$\text{shear stress } (\tau) = \frac{V Q}{I b}$$

V: shear force

I: second moment of area $\int y^2 da$

b: base (perpindicular to the axis which is about)

Q: first moment of area $\int y\, da$



$$\tau = \frac{V Q}{I b}$$ for A'

↳ for all of the c.s

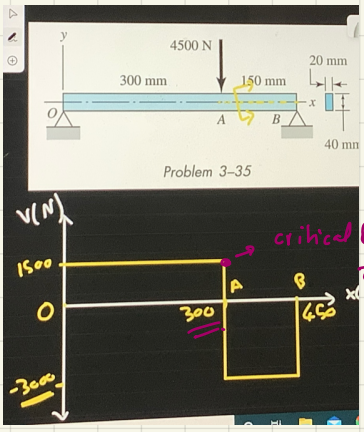
$$Q = \int \bar{y}\, dA$$

$$Q = A' \bar{y}'$$

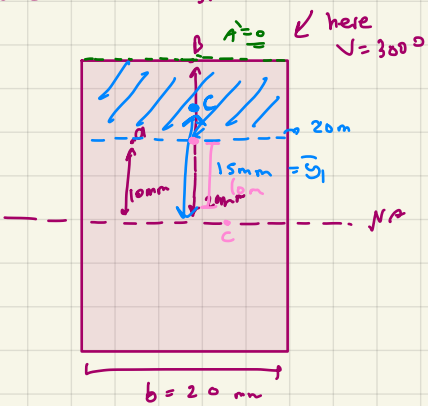
- first step:

① decide critical c.s

②



first step : decide critical cross sect. by shear force diagram



$Q_a = \bar{y}' A'$

$(10)(20) \times 15 = 3000\ mm^3$

$\tau_a = \frac{3000 \cdot 3000}{\frac{1}{12}(20)(40)^3(20)} = 4.22$

So $Q = \bar{y}' A'$

And I is the moment of inertia for the whole cross section

$Q_B = \bar{y}' A'$

$= 0$

$\tau_B = 0$

- so the shear stress at the outer surface is ZERO
- And MAXIMUM at the N.A

$Q_C = 20 \times 20 \times 10 = 4000$

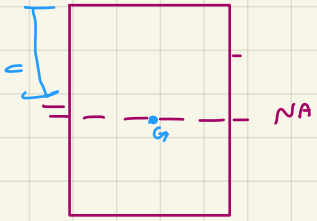
$\tau_C = \frac{3000 \times 4000}{\frac{1}{12}(20)(40)^3(20)} = 5.63$

$Q_C > Q_B$

∴ the transverse shear force because the A' is bigger

For rectangular s-

$$\tau = \frac{3V}{2\theta}\left(\frac{1-y_1^2}{c^2}\right)$$



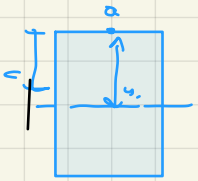
$y_1 = 0$

$\tau = \frac{3V}{2A} \rightarrow$ max

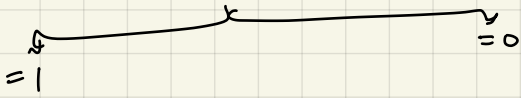
$y_1 = c$

$\tau = \frac{2V}{3A}\left(\frac{1-c^2}{c^2}\right)$ ← zero

$\tau = 0$



$$\tau = \frac{2V}{3A}\left(\frac{1-y^2}{c^2}\right)$$



$= 1$

$y_1 = 0$

on the N.A

$= 0$

$y_1 = c$

outer surface

bending & Torsion → max on outer surface

zero on N.A

$$\tau_{max} = \frac{VQ}{Ib} \quad \text{----}$$

$$\tau_{max} = \frac{2V}{3\theta}$$

Summary :-

: بعد، ترى ـ
applied stres
shearh principle stres
applications

① Axial load :

$\sigma_{avg} = \frac{P}{A}$ (uniformly distributed)

- tension ⇒ positive
- compression ⇒ negative

---

② bending moment : → normal stress

rotation about parallel axis of the c.s

$\sigma = \frac{Mc}{I}$, c : perpindicular distance from N.A

σ_{max} → when c is max

so σ_{max} is on outer surface → critical point, critical stress element

$\sigma = 0$ on the N.A

① bending moment diagram → cont' c.s

- we do the diagram to find the critical c.s then

head → Compression

tail → tension

* two plane bendings-

→ rectangle ⇒ superposition

→ circle ⇒ resultant bending moment

I for :

① rectangle: $I = \frac{1}{12} b h^3$

② Circle: $I = \frac{\pi D^4}{64}$

---

③ Torque → shear stress

moment about the normal axis

$\tau = \frac{T \rho}{J}$ (T: torque, ρ: r)

τ_{max} → outer surface

$\tau = 0$ → on the N.A

τ in outer surface

J for :

① circular: $J = \frac{\pi}{32} D^4$

② Hollow circular: $J = \frac{\pi (D_o^4 - D_i^4)}{32}$

D_o: outer

D_i: inner

- angle of twist :

→ the torque sign ⇒ R.H.R

$$\theta = \frac{TL}{GJ}$$

→ in radians ~~visualizable~~

rad → deg
$* \frac{180}{\pi}$

deg → rad
$* \frac{\pi}{180}$

---

④ Transverse shear :

① Shear force diagram → first thing

$$\tau_{avg} = \frac{V}{A}$$

$$\tau = \frac{VQ}{Ib}$$ → For all cross sections , for rectangular c.s

$$\tau = \frac{2V}{3A}\left(1 - \frac{y^2}{c^2}\right)$$

, $Q = A' \bar{y}$

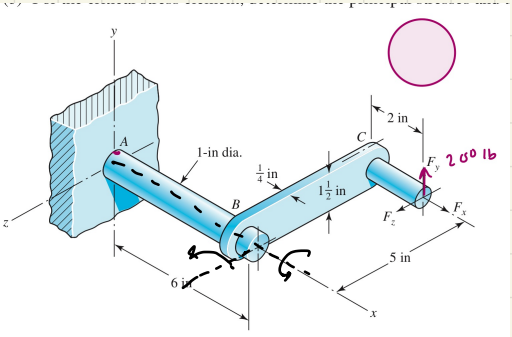
$$\tau_{max} = \frac{2V}{3A}$$

τ_{max} on N.A

$\tau = 0$ on outer surface

$\tau = 0$ on the outer surface

-table 3-2 page 111



$M_x = 200 \times 5 = 1000$ lb.in

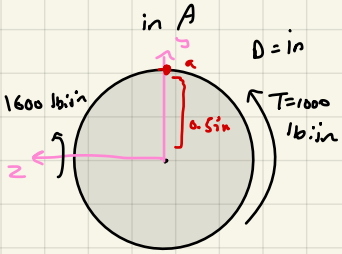
$T = 1000$ lb.in → cross section ↪

$M_z = 200 \times 8 = 1600$ lb.in

bending moment

axial	bending
F_a ✗	$M_z = 1600$ lb.in
Torsion	**Shear**
$M_x = 1000$ lb.in	$V = -200$ lb

in A



$$\sigma = \frac{Mc}{I} = \frac{1600(0.5)}{\frac{\pi}{64}(1)^4} = 16297 \text{ psi} = 16.3 \text{ kpsi}$$

$$\tau = \frac{Tp}{J} = \frac{1000(0.5)}{\frac{\pi}{32}(1)^4} = 5092 \text{ psi} = 5.09 \text{ kpsi}$$

max σ from bending moment is on outer surface & max τ surface is on outer surface also so max point is outer surface

at point a:

$\sigma = 16.3$ kpsi;

$\tau = 5.09$ kpsi;

$\sigma \rightarrow N/mm^2 = Mpa$

$\sigma \rightarrow lb/in^2 = psi$

- another way to calculate $M = r \times F$

$\vec{r} = (1.8 \rightarrow \hat{j} - 5\hat{k})$ in

$\vec{F} = -200\hat{j}$

$M = 1000\hat{i} + 1600\hat{k}$

— we finished part ①

---

part ② :

- stress element
- stress transformation
  - Principle stress / max shear stress / principle angle
- mohr's circle
- failure theories
- stress contransformation

Stress element :-

is a tool to represent the stresses at a specific point

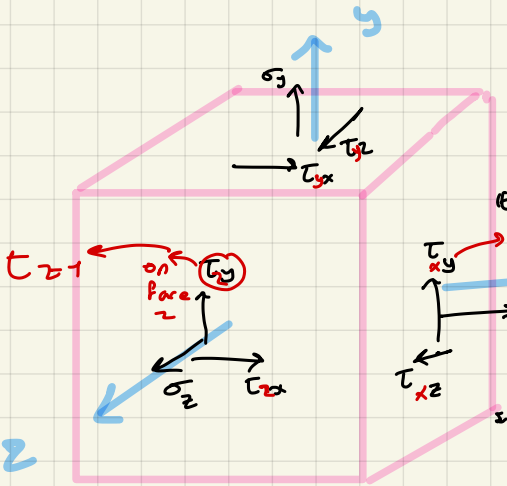
يعني هو نوع من الـ stresses اللي بتأثر على نقطة معينة

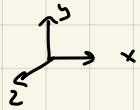
in questions :

at critical point: — بحيث : بتشتاء / بتشتاء

σ = ... Mpa

τ = ... Mpa





(E) parallel to surface

τ_{xy} on face x

τ_{xy}

σ_x (...)

τ_{xz}

x has no size

σ_x , σ_y , σ_z

τ → two cubic subs.....

- 9 stress components

τ_{xy}

surface normal ↓ direction

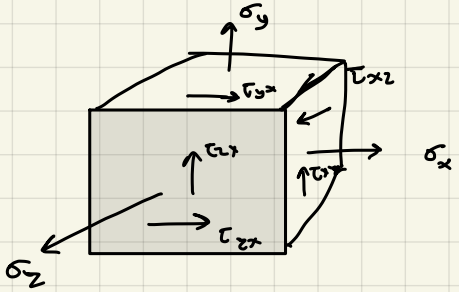
- the box is part of a beam in equilibrium
so this stress element should be in equilibrium

- غيّر شكل الـ stress باتجاهات ثانية.

$$\tau_{zx} = \tau_{xz}$$
$$\tau_{xy} = \tau_{yx}$$
$$\tau_{yz} = \tau_{zy}$$

9 components
↓
6 independent

different direction



Stress transformation :-

→ if we rotate the stress element by an angle "θ" we'll get a new stress state that is different from the original one

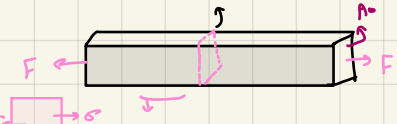
$\sigma \neq \bar{\sigma}$

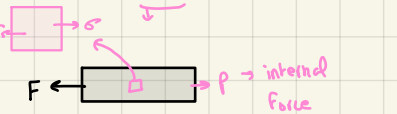
$\tau \neq \bar{\tau}$

stress transformation

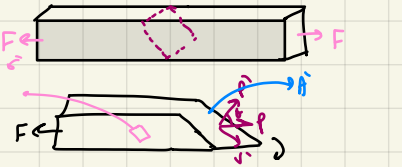
ex :-

imaginary cut.





$$\sigma = \frac{P}{A_0} \quad *$$



$P' = P\cos\theta$

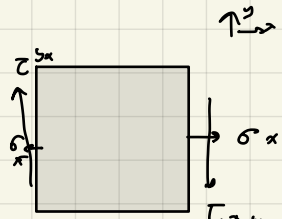
$V' = P\sin\theta$

resultant force into two components

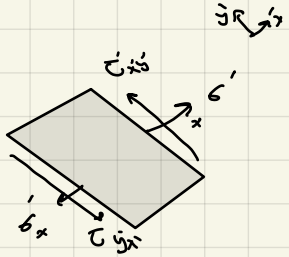
$$\sigma' = \frac{P\cos\theta}{\frac{A_0}{\cos\theta}} = \frac{P}{A_0}\cos^2\theta$$

$$A' = \frac{A_0}{\cos\theta}$$

$$\tau' = \frac{P\sin\theta}{\frac{A_0}{\cos\theta}} = \frac{P}{A_0}\sin\cos \quad (A' > A_0)$$



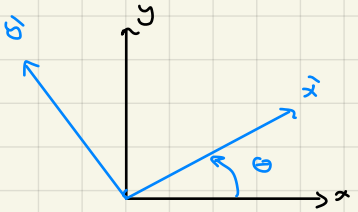
original



new

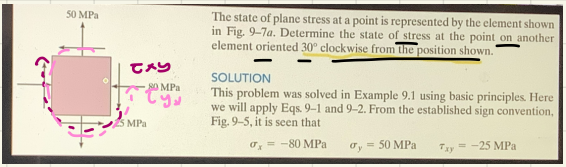
what do we have to do?

given the original stress state, find the stresses of the new element



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad *$$

$$\tau_{y'x'} = \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad *$$



The state of plane stress at a point is represented by the element shown in Fig. 9–7*a*. Determine the state of stress at the point on another element oriented 30° clockwise from the position shown.

SOLUTION

This problem was solved in Example 9.1 using basic principles. Here we will apply Eqs. 9–1 and 9–2. From the established sign convention, Fig. 9–5, it is seen that

$\sigma_x = -80$ MPa $\quad \sigma_y = 50$ MPa $\quad \tau_{xy} = -25$ MPa

C.C.W

$\theta = -30°$

$\theta = -30°$

σ (+ve) in tension; (-ve) in compression

τ (+ve) rotate C.C.W; (-ve) rotate C.W

θ (+ve) C.C.W; (-ve) C.W

$\sigma_x = -80$ MPa $\quad \sigma_y = +50$ MPa $\quad \tau_{xy} = -25$ MPa

but $\tau_{yx} = 25$ MPa

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2} + \sin 2\theta + \tau_{xy} \cos 2\theta$$

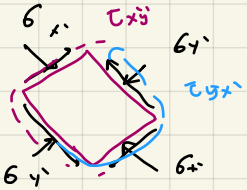
$$\sigma_{x'} = \frac{-80+50}{2} + -\frac{80-50}{2} + \cos -60 + (-25) \sin -60$$

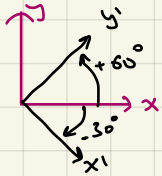
$= -25.9$ MPa comp

$$\tau_{x'y'} = -\frac{-80-50}{2} + \sin -60 + (25)(\cos(-60)^\circ) = -68.3 \text{ C.W}$$

$$\sigma_{y'} = \frac{-80+50}{2} + \frac{-80-50}{2} + \cos(120) + -25 \sin 120$$

$= -4.15$ Compress





α between x & $x' = \theta$

β between x & $y' = \theta + 90$

* principle stresses :-

→ the magnitude of σ_x & $\tau_{x'y'}$ depend on the angle of incilation "ϕ" (σ_x, τ_{xy} → function "ϕ")

ϕ_p : principle angle
σ_1, σ_2 : principle stresses
(σ_1 : σ_{max}, σ_2 : σ_{min})

} when a certain orientation σ_1, τ → max

---

* $\tan 2\phi_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

* $\sigma_1, \sigma_2 = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

σ_1 : max
σ_2 : min

---

* max shear stress "ϕ_s" (max in-plane shear stress)

$\tau \leftarrow \max \tau$; σ ⇐ $\phi_s = \phi$ ⇐ .

* $\tan 2\phi_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{1}{\tan 2\phi_p}$

* $\tau_{\substack{max \\ in\text{-}plane \\ shear \\ stress}} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$

Rules to work with principle stress :-

① no shear stresses act on the principle plane

② there is an avg normal on the planes & max. in-plane shear stress

$\frac{\sigma_x + \sigma_y}{2}$

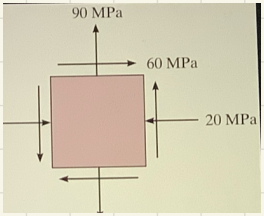
③ An element subjected to max shear stress will be 45° from the element that is subjected to the principle stresses

→ the difference between ϕ_p , $\phi_s = 45°$

$\phi_p - \phi_s = 45°$

---

example:-



$\sigma_x = -20$ Mpa , $\sigma_y = 90$ Mpa $\quad \tau_{xy} = 60$ Mpa

$$* \quad \sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = \frac{-20+90}{2} + \sqrt{\left(\frac{-20-90}{2}\right)^2 + 60^2}$$

$\sigma_1, \sigma_2 = 35 \pm 81.4$ → τ_{max}

$\sigma_1 = 116.4$ Mpa

$\sigma_2 = -46.4$ Mpa

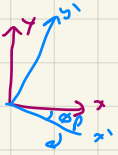
$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

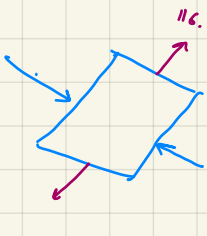
$$= \frac{2 \times 60}{-20-90} = \frac{-12}{11}$$

$2\phi_p = \tan^{-1}\left(\frac{-12}{11}\right)$

$2\phi_p = -47.4$

$\phi_p = -23.7°$





σ_1, σ_2 هاد كيف بدأت اوجد

مين $\sigma_{x'}$ و مين σ_x

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi_p + \tau_{xy} \sin 2\phi_p$$

because ← shear stress هاد ما فيه

not τ on principle plane

$\phi_p = \phi$ بعوض بهي بالقانون

$\sigma_x = \sigma_1$ ← اذا طلع $\sigma_{x'} = \sigma_2$

$\sigma_x = \sigma_2$ $\sigma_{x'} = \sigma_1$

$$\sigma_{x'} = \frac{-20+90}{2} + \frac{-20-90}{2}\cos -47.49 + 60 \times \sin(-47.49)$$

$$\sigma_{x'} = -46.4 \text{ Mpa} \qquad \text{So} \quad \sigma_2 = \sigma_{x'}$$

اذا كان $\tau_{x'y'}$ و ϕ_p بدل τ_{xy} و ϕ كمان

---

max inplane shear stress :

ϕ_s

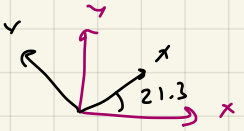
$$\tau_{max\ in} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 81.4$$

σ_1, σ_2 بدل $\sigma_{x'}, \sigma_{y'}$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \tau_{max\ in}$$

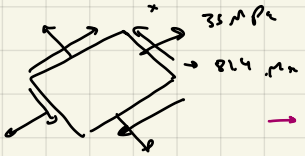
$$\phi_s = \phi_p + 45$$

$$\phi_s = -23.7 + 45 = 21.3°$$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$= 35 \text{ MPa}$$



← max inplane shear stress plane

Mohr's Circle

- relationship between σ and τ (in a circle equation)

- normal circle

$(x-a)^2 + (y-b)^2 = r^2$

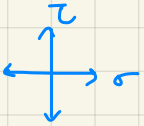
① center (a,b)

② radius $\rightarrow r$

$x^2 + y^2 = r^2$ (if $a=0, b=0$)

But in mohr's Circle:-

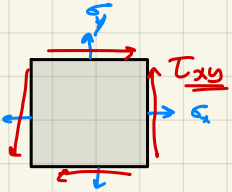
$$\left(\sigma_x - \sigma_{avg}\right)^2 + \tau_{xy}^2 = R^2$$



① center $(c) = (\sigma_{avg}, 0)$

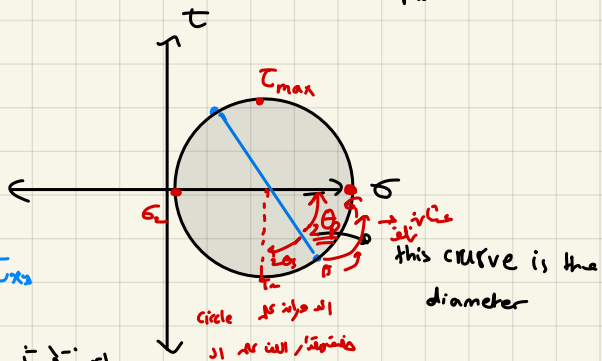
② radius $= R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2}$

⇒ how to draw the mohr's circle:-



↳ we put σ_x , and τ_{xy}

critical إذا كانت الدائرة - Pla

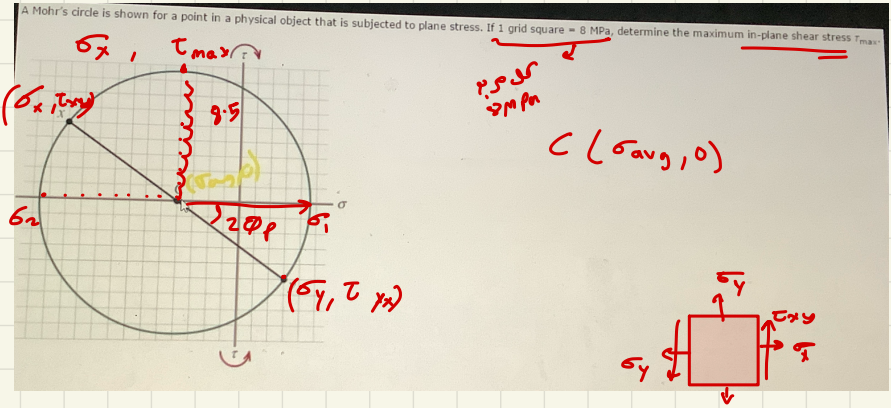


← stress state on the same

the extreme on the right → σ_1

" " " " left → σ_2

Principle elements $\{\sigma_1, \sigma_2, \tau_{max}$ in plane$\}$ → تقع على الدائرة من mohr circle



① σ_y , τ_{xy}

$\sigma_x = -11ksi = -89MPa$

$\sigma_y = 24\ MPa$

$\tau_{xy} = 40MPa$

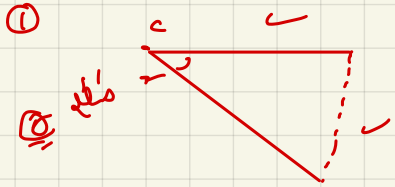
② σ_1 : . —

σ_2 : . —.

τ_{max} : . . —.

- كيف نجد $2\theta_p$.

①



②

or $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

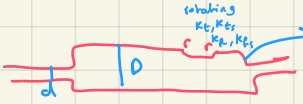
- سبب تسمية الدائرة بـ Mohr circle هو اختصار لاسم العالم الذي اكتشفها

---

step shaft

(rotating) → ماشي طبيعي

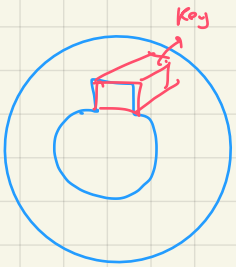
بيكلل لانه دايماً



concentration

K_f , K_{fs}

K_t , K_{ts}



$d < 3''$ $d \geq 3''$

AISI1040CD HR

to push the key we have key seat

fatigue analysis

fatigue بال تعامل مع fatigue بال

بس بدك shaft ← بيكلل معلومات و بس

* the most important & first step is given d ⇒ you can pick the material

Bending moment diagram

M_{max}

M_{max} اذا طلعت عند نقطة غلط بعين bending moment diagram ماشكل صح الاكتر بيعطيني علامتين على السؤال كامل وخلص

$\frac{r}{d}$ } انها دائما كتير بيجي equals = 0.02

Table 7-1 ⇐ we use it if we have the information to get K_t, K_{ts} from the table

* no need to do an analysis about the axial we just work bending moment & torsion

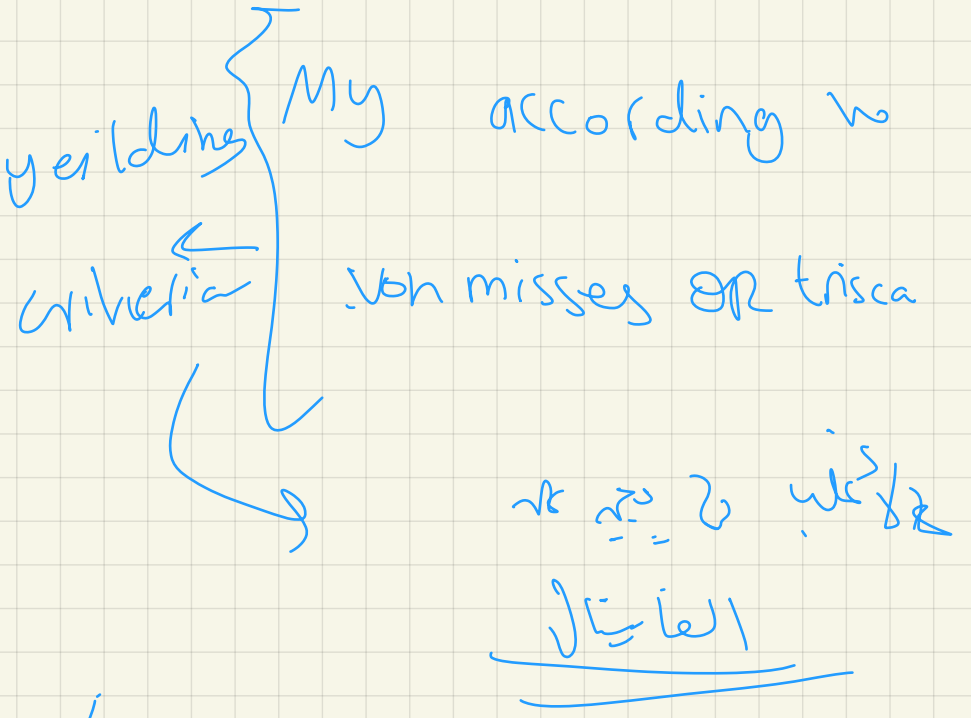
End-mill ⇒ هي بيعكي عنها لانها بتعمل Key seat

the critical point is either key seat or shoulder

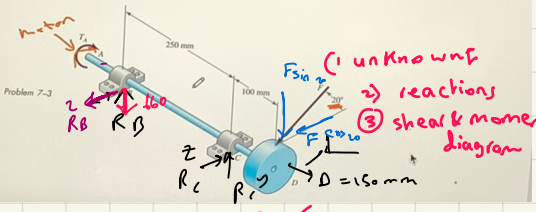
* very important notes- I have to restudy the subjects that were in the midterm.

- the subjects:
  1) K_t, K_{ts}
  2) bending moment diagram ⇒ critical point
  3) σ, τ } how to get them
  4) reactions } → sooo important

  and alot more I don't remember but study all of them.



7-3 The rotating solid steel shaft is simply supported by bearings at points B and C and is driven by a gear (not shown) which meshes with the spur gear at D, which has a 250-mm pitch diameter. The force F from the drive gear acts at a pressure angle of 20°. The shaft transmits a torque to point A of $T_A = 340$ N · m. The shaft is machined from steel with $S_y = 420$ MPa and $S_{ut} = 560$ MPa. Using a factor of safety of 2.5, determine the minimum allowable diameter of the 250-mm section of the shaft based on (*a*) a static yield analysis using the distortion energy theory and (*b*) a fatigue-failure analysis. Assume sharp fillet radii at the bearing shoulders for estimating stress-concentration factors.



Ch7 steps:-

1- UTS, Sy
2- Se, Se
3- Kf, Kfs
4- fatigue failure criteria
5- check yielding if neccesary

according to which criterion you're choosing

n = 2.5, d = ??

d = 50 mm given

Find n = ??

→ load analysis :

① unknowns → $\Sigma T = 0$

② reactions → $\Sigma M = 0$, $\Sigma F = 0$

---

solution :

$\Sigma T = 0$

$-340 \text{ N·m} + F\cos 20 \times 77 \times 10^{-3} = 0$

mm → m

$F = 4824$ N

$-z \times -x = (-j)$

$\Sigma M_{C,B} = 0$

$\Sigma M_C^y = 0$

$-4824 \cos 20 \times 160 + R_B^z \times 250 = 0$

$-z \times -x \quad z \times x$

$R_B^z = -1813.2$ N

$= 1813.2$ ←

$$\Sigma M_c^z = -4324 \times \sin 20 \times 100 - R_B^y \times 250 = 0$$

$-y \times -x = -z$ ، $y \cdot x = -z$



$$R_B^y = \boxed{-660}\ N$$

$$= 660\ N \downarrow$$

$$\Sigma F^z = 0 \qquad 4324 \cos 20 + 1913.2 + R_C^z = 0$$

$$R_C^z = 6346.3\ N \rightarrow$$

$$\Sigma F^y = 0 \qquad -4324 \sin 20 + R_C^y - 660 = 0$$

$$R_C^y = 2310\ N$$

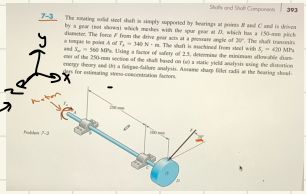
shear + bending diagram:

1- free body

x-y planes

x-z

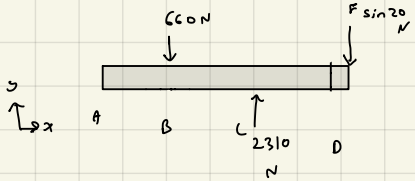
FBD x-y plane

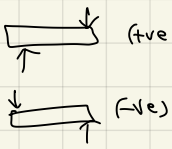


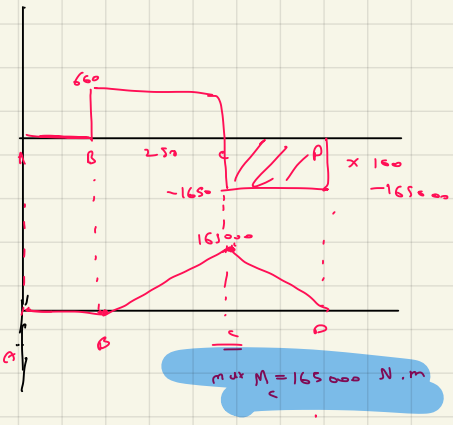
Shafts and Shaft Components 393

7–3 The rotating solid steel shaft is simply supported by bearings at points B and C and is driven by a gear (not shown) which meshes with the spur gear at D, which has a 150-mm pitch diameter. The force F from the drive gear acts at a pressure angle of 20°. The shaft transmits a torque to point A of $T_1 = 340$ N · m. The shaft is machined from steel with $S_y = 420$ MPa and $S_u = 560$ MPa. Using a factor of safety of 2.5, determine the minimum allowable diameter of the 250-mm section of the shaft based on (a) a static yield analysis using the distortion-energy theory and (b) a fatigue-failure analysis. Assume sharp fillet radii at the bearing shoulders for estimating stress-concentration factors.

Problem 7–3







$660 \times 250 = 165\,000$

$M_{max} = 165\,000\ N \cdot m$ in x-y plane

V → linear

M → تربيعي

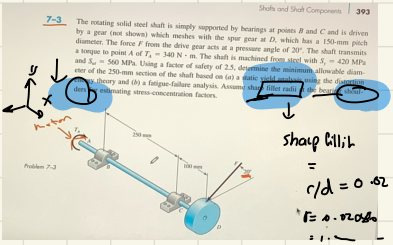
V → ثابت

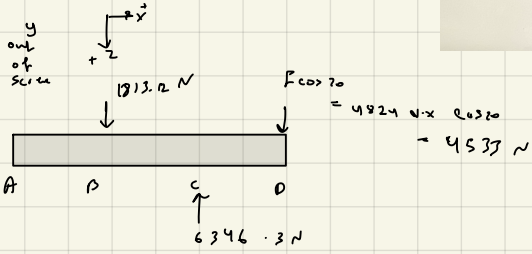
M = linear

$\Delta M = \int V\,dx$

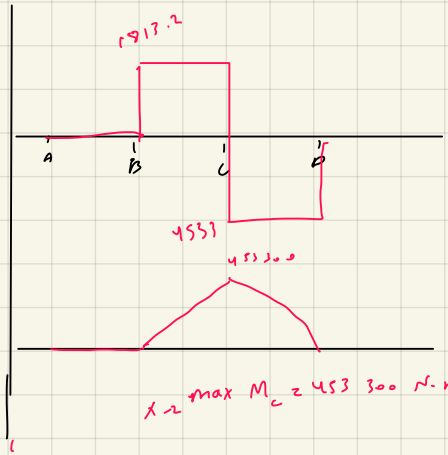
area

FBD in X-Z plane





Fcos20 = 4824 N × cos20 = 4533 N



1913.2 × 250 = 453 300

x → max M_c = 453 300 N.m → results → shaft analysis

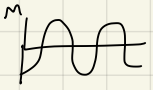
→ Max Mc = 165 N.m

$$(M_c) = \sqrt{165^2 + 453.3^2}$$

$$M_c = 482.4 \text{ N.m}$$

shaft design

max M at (c) = 482.4 N.m

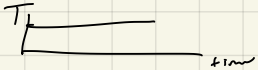


T = 340 N.m

M = 482.4 N.m → rotating & completely reversed

oil pump → load analysis → fluctuating (rotating)





M_a = 482.4 N-m, M_m = 0

T_m = 340, T_a = 0

وحدة متين الاسم

shaft design

1- UTS = 880 mpa

Sy = 5420 mpa

2- S_e' = 280 mpa

S_e = 192.9 mpa

$K_a = 0.84$

$K_b = 0.82$

$q = 0.7$

$q_s = 0.75$

3- K_f ← $K_t = 2.1$ = 1.77

K_{fs} = 1.375

↳ $K_{ts} = 1.5$

q = — —

$r = 1$

$d = 50$

K_t, K_{ts} → A-15

$D = 1.1d = 55$

4- ASME eliptic

وسيتم تعويض بـ بيانات